03 Modern Physics

Modern Physics-1 [Dual Nature of Radiation and Matter]

Dual Nature of Light

Experimental phenomena of light reflection, refraction, interference, diffraction are explained only on the basis of wave theory of light. These phenomena verify the wave nature of light.

Experimental phenomena of light photoelectric effect and Compton effect, pair production and pair annihilation can be explained only on the basis of the particle nature of light. These phenomena verify the particle nature of light. it is inferred that light does not have any definite nature, rather its nature depends on its experimental phenomenon. This is known as the dual nature of light. The wave nature and particle nature both cannot be possible simultaneously.

Electron Emission

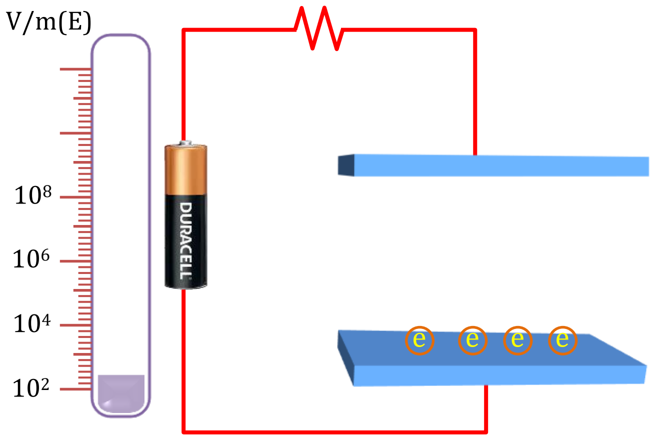
- Emission of electron from a metal surface, when energy is incident upon the surface.
- Minimum energy required to bring out the electron from metal surface is called work function (ϕ_0).
- It depends on nature of material and nature of surface.

Types of Electron Emission

1. Thermionic emission   2. Field emission   3. Photoelectric emission

1. **Thermionic emission :** The phenomenon of emission of electrons from the metal surface when it is heated is called thermionic emission.

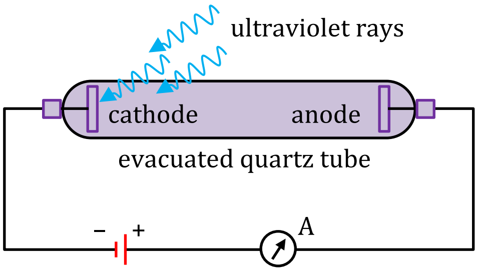
2. **Field emission :** In this process strong electric field ($\approx 10^8$ V/m) is applied to a metal, and the electrons are emitted from the surface.



3. **Photoelectric emission:** When light of suitable frequency illuminates a metal surface, electrons are emitted from the metal surface. This phenomenon is called photo electric emission.

Different Experiments

Hertz Experiment: Hertz observed that when ultraviolet rays are incident on a negative plate of electric discharge tube then conduction takes place easily in the tube.



Hallwach Experiment

Hallwach observed that if negatively charged Zn plate of electroscope is illuminated by ultra violet light, its negative charge decreases and becomes neutral and after some time, it gains positive charge. It indicates that under the action of ultra violet light, some negative charged particles are emitted from the metal.

Lenard Experiment

He told that when ultraviolet rays are incident on cathode, electrons are ejected, these electrons are attracted by anode and due to complete path of photo electrons, photo current flows. When ultra violet rays are incident on anode, electrons are ejected but current does not flow.

Work Function (ϕ)

It is the minimum energy required by an electron to escape from metal surface.

Work function depends on the nature of metal and its surface.

Minimum : Cs = 2.14 eV ; Maximum : Pt = 5.65 eV ($1\text{eV} = 1.6 \times 10^{-19}$ J)

Einstein's Quantum theory of Light

- Light behaves as Quanta, when it interacts with the matter.
  These energy quanta are called photons, having definite Energy and Momentum.
- Photons always travel with speed of light in vacuum. ($c = 3 \times 10^8$ m/s)
- Photons are electrically neutral and are not affected by electric and magnetic fields.

Energy of photon

Energy radiated from a source propagates (microscopically) in the form of small packets and these are known as photons. according to Planck the energy of a photon is directly proportional to the frequency of the radiation.

$E \propto \nu \Rightarrow E = h\nu$

$$E = \frac{hc}{\lambda} \quad \left(\because \nu = \frac{c}{\lambda}\right) \quad \Rightarrow \quad E = \frac{hc}{\lambda} = \frac{12400}{\lambda(\text{Å})} \text{ eV – Å} \qquad [\because hc = 12400 \text{ (Å – eV)}]$$

Here; E = energy of photon, c = speed of light, h = Planck's constant ($h = 6.62 \times 10^{-34}$ J-s),
e = charge of electron, ν = frequency of photon, λ = wavelength of photon

Linear momentum of photon

Linear momentum of photon $p = \frac{E}{c} = \frac{h\nu}{c} = \frac{h}{\lambda}$

Effective mass of photon

Effective mass of photon $m = \frac{E}{c^2} = \frac{hc}{c^2\lambda} = \frac{h}{c\lambda}$ i.e. effective mass $m \propto \frac{1}{\lambda}$

So, mass of violet light photon is greater than the mass of red light photon. ($\because \lambda_R > \lambda_V$)

Rest mass of a photon is always zero.

Intensity of light

$$I = \frac{E}{At} = \frac{P}{A} \quad ...(i)$$

SI Unit : $\frac{\text{joule}}{m^2 - s}$ or $\frac{\text{watt}}{m^2}$

Here; P = power of source, A = Area, t = time taken

E = energy incident in t time = Nhν, N = number of photon incident in t time

Intensity $I = \frac{N(h\nu)}{At} = \frac{n(h\nu)}{A}$ ...(ii) $\left[\because \quad n = \frac{N}{t} = \text{no. of photon per sec.}\right]$

from equation (i) and (ii), $\frac{P}{A} = \frac{n(h\nu)}{A}$

$n = \frac{P}{h\nu} = \frac{P\lambda}{hc} \Rightarrow n = (5 \times 10^{24}\ J^{-1}\ m^{-1})\ P \times \lambda$

Illustration 1

Power of a radiator is 100 watt and its λ is 400 nm. Calculate No. of photon emitted in 10 hrs.

Solution:

$n = (5 \times 10^{24}\ J^{-1}\ m^{-1})\ P \times \lambda$

$n = 5 \times 10^{24} \times 100 \times 4 \times 10^{-7} = 2 \times 10^{20}$

$N = n \times t = 2 \times 10^{20} \times 10 \times 3600 = 7.2 \times 10^{24}$

Illustration 2

A source S_1 is producing 10^{15} photon/s of wavelength 5000 Å. Another source S_2 is producing 1.02×10^{15} photon/s of wavelength 5100 Å. Calculate ratio of power of S_2 to power of S_1.

Solution:

$n = \frac{P\lambda}{hc}$ or $P = \frac{nhc}{\lambda}$

$$\frac{P_2}{P_1} = \frac{n_2}{\lambda_2} \times \frac{\lambda_1}{n_1} = \frac{1.02 \times 10^{15} \times 5000}{5100 \times 10^{15}} = \frac{1}{1}$$

Illustration 3

In an photo-electric experiment frequency of incident light is doubled and its intensity is trippled. How much time the photon current is obtained?

Solution:

$I = nh\nu \quad ; \quad n = \frac{3}{2}$ times

$i \propto n$

So, $i = \frac{3}{2}$ times

Illustration 4

The energy flux of sunlight reaching the Earth's surface is $1.388 \times 10^3\ Wm^{-2}$. How many photons (nearly) per square meter are incident on the earth per second? Assume that the photons in the sunlight have an average wavelength of 1550 nm.

Solution:

$$n = \frac{IA}{E} = \frac{IA\lambda}{hc}$$

Illustration 5

The power of a bulb is 60 milliwatt and the wavelength of light is 6000 Å. Calculate the number of photons/second emitted by the bulb?

Solution:

Energy released per second

$nh\nu = \text{power} \Rightarrow \frac{nhc}{\lambda} = P \Rightarrow n = \frac{P\lambda}{hc} = \frac{60\times10^{-3}\times6000\times10^{-10}}{6.62\times10^{-34}\times3\times10^{8}}$ photon/sec = 1.8×10^{17} photon/sec

Illustration 6

The energy flux of sunlight reaching on the Earth's surface is 1.388×10^3 W/m². How many photons (nearly) per square metre are incident on the Earth per second? Assume that the photons in the sunlight have an average length of 550 nm.

Solution:

Intensity, $I = \frac{\text{Power emitted}}{\text{Area}} = \frac{P}{A} = 1.388 \times 10^3$ W/m²

Also, Energy of a photon, $E = \frac{hc}{\lambda} = \frac{6.63\times10^{-34}\times3\times10^{8}}{550\times10^{-9}} = 3.616 \times 10^{-19}$ J

Let n be the total number of photon/area then, $n = \frac{P/A}{E} = \frac{I}{E} = \frac{1.388\times10^{3}}{3.616\times10^{-19}} = 3.83 \times 10^{21}$ photon/m²s

Radiation Force and Pressure

(i) When radiations are falling normally on a perfectly reflecting surface -

Let 'N' photons are falling in time t,

momentum before striking the surface $(p_1) = \frac{Nh}{\lambda}$

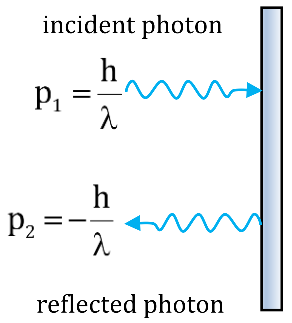
momentum after striking the surface $(p_2) = -\frac{Nh}{\lambda}$

change in momentum of photons = $p_2 - p_1 = \frac{-2Nh}{\lambda}$

momentum transferred to the surface = $\Delta p = \frac{2Nh}{\lambda}$

radiation force on the surface $F = \frac{\Delta p}{\Delta t} = \frac{2Nh}{t\lambda} = n\left[\frac{2h}{\lambda}\right]$ but $n = \frac{P\lambda}{hc}$

$\therefore \quad F = \frac{2h}{\lambda}\times\frac{P\lambda}{hc} = \frac{2P}{c}$ and Radiation pressure = $\frac{F}{A} = \frac{2P}{cA} = \frac{2I}{c}$ $\left[\because I = \frac{P}{A}\right]$



(ii) When radiations are falling normally on a perfectly absorbing surface –

Radiation force $F = \frac{p_1 - p_2}{t} = \frac{\frac{Nh}{\lambda} - 0}{t} = \frac{Nh}{t\lambda} \Rightarrow F = n\frac{h}{\lambda}$ $\left[\because n = \frac{P\lambda}{hc}\right]$

$F = \frac{P}{c}$ and Pressure = $\frac{F}{A} = \frac{P}{Ac} = \frac{I}{c}$

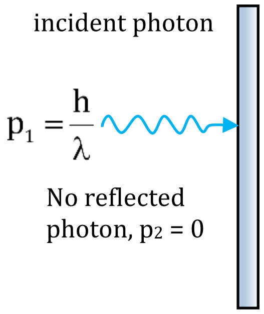


Illustration 7

A beam of electromagnetic wave of power 10 watt is incident normally on a surface, which absorb 40% power and remaining power is reflected. Calculate force exerted on a surface.

Solution:

$F = \frac{P_i + P_r}{c} = \frac{10+6}{3\times10^8} = \frac{16}{3\times10^8} = 5.33\times10^{-8}N$

Illustration 8

A 100 W sodium lamp radiates energy uniformly in all directions. The lamp is located at the centre of a large sphere that absorbs all the sodium light, which is incident on it. The wavelength of the sodium light is 589 nm. (a) What is the energy per photon associated with the sodium light?

(b) At what rate are the photons delivered to the sphere? [NCERT]

Solution:

(a) $E = \frac{hc}{\lambda}$ (b) $n = \frac{P\lambda}{hc} = \frac{P}{E}$

Illustration 9

How many photons of wavelength λ = 6600 nm must strike a totally reflecting screen per second at normal incidence so as to exert a force of 1N?

Solution:

Momentum of the incident photon, p = $\frac{h}{\lambda}$, Momentum after reflection = $-\frac{h}{\lambda}$

Change in momentum = Δp = $\frac{2h}{\lambda}$

If n is the number of photons falling per second on the screen, then force

F = $\frac{\Delta p}{\Delta t} = \frac{2nh}{\lambda} \Rightarrow n = \frac{F\lambda}{2h} = \frac{1\times6600\times10^{-9}}{2\times6.6\times10^{-34}} = 5\times10^{27}$ photons s^{-1}

BEGINNER'S BOX-1

1. Who discovered photo electric effect?
(1) Hertz (2) Lenard (3) Hallwach (4) Einstein

PMD104

2. The energy of a photon is equal to 3 kilo eV. Calculate its linear momentum?

PMD105

3. A parallel beam of monochromatic light of wavelength 500 nm is incident normally on a perfectly absorbing surface. The power through any cross-section of the beam is 10 W. Find
(a) Number of photon absorbed by the surface per second.
(b) The force exerted by light beam on the surface.

