Oscillation

SOLUTIONS

Exercise-I (Conceptual Questions)

1. Force constant
$K = m\omega^2$
$K = m(2\pi n)^2$
$K = m4\pi^2n^2$

2. $\frac{d^2x}{dt^2} = -\left(\frac{k}{m}\right)x$

3. $\frac{d^2x}{dt^2} = -kx$

compare this given equation with $\frac{d^2x}{dt^2} = -\omega^2 x$

$\omega^2 = k$ $\qquad$ $\omega = \sqrt{k}$

$\frac{2\pi}{T} = \sqrt{k}$ $\qquad$ $T = \frac{2\pi}{\sqrt{k}}$

4. Maximum value of tan is ∞

5. $\omega = 20$

$\frac{2\pi}{T} = 20$ $\qquad$ $T = \frac{2\pi}{20} = \frac{\pi}{10}\,s$

6. At $x = +A$, $\theta = \frac{\pi}{2}$

7. $\omega = 3000 \Rightarrow n = \frac{\omega}{2\pi}$

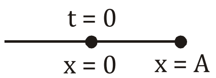
8. acceleration is always towards equilibrium position (mean position) where as displacement is away from mean position.

9. Four times of amplitude

10. At extreme position displacement is maximum and phase $\theta = \frac{\pi}{2}$

11. $V_{max} = A\omega = 10(20) = 200$ m/s

12. Phase constant $= \frac{\pi}{3} = 60°$

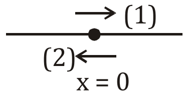
13. 

$t = \frac{T}{4}\left(\frac{\pi}{2}\text{Angle}\right)$

$\frac{\pi}{6}$ phase will develop when particle move from mean position towards amplitude position. It mean particle move in +x direction

$x = a \sin \omega t$
$x = a \sin \theta$

$x = a \sin\frac{\pi}{6} = a \sin 30° = \frac{a}{2}$

14. 

Particle (1) start its motion from mean position so phase angle is 0.
Particle (2) move in –ve x direction at mean position the phase angle becomes π so phase difference between the two particle is $\pi - 0 = \pi$

15. $x = A \sin \omega t$

$\frac{\sqrt{3}}{2}A = A\sin\left(\frac{2\pi\times 3}{T}\right)$

$\frac{\sqrt{3}}{2} = \sin\left(\frac{6\pi}{T}\right)$

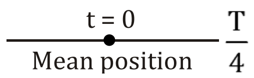
$\frac{\pi}{3} = \frac{6\pi}{T}$, T = 18 seconds

16. Time required between mean to half of amplitude $\left(\frac{a}{2}\right)$ position in t_1 is $= \frac{T}{12}$

Time required between $\left(\frac{a}{2}\right)$ to a position is

$t_2 = \frac{T}{6}$

$\frac{t_1}{t_2} = \frac{\frac{T}{12}}{\frac{T}{6}} = \frac{1}{2} = 1:2$

17. 

Maximum displacement (amplitude position)

18. Required time from mean to half of amplitude position is $= \frac{T}{12} = \frac{6}{12} = \frac{1}{2}$

21. $v_0 = A\omega,\ x = \frac{A}{2}$

$v = \omega\sqrt{A^2 - x^2} \Rightarrow v = \omega\sqrt{A^2 - \frac{A^2}{4}}$

$v = \omega\sqrt{\frac{3}{4}A^2} \Rightarrow \frac{\sqrt{3}}{2}A\omega = \frac{\sqrt{3}}{2}v_0$

22. $v = \frac{\sqrt{3}}{2}$ (Velocity at mean position)

$v = \frac{\sqrt{3}}{2} a\omega$

$\Rightarrow \quad v = \omega\sqrt{a^2 - x^2}$

$\Rightarrow \quad \frac{\sqrt{3}}{2} a\omega = \omega\sqrt{a^2 - x^2}$

$\Rightarrow \quad x^2 = a^2 - \frac{3}{4} a^2$

$\Rightarrow \quad x^2 = \frac{a^2}{4} \Rightarrow x = \frac{a}{2}$

23. $a = \omega^2 x$ (magnitude)

$20 = \omega^2(5) \Rightarrow \omega^2 = 4 \Rightarrow \omega = 2 \frac{\text{Rad}}{\text{s}}$

24. $V = \omega\sqrt{A^2 - x^2}$

$= \frac{2\pi}{T}\sqrt{5^2 - 3^2} = \frac{2\pi}{\pi}\sqrt{16} = 8$ cm/sec.

