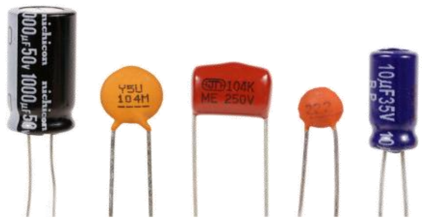
02 Capacitor

Concept of Capacity, Capacitors and Capacitance



Definition: A capacitor or condenser consists of two conductors separated by an insulator or dielectric. Having opposite charges on which sufficient quantity of charge may be accommodated.

It is a device which is used to store energy in form of Electric field by storing charge.

Conductors are used to form capacitors.

Theoretically, infinite amount of charge can be given to any conductor

Practically, only a certain amount of charge can be given because of dielectric strength of air

Dielectric strength: The maximum value of electric field which can be tolerated by a medium without breakdown

Dielectric strength of air, $E = 3 \times 10^6$ N/C

Corona discharge: leakage of charge from sharp edges of conductor.

Electrical Capacitance

It shows the capacity of a conductor to store electric energy in the form of electric field. If charge(Q) is given to an isolated conducting body and it's potential increases by V, then

$Q \propto V$, $Q = CV$

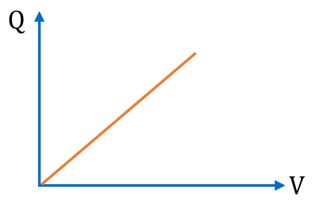
$C = \frac{Q}{V} \Rightarrow$ constant [electrical capacitance of a conductor]

Capacitance of conductor depends upon shape, size, presence of medium and nearness of other conductor.

Electrical capacitance is a Scalar quantity.

Note: Capacitance of a conductor does not depend upon charge (Q) and potential difference (V).

Graph between Q and V



$\text{Slope} = \frac{Q}{V} = C$

Unit
- SI → Coulomb/volt or Farad (F)
- CGS → Stat Farad

$$1F = \frac{1C}{1V} = \frac{3\times10^9 \text{stat}-C}{\frac{1}{300}\text{stat}-\text{volt}} = 9\times10^{11}\text{stat}-F$$

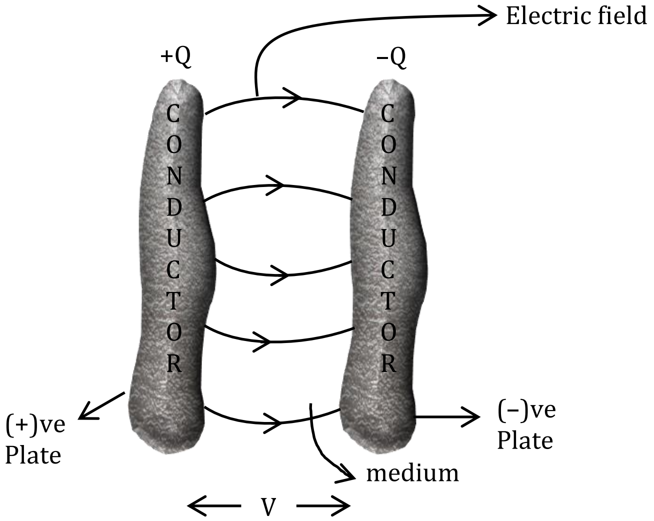
$1F = 9 \times 10^{11}$ state – F

Dimensional formula of capacitance $\Rightarrow [M^{-1}L^{-2}T^4A^2]$

Capacitor

When 2 conductors which carry equal and opposite charges are separated by some distance having some medium between them, then this arrangement is known as capacitor.

Capacitor is electrically neutral.



$$C = \frac{Q}{V}$$

$Q \rightarrow$ Charge transferred

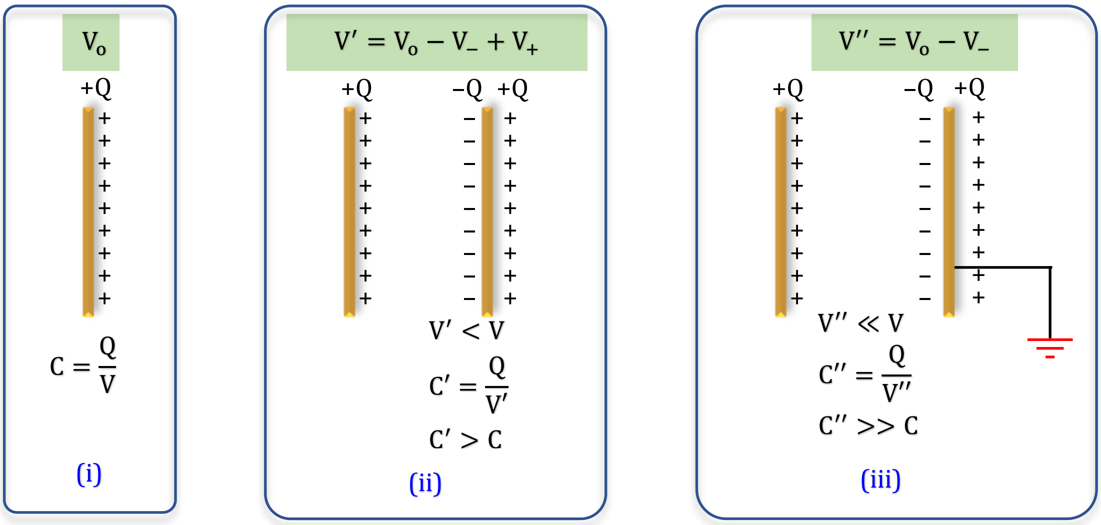
Or

Charge on (+)ve plate

$V \rightarrow$ Potential difference b/w plates

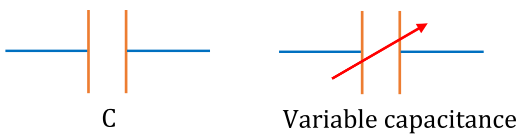
Principle of Capacitor

When uncharged conductor is placed nearer to the charged conductor and uncharged conductor is connected to earth, then capacitance of charged conductor is increased.



Circuit Symbol of Capacitor

The capacitor is represented as following:



Types of Capacitors

Based on shape and arrangement of capacitor plates there are various types of capacitors.

(a) Parallel plate capacitor

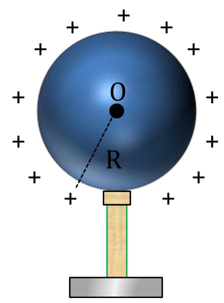
(b) Spherical capacitor

(c) Cylindrical capacitor

Capacity of an Isolated Conductor and Spherical Capacitor

Spherical Capacitance

Capacitance of an Isolated Spherical Conductor



Let there is charge Q on sphere of radius R.

$\because$ Potential $V = \dfrac{KQ}{R}$

Hence by formula: $Q = CV \Rightarrow Q = \dfrac{CKQ}{R} \Rightarrow C = 4\pi\epsilon_0 R$

Capacitance of an isolated spherical conductor $\rightarrow C = 4\pi\epsilon_0 R$

(a) If the medium around the conductor is vacuum or air.

$C_{Vacuum} = 4\pi\epsilon_0 R$ (R = Radius of spherical conductor. (may be solid or hollow.))

If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity then $C_{medium} = 4\pi\epsilon_0 KR$

(b) $\dfrac{C_{medium}}{C_{air/vaccum}} = K$ = dielectric constant.

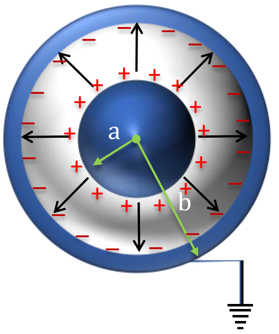
Illustration 1:

Find out the capacitance of the earth? (Radius of the earth = 6400 km)

Solution:

$C = 4\pi\epsilon_0 R = \dfrac{6400\times10^3}{9\times10^9} = 711\ \mu F.$

Capacitance of spherical capacitor

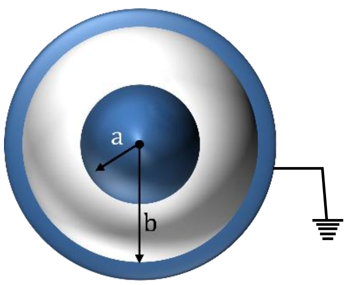


(a) Outer sphere is earthed

When a charge Q is given to inner sphere it is uniformly distributed on its surface A charge –Q is induced on inner surface of outer sphere. The charge +Q induced on outer surface of outer sphere flows to earth as it is grounded.

E = 0 for $r < a$ and E = 0 for $r > b$

This arrangement is known as spherical capacitor.



Potential of inner sphere $V_1 = \dfrac{Q}{4\pi\varepsilon_0 a} + \dfrac{-Q}{4\pi\varepsilon_0 a} \Rightarrow \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{b-a}{ab}\right)$

As outer surface is earthed so potential

$V_2 = 0 \Rightarrow V_1 - V_2 = \left[\dfrac{kQ}{a} - \dfrac{kQ}{b}\right] - \left[\dfrac{kQ}{b} - \dfrac{kQ}{b}\right] = \dfrac{kQ}{a} - \dfrac{kQ}{b}$

$$C = \frac{Q}{V_1 - V_2} = \frac{Q}{\frac{kQ}{a} - \frac{kQ}{b}} = \frac{ab}{k(b-a)} = \frac{4\pi\epsilon_0 ab}{b-a} = \frac{4\pi\epsilon_0 ab}{b-a}$$

If b >> a then $C = 4\pi\epsilon_0 a$ (Like isolated spherical capacitor)

If dielectric mediums are filled as shown then: $C = \dfrac{4\pi\epsilon_0\epsilon_r ab}{b-a}$

Illustration 2:

The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km.

Solution:

The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

b = radius of the top of stratosphere layer = 6400 km + 50 km = 6450 km = 6.45×10^6 m

a = radius of earth = 6400 km = 6.4×10^6 m

$$\therefore \quad C = \frac{1}{9\times10^9} \times \frac{6.45\times10^6 \times 6.4\times10^6}{6.45\times10^6 - 6.4\times10^6} = 0.092\text{F}$$

(b) Inner sphere is earthed

Here the system is equivalent to a spherical capacitor of inner and outer radii a and b respectively and a spherical conductor of radius b in parallel. This is because charge Q given to outer sphere distributes in such a way that for the outer sphere

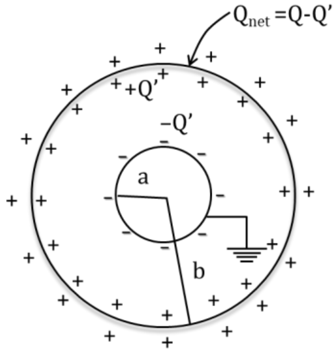
charge on the inner side is $\frac{a}{b}Q$ and

charge on the outer side is $Q - \frac{a}{b}Q = \frac{(b-a)}{b}Q$

So total capacity of the system.

$C = 4\pi\varepsilon_0 \frac{ab}{b-a} + 4\pi\varepsilon_0 b$

$$\boxed{C = \frac{4\pi\varepsilon_0 b^2}{b-a}}$$



Cylindrical Capacitor

There are two co-axial conducting cylindrical surfaces where $\ell >> a$ and $\ell >> b$, where a and b are radius of cylinders. When a charge Q is given to inner cylinder it is uniformly distributed on its surface. A charge –Q is induced on inner surface of outer cylinder. The charge +Q induced on outer surface of outer cylinder flows to earth as it is grounded

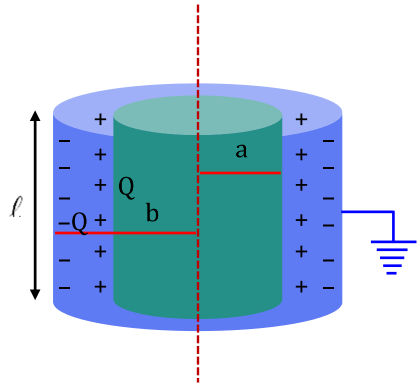
Electrical field between cylinders $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{Q/\ell}{2\pi\varepsilon_0 r}$

Potential difference between plates

$$V = \int_a^b \frac{Q}{2\pi\varepsilon_0 r\ell}\,dr = \frac{Q}{2\pi\varepsilon_0\ell}\ell n\left(\frac{b}{a}\right)$$

Capacitance per unit length

$$C = \frac{\lambda}{V} = \frac{\lambda}{2K\lambda\ell n\frac{b}{a}} = \frac{4\pi\epsilon_0}{2\ell n\frac{b}{a}} = \frac{2\pi\epsilon_0}{\ell n\frac{b}{a}}$$



Capacitance per unit length $= \frac{2\pi\epsilon_0}{\ell n\frac{b}{a}}$ F / m

Capacitance, $C = \frac{2\pi\varepsilon_0\ell}{\ell n(b/a)}$

Illustration 3:
A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder.

