Modern Physics-1 [Dual Nature of Radiation and Matter]

SOLUTIONS

Exercise-I (Conceptual Questions)

1. Maximum wavelength in visible light is 7800 Å. So energy of photon of this wavelength is

$$E = \frac{hc}{\lambda} = \frac{6.62\times10^{-34}\times3\times10^{8}}{7800\times10^{-10}\times1.6\times10^{-19}} = 1.6 \text{ eV}$$

2. Momentum of photon $p = \frac{h\nu}{c}$

$$= \frac{6.6\times10^{-34}\times1.5\times10^{13}}{3\times10^{8}} = 3.3\times10^{-29} \text{ kg m/s}$$

6. $W_0 = \frac{hc}{\lambda_0} = \frac{12400}{\lambda_0}$ eV–Å

or $\lambda_0 = \frac{12400}{4} = 3100$ Å $= 310$ nm

7. For element A , $\lambda_0 = \lambda$

$\therefore \quad W_A = \frac{hc}{\lambda_0} = \frac{hc}{\lambda} = 2.5$ eV ....(1)

For element B, $\quad W_B = \frac{hc}{\lambda_B} = 5$ eV......(2)

Dividing (1) & (2), we get

$\frac{hc}{\lambda}\times\frac{\lambda_B}{hc} = \frac{2.5}{5} \quad \Rightarrow \quad \lambda_B = \frac{\lambda}{2}$

8. $W_0 = \frac{hc}{\lambda_0} = \frac{12400}{\lambda_0}$ eV–Å $= \frac{12400}{6000}$

$= 2.06 \text{ eV} \approx 2 \text{ eV}$

9. Minimum kinetic energy is always zero.

10. $K.E._{max} = eV_0 = 4eV$

$\therefore \quad V_0 = 4$ volt

11. Stopping potential does not depend on relative distance between the source and cell and saturation current $\propto \frac{1}{d^2}$.

12. $\lambda_0 = 200$ nm $= 2000$ Å

$\lambda = 100$ nm $= 1000$ Å

$$K.E._{max} = \left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) 12400 \text{eV – Å}$$

$$= 12400\left(\frac{1}{1000} - \frac{1}{2000}\right) \text{eV} = \frac{12400}{2000} = 6.2 \text{ eV}$$

17. $n_e \propto \frac{1}{d^2}$

19. $(KE)_{max} = hv - \phi_0 = 6eV - 4 eV = 2 eV$

21. $\nu_0 = \frac{\phi_0}{h} = \frac{3.3\times1.6\times10^{-19}}{6.6\times10^{-34}} = 8\times10^{14}$ Hz

25. The stopping potential depends on the substance of the cathode. Since it is same for A and B, the cathodes are made of the same substance. Since the current of A is greater than the current of B, the lights used are of different intensities. Light of A is more intense.

28. $(K.E.)_{max} = h\upsilon - \phi$

$= 4$ eV $- 2$ eV

K.E. = 2 eV; V = 2 volt

29. $\lambda = \frac{hc}{E} = \frac{h}{p}$

on decreasing λ, both p and E will increase

31. E = 100 eV

V = 100 volt

$\lambda = \frac{12.27}{\sqrt{V}}$ Å $= \frac{12.27}{\sqrt{100}} = 1.227$ Å

32. $\lambda = \frac{h}{\sqrt{2mqV}} \quad \therefore \quad \lambda \propto \frac{1}{\sqrt{mq}} \quad (\because V \text{ is constant})$

$$\frac{\lambda_d}{\lambda_p} = \sqrt{\frac{m_p q_p}{m_d q_d}} = \sqrt{\frac{m_p q_p}{2m_p q_p}} = \frac{1}{\sqrt{2}}$$

33. V = 40 – 20 = 20 volt

$\lambda = \frac{12.27}{\sqrt{V}}$ Å $= \frac{12.27}{\sqrt{20}}$ Å $= 2.75$ Å

34. $\lambda = \frac{h}{mv} = \frac{6.62\times10^{-34}}{9\times10^{-31}\times6.6\times10^{3}} = 1.1\times10^{-7}$ m

35. Here $\lambda_2 = \lambda_1/2$

$\lambda = \frac{h}{\sqrt{2mE}} \quad \Rightarrow \quad \lambda \propto \frac{1}{\sqrt{E}}$

$E \propto \frac{1}{\lambda^2} \quad \Rightarrow \quad \frac{E_1}{E_2} = \left(\frac{\lambda_2}{\lambda_1}\right)^2$

$$E_2 = E_1 \times \left(\frac{\lambda_1}{\lambda_2}\right)^2 = E_1\left(\frac{\lambda_1}{\lambda_1/2}\right)^2 = 4E_1 = E_1 + 3E_1$$

energy added should be thrice initial energy.

