01 Magnetic Effect of Current and Magnetism

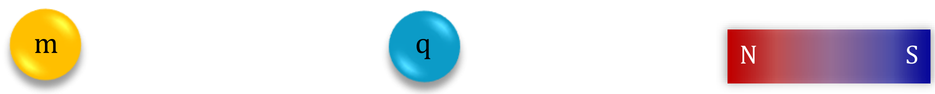
Moving Charges and Magnetism

The branch of physics which deals with the magnetism due to electric current or moving charge (i.e. electric current is equivalent to the charges or electrons in motion) is called electromagnetism.

Concept of Field and Oersted Experiment

Field

Region around any physical quantity where another similar physical quantity experiences force or torque is called field.



A mass has Gravitational field around it | A charge has Electric field around it | A Magnet has Magnetic field around it

The nature of Force

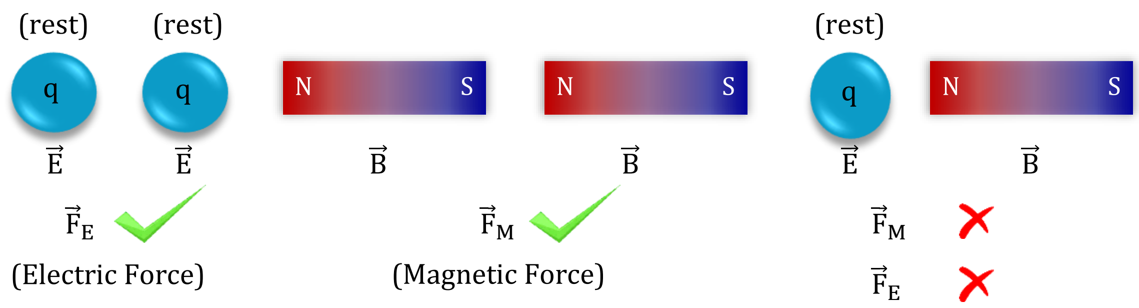


Illustration 1

A charge particle at rest placed near a current carrying wire then what is the nature of force on the charge?

Solution:

A current carrying wire produce magnetic field but charge at rest produces electric field. So no force on charge at rest.

Magnetic Field

Region around a magnet or magnetic substance where another magnetic substance experiences force or torque is called Magnetic field.

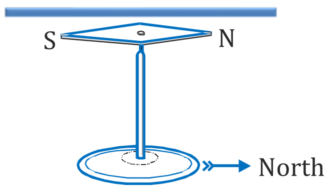
Unit : SI : weber/(meter)^2 or tesla (T)

C.G.S. : gauss (1 Tesla = 10^4 gauss)

Dimensions : $[MT^{-2}A^{-1}]$

Oersted's Discovery

The relation between electricity and magnetism was discovered by Oersted in 1820. Oersted showed that the electric current through the conducting wire deflects the magnetic needle held near the wire.



- When the direction of current in conductor is reversed then deflection of magnetic needle is also reversed.
- On increasing the current in conductor or bringing the needle closer to the conductor, the deflection of magnetic needle increases.

Biot Savart Law

With the help of experimental results, Biot and Savart arrived at a mathematical expression that gives the magnetic field at some point in space in terms of the current that produces the field.

Current element: A very small element of length dl of a thin current carrying conductor (wire). It is represented by $i(d\vec{\ell})$. It is a vector quantity and its direction is same as that of current.

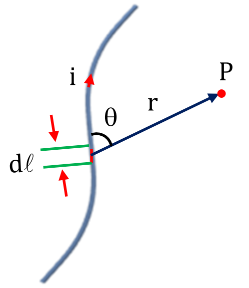
Experimentally, it was found, magnetic field dB at point P due to current element varies as:

$dB \propto i$

$dB \propto d\ell$

$dB \propto \sin\theta$

$dB \propto \dfrac{1}{r^2}$



So, $\quad dB \propto \dfrac{i d\ell \sin\theta}{r^2}$

$$dB = \frac{\mu_0}{4\pi}\frac{i d\ell \sin\theta}{r^2} \qquad \left(\text{here } \frac{\mu_0}{4\pi} = 10^{-7}\frac{T-m}{A}\right)$$

μ_0 : Permeability of free space (or vacuum)

Permeability

Also known as magnetic permeability. It indicates to what extent a material/medium can be magnetized when placed inside an externally applied magnetizing field (to be discussed in Magnetic Materials)

μ_0 : Permeability of free space

μ : Permeability of a medium

μ_r : Relative Permeability of a medium

$$\mu_r = \frac{\mu}{\mu_0} \qquad \text{where } \mu_0 = 4\pi\times10^{-7}\frac{Tm}{A}$$

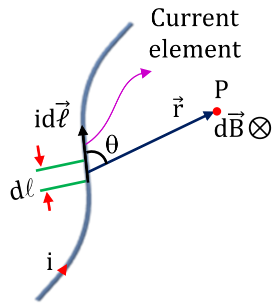
Vector Form

Experimentally, it is found that direction of magnetic field due to a current element is perpendicular to plane containing $d\vec{\ell}$ and $\vec{r}$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i d\ell \sin\theta}{r^2}\hat{n} \qquad (\hat{n}: \text{Unit vector normal to plane containing } d\vec{\ell} \text{ and } \vec{r})$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{(i d\ell)(r)\sin\theta}{r^3}\hat{n}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\left(d\vec{\ell}\times\vec{r}\right)}{r^3}$$



$\otimes$ Inside the plane of paper $\qquad$ $\odot$ Outside the plane of paper

Illustration 2

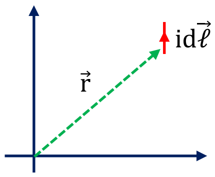
Calculate magnetic field at P

Solution:

Here $Id\vec{\ell}\,||\,\vec{r}$. So, $\sin\theta = 0$. Hence, magnetic field at P is zero.

Illustration 3

A small current element $\left(id\vec{\ell}\right)$ is placed at a point P and its position vector is $\vec{r}$.



Then find magnetic field at origin.

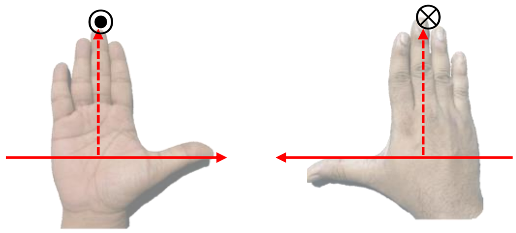
Solution:

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{\left(id\vec{l}\times(-\vec{r})\right)}{r^3}$$

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{\left(\vec{r}\times id\vec{l}\right)}{r^3}$$

Direction of Magnetic Field

Right Hand Palm Rule

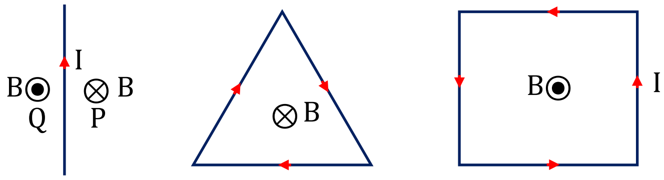


Thumb: along the direction of current

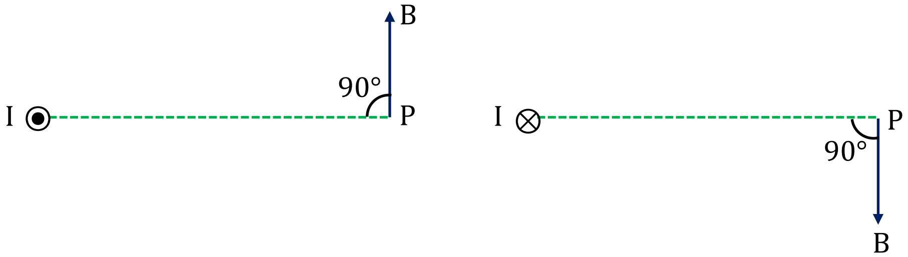
Fingers: along the direction of point where B is to be found

Palm: indicates direction of magnetic field

Case I :– Wire is in the plane of the paper :-

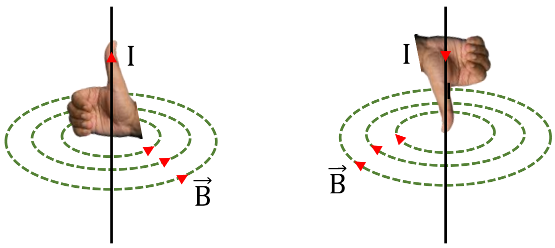


Case II :– Wire is perpendicular to the plane of the paper :-



Right Hand Thumb Rule

Point the thumb of Right Hand along the direction of current then curling of fingers represents direction of the magnetic field.



Note: For a straight current carrying wire M.F. lines are in form of concentric circles.

Remember!!

East, West, North and South are in Horizontal Plane

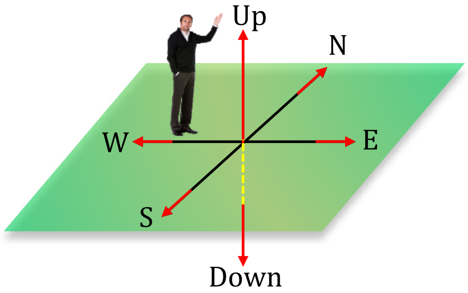


Illustration 4

A wire has current in south direction. Find direction of magnetic field due to it just below the wire.

Solution:

The direction of magnetic field is towards east.

Magnetic Field due to Moving Charge

MF at a point due to a charge can be found in a way similar to that of a current element.

MF due to current element $d\vec{B}=\frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3}$

$$\boxed{Id\vec{\ell}=\left(\frac{dq}{dt}\right)d\vec{\ell}=(dq)\frac{d\vec{\ell}}{dt}=(dq)\vec{v}}$$

MF due to moving charge $d\vec{B}=\frac{\mu_0}{4\pi}\frac{(dq)\vec{v}\times\vec{r}}{r^3}$

For moving charge $\vec{B}=\frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$ (in vector form) $B=\frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$ (in scalar form)

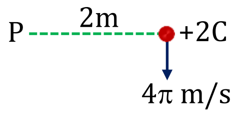
Direction:

For Positive charge flow :- same as current element (Right Hand Rule)

For Negative charge flow :- Opposite to current element.

Illustration 5

A charge +2C moves vertically downward with speed 4πm/s. Find magnetic field at point P.

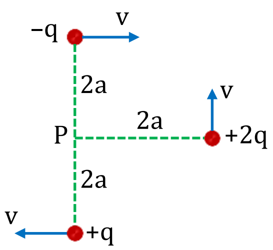


Solution:

$$B=\frac{\mu_0 qv}{4\pi r^2}\Rightarrow B=\frac{\mu_0}{4\pi}\frac{(2)(4\pi)}{4}\otimes \Rightarrow B=\frac{\mu_0}{2}\otimes \text{ Tesla}$$

Illustration 6

A charge +2q moves vertically upwards with sped v, a second charge −q moves horizontally to the right with the same speed v, and a third charge +q moves horizontally to the left with the same speed v. The point P is located at a perpendicular distance a away from each charge as shown in the figure. Find the magnetic field at point P.



Solution:

We know that magnetic field due to a moving charge

$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v}\times\vec{r})}{r^3}$ $B = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$

$\vec{B}_P = \vec{B}_q + \vec{B}_{2q} + \vec{B}_{-q}$

$= \frac{\mu_0}{4\pi}\frac{qv}{(2a)^2}\otimes + \frac{\mu_0}{4\pi}\frac{2qv}{(2a)^2}\odot + \frac{\mu_0}{4\pi}\frac{qv}{(2a)^2}\odot = \frac{\mu_0}{8\pi}\frac{qv}{a^2}\odot$

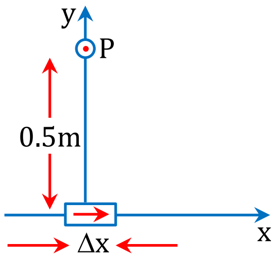
BEGINNER'S BOX-1

1. The direction of magnetic field $d\vec{B}$ due to a current element $i\vec{d\ell}$ at a point, distance r from it, when a current i passes through a long conductor is in the direction: -

(1) of position vector $\vec{r}$ of the point
(2) of current element $\vec{d\ell}$
(3) Perpendicular to both $\vec{d\ell}$ and $\vec{r}$
(4) Perpendicular to $\vec{d\ell}$ only

PME193

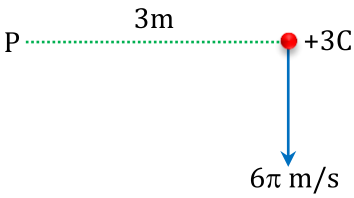
2. An element $\Delta\ell = \Delta x\,\hat{i}$ is placed at the origin and carries a large current I = 10 A. What is the magnetic field on the y - axis at a distance of 0.5 m. Δx = 1 cm :-



(1) 4×10^{-8} T in +z direction
(2) 4×10^{-8} T in –z direction
(3) 2×10^{-8}T in +z direction
(4) 2×10^{-8} T in –z direction

PME194

3. A charge +2C moves vertically downward with speed 6π m/s. Find magnetic field at point P.

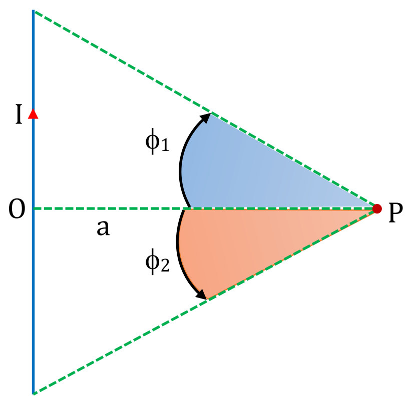


(1) $2\mu_0$T
(2) μ_0T
(3) $\frac{\mu_0}{3}$ T
(4) $\frac{\mu_0}{2}$ T

PME195

Application of Biot-Savart Law

Magnetic Field due to Thin Wire of Finite Length



AIM: To find magnetic field due to a current carrying wire of finite length at a point P.

Geometry: Point P is at a perpendicular/shortest distance 'a' from the wire.

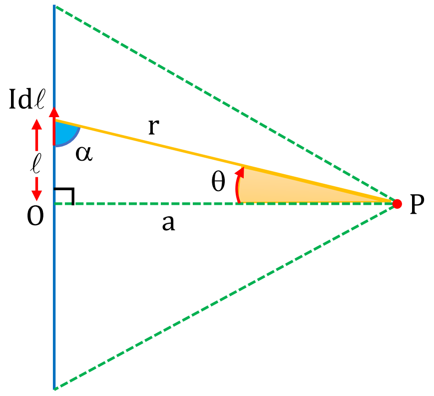
Ends of the wire make an angle ϕ_1 and ϕ_2 with line OP

Result: $B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$

$\phi_2 \to$ Start Point of current

$\phi_1 \to$ End Point of current

Let's take a current element $Id\ell$ at a distance ℓ from point O of the wire.



$$\tan\theta = \frac{\ell}{a} \Rightarrow \ell = a\tan\theta$$

differentiating on both sides,

$$\Rightarrow d\ell = a\sec^2\theta d\theta$$

If distance of current element from point P is r,

$$\cos\theta = \frac{a}{r} \Rightarrow r = \frac{a}{\cos\theta} \Rightarrow r = a\sec\theta$$

and, $\alpha = 90° - \theta$

M.F. at P due to current element: -

$$dB = \frac{\mu_0}{4\pi}\frac{(Id\ell)\sin(180° - \alpha)}{r^2} = \frac{\mu_0}{4\pi}\frac{(Id\ell)\sin\alpha}{r^2}$$

$$\sin\alpha = \sin(90° - \theta) = \cos\theta$$

$$d\ell = a\sec^2\theta d\theta$$

$$r = a\sec\theta$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I(a\sec^2\theta d\theta)\cos\theta}{a^2\sec^2\theta}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\cos\theta d\theta}{a}$$

M.F. at P due to current element: -

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{I\cos\theta d\theta}{a}$$

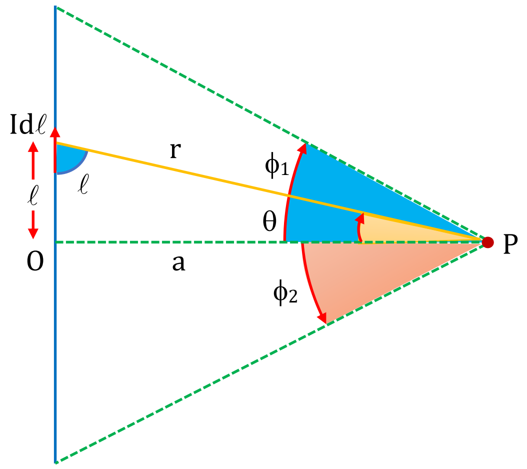
Due to all such current elements, M.F. will be inward to plane of paper,

So, net M.F. will be sum of all such elements

$$\Rightarrow B_{net} = \int\frac{\mu_0}{4\pi}\frac{I\cos\theta d\theta}{a}$$

here, θ varies from $-\phi_2$ to ϕ_1

$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}\int_{-\phi_2}^{\phi_1}\cos\theta d\theta$$



M.F. at P due to straight current carrying wire

$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}\int_{-\phi_2}^{\phi_1}\cos\theta d\theta$$

$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}(\sin\theta)\Big|_{-\phi_2}^{\phi_1}$$

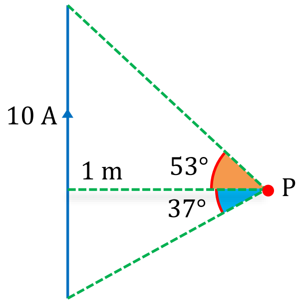
$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 - \sin(-\phi_2))$$

$$\Rightarrow B_{net} = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

Direction : By right hand palm rule, inwards

Illustration 7

Find magnetic field at point P.



Solution:

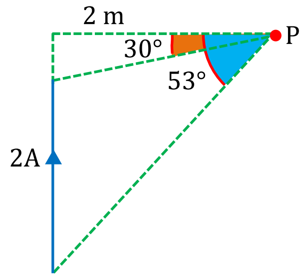
$$B=\frac{\mu_0 I}{4\pi a}(\sin\phi_1+\sin\phi_2)$$

$$=\frac{10^{-7}(10)}{(1)}\left[\frac{4}{5}+\frac{3}{5}\right]$$

$$=10^{-6}\left(\frac{7}{5}\right)=\frac{7}{5}\times10^{-6}\,T\otimes$$

Illustration 8

Find magnetic field at point P.



Solution:

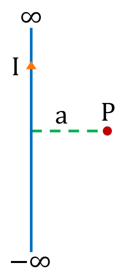
$\phi_2 = 53°$, $\phi_1 = -30°$

$$B=\frac{\mu_0 I}{4\pi a}(\sin\phi_1+\sin\phi_2)$$

$$=\frac{10^{-7}(20)}{(2)}\left[-\frac{1}{2}+\frac{4}{5}\right]$$

$= 10^{-6}(0.3) = 3 \times 10^{-7}\ T\otimes$

Magnetic Field due to Infinite Straight Wire



$$B=\frac{\mu_0 I}{4\pi a}(\sin\phi_1+\sin\phi_2)$$

$\phi_2 \rightarrow$ Start Point of current, $\phi_1 \rightarrow$ End Point of current

Here, $\phi_1 \rightarrow 90°$, $\phi_2 \rightarrow 90°$

$$B=\frac{\mu_0 I}{4\pi a}(\sin\phi_1+\sin\phi_2)\ \Rightarrow B=\frac{\mu_0 I}{4\pi a}(\sin 90°+\sin 90°)$$

$$\Rightarrow B=\frac{\mu_0 I}{4\pi a}(1+1)=\frac{\mu_0 I}{4\pi a}(2)\Rightarrow B_\infty=\frac{\mu_0 I}{2\pi a}$$

Illustration 9

A current of 20 A is established in a long wire along positive z-direction. The magnetic field B at the point (1m, 0, 0) is :-

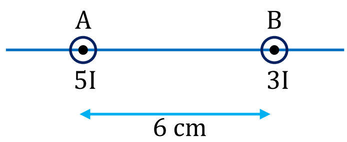
Solution:

$$B=\frac{\mu_0 I}{2\pi r}=\frac{\mu_0 20}{2\pi(1)}=2\times10^{-7}\times20=4\times10^{-6}\ T=4\mu T$$

direction of $\vec{B}\Rightarrow i\vec{d\ell}\times\vec{r}=\hat{k}\times\hat{i}=+\hat{j}$

Illustration 10

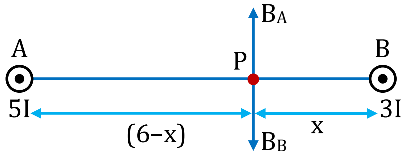
The position of point from wire 'B', where net magnetic field is zero due to following current distribution.



Solution:

Suppose magnetic field at the distance x from 'B' is zero so at that point magnetic field due to wires A and B will be equal in magnitude.

$B_A = B_B$



$$\frac{\mu_0(5I)}{2\pi(6-x)} = \frac{\mu_0(3I)}{2\pi x}$$

$$\frac{5}{(6-x)} = \frac{3}{x} \; ; \; x = \frac{9}{4} \text{ cm}$$

So, $\frac{9}{4}$ cm from B in left side magnetic field will be zero.

Illustration 11

Same current i is flowing in three infinitely long wires along positive x, y and z directions. The magnetic field at a point (0, 0, –a) would be

Solution:

Point (0, 0, –a) lies on z-axis, therefore magnetic field due to current along z-axis is zero and due to rest two wires is $\frac{\mu_0 I}{2\pi a}$ in mutually perpendicular directions along positive y direction and negative x direction.

$$\therefore \; \vec{B} = \frac{\mu_0}{2\pi}\frac{i}{a}(\hat{j} - \hat{i})$$

Magnetic Field due to Semi Infinite Straight Wire

Case-1

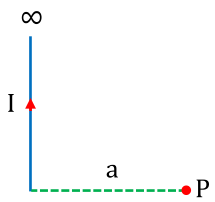
Here,

$\phi_1 \to 90^\circ$

$\phi_2 = 0^\circ$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}(\sin 90^\circ + \sin 0^\circ) \Rightarrow B = \frac{\mu_0 I}{4\pi a} \otimes$$



Case-2

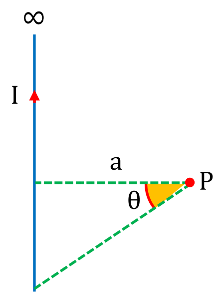
Here,

$\phi_1 \to 90^\circ$

$\phi_2 = \theta$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}(\sin 90^\circ + \sin\theta) \Rightarrow B = \frac{\mu_0 I}{4\pi a}(1 + \sin\theta) \otimes$$



Case-3

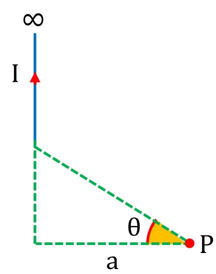
Here,

$\phi_1 \to 90^\circ$

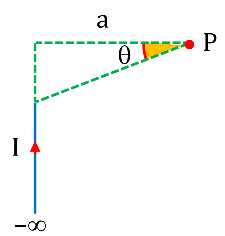
$\phi_2 = -\theta$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}(\sin 90^\circ + \sin(-\theta)) \Rightarrow B = \frac{\mu_0 I}{4\pi a}(1 - \sin\theta) \otimes$$



Case-4



here,

$\phi_1 = -\theta$

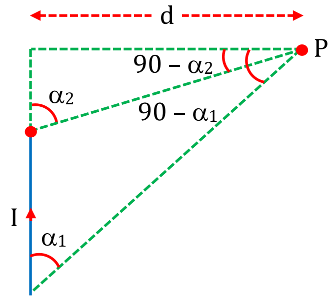
$\phi_2 \to 90°$

$$B = \frac{\mu_0 I}{4\pi a}(\sin\phi_1 + \sin\phi_2)$$

$$\Rightarrow \quad B = \frac{\mu_0 I}{4\pi a}(\sin(-\theta) + \sin 90°) \Rightarrow B = \frac{\mu_0 I}{4\pi a}(1 - \sin\theta) \otimes$$

Illustration 12

If point 'P' lies out-side the line of wire then magnetic field at point 'P' will be :



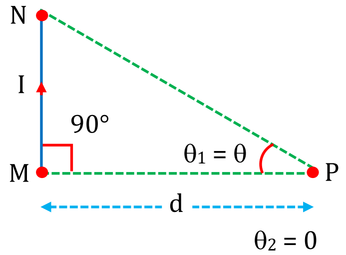
Solution:

Here $\phi_1 = (90° - \alpha_1)$ $\qquad \phi_2 = (90° - \alpha_2)$

$$B_P = \frac{\mu_0 I}{4\pi d}\left[\sin(90 - \alpha_1) - \sin(90 - \alpha_2)\right] = \frac{\mu_0 I}{4\pi d}(\cos\alpha_1 - \cos\alpha_2)$$

Illustration 13

Find magnetic field due to finite length wire at point 'P'.

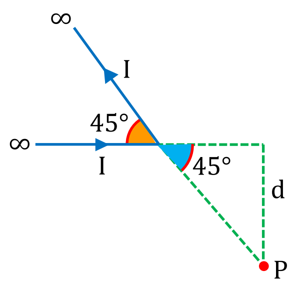


Solution:

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin\theta + \sin 0°]; B_P = \frac{\mu_0 I}{4\pi d}\sin\theta$$

Illustration 14

Find net magnetic field at point P.