Solution:

ℓ = 15 cm = 15 × 10^{-2} m; a = 1.4cm = 1.4 × 10^{-2} m; b =1.5 cm = 1.5 × 10^{-2}m; q = 3.5 μC = 3.5 × 10^{-6}C

$$\text{Capacitance } C=\frac{2\pi\varepsilon_0\ell}{2.303\,\log_{10}\left(\frac{b}{a}\right)}=\frac{2\pi\times 8.854\times 10^{-12}\times 15\times 10^{-2}}{2.303\,\log_{10}\frac{1.5\times 10^{-2}}{1.4\times 10^{-2}}}=1.21\times 10^{-10}\,F$$

Since the outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them. Potential of inner cylinder, is

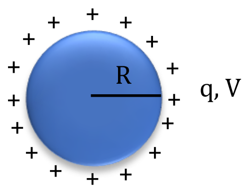
$$V=\frac{q}{C}=\frac{3.5\times 10^{-6}}{1.2\times 10^{-10}}=2.89\times 10^{4}\,V$$

Work Done by External Agent to Charge a Conductor

Work done by external agent to bring small charged element dq from infinity to surface of conductor.

dw = V dq

$$\int dw=\int V.dq \qquad V=\frac{q}{C}$$



Work done by external agent to charge the conductor from q_1 to q_2 is

$$W=\int_{q_1}^{q_2}\frac{q}{C}.dq\Rightarrow W=\frac{1}{C}\left(\frac{q^2}{2}\right)_{q_1}^{q_2}\Rightarrow W=\frac{1}{C}\left(\frac{q^2}{2}\right)_{q_1}^{q_2}$$

As $q_1 = CV_1$, $q_2 = CV_2$

work done by external agent to increase the potential of conductor from V_1 to V_2

$$W=\frac{1}{2}C\left(V_2^2-V_1^2\right)$$

Illustration 4:
The work done against electric forces in increasing the potential difference of a condenser from 20 V to 40 V is W. The work done in increasing its potential difference from 40 V to 50 V will be

Solution:

$$W_1 = U_f - U_i = \frac{1}{2}CV_f^2 - \frac{1}{2}CV_i^2 = \frac{1}{2}C\,(40^2 - 20^2) = 600\,C = W \qquad \text{...(i)}$$

$$W_2 = \frac{1}{2}C\,(50^2 - 40^2) = \frac{900}{2}\,C \qquad \text{...(ii)}$$

From equation (i) and (ii)

$$W_2=\frac{900}{2}\cdot\frac{W}{600}=\frac{3}{4}W$$

Illustration 5:
Energy stored in 2000 mF condenser charge to a potential difference of 10V is?

Solution:

$$\text{W.D. = Energy stored} = \frac{1}{2}CV^2 \quad \{C = 2000\text{ mF} = 2F\} =\frac{1}{2}\times 2\times 10^2=100\,J$$

Illustration 6:
The energy stored in a sphere of 10cm radius when the sphere is charged to a potential difference of 300V is

Solution:

$C = 4\pi \in_0 r$

For r = 10 cm; C = $\dfrac{0.1}{9\times10^{-9}}$

The energy stored in a capacitor is given as : E = $\dfrac{1}{2}CV^2$

$E = \dfrac{1}{2}CV^2 = \dfrac{1}{2}\times\dfrac{10^{-10}}{9}\times300^2 = 5\times10^{-7}\,J$

Energy Stored in a Capacitor
Potential energy of a charged conductor
Potential energy of conductor will be stored in the form of electric field.
Potential energy of a conductor which charged by V potential is given by

P.E. = $W_{ext.}$ $\quad U = \dfrac{1}{2}CV^2 = \dfrac{1}{2}QV = \dfrac{Q^2}{2C}$

Potential energy of conducting sphere is given by $U = \dfrac{kQ^2}{2R} = \dfrac{1}{2}\dfrac{Q^2}{4\pi\varepsilon_0 R}$

Illustration 7:
A capacitor of capacitance $\dfrac{1}{3}\mu F$ is connected to a battery of 300 volt and charged. Then the energy stored in capacitor is :

Solution:

U = $\dfrac{1}{2}CV^2 = 1.5\times10^{-2} J$

BEGINNER'S BOX-1

1. A capacitor of capacitance C is charged to a potential V. The flux of the electric field through a closed surface enclosing the capacitor is ....

PCP135

2. A capacitor of capacitance C has a charge Q. The net charge on a capacitor is always ......It stores...... energy.

PCP136

3. A capacitor of capacity C has charge Q and stored energy is W. If the charge is increased to 2Q then what will be the stored energy?

PCP137

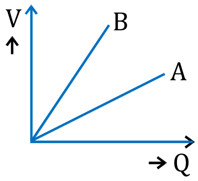
4. Eight drops of mercury of equal radii and possessing equal charges combine to form a big drop. Then the capacitance of the bigger drop compared to each individual drop is ....

PCP138

5. The capacitance of a spherical condenser whose inner sphere is grounded is 1μF. If the spacing between the two spheres is 1 mm then what is the radius of the outer sphere?

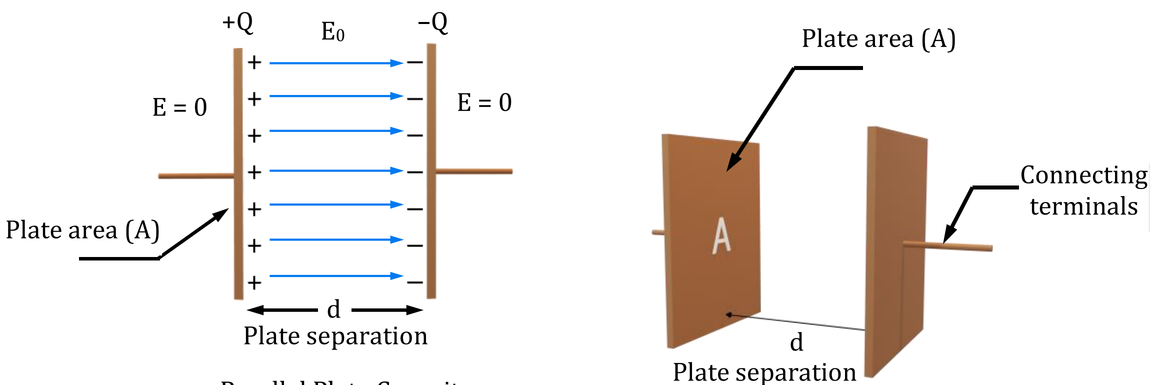
PCP139

6.	When 2×10^{16} electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Calculate the capacitance of the two conductors system. **PCP140**
7.	The graph shows the variation of voltage V across the plates of two capacitors A & B with charge Q. Which of the two capacitors has larger capacitance? **PCP141**
8.	For flash pictures, a photographer uses a 30 μF capacitor and a charger that supplies 3×10^3 volt. Calculate the charge and the energy spent for each flash. **PCP142**
9.	Two capacitors C_1 and C_2 have equal amount of energy stored in them. What is the ratio of potential differences across their plates? **PCP143**



Parallel Plate Capacitor

It consists of two large plane parallel conducting plates separated by a small distance.



Parallel Plate Capacitor

Capacitance of Parallel Plate Capacitor

Surface charge density $\sigma = Q/A$

Electric field intensity between plates, $E_0 = \frac{\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

Potential difference between the plates, $V = E_0 \times d = \frac{Qd}{\varepsilon_0 A}$

Capacitance of parallel plate capacitor, $C_0 = \frac{Q}{V} = \frac{Q}{\left(\frac{Qd}{\varepsilon_0 A}\right)} = \frac{\varepsilon_0 A}{d}$

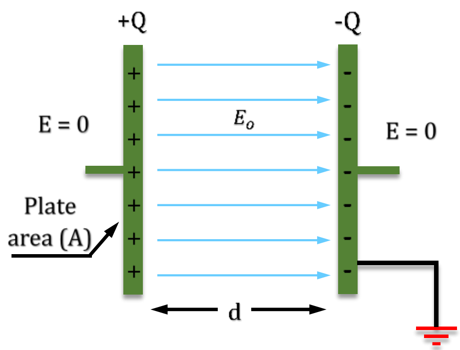


Illustration 8:

One plate of parallel plate capacitor is smaller than other, then charge on smaller plate will be

(A) Less than other

(B) More than other

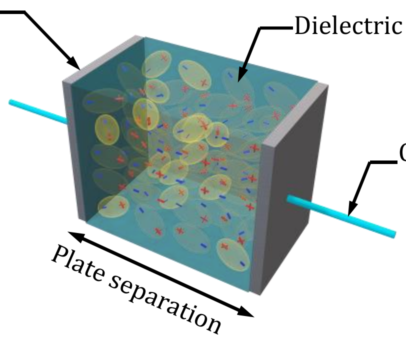
(C) Equal to other

(D) Will depend upon the medium between them

Solution:

Equal to other, because the charges are produced due to induction and moreover the net charge of the capacitor should be zero.

Capacitance of Parallel Plate Capacitor with dielectric



Electric field intensity in vacuum $\Rightarrow E_0$

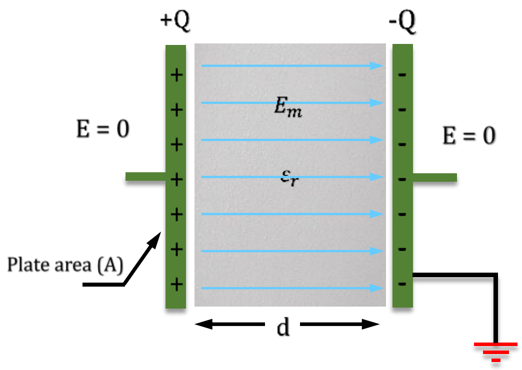
Electric field intensity in medium $\Rightarrow E_m$

$$E_m = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r} = \frac{Q}{\varepsilon_0\varepsilon_r A} \Rightarrow E_m = \frac{E_0}{\varepsilon_r}$$

Potential difference between the plates $V = E_m \times d = \frac{Qd}{\varepsilon_0\varepsilon_r A}$

Capacitance of a capacitor in presence of medium

$$C_m = \frac{Q}{V} = \frac{\varepsilon_0\varepsilon_r A(Q)}{Qd} = \frac{\varepsilon_0\varepsilon_r A}{d} = \varepsilon_r C_0$$



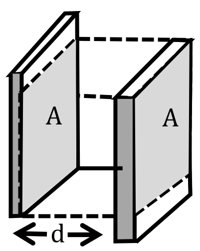
Capacitance of Parallel Plate Capacitor (PPC) depends on

(i) Overlapping area $\Rightarrow$ $C \propto A$

(ii) Distance between plates $\Rightarrow$ $C \propto \frac{1}{d}$

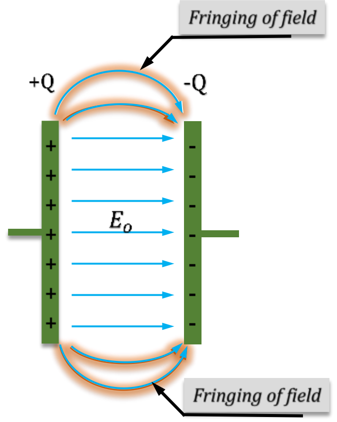
(iii) Medium between plates $\Rightarrow$ $C \propto \varepsilon_r$

* Capacitance of a parallel plate capacitor does not depend on thickness and nature of metal of plates.

