Wave Motion

SOLUTIONS

Exercise-I (Conceptual Questions)

3. distance = speed × time

6. Velocity of wave = $\frac{600}{2}$ = 300 m/s

Frequency = 500 Hz, Wave length $\lambda = \frac{3}{5}$ m

Number of wavelength = $\frac{600}{3/5}$ = 1000

7. Speed of sound wave is medium dependent term.

8. $y = 4\sin\frac{\pi}{2}\left(8t - \frac{x}{8}\right) = 4\sin\left(4\pi t - \frac{\pi}{16}x\right)$

Compare by $y = A\sin(\omega t - kx)$, $\omega = 4\pi$

$k = \frac{\pi}{16} \Rightarrow v = \frac{\omega}{k}$

v = 64 cm / sec. → +ve x-direction.

9. $y = 4\sin\left[\pi\left(\frac{t}{5} - \frac{x}{9}\right) + \frac{\pi}{6}\right]$ compare this equation by

$y = a\sin(\omega t - kx + \phi)$

$\omega = \frac{\pi}{5}$ $\quad k = \frac{\pi}{9}$

a = 4 $\quad \frac{T}{2} = 5 \quad \frac{\lambda}{2} = 9$

T = 10 s, λ = 18 cm, v = nλ = 1.8 cm/s

10. Amplitude becomes double

Hence it becomes = 0.5

and frequency becomes half

hence ω becomes = π (half) and travelling in opposite direction with same speed so K will also become half

$\left\{v = \frac{\omega}{K} = \text{const.}\right\}$

∴ $y = 0.5\cos(\pi t + \pi x)$

11. $v_{max} = A\omega$

$v_{max} = 3 \times 10$

= 30 units

12. $v = \frac{\omega}{k} = \frac{6}{3} = 2$ units

13. Wave is a disturbance, which carries energy and momentum without transport of mass of matter.

15. $y_1 = a\sin\omega t$, $y_2 = a\cos\omega t = a\sin(\omega t + \frac{\pi}{2})$

16. Distance between two consecutive crests is 5 m which is equal to λ = 5m, freq. n = 2 Hz

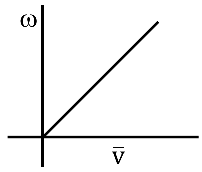
v = nλ = 10 m/s

17. $\omega = 2\pi n$

$\omega \propto n \qquad (2\pi = \text{const})$

$\omega \propto \frac{1}{\lambda} \qquad \bar{v} = \frac{1}{\lambda}$

$\Rightarrow \quad \omega \propto \bar{v}$



19. v = 10

n = 100 $\quad \therefore \lambda = \frac{10}{100}$ = 0.1 m = 10 cm

$\frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} \Rightarrow \frac{\Delta\phi}{2\pi} = \frac{2.5}{10} \Rightarrow \Delta\phi = \frac{\pi}{2}$

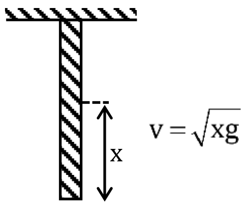
20. At the upper end –

$v = \sqrt{2.5 \times 10}$ = 5 m/s

at a point 0.5 m distance from bottom -

$v = \sqrt{0.5 \times 10} = \sqrt{5}$

= 2.24 m/s



21. $v = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{\omega}{K} = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{\left(\frac{2\pi}{0.04}\right)}{\left(\frac{2\pi}{0.50}\right)} = \sqrt{\frac{T}{0.04}}$

$\Rightarrow \quad \frac{0.50}{0.04} \times \frac{0.50}{0.04} = \frac{T}{0.04}$

∴ $T = \frac{50}{4} \times 0.5$ = 6.25 N

22. $v_{wave} = \frac{\omega}{k}$

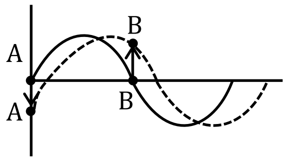
$v_{particle} = A\omega$

On comparing

v_{w1} = 2m/s, v_{w2} = 3m/s, $v_{w3} = \frac{11}{5}$ m/s

v_{p1} = 12cm/s, v_{p2} = 36cm/s, v_{p3} = 44cm/s

23.



from fig. A is moving down & B is moving up and the phase at A is greater the phase at B.