Solution:

$$B = \frac{\mu_0 I}{4\pi d}(1 - \sin\theta)$$

$$= \frac{\mu_0 I}{4\pi d}\left(1 - \frac{1}{\sqrt{2}}\right)\otimes$$

BEGINNER'S BOX-2

1. A proton moves along horizontal line and towards observer, the pattern of concentric circular field lines of magnetic field which produced due to its motion :-

(1) ACW, In horizontal plane (2) ACW, In vertical plane
(3) CW, In horizontal plane (4) CW, In vertical plane

PME196

2. An electron moves along vertical line and towards observer, the pattern of concentric circular field lines of magnetic field which is produced due to its motion :-

(1) ACW, In horizontal plane (2) ACW, In vertical plane
(3) CW, In horizontal plane (4) CW, In vertical plane

PME197

3. A current is flowing in electricity line towards north, the direction of magnetic field at a point which is just below the line :-

(1) towards north (2) towards south
(3) towards east (4) towards west

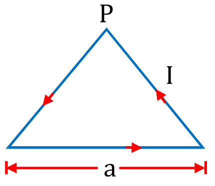
PME198

4. A current is flowing in a vertical wire in downward direction, the direction of magnetic field of a point which is just right side to the wire :-

(1) towards north (2) towards south
(3) towards east (4) towards west

PME199

5. An equilateral triangle of side 'a' carries a current 'I'. Magnetic field at point 'P' which is vertex of triangle. :-

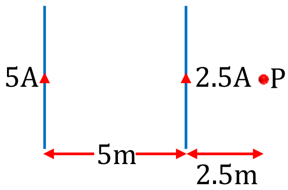


(1) $\frac{\mu_0 I}{2\sqrt{3}\pi a}\odot$ (2) $\frac{\mu_0 I}{2\sqrt{3}\pi a}\otimes$

(3) $\frac{9}{2}\left(\frac{\mu_0 I}{\pi a}\right)\odot$ (4) $\frac{9}{2}\left(\frac{\mu_0 I}{\pi a}\right)\otimes$

PME200

6. Magnetic field at point 'P' due to both infinite long current carrying wires is :-

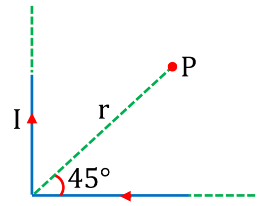


(1) $\frac{\mu_0}{2\pi}\otimes$ (2) $\frac{5\mu_0}{6\pi}\otimes$

(3) $\frac{5\mu_0}{6\pi}\odot$ (4) $\frac{\mu_0}{2\pi}\odot$

PME201

7. Magnetic field at point 'P' due to given current distribution is :-

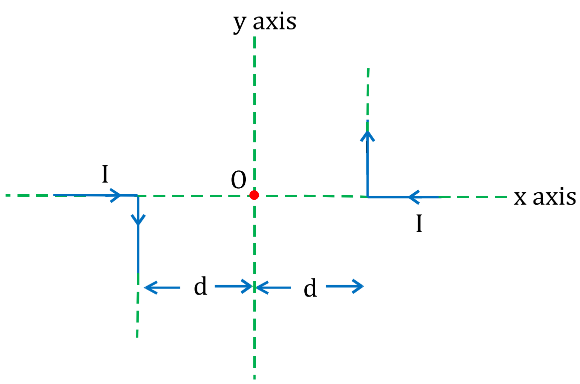


(1) $\frac{\mu_0 I}{4\pi r}\left(1+\sqrt{2}\right)\odot$ (2) $\frac{\mu_0 I}{2\pi r}\left(1+\sqrt{2}\right)\odot$

(3) $\frac{\mu_0 I}{4\pi r}\left(1+\sqrt{2}\right)\otimes$ (4) $\frac{\mu_0 I}{2\pi r}\left(1+\sqrt{2}\right)\otimes$

PME202

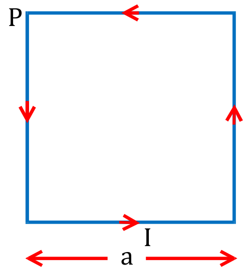
8. Magnetic field at origin 'O' due to given current distribution is :-



(1) $\frac{\mu_0 I}{2\pi d}\odot$ (2) $\frac{\mu_0 I}{2\pi d}\otimes$ (3) $\frac{\mu_0 I}{4\pi d}\odot$ (4) $\frac{\mu_0 I}{4\pi d}\otimes$

PME203

9. A square loop of side 'a' is made by a current carrying wire. Magnetic field at its vertex 'P' is :-

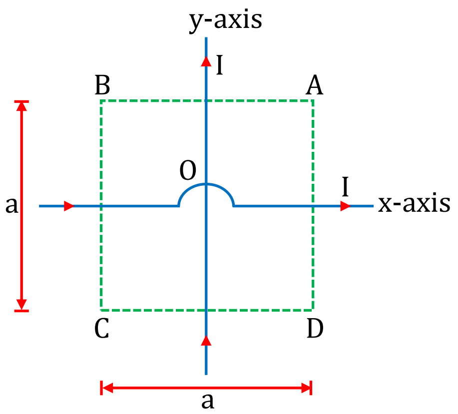


(1) $\frac{\mu_0 I}{2\sqrt{2}\pi a} \odot$ (2) $\frac{\mu_0 I}{2\sqrt{2}\pi a} \otimes$

(3) $2\sqrt{2}\frac{\mu_0 I}{\pi a} \otimes$ (4) $2\sqrt{2}\frac{\mu_0 I}{\pi a} \odot$

PME204

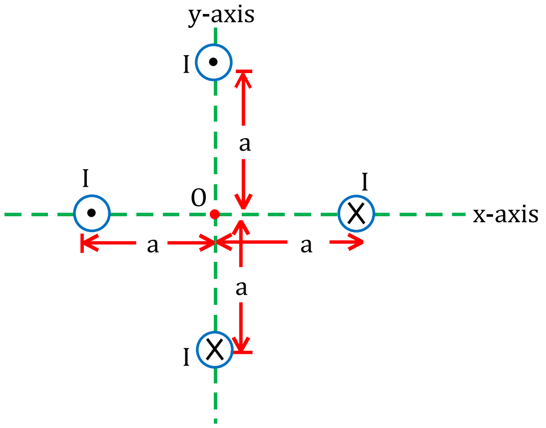
10. Two infinite length wires carry equal current and placed along x and y axis respectively. At which points the resultant magnetic field is zero?



(1) A, B (2) B, D (3) A, C (4) C, D

PME205

11. For given current distribution, each infinite length wire produces magnetic field 'B' at origin then resultant magnetic field at origin 'O' is :-

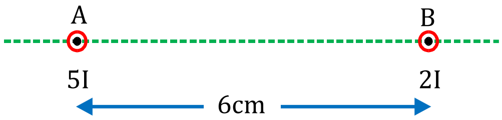


(1) 4B (2) $\sqrt{2}$ B (3) $2\sqrt{2}$ B (4) zero

PME206

12. The position of point from wire 'B', where net magnetic field is zero due to following current distribution.

(a)

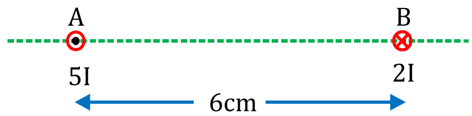


(1) 6/7 cm (2) 12/7 cm

(3) 18/17 cm (4) 16/7 cm

PME207

(b)



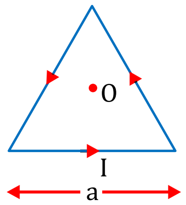
(1) 4 cm (2) 2 cm

(3) 8 cm (4) 12 cm

PME208

13. Magnetic field at the centre of various **regular polygons,** which are formed by current carrying wires.

(a) Equilateral triangle :

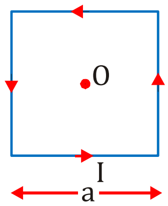


(1) $\frac{\mu_0 I}{2\sqrt{3}\pi a}$ (2) $\frac{27\mu_0 I}{2\pi a}$

(3) $\frac{9\mu_0 I}{2\pi a}$ (4) $\sqrt{3}\frac{\mu_0 I}{\pi a}$

PME209

(b) Square :

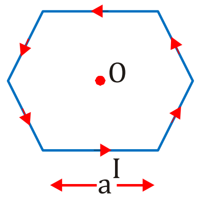


(1) $\frac{2\sqrt{2}\mu_0 I}{\pi a}$ (2) $\frac{\mu_0 I}{2\sqrt{2}\pi a}$

(3) $8\sqrt{2}\frac{\mu_0 I}{\pi a}$ (4) $\sqrt{2}\frac{\mu_0 I}{\pi a}$

PME210

(c) Regular hexagon



(1) $\frac{6\mu_0 I}{\pi a}$ (2) $\frac{\sqrt{3}\mu_0 I}{\pi a}$

(3) $6\sqrt{3}\frac{\mu_0 I}{\pi a}$ (4) $\frac{\mu_0 I}{\sqrt{3}\pi a}$

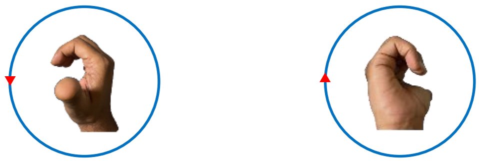
PME211

Magnetic Field due to Circular Loop

Direction: Can be obtained by right hand thumb rule

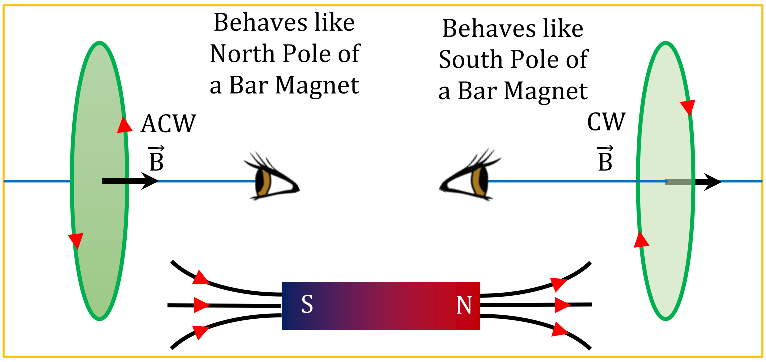
(i) Curl the fingers of right hand in the direction of current in the loop then thumb will indicated the direction of magnetic field

(ii)



Anti-Clockwise MF : Outwards

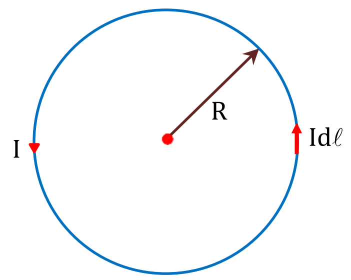
Clockwise MF : Inwards



Magnetic Field due to Circular Coil

M.F. at the center due to a current element

$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell}{R^2}$$



Net M.F. at the center

$$B_0 = \frac{\mu_0}{4\pi}\frac{I}{R^2}\int d\ell = \frac{\mu_0}{4\pi}\frac{I}{R^2}(2\pi R)$$

If number of loops is not given, take single loop.

For single loop : - $B_0 = \frac{\mu_0 I}{2R}$

For N loops : - $B_0 = \frac{\mu_0 NI}{2R}$

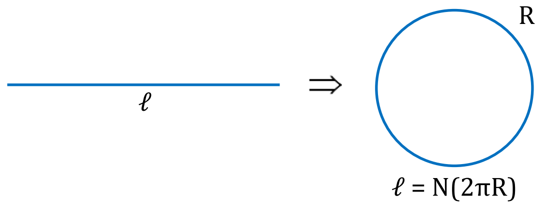
Illustration 15

Circular loop is made by a wire of length 7.5m. If current 5A is flowing in the loop, then find magnetic field at centre.

Solution:

$$2\pi R = 7.5 \Rightarrow R = \frac{7.5}{2\pi} \Rightarrow B_C = \frac{\mu_0 I}{2R} \Rightarrow \frac{4\pi\times10^{-7}\times5}{2\times7.5/2\pi} = \frac{40\pi^2\times10^{-7}}{15}\,T$$

Note : If a single wire of constant length ℓ is bent in form of N circular loops,



Then, M.F. at the center due to current I flowing in the loops will be

$$B_0 = \frac{\mu_0 NI}{2R} = \frac{\mu_0 NI}{2}\times\frac{1}{R} = \frac{\mu_0 NI}{2}\times\frac{2\pi N}{\ell} \Rightarrow B \propto N^2$$

Illustration 16

A circular ring of 2 turns is made by a conducting wire. The magnetic field at the center is B. Now, this same wire is bent to form circular ring of 4 turns, find magnitude of new magnetic field at the center (current is same).

Solution:

$$B_1 \propto (N_1)^2 = B \propto 2^2 \Rightarrow B_2 \propto (N_2)^2 = B_2 \propto 4^2$$

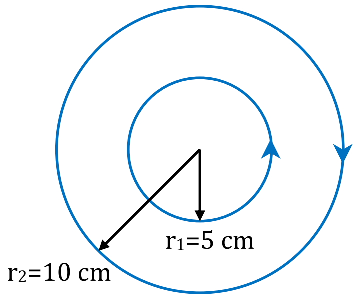
From above equation

$$\frac{B_1}{B_2} = \frac{2^2}{4^2} \Rightarrow B_2 = 4B_1 = 4B$$

Illustration 17

Two concentric circular coils of radius 5 cm and 10 cm are placed in same plane. If current 0.2 A is flowing in both coils in opposite direction, then find magnetic field at the centre.

Solution:



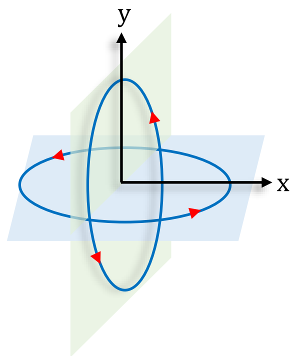
$$B_1 = \frac{\mu_0 I}{2r_1} \odot \quad ; \quad B_2 = \frac{\mu_0 I}{2r_2} \otimes$$

Net magnetic field at the centre $(B_0) = B_1 - B_2 = \dfrac{\mu_0 I}{2}\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$

$$B_0 = \frac{4\pi \times 10^{-7} \times 0.2}{2}\left(\frac{1}{0.05} - \frac{1}{0.1}\right) \Rightarrow B_0 = 4\pi \times 10^{-7} \odot T$$

Illustration 18

Two symmetrical current carrying rings are placed perpendicular to each other with common centre. If magnetic field at the centre due to one coil is B, then find net magnetic field.



Solution:

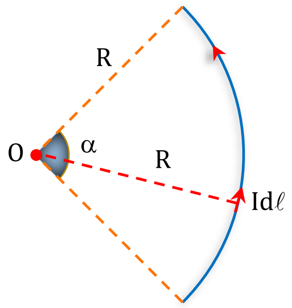
$$B_H = \frac{\mu_0 I}{2R} = B\hat{j} \Rightarrow B_V = \frac{\mu_0 I}{2R} = B\hat{i}$$

$$\Rightarrow B_{net} = \sqrt{B_H^2 + B_V^2} \Rightarrow B\sqrt{2}$$

Magnetic Field due to Circular Arc

As per Biot Savart Law,

Magnetic field due to current element $Id\ell$ at the centre of arc



$$dB = \frac{\mu_0}{4\pi}\frac{Id\ell}{R^2}$$

Due to complete arc, M.E. will be –

$$B = \frac{\mu_0}{4\pi}\frac{1}{R^2}\int d\ell \Rightarrow B = \frac{\mu_0}{4\pi}\frac{I}{R^2}(R\alpha) \Rightarrow B = \frac{\mu_0}{4\pi}\frac{I\alpha}{R}$$

Note: α must be in radian only

$$\theta^\circ \times \frac{\pi}{180^\circ} \to \theta \text{ rad}$$

BEGINNER'S BOX-3

1. The magnetic field at the centre of a circular coil of radius r carrying current is B_1. The field at the centre of another coil of radius $\dfrac{r}{2}$ carrying same current is B_2 then ratio of B_1/B_2 is :-

(1) 1 : 2 (2) 2 : 1 (3) 1 : 1 (4) 4 : 1

PME212

2. A circular coil of one turn in formed by a 6.28m length wire, which carries a current of 3.14A. The magnetic field at the centre of coil is :-

(1) 1×10^{-6} T (2) 4×10^{-6} T (3) 0.5×10^{-6} T (4) 2×10^{-6} T

PME213

3. A length of wire carries a steady current, is bent first to form a plane circular coil of one turn, same length now bent more sharply to give three turns of smaller radius. Magnetic field becomes :-

(1) 3 times (2) 1/3 times (3) 9 times (4) unchanged

PME214

4. Two concentric coplanner coils of equal turns have radii 10 cm and 30 cm respectively. Same current flowing in both the coils in same direction. Now direction of current is reversed in one coil then ratio of magnetic field at their common centre in two conditions respectively :-

(1) 2 : 1 (2) 1 : 2 (3) 1 : 1 (4) 4 : 1

PME215

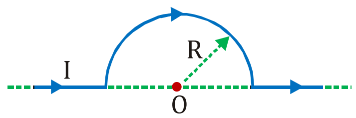
5. Two concentric coplanner coils of turns n_1 and n_2 have radii ratio 2 : 1 respectively. Equal current in both the coils flows in opposite direction. If net magnetic field is zero at their common centre then $n_1 : n_2$ is :-

(1) 2 : 1 (2) 1 : 2 (3) 1 : 1 (4) 4 : 1

PME216

Find out magnetic field at point 'O' for the following current distributions.

6. (a)

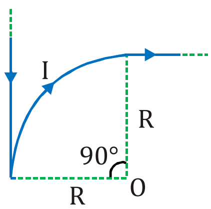


(1) $\frac{\mu_0 I}{8R}\odot$ (2) $\frac{\mu_0 I}{8R}\otimes$

(3) $\frac{\mu_0 I}{4R}\odot$ (4) $\frac{\mu_0 I}{4R}\otimes$

PME217

(b)

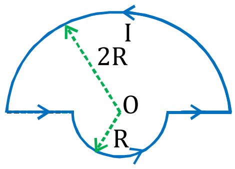


(1) $\frac{\mu_0 I}{8R}\otimes$ (2) $\frac{\mu_0 I}{16R}\otimes$

(3) $\frac{\mu_0 I}{8R}\odot$ (4) $\frac{\mu_0 I}{16R}\odot$

PME218

7. (a)

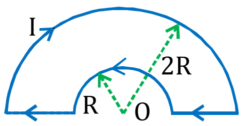


(1) $\frac{3\mu_0 I}{16R}\odot$ (2) $\frac{3\mu_0 I}{8R}\odot$

(3) $\frac{4\mu_0 I}{3R}\otimes$ (4) $\frac{\mu_0 I}{8R}\otimes$

PME219

(b)

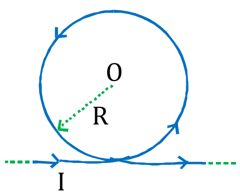


(1) $\frac{\mu_0 I}{4R}\odot$ (2) $\frac{\mu_0 I}{2R}\odot$

(3) $\frac{\mu_0 I}{8R}\odot$ (4) $\frac{\mu_0 I}{16R}\otimes$

PME220

8. (a)

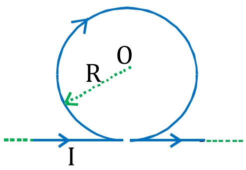


(1) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)\otimes$ (2) $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)\odot$

(3) $\frac{\mu_0 I}{2R}\left(1+\frac{1}{\pi}\right)\otimes$ (4) $\frac{\mu_0 I}{4R}\left(1+\frac{1}{\pi}\right)\odot$

PME221

(b)

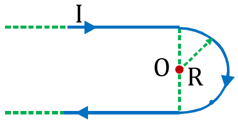


(1) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)\otimes$ (2) $\frac{\mu_0 I}{4R}\left(1-\frac{1}{\pi}\right)\otimes$

(3) $\frac{\mu_0 I}{2R}\left(1-\frac{1}{\pi}\right)\odot$ (4) $\frac{\mu_0 I}{4R}\left(1-\frac{1}{\pi}\right)\odot$

PME222

9. (a)

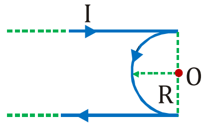


(1) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}+\frac{1}{\pi}\right)\otimes$ (2) $\frac{\mu_0 I}{2R}\left(\frac{1}{4}-\frac{1}{\pi}\right)\otimes$

(3) $\frac{\mu_0 I}{8R}\left(\frac{1}{4}-\frac{1}{\pi}\right)\otimes$ (4) $\frac{\mu_0 I}{R}\left(\frac{1}{4}-\frac{1}{\pi}\right)\otimes$

PME223

(b)

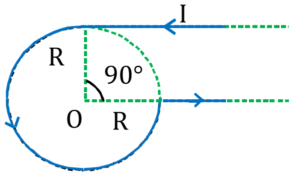


(1) $\frac{\mu_0 I}{3R}\left(\frac{1}{4}-\frac{1}{\pi}\right)\odot$ (2) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}-\frac{1}{\pi}\right)\odot$

(3) $\frac{4\mu_0 I}{3R}\otimes$ (4) $\frac{\mu_0 I}{8R}\otimes$

PME224

10. (a)

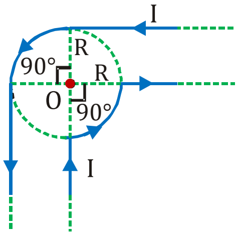


(1) $\frac{4\mu_0 I}{3R}\otimes$ (2) $\frac{\mu_0 I}{8R}\otimes$

(3) $\frac{\mu_0 I}{8R}\left(3+\frac{2}{\pi}\right)\odot$ (4) $\frac{\mu_0 I}{4R}\left(3+\frac{2}{\pi}\right)\otimes$

PME225

(b)

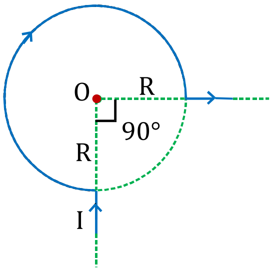


(1) $\frac{\mu_0 I}{8R}\odot$ (2) $\frac{\mu_0 I}{4R}\left(1+\frac{2}{\pi}\right)\odot$

(3) $\frac{\mu_0 I}{4R}\odot$ (4) $\frac{\mu_0 I}{4R}\otimes$

PME226

11. (a)

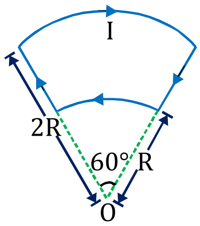


(1) $\frac{3\mu_0 I}{8R}\otimes$ (2) $\frac{3\mu_0 I}{8R}\odot$

(3) $\frac{4\mu_0 I}{3R}\otimes$ (4) $\frac{\mu_0 I}{8R}\odot$

PME227

(b)



(1) $\frac{\mu_0 I}{3R}\left(\frac{1}{4}-\frac{1}{\pi}\right)\odot$ (2) $\frac{\mu_0 I}{2R}\left(\frac{1}{2}-\frac{1}{\pi}\right)\odot$

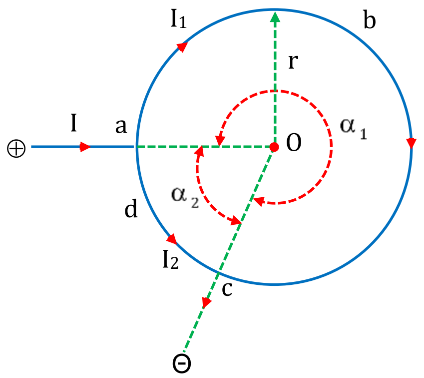
(3) $\frac{4\mu_0 I}{3R}\otimes$ (4) $\frac{\mu_0 I}{24R}\odot$

PME228

Magnetic field at centre of circular wire

(i) Wire of circuit loop is uniform:

Magnetic field at the centre of arc abc and adc is



$$\left.\begin{aligned} B_{abc} &= \frac{\mu_0 I_1 \alpha_1}{4\pi r} \\ B_{adc} &= \frac{\mu_0 I_2 \alpha_2}{4\pi r} \end{aligned}\right\} \Rightarrow \frac{B_{abc}}{B_{adc}} = \frac{I_1\alpha_1}{I_2\alpha_2} \qquad ...(1)$$

$$\because \text{angle} = \frac{\text{arc length}}{\text{Radius}} \Rightarrow \frac{\alpha_1}{\alpha_2} = \frac{\ell_1}{\ell_2} \qquad ...(2)$$

$$\text{and} \quad \left.\begin{aligned} V &= IR \Rightarrow I \propto \frac{1}{R} \\ R &= \frac{\rho\ell}{A} \Rightarrow R \propto \ell \end{aligned}\right\} \Rightarrow \frac{I_1}{I_2} = \frac{\ell_2}{\ell_1} \qquad ...(3)$$

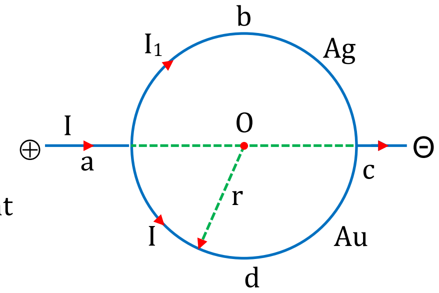
put the value of (2) & (3) in (1)

$$\frac{B_{abc}}{B_{adc}}=\left(\frac{\ell_2}{\ell_1}\right)\left(\frac{\ell_1}{\ell_2}\right) \quad \Rightarrow \boxed{\frac{B_{abc}}{B_{adc}}=\frac{1}{1}}$$

- Magnetic field produced by both the arc is at the centre of the circuit loop is equal in magnitude and opposite in direction. So $(B_0)_{net} = 0$ (always and it is free from angle of connection of terminals)

(ii) Wire of circuit loop is non-uniform:

Case I : Thickness $\longrightarrow$ Same, Material $\longrightarrow$ different: -



Magnetic field at the centre of arc abc and adc is –

$$B_{abc}=\frac{\mu_0 I_1}{4r}\otimes, B_{adc}=\frac{\mu_0 I_2}{4r}\odot$$

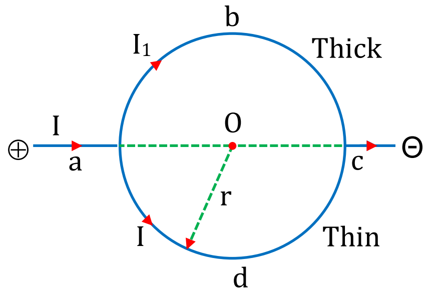
Both arc connected in parallel with source, so V = IR = constant

$$I \propto \frac{1}{R} \Rightarrow R_{A_u} > R_{A_g} \Rightarrow I_{A_u} < I_{A_g}$$

Net magnetic field at centre

$$(B_0)_{net}=\frac{\mu_0}{4r}(I_1-I_2)\otimes \text{ (where } I_1 > I_2)$$

Case II : Thickness $\longrightarrow$ different, Material $\longrightarrow$ same :–



Magnetic field at the centre of arc abc and adc

$$B_{abc}=\frac{\mu_0 I_1}{4r}\otimes, B_{adc}=\frac{\mu_0 I_2}{4r}\odot$$

Both arc are connected in parallel with source, so

V = IR = constant and resistance of wire $R=\frac{\rho\ell}{A}$

$$I \propto \frac{1}{R} \quad \text{...(1)}$$

$$R \propto \frac{1}{A} \quad \text{...(2)}$$

From (1) & (2) $I \propto A \Rightarrow I_{thick} > I_{thin}$

Net magnetic field at centre

$$(B_0)_{net}=\frac{\mu_0}{4r}(I_1-I_2)\otimes \text{ (where } I_1 > I_2)$$

Note: When current divides in any symmetrical planar loop made with uniform wire, then magnetic field at the centre due to this loop is ZERO.