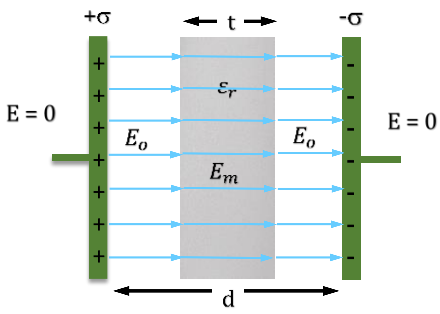


Fringing of Electric Field

For the plates of finite area the electric field between the two plates will not be uniform and the field lines bend out ward at the edges. This is called "**fringing of electric field**"



Capacitance of a parallel plate capacitor when dielectric is partially filled



Surface charge density, $\sigma = \dfrac{Q}{V}$

Electric field in air or Vacuum, $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0 A}$

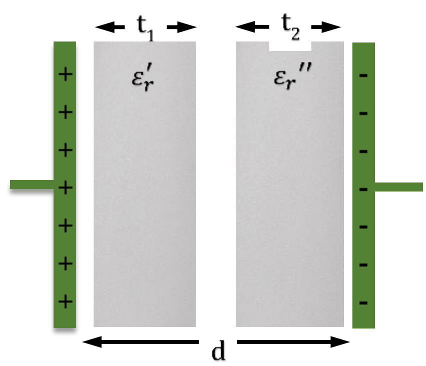
Electric field in dielectric medium, $E_m = \dfrac{\sigma}{\varepsilon} = \dfrac{Q}{\varepsilon_0 \varepsilon_r A}$

Potential difference between the plates of capacitor

$$V = E_0(d-t) + E_m t = \frac{Q}{\varepsilon_0 A}(d-t) + \frac{Q}{\varepsilon_0 \varepsilon_r A} t = \frac{Q}{\varepsilon_0 A}\left[(d-t) + \frac{t}{\varepsilon_r}\right]$$

Capacitance, $C = \dfrac{Q}{V}$

$$\Rightarrow \quad C = \frac{Q}{\dfrac{Q}{\varepsilon_0 A}\left[(d-t) + \dfrac{t}{\varepsilon_r}\right]} = \frac{\varepsilon_0 A}{(d-t) + \dfrac{t}{\varepsilon_r}}$$



In case of multiple slabs,

$$C = \frac{\varepsilon_0 A}{(d - (t_1 + t_2 + ...)) + \dfrac{t_1}{\varepsilon_r'} + \dfrac{t_2}{\varepsilon_r''} + ...}$$

Illustration 9:

A parallel plate capacitor with air between the plates has a capacitance of 8pF. What will be the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant 8?

Solution:

For air, $C_0 = \dfrac{A\varepsilon_0}{d} = 8 \times 10^{-12} F$

Now d' = d/2 and K = 8

$C' = \dfrac{A\varepsilon_0}{d'} \times K = \dfrac{2A\varepsilon_0}{d} \times K = 2 \times 8 \times 10^{-12} \times 8 = 128 \times 10^{-12}\ F$

Illustration 10:

A slab of material of dielectric constant ε_r has the same area as the plates of a parallel plate capacitor but has a thickness (2d/3), where d is the separation of the plates. How is the capacitance changed when the slab is inserted between the plates?

Solution:

$$V = E\times\frac{d}{3}+\frac{E}{\varepsilon_r}\times\frac{2d}{3} = \frac{\sigma d}{3\varepsilon_0}\left(1+\frac{2}{\varepsilon_r}\right)$$

Capacitance of parallel plate capacitor $C=\dfrac{Q}{v}=\dfrac{Q}{\dfrac{Qd}{3A\varepsilon_0}\left(1+\dfrac{2}{\varepsilon_r}\right)}=\dfrac{3A\varepsilon_0}{d\left(1+\dfrac{2}{\varepsilon_r}\right)}$

Illustration 11:

If the distance between the plates of a capacitor of capacitance C_1 is halved and the area of plates is doubled then what will be the capacitance?

Solution:

$$C=\frac{\varepsilon_0 A}{d} \Rightarrow \frac{C_1}{C_2}=\frac{A_1}{A_2}\frac{d_2}{d_1}=\frac{A_1}{2A_1}\times\left(\frac{1}{d_1}\right)\left(\frac{d_1}{2}\right)=\frac{1}{4}\Rightarrow C_2=4C_1$$

Illustration 12:

A capacitor has two circular plates whose radii are 8 cm each and distance between them is 1mm. When mica slab (dielectric constant = 6) is placed between the plates, calculate the capacitance and the energy stored when it is given a potential of 150 volts.

Solution:

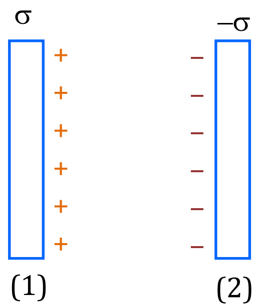
Area of each plate $\pi r^2=\pi\times\left(8\times10^{-2}\right)^2=0.0201\text{m}^2$ and $d=1\text{mm}=1\times10^{-3}$ m

Capacity of capacitor $C=\dfrac{\varepsilon_0\varepsilon_r A}{d}=\dfrac{8.85\times10^{-12}\times6\times0.0201}{1\times10^{-3}}$ = 1.068×10^{-9} F

Potential difference V = 150 volt

Energy stored $U=\dfrac{1}{2}CV^2=\dfrac{1}{2}\times(1.068\times10^{-9})\times(150)^2$ = 1.2×10^{-5} J

Force between plates of PPC



Force between two plates means force on a plate

F = Electric Field due to plate (1) × charge of plate (2)

Here, $F=\left(\dfrac{\sigma}{2\,\varepsilon_0}\right)Q \Rightarrow \sigma=\dfrac{Q}{A}$ $\quad \boxed{F=\dfrac{Q^2}{2A\varepsilon_0}=\dfrac{\sigma^2 A}{2\varepsilon_0}}$

Again $F=\dfrac{(CV)^2}{2A\varepsilon_0}\Rightarrow C=\dfrac{A\varepsilon_0}{d}\Rightarrow A\varepsilon_0=Cd$

$F=\dfrac{C^2V^2}{2Cd}=\dfrac{CV^2}{2d} \quad\Rightarrow\quad \boxed{F=\dfrac{CV^2}{2d}}$

Pressure on each plate of a capacitor

As $\quad F=\dfrac{\sigma^2 A}{2\,\varepsilon_0} \quad$ or $\quad P=\dfrac{F}{A}=\dfrac{\sigma^2}{2\,\varepsilon_0}$

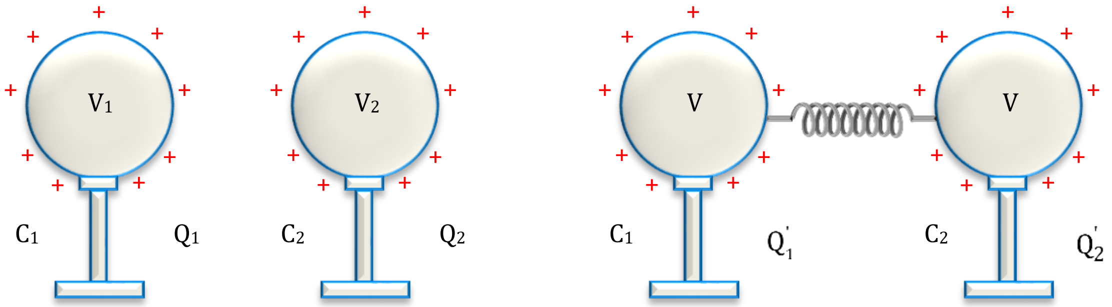
This is known as electrostatic pressure.

* Electrostatic pressure always act perpendicular to surface and outwards.

Sharing of Charges

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of both conductors become equal.

Let the amounts of charges after the conductors are connected be Q_1' and Q_2' respectively and their common potential be V then



(Before connection) (After connection)

Common potential

According to the law of conservation of charge $Q_{\text{before connection}} = Q_{\text{after connection}}$

$$C_1V_1 + C_2V_2 = C_1V + C_2V$$

Common potential after connection $\boxed{V = \dfrac{C_1V_1 + C_2V_2}{C_1 + C_2}}$

Charges after connection

$$Q_1' = C_1V = C_1\left(\frac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\frac{C_1}{C_1 + C_2}\right)Q \qquad \text{(Q = Total charge on the system)}$$

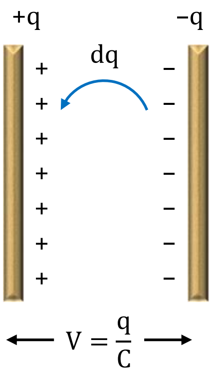
$$Q_2' = C_2V = C_2\left(\frac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\frac{C_2}{C_1 + C_2}\right)Q$$

Ratio of the charges after redistribution $\boxed{\dfrac{Q_1'}{Q_2'} = \dfrac{C_1}{C_2} = \dfrac{R_1}{R_2} \text{ (in case of spherical conductors)}}$

Energy Stored in a Capacitor

Imagine a process of transferring charge from conductor 2 to conductor 1 bit by bit, so that at the end, conductor 1 gets charge Q. By charge conservation, conductor 2 has charge –Q at the end.

At any instant, work done in moving dq charge



$dW = dq.V$

$dW = dq.\dfrac{q}{C}$ $\qquad \left\{P.D = V = \dfrac{q}{C}\right\}$

Total work done

$$(W.D.)_{ext} = \int_0^Q \frac{q}{C}dq$$

Let assume Capacitance = C $\quad (W.D.)_{ext} = \dfrac{Q^2}{2C}$

As we know that $W.D._{ext} = \Delta U$

$\left[U = \dfrac{Q^2}{2C}\right] \qquad \left[U = \dfrac{1}{2}CV^2\right] \qquad \left[U = \dfrac{QV}{2}\right]$

Energy density : In PPC, potential energy stored in the form of electric field i.e. in the space between two plates and volume of this space is (A × d).

Energy density = $\dfrac{\text{Energy}}{\text{volume}} = \dfrac{\frac{1}{2}CV^2}{Ad}$

Here, $\quad C = \dfrac{A\varepsilon_0}{d}$, V = Ed

Energy density = $\dfrac{1}{2}\dfrac{A\varepsilon_0}{d}\dfrac{(Ed)^2}{Ad}$

$\Rightarrow \qquad \boxed{\text{Energy density} = \frac{1}{2}\varepsilon_0 E^2} \qquad$ As, $E = \dfrac{\sigma}{\varepsilon_0}$ $\qquad$ So, $\boxed{\text{Energy density} = \frac{1}{2}\frac{\sigma^2}{\varepsilon_0}}$

Illustration 13:

If the charge in a capacitor is 4C and the energy stored in it is 4J, calculate voltage across its plates.

Solution:

$U = \dfrac{Q^2}{2C} \Rightarrow 4 = \dfrac{4\times 4}{2\times C} \Rightarrow C = 2F \quad \therefore \; V = \dfrac{Q}{C} = \dfrac{4}{2} = 2V$

Illustration 14:

The plate separation in a parallel plate condenser is d and plate area is A. If it is charged to V volt & battery is disconnected then the work done in increasing the plate separation to 2d will be :

Solution:

As battery is disconnected, charge remains constant in the work process.

Work done = final potential energy – initial potential energy = $\dfrac{Q^2}{2C'} - \dfrac{Q^2}{2C^o} = \dfrac{Q^2}{2}\left\{\dfrac{1}{C'} - \dfrac{1}{C^o}\right\}$

Where, Q = CV = $\dfrac{A\varepsilon_0 V}{d}$

C = $\dfrac{A\varepsilon_0}{d}$

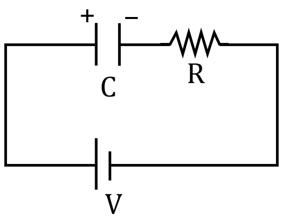
C' = $\dfrac{A\varepsilon_0}{2d}$

Now, work done = $\dfrac{\varepsilon_0 V^2 A}{2d}$

Charging of PPC by battery

When a capacitor is charged by a battery then battery charges it till than its potential difference becomes equal to EMF of battery.

Whenever any capacitor (Initially charged or uncharged) connected to battery then its final voltage is always equal to emf of battery.



Final potential energy of capacitor = $\frac{1}{2}CV^2$

Work done by battery

W = Charge supplied by battery × EMF

W = qV

W = CV^2

* Energy supplied by battery = CV^2

From conservation of energy

$$W_{\text{by battery}} = \underbrace{\Delta U}_{\text{Change in P.E.}} + \text{Heat loss}$$

$$CV^2 = \left(\frac{1}{2}CV^2 - 0\right) + \text{heat loss} \Rightarrow \boxed{\text{Heat loss} = \frac{1}{2}CV^2}$$

Note :

(1) When uncharged capacitor is connected to battery then 50% of energy supply by battery is stored in capacitor and remaining 50% will be lost.

(2) Energy loss does not depends on resistance of circuit.

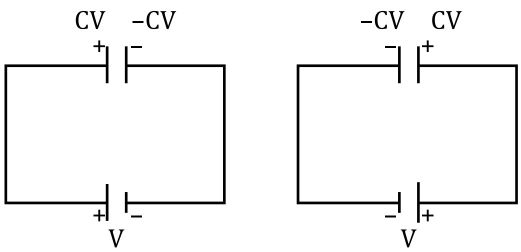
Note: When initially capacitor is charged then heat loss is not equal to $\frac{1}{2}CV^2$, find heat loss by use of following concept

$W_{\text{by battery}} = \Delta U$ + Heat loss

Illustration 15:

Initially a uncharged capacitor having capacitance C is connected across a battery of emf V. Now the capacitor is disconnected and then reconnected across the same battery but with reversed polarity. Find heat loss in this process.