24. $v = \frac{\omega}{k} = \frac{30}{1} = 30$ m/s

$d = 1.3 \times 10^{-4}$ kg/m, $v^2 d = T$

then $T = 1.17 \times 10^{-1}$N

26. $v_{O_2} = \sqrt{\frac{\gamma RT}{M_w}} = V$ $v_{H_2+O_2} = \sqrt{\frac{\gamma RT}{(M_w)_{mix}}}$

$(M_w)_{mix} < M_w$ so $v_{H_2+O_2} > V_{O_2}$

27. $v = \sqrt{\frac{E}{\rho}}$

28. $I = \frac{1}{2}\rho v a^2 \omega^2 \Rightarrow I \propto n^2 a^2$

29. frequency does not depends upon medium, velocity of sound decreases when it goes from water to air. $v = f\lambda$ ($\because$ f = const.)

so λ also decreases

30. Frequency doesn't change on changing the medium

32. $L = 10\log_{10}\frac{I}{I_0} \Rightarrow \log_{10}\frac{I}{I_0} = 3 \Rightarrow \frac{I}{I_0} = 10^3$

33. Velocity of sound

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

as $\rho_d > \rho_m$

$\therefore \upsilon_m > \upsilon_d$

34. $V = v \Rightarrow \frac{\omega}{k} = \omega A \Rightarrow A = \frac{1}{k} = \frac{1}{(2\pi/\lambda)}$

$$\Rightarrow A = \frac{\lambda}{2\pi}$$

35. Energy same so $a_1^2 n_1^2 = a_2^2 n_2^2$

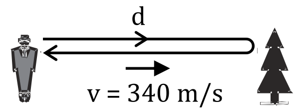
36. $\frac{V_1}{V_2} = \sqrt{\frac{\rho_2}{\rho_1}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$

37. $\frac{\Delta\phi}{2\pi} = \frac{\Delta t}{T} \Rightarrow \Delta\phi = \frac{1}{12} \times 2\pi = \frac{\pi}{6}$

38. $v \propto \frac{1}{\sqrt{\rho}}$

39. Let distance between cliff and mountain be d

$t = \frac{2d}{v} \Rightarrow 1 = \frac{2d}{340} \Rightarrow \text{distance}(d) = \frac{340}{2} = 170\text{m}$



40. $A_R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi}$

41. In superposition, only redistribution of energy occurs.

42. (1) Same phase $I_R = 4I$

(2) $\phi = 90°$, $I_R = I + I + 2\sqrt{I}\sqrt{I}\cos 90° = 2I$

43. $I_1 = I$ $I_2 = I$ $\phi = 0$

$I_R = I + I + 2\sqrt{I}\sqrt{I}\cos 0 = 4I$

44. $\frac{a_1}{a_2} = \frac{3}{4}$

$$\frac{I_{max}}{I_{min}} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2 = \frac{(3+4)^2}{(3-4)^2} = \frac{49}{1}$$

45. $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

for $I_{max} \Rightarrow \phi = 0 \Rightarrow I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$

46. $\frac{I_{max.}}{I_{min.}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \frac{(3+1)^2}{(3-1)^2} = \frac{16}{4} = \frac{4}{1}$

47. A B

n $n \pm x$

n + x or n - x

If frequency of B is n + x then on waxing beats decrease.

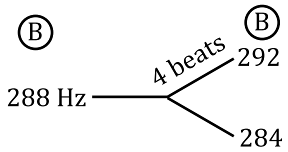
48. 250 ± 8 → 242 or $\boxed{258}$ 270 ± 12 → 282 or $\boxed{258}$

common → 258 Hz

49. Number of beats $\Delta n = n_1 - n_2$

$2 = \frac{v}{\lambda_1} - \frac{v}{\lambda_2} \Rightarrow 2 = \frac{v}{2} - \frac{v}{2.02}$

$2 = \frac{v}{202} \Rightarrow v = 404$

50. 