36. $\lambda = \frac{h}{\sqrt{2mE}}$ Here E is same. So $\lambda \propto \frac{1}{\sqrt{m}}$

$m_\alpha > m_n > m_P > m_e \quad \therefore \quad \lambda_\alpha < \lambda_n < \lambda_P < \lambda_e$

37. $\lambda = \frac{12.27}{\sqrt{V}}$ Å $\Rightarrow \lambda = \frac{12.27}{\sqrt{50000}}$ = 0.055 Å

38. For a neutron m = 1.7×10^{-27} kg

$\lambda = \frac{h}{\sqrt{2mE}} = \frac{6.62\times10^{-34}}{\sqrt{2\times1.7\times10^{-27}\times E}} = \frac{0.284}{\sqrt{E(eV)}}$ Å

$= \frac{0.286}{\sqrt{3}} \simeq 0.16$ Å = 1.6×10^{-11} m

39. $\lambda = \frac{h}{\sqrt{2mqV}}$ Here V is same. So $\lambda \propto \frac{1}{\sqrt{mq}}$

Or $\frac{\lambda_P}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_P q_P}} = \sqrt{\frac{4m_P \times 2q_P}{m_P q_P}} = \frac{2\sqrt{2}}{1}$

41. Energy in ground state = 13.6 eV

Ist Method : $\lambda = \frac{h}{\sqrt{2mE}}$

$= \frac{6.62\times10^{-34}}{\sqrt{2\times9.1\times10^{-31}\times13.6\times1.6\times10^{-19}}}$

= 3.3×10^{-10} m = 3.3 Å

IInd Method : $\lambda = \frac{12.27}{\sqrt{V}}$ Å

$\Rightarrow \lambda = \frac{12.27}{\sqrt{13.6}} = 3.3$ Å

47. By de Broglie formula

$p = \frac{h}{\lambda} = \frac{6.6\times10^{-34}}{4400\times10^{-10}} = 1.5\times10^{-27}$ kgm/s

For photon, velocity = c = 3×10^8 m/s

$\therefore m = \frac{p}{c} = \frac{1.5\times10^{-27}}{3\times10^8}$ or m = 5×10^{-36} kg

50. $\because$ $2\pi r = n\lambda$

Here n = 1

$\therefore$ $2\pi r = \lambda$

52. K.E = 1 KeV

54. $2\pi r_n = n\lambda \Rightarrow 2\pi r_n = 2 \times 10^{-9}$m

55. $\lambda = \frac{h}{mv}$

$\therefore \frac{\lambda_2}{\lambda_1} = \frac{m_1 v_1}{m_2 v_2} = \frac{2m_1}{m_1}\frac{2v_1}{v_1} = \frac{4}{1}$

$\therefore \lambda_2 = 4\lambda_1$

New wavelength = 4 × original wavelength

$\therefore$ The wavelength increases by a factor more than 2.

56. For charged particle

$\lambda = \frac{h}{\sqrt{2mqV}}$

for proton and electron

q $\rightarrow$ same, V $\rightarrow$ same

So, $\lambda \propto \frac{1}{\sqrt{m}} \Rightarrow \frac{\lambda_e}{\lambda_p} = \sqrt{\frac{m_p}{m_e}}$

57. de-Broglie wavelength $\lambda = \frac{h}{\sqrt{2mE}}$

where E is the kinetic energy

Since E is the same, $\therefore \lambda \propto \frac{1}{\sqrt{m}}$

Since $m_\alpha > m_p > m_e$, $\therefore \lambda_e > \lambda_p > \lambda_\alpha$.

58. For a particle, de Broglie wavelength $\lambda = \frac{h}{p}$

Kinetic energy K = $\frac{p^2}{2m}$

$\therefore \lambda = \frac{h}{\sqrt{2mK}}$

For an electron $\lambda = \frac{h}{\sqrt{2m_e K_e}}$

$= \frac{(6.63\times10^{-34} J)}{\sqrt{2\times(9.11\times10^{-31} kg)\times(120\times1.6\times10^{-19} J)}}$

= 112×10^{-12} m = 112 pm [$\therefore$ 1 pm = 10^{-12} m]

OR

K.E. = 120 eV V = 120 volt

For electron

$\lambda = \frac{12.27}{\sqrt{120}}$ = 112 pm.

59. $\lambda = \frac{h}{\sqrt{2mk}}$

For protons $\lambda_p = \frac{h}{\sqrt{2m_p.k_p}}$ ....(i)

For α – particle $\lambda_\alpha = \frac{h}{\sqrt{2m_\alpha.k_\alpha}}$

$\lambda_\alpha = \frac{h}{\sqrt{2.(4m_p).k_p}}$ ($\because$ $k_p = k_\alpha$) given

$\lambda_\alpha = \frac{h}{2\sqrt{2m_p k_p}}$ ....(ii)

Dividing (i) by (ii) we get $\frac{\lambda_p}{\lambda_\alpha} = \frac{\frac{h}{\sqrt{2m_pk_p}}}{\frac{h}{2\sqrt{2m_pk_p}}}$

$\frac{\lambda_p}{\lambda_\alpha} = 2$, $\lambda_p = 2\lambda_\alpha$

Exercise-II (Previous Year Questions)

1. Average force $F_{av} = \frac{\Delta p}{\Delta t} = \frac{2IA}{c}$

$= \frac{2\times25\times10^4\times15\times10^{-4}}{3\times10^8} = 2.50 \times 10^{-6}$ N

