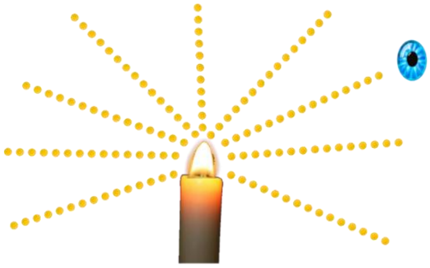
02 Wave Optics

Introduction

- The **Greeks** believed that light consisted of tiny particles (corpuscles) that were emitted by a light source and these particles stimulated the perception of vision upon striking the observer's eye. **Newton used this particle theory** to explain the reflection and refraction (bending) of light.

Newton's Corpuscular Theory

- Light consists of **little, invisible particles** known as **corpuscles.**
- These are **constantly emitted** by all luminous light sources in all directions.
- When corpuscles strike the retina of our eye then they produce the **sensation of vision**.
- These corpuscles travel with the **velocity of light** in straight lines.
- Their **velocity changes** with the **change of medium.**
- The **different colors** of light are due to **different size** of these corpuscles.
- The **rest mass** of these corpuscles is **zero**.



Corpuscles

The phenomena explained by this theory

(i) Reflection (ii) Refraction (iii) Rectilinear propagation of light.

The phenomena not explained by this theory

(i) Interference (ii) Diffraction (iii) Polarisation (iv) Photoelectric effect

Note: Newton proposed in his theory that light travels faster in denser medium than in rarer medium but he was proved wrong later.

Wave front and Huygens' Principle

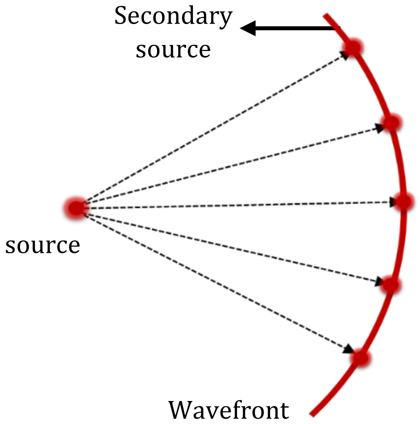
In 1678, one of Newton's contemporaries, the Dutch scientist **Christian Huygens**, was able to explain many other properties of light by proposing that light is a wave.

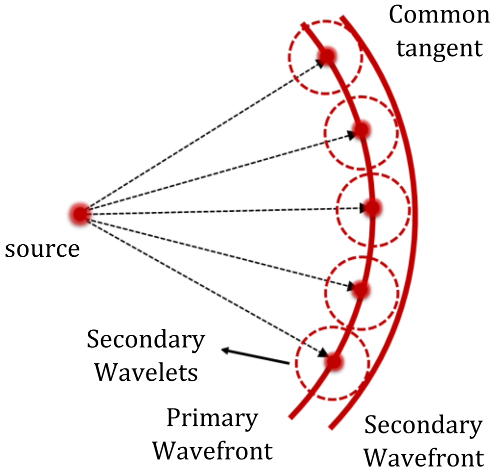
Huygens showed that a wave theory of light could also explain reflection and refraction.

Huygens' Wave theory of light

- The locus of all particles vibrating in same phase is known as **wavefront.**
- **Light travels** in a medium in the **form of wavefront**.
- When light travels in a medium then the particles of medium start vibrating and consequently a disturbance is created in the medium.
- Every point on the wavefront becomes the source of secondary wavelets. It emits secondary wavelets in all directions which travel with the speed of light.

➢ The tangent plane to these secondary wavelets represents the new position of wavefront.





The phenomena explained by this theory

➢ Reflection, Refraction, interference and diffraction

➢ Rectilinear propagation of light.

➢ Velocity of light in rarer medium being greater than that in denser medium.

The phenomena not explained by this theory

(1) Photoelectric effect (2) Polarisation

Note: Huygens considered that the environment is filled with anisotropic luminiferous ether but later he was proved wrong, later.

Reflection and Refraction of plane waves at a plane surface using wave fronts

Types of Wavefront: The shape of wavefront depends upon the shape of the light source from which the wavefront originates. On this basis there are three types of wavefronts.

(1) Spherical Wavefront **(2) Cylindrical Wavefront** **(3) Plane Wavefront**

(1) Spherical Wavefront

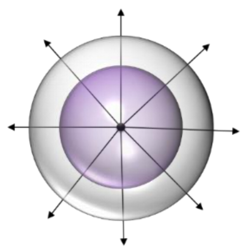
→ Spherical wavefront originates from **point source.**

→ Intensity $(I) \propto \frac{1}{\text{Area}}$

$\therefore I \propto \frac{1}{r^2}$ $(\text{Area} = 4\pi r^2)$

→ Intensity ∝ (Amplitude)^2

⇒ $\frac{1}{r^2} \propto A^2 \Rightarrow A \propto \frac{1}{r}$



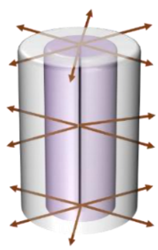
(2) Cylindrical Wavefront

→ Cylindrical wavefront originates from **linear source.**

→ Intensity $(I) \propto \frac{1}{\text{Area}} \Rightarrow I \propto \frac{1}{r}$ $(\text{Area} = 2\pi rh)$

→ Intensity ∝ (Amplitude)^2

⇒ $\frac{1}{r} \propto A^2 \Rightarrow A \propto \frac{1}{\sqrt{r}}$



(3) Plane Wavefront

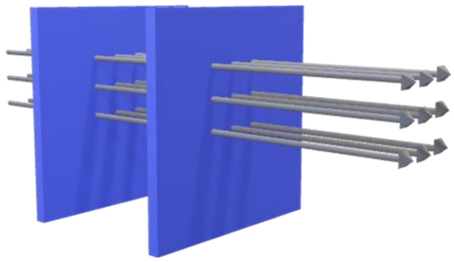
→ Plane wavefront originates from the **source situated at very large distance.**

→ Intensity $(I) \propto \dfrac{1}{\text{Area}}$

I = constant (Area is constant)

→ Intensity $\propto$ (Amplitude)^2

A = constant



Summary

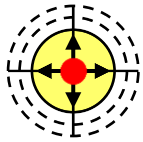
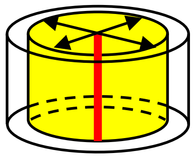
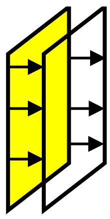
Sr. No.	Wavefront	Shape of Light Source	Diagram or shape of wavefront	Variation of amplitude (A) with distance	Variation of Intensity (I) with distance
1.	Spherical	Point source		$A \propto \dfrac{1}{r}$	$I \propto \dfrac{1}{r^2}$
2.	Cylindrical	Linear source/ Slit		$A \propto \dfrac{1}{\sqrt{r}}$	$I \propto \dfrac{1}{r}$
3.	Plane	Extended Large source / Point source at very large distance		A: constant $A \propto r^0$	I: constant $I \propto r^0$

Illustration 1

If amplitude of light at 10 m from a small light bulb is A_0 then find amplitude of light at a distance 50 m from the same light bulb?

Solution:

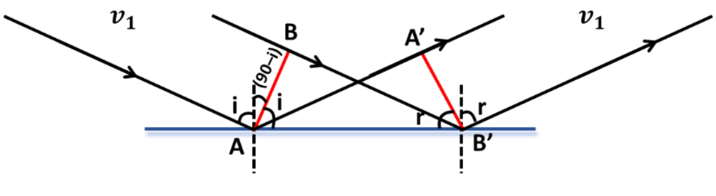
$$A \propto \frac{1}{r} \Rightarrow \frac{A_1}{A_2} = \frac{r_2}{r_1} \Rightarrow \frac{A_0}{A_2} = \frac{50}{10} \Rightarrow A_2 = \frac{A_0}{5}$$

Law of reflection by huygen's wave theory

$BB' = v_1 t \Rightarrow AA' = v_1 t$

$AA' = BB' \Rightarrow \text{In } \Delta ABB'$

$$\sin i = \frac{BB'}{AB'} \qquad \text{...(i)}$$



In $\Delta B'A'A$

$$\sin r = \frac{AA'}{AB'} \qquad \text{...(ii)}$$

Divide (i) by (ii)

$$\frac{\sin i}{\sin r} = \frac{BB'}{AA'} = 1 \Rightarrow \sin i = \sin r \Rightarrow i = r$$

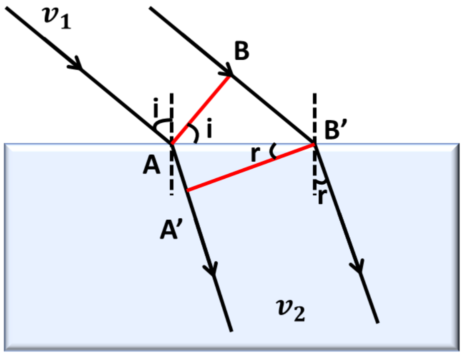
Law of refraction by huygen's wave theory

$BB' = v_1 t$

$AA' = v_2 t$

In $\Delta ABB'$

$\sin i = \frac{BB'}{AB'}$ ...(i)



In $\Delta AA'B'$

$\sin r = \frac{AA'}{AB'}$ ...(ii)

Divide (i) by (ii)

$\frac{\sin i}{\sin r} = \frac{BB'}{AA'}$

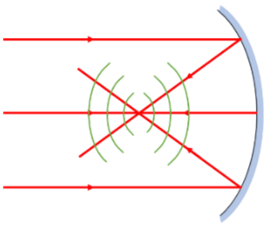
$\frac{\sin i}{\sin r} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} = \frac{\mu_2}{\mu_1}$

$\mu_1 \sin i = \mu_2 \sin r$

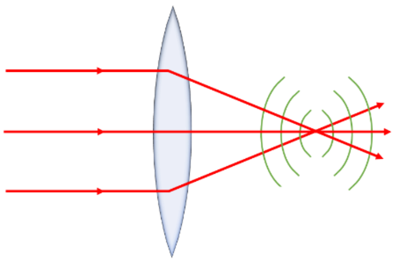
Illustration 2

A plane wavefront is incident on given optical device in each case. Draw the correct wavefront after interaction of light ray with the Optical device.

Solution:



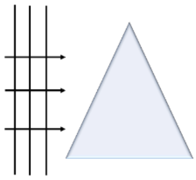
Spherical wavefront



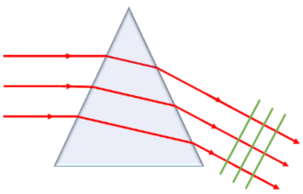
Spherical wavefront

Illustration 3

A plane wavefront of monochromatic light is incident on given optical device. Draw the correct wavefront after interaction of light ray with the Optical device.



Solution:



Plane wavefront

Characteristics of Wavefront

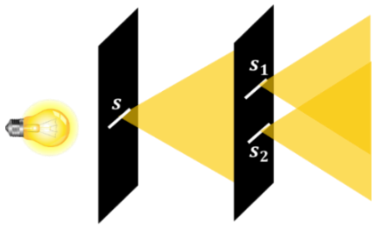
- The phase difference between various particles on the wavefront is zero.
- These wavefronts travel with the speed of light in all directions in an isotropic medium.
- A point source of light always gives rise to a spherical wavefront in an isotropic medium.
- Normal to the wavefront represents a ray of light.
- It always travels in the forward direction in the medium.

Golden Key Points

- Law of reflection by Huygen's wave Theory: sin i = sin r
- Law of refraction by Huygen's wave Theory: $\mu_1 \sin i = \mu_2 \sin r$

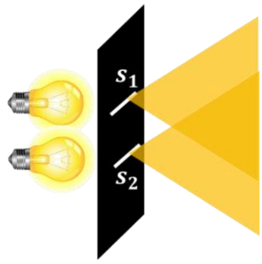
Coherent and Incoherent sources

(1) Coherent sources :-



- Two light sources are said to be coherent if they emit light waves of the same frequency and have a constant phase difference. They are obtained from a single source.

(2) Incoherent sources :-

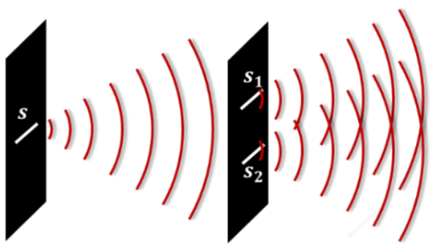


- Two sources are said to be incoherent if their phase difference changes with time.
- Two independent monochromatic sources of same frequency are incoherent because atoms cannot emit light waves in same phase and these sources are said to be incoherent sources.

