# Alternating Current

## SOLUTIONS

### Exercise-I (Conceptual Questions)

**1.** Given that,

$H_{AC} = 3\ H_{DC}$

$I_{rms}^2\ Rt = 3\ (I^2Rt)$

$I_{rms}^2 = 3I^2$

$I_{rms} = \sqrt{3}I = \sqrt{3}\ 2$

$I_{rms} = 3.46$ A

**2.** $E = E_0 \cos \omega t$

Given, $E_0 = 10V$, $t = \frac{1}{600}$ sec., $f = 50Hz$

$E = 10 \cos (2\pi \times 50 \times \frac{1}{600})$

$E = 10 \cos \frac{\pi}{6} \rightarrow E = 5\sqrt{3}$ V

**3.** $\because$ $2\pi$ (phase difference) $\equiv$ T(Time difference)

1 rad (PD) $\equiv \frac{T}{2\pi}$ (Time difference)

So, $\frac{\pi}{4}$ (PD) $\equiv \frac{T}{2\pi} \times \frac{\pi}{4}$ (Time difference)

$= \frac{T}{8} \Rightarrow$ time difference $= \frac{1}{50\times 8}$ s

or time difference = 2.5 ms.

**4.** $I = 3 + 4 \sin \omega t$

(i) square $= 9 + 16 \sin^2\omega t + 24 \sin\omega t$

(ii) mean value $= 9 + 16 \times \frac{1}{2} + 0 = 17$

(iii) Root $= \sqrt{17}$

(iv) Rms value $= \sqrt{17}$

OR

$I = a + b \sin \omega t$

$\therefore \boxed{I_{rms} = \sqrt{a^2 + \frac{b^2}{2}}} = \sqrt{9 + \frac{16}{2}} = \sqrt{9+8} = \sqrt{17}$

**6.** $i = i_1 \cos \omega t + i_2 \sin \omega t$

Let $i_1 = a\sin \phi$, and $i_2 = a \cos \phi$

than $i = a \sin\phi \cos \omega t + a \cos \phi \cos \omega t$

$i = a \sin (\phi + \omega t)$

[$\because$ $\sin A \cos B + \sin B \cos A = \sin (A+B)$]

So $I_{rms} = \frac{a}{\sqrt{2}}$

$a = \sqrt{i_1^2 + i_2^2}$ $\left(\because a = \sqrt{i_1^2 + i_2^2}\right)$

OR

$i = i_1 \cos \omega t + i_2 \sin \omega t$

$I_{max} = \sqrt{i_1^2 + i_2^2} \Rightarrow I_{rms} = \frac{\sqrt{i_1^2 + i_2^2}}{\sqrt{2}} = \sqrt{\frac{i_1^2 + i_2^2}{2}}$

**7.** $V = 120 \sin (100\pi\ t) \cos (100\pi t)$

$\because$ $2 \sin\theta \cos\theta = \sin 2\theta$

$\therefore$ $\sin (100\pi t) \cos (100\pi t) = \frac{\sin(2\times 100\pi t)}{2}$

$V = 60 \sin (200\pi\ t)$

compare from

$V = V_0 \sin \omega t$

$V_0 = 60$ volt,

$\omega = 200\pi$

$2\pi f = 200\pi$

$f = 100$ Hz

**8.** $V_{avg} = \frac{2V_0}{\pi} = \frac{2\sqrt{2}V_{rms}}{\pi} = \frac{2\sqrt{2}\times 220}{3.14} = 198$Volt

**12.** Peak voltage in primary $V_0 = \sqrt{2}\ V_{rms} = 100\sqrt{2}$

$\frac{V_s}{V_p} = \frac{N_2}{N_1} \Rightarrow V_s = \frac{N_s}{N_p} \times V_p$

$V_s = 5 \times 100\sqrt{2} = 500\sqrt{2}$Volt

**15.** $X_L = 2\pi fL$

$X_L \propto f$(graph straight line)

$X_C = \frac{1}{2\pi fC}$; $X_C \propto \frac{1}{C}$

(graph rectangular hyperbola)

**16.** $\tan \phi = \frac{X_L}{R}$

$\tan \phi = \frac{2\pi\times 50\times 0.21}{12}$

$\tan \phi \approx 5.5$

Now $\tan 60° = \sqrt{3}$ and $5.5 > \sqrt{3}$

So, angle will be greater 60°.