PMD106

4. Which colour of photon has greater energy either red or violet?

PMD107

5. A TV station is operated at 100 MW with a signal frequency of 10 MHz. Calculate the number of photons radiated per second by its antenna.

PMD108

6. Calculate number of photons passing through a ring of unit area in unit time if light of intensity 100 Wm^{-2} and of wavelength 400 nm is falling normally on the ring.

PMD109

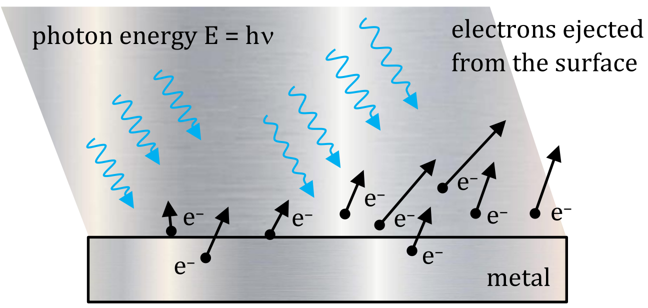
7. A special kind of light bulb emits monochromatic light of wavelength 700 nm. Electrical energy supply to it at the rate of 60 W and the bulb is 50% efficient at converting that energy to light energy. How many photons are emitted by the bulb during its life time of 1 day.

PMD110

Photoelectric Effect

It is a phenomenon of ejecting electrons by falling light of suitable frequency or suitable wavelength on a metal. Ejected electron are called photoelectrons and current flowing due to the photoelectrons is called photoelectric current. This effect was discovered by Hertz.

Laws of photo electric effect were given by Lenard. It was explained by Einstein using Quantum theory of light.

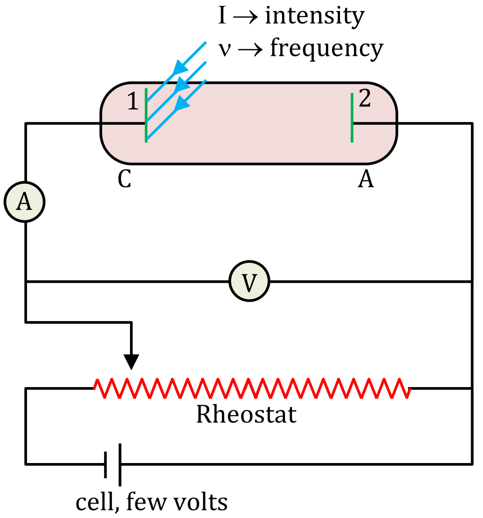


The work function (ϕ) represents the energy needed to remove the least tightly bound electrons from the surface. It depends only on nature of the metal surface and independent of any other factors.

➢ Condition for PEE ; Energy of photon ≥ ϕ

Experimental Setup of Photoelectric Effect by Lenard

When light of frequency ν and intensity I falls on the cathode, electrons are emitted from it. The electrons are collected by the anode and a current flows in the circuit. This current is called photoelectric current. This experiment is used to study the variation of photoelectric current with different factors like intensity, frequency and the potential difference between the anode and cathode.



Photoelectron: The electron emitted in photoelectric effect is called photoelectron.

Photoelectric current: If current passes through the circuit in photoelectric effect then the current is called photoelectric current.

Work function: The minimum energy required to make an electron free from the metal is called work function. It is constant for a metal and denoted by ϕ or W. It is the minimum for Cesium. It is relatively less for alkali metals.

Saturation current: When all the photo electrons emitted by cathode reach the anode then current flowing in the circuit at that instant is known as saturated current, this is the maximum value of photoelectric current.

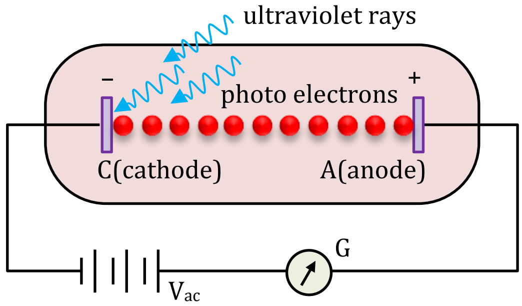
Stopping potential: Minimum magnitude of negative potential of anode with respect to cathode for which current is zero is called stopping potential. This is also known as cutoff voltage. This voltage is independent of intensity.

Retarding potential: Negative potential of anode with respect to cathode which is less than stopping potential is called retarding potential.

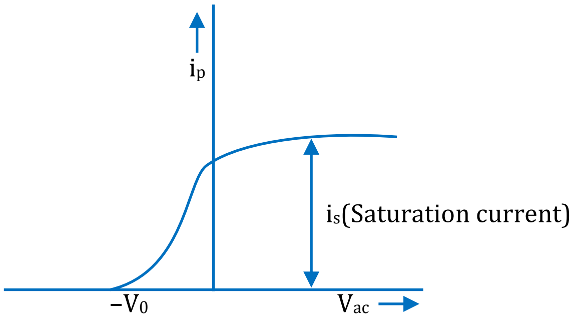
Observations of Lenard's Experiment of Photoelectric Effect

The effect of potential difference between A and C: Light of fixed frequency (ν) and intensity (I) is incident on photo emissive plate C, keeping A positive w.r.t. C. If positive potential of A gradually increased, it is found that first photo current increases then becomes maximum called saturation current.

Now we apply negative potential to plate A w.r.t. C and on increasing it gradually, current decreases and becomes zero at a certain negative potential.



Magnitude of the minimum negative potential at which i_p becomes zero, called stopping potential (V_0).



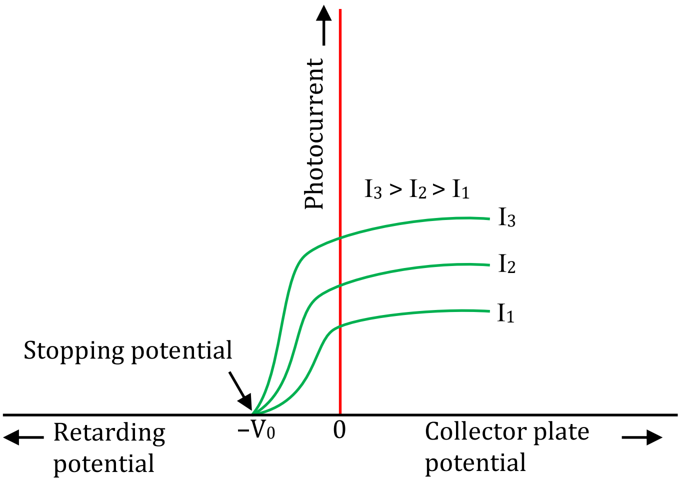
At stopping potential most energetic electrons are stopped.

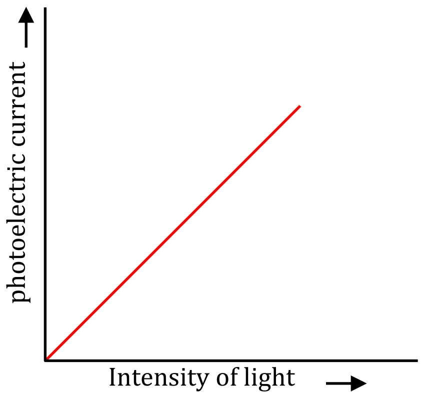
It means stopping potential measures maximum kinetic energy of photo electrons. i.e. $K_{max.} = eV_0$

Stopping potential does not depend on the distance between emitter and collector.

Effect of intensity of light

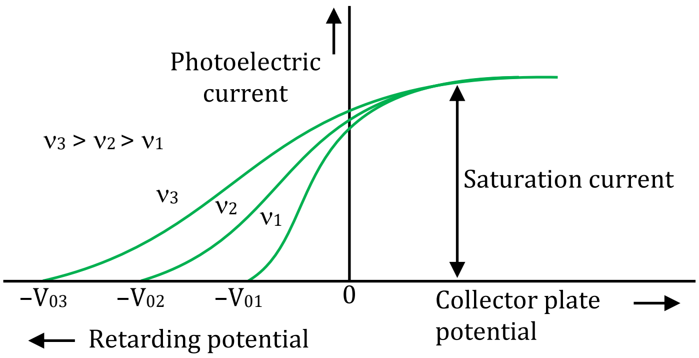
When intensity of given source able to produce PEE, is increased, i_s increases but V_0 remains unchanged.

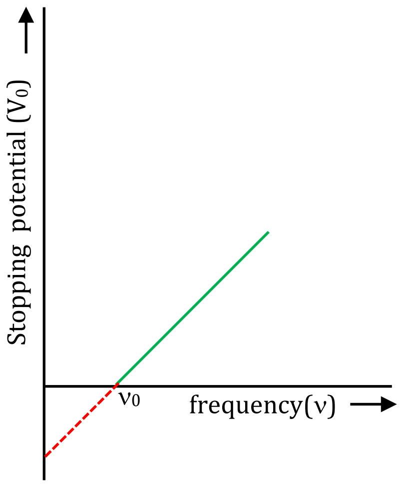




Effect of frequency

When frequency of radiations is increased, V_0 increases but i_s is almost unchanged. It is also observed that below a certain frequency photoelectrons don't come out. The minimum frequency which can eject the photo electrons is called cut off or threshold frequency. Similarly corresponding maximum wavelength is called threshold wavelength.





Time lag

There is no time lag between the incidence of radiations and emission of electrons. The photoelectric emission is an instantaneous process without any apparent time lag ($\sim 10^{-9}$ s or less), even when the incident radiation is made exceedingly dim.

Einstein's Photoelectric Equation

Radiations absorbed by the surface are in the form of quanta (photon). Energy of each photon depends on frequency. One photon can interact with one electron at a time. In the interaction between photon and electron incident photon transfers its whole energy to the electron.

If energy is sufficient then electron comes out without any time delay. It means phot electric effect is an instantaneous process.

If intensity of the given source is increased then number of photon increases. So that, more number of electrons are emitted and greater saturation current is obtained. It means saturation current depends upon intensity of the given source, $i_s \propto I$

At a time, only one photon can interact with one electron.

Energy of photon used by the electron is,

$h\nu$ = Kinetic energy of electron + Energy required to make electron free from the metal surface (ϕ_0) + Energy lost in collision before emission (Q)

If Q = 0, means there is no heat loss, then kinetic energy of electron is maximum.

Now, $h\nu = (K.E._{max}) + \phi_0$ It is known as Einstein's equation of P.E.E.

$\mathbf{(K.E._{max}) = h\nu - \phi_0}$ or $eV_0 = h\nu - \phi_0$ or $eV_0 = h\nu - h\nu_0$

Here, ν_0 is threshold frequency for that, $V_0 = 0$.

It means maximum K.E. and stopping potential (V_0) depends on frequency. It is independent of intensity of the given source.

Kinetic energy cannot be negative so that, $h\nu > \phi_0$

$h\nu > h\nu_0$ $\left[\text{Here} \quad \phi_0 = h\nu_0 = \frac{hc}{\lambda_0},\ \phi_0 = \frac{12400}{\lambda_0} eV - \mathring{A}\right]$

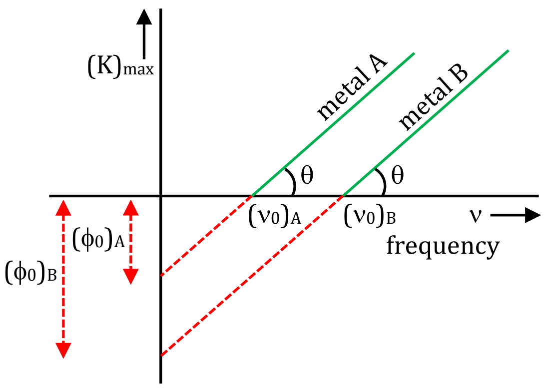
$\nu > \nu_0$ It means if frequency is less than 'ν_0', electron does not come out.