Methods of Obtaining Coherent Source

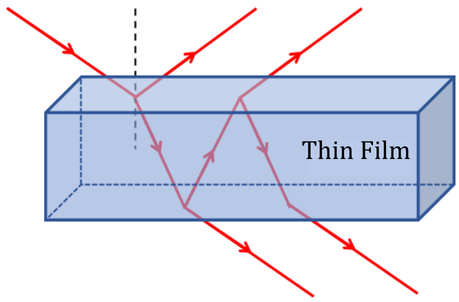
(1) Division of wavefront :-

- In this method, the wavefront is divided into two or more parts by reflection or refraction using mirrors, lenses, prisms, or slits.



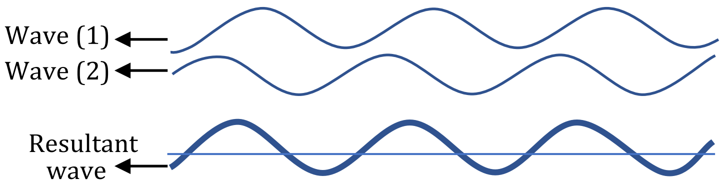
(2) Division of Amplitude :-

- The amplitude of the incoming beam is divided in two parts either by parallel reflection or refraction. These divided parts reunite after traversing different paths and produce interference.



Interference of Light

- When the waves emitting from the coherent sources propagating in same direction are superposed over one another, then redistribution of energy takes place in the space and this phenomenon is called as interference.
- It is based on energy conservation principle. Total energy of the waves remains constant, only redistribution of energy takes place.



Mathematical Analysis: Let two waves having amplitude A_1 and A_2 and same frequency, and constant phase difference ϕ superimpose. Let their displacements are:

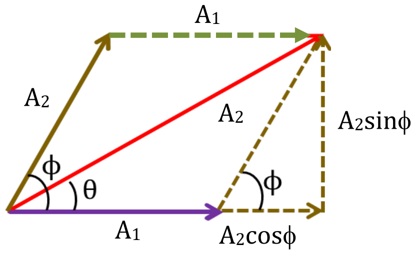
$y_1 = A_1 \sin(\omega t)$

$y_2 = A_2 \sin(\omega t + \phi)$

$\Delta\phi = (\omega t + \phi) - (\omega t) = \phi$

$A_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos\Delta\phi}$

Phase angle $\theta = \tan^{-1}\left(\dfrac{A_2 \sin\phi}{A_1 + A_2\cos\phi}\right)$



$\boxed{A_{res}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\phi}$

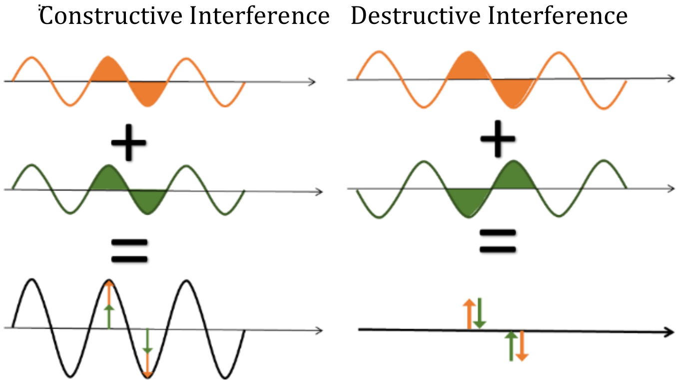
we know that, $I \propto A^2$

$\boxed{I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi}$

$2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$ is known as interference factor.

Constructive Interference	Destructive Interference
$\Delta x = 0, \lambda, 2\lambda, 3\lambda \ldots\ldots n\lambda$ n = 0, 1, 2, 3.........	$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots\ldots \frac{(2n-1)\lambda}{2}$ n = 1, 2, 3.........
$\Delta\phi = 0, 2\pi, 4\pi \ldots\ldots 2n\pi$	$\Delta\phi = \pi, 3\pi, 5\pi \ldots\ldots (2n-1)\pi$
$A_{max} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos 0}$ $A_{max} = A_1 + A_2$	$A_{min} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\pi}$ $A_{min} = A_1 - A_2$
$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$	$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

- Here intensity varies as the function of position.
- Existence of one wave is not affected due to presence or absence of another wave in the medium, waves propagate independently in the medium.



Conditions for sustained interference

- The two sources should be coherent.
- The separation between two coherent sources should be small.
- The distance of the screen from the two sources should be large.
- For good contrast between maxima and minima, the amplitude of two interfering waves should be as nearly equal as possible. The two sources should be

Golden Key Points

- Interference follows the law of conservation of energy.
- Average intensity of interference pattern $=\dfrac{I_{max}+I_{min}}{2}=I_1+I_2$
- Intensity $\propto$ width of slit $\propto$ (amplitude)² $\Rightarrow I \propto W \propto A^2 \Rightarrow \dfrac{I_1}{I_2}=\dfrac{w_1}{w_2}=\dfrac{A_1^2}{A_2^2}$
- $\dfrac{I_{max}}{I_{min}}=\left[\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right]^2=\left[\dfrac{A_1+A_2}{A_1-A_2}\right]^2=\left[\dfrac{A_{max}}{A_{min}}\right]^2$
- Fringe visibility $V=\dfrac{I_{max}-I_{min}}{I_{max}+I_{min}}\times 100\%$ when I_{min} = 0 then fringe visibility is maximum. i.e. when both slits are of equal width the fringe visibility is the best and equal to 100%.
- Intensity is proportional to width of slit.

Illustration 4

Two waves having the intensities in the ratio of 16 : 9 produce interference. Find the ratio of maximum to minimum intensity?

Solution:

$$\frac{I_1}{I_2}=\frac{16}{9}\ ;\ \frac{I_{max}}{I_{min}}=\frac{\left(\sqrt{I_1}+\sqrt{I_2}\right)^2}{\left(\sqrt{I_1}-\sqrt{I_2}\right)^2}=\frac{\left(\sqrt{16}+\sqrt{9}\right)^2}{\left(\sqrt{16}-\sqrt{9}\right)^2}=\frac{49}{1}$$

Illustration 5

The ratio of maximum to minimum intensity when two waves interfere is equal to 16 : 9, find the ratio of Amplitudes of superimposing waves.

Solution:

$$\frac{I_{max}}{I_{min}}=\frac{16}{9}\Rightarrow\left(\frac{A_{max}}{A_{min}}\right)^2=\frac{16}{9}$$

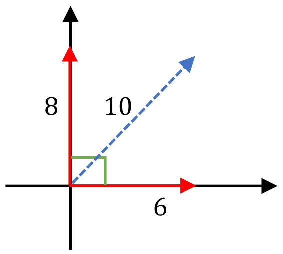
$$\frac{A_{max}}{A_{min}}=\frac{4}{3}\Rightarrow\frac{A_1+A_2}{A_1-A_2}=\frac{4}{3}$$

$$3A_1+3A_2=4A_1-4A_2\Rightarrow\frac{A_1}{A_2}=\frac{7}{1}$$

Illustration 6

Find resultant amplitude of two superimposing waves given by $y_1 = 6\sin(\omega t)$, $y_2 = 8\cos(\omega t)$

Solution:



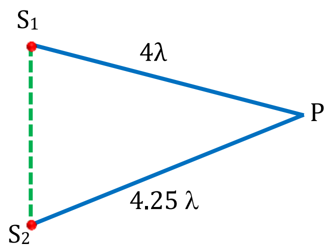
$$y_1=6\sin(\omega t)\text{ and }y_2=8\cos(\omega t)=8\sin(\omega t+90°)$$

$$\Delta\phi=(\omega t+90)-(\omega t)=90°$$

$$A_{Res}=\sqrt{6^2+8^2+2(6)(8)\cos(90°)}=\sqrt{100}=10$$

Illustration 7

Two coherent sources, each producing identical waves of intensity I_0 and wavelength λ are at some distance from point P as shown. Find the intensity of resultant wave at point P.



Solution:

$$\Delta x \text{ at point P} = 4.25\,\lambda - 4\lambda = 0.25\lambda=\frac{\lambda}{4}$$

$$\Delta\phi=\frac{2\pi}{4}=\frac{\pi}{2}\qquad\left(\because\frac{\Delta\phi}{2\pi}=\frac{\Delta x}{\lambda}\right)$$

$$I_{Res}=4I_o\cos^2\left(\frac{\Delta\phi}{2}\right)=4I_o\left(\frac{1}{2}\right)=2I_o$$

BEGINNER'S BOX-1

1. Consider interference between two sources of intensities I and 4I. Obtain intensity at a point where phase difference is $\frac{\pi}{2}$.

PW0171

2. Two coherent sources whose intensity ratio is 81 : 1 produce interference fringes. Calculate the ratio of intensity of maxima and minima in the fringe system.

PW0172

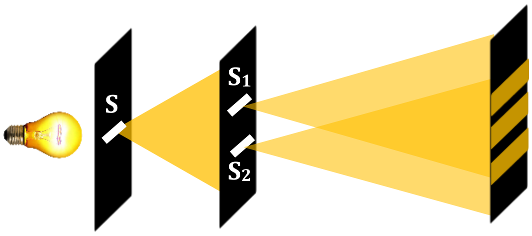
3. Consider interference between two sources of intensity I and 4I. Find the resultant intensity. where phase difference is :-

(a) $\frac{\pi}{4}$ (b) π (c) 4π

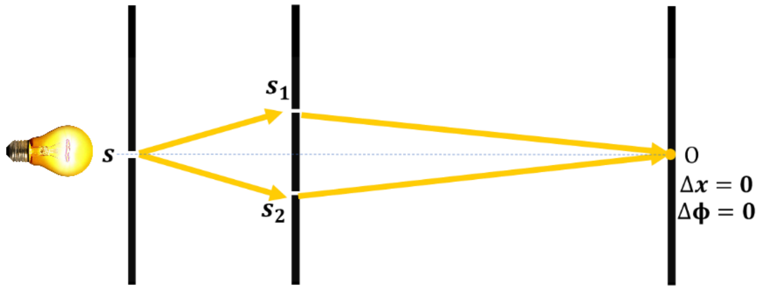
PW0173

Young's Double Slit Experiment (Y.D.S.E.)

According to Huygens, light is a wave. It is proved experimentally by YDSE. In YDSE division of wavefront takes place.



- At point O path difference and phase difference of the waves coming from the source via both the slit is zero.
- The fringe formed at the point O is known as central maxima or central bright fringe.



At Point 'P'

$\Delta x = SS_2P - SS_1P = S_2P - S_1P$

$\Delta x = (S_2M + MP) - S_1P$

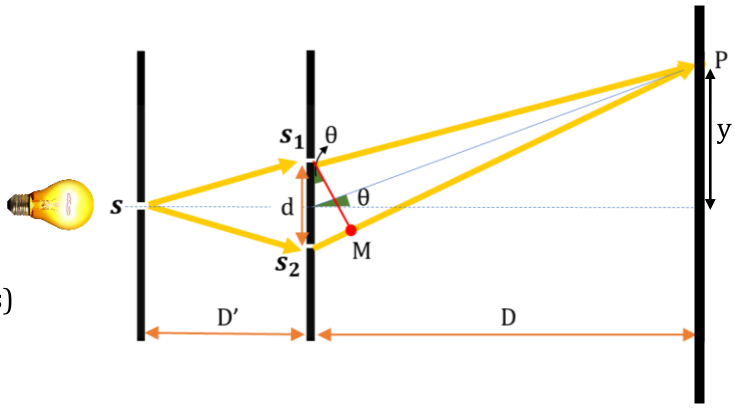
$\Delta x = S_2M \qquad (\because MP = S_1P)$

$\Delta x = d \sin\theta$

if angle θ is small then

$\Delta x = d \sin\theta \approx d \tan\theta$ (for small angles)

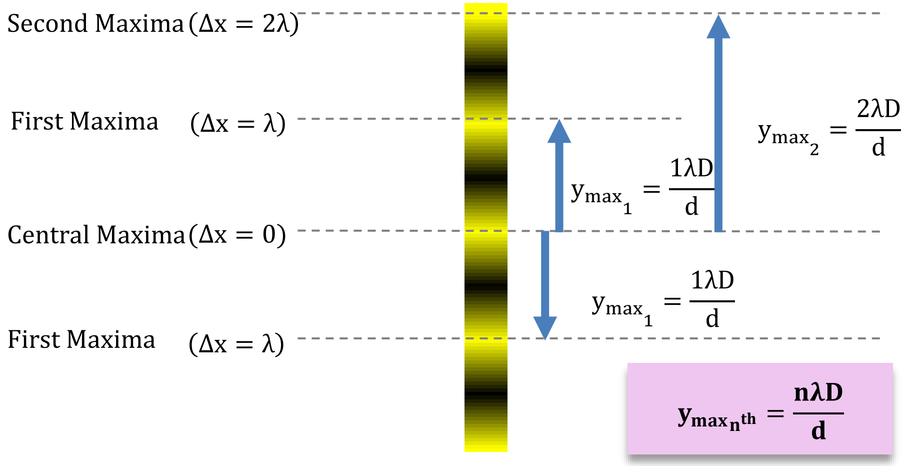
$\Delta x = \frac{dy}{D}$.......(1) $\qquad \left(\tan\theta = \frac{y}{D}\right)$