2. By using $h\nu = \phi_0 + K_{max}$

We have

$h\nu = \phi_0 + 0.5$ ...... (i)

and $1.2h\nu = \phi_0 + 0.8$ ...... (ii)

Therefore $\phi_0 = 1.0$ eV

3. $\lambda = \frac{h}{\sqrt{2mK}}$

$\frac{\lambda_1}{\lambda_2} = \sqrt{\frac{K_2}{K_1}} = \sqrt{\frac{16K}{K}} = \frac{4}{1}$

% change $= \frac{1-4}{4}\times100 = -75\%$

4. $eV_S = E - \phi \Rightarrow V_S = \frac{hc}{\lambda e} - \frac{hc}{\lambda_0 e}$

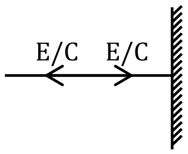
here $3V_0 = \frac{hc}{\lambda e} - \frac{hc}{\lambda_0 e}$ ...(1)

and $V_0 = \frac{hc}{2\lambda e} - \frac{hc}{\lambda_0 e}$ ...(2)

equation (1) – 3 × equation (2)

$\Rightarrow 0 = -\frac{hc}{2\lambda e} + \frac{2hc}{\lambda_0 e} \Rightarrow \lambda_0 = 4\lambda$

5.



Momentum of light $p = \frac{E}{C}$

So momentum transferred to the surface

$= p_f - p_i = \frac{2E}{C}$

6. $p = \frac{h}{\lambda} \Rightarrow p \propto \frac{1}{\lambda}$ (Rectangular hyperbola)

7. Energy of photon (E) = $\frac{12400}{5000}$ = 2.48 eV

Work function (ϕ_0) = 2.28 eV

According to Einstein equation

$E = \phi_0 + (K.E.)_{max}$

$\Rightarrow$ 2.,48 = 2.28 + $(K.E.)_{max}$

$\Rightarrow$ $(K.E.)_{max}$ = 0.20 eV

For electron $\lambda = \frac{h}{\sqrt{2mE}} \Rightarrow \lambda \approx 28$ Å

So $\lambda \geq 2.8\times10^{-9}$ m

8. $KE_1 = \frac{hc}{\lambda} - \phi$

$KE_2 = \frac{hc}{\lambda/2} - \phi = \frac{2hc}{\lambda} - \phi$

$KE_2 = 3KE_1$

$\Rightarrow \frac{2hc}{\lambda} - \phi = 3\left(\frac{hc}{\lambda} - \phi\right)$

$\Rightarrow 2\phi = \frac{hc}{\lambda}$

$\Rightarrow \phi = \frac{hc}{2\lambda}$

9. For electron $\lambda_e = \frac{h}{\sqrt{2mE}}$

for Photon E = pc

$\Rightarrow \lambda_{Ph} = \frac{hc}{E}$

$\Rightarrow \frac{\lambda_e}{\lambda_{Ph}} = \frac{h}{\sqrt{2mE}}\times\frac{E}{hc} = \left(\frac{E}{2m}\right)^{1/2}\frac{1}{c}$

10. $eV = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ ...(i)

$\frac{eV}{4} = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$ ...(ii)

From equation (i) and (ii)

$\Rightarrow 4 = \frac{\frac{1}{\lambda} - \frac{1}{\lambda_0}}{\frac{1}{2\lambda} - \frac{1}{\lambda_0}}$ On solving $\lambda_0 = 3\lambda$

11. $eV_s = \frac{1}{2}mv_{max}^2 = h\nu - \phi_0$

$2 = 5 - \phi_0 \Rightarrow \phi_0 = 3$ eV

In second case

$eV_s = 6 - 3 = 3$ eV $\Rightarrow V_s = 3$ V.

$\therefore$ $V_{AC} = -3$ V

12. Kinetic energy of thermal neutron with equilibrium is $\frac{3}{2}KT$

$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2m\,K.E}} = \frac{h}{\sqrt{2m\left(\frac{3}{2}KT\right)}}$$

$$= \frac{h}{\sqrt{3\,mKT}}$$

13. $\lambda_0 = 3250$ Å

$\lambda = 2536$ Å

$$\frac{1}{2}mv^2 = hc\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]$$

$$v = \sqrt{\frac{2hc}{m}\left[\frac{1}{\lambda} - \frac{1}{\lambda_0}\right]}$$

$$= \sqrt{\frac{2\times12400\times1.6\times10^{-19}}{9.1\times10^{-31}}\left[\frac{714}{2536\times3250}\right]}$$

$= 0.6 \times 10^6$ m/s $= 6 \times 10^5$ m/s

14. $$\vec{a} = \frac{\vec{F}}{m} = \frac{q\vec{E}}{m} = \frac{(-e)(-E_0)\hat{i}}{m} = \frac{eE_0}{m}\hat{i}$$

$$V = V_0 + \frac{eE_0}{m}t$$

$$\lambda = \frac{h}{mv} = \frac{h}{m\left[V_0 + \frac{eE_0 t}{m}\right]} = \frac{h}{mV_0\left[1 + \frac{eE_0 t}{mV_0}\right]}$$

$$= \frac{\lambda_0}{1 + \frac{eE_0 t}{mV_0}}$$

15. From Einstein's equation of PEE

$\frac{1}{2}mv_1^2 = 2h\nu_0 - h\nu_0 \Rightarrow \frac{1}{2}mv_1^2 = h\nu_0$ ...(1)