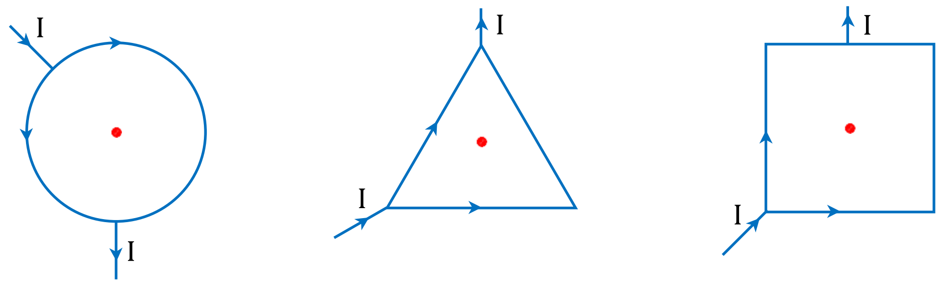
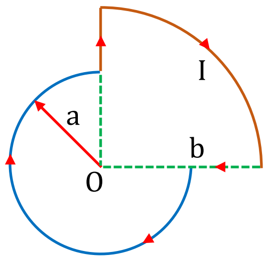


Illustration 19

The magnetic induction at the centre O is?



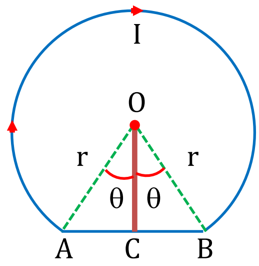
Solution:

$$B=\frac{3}{4}\left[\frac{\mu_0 I}{2a}\right]\otimes+\frac{1}{4}\left[\frac{\mu_0 I}{2b}\right]\otimes$$

$$B=\left[\frac{3\mu_0 I}{8a}+\frac{\mu_0 I}{8b}\right]\otimes$$

Illustration 20

A wire is bent in the form of a circular arc with a straight portion AB. Magnetic induction at O when current I flowing in the wire, is?



Solution:

$$B_{AB}=\frac{\mu_0 I}{4\pi(OC)}[2\sin\theta]$$

But $OC=r\cos\theta$ or $B_{AB}=\frac{\mu_0 I}{2\pi r}\tan\theta\ \otimes$

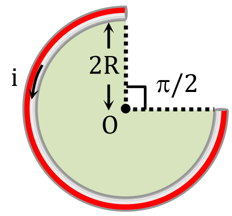
Magnetic field due to circular portion,

$$B_{AB}=\frac{\mu_0 I}{2r}\left(\frac{2\pi-2\theta}{2\pi}\right)=\frac{\mu_0 I}{2\pi r}(\pi-\theta)\ \otimes$$

Total magnetic field = $\frac{\mu_0 I}{2\pi r}\tan\theta+\frac{\mu_0 I}{2\pi r}(\pi-\theta)=\frac{\mu_0 I}{2\pi r}[\tan\theta+\pi-\theta]\ \otimes$

Illustration 21

A current i ampere flows in a circular arc of wire whose radius is 2R, which subtend an angle $3\pi/2$ radian at its centre. The magnetic induction B at the centre is

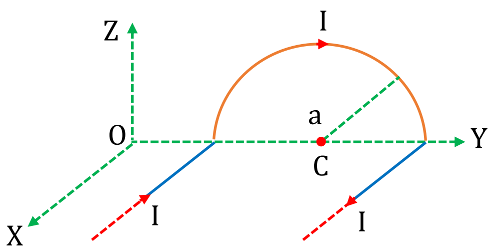


Solution:

$$B=\frac{\mu_0}{4\pi}\frac{(2\pi-\theta)i}{R'}=\frac{\mu_0}{4\pi}\frac{\left(2\pi-\frac{\pi}{2}\right)\times i}{2R}=\frac{3\mu_0 i}{16R}$$

Illustration 22

A long wire bent as shown in the figure carries current I. If the radius of the semi-circular portion is "a" then find the magnetic induction at the centre C.



Solution:

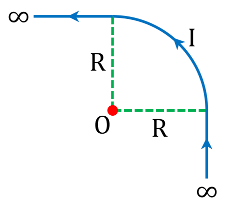
Due to semi-circular part $\vec{B}_1=\frac{\mu_0 I}{4a}\left(-\hat{i}\right)$

Due to parallel parts of currents $\vec{B}_2=2\times\frac{\mu_0 I}{4\pi a}(-\hat{k}),\vec{B}_{net}=\vec{B}_C=\vec{B}_1+\vec{B}_2=\frac{\mu_0 I}{4a}(-\hat{i})+\frac{\mu_0 I}{2\pi a}(-\hat{k})$

Magnitude of resultant field $B_{net}=\sqrt{B_1^2+B_2^2}=\frac{\mu_0 I}{4\pi a}\sqrt{\pi^2+4}$

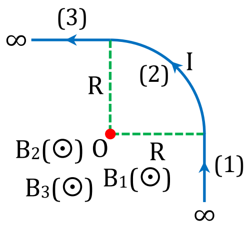
Illustration 23

Find magnetic field at O.



Solution:

$B_{net} = B_1 + B_2 + B_3$

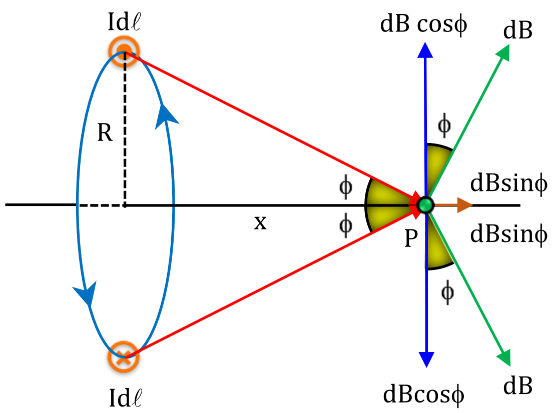


$$B_{net} = \frac{\mu_0 I}{4\pi R} + \frac{\mu_0 I}{4\pi R}\left(\frac{\pi}{2}\right) + \frac{\mu_0 I}{4\pi R}$$

$$= \frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{8R}$$

$$= \frac{\mu_0 I}{2R}\left[\frac{1}{\pi} + \frac{1}{4}\right] \odot$$

Magnetic Field at the Axis of Circular loop



Component of magnetic field perpendicular to axis of ring will be zero because the components of magnetic field produced due to opposite current elements on the ring cancel each other. Net magnetic field due to ring at any point on the axis will be along the axis.

$$B_p = \int dB\sin\phi$$

$$B_p = \int \frac{\mu_0}{4\pi}\frac{Idl\sin(90°)}{r^2}\left(\frac{R}{r}\right)$$

$$B_p = \frac{\mu_0}{4\pi}\frac{IR}{r^3}\int d\ell = \frac{\mu_0}{4\pi}\frac{IR}{r^3}(2\pi R) = \frac{\mu_0}{2}\frac{IR^2}{r^3}$$

In terms of distance from center,

$$B_p = \frac{\mu_0}{2}\frac{IR^2}{\left(\sqrt{R^2+x^2}\right)^3} = \frac{\mu_0 IR^2}{2\left(R^2+x^2\right)^{2/3}}$$

To find magnetic field at the center of circular current loop(B_o),

At center, $(x) = 0$

$$B_o = \frac{\mu_0 IR^2}{2\left(R^2+0\right)^{3/2}} = \frac{\mu_0 IR^2}{2R^3} = \frac{\mu_0 I}{2R}$$

Now,

$$B_P = \frac{\mu_0 IR^2}{2\left(R^2+x^2\right)^{3/2}} = \frac{\mu_0 IR^2}{2\left(R^2\right)^{3/2}\left(1+\frac{x^2}{R^2}\right)^{3/2}} = \frac{\mu_0 I}{2R\left(1+\frac{x^2}{R^2}\right)^{3/2}} = \frac{B_0}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

Case 1 : At very large distance from centre

$x >> R$

$x^2 + R^2 \approx x^2$

$$B \approx \frac{\mu_0 I R^2}{2\left(x^2\right)^{3/2}}$$

$$\Rightarrow B \approx \frac{\mu_0 I R^2}{2x^3}$$

Case 2 : Near centre of the ring

$x << R$

$$B_P = \frac{B_0}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

$$\Rightarrow B_P = B_0\left(1+\frac{x^2}{R^2}\right)^{-3/2}$$

$$B_P \approx B_0\left(1-\frac{3x^2}{2R^2}\right)$$

$$\frac{B_P}{B_0} = 1 - \frac{3x^2}{2R^2}$$

$$\frac{B_0 - B_P}{B_0} = \frac{3x^2}{2R^2}$$

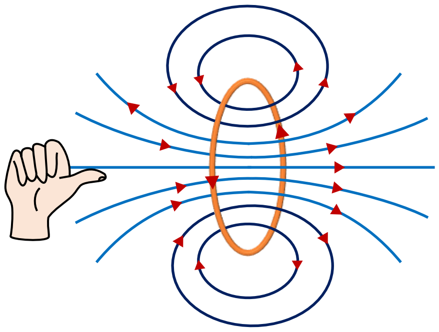
$$\frac{\Delta B}{B_0} = \frac{3x^2}{2R^2} \text{ (Fractional change)}$$

$$\Delta B = B_0 \frac{3x^2}{2R^2} \text{(Valid upto only 5\%)}$$

Note: 1. At $x = 0$, B is maximum (B_0)

2. At $x = \infty$, $B = 0$

3. On any point on the axis of current carrying coil, magnetic field is always along the axis (on both sides), direction can be found using RHTR.



Graph : B vs x

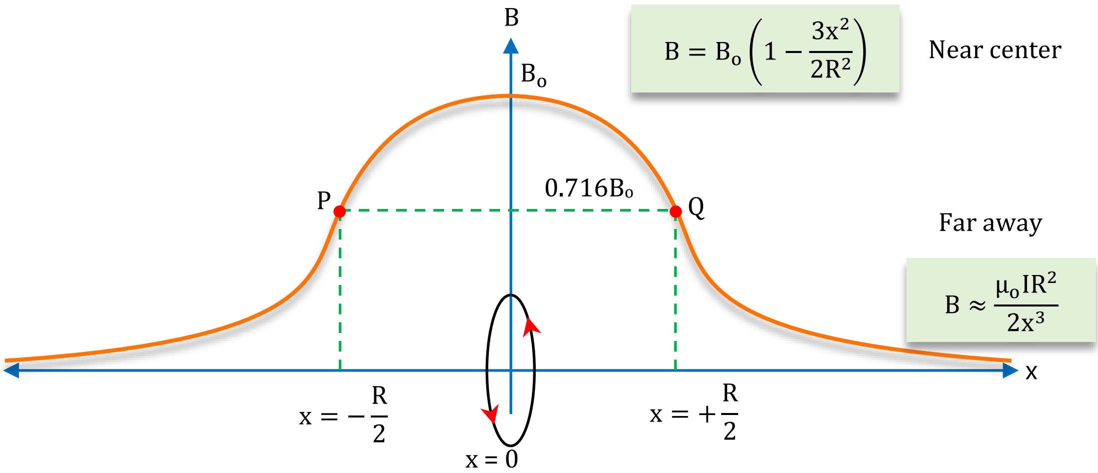


Illustration 24

Find the magnetic field at axis (x = R/2) of circular current carrying loop in terms of B_o.

Solution:

$$B_P = \frac{B_0}{\left(1+\frac{x^2}{R^2}\right)^{3/2}} = \frac{B_0}{\left(1+\frac{R^2}{4R^2}\right)^{3/2}} = \frac{B_0}{\left(\frac{5}{4}\right)^{3/2}} = \frac{8B_0}{5\sqrt{5}} = 0.716B_0$$

Illustration 25

Radius of a current carrying coil is 'R'. The ratio of magnetic field at a axial point which is 3R distance away from the centre of the coil to the magnetic field at the centre of the coil: -

Solution:

$$B_x = \frac{B_0}{\left(1+\frac{x^2}{R^2}\right)^{3/2}}$$

$\because$ x = 3R

$$\therefore B_x = \frac{B_0}{(1+9)^{3/2}} = \frac{B_0}{(10)^{3/2}} \Rightarrow \frac{B_x}{B_0} = \left(\frac{1}{10}\right)^{3/2} \Rightarrow B_x = B_0\left(\frac{1}{10}\right)^{3/2}$$

Illustration 26

Magnetic fields at two points on the axis of a circular coil at a distance of 0.04m and 0.2m from the centre are in the ratio 8 : 1. The radius of the coil is

Solution:

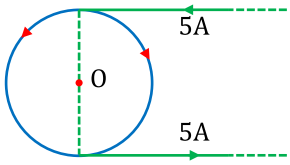
$$B = \frac{\mu_0}{4\pi} \times \frac{2\pi NiR^2}{(R^2+x^2)^{3/2}} \Rightarrow B \propto \frac{1}{(r^2+x^2)^{3/2}}$$

$$\Rightarrow \frac{8}{1} = \frac{(R^2+x_2^2)^{3/2}}{(R^2+x_1^2)^{3/2}} \Rightarrow \left(\frac{8}{1}\right)^{2/3} = \frac{R^2+0.04}{R^2+0.0016}$$

$$\Rightarrow \frac{4}{1} = \frac{R^2+0.04}{R^2+0.0016}$$ On solving R = 0.10 m

BEGINNER'S BOX-4

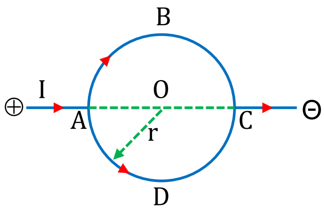
1. Magnetic field at point 'O' due to given current distribution. If 5A current is flowing in this system and the diameter of the loop is 10cm.



(1) 2×10^{-5} T, $\otimes$ (2) 10^{-5}T, $\odot$
(3) 10^{-5} T, $\otimes$ (4) 2×10^{-5} T, $\odot$

PME229

2. Figure shows a circular loop with radius 'r'. The resistance of arc ABC is 5Ω and that of ADC is 10Ω. Magnetic field at the centre of the loop is :-



(1) $\frac{\mu_0 I}{6r} \otimes$ (2) $\frac{\mu_0 I}{12r} \otimes$ (3) $\frac{\mu_0 I}{12r} \odot$ (4) $\frac{\mu_0 I}{6r} \odot$

PME230

3. A circular current carrying coil has a radius R. The distance from the centre of the coil on the axis where the magnetic induction will be (1/8)th of its value at the centre of the coil, is :-

(1) $R/\sqrt{3}$ (2) $R\sqrt{3}$ (3) $2R\sqrt{3}$ (4) $(2/\sqrt{3})R$

PME231

4. The magnetic field due to a current carrying circular loop of radius 3 cm at a point on the axis at a distance of 4 cm from the centre is 54 μT. What will be its value at the centre of the loop

(1) 250μT (2) 150μT (3) 125μT (4) 75μT

PME232

5. Consider a circular coil with radius R carrying an electric current. The magnetic field at a point located on the axis of the coil, at a distance r from the center (with $r \gg R$), follows which of these relationships with r?

(1) $1/r$ (2) $1/r^{3/2}$ (3) $1/r^2$ (4) $1/r^3$

PME233

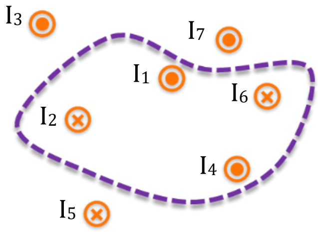
6. Radius of current carrying coil is 'R'. If fractional decreases in field value with respect to centre of the coil for a nearby axial point is 1% then find axial position of that point.

(1) $\frac{R}{\sqrt{150}}$ (2) $\frac{R}{\sqrt{100}}$ (3) $\frac{R}{\sqrt{50}}$ (4) $\frac{R}{100}$

PME234

Ampere's Circuital Law and it's Applications

Ampere Circuital Law



It gives another method to calculate the magnetic field due to given current distribution.

Line integral of magnetic field along any closed loop is equal to μ_0 times the net current crossing the surface bounded by the loop.

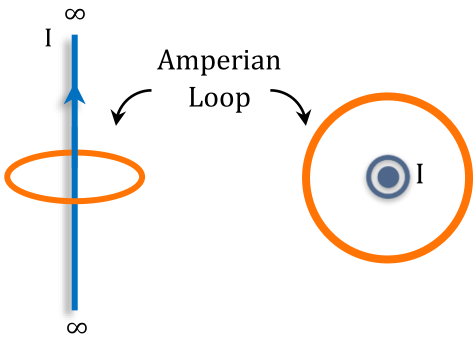
Mathematically,

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

Note: 1. It is applicable only for steady/constant current

2. It can be applied for any distribution of current **but**, it is applied for **symmetric distribution** for calculation purpose.

Magnetic Field due to Long Thin wire



$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

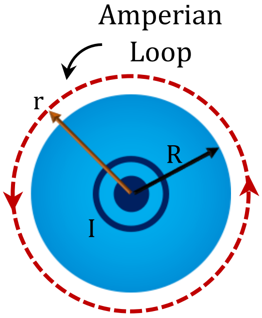
$$B(2\pi r) = \mu_0(I)$$

$$B = \frac{\mu_0(I)}{2\pi r}$$

$$B = \frac{\mu_0 I}{2\pi r}$$

Magnetic Field due to Long Thick wire

At a point outside wire



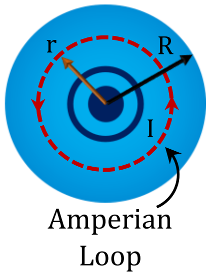
$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0(I)$$

$$B = \frac{\mu_0(I)}{2\pi r}$$

$$B_{out} = \frac{\mu_0 I}{2\pi r},\ B_{sur} = \frac{\mu_0 I}{2\pi R}$$

At a point inside wire



$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0\left(\frac{I}{\pi R^2}\times \pi r^2\right)$$

$$B = \frac{\mu_0(Ir)}{2\pi R^2}$$

$$B_{in} = \frac{\mu_0 I}{2\pi R^2} r = \frac{\mu_0 J r}{2} \qquad \left(\text{Current density, } J = \frac{I}{\pi R^2}\right)$$

A : Outside the wire surface

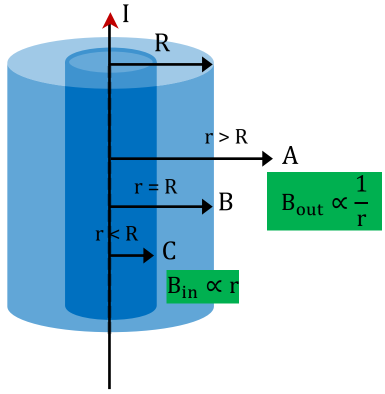
$$B_{out} = \frac{\mu_0 I}{2\pi r},\ B_{out} \propto \frac{1}{r}$$

B : On the wire surface

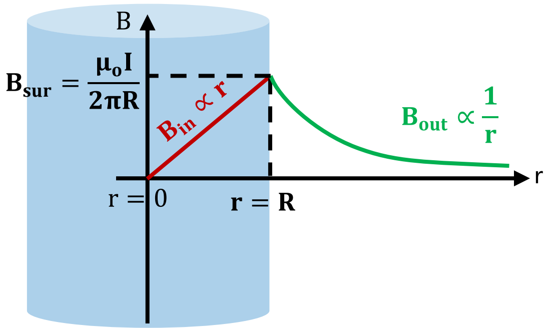
$$B_{sur} = \frac{\mu_0 I}{2\pi R}$$

C : Inside the wire surface

$$B_{in} = \frac{\mu_0 I}{2\pi R^2} r,\ \ B_{in} \propto r$$

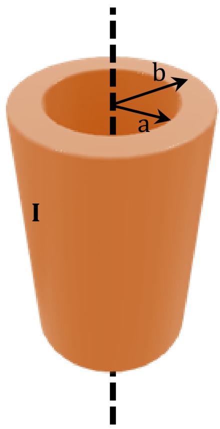


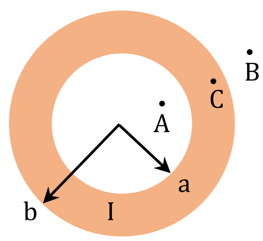
Graph



Magnetic field is maximum at the surface of wire.

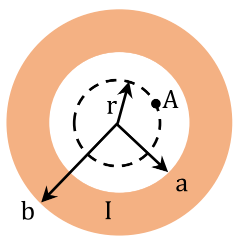
Magnetic Field due to Long Hollow Cylindrical Conductor





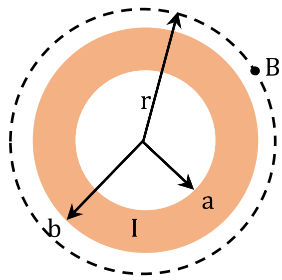
At point A, r < a

$I_{enc} = 0 \Rightarrow B_{in} = 0$



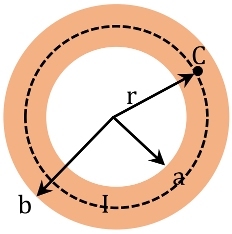
At point B, r ≥ b

$I_{enc} = I \Rightarrow B_{out} = \frac{\mu_0 I}{2\pi r}$



At point C, $b > r \geq a$

$$I_{enc} = \frac{I}{\pi(b^2 - a^2)} \times \left(\pi(r^2 - a^2)\right)$$



$$I_{enc} = I\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{enc}$$

$$B(2\pi r) = \mu_0 I_{enc}$$

$$B_{mid} = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

Illustration 27

A current of $1/(2\pi)$ ampere is flowing in a long straight conductor. The line integral of magnetic induction around a closed path enclosing the current carrying conductor is.

Solution:

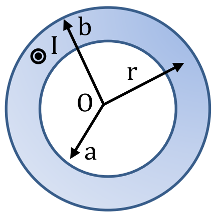
$$\int \vec{B} \cdot d\vec{l} = \mu_0 I = 4\pi \times 10^{-7} \times \frac{1}{2\pi} = 2 \times 10^{-7} \text{ weber/metre}$$

Illustration 28

A hollow cylindrical wire carries a current 5A, having inner and outer radii 'R' and 2R respectively. Magnetic field at a point which 3R/2 distance away from its axis is (R = 5m) : -

Solution:

Field inside cross-section of conductor



$$B_p = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

[a = R, b = 2R, r = 3R/2]

$$B_p = \frac{\mu_0 I}{2\pi\left(\frac{3R}{2}\right)}\left[\frac{(3R/2)^2 - (R)^2}{(2R)^2 - (R)^2}\right] = \frac{5}{36} \cdot \frac{\mu_0 I}{\pi R} = \frac{5}{36} \cdot \frac{5 \times \mu_0}{5 \times \pi} = \frac{5}{9} \times 10^{-7} \text{T}$$

BEGINNER'S BOX-5

1. A solid cylindrical wire of radius 'R' carries a current 'I'. The ratio of magnetic fields at points which are located at R/2 and 2R distance away from the axis of the wire :-

(1) 1 : 1 (2) 1 : 2 (3) 2 : 1 (4) 1 : 4

PME235

2. A solid cylindrical wire of radius 'R' carries a current 'I'. The magnetic field is 5μT at a point, which is '2R' distance away from the axis of wire. Magnetic field at a point which is R/3 distance inside from the surface of the wire is :-

(1) $\frac{10}{3}\mu T$ (2) $\frac{20}{3}\mu T$ (3) $\frac{5}{3}\mu T$ (4) $\frac{40}{3}\mu T$

PME236

3. A long, straight and solid metal wire of radius 2mm carries a current uniformly distributed over its circular cross section. The magnetic field at a distance 2 mm from its axis is B. Magnetic field at a distance 1 mm from the axis of the wire is :-

(1) B (2) 4B (3) 2B (4) B/2

PME237

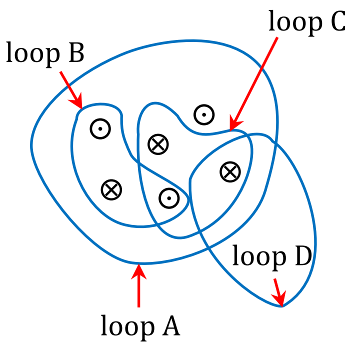
4. A long straight wire (radius = 3.0 mm) carries a constant current distributed uniformly over a cross section perpendicular to the axis of the wire. If the current density is 100 A/m^2. The magnitudes of the magnetic field at (a) 2.0 mm from the axis of the wire and (b) 4.0 mm from the axis of the wire is :-

(1) $2\pi \times 10^{-8}$ T, $\frac{9\pi}{2}\times10^{-8}$T (2) $4\pi \times 10^{-8}$ T, $\frac{\pi}{2}\times10^{-8}$T

(3) $4\pi \times 10^{-8}$ T, $\frac{9\pi}{2}\times10^{-8}$T (4) $\pi \times 10^{-4}$ T, $\frac{9\pi}{2}\times10^{-8}$T

PME238

5. Consider six wires coming into or out of the page, all with the same current. Rank the line integral of the magnetic field (from most positive to most negative) considering the sense of circulation in counterclockwise around each loop shown.