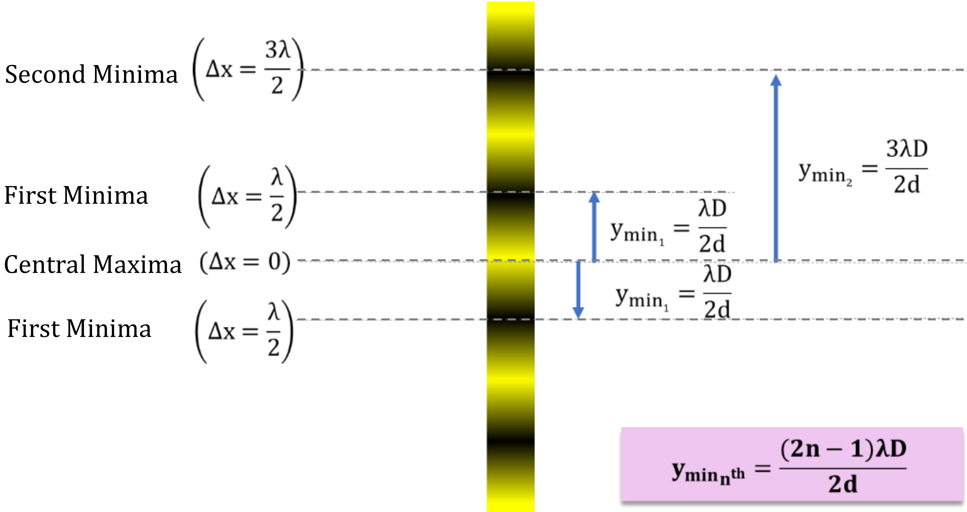
If bright fringe is formed at point P	If dark fringe is formed at point P
$\Delta x = n\lambda$	$\Delta x = \frac{(2n-1)\lambda}{2}$
$\frac{dy}{D} = n\lambda$, from equation	$\frac{dy}{D} = \frac{(2n-1)\lambda}{2}$, from equation
$y = \frac{n\lambda D}{d}$	$y = \frac{(2n-1)\lambda D}{2d}$
Distance of n^{th} maxima from central maxima $\boxed{y_{max_{n^{th}}} = \frac{n\lambda D}{d}}$	Distance of n^{th} minima from central maxima $\boxed{y_{min_{n^{th}}} = \frac{(2n-1)\lambda D}{2d}}$

YDSE Pattern on Screen

For maxima



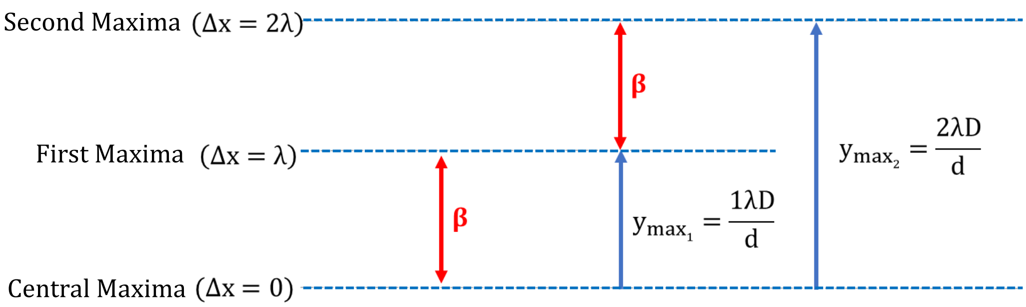
For minima



Fringe Width (β)

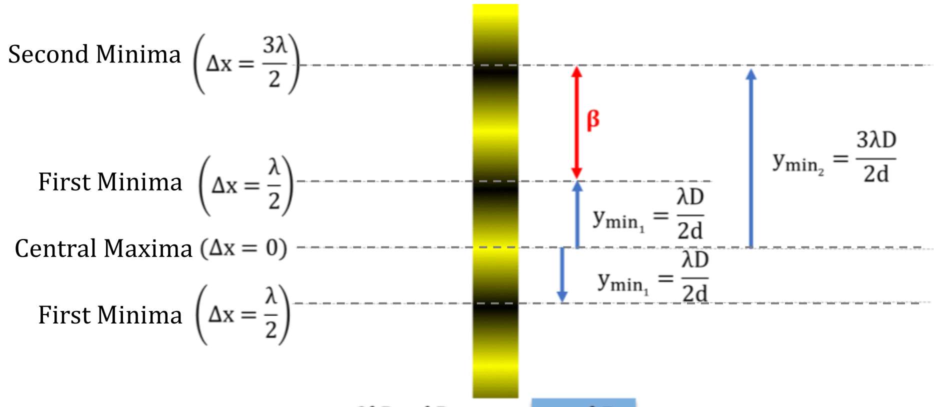
Distance between two successive dark fringes or two successive bright fringes is known as fringe width (β).

Dark fringe width



$$\beta = \frac{2\lambda D}{d} - \frac{1\lambda D}{d} \Rightarrow \boxed{\boldsymbol{\beta = \frac{\lambda D}{d}}}$$

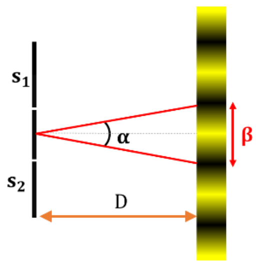
Bright fringe width



$$\beta = \frac{3\lambda D}{2d} - \frac{\lambda D}{2d} \Rightarrow \boxed{\beta = \frac{\lambda D}{d}}$$

Angular Fringe Width (α)

It is the angle subtended by linear fringe width at the centre of the plane of slit.



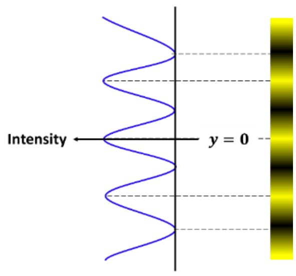
$$\alpha = \frac{\beta}{D}$$

$$\alpha = \frac{\lambda D}{dD}$$

$$\alpha = \frac{\lambda}{d}$$

Angular fringe width does not depend on the distance between plane of slit and screen.

Variation of intensity with phase difference



If slits are of equal width then intensity is also equal.

$$I = 4I_o \cos^2\left(\frac{\Delta\phi}{2}\right)$$

Illustration 8

In YDSE two narrow slits are 1 mm apart are illuminated by a source of light of wavelength 500 nm. How far apart adjacent bright bands in the interference pattern observed on a screen 2 m away and also find distance of third bright fringe from center of screen?

Solution:

$$\beta = \frac{\lambda D}{d} = \frac{5\times10^{-7}\times2}{10^{-3}} = 10^{-3}\,m = 1mm \;\Rightarrow y_{max_3} = \frac{3\lambda D}{d} = 3\beta = 3 \text{ mm}$$

Illustration 9

In the above question find the distance of third dark fringe from central maxima and also find distance between 4^{th} dark and 6^{th} bright fringe on same side of central maxima?

Solution:

$$y_{min_3} = \frac{(2(3)-1)\lambda D}{2d} = \frac{5\lambda D}{2d} = \frac{5}{2}\times\frac{5\times10^{-7}\times2}{10^{-3}} = 2.5 \text{ mm}$$

$$D = y_{max_6} - y_{min_4} = \frac{6\lambda D}{d} - \frac{7\lambda D}{2d} = 6\beta - \frac{7}{2}\beta = 2.5\,\beta = 2.5 \text{ mm}$$

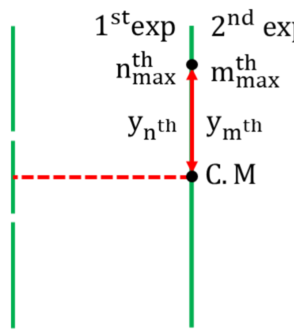
Illustration 10

YDSE is performed twice, firstly with the wavelength 300 nm and secondly with wavelength 500 nm.

(i) Find minimum order of maxima of 1st experiment that coincide with maxima of another experiment.

(ii) Distance between consecutive overlapping region of maxima? (D = 1 m, d = 2 mm)

Solution:



(i) $y_{max_{n^{th}}} = y_{max_{m^{th}}}$

$$\frac{n\lambda_1 D}{d} = \frac{m\lambda_2 D}{d}$$

$$n \times 300 \times 10^{-10} = m \times 500 \times 10^{-10}$$

$$\boxed{\frac{n}{m} = \frac{5}{3}}$$

5^{th} maxima of 1^{st} experiment coincide with 3^{rd} maxima of 2^{nd} experiment.

(ii) $y_{n^{th}} = y_{m^{th}}$

$$\therefore\ y_{n^{th}} = \frac{n\lambda D}{d} = \frac{5(3\times10^{-7})\times1}{2\times10^{-3}} = 7.5 \times 10^{-4}\ \text{m}$$

Illustration 11

In a Young's double slit experiment, light has a frequency of 6×10^{14} Hz. The distance between the centre of adjacent bright fringes is 1 mm. If the screen is 2 m away then find the distance between the slits?

Solution:

$v = f\lambda \Rightarrow 3 \times 10^8 = 6 \times 10^{14} \times \lambda \Rightarrow \lambda = 0.5 \times 10^{-6}$m

$$\beta = \frac{\lambda D}{d} \Rightarrow 1\times10^{-3} = \frac{0.5\times10^{-6}\times2}{d} \Rightarrow d = 10^{-3}\text{m} = 1\text{ mm}$$

Illustration 12

The distance between the coherent source is 0.3 mm and the screen is 90 cm from the sources. The second dark band is 0.3 cm away from central bright fringe. Find the wavelength and the distance of the fourth bright fringe from central bright fringe?

Solution:

As $y_{min_2} = 0.3\times10^{-2}$m

$$\frac{3\lambda D}{2d} = 0.3\times10^{-2} \Rightarrow \frac{3\times\lambda\times90\times10^{-2}}{2\times3\times10^{-4}} = 3\times10^{-3} \Rightarrow \lambda = \frac{6\times10^{-7}}{90\times10^{-2}} = 666\text{ nm}$$

Illustration 13

In Young's double slit experiment, wavelength of light is 6000 Å. Then the phase difference between the light waves reaching the seventh bright fringe from the central fringe will be :

Solution:

Phase difference, $\Delta x = 7\lambda \Rightarrow$ So, path difference $\Delta\phi = \frac{\Delta x}{\lambda}\times2\pi = 14\pi$

Illustration 14

In Y.D.S.E. the fringe width is 0.2 mm. If the wavelength of light is increased by 10% and separation between the slits is increased by 10 % then fringe width will be?

Solution:

$$\beta = \frac{\lambda D}{d} = 0.2 \text{ mm} \quad \& \quad \lambda_{new} = \lambda + \frac{10}{100}\lambda = 1.1\,\lambda \quad \& \quad d_{new} = d + \frac{10}{100}d = 1.1\,d$$

$$\beta_{new} = \frac{\lambda_{new} D}{d_{new}} = \frac{1.1\,\lambda D}{1.1\,d} = \frac{\lambda D}{d} = 0.2 \text{ mm}$$

Illustration 15

A YDSE produces interference fringe for sodium light of wavelength 5890 A° that are 0.40° a part. What is the angular fringe separation if the entire arrangement is immersed in water ($\mu = 4/3$)

Solution:

$$\alpha_{new} = \frac{\alpha}{\mu} = \frac{0.40°}{\frac{4}{3}} = 0.30°$$

Illustration 16

In YDSE the intensity of light at a point on the screen where the path difference is λ is K (λ being the wavelength of light used). Find intensity at a point where the path difference is $\lambda/4$.