$\frac{1}{2}mv_2^2 = 5h\nu_0 - h\nu_0 \Rightarrow \frac{1}{2}mv_2^2 = 4h\nu_0$ ...(2)

(1) ÷ (2)

$$\frac{v_1^2}{v_2^2} = \frac{1}{4} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

16. $\lambda = \sqrt{\frac{150}{V}}$ Å

$\lambda = \sqrt{\frac{150}{10^4}}$ Å $= 12.27 \times 10^{-12}$ m

17. $\phi = \frac{hc}{\lambda}$

$\Rightarrow 4$ eV $= \frac{1240}{\lambda}$

$\Rightarrow \lambda = \frac{1240}{4} = 310$ nm

18. $\lambda = \frac{h}{\sqrt{2mE_k}}$ ($\because$ E_k is same for both)

$$\Rightarrow \frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{4m}{m}} = 2:1$$

19. $I = \frac{E}{At} \Rightarrow E = IAt = \frac{20}{10^{-4}} \times 20 \times 10^{-4} \times 60$

$= 24 \times 10^3$ J

20. $E = \frac{10^{-20}}{1.6\times10^{-19}}$ eV $= 0.0625$ eV

21. $\lambda = 1.227 \times 10^{-2}$ nm $= 0.1227$ Å

$\lambda = \frac{12.27}{\sqrt{V}}$ Å $\Rightarrow 0.1227 = \frac{12.27}{\sqrt{V}}$ Å

$\Rightarrow \sqrt{V} = 10^2 \Rightarrow V = 10^4$ volt

22. $K_1 = 1.5\,h\nu_0 - \phi_0 = 0.5\,h\nu_0$

$K_2 = \frac{1.5}{2}h\nu_0 - h\nu_0 = -0.25\,h\nu_0$

$\because$ Kinetic energy can never be negative

So, no emission and i = 0

OR

In second case the incident frequency is halved

Incident frequency $= \frac{1.5}{2}\nu_0 = 0.75\,\nu_0$

Now the incident frequency is less than threshold frequency so no emission of electron take place therefore no current. (i = 0)

23. $\lambda = \frac{12.27}{\sqrt{V}}$ Å $= \frac{12.27}{\sqrt{144}} \times 10^{-10}$

$= 1.02 \times 10^{-10}$ m $= 102 \times 10^{-3}$ nm

24. The wave nature of electrons was experimentally verified by Davission and Germer.

25. $P = \frac{nhc}{\lambda} \Rightarrow n = \frac{P\lambda}{hc}$

$n = \frac{3.3 \times 10^{-3} \times 600 \times 10^{-9}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 10^{16}$

26. $\frac{hc}{\lambda} = K_{max} + \phi$ [given ϕ is negligible]

so, $\frac{hc}{\lambda} = K_{max}$

and $\lambda_d = \frac{h}{\sqrt{2m\,K_{max}}} \Rightarrow K_{max} = \frac{h^2}{2m\lambda_d^2}$

$\left(\frac{hc}{\lambda}\right) = \frac{h^2}{2m\lambda_d^2} \Rightarrow \lambda = \left(\frac{2mc}{h}\right)\lambda_d^2$

30. Given energy of photon E = 2.20 eV

Work function of Cs ϕ_0 = 2.14 eV, K ϕ_0 = 2.30 eV, Na ϕ_0 = 2.75 eV

We know that e^- emitts when $h\nu > \phi_0$

here it is clear that energy of photon is more than the work function of Cs [Caesium] only so Ans. only (Cs).

31. $\lambda_e = \frac{12.27}{\sqrt{V}}$ Å $= \frac{12.27}{\sqrt{81}}$ Å $= \frac{12.27}{9}$ Å

= 1.36 Å $\left(\because 1\text{Å} = \frac{1}{10}\text{nm}\right)$

= 0.136 nm

32. Maximum kinetic energy of emitted electron is independent of intensity of radiation.

33. For a photon,

(i) Energy E = $h\nu$ $\Rightarrow$ (statement A is correct)

(ii) All photons travel with speed of light (= c in free space)

$\Rightarrow$ statement B is correct

(iii) Momentum of a photon. $p = \frac{E}{c} = \frac{h\nu}{c}$

$\Rightarrow$ Statement C is correct.

(iv) In a photon-electron collision, total energy and total momentum are conserved.

$\Rightarrow$ statement D is also correct.

(v) Photons are massless and do not carry any charge.

$\Rightarrow$ statement E is incorrect.

Correct choice (2)

A, B, C, & D are correct.

