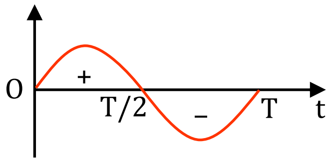
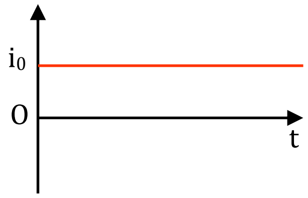
03 Alternating Current

Alternating current and Voltage

Alternating Quantity

An alternating quantity (current I or voltage V) whose direction reverses periodically.

Comparison of AC and DC

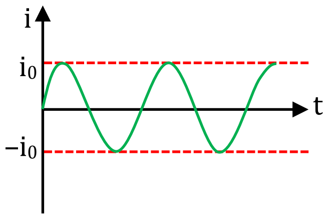
Alternating Current **AC**	**Direct Current** **DC**
$i = i_0 \sin\omega t$  Changes direction periodically	 Flows only in one direction
Can be Generated by using AC Generator.	Can be Generated by using DC Generator, Battery, solar panels.
Rectifier converts AC into DC.	Inverter converts DC into AC.
Can be controlled using Transformer.	Cannot be controlled using Transformer.

Advantages of AC

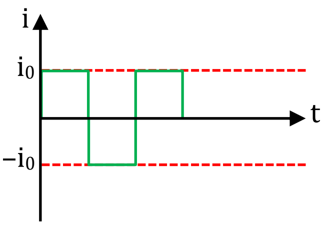
- A.C. is cheaper than D.C
- It can be easily converted into D.C. (by rectifier).
- It can be controlled easily (by choke coil).
- It can be transmitted over long distance at low power loss.
- It can be stepped up or stepped down with the help of transformer.

Types of current

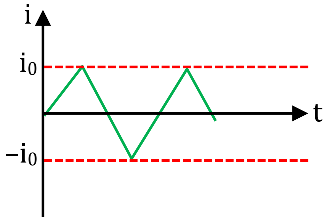
Examples of AC :

(1) 

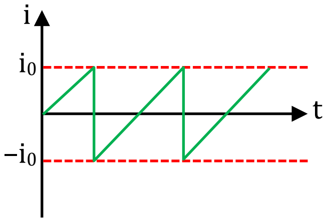
Sinusoidal AC

(2) 

Square AC

(3) 

Triangular AC

(4) 

Saw-tooth AC

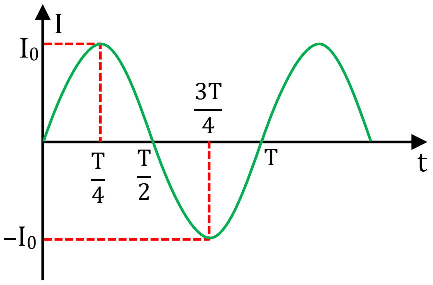
Equation for I and V

Alternating current or voltage varying as sine or cosine function can be written as

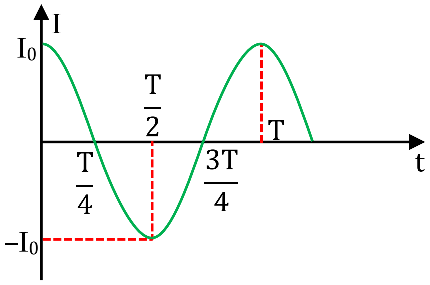
$I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$

where I = Instantaneous value of current at time t,

I_0 = Amplitude or peak value



I as a sine function of t



I as a cosine function of t

Standard definitions

1. **Amplitude of AC :** The maximum value of alternating current in either direction is called the amplitude of current. It is represented by I_0
2. **Time Period :** The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.
3. **Frequency :** The number of cycle completed by an alternating current in one second is called the frequency of the current.

   UNIT : (cycle/s) or (Hz)

   In India : f = 50 Hz , supply voltage = 220 volt

   In USA : f = 60 Hz , supply voltage = 110 volt

Important values of alternating quantities

Instantaneous Values

The instantaneous value is "the value of an alternating quantity at a particular instant of time in the cycle."

Illustration 1

Find out instantaneous current value for $I = I_0 \sin\omega t$ at $t = \frac{T}{8}$

Solution:

As $I = I_0 \sin\omega t$

At $t = \frac{T}{8} \Rightarrow I = I_0 \sin\left(\frac{2\pi}{T}\right)\left(\frac{T}{8}\right) = I_0 \sin\frac{\pi}{4} \Rightarrow I = \frac{I_0}{\sqrt{2}}$

Illustration 2

Find out instantaneous voltage for $V = 200 \sin (400\,\pi t)$ at $t = \frac{1}{800}$ sec

Solution:

As $V = 200 \sin 400\,\pi t$

At $t = \frac{1}{800}$ sec $\Rightarrow V = 200 \sin\left(400\pi \times \frac{1}{800}\right) \Rightarrow V = 200$ volt

Illustration 3

Find out instantaneous value for $I = I_0 \sin\left(\omega t - \frac{\pi}{6}\right)$ at $t = \frac{T}{2}$

Solution:

$$I = I_0 \sin\left[\frac{2\pi}{T} \times \frac{T}{2} - \frac{\pi}{6}\right] = I_0 \sin\frac{\pi}{6} = \frac{I_0}{2}$$

Peak value/Maximum value

The maximum value of current or voltage (I or V) is defined as peak value. It may or may not be equal to amplitude.

Some common examples:

1.	$I = I_0 \sin\omega t$	peak value = I_0	Amplitude = I_0
2.	$I = I_0 \cos\omega t$	peak value = I_0	Amplitude = I_0
3.	$I = I_0 \sin(\omega t+\phi)$	peak value = I_0	Amplitude = I_0
4.	$I = I_1 + I_0 \sin\omega t$	peak value = $I_1 + I_0$	Amplitude = I_0
5.	$I = I_1 \sin\omega t + I_2 \cos\omega t$	peak value = $\sqrt{\left(I_1^2 + I_2^2\right)}$	Amplitude = $\sqrt{I_1^2 + I_2^2}$
6.	$I = I_1 + I_2 \sin\omega t + I_3 \cos\omega t$	peak value = $I_0 + \sqrt{I_2^2 + I_3^2}$	Amplitude = $\sqrt{I_2^2 + I_3^2}$
7.	$I = I_0 \sin\omega t \cos\omega t$	peak value = $\frac{I_0}{2}$	Amplitude = $\frac{I_0}{2}$

Illustration 4

Find the time taken by current to reach $\frac{I_0}{\sqrt{2}}$, if the frequency of current is 50Hz.

Solution:

At $I = 0, \quad \phi = 0°$

At $I = \frac{I_0}{\sqrt{2}}, \ \phi = \frac{\pi}{4}$

As $\Delta t = \frac{T}{2\pi} \times \frac{\pi}{4} = \frac{T}{8}$

or $\Delta t = \frac{1}{8f} = \frac{1}{8 \times 50} = 2.5\text{ms}$

Average Value

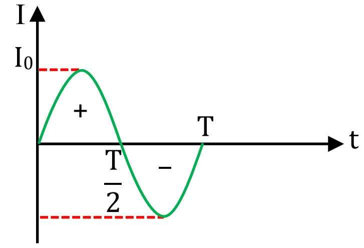
The mean value of A.C. over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

$$I_{avg} = \langle I \rangle = \frac{\int_{t_1}^{t_2} I dt}{\int_{t_1}^{t_2} dt} = \frac{\text{Area under } I - t \text{ graph}}{\text{time interval}}$$

Important formulae :

1. $\langle\sin\theta\rangle = \langle\cos\theta\rangle = 0$ (for full cycle)
2. $\langle\sin\theta\rangle = \langle\cos\theta\rangle = \frac{2}{\pi}$ (for half cycle)
3. $\langle\sin^2\theta\rangle = \langle\cos^2\theta\rangle = \frac{1}{2}$ (for full / half cycle)

1. Average value of sinusoidal AC



Let $I = I_0 \sin \omega t$

- **For one complete cycle**

$<I> = <I_0 \sin \omega t> = I_0 <\sin \omega t>$

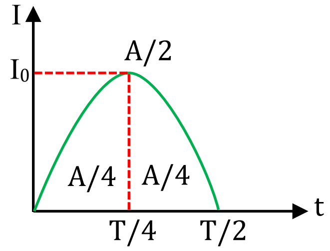
$<I> = 0$

- For half cycle :

$<I> = <I_0 \sin \omega t> = I_0 <\sin \omega t>$

$$<I> = \frac{2I_0}{\pi}$$

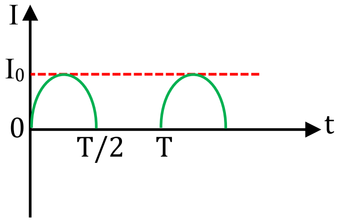
- For Quarter cycle :



$$<I> = \frac{A_{1/4}}{t_{1/4}} = \frac{\left(\frac{A_{1/2}}{2}\right)}{\left(\frac{t_{1/2}}{2}\right)} = \frac{A_{1/2}}{t_{1/2}}$$

Hence $<I> = \dfrac{2I_0}{\pi}$

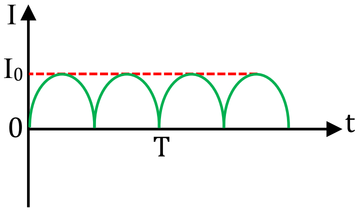
2. Average value for half wave rectifier



$$<I> = \frac{A}{t} = \frac{A_{1/2} + 0}{2t_{1/2}} = \frac{1}{2}\frac{A_{1/2}}{t_{1/2}} = \frac{1}{2}<I>_{1/2}$$

Hence $<I> = \dfrac{1}{2}\left(\dfrac{2I_0}{\pi}\right) = \dfrac{I_0}{\pi}$

3. Average value for full wave rectifier



$$<I> = \frac{A}{T} = \frac{2A_{1/2}}{2T_{1/2}} = \frac{A_{1/2}}{T_{1/2}}$$

Hence $<I>_{\text{full cycle}} = <I>_{\text{Half cycle}} = \dfrac{2I_0}{\pi}$

Note:

(a) Since average value of AC over complete cycle is zero therefore, batteries cannot be charged by using ac.

(b) Electrolysis and electroplating cannot be done by using ac.

Physical significance of Average Current

$$i_{av} = \frac{\int_0^t i.dt}{\int_0^t dt}$$

$i_{av} \times \Delta t = \int_0^t i.dt$ $\quad\therefore$ Charge flown by DC of value i_{av} = Charge flown by AC of value i

Illustration 5

If an A.C. main supply is given to be 220 V. What would be the average e.m.f. during a positive half cycle?