Graph between $(K.E.)_{max.}$ and frequency;

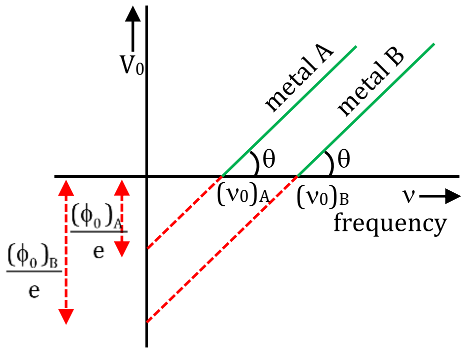
$(K)_{max.} = h\nu - \phi_0$ $[\because \quad Y = mx + c]$

slope = m = $\tan\theta$ = h (same for all metals)

$(\phi_0)_B > (\phi_0)_A$



Graph between stopping potential (V_0) and frequency (ν)



$\because \quad eV_0 = h\nu - \phi_0$

$V_0 = \left[\frac{h}{e}\right]\nu - \left[\frac{\phi_0}{e}\right]$

slope = m = $\tan\theta = \frac{h}{e}$ (same for all metals)

Illustration 10

The threshold wavelength of a metal is 400 nm. Photo electrons have kinetic energy maximum 1.5 eV. Find the wavelength of incident photon.

Solution:

$\lambda_0 = 400 \text{ nm} = 4000 \text{ Å}$ $\quad$ $K.E._{max.} = \frac{12400 \text{ eVÅ}}{\lambda} - \frac{12400 \text{ eVÅ}}{\lambda_0}$

$1.5 \text{ eV} = \frac{12400 \text{ eVÅ}}{\lambda} - \frac{12400 \text{ eV}}{4000} \Rightarrow 1.5 \text{ eV} = \frac{12400 \text{ eVÅ}}{\lambda} - 3.1 \text{ eV}$

$(1.5 + 3.1) \text{ eV} = \frac{12400 \text{ eVÅ}}{\lambda} \Rightarrow \lambda = \frac{12400 \text{Å}}{4.6} \Rightarrow \lambda = 2696 \text{ Å}$

Illustration 11

The work function of a metal is 2.3 eV and the wavelength of incident photon is 4.8×10^{-7}m. Find maximum kinetic energy of photo electrons.

Solution:

$\phi = 2.3 \text{ eV}$ and $\lambda = 4.8 \times 10^{-7} \text{ m} = 4800 \text{ Å}$

$K.E._{max.} = \frac{hc}{\lambda} - \phi = \frac{(6.62\times10^{-34} \text{ Js})(3\times10^8 \text{ ms}^{-1})}{\lambda} - \phi = \frac{12400 \text{ eVÅ}}{\lambda} - \phi$

$K.E._{max.} = \left[\frac{12400}{4800} - 2.3\right] \text{eV} = 0.28 \text{ eV}$

Illustration 12

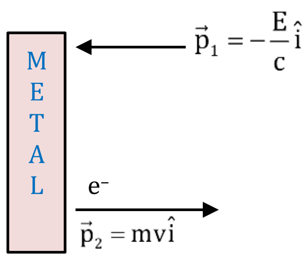
Light quanta with an energy 4.9 eV eject photoelectrons from metal with work function 4.5 eV. Find the maximum impulse transmitted to the surface of the metal when each electron flies out.

Solution:

According to Einstein's photoelectric equation

$K_{max} = \frac{1}{2} mv^2_{max} = h\nu - \phi_0 = 4.9 - 4.5 = 0.4 \text{ eV}$

Change of momentum ($p_2 - p_1$) = impulse $\therefore$ impulse = $mv - \left(-\frac{E}{c}\right) = mv + E/c$



Maximum impulse = $\sqrt{2mK_{max}} + \frac{E}{c}$

$= \sqrt{2\times0.4\times1.6\times10^{-19}\times9.1\times10^{-31}} + \frac{4.9\times1.6\times10^{-19}}{3\times10^8} = 3.43\times10^{-25} \text{ kg m/s}$

Illustration 13

The stopping potential for the photoelectrons emitted from a metal surface of work function 1.7 eV is 10.4V. Find the wavelength of the radiation used. Also identify the energy levels in hydrogen atom which will emit this wavelength.

Solution:

Energy of radiation, $h\nu = KE_{max} + \phi_0 = eV_0 + \phi_0 = 10.4 \text{ eV} + 1.7 \text{ eV} = 12.1 \text{ eV}$

Wavelength corresponding to this energy, $\lambda = \frac{hc}{E} = \frac{12400}{12.1\text{eV}} \text{eV–Å} = 1024 \text{ Å}$

As $\Delta E = 13.6\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 12.1 \text{ eV}$

So this radiation will be emitted by transition; $n = 3 \rightarrow n = 1$

Quantum Efficiency

Quantum efficiency $(\eta) = \frac{\text{number of electrons emitted per second}}{\text{total number of photons incident per second}} = \frac{n_e}{n_{ph}}$

If quantum efficiency is $\eta\%$ then $n_e = \frac{\eta}{100} n_{ph}$ [Here, $n_{ph} = (5 \times 10^{24} \text{ J}^{-1} \text{ m}^{-1}) P\lambda$]

Photoelectric current

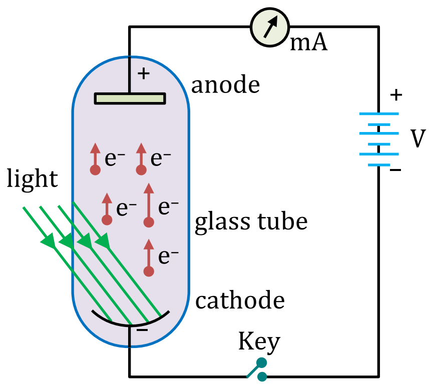
Photoelectric current, $I_e = \frac{\text{charge}}{\text{time}} = \frac{Q}{t} = n_e e = 1.6 \times 10^{-19} n_e$

Photocell

A photo cell is a practical application of the phenomenon of photo electric effect, It converts light energy into electrical energy.

Construction: A photo cell consists of an evacuated sealed glass tube containing anode and a concave cathode of suitable emitting material such as Cesium (Cs).

Working: When light of frequency greater than the threshold frequency of cathode material falls on the cathode, emitted photoelectrons are collected by the anode and an electric current starts flowing in the external circuit. The current increases with the increase in the intensity of light. The current would stop, if the light does not fall on the cathode.



Application

(i) In television camera. (ii) In automatic door (iii) Burglar's alarm

(iv) Automatic switching of street light and traffic signals.

Failure of Wave Theory in Explanation of Photoelectric Effect

(i) According to wave theory when light incident on a surface, energy is distributed continuously over the surface. So that electron must take a time interval to accumulate sufficient energy to come out. But in experiment there is no time lag.

(ii) When intensity is increased, more energetic electrons should be emitted. So that stopping potential should be intensity dependent. But it is not observed.

(iii) According to wave theory, if intensity is sufficient then, at each frequency, electron emission is possible. It means there should not be existence of threshold frequency.

BEGINNER'S BOX-2

1. The work function of a metal is 4 eV if 5000Å wavelength of light is incident on the metal. Is there any photo electric effect?

PMD111

2. In a photo cell 4 unit photo electric current is flowing, the distance between source and cathode is 4 unit. Now distance between source and cathode becomes 1 unit. What will be photo electric current now?

PMD112

3. The wavelength of photons in two cases are 4000 Å and 3600 Å respectively what is difference in stopping potential for these two?

PMD113

4. Threshold frequency of a surface is ν_0. It is illuminated by $3\nu_0$ frequency, then maximum speed of photo electrons is V m/sec. What will be maximum speed if incident frequency is $9\nu_0$?

PMD114

5. When incident wavelength is λ, stopping potential is $3V_0$. If incident wavelength is 2λ then stopping potential is V_0. Find out threshold wavelength in terms of λ.

PMD115

6. A light beam of power 1.5mW and 400nm wavelength incident on a cathode. If quantum efficiency is 0.1% then, find out obtained photo current and number of photoelectron per second.

PMD116

7. A metallic surface of work function $h\nu$ is illuminated by a radiation beam of frequency 5ν. Stopping potential observed is X. What will be stopping potential if the surface is illuminated by radiations of 7ν frequency?

PMD117

8. The kinetic energy of the fastest moving photo electron from a metal of work function 2.8 eV is 2eV. If the frequency of light is doubled, then find the maximum kinetic energy of photo electron.

PMD118

9. A monochromatic light incident on metal 'A' having threshold frequency ν_0. It emits photo electrons of maximum kinetic energy K. Now incident frequency is made three times and fall on a metal 'B' having threshold frequency $2\nu_0$. What will maximum kinetic energy of photo electrons emitted by metal 'B'?

PMD119

10. If light of wavelength 4000 Å falls on a metal which has a stopping potential 1·4 volt against photoelectric emission then what is the work function of the metal. [Take $h = 6{\cdot}6 \times 10^{-34}$ Js and $c = 3 \times 10^8$ ms^{-1}]

PMD120

Matter Wave-De Broglie Hypothesis

Dual nature of light

1. Sometimes it behaves like a particle (called a photon), which explains how light travels in straight lines and interacts with matter. For example, in experiments of PEE, Compton effect, Raman effect light behaves as particle.
2. Sometimes it behaves like a wave, which explains how light show interference, diffraction and polarisation.

De-Broglie Hypothesis

Moving particles of matter should display wave like properties under suitable conditions. Because nature loves symmetry and the two basic physical entities- matter and energy, must have symmetrical character. If radiation shows dual aspects, so matter should also show dual nature (particle as well as wave).

Introduction of matter waves

1. Waves associated with a moving particles are matter waves. (not with stationary particles).
2. Every wave has a wave function, for example EM wave has electric and magnetic field as a functions of time. Mechanical wave has medium particle's displacement as a function of time. But matter waves can not be expressed by such functions therefore it is stated that, "matter waves are neither EM waves nor mechanical waves."
3. Matter waves are probabilistic waves because these are based on concept of probability and can be expressed by probability density functions.
4. Matter waves are also known as de-Broglie waves. Because their concept was given by physicist Louis Victor de Broglie.
5. Macroscopic objects in our daily life do not show wave-like properties because their wavelength are so small (beyond any measurement). But the wave character of microscopic particles (sub-atomic particles) is significant and measurable.

Formula for de-Broglie wavelength

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{h}{\sqrt{2mK}}$$

Where : λ = de-Broglie wavelength associated with the moving particles.

h = Planck's constant = 6.63×10^{-34} J-s = 4.132×10^{-15} eV-s

p = Linear momentum = mv

K = Kinetic energy = $\frac{1}{2}mv^2 = \frac{p^2}{2m} \Rightarrow p^2 = 2mK \Rightarrow p = \sqrt{2mK}$

De-Broglie wavelength for a charged particle

Consider a charged particle of mass 'm' and charge 'q' accelerated from rest through a potential V, then its

(a) Kinetic energy K = qV

(b) Momentum $p = \sqrt{2mK} = \sqrt{2mqV}$

(c) Velocity $v = \sqrt{\frac{2qV}{m}}$ $\left[\because K = \frac{1}{2}mv^2 \Rightarrow v = \sqrt{\frac{2K}{m}}\right]$

(d) de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mqV}}$

Examples of different charged particles

1. Electron ($_{-1}e^0$): $m_e = 9.1 \times 10^{-31}$ kg ; $q_e = e = 1.6 \times 10^{-19}$ C

$$\lambda_e = \frac{12.27}{\sqrt{V}} \text{Å}\sqrt{\text{volt}} = \sqrt{\frac{150}{V}}\text{Å}\sqrt{\text{volt}}$$

2. Proton ($_1p^1$): $m_p = 1.67 \times 10^{-27}$ kg ; $q_p = e = 1.6 \times 10^{-19}$ C

$$\lambda_p = \frac{0.2863}{\sqrt{V}} \text{Å}\sqrt{\text{volt}}$$

3. Deutron ($_1H^2$ or D): $m_D = m_p + m_n \approx 2m_p$; $q_D = q_p + q_n = e + 0 = e$

$$\lambda_d = \frac{0.2021}{\sqrt{V}} \text{Å}\sqrt{\text{volt}}$$

4. Alpha particle (α or $_2He^4$) : $m_\alpha = 2m_p + 2m_n \approx 4m_p$; $q_\alpha = 2q_p + 2q_n = 2e + 0 = 2e$

$$\lambda_\alpha = \frac{0.1012}{\sqrt{V}} \text{Å}\sqrt{\text{volt}}$$

Illustration 14

With what potential an electron should be accelerated so that its de-Broglie wavelength becomes 10Å.

Solution:

$$\lambda_e = \sqrt{\frac{150}{V}}\text{Å}\sqrt{\text{volt}} = 10\text{Å} \Rightarrow V = \frac{150}{100} \text{ volt} = 1.5 \text{ volt}$$

Illustration 15

Find the ratio of de-Broglie wavelengths for a proton and an alpha particle, if both have same speed.

Solution:

$$\lambda = \frac{h}{mv} \Rightarrow \lambda \propto \frac{1}{m}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p} = \frac{4}{1}$$

Illustration 16

Find the ratio of de-Broglie wavelength for a proton and a deutron, if both have same kinetic energy.