OR

$x = A \sin \omega t \Rightarrow 3 = 5 \sin \omega t \Rightarrow \sin \omega t = \frac{3}{5}$

$\therefore \quad \cos \omega t = \frac{4}{5} \quad \therefore V = A\omega \cos \omega t$

so $\quad V = 5 \times \frac{2\pi}{\pi} \times \frac{4}{5}$

25. $V_{max} = A\omega = 100$ ...(1)

$a_{max} = A\omega^2 = 157$ ...(2)

Divided (1) by (2)

$\frac{1}{\omega} = \frac{100}{157}$

$\frac{T}{2\pi} = \frac{100}{157} \qquad T = \frac{2 \times 3.14 \times 100}{157} = 4$

26. $x = 1$ cm $\qquad a = 1$ cm/s^2

$V = 1$ cm/s

$a = \omega^2 x \Rightarrow \omega^2 = 1 \Rightarrow \omega = 1 \qquad T = 2\pi$

27. Length of SHM path (2A) is 4cm

$2A = 4$ So amp $A = 2$ cm

$V = A\omega \Rightarrow 12 = 2(2\pi n)$

$n = \frac{12}{4\pi} \Rightarrow \frac{3}{\pi}$ Hz

28. The velocity at mean position is non zero.

29. $a = -\omega^2 x$,

So graph is straight line with negative slope

30. $V_{max} = A\omega \Rightarrow A\left(\frac{2\pi}{T}\right) = 0.40\left(\frac{2\pi}{0.05}\right)$

$\Rightarrow 16\pi$ m/s

31. $V = \omega\sqrt{A^2 - x^2} \qquad V_{max} = 100 = A\omega$

$50 = 10\sqrt{100 - x^2} \quad 100 = 10\omega$

$25 = 100 - x^2 \qquad \omega = 10$

$x^2 = 75 = 5$ cm

OR

$100 = 50 \cos \omega t$

$\cos \omega t = \frac{1}{2} \qquad \therefore \qquad \sin \omega t = \frac{\sqrt{3}}{2}$

so $\quad x = A \sin \omega t$

$x = 5\sqrt{3}$ cm

32. Time period T = 0.04 sec

So frequency $n = \frac{1}{T} = \frac{1}{0.04} = 25$ Hz

33. $V_{max} \propto A$

34. At extreme position V is zero. But acceleration (a) is maximum

At mean position V is maximum but acceleration (a) is zero.

35. The force and acceleration of particle is zero at mean position.

36. In SHM the velocity is maximum at central (mean) position.

37. $x = 5.0 \cos(2\pi t + \pi) = -5.0 \cos 2\pi t$

Velocity = $\frac{dx}{dt} = 10\pi \sin 2\pi t$

Acceleration = $\frac{d^2x}{dt^2} = 20\pi^2 \cos 2\pi t$

$\therefore \quad$ At t = 1.5 sec,

$x = -5.0 \cos 3\pi = -5.0(-1) = 5$

Velocity = $\frac{dx}{dt} = 10\pi(\text{zero}) = 0$

Acceleration = $\frac{d^2x}{dt^2} = 20\pi^2(-1) = -20\pi^2$

38. $Y = 3 \sin(100 t + \frac{\pi}{6})$

$Y = A \sin(\omega t + \phi)$

$A = 3, \quad \omega = 100$

maximum velocity

$V_{max} = A\omega = 3 \times 100 = 300$

39. $v_{max} = A\omega \;\Rightarrow\; 4.4 = 7 \times 10^{-3}\omega \;\Rightarrow\; \omega = \dfrac{4400}{7}$

So $T = \dfrac{14\pi}{4400} = 0.01$ second

40. In one complete oscillation the displacement is zero, so average velocity is also zero.

41. If particle starts oscillation from extreme position then $x = A\cos\omega t$

$\dfrac{A}{2} = A\cos\omega t \Rightarrow \cos\omega t = \cos\dfrac{\pi}{3} \;\Rightarrow\; \omega t = \dfrac{\pi}{3}$

$\dfrac{2\pi}{T}t = \dfrac{\pi}{3} \;\Rightarrow\; t = \dfrac{T}{6}$

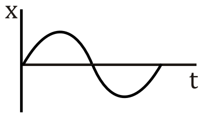
42. In one complete oscillation the displacement is zero so total work done is also zero.

