Electromagnetic Induction

SOLUTIONS

Exercise-I (Conceptual Questions)

2. $\phi = BA \cos 0° = BA = 0.01 \times 10^3 = 10$ Wb

4. $e = A\dfrac{\Delta B}{\Delta t} = 2\left(\dfrac{4-1}{2}\right) = 3$ volt

7. $q = \dfrac{\Delta\phi}{R} = \dfrac{5.5\times10^{-4} - 5\times10^{-4}}{10} = 5\times10^{-6}C$

8. $q = \dfrac{\Delta\phi}{R} \Rightarrow q = \dfrac{NBA}{R}$

$B = \dfrac{qR}{NA} = \dfrac{32\times10^{-6}\times200}{100\times\pi(6\times10^{-3})^2} = 0.566T$

9. $\varepsilon = \dfrac{-\Delta\phi}{\Delta t} = -B\dfrac{(A_2 - A_1)}{\Delta t}$

$(2\pi r = 4\ell \Rightarrow r = \dfrac{2\ell}{\pi})$

$A_1 = A_{square} = (22 \times 10^{-2})^2$

$A_2 = A_{circular} = 3.14 \times \left(\dfrac{2\times 22}{3\cdot14}\times10^{-2}\right)^2$

$\varepsilon = -6\cdot6$ mV

10. $\varepsilon = \dfrac{\Delta\phi}{\Delta t}$ (1Wb = 10^8 Maxwell)

$= \dfrac{12\times10^{+3}\times10^{-8}}{0\cdot2} = 0\cdot6$ mV

11. $q = \dfrac{\Delta\phi}{R} = \dfrac{2NBA}{R}$

12. $\varepsilon = \dfrac{2NBA}{t}$

$= \dfrac{1000\times(0.4\times10^{-4})\times(500\times10^{-4})\times2}{0.1}$

$= 0.04V$

15. Speed of beam is constant so current is also constant. Due to this no current induced.

($\because$ v = constant $\Rightarrow$ I = constant ϕ = constant

$\Rightarrow \dfrac{d\phi}{dt} = 0 \Rightarrow \varepsilon = 0$)

16. Current in wire is constant so flux is also constant I = const $\Rightarrow$ B = const $\Rightarrow$ ϕ = constant

$\Rightarrow$ NO EMI

20. $q = \dfrac{\Delta\phi}{R}$ $\quad\therefore\ q \propto (\Delta t)^\circ$

21. $\phi_1 = NBA \qquad \phi_2 = -NBA$

$\Delta\phi = -2NBA$

$e = -\dfrac{d\phi}{dt} = -\left(\dfrac{-2NBA}{t}\right) = \dfrac{2NBA}{t}$

(Only magnitude)

$= \dfrac{2\times200\times0.8\times70\times10^{-4}}{0.1} = 22.4$ volt

25. Magnetic energy

$U_B = \dfrac{1}{2}LI^2 \propto I^2 \Rightarrow I \propto \sqrt{U_B}$

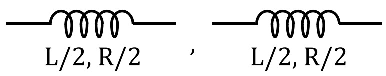
$\dfrac{I_2}{I_1} = \sqrt{\dfrac{U_2}{U_1}} = \sqrt{\dfrac{1}{4}} = \dfrac{1}{2} \Rightarrow I_2 = \dfrac{I_1}{2}$

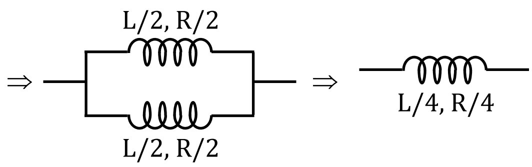
t is equal to Half life period therefore :

$t = 0{\cdot}693\dfrac{L}{R} = 0{\cdot}693 \times \dfrac{10}{2} = 3{\cdot}5$ sec.