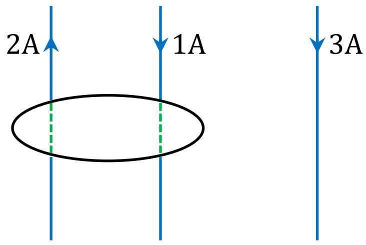


(1) B > C > D > A (2) B > C = D > A

(3) B > A > C = D (4) C > B = D > A

PME239

6. Two wires with currents 2 A & 1 A are enclosed in circular loop. Another wire with current 3 A is situated outside the loop as shown. The $\oint \vec{B}.\vec{d\ell}$ around the loop is :-



(1) μ_0 (2) $3\mu_0$ (3) $6\mu_0$ (4) $2\mu_0$

PME240

Magnetic field due to Solenoid

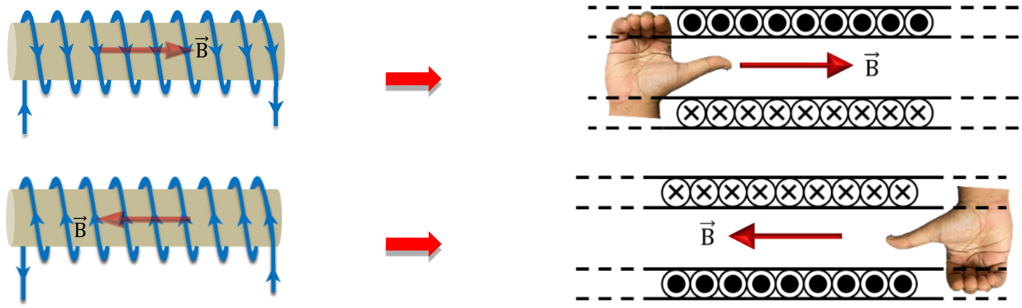
(i) It consists of a conducting wire which is tightly wound over a cylindrical frame in the form of helix. All the adjacent turns are electrically insulated to each other. The magnetic field at a point on the axis of a solenoid can be obtained by superposition of field due to large number of identical circular turns having their centres on the axis of solenoid.

(ii) Magnetic field is produced along axis of solenoid. Outside a long solenoid, magnetic field is almost Zero.

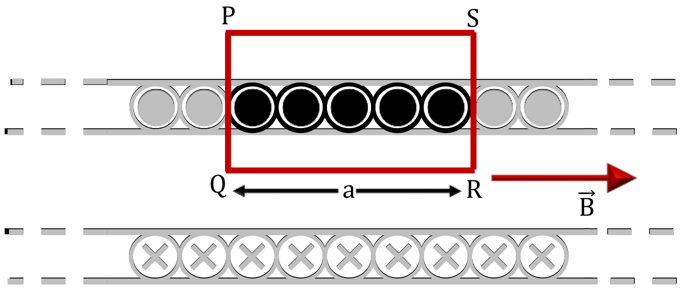
(iii) Direction of magnetic field inside a solenoid can be obtained by using right hand thumb rule.

(a)

(b)



Magnetic Field Inside a Long Solenoid



General Parameters of Solenoid

N : Number of turns

ℓ : Length of solenoid

n : Number of turns per unit length

Hence $n=\frac{N}{\ell}$

R : Radius of solenoid coil

For the loop PQRS, we have

$$\oint \vec{B}\cdot d\vec{\ell}=\int_{PQ}\vec{B}\cdot d\vec{\ell}+\int_{QR}\vec{B}\cdot d\vec{\ell}+\int_{RS}\vec{B}\cdot d\vec{\ell}+\int_{SP}\vec{B}\cdot d\vec{\ell}$$

Here, $\int_{PQ}\vec{B}.d\vec{\ell}=\int_{RS}\vec{B}.d\vec{\ell}=0$ $\left(\because \vec{B}\perp d\vec{\ell}\right)$ and $\int_{SP}\vec{B}.d\vec{\ell}=0$ $(\because \vec{B}$ outside the solenoid is negligible)

$$\int_{QR}\vec{B}\cdot d\vec{\ell}=B\int_{QR}d\ell=Ba$$

Now, $\oint \vec{B}\cdot d\vec{\ell}=Ba$

Current enclosed for loop PQRS, $\Sigma i=(n\times a)\times i$

From Ampere's Circuital Law for the loop PQRS, we have

$$\oint \vec{B}\cdot d\vec{\ell}=\mu_0\Sigma i$$

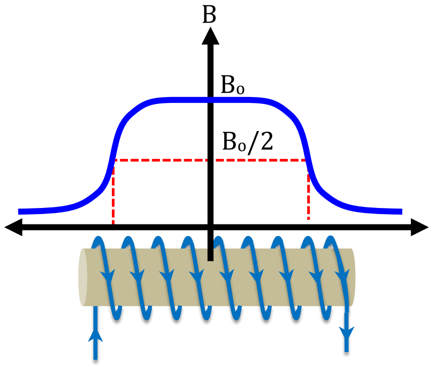
$$B\times a=\mu_0(n\times a\times i)$$

$$B=\mu_0 ni$$

Note: For a long solenoid length of solenoid is very large as compared to radius of solenoid i.e. ℓ>>r

Magnetic Field due to Long Solenoid

For a long solenoid, carrying current I



M.F. in the central region $B_0 = \mu_0 n I$ $\quad B_0 = \mu_0\left(\frac{N}{\ell}\right)I$

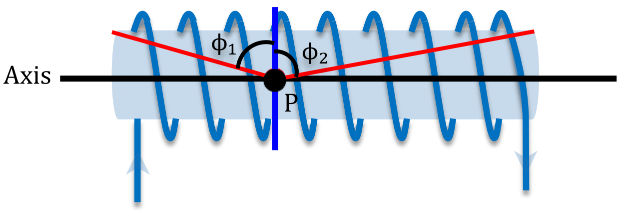
M.F. outside the solenoid $B = 0$

M.F. at the edges/end points $B = \frac{\mu_0 n I}{2}$

In presence of Core, $\mu \rightarrow \mu_0 \mu_r$

Where μ_r is the relative permeability of core material.

Magnetic Field due to Finite Length Solenoid



$$B_P = \frac{\mu_0 n I}{2}(\sin\phi_1 + \sin\phi_2)$$

Note: Magnetic field outside a finite length solenoid is not zero.

For long solenoid, in the middle region

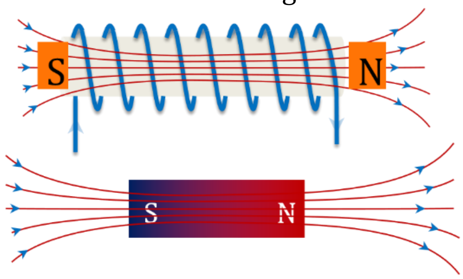
$$\phi_1, \phi_2 \rightarrow 90^\circ \quad \Rightarrow \quad B = \mu_0 n I\frac{(\sin 90^\circ + \sin 90^\circ)}{2} \quad \Rightarrow \quad B = \mu_0 n I$$

For long solenoid, in the end region

$$\phi_1, \phi_2 \rightarrow 0^\circ, 90^\circ \quad \Rightarrow \quad B = \mu_0 n I\frac{(\sin 0^\circ + \sin 90^\circ)}{2} \quad \Rightarrow \quad B = \frac{\mu_0 n I}{2}$$

Key Note

Magnetic Field produced by a solenoid resembles the Magnetic field of a Bar Magnet



So, a solenoid acts as a Magnetic Dipole and is used as electromagnet in many devices.

Illustration 29

The length of solenoid is 0.1m and its diameter is very small. A wire is wound over it in two layers. The number of turns in inner layer is 50 and that of outer layer is 40. The strength of current flowing in two layers in opposite direction is 3A. Then find magnetic induction at the middle of the solenoid.

Solution:

Direction of magnetic field due to both layers is opposite as direction of current is opposite, so

$$B_{net} = B_1 - B_2 = \mu_0 n_1 I_1 - \mu_0 n_2 I_2 = \mu_0\frac{N_1}{\ell}I - \mu_0\frac{N_2}{\ell}I \quad (\because I_1 = I_2 = I)$$

$$B_{net} = \frac{\mu_0 I}{\ell}(N_1 - N_2) = \frac{4\pi\times10^{-7}\times3}{0.1}(50-40) = 12\pi\times10^{-5}\,T$$

Illustration 30

A closely wound, solenoid 80 cm long has 5 layers of winding of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0A. Estimate the magnetic field (a) Inside the solenoid (b) Axial end points of the solenoid

Solution:

(a) Magnetic field inside the solenoid

$$B_{in} = \mu_0 nI = \mu_0 \frac{N}{\ell} I, (N = 400 \times 5 = 2000) = \frac{4\pi \times 10^{-7} \times 2000 \times 8}{(80 \times 10^{-2})} = 8\pi \times 10^{-3} T$$

(b) Magnetic field at axial end points of solenoid $B_{ends} = \frac{\mu_0 nI}{2} = \frac{8\pi \times 10^{-3}}{2} = 4\pi \times 10^{-3} T$

Illustration 31

A straight long solenoid produces magnetic field 'B' at its centre. If it is cut into two equal parts and same number of turns are wound on one part in double layer. Find magnetic field produced by new solenoid at its centre.

Solution:

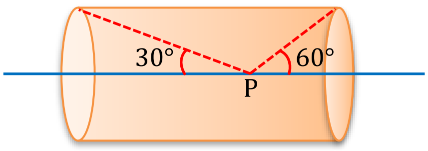
Magnetic field produced by a long solenoid is $B = \mu_0 nI$, where $n = N/\ell$

$\because$ Same number of turns wound over half length

$\therefore$ Magnetic field produced by new solenoid is $B' = \mu_0 \left(\frac{N}{\ell/2}\right) I = 2\left(\frac{\mu_0 NI}{\ell}\right) = 2B$

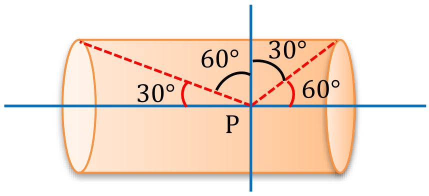
Illustration 32

Find out magnetic field at axial point 'P' of solenoid shown in figure(where turn density 'n' and current through it is I)



Solution:

Magnetic field at point 'P' due to finite length solenoid



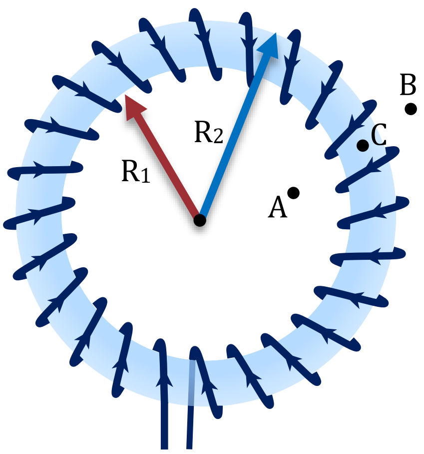
$$B_P = \frac{\mu_0 nI}{2}(\sin\phi_1 + \sin\phi_2)$$

$$= \frac{\mu_0 nI}{2}[\sin 30° + \sin 60°]$$

$$= \frac{\mu_0 nI}{2}\left[\frac{1}{2} + \frac{\sqrt{3}}{2}\right] = \frac{\mu_0 nI}{4}(1 + \sqrt{3})$$

Magnetic field due to Toroid

A toroid can be considered as a ring-shaped closed solenoid also called endless solenoid.
Toroid is a solenoid bent in ring shape.



Magnetic field inside a toroid by A.C.L. given as: –

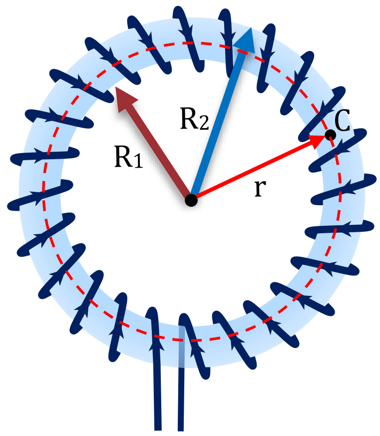
Point A: Magnetic field in the empty space surrounded by toroid ($r < R_1$)

Since $I_{enc} = 0 \Rightarrow B = 0$

Point B: Magnetic field outside the toroid ($r > R_2$)

Since $I_{enc} = NI - NI = 0 \Rightarrow B = 0$

Point C: Inside the Toroid ($R_1 \le r \le R_2$)



$$\oint \vec{B} \cdot d\vec{l} = \mu_o I_{enc}$$

$$B(2\pi r) = \mu_o(NI)$$

$$B = \mu_o \frac{N}{2\pi r} I$$

$$B = \mu_o nI$$

If r is not given then for magnetic field inside the toroid

$$n = \frac{N}{2\pi r_m}$$

$$\left[R_m = \frac{R_1 + R_2}{2} = \text{mean radius of toroid}\right]$$

Illustration 33

A toroid has a core (nonferromagnetic) of inner radius (R_1) 25 cm and outer radius (R_2) 26 cm around which 3500 turns of a wire are wound. If the current in the wire is 11 A, what is the magnetic field: (a) Outside the toroid (b) Inside the core of the toroid (c) In the empty space surrounded by the toroid.

Solution:

We are given that, mean radius of the toroid, i.e.,

$$r = \frac{25\text{cm} + 26\text{cm}}{2} = 25.5 \text{ cm} = 0.255 \text{ m}$$

total number of turns, N = 3500

current through the toroid, I = 11 A

Clearly number of turns per unit length of the toroid,

$$n = \frac{N}{2\pi r} = \frac{3500}{2\pi \times 0.255}$$

(a) Magnetic field outside the toroid is zero as it is non-zero inside its core.

(b) Magnetic field inside the core of the toroid, i.e.,

$$B = \mu_0 nI = \left[(4\pi \times 10^{-7})\left(\frac{3500}{2\pi \times 0.255}\right) \times 11\right] T = 3.0 \times 10^{-2} T$$

Note : Inside the toroid, $B \propto \frac{1}{r}$. The above value of B corresponds to mean radius 25.5 cm.

(c) In the empty space surrounded by the toroid, B = 0

Key Note

- Magnetic field of toroid is non-uniform because at each point direction of magnetic field is not same.
- It is not a Magnetic Dipole
- Magnetic field produced by a toroid is directed along its circular axis and constant in magnitude over its entire cross section.
- Magnetic field at the centre of toroid is always zero.

BEGINNER'S BOX-6

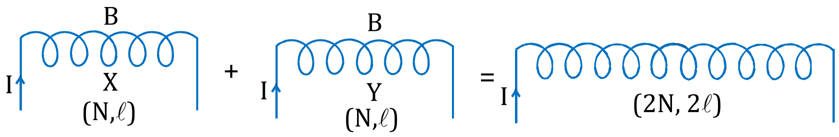
1. State the following statements are true or false :

(a) The magnetic field inside straight solenoid of finite length is always less than $\mu_0 nI$.

(b) The magnetic field of an infinitely long ideal solenoid of radius 'R' carrying current I is constant inside and zero outside.

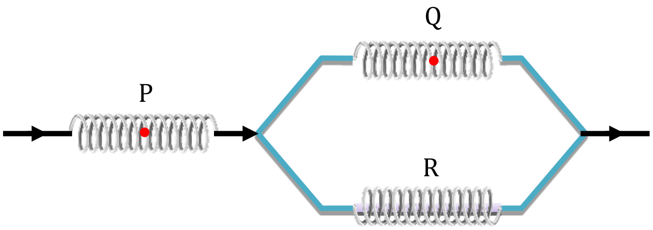
PME241

2. Two identical long solenoids X and Y carries equal current. The magnetic field produced at their axial mid point is B. If both the solenoids joined to each other according to figure. Find magnetic field at axial mid point of new solenoid.



PME242

3. Three identical long solenoids P, Q and R connected to each other as shown in figure. If the magnetic field at the centre of P is 2.0 T, what would be the field at the centre of Q?



PME243

4. For toroid of mean radius 20 cm, turns = 240 and a current of 2A. What is the value of magnetic field within toroid ?

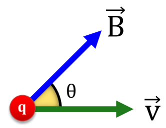
(1) 4.8×10^{-4} T (2) 2.4×10^{-4} T (3) 1.2×10^{-4} T (4) zero

PME244

Motion of Charge in Magnetic Field

Magnetic Force on Moving Charge

Consider a charge q moving with velocity $\vec{v}$ in uniform magnetic field $\vec{B}$ such that $\vec{v}$ makes an angle θ with $\vec{B}$, then magnetic force experienced by the charge is given by



$\vec{F} = q(\vec{v} \times \vec{B})$

$F = qvB \sin\theta$

Important points

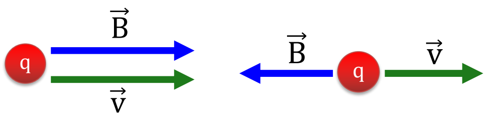
(1) Since magnetic force, $\vec{F} = q(\vec{v} \times \vec{B})$

$\vec{F} \perp \vec{v}$ $\quad \vec{F} \perp \vec{B}$

(2) If v = 0, then F = qvB sinθ = 0. So, there will be no magnetic force on stationary charge.

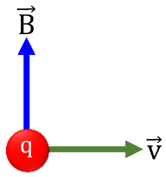
(3) Magnetic force depends on angle between $\vec{v}$ and $\vec{B}$

Case-1: θ = 0° or 180°



$\Rightarrow F = 0$

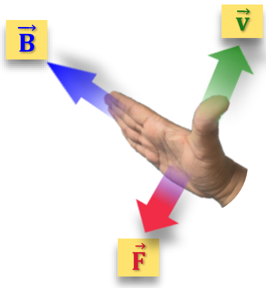
Case-2: θ = 90°



$\Rightarrow F_{max} = qvB$

(θ can vary from 0° to 180°)

(4) Direction of force can be identified by right hand thumb rule or right hand palm rule.

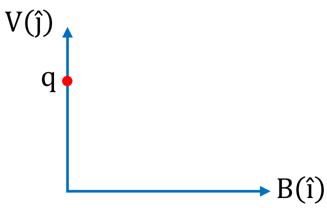


Right hand thumb rule : Point the fingers of right hand in the direction of $\vec{v}$, and curl them towards $\vec{B}$ then thumb will give the direction of $\vec{v} \times \vec{B}$

Note: For force on negative charge, just reverse the final direction.

Illustration 34

Find direction of & plane of motion.

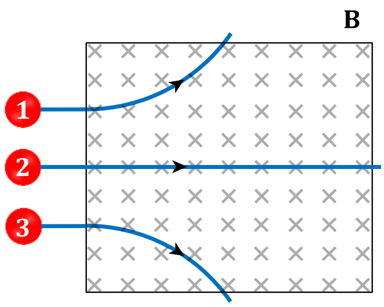


Solution:

$\vec{F} = q\left(\vec{v} \times \vec{B}\right) \Rightarrow \hat{j} \times \hat{i} = \left(-\hat{k}\right) \Rightarrow \vec{F} \rightarrow \left(-\hat{k}\right)$ $\quad \otimes$ y z plane

Illustration 35

From the path followed, identify nature of charge on the particle.



Solution:

(1) Positive charge = According to Right-Hand Rule positive charge will move in circular upward direction.

(2) Since F = 0 (No magnetic force, No deviation) $\Rightarrow$ Neutral (q = 0)

(3) Negative charge = According to Right-Hand Rule negative charge will move in circular downward direction.

Important

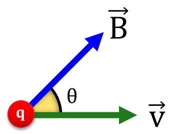
Work done by magnetic field on any charge is always **ZERO**!!

Quick questions: In presence of external Magnetic field(only)-

1.	Can direction of a moving charge change	Yes
2.	Can velocity of a moving charge change	Yes
3.	Can a stationary charge start moving	No

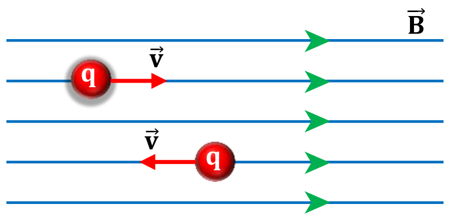
Motion of Charged Particle in Uniform Magnetic Field

Based on angle between $\vec{v}$ and $\vec{B}$, charge can move on three type of paths in Uniform Magnetic Field



Case 1	**Case 2**	**Case 3**
$\theta = 0°/180°$	$\theta = 90°$	$\theta \neq 0°, 90°, 180°$
Straight Line Path	Circular Path	Helical Path

Case 1:

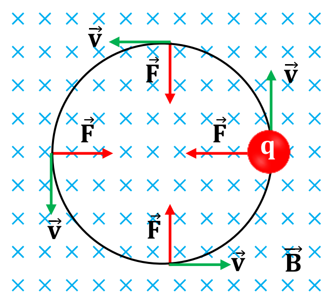


$\theta = 0°$	$\theta = 180°$
$F = qvB \sin 0° = 0$	$F = qvB \sin 180° = 0$
Straight Line Path	Straight Line Path

Case 2: Motion of charge particle in uniform transverse magnetic field

When a charge +q projected in uniform transverse magnetic field $\left(\theta = 90°, \vec{v} \perp \vec{B}\right)$ then maximum magnetic force of constant magnitude always acts perpendicular to its direction of motion so the charge moves along circular path and required centripetal force is provided by the magnetic force.

Magnetic force on moving charge given as



$$\vec{F}_m = q\left(\vec{v} \times \vec{B}\right)$$

$$F_m = qvB\sin 90° = qvB$$

Since centripetal force is provided by magnetic force

$$F_m = F_{cp}$$

$$qvB = \frac{mv^2}{r}$$

$$R = \frac{mv}{qB} \qquad \text{(R : Radius of curvature)}$$

Now, momentum, p = mv

$$R = \frac{p}{qB} = \frac{\sqrt{2mK}}{qB} \qquad \text{(K : Kinetic energy)}$$

If a charge particle comes in motion from rest, due to an accelerating potential difference V_{acc}

Then, its KE, $K = qV_{acc}$

$$\text{So, } R = \frac{\sqrt{2mqV_{acc}}}{qB} \quad \Rightarrow \quad R = \frac{1}{B}\sqrt{\frac{2mV_{acc}}{q}}$$

Time period (T) :–

Time period of charge particle of a circular motion in uniform transverse magnetic field given as

$$T=\frac{2\pi R}{v} \text{ where } R=\frac{mv}{qB}$$

$$\text{So, } T=\frac{2\pi}{v}\left(\frac{mv}{qB}\right)=\frac{2\pi m}{qB}$$

$$\text{Frequency, } f=\frac{1}{T}=\frac{qB}{2\pi m}$$

$$\text{Angular frequency, } \omega=\frac{qB}{m}$$

Note:

- Time period does not depend on velocity or Radius!!
- It depends on ratio of charge to mass of the particle

Particle	Charge	Mass	q/m
electron	e	$m_e = 9.1 \times 10^{-31}$ kg	0.17×10^{12} C/kg
proton	e	$m_p = 1.67 \times 10^{-27}$ kg	0.95×10^{8} C/kg
neutron	0	$m_p \approx m_p$	0
deuteron	e	$2m_p$	0.47×10^{8} C/kg
α-particle	2e	$4m_p$	0.47×10^{8} C/kg

Charge to mass ratio is known as **specific charge.**

Illustration 36

Proton, deuteron and an α particle are moving $\perp$ to B then find the ratio of their radii if they are moving :-

1. with Same speed
2. with Same momentum
3. with Same Kinetic Energy
4. through Same accelerating potential

Solution:

Particle	Mass	Charge
Proton	m_p	+e
Deutron	$2m_p$	+e
Alpha	$4m_p$	+2e

1. $R=\frac{mv}{qB}$; $R\propto\frac{m}{q}$

$$R_1:R_2:R_3=\frac{m_1}{q_1}:\frac{m_2}{q_2}:\frac{m_3}{q_3}=1:2:2$$

2. $R=\frac{p}{qB}$; $R\propto\frac{1}{q}$ $(p\rightarrow \text{same})$

$$R_1:R_2:R_3=\frac{1}{q_1}:\frac{1}{q_2}:\frac{1}{q_3}=1:1:\frac{1}{2}=2:2:1$$

3. $R=\frac{\sqrt{2mK}}{qB}$; $R\propto\frac{\sqrt{m}}{q}$ $(K\rightarrow \text{same})$

$$R_1:R_2:R_3=\frac{\sqrt{m_1}}{q_1}:\frac{\sqrt{m_2}}{q_2}:\frac{\sqrt{m_3}}{q_3}=1:\sqrt{2}:1$$

4. $R=\frac{1}{B}\sqrt{\frac{2mV_{acc}}{q}}$; $R\propto\sqrt{\frac{m}{q}}$ $(V_{acc.}\rightarrow \text{same})$

$$R_1:R_2:R_3=\sqrt{\frac{m_1}{q_1}}:\sqrt{\frac{m_2}{q_2}}:\sqrt{\frac{m_3}{q_3}}=1:\sqrt{2}:\sqrt{2}=1:\sqrt{2}:\sqrt{2}$$

Illustration 37

A Deuteron of kinetic energy 100 keV is describing a circular orbit of radius 0.5 metre in a plane perpendicular to magnetic field $\vec{B}$. The kinetic energy of the proton that describes a circular orbit of radius 0.5 metre in the same plane with the same $\vec{B}$ is

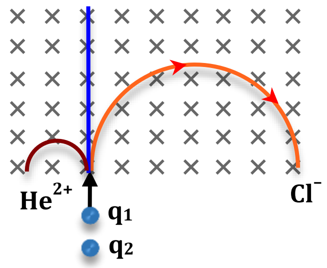
Solution:

$$r=\frac{\sqrt{2mK}}{qB}\Rightarrow K\propto\frac{q^2}{m}\Rightarrow\frac{K_p}{K_d}=\left(\frac{q_p}{q_d}\right)^2\times\frac{m_d}{m_p}=\left(\frac{1}{1}\right)^2\times\frac{2}{1}=\frac{2}{1}\Rightarrow k_p=2\times100=200\text{ keV}.$$

Illustration 38

He^{2+} and Cl^-, both are projected with same momentum in a transverse magnetic field. Which one will have more curvature?

Solution:



$$R\propto\frac{m}{q}$$

$$\text{Curvature (C)}\propto\frac{1}{\text{Radius of Curvature (R)}}$$

$$\left(\frac{m}{q}\right)_{Cl^-}>\left(\frac{m}{q}\right)_{He^{2+}}$$

So, He^{2+} has more curvature.

Illustration 39

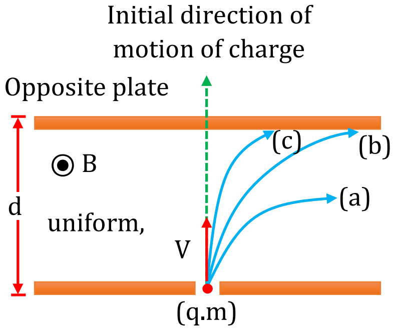
Proton, deuteron and an α particle are moving perpendicular to B then find the ratio of their time periods.