43. $a = -\omega^2 x \;\Rightarrow\; \dfrac{\text{acceleration}}{\text{displacement}} = \text{constant}$

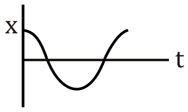
44. $Y = 2\cos(\pi t + \pi/6)$

here $\omega = \pi \Rightarrow \dfrac{2\pi}{T} = \pi \;\Rightarrow\; T = 2$ second

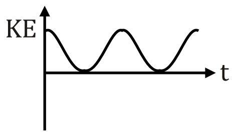
45. $x = A\sin\omega t$



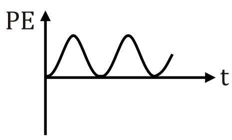
46. $V = \dfrac{dx}{dt} = A\omega\cos\omega t$



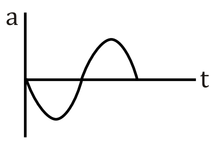
47. K.E. $= \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\omega^2A^2\cos^2\omega t$



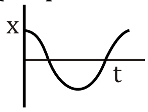
48. PE $= \dfrac{1}{2}Kx^2 = \dfrac{1}{2}KA^2\sin^2\omega t$



49. $a = -\omega^2 x \;\Rightarrow\; a = -\omega^2(A\sin\omega t)$

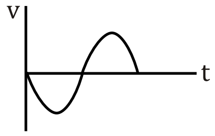


50. $x = A\cos\omega t$ (Displacement – time)



51. $\dfrac{dx}{dt} = -A\omega\sin\omega t$

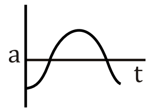
$V = -A\omega\sin\omega t$



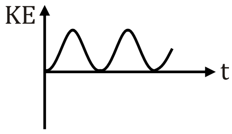
52. $V = -A\omega\sin\omega t$

$\dfrac{dv}{dt} = -A\omega^2\cos\omega t$

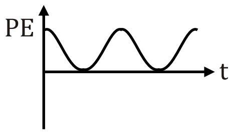
$a = -A\omega^2\cos\omega t$



53. $KE = \dfrac{1}{2}mv^2 = \dfrac{1}{2}m\omega^2A^2\sin^2\omega t$



54. $PE = \dfrac{1}{2}kx^2 = \dfrac{1}{2}kA^2\cos^2\omega t$



55. $E = \dfrac{1}{2}kA^2$

56. T.E. $= \dfrac{1}{2}m\omega^2A^2 = 2\pi^2 m\dfrac{A^2}{T^2}$

57. $E = \dfrac{1}{2}m\omega^2A^2$

K.E. $= \dfrac{1}{2}m\omega^2(A^2 - x^2)$ $\qquad x = \dfrac{A}{2}$

$= \dfrac{1}{2}m\omega^2\left(A^2 - \dfrac{A^2}{4}\right)$

$(K.E.) = \left(\dfrac{1}{2}m\omega^2A^2\right) = \dfrac{3E}{4}$

OR

$\sin\omega t = \dfrac{1}{2} \quad \therefore \quad \cos\omega t = \dfrac{\sqrt{3}}{2}$

$KE = E_0\cos^2\omega t$

58. $E = \dfrac{1}{2}m\omega^2A^2$ $\qquad x = \dfrac{A}{2}$

P.E. $= \dfrac{1}{2}m\omega^2x^2 = \dfrac{1}{2}m\omega^2\dfrac{A^2}{4} = \dfrac{E}{4}$

OR

$\because \quad \sin\omega t = \dfrac{1}{2} \qquad \therefore \quad U = E_0\sin^2\omega t$

59. Length of SHM is 8 cm

So 2A = 8

Amp of SHM A = 4 cm

K.E. = P.E.

$\frac{1}{2}m^2(A^2 - x^2) = \frac{1}{2}m\omega^2x^2$

$A^2 - x^2 = x^2$

$2x^2 = A^2 \quad \Rightarrow \quad x = \frac{A}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ cm

60. $(PE.)_{AV} = \frac{1}{2}(P.E.)_{max} = \frac{1}{2}\left(\frac{1}{2}Ka^2\right) = \frac{1}{4}Ka^2.$

61. T.E. = $\frac{1}{2}m\omega^2A^2.$

Does not depend on time & displacement.