After loading wax, frequency of A decrease. So beats frequency will also decrease.

Ans. would be 292

52. for minima/destructive interferance

$\Delta\phi = (2n+1)\pi$

$\frac{2\pi}{\lambda}\ \Delta\lambda = (2n+1)\pi$

$\Delta\lambda = (2n+1)\ \frac{\lambda}{2}$

53. $I_{max} = 4I,\ \frac{I_{max}}{I} = 4$

54. $n_1 = 202$ Hz

$n_2 = 200$ Hz

$b = n_1 - n_2 = 2$

55. $a = \sqrt{a^2 + a^2 + 2a^2\cos\phi}$

$\Rightarrow \cos\phi = -\frac{1}{2} \qquad \Rightarrow \phi = \frac{2\pi}{3}$

56. $T = \frac{1}{|v_1 - v_2|}$

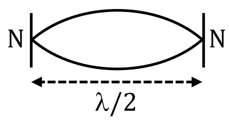
57. beats per second = $n_2 - n_1 = 253 - 250 = 3$

beats per minuts = $3 \times 60 = 180$

59. Fundamental frequency

$n = \frac{1}{2L}\sqrt{\frac{T}{\mu}} \quad [\because\ \mu = \frac{M}{L}\ ] \quad \therefore\ n = \frac{1}{2}\sqrt{\frac{T}{ML}}$

60. v = 700 cm/sec = 7 m/sec, L = 2 m



$n = \frac{V}{2\ell} = \frac{7}{2\times 2} \Rightarrow n = \frac{7}{4}$ Hz

61. $y = a\cos(\omega t - kx)$

x = 0 is a node

So rigid end reflection

$y = a\cos(\omega t + kx + \pi)$

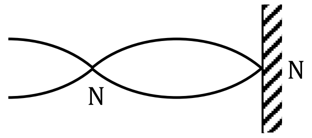
$y = -a\cos(\omega t + kx)$ $[\cos(180 + \theta) = -\cos\theta]$

62. Here $\lambda = 4$ cm

Now distance between two consecutive nodes

$= \frac{\lambda}{2} = 2$cm

63. n = 100 Hz., Given $\frac{\lambda}{2} = 10$ cm



$v = n\lambda = 100 \times 0.2 \qquad v = 20$ m/ s

65. $n \propto \sqrt{T}$

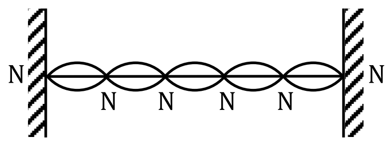
66. At earth $n = \frac{P}{2L}\sqrt{\frac{Mg}{\mu}} \Rightarrow n = \frac{P}{2L}\sqrt{\frac{g}{\mu}}$

(T = Mg M=1 Kg)

At moon $n' = \frac{P}{2L}\sqrt{\frac{M'g}{6\mu}}$

n' = n When M' = 6 Kg

67. p = 5, $\ell = 10$ m



$v = \sqrt{\frac{T}{\mu}} = 20$ m/s, $n = \frac{P}{2\ell}\sqrt{\frac{T}{\mu}}$

68. $N\underset{\lambda/4}{\text{———}}A\underset{\lambda/4}{\text{———}}N\underset{\lambda/4}{\text{———}}A\underset{\lambda/4}{\text{———}}N$

$4\left(\frac{\lambda}{4}\right) = 1.21$Å $\Rightarrow \lambda = 1.21$ Å

69. $n \propto \sqrt{T} \Rightarrow \frac{n_1}{n_2} = \sqrt{\frac{T_1}{T_2}}$

$\Rightarrow \frac{n-10}{n} = \sqrt{\frac{129.6}{160}} = \frac{36}{40}$

$40n - 400 = 36\,n \Rightarrow n = 100$ Hz

70. $v \propto \sqrt{T}$

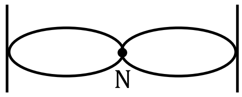
71. In stationary wave, energy does not propogate.