Solution:

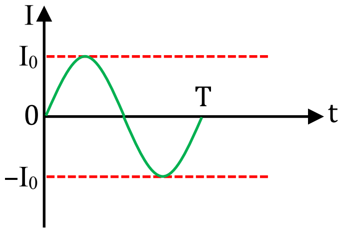
$$V_{avg}=\frac{2V_0}{\pi}=\frac{2\sqrt{2}V_{rms}}{\pi}=\frac{2\sqrt{2}\times 220}{3.14}=198\,V$$

Root Mean Square (RMS) value of current

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms}=\sqrt{\frac{\int_{t_1}^{t_2} I^2 dt}{\int_{t_1}^{t_2} dt}} \text{ or } I_{rms}^2=<I^2>$$

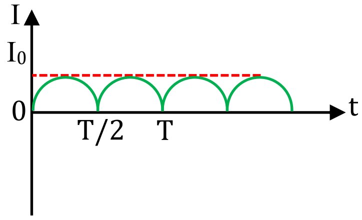
1. RMS value of sinusoidal ac



$$I_{rms}^2=<I^2>=<I_0^2\sin^2\omega t>=I_0^2<\sin^2\omega t>=I_0^2\times\frac{1}{2}$$

Hence $I_{rms}=\frac{I_0}{\sqrt{2}}$ or I_{rms} = $0.707I_0$

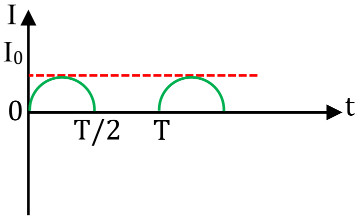
2. RMS value for full wave rectifier



$$I_{rms}^2=\frac{\left\{(I_{rms})^2_{0-\frac{T}{2}}\right\}\frac{T}{2}+\left\{(I_{rms})^2_{\frac{T}{2}-T}\right\}\frac{T}{2}}{\frac{T}{2}+\frac{T}{2}}$$

$$I_{rms}^2=\frac{\left(\frac{I_0}{\sqrt{2}}\right)^2\frac{T}{2}+\left(\frac{I_0}{\sqrt{2}}\right)^2\frac{T}{2}}{T}=\frac{\frac{I_0^2}{2}\times\frac{T}{2}\times 2}{T}=\frac{I_0^2}{2}\text{. Hence } I_{rms}=\frac{I_0}{\sqrt{2}}$$

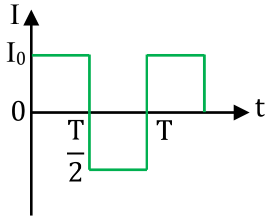
3. RMS value for half wave rectifier



$$I_{rms}^2=\frac{\left\{(I_{rms})^2_{0-\frac{T}{2}}\right\}\frac{T}{2}+\left\{(I_{rms})^2_{\frac{T}{2}-T}\right\}\frac{T}{2}}{\frac{T}{2}+\frac{T}{2}}$$

$$I_{rms}^2=\frac{\left(\frac{I_0}{\sqrt{2}}\right)^2\frac{T}{2}+(0)^2\frac{T}{2}}{T}=\frac{I_0^2}{4}\text{. Hence } I_{rms}=\frac{I}{2}$$

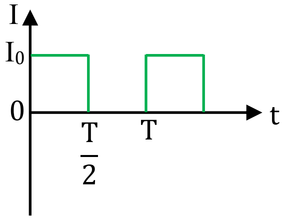
4. RMS value for a square wave AC



$$I_{rms}^2=\frac{\left\{(I_{rms})^2_{0-\frac{T}{2}}\right\}\frac{T}{2}+\left\{(I_{rms})^2_{\frac{T}{2}-T}\right\}\frac{T}{2}}{\frac{T}{2}+\frac{T}{2}}=\frac{I_0^2\frac{T}{2}+I_0^2\frac{T}{2}}{T}=I_0^2$$

Hence $I_{rms}=I_0$

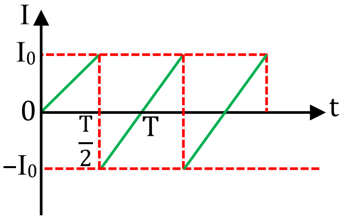
5. $$I_{rms}^2 = \frac{\left\{(I_{rms})^2_{0-\frac{T}{2}}\right\}\frac{T}{2} + \left\{(I_{rms})^2_{\frac{T}{2}-T}\right\}\frac{T}{2}}{\frac{T}{2}+\frac{T}{2}} = \frac{I_0^2\frac{T}{2}+(0)^2\frac{T}{2}}{T} = \frac{I_0^2}{2}$$



Hence $I_{rms} = \frac{I_0}{\sqrt{2}}$

6. As y = mx

$$I = \left(\frac{2I_0}{T}\right)t$$



$$I_{rms}^2 = \frac{\int_0^{\frac{T}{2}}\left(\frac{2I_0}{T}t\right)^2 dt}{\frac{T}{2}} = \frac{2}{T}\times\frac{4I_0^2}{T^2}\int_0^{\frac{T}{2}} t^2 dt$$

$$I_{rms}^2 = \frac{8I_0^2}{T^3}\left[\frac{1}{3}\left(\frac{T}{2}\right)^3\right] = \frac{8I_0^2}{3T^3}\times\frac{T^3}{8} = \frac{I_0^2}{3}$$

Hence $I_{rms} = \frac{I_0}{\sqrt{3}}$

Significance of RMS value

$$i_{rms}^2 = \frac{\int_0^T i^2 dt}{\Delta T}$$

$$i_{rms}^2 \times \Delta T = \int_0^T i^2 dt$$

$$i_{rms}^2 \times R \times \Delta T = \int_0^T i^2 R dt$$

$\therefore$ Heat produced by DC = Heat produced by AC

Important Note :

(i) The r.m.s. value of alternating current is also called virtual value or effective value.

(ii) In general when values of voltage or current for alternating circuits are given, these are r.m.s. value.

(iii) AC ammeter and voltmeter always measure r.m.s. value. Values printed on ac circuits are r.m.s. values.

(iv) In our houses ac is supplied at 220 V, which is the r.m.s. value of voltage. It's peak value is $\sqrt{2} \times 220 = 311$ V.

(v) Alternating current and voltages are measured by a.c. ammeter and a.c. voltmeter respectively. Working of these instruments is based on heating effect of current, hence they are also called hot wire instruments

	D.C. meter	**A.C. meter**
Name	Moving coil instrument	Hot wire instrument
Based on	Magnetic effect of current	Heating effect of current
Reads	Average value	r.m.s. value
If used in	A.C. circuit then they reads zero because average value of A.C is zero	A.C. or D.C. then meter works properly as it measures rms value
Deflection	Deflection $\propto$ current $\phi \propto I$ (linear)	Deflection $\propto$ heat $\phi \propto I_{rms}^2$ (Non Linear)
Scale	Uniform Separation	Non-Uniform Separation

Illustration 6

Find RMS value of $I = 3 + 4\sin(\omega t+\frac{\pi}{3})$.

Solution:

$i = 3 + 4\sin(\omega t + \frac{\pi}{3})$

$i^2 = 9 + 16\sin^2(\omega t + \pi/3) + 24\sin(\omega t + \frac{\pi}{3})$

$<i^2> = 9 + 16\left[\frac{1}{2}\right] + 24\,[0]$

$<i^2> = 9 + 16\left(\frac{1}{2}\right)$

$<i^2> = 17$

$\therefore$ r.m.s value = $\sqrt{17}$

Illustration 7

Find out RMS value of following currents

(i) $I = a + b\sin\omega t$

(ii) $I = a\sin\omega t + b\cos\omega t$

Solution:

(i) $I = a + b\sin\omega t$

$I_{rms}^2 = <I^2> = <(a + b\sin\omega t)^2>$

$= <a^2 + b^2\sin^2\omega t + 2ab\sin\omega t>$

$= <a^2> + b^2 <\sin^2\omega t> + 2ab<\sin\omega t>$

$= a^2 + b^2 \times \frac{1}{2} + 2ab \times 0 = a^2 + \frac{b^2}{2}$

Hence $I_{rms} = \sqrt{a^2 + \frac{b^2}{2}}$

(ii) $I = a\sin\omega t + b\cos\omega t$

$I_{rms} = \sqrt{a^2 + b^2}$

$< I > = < a\sin\omega t + b\cos\omega t > = a < \sin\omega t > + b < \cos\omega t >$

$\Rightarrow < I > = 0$

$I_{rms}^2 = <I^2> = a^2 < \sin^2\omega t > + b^2 < \cos^2\omega t > + ab < \sin^2\omega t > = \frac{a^2}{2} + \frac{b^2}{2} + 0$

$\Rightarrow I_{rms} = \sqrt{\frac{a^2}{2} + \frac{b^2}{2}}$

BEGINNER'S BOX-1

1. Explain why A.C. is more dangerous than D.C.?

PAC137

2. Show that average heat produced during a cycle of AC is same as produced by DC with $I = I_{rms}$.

PAC138

3. An ordinary moving coil ammeter used for d.c., cannot be used to measure a.c. even if its frequency is low why?

PAC139

4. Find the time required for a 50Hz alternating current to change its value from zero to rms value.

PAC140

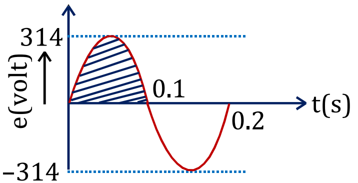
5. The current and voltage in a circuit is given by
$i = 3.5 \sin(628t + 30°)$ A, $V = 28 \sin(628t – 30°)$ volt. Find

(a) time period of current

(b) phase difference between voltage and current.

PAC141

6. The figure given below shows the variation of an alternating emf with time. What is the average value of the emf for the shaded part of the graph?



PAC142

Concept of Phasor

Phase:

Physical quantity which represents both the instantaneous value and direction of alternating quantity at any instant is called its phase.

It's SI unit is radian. It is a dimensionless quantity.

If an alternating quantity is expressed as $I = I_0 \sin(\omega t + \phi)$ then the argument of sine is called it's phase.

where ωt = instantaneous phase (changes with time)

ϕ = initial phase or phase constant(constant w.r.t. time)

- **Phase difference :** The difference between the phases of currents and voltage is called phase difference. If alternating voltage and current are given by $V = V_0 \sin(\omega t + \phi_1)$ and $i = i_0 \sin(\omega t + \phi_2)$ then phase difference $\Delta\phi = \phi_1 - \phi_2$ (relative to current) or (relative to voltage)
- **Time difference :** If phase difference between alternating current and voltage is ϕ then time difference between them is given as :-

$$\frac{\Delta\phi}{2\pi} = \left(\frac{\Delta t}{T}\right)$$

Illustration 8

If phase difference between E and I is $\frac{\pi}{4}$ and f = 50 Hz then calculate time difference.

Solution:

$$\frac{\text{Phase difference}}{2\pi} = \frac{\text{time difference}}{T} \Rightarrow \text{Time difference} = \frac{T}{2\pi} \times \frac{\pi}{4} = \frac{T}{8} = \frac{1}{50 \times 8} = 2.5\text{ms}$$

- ## Phasor Diagram

A diagram representing alternating current and voltage (of same frequency) as vectors (phasor) with the phase angle between them is called phasor diagram.

Properties :

(1) They rotate anticlockwise

(2) The length of phasors represents the maximum value (amplitude) of quantity.

(3) Vertical component represents its instantaneous value.