Ans. is (1) option

62. $(KE)_{max} = \frac{1}{2}KA^2$

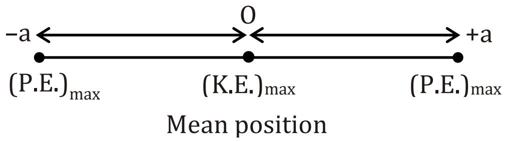
$5 = \frac{1}{2}K(10 \times 10^{-2})^2$

$K = \frac{10}{(10 \times 10^{-2})^2} = 1000$ N/m

63. $F = -\frac{dU}{dx} = \frac{-d}{dx}(8x^2) \quad \Rightarrow -16x$

(F) = –16(–2) = 32 Dyne.

64.



displacement is ±a

65. $U = E_0 \sin^2 \omega t = \frac{E_0}{4}$

$\sin \omega t = \frac{1}{2}$

$x = a \sin \omega t = \frac{a}{2}$

66. $\frac{\frac{1}{2}KA^2}{\frac{1}{2}K(A^2 - x^2)} = \frac{A^2}{A^2 - \frac{A^2}{4}} = \frac{4}{3}$

Ans. is (3) option

67. $\frac{1}{2}Kx^2 = \frac{1}{2}K(A^2 - x^2)$

$\Rightarrow x^2 = \frac{A^2}{2} \quad \Rightarrow \frac{x}{A} = \frac{1}{\sqrt{2}}$

68. In this conservative field total energy remains constant.

69. $\langle E \rangle = \langle V \rangle$

70. $U = \frac{1}{2}Kx^2 \Rightarrow U \propto x^2$

or $\frac{U'}{U} = \left(\frac{x'}{x}\right)^2 = \left(\frac{3}{1}\right)^2 = 9 \Rightarrow U' = 9U$

71. $E = \frac{1}{2}Kx^2$

$\frac{E'}{E} = \frac{(x/2)}{x} = \frac{1}{4} \Rightarrow E' = \frac{E}{4}$

72. $f = \frac{1}{2\pi}\sqrt{\frac{K}{m}} \propto \sqrt{K}$

Both springs are in parallel.

So $f' \propto \sqrt{2K} \propto \sqrt{2}f$

73. $f \propto \sqrt{K'}$,

both springs are in parallel so $K' = K_1 + K_2$

74. $\upsilon = \frac{1}{2\pi}\sqrt{\frac{K}{m}}$

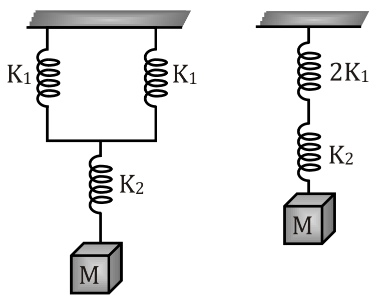
$\upsilon' = \frac{1}{2\pi}\sqrt{\frac{K}{4m}} = \frac{1}{2}\left(\frac{1}{2\pi}\sqrt{\frac{K}{m}}\right) = \frac{\upsilon}{2}$

75. In series combination

$\frac{1}{K_S} = \frac{1}{K_1} + \frac{1}{K_2} \Rightarrow K_S = \frac{K_1K_2}{K_1 + K_2}$

76. $T = 2\pi\sqrt{\frac{M}{K_1 + K_2}}$

77.



$\frac{1}{K_{eq}} = \frac{1}{2K_1} + \frac{1}{K_2}$

$K_{eq} = \left[\frac{1}{2K_1} + \frac{1}{K_2}\right]^{-1}$

78. $T = 2\pi\sqrt{\frac{m}{K}}$

$2 = 2\pi\sqrt{\frac{m}{K}}$

$2+1 = 2\pi\sqrt{\frac{m+2}{K}} \Rightarrow \frac{4}{9} = \frac{m}{m+2}$

4m + 8 = 9m   5m = 8   m = 1.6 kg

79. Time period of spring pendulum does not depends on 'g'.

80. Time period will remain same

81. In form of P.E.

82. Spring watch (independent of g)

83. $\nu \propto \sqrt{K}$

Now K' = 2K So $\frac{\nu'}{\nu} = \frac{\sqrt{2K}}{\sqrt{K}} = \sqrt{2} \Rightarrow \nu' = \sqrt{2}\nu$