72. $n \propto \sqrt{T} \qquad T\downarrow\ n\downarrow$

73. Plucking distance

$$x=\frac{\ell}{2p}=\frac{\ell}{4}$$

touching distance = $\ell/2$



74. $n=\frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2 d}} \Rightarrow \frac{n_2}{n_1}=\sqrt{\frac{T_2}{T_1}\times\frac{r_1^2}{r_2^2}\times\frac{d_1}{d_2}}$

$$=\sqrt{\frac{2}{1}\times\frac{1}{4}\times\frac{2}{1}}=1$$

$$\Rightarrow n_2 = n_1 = n$$

75. $n\propto\sqrt{T}$

76. $n=\frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2 d}}$

$$n'=\frac{1}{2\times 3\ell}\sqrt{\frac{3T}{9\pi r^2 d}}=\frac{n}{3\sqrt{3}}$$

77. $n\propto\frac{1}{\ell}$

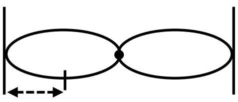
$$\frac{n_1}{n_2}=\frac{\ell_2}{\ell_1}\Rightarrow\frac{800}{1000}=\frac{\ell_2}{50}$$

ℓ_2 = 40 cm

78. $n\propto\sqrt{T}$

79. y = A coskx cosωt

80.



0.25m

$\frac{\lambda}{4}=0.25$ λ = 1m

$$n=\frac{V}{\lambda}=\frac{1}{1}\sqrt{\frac{T}{m}}=\sqrt{\frac{20}{0.5\times10^{-3}}}=200\text{ Hz}$$

81. When T↑ and velocity (v)↑ Then n↑

82. Length of air column decreases,

$\therefore$ frequency increases.

83. e = 1 cm ℓ_1 = 15 cm

$$e=\frac{\ell_2-3\ell_1}{2}$$

$$1=\frac{\ell_2-3\times15}{2}$$

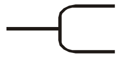
2 = ℓ_2 – 45

ℓ_2 = 47 cm

84. Next 3 overtones are in ratio of 3 : 5 : 7

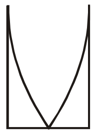
Only odd harmonic are present.

85. For closed pipe



$$n=\frac{v}{4\ell}=512$$

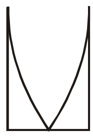
For open pipe



$$n=\frac{v}{2\ell}=1024$$

n = 1024 Hz.

86. $n=\frac{v}{4\ell}$



n = 264 Hz

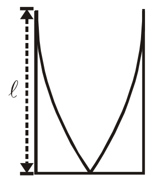
v = 330 m/s

Then

ℓ = 31.25 m

87. Fundamental frequency

$$n=\frac{v}{4\ell}$$



v = 320 m/s

ℓ = 1 m., n = 80 Hz

For closed pipe

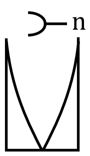
$n_1 : n_2 : n_3$ = ... = 1 : 3 : 5 : ....

frequency 80, 240, 400, ...

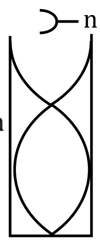
88. Fundamental frequency of open organ pipe

$$n=\frac{v}{2\ell}=\frac{330}{2\times0.3}=550\text{Hz}$$

89.