Solution:

Since, $T=\dfrac{2\pi m}{qB}, T\propto\dfrac{m}{q}$

$$T_P:T_D:T_\alpha=\frac{m_P}{q_P}:\frac{m_D}{q_D}:\frac{m_\alpha}{q_\alpha}=\frac{m}{q}:\frac{2m}{q}:\frac{4m}{2q}=1:2:2$$

A charge (q,m) is projected from **field free region to field region** in such a way that its velocity (v) is perpendicular to the magnetic field (B). The charge deflected from its initial direction of motion and trace a part of circle according to following conditions: -



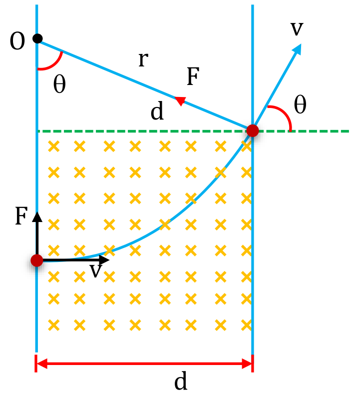
(a) Charge does not strike to the opposite plate and completes the semi-circle ⇒ r < d

(b) Charge does not strike to the opposite plate and just completes the semi-circle ⇒ r = d

(c) Charge strikes to the opposite plate and does not completes the semi-circle ⇒ r > d

Illustration 40

A particle of mass m and charge q is projected into a region having a perpendicular uniform magnetic field B of width $d < \frac{mv}{qB}$. Find the angle of deviation θ of the particle as it comes out of the magnetic field.



Solution:

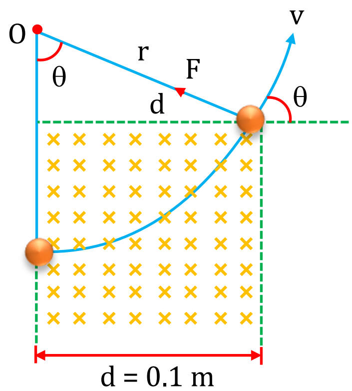
The radius of the circular orbit is $r = \frac{mv}{qB}$

The deviation θ may be obtained from the fig. as

$\sin\theta = \frac{d}{r} = \frac{dBq}{mv}$ or $\theta = \sin^{-1}\left(\frac{dBq}{mv}\right)$

Illustration 41

A proton accelerated by a potential difference 500 kV moves though a transverse magnetic field of 0.51T as shown in figure. The angle θ through which the proton deviates from the initial direction of its motion, is



Solution:

According to following figure, $\sin\theta = \frac{d}{r}$

Also, $r = \frac{\sqrt{2mK}}{qB} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$

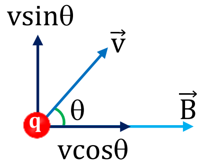
$\therefore\ \sin\theta = Bd\sqrt{\frac{q}{2mV}}$

$$= 0.51 \times 0.1\sqrt{\frac{1.6\times10^{-19}}{2\times1.67\times10^{-27}\times500\times10^{3}}} = \frac{1}{2} \Rightarrow \theta = 30°$$

Case 3: Motion of Charge Particle in Oblique Magnetic Field ($\theta \neq 0°, 90°, 180°$)

Consider a charge 'q' of mass 'm' moving with velocity $\vec{v}$ at an angle θ with the direction of magnetic field $\vec{B}$.

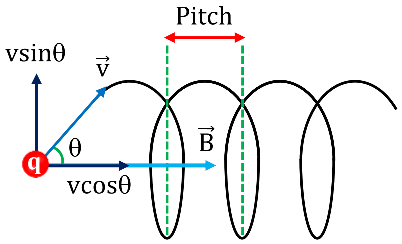
In this situation resolving the velocity of charge along and perpendicular to the field, we find that the particle moves with constant velocity vcosθ along the field (along x-axis) and at the same time it is also moving with velocity vsinθ perpendicular to the field due to which it will describe a circle in a plane perpendicular to the field (y–z plane).

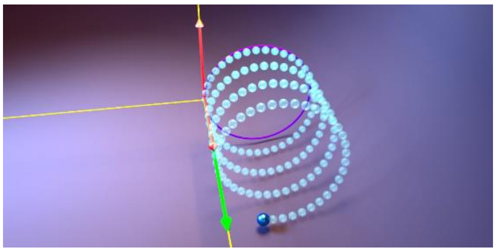


$v\sin\theta \perp \vec{B} \ \Rightarrow$ Circular motion

$v\cos\theta \,||\, \vec{B} \ \Rightarrow$ Straight Line motion

So in the combined effect of both the component of velocity, charge will move along 'helical path' and the curve is called 'helix'.





Axis of helix

It is a straight line along external magnetic field which joins all the centres of circular turns, called axis of helix.

Radius of Circular Path

Centripetal force required to move the charge in a circle is provided by the magnetic force $F_m = F_{cp}$

$$q(v\sin\theta)B\sin 90° = \frac{m(v\sin\theta)^2}{R} \Rightarrow \boxed{R = \frac{mv\sin\theta}{qB}}$$

or

$$R = \frac{mv_\perp}{qB} \Rightarrow R = \frac{mv\sin\theta}{qB}$$

Time period of circular motion

$$T = \frac{2\pi R}{v\sin\theta}, \text{ where } R = \frac{mv\sin\theta}{qB}$$

$$T = \frac{2\pi}{v\sin\theta}\left(\frac{mv\sin\theta}{qB}\right) \Rightarrow \boxed{T = \frac{2\pi m}{qB}}$$ (Time does not depend on v)

Pitch of helix (p)

The linear distance travelled by the charge particle in one revolution or in one time period along external magnetic field direction is called 'pitch of helix'.

Displacement in T time

$$p = (v\cos\theta)T, \text{ where } T = \frac{2\pi m}{qB} \Rightarrow \boxed{p = \frac{2\pi mv\cos\theta}{qB}}$$

Illustration 42

A charge (q, m) projected with velocity $\vec{v} = 3\hat{i} + 4\hat{j}$ m/s in external magnetic field $\vec{B} = 6\hat{i}\,T$. Find

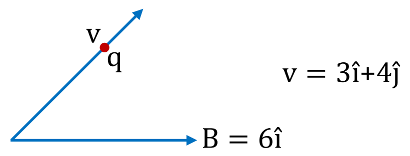
(i) Radius of circular path (ii) Pitch (iii) Time period

Solution:

$$\sin\theta = \frac{4}{5}$$

Radius of circular path.

$$R = \frac{mv\sin\theta}{qB} \Rightarrow R = \frac{m \times 5 \times \frac{4}{5}}{q \times 6}$$



$$\text{Time period} = \frac{2\pi m}{6q}$$

Pitch = p = $(v\cos\theta)T$

$$p = 3 \times \frac{2\pi m}{6q}$$

Illustration 43

A charged particle enters a uniform magnetic field with velocity vector at an angle of 45° with the magnetic field. The pitch of the helical path followed by the particle is p.

The radius of the helix will be

Solution:

$$p=\frac{2\pi m}{Bq}(v\cos 45°)=\frac{2\pi m}{Bq}(v\sin 45°)$$

$$\therefore\ \frac{mv\sin 45°}{Bq}=\frac{p}{2\pi} = \text{radius of helix}$$

Lorentz Force

When a charge is moving in a region, where both electric field $\vec{E}$ and magnetic field $\vec{B}$ exist, then electric and magnetic forces are acting on it. The resultant of these forces called electromagnetic or Lorentz force on charge. (zero gravity)

Net Electromagnetic force is known as Lorentz force.

$$\vec{F}_{net}=\vec{F}_m+\vec{F}_e$$

$$\vec{F}_{net}=q\left(\vec{v}\times\vec{B}\right)+\left(q\vec{E}\right)$$

- If $F_{net} = 0$

  Then v = constant

Note : If $\vec{E}\neq 0$, $\vec{B}\neq 0$

and $\vec{F}_{net}=\vec{0}$

$$q\left(\vec{v}\times\vec{B}\right)+\left(q\vec{E}\right)=\vec{0}$$

$$\vec{E}=-\left(\vec{v}\times\vec{B}\right)$$

for both positive and negative charges

Illustration 44

A particle of charge –1.6 × 10^{-18} coulomb moving with velocity 10 ms^{-1} along the x-axis enters a region where a magnetic field of induction B is along the y-axis, and an electric field of magnitude 10^4 V/m is along the negative z-axis. If the charged particle continues moving along the x-axis, the magnitude of B is

Solution:

Since particle is moving undeflected.

So $qE=qvB\Rightarrow B=E/v=\frac{10^4}{10}=10^3\,Wb/m^2$

Illustration 45

An electron (mass = 9.1 × 10^{-31} kg; charge = 1.6 × 10^{-19} C) experiences no deflection if subjected to an electric field of 3.2 × 10^5 V/m, and a magnetic field of 2.0 × 10^{-3} Wb/m^2. Both the fields are normal to the path of electron and to each other. If the electric field is removed, then the electron will revolve in an orbit of radius

Solution:

For no deflection in mutually perpendicular electric and magnetic field $v=\frac{E}{B}=\frac{3.2\times10^5}{2\times10^{-3}}=1.6\times10^8\,m/s$.

If electric field is removed then due to only magnetic field radius of the path described by electron

$$r=\frac{mv}{qB}=\frac{9.1\times10^{-31}\times1.6\times10^8}{1.6\times10^{-19}\times2\times10^{-3}}=0.45\,m$$

Illustration 46

A particle having charge +5C moves with $\vec{v}=3\hat{j}$ m/s in an electromagnetic field. If $\vec{B}=3\hat{i}+2\hat{j}$ T, and $\vec{E}=2\hat{k}$, then find net electromagnetic force on the particle.

Solution:

Net Electromagnetic Force = Magnetic Force + Electric Force = $q\left(\vec{v}\times\vec{B}\right)+q\vec{E}$

$$= 5(-9\hat{k})+5(2\hat{k}) = -35\hat{k}$$

BEGINNER'S BOX-7

1. Three charge proton, deutron and α–particle are projected in uniform transverse magnetic field in such a way that their radii of circular tracks are equal then find their following ratio respectively :-

(a) $v \rightarrow$ ratio (b) $P \rightarrow$ ratio (c) $E_K \rightarrow$ ratio (d) $V_{acc.} \rightarrow$ ratio

PME245

2. A proton and an electron are projected in uniform transverse magnetic field, then trajectory of which charge is more curved or deflection of which charge is more from its initial direction of motion, if their

(a) $v \rightarrow$ same (b) $P \rightarrow$ same (c) $E_K \rightarrow$ same (d) $V_{acc.} \rightarrow$ same

PME246

3. Three charge proton, deutron and α–particle are projected in uniform transverse magnetic field then find ratio of following parameters of circular motion respectively.

(a) Time period (b) Frequency (c) Angular frequency

PME247

4. A charge is released from rest in a region of steady and uniform electric and magnetic fields which are parallel to each other. The charge will moves along which path :-

(1) Circular (2) Helical (3) Parabola (4) Straight line

PME248

5. A beam of proton moving with velocity 4×10^5 m/sec enters in a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Calculate radius of helical path and pitch of helix.

PME249

6. A proton is projected with a speed of 2×10^6 m/sec at an angle of 60° to the x-axis. If a uniform magnetic field of 0.104T is applied along y axis. Calculate radius of helix and time period of proton.

PME250

7. A charge projected in uniform magnetic field at angle 30° to the field direction. The time period of circular motion of helical path is 'T'. If angle of projection becomes 60° then what is the new time period.

PME251

8. In a region a uniform magnetic field acts in horizontal plane towards north. If cosmic particles (80% protons) falling vertically downwards, then they are deflected towards–

(1) North (2) South (3) East (4) West

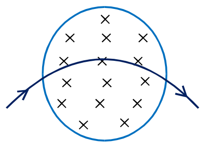
PME252

9. An electron is moving along +x direction. To get it moving along an anticlockwise circular path in x–y plane, magnetic field applied along :-

(1) +y–direction (2) +z–direction (3) –y–direction (4) –z–direction

PME253

10. There is a magnetic field acting in a plane perpendicular downwards. A particle in vacuum moves in the plane of paper from left to right as shown in figure. The path indicated by the arrow could be due to –



(1) Proton (2) Neutron

(3) Electron (4) Alpha particle

PME254

11. A proton is moving towards north along horizontal line passes through a zero gravity region, where electric and magnetic field are mutually perpendicular to each other. The path of particle remains undeviated or undeflected. If the direction of electric field is towards east then direction of magnetic field is

(1) Towards east (2) Vertically downwards

(3) Towards west (4) Vertically upwards

PME255

12. A proton moving along z-axis with constant velocity. If a magnetic field is applied along x-axis then direction of magnetic force on proton

(1) along z-axis (2) along y-axis

(3) along x-axis (4) zero force

PME256

13. A vertical wire carries a current in upward direction. If an electron beam sent horizontally towards the wire, then it will deflected

(1) vertically downwards and perpendicular to the plane of the paper

(2) vertically upwards and perpendicular to the plane of the paper

(3) In the plane of the paper

(4) No deflection

PME257

14. If a charge projected in a zero gravity region. Find possible cases of electric and magnetic field in that region so no net force exerts on charge.

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$ (c) $E \neq 0, B \neq 0$ (d) $E \neq 0, B = 0$

PME258

15. If a projected charge passes through a zero gravity region without change in its velocity, then find possible cases of electric and magnetic field in that region

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$ (c) $E \neq 0, B \neq 0$ (d) $E \neq 0, B = 0$

PME259

16. If a projected charge passes through a zero gravity region, remains undeviated or undeflected, then find possible cases of electric field and magnetic field in that region

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$ (c) $E \neq 0, B \neq 0$ (d) $E \neq 0, B = 0$

PME260

17. If a projected charge passes through a zero gravity region, its speed may be constant, then find possible cases of electric and magnetic field in that region

(a) $E = 0, B = 0$ (b) $E = 0, B \neq 0$ (c) $E \neq 0, B \neq 0$ (d) $E \neq 0, B = 0$

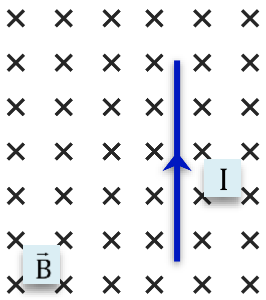
PME261

Magnetic Force on a Current Carrying Wire

Current Carrying wire in Uniform Magnetic Field

When a current carrying conductor placed in magnetic field, a magnetic force exerts on each free electron which are present inside the conductor. The resultant of these forces on all the free electrons is called magnetic force on conductor.

Force on a charge element dq moving in MF.



$$d\vec{F} = dq\left(\vec{v}\times\vec{B}\right)$$

$$d\vec{F} = dq\left(\frac{d\vec{\ell}}{dt}\times\vec{B}\right)$$

$$d\vec{F} = \frac{dq}{dt}\left(d\vec{\ell}\times\vec{B}\right)$$

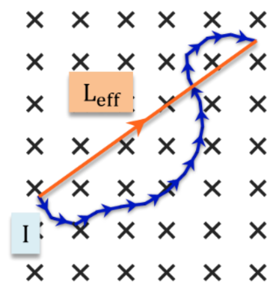
$$d\vec{F} = I\left(d\vec{\ell}\times\vec{B}\right)$$

When current element $Id\vec{\ell}$ is placed in magnetic field $\vec{B}$, it experiences a magnetic force

$$d\vec{F} = I(d\vec{\ell}\times\vec{B})$$

- Net force on wire due to all such current elements

  $$\vec{F} = I\int\left(d\vec{\ell}\times\vec{B}\right)$$



- If B is uniform,

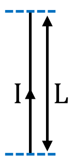
  $$\vec{F} = I\left(\int d\vec{\ell}\right)\times\vec{B}$$

  $$\vec{F} = I\left(\vec{L}_{eff}\times\vec{B}\right)$$

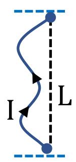
  L_{eff} is the displacement vector from starting point of current to end point of current.

  Example

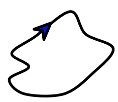
  For straight wire, $|L_{eff}| = L$

  Arbitrary shape current carrying conductor $|L_{eff}| = L$

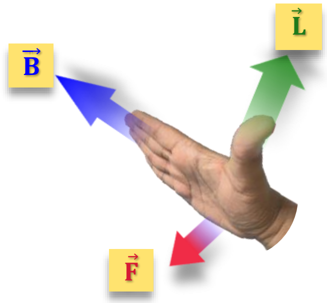
  

  For loop, $|L_{eff}| = 0$

- Current element in a magnetic field does not experience any force if the current in it is parallel or anti–parallel with the field $\theta = 0°$ or $180°$ $dF = 0$ (min.)
- Current element in a magnetic field experiences maximum force if the current in it is perpendicular with the field $\theta = 90°$ $dF = BId\ell$ (max.)
- Magnetic force on current element is always perpendicular to the current element vector and magnetic field vector. $d\vec{F}\perp Id\vec{\ell}$ and $d\vec{F}\perp\vec{B}$ (always)

Direction of magnetic force



Magnetic force on the wire we can find by Right Hand Palm Rule

Fingers $\rightarrow$ In direction of external Magnetic field

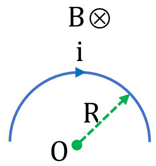
Thumb $\rightarrow$ In direction of $\vec{L}$

$\perp$ to Palm $\rightarrow$ Direction of magnetic force on wire

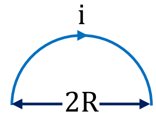
Note : For straight wire, direction of $\vec{L}$ is in the direction of current flow.

Illustration 47

Find force on given wire in uniform magnetic field



Solution:



$\Rightarrow \quad \vec{F}_m = I\left(\vec{L}_{eff} \times \vec{B}\right) = I(L_{eff})B$

$= I(2R)B \qquad$ (Upward)

Illustration 48

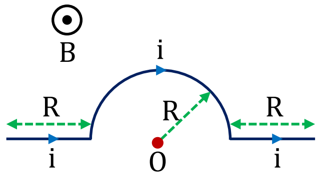
A current of 10 amperes is flowing in a wire of length 1.5 m. A force of 15 N acts on it when it is placed in a uniform magnetic field of 2 tesla. The angle between the magnetic field and the direction of the current is

Solution:

$$F = Bil\sin\theta \Rightarrow \sin\theta = \frac{F}{Bil} = \frac{15}{2\times10\times1.5} = \frac{1}{2} \Rightarrow \theta = 30°$$

Illustration 49

A wire bent as shown in fig carries a current i and is placed in a uniform field of magnetic induction $\vec{B}$ that emerges from the plane of the figure. Calculate the force acting on the wire.

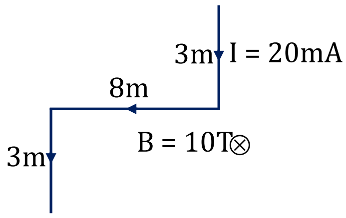


Solution:

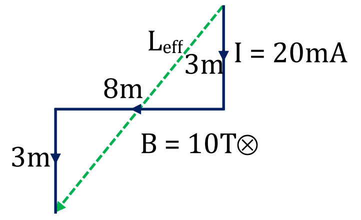
The total force on the whole wire is $F_m = I|\vec{L}|B = I(R + 2R + R)B = 4RIB$

Illustration 50

Find force on given wire.



Solution:



L_{eff} = 10 m

$F_m = IL_{eff}\,B = 20 \times 10^{-3} \times 10 \times 10 = 2N$

Current Carrying Loop in Uniform Magnetic Field

Force : Net force on a closed loop(any shape) in uniform magnetic field is always ZERO.

Tension Developed in the wire :

T = RBI

If length of wire is given

$$\ell = 2\pi R \quad \Rightarrow \quad R = \frac{\ell}{2\pi}$$

Note: Due to Tension in wire, loop of any shape becomes circular in shape.

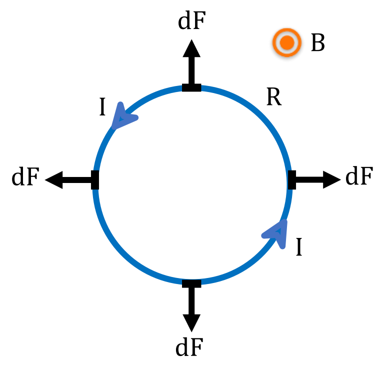


Illustration 51

A loop of flexible conducting wire of length 6.28 cm lies in magnetic field 2T is normal to plane of loop. A current 2A is passed through loop. The tension developed in wire to open up is

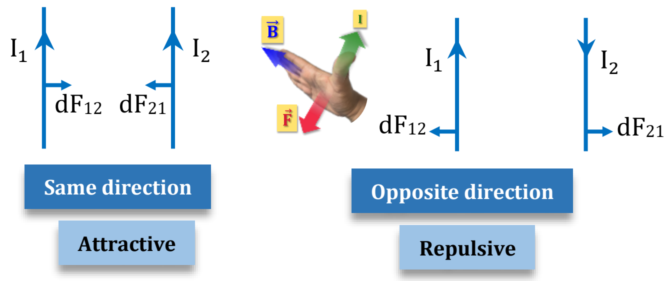
Solution:

T = RBI

$$\therefore \{\ell = 2\pi R\} = \frac{\ell}{2\pi}BI = \frac{6.28 \times 10^{-2} \times 2 \times 2}{2\pi} = 4 \times 10^{-2}\,N$$

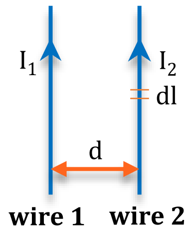
Magnetic Force Between Two Parallel Current Carrying Wires

Nature of Force



Magnitude of Force

The net magnetic force acts on a current carrying conductor due to its own field is zero. So consider two infinite long parallel conductors separated by distance 'd' carrying currents I_1 and I_2.



Magnetic field at each point on conductor (ii) due to current I_1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi d} \text{ [uniform field for conductor (2)]}$$

Magnetic field at each point on conductor (i) due to current I_2 is

$$B_2 = \frac{\mu_0 I_2}{2\pi d} \text{ [Uniform field for conductor (1)]}$$

Consider a small element of length '$d\ell$' on each conductor. These elements are right angle to the external magnetic field, so magnetic force experienced by elements of each conductor given as

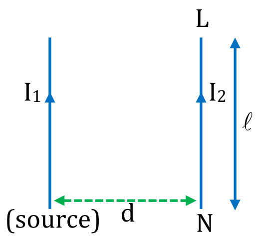
$$dF_{12} = B_2 I_1 \, d\ell = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1 d\ell \qquad ...(i) \qquad (\text{Where } I_1 d\ell \perp B_2)$$

$$dF_{21} = B_1 I_2 \, d\ell = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2 \, d\ell \qquad ...(ii) \qquad (\text{Where } I_2 \, d\ell \perp B_1)$$

Where dF_{12} is magnetic force on element of conductor (i), due field of conductor (ii) and dF_{21} is magnetic force on element of conductor (ii), due to field of conductor (i).

Magnetic force per unit length of each conductor is $\frac{dF_{12}}{d\ell} = \frac{dF_{21}}{d\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$

$f = \frac{\mu_0 I_1 I_2}{2\pi d}$ N/m (in S.I.) $\quad f = \frac{2I_1 I_2}{d}$ dyne/cm (In C.G.S.)



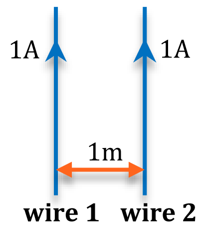
Force scale $f = \frac{\mu_0 I_1 I_2}{2\pi d}$ is applicable when at least one conductor must be of infinite length, so it behaves like source of uniform magnetic field for other conductor.

Magnetic force on conductor 'LN' is $F_{LN} = f \times \ell \Rightarrow F_{LN} = \left(\frac{\mu_0 I_1 I_2}{2\pi d}\right)\ell$

Definition of Ampere

Magnetic force/unit length for both infinite length conductor gives as

$$f = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{(4\pi \times 10^{-7})(1)(1)}{2\pi(1)} = 2 \times 10^{-7} \text{ N/m}$$



'Ampere' is the current which, when passed through each of two parallel infinite long straight conductors placed in free space at a distance of 1 m from each other, produces between them a force of 2×10^{-7} N/m

Illustration 52

Two long and parallel wires are at a distance of 0.1 m and a current of 5 A is flowing in each of these wires. The force per unit length due to these wires will be

Solution:

$$F = \frac{\mu_0}{4\pi}\frac{2 \times i_1 i_2}{a} = \frac{10^{-7} \times 2 \times 5 \times 5}{0.1} = 5 \times 10^{-5} \text{ N/m}$$

Illustration 53

A current of 5A flows through two long parallel wires. The magnetic force on each wire is 4 × 10^{-3} N/m. If their currents makes half and separation between them makes double then magnetic force per unit length of each wire becomes :-

Solution:

Magnetic force per unit length

$$f_m = \frac{\mu_0 I_1 I_2}{2\pi d} \qquad ...(1)$$

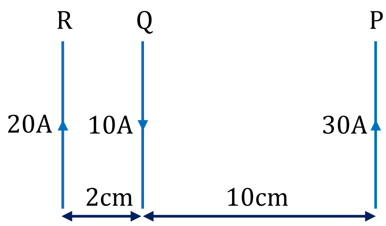
$$f_m' = \frac{\mu_0\left(\frac{I_1}{2}\right)\left(\frac{I_2}{2}\right)}{2\pi(2d)} = \frac{\mu_0 I_1 I_2}{2\pi d}\times\frac{1}{8} \qquad ...(2)$$

From (1) & (2)

$$f'_m = \frac{f_m}{8} = \frac{4\times10^{-3}}{8} = 0.5\times10^{-3}\,N/m$$

Illustration 54

Three long, straight and parallel wires carrying currents are arranged as shown in figure. The force experienced by 10 cm length of wire Q is



Solution:

Force on wire Q due to wire P is

$$F_P = 10^{-7}\times\frac{2\times30\times10}{0.1}\times0.1 = 6\times10^{-5}N \quad \text{(Towards left)}$$

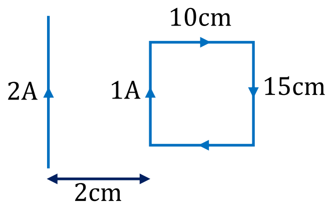
Force on wire Q due to wire R is

$$F_R = 10^{-7}\times\frac{2\times20\times10}{0.02}\times0.1 = 20\times10^{-5}N \quad \text{(Towards right)}$$

Hence $F_{net} = F_R - F_P = 14 \times 10^{-5}$ N = 1.4 × 10^{-4} N (Towards right)

Illustration 55

What is the net force on the square coil

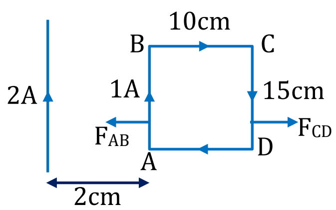


Solution:

Force on side BC and AD are equal but opposite so their net will be zero.