84. $U_1 = U_2$

$\frac{1}{2}K_1x_1^2 = \frac{1}{2}K_2x_2^2 \Rightarrow \frac{F_1^2}{2K_1} = \frac{F_2^2}{2K_2}$

$\frac{F_1^2}{F_2^2} = \frac{K_1}{K_2}$ or $\frac{F_1}{F_2} = \sqrt{\frac{K_1}{K_2}}$

85. Length of one part is 3 times of other

$\ell_1 = 3\ell_2$

$\frac{K_1}{K} = \frac{\ell}{\ell_1} = \frac{\ell_1+\ell_2}{\ell_1} = \frac{4}{3}$

$K_1 = \frac{4}{3}K$

86. $K \propto \frac{1}{\ell}$

$\because \quad \ell' = \frac{\ell}{n} \qquad \therefore \quad K' = nK$

87. $T \propto \sqrt{m}$ for spring oscillation

$\frac{T'}{T} = \sqrt{\frac{m'}{m}} = \sqrt{\frac{40}{10}} = 2 \Rightarrow T' = 2T$

88. T of simple pendulum does not depend on mass of bob

89. $V_{max} = A\omega$

90. $T = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{39.2}{9.8\times\pi^2}} = 2\pi\sqrt{\frac{4}{\pi^2}} = 2\pi\times\frac{2}{\pi}$

$\Rightarrow 4$

91. $T = 2\pi\sqrt{\frac{\ell}{g}} \propto \sqrt{\ell}$

$T' \propto \sqrt{4\ell} \propto 2\sqrt{\ell} \propto 2T$ (Double)

92. T = 2 sec

One extreme to other extreme is

$= \frac{T}{2} = \frac{2}{2} = 1\,\text{sec}$

93. $T = 2\pi\sqrt{\frac{\ell}{g}}$, T = 2 seconds so $\ell \approx 1\text{m}$

94. In artificial satellite time period becomes ∞.

95. Due to frictional forces the mechanical energy of oscillator converts in heat energy and finally oscillation stops.

96. $T = 2\pi\sqrt{\frac{\ell}{g}\left(\frac{R_E}{\ell+R_E}\right)}$, if $\ell = R_e$ then T = 59.8

97. If ℓ is large $\ell + R_E \approx \ell$

then t = 84.6 min $\approx 1\frac{1}{2}$ hour

98. W = 0, as tension force is always perpendicular to displacement.

99. When lift is at rest T = $\sqrt{\frac{\ell}{g}}\,2\pi$

When lift is ascending

T = $2\pi\sqrt{\frac{\ell}{g+g/3}} = 2\pi\sqrt{\frac{3\ell}{4g}} = \frac{\sqrt{3}}{2}\left(2\pi\sqrt{\frac{\ell}{g}}\right) = \frac{\sqrt{3}}{2}T$

100. T = $2\pi\sqrt{\frac{\ell}{g}}$ $\ell\downarrow \therefore T\downarrow$

101. If velocity is same then acceleration will be zero and there will be no variation in time period.

Exercise-II (Previous Year Questions)

1. Displacement, x = $A\cos(\omega t)$

Velocity, v = $\frac{dx}{dt} = -A\omega\sin(\omega t)$

Acceleration, a = $\frac{dv}{dt} = -A\omega^2\cos(\omega t)$

2. $Y = y_1 + y_2$

$= a\sin\omega t + b\cos\omega t$

$$= \sqrt{a^2+b^2}\left[\frac{a}{\sqrt{a^2+b^2}}\sin\omega t + \frac{b}{\sqrt{a^2+b^2}}\cos\omega t\right]$$

$$= \sqrt{a^2+b^2}\,[\cos\phi\sin\omega t + \sin\phi\cos\omega t]$$

$$\left[\text{Let} \quad \cos\phi = \frac{a}{\sqrt{a^2+b^2}}, \quad \sin\phi = \frac{b}{\sqrt{a^2+b^2}}\right]$$

$$Y = \sqrt{a^2+b^2}\,\sin(\omega t + \phi)$$

3. $v = \omega\sqrt{A^2 - x^2}$

$v_1 = \omega\sqrt{A^2 - x_1^2} \Rightarrow v_1^2 = \omega^2(A^2 - x_1^2)$ ...(1)