Solution:

$$\lambda = \frac{h}{\sqrt{2mK}} \Rightarrow \lambda \propto \frac{1}{\sqrt{m}}$$

$$\frac{\lambda_p}{\lambda_d} = \sqrt{\frac{m_d}{m_p}} \simeq \sqrt{\frac{2m_p}{m_p}} = \frac{\sqrt{2}}{1}$$

De-Broglie Wavelength associated with Uncharged Particles

De-Broglie wavelength of neutral particles

1. Neutron ($_0n^1$):

$$\lambda_n = \frac{h}{P_n} = \frac{h}{m_n v_n} = \frac{h}{\sqrt{2m_n K}}$$ Here K must be in joule (J)

$$\lambda_n = \frac{0.2863}{\sqrt{K}} \text{Å}\sqrt{eV}$$ Here K must be in eV

Special Note: For thermal neutrons, average kinetic energy is given by $\frac{3}{2}kT$.

$$\therefore \lambda_n = \frac{h}{\sqrt{2m_n\left(\frac{3}{2}kT\right)}} = \frac{h}{\sqrt{3m_n kT}} \approx \frac{25.6}{\sqrt{T}} \text{Å}\sqrt{K}$$ (Temperature is in Kelvin)

2. Gas molecule: Total kinetic energy of any gas molecule is $\frac{f}{2}kT$, where ; f = degree of freedom.

But total translatory kinetic energy of any gas molecule is $\frac{3}{2}kT$. Because de-Broglie wavelength is associated with translational motion only. Therefore, for any gas molecule

$$\lambda = \frac{h}{\sqrt{2m\left(\frac{3}{2}kT\right)}} = \frac{h}{\sqrt{3mkT}}$$ (Here T is in kelvin)

Illustration 17

Compute the typical de-Broglie wavelength of an electron in a metal at 27°C and compare it with the mean separation between two electrons in a metal, which is given to be about 2×10^{-10}m.

Solution:

$$\lambda = \frac{h}{\sqrt{3mkT}} = \frac{6.6\times10^{-34}}{\sqrt{3(9.1\times10^{-31})\times(1.38\times10^{-23})\times(273+27)}} = 6.2\times10^{-9}\,m$$

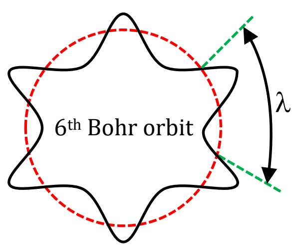
As mean separation between two electrons $= \frac{r}{\lambda} = \frac{2\times10^{-10}\,m}{6.2\times10^{-9}\,m} \approx 0.03$

Explanation of Bohr Quantisation Condition

According to De Broglie electron revolves round the nucleus in the form of stationary waves (i. e. wave packet) in the similar fashion as stationary waves in a vibarting string. Electron revolves in those circular orbits whose circumference is an integral multiple of de–Broglie wavelength associated with the electron, $2\pi r = n\lambda$

$\because \quad \lambda = \frac{h}{mv}$ and $2\pi r = n\lambda$

$\therefore \quad mvr = \frac{nh}{2\pi}$



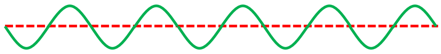
This is the Bohr quantisation condition.

General form, $2\pi r_n = n\lambda$

Where, n = order of the orbit

r_n = radius of the n^{th} Bohr orbit

$r_n = (0.529 \text{ Å}) \frac{n^2}{Z}$ [Z = atomic number]



equivalent straightened orbit

Illustration 18

Calculate de-Broglie wavelength of an e^- revolving in first excited state in hydrogen atom.

Solution:

Given, n = 2 (first excited state) ; Z = 1 (for hydrogen)

$$2\pi r_2 = 2\lambda \Rightarrow \lambda = \pi r_2 = \pi(0.53\text{Å})\frac{(2)^2}{(1)} = 2.116 \text{ Å}$$

Illustration 19

In third orbit of hydrogen atom wavelength of revolving electron is λ. Calculate the circumference of third Bohr orbit.

Solution:

$2\pi r_3 = 3\lambda$

Circumference of 3^{rd} Bohr orbit = 3λ

BEGINNER'S BOX-3

1. de-Broglie wavelength of an electron, accelerated by potential V is λ . What will be de-Broglie wavelength of the electron which is accelerated by 4V potential ?

PMD121

2. Find the ratio of de Broglie wavelength of molecules of hydrogen and helium which are at temperatures 27°C and 127°C respectively.

PMD122

3. A particle of mass M at rest decays into two particles of masses m_1 and m_2 having non zero velocities. Find out the ratio of the de–Broglie wavelengths of the two particles.

PMD123

4. Find out velocity of an electron so that its momentum is equal to that of photon with a wavelength of λ = 5200Å.

PMD124

5. Find out voltage applied to an electron microscope to produce electron of wavelength 0.6Å.

PMD125

6. What is the effective mass of a photon having wavelength λ ? **[AIPMT 2006]**

PMD126

BEGINNER'S BOX ANSWER KEY

BEGINNER'S BOX-1

1.	Hertz	**2.**	1.6×10^{-24} kg-m/s	**3.**	(a) 2.5×10^{19} (b) 3.33×10^{-8} N		
4.	Violet	**5.**	1.5×10^{34}	**6.**	2×10^{20}	**7.**	9×10^{24}

BEGINNER'S BOX-2

1.	No	**2.**	64 unit	**3.**	0.34 V	**4.**	2V
5.	4λ	**6.**	0.48 μA, 3×10^{12}	**7.**	1.5 X	**8.**	6.8 eV
9.	$3K + h\nu_0$	**10.**	1.7 eV				

BEGINNER'S BOX-3

1.	$\frac{\lambda}{2}$	**2.**	$\lambda_{H_2} : \lambda_{He} = \sqrt{8} : \sqrt{3}$	**3.**	1 : 1	**4.**	1400 m/s
5.	416.6 volt	**6.**	$m = \frac{h}{\lambda c}$				

Important Notes

Modern Physics-2 [Nuclear Physics & Radioactivity]

Nucleus and Nuclear Force

(i) Central core of every atom.

(ii) Discovered by Rutherford in α-scattering experiment.

(iii) The order of nucleus size = 10^{-15} m or fm while the order of atomic size = 10^{-10} m or Å

(iv) Protons and neutrons, together referred as nucleons.

(v) A nuclide is represented by ${}_Z^A X$

Z = atomic number = p (no. of protons)

A = mass number = total no. of nucleons = n + p

(vi) Atomic masses are generally represented by atomic mass unit (u)

$$1u = \frac{\text{mass of } C^{12} \text{ atom}}{12} = 1.66 \times 10^{-27} \text{ kg}$$

$m_p = 1.6726 \times 10^{-27}$ kg = 1.00727 u ; $m_n = 1.6749 \times 10^{-27}$ kg = 1.00866 u

$m_e = 9.1 \times 10^{-31}$ kg = 0.00055 u

Types of Nuclei

(i) Isotope: same Z (ii) Isobar: same A (iii) Isotone: same (A – Z)

Properties of Nuclei

Size of Nucleus : (Order is fermi)

As the number of nucleons in nucleus increases its size also increases and relation between its radius and mass number is $R \propto A^{1/3} \rightarrow R = R_0 A^{1/3}$

Here R_0 is a constant and its value $R_0 \approx 1.2$ fm.

Volume of Nucleus

Volume $\propto R^3$ (But $R \propto A^{1/3}$) or volume $\propto A$

Mass of Nucleus

Its mass is quite small compare to gm or kg. Therefore it is measured in another unit – amu (Atomic Mass Unit)

Mass of an nucleus of mass number A is $\approx Am_p \approx$ amu or mass of an nucleus, $m \propto A$

Density of Nucleus (ρ)

$$\rho = \frac{\text{mass}}{\text{volume}} \cong \frac{Am_p}{\frac{4}{3}\pi R^3} = \frac{Am_p}{\frac{4}{3}\pi R_0^3 A} = \frac{3m_p}{4\pi R_0^3} = 2.3 \times 10^{17} \text{ kg/m}^3$$

It means ρ is independent of A. Density of nuclei of all types of element is same and its order is 10^{17} kg/m^3 or 10^{14} g/cm^3

Forces acting inside the nucleus

There are three forces interacting between nucleons, these are

(i) Gravitational force - weakest force of nature.

(ii) Electrostatic repulsive (Colombian) force $\rightarrow$ only works between proton. This is stronger than gravitational force.

(iii) Nuclear force $\rightarrow$ strongest interaction that holds nucleons together to form nuclei and it is powerful enough to overcome the electric repulsion of proton and proton.

Features of Nuclear Force (F_n)

1. The strongest force in the universe.
2. Works only between the nucleons.
3. **Very short range :** only upto size of nucleus (3 or 4 fermi). More than this distance, nuclear force is almost zero.
4. **Very much depends upon distance :–** Small variation in distance may cause of large change in nuclear force while electrostatic force remains almost unaffected.
5. **Independent of charge :–** Interacts between n–n as well as between p–p and also between n–p.
6. **Spin dependent :–** It is stronger between nucleons having same sense of spin than between nucleons having opposite sense of spin.
7. **It is not a central force: –** Definition of central force (F_c) : Whose line of action always passes through a fixed point and its magnitude depends only on distance, if medium is same.

$$\vec{F}_c = \frac{K}{r^n}(\pm\hat{r}) \text{ is central force.}$$

Electrostatic and gravitational forces are central forces.

8. **Nature: – (i) Attractive –** If distance is greater than 0.8 fm or above.
   (ii) Repulsive – If distance is lesser than 0.8 fm.

Illustration 1

Calculate mass no. of that nucleus whose radius is half of Ge^{72}.

Solution:

$$r \propto A^{1/3} \rightarrow \frac{\frac{r}{2}}{r} = \left(\frac{A}{72}\right)^{1/3} \Rightarrow \frac{1}{8} = \frac{A}{72} \Rightarrow A = 9$$

Illustration 2

Find the density of ${}^{12}_{6}C$.

Solution:

Mass of nucleus ≈ $12m_P = 12 \times 1.66 \times 10^{-27}$ kg {m_P = mass of proton}

$$\rho = \frac{M}{\frac{4}{3}\pi R^3} = \frac{12\times1.66\times10^{-27}\,\text{kg}}{\frac{4}{3}\pi\left[1.2\times10^{-15}(12)^{1/3}\right]^3 \text{m}^3} = 2.4\times10^{17}\,\text{kg m}^{-3}$$

Einstein's Mass - Energy Relation

According to Einstein, mass can be converted into energy and energy into mass.

This relation is given by; $E = mc^2$

Here, E = total energy associated with mass m; c^2 is used as a conversion coefficient

Mass Defect and Nuclear Binding Energy

Mass defect:

(i) Mass of a nucleus is always less than the sum of masses of its constituent nucleons. This difference is called mass defect.

(ii) If observed mass of nucleus ${}_ZX^A$ be M, mass of proton is M_p and mass of neutron is M_n then mass defect = $\Delta m = [ZM_p + (A - Z)M_n] - M$.

(iii) If M is taken as mass atom of ${}_ZX^A$ instead of mass of nucleus then

$\Delta m = [Z(M_p + M_e) + (A - Z)M_n] - M_{atom}$

Binding energy (E_b)

(i) Binding energy of a nucleus is the energy required to split it into its nucleons (free).

(ii) $\Delta E_b = \Delta m.c^2$

(iii) It is always positive and numerically equal to the energy equivalent of mass defect (or equal to the energy liberated when it was formed)

Illustration 3

The mass defect in a nuclear fusion reaction is 0.05%. What amount of energy will be liberated in one kg fusion reaction?

Solution:

Mass defect = Δm = 0.05% of 1 kg = $\frac{0.05}{100}$ kg = 5×10^{-4} kg

Energy liberated = $(\Delta m)c^2 = (5 \times 10^{-4})(9 \times 10^{16})J = 45 \times 10^{12}$ J

Illustration 4

What is energy released by fission of 1 gm U^{235}?

Solution:

Number of atom in 1 gm of $U^{235} = \frac{N_A}{235}$

Energy released = $\frac{N_A}{235} \times 200 \text{ MeV} = \frac{6.023 \times 10^{23}}{235} \times 200 \text{ MeV} = 5 \times 10^{23}$ MeV

$= (5 \times 10^{23})(1.6 \times 10^{-13} \text{ J}) = 8 \times 10^{10}$ J

$= \frac{8 \times 10^{10}}{3.6 \times 10^{6}}$ kWh = 2.22×10^4 kWh

Illustration 5

What is the power output of ${}^{235}_{92}U$ reactor if it takes 30 days to use up 2 kg of fuel and if each fission gives 185 MeV of usable energy?