26. $\lambda = \dfrac{L}{R} = 4$ sec. $\qquad L = \mu_0 n^2 A\ell \Rightarrow L \propto \ell$

$R = \dfrac{\rho\ell}{A} \Rightarrow R \propto \ell$





$L_P = \dfrac{L_1L_2}{L_1+L_2} = \dfrac{L}{4},\ R_P = \dfrac{R_1R_2}{R_1+R_2} = \dfrac{R}{4}$

$\lambda' = \dfrac{L/4}{R/4} = \dfrac{L}{R} = 4\sec$

28. $N\phi = LI$

$L = \dfrac{N\phi}{I} = \dfrac{100\times5\times10^{-5}}{2} = 2.5 \times 10^{-3}$ H

$U = \dfrac{1}{2}LI^2 = \dfrac{1}{2}(2.5\times10^{-3})(2)^2 = 5\times10^{-3}$ J

OR

$U = (\phi_{total})I/2$

29. $L = \frac{\mu_0 N^2 \pi r^2}{\ell} = \mu_0 n^2 \ell \pi r^2$

$L' = \mu_0 n^2 \frac{\ell}{2} \pi (2r)^2 = 2L \Rightarrow$ two times

$\Rightarrow$ inductance increases by 100%.

31. $I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$

$\frac{9I_0}{10} = I_0\left(1 - e^{-\frac{5}{5}t}\right) \Rightarrow e^{-t} = \frac{1}{10}$

After taking log both the sides

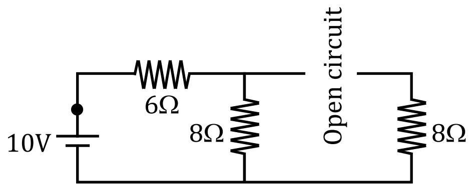
$t = \log_e 10 = 2.3$ sec.

33. $\varepsilon = L\frac{\Delta I}{\Delta t} = 60 \times 10^{-6} \times \frac{(1.5)-(1)}{0.1}$

$= 300 \times 10^{-6}$ V

$i = \frac{\varepsilon}{R} = \frac{300\times10^{-6}}{600\times10^{-6}} = 0.5$ Amp.

34. At t = 0



$I = \frac{V}{R_{net}} = \frac{10}{6+8} = \frac{10}{14} = \frac{5}{7}A$

35. In parallel combination

$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} = \frac{3}{3} = 1H$

36. $\lambda = \frac{L}{R} = 2 \times 10^{-3}$ sec

$\frac{L}{R+90} = 0.5 \times 10^{-3}$ sec.

$(2 \times 10^{-3})R = (0.5 \times 10^{-3})(R + 90)$

$R = 30\Omega$

$L = 2 \times 10^{-3} \times 30 = 60$ mH

37. Magnetic energy

$U_B = \frac{1}{2}LI^2 = \frac{1}{2}(LI)I \quad [\because \phi = LI] = \frac{1}{2}\phi I$

38. Self inductance of toroid is given by

$L = \frac{\mu_0 N^2 A}{2\pi R_m} = \frac{\mu_0 N^2 \pi r^2}{2\pi R_m} = \frac{\mu_0 N^2 r^2}{2R_m}$

41. $e = -\frac{L\Delta I}{\Delta t} \Rightarrow L = -\frac{e}{(\Delta I/\Delta t)}$

$L = -\frac{8}{(2/0.05)} = -0.2H$

= 0.2 H (only positive value)

42. $e = \frac{LdI}{dt} \Rightarrow L = e\frac{dt}{dI}$

$L = \frac{12}{48} \times \frac{60}{1} = 15H$

44. $U = \frac{1}{2}LI^2$, where $I_0 = \frac{E}{R}$

46. Self inductance of solenoid

$L = \frac{\mu_0 N^2 A}{\ell} \Rightarrow L \propto N^2$

47. $\lambda = \frac{L}{R} = \frac{40}{8} = 5$ second

48. $e = \frac{LdI}{dt} \Rightarrow L = \frac{e}{(dI/dt)} = \frac{5}{(1/10^{-3})}$

$= 5 \times 10^{-3} = 5$mH

49. Current becomes half of its peak value in half life time

$T = \frac{L}{R} \times 0{\cdot}693 = \frac{300 \times 10^{-3}}{2} \times 0.693 = 0.1$ sec.