Let $V = V_0 \sin \omega t$ and $I = I_0 \sin (\omega t + \phi)$

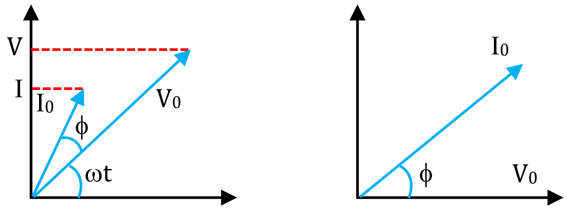


fig (a) fig (b)

In figure (a) two arrows represents phasors. The length of phasors represents the maximum value of quantity. The projection of a phasor on y-axis represents the instantaneous value of quantity.

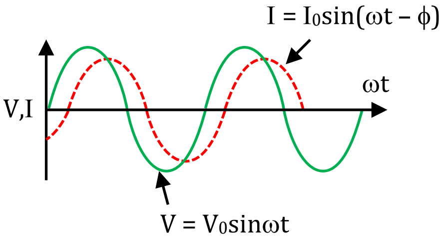
In figure (b) two arrows represents phasor. Their length represents maximum value.

Lagging and leading Concept

(a) V leads I or I lags V → It means, V reach maximum before I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin (\omega t - \phi)$

and if $V = V_0 \sin (\omega t + \phi)$ then $I = I_0 \sin \omega t$



(b) V lags I or I leads V → It means V reach maximum after I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin (\omega t + \phi)$

and if $V = V_0 \sin (\omega t - \phi)$ then $I = I_0 \sin \omega t$

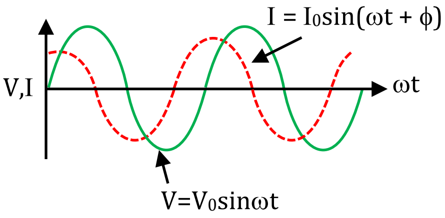
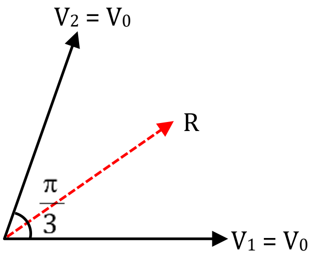


Illustration 9

If $V_1 = V_0 \sin\omega t$ and $V_2 = V_0 \sin\left(\omega t + \frac{\pi}{3}\right)$ than find resultant of the two voltages.



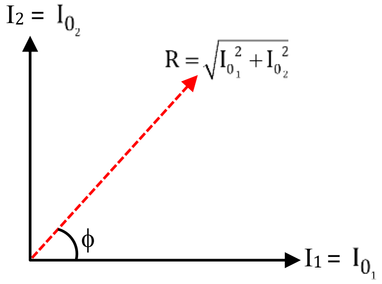
Solution:

$R = \sqrt{3}V_0$

Resultant voltage equation $= R = \sqrt{3}V_0 \sin\left(\omega t + \frac{\pi}{6}\right)$

Illustration 10

Find resultant of $I_1 = I_{0_1}\sin\omega t$ & $I_2 = I_{0_2}\cos\omega t$



Solution:

$$\tan\phi=\left(\frac{I_{0_2}}{I_{0_1}}\right)$$

$$\text{Final equation of current}=\sqrt{I_{0_1}^2+I_{0_2}^2}\ \sin\left[\omega t+\tan^{-1}\left(\frac{I_{0_2}}{I_{0_1}}\right)\right]$$

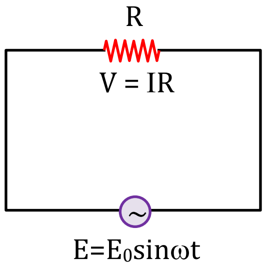
AC circuits

In order to study the behaviour of A.C. circuits we classify them into two categories :

(a) Simple circuits containing only one basic element i.e. resistor (R) or inductor (L) or capacitor (C) only.

(b) Complicated circuit containing any two of the three circuit elements R, L and C or all of the three elements.

AC circuit containing pure Resistor



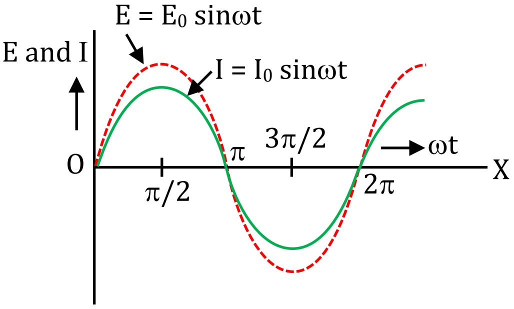
Let at any instant t, the current in the circuit is I.

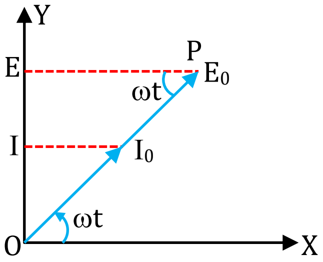
Potential difference across the resistance = I R

with the help of Kirchhoff's circuital law $E - IR = 0 \Rightarrow E_0\sin\omega t = IR$

$$I=\frac{E_0}{R}\sin\omega t=I_0\sin\omega t\ \ \left(I_0=\frac{E_0}{R}=\text{peak or maximum value of current}\right)$$

Alternating current developed in a pure resistance is also of the sinusoidal nature. In a.c. circuits containing pure resistance, the voltage and current are in the same phase. The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. are as shown in figure.





In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors E_0 and I_0 are in the same direction, making an angle ωt with OX. Their projections on Y-axis represent the instantaneous values of alternating current and voltage.

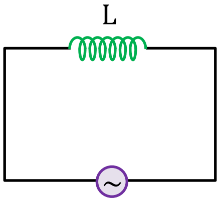
i.e. $I = I_0\sin\omega t$ and $E = E_0\sin\omega t$.

$$\text{Since } I_0=\frac{E_0}{R}\text{, hence }\frac{I_0}{\sqrt{2}}=\frac{E_0}{R\sqrt{2}}\Rightarrow I_{rms}=\frac{E_{rms}}{R}$$

AC circuit containing pure Inductor

A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf. Let the alternating e.m.f. $E = E_0 \sin\omega t$

When a.c. flows through the circuit, emf induced across inductance $= -L\frac{dI}{dt}$

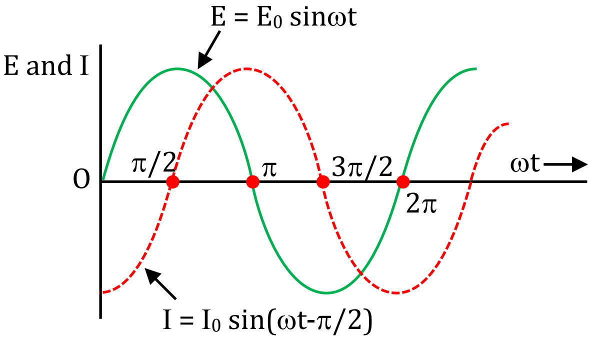


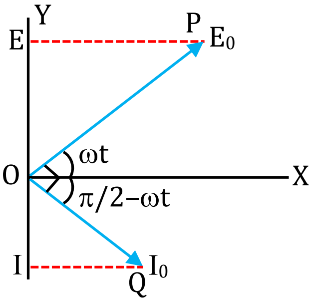
$E=E_0\sin\omega t$

Note : Negative sign indicates that induced emf acts in opposite direction to that of applied emf. Because there is no other circuit element present in the circuit other than inductance so with the help of Kirchhoff's circuital law $E+\left(-L\frac{dI}{dt}\right)=0 \Rightarrow E=L\frac{dI}{dt}$ so we get $I=\frac{E_0}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right)$

Maximum current $I_0=\frac{E_0}{\omega L}\times 1=\frac{E_0}{\omega L}$, Hence, $I=I_0\sin\left(\omega t-\frac{\pi}{2}\right)$

In a pure inductive circuit current always lags behind the emf by $\frac{\pi}{2}$ or alternating emf leads the a. c. by a phase angle of $\frac{\pi}{2}$.





Expression $I_0=\frac{E_0}{\omega L}$ resembles the expression $\frac{E}{I}=R$.

This non-resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circuit.

$X_L = \omega L = 2\pi f L$ where f = frequency of A.C.

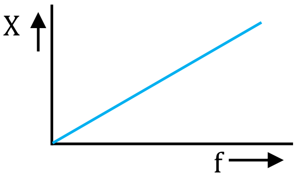
Unit of X_L : ohm

(ωL) = Unit of L × Unit of $(\omega = 2\pi f)$ = henry × $\sec^{-1}$

$$=\frac{\text{volt}}{\text{ampere / sec}}\times\sec^{-1}=\frac{\text{volt}}{\text{ampere}}=\text{ohm}$$

Inductive reactance $X_L \propto f$

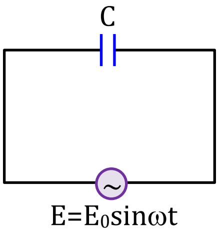
Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.



For d.c. circuit, f = 0 $\therefore X_L = \omega L = 2\pi f L = 0$

Hence, inductor offers no opposition to the flow of d.c. whereas a resistive path to a.c.

AC circuit containing pure Capacitor



A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit $E = E_0 \sin \omega t$. When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.

The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also the electric field between the plates of the capacitor varies sinusoidally with time. Let at any instant t charge on the capacitor = q

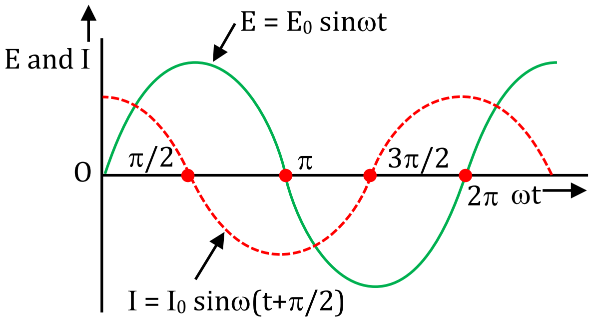
Instantaneous potential difference across the capacitor $E = q/C$

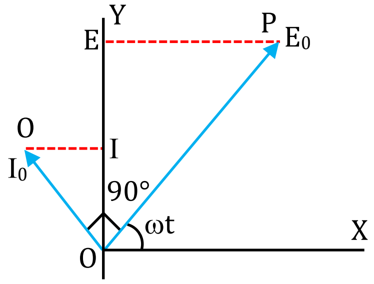
$q = C E \Rightarrow q = CE_0 \sin \omega t$

The instantaneous value of current $I = \frac{dq}{dt} = \frac{d}{dt}(CE_0 \sin \omega t) = CE_0 \omega \cos \omega t$

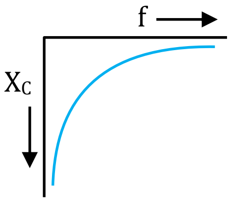
$\Rightarrow = I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$ where $I_0 = \omega CE_0$

In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of $\pi/2$. The alternating emf lags behinds the alternating current by a phase angle of $\pi/2$.