Solution:

When path difference (Δx) = λ then constructive interference takes place (I_{max} = K)

$$I_{res} = I_{max}\cos^2\left(\frac{\Delta\phi}{2}\right); \text{ When } \Delta x = \frac{\lambda}{4}, \quad \Delta\phi = \frac{2\pi}{4} = \frac{\pi}{2}$$

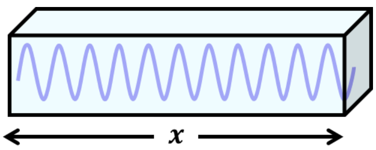
$$I_{res} = K\cos^2\left(\frac{\frac{\pi}{2}}{2}\right) = K\cos^2\left(\frac{\pi}{4}\right) = \frac{K}{2}$$

Variations in YDSE

- When a wave travels through a distance Δx in a medium of refractive index μ, then it suffers the same phase change as it travels a distance $\mu\Delta x$ in vacuum. The quantity $\mu\Delta x$ is called the optical path of the light.
- Consider a wave travelling in a medium of refractive index μ, then time taken by wave to cover x distance is calculated as-

$$t = \frac{x}{v}$$

$$t = \frac{\mu x}{c} \qquad \left(\because\ v = \frac{c}{\mu}\right)$$



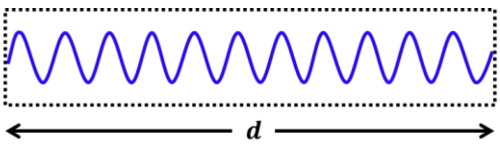
Optical Path

- Distance covered by the wave in same time t in vacuum is calculated as,

  d = c × t

  $d = c \times \frac{\mu x}{c}$

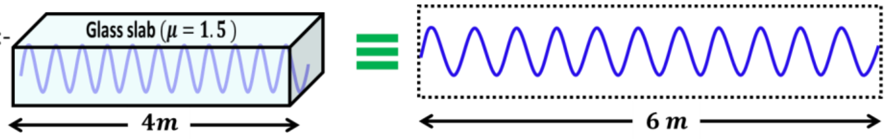
  d = μx



- This distance d is the optical path length of light.
- Here we considered the same time interval so that the wave in medium and vacuum suffer the same phase change.

Note: (1) In other words, a path length of Δx in a medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.

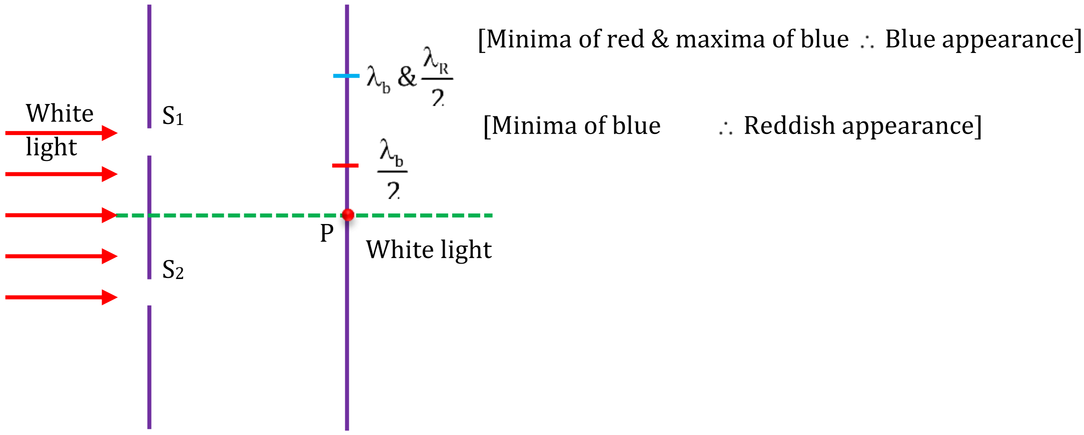
e.g. :-



(2) Number of waves remain same in both the medium and vacuum only wavelength changes.

(3) Optical path lengths of all paths between an object point and its image are exactly equal.

Effect of white light in YDSE



- The interference patterns due to different component colours of white light overlap (incoherently). The central bright fringes for different colour are at the same position. Therefore, the central fringe is white.
- For a point P for which $S_2P - S_1P = \lambda_b/2$. Where λ_b ($\approx$ 4000 Å) represents the wavelength for the blue colour, the blue component will be absent and the fringe will appear red in colour.
- Slightly farther away where $S_2Q - S_1Q = \lambda_b = \lambda_r/2$ where λ_r ($\approx$ 8000 Å) is the wavelength for the red colour, the fringe will be predominantly blue because red component will be absent.
- Thus, the fringe closest on either side of the central white fringe is red and the farthest will appear blue. After a few fringes, no clear fringe pattern is seen.

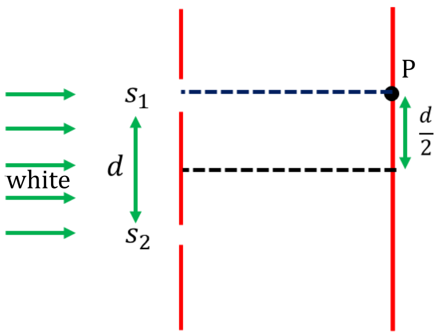
Illustration 17

White light is used in YDSE, D >> d, then find the missing wavelengths in front of any slit on screen?

Solution:

$$\Delta x = \frac{dy}{D} \qquad \left(y = \frac{d}{2}\right)$$

$$\Delta x = \frac{d^2}{2D}$$



Missing wavelength means minima

$$(2n-1)\frac{\lambda}{2} = \frac{d^2}{2D}$$

$$\lambda = \frac{d^2}{(2n-1)D}$$

For, n = 1; $\lambda = \frac{d^2}{D}$

n = 2; $\lambda = \frac{d^2}{3D}$

n = 3; $\lambda = \frac{d^2}{5D}$ and so on.........

Illustration 18

White light is used in YDSE, where D = $\frac{1}{3}$ m, d = 1 mm. Find the missing wavelengths of visible region in front of the slits on screen?

Solution:

$$\Delta x = \frac{dy}{D} = \frac{d}{D}\left(\frac{d}{2}\right) = \frac{d^2}{2D}$$

$$(2n-1)\frac{\lambda}{2} = \frac{d^2}{2D} \Rightarrow \lambda = \frac{d^2}{(2n-1)D}$$

On putting values, we get.

$$\lambda = \frac{\left(10^{-3}\right)^2}{(2n-1)\frac{1}{3}} = \frac{3\times10^{-6}}{(2n-1)} = \frac{3000}{(2n-1)}\text{nm}$$

When n = 1, $\lambda = \frac{3000}{1}$ = 3000 nm (Not in range)

When n = 2, $\lambda = \frac{3000}{(3)} = 1000\text{nm}$ (Not in range)

When n = 3, $\lambda = \frac{3000}{5} = 600\text{nm}$ (possible)

When n = 4, $\lambda = \frac{3000}{7} = 428\text{nm}$ (possible)

Thus, 428 nm and 600 nm are the missing wavelengths, because visible region have 400-700 nm range.

Golden Key Points

- Central Fringe is white.
- The fringe closest on either side of the central white fringe is red and the farthest will appear blue.
- After a few fringes, no clear fringe pattern is seen.
- Fringe width of each colour is different because of different wavelength.

BEGINNER'S BOX-2

1. In Young's double slit experiment, the slits are 2mm apart and are illuminated with a mixture of two wavelength, λ = 7500 Å and λ' = 9000 Å. At what minimum distance from the common central bright fringe on a screen 2 m from the slits will a bright fringe from one interference pattern coincide with a bright fringe from the other?

PW0174

2. In a Young's double slit experiment the angular width of fringe formed on a distant screen is 0·1 radian. Find the distance between the two slits if wavelength of light used is 6000Å.

PW0175

3. Two slits in Youngs experiment are 0·02 cm apart. The interference fringes for light of wavelength 6000Å are formed on a screen 80cm away. Calculate the distance of the fifth bright fringe.

PW0176

4. In Young's double slit experiment, two slits are separated by 3mm distance and illuminated by light of wavelength 480 nm. The screen is at 2m from the plane of the slits. Calculate the separation between the 8^{th} bright fringe and the 3^{rd} dark fringe obtained with respect to central Bright fringe.

PW0177

5. In a double slit experiment with monochromatic light, fringes are obtained on a screen placed at some distance from slits. If the screen is moved by 5×10^{-2} m towards the slits, the change in fringe width is 3×10^{-5} m. If the distance between slits is 10^{-3} m. Calculate the wavelength of light used.

PW0178

6. In young's experiment for interference of light the slits 0.2 cm apart are illuminated by yellow light (λ = 5896 Å). What would be the fringe width on a screen placed 1m from the plane of slits. What will be the fringe width if the system is immersed in water. (μ_w = 4/3)?

PW0179

7. The distance between the coherent source is 0.3 mm and the screen is 90 cm from the sources. The second dark band is 0.3 cm away from central bright fringe. Find the wavelength and the distance of the fourth bright fringe from central bright fringe.

PW0180

8. State two conditions to obtain sustained interference of light. In Young's double slit experiment, using light of wavelength 400 nm, interference fringes of width 'X' are obtained. The wavelength of light is increased to 600 nm and the separation between the slits is halved. If one wants the observed fringe width on the screen to be the same in the two cases, find the ratio of the distance between the screen and the plane of the slits in the two arrangements.

PW0181

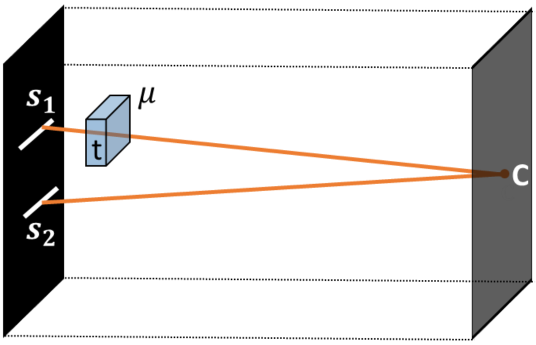
9. In a Young's double slit experiment, light has a frequency of 6×10^{14} Hz. The distance between the centres of adjacent bright fringes is 0.75 mm. If the screen is 1.5 m away then find the distance between the slits.

PW0182

10. In a Young's experiment, the width of the fringes obtained with light of wavelength 6000 Å is 2.0 mm. What will be the fringe width, if the entire apparatus is immersed in a liquid of refractive index 1.33?

PW0183

Thin Film in Front of a Slit in YDSE



- Central maxima is not at the center of the screen.
- Path difference at center of the screen is $(\mu - 1)t$ if a thin film is introduced in front of any of the slit.
- Fringe width doesn't get affected due to the introduction of the thin film in front of slit.
- Central maxima is shifted on the same side where thin film is placed.

  Path difference at point P,

  $\Delta x = S_2P - S_1 P_{eff}$

  $\Delta x = S_2P - (S_1P - t + \mu t)$

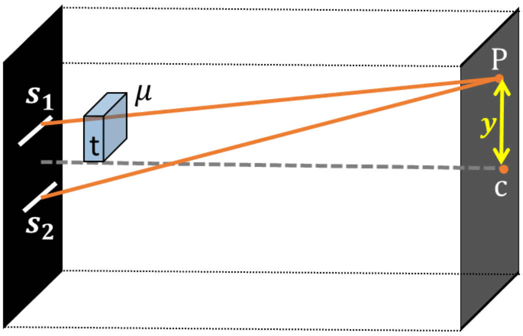
  $\Delta x = S_2P - S_1P - (\mu - 1) t$

  $\Delta x = d\sin\theta - (\mu - 1) t$

- If angle is small, then

  $\Delta x = d\tan\theta - (\mu - 1)t$

  $$\Delta x = \frac{dy}{D} - (\mu - 1)t$$



- If Maxima is formed at P,

  $$n\lambda = \frac{dy}{D} - (\mu - 1)t$$

- If Minima is formed at P,

  $$\frac{(2n-1)\lambda}{2} = \frac{dy}{D} - (\mu - 1)t$$

- If central maxima is formed at P then $\Delta x = 0$

  $$0 = \frac{dy}{D} - (\mu - 1)t \Rightarrow \frac{dy}{D} = (\mu - 1)t$$

  $$y = \frac{(\mu - 1)tD}{d} = y_{cm}$$

- When central maxima shifts its position from center then all the fringes also get shifted in that direction and by the same value.
- The number of fringes crossing the center line $(n) = \frac{y_{cm}}{\beta}$

  $$n = \frac{(\mu-1)tD}{d\beta} = \frac{(\mu-1)tD}{d\frac{\lambda D}{d}} = \frac{(\mu-1)t}{\lambda}$$

When thin film is placed in front of both the slit

- If thin film is placed in front of both the slit then central maxima is shifted to that side where the value of $(\mu - 1)t$ is more or path difference due to slab is more.
- For CBF to shift upwards, $(\mu_1 - 1)t_1 > (\mu_2 - 1)t_2$
- For CBF to shift downwards,
- $(\mu_1 - 1)t_1 < (\mu_2 - 1)t_2$
- For no shift in CBF

  $(\mu_1 - 1)t_1 = (\mu_2 - 1)t_2$

  $$y_{cm} = \frac{((\mu_2 - 1)t_2 - (\mu_1 - 1)t_1)D}{d}$$

  $$n = \frac{((\mu_2 - 1)t_2 - (\mu_1 - 1)t_1)}{\lambda}$$

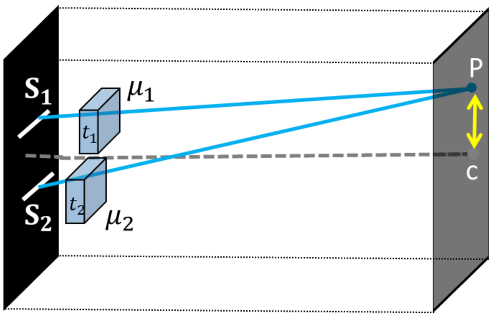


Illustration 19

In YDSE, λ = 6000 Å, D = 2 m, d = 6 mm. When a film of refractive index 1.5 is introduced in front of the lower slit, the fourth maxima shifts to the origin. Find thickness of the film.