**17.** $V^2 = V_R^2 + V_C^2$

$(220)^2 = (110)^2 + V_C^2$

$V_C = 190$ Volt.

**19.** For DC. V = IR

$R = \frac{V}{I} = \frac{100}{25}$

R = 4Ω (Resistance of coil)

For AC. V = IZ

$Z = \frac{V}{I} = \frac{100}{20}$   Z = 5 Ω

$Z^2 = R^2 + X^2 \rightarrow X^2 = Z^2 - R^2$

$= 5^2 - 4^2$

Reactance of coil X = 3Ω.

**24.** $I = \frac{V}{Z}$

Z = ωL = 2π × 50 × 2 = 200π

$I_1 = \frac{V}{200\pi}$

on frequency 400 Hz

Z = ωL = 2π × 400 × 2

$I_2 = \frac{V}{1600\pi}$ ; $\frac{I_1}{I_2} = \frac{V}{200\pi} \times \frac{1600\pi}{V} = 8$

$I_2 = \frac{I_1}{8}$ (lagging by 90° from V due to purely inductive circuit)

OR

$X_L \propto f$, f becomes 8 times, so $X_L$ also becomes 8 times, so current becomes 1/8 times and lagging by 90° from V (pure inductive circuit)

∴

**27.** Use $X_c = \frac{1}{\omega C}$

**31.** $\tan\phi = \frac{X_L}{R}$

$\tan\phi = \frac{40\pi}{100}$   $\begin{cases} X_L = \omega L \\ = 2\pi \times 50 \times 0.4 \\ = 40\pi\Omega \\ R = 100\Omega \end{cases}$

tan ϕ = 0.4π ,

ϕ = tan⁻¹ (0.4π)

**32.** $\tan\phi = \frac{X_C}{R} = \frac{1}{\omega RC}$

$= \frac{1}{2\pi \times 50 \times 100 \times 100 \times 10^{-6}} = \frac{1}{\pi}$

$\rightarrow \phi = \tan^{-1}\left(\frac{1}{\pi}\right)$

**33.** $V_L = L\frac{dI}{dt} = L\frac{d}{dt}(I_0 \sin\omega t)$

$= I_0\omega L\cos\omega t = I_0\omega L\sin(\omega t + \pi/2)$

**35.** Use $V^2 = V_R^2 + V_C^2$

**39.** $V_L = I \times X_L$

$I = \frac{0.6}{2\pi \times 60 \times 10^{-3}}$

**40.** ∵ $X_L = X_C$

$f = \frac{1}{2\pi\sqrt{LC}} \rightarrow X_L = 2\pi fL$

$X_L = 2\pi L \times \frac{1}{2\pi\sqrt{LC}} = \sqrt{\frac{L}{C}}$

$X_L = \sqrt{\frac{10^{-3}}{10 \times 10^{-6}}} \Rightarrow X_L = 10\Omega.$

OR

At resonance, $\omega = \frac{1}{\sqrt{LC}}$

ω = $10^4$ Rad/sec.

∴ $X_L = \omega L = 10^4 \times 10^{-3} = 10\ \Omega$

**42.** Impedance of series circuit

$Z = \sqrt{R^2 + (X_L - X_C)^2}$

If $X_L$ and $X_C$ are interchanged then there is no effect on Z.

**47.** Power dissipation = VI cosϕ

$= V.\frac{V}{Z}.\frac{R}{Z} = \frac{V^2R}{Z^2}$

Now $Z = \sqrt{R^2 + (X_L - X_C)^2}$

Z = R (∴ $X_L = X_C$)

Power dissipations = $\frac{V^2R}{R^2} = \frac{V^2}{R}$

$V = \frac{V_0}{\sqrt{2}} = \frac{10}{\sqrt{2}} \Rightarrow R = 10\Omega$

Power dissipation = $\frac{10 \times 10}{\sqrt{2} \times \sqrt{2} \times 10}$ = 5 Watt.