$v_2 = \omega\sqrt{A^2 - x_2^2} \Rightarrow v_2^2 = \omega^2[A^2 - x_2^2]$

$v_1^2 - v_2^2 = \omega^2[A^2 - x_1^2 - A^2 + x_2^2]$

$$\omega^2 = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}$$

$$\omega = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}} \Rightarrow \frac{2\pi}{T} = \sqrt{\frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}}$$

$$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

4. **Case-A :** When stretched by same amount

$$\frac{W_P}{W_Q} = \frac{\frac{1}{2}K_P.x^2}{\frac{1}{2}K_Q.x^2} = \frac{K_P}{K_Q} > 1$$

$\therefore$ $W_P > W_Q$

Case-B : When stretched by same force

$$\frac{W_P}{W_Q} = \frac{F^2/2K_P}{F^2/2K_Q} = \frac{K_Q}{K_P} < 1$$

$\therefore$ $W_P < W_Q$

5. $\alpha = \omega^2 A$

$\beta = \omega A$

$$\therefore \frac{\alpha}{\beta} = \omega \Rightarrow \frac{\alpha}{\beta} = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi\beta}{\alpha}$$

6. $T = 2\pi\sqrt{\frac{m}{k}}$

$3 = 2\pi\sqrt{\frac{m}{k}}$ ...(1)

$5 = 2\pi\sqrt{\frac{m+1}{k}}$ ...(2)

$$\frac{(1)^2}{(2)^2} \Rightarrow \frac{9}{25} = \frac{m}{m+1} \Rightarrow m = \frac{9}{16}$$

7. Length of the spring segments $= \frac{\ell}{6}, \frac{\ell}{3}, \frac{\ell}{2}$

As we know $K \propto \frac{1}{\ell}$

so spring constants for spring segments will be

$K_1 = 6K,\ K_2 = 3K,\ K_3 = 2K$

so in parallel combination

$K'' = K_1 + K_2 + K_3 = 11K$

in series combination

$K' = K$ (As it will become original spring)

so $K' : K'' = 1 : 11$

8. Amplitude A = 3 cm

When particle is at x = 2 cm,

its |velocity| = |acceleration|

i.e., $\omega\sqrt{A^2 - x^2} = \omega^2 x \Rightarrow \omega = \frac{\sqrt{A^2 - x^2}}{x}$

$$T = \frac{2\pi}{\omega} = 2\pi\left(\frac{2}{\sqrt{5}}\right) = \frac{4\pi}{\sqrt{5}}$$

9. $|a| = \omega^2 x$

$\Rightarrow 20 = \omega^2\, 5$

$\Rightarrow \omega^2 = 4$

$\Rightarrow \omega = 2 \Rightarrow \frac{2\pi}{T} = 2$

$\Rightarrow T = \pi$ s

10. $y = A_0 + A\sin\omega t + B\cos\omega t$

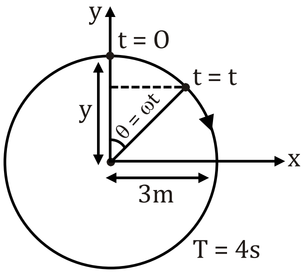
$y = A_0 + \sqrt{A^2 + B^2}\sin(\omega t + \phi)$

A_0 is mean position, and $\sqrt{A^2+B^2}$ is amplitude

11. Displacement = zero in one complete oscillation

$\Rightarrow$ Average velocity $= \frac{\text{Displacement}}{T} = 0$

12.



$\omega = \frac{2\pi}{4} = \frac{\pi}{2}$

For y-projection,

y = A cos ωt

$\Rightarrow\ y = 3\cos\left(\frac{\pi t}{2}\right)$

13. Distance in one oscillation = 4A

14. Time of flight = $\sqrt{\frac{2h}{g}} \propto \frac{1}{\sqrt{g}}$

Time period of pendulum = $2\pi\sqrt{\frac{l}{g}} \propto \frac{1}{\sqrt{g}}$

Ratio of time of flight & time period of pendulum is independent of g. Hence t' = 2T'

15. Displacement (x) equation of SHM

$x = A\sin(\omega t + \phi)$ ...(1)

$\frac{dx}{dt} = A\omega\cos(\omega t + \phi)$

acceleration (a) = $\frac{d^2x}{dt^2}$

$a = -\omega^2 A\sin(\omega t + \phi)$

$a = \omega^2 A\sin(\omega t + \phi + \pi)$ ...(2)

from (1) & (2), phase difference between displacement and acceleration is π.