Solution:



Total charge flow $Q_f - Q_i = (CV - (-CV)) = 2CV$

W.D. by battery = (2CV)(V) = $2CV^2$

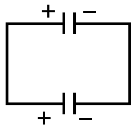
Heat loss = W.D. by battery – (Change in Energy stored)

$$= 2CV^2 - \left(\frac{1}{2}CV^2 - \frac{1}{2}CV^2\right) = 2CV^2$$

For Parallel Plate Capacitor

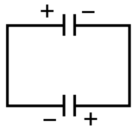
* If like plate are connected

$$V_{cm} = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$



* If unlike plate are connected

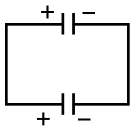
$$V_{cm} = \frac{Q_1 - Q_2}{C_1 + C_2} = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$$



Heat loss

When two capacitors or two metallic spheres are connected without battery then to find heat loss use following formula.

$$\text{Heat} = \frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}(V_1 - V_2)^2$$



* If unlike plates are connected together then

$$\text{Heat} = \frac{1}{2}\frac{C_1C_2}{(C_1 + C_2)}(V_1 + V_2)^2$$

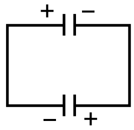


Illustration 16:

A capacitor having a capacitance of 200 μF is charged to a potential difference of 20V. The charging battery is disconnected and the capacitor is connected to another battery of emf 10V with the positive plate of the capacitor joined with the positive terminal of the battery.

(a) Find the charges on the capacitor before and after the reconnection in steady state.

(b) Find the net charge flown through the 10 V battery

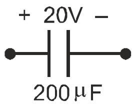
(c) Is work done by the battery or is it done on the battery? Find its magnitude.

(d) Find the decrease in electrostatic field energy.

(e) Find the heat developed during the flow of charge after reconnection.

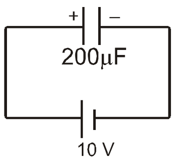
Solution:

(a) Charge on capacitor before connection



$Q_1 = CV_1 = 4000\ \mu C$

Charge on capacitor after connection



$Q_2 = CV_2 = 200 \times 10 = 2000\ \mu C$

(b) Charge flown through the 10V battery = 4000 – 2000 = 2000 μC

(c) Work is done on the battery

$$W = q_{flow}(V) = 2000 \times 10^{-6} \times 10 = 2 \times 10^{-2}\ J$$

(d) The decrease in electrostatic field energy.

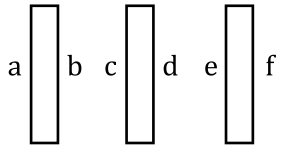
$$= U_i - U_f = \frac{1}{2}CV_1^2 - \frac{1}{2}CV_2^2 = \frac{1}{2} \times 200 \times 10^{-6} \times (20)^2 - \frac{1}{2} \times 200 \times 10^{-6} \times (10)^2 = 30\ mJ$$

(e) $W = \Delta U + \Delta H \Rightarrow -20\ mJ = -30\ mJ + \Delta H \Rightarrow [\Delta H = 10\ mJ]$

BEGINNER'S BOX-2

1. The plate separation in a parallel plate capacitor is d and plate area is A. If it is charged to V volts then calculate the work done in increasing the plate separation to 2d. **PCP144**

2. Three parallel metallic plates, each of area A are kept as shown in the figure and charges Q_1, Q_2 and Q_3 are given to them. Edge effects are negligible. Calculate the charges on the two outermost surfaces 'a' and 'f'.



(A) $\dfrac{Q_1+Q_2+Q_3}{2}$ (B) $\dfrac{Q_1+Q_2+Q_3}{3}$

(C) $\dfrac{Q_1-Q_2+Q_3}{3}$ (D) $\dfrac{Q_1-Q_2+Q_3}{2}$

PCP145

3. A capacitor has a capacitance of 50 pF, which increases to 175 pF with a dielectric material between its plates. What is the dielectric constant of the material? **PCP146**

4. A parallel plate capacitor has rectangular plates with dimensions 6.0 cm × 8.0 cm. If the plates are separated by a sheet of teflon (K = 2.1) 1.5 mm thick, how much energy is stored in the capacitor when it is connected to a 12 V battery? **PCP147**

5. The distance between the plates of a parallel plate capacitor is 'd'. Another thick metal plate of thickness d/2 and area same as that of plates is so placed between the plates, that it does not touch them. The capacity of the resulting capacitor :–

(A) remains the same (B) becomes double (C) becomes half (D) becomes one fourth

PCP148

6. The capacity and the energy stored in a parallel plate condenser with air between its plates are respectively C_0 and W_0. If the air between the plates is replaced by glass (dielectric constant = 5) find the capacitance of the condenser and the energy stored in it.

PCP149

7. A parallel plate capacitor is to be designed with a voltage rating 1 kV using a material of dielectric constant 10 and dielectric strength 10^6 Vm^{-1}. What minimum area of the plates is required to have a capacitance of 88.5 pF? **PCP150**

8. A capacitor of capacitance 10 μF is connected to battery of emf 20 V. Without disconnecting the source a dielectric (K = 4) is introduced to fill the space between the two plates of the capacitor. Calculate the –

(a) charge before the dielectric was introduced. (b) charge after the dielectric is introduced.

PCP151

9. An air capacitor of capacity C = 10 μF is connected to a constant voltage battery of 10 V. Now the space between the plates is filled with a liquid of dielectric constant 5. Calculate additional charge which flows from the battery to the capacitor. **PCP152**

10. If the distance between the plates of a capacitor is d and potential difference is V then what is the energy density between the plates? **PCP153**

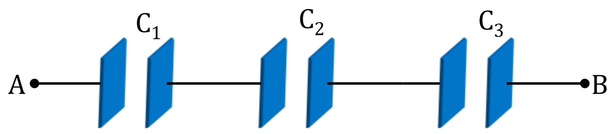
11. 64 droplets of mercury each of radius r and carrying charge q, coalesce to form a big drop. Compare the surface density of charge of each drop with that of the big drop.

PCP154

Combination of Capacitors - Series and Parallel

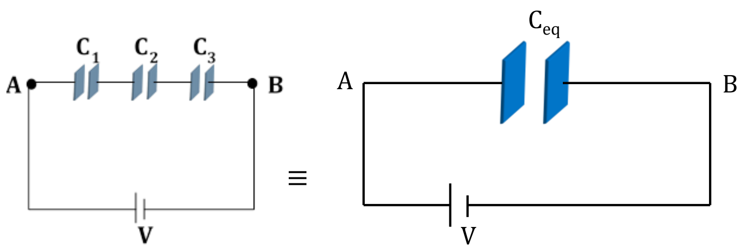
(1) Series Combination

When initially uncharged capacitors are connected as shown so that charges do not have any alternative path(s) to flow then the combination is called series combination.

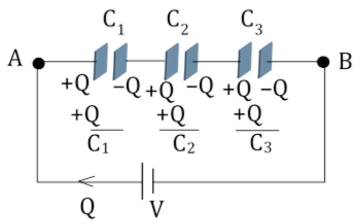


Derivation:

To find equivalent capacitance of this combination lets connect a battery across its terminals.



$$C_{eq} = \frac{Q}{V}$$

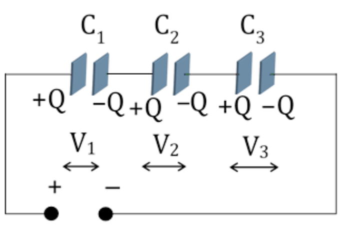


Lets assume that initially, the capacitors were uncharged and after connecting to battery, Q charge flows through the battery as shown in above figure.

Applying Kirchhoff's voltage law

$$-\left(\frac{Q}{C_1}\right)-\left(\frac{Q}{C_2}\right)-\left(\frac{Q}{C_3}\right)+V=0.\ ;\ \frac{V}{Q}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3} \Rightarrow \frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

Salient features of series combination



(a) All capacitors will have same charge but can have different potential difference across them.

(b) We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination $\propto \frac{1}{C}$

$$V_1=\frac{Q}{C_1}, V_2=\frac{Q}{C_2}, V_3=\frac{Q}{C_3}......$$

$$V_1 : V_2 : V_3 = \frac{1}{C_1}:\frac{1}{C_2}:\frac{1}{C_3}$$

(c) In series combination the smallest capacitor gets maximum potential.

$$V_1 = \frac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\dots\dots}V, V_2 = \frac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\dots\dots}V, V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\dots\dots}V$$

Where $V = V_1 + V_2 + V_3$

(d) In series : $\frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\dots\dots$

In series combination equivalent capacitance is always less than the smallest capacitor of combination.

(e) Energy stored in the combination $U_{combination} = \frac{Q^2}{2C_1}+\frac{Q^2}{2C_2}+\frac{Q^2}{2C_3} = \frac{Q^2}{2C_{eq}}$

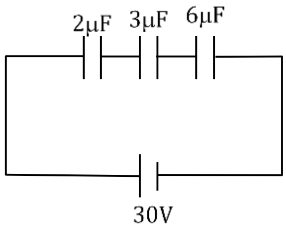
Energy supplied by the battery in charging the combination $U_{battery} = Q \times V = Q.\frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$

$$\frac{U_{combination}}{U_{battery}} = \frac{1}{2}$$

Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (if capacitors are initially uncharged)

Illustration 17:

Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30 V. Find out following:



(i) charge flow through the battery,

(ii) potential energy in 3μF capacitor.

(iii) U_{total} in capacitors

(iv) heat produced in the circuit

Solution:

$$\frac{1}{C_{eq}} = \frac{1}{2}+\frac{1}{3}+\frac{1}{6} = \frac{3+2+1}{6} = 1$$

$C_{eq} = 1\mu F$.

(i) $Q = C_{eq} V = 30\ \mu C$.

(ii) Charge on 3μF capacitor = 30 μC ; Energy = $\frac{Q^2}{2C} = \frac{30\times30}{2\times3}$ = 150μJ

(iii) $U_{total} = \frac{30\times30}{2}$ μJ = 450 μJ

(iv) Heat produced = (30 μC) (30) – 450 μJ = 450 μJ.

(2) Parallel Combination

When one plate of each capacitors (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.

Derivation

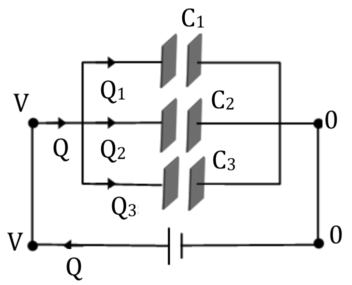
$Q = Q_1 + Q_2 + Q_3$

$Q = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$

$\frac{Q}{V} = C_1 + C_2 + C_3$

$C_{eq} = C_1 + C_2 + C_3$

In general $C_{eq} = \sum_{n=1}^{n} C_n$



Salient Features of Parallel Combination

(a) All capacitors have same potential difference but can have different charges.

(b) We can say that:

$Q_1 = C_1V$

Q_1 = Charge on capacitor C_1

C_1 = Capacitance of capacitor C_1

V = Potential across capacitor C_1

- $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

  The charge on the capacitor is proportional to its capacitance

  $Q \propto C$

  $Q_1 = \frac{C_1}{C_1+C_2+C_3}Q \quad \Rightarrow \quad Q_2 = \frac{C_2}{C_1+C_2+C_3}Q \quad \Rightarrow \quad Q_3 = \frac{C_3}{C_1+C_2+C_3}Q$

  Where total charge = $Q = Q_1 + Q_2 + Q_3$ ......

  Maximum charge will flow through the capacitor of largest value.

- $C_{eq} = C_1 + C_2 + C_3$

  Equivalent capacitance is always greater than the largest capacitor of combination.

- Energy stored in the combination :

  $U_{combination} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \ldots = \frac{1}{2}(C_1 + C_2 + C_3 \ldots\ldots)V^2 = \frac{1}{2}C_{eq}V^2$

Illustration 18:

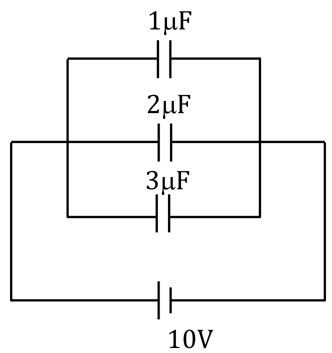
Three initially uncharged capacitors are connected in a parallel combination to a battery of 10 V. Find out following

(i) charge flow from the battery

(ii) total energy stored in the capacitors

(iii) heat produced in the circuit

(iv) potential energy in the $3\mu F$ capacitor.