OR

At resonance

$I_0 = \frac{10}{10} = 1\ A$

$I_{rms} = \frac{1}{\sqrt{2}}A$

$P_{av} = V_{rms} I_{rms} = \frac{10}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$ = 5 Watt.

**48.** At resonance, maximum power impart to circuit.

$$f = \frac{1}{2\pi\sqrt{LC}} \Rightarrow 50 = \frac{1}{2\pi\sqrt{10\times C}}$$

$\rightarrow C = 10^{-6}$ F

**49.** $f = \dfrac{1}{2\pi\sqrt{LC}}$

Now $L' = L + \dfrac{L}{4} = \dfrac{5L}{4}$

$C' = C - \dfrac{C}{5} = \dfrac{4C}{5}$

$$f' = \frac{1}{2\pi\sqrt{L'C'}} = \frac{1}{2\pi\sqrt{\frac{5L}{4}\times\frac{4C}{5}}}$$

$$f' = \frac{1}{2\pi\sqrt{LC}} = f$$

**51.** R Changed $\Rightarrow$ Z Changed $\Rightarrow$ I Changed

$V_L = I X_L$ (Changed)

$V_C = I X_C$ (Changed)

$V_L$ and $V_C$ are changed by same amount

So Voltage across LC combination $(V_L - V_C)$ remains same.

**52.** $\tan\phi = \dfrac{V_L - V_C}{V_R} = \dfrac{1}{1} = 45°$

$V_L > V_C$

So current lags by $\dfrac{\pi}{4}$

$\therefore I = I_0 \cos\left(\omega t - \dfrac{\pi}{4}\right)$

**53.** $Z = \sqrt{R^2 + X^2}$ [X = reactance]

if $\omega\uparrow$ than $X_C\downarrow$, $Z\downarrow$

So circuit contain resistance and capacitance

**54.** $V_L = V_C$

$\therefore X_L = X_C$ (Resonance condition)

$i = \dfrac{V}{R} = \dfrac{100}{50} = 2A$

$V_R = iR = 2 \times 50 = 100$ V

**56.** $V^2 = V_R^2 + (V_L - V_C)^2$

**59.** $P_{loss} = VI\cos\phi$

$\cos\phi = \dfrac{P_{loss}}{VI} = \dfrac{600}{5\times 160}$

$\cos\phi = 0.75$

**64.** $E = E_0 \sin\omega t$

$I = I_0 \sin(\omega t + \dfrac{\pi}{2}) \Rightarrow \phi = \dfrac{\pi}{2}$

$\cos\phi = 0$ so $P = 0$

**65.** Power factor $\cos\phi = \dfrac{R}{Z}$

For negligible power factor $\Rightarrow$ R Low

Z high (L high)

**66.** Wattless power = V I $\sin\phi$,

Wattless power

$$= \frac{100}{\sqrt{2}} \times \frac{100}{\sqrt{2}} \times \sin\frac{\pi}{6} \begin{cases} V = \dfrac{100}{\sqrt{2}} V \\ I = \dfrac{100}{\sqrt{2}} A \\ \phi = \dfrac{\pi}{6} \end{cases}$$

$= 2.5 \times 10^3$ Watt

**69.** Power factor $= \cos\phi = \dfrac{R}{Z} = \dfrac{R}{\sqrt{R^2 + X_L^2}}$

$$\cos\phi = \frac{1100}{\sqrt{(1100)^2 + \left(2\times\frac{22}{7}\times 50\times\frac{7}{2}\right)^2}}$$

$$= \frac{1100}{\sqrt{(1100)^2 + (1100)^2}};\ \cos\phi = \frac{1}{\sqrt{2}}$$

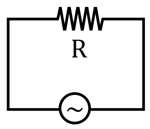
## Exercise-II (Previous Year Questions)

**1.** When capacitor is filled with mica then capacitance C increases so $X_C$ decreases

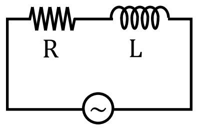
In case (b) $X_C\downarrow$ so voltage across capacitor decreases. so $V_a > V_b$

**2.**

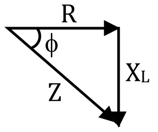
Pure resistor circuit




Resistor & Inductor circuit




(Phasor diagram)