But $F_{AB} = 10^{-7}\times\frac{2\times2\times1}{2\times10^{-2}}\times15\times10^{-2} = 3\times10^{-6}N$

and $F_{CD} = 10^{-7}\times\frac{2\times2\times1}{(12\times10^{-2})}\times15\times10^{-2} = 0.5\times10^{-6}N$

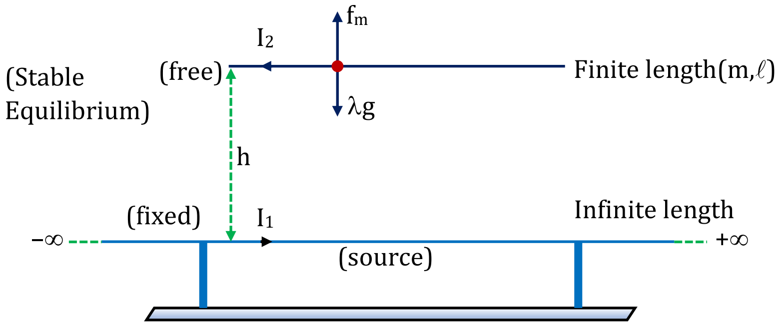


$\Rightarrow F_{net} = F_{AB} - F_{CD} = 2.5 \times 10^{-6}$ N

$= 25\times10^{-7}N$, towards the wire.

Equilibrium of free wire

Case I : Upper wire is free : Consider a long horizontal wire which is rigidly fixed another wire is placed directly above and parallel to fixed wire.

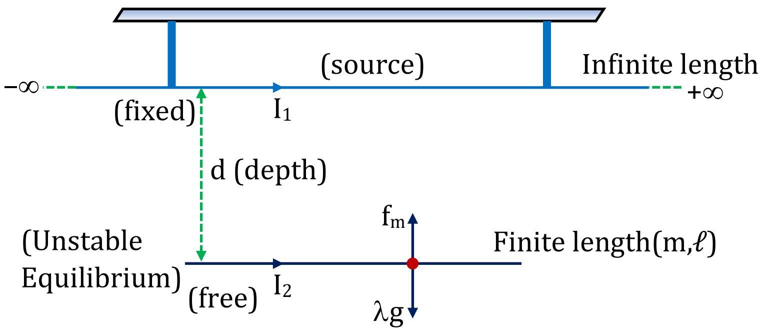


Magnetic force per unit length of free wire $f_m = \frac{\mu_0 I_1 I_2}{2\pi h}$, and it is repulsive in nature because currents are unlike.

Free wire may remains suspended if the magnetic force per unit length is equal to weight of its unit length

At balanced condition $f_m = W'$. Weight per unit length of free wire $= \frac{\mu_0 I_1 I_2}{2\pi h} = \frac{m}{\ell} g$ (stable equilibrium condition)

Case II : Lower wire is free : Consider a long horizontal wire which is rigidly fixed. Another wire is placed directly below and parallel to the fixed wire.



Magnetic force per unit length of free wire is $f_m = \frac{\mu_0 I_1 I_2}{2\pi d}$, and it is attractive in nature because currents are like.

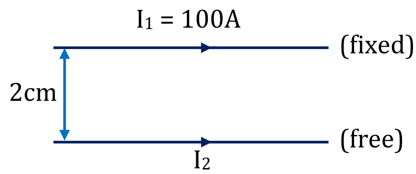
Free wire may remains suspended if the magnetic force per unit length is equal to weight of its unit length

At balanced condition $f_m = W'$

Weight per unit length of free wire $\frac{\mu_0 I_1 I_2}{2\pi d} = \frac{m}{\ell} g$ (unstable equilibrium condition)

Illustration 56

A long horizontal wire is rigidly fixed and carries 100A current. Another wire of linear mass density 2×10^{-3} kg/m placed below and parallel to the fixed wire. If the free wire kept 2cm below and hangs in air, then current in free wire is:-



Solution:

At balanced condition of free wire :-

$f_m = \lambda g$

$\frac{\mu_0 I_1 I_2}{2\pi d} = \lambda g$

$I_2 = \frac{2\pi\lambda g d}{\mu_0 I_1}$ $(\lambda = 2 \times 10^{-3}$ kg/m$) = \frac{2 \times 3.14 \times 2 \times 10^{-3} \times 9.8 \times 2 \times 10^{-2}}{4\pi \times 10^{-7} \times 100}$ = 19.6 A (in free wire)

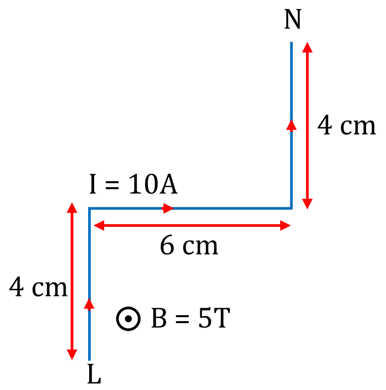
BEGINNER'S BOX-8

1. A wire of length 5 cm is placed inside the solenoid near its centre such that it makes an angle of 30° with the axis of solenoid. The wire carries a current of 5A and the magnetic field due to solenoid is 2.5×10^{-2} T. Magnetic force on the wire is :-

(1) 3.12×10^{-4} N (2) 31.2×10^{-4} N (3) 312×10^{-4} N (4) 0.312×10^{-4} N

PME262

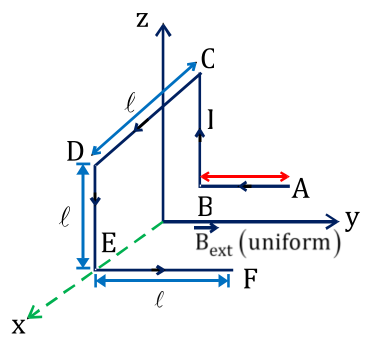
2. A wire 'LN' bent as shown in figure is placed in uniform perpendicular magnetic field of 5T. A 10A current flows through the wire. Magnetic force experienced by the wire is :-



(1) 5N (2) 10N (3) 2.5 N (4) 1.25 N

PME263

3. A wire ABCDEF with each side of length 'ℓ' bent as shown in figure and carrying a current I. If it is placed in a uniform magnetic field B which is parallel to +y direction. Magnetic force experienced by the wire is:-

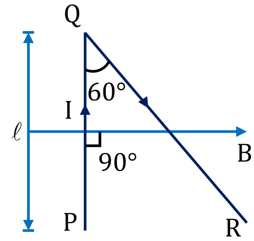


(1) $BI\ell$, along + z direction (2) $BI\ell$, along –z direction

(3) $2BI\ell$, along + z direction (4) $2BI\ell$, along –z direction

PME264

4. A current 'I' is flowing through wire PQR. This wire is bent in form of an angle and placed is uniform magnetic field 'B' according to figure. If PQ = ℓ and $\angle PQR = 60°$. The ratio of magnetic forces on PQ to QR respectively:-



(1) 1 : 2 (2) $\sqrt{3}:2$ (3) $2:\sqrt{3}$ (4) 1 : 1

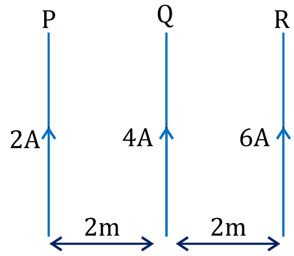
PME265

5. A current of 10A flows through two long parallel wires. The magnetic force on each wire is 2×10^{-3}N/m. If their currents makes doubled and separation between them makes half then magnetic force per unit length of each wire becomes :-

(1) 16×10^{-3} N/m (2) 8×10^{-3} N/m (3) 4×10^{-3} N/m (4) 32×10^{-3} N/m

PME266

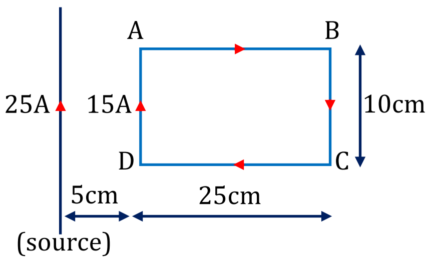
6. Three long straight wires, carrying currents are arranged according to figure. Magnetic force on 10cm part of the wire Q is :-



(1) 16×10^{-9} N, towards right (2) 16×10^{-8} N, towards right
(3) 16×10^{-8} N, towards left (4) 16×10^{-9} N, towards left

PME267

7. A rectangular loop ABCD is placed near on infinite length current carrying wire. Magnetic force on the loop is :-



(1) 1.25×10^{-4} N, Attraction (2) 1.25×10^{-4} N, Repulsion
(3) 12.5×10^{-4} N, Repulsion (4) 12.5×10^{-4} N, Attraction

PME268

8. A long horizontal wire 'P' which is rigidly fixed, carries a current 50A. An another wire 'Q' is placed directly above and parallel to 'P'. The weight of wire Q is 0.075 N/m and carries a current of 25A. The position of wire 'Q' from P so that the wire 'Q' remains suspended due to magnetic repulsion :-

(1) 33.3 mm (2) 0.33 mm (3) 333 mm (4) 3.33 mm

PME269

9. A long horizontal wire is rigidly fixed and carries 100A current. An another wire of linear mass density 2×10^{-3} kg/m placed below and parallel to the fixed wire. If the free wire kept 2cm below and hangs in air, then current in free wire is :-

(1) 19.6 A (2) 9.8 A (3) 4.9 A (4) 100 A

PME270

Magnetism and Matter

Bar Magnet and Magnetic Dipole

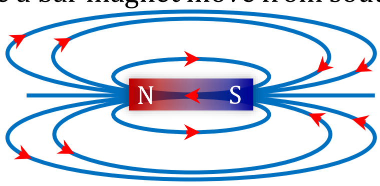
Bar Magnet

It is made up of iron, steel or any other ferromagnetic substance or ferromagnetic composite, having permanent magnetic properties.

Two poles are present in a Bar magnet, North Pole and South Pole.

Properties of bar magnets

1. When a bar magnet is suspended freely near the surface of the earth, it tends to align always in the nearly North-South direction. One end of the bar magnet always pointing towards the geographic North is called the North pole of the magnet. The other end that always point towards the geographic South is called the South pole of the magnet.
2. Like poles repel and unlike poles attract each other.
3. The North and South poles of a bar magnet cannot be separated. The magnetic monopoles do not exist.
4. Bar magnets are made up of ferromagnetic materials, like iron, cobalt, nickel etc.
5. The magnetic field lines outside a bar magnet move from north pole to south pole.
6. The magnetic field lines inside a bar magnet move from south pole to north pole.



Characteristics of Magnetic Field lines

1. The tangent to a magnetic field line at a point gives the direction of the net magnetic field acting at that point
2. Magnetic field lines are always closed paths, whatever may be the source of magnetic field. Hence, there is no beginning or end point for a magnetic field line. There are no sources or sinks for magnetic field lines in a magnetic field. That is, there is no isolated magnetic pole.
3. The magnetic field lines can never intersect to each other.
4. The number of magnetic field lines across unit area in a region of space is a measure of the strength of the magnetic field in that region.

Magnetic Dipole

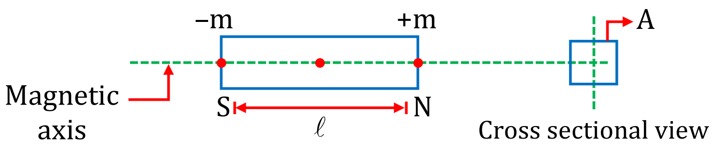
A magnetic dipole consists of a pair of magnetic poles of equal and opposite strength separated by small distance.

Example: Magnetic needle, Bar Magnet, Current carrying coil/solenoid etc.

Dipole Moment of a bar magnet

Dipole Moment $\rightarrow \vec{M}$

- Vector quantity
- Direction: South → North
- Unit: A-m^2



The magnetic dipole moment of a bar magnet is defined as a vector quantity having magnitude equal to the product of pole strength (m) with effective length (ℓ) and directed along the axis of the magnet from south pole to north pole. $\vec{M} = m\vec{\ell}$

Where m : Pole Strength (Unit: A-m)

ℓ : effective distance between North pole and South pole

M : Dipole Moment of bar magnet

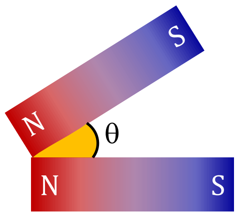
Note:

- Number of field lines emerging from each pole indicate the pole strength of pole.
  Pole strength (m) of each pole $\propto$ area of cross section for a given bar magnet.
- As magnetic moment is a vector, in case of two magnets having magnetic moments M_1 and M_2 with angle θ between them, the resulting magnetic moment.

$$M = \left[M_1^2 + M_2^2 + 2M_1M_2\cos\theta\right]^{1/2} \text{ with } \tan\phi = \left[\frac{M_2\sin\theta}{M_1 + M_2\cos\theta}\right]$$

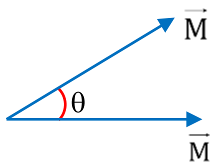
Illustration 57

Dipole moment of a bar magnet is M. Find effective dipole moment of the given combination of two such magnets.



Solution:

Here

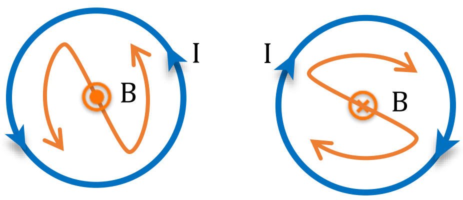


$$\text{So, } M_{net} = 2M\cos\frac{\theta}{2}$$

Magnetic Moment of Current Carrying Coil (Loop)

Current carrying coil (or loop) behaves like magnetic dipole. The face of coil in which current appears to flow anticlockwise acts as north pole while face of coil in which current appears to flow clockwise acts as south pole.

A current carrying coil acts as a Magnetic Dipole.

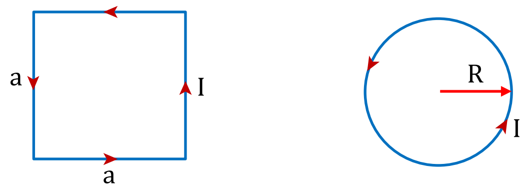


ACW $\rightarrow$ North Pole CW $\rightarrow$ South Pole

- For a coil of turns 'N', geometrical area 'A', carries a current 'I'
  Magnetic Moment, $\vec{M} = N I \vec{A}$
  where $\vec{A}$ is area vector perpendicular to the plane of the coil and along its axis.
  SI unit : A-m^2 or J/T
- Direction of $\vec{M}$ can be find out by right hand thumb rule
  Curling fingers - In the direction of current
  Thumb - Gives the direction of $\vec{M}$
  For a current carrying coil, its magnetic moment and magnetic field vectors both are parallel axial vectors.

Illustration 58

Calculate Magnetic Moment of given loops.



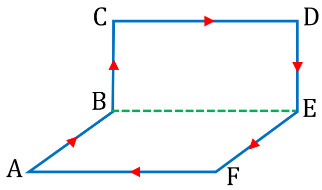
(A) (B)

Solution:

(A) $M = IA = I(a^2)$, (B) $M = IA = I(\pi R^2)$

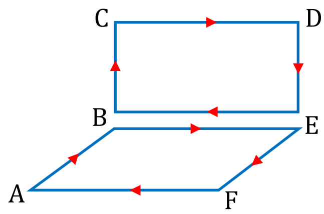
Illustration 59

Find the magnitude of magnetic moment of the current carrying loop ABCDEFA. Each side of the loop is 10 cm long and current in the loop is i = 2.0 A



Solution:

By assuming two equal and opposite currents in BE, two current carrying loops (ABEFA and BCDEB) are formed. Their magnetic moments are equal in magnitude but perpendicular to each other.

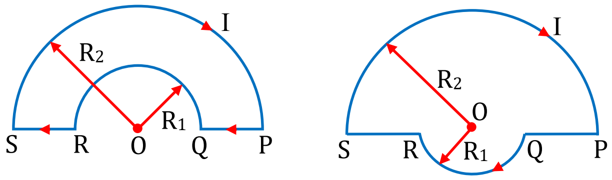


Hence, $M_{net} = \sqrt{M^2 + M^2} = \sqrt{2}M$ where $M = iA = (2.0)(0.1)(0.1) = 0.02$ A-m²

$M_{net} = (\sqrt{2})(0.02)$ A-m² = 0.028 A-m²

Illustration 60

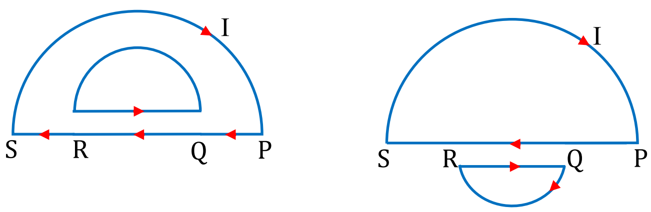
The wire loop PQRSP formed by joining two semi-circular wires of radii R_1 and R_2 carries a current I as shown in fig. What is the magnetic moment of the loop in cases (A) and (B)?



(A) (B)

Solution:

By assuming two equal and opposite currents in QR two semi-circular current carrying loops are formed

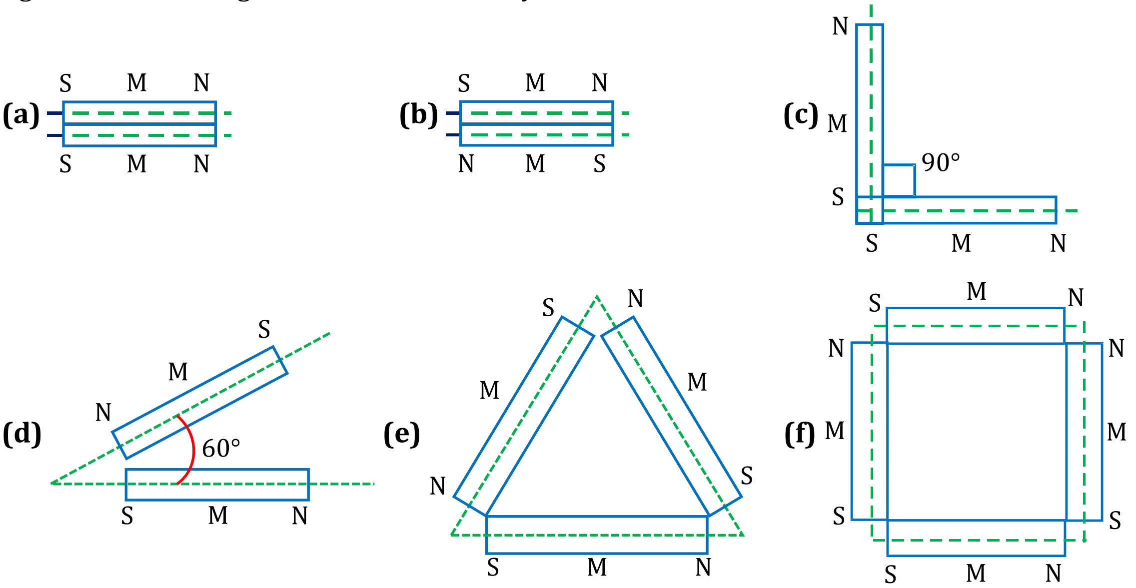


(A) $\vec{M} = NI\vec{A} = 1 \times I\left[\frac{1}{2}\pi R_2^2 \otimes + \frac{1}{2}\pi R_1^2 \odot\right] = \frac{1}{2}\pi I\left[R_2^2 - R_1^2\right]\otimes$

(B) $\vec{M} = NI\vec{A} = 1 \times I\left[\frac{1}{2}\pi R_2^2 \otimes + \frac{1}{2}\pi R_1^2 \otimes\right] = \frac{1}{2}\pi I\left[R_2^2 + R_1^2\right]\otimes$

BEGINNER'S BOX-9

1. Two/three/four identical bar magnets of magnetic moment 'M' are combined according to figure. Find net magnetic moment of the system.

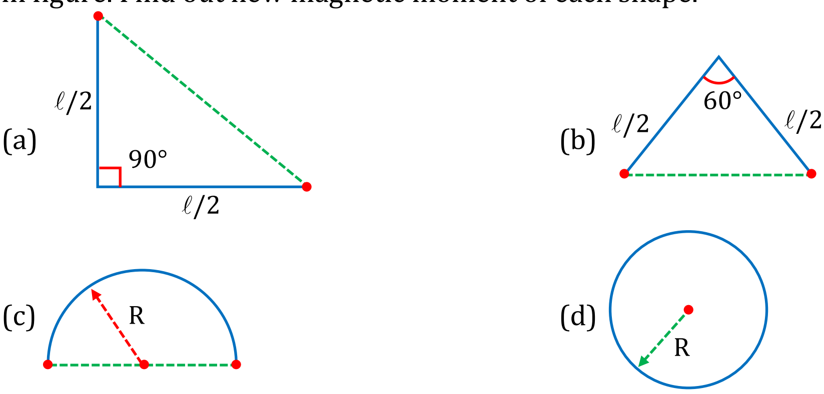


PME271

2. A bar magnet of magnetic moment 'M' is cuts into two equal parts in following two fashions (a) Parallel to its length (b) Perpendicular to its length, then what happens to the magnetic moment of each part.

PME272

3. A straight steel wire of length 'ℓ' has a magnetic moment 'M'. It is bent into various shapes shown in figure. Find out new magnetic moment of each shape.

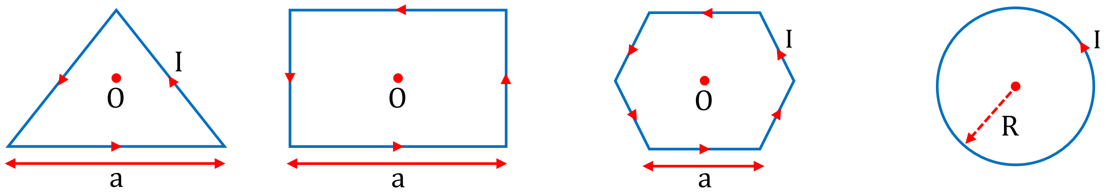


PME273

4. A wire of length 'ℓ' carries a current bent into various regular polygons shown below. Find magnetic moment in terms of length of the wire.