Solution:

$C_{eq} = C_1 + C_2 + C_3 = 1 + 2 + 3 = 6\,\mu F$

(i) $Q = (C_{eq}V) = 6 \times 10 = 60\ \mu C$

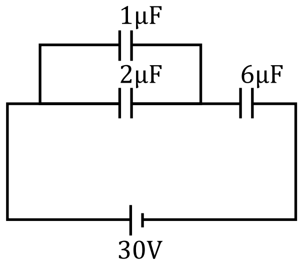
(ii) $U_{total} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

(iii) heat produced = $60 \times 10 - 300 = 300\ \mu J$

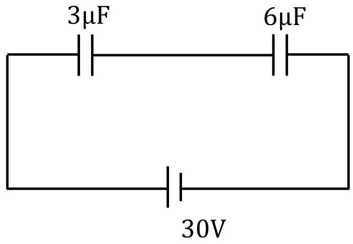
(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ \mu J$

Illustration 19:

In the given circuit find out charge on 6μF and 1μF capacitor.



Solution:



It can be simplified as $C_{eq} = \frac{18}{9} = 2\mu F$

charge flow through the battery = 30 × 2μC

$Q = 60\mu C$

Now charge on 3μF = Charge on 6μF= 60μC

Potential difference across 3μF = 60/ 3= 20V

$\therefore$ Charge on 1μF = 20 μC.

Illustration 20:

An infinite number of capacitors of capacitance C, 4C, 16C... ∞ are connected in series then what will be their resultant capacitance?

Solution:

Let the equivalent capacitance of the combination = C_{eq}

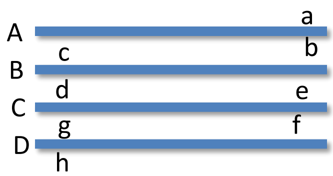
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{4C} + \frac{1}{16C} + ...\infty = \left[1 + \frac{1}{4} + \frac{1}{16} + ...\infty\right]\frac{1}{C} \text{ (this is G.P. series)}$$

$$\Rightarrow \quad S_\infty = \frac{a}{1-r} \; ; \quad \text{first term a = 1, common ratio r = } \frac{1}{4}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{1-\frac{1}{4}} \times \frac{1}{C} \Rightarrow C_{eq} = \frac{3}{4}Ce$$

Problems based on Combination of Parallel Plates

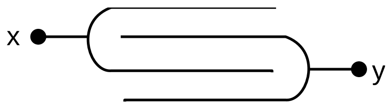
Combinations of parallel plates can be grouped and equivalent capacitance can be found easily by redrawing the circuit by connecting plates to common terminals. This has been illustrated by following example.



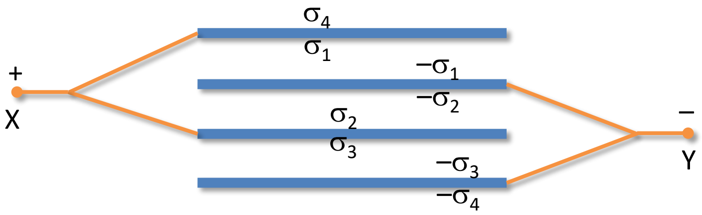
Four parallel plates each of area A and plate separation d. These four plates are forming three parallel plate capacitors, one between plates A and B, one between plates B and C and another between plates C and D.

Capacitance of each of these capacitors is given as $C = \frac{\epsilon_0 A}{d}$.

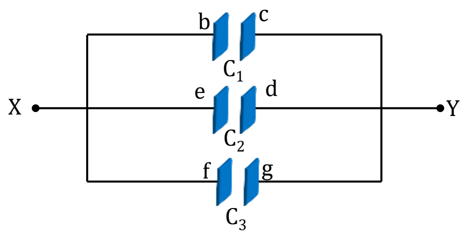
If plates A and C are connected as terminal X and plates B and D are connected as terminal Y. Then calculation of equivalent capacitance across terminal X and Y can be done by following method.



Let us assume that we have connected a battery between terminal X and Y. This battery supplies equal and opposite charges to terminal X and Y. For the detailed analysis let us say σ_1 and σ_2 are charge density on faces b and e. Similarly σ_3 and σ_4 are charge density on faces f and a.



Now equal opposite charges will be there on opposite faces as shown in figure (as per Gauss Law). Also electric field at any point inside conductor must be zero therefore σ_4 must be zero.

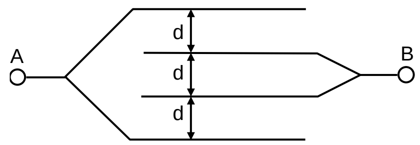


$V_{bc} = V_{ed} = V_{fg}$

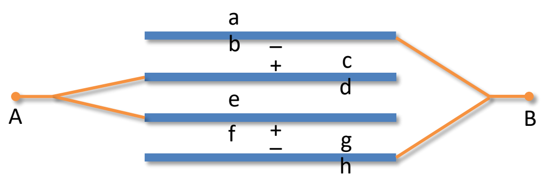
So all the capacitors are in parallel combination.

$$C_{eq} = C_1 + C_2 + C_3 = \frac{3A\epsilon_0}{d}$$

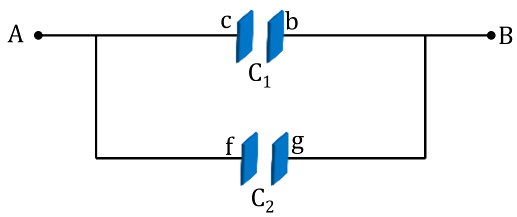
Let us try to find out equivalent capacitance between A and B. (take each plate Area = A) in following case by using the same above concept.



Assuming a battery between terminals A and B the positive and negative charges on the plates will be as shown in the figure fellow.



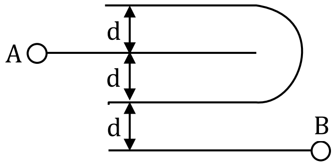
Here charges on plates d and e is zero because there potential must be same so no electric field should be there in between these faces, so charge on faces d and e must be zero.



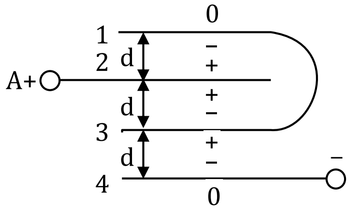
These are only two capacitors $C_{eq} = C_1 + C_2 = \dfrac{2A\epsilon_0}{d}$

Illustration 21:

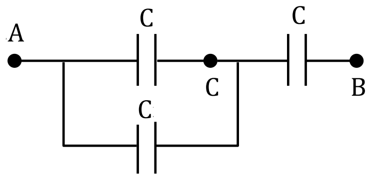
Find out equivalent capacitance between A and B. (take each plate Area = A)



Solution:



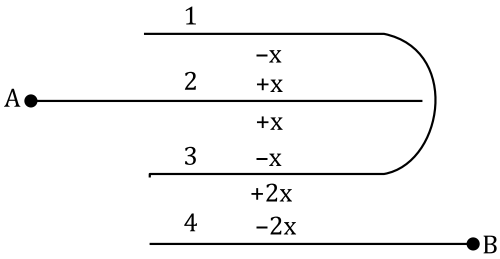
The modified circuit is



$$C_{eq} = \frac{2C}{3} = \frac{2A\epsilon_0}{3d}$$

Other method:

Let charge density as shown



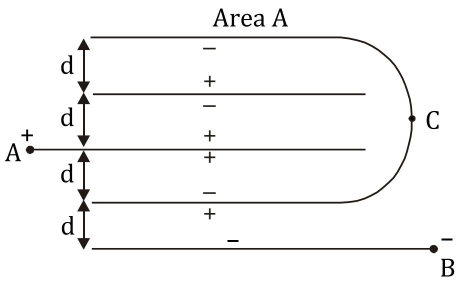
$$C_{eq} = \frac{Q}{V} = \frac{2 \times A}{V}$$

$$V = V_2 - V_4 = (V_2 - V_3) + (V_3 - V_4) = \frac{xd}{\epsilon_0} + \frac{2xd}{\epsilon_0} = \frac{3xd}{\epsilon_0}$$

$$\therefore \qquad C_{eq} = \frac{2Ax\epsilon_0}{3xd} = \frac{2A\epsilon_0}{3d} = \frac{2C}{3}$$

Illustration 22:

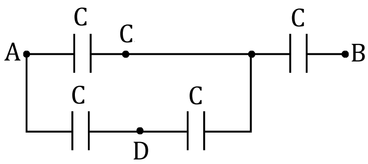
Find out equivalent capacitance between A and B.



Solution:

Let $\qquad C = \dfrac{A\epsilon_0}{d}$

Equivalent circuit :

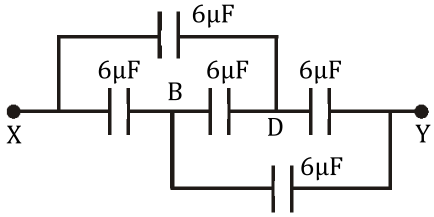


$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$$

$$C_{eq} = \frac{3C}{5} = \frac{3A\epsilon_0}{5d}$$

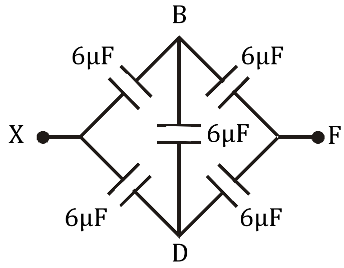
Illustration 23:

What is the effective capacitance between the points X and Y?



Solution:

This is a balanced Wheatstone bridge, so BD can be removed.



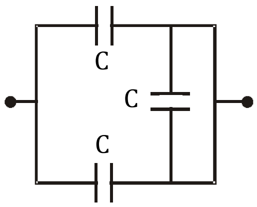
equivalent circuit

$6\mu F$ and $6\mu F$ are in series so equivalent capacitance = $3\mu F$

then $3\mu F$ and $3\mu F$ are in parallel so equivalent capacitance between X and Y is $6\mu F$.

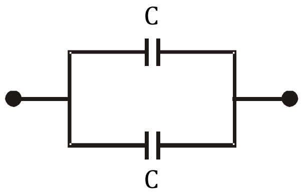
Illustration 24:

What is the equivalent capacitance of the combination?



Solution:

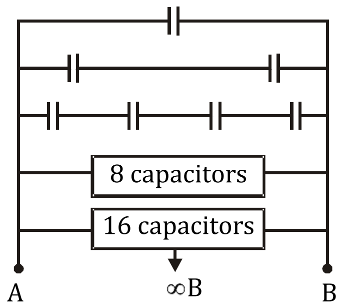
The capacitor shown as a vertical element is shorted, so it can be removed.



$\Rightarrow C_{eq} = C + C = 2C$

Illustration 25:

An infinite number of identical capacitors each of capacitance $1\mu F$ are connected as in the adjoining figure. Then the equivalent capacitance between A and B is:



Solution:

$C = 1\mu F$

$$C_{eq.} = C + \frac{C}{2} + \frac{C}{4} + \frac{C}{8} + \frac{C}{16} + ...\infty \Rightarrow C\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\infty\right]$$

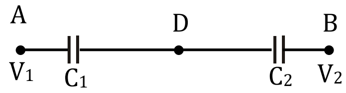
It is Geometric progression sum of infinite terms

$S_{\infty} = \dfrac{a}{1-r}$ Here a = first term = 1, r = common ratio = $\dfrac{1}{2}$

$$= C\left[\frac{1}{\left(1 - \frac{1}{2}\right)}\right] = 2C = 2\mu F$$

Illustration 26:

Two capacitances C_1 and C_2 in a circuit are joined as shown in figure. The potential of point A is V_1 and that of B is V_2. The potential of point D will be:



Solution:

Q = same $\Rightarrow Q = C_1V_{AD} = C_1(V_1 - V_D)$ and $Q = C_2V_{DB} = C_2(V_D - V_2)$

$C_1(V_1 - V_D) = C_2(V_D - V_2)$; $C_1V_1 + C_2V_2 = (C_1 + C_2)V_D$

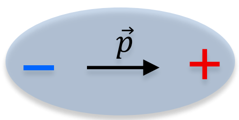
$$V_D = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

Dielectrics

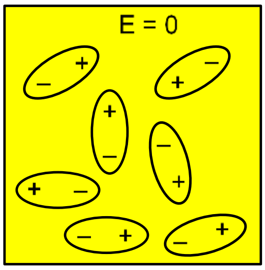
- The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as dielectrics.
- Dielectrics are non-conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called dielectric strength they lose their insulating property and begin to conduct.
- Dielectric strength is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre. Dimensions $M^1 L^1 T^{-3} A^{-1}$.