AC source Impedance = Z

$P' = V.I.\cos\phi$

$P = \dfrac{V^2}{R}$ $\quad P' = V.\left[\dfrac{V}{Z}\right].\cos\phi$

$\Rightarrow V^2 = PR$ $\quad P' = \dfrac{V^2}{Z}.\dfrac{R}{Z}$

(From phasor diagram)

$P' = \dfrac{(PR)R}{Z^2}$

$P' = \left(\dfrac{R}{Z}\right)^2 P$

**3.** $\frac{1}{\omega C}=\frac{1}{340\times50\times10^{-6}}$ $X_C$ = = 58.8 Ω

$X_L = \omega L = 340 \times 20 \times 10^{-3} = 6.8\ \Omega$

$Z = \sqrt{R^2+(X_C-X_L)^2}$

$= \sqrt{40^2+(58.8-6.8)^2} = \sqrt{4304}\,\Omega$

$P = i_{rms}^2 R = \left(\frac{V_{rms}}{Z}\right)^2 R$

$= \left(\frac{10/\sqrt{2}}{\sqrt{4304}}\right)^2 \times 40 = \frac{50\times40}{4304} = 0.47\ W$

**So best answer (nearest answer) will be (1)**

**4.** Input voltage, $V(t) = V_0\sin\omega t$

For capacitor, $I(t) = \frac{dq}{dt}=C\frac{dV}{dt}=\omega CV_0\cos\omega t$

$\Rightarrow$ Current I(t) leads voltage V (t) by 90°

Also, Capacitor does not consume any energy over a full cycle.

**5.** For better tuning, Q-factor must be high.

$\therefore\ Q=\frac{\omega_0 L}{R}=\frac{1}{\sqrt{LC}}\left(\frac{L}{R}\right)=\frac{1}{R}\sqrt{\frac{L}{C}}$

R and C should be small and L should be high.

**6.** $\tan\phi=\frac{V_L-V_C}{V_R}=\frac{100-40}{80}=\frac{3}{4}$ or $\phi$ = 37°

Power factor = $\cos\phi$ = cos 37° = $\frac{4}{5}$ or 0.8

**7.** Here

$X_L = \omega L = (314)(20 \times 10^{-3}) = 6.280$

$X_C = \frac{1}{\omega C} = \frac{1}{314\times100\times10^{-6}} = 31.84\Omega$

$R = 50\Omega$

$Z=\sqrt{(X_C-X_L)^2+R^2}$

$=\sqrt{(31.84-6.28)^2+(50)^2} = 56\Omega$

$\Rightarrow P=\left(\frac{V_{rms}}{Z}\right)^2 R=\left(\frac{10}{\sqrt{2}\times56}\right)^2\times50$

$=0.79\text{watt}$

**8.** Changing polarity is termed as AC.

**9.** $Z = \frac{12}{0.2}=60\,\Omega$ and $R = \frac{12}{0.4}=30\,\Omega$

**10.** $I=\frac{V}{X_C}=\frac{V}{1/C\omega}=VC\omega$

$= 200 \times 40 \times 10^{-6} \times 2\pi \times 50$

$= 2.5$ A

**11.** When L removed $\tan\phi=\frac{X_C}{R}$

When L removed $\tan\phi=\frac{X_L}{R}$

$\frac{X_C}{R}=\frac{X_L}{R}\Rightarrow$ Resonance $\Rightarrow Z = R$

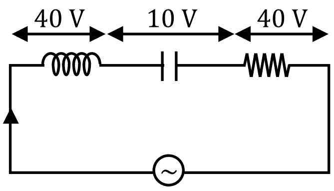
$\cos\phi=\frac{R}{Z}=\frac{R}{R}=1$

**12.** Impedance, $z = \sqrt{R^2+X_L^2}$

$X_L\uparrow, Z\uparrow, I\downarrow$

**13.** $I_0=10\sqrt{2}$ A

$I_{RMS}=\frac{I_0}{\sqrt{2}}=10A$




$V_{RMS}=\sqrt{V_R^2+(V_L-V_C)^2}$

$=\sqrt{(40)^2+(40-10)^2}$

$= 50$ V

$Z=\frac{V_{RMS}}{I_{RMS}}=\frac{50\ V}{10\ V}=5\Omega$

**14.** $Q=\frac{\omega}{\Delta\omega}=\frac{\omega L}{R}\Rightarrow\Delta\omega=R/L=\frac{50}{4}=8\text{ rad/sec}$

$\omega_0=\frac{1}{\sqrt{LC}}=\frac{1}{\sqrt{5\times80\times10^{-6}}}=50\text{ rad/sec.}$