Important Points



E/I is the resistance R when both E and I are in phase, in present case they differ in phase by $\frac{\pi}{2}$, hence $\frac{1}{\omega C}$ is not the resistance of the capacitor, the capacitor offer opposition to the flow of A.C. This non-resistive opposition to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

Unit of X_C : ohm

Capacitive reactance X_C is inversely proportional to frequency of A.C. X_C decreases as the frequency increases.

For d.c. circuit f = 0 $\therefore$ $X_C = \frac{1}{2\pi fC} = \infty$ but has a very small value for a.c.

This shows that capacitor blocks the flow of d.c. but provides an easy path for a.c. individual Components (R or L or C)

Illustration 11

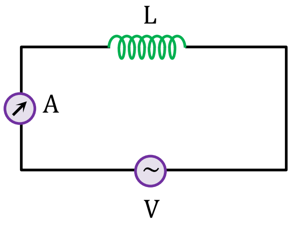
A capacitor of 50 pF is connected to an a.c. source of frequency 1kHz. Calculate its reactance.

Solution:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi\times10^3\times50\times10^{-12}} = \frac{10^7}{\pi}\,\Omega$$

Illustration 12

In given circuit applied voltage V = 50 $\sqrt{2}$ sin (100πt) volt and ammeter reading is 2A then calculate value of L?



Solution:

$V_{rms} = I_{rms} X_L$ ∵ Reading of ammeter = I_{rms}

$$X_L = \frac{V_{rms}}{I_{rms}} = \frac{V_0}{\sqrt{2}\,I_{rms}} = \frac{50\sqrt{2}}{\sqrt{2}\times2} = 25\,\Omega \Rightarrow L = \frac{X_L}{\omega} = \frac{25}{100\pi} = \frac{1}{4\pi}\,H$$

Illustration 13

An Inductor $\frac{10}{\pi}$ mH is connected across an ac-source of 50 V, 50 Hz. Find out.

(i) Impendence (ii) I_{rms} (iii) I_0 (iv) Equation of current

Solution:

Given $L = \frac{10}{\pi}\text{mH}$, f = 50 Hz

(i) $Z = \omega L = 2\pi\times50\times\frac{10}{\pi}\times10^{-3} = 1\Omega$

(ii) $I_{rms} = \frac{V_{rms}}{Z} = \frac{50}{1} = 50A$

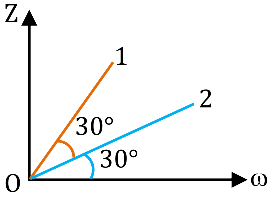
(iii) $I_0 = I_{rms}\sqrt{2} = 50\sqrt{2}A$

(iv) $I = 50\sqrt{2}\sin\left(100\pi t - \frac{\pi}{2}\right) = -50\sqrt{2}\cos(100\pi)t$

Illustration 14

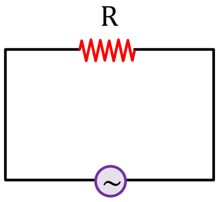
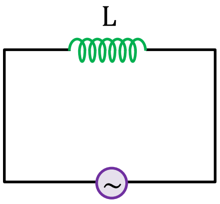
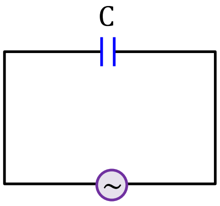
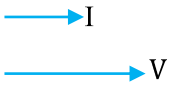
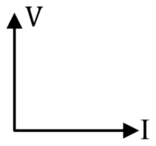
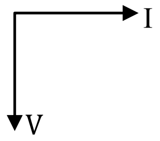
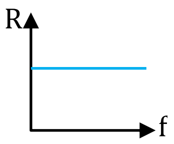
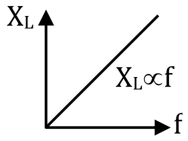
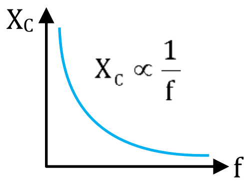
Across two different inductors same alternating source is connected. For this Z v/s ω graph is given. Find out $\frac{L_1}{L_2}$?

Solution:



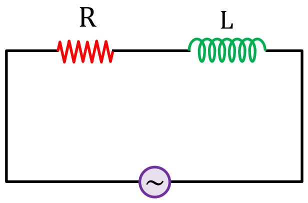
As $\tan\phi = \frac{Z}{\omega} = \frac{\omega L}{\omega} = L$

So $\frac{L_1}{L_2} = \frac{\tan60^\circ}{\tan30^\circ} = \frac{\sqrt{3}}{\frac{1}{\sqrt{3}}} = 3$

TERM	R	L	C
Circuit			
Supply Voltage	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
Current	$I = I_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$
Peak Current	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0 \omega C$
Impedance (Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$	R = Resistance	X_L = Inductive reactance.	X_C = Capacitive reactance.
Phase difference	zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
Phasor diagram			
Variation of Z with f			
G, S_L, S_C (mho, seiman)	G=1/R conductance.	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
Behaviour of device in D.C. and A.C	Same in A C and D C	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f}\right]$
Ohm's law	$V_R = IX_R$	$V_L = IX_L$	$V_C = IX_C$

AC circuit containing resistance and inductance in series (R-L circuit)

A circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in figure.

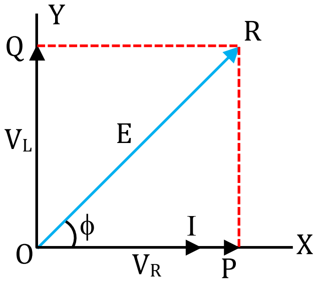


$E=E_0 \sin\omega t$

- Phasor diagram For L-R circuit

  Let in a L-R series circuit, applied alternating emf is $E = E_0 \sin\omega t$. As R and L are joined in series, hence current flowing through both will be same at each instant. Let I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant.

  Then $V_L = IX_L$ and $V_R = IR$

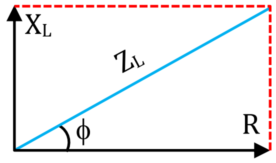
  Now, V_R is in phase with the current while V_L leads the current by $\frac{\pi}{2}$.

  So V_R and V_L are mutually perpendicular (Note : $E \neq V_R + V_L$)

  The vector OP represents V_R (which is in phase with I), while OQ represents V_L (which leads I by 90°).

  The resultant of V_R and V_L = the magnitude of vector OR $E=\sqrt{V_R^2+V_L^2}$

  Thus $E^2=V_R^2+V_L^2=I^2(R^2+X_L^2) \Rightarrow I=\dfrac{E}{\sqrt{R^2+X_L^2}}$

  The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind the e.m.f. E by a phase angle ϕ

  $$\tan\phi=\frac{V_L}{V_R}=\frac{IX_L}{IR}=\frac{X_L}{R}=\frac{\omega L}{R} \Rightarrow \phi=\tan^{-1}\left(\frac{\omega L}{R}\right)$$

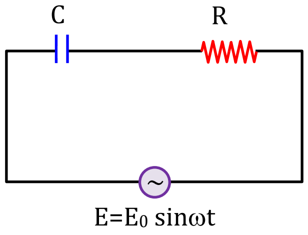
- Inductive Impedance Z_L :

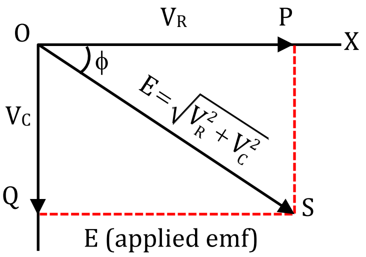
  In L-R circuit the maximum value of current $I_0=\dfrac{E_0}{\sqrt{R^2+\omega^2L^2}}$ Here $\sqrt{R^2+\omega^2L^2}$ represents the effective opposition offered by L-R circuit to the flow of a.c. through it. It is known as impedance of L-R circuit and is represented by Z_L. $Z_L=\sqrt{R^2+\omega^2L^2}=\sqrt{R^2+(2\pi fL)^2}$

  The reciprocal of impedance is called admittance $Y_L=\dfrac{1}{Z_L}=\dfrac{1}{\sqrt{R^2+\omega^2L^2}}$

AC circuit containing resistance and capacitor in series (R-C circuit)

A circuit containing a series combination of a resistance R and a capacitor C, connected with a source of e.m.f. of peak value E_0 as shown in fig.





- phasor diagram For R-C circuit

  Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

  $V_C = I\,X_C$ and $V_R = I\,R$

  where X_C = capacitive reactance and I = instantaneous current.

  Now, V_R is in phase with I, while V_C lags behind I by 90°.

  The vector OP represents V_R (which is in phase with I)

  and the vector OQ represents V_C (which lags behind I by $\frac{\pi}{2}$).

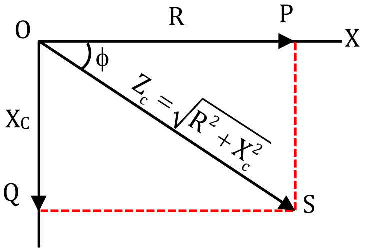
  The vector OS represents the resultant of V_R and

  V_C = the applied e.m.f. E.

  Hence $V_R^2+V_C^2=E^2 \Rightarrow E=\sqrt{V_R^2+V_C^2} \Rightarrow E^2=I^2(R^2+X_C^2) \Rightarrow I=\frac{E}{\sqrt{R^2+X_C^2}}$

  The term $\sqrt{(R^2+X_C^2)}$ represents the effective resistance of the R-C circuit and called the capacitive impedance Z_C of the circuit.