$\frac{\lambda}{4}=\ell_1=50\text{cm}$



$\frac{3\lambda}{4}=\ell_2=150\text{cm}$

length will be 3 times

90. $\frac{v}{4\ell} = 260$, $\ell = 31.7$ cm

91. $\lambda = \frac{v}{n} = \frac{330}{330} = 1$ m

particle vibrate maximum at a distance

$\lambda/4$. So $\frac{\lambda}{4} = 0.25$m

92. $L_0 = 33$ cm

n = 1000 Hz $1000 = \frac{N \times 330}{2 \times .33} \Rightarrow N = 2$

i.e. 1st overtone

2nd harmonic

93. $n = (2m+1)\frac{V}{4L}$

$L = (2m+1)\frac{V}{4n}$

For L to be minima,
mode should be fundamental i.e. m = 0

$L = \frac{V}{4n}$

94. $n_1 = \frac{v}{4\ell_1}$, $n_2 = \frac{v}{4\ell_2}$

$\ell_1 = 0.75$ m $\ell_2 = 0.770$ m

$n_1 - n_2 = 3 \Rightarrow \frac{v}{4}\left(\frac{1}{\ell_1} - \frac{1}{\ell_2}\right) = 3 \Rightarrow v = 346.5$ m/s

95. $n = \frac{Nv}{2L} \Rightarrow L = \frac{Nv}{2n}$

for L to be minm N = 1

$L = \frac{v}{2n} = \frac{350}{2 \times 350} = 50$ cm

96. Frequency remains unchanged on change medium
n = 60 kHz

$\lambda = \frac{v}{n} = \frac{330}{60 \times 10^3} = 5.5 \times 10^{-3}$ m = 5.5 mm

97. $\frac{v}{4\ell}$ = fundamental frequency = 1500 Hz

Normal person can listen frequency 20,000 Hz.
1500 × x = 20,000, x = 13 (No. of Harmonic)
Then overtones and frequency in case of closed organ pipe, ratio of frequency

1,	3,	5,	7,	9,	11,	13
I	II	III	IV	V	VI	

Then 6 overtones.

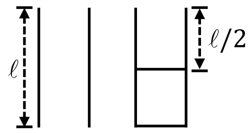
98. $n_N = (2n+1)\frac{v}{4L}$

$n_N = (2n+1)\frac{320}{4 \times 1}$

$n_N = (2n + 1) \times 80$ = odd multiple of 80
At 300 Hz resonance will not occur.

99. For open organ pipe

$n_1 = \frac{v}{2\ell}$



for COP $n_2 = \frac{v}{4(\ell/2)} = \frac{2v}{4\ell} \Rightarrow n_2 = \frac{v}{2\ell}$, $n_1 = n_2$

100. It occurs due to end correction.

Exercise-II (Previous Year Questions)

1. Total length of string $\ell = \ell_1 + \ell_2 + \ell_3$

But frequency $\propto \frac{1}{\text{length}}$

so $\frac{1}{n} = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$

2. Frequency COP, $n_N = (2N + 1)\frac{v}{4L}$

for N = 0, $n_0 = 100$ Hz
N = 1, $n_1 = 300$ Hz
N = 2, $n_2 = 500$ Hz
N = 3, $n_3 = 700$ Hz
N = 4, $n_4 = 900$ Hz
N = 5, $n_5 = 1100$ Hz Total = 6
Which are less than 1250 Hz.

3. $\frac{V}{4L_C} = \frac{3V}{2L_0}$

$L_0 = 4L_C \times \frac{3}{2}$

$L_0 = 6 \times 20 = 120$ cm

4. Two consecutive resonant frequencies for a string fixed at both ends will be

$\frac{nv}{2\ell}$ and $\frac{(n+1)v}{2\ell} \Rightarrow \frac{(n+1)v}{2\ell} - \frac{nv}{2\ell} = 420 - 315$

$\frac{v}{2\ell} = 105$ Hz

Which is the minimum resonant frequency

5. $T_1 = m_2 g$

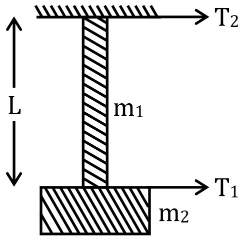
$T_2 = (m_1 + m_2)g$

Velocity $\propto \sqrt{T}$ (n=const.)