50. $i = i_0(1 - e^{-t/\lambda})$

$i_0 = \frac{5V}{5\Omega} = 1$ amp.

$l = \frac{L}{R} = \frac{10}{5} = 2$ sec.

at t = 2 sec.

$i = 1(1 - e^{-2/2})$

$i = (1 - e^{-1})$

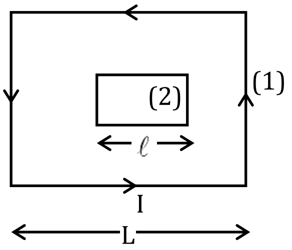
52. In both cases (a) & (b) M is constant

For (a) $\varepsilon_2 = M\frac{dI_1}{dt}$

$M = \frac{\varepsilon_2}{(dI_1/dt)} = \frac{2\times10^{-3}}{5} = 4 \times 10^{-4}$ H

For (b) $e_1 = M\frac{dI_2}{dt} = 4 \times 10^{-4} \times 2 = 8 \times 10^{-4}$ V

53. $M = \frac{N_2 B_1 A_2}{I_1}$



$= (1)\left(\frac{2\sqrt{2}\,\mu_0 I}{\pi L}\right)\frac{\ell^2}{I} = \frac{2\sqrt{2}\,\mu_0 \ell^2}{\pi L}$

$M \propto \frac{\ell^2}{L}$

61. $\phi = MI \Rightarrow \frac{d\phi}{dt} = M\frac{dI}{dt} \Rightarrow M = \frac{d\phi}{dI} = \frac{2\times10^{-2}}{0.01} = 2H$

64. $M = K\sqrt{L_1 L_2}$ here $(K = 1)$

so $M = \sqrt{L_1 L_2}$

66. $\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{1}{20} \Rightarrow V_s = \frac{1}{20}\times V_P$

$V_s = \frac{2500}{20}$ volt

67. Transfarmer cannot works on dc supply.

68. $e = -\frac{d\phi}{dt} = -\frac{d}{dt}(10t^2 - 50t + 250) = -(20t - 50)$

Put t = 3 sec

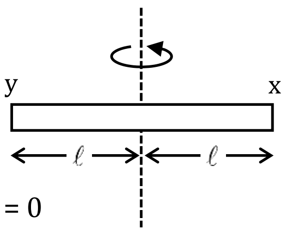
$e = -(20 \times 3 - 50) = -10V$

69. $M = \frac{\mu_0 N_1 N_2 A}{\ell}$

70. $e_{ox} = \frac{1}{2}B\omega\ell^2$

$e_{oy} = \frac{1}{2}B\omega\ell^2$

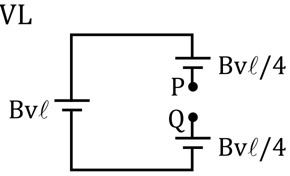
$e_{xy} = \frac{1}{2}B\omega\ell^2 - \frac{1}{2}B\omega\ell^2 = 0$



71. $e = \frac{1}{2}B\omega\ell^2 = \frac{1}{2}B\,(2\pi f)\ell^2$

$= \frac{1}{2}(6.28 \times 10^{-3})\,(2 \times 3.14 \times 50)(1)^2 = 1\text{ V}$