Solution:

Number of atoms in 2 kg of ${}^{235}_{92}U = \frac{6.02 \times 10^{23} \times 2 \times 10^3}{235} = 5.12 \times 10^{24}$

Therefore, energy released in 30 day = $5.12 \times 10^{24} \times 185$ MeV = 1.51×10^{14} J

$\therefore$ Energy released per second = $\frac{1{\cdot}51 \times 10^{14}}{30 \times 24 \times 60 \times 60}$ = 58.4 MW

Illustration 6

Obtain the binding energy of a nitrogen nucleus $\left({}^{14}_{7}N\right)$ in MeV from the following data.

$m_H = 1{\cdot}00783u$, $m_n = 1{\cdot}00867$ u, $m_N = 14{\cdot}00307$ u

Solution:

Mass defect, $\Delta m = 7m_p + 7m_n - m_N = 7 \times 1{\cdot}00783 + 7 \times 1{\cdot}00867 - 14{\cdot}00307 = 0{\cdot}11243$ amu

Binding energy = $\Delta m \times 931$ MeV = $0{\cdot}11243 \times 931$ MeV = $104{\cdot}67$ MeV

Binding Energy per Nucleon and its Curve $\left[\frac{\Delta E_b}{A}\right]$

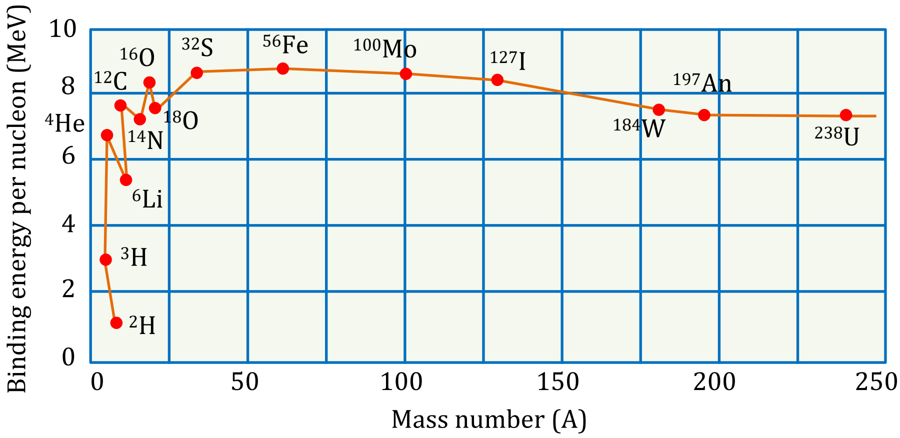


Figure: The binding energy per nucleon as a function of mass number.

(i) The value of binding energy per nucleon decides the stability of a nucleus. It is obtained by dividing binding energy by the mass number of given nucleus.

(ii) The following figure shows the binding energy per nucleon plotted against the mass number of various atoms nuclei

Greater the binding energy per nucleon, the more stable the nucleus.

(iii) It is maximum for isotope of iron – $^{56}_{26}Fe$ and is 8.8 MeV/nucleon. It is the most stable nucleus.

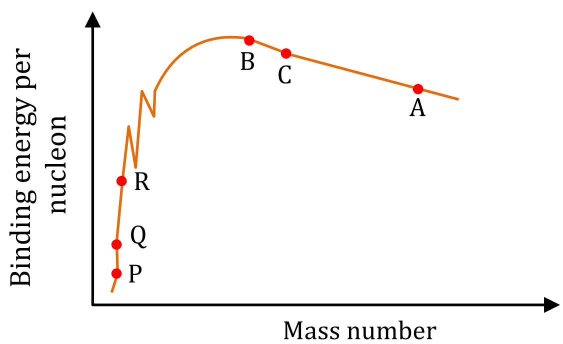
(iv) For Uranium, binding energy per nucleon is about 7.7 MeV/nucleon and it is unstable.

(v) The medium size nuclei are more stable than light or heavy nuclei.

Illustration 7

Explain nuclear fission & fusion on the basis of binding energy of nucleus.

Solution:



In fission : nucleus A breaks into B & C
In fussion : P & Q fuse to result in nucleus R

In both cases the net B.E. increases resulting in energy release.

Nuclear Fission and Chain Reaction

Nuclear Fission: Splitting of a heavy nucleus (A > 230) into two or more lighter nuclei when struck by a neutron.

In this process certain mass disappears which is obtained in the form of energy (enormous amount)

A + n $\rightarrow$ excited nucleus $\rightarrow$ B + C + Q

Hahn and Strassmann done the first fission (fission of nucleus of U^{235}).

When U^{235} is bombarded by a neutron it splits into two fragments and 2 or 3 secondary neutrons and releases about 200 MeV energy per fission (or from single nucleus)

Fragments are uncertain but each time energy released is almost same.

Possible reactions are -

$U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3\,{}_0n^1 + 200$ MeV

or $U^{235} + {}_0n^1 \rightarrow Xe + Sr + 2\,{}_0n^1 + 200$ MeV

and many other reactions are possible.

(i) The average number of secondary neutrons is 2.5.

(ii) Nuclear fission can be explained by using " liquid drop model" also.

(iii) The mass defect Dm is about 0.1% of mass of fissioned nucleus

(iv) About 93% of released energy (Q) is appear in the form of kinetic energies of products and about 7% part in the form of γ - rays.

Nuclear Chain Reaction

The equation of fission of U^{235} is, ${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3\,{}_0n^1 + Q$

These three secondary neutrons produced in the reaction may cause of fission of three more U^{235} and give 9 neutrons, which in turn, may cause of nine more fission of U^{235} and so on.

Thus a continuous 'Nuclear Chain reaction' would start.

If there is no control on chain reaction then in a short time ($\approx 10^{-6}$ sec.) a huge amount of energy will be released. (This is the principle of 'Atom bomb')

If chain reaction is controlled then produced energy can be used for peaceful purposes. For example nuclear reactor (Based on fission) generates electricity.

Natural Uranium

It is mixture of U^{235} (0.7%) and U^{238} (99.3%)

U^{235} is easily fissionable, by slow neutron (or thermal neutrons) having K.E. of the order of 0.03 eV.

To improve the quality, percentage of U^{235} is increased to 3%. The improved uranium is called 'Enriched Uranium' (97% U^{238} and 3% U^{235}).

Losses of Secondary Neutrons

Leakage of neutrons from the system: Due to their high K.E. some neutrons escape from the system.

Absorption of neutrons by U^{238}: U^{238} is not fissionable by these secondary fast neutrons. But U^{238} absorbs some fast neutrons.

Critical Size (or mass)

In order to sustain chain reaction in a sample of enriched uranium, it is required that the number of lost neutrons should be much smaller than the number of neutrons produced in a fission process. For it the size of uranium block should be equal or greater than a certain size called critical size.

Reproduction factor (K) = $\frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$

(i) If size of Uranium used is 'Critical' then K = 1 and the chain reaction will be steady or sustained (As in nuclear reaction).

(ii) If size of Uranium used is 'Super critical' then K > 1 and chain reaction will accelerate resulting in a explosion (As in atom bomb).

(iii) If size of Uranium used is 'Sub Critical' then K < 1 and chain reaction will retard and ultimately stop.

Illustration 8

Show that the nucleus $^{238}_{92}U$ emitting a proton through the decay process $^{238}_{92}U \rightarrow ^{237}_{91}Pa + ^{1}_{1}H$ cannot proceed spontaneously.

Mass of uranium = 238.05079 a.m.u.; Mass of paladium = 237.05121 a.m.u.; Mass of proton = 1.00783 a.m.u.

Solution:

Here, Q = (238.05079 – 237.05121 – 1.00783)c^2 = (– 0.00825u)c^2

As the Q for this process is negative, the decay can not proceed spontaneously.

Illustration 9

The radionuclide $^{11}_{6}C$ decays by β^+ emission. Write symbolically this decay process.

Given that, $m\left(^{11}_{6}C\right) = 11.011434$ u $\qquad m\left(^{11}_{6}B\right) = 11.009305$ u

$m_e = 0.000548$ u, 1u = 931.5 MeV/c^2

Calculate the Q-value.

Solution:

Equation of β^+-decay of $_6C^{11}$; $_6C^{11} \longrightarrow {}_5B^{11} + {}_{+1}\beta^0 + \nu + Q$

Q-value of reaction = Δmc^2

$$Q = \left[m\left({}_6C^{11}\right) - 6m_e - m\left({}_5B^{11}\right) + 5m_e - m_e\right]c^2 = \left[m\left({}_6C^{11}\right) - m\left({}_5B^{11}\right) - 2m_e\right]c^2$$

$$Q = [11.011434 - 11.009305 - 2 \times 0.000548]uc^2$$

$$Q = [0.001033]\ uc^2 = 0.001033 \times 931.5\ \text{MeV} = 0.962\ \text{MeV}$$

Illustration 10

Calculate the percent increase in mass of an electron accelerated by a potential difference of 500 kV.

Solution:

Kinetic energy of electron = 500 keV & Rest mass energy of electron = 511 keV

Total energy = $mc^2 = m_0c^2$ + KE = (511 + 500) keV

Percent increase in mass = $\frac{m - m_0}{m_0} \times 100 = \frac{500}{511} \times 100 = 97.8\ \%$

Nuclear Reactor and Fast Breeder Reactors

Nuclear Reactor (K = 1)

Its main constituents are -

(i) **Nuclear Fuel:** Commonly used are U^{235}, Pu^{239}.

Pu^{239} is the best. But Pu^{239} is not naturally available and U^{235} is used in most of the reactors.

(ii) **Moderator:** Its function is to slow down the fast-secondary neutrons. Because only slow neutrons are capable for the fission of U^{235}. The moderator should be light and it should not absorb neutrons. Commonly, Heavy water (D_2O, molecular weight 20 gm.) are used.

(iii) **Control rods:** They have the ability to capture the slow neutrons and can control the rate of chain reaction at any stage. Boron and Cadmium are best absorber of neutrons.

(iv) **Coolant:** A substance which absorb the produced heat and transfers it to water for further use. Generaly coolant is water at high pressure.

Fast breeder reactors

The atomic reactor in which fresh fissionable fuel (Pu^{239}) is produced along with energy.

Fuel : Natural Uranium.

During fission of U^{235}, energy and secondary neutrons are produced. These secondary neutrons are absorbed by U^{238} and U^{239} is formed. This U^{239} converts into Pu^{239} after two beta decay. This Pu^{239} can be separated, its half life is 2400 years.

$$_{92}U^{238} + {}_0n^1 \rightarrow {}_{92}U^{239} \xrightarrow{2\beta^-} {}_{94}Pu^{239} \text{ (best fuel of fission)}$$

This Pu^{239} can be used in nuclear weapons because of its small critical size than U^{235}.

Nuclear Fusion

It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

$A + B \rightarrow C + Q$ (Fusion)

The product (C) is more stable than reactants (A and B).

and $m_c < (m_a + m_b)$

and mass defect $\Delta m = [(m_a + m_b) - m_c]$ amu

Energy released is $E = \Delta m \times 931$ MeV/amu

The total binding energy and binding energy per nucleon C both are more than of A and B.

$$\Delta E = E_c - (E_a + E_b)$$

Fusion of four hydrogen nuclei into helium nucleus -

$$4({}_1H^1) \rightarrow {}_2He^4 + 2\ {}_{+1}\beta^0 + 2\nu + 26 \text{ MeV}$$

(i) Energy released per fission >> Energy released per fusion

(ii) Energy per nucleon in fission $\left[=\frac{200}{235} \simeq 0.85\text{MeV}\right]$ << energy per nucleon in fusion $\left[=\frac{26}{4} \simeq 6.5\text{MeV}\right]$

Required condition for nuclear fusion

(a) **High Temperature:** Which provide kinetic energy to nuclei to overcome the repulsive electrostatic force between them.

(b) **High Pressure (or density):** Which ensure frequent collision and increases the probability of fusion.

These condition exist in the sun and in many other stars. The source of energy in the sun is nuclear fusion, where hydrogen is in plasma state and protons fuse to form helium nuclei.

Pair Production and Pair Annihilation

Pair Production: A γ-photon of energy more than ≥ 1.02 MeV, when interact with a nucleus produces pair of electron (e^-) and positron (e^+).

The energy equivalent to rest mass of e^- (or e^+) = 0.51 MeV. The energy equivalent to rest mass of pair ($e^- + e^+$) = 1.02 MeV.

For pair production Energy of photon 1.02 MeV.

If energy of photon is more than 1.02 MeV, the extra energy (E – 1.02) MeV divides approximately in equal amount to each particle as the kinetic energy. $(\text{K.E.})_{e^- \text{ or } e^+} = \left[\dfrac{E_{Ph} - 1.02}{2}\right] \text{MeV}$

If E < 1.02 MeV, pair will not produce.

Pair Annihilation: When electron and positron combines they annihilates to each other and only energy is released in the form of two γ-photons.