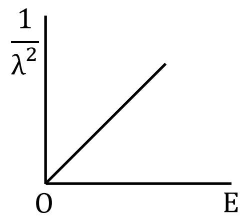
34. de-Broglie wavelength and energy relation of a free particle

$\lambda = \frac{h}{\sqrt{2mE}}$

$\lambda^2 = \frac{h^2}{2mE}$

$\frac{1}{\lambda^2} = \frac{2m}{h^2}E$

Graph $\frac{1}{\lambda^2}$ v/s E



35. Ve = Vx

V $\propto$ Same

$\lambda = \frac{h}{\sqrt{2mqv}}$

$\lambda \propto \frac{1}{\sqrt{mq}}$

$\frac{\lambda_e}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_e q_e}} = \sqrt{\frac{4m_p \times 2q_e}{m_e \times q_e}}$ $(q_e)=(q_p)$

$\frac{\lambda_e}{\lambda_\alpha} = \sqrt{8}\sqrt{\frac{m_p}{m_e}}$ $\begin{cases} m_p > me \\ \sqrt{8} = 2.82 \end{cases}$

$\lambda_e > \lambda_\alpha$

36. $\lambda = \frac{hc}{e}$

$K_{max} = \frac{hc}{\lambda} - \phi$

$K_{max} = \frac{ehc}{hc} - \phi$

$K_{max} = e - \phi$

Exercise-III (Analytical Questions)

1. Photo electric equation

$KE_{max} = E - \phi = h\nu - \phi$

$(4.14 \times 10^{-15}\ eV\text{–}s)\ (6 \times 10^{14}\ Hz) - 2.0\ eV$

$= 2.48 - 2.0 = 0.48\ eV \approx 0.5 eV$

$= 0.5 \times 1.6 \times 10^{-19} = 8 \times 10^{-20}\ J$

Stopping potential

$eV_s = KE_{max} = 0.48eV$

$V_s = 0.48$ volt

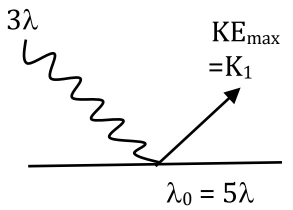
$$n = \frac{I\lambda}{hc} \times A = \frac{I}{h\nu} \times A$$

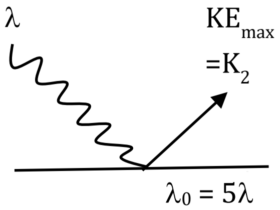
$$= \frac{60 \times 10^3 \times 6.6 \times 10^{-4}}{6.6 \times 10^{-34} \times 6 \times 10^{14}}$$

$n = 10^{20}$ per second

$i = ne = 10^{20} \times 1.6 \times 10^{-19} = 16\ A$

4.





$$K_1 = \frac{hc}{3\lambda} - \frac{hc}{5\lambda} = \frac{hc}{\lambda}\left[\frac{1}{3} - \frac{1}{5}\right] = \frac{hc}{\lambda}\left[\frac{2}{15}\right]$$

$$K_2 = \frac{hc}{\lambda} - \frac{hc}{5\lambda} = \frac{hc}{\lambda}\left[1 - \frac{1}{5}\right] = \frac{hc}{\lambda}\left[\frac{4}{5}\right]$$

(a) Ratio of KE

$$\frac{K_1}{K_2} = \frac{\frac{hc}{\lambda}\left[\frac{2}{15}\right]}{\frac{hc}{\lambda}\left[\frac{4}{5}\right]} = \frac{1}{6} = 1:6$$

(b) Ratio of speeds of incident photons

$$\frac{c_1}{c_2} = \frac{1}{1} = 1:1$$

Ratio of momentum of incident photons

$$\frac{p_1}{p_2} = \frac{\lambda_2}{\lambda_1} = \frac{\lambda}{3\lambda} = 1:3$$

(c) Ratio of fastest speed of photo electrons

$$\frac{v_1}{v_2} = \sqrt{\frac{K_1}{K_2}} = \sqrt{\frac{1}{6}} = 1:\sqrt{6}$$

6. Statement-I is true, statement-II is false.

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$\lambda \propto \frac{1}{\sqrt{m}}$ $(\therefore$ KE = same)

7. (A) $\lambda = \frac{h}{mv}$ or $\lambda \propto \frac{1}{m}$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{m_\alpha}{m_p} = \frac{4}{1}\ (\therefore m_\alpha \approx 4m_p)$$

(B) $\lambda = \frac{h}{\sqrt{2mqV}}$ or $\lambda \propto \frac{1}{\sqrt{m.q}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\left(\frac{m_\alpha}{m_p}\right)\left(\frac{q_\alpha}{q_p}\right)} = \sqrt{\frac{4}{1} \times \frac{2}{1}} = \frac{2\sqrt{2}}{1}$$

$(\therefore\ q_\alpha = 2q_p)$

(C) $\lambda = \frac{h}{\sqrt{2mE_k}}$ or $\lambda \propto \frac{1}{\sqrt{m}}$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha}{m_p}} = \sqrt{\frac{4}{1}} = \frac{2}{1}$$