73. Using KVL



$V_P + \frac{Bv\ell}{4} - Bv\ell + \frac{Bv\ell}{4} = V_Q$

$V_P - V_Q = \frac{Bv\ell}{2}$

$V_P - V_Q$ is positive so $V_P > V_Q$

75. Effective length of the loop = 2R

perpendicular component of velocity to effective length vsinθ

$e = B|(\vec{\ell}\times\vec{v})|$

$e = B2R\,v\sin\theta$

76. For rotating conducting rod emf across its end is

$\varepsilon = \frac{1}{2}B\omega\ell^2$

So $\varepsilon_{OP} = \frac{1}{2}B\omega(4\ell)^2 = 8B\omega\ell^2$

$\therefore$ $\varepsilon_{OQ} = \frac{1}{2}B\omega(2\ell)^2 = 2B\omega\ell^2$

therefore $e_{PQ} = e_{OP} - e_{OQ}$

$\varepsilon_{PQ} = 6B\omega\ell^2$

77. An emf will induce in that close loop in such a way that it opposes the motion of rod.

79. $\Delta\phi = 0$ (due to motion inside a uniform magnetic field)

$\therefore \varepsilon = 0$

87. $\varepsilon_0 = NBA\omega$

91. Induced electric field in this loop

$E(2\pi R) = \pi R^2 \frac{dB}{dt}$ (r = R, on the surface)

$E = \frac{R}{2}\frac{dB}{dt} = 2 \times 10^{-2} \times 0.2 = 4 \times 10^{-3}$ v/m

Force applied by this electric field

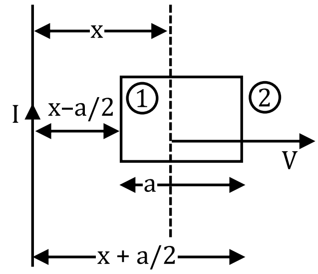
$F = qE = 2 \times 4 \times 10^{-3} = 8 \times 10^{-3}$ N

Exercise-II (Previous Year Questions)

1. $\eta = \frac{V_S I_S}{V_P I_P} \Rightarrow 0.9 = \frac{V_S(6)}{3\times10^3} \Rightarrow V_S = 450V$

As $V_P I_P = 3000$ so $I_P = \frac{3000}{200}A = 15A$

2.



emf Induced in side (1)

$\varepsilon_1 = B_1 V\ell$

emf Induced is side (2)

$\varepsilon_2 = B_2 V\ell$

emf in the frame

$= B_1 V\ell - B_2 V\ell$

$\varepsilon = V\ell\,[B_1 - B_2]$

$\Rightarrow \quad \varepsilon \propto B_1 - B_2 \qquad \text{Since } B \propto \dfrac{1}{r}$

$$\text{So} \quad \varepsilon \propto \left[\frac{1}{x-\frac{a}{2}} - \frac{1}{x+\frac{a}{2}}\right]$$

$$\Rightarrow \quad \varepsilon \propto \left[\frac{1}{(2x-a)} - \frac{1}{(2x+a)}\right]$$

3. First current develops in direction of abcd but when electron moves away, then magnetic field inside loop decreases & current will change its direction.

4. Flux linked with each turn = 4×10^{-3} Wb

$\therefore$ Total flux linked

$= 1000[4\times10^{-3}]$ Wb = 4 Wb

$\phi_{total} = L\,i = 4 \Rightarrow L = 1H$

5. For Loop 1

$$\varepsilon_{ind} = -\frac{d\phi}{dt} = -A\left(\frac{dB}{dt}\right)\cos 0^\circ = -\pi r^2\left(\frac{dB}{dt}\right)$$

For Loop 2, $\varepsilon_{ind} = 0$ as no flux linkage

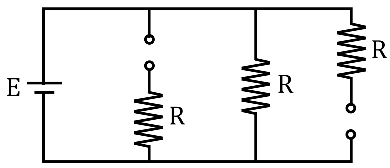
6. $$q = \left[\left(\frac{\Delta\phi}{\Delta t}\right)\cdot\frac{1}{R}\right]\Delta t$$

$$q = \left[\mu_0 n N \pi r^2 \frac{\Delta i}{\Delta t}\right]\frac{1}{R}\Delta t$$

$$q = \left[\begin{matrix} 4\pi\times10^{-7}\times2\times10^{4}\times \\ 100\times\pi\times(10^{-2})^2\times\left(\dfrac{4}{.05}\right)\end{matrix}\right]\frac{1}{10\pi^2}\times0.05$$