16. Option (3) is a combination of SHM of same ω and same axis so its resultant is also a SHM which is periodic.

17. Displacement equation of SHM of frequency 'n'

x = Asin(ωt) = Asin(2πnt)

Now,

Potential energy

$U = \frac{1}{2}kx^2 = \frac{1}{2}KA^2\sin^2(2\pi nt)$

$U = \frac{1}{2}kA^2\left[\frac{1-\cos(2\pi(2n)t)}{2}\right]$

So frequency of potential energy = 2n

18. F = kx

$10 = k(5 \times 10^{-2})$

$k = \frac{10}{5\times10^{-2}} = 2 \times 10^2 = 200$ N/m

Now $T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{2}{200}} = \frac{2\pi}{10}$

23. x = A sin (ωt)

$\frac{dx}{dt} = v = A\omega\cos(\omega t)$

$\frac{dv}{dt} = a = -\omega^2 A\sin(\omega t)$

$a = -\left(\frac{2\pi}{8}\right)^2 \times 1\sin\left(\frac{2\pi}{8}\times 2\right)$

$\Rightarrow a = -\frac{\pi^2}{16}\times\sin\left(\frac{\pi}{2}\right)$

$\therefore \boxed{a = \frac{-\pi^2}{16}\,m/s^2}$

24. $x = 5\sin\left(\pi t + \frac{\pi}{3}\right)$

comparing with x = A sin (ωt + ϕ)

we get A = 5 m and ω = π

$\Rightarrow\ T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\,\text{sec.}$

25. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$T' = 2\pi\sqrt{\frac{\ell'}{g}} = 2\pi\sqrt{\frac{\ell/2}{g}}$

$T' = \frac{T}{\sqrt{2}} = \frac{x}{2}T$

$\Rightarrow x = \sqrt{2}$

26. PE = KE

$\frac{1}{2}kx^2 = \frac{1}{2}k(A^2 - x^2) \quad \Rightarrow x = \pm\frac{A}{\sqrt{2}}$

Exercise-III (Analytical Questions)

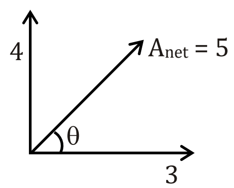
1. x = 3 sin 4πt + 4cos 4πt

$\tan\theta = \frac{4}{3}$

θ = 53°

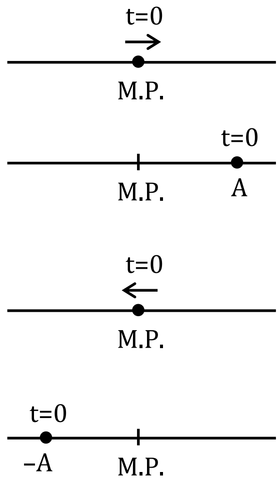
x = 5sin (4πt + 53°)

(SHM)



2. Theory

3.



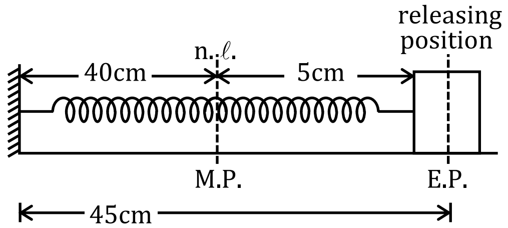
$\phi = 0°$
$x = A\sin \omega t$

$\phi = 90°$
$x = A\sin(\omega t + 90°)$
$x = A\cos \omega t$

$\phi = \pi$
$x = A\sin(\omega t + \pi)$
$x = -A\sin \omega t$

$\phi = 3\pi/2$
$x = A\sin(\omega t + 3\pi/2)$
$x = -A\cos \omega t$

4.



elongation in spring = 5 cm
amplitude = 5 cm

$v_{max} = A\omega = 5\sqrt{\frac{500}{1}} \quad \therefore\ \omega = \sqrt{\frac{k}{m}}$