$\Rightarrow \lambda \propto \sqrt{T}$

$$\Rightarrow \frac{\lambda_1}{\lambda_2} = \frac{\sqrt{T_1}}{\sqrt{T_2}}$$

$$\Rightarrow \frac{\lambda_2}{\lambda_1} = \sqrt{\frac{m_1 + m_2}{m_2}}$$



6. First minimum resonating length for closed organ pipe = $\frac{\lambda}{4} = 50\,cm$

$\therefore$ Next larger length of air column

$= \frac{3\lambda}{4} = 150\,cm$

7. For second overtone (3^{rd} harmonic) in open organ pipe,

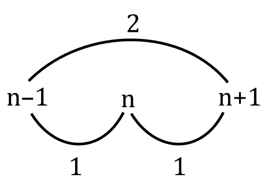
$$\frac{3\lambda}{2} = \ell_0 \Rightarrow \lambda = \frac{2\ell_0}{3}$$

for first overtone (3^{rd} harmonic) in closed organ pipe,

$$\frac{3\lambda}{4} = \ell_C \Rightarrow \lambda = \frac{4\ell_C}{3} = \frac{4L}{3}$$

So, $\frac{2\ell_0}{3} = \frac{4L}{3} \Rightarrow \ell_0 = 2L$

8.



Now divide 1 second into 1, 1, 2 equal divisions

$\frac{1}{1}$

$\frac{1}{1}$

$\frac{1}{2}$ $\frac{2}{2}$

By eliminating common time instants, total maxima in one second is 2.

So, two beats per second will be heard.

9. Difference between any two consecutive frequencies of COP = $\frac{2v}{4\ell} = 260 - 220 = 40$ Hz

$\Rightarrow \frac{v}{4\ell} = 20\,Hz$

So fundamental frequency = 20 Hz

10. $V = 2f(\ell_2 - \ell_1)$ $\because \lambda = 2(\ell_2 - \ell_1)$

$= 2(320) \times (0.73 - 0.20)$ $V = f\lambda$

$= (640) \times (0.53) = 339.2$ m/s

11. $f_{oop} = 3f_{cop} \Rightarrow \frac{v}{2\ell_{oop}} = \frac{3v}{4\ell_{cop}}$

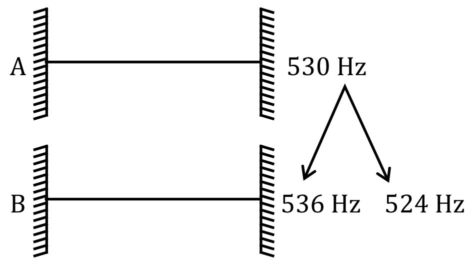
$\Rightarrow \ell_{oop} = \frac{2}{3} \times \ell_{cop} = \frac{2}{3} \times 20 = 13.3$ cm

12. Here $\lambda = 2(l_2 - l_1)$

where $l_1 = 9.75$ cm, $l_2 = 31.25$ cm

so $v = 2n(l_2 - l_1) = 2 \times 800\,(31.25 - 9.75) = 344$ m/s

13. Guitar string i.e. string is fixed from both ends



Frequency $\propto \sqrt{\text{Tension}}$

If tension in B slightly decreases then frequency of B decreases.

If B is 536 Hz, as the frequency decreases, beats with A also decreases.

If B is 524 Hz, as the frequency decreases, beats with A increases.

$\therefore$ Original frequency of B will be 524 Hz.

14. Frequency of stretched string

$$n=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

Here T and μ are constant so $n\propto\frac{1}{\ell}$

Therefore $\frac{n'}{n}=\frac{\ell}{\ell'}$

$$\Rightarrow \frac{180}{120}=\frac{90}{\ell}\Rightarrow \ell' = 60\text{ cm}$$

17. $$\frac{n_{oop}}{n_{cop}}=\frac{\frac{V}{2L}}{\frac{V}{4L}}$$

$$\Rightarrow \frac{n_{oop}}{n_{cop}}=\frac{2}{1}$$

18. Interference is obtained by coherent waves superposition.
A and C has same frequency.

19. Coefficient of t in equation y = Asin(ωt–kx) is angular frequency.

So, comparing $\omega=2\pi\frac{a}{\lambda}$

$$2\pi f=2\pi\frac{a}{\lambda}$$

$$f=\frac{a}{\lambda}$$

Exercise-III (Analytical Questions)

1. Same length of organ pipe means wavelength in both the pipe are same but velocity of sound is different in both the pipes due to different gases inside it.