(D) $\lambda = \frac{h}{p}$

$$\frac{\lambda_p}{\lambda_\alpha} = \frac{1}{1}$$

8. Give $\lambda_p = 2\lambda_e$

So $\frac{p_p}{p_e} = \frac{\lambda_e}{\lambda_p} = \frac{1}{2}\ \left(\because \lambda = \frac{h}{p}\right)$

$$\frac{E_e}{E_p} = \frac{\frac{1}{2}m_e v^2}{\frac{hc}{\lambda}} = \frac{\frac{1}{2}(mv)v}{\left(\frac{h}{\lambda}\right)c}$$

$$= \frac{1}{2}\frac{p_e.v}{p_p c} = \frac{1}{2}\left(\frac{p_e}{p_p}\right)\frac{v}{c}$$

$$\frac{E_e}{E_p} = \frac{1}{2}\left(\frac{2}{1}\right)\left(\frac{1}{100}\right) = 10^{-2}$$

and $\frac{p_e}{m_e c} = \frac{m_e v_e}{m_e c} = \frac{v_e}{c} = \frac{1}{100} = 10^{-2}$

Modern Physics-2 [Nuclear Physics & Radioactivity]

SOLUTIONS

Exercise-I (Conceptual Questions)

2. B.E. of A = 240 × 7.6 = 1824 MeV
B.E. of B = 100 × 8.1 = 810 MeV
B.E. of C = 140 × 8.1 = 1134 MeV
So Q = (810 + 1134) MeV – 1824 MeV
= 120 MeV

3. Energy released in 10 kg of Uranium
$E = mc^2 = 10 \times (3 \times 10^8)^2 = 9 \times 10^{17}$ J

4. Energy released in 2 mole of U^{235}
$= \dfrac{6.02\times10^{23}\times2\times235}{235}\times200$ MeV
$= 24 \times 10^{25}$ MeV

22. Released power 1 kW or 1×10^3 W
and per fission released energy 200 MeV
or $200 \times 1.6 \times 10^{-13}$ J
Number of fission $= \dfrac{10^3}{200\times1.6\times10^{-13}}$
per sec $= 3.125 \times 10^{13}$

29. Binding energy $= [Zm_p + (A - Z)m_n] - M_n$
$= [2 \times 1.0073 + 2 \times 1.0087] - 4.0015$
$= 4.032 - 4.0015 = 0.0305$ u
or $0.0305 \times 931 = 28.4$ MeV

33. Nuclear density is of the order of 10^{17} kg/m^3.

36. Mass of nucleus is slightly less than sum of masses of its constituents. This mass difference is equivalent to binding energy.
$$\Delta m = \frac{B}{c^2} = (Zm_p + NM_n) - M(N,Z)$$
Hence $M(N, Z) = NM_n + ZM_P - B/c^2$

38. m = 1mg = 10^{-6} kg
$E = mc^2 = 10^{-6} \times (3 \times 10^8)^2 = 9 \times 10^{10}$ J

41. Isotones $\rightarrow$ number of neutrons are same(A-Z)

42. In option (1) charge is not conserved
i.e. $(5 + 2 \neq 7 + 1) \Rightarrow$ not possible
In (2) this reaction is possible only at very high temperature (or ${}^{23}_{11}Na$ is a very stable isotope and given nuclear reaction is very difficult)
In (3) & (4) $\bar{\nu}$ is emitted with β^- so option (4) is also wrong so correct answer is (3)

43. Energy released = binding energy of products – binding energy of reactants
= c – (a + b) = c – a – b

52. $N_\alpha = \dfrac{A_i - A_t}{4} = \dfrac{238-222}{4} = 4$
$N_\beta = Z_f - Z_i + 2N_\alpha = 83 - 90 + 2(4) = 1$

53. $N_\alpha = \dfrac{A_i - A_f}{4} = \dfrac{238-206}{4} = 8$
$N_\beta = Z_f - Z_i + 2N_\alpha = 82 - 92 + 2(8) = 6$

Exercise-II (Previous Year Questions)

1. BE of ${}_2He^4 = 4 \times 7.06 = 28.24$ MeV
BE of ${}_3{}^7Li = 7 \times 5.60 = 39.20$ MeV
${}_3{}^7Li + {}_1{}^1H \longrightarrow {}_2He^4 + {}_2He^4 + Q$
39.20 $\quad$ 28.24 × 2
Q = 56.48 – 39.20 = 17.28 MeV

2. $R \propto A^{1/3}$
$$\frac{R_{Al}}{R_{Te}} = \left(\frac{27}{125}\right)^{1/3}$$
$\Rightarrow R_{Te} = \dfrac{5}{3} R_{Al}$

3. By COLM :
$p_f - p_i = 0$
$\Rightarrow p_{He} - p_{Th} = 0 \qquad \left[K.E. = \dfrac{p^2}{2m}\right]$
$\Rightarrow p_{He} = p_{Th}$
but $K \propto \dfrac{1}{m}$ and $m_{He} < m_{Th}$ So $K_{He} > K_{Th}$

4. $\alpha = {}^4_2He^{2+}$ = Helium Nucleus
2 protons and 2 neutrons

5. ${}^{235}_{92}U + {}^1_0n \rightarrow {}^{89}_{36}Kr + {}^{144}_{56}Ba + 3{}^1_0n + Q$

6. $E = mc^2$
$= 0.5 \times 10^{-3} \times 9 \times 10^{16}$
$= 4.5 \times 10^{13}$ J

7. ${}_ZX^A \xrightarrow{\gamma-\text{decay}} {}_ZZ^A$
Due to gamma emission, there is no change in mass number and atomic number.