$50\sqrt{5}$ cm/s

5. (a) Lift is at rest $a = 0$
(b) lift is moving downward with constant speed
$\Rightarrow$ constant velocity $\Rightarrow$ $a = 0$
in both cases $a = 0$ so $g_{eff} = g$

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

(c) In this case lift is moving with increasing speed so acceleration is downward.
$g_{eff} = g - a.$

6. Energy $E = \frac{1}{2}KA^2 = \frac{1}{2}m\omega^2A^2$

$v_{max} = A\omega = A\sqrt{\frac{K}{m}}$

$T = 2\pi\sqrt{\frac{m}{K}}$

$(P.E.)_{max} = \frac{1}{2}KA^2$

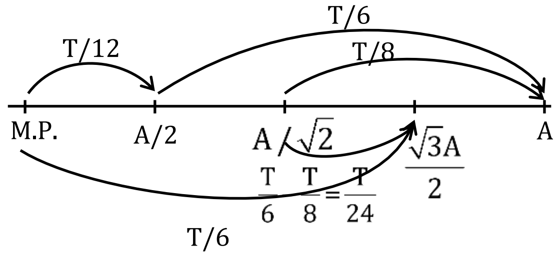
(i) Mass is doubled
$\Rightarrow$ Time period become $\sqrt{2}$ times.

(ii) Amplitude $\Rightarrow$ double
Energy $\Rightarrow$ 4 times
$v_{max} \Rightarrow$ 2 times
$(P.E.)_{max} \Rightarrow$ 4 times

(iii) $K \Rightarrow$ double
Energy = double
$v_{max} = \sqrt{2}$ times
$T \Rightarrow \frac{1}{\sqrt{2}}$ times
$(P.E.)_{max}$ = 2 times

(iv) Angular frequency $\left(\omega = \sqrt{\frac{k}{m}}\right)$ is halfed means k becomes 1/4 times so maximum speed is halfed.

7.

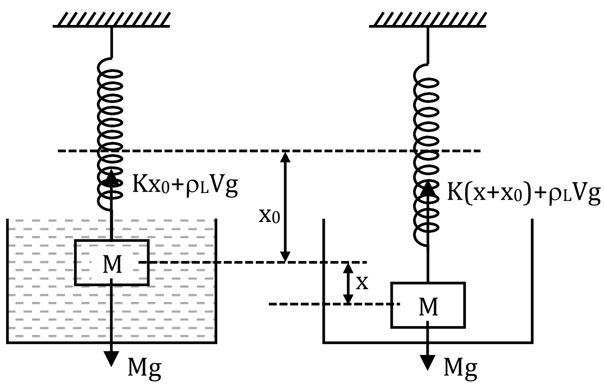


11. $a = -\omega^2 x$

13. $A = A_0 e^{-\frac{bt}{2m}}$

$$A = \frac{F_0}{m\sqrt{\left(\omega_d^2 - \omega^2\right)^2 + \left(\frac{b\omega_d}{m}\right)^2}}$$

14.



(1) $\rho_L Vg + Kx_0 = Mg$

(2) $F_R = -[K(x + x_0) + \rho_L Vg - Mg]$
$= -[Kx + Kx_0 + \rho_L Vg - Mg]$
$= -Kx$

(3) $a = \frac{F_R}{M} = -\frac{Kx}{M} \Rightarrow \omega = \sqrt{\frac{K}{M}} \Rightarrow T = 2\pi\sqrt{\frac{M}{K}}$

15. It is a periodic oscillatory and since released from small height it is SHM

16. $x = 2 + 3\,\frac{1-\cos 2\omega t}{2} = 2 + \frac{3}{2} - \frac{3}{2}\cos 2\omega t$

$= \frac{7}{2} - \frac{3}{2}\cos 2\omega t \Rightarrow \left(x - \frac{7}{2}\right) = \frac{3}{2}\cos 2\omega t$

17. $\frac{xy}{z} = \frac{A\omega.f}{A\omega^2} = \frac{f}{\omega} = \frac{f}{2\pi f} = \frac{1}{2\pi}$

π constant and dimension less.

18. At the mean position if $U_0 < 0$, then $K_{max} > E_{total}$

19. $T = 2\pi\sqrt{\frac{m}{k}}$

22. Here $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow \frac{\ell}{g} = \frac{1}{\pi^2}$ But $g = \pi^2$ therefore $\ell = 1$ m

23. It is due to resonance i.e. bridge may collapse.