  Hence, in C-R circuit $Z_C=\sqrt{R^2+X_C^2}=\sqrt{R^2+\left(\frac{1}{\omega C}\right)^2}$



- **Capacitive Impedance Z_C :**

  In R-C circuit the term $\sqrt{R^2+X_C^2}$ effective opposition offered by R-C circuit to the flow of a.c. through it. It is known as impedance of R-C circuit and is represented by Z_C. The phasor diagram also shows that in R-C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

  $\tan\phi=\frac{V_C}{V_R}=\frac{X_C}{R}=\frac{1/\omega C}{R}=\frac{1}{\omega CR}$, $\tan\ \phi=\frac{X_C}{R}=\frac{1}{\omega CR}\Rightarrow\phi=\tan^{-1}\left(\frac{1}{\omega CR}\right)$

Combination of components (R-L or R-C or L-C)

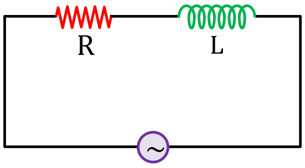
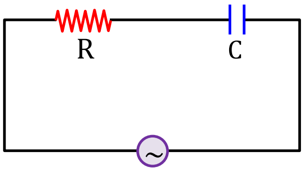
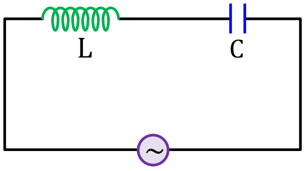
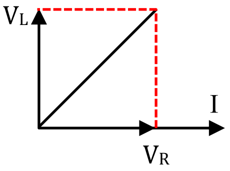
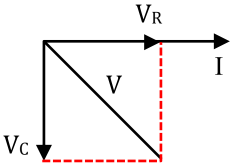
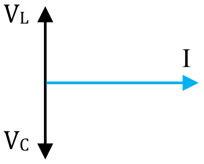
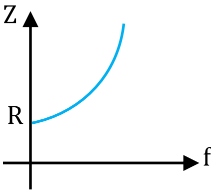
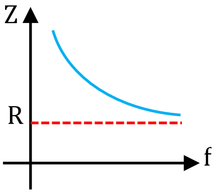
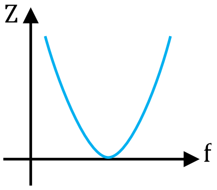
TERM	R-L	R-C	L-C
Circuit	 I is same in R & L	 I is same in R & C	 I is same in L & C
Phasor diagram	 $V^2 = V_R^2 + V_L^2$	 $V^2 = V_R^2 + V_C^2$	 $V = V_L - V_C$ ($V_L > V_C$) $V = V_C - V_L$ ($V_C > V_L$)
Phase difference in between V & I	V leads I ($\phi = 0$ to $\frac{\pi}{2}$)	V lags I ($\phi = -\frac{\pi}{2}$ to 0)	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C > X_L$) V leads I ($\phi = +\frac{\pi}{2}$,if $X_L > X_C$)
Impedance	$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + (X_C)^2}$	$Z = \lvert X_L - X_C \rvert$
Variation of Z With f	as f↑, Z↑ 	as f↑, Z↓ 	as f↑, Z first ↓ then ↑ 
At very low f	$Z \simeq R$ ($X_L \to 0$)	$Z \approx X_C$	$Z \simeq X_C$
At very high f	$Z \simeq X_L$	$Z \simeq R$ ($X_C \to 0$)	$Z \simeq X_L$

Illustration 15

What is the inductive reactance of a coil if the current through it is 20 mA and voltage across it is 100 V.

Solution:

$\because \quad V_L = IX_L$

$$\therefore \quad X_L = \frac{V_L}{I} = \frac{100}{20 \times 10^{-3}} = 5\ k\Omega$$

Illustration 16

The reactance of capacitor is 20 ohm. What does it mean? What will be its reactance if frequency of AC is doubled? What will be its, reactance when connected in DC circuit? What is its consequence?

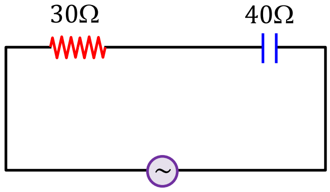
Solution:

The reactance of capacitor is 20 ohm. It means that the hinderance offered by it to the flow of AC at a specific frequency is equivalent to a resistance of 20 ohm. The reactance of capacitance $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

Therefore, by doubling frequency, the reactance is halved i.e., it becomes 10 ohm. In DC circuit f = 0. Therefore, reactance of capacitor = ∞ (infinite). Hence the capacitor can not be used to control DC.

Illustration 17

Calculate the impedance of the circuit shown in the figure.



Solution:

$z = \sqrt{R^2 + (X_c)^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = 50\ \Omega$

Illustration 18

When 10V, dc is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduces to 2 A. Calculate reactance of the coil.

Solution:

For 10 V D.C. $\because$ V = IR

$\therefore$ Resistance of coil R = $\frac{10}{2.5} = 4\Omega$

For 10 V A.C. V = I Z $\Rightarrow$ $Z = \frac{V}{I} = \frac{10}{2} = 5\Omega$

$\because\ Z = \sqrt{R^2 + X_L^2} = 5 \Rightarrow R^2 + X_L^2 = 25 \Rightarrow X_L^2 = 5^2 - 4^2 \Rightarrow X_L = 3\Omega$

Illustration 19

When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by π/2 radians.

(a) Name the devices X and Y.

(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.

Solution:

(a) X is resistor and Y is a capacitor

(b) Since the current in the two devices is the same (0.5A at 220 volt)

When R and C are in series across the same voltage then

$R = X_C = \frac{220}{0.5} = 440\ \Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{220}{\sqrt{(440)^2 + (440)^2}} = \frac{220}{440\sqrt{2}} = 0.35A$

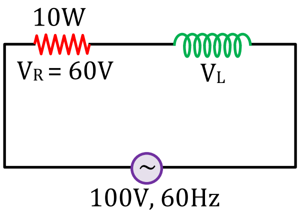
Illustration 20

Find out the required inductance to put in series of bulb (10W, 60V) to run it safely across an alternating supply of 100V, 60Hz.

Solution:

For LR-circuit

$V = \sqrt{V_R^2 + V_L^2}$



$$R=\frac{60\times60}{10}=360\Omega$$

$$V_L=\sqrt{(100)^2-(60)^2}=80V$$

P = VI

$$I=\frac{P}{V}=\frac{10W}{60V}=\frac{1}{6}A(\text{in bulb})$$

$$X_L=\frac{V_L}{I}=\frac{80}{\frac{1}{6}}=480\Omega$$

$$\therefore\ L=\frac{X_L}{\omega}=\frac{480}{2\pi(60)}=\frac{4}{\pi}H$$

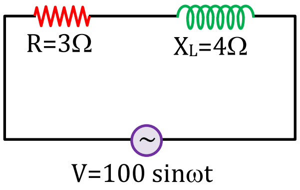
Illustration 21

For the RL-circuit calculate the following :

(i) Impedance of circuit

(ii) Phase difference between V & I

(iii) I_{rms}



Solution:

(i) Impedance of circuit : $Z=\sqrt{R^2+X_L^2}=\sqrt{9+16}=5\Omega$

(ii) $\tan\phi=\frac{X_L}{R}=\frac{4}{3}\Rightarrow\phi=53°$

(iii) $I_{rms}=\frac{V_{rms}}{Z}=\frac{\left(\frac{100}{\sqrt{2}}\right)}{5}10\sqrt{2}A$

BEGINNER'S BOX-2

1. A voltage V = 60 sin πt volt is applied across a 20 Ω resistor. What will an ac ammeter in series with the resistor read?

PAC143

2. An alternating current source E = 100 sin (1000t) volt is connected through a inductor of 10 μH then write down the equation of current.

PAC144

3. An alternating voltage E = 200 $\sqrt{2}$ sin (100t) volt is applied to 2H inductor through an a.c. ammeter. What will be reading of the ammeter?

PAC145

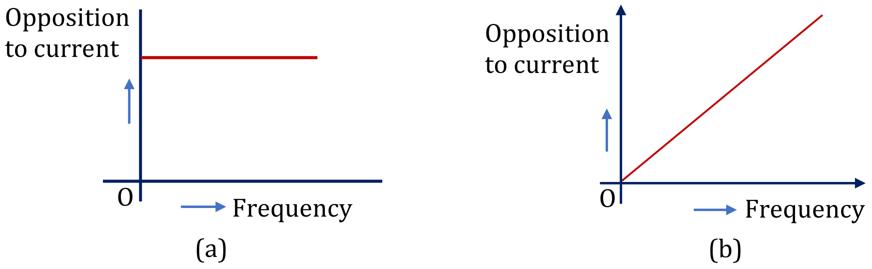
4. A 15.0 μF capacitor is connected to a 220 V, 50 Hz source. Find the capacitive reactance and the current (rms and peak) in the circuit. If the frequency is doubled, what happens to the capacitive reactance and the current?

PAC146

5. A 60 μF capacitor is connected to a 110 V. 60 Hz a.c. supply. Determine the rms value of the current in the circuit.

PAC147

6. The given graphs (a) and (b) represent the variation of the opposition offered by the circuit element to the flow of alternating current, with frequency of the applied emf. Identify the circuit element corresponding to each graph.

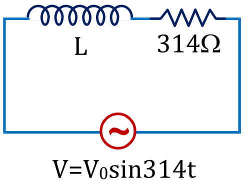


PAC148

7. When a series combination of inductance and resistance are connected with a 10V, 50 Hz a.c. source, a current of 1A flows in the circuit. The voltage leads the current by a phase angle of $\frac{\pi}{3}$ radian. Calculate the values of resistance and inductive reactance.

PAC149

8. The current in the shown circuit is found to be $4\sin\left(314t-\frac{\pi}{4}\right)$A.



Find the value of inductance.

PAC150

9. A current of 4A flows in a coil when connected to a 12V dc source. If the same coil is connected to a 12V, 50 rad/s a.c. source, a current of 2.4A flows in the circuit. Determine the inductance of the coil.

PAC151

10. An alternating voltage E=200 sin (300t) volt is applied across a series combination of R=10Ω and inductance of 800mH. Calculate the impedance of the circuit.

PAC152

11. A coil of reactance 100 Ω and resistance 100 Ω is connected to a 240 V, 50 Hz a.c. supply.

(a) What is the maximum current in the coil?

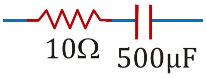
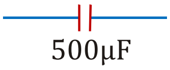
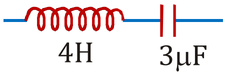
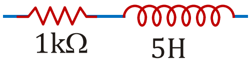
(b) What is the time lag between the voltage maximum and the current maximum?

PAC153

12. An inductance has a resistance of 100 Ω. When a.c. signal of frequency 1000 Hz is applied to the coil, the applied voltage leads the current by 45°. Calculate the self inductance of the coil.

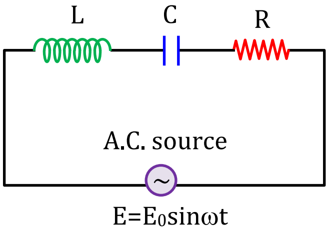
PAC154

13. Match the following options –

Circuit component across an ac source (ω = 200 rad/sec)		Phase difference between current and source voltage	
(A)		(p)	$\frac{\pi}{2}$
(B)		(q)	$\frac{\pi}{6}$
(C)		(r)	$\frac{\pi}{4}$
(D)		(s)	$\frac{\pi}{3}$
(E)		(t)	None of the above

PAC155

Series LCR Circuit



A circuit containing a series combination of an resistance R, a coil of inductance L and a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value E_0, as shown in figure.

Phasor Diagram For Series L-C-R circuit

Let in series LCR circuit applied alternating emf is $E = E_0 \sin\omega t$. As L, C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

However voltage across each element bears a different phase relationship with the current.

Let at any instant of time t the current in the circuit is I.

Let at this time t the potential differences across L, C, and R

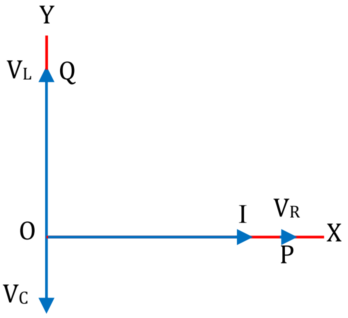
$V_L = I X_L$, $V_C = I X_C$ and $V_R = I R$

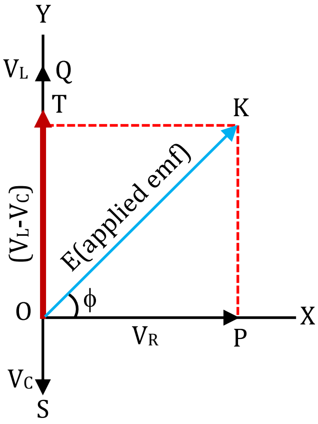
Now, V_R is in phase with current I but V_L leads I by 90°

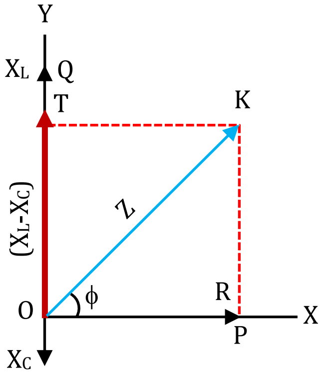
While V_C lags behind I by 90°.