3. $$v=\sqrt{\frac{\gamma RT}{M_W}}=\sqrt{\frac{\gamma P}{\rho}}$$

$(M_W)_{\text{dry air}} > (M_W)_{\text{moist air}}$

$(\rho)_{\text{dry air}} > (\rho)_{\text{moist air}}$

4. y = 0.02 sin 2π (10t – 5x)

compare the about equation with

y = A sin (ωt – kx)

A = 0.02 m

$\omega = 20\pi$ rod/s

k = 10π m^{-1}

$$v=\frac{\omega}{k}=\frac{20\pi}{10\pi}=2$$

$\omega = 20\pi = 2\pi f$

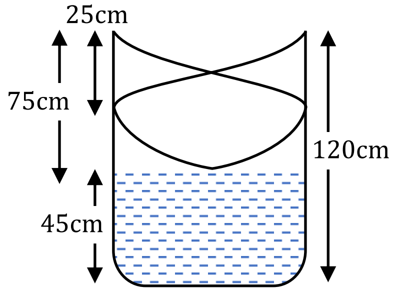
f = 10 Hz

$k = 10\pi = \frac{2\pi}{\lambda} \Rightarrow \lambda = 0.2$ m

$v_{max} = A\omega = 0.02 \times 20\pi = 0.4\pi$ m/s

5. As V = fλ

$$\lambda=\frac{V}{f}=\frac{340}{340}=1\text{m}$$



first Resonance length

$$R_1=\frac{\lambda}{4}=\frac{1}{4}\text{m}=25\text{cm}$$

$$\therefore R_2=\frac{3\lambda}{4}=\frac{3}{4}\text{m}=75\text{cm}$$

$$\therefore R_3=\frac{5\lambda}{4}=\frac{5}{4}\text{m}=125\text{cm}$$

i.e. third resonance does not establish

$\therefore$ minimum length of H_2O column to have the resonance = 45 cm

$\therefore$ Distance between two successive nodes = $\frac{\lambda}{2}$

$$=\frac{1}{2}\text{m}=50\text{cm}$$

and maximum length of H_2O column to create resonance i.e. 120 – 25 = 95 cm

OR

h = 120 cm

f = 340 Hz

v = 340 m/s

$\lambda = \frac{v}{f}=\frac{340}{340}$ = 100 cm

(1) $\frac{\lambda}{4}=\frac{100}{4}$ = 25cm (air) $\Rightarrow$ water level = 95 cm

(2) $\frac{3\lambda}{4}$ = 75cm (air) $\Rightarrow$ water level = 45 cm

(3) $\frac{5\lambda}{4}$ = 125 cm (not possible)

So maximum water level = 95 cm

Minimum water level = 45 cm

Distance between two successive nodes

$= \frac{\lambda}{2} = \frac{100}{2}$ = 50 cm

6. From $\sqrt{\frac{\gamma RT}{M_W}} = \sqrt{\frac{\gamma P}{\rho}}$

(i) Temperature 2 times $\Rightarrow$ speed double

(ii) Temperature constant there

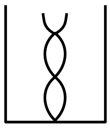
$P\uparrow \quad \rho\uparrow \quad v_{sound}$ = const

(iv) M_W is 4 times $\Rightarrow$ speed half.