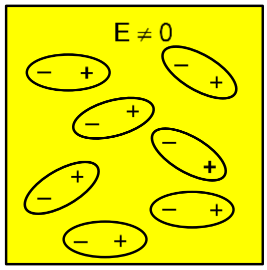
Polar dielectrics

$E = 0; \quad p \neq 0$



Permanent Dipole moment ($\vec{p}$)



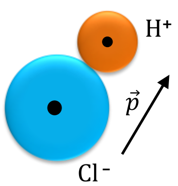


- In absence of external field the centres of positive and negative charge do not coincide-due to asymmetric shape of molecules.
- Each molecule has permanent dipole moment.
- The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.

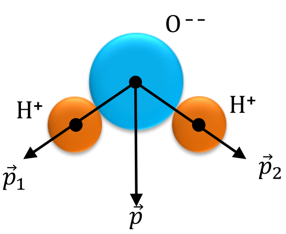
  In presence of external field dipoles tends to align in direction of field.

  Example: Water, Alcohol CO_2, $EC\ell$, NH_3

HCl molecule



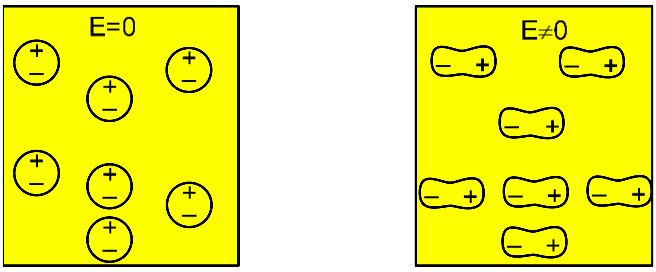
H_2O molecule



$\vec{p} = \vec{p}_1 + \vec{p}_2$

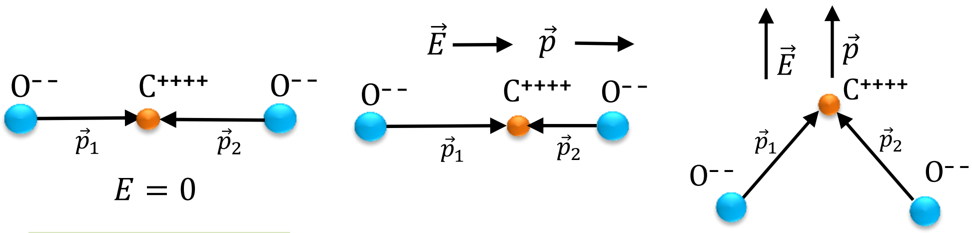
Note: Dipole moment of polar molecules depends on temperature.

Non-polar dielectrics

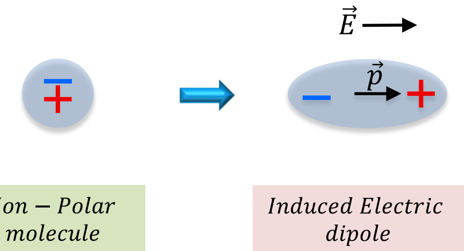


- In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In presence of external field they acquire induced dipole moment.

  Example: Nitrogen, Oxygen, Benzene, Methane



$p = p_1 - p_2 = 0$ $\quad$ $p = p_1 - p_2$ $\quad$ $\vec{p} = \vec{p}_1 + \vec{p}_2$



$E = 0$ $\quad$ $E \neq 0$

Note: Induced electric dipole moment of non-polar molecules is independent of temperature.

Polarisation: The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

- **Polarisation vector $\vec{P}$**

  This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

  $\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

  where n is number of atoms per unit volume of dielectric and $\vec{P}$ is dipole moment of an atom or molecule.

  $\vec{P} = n\vec{p} = \frac{qib}{Ad} = \left(\frac{qi}{A}\right) = \sigma_i$ = induced surface charge density.

Capacitors with Dielectric

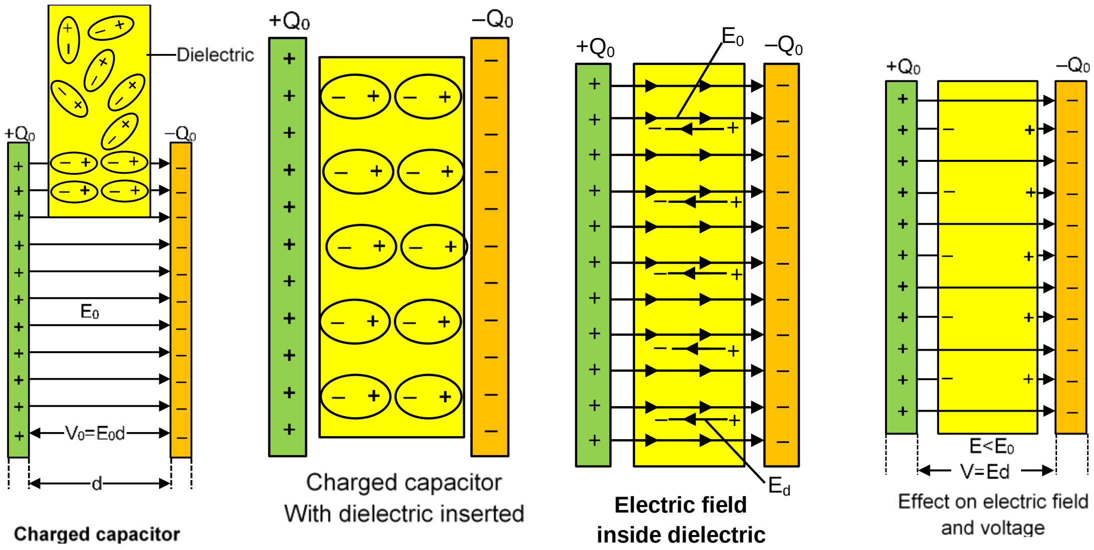
(i) In absence of dielectric $E = \dfrac{\sigma}{\epsilon_0}$

(ii) When a dielectric fills the space between the plates then molecules having dipole moment align themselves in the direction of electric field.

$\sigma_b = \sigma_i$ induced (bound) charge density (called bound charge because it is not due to free electrons).

The induced charge also produce electric field.

Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric and E, V, C are electric field, potential difference and capacitance respectively in presence of dielectric.



Electric field in absence of dielectric $E_0 = \dfrac{V_0}{d} = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric $E = E_0 - E_i = \dfrac{\sigma - \sigma_b}{\varepsilon_0} = \dfrac{Q - Q_b}{\varepsilon_0} = \dfrac{V}{d}$

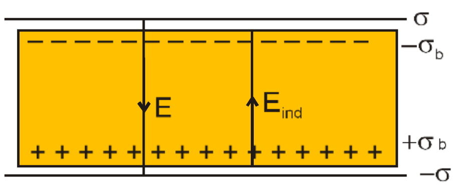
Capacitance in absence of dielectric $C_0 = \dfrac{Q}{V_0}$

Capacitance in presence of dielectric $C = \dfrac{Q - Q_b}{V}$

The dielectric constant or relative permittivity K or $\varepsilon_r = \dfrac{E_0}{E} = \dfrac{V_0}{V} = \dfrac{C}{C_0} = \dfrac{Q}{Q - Q_b} = \dfrac{\sigma}{\sigma - \sigma_b} = \dfrac{\varepsilon}{\varepsilon_0}$

Form $K = \dfrac{Q}{Q - Q_b} \Rightarrow Q_b = Q\left(1 - \dfrac{1}{K}\right)$ and $K = \dfrac{\sigma}{\sigma - \sigma_b} \Rightarrow \sigma_b = \sigma\left(1 - \dfrac{1}{K}\right)$

(iii) Capacitance in the presence of dielectric



$$C = \frac{\sigma A}{V} = \frac{\sigma A}{\dfrac{\sigma}{K \epsilon_0} . d} = \frac{AK \epsilon_0}{d} = \frac{AK \epsilon_0}{d}$$

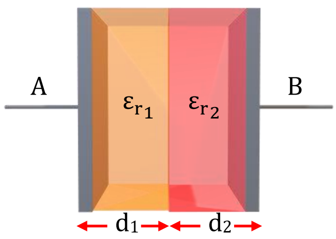
Here capacitance is increased by a factor K.

$$C = \frac{AK \epsilon_0}{d} = C_0 K$$

Relation between polarisation and surface charge density of induced charge

Equivalent dipole moment of dielectric slab = $q_i \times d$

Electric polarization



$(\vec{P}) = \dfrac{q_i \times d}{V}$ (V $\Rightarrow$ volume of dielectric slab)

$\vec{P} = \dfrac{q_i \times d}{A \times d} \Rightarrow \vec{P} = \dfrac{q_i}{A} = \sigma_i$ $\qquad P = \sigma_i$

It is clear that polarisation is equal to induced surface charge density.

Electric susceptibility (χ_e)

Polarisation (P) of dielectrics is directly proportional to the electric field (E), i.e.,

$$P \propto E$$

$$P = \varepsilon_0 \chi_e E$$

Where, $\chi_e \Rightarrow$ electric susceptibility (constant) For vacuum, $\chi_e = 0$

Dielectric strength

The maximum value of the electric field intensity that can be applied to the dielectric without its electric breakdown.

$E_{break} = \dfrac{V_{break}}{d}$

For air, $E_{break} = 3 \times 10^6$ V/m

Effects of Dielectrics in Capacitor

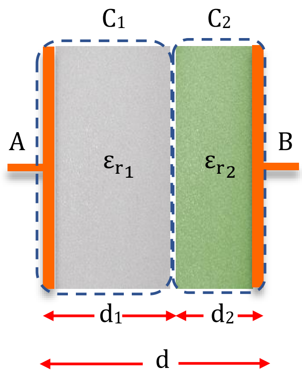
Distance and Area Division by Dielectric

(1) Distance Division

(a) Distance is divided and area remains same

(b) Capacitors are in series

(c) Individual capacitances are



$C_1 = \dfrac{\varepsilon_0 \varepsilon_{r_1} A}{d_1} \qquad C_2 = \dfrac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}$

These two are in series

$\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \Rightarrow \dfrac{1}{C} = \dfrac{d_1}{\varepsilon_0 \varepsilon_{r_1} A} + \dfrac{d_2}{\varepsilon_0 \varepsilon_{r_2} A}$

$\Rightarrow \dfrac{1}{C} = \dfrac{1}{\varepsilon_0 A}\left[\dfrac{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}{\varepsilon_{r_1} \varepsilon_{r_2}}\right] \Rightarrow C = \varepsilon_0 A \left[\dfrac{\varepsilon_{r_1} \varepsilon_{r_2}}{d_1 \varepsilon_{r_2} + d_2 \varepsilon_{r_1}}\right]$

Special case: If $d_1 = d_2 = \dfrac{d}{2} \Rightarrow C = \dfrac{\varepsilon_0 A}{d}\left[\dfrac{2\varepsilon_{r_1} \varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right]$

(2) Area Division

(a) Area is divided and distance remains same.

(b) Capacitors are in parallel.

(c) Individual capacitances are

$$C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d} \qquad C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d}$$

These two are in parallel so,

$C = C_1 + C_2$

$$C = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d} + \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d}$$

Special case : If $A_1 = A_2 = \frac{A}{2}$

Then, $C = \frac{\varepsilon_0 A}{d}\left(\frac{\varepsilon_{r_1} + \varepsilon_{r_2}}{2}\right)$

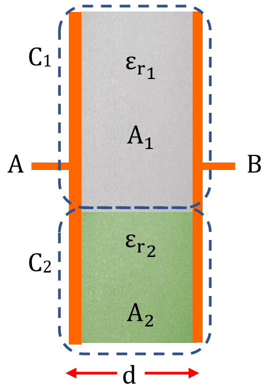
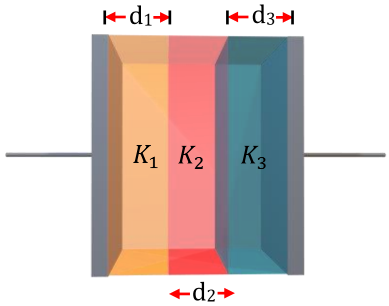


Illustration 27:

Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A and thickness d_1, K_2 of area A and thickness d_2 and K_3 of area A and thickness d_3 are inserted between the plates of series plate capacitor of plate area A as shown in figure. (Given distance between the two plates d).