The vector OP represents V_R (which is in phase with I) the vector OQ represent V_L (which leads I by 90°)

and the vector OS represents V_C (which legs behind I by 90°) V_L and V_C are opposite to each other.







If $V_L > V_C$ (as shown in figure) then their resultant will be $(V_L - V_C)$ which is represented by OT. Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three applied e.m.f.

Thus $$E=\sqrt{V_R^2+(V_L-V_C)^2} = I\ \sqrt{R^2+(X_L-X_C)^2}$$

$$\Rightarrow I=\frac{E}{\sqrt{R^2+(X_L-X_C)^2}}$$

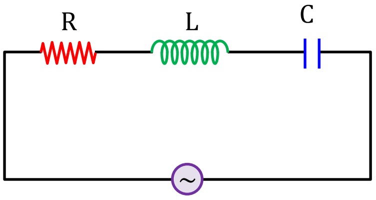
Impedance $$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$$

The phasor diagram also shown that in LCR circuit the applied e.m.f. leads the current I by a phase angle ϕ.

$$\tan\phi = \frac{X_L-X_C}{R}$$

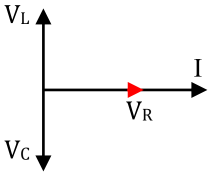
SERIES L-C-R CIRCUIT

1. Circuit diagram

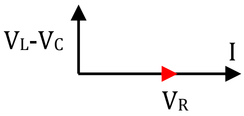


I same for R, L & C

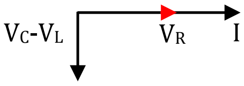
2. Phasor diagram



(i) If $V_L > V_C$ then



(ii) If $V_C > V_L$ then



(iii) $V = \sqrt{V_R^2+(V_L-V_C)^2}$

Impedance $Z=\sqrt{R^2+(X_L-X_C)^2}$

$\tan\phi = \frac{X_L-X_C}{R} = \frac{V_L-V_C}{V_R}$

(iv) Impedance triangle

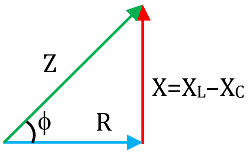
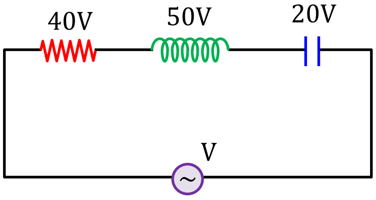


Illustration 22

For the given LCR-series circuit. Find out applied source voltage of ac.

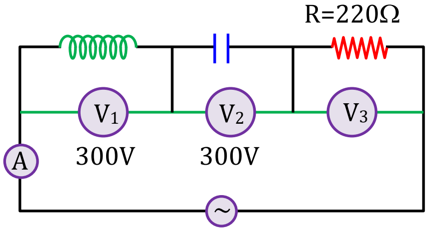


Solution:

$$V=\sqrt{V_R^2+(V_L-V_C)^2}=\sqrt{(40)^2+(50-20)^2}=50V$$

Illustration 23

A capacitor, a resistor and a inductor are connected in series to an ac-source of 110 V and frequency 60Hz. Find reading of voltmeter V_3 and ammeter in the given LCR-series circuit.



110V, 60Hz

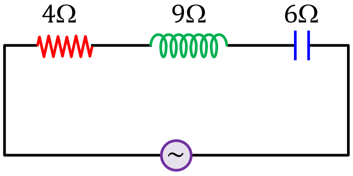
Solution:

∵ $V_1 = V_2$

therefore $V_3 = V_R = 110$ volt and $I=\frac{V}{R}=\frac{110}{220}=0.5\,A$

Illustration 24

Find out the impedance of given circuit.



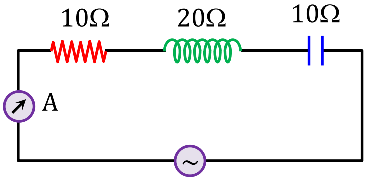
$\because X_L > X_C \Rightarrow$ Inductive

Solution:

$$Z=\sqrt{R^2+(X_L-X_C)^2}=\sqrt{4^2+(9-6)^2}=\sqrt{4^2+3^2}=\sqrt{25}=5\Omega$$

Illustration 25

Find out reading of A.C. ammeter and also calculate the potential difference across, resistance and capacitor.



$E = 100\sin\omega t100\pi t$ volt

Solution:

$$Z=\sqrt{R^2+(X_L-X_C)^2}=10\sqrt{2}\,\Omega \Rightarrow I_0=\frac{V_0}{Z}=\frac{100}{10\sqrt{2}}=\frac{10}{\sqrt{2}}A$$

∵ ammeter reads RMS value, so its reading = $\frac{10}{\sqrt{2}\sqrt{2}}=5A$

So, $V_R = 5 \times 10 = 50$ V and $V_C = 5 \times 10 = 50$ V

Illustration 26

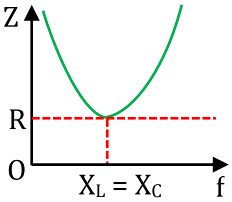
In LCR circuit with an AC source R = 300 Ω, C = 20 μF, L = 1.0 H, E_{rms} = 50V and f = 50/π Hz. Find RMS current in the circuit.

Solution:

$$I_{rms}=\frac{E_{rms}}{Z}=\frac{E_{rms}}{\sqrt{R^2+\left[\omega L-\frac{1}{\omega C}\right]^2}}=\frac{50}{\sqrt{300^2+\left[2\pi\times\frac{50}{\pi}\times 1-\frac{1}{20\times10^{-6}\times2\pi\times\frac{50}{\pi}}\right]^2}}$$

$$\Rightarrow \quad I_{rms}=\frac{50}{\sqrt{(300)^2+\left[100-\frac{10^3}{2}\right]^2}}=\frac{50}{100\sqrt{9+16}}=\frac{1}{10}=0.1A$$

Impedance vs Frequency Graph



$$Z=\sqrt{(R)^2+(X_L-X_C)^2}$$

$$Z=\sqrt{(R)^2+\left(2\pi fL-\frac{1}{2\pi fC}\right)^2}$$

Series LCR Resonance Circuit

Resonance

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage.

For resonance both L and C must be present in circuit.

Series Resonance

(a) At Resonance

(i) $X_L = X_C$ (ii) $V_L = V_C$

(iii) $\phi = 0$ (V and I in same phase) (iv) $Z_{min} = R$ (impedance minimum)

(v) $I_{max} = \frac{V}{R}$ (current maximum)

(b) Resonance frequency

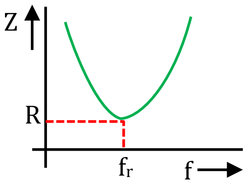
$$\because\ X_L = X_C \Rightarrow \omega_r L = \frac{1}{\omega_r C} \Rightarrow \omega_r^2 = \frac{1}{LC} \Rightarrow \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow f_r = \frac{1}{2\pi\sqrt{LC}}$$

(c) Variation of Z with f

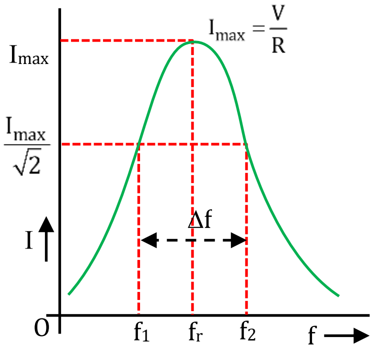
(i) If $f < f_r$ then $X_L < X_C$ $\Rightarrow$ circuit nature capacitive, ϕ (negative)

(ii) At $f = f_r$ then $X_L = X_C$ $\Rightarrow$ circuit nature, Resistive, ϕ = zero

(iii) If $f > f_r$ then $X_L > X_C$ $\Rightarrow$ circuit nature is inductive, ϕ (positive)



(d) Variation of I with f



as f increase, Z first decreases then increase

as f increase, I first increase then decreases

At resonance, impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

Illustration 27

In a series resonant R-L-C circuit, if L is increased by 25% and C is decreased by 20%, then the resonant frequency will :

Solution:

$$f=\frac{1}{2\pi\sqrt{LC}}$$

Now $L'=L+\frac{L}{4}=\frac{5L}{4}$

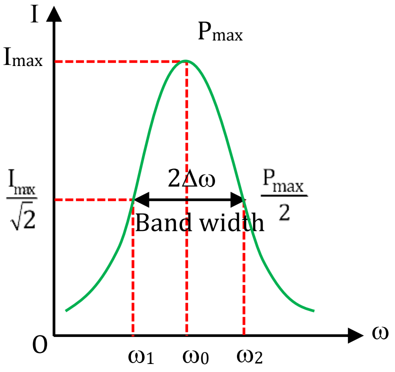
$$C'=C+\frac{C}{5}=\frac{4C}{5}\ \Rightarrow f'=\frac{1}{2\pi\sqrt{L'C'}}=\frac{1}{2\pi\sqrt{\frac{5L}{4}\times\frac{4C}{5}}}$$

$f'=\frac{1}{2\pi\sqrt{LC}}=f\ \Rightarrow$ frequency will remains unchanged

Half Power Frequency and Quality Factor

Half power frequency

There are two such frequencies of applied ac-source in LCR-series circuit where power consumption is exactly half of the maximum at resonance such frequencies are called half power frequencies.