(a) Equilateral triangle (b) Square

(c) Regular hexagon (d) Circular loop



PME274

5. A current carrying wire of length 'ℓ' is bent to form a circular coil of one turn. After that it is again bent to form a circular coil of two turns. Find ratio of magnetic moment of coils respectively.
PME275

6. Circular coils of different turns are made by a constant length current carrying wire. Find number of turns in coil for its maximum magnetic moment.
PME276

7. Two identical current carrying coils are concentric and perpendicular to each other. If magnetic moment of each coil is 'M'. Find resultant magnetic moment of two coils.
PME277

Torque and Force on Magnetic Dipole

Torque on a Magnetic Dipole in Uniform Magnetic Field

When a bar magnet is placed in a uniform magnetic field the two poles experience a force. They are equal in magnitude and opposite in direction and do not have same line of action. They constitute a couple of forces which produces a torque. The torque tries to rotate the magnet so as to align it parallel too direction of field

Torque due to force couple

(a) Bar magnet

Consider a bar magnet of magnetic moment $\vec{M}$ held in a uniform magnetic field $\vec{B}$ making an angle θ with the field. Then the magnet experiences a torque given by

τ = force × perpendicular distance between force couple

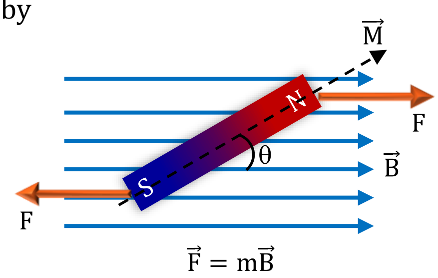
$\tau = F(\ell \sin\theta)$

$\tau = (mB)(\ell \sin\theta)$

$\tau = (m\ell)B\sin\theta$

$\tau = MB\sin\theta$, where $M = m\ell$

$\vec{\tau} = \vec{M} \times \vec{B}$



$\tau = MB\sin\theta$ → $\theta = 90°$ $\Rightarrow$ $\tau = MB$ (maximum)

$\tau = MB\sin\theta$ → $\theta = 0°$ or $180°$ $\Rightarrow$ $\tau = 0$ (minimum)

(b) Coil or Loop

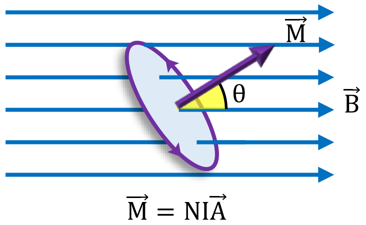
If a loop carries a current of magnitude I and magnetic moment $\vec{M}$ held in a uniform magnetic field $\vec{B}$ making an angle θ

Then the loop experiences a torque given by

$\vec{\tau} = \vec{M} \times \vec{B}$

$\tau = MB\sin\theta$

$\vec{\tau} = I(\vec{A} \times \vec{B})$



If loop have N turns then

$\vec{\tau} = NI(\vec{A} \times \vec{B})$

$\tau = NIAB\sin\theta$

$\tau = NIAB\sin\theta$ → $\theta = 90°$ $\Rightarrow$ $\tau = NIAB$ (maximum)

$\tau = NIAB\sin\theta$ → $\theta = 0°$ or $180°$ $\Rightarrow$ $\tau = 0$ (minimum)

Note:

- Torque on dipole is an axial vector and it is directed along axis of rotation of dipole.
- Tendency of torque on dipole is try to align the $\vec{M}$ in the direction of $\vec{B}$ or tries to makes the axis of dipole parallel to $\vec{B}$ or makes the plane of coil (or loop) perpendicular to $\vec{B}$.
- Dipole in uniform magnetic field → $F_{net} = 0$ (no translatory motion); → τ may or may not be zero (decides by θ)
- Dipole in non-uniform magnetic field → F_{net} may or may not be zero; → τ may or may not be zero (decides by θ)

Illustration 61

A magnet of magnetic moment $50\hat{i}$ A–m^2 placed along x–axis. Where magnetic field is $\vec{B} = (0.5\,\hat{i} + 3.0\,\hat{j})$ tesla. The torque acting on magnet is : –

Solution:

$$\vec{\tau} = \vec{M} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 50 & 0 & 0 \\ 0.5 & 3 & 0 \end{vmatrix} = (0)\,\hat{i} - (0)\,\hat{j} + (150)\,\hat{k} \text{ N–m} = 150\,\hat{k} \text{ N–m}$$

Illustration 62

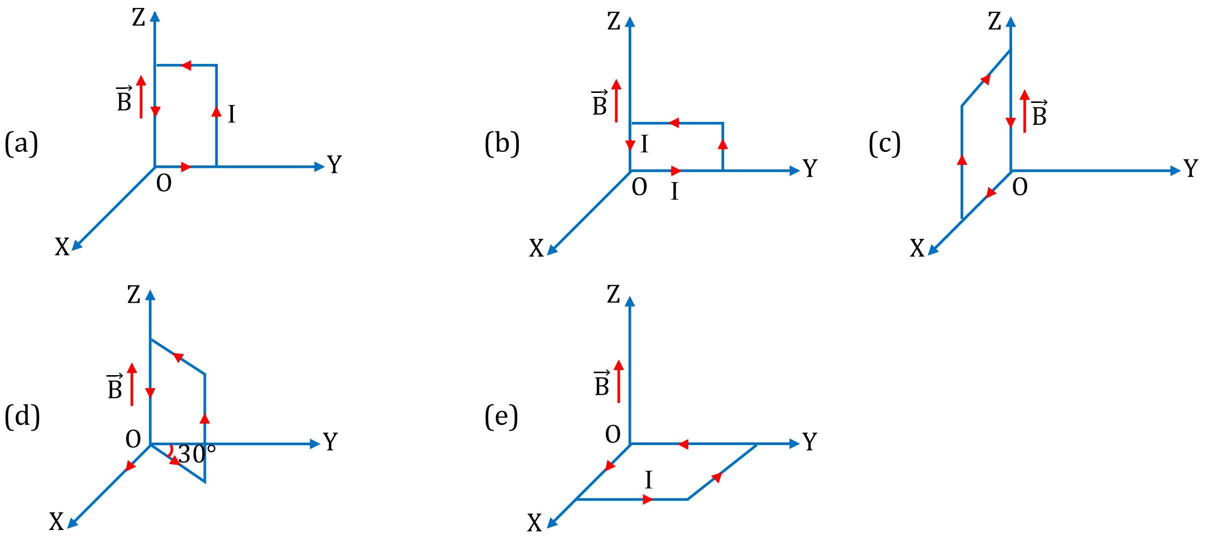
A bar magnet of length 10 cm and having the pole strength equal to 10^{-3} Am is kept in a magnetic field having magnetic induction (B) equal to $4\pi \times 10^{-3}$ T. It makes an angle of 30° with the direction of magnetic induction. The value of the torque acting on the magnet is

Solution:

Torque, $\tau = MB\sin\theta = 0.1 \times 10^{-3} \times 4\pi \times 10^{-3} \times \sin 30° = 10^{-7} \times 4\pi \times \frac{1}{2} = 2\pi \times 10^{-7}$ Nm

Illustration 63

A uniform magnetic field of 5000 gauss is established along the positive z-direction. A rectangular loop of side 20 cm and 5 cm carries a current of 10 A is suspended in this magnetic field. What is the torque on the loop in the different cases shown in the following figures? What is the force in each case? Which case corresponds to stable equilibrium?



Solution:

(a) Torque on loop, $\tau = BIA \sin\theta$

Here, $\theta = 90°$; B=5000 gauss = 5000×10^{-4} tesla = 0.5 tesla

I = 10 ampere, A = $20 \times 5 cm^2 = 100 \times 10^{-4} = 10^{-2} m^2$

Now, $\tau = 0.5 \times 10 \times 10^{-2} = 5 \times 10^{-2}$ Nm It is directed along –Y axis

(b) Same as (a).

(c) $\tau = 5 \times 10^{-2}$ Nm along –x-direction

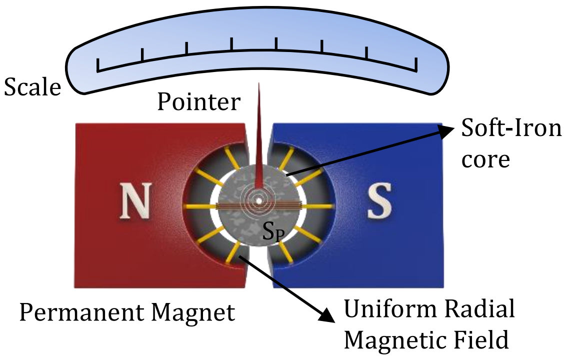
(d) $\tau = 5 \times 10^{-2}$ N m at an angle of 240° with +x direction.

(e) τ is zero. [$\because$ Angle between plane of loop and direction of magnetic field is 90°]

Resultant force is zero in each case. Case (e) corresponds to stable equilibrium.

Moving Coil Galvanometer

Construction



The galvanometer consists of a coil, with many turns, free to rotate about a fixed axis, in a uniform radial magnetic field. There is a cylindrical soft iron core which not only makes the field radial but also increases the strength of the magnetic field. A spiral spring is also attached to the coil which resists the rotation of coil.

Working Principle

When a current carrying coil is placed in a magnetic field, it experiences a torque

$\vec{\tau} = NI(\vec{A} \times \vec{B})$

$\tau = NIAB\sin\theta$

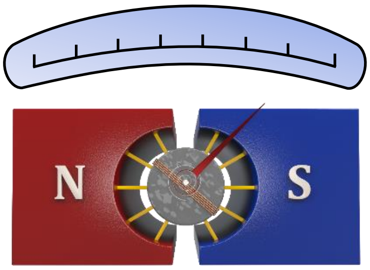
$\tau = NIAB$ ($\theta = 90°$ due to radial magnetic field)

In this case I = current through coil,

A = Area of coil,

B = Magnetic field

N = no. of turns,



This torque tends to rotate the coil by ϕ angle.

The spring S provides a counter torque $C\phi$ that balances the magnetic torque. In equilibrium,

$$C\phi = NIAB \quad \Rightarrow \quad I = \frac{C}{NAB}\phi$$

$I \propto \phi$

Hence, it means the deflection produced is proportional to the current flowing through the galvanometer.

Current Sensitivity

Current sensitivity is defined as the deflection produced in the galvanometer when a unit current flows through it.

$$C.S. = \frac{\phi}{I} = \frac{NAB}{C}(rad/A)$$

Voltage Sensitivity

Voltage sensitivity is equal to the deflection per unit voltage applied across voltmeter.

$$V.S. = \frac{\phi}{V} = \frac{\phi}{IR} = \frac{NAB}{CR}(\text{rad}/V)$$

Advantage of a Moving Coil Galvanometer

1. As the deflection of the coil is proportional to the current passed through it, so a linear scale can be used to measure the deflection.
2. A moving coil galvanometer can be made highly sensitive by increasing N, B, A and decreasing C.
3. As the coil is placed in a strong magnetic field of a powerful magnet, its deflection is not affected by external magnetic fields. This enables us to use the galvanometer in any position.
4. As the coil is wound over a metallic frame, the eddy currents produced in the frame bring the coil to rest quickly.

Disadvantage of a Moving Coil Galvanometer

1. The main disadvantage is that its sensitivity cannot be changed at will.
2. All types of moving coil galvanometer are easily damaged by overloading. A current greater than that which the instrument is intended to measure will burn out its hair springs or suspension.

Illustration 64

A moving coil galvanometer has a coil with 175 turns and area 1cm². It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a magnetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B(in Tesla) is approximately.

Solution:

$$\tau = \vec{M} \times \vec{B}$$

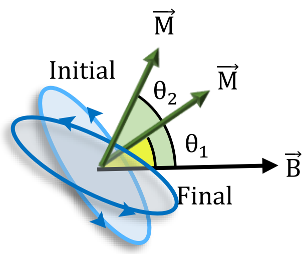
$C\phi$ = INAB

$$10^{-6} \times \frac{\pi}{180} = 10^{-3} \times 10^{-4} \times 175 \times B$$

B = 10^{-3} T.

Potential Energy of Magnetic Dipole

Work Done in Rotating the Coil in Uniform Magnetic Field



When a bar magnet or coil of dipole moment M is kept in a uniform magnetic field B it experiences a torque which tries to align it in the direction of external magnetic field $\vec{B}$. Work has to be done in rotation the magnet or coil.

The work done in rotating dipole by small-angle $d\theta$ is $dW = \tau d\theta$

The work done in rotating the magnet or coil from $\theta = \theta_1$ to $\theta = \theta_2$ is given by

$$W = \int_{\theta_1}^{\theta_2} \tau d\theta$$

$$W = \int_{\theta_1}^{\theta_2} MB\sin\theta d\theta$$

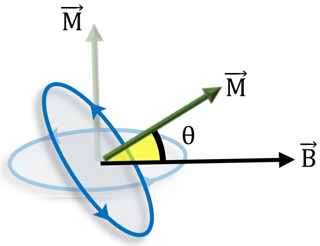
$$W = MB[-\cos\theta]_{\theta_1}^{\theta_2}$$

$$W = -MB(\cos\theta_2 - \cos\theta_1)$$

$$W = MB(\cos\theta_1 - \cos\theta_2)$$

Potential Energy of the Coil in Uniform Magnetic Field

The potential energy of dipole defined as work done in rotating the dipole from a direction perpendicular to the given direction.



Work done in rotating the dipole from $\theta_1 = 90°$ to $\theta_2 = 0°$

$$W = MB(\cos 90^{o} - \cos\theta)$$

$$U = -MB\cos\theta$$

In vector form $\boxed{U = -\vec{M}.\vec{B}}$

Important Note

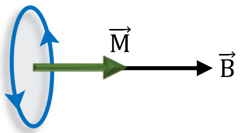
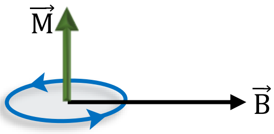
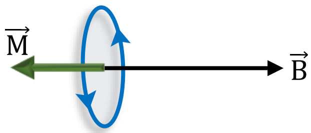
Case-I	Case-II	Case-III
 When $\vec{M}$ and $\vec{B}$ are parallel $\theta = 0°$ $\tau = 0$ $U_{min} = -MB$ The dipole has minimum potential energy and it is in stable equilibrium.	 When $\vec{M}$ and $\vec{B}$ are perpendicular $\theta = 90°$ $\tau = MB$ $U = 0$ No Equilibrium as $\tau \neq 0$	 When $\vec{M}$ and $\vec{B}$ are anti-parallel $\theta = 180°$ $\tau = 0$ $U_{max} = MB$ The dipole has maximum potential energy and it is in unstable equilibrium.

Illustration 65

A circular coil of 100 turns and having a radius of 0.05 m carries a current of 0.1 A. Calculate the work required to turn the coil in an external field of 1.5 T through 180° about an axis perpendicular to the magnetic field? The plane of coil is initially at right angles to magnetic field.

Solution:

Work done $W = MB(\cos\theta_1 - \cos\theta_2) = NIAB(\cos\theta_1 - \cos\theta_2)$

$W = NI(\pi r^2)B(\cos\theta_1 - \cos\theta_2) = 100 \times 0.1 \times 3.14 \times (0.05)^2 \times 1.5(\cos 0° - \cos\pi) = 0.2355J$

Illustration 66

A bar magnet of magnetic moment 1.5 JT^{-1} lies aligned with the direction of a uniform magnetic field of 0.22 T. What is the amount of work required to turn the magnet so as to align its magnetic moment.

(i) Normal to the field direction? (ii) Opposite to the field direction?

Solution:

Here, $M = 1.5\ JT^{-1}$, $B = 0.22$ T.

P.E. with magnetic moment aligned to field = – MB

P.E. with magnetic moment normal to field = 0

P.E. with magnetic moment antiparallel to field = + MB

(i) Work done = increase in P.E. = 0 – (–MB) = MB = 1.5 × 0.22 = 0.33 J.

(ii) Work done = increase in P.E. = MB – (–MB) = 2MB = 2 × 1.5 × 0.22 = 0.66 J.

Illustration 67

A short bar magnet of magnetic moment 0.32 J/T is placed in uniform field of 0.15 T. If the bar is free to rotate in plane of field then which orientation would correspond to its (i) stable and (ii) unstable equilibrium? What is potential energy of magnet in each case?

Solution:

(i) If M is parallel to B then $\theta = 0°$. So potential energy $U = U_{min} = -MB$

$U_{min} = -MB = -0.32 \times 0.15$ J$= -4.8 \times 10^{-2}$ J (stable equilibrium)

(ii) If M is antiparallel to B then $\theta = \pi$ So potential energy

$U = U_{max} = +MB = +0.32 \times 0.15 = 4.8 \times 10^{-2}$ J (unstable equilibrium.)

BEGINNER'S BOX-10

1. When a magnetic needle placed in non uniform field the needle may experiences a :-

(1) Force (2) Torque

(3) Force and torque (4) No force and no torque

PME278

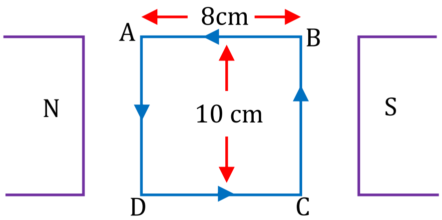
2. A magnet of magnetic moment $50\,\hat{i}$ A–m² placed along x–axis. Where magnetic field is $\vec{B} = (0.5\,\hat{i} + 3.0\,\hat{j})$ tesla. The torque acting on magnet is :–

(1) $175\,\hat{k}$ N–m (2) $150\,\hat{k}$ N–m

(3) $75\,\hat{k}$ N–m (4) $25\sqrt{37}\,\hat{k}$ N–m

PME279

3. A coil of 100 turns kept in magnetic field B = 0.2T, carries a current of 2A as shown in figure. Torque on coil and which side of coil comes out from the plane of the paper :-



(1) 0.16 N-m, BC side comes out from plane of paper

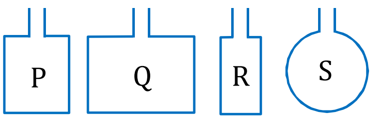
(2) 0.16 N–m, AD side comes out from plane of paper

(3) 0.32 N, BC side comes out from plane of paper

(4) 0.32 N–m, AD side comes out from plane of paper

PME280

4. Four wires each of length 2.0 m, are bent in four loop P,Q,R and S then suspended in uniform magnetic field. All loop carries equal currents, which statement of the following is true :



(1) Couple acting on the loop P is greatest.

(2) Couple acting on the loop Q is greatest.

(3) Couple acting on the loop R is greatest

(4) Couple acting on the loop S is greatest.

PME281

5. A current carrying wire is bent to form a circular coil. If this coil is placed in any other magnetic field then for maximum torque on the coil, the number of turns will be -

(1) 1 (2) 2 (3) 4 (4) 8

PME282

6. A magnet of magnetic moment 4A–m² is held in a uniform magnetic field 5×10^{-4}T with the magnetic moment vector makes an angle 30° with the field. Work done in increasing the angle from 30° to 45°:-

(1) 3.2×10^{-4} J (2) 1.6×10^{-4} J (3) 1.6×10^{-3} J (4) 3.2×10^{-3} J

PME283

7. A magnetic needle laying parallel to the magnetic field requires W units of work to turn it through 60°. Torque needed to maintain the needle in this position :-

(1) W (2) $\sqrt{3}$ W (3) $\frac{W}{2}$ (4) $\frac{\sqrt{3}W}{2}$

PME284

8. A bar magnet has a magnetic moment 2.5 A–m² and it is placed in a magnetic field of 0.2T. Calculate the Work done in turning the magnet from parallel to antiparallel position relative to the field direction:-

(1) 0.1 J (2) 1J (3) 2J (4) 5J

PME285

9. A circular coil of 'N' turns, 'R' radius carries a 'i' current. Work done in rotating this coil in an external magnetic field from $\theta_1 = 0°$ to $\theta_2 = 90°$:-

(1) $2\pi NiR^2B$ (2) $\frac{\pi NiR^2B}{2}$ (3) πNiR^2B (4) $\frac{\sqrt{3}\pi NiR^2B}{2}$

PME286

10. A short bar magnet placed with its axis at 30° with uniform magnetic field of 0.16T, experiences a torque of magnitude 0.032 N–m. The potential energy of bar magnet in stable equilibrium and in unstable equilibrium respectively :-

(1) –0.064 J, + 0.064 J (2) –0.032 J, + 0.032 J

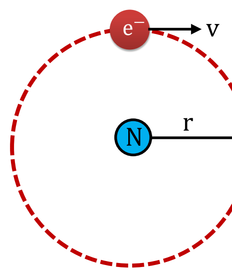
(3) –0.016 J, + 0.016 J (4) zero, zero

PME287

Atomic Magnetism

Consider an electron revolving around a nucleus in an atom. Due to motion of electron, orbit behaves like a current carrying loop. Due to this, magnetic field is produced at center of orbit. This is called 'atomic magnetism'.

(a) Effective current



$$I = \frac{q}{T} = \frac{e}{T} = ef = \frac{e\omega}{2\pi} = \frac{ev}{2\pi r}$$

$$I = \frac{qv}{2\pi r} \text{ (In general)}$$

(b) Induced Magnetic Field at center

$$B_{in} = \frac{\mu_0 I}{2r} = \frac{\mu_0 e}{2Tr} = \frac{\mu_0 ef}{2r} = \frac{\mu_0}{2r}\left(\frac{ev}{2\pi r}\right) = \frac{\mu_0 ev}{4\pi r^2}$$

$$B_{in} = \frac{\mu_0 qv}{4\pi r^2} \text{ (In general)}$$

(c) Magnetic Moment

$$M = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \frac{e\omega r^2}{2} = \frac{evr}{2}$$

$$M = \frac{qvr}{2} \text{ (In general)}$$

Illustration 68

A helium nucleus is moving in a circular path of radius 0.8m. If it takes 2 sec to complete one revolution. Magnetic field produced at the centre of the circle :-

(1) $\mu_0 \times 10^{-19}$ T (2) $\frac{10^{-19}}{\mu_0}$T (3) 2×10^{-19} T (4) $\frac{2\times10^{-19}}{\mu_0}$T

Solution:

$$B = \frac{\mu_0 i}{2r} = \frac{\mu_0}{2r}\left(\frac{q}{T}\right) = \frac{\mu_0 \times 2\times1.6\times10^{-19}}{2\times2\times0.8} = \mu_0 \times 10^{-19}\ \text{T (where } q = +2e)$$

BEGINNER'S BOX-11

1. Magnetic field produced by α-particle at centre, when it moves on circular path of radius 'r' with velocity 'v' will be :-

(1) $B = \frac{\mu_0}{4\pi}\frac{ev}{r^2}$ (2) $B = \frac{\mu_0}{2\pi}\frac{ev}{r^2}$ (3) $B = \frac{\mu_0}{2\pi}\frac{(2e)v}{r^2}$ (4) $B = \frac{\mu_0}{\pi}\frac{ev}{r^2}$

PME288

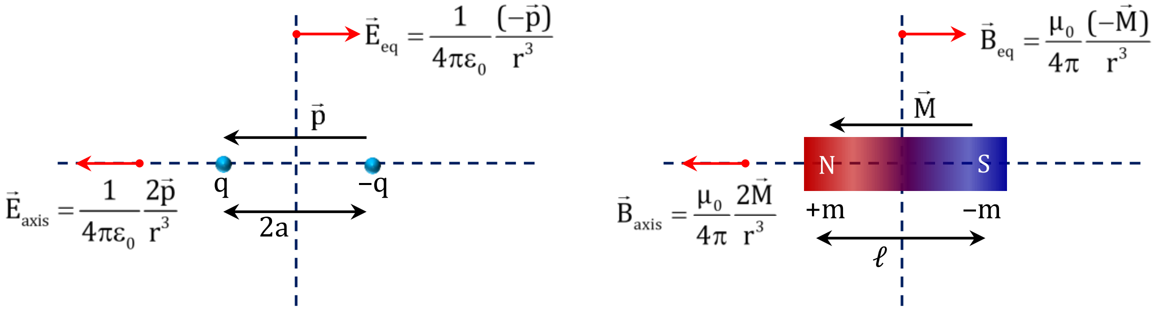
2. A charge q moving in a circular orbit of radius r makes n revolution per second, the magnetic field produced at centre will be

(1) $\frac{\mu_0 qn}{2r}$ (2) $\frac{\mu_0 n^2 q}{2r}$ (3) Zero (4) $\frac{\mu_0 nq}{2\pi r}$

PME289

Field of a Bar Magnet

Magnetic field at large distance due to bar magnet of magnetic moment $\vec{M}$ can be obtained from the equation for electric field due to an electric dipole of dipole moment $\vec{p}$.



Angular SHM of Magnetic Dipole

When a dipole is suspended in a uniform magnetic field it will align itself parallel to field. Now if it is given a small angular displacement θ with respect to its equilibrium position. The restoring torque acts on it :-

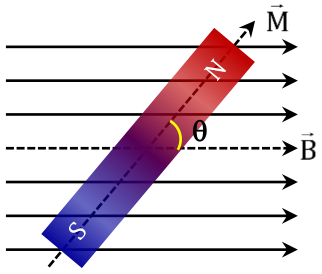
$\tau = -MB\sin\theta$

For small angle, $\sin\theta \approx \theta$

$\tau = -MB\theta$

For angular SHM, $\tau = -I\omega^2\theta$

$-I\omega^2\theta = -MB\theta$



$$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T}$$

$T = 2\pi\sqrt{\frac{I}{MB}}$ where I : Moment of Inertia $= \frac{ml^2}{12}$ {m : mass of magnet, ℓ : length of magnet}

Illustration 69

Time period of thin bar magnet of vibration magnetometer is 'T'. If it cuts into two equal parts in then find the time period of each part if it is cut

(a) Along or parallel to its length (b) Perpendicular to its length,

Solution:

(a) For each part

$$\left\{\begin{array}{l} \text{Mass} \to \text{half and length} \to \text{same} \\ \text{Moment of inertia} \to I' = \frac{I}{2} \\ \text{Magnetic moment} \to M' = \frac{M}{2} \end{array}\right. \qquad T' \propto \sqrt{\frac{I/2}{M/2}}$$

Time period of each part, $T' = T$

So, if bar magnet cuts into 'n' equal parts parallel to its length then time period of each part becomes $T' = T$

(b) For each part

$$\left\{\begin{array}{l} \text{Mass} \to \text{half , length} \to \text{half} \\ \text{Moment of inertia} \to I' = \frac{I}{8} \\ \text{Magnetic moment} \to M' = \frac{M}{2} \end{array}\right. \qquad T' \propto \sqrt{\frac{I/8}{M/2}}$$

Time period of each part, $T' = \frac{T}{2}$

So, if bar magnet broken into 'n' equal parts perpendicular to its length then time period of each part becomes $T' = \frac{T}{n}$

BEGINNER'S BOX-12

1. The ratio of the magnetic field due to a small bar magnet in end-on position to broadside-on position is :-

(1) $\frac{1}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

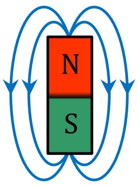
PME290

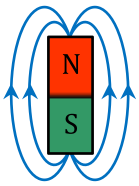
2. The magnetic field at a point X on the axis of a small bar magnet is equal to the field at a point Y on the equator of the same magnet. The ratio of the distances of point X and Y from the centre of the magnet is :-

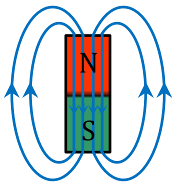
(1) 2^{-3} (2) $2^{-1/3}$ (3) 2^{3} (4) $2^{1/3}$

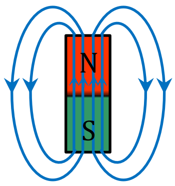
PME291

3. The magnetic field lines due to a bar magnet are correctly shown in :

(1) 

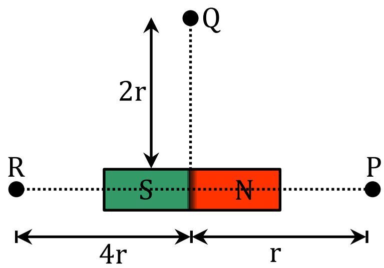
(2) 

(3) 

(4) 

PME292

4. In the surrounding region of a small bar magnet as shown in figure, the ratio of magnetic fields at P, Q and R will be



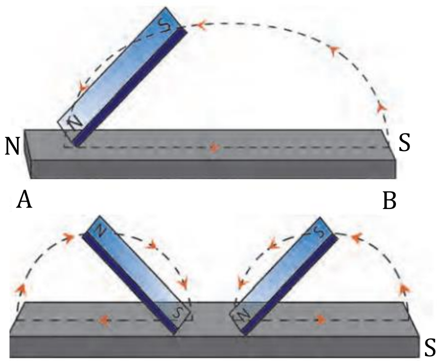
(1) 1 : 1 : 4 (2) 1 : 2 : 4 (3) 1 : 8 : 64 (4) 64 : 4 : 1

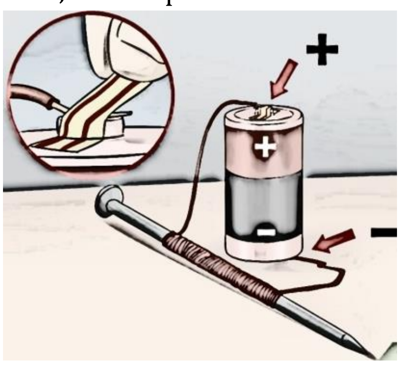
PME293

Magnetization

Magnetization, also termed as magnetic polarization, is a vector quantity that gives the measure of the density of permanent or induced dipole moment in a given magnetic material.