Solution:

$$y=\frac{(\mu-1)tD}{d}\Rightarrow 4\beta=\frac{(1.5-1)tD}{d}\Rightarrow\frac{4\lambda D}{d}=\frac{0.5tD}{d}\Rightarrow 4\times6\times10^{-7}=0.5\,t\Rightarrow t=4.8\ \mu m$$

Illustration 20

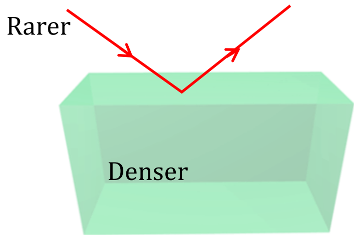
In Y.D.S.E, λ = 400 nm is used when a thin film of refractive index 1.5 and thickness 4 μm is placed in front of upper slit, find the number of maxima that crossed the centre line?

Solution:

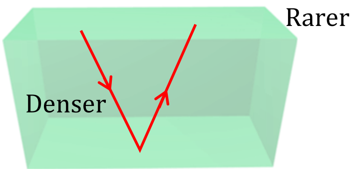
$$n=\frac{(\mu-1)t}{\lambda}=\frac{(1.5-1)}{4\times10^{-7}}\times4\times10^{-6}=5$$

Note:

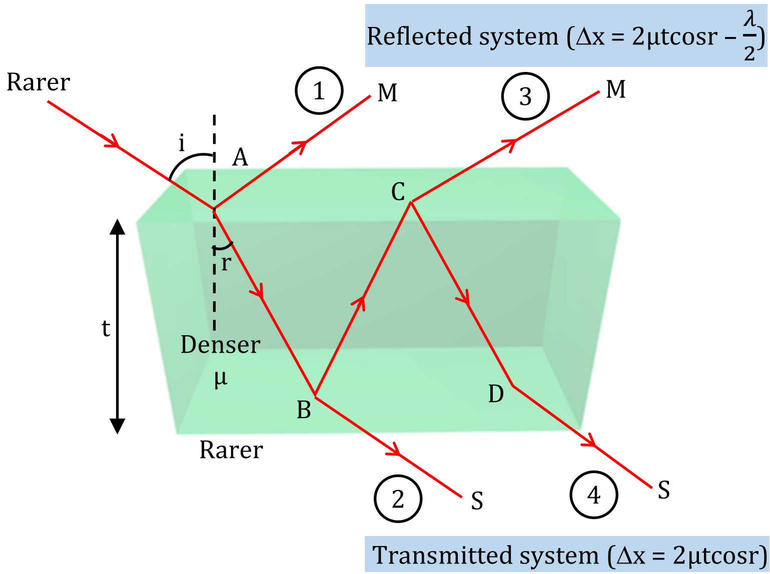
- Whenever a wave is reflected from a denser medium, an extra phase difference of π or path difference of $\lambda/2$ is introduced.



- Whenever a wave is reflected from a rarer medium, no extra phase difference or path difference is introduced.



Path difference in Thin Film Interference



System	Maxima	Minima
Reflected	$2\mu t\cos r-\frac{\lambda}{2}=n\lambda$	$2\mu t\cos r-\frac{\lambda}{2}=\frac{(2n-1)\lambda}{2}$
Transmitted	$2\mu t\cos r=n\lambda$	$2\mu t\cos r=\frac{(2n-1)\lambda}{2}$

For thin film interference to be observable, thickness of film must be in order of wavelength of light ($\approx 10^{-7}$ m).

Note: For the reflected system if thickness of the thin film tends to zero then thin film appears dark.

$$\Delta x = 2\mu t \cos r - \frac{\lambda}{2}$$

$$|\Delta x| = \frac{\lambda}{2} \qquad (\text{When, } t \approx 0)$$

This condition represents destructive interference therefore dark fringes are formed.

Illustration 21

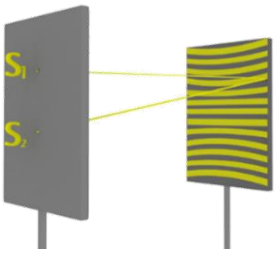
Light of wavelength 6000 Å is incident on a thin glass plate of refractive index 1.5 such that angle of refraction into the plate is 60°. Calculate the smallest thickness of plate which will make it appear dark by reflection.

Solution:

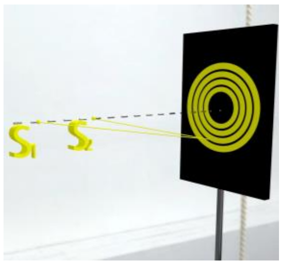
$$2\mu t \cos r - \frac{\lambda}{2} = (2n-1)\frac{\lambda}{2}$$

$$2\mu t \cos r = n\lambda \Rightarrow 2\times1.5\times t \cos 60° = 1\times6\times10^{-7} \Rightarrow t = 4\times10^{-7} = 0.4\ \mu m$$

Shape of fringe in YDSE



Shape of fringe is Hyperbolic



Golden Key Points

- Optical path length of light, d = μx.
- A path length of Δx in a medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.
- When thin film is placed in front of a slit in YDSE,
  - (i) Fringe width doesn't get affected due to the introduction of the thin film in front of slit.
  - (ii) Central maxima is shifted on the same side where thin film is placed.
  - (iii) Shift of central maxima (C.M.) = $y = \frac{(\mu-1)tD}{d} = y_{cm}$
  - (iv) Number of fringes crossing centre line = n = $\frac{(\mu-1)t}{\lambda}$
- If thin film is placed in front of both the slits then central maxima is shifted to that side where the value of $(\mu - 1)t$ is more or path difference due to slab is more.
- Strongly reflected means maxima is formed by reflected system.
- Strongly transmitted means maxima is formed by transmitted system.

BEGINNER'S BOX-3

1. On placing a thin sheet of mica of thickness 12×10^{-7} m in the path of one of interfering beams in a Young's experiment, it is found that the central bright band shifts a distance equal to the width of a bright fringe. If the wavelength of light used is 6×10^{-7} m. then find refractive index of mica.

PW0184

2. A central fringe of the interference produced by light of wavelength 6000Å is shifted to the position of 5^{th} bright fringes by introducing a thin glass plate of refractive index 1.5. Calculate the thickness of the plate.

PW0185

3. White light is incident on a soap film of refractive index $\frac{4}{3}$ at an angle of refraction 30°. The reflected light is observed to have a dark band for wavelength 6×10^{-5} cm. Calculate the minimum thickness of the film.

PW0186

4. A soap solution film of $\mu = \frac{4}{3}$ is illuminated by white light incident with angle of refraction is 60°. In reflected light, dark band was found corresponding to wavelength 5500 Å. Calculate the minimum thickness of the film.

PW0187

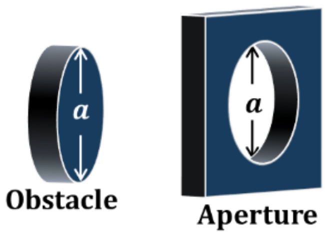
Diffraction

Single slit Diffraction

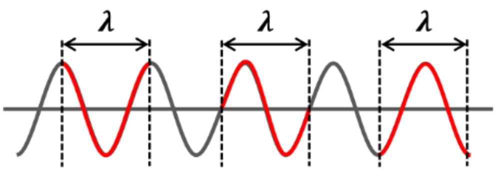
- Bending of light rays from sharp edges of an opaque obstacle or aperture and its spreading in the geometrical shadow region is defined as diffraction of light or deviation of light from its rectilinear propagation tendency is defined as diffraction of light.
- The phenomenon of diffraction of light was discovered by Grimaldi.

Diffraction depends on two factors:

(i) Size of Obstacle/aperture,



(ii) Wavelength of wave,



Conditions of Diffraction

- Size of obstacle or aperture should be nearly equal (Comparable) to the wavelength.
- If size of obstacle/aperture is much larger than wavelength, then diffraction is not observable. (It is practically observed when size of aperture or obstacle is greater than 50λ. The obstacle or aperture does not show diffraction.)

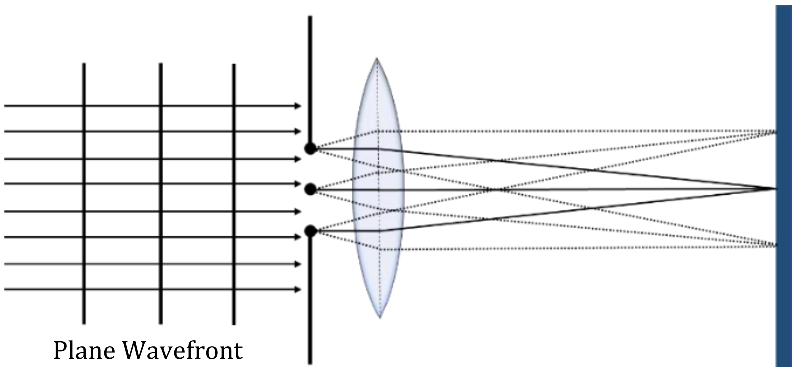
Note: (1) Wavelength of light is of the order 10^{-7} m. In general obstacle of this wavelength is not present so light rays does not show diffraction and it appears to travel in straight line.

(2) Sound wave shows more diffraction as compared to light rays because wavelength of sound is large (16 mm to 16m). So, it is generally diffracted by the objects of our daily life.

Types of Diffraction

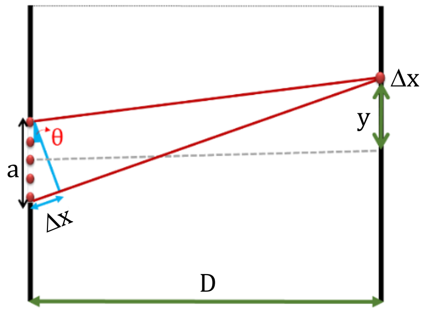
Fresnel diffraction: In Fresnel diffraction both source and screen are near from the diffraction device.

Fraunhofer diffraction: In Fraunhofer diffraction both source and screen are effectively at infinite distance from the diffracting device.