BEGINNER'S BOX-1

1. The Q value of a nuclear reaction A + b → C + d is defined by

$Q = [m_A + m_b - m_C - m_d]c^2$

where the masses refer to the respective nuclei. Determine from the given data the Q-value of the following reactions and state whether the reactions are exothermic or endothermic.

(i) ${}^{1}_{1}H + {}^{3}_{1}H \rightarrow {}^{2}_{1}H + {}^{2}_{1}H$ (ii) ${}^{12}_{6}C + {}^{12}_{6}C \rightarrow {}^{20}_{10}Ne + {}^{4}_{2}He$

Atomic masses are given to be

$m({}^{1}_{1}H) = 1.007825$ u

$m({}^{2}_{1}H) = 2.014102$ u

$m({}^{3}_{1}H) = 3.016049$ u

$m({}^{12}_{6}C) = 12.000000$ u

$m({}^{20}_{10}Ne) = 19.992439$ u

mass of He atom is 4.0015 amu

PMD207

2. Calculate the binding energy of ${}_{17}Cl^{35}$ if mass of ${}_{17}Cl^{35}$ nucleus is 34.98 amu, mass of neutron is 1.008665 amu and mass of proton is 1.007277 amu.

PMD208

3. Calculate the energy released by the fission of 2 g of ${}_{92}U^{235}$ in kWh. Given that the energy released per fission is 200 MeV.

PMD209

4. If the energy released in the fission of one nucleus is $3{\cdot}2 \times 10^{-11}$J, then find number of nuclei required per second in a power plant of 16 kW.

PMD210

5. Find out the mass of Uranium required per day to generate 10 MW power from the fission of ${}_{92}U^{235}$.

PMD211

6. The mass defect in a nuclear fusion reaction is 0.3 percent. What amount of energy will be liberated in one kg fusion reaction ?

PMD212

7. Two nuclei have their mass numbers in ratio 1 : 3. What is the ratio of nuclear densities ?

[AIPMT 2006]

PMD213

Radioactivity

(i) Spontaneous emission of radiations from the nucleus is known as **radioactivity** and substances showing this property are called **radioactive substances**.

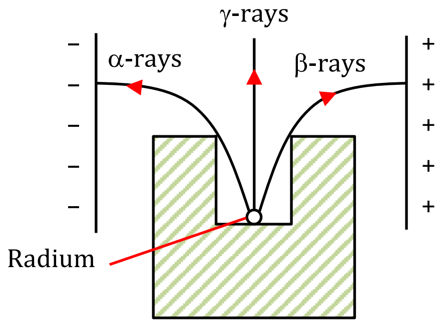
(ii) Only unstable nuclei exhibit this property.

(iii) A particular nuclide (element) can radiate only a particular type of radiations at a time, according to its requirement of stability.

(iv) This phenomenon (Radioactivity) was discovered by ***Becquerel*** therefore the radioactive radiations are also called Becquerel radiations.

(v) Later on Curie couple (Merrie Curie and Pierre Curie) discovered many other radioactive substances.

Rutherford's Experiment



Pb box (best absorber of nuclear radiations)

He put a sample of radioactive substance in a lead box and allow the emission of radiations through a small hole only. When the radiation enter into the external electric field, they split into three parts.

Radiations which deflect towards negative plate are called α–rays.

Radiations which deflect towards positive plate are called β–rays.

Radiations which are undeflected, called γ–rays.

(i) **Alpha rays :–** These are stream of positive charged particles i.e. particle nature.

(ii) **Beta rays :–** These are stream of negative charged particles i.e. particle nature.

(iii) **Gamma rays :–** These are electromagnetic waves.

Type of Radioactive Decay

Properties of Alpha, Beta and Gamma Rays

S.N.	Features	α–particles	β–particles	γ–rays
(1)	Identity	Helium nucleus or doubly ionised helium ion ($_2He^4$)	Fast moving electrons ($_{-}\beta^0$ or β^-)	Electromagnetic wave (photons)
(2)	Charge	Twice of proton (+2e)	Electronic (– e)	Neutral
(3)	Mass	$\approx 4m_p$, m_p–mass of proton	(rest mass of β) = (rest mass of ele.)	rest mass = 0
(4)	Speed	$\approx 10^7$ m/s Their speed depends on nature of the nucleus. So, it is a characteristic speed.	$\approx 10^7$ m/s β-particles come out with different speeds from the same type of nucleus. therefore can not be a characteristic speed.	Only $c = 3 \times 10^8$ m/s γ-photons come out with same speed from all types of nucleus. So, can not be a characteristic speed.

(5)	K.E.	$\approx$ MeV	$\approx$ MeV	$\approx$ MeV
(6)	Energy spectrum	Line and discrete	Continuous	Line and discrete
(7)	Ionization power ($\alpha>\beta>\gamma$)	10,000 times of γ-rays	100 times of γ-rays (or $\frac{1}{100}$ times of α)	1 (or $\frac{1}{100}$ times of β)
(8)	Penetration power ($\gamma > \beta > \alpha$)	$\frac{1}{10000}$ times of γ-rays	$\frac{1}{100}$ times of γ-rays (100 times of α)	1 (100 times of β)
(9)	Effect of electric or magnetic field	Deflection	Deflection (More than α)	No deflection

1. Alpha Decay Process

In this decay, mass number decreases by 4 and atomic number decreases by 2. Its decay equation is

$$\underset{\text{(Parent)}}{{}^{A}_{Z}X} \longrightarrow \underset{\text{(Daughter)}}{{}^{A-4}_{Z-2}Y} + \underset{\text{Alpha}}{{}^{4}_{2}He}$$

In this decay total mass of product is less than the mass of parent. This difference in mass appears as kinetic energy of the products.

The disintegration energy or Q value for α-decay.

$$Q = (m_X - m_Y - m_{He})c^2$$

2. Beta Decay Process

(A) The basic nuclear process underlying β^- -decay is the conversion of neutron to proton.

$$n \longrightarrow p + e^- + \bar{\nu}$$

(i) Its decay equation

$$^{A}_{Z}X \longrightarrow {}_{Z+1}^{\;\;A}Y + {}^{0}_{-1}\beta + \bar{\nu}$$

(ii) After $_{-}\beta^{\circ}$ decay, n/p ratio decreases.

(iii) $_{-}\beta^{\circ}$ always comes out from the nucleus along with antineutrino.

(B) The basic nuclear process underlying β^+-decay is the conversion of proton into neutron.

$$p \longrightarrow n + e^+ + \nu$$

(i) Its decay equation

$$^{A}_{Z}X \longrightarrow {}_{Z-1}^{\;\;A}Y + {}^{0}_{+1}\beta + \nu$$

(ii) After $_{+1}\beta^{\circ}$ decay, n/p ratio increases.

(iii) $_{+1}\beta^{\circ}$ comes out from the nucleus along with neutrino.

3. Gamma Decay Process

Similar to an atom, nucleus also have certain energy levels and nucleons occupy them. After α-decay (or β-decay), daughter nucleus may be in excited state and return to ground state by emitting photons of high energy (MeV order) called γ-photons.

Equation of γ–decay :– $({}_ZX^A)^* \rightarrow {}_ZX^A + \gamma$-photons (or $h\nu$)

*Shows excited nucleus.

(i) γ emission don't change the structure of nucleus (ii) No change in Z and A

Illustration 11

$_{90}Th^{234} \xrightarrow{\alpha} {}_aX^b \xrightarrow{_-\beta^0} {}_cY^d \xrightarrow{(p)} {}_{90}Th^{230}$

Find a, b, c and d and identify particle p

Solution:

$$_{90}Th^{234} \xrightarrow{\alpha} {}_{88}X^{230} \xrightarrow{_-\beta^0} {}_{89}Y^{230} \xrightarrow{P=(_-\beta^0)} {}_{90}Th^{230}$$

Illustration 12

$_{90}Th^{234} \xrightarrow{n\alpha,\, n'{}_-\beta^0} {}_{83}Bi^{214}$, Find n and n'

Solution:

Number of α particle, n = $N_\alpha = \frac{234-214}{4} = \frac{20}{4} = 5$

Number of β^- particle, n' = $N_{-\beta} = Z_f - [Z_i - 2 \times \text{No. of } \alpha] = 83 - (90 - 2 \times 5) = 83 - 80 = 3$

BEGINNER'S BOX-2

1. $_{92}U^{238} \xrightarrow{\alpha} \xrightarrow{_+\beta^0} {}_aX^b$, find a & b.

PMD214

2. $_aX^b \xrightarrow{_-\beta^0} \xrightarrow{\alpha} {}_cY^{215} \xrightarrow{_-\beta^0} {}_{110}Y^d$ Find a, b, c and d.

PMD215

3. $_{92}U^{238} \xrightarrow{n\alpha,n'{}_-\beta^0} {}_{82}Pb^{206}$. Find n & n'

PMD216

4. Thorium isotope $_{90}Th^{232}$ emits some α–particles and some β–particles and gets transformed into lead isotope $_{82}Pb^{200}$. Find the number of α and β particles emitted.

PMD217

5. A radioactive nucleus undergoes a series of decays according to the following scheme :

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta^-} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

If the mass number and atomic number of A are 180 and 72 respectively, what are these numbers for A_4?

PMD218

6. Write nuclear reaction equations for

(i) α-decay of $^{226}_{88}Ra$ (ii) α-decay of $^{242}_{94}Pu$ (iii) β^--decay of $^{32}_{15}P$

(iv) β^--decay of $^{210}_{83}Bi$ (v) β^+-decay of $^{11}_{6}C$ (vi) β^+-decay of $^{97}_{43}Tc$

(vii) Electron capture of $^{120}_{54}Xe$

PMD219

K-Electron Capture and Artificial Transmission

In K-capture a nucleus captures one of the inner orbital electrons and a proton transforms into a neutron. Hence K capture is like positron decay, in both n/p ratio increases. In this event a vacancy is created in K-shell to fill up the vacancy, electron transition takes place and X-rays are emitted.

$$_ZX^A + {}_{-1}e^0 \text{ (K-capture)} \longrightarrow {}_{Z-1}Y^A + \text{X-rays} + \upsilon$$

BEGINNER'S BOX ANSWER KEY

BEGINNER'S BOX-1

1. (i) –4.031 MeV, endothermic (ii) 5.64 MeV, exothermic

2. 278.92 MeV **3.** 4.55×10^4 kWh. **4.** 5×10^{14}

5. 10.5 g **6.** 2.7×10^{14} J **7.** 1 : 1

BEGINNER'S BOX-2

1. a = 89, b = 234 **2.** a = 110, b = 219, c = 109, d = 215 **3.** n = 8, n' = 6

4. $N_\alpha = 8$, $N_\beta = 8$ **5.** Mass number = 172 and Atomic number = 69

6. (i) ${}^{226}_{88}Ra \xrightarrow{-\alpha} {}^{222}_{86}Rn + {}^{4}_{2}He$ (ii) ${}^{242}_{94}Pu \xrightarrow{-\alpha} {}^{238}_{92}U + {}^{4}_{2}He$ (iii) ${}^{32}_{15}P \xrightarrow{-\beta^-} {}^{32}_{16}S + {}^{0}_{-1}\beta + \overline{\upsilon}$

(iv) ${}^{210}_{83}Bi \xrightarrow{-\beta^-} {}^{210}_{84}Po + {}^{0}_{-1}\beta + \overline{\upsilon}$ (v) ${}^{11}_{6}C \xrightarrow{-\beta^+} {}^{11}_{5}B + {}^{0}_{+1}\beta + \upsilon$ (vi) ${}^{97}_{43}Tc \xrightarrow{-\beta^+} {}^{97}_{42}Mo + {}^{0}_{+1}\beta + \upsilon$

(vii) ${}^{120}_{54}Xe + {}_{-1}e^0 \xrightarrow{\text{electron capture}} {}^{120}_{53}I + \text{X-Ray} + \upsilon$

Modern Physics-3 [Atomic Structure]

Various Models for Structure of Atom

Dalton's Atomic Theory

Every material is composed of minute particles known as atom. Atom is indivisible i.e. it cannot be subdivided. It can neither be created nor be destroyed.

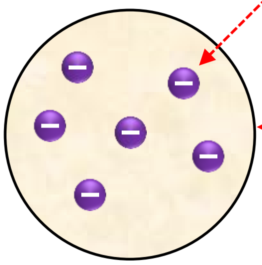
All atoms of same element are identical physically as well as chemically, whereas atoms of different elements are different in properties.

The atoms of different elements are made up of hydrogen atoms. (The radius of the heaviest atom is about 10 times that of hydrogen atom and its mass is about 250 times that of hydrogen).

The atom is stable and electrically neutral.