8. $X^{240} \rightarrow Y^{120} + Z^{120}$

given binding energy per nucleon of X, Y & Z are 7.6 MeV, 8.5 MeV & 8.5 MeV respectively.

Gain in binding energy is :-

Q = Binding Energy of products – Binding energy of reactants

= (120 × 8.5 × 2) – (240 × 7.6) MeV

= 216 MeV

9. ${}_{Z}^{A}X \xrightarrow{\beta^+} {}_{Z-1}B \xrightarrow{\alpha} {}_{Z-3}C \xrightarrow{\beta^-} {}_{Z-2}D$

β^+ decreases atomic number by 1

α decreases atomic number by 2

β^- increases atomic number by 1

12. ${}_{82}^{290}X \xrightarrow{\alpha} Y \xrightarrow{e^+} Z \xrightarrow{\beta^-} P \xrightarrow{e^-} {}_{81}Q^{286}$

13. Slow neutron can cause fission in ${}_{92}U^{235}$ than fast neutrons.

α-rays are Helium nuclei

β-rays are fast moving electron or positrons.

Exercise-III (Analytical Questions)

1. Assertion is true but reason is false.

4. Informative facts. At distance less than 0.7 fm, the nuclear force becomes repulsive.

5. (i) $R = R_o A^{1/3}$

$$\frac{R_1}{R_2} = \left(\frac{2}{16}\right)^{1/3} = \frac{1}{2}$$

(ii) ρ = constant $\frac{\rho_1}{\rho_2} = \frac{1}{1}$

(iii) $v \propto A$, $\frac{v_1}{v_2} = \frac{2}{16} = \frac{1}{8}$

(iv) $\frac{m_1}{m_2} = \frac{v_1}{v_2} = \frac{2}{16} = \frac{1}{8}$

7. larger is the binding energy per nucleon more is the stability of nucleus.

Modern Physics-3 [Atomic Structure]

SOLUTIONS

Exercise-I (Conceptual Questions)

1. For a Lyman series

$$\frac{1}{\lambda}=R\left[\frac{1}{1^2}-\frac{1}{n^2}\right] \quad ...(i)$$

where n = 2, 3, 4 ....

By putting n = 2, is equation (i), we obtain the longest wavelength of a Lyman series

$\frac{1}{\lambda}=R\left[\frac{1}{1^2}-\frac{1}{2^2}\right]$ or λ = 121.5 nm. This is the longest wavelength of the Lyman series.

Energy of photon E = $\frac{hc}{\lambda}$ or $E \propto \frac{1}{\lambda}$

For the longest wavelength, energy of the photon is the least.

2. For a Balmer series

$$\frac{1}{\lambda_B}=R\left[\frac{1}{2^2}-\frac{1}{n^2}\right] \quad ...(i)$$

where n = 3, 4, ...

By putting n = ∞ in equation (i), we obtain the series limit of the Balmer series. This is the shortest wavelength of the Balmer series. or

$\lambda_B = \frac{4}{R}$

For a Lyman series

$$\frac{1}{\lambda_L}=R\left[\frac{1}{1^2}-\frac{1}{n^2}\right] \quad ...(i)$$

where n = 2, 3, 4.........

By putting n = ∞ in equation (i), we obtain the series limit of the Lyman series. This is the shortest wavelength of the Lyman series.

or $\lambda_L = \frac{1}{R}$ ...(ii)

From (i) and (ii), we get $\frac{\lambda_B}{\lambda_L}=\frac{4}{1}$

3. KE = – TE = –(–13.6 eV) = 13.6 eV

8. According to the Bohr's theory of hydrogen atom

(i) The speed of the electron in the n^{th} orbit

$V_n = 2.2 \times 10^6 \times \frac{Z}{n} \Rightarrow \boxed{V_n \alpha \frac{1}{n}}$

(ii) The energy of the electron in the n^{th} orbit is

$$E_n=\frac{-13.6Z^2}{n^2} \Rightarrow E_n \alpha \frac{1}{n^2}$$

(iii) The radius of the electron in the n^{th} orbit is

$$r_n=\frac{0.53n^2}{Z}\text{Å} \Rightarrow r_n \alpha n^2$$

9. $E_n = -\frac{13.6}{n^2}\text{eV} = -\frac{13.6}{(3)^2}\text{ eV} = -1.51\text{ eV}$

Exercise-II (Previous Year Questions)

1. $\frac{1}{\lambda}=R\left(\frac{1}{1^2}-\frac{1}{n^2}\right)$

$$\frac{912}{975}=\frac{1}{1}-\frac{1}{n^2} \Rightarrow n=4$$

Number of spectral lines = $\frac{n(n-1)}{2}=6$

2. For H-like atoms

$v = \frac{Z}{n} \times 2.188 \times 10^6$ m/s

here Z = 2, n = 3

$v = 1.46 \times 10^6$ m/s

3. For Lyman series

$$\left(\frac{1}{\lambda_{max}}\right)_L = R(1)^2\left[\frac{1}{(1)^2}-\frac{1}{(2)^2}\right]$$

$$(\lambda_{max})_L=\frac{4}{3R}$$

For Balmer series

$$\left(\frac{1}{\lambda_{max}}\right)_B = R(1)^2\left[\frac{1}{(2)^2}-\frac{1}{(3)^2}\right]$$

$$(\lambda_{max})_B=\frac{36}{5R}$$

$$\frac{(\lambda_{max})_L}{(\lambda_{max})_B}=\frac{4}{3R}\times\frac{5R}{36}=\frac{5}{27}$$