Hence ω_1 & ω_2 are called half power frequencies where

$\omega_1 = \omega_0 - \Delta\omega$

$\omega_2 = \omega_0 + \Delta\omega$

$\omega_2 - \omega_1 = 2\Delta\omega$ = Bandwidth

At ω_1 & $\omega_2 \Rightarrow P=\frac{P_{max}}{2}$

$$I^2R=\frac{I^2_{max}R}{2}\ \Rightarrow I=\frac{I_{max}}{\sqrt{2}}$$

So, at ω_1 & $\omega_2 \Rightarrow I=\frac{I_{max}}{\sqrt{2}}$

Now, $I=\frac{V}{Z}=\frac{1}{\sqrt{2}}\frac{V}{R}$

$Z=\sqrt{2}R$

$Z^2 = 2R^2$

$R^2 + (X_L - X_C)^2 = 2R^2$

$X_L - X_C = \pm R$

For $\omega > \omega_0$ $\quad X_L > X_C \ \Rightarrow\ \omega = \omega_2$

For $\omega < \omega_0$ $\quad X_L < X_C \ \Rightarrow\ \omega = \omega_1$

So, $\quad \omega_2L-\frac{1}{\omega_2C}=R \quad$ ...(i)

$\quad \omega_1L-\frac{1}{\omega_1C}=-R \quad$ ...(ii)

Adding (i) & (ii), $\omega_1\omega_2=\frac{1}{LC}\Rightarrow\omega_0=\sqrt{\omega_1\omega_2}$

Subtracting (i) & (ii)

$$\omega_2-\omega_1=\frac{R}{L}\ \Rightarrow\ 2\Delta\omega=\frac{R}{L}$$

Quality factor or Q-factor

This factor gives relative information about stored energy and lost energy per cycle. Hence

$$Q = 2\pi\left[\frac{\text{Maximum energy stored per cycle}}{\text{Maximum energy lost per cycle}}\right] = \frac{\omega_0}{\text{Bandwidth}} = \frac{\omega_0}{2\Delta\omega}$$

Since $\Delta\omega = \frac{R}{2L}$ then $Q = \frac{\omega_0 L}{R} = \frac{1}{\omega_0 RC}$

$$Q = \frac{1}{\sqrt{LC}} \times \frac{L}{R} \Rightarrow Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Magnification

At resonance V_L or V_C = Q × (supplied voltage)

Hence at resonance magnification factor is the Q-factor

Q - factor $\propto$ Magnification factor $\propto$ Sharpness

According to figure, if 'R' is decreasing means Q-factor or sharpness will increase in the circuit.

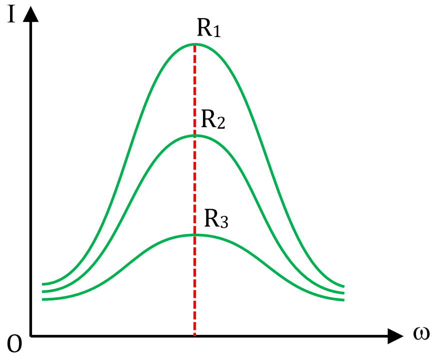
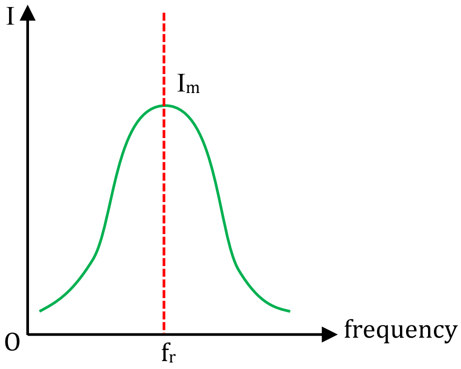


Illustration 28

The graph shows variation of I with f for a series R-L-C network. Keeping L and C constant. If R decreases. What is its effect on : (a) Maximum Current (b) Sharpness of graph (c) Quality factor.



Solution:

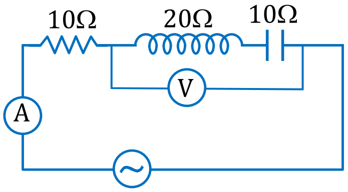
(a) Maximum current (I_m) increases

(b) Sharpness of the graph increases

(c) Quality factor increases

BEGINNER'S BOX-3

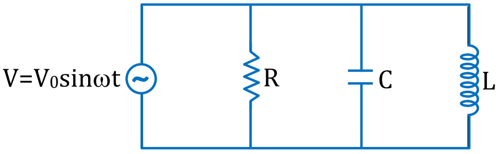
1. In given circuit find the reading of ammeter and voltmeter.



E=50 sin100πt volt

PAC156

2. For the circuit shown in figure, write down the instantaneous current through each element.



PAC157

3. A variable frequency 230V alternating voltage source is connected across a series combination of L = 5.0H C = 80μF and R = 40Ω. Calculate

(a) The angular frequency of the source at resonance.

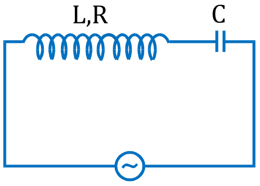
(b) Amplitude of current at resonance frequency

PAC158

4. A coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c.

Voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.

Find out relation between ω_0, L and C



V=V₀ sinωt(volt)

PAC159

5. Find the phase difference between voltage and current in series LCR circuit at half power frequencies.

PAC160

6. A series LCR circuit with L = 0.12H, C = 480 nF, R = 23Ω is connected to a 230 V variable frequency supply Find –

(a) Source frequency for which current is maximum.

(b) Q–factor of the given circuit.

PAC161

Power in AC

Power in ac-circuit

The rate of doing work or the amount of energy transferred by a circuit per unit time is known as power in AC circuits. It is used to calculate the total power required to supply a load.

Instantaneous power

As $P_{inst} = VI$

$= (V_0 \sin \omega t)[(I_0 \sin (\omega t + \phi)]$

$= V_0I_0 \sin \omega t \sin(\omega t + \phi) = \frac{V_0I_0}{2}$ [2sin ωt sin(ωt + ϕ)]

Hence $P_{inst} = \frac{V_0I_0}{2}$ [cosϕ – cos(2ωt + ϕ)] {Since 2sinA sinB = cos(A – B) –cos(A + B)}

Note : Therefore frequency of power fluctuation is twice the frequency of applied ac-source.

Average power :

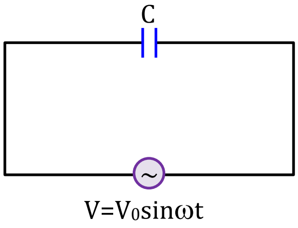
$$P_{av} = \frac{\int_0^T (V_0 \sin \omega t)(I_0 \sin(\omega t + \phi))dt}{\int_0^T dt} = \frac{V_0I_0}{T}\left[\cos\phi \int_0^T \sin^2 \omega t dt + \frac{\sin\phi}{2}\int_0^T \sin^2 \omega t dt\right]$$

$$P_{av} = V_0I_0\left[\cos\phi \frac{\int_0^T \sin^2 \omega t\, dt}{T} + \frac{\sin\phi}{2}\frac{\int_0^T \sin^2 \omega t\, dt}{T}\right] = V_0I_0\left[\cos\phi \times \frac{1}{2} + 0\right]$$

$P_{av} = \frac{1}{2}V_0I_0 \cos\phi$ or $P_{av} = V_{rms}\, I_{rms} \cos\phi$

Note : Hence $\cos\phi = \frac{R}{Z}$ = Power factor of ac-circuit.

Average power in capacitive circuit

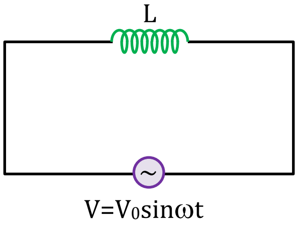


$I = I_0 \cos\omega t$

$P_{av} = V_{rms}I_{rms}\cos 90°$

$P_{av} = 0$

Average power in Inductive circuit



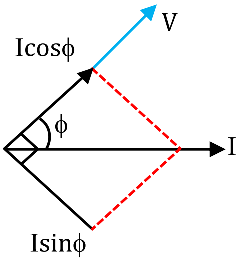
$I = I_0 \cos\omega t$

$P_{av} = V_{rms}I_{rms}\cos 90°$

$P_{av} = 0$

RMS Power:

$P_{rms} = V_{rms}\, I_{rms}$



Wattless current : That component of current in ac-circuit which is not active.

Hence, Icosϕ is the activity component of current because it is in phase with applied voltage.

But Isinϕ is the component which is inactive, called as wattless current because it is in $\frac{\pi}{2}$ phase with applied voltage.

Power factor

Average power $\overline{P}=V_{rms}I_{rms}\cos\phi$ = rms power × $\cos\phi$

Power factor $(\cos\phi)$ = $\dfrac{\text{Average power}}{\text{r.m.s. Power}}$ and $\cos\phi=\dfrac{R}{Z}$

1. Watt full Power:
   Average power is also known as watt full power
   $P_{av} = V_{rms}I_{rms}\cos\phi$
2. Wattless Power:
   That component of current in ac-circuit which is not active.
   $P_{av} = V_{rms}I_{rms}\sin\phi$

Illustration 29

In an A.C. circuit V and I are given by V = 100 sin(100 t) V and I = 100 sin(100t + π/3) mA

Find power dissipated in the circuit?

Solution:

$$V_{rms}=\frac{100}{\sqrt{2}}V;I_{rms}=\frac{100}{\sqrt{2}}\times10^{-3}A;\phi=\frac{\pi}{3}$$

$P_{av} = V_{rms}\,I_{rms}\cos\phi$

$$P_{av}=\frac{100}{\sqrt{2}}\times\frac{100}{\sqrt{2}}\times10^{-3}\times\cos\frac{\pi}{3}$$

$$P_{av}=\frac{10}{2}\times\frac{1}{2}=2.5\,W$$

Illustration 30

A series R-L-C (R =10Ω, X_L= 20Ω, X_C = 20Ω) circuit is supplied by V=10 sinωt V.

Find power dissipation in circuit.

Solution:

$$V_{rms}=\frac{10}{\sqrt{2}}V\rightarrow I_{rms}=\frac{V_{rms}}{Z} \quad\Rightarrow\quad Z=\sqrt{(R)^2+(X_L-X_C)^2}$$

$$I_{rms}=\frac{V_{rms}}{Z}=\frac{\frac{10}{\sqrt{2}}}{10}=\frac{1}{\sqrt{2}}A \quad\Rightarrow\quad P_{av}=I_{rms}^2R=\frac{1}{2}\times10=5W$$

Illustration 31

A voltage of 10 V and frequency 10^3 Hz is applied to $\frac{1}{\pi}$ μF capacitor in series with a resistor of 500Ω. Find the power factor of the circuit and the power dissipated.

Solution:

$$\because \quad X_C=\frac{1}{2\pi fC}=\frac{1}{2\pi\times10^3\times\frac{10^{-6}}{\pi}}=500\Omega$$

$$\therefore \quad Z=\sqrt{R^2+X_C^2}=\sqrt{(500)^2+(500)^2}=500\sqrt{2}\,\Omega$$

Power factor $\cos\phi=\dfrac{R}{Z}=\dfrac{500}{500\sqrt{2}}=\dfrac{1}{\sqrt{2}}$

Power dissipated = $V_{rms}\,I_{rms}\cos\phi$ = $\dfrac{V_{rms}^2}{Z}$ $\cos\phi$ = $\dfrac{(10)^2}{500\sqrt{2}}\times\dfrac{1}{\sqrt{2}}=\dfrac{1}{10}$ W

Illustration 32

If V = 100 sin 100 t volt and I = 100 sin (100 t + $\frac{\pi}{3}$) mA for an A.C. circuit then find out

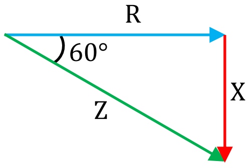
(a) phase difference between V and I
(b) total impedance, reactance, resistance
(c) power factor and power dissipated
(d) components contain by circuits

Solution:

(a) Phase difference $\phi = -\frac{\pi}{3}$ (I leads V)

(b) Total impedance $Z = \frac{V_0}{I_0} = \frac{100}{100\times10^{-3}} = 1k\Omega$ Now resistance $R = Z\cos 60° = 1000\times\frac{1}{2} = 500\Omega$

reactance $X = Z\sin 60° = 1000\times\frac{\sqrt{3}}{2} = 500\sqrt{3}\Omega$



(c) $\phi = -60°$ ⇒ Power factor = cosϕ = cos (–60°) = 0.5 (leading)

Power dissipated $P = V_{rms}I_{rms}\cos\phi = \frac{100}{\sqrt{2}}\times\frac{0.1}{\sqrt{2}}\times\frac{1}{2} = 2.5\,W$

(d) Circuit must contain R as $\phi \neq \frac{\pi}{2}$ and as ϕ is negative so C must be there,

(L may exist but $X_C > X_L$)

Illustration 33

If power factor of a R-L series circuit is $\frac{1}{2}$ when applied voltage is V = 100 sin 100πt volt and resistance of circuit is 200Ω then calculate the inductance of the circuit.