Thomson's Atom Model

The atom as a whole is electrically neutral because the positive charge present on the atom (sphere) is equal to the negative charge of electrons present in the sphere. Atom is a positively charged sphere of radius 10^{-10} m in which electrons are embedded in between. The positive charge and the whole mass of the atom is uniformly distributed throughout the sphere.



Shortcomings of Thomson's model:

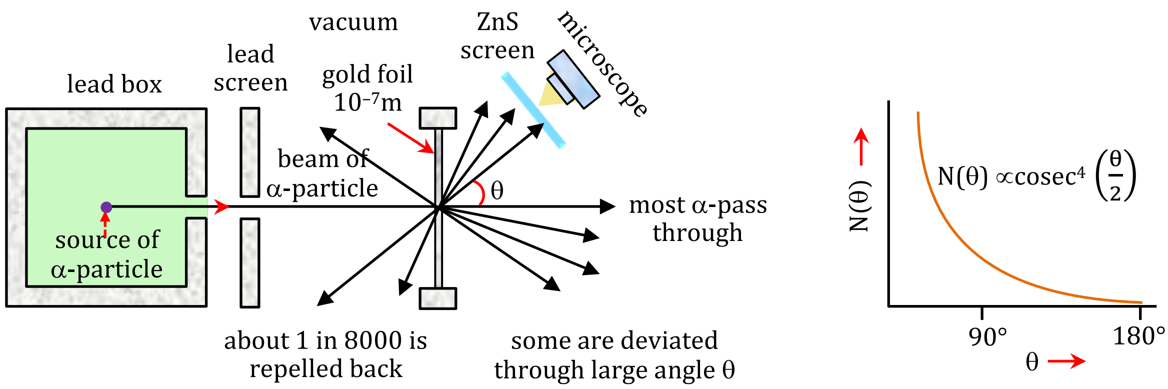
(i) The spectrum of atoms cannot be explained with the help of this model.

(ii) Scattering of α–particles cannot be explained with the help of this model.

Rutherford atomic model

Rutherford experiments on scattering of α–particles by thin gold foil: Ernst Rutherford, H. Geiger and E. Marsden Performed this experiment. In this experiment a beam of α-Particles (Helium nucleus) of energy 5.5 MeV, emitted by a $^{214}_{83}\text{Bi}$ radioactive source was directed at a thin metal foil made of gold. They studied the α-particles scattered at various angles.

The experimental arrangement is shown in figure. α–particles are emitted by some radioactive material (polonium), kept inside a thick lead box. A very fine beam of α–particles passes through a small hole in the lead screen. This well collimated beam is then allowed to fall on a thin gold foil. While passing through the gold foil, α–particles are scattered through different angles. A zinc sulphide screen was placed out the other side of the gold foil. This screen was movable, so as to receive the α–particles, scattered from the gold foil at angles varying from 0° to 180°. When an α–particle strikes the screen, it produces a flash of light and it is observed by the microscope. It was found that :



- Most of the α–particles went straight through the gold foil and produced flashes on the screen as if there were nothing inside gold foil. Thus the atom is hollow.
- Few particles collided with the atoms of the foil which have scattered or deflected through considerable large angles. Few particles even turned back towards source itself.
- The entire positive charge and almost whole mass of the atom is concentrated in small centre called a nucleus.
- The electrons could not deflected the path of a α–particles i.e. electrons are very light.
- Electrons revolve round the nucleus in circular orbits. So, Rutherford 1911, proposed a new type of model of the atom. According to this model, the positive charge of the atom, instead of being uniformly distributed throughout a sphere of atomic dimension is concentrated in a very small volume (Less than 10^{-13}cm is diameter) at it centre. This central core, now called nucleus, is surrounded by clouds of electron makes the entire atom electrically neutral. According to Rutherford scattering formula, the number of α–particle scattered at an angle θ by a target are given by,

$$N_\theta = \frac{N_0 nt(2Ze^2)^2}{4(4\pi\varepsilon_0)^2 r^2 (mv_0^2)^2} \times \frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$$

Where, N_0 = number of α–particles that strike the unit area of the scatter

n = Number of target atom per m^3

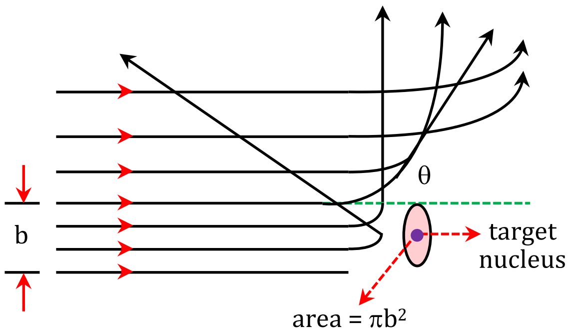
t = Thickness of target

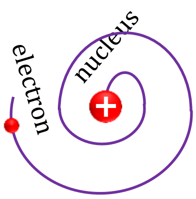
Ze = Charge on the target nucleus

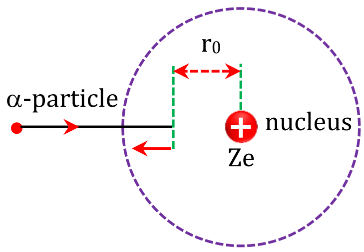
2e = Charge on α–particle

r = Distance of the screen from target

v_0 = Velocity of α–particles







At nearer distance of approach the size of a nucleus or the distance of nearer approach is given by,

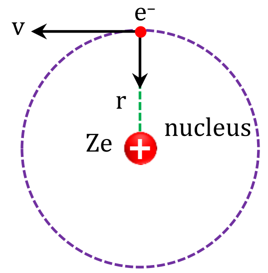
$$r_0 = \frac{1}{4\pi\varepsilon_0} \times \frac{(2Ze)^2}{\left[\frac{1}{2}mv_0^2\right]} = \frac{1}{4\pi\in_0}\frac{(2Ze)^2}{E_K} \quad \text{(Where, } E_K = \text{K.E. of α–Particle)}$$

Failure of Rutherford's Atomic model

(i) It couldn't explain the stability of atom.

(ii) It couldn't explain discrete nature of hydrogen spectra.

Bohr Model of Hydrogen Atom



Bohr's theory of hydrogen atom is based on the following assumption: An electron in an atom moves in a circular orbit about the nucleus under the influence of coulomb's force of attraction between the electron and nucleus. As the atom as a whole is stable the coulombian force of attraction provides necessary centripetal force.

$$\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{mv^2}{r} \quad \text{...(i)}$$

Only those orbits are possible for which the angular momentum of the electron is equal to an integral multiple of $\frac{h}{2\pi}$ i.e.

$$mvr = n\frac{h}{2\pi} \quad \text{...(ii)}$$

Where, h is Planck's constant.

The electron moving in such allowed orbits does not radiate electromagnetic radiations. Thus, the total energy of the electron revolving in any of the stationary orbits remains constant.

Electromagnetic radiations are emitted if an electron jumps from stationary orbit of higher energy E_2 to another stationary orbit of lower energy E_1. The frequency n of the emitted radiation is related by the equation.

$$E_2 - E_1 = h\nu \quad \text{...(iii)}$$

Drawbacks of Bohr Model

- This model could not explain the fine structure of spectral lines, Zeeman effect and Stark effect.
- This model is valid only for single electron systems. (can not explain electron-electron interaction)
- This model is based on circular orbits of electrons whereas in reality there is no orbit.
- Electron is presumed to revolve round the nucleus only whereas in reality motion of electron can not be described.
- This model could not explain the intensity of spectral lines.
- It could not explain the doublets obtained in the spectra of some of the atoms.
- Bohr's model is semi quantum model, it means, it includes two quantum numbers (E and L) but unfortunately it consider circular motion of electron.

Radii of Orbit and Velocity of Revolving Electron

Radii of orbits

From equation, $v = \frac{nh}{2\pi mr}$, Here n is number of orbit.

Substituting value of v in equation $\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m}{r}\left[\frac{nh}{2\pi mr}\right]^2 \Rightarrow r = \frac{mn^2h^2.4\pi\varepsilon_0 r^2}{4\pi^2 m^2 r^2 e^2} = \frac{n^2h^2\varepsilon_0}{\pi me^2}$

In general, $$r_n = \frac{n^2h^2\varepsilon_0}{\pi me^2} \quad \text{...(iv)}$$

Equation (iv) shows that the radii of the permitted orbits vary as the square of n. For the smallest orbit n = 1 substituting the values of h, ε_0, m and e we have

radius of first orbit $r_1 = 0.529 \times 10^{-10}$ m = 0.529 Å

This calculations shows that the atom is about 10^{-10} meter in diameter.

Velocity of Revolving Electron

To obtain the velocity of the revolving electron, we substitute the value of r from eq. (iv) in eq. (ii), we have

$$mv\left[\frac{n^2h^2\varepsilon_0}{\pi me^2}\right]=n\frac{h}{2\pi}\Rightarrow v=\frac{nh}{2\pi}.\frac{\pi me^2}{n^2h^2\varepsilon_0}.\frac{1}{m}=\frac{e^2}{2nh\varepsilon_0} \qquad ...(v)$$

This expression shows that the velocity of the electron is inversely proportional to n i.e. the electron in the inner most orbit has the highest velocity.

Frequency of Electron in an orbit: Frequency of electron is given by,

$$\nu=\frac{1}{T}=\frac{v}{2\pi r} \quad \Rightarrow \quad \nu=\frac{e^2}{2nh\varepsilon_0}\times\frac{1}{2\pi}\times\frac{\pi me^2}{n^2h^2\varepsilon_0}=\frac{me^4}{4\varepsilon_0^2h^3n^3} \qquad ...(vi)$$

This expression shows that the frequency of an electron is inversely proportional to the cube of n.

Electron Energy

The electron energy consist of two types:

(i) Kinetic energy and (ii) Potential energy

(i) Kinetic energy is due to the motion of electron and its value is $\frac{1}{2}mv^2$ where v is the velocity of the electron,

$\therefore$ K.E $=\frac{1}{2}mv^2=\frac{1}{2}m\left[\frac{e^2}{2nh\varepsilon_0}\right]^2$ from equation $\therefore$ K.E. $=\frac{me^4}{8n^2h^2\varepsilon_0^2}$

(ii) Potential energy is due to the fact that electron lies in the electric field of positive nucleus. We know that potential at a distance r from the nucleus is :- $V=\frac{e}{4\pi\varepsilon_0 r}$

The potential energy of electron of charge e is. P.E. = V × (–e) = $\frac{-e^2}{4\pi\varepsilon_0 r}=\frac{-e^2\times\pi me^2}{4\pi\varepsilon_0 n^2h^2\varepsilon_0}=\frac{-me^4}{4n^2h^2\varepsilon_0^2}$

So, total energy in n^{th} orbit, E_n = K.E. + P.E.

$\Rightarrow E_n=\frac{me^4}{8n^2h^2\varepsilon_0^2}-\frac{me^4}{4n^2h^2\varepsilon_0^2} \quad \Rightarrow E_n=\frac{-me^4}{8n^2h^2\varepsilon_0^2}$

Frequency of emitted radiation

The frequency of emitted radiations can be found from the following relation when electron jumps from higher orbit n_2 to lower orbit n_1.

$$h\nu=E_{n_2}-E_{n_1} \quad \Rightarrow \quad \nu=\frac{me^4}{8\varepsilon_0^2h^3}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right] \qquad ...(vii)$$

$\frac{1}{\lambda}=\frac{me^4}{8\varepsilon_0^2h^3c}\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$ where R = $\frac{me^4}{8\varepsilon_0^2h^3c}$; R = Rydberg's constant = $10.97\times10^6\ m^{-1}\approx1.1\times10^7\ m^{-1}$

$$\bar{\nu}=\frac{1}{\lambda}=R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

Electron energy levels in hydrogen atom

Energy of an electron revolving in n^{th} orbit is given by,

$$E_n = \frac{-me^4}{8\varepsilon_0^2 h^2 n^2} = -\frac{(9.11\times10^{-31})(1.6\times10^{-19})^4}{8(8.854\times10^{-12})^2(6.62\times10^{-34})^2 n^2}$$

$$= -\frac{21.7\times10^{-19}}{n^2}\text{joule} = -\frac{21.7\times10^{-19}}{1.6\times10^{-19}}\times\frac{1}{n^2}\text{eV} \qquad (\because 1\text{eV} = 1.6\times10^{-19}\text{ J})$$

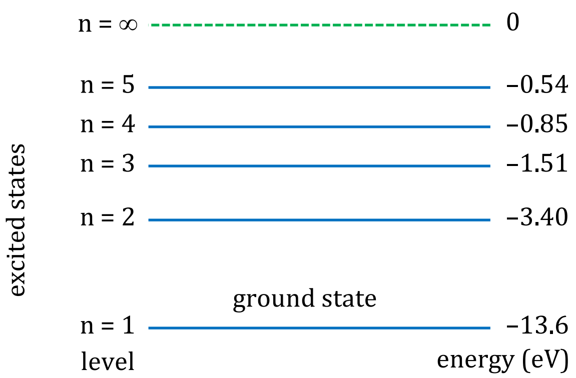
$$\therefore \qquad E_n = \frac{-13.6}{n^2}\text{ eV}$$

The negative sign in energy shows that is thus electron is bound to the nucleus by attractive forces and to separate the electron from the nucleus energy must be supplied to it. Giving different values to n, we can calculate the energy of the electron in different orbits.