4. At closest distance of approach, the kinetic energy of the particle will convert completely into electrostatic potential energy.

$$\Rightarrow \frac{1}{2}mv^2=\frac{KZe^2}{d_{min}} \Rightarrow d_{min} \propto \frac{1}{m}$$

5. Wave number, $\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_2^2}-\frac{1}{n_1^2}\right)$

$= 10^7 \times 1^2\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right) = 0.25 \times 10^7\ m^{-1}$

6. Transition : $3 \rightarrow 2 \Rightarrow$ Wavelength λ.

Transition : $4 \rightarrow 3 \Rightarrow$ Wavelength $\lambda' = ?$

$$\frac{\frac{1}{\lambda}=RZ^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)}{\frac{1}{\lambda'}=RZ^2\left(\frac{1}{3^2}-\frac{1}{4^2}\right)} \Rightarrow \frac{\lambda'}{\lambda}=\frac{20}{7} \Rightarrow \lambda'=\frac{20\lambda}{7}$$

8. $E_{total} = -KE \Rightarrow \frac{KE}{E_{Total}} = \frac{1}{-1}$

9. TE = –3.4 eV

KE = –T.E PE = 2T.E

$\Rightarrow$ KE = +3.4 eV $\Rightarrow$ PE = –6.8 eV

10. $m_\mu = 207m_e$, $q_\mu = q_{e^-}$, $M_{nucleus} = 1836\ m_e$

Reduced mass

$$\mu = \frac{mM}{M+m} = \frac{207m_e \times 1836m_e}{207m_e + 1836m_e} = 186m_e$$

$\because\ r_1 = \frac{n^2h^2}{4\pi^2mkze^2} = 0.51\ Å$

(Given in Question)

Radius of first orbit of new atom

$$r_1' = \frac{m_e r_1}{\mu} = \frac{m_e}{186m_e} \times 0.51\,Å = 2.56 \times 10^{-13}\ m$$

$$E_1' = \frac{\mu}{m}E_1 = \frac{186\ m_e}{m_e}(-13.6\ eV)$$

$= -2.5\ keV \approx -2.8\,keV$

(according to given options)

11. Bohr model is applicable for only single electron species.

12. For hydrogen atom

$E = -\frac{13.6}{n^2}\ eV$

15. Radius of n^{th} orbit in Hydrogen Atom

$r_n = 0.53 \times \frac{n^2}{Z}\,Å$

So, radius of third orbit

$$r_3 = 0.53 \times \frac{(3)^2}{(1)}\,Å = 4.77\,Å$$

16. $E_n = -13.6\frac{Z^2}{n^2}$

for ground state n = 1 and for second excited state n = 3

So $\frac{E_3}{E_1} = \left(\frac{n_1}{n_3}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$E_3 = \frac{E_1}{9} = \frac{13.6\,eV}{9} = 1.51\,eV$

17. $\frac{1}{\lambda} = R\left(\frac{1}{(1)^2}-\frac{1}{n^2}\right)$ n = 2, 3, 4, ...

$\left(\frac{1}{\lambda_L}\right)_{max} = R\left(\frac{1}{(1)^2}-\frac{1}{(2)^2}\right)$ $\left(\because \frac{1}{R} \simeq 912Å\right)$

$(\lambda_L)_{max} = \frac{4}{3}\frac{L}{R}$

$(\lambda_L)_{max} = \frac{4}{3} \times 912Å = 4 \times 304\ Å = 1216Å$

$\left(\frac{1}{\lambda_L}\right)_{min} = R\left(\frac{1}{(1)^2}-\frac{1}{(\infty)^2}\right)$

$(\lambda_L)_{min} = \frac{1}{R} \simeq 912Å$

Range of λ is 912Å to 1216Å which lies in U.V. region.

18. Statement I is correct.

Statement II is incorrect because atom of radioactive elements are not stable.

19. $\Delta E = \frac{hc}{\lambda}$

$\Delta E \rightarrow$ less

$\lambda \rightarrow$ large

$E_A < E_B < E_C < E_D$

$\Rightarrow$ 656.3 > 486.1 > 434.1 > 410.2

III IV II I

$\Rightarrow$ A-III B-IV C-II D-I

20. Transition from n_2 = 5, 6..... to n_1 = 4 is Brackett series.

21. Energy = $\frac{hc}{\lambda}$

(a) $E = \frac{hc}{\lambda_2} \Rightarrow \lambda_2 \propto \frac{1}{E}$

(b) $1.5E = \frac{hc}{\lambda_1} \Rightarrow \lambda_1 \propto \frac{1}{1.5E}$

(c) $ZE = \frac{hc}{\lambda_3} \Rightarrow \lambda_3 \propto \frac{1}{2E}$

$\lambda_2 > \lambda_1$, $\lambda_2 = 2\lambda_3$