$\omega_{min}=\omega_0-\frac{\Delta\omega}{2}=46\text{ rad/sec}$

$\omega_{max}=\omega_0+\frac{\Delta\omega}{2}=54\text{ rad/sec}$

**19.** $L = 10 \times 10^{-3}$ H

$C = 1 \times 10^{-6}$ F

$R = 100\ \Omega$

At resonance $X_L = X_C$

$\omega L=\frac{1}{\omega C}$

$f=\frac{1}{2\pi\sqrt{LC}}=\frac{1}{2\pi\sqrt{10\times10^{-3}\times10^{-6}}}=1.59\text{ KHz}$

**20.** $i_C = i_D = \frac{V_0}{X_C}\sin\omega t$

$i_C = i_D = (V_0\omega C)\sin\omega t$

On decreasing frequency $i_C \downarrow$

**21.** $X_L = \frac{50}{L}\times10^{-3}\times2\pi\times50 = 5\Omega$

$X_C = \frac{1}{2\pi\times50\times\frac{10^3}{\pi}\times10^{-6}} = 10\Omega$

$Z = \sqrt{R^2+(X_L-X_C)^2}$

$= \sqrt{(10)^2+(5)^2}$

$= 5\sqrt{5}\,\Omega$

**22.** $\tan\phi = \frac{X_L}{R}$

$X_L = \omega l = 100\pi \times \frac{1}{\pi} = 100\Omega$

$R = 100\Omega$

$\tan\phi = \frac{100}{100} = 1$

$\phi = 45°$

**23.** $Z_1 = \sqrt{X_L^2+R^2}$ $X_L = 0$ (D.C. circuit)

$Z_1 = 10\Omega$

$Z_2 = \sqrt{X_C^2+R^2}$

$X_C = \frac{1}{2\pi\times50\times\frac{10^3}{\pi}\times10^{-6}} = 10\Omega$

$Z_2 = \sqrt{(10)^2+(10)^2}$

$= 10\sqrt{2}$

$Z_1 < Z_2$

**24.** Power dissipated is maximum of purely resistive circuit.

**25.** $\omega = 2\pi\frac{\text{rad}}{\text{sec}}$

$E_{max} = NBA\omega$

$= 100 \times 0.05 \times 1 \times 2\pi$

$= 10 \times \pi$

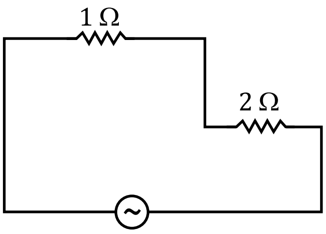
$= 31.4$ V

**26.** as frequency is very high

$X_C \approx 0$

$X_L \to \infty$

Effective circuit will be




Effective impedance of circuit will be = $3\Omega$

**27.** $I_{Peak} = \frac{V_{Peak}}{X_c} = V_{Peak}(\omega c)$

$= V_{Peak}(2\pi fc)$

$= (210\sqrt{2})\,(2\pi \times 50 \times 10 \times 10^{-6})$

= 0.93 Ampere

**28.** $V_L = i\,X_L$

$V_L = \omega L\, i_0 \cos\omega t$

**29.** $P = Vi$

$88 = 11000\, i$

$i = 8 \times 10^{-3}$ A

$i = 8$ mA

**30.** $Q = Q_0 e^{\frac{-Rt}{2L}}$

putting $Q = 0.50\,Q_0$;

$0.50\,Q_0 = Q_0 e^{\frac{-Rt}{2L}}$

$\Rightarrow e^{\frac{Rt}{2L}} = 2$

$\Rightarrow \frac{Rt}{2L} = \ln(2)$

$\Rightarrow t = \frac{2L}{R}(\ln 2) = \frac{2\times12\times10^{-3}}{1.5}.(0.693)$

$= 11.088\times10^{-3} = 11.09$ ms

## Exercise-III (Analytical Questions)

**1.** At frequency lower than resonant frequency since $X_C > X_L$

$\therefore$ Current leads voltage

$\rightarrow$ At resonance, impedance is minimum.