7. For closed organ pipe fundament frequency

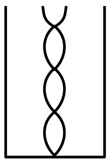
$f = \frac{v}{4\ell}$

Third overtone $f = \frac{7v}{4\ell}$ (7 time of fundamental frequency)



Second overtone $f = \frac{5v}{4\ell}$

No. of nodes = 3



Third overtone $f = \frac{7v}{4\ell}$

No. of A.N. = 4

8. Theory

9. $v = \sqrt{\frac{\gamma RT}{M\omega}} = \sqrt{\frac{\gamma \rho}{\rho}}$

$(M_\omega)_{\text{dry air}} > (M_\omega)_{\text{moist air}}$

$(\rho)_{\text{dry air}} > (\rho)_{\text{moist air}}$

10. First resonating length $\ell_1 = \frac{\lambda}{4}$

Second resonating length $\ell_2 = \frac{3\lambda}{4}$

11. Theory

13. Wavelength depends on length which is fixed. Therefore wavelength does not change.

Further $v = \sqrt{T/m}$ or $v \propto T^{1/2}$

$\therefore$ % change in $v = \frac{1}{2}$ (% change in T)

$= \frac{1}{2}(2) = 1\%$

i.e., Speed and hence frequency will change by 1%.

Change in frequency is 15 Hz which is 1% of 1500 Hz.

Therefore, original frequency should be 1500 Hz.

OR

$f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$

$\frac{\Delta t}{f} = \frac{1}{2}\frac{\Delta T}{T}$

$\frac{15}{f} = \frac{1}{2}\frac{2}{100}$

f = 1500 Hz (original frequency)

$v = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$

$\frac{\Delta v}{v} \times 100 = \frac{1}{2} \times 2\% = 1\%$

$\because$ Length of string is fixed that means wavelength remain same.

14. For closed pipe,

$f = n\left(\frac{v}{4\ell}\right) m = 1,3,5,.....$

For n = 1, $f_1 = \left(\frac{v}{4\ell}\right)$ n = 1, 3, 5.....

For n = 1, $f_1 = \frac{v}{4\ell} = \frac{320}{4 \times 1} = 80\text{Hz}$

For n = 3, $f_3 = 3f_1$ = 240 Hz

For n = 5, $f_5 = 5f_1$ = 400 Hz

OR

L = 1m = 100 cm

v = 320 m/s

fundament freq. $f = \frac{v}{4\ell} = \frac{320}{4 \times 1}$ = 80 Hz

3rd harmonic = 3 × 80 = 240 Hz

5th harmonic = 5 × 80 = 400 Hz

15. $y = 8\sin^2\pi\left[\frac{t}{0.02} - \frac{x}{100}\right]$

$= 4\left[1 - \cos 2\pi\left(\frac{t}{0.02} - \frac{x}{100}\right)\right]$

$\Rightarrow y - 4 = -4\cos\left(\frac{2\pi t}{0.02} - \frac{2\pi x}{100}\right)$

For (A) : Amplitude = 4 cm

For (B) :

Frequency $f = \frac{\omega}{2\pi} = \left(\frac{2\pi}{0.02}\right)\frac{1}{2\pi} = 50\text{Hz}$

For (C) : Velocity of wave = $\frac{\omega}{k}$ = 5000cm/s

= $50 ms^{-1}$

For (D) : Maximum particle velocity

$= A\omega = (4 \times 10^{-2})\left(\frac{2\pi}{0.02}\right) = 4\pi\ ms^{-1}$

$= 400\ \pi$ cm-s^{-1}

16. The wave speed depends on properties of the medium, not on how you generate the wave. For a string. $v = \sqrt{T_s/\mu}$. Increasing the tension or decreasing the linear density (lighter string) will increase the wave speed.

17. $v_1 = \frac{\omega}{k} = \frac{4}{2} = 2m/s$,

$v_2 = \frac{10}{4} = 2.5 m/s$,

$v_3 = \frac{12}{6} = 2m/s$,

$v_4 = \frac{16}{8} = 2m/s$,

$v_5 = \frac{20}{10} = 2\ m/s$

as $v = \sqrt{\frac{T}{\mu}}$ so T_2 is different.

18. At node $\cos(10\pi x) = 0$ &

at antinode $\cos(10\pi x) = 1$

& $\omega = 50\ \pi$, $k = 10\pi$ & $v = \frac{\omega}{k}, k = \frac{2\pi}{\lambda}$