Solution:

$\cos\phi = \frac{R}{Z}$ ⇒ $\frac{1}{2} = \frac{R}{Z} \Rightarrow z = 2R \Rightarrow \sqrt{R^2 + X_L^2} = 2R$

⇒ $X_L = \sqrt{3}R$

$\omega L = \sqrt{3}R$ ⇒ $L = \frac{\sqrt{3}R}{\omega} = \frac{\sqrt{3}\times200}{100\pi} = \frac{2\sqrt{3}}{\pi}H$

Illustration 34

A circuit consisting of an inductance and a resistance joined to a 200 volt supply (A.C.). It draws a current of 10 ampere. If the power used in the circuit is 1500 watt. Calculate the wattless current

Solution:

Apparent power = 200 × 10 = 2000 W

∴ Power factor : $\cos\phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{1500}{2000} = \frac{3}{4}$

Wattless current = $I_{rms}\sin\phi = 10\sqrt{1-\left(\frac{3}{4}\right)^2} = \frac{10\sqrt{7}}{4}A$

Choke Coil

A choke coil is an inductance of very small resistance used for controlling current in an a.c. circuit in a direct current circuit, current is reduced with the help of a resistance. Hence there is a loss of electrical energy I^2R per second in the form of heat in the resistance. But in an AC circuit the current can be reduced by choke coil which involves very small amount of loss of energy. Choke coil is a copper coil would over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced.

Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)

The current in the circuit $I=\frac{E}{Z}$ with $Z=\sqrt{(R+r)^2+(\omega L)^2}$

So due to large inductance L of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil r,

The power loss in the choke $P_{av} = V_{rms} I_{rms} \cos\phi \to 0$

$$\because \quad \cos\phi=\frac{r}{Z}=\frac{r}{\sqrt{r^2+\omega^2L^2}}\approx\frac{r}{\omega L}\to 0$$

Note: A capacitor of suitable capacitance replace a choke coil in an AC circuit, the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in AC circuit without power dissipation.

Cost of capacitor is much more than the cost of inductance of same reactance that's why choke coil is used.

Illustration 35

An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V – 50 Hz ac source compute the inductance of the choke required.

Solution:

Resistance of lamp $R=\frac{V}{I}=\frac{80}{10}=8\Omega$

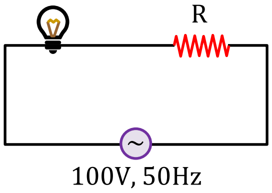
Let Z be the impedance which would maintain a current of 10 A through the Lamp when it is run on 100 Volt a.c. then. $Z = \frac{V}{I}=\frac{100}{10} = 10\ \Omega$ but $Z = \sqrt{R^2+(\omega L)^2}$

$$\Rightarrow \quad (\omega L)^2 = Z^2 - R^2 \Rightarrow (10)^2 - (8)^2 = 36 \Rightarrow \omega L = 6 \Rightarrow L = \frac{6}{\omega} = \frac{6}{2\pi\times 50} = 0.02H$$

Illustration 36

Calculate the resistance or inductance required to operate a lamp (60V, 10W) from a source of (100 V, 50 Hz)?

Solution:



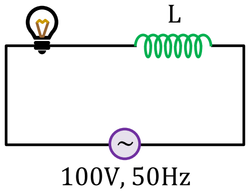
(a) Maximum voltage across lamp = 60V

$\because \quad V_{Lamp} + V_R = 100$

$\therefore \quad V_R = 40V$

Now current through Lamp is $=\frac{\text{Power}}{\text{voltage}}=\frac{10}{60}=\frac{1}{6}A$

But, $V_R = IR \Rightarrow 40 = \frac{1}{6}(R) \Rightarrow R = 240\ \Omega$



(b) Now in this case $(V_{Lamp})^2 + (V_L)^2 = (V)^2$

$(60)^2 + (V_L)^2 = (100)^2 \Rightarrow V_L = 80\ V$

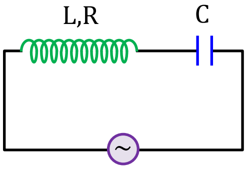
Also $V_L = IX_L = \frac{1}{6}X_L$ so $X_L = 80 \times 6 = 480\ \Omega = L(2\pi f) \Rightarrow L = 1.5\ H$

Illustration 37

A choke coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.

(a) Find out relation between ω_0, L and C.

(b) What is phase difference between V and I at resonance, does it changes when resistance of choke coil is zero.



Solution:

(a) At resonance condition $X_L = X_C \Rightarrow \omega_0 L = \dfrac{1}{\omega_0 C} \Rightarrow \omega_0 = \dfrac{1}{\sqrt{LC}}$

(b) $\because \cos\phi = \dfrac{R}{Z} = \dfrac{R}{R} = 1 \quad \therefore \phi = 0°$ No, It is always zero.

BEGINNER'S BOX-4

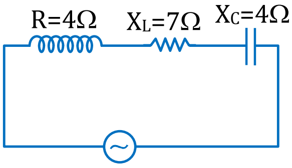
1. What is the power factor of a circuit that draws 5A at 160 V and whose power consumption is 600W?

PAC162

2. In a series LCR circuit as shown in fig.

(a) Find heat developed in 80 seconds

(b) Find wattless current



PAC163

3. For a series LCR circuit

$I = 100\sin(100\pi t - \pi/3)$mA and $V = 100\sin(100\pi t)$volt, then

(a) Calculate resistance and reactance of circuit.

(b) Find average power loss.

PAC164

4. The source voltage and current in the circuit are represented by the following equations –

$E = 110 \sin(\omega t + \dfrac{\pi}{6})$ volt, $I = 5\sin(\omega t - \dfrac{\pi}{6})$ ampere

Find :–

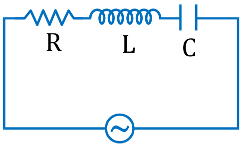
(a) Impedance of circuit. (b) Power factor with nature

PAC165

5. In given circuit R = 100Ω. If voltage leads current by 60° then find –

(a) Current supply by source.

(b) Average power



PAC166

6. An inductor of reactance 4Ω and a resistor of resistance 3Ω are connected in series with 100V ac supply, calculate wattless current in circuit.

PAC167

7. A 100 Ω resistor is connected to a 220 V, 50 Hz a.c. supply.

(a) What is the rms value of current in the circuit?

(b) What is the net power consumed over a full cycle?

PAC168

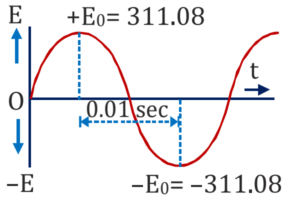
8. A choke coil and a resistance are connected in series in an a.c. circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50V. What would be the potential difference across the choke coil.

PAC169

BEGINNER'S BOX ANSWER KEY

BEGINNER'S BOX-1

1. There are two reasons for it :



2. For AC, $I = I_0 \sin\omega t$, the instantaneous value of heat produced (per second) in a resistance R is,

$H = I^2R = I_0^2\sin^2\omega t \times R$ the average value of heat produced during a cycle is :

$$H_{av} = \frac{\int_0^T H\,dt}{\int_0^T dt} = \frac{\int_0^T (I_0^2 \sin^2 \omega t \times R)dt}{\int_0^T dt} = \frac{1}{2} I_0^2 R$$

$$\left[\because \int_0^T I_0^2 \sin^2 \omega t\,dt = \frac{1}{2} I_0^2 T\right]$$

$$\Rightarrow H_{av} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = I_{rms}^2 R \qquad ...(i)$$

However, in case of DC, $H_{DC} = I^2 R$ ...(ii)

$\because$ $I = I_{rms}$ so from equation (i) and (ii) $H_{DC} = H_{av}$

AC produces same heating effects as DC of value Based on heating effect $I = I_{rms}$. This is also why AC instruments which are based on heating effect of current give rms value.

3. The average value of a.c. for a cycle is zero. So a d.c. ammeter will always read zero in a.c. circuit.

4. 2.5 m s **5.** (a) 0.01 s, (b) 60° **6.** 200 V

BEGINNER'S BOX-2

1. 2.1 A **2.** $10000 \sin\left(1000t - \frac{\pi}{2}\right)$A **3.** 1A

4. The capacitive reactance $X_C = 212\ \Omega$

$I_{rms} = 1.03$ A

The peak current $I_0 = 1.46$ A

If the frequency is doubled the capacitive reactances is halved, the consequently, the current is doubled.

5. 2.49 A **6.** (a) resistor (b) inductor **7.** $R = 5\Omega$ and $X_L = 5\sqrt{3}\ \Omega$

8. 1H **9.** 0.08 henry **10.** 240.2 Ω **11.** (a) 2.4 A (b) 2.5 ms

12. 15.9 mH **13.** (A) – r, (B) – p, (C) – p, (D) – p, (E) – r

BEGINNER'S BOX-3

1. Reading of ammeter = 2·5A
Reading of voltmeter = 25V

2. The three current equations are,

$$V = i_R R,\quad V = L\frac{di_L}{dt} \text{ and } \frac{dV}{dt} = \frac{1}{C} i_C$$

$$\text{so } i_R = \frac{V_0}{R}\sin\omega t,$$

$$i_L = -\frac{V_0}{\omega L}\cos\omega t \text{ and } i_C = V_0\omega C\cos\omega t$$

3. (a) angular frequency at resonance $\omega_r = 50$ rad/s
(b) amplitude of current at resonance $I_m = 8.13$ A

4. (a) $\omega_0 = \frac{1}{\sqrt{LC}}$, (b) $\phi = 0°$ No, It is always zero.

5. $\phi = \frac{\pi}{4}$

6. (a) 6.63×10^2 Hz (b) Quality factor Q = 21.7

BEGINNER'S BOX-4

1. 0.75

2. (a) 4000 joule (b) 2.12 A

3. (a) R = 500 ohm, X = $500\sqrt{3}$ ohm
(b) 2.5 watts

4. (a) Impedance Z = 22Ω
(b) Power factor = $\frac{1}{2}$ (lagging)

5. (a) $\frac{1}{\sqrt{2}}$ A, (b) 50W

6. Wattless current = 16A

7. (a) 2.2 A (b) 484 watt

8. 120V