Solution:

$$\frac{1}{C_{AB}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}\left(C_1 = \frac{\epsilon_0 AK_1}{d_1}; C_2 = \frac{\epsilon_0 AK_2}{d_2}; C_3 = \frac{\epsilon_0 AK_3}{d_3}\right)$$

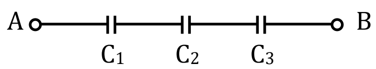
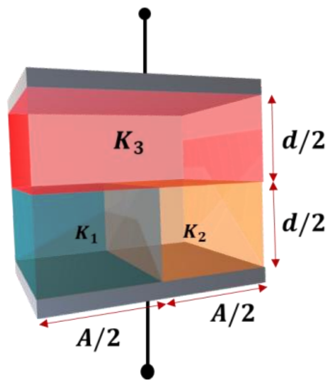
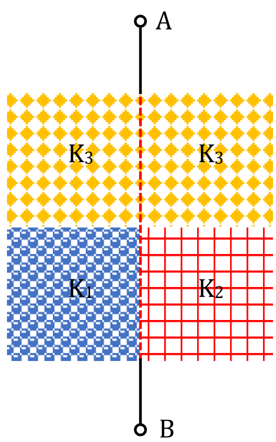


Illustration 28:

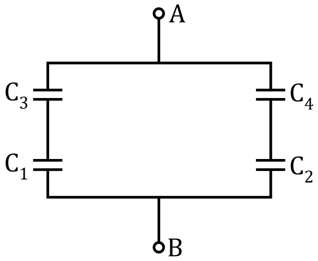
Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A_1 and thickness d, K_2 of area A_2 and thickness d_1 and K_3 of area A_2 and thickness d_2 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates d).





Solution:

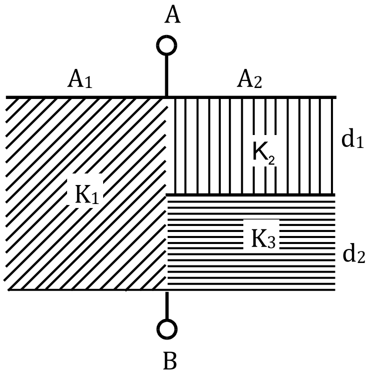
$$C_1 = \frac{\frac{A}{2}K_1 \epsilon_0}{\frac{d}{2}} \qquad C_2 = \frac{\frac{A}{2}K_2 \epsilon_0}{\frac{d}{2}} \qquad C_3 = \frac{\frac{A}{2}K_3 \epsilon_0}{\frac{d}{2}} \qquad C_4 = \frac{\frac{A}{2}K_3 \epsilon_0}{\frac{d}{2}}$$



$$C_{eq} = \frac{C_1 \times C_3}{C_1 + C_3} + \frac{C_2 \times C_4}{C_2 + C_4}$$

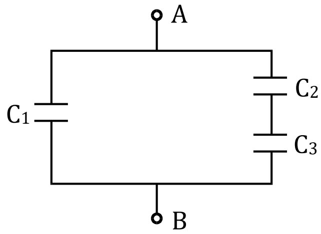
Illustration 29:

Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A_1 and thickness d, K_2 of area A_2 and thickness d_1 and K_3 of area A_2 and thickness d_2 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. (Given distance between the two plates $d = d_1+d_2$).



Solution:

It is equivalent to $C = C_1 + \frac{C_2C_3}{C_2+C_3}$

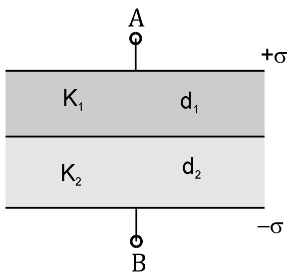


$$C = \frac{A_1K_1 \epsilon_0}{d_1+d_2} + \frac{\frac{A_2K_2 \epsilon_0}{d_1} \cdot \frac{A_2K_3 \epsilon_0}{d_2}}{\frac{A_2K_2 \epsilon_0}{d_1} + \frac{A_2K_3 \epsilon_0}{d_2}}$$

$$= \frac{A_1K_1 \epsilon_0}{d_1+d_2} + \frac{A_2^2K_2K_3 \epsilon_0^2}{A_2K_2 \epsilon_0 d_2 + A_2K_3 \epsilon_0 d_1} = \frac{A_1K_1 \epsilon_0}{d_1+d_2} + \frac{A_2K_2K_3 \epsilon_0}{K_2d_2 + k_3d_1}$$

Illustration 30:

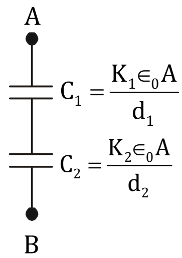
Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of thickness d_1 and d_2 and each of area A are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



Solution:

$$C = \frac{\sigma A}{V}\ ;\ V = E_1d_1 + E_2d_2 = \frac{\sigma d_1}{K_1 \epsilon_0} + \frac{\sigma d_2}{K_2 \epsilon_0} = \frac{\sigma}{\epsilon_0}\left(\frac{d_1}{k_1} + \frac{d_2}{k_2}\right)$$

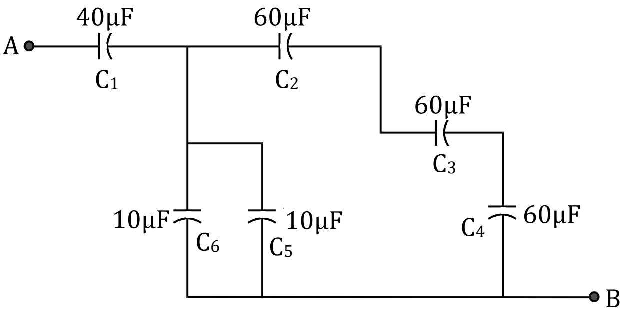
$$\therefore \quad C = \frac{A \epsilon_0}{\frac{d_1}{K_1} + \frac{d_2}{K_2}} \Rightarrow \frac{1}{C} = \frac{d_1}{AK_1 \epsilon_0} + \frac{d_2}{AK_2 \epsilon_0}$$



This formula suggests that the system between A and B can be considered as series combination of two capacitors.

BEGINNER'S BOX-3

1. Find the equivalent capacitance of the combination of capacitors between the points A and B as shown in figure. Also calculate the total charge that flows in the circuit when a 100 V battery is connected between the points A and B **PCP155**

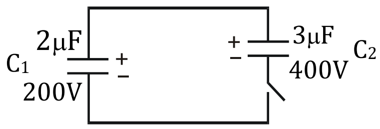


2. Three capacitors each of capacitance 9 pF are connected in series.

(a) What is the total capacitance of the combination?

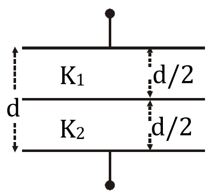
(b) What is the potential difference across each capacitor if the combination is connected to a 120 V supply? **PCP156**

3. Two capacitors of capacity C_1 and C_2 are connected as shown in figure.

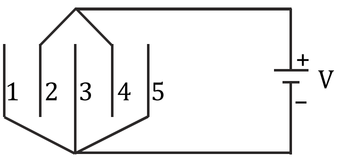


Now the switch is closed. Calculate the charge on each capacitor. **PCP157**

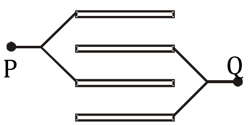
4. Two dielectric slabs of dielectric constants K_1 and K_2 have been inserted in between the plates of a capacitor as shown below. What will be the capacitance of the capacitor inserted? (Plate area = A) **PCP158**



5. Five identical plates each of area A are joined as shown in the figure. The distance between successive plates is d. The plates are connected to potential difference of V volt. Find the charges of plates 1 and 4. **PCP159**



6. Four plates of the same area A are joined as shown in the figure. The distance between successive plates is d. Find the equivalent capacity across PQ will be. **PCP160**



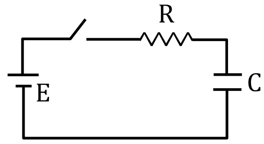
7.	Two identical capacitors each of capacity C are charged upto same potential V. Now their oppositely charged plates are connected together then calculate the – (a) energy of each capacitor before connection. (b) potential of each capacitor after connection. (c) charge of each capacitor after connection. (d) energy stored in each capacitor after connection. (e) energy loss in the form of heat.	**PCP161**

RC Circuits

An electric circuit consisting of resistor(s) and capacitor(s) only is known as RC circuit.

Charging of a capacitor/condenser:

(i) In the following circuit, if key is closed then the condenser gets charged. Finite time is taken in the charging process.



The quantity of charge at any instant of time t is given by $q = q_0[1 - e^{-(t/RC)}]$

Where q_0 = maximum/final value of charge = value of charge at t = ∞

According to the above equation, the quantity of charge on the condenser increases exponentially with increase in time.

(ii) If $t = RC = \tau$ then

$$q = q_0\,[1 - e^{-(RC/RC)}] = q_0\left[1 - \frac{1}{e}\right]$$

$\Rightarrow\ q = q_0\,(1 - 0.37) = 0.63\,q_0 = 63\%$ of q_0

(iii) Time t = RC is known as time constant.

i.e. the time constant is that time during which the charge rises on the condenser plate to 63% of its maximum value.

(iv) The potential difference across the condenser plate at any instant of time is given by

$V_C = V_0[1 - e^{-(t/RC)}]$

Where V_0 = maximum/final value of potential difference across the condenser plate = value of potential difference across the condenser plate at t = ∞

(v) The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$I = I_0[e^{-(t/RC)}]$

Where I_0 = maximum/initial value of current in the circuit = value of current in the circuit at t = 0

(vi) If $t = RC = \tau$ = Time constant

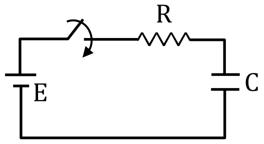
$$I = I_0e^{(-RC/RC)} = \frac{I_0}{e} = 0.37\ I_0$$

$= 37\%$ of I_0

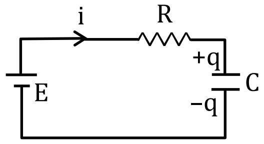
i.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.

Derivation of formqulae for charging of capacitor

It is given that initially capacitor is uncharged. Let at any time charge on capacitor is q.



At t = 0



At any time t

Applying Kirchhoff's voltage law at time t

$E - iR - \frac{q}{C} = 0$

$iR = \frac{EC-q}{C} \Rightarrow i = \frac{EC-q}{RC}$

$\frac{dq}{dt} = \frac{EC-q}{RC} \quad \left[\because \; i = \frac{dq}{dt}\right]$

$\frac{RC}{EC-q}\,dq = dt$

$\int_0^q \frac{dq}{EC-q} = \int_0^t \frac{dt}{RC}$

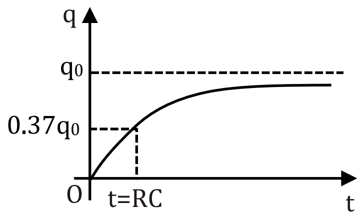
$-\ln(EC-q) + \ln EC = \frac{t}{RC}$

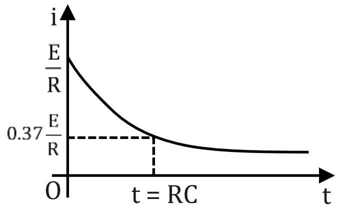
$\ln \frac{EC}{EC-q} = \frac{t}{RC}$

$EC - q = ECe^{-t/RC}$

$\boxed{q = EC\left(1 - e^{-t/RC}\right)}$

$\boxed{q = EC\left(1 - e^{-t/\tau}\right)}$ here τ = RC is known as time constant.

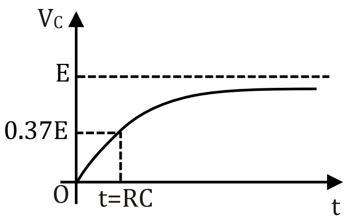




After one time constant i.e. at t = τ

$q = EC\left(1 - \frac{1}{e}\right)$

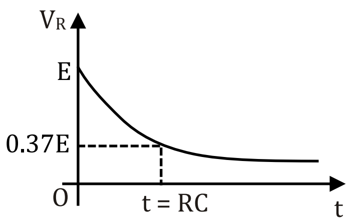
$= EC\,(1 - 0.37) = 0.63\,EC$



Current at any time t

$i = \frac{dq}{dt} = EC\left[-e^{-t/RC}\left(-\frac{1}{RC}\right)\right]$

$\boxed{i = \frac{E}{R}e^{-t/RC}}$



Potential difference across condenser plates at any time t

$\boxed{V_C = \frac{q}{C} = E\left(1 - e^{-t/RC}\right)}$

Potential difference across resistor at any time t

$\boxed{V_R = iR = Ee^{-t/RC}}$

Heat dissipated

From energy conservation,

Heat dissipated = work done by battery − $\Delta U_{capacitor}$

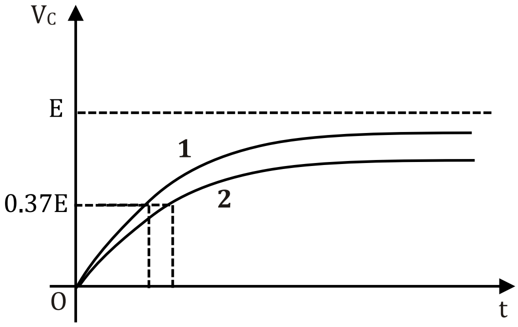
$= CE(E) - \left(\frac{1}{2}CE^2 - 0\right) = \frac{1}{2}CE^2$

Alternatively:

Heat dissipated, $H = \int_0^{\infty} i^2 R dt$

$$= \int_0^{\infty} \frac{E^2}{R^2} e^{-\frac{2t}{RC}} R\,dt = \frac{E^2}{R}\int_0^{\infty} e^{-2t/RC}\,dt = \frac{E^2}{R}\left[\frac{e^{-\frac{2t}{RC}}}{-2/RC}\right]_0^{\infty} = -\frac{E^2RC}{2R}\left[e^{-\frac{2t}{RC}}\right]_0^{\infty} = \frac{1}{2}E^2C$$

Note:



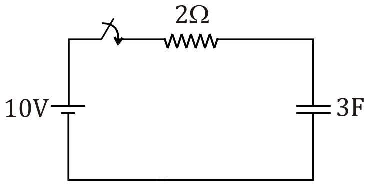
In the figure time constant of (2) is more than (1).