$q = 32\ \mu C$

7. Just after switch closed then



Here E = 18V, R = 9Ω

$I = E/R = \dfrac{18}{9} = 2A$

8. Magnetic potential energy $= \dfrac{1}{2}LI^2$

$\Rightarrow 25\times10^{-3} = \dfrac{1}{2}L(60\times10^{-3})^2$

$\Rightarrow L = 13.89$ H

9. Given

N = 800, A = 0.05 m², B = 5×10^{-5} T

$\Delta t = 0.15$ s

$$\text{As} \quad e = -\frac{(\phi_f - \phi_i)}{\Delta t} = -\frac{(0 - NBA)}{\Delta t}$$

$$= \frac{800\times5\times10^{-5}\times5\times10^{-2}}{0.1} = 0.02\ V$$

10. Eddy current effect is not used in electric heater

11. $$E = \frac{B\omega\ell^2}{2} = \frac{0.1(10)(0.5)^2}{2} = 0.125\ V$$

12. $\phi = 5t^2 + 3t + 60$

$$|\varepsilon| = \left|\frac{d\phi}{dt}\right| = 10t + 3$$

At t = 4 sec.

$|\varepsilon| = 40 + 3 = 43$ volt

13. $$\varepsilon = \frac{1}{2}B\omega r^2$$

$$\varepsilon = \frac{1}{2}\times(0.4\times10^{-4})\times\left(2\pi\left[\frac{120}{60}\right]\right)(1)^2$$

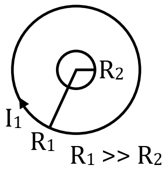
$\varepsilon = 0.8\pi\times10^{-4} = 2.512\times10^{-4}$ V

14. $220\times i_p = 44$

$$\Rightarrow i_p = \frac{44}{220} = \frac{1}{5} = 0.2A$$

15. $M = \dfrac{\phi_{12}}{I_1} = \dfrac{B_1 A_2}{I_1} = \dfrac{\left(\dfrac{\mu_0 I_1}{2R_1}\right)(\pi R_2^2)}{I_1}$

$M = \dfrac{\mu_0 \pi R_2^2}{2R_1}$



$M \propto \dfrac{R_2^2}{R_1}$

18. $V_S I_S = V_P I_P$ (ideal Transformer)

$\Rightarrow P_{out} = P_{in}$

$\Rightarrow 60 = 220 \times I_P$

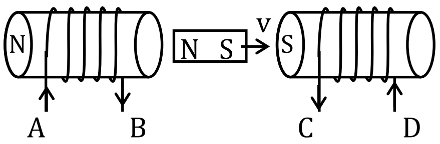
$I_P = \dfrac{60}{220} = 0.27A$

19. Energy $= \dfrac{1}{2}Li^2$

$= \dfrac{1}{2}4 \times 10^{-6} \times 2^2$

$= 8 \times 10^{-6}$ J

energy = 8 μJ

20. 

A to B & D to C

21. For ideal transformer

$\dfrac{V_S}{V_P} = \dfrac{N_S}{N_P} = 2:1$

22. $L = \mu_0 n^2 A\ell$

$\dfrac{L_A}{L_B} = \dfrac{\mu_r \mu_0 (n/2)^2 A(2\ell)}{\mu_r \mu_0 n^2 A\ell}$

$\dfrac{L_A}{L_B} = \dfrac{1}{4} \times 2 = \dfrac{1}{2}$

Exercise-III (Analytical Questions)

1. $e = (vB\,\ell_{eff})$

$\ell_{eff} = L/2$

$\Rightarrow e = \dfrac{1}{2}$ BLv [P is at higher potential]

2. (a) $\phi = LI$ = Henry – ampere

(b) Energy $= \dfrac{1}{2}LI^2$

$L \rightarrow \left(\dfrac{J}{A^2}\right)$

(c) emf $= -L\dfrac{dI}{dt}$

$L \rightarrow \dfrac{\text{volt} - \text{second}}{\text{ampere}}$

(d) S.I, unit of magnetic induction is Tesla.