Path difference at any point on screen due to Fraunhofer diffraction

Path difference, $\Delta x = a\sin\theta$

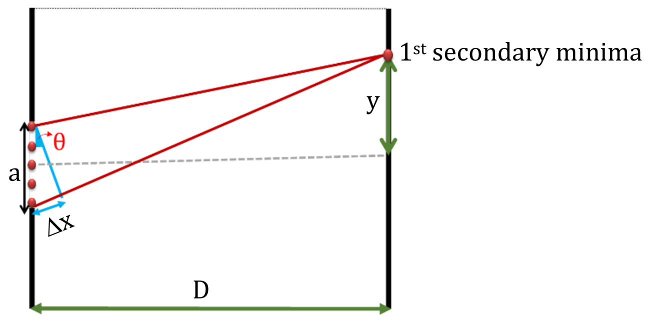


1st secondary Minima

$\Delta\phi = 2\pi$

$\Delta x = \lambda$

$a\sin\theta = \lambda$

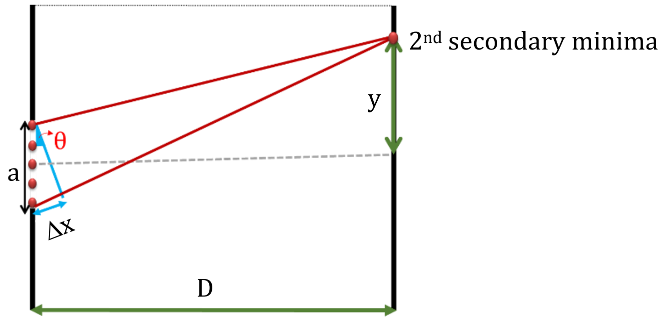


2^{nd} secondary Minima

$\Delta\phi = 4\pi$

$\Delta x = 2\lambda$

$a \sin\theta = 2\lambda$



n^{th} secondary Minima

$\Delta\phi = 2n\pi$

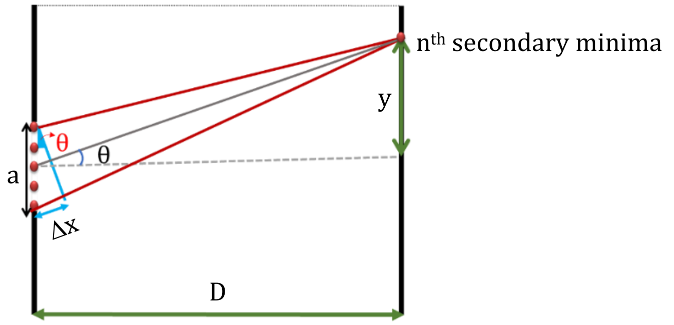
$a \sin\theta = n\lambda$

If angle is small then,

$a \tan\theta = n\lambda$

$$a\frac{y}{D} = n\lambda$$

$$y = \frac{n\lambda D}{a}$$

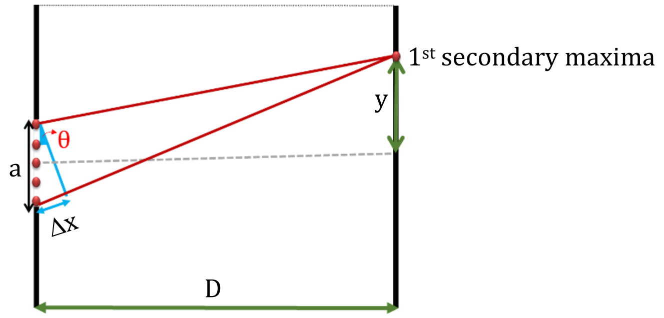


1^{st} secondary Maxima

$\Delta\phi = 3\pi$

$$\Delta x = \frac{3}{2}\lambda$$

$$a\sin\theta = \frac{3}{2}\lambda$$

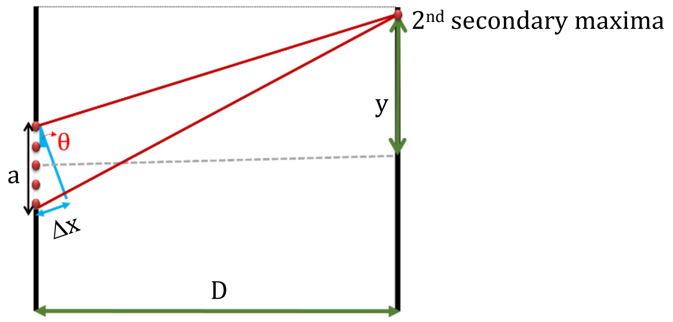


2^{nd} secondary Maxima

$\Delta\phi = 5\pi$

$$\Delta x = \frac{5}{2}\lambda$$

$$a\sin\theta = \frac{5}{2}\lambda$$



n^{th} secondary Maxima

$\Delta\phi = (2n + 1)\pi$

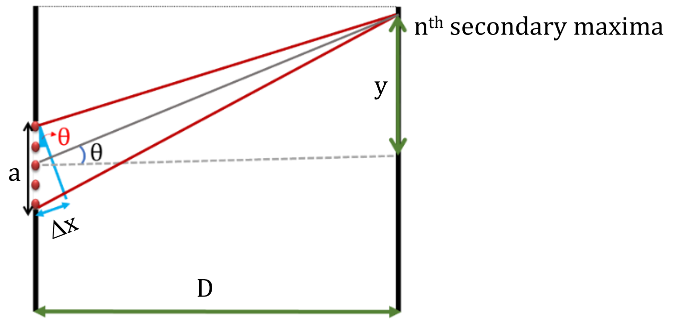
$$\Delta x = \frac{(2n+1)\lambda}{2}$$

$$a\sin\theta = \frac{(2n+1)\lambda}{2}$$

If angle is small then,

$$a\tan\theta = \frac{(2n+1)\lambda}{2}$$

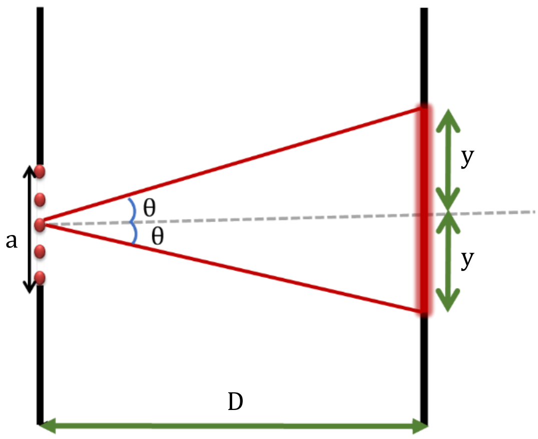
$$a\frac{y}{D} = \frac{(2n+1)\lambda}{2} \Rightarrow y = \frac{(2n+1)\lambda D}{2a}$$



Central Bright Fringe (CBF)

- It is the region between first minima on both the sides of centre of screen.
- Linear width of CBF = 2y
- Linear width of CBF = $\frac{2\lambda D}{a}$
- Angular width of CBF = 2θ
- Angular width of CBF = $2\sin^{-1}\left(\frac{\lambda}{a}\right)$
- For small angle,

  Angular width of CBF = $\frac{2\lambda}{a}$



Golden Key Points

- **Condition of diffraction:** It occurs when size of obstacle or aperture should be nearly equal (Comparable) to the wavelength.
- For n^{th} secondary minima; $a\sin\theta = n\lambda \Rightarrow y = \frac{n\lambda D}{a}$
- For n^{th} secondary maxima; $a\sin\theta = \frac{(2n+1)\lambda}{2} \Rightarrow y = \frac{(2n+1)\lambda D}{2a}$
- For Central Bright Fringe,

  Linear width of CBF = $\frac{2\lambda D}{a}$; Angular width of CBF = $2\sin^{-1}\left(\frac{\lambda}{a}\right)$

Illustration 22

Light of wavelength 6000 Å is incident normally on a slit of width 0.3 mm. Calculate the distance of 1^{st} minimum on a screen placed at a distance 8 m?

Solution:

$$y_{min_1} = \frac{n\lambda D}{a} = \frac{(1)(6000\times10^{-10})\times 8}{0.3\times10^{-3}} = 16 \times 10^{-3}\ \text{m}$$

Illustration 23

If slit width is 2λ and distance of screen is D. What is

(i) Angular position of first minimum? (ii) Linear position of first minimum?

Solution:

(i) $a\sin\theta = n\lambda \Rightarrow 2\lambda \sin\theta = \lambda \Rightarrow 2\sin\theta = 1 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

(ii) $y = \frac{n\lambda D}{a} = \frac{(1)\lambda D}{2\lambda} = \frac{D}{2}$

Illustration 24

Red light of wavelength 6000Å from a distant source falls on a slit 0.75 mm wide. What is the distance between the first two dark bands on each side of the central bright of the diffraction pattern observed on a screen placed 1.5 m from the slit?

Solution:

Distance between the first two dark bands is width of Central Bright Fringe

$$=\frac{2\lambda D}{a}=\frac{2\times 6000\times 10^{-10}\times 1.5}{0.75\times 10^{-3}}=2.4\times 10^{-3}\text{ m}$$

Illustration 25

Light of wavelength 3000 Å is incident normally on a slit of width 15 × 10^{-5} cm. Find out the angular width of first secondary maxima.

Solution:

$$\because\ a\sin\theta=(2n+1)\frac{\lambda}{2}$$

$$\therefore\ (15\times 10^{-7})\sin\theta=(2\times 1+1)\frac{3000\times 10^{-10}}{2}\Rightarrow \sin\theta=0.3\Rightarrow\theta=\sin^{-1}(0.3)$$

Illustration 26

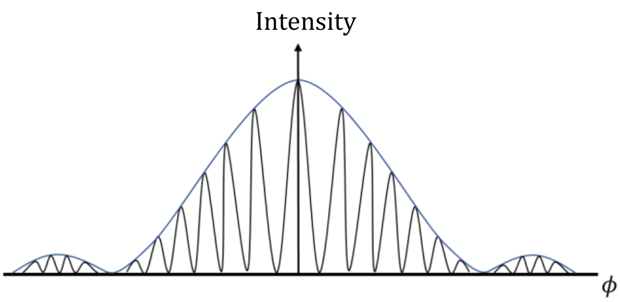
A linear aperture whose width is 0.04 cm is placed immediately in front of a lens of focal length 30 cm. The aperture is illuminated normally by a parallel beam of wavelength 10 × 10^{-5} cm. The distance of first dark band of diffraction pattern from the center of screen is:-

Solution:

$$y_{\min_1}=\frac{n\lambda D}{a}=\frac{1\lambda D}{a}=\frac{1\times 10\times 10^{-7}\times 30\times 10^{-2}}{4\times 10^{-4}}=75\times 10^{-5}\text{m} = 0.075\text{ cm}$$

Double Slit Diffraction

- In the double-slit experiment, we must note that the pattern on the screen is actually a superposition of single-slit diffraction from each slit or hole, and the double-slit interference pattern.
- It shows a broader diffraction peak in which there appear several fringes of smaller width due to double-slit interference.
- The number of interference fringes occurring in the broad diffraction peak depends on the ratio d/a, that is the ratio of the distance between the two slits to the width of a slit.
- In the limit of a becoming very small, the diffraction pattern will become very flat and we will observe the two-slit interference pattern



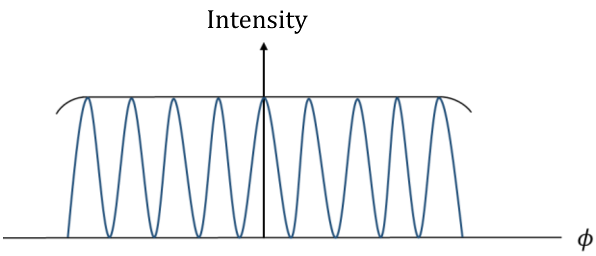


Illustration 27

In a double slit experiment, the two slits are 2 mm apart and the screen is placed 1 m away. What should be the width of each slit for obtaining 8 maxima of double slit within the central maxima of single slit pattern for same distance of screen. A monochromatic light of wavelength 500 nm is used to illuminate the slits.

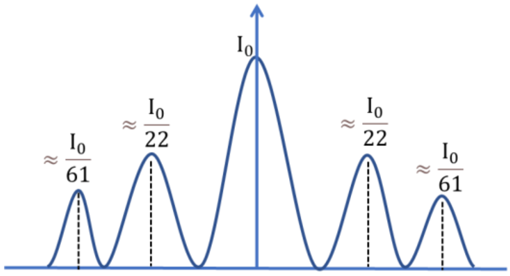
Solution:

8β = width of CBF

$$\frac{8\lambda D}{d} = \frac{2\lambda D}{a} \Rightarrow a = \frac{2\times 2}{8} \Rightarrow a = 0.5 \text{ mm}$$

Intensity Distribution Curve in Diffraction:

- Intensity of CBF = I_o
- Intensity of 1st maxima $\approx \frac{I_o}{22} \approx 4\%$ of I_o
- Intensity of 2nd maxima $\approx \frac{I_o}{61} \approx 1.6\%$ of I_o



Fresnel distance (Validity of Ray Optics)

It is a distance up to which ray optics is a good approximation.

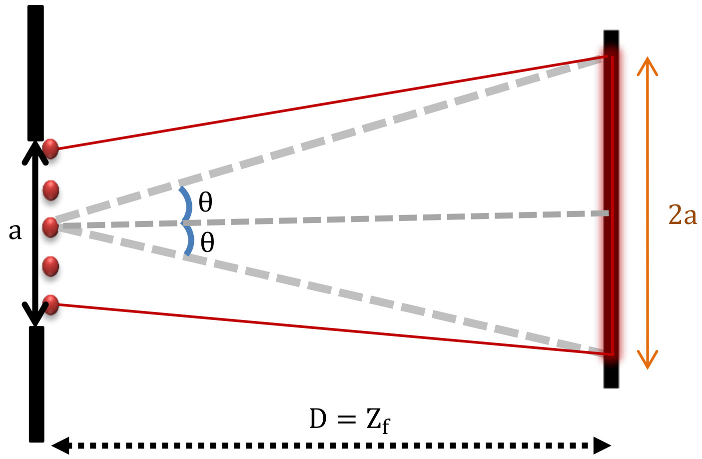
OR

It is the minimum distance a beam of light can travel before its deviation from straight line path becomes significant/noticeable.

$$\frac{2D\lambda}{a} = 2a$$

$$\frac{2Z_f\lambda}{a} = 2a$$

$$Z_f = \frac{a^2}{\lambda} \quad [Z_f = \text{Fresnel distance}]$$



Note: (1) Up to the Fresnel distance diffraction is not observable, ray optics is valid there.

(2) After Fresnel distance spreading due to diffraction dominates over that due to ray optics.

Illustration 28

For what distance is ray optics a good approximation when the aperture is 3 mm wide and the wavelength is 500 nm?

Solution:

$$Z_f = \frac{a^2}{\lambda} = \frac{9\times 10^{-6}}{5\times 10^{-7}} = 18\text{m};$$ Up to 18m Ray optics is valid.

Golden Key Points

- $Z_f = \frac{a^2}{\lambda}$ [Z_f = Fresnel distance]
- Up to the Fresnel distance diffraction is not observable, ray optics is valid there.
- After Fresnel distance spreading due to diffraction dominates over that due to ray optics.