$E_1 = -13.6$ eV When $n = 1$ (K - Shell)
$E_2 = -3.4$ eV $n = 2$ (L - Shell)
$E_3 = -1.5$ eV $n = 3$ (M - Shell)

$E_\infty = 0$ eV $n = \infty$ (Limiting case)

	Energy of electron	Binding energy or Ionisation energy
$n = \infty$	0	0
$n = 4$	−0.85 eV	+0.85 eV
$n = 3$	−1.51 eV	+1.51 eV
$n = 2$	−3.4 eV	+3.4 eV
$n = 1$	−13.6 eV	+13.6 eV



The diagram is known as energy level diagram. The lowest energy level (n = 1) correspond to normal unexcited state of hydrogen. This state is also called as ground state. In energy level diagram the lower energy (more negative) are at the bottom while higher energies (Less negative) are at the top. By such a consideration the various electron jumps between allowed orbit will be vertical arrows between different energy level. The energy of radiated photon is greater when the length of arrow is greater.

Spectral Series of Hydrogen Atom

It has been shown that the energy of the outer orbit is greater than the energy of the inner ones. When the Hydrogen atom is subjected to external energy, the electron jumps from lower energy State i.e. the hydrogen atom is excited. The excited state is not stable hence the electron return to its ground state in about 10^{-8} seconds. The excess of energy is now radiated in the form of radiations of different wavelength. The different wavelength constitute spectral series. Which are characteristic of atom emitting, then the wavelength of different members of series can be found from the following relations,

$$\bar{\nu} = \frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

This relation explain the complete spectrum of hydrogen. A detailed account of the important radiations are listed below.

(i) Lyman Series

The series consist of wavelength which are emitted when electron jumps from an outer orbits to the first orbit i. e. the electron jumps to K orbit give rise to Lyman series.

Here $n_1 = 1$ and $n_2 = 2, 3, 4, ...\infty$

The wavelengths of different members of Lyman series are:

(a) First member

In this case $n_1 = 1$ and $n_2 = 2$ hence

$$\frac{1}{\lambda}=R\left[\frac{1}{1^2}-\frac{1}{2^2}\right]=\frac{3R}{4} \quad \text{or} \quad \lambda=\frac{4}{3R} \quad \text{or} \quad \lambda=\frac{4}{3\times10.97\times10^6} = 1216\times10^{-10}\text{ m} = 1216\text{Å}$$

(b) Second member

In this case $n_1 = 1$ and $n_2 = 3$ hence

$$\frac{1}{\lambda}=R\left[\frac{1}{1^2}-\frac{1}{3^2}\right]=\frac{8R}{9} \quad \text{or} \quad \lambda=\frac{9}{8R} \quad \text{or} \quad \lambda=\frac{9}{8\times10.97\times10^6} = 1026\times10^{-10}\text{ m} =1026\text{Å}$$

similarly, the wavelength of the other members can be calculated.

(c) Limiting member

In this case $n_1 = 1$ and $n_2 = \infty$, hence

$$\frac{1}{\lambda}=R\left[\frac{1}{1^2}-\frac{1}{\infty^2}\right]=R \quad \text{or} \quad \lambda=\frac{1}{R}$$

or $\lambda=\frac{1}{10.97\times10^6}=912\times10^{-10}\text{m}=912\text{Å}$ This series lies in ultraviolet region.

(ii) Balmer Series

This series is consist of all wavelengths which are emitted when an electron jumps from an outer orbit to the second orbit i. e. the electron jumps to L orbit give rise to Balmer series.

Here $n_1 = 2$ and $n_2 = 3, 4, 5 ....\infty$

The wavelength of different members of Balmer series.

(a) First member

In this case $n_1 = 2$ and $n_2 = 3$, hence

$$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{3^2}\right]=\frac{5R}{36} \quad \text{or} \quad \lambda=\frac{36}{5R}$$

or $\lambda=\frac{36}{5\times10.97\times10^6} = 6563\times10^{-10}\text{m} = 6563\text{Å}$

(b) Second member

In this case $n_1 = 2$ and $n_2 = 4$, hence.

$$\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{4^2}\right]=\frac{3R}{16} \quad \text{or} \quad \lambda=\frac{16}{3R}$$

or $\lambda=\frac{16}{3\times10.97\times10^6} = 4861\times10^{-10}\text{m} = 4861\text{Å}$

(c) Limiting member

In this case $n_1 = 2$ and $n_2 = \infty$, hence $\frac{1}{\lambda}=R\left[\frac{1}{2^2}-\frac{1}{\infty}\right]=\frac{R}{4}= \lambda=\frac{4}{R} = 3646\text{Å}$

This series lies in visible and near ultraviolet region.

(iii) Paschen Series

This series consist of all wavelengths are emitted when an electron jumps from an outer orbit to the third orbit i. e. the electron jumps to M orbit give rise to paschen series.

Here $n_1 = 3$ and $n_2 = 4, 5, 6 \,.......\infty$

The different wavelengths of this series can be obtained from the formula

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{n_2^2}\right] \qquad \text{where } n_2 = 4, 5, 6 \,........\infty$$

For the first member, the wavelength is 18750Å. This series lies in infra-red region.

(iv) Brackett Series

This series is consist of all wavelengths which are emitted when an electron jumps from an outer orbits to the fourth orbit i. e. the electron jumps to N orbit give rise to Brackett series.

Here $n_1 = 4$ and $n_2 = 5, 6, 7, ...\infty$

The different wavelengths of this series can be obtained from the formula

$$\frac{1}{\lambda} = R\left[\frac{1}{4^2} - \frac{1}{n_2^2}\right] \qquad \text{where } n_2 = 5, 6, 7 \,..........\infty$$

This series lies in infra-red region of spectrum.

(v) Pfund series

The series consist of all wavelengths which are emitted when an electron jumps from an outer orbit to the fifth orbit i. e. the electron jumps to O orbit give right to Pfund series.

Here $n_1 = 5$ and $n_2 = 6, 7, 8 \,.........\infty$

The different wavelengths of this series can be obtained from the formula

$$\frac{1}{\lambda} = R\left[\frac{1}{5^2} - \frac{1}{n_2^2}\right] \qquad \text{where } n_2 = 6, 7, 8 \,.......\infty$$

This series lies in infra-red region of spectrum.

Conclusion

S. No.	Series Observed	Value of n_1	Value of n_2	Position in the Spectrum
1.	Lyman Series	1	$2,3,4...\infty$	Ultra Violet
2.	Balmer Series	2	$3,4,5...\infty$	Visible
3.	Paschen Series	3	$4,5,6....\infty$	Infra-red
4.	Brackett Series	4	$5,6,7....\infty$	Infra-red
5.	Pfund Series	5	$6,7,8....\infty$	Infra-red

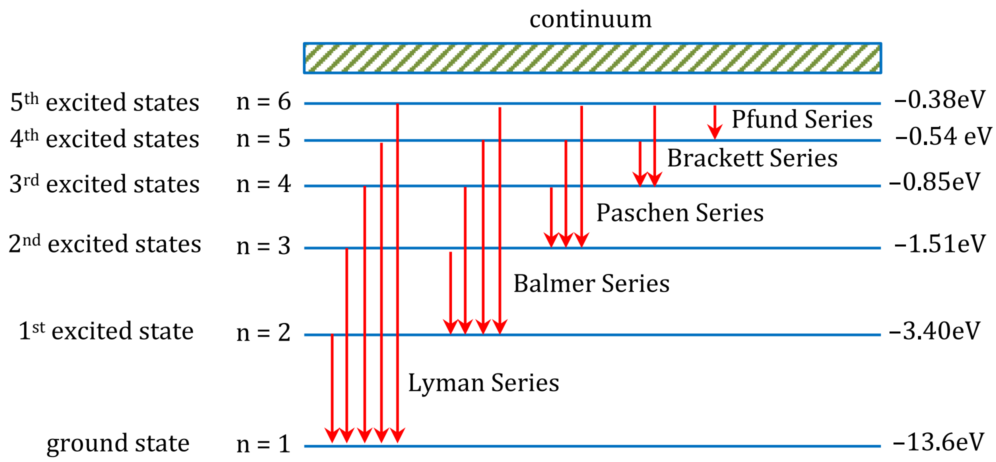


Figure: Conclusion in spectral form

Illustration 1

Find the first and second excitation potentials of an atom when its ionisation potential is 122.4 V.

Solution:

I.P. = 122.4 V $\qquad E_{ex1} = 122.4 - \dfrac{122.4}{4} = 91.8\text{ V}$

$\therefore \qquad E_{ex2} = 122.4 - \dfrac{122.4}{9} = 108.8\text{ V}$

Illustration 2

Find the atomic number of atom when given that its ionisation potential is equal to 122.4 V.

Solution:

I.P. = 122.4 V $\qquad E = Z^2E_H \qquad \because \qquad Z = \sqrt{\dfrac{E}{E_{H_1}}} = \sqrt{\dfrac{122.4}{13.6}} = 3$

Illustration 3

Find the maximum wavelength of Brakett series of hydrogen atom.

Solution:

$n_1 = 4$ and $n_2 = 5 \qquad \therefore \dfrac{1}{\lambda_{max}} = R\left[\dfrac{1}{4^2} - \dfrac{1}{5^2}\right]$ or $\qquad \lambda_{max} = \dfrac{25\times16\times10^{10}}{9\times1.1\times10^{7}} = 40400$ Å

Illustration 4

Find the value of magnetic induction at the proton due to electron motion, if the radius of the first orbit of hydrogen atom is 0.5 Å and the speed of electron in it is 2.2×10^6 m/sec.

Solution:

r = 0.5 Å and V = 2.2×10^6 m/sec

$$B = \frac{\mu_0}{4\pi}\frac{ev}{r^2} = \frac{10^{-7}\times1.6\times10^{-19}\times2.2\times10^{6}}{25\times10^{-22}} = 14.08\text{ Tesla}$$

Illustration 5

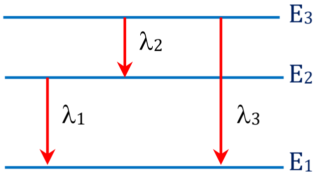
Find the ratio of equivalents current due to electron motion in first and second orbits of hydrogen atom.

Solution:

$I_n \propto \dfrac{1}{n^3} \qquad \therefore \dfrac{I_1}{I_2} = \left[\dfrac{n_2}{n_1}\right]^3 = \left[\dfrac{2}{1}\right]^3 = 8:1$

Illustration 6

For the given transitions of electron, obtain the relation between λ_1, λ_2 & λ_3.



Solution:

For given condition $\quad E_3 - E_1 = (E_3 - E_2) + (E_2 - E_1)$

$$\Rightarrow \frac{hc}{\lambda_3} = \frac{hc}{\lambda_2} + \frac{hc}{\lambda_1} \Rightarrow \frac{1}{\lambda_3} = \frac{1}{\lambda_2} + \frac{1}{\lambda_1} \text{ Therefore } \lambda_3 = \frac{\lambda_2\lambda_1}{\lambda_2 + \lambda_1}$$

BEGINNER'S BOX-1

1. Find the longest and shortest wavelength when a hydrogen atom in the ground state is excited by radiations of wavelength 975 Å.

PMD250

2. How many lines will be possible in the absorption spectrum when a hydrogen atom in the ground state is excited by radiations of wavelength 975 Å.

PMD251

3. Find the ratio of wavelength of first line of Lyman series of doubly ionised lithium atom to that of the first line of Lyman series of deuterium ($_1H^2$).

PMD252

4. Find the ratio of the area of orbit of first excited state of electron to the area of orbit of ground level for hydrogen atom.

PMD253

5. If the ionisation potential in the ground state for hydrogen is 13.6 eV., then find the excitation potential of third orbit.

PMD254

6. When an electron jump from second orbit to ground state of hydrogen atom then calculate the wavelength of emitted photon.

PMD255

BEGINNER'S BOX ANSWER KEY

BEGINNER'S BOX-1

1.	λ_{max} = 18787.8 Å, λ_{min} = 973 Å			**2.**	n = 4		
3.	1 : 9	**4.**	16 : 1	**5.**	0.66 eV	**6.**	122 nm.