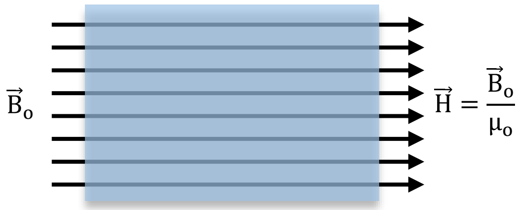
The magnetic effects of a material can also be induced by passing an electrical current through the material; the magnetic effect is caused by the motion of electrons in atoms, or the spin of the electrons or the nuclei.





Magnetizing field or Magnetic Intensity $(\vec{H})$

Field in which a material is placed for magnetization, called as magnetizing field.



$\vec{H} = \frac{\vec{B}_o}{\mu_o}$

unit of $\vec{H}$: A/m

$\vec{B}_o$: Magnetic Field

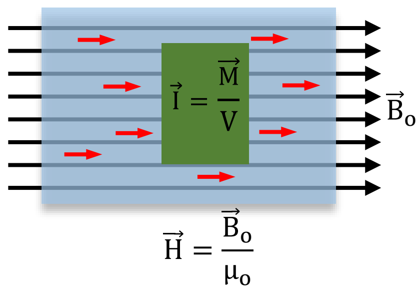
μ_o : Permeability of free space

Intensity of magnetization $(\vec{I})$

When a magnetic material is placed in magnetising field then induced dipole moment per unit volume of that material is known as intensity of magnetisation.

$\vec{I} = \frac{\vec{M}}{V}$

unit of $\vec{I}$: A/m $\left\{\because \frac{\vec{M}}{V} = \frac{I\vec{A}}{V} = \frac{\text{ampere} \times \text{meter}^2}{\text{meter}^3}\right\}$



Induced magnetic field due to these induced dipoles in the material is given by:

$\vec{B}_{in} = \mu_o \vec{I}$

Magnetic susceptibility (χ_m)

$\chi_m = \frac{I}{H}$ [It is a scalar with no units & dimensions]

Physically it represents the ease with which a magnetic material can be magnetised.

Note:- A material with more χ_m , can be change into magnet easily.

Magnetic Permeability (μ)

$\mu = \frac{B}{H} = \frac{\text{Total magnetic field inside a material}}{\text{Magnetizing field}}$

Unit of μ : $\mu = \frac{B_m}{H} \equiv \frac{\text{Wb}/\text{m}^2}{\text{A}/\text{m}} = \frac{\text{Weber}}{\text{A}-\text{m}} = \frac{\text{H}-\text{A}}{\text{A}-\text{m}} = \frac{\text{H}}{\text{m}}$

It represents the degree up to which a material can be penetrated by the magnetic field lines

Relative Permeability (μ_r)

$\mu_r = \frac{\mu}{\mu_0}$

It has no units and dimensions.

Relation between μ and χ_m

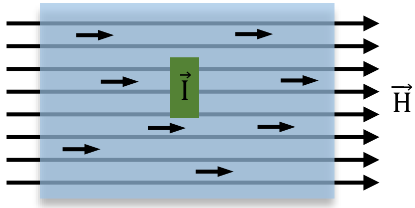
When a magnetic material is placed in a magnetic field $\vec{B}_o$ for magnetization, then total magnetic field in the material is

$\vec{B}_m = \vec{B}_o + \vec{B}_{in}$ {$\vec{B}_{in}$= induced field.}

$\because$ $\vec{B}_0 = \mu_0 \vec{H}$; $\vec{B}_{in} = \mu_0 \vec{I}$

$\therefore$ $\vec{B}_m = \mu_o(\vec{H} + \vec{I})$

$B_m = \mu_o(H + I)$



$\mu H = \mu_o H\left(1+\frac{I}{H}\right)$

$\mu = \mu_o(1+\chi_m)$

$\frac{\mu}{\mu_o} = (1+\chi_m)$

$\mu_r = (1+\chi_m)$

For vacuum, $\mu_r = 1$, $\chi_m = 0$

For air, $\mu_r = 1.04$, $\chi_m = 0.04$

Illustration 70

A solenoid of 200 turns/m is carrying a current of 5A. Relative permeability of core material of solenoid is 3000. Determine the magnitudes of the magnetic intensity, magnetization and the magnetic field inside the core.

Solution:

magnetic intensity $H = ni = 200\ m^{-1} \times 5A = 1000\ Am^{-1}$

$\mu_r = 1 + \chi$, i.e., $\chi = 3000 - 1 = 2999$

Hence, the magnetization $I = \chi H = 2.999 \times 10^6\ Am^{-1}$

The magnetic field $B = \mu_0\mu_r H$

$B = 3000\ \mu_0 H = 3000 \times 4\pi \times 10^{-7} \times 1000 = 3.769$ T

Illustration 71

When a rod of magnetic material of size 10 cm × 0.5 cm × 0.2 cm is located in magnetising field of 0.5×10^4 A/m then a magnetic moment of 5 A-m² is induced in it. Find out magnetic induction in rod.

Solution:

Total magnetic induction $B = \mu_0 (I + H) = \mu_0\left(\frac{M}{V}+H\right)$ $\quad\left(\because I = \frac{M}{V}\right)$

$$= 4\pi \times 10^{-7}\left(\frac{5}{10^{-6}}+0.5\times10^4\right) = 6.28\ Wb/m^2$$

Illustration 72

A rod of magnetic material of cross section 0.25 cm² is located in 4000 A/m magnetising field. Magnetic flux passes through the rod is 25×10^{-6} Wb. Find out for rod

(i) permeability (ii) magnetic susceptibility

(iii) magnetisation

Solution:

(i) Magnetic flux $\phi = BA \Rightarrow B = \phi/A = \frac{25\times10^{-6}}{0.25\times10^{-4}} = 1\ Wb/m^2$

$B = \mu H \Rightarrow \mu = \frac{B}{H} = \frac{1}{4000} = 2.25 \times 10^{-4}$ Wb/A-m

(ii) $\mu_r = 1 + \chi_m$ $\qquad \chi_m = \mu_r - 1\left(\because \mu_r = \frac{\mu}{\mu_0}\right)$

$\chi_m = \frac{\mu}{\mu_0} - 1 = \left(\frac{2.5\times10^{-4}}{4\pi\times10^{-7}}-1\right) = 199 - 1 = 198$

(iii) Magnetization $I = \chi_m H = 198 \times 4000 = 7.92 \times 10^5$ A/m

Magnetic Materials

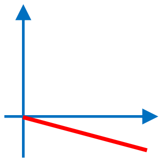
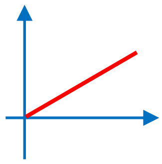
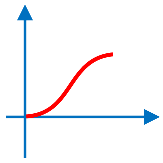
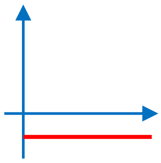
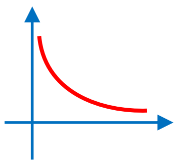
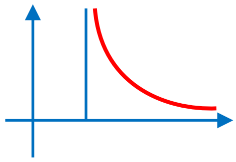
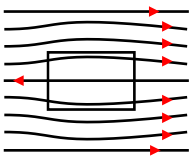
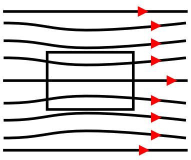
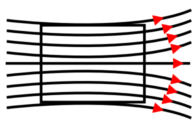
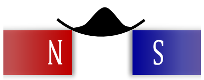
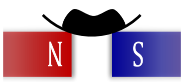
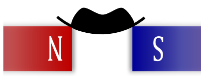
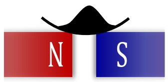
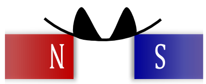
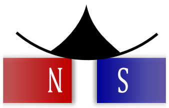
Types of Magnetic Materials

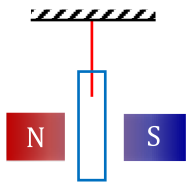
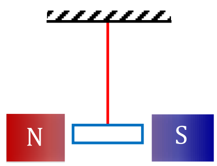
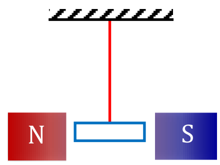
On the basis of magnetic properties of the materials [as magnetization intensity (I), Susceptibility (χ_m) and relative permeability (μ_r)] Faraday divide these materials in three classes –

Based on the observation of how materials respond to external magnetizing field

	Diamagnetic	**Paramagnetic**	**Ferromagnetic**
Cause of Magnetism	Orbital Motion of electrons	Spin Motion of electrons	formation of Domains

Classification of Magnetic Materials

Properties	**Diamagnetic**	**Paramagnetic**	**Ferromagnetic**
I	Small, negative	Small, positive	Very large, positive
χ_m	Small, negative	Small, positive	Very Large, positive
μ_r	$1 > \mu_r > 0$	$2 > \mu_r > 1$	$\mu_r \gg 1$
I – H curve	I $\rightarrow$ Small, negative varies linearly with field 	I $\rightarrow$ Small, positive, varies linearly with field 	I $\rightarrow$ very large, positive varies non-linearly with field 
Variation of χ_m with Temp	$\chi_m \rightarrow$ small, negative & temperature independent $\chi_m \propto T^0$ 	$\chi_m \rightarrow$ small, positive & varies inversely with temp. Curie law $\chi_m \propto \frac{1}{T}$ 	$\chi_m \rightarrow$ very large, positive & temp. dependent Curie Weiss law (for $T > T_C$) $\chi_m \propto \frac{1}{T-T_c}$ (T_C = Curie temperature) 
Degree of magnetization			
Behavior in non-uniform magnetizing field	 Weak Field  Strong Field	 Weak Field  Strong Field	 Weak Field  Strong Field

Magnetic moment of single atom	Atoms do not have any permanent magnetic moment	Atoms have permanent magnetic moment which are randomly oriented. (i.e. in absence of external magnetic field the magnetic moment of whole material is zero)	Atoms have permanent magnetic moment which are organised in domains.
When rod of material is suspended between poles of magnet.	It becomes perpendicular to the direction of external magnetic field.  	If there is strong magnetic field in between the poles then rod becomes parallel to the magnetic field.  	Weak magnetic field between magnetic poles can made rod parallel to field direction. 
Magnetic moment of substance in presence of external magnetic field	Value $\vec{M}$ is very less and opposite to $\vec{H}$.	Value $\vec{M}$ is low but in direction of $\vec{H}$.	$\vec{M}$ is very high and in direction of $\vec{H}$.
Examples	Bi, Cu, Ag, Pb, H_2O, Hg, H_2, He, Ne, Au, Zn, Sb, NaCl, Diamond. (May be found in solid, liquid or gas).	Na, K, Mg, Mn, Sn, Pt, Al, O_2 (May be found in solid, liquid or gas.)	Fe, Co, Ni all their alloys, Fe_3O_4 Gd, Alnico, etc. (Normally found only in solids) (crystalline solids)

Note: Above Curie temperature, ferromagnetic materials lose their permanent magnetic properties and behave as paramagnetic materials.

Illustration 73

If a ferromagnetic material is inserted in a current carrying solenoid, the magnetic field of solenoid.

Solution:

The flux inside the solenoid increases due to the presence of ferromagnetic material and hence the field of solenoid increases largely.

Illustration 74

Susceptibility of Mg at 300 K is 1.2×10^{-5}. The temperature at which susceptibility will be 1.8×10^{-5} is

Solution:

$$\chi \propto \frac{1}{T}$$

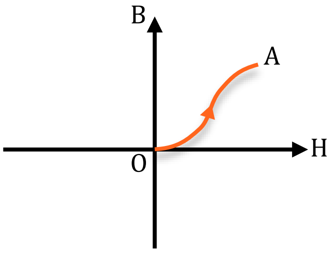
$\therefore \quad \chi_1 T_1 = \chi_2 T_2$

Hence, $T_2 = \dfrac{1.2\times10^{-5}\times300}{1.8\times10^{-5}} = 200\text{K}$

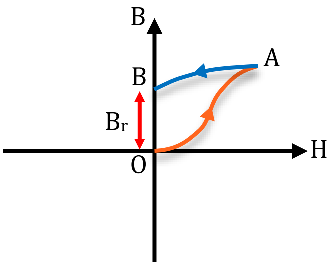
Magnetic Hysteresis

Only Ferromagnetic materials show magnetic hysteresis, when Ferromagnetic material is placed in external magnetic field for magnetization.

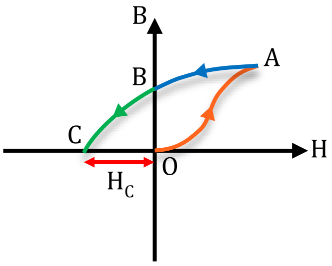
- When magnetizing field is increased across a ferromagnetic material, the magnetic field inside the material also increases.
  If magnetising field is increased further there will be no change in the magnetic field inside the material.



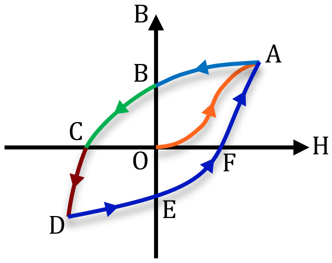
- When this magnetising field is reduced, magnetic field inside the material decreases (A to B).
  When this magnetizing field is removed, there is some residual magnetic field in the material known as **Retentivity (B_r)**



- To remove this residual magnetic field inside the material, external magnetizing field is again applied but in opposite direction till net magnetic field inside the material becomes zero (B to C).
  This magnetizing field required to destroy residual magnetic field is known as **Coercivity (H_C)**.



- As external magnetizing field is further increased, magnetic field again develops in the material (C to D).
  Now if magnetising field is again decreased then graph repeats itself again i.e., magnetic field decreases, and it achieves the residual value (D to E). Now if magnetizing field is reversed and increased then magnetic field becomes zero (E to F). If the magnetizing field is further increased then again magnetic field starts rising (F to A).



- Due to lagging behind of B with H this curve is known as hysteresis curve.
  [Lagging of B behind H is called hysteresis]

Cause of hysteresis :

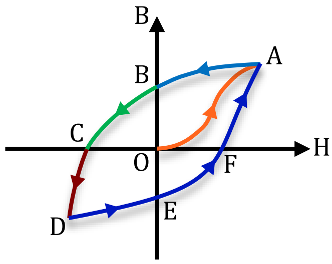
By removing external magnetizing field (H = 0), the magnetic moment of some domains remains aligned in the applied direction of previous magnetizing field which results into a residual magnetism.

Residual magnetism (OB) = B_r (retentivity/remanence) :

Retentivity of a specimen is a measure of the magnetic field remaining in the ferromagnetic specimen when the magnetizing field is removed.

Coercivity (OC) = H_c :

Coercivity is a measure of magnetizing field required to destroy the residual magnetism of the ferromagnetic specimen.



Hysteresis Loss

- The area of hysteresis loop is equal to the energy loss per cycle per unit volume.
- Its value is different for different materials.
- The work done per cycle per unit volume of material is equal to the area of hysteresis loop.
  Total energy loss in material = W_H
  W_H = volume of material × area of hysteresis curve × frequency × time.
  $W_H = V \times A \times n \times t$ joule
  $W_H = \dfrac{V \times A \times n \times t}{J}$ calorie

Soft and Hard Magnetic Materials

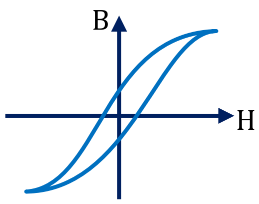
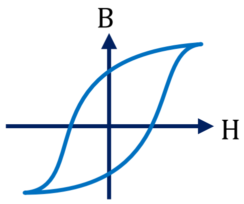
Soft magnetic materials	Hard magnetic materials
1. Low retentivity	1. High retentivity
2. Low coercivity	2. High coercivity
3. Small hysteresis Loss	3. Large hysteresis Loss
4. Suitable for making electromagnets, cores of transformers etc. e.g. Soft Iron	4. suitable for permanent magnet e.g. Steel, Alnico
	

Illustration 75

Which of the following statement is True of False about hysteresis

(i) This effect is common to all ferromagnetic substances.

(ii) The hysteresis loop area is proportional to the thermal energy developed per unit volume of the material

(iii) The hysteresis loop area is independent of the thermal energy developed per unit volume of the material.

(iv) The shape of the hysteresis loop is characteristic of the material.

Solution:

(i) True (ii) True (iii) False (iv) True

The energy lost per unit volume of a substance in a complete cycle of magnetisation is equal to the area of the hysteresis loop.

Illustration 76

The area of hysteresis loop of a material is equivalent to 250 Joule/m^3. When 10 kg material is magnetised by an alternating field of 50Hz then energy lost in one hour will be if the density of material is 7.5 gm/cm^3.

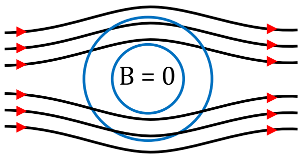
Solution:

Energy loss in 1 hour = Area × Volume × time × frequency $= \frac{250\times10\times3600\times50}{7500}$ = 60000 J = 6 × 10^4 J

Magnetic Shielding and Electromagnet

Magnetic Shielding

When a soft iron ring is placed in magnetic field, most of the lines are found to pass through the ring and no lines pass through the space inside the ring. The inside of the ring is thus protected against any external magnetic effect. This phenomenon is called magnetic screening or shielding.



Iron Ring in external magnetic field

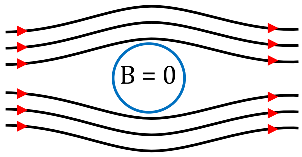
It is used to protect costly wrist–watches and other instruments from external magnetic fields by enclosing them in a soft–iron case or box.

Special Note : -

(i) Superconductors also provide perfect magnetic screening due to exclusion of lines of force. This effect is called **'Meissner effect'**

(ii) Relative magnetic permeability of super conductor is zero as the susceptibility is $\chi = -1$.

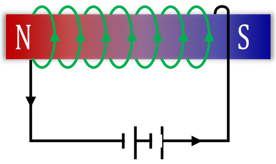
So, we can say that super conductors behave like **perfect diamagnetic material**.



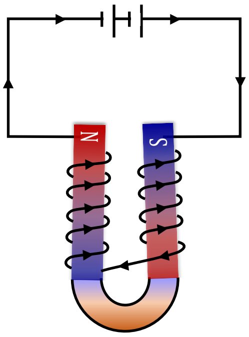
Super conductor in a field

Electromagnet

- An electromagnet can be defined as a magnet which functions on electricity.
- A current carrying solenoid behaves like a bar magnet.



(a)



(b)

- On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.
- If we place a soft iron rod in the solenoid, the magnetism of the solenoid increases hundreds of times and the solenoid is called an electromagnet.
- It is a temporary magnet.
- An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft iron straight rod or a U-shaped rod.

Applications of Electromagnet

(i) Electromagnets are used in electric bell, electric lock, telephone diaphragms etc.

(ii) In medical field they are used in extracting bullets from the human body.

(iii) Large electromagnets are used in cranes for lifting and transferring big machines and large objects.

Illustration 77

For protecting a magnetic needle, it should be placed in an iron box or in wooden box?

Solution:

In an iron box

BEGINNER'S BOX-13

1. Fill in the blanks

Diamagnetism is the .... property of every material and it is generated due to mutual interaction between the applied magnetic field and .... motion of electrons.

PME294

2. Match the column I with column II -

Column-I		Column-II	
(A)	Paramagnetic substance	(p)	$\chi_m = -1$
(B)	Diamagnetic substance	(q)	$\chi_m < 0$
(C)	Super conductor	(r)	$\chi_m >> 1$
(D)	Ferromagnetic substance	(s)	$\mu_r > 1$

PME295

3. How the magnetic susceptibility (χ_m) of diamagnetic, paramagnetic and ferromagnetic substances depends on temperature (T).

PME296

4. A solenoid has 10^3 turns per unit length. On passing a current of 2A, magnetic induction is measured to be 4π Wb/m^2. Calculate magnetic susceptibility of core.

PME297

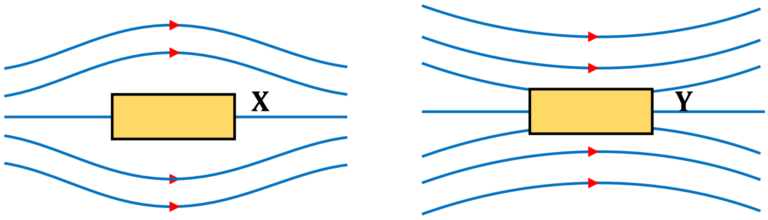
5. A magnetising field of 1600 A-m^{-1} produces a magnetic flux of 3.14×10^{-5} weber in a bar of iron of cross-section 0.2 cm^2 . Calculate permeability and susceptibility of the bar.

PME298

6. A uniform magnetic field gets modified as shown below, when the specimans X and Y are placed in it.

(a) Identify the two specimans X and Y

(b) State the reason for the behaviour of the field lines in X and Y.



PME299

7. How will a dia., para. and a ferromagnetic material behave when kept in a non–uniform external magnetic field? Give two examples of each of these materials. Name two main characteristics of a ferromagnetic material which help us to decide its suitability for making

(a) a permanent magnet (b) an electromagnet.

Which of these two characteristics should have high or low values for each of these two types of magnets?

PME300

8. An iron rod of cross sectional area 4 cm^2 is placed with its length parallel to a magnetising field 1600 Am^{-1}. The flux through the rod is 4×10^{-4} weber. Find out permeability of the material of the rod?

PME301

BEGINNER'S BOX — ANSWER KEY

BEGINNER'S BOX-1

1.	(3)	**2.**	(1)	**3.**	(4)

BEGINNER'S BOX-2

1.	(2)	**2.**	(3)	**3.**	(4)	**4.**	(2)	**5.**	(1)
6.	(2)	**7.**	(4)	**8.**	(1)	**9.**	(1)	**10.**	(3)
11.	(3)	**12.**	**(a)** 2 **(b)** 1	**13.**	**(a)** 3 **(b)** 1	**(c)** 2			

BEGINNER'S BOX-3

1.	(1)	**2.**	(4)	**3.**	(3)	**4.**	(1)	**5.**	(1)
6.	**(a)** (4)	**(b)** (1)		**7.**	**(a)** (2)	**(b)** (3)			
8.	**(a)** (2)	**(b)** (1)		**9.**	**(a)** (1)	**(b)** (2)			
10.	**(a)** (3)	**(b)** (2)		**11.**	**(a)** (1)	**(b)** (4)			

BEGINNER'S BOX-4

1.	(4)	**2.**	(2)	**3.**	(2)	**4.**	(1)	**5.**	(4)
6.	(1)								

BEGINNER'S BOX-5

1.	(1)	**2.**	(2)	**3.**	(4)	**4.**	(3)	**5.**	(3)
6.	(1)								

BEGINNER'S BOX-6

1.	(a) True (b) True	**2.**	B	**3.**	1T	**4.**	(1)

BEGINNER'S BOX-7

1.	(a) 2 : 1 : 1	(b) 1 : 1 : 2	(c) 2 : 1 : 2	(d) 2 : 1 : 1					
2.	(a) $C_p < C_e$	(b) $C_p = C_e$	(c) $C_p < C_e$	(d) $C_p < C_e$	(C = curvature)				
3.	(a) 1 : 2 : 2	(b) 2 : 1 : 1	(c) 2 : 1 : 1						
4.	(4)	**5.**	1.2 cm, 4.35 cm			**6.**	0.1m, 6.28×10^{-7} sec		
7.	T	**8.**	(3)	**9.**	(2)	**10.**	(3)	**11.**	(2)
12.	(2)	**13.**	(2)	**14.**	(a), (b) and (c)	**15.**	(a), (b) and (c)		
16.	(a), (b), (c) and (d)			**17.**	(a), (b) and (c)				

BEGINNER'S BOX-8

1.	(2)	**2.**	(1)	**3.**	(1)	**4.**	(4)	**5.**	(1)
6.	(2)	**7.**	(1)	**8.**	(4)	**9.**	(1)		

BEGINNER'S BOX-9

1. (a) 2M, (b) Zero, (c) $\sqrt{2}$ M, (d) M,(e) Zero, (f) $2\sqrt{2}$M

2. (a) $\frac{M}{2}$, (b) $\frac{M}{2}$

3. (a) $\frac{M}{\sqrt{2}}$, (b) $\frac{M}{2}$, (c) $\frac{2M}{\pi}$, (d) Zero

4. (a) $\sqrt{3}\frac{I\ell^2}{36}$, (b) $\frac{I\ell^2}{16}$, (c) $\frac{\sqrt{3}}{24}I\ell^2$, (d) $\frac{I\ell^2}{4\pi}$

5. 2 : 1

6. One turn

7. $\sqrt{2}$M

BEGINNER'S BOX-10

1.	(3)	**2.**	(2)	**3.**	(4)	**4.**	(4)	**5.**	(1)
6.	(1)	**7.**	(2)	**8.**	(2)	**9.**	(3)	**10.**	(1)

BEGINNER'S BOX-11

1.	(2)	**2.**	(1)

BEGINNER'S BOX-12

1.	(4)	**2.**	(4)	**3.**	(4)	**4.**	(4)

BEGINNER'S BOX-13

1. Intrinsic, orbital

2. A – s, B – q, C – p, D – r

3. For diamagnetic $\chi_m \propto T^0$

For paramagnetic $\chi_m \propto \frac{1}{T}$

For ferromagnetic $\chi_m \propto \frac{1}{T-T_c}$ here $T_c \to$ Curie temperature

4. 4999

5. Permeability (μ) = $9.8\times 10^{-4}\frac{Wb}{A-m}$ Susceptibility (χ_m) = 779.25

6. (a) X $\to$ Dia-magnetic Y $\to$ Para-magnetic

(b) Magnetic permeability of dia-magnetic speciman is slightly less than vacuum and magnetic permeability of para–magnetic speciman is slightly more than vacuum.

7. **Diamagnetic :** Moves (very weakly) away from strong field region towards the weak field region.

Para magnetic : Moves (weakly) towards the strong field region.

Ferromagnetic : Moves (strongly) towards the strong field region.

Two examples : Diamagnetic (Bismuth, Copper)

Paramagnetic (Aluminium, Oxygen)

Ferromagnetic (Iron, Nickel)

Two characteristics of ferromagnetic materials : Coercivity and Retentivity

For Permanent Magnets : High Coercivity and High Retentivity

For Electromagnets : Low Coercivity and Low Retentivity

8. $\mu = 0.625 \times 10^{-3}$ Wb /A-m