BEGINNER'S BOX-4

1. A slit of width 0·15 cm is illuminated by light of wavelength 5×10^{-5} cm and a diffraction pattern is obtained on a screen 2·1m away. Calculate the width of central maxima.

PW0188

2. The light of wavelength 600nm is incident normally on a slit of width 3mm. Calculate the angular width of central maximum on a screen kept 3m away from the slit.

PW0189

3. Red light of wavelength 6500Å from a distant source falls on a slit 0.50 mm wide. What is the distance between the first two dark bands on each side of the central bright of the diffraction pattern observed on a screen placed 1.8 m from the slit.

PW0190

4. In a single slit diffraction experiment first minimum for λ_1 = 660 nm coincides with first maxima for wavelength λ_2. Calculate λ_2.

PW0191

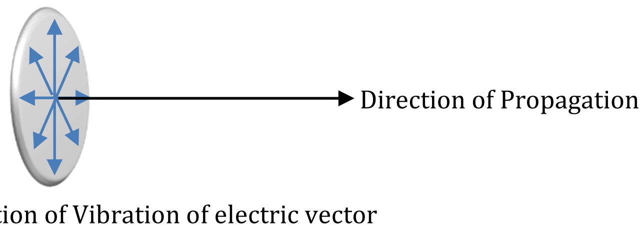
Polarisation of Light Wave

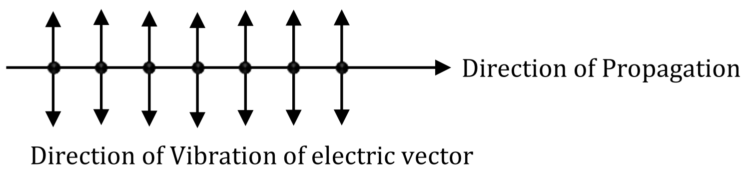
The phenomenon of **restricting the vibration of** light (**electric vector**) in a particular direction perpendicular to the direction of propagation of wave is called polarisation of light.

- Experiments like YDSE and Diffraction tell us that light is a wave.
- Polarisation confirms the transverse nature of wave.
- Longitudinal waves (Ultrasonic wave, sonic wave, infrasonic wave etc.) cannot be polarised.

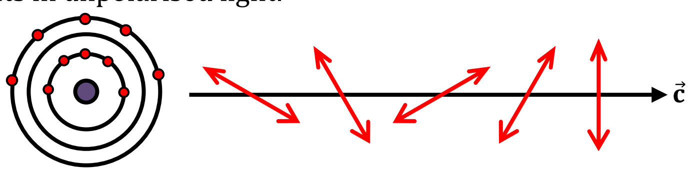
Unpolarised light

- A beam of unpolarised light consists of waves moving in the same direction with their electric vectors pointed in random orientations about the axis of propagation.



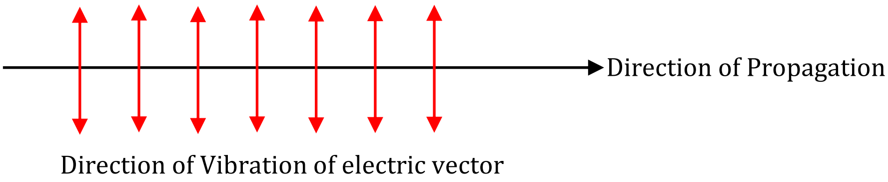


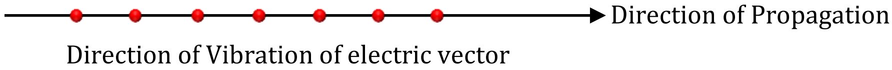
- The atoms on the surface of a heated filaments, which generate light, act independently on one another. Each of their emissions can be approximately modeled as a short-wave train lasting from about 10^{-9} to 10^{-8} sec. The electromagnetic wave emanating from the filament is a superposition of these wave trains, each having its own polarisation direction. The sum of the randomly oriented wave trains results in unpolarised light.



Polarised light

- Plane polarised light consists of waves in which the direction of vibration is the same for all waves.

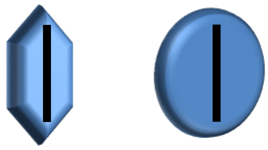




Polariser

Polariser is any device or crystal that can convert an unpolarised light into Polarised light.

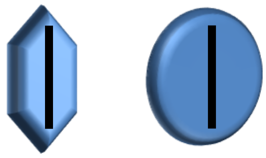
e.g. : Tourmaline crystal , Nicol Prism etc.



Analyser

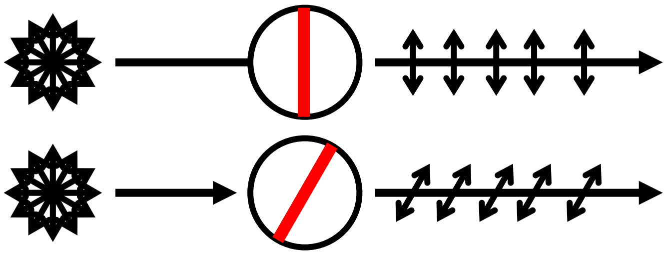
Analyser is a device which is used to determine whether the light is Plane polarised or not.

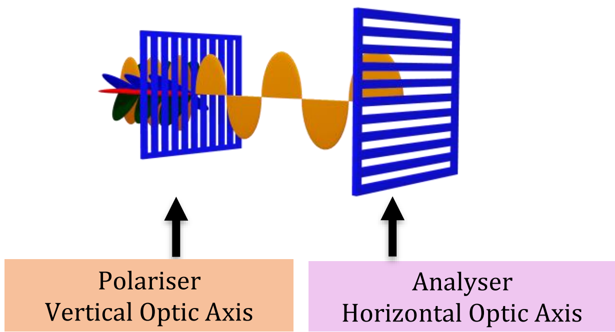
Analyser and Polariser are made up of same material only placement position is different.



Optic axis/Transmission axis

- The axis along which light gets polarised after passing through a polariser.

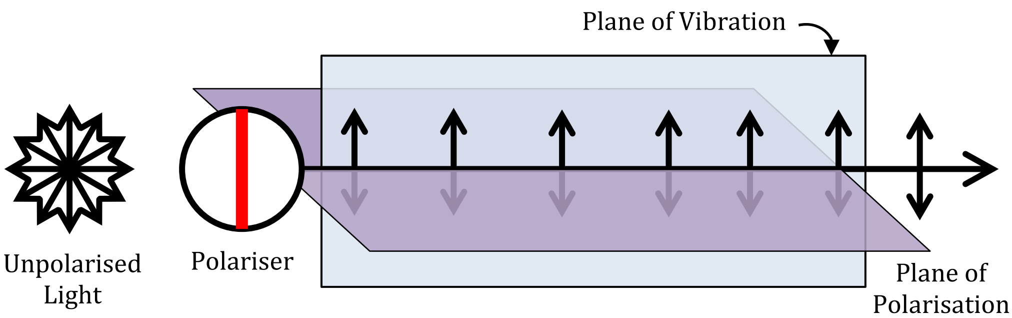




Plane Polarised Light

Plane of Vibration: Plane in which $\vec{E}$ vibrates/oscillates is known as plane of vibration.

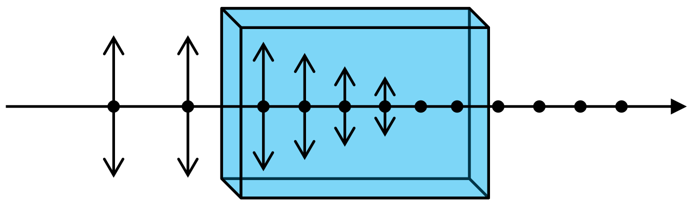
Plane of Polarisation: Plane which is perpendicular to plane of vibration.



Methods of Obtaining Plane Polarised light

- ➢ Polarisation by Dichroism/Selective absorption.
- ➢ Polarisation by Reflection.
- ➢ Polarisation by Refraction.
- ➢ Polarisation by Scattering.
- ➢ Polarisation by Double Refraction.

Polarisation by Dichroism/Selective absorption

- Some crystals such as tourmaline and sheets of iodosulphate of quinone have the property of strongly absorbing the light with vibrations perpendicular of a specific direction (called transmission axis) and transmitting the light with vibration parallel to it. This selective absorption of light is called dichroism.
- 
- So, if unpolarised light passes through proper thickness of these crystals, the transmitted light will plane polarised with vibrations parallel to transmission axis.

Golden Key Points

- Polarisation confirms the transverse nature of wave.
- Longitudinal waves (Ultrasonic wave, sonic wave, infrasonic wave etc.) cannot be polarised.
- Plane of polarisation is perpendicular to plane of vibration.

Malus' Law

When Plane Polarised Light (PPL) of intensity I_o passes through a polaroid such that angle between Axis of vibration of PPL and Optic Axis of Polaroid is θ, then Intensity of Emergent Light after passing through Polaroid becomes, $I = I_o \cos^2\theta$

Reasons:

- A Polarised light has $\vec{E}$ confined to one axis. When it passes through a polaroid.
- Component of $\vec{E}$ perpendicular to Optic Axis (E sinθ) is absorbed by the polaroid.
- Only component of $\vec{E}$ along Optic Axis (E cosθ) passes through the polaroid.

Now, $I \propto A^2$

$$\frac{I}{I_o} = \left(\frac{E_o\cos\theta}{E_o}\right)^2$$

$$I = I_o\cos^2\theta$$

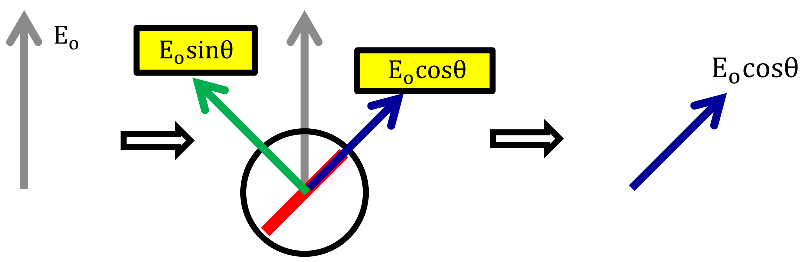
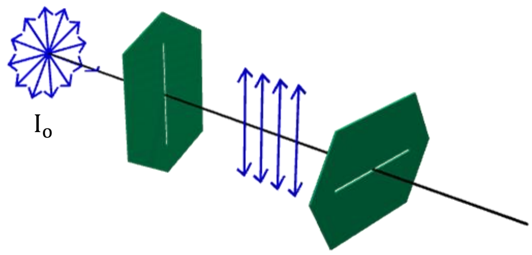


Illustration 29

Find the output intensity of light if angle between analyser and Polariser is 90°.

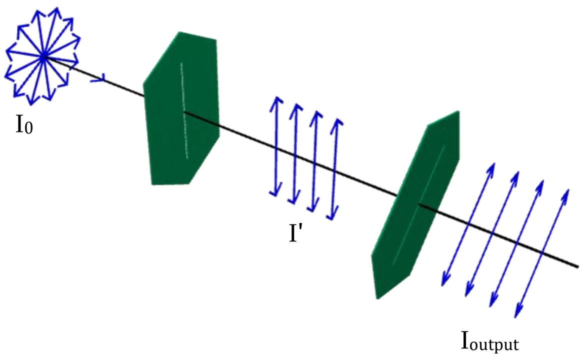


Solution:

$I_{output} = 0$ because angle between analyser and Polariser is 90°.

Illustration 30

Find output intensity if angle between analyser and Polariser is 30°.



Solution:

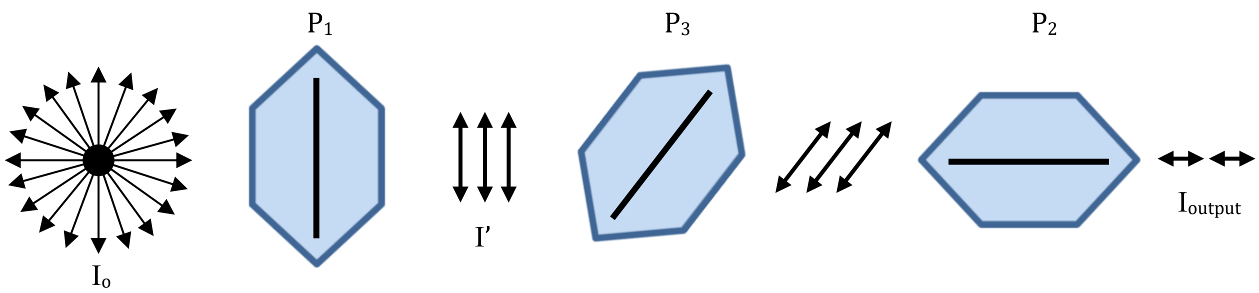
$I' = \frac{I}{2}$ (When unpolarised light get Polarised then intensity reduced to half)

$$I_{output} = I^{'}\cos^2\theta = \frac{I}{2}\cos^2(30°) = \frac{3I}{8}$$

Illustration 31

Two Polaroids P_1 and P_2 are placed with their axis perpendicular to each other. Unpolarised light I_0 is incident on P_1. A third polaroid P_3 is kept in between P_1 and P_2 such that its axis makes an angle 45° with that of P_1. Find the intensity of transmitted light through P_2?