Illustration 31:

Find out current in the circuit and charge on capacitor which is initially uncharged in the following situations.

(a) Just after the switch is closed.

(b) A long time after the switch was closed.



Solution:

(a) Here t = 0 [Just after the switch of closed]

$$i = \frac{E}{R}e^{-t/RC} = \frac{E}{R} = \frac{10}{2} = 5A$$

$q = EC(1 - e^{-t/RC}) = 0$

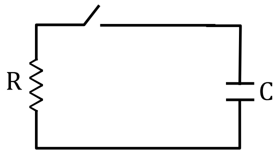
(b) Here $t = \infty$ (A long time after the switch was closed)

$$i = \frac{E}{R}e^{-t/RC} = 0$$

$q = EC(1 - e^{-t/RC}) = EC = 10 \times 3 = 30$ C

Discharging of a condenser

(i) In the circuit shown below, if key is closed then the condenser gets discharged by the time.



(Initial charge on capacitor = q_0)

(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0\, e^{-(t/RC)}$

Where q_0 = maximum/initial value of charge = value of charge at t = 0

According to the above equation, the quantity of charge on the condenser decreases exponentially with increase in time.

(iii) If $t = RC = \tau$ = time constant, then $q = \frac{q_0}{e} = 0.37q_0 = 37\%$ of q_0

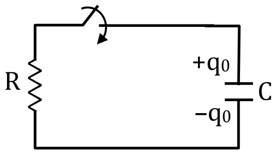
i.e., the time constant is that time during which the charge on condenser plates in discharge process, falls to 37% of initial charge.

(iv) The dimensions of RC are those of time i.e. $M^0L^0T^1$ and the dimensions of $\frac{1}{RC}$ are those of frequency i.e. $M^0L^0T^{-1}$.

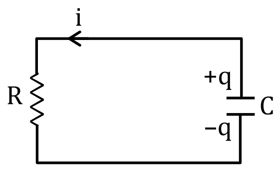
(v) The potential difference across the condenser plates at any instant of time t is given by $V = V_0e^{-(t/RC)}$
Where V_0 = maximum/initial value of potential difference across the condenser plate = value of potential difference across the condenser plate at t = 0

(vi) The transient current at any instant of time is given by $I = I_0e^{-(t/RC)}$
Where I_0 = maximum/initial value of current in the circuit = value of current in the circuit at t = 0

Derivation of equation of discharging circuit



At time t = 0



At any time t

Applying Kirchhoff's voltage law at any time t

$$+\frac{q}{C} - iR = 0$$

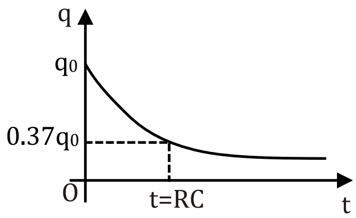
$\Rightarrow \quad i = \frac{q}{RC}$

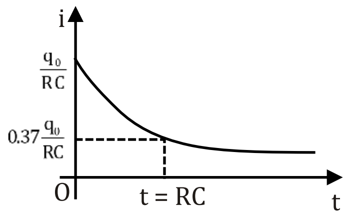
$\Rightarrow \quad -\frac{dq}{dt} = \frac{q}{RC} \quad (\because \text{ current is decreasing } \therefore\ i = -\frac{dq}{dt})$

$\Rightarrow \quad \int_{q_0}^{q} \frac{-dq}{q} = \int_0^t \frac{dt}{RC}$

$\Rightarrow \quad -\ln\frac{q}{q_0} = +\frac{t}{RC}$

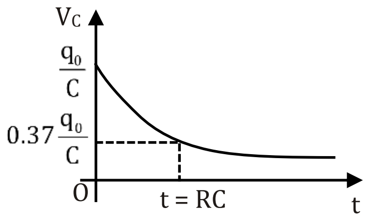
$\Rightarrow \quad \boxed{q = q_0e^{-t/RC}}$





Current at any time, $i = -\frac{dq}{dt}$ $\boxed{i = \frac{q_0}{RC}e^{-t/RC}}$

Potential difference across the condenser at any time, $\boxed{V_C = \frac{q}{C} = \frac{q_0}{C}e^{-t/RC}}$



Potential difference across the resistor at any time, $\boxed{V_R = iR = \frac{q_0}{C}e^{-t/RC}}$

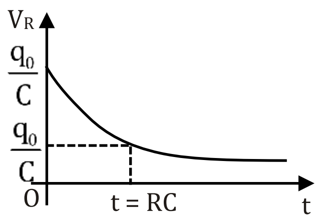


Illustration 32:

A capacitor of capacitance 1μF is connected in a closed series circuit with a resistance of 10^7 ohms, an open key and a cell of 2 V with negligible internal resistance:

(i) When the key is switched on at time t = 0, find;

(a) The time constant for the circuit.

(b) The charge on the capacitor at steady state.

(ii) If after completely charging the capacitor, the cell is shorted by zero resistance at time t = 0, find the charge on the capacitor at t = 50 s. (Given: $e^{-5} = 6.73 \times 10^{-3}$, ln2 = 0.693)

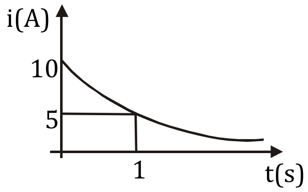
Solution:

(i) (a) Time constant $\tau = RC = 10^7 \times 1 \times 10^{-6} = 10$ sec. (b) $q_0 = CV = 1 \times 10^{-6} \times 2 = 2\ \mu C$

(ii) $q = q_0 e^{-t/RC} = 2 \times 10^{-6}\ e^{-50/10} = 2 \times 10^{-6}\ e^{-5} = 1.348 \times 10^{-8}$ C.

BEGINNER'S BOX-4

1. Figure shows, the graph of the current in a discharging circuit of a capacitor through a resistor of resistance 10 Ω :



(i) Find the initial potential difference across the capacitor.

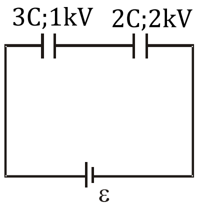
(ii) Find the capacitance of the capacitor.

(iii) Find the total heat produced in the circuit.

(iv) Find the time constant of the circuit.

PCP162

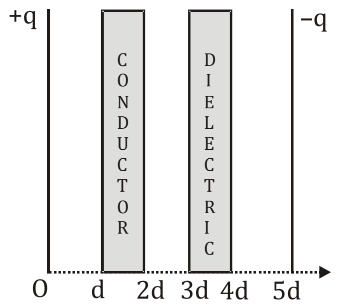
2. The diagram shows two capacitors with capacitance and breakdown voltages as mentioned. What should be the maximum value of the external emf source such that no capacitor undergoes breaks down?



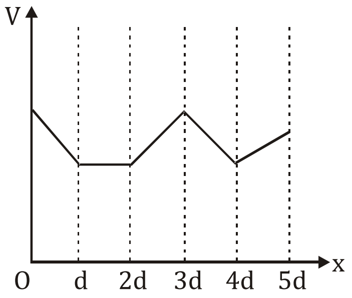
(A) 2.5 kV (B) 10/3 kV (C) 3 kV (D) 1 kV

PCP163

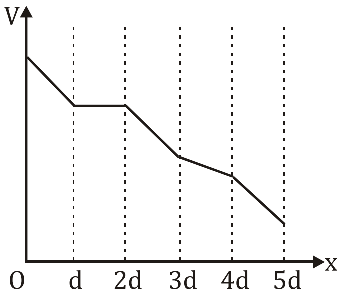
3. The distance between plates of a parallel plate capacitor is 5d. The positively charged plate is at x = 0 and negatively charged plates is at x = 5d. Two slabs – one of conductor and the other of a dielectric, both of same thickness d are inserted between the plates as shown in figure. Potential (V) versus distance x graph will be



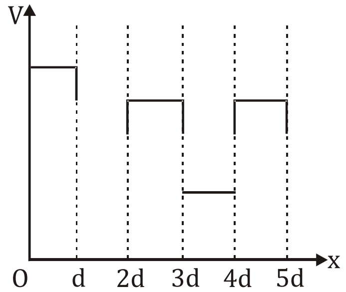
(A)



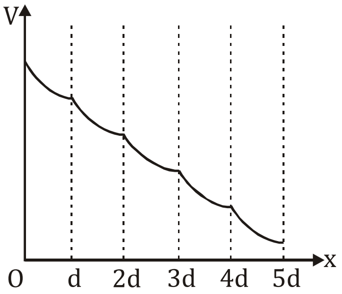
(B)



(C)



(D)

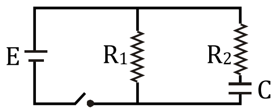


PCP164

4. A 400 μF condenser is charged at the steady rate of 100 μC per second. Calculate the time required to establish a potential difference of 100 volts between its plates.

PCP165

5. For the circuit shown in figure, find



(a) the initial currents through each resistor after the switch is closed.

(b) steady state currents through each resistor after the switch is closed.

(c) final energy stored in the capacitor after the switch is closed.

(d) time constant of the circuit when switch is opened.

(e) time constant of the circuit when switch is closed.

PCP166

6. A condenser of capacitance 2 μF has been charged to 200 V. It is now discharged through a resistance; the heat produced in the wire is ....

PCP167

BEGINNER'S BOX — ANSWER KEY

BEGINNER'S BOX-1

1.	Zero	**2.**	Zero, $\frac{1}{2}\frac{Q^2}{C}$	**3.**	4W	**4.**	2 times
5.	3 m	**6.**	3.2×10^{-4} F	**7.**	$C_A > C_B$	**8.**	0.09 C, 135 J
9.	$\sqrt{\frac{C_2}{C_1}}$						

BEGINNER'S BOX-2

1.	$\frac{\epsilon_0 AV^2}{2d}$	**2.**	A	**3.**	3.5	**4.**	4.3×10^{-9} J
5.	B	**6.**	$5C_0$, $W_0/5$	**7.**	10^{-3} m^2	**8.**	200 μC; 800 μC
9.	400 μC	**10.**	$\frac{1}{2}\varepsilon_0\frac{V^2}{d^2}$	**11.**	$\sigma_{Big} = 4\sigma_{Small}$		

BEGINNER'S BOX-3

1.	20 μF ; 2×10^{-3} C	**2.**	(a) 3 pF; (b) 40 V
3.	640 μC ; 960 μC	**4.**	$\frac{2\varepsilon_0 A}{d}\left(\frac{K_1K_2}{K_1+K_2}\right)$
5.	$\frac{-\varepsilon_0 AV}{d}, \frac{2\varepsilon_0 AV}{d}$	**6.**	$\frac{3\varepsilon_0 A}{d}$

7. (a) $\frac{1}{2}CV^2$ (b) Zero; (c) Zero & Zero; (d) Zero & Zero; (e) CV^2

BEGINNER'S BOX-4

1. (i) 100 volts ; (ii) $\frac{1}{10\ell n2}$ F; (iii) $\frac{500}{\ell n2}$ joules; (iv) $\frac{1}{\ell n2}$ seconds

2.	(A)	**3.**	(B)	**4.**	400 s

5. (a) $i_1 = \frac{E}{R_1}$ and $i_2 = \frac{E}{R_2}$; (b) $i_1 = \frac{E}{R_1}$, $i_2 = 0$; (c) $U = \frac{1}{2}CE^2$; (d) $C(R_1+R_2)$; (e) $\tau = R_2C$

6. 0.04 J