$\rightarrow$ At resonance, phase diff. between voltage across L and current in C is $\frac{\pi}{2}$

$\rightarrow f = \frac{1}{2\pi\sqrt{LC}}$

**2.** $\tau = RC \Rightarrow R = \dfrac{\tau}{C}$

$[R] = \left[\dfrac{1}{fC}\right]$

$\tau = \dfrac{L}{R} \Rightarrow R = \dfrac{L}{\tau} = (f.L)$

$\therefore$ b & c are correct.

**3.** $V_1 = \sqrt{V_R^2 + V_L^2} = 100\text{Volt}$ ...(1)

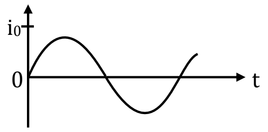
$V_C - V_L = 120$ Volt ...(2)

$130 = \sqrt{V_R^2 + (V_L - V_C)^2} \Rightarrow V_R = 50\text{Volt}$

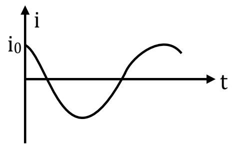
$V_L = 50\sqrt{3}$ Volt = 86.6 Volt & $V_C$ = 206.6 Volt

$\cos\phi = \dfrac{V_R}{130} = \dfrac{5}{13}$

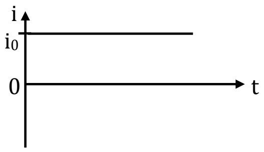
**4.**



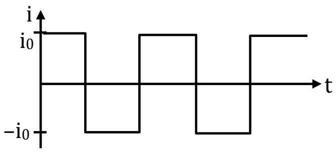
$i_{AVg} = 0$, $i_{rms} = \dfrac{i_0}{\sqrt{2}}$



$i_{avg} = 0$, $i_{rms} = \dfrac{i_0}{\sqrt{2}}$



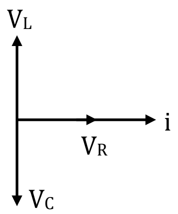
$i_{AVg} = i_{rms} = i_0$



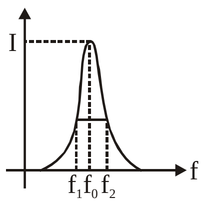
$i_{AVg} = 0$ $i_{rms} = i_0$

**5.** $Q_{factor} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$

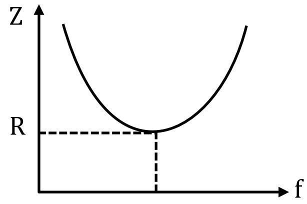
**6.**



**7.** $\boxed{f_0 = \sqrt{f_1 f_2}}$

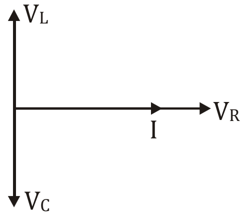


**8.** $z = \sqrt{R^2 + (x_L - x_C)^2}$



Impedance can be greater than or equal to resistance, but it can't be less than resistance of circuit.

**9.**



At resonance

$V_L = V_C$ & Z = R

there $V_L$ & $V_C$ are out of phase

**10.** For capacitor and inductor

$\phi = 90° \Rightarrow \cos\phi = 0 \Rightarrow P_{avg} = 0$

**11.** **For (A)** : Current and voltage are in phase

$\Rightarrow$ only R or RLC in resonance.

**For (B)**: Current leads voltage by $\dfrac{\pi}{2} \Rightarrow$ only C

$\Rightarrow$ power factor = 0

**For (C)** : Current leads voltage $\Rightarrow$ RC or RLC with $X_C > X_L$

**For (D)** : Current leads voltage

$\Rightarrow$ RL or RLC with $X_L > X_C$