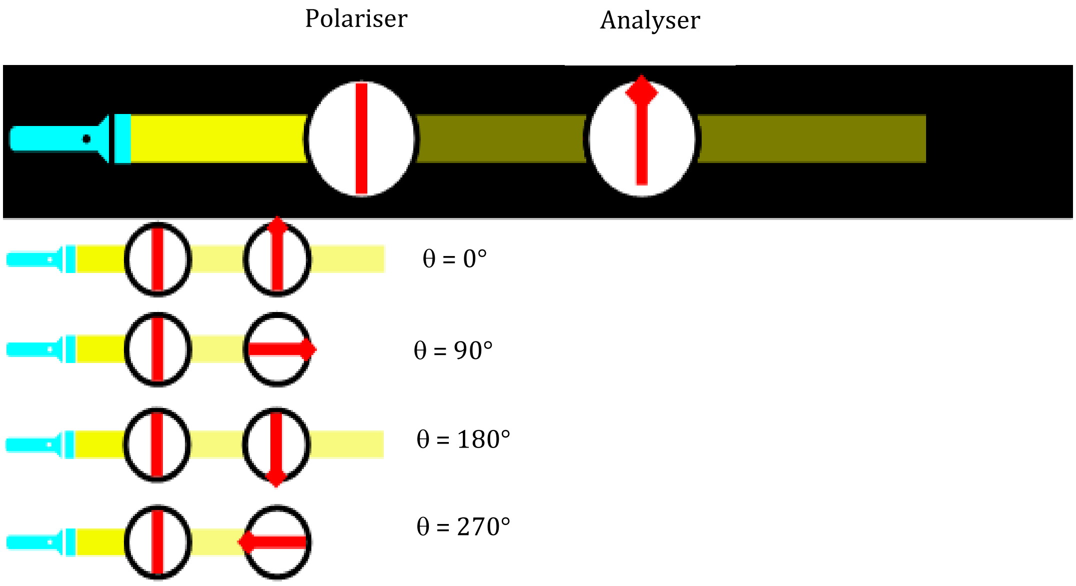
Solution:



$$I' = \frac{I_0}{2} \text{ and } I'' = I'\cos^2 45° = \frac{I_o}{2}\left(\frac{1}{\sqrt{2}}\right)^2 = \frac{I_o}{4}$$

$$I_{output} = I''\cos^2 45 = \frac{I_o}{4}\cos^2(45) = \frac{I_0}{8}$$

When analyser is rotated by 360° in front of polariser



Observations

- After passing through polariser, intensity of light becomes half.
- Final intensity is maximum two times.
- Final intensity becomes zero two times.

Illustration 32

Plane Polarised light passes through a Polaroid. On viewing through the polaroid, we find that when Polaroid is given one complete rotation about the direction of light, one of the following is observed.

(1) The intensity of light gradually decreases to zero and remains at zero

(2) The intensity of light gradually increases to a maximum and remains at maximum

(3) There is no change in intensity

(4) The intensity of light is twice maximum and twice zero

Solution:

The Intensity of light coming out of a polaroid is given by, $I = I_o \cos^2\theta$

In one complete rotation; the angle between polaroid and plane polaroid light will be zero twice and $\pi/2$ twice.

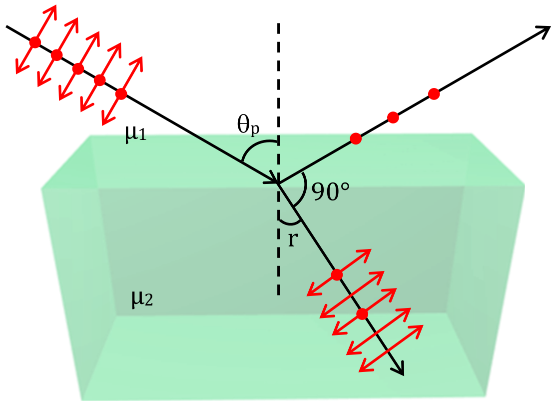
The Intensity of light varies such that, it is twice maximum and twice zero. Thus, option D is correct.

Golden Key Points

- Malus law; $I = I_0 \cos^2\theta$
- When analyser is rotated by 360° in front of Polariser,
  Final intensity is maximum two times, Final intensity becomes zero two times.

Brewster's Law

Polarisation by Reflection: When unpolarised light is incident on the interface of two medium such that reflected and refracted ray are mutually perpendicular to each other than reflected ray is perfectly polarised normal to plane of incidence and refracted ray in the same plane is partially polarised.



$\theta_P + 90° + r = 180°$

$r = 90° - \theta_P$

By Snell's Law, $\mu_1 \times \sin\theta_P = \mu_2 \times \sin r$

$\mu_1 \times \sin\theta_P = \mu_2 \times \sin(90° - \theta_P)$

$\mu_1 \times \sin\theta_P = \mu_2 \times \cos(\theta_P)$

$$\frac{\sin\theta_P}{\cos\theta_P} = \frac{\mu_2}{\mu_1}$$

$$\tan\theta_P = \frac{\mu_2}{\mu_1} \left(\theta_P = \text{Polarising angle (Brewster's angle)}\right)$$

Note:

(1) If incident light medium is denser than the other considered medium $(\mu_1 > \mu_2)$ then $\theta < 45°$.

(2) If incident light medium is rarer than the other considered medium $(\mu_1 < \mu_2)$ then $\theta > 45°$.

(3) If incident light medium is air $(\mu_1 = 1)$ then $\tan\theta_p = \mu$.

(4) If it is mentioned that light is incident at polarising angle or $\tan\theta_p = \mu$, then consider that reflected ray is completely polarised.

(5) If it is mentioned that reflected and refracted ray are perpendicular to each other then consider reflected ray is completely polarised.

(6) If above 2 points are not mentioned, and nothing is written about polarisation then consider reflected and refracted ray both as partially polarised

Illustration 33

For a given medium, the polarising angle is 53°. What will be the critical angle for this medium?

Solution:

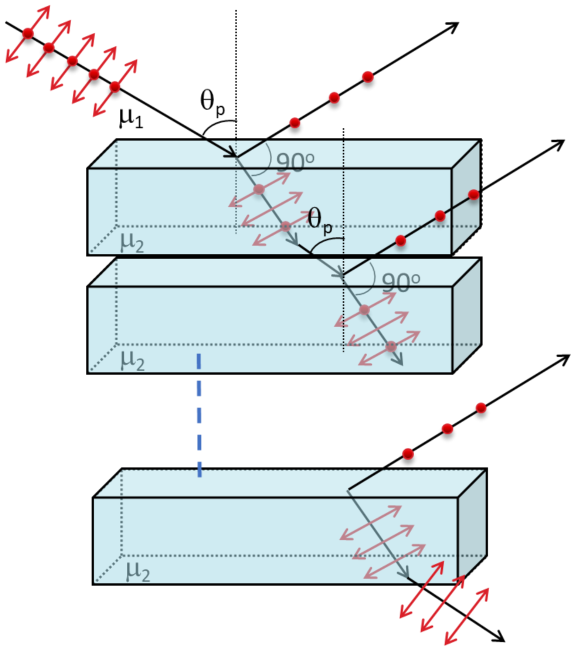
$$\mu = \tan 53° \Rightarrow \mu = \frac{4}{3}$$

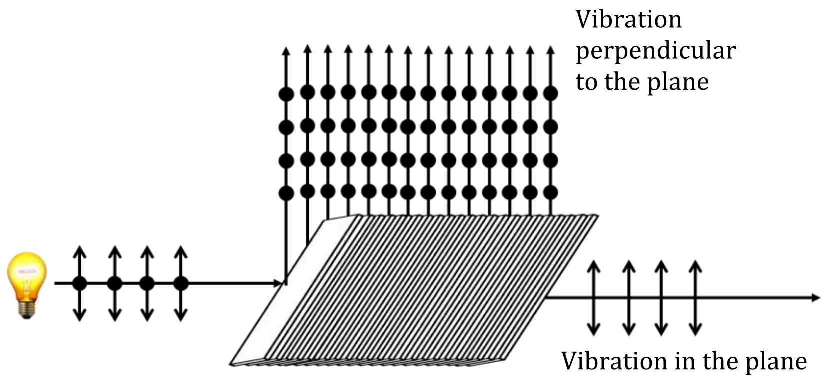
$$\sin\theta_C = \frac{1}{\mu} \Rightarrow \theta_C = \sin^{-1}\left(\frac{3}{4}\right)$$

Other methods of polarisation

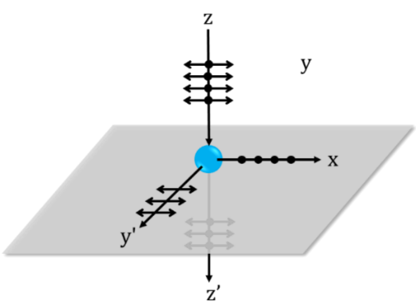
1. Polarisation by Refraction

- According to Brewster's law, the reflected light will be plane polarised with vibrations perpendicular to the plane of incidence and the transmitted light will be partially polarised. Since in one reflection about 15% of the light with vibration perpendicular to plane of paper is reflected therefore after passing through a number of plates emerging light will become plane polarised with vibrations in the plane of paper.





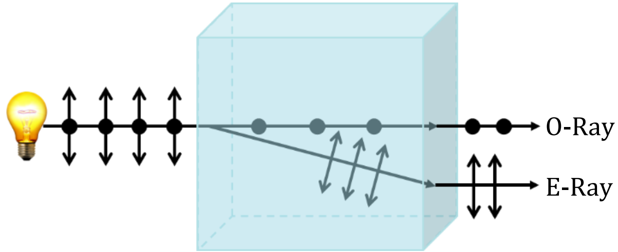
2. Polarisation by Scattering



- When light scatters on small particles such as dust, air molecules, it gets absorbed by electrons in the atoms and re-radiated in all directions.

- Light scattered in a plane at right angle to the incident light is always plane polarised.

3. Polarisation by Double Refraction



- It was found that in certain crystals such as calcite, quartz and tourmaline, etc., incident unpolarised light splits into two light beams of equal intensities with perpendicular polarisations.
- One of the rays behaves as ordinary light (obeys law of refraction) is called O-ray (ordinary ray) while the other which does not obey laws of refraction is called E-ray (extra-ordinary ray).
- When an object is seen through these crystals is rotated, one image (due to E-ray) rotates around the other (due to O-ray).

BEGINNER'S BOX-5

1. Refractive index of water is 1.33. Calculate the angle of polarisation for light reflected from the surface of a lake. ($\tan^{-1} 1.33 = 53°.4'$)

PW0192

2. A ray of light strikes a glass plate at an angle of 60°. If the reflected and the refracted rays are perpendicular to each other, find the index of refraction of glass.

PW0193

3. A parallel beam of light is incident at an angle of 60° on a plane glass surface and the reflected beam is completely polarised.

(a) What is the angle of refraction in glass?

(b) What is the refraction index of glass?

PW0194

4. Light reflected from the surface of a glass plate of refractive index 1.732 is linearly polarised. Calculate the angle of refraction in glass.

PW0195

5. Critical angle for a certain wavelength of light in case of glass is 40°. Find the polarising angle and angle of refraction in glass corresponding to this. ($\sin 40° = \frac{2}{3}$)

PW0196

BEGINNER'S BOX — ANSWER KEY

BEGINNER'S BOX-1

1.	5I	**2.**	$\frac{25}{16}$	**3.**	(a) 7.8I, (b) I (c) 9I

BEGINNER'S BOX-2

1.	4.5 mm	**2.**	6μm	**3.**	1·2 cm	**4.**	1.76×10^{-3} m
5.	6×10^{-7} m	**6.**	0.2948 mm, 0.2211 mm			**7.**	8×10^{-3} m, $\lambda = 0.66 \times 10^{-6}$ m
8.	$\frac{3}{1}$	**9.**	10^{-3} m.	**10.**	1.5 mm		

BEGINNER'S BOX-3

1.	1·5	**2.**	6×10^{-6} m	**3.**	2.59×10^{-5} cm	**4.**	4125 Å

BEGINNER'S BOX-4

1.	1·4 mm	**2.**	0.023°	**3.**	4.68 mm	**4.**	440 nm.

BEGINNER'S BOX-5

1.	53°4'	**2.**	1.732	**3.**	(a) 30° (b) 1.732
4.	30°	**5.**	57.3°, 32.7°		