3. $\left.\begin{array}{l} q = 2t^2 \\ I = \dfrac{dq}{dt} = 4t \\ \dfrac{dI}{dt} = 4 \end{array}\right\}$ $\begin{array}{l} \text{At } t = 1\text{sec} \\ q = 2C \\ I = 4A \\ \dfrac{dI}{dt} = 4A/s \end{array}$

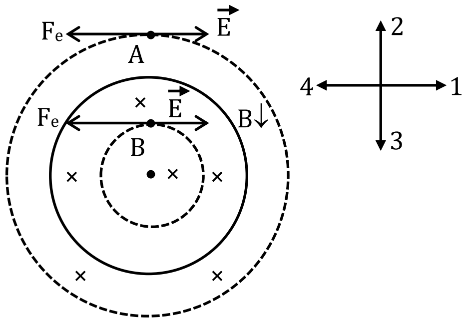
(i) $V_a - L.\dfrac{dI}{dt} = V_b \Rightarrow V_a - V_b = L.\dfrac{dI}{dt} = 4V$

(ii) $V_b - \dfrac{q}{C} = V_c \Rightarrow V_b - V_c = \dfrac{q}{C} = 1V$

(iii) $V_c - IR = V_d \Rightarrow V_c - V_d = IR = 16V$

(iv) $V_a - L.\dfrac{dI}{dt} - \dfrac{q}{c} - 4I = V_d$

$\Rightarrow V_a - V_d = 21$ V

4. 

5. $B = \dfrac{F}{i\ell} = \dfrac{MLT^{-2}}{AL}$

$L = \dfrac{\text{Energy}}{i^2} = [ML^2T^{-2}A^{-2}]$

$T = 2\pi\sqrt{LC} \Rightarrow LC = [T^2]$

$\phi = LI = [M^1L^2T^{-2}A^{-1}]$

6. At t = 0 $\quad V_L = V_0 = 10V$

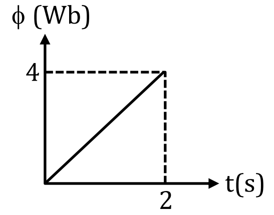
$V_R = V_0 = 0V$

At t = 1 $\quad V_L = V_0\, e^{-t/\tau}$ | $V_R = V_0\,[1 - e^{-t/\tau}]$

$V_L = 10\, e^{-1}$ | $V_R = 10\left[1-\frac{1}{e}\right]$

$V_L = \frac{10}{e}$

7.



$R = 2\Omega$

$\Delta\phi = 4$

$e_{in} = \frac{\Delta\phi}{\Delta t} = \frac{4}{2} = 2\,\text{volt}$

$i_{in} = \frac{\varepsilon_{in}}{R} = \frac{2}{2} = 1\,A$

$\Delta q = \frac{\Delta\phi}{R} = \frac{4}{2} = 2C$

$H = I^2Rt$

$H = (1)^2 \times 2 \times 2 = 4\,J$

8. D.C. voltage is steady with time.

9. According to lezs law induced current always opposes the change in flux.

10. Inductor tends to keep flux constant.

11. $M_{1,2} = M_{2,1}$

Independent of current.

12. $\oint \vec{E}\cdot\vec{d\ell} = -\,A\frac{dB}{dt}$

13. $\phi = \vec{B}.\vec{A}$

14. When flux is zero, at that instant rate of flux change is maximum.