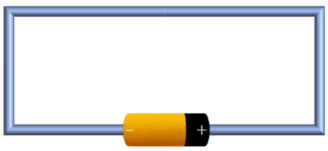
03 Current Electricity

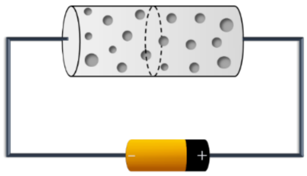
In previous chapters we have dealt largely with electrostatics that is, charges at rest. In this chapter we will focus on electric currents, that is, charges in motion.

Introduction of current

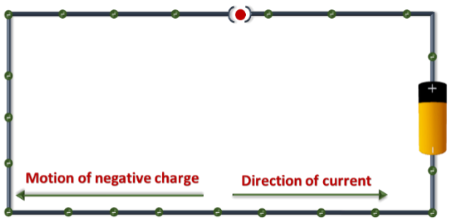
Current

- Rate of flow of charge through cross-section of a conductor is called electric current.

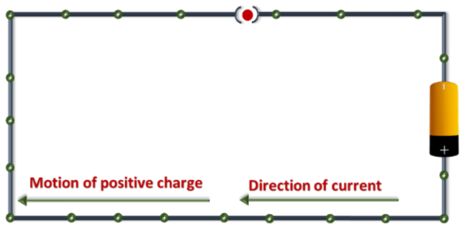




- If moving charges are negative, the current is opposite to the direction of motion of negative charges.

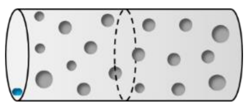


- If the moving charges are positive, the current is in the direction of motion of positive charge.



- If a charge ΔQ crosses an area in time Δt then the average electric current through the area, during this time is

$$i_{av} = \frac{\Delta Q}{\Delta t} = \frac{\text{Average charge flow}}{\text{time taken}} = \frac{ne}{t} = \frac{C}{s} = \text{Ampere}$$

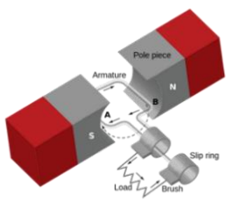


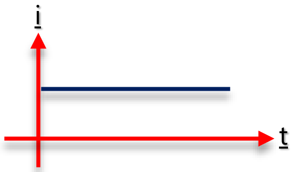
Classification of Current

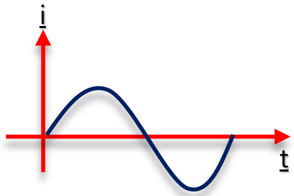
On the basis of source

Direct current
Source: (Battery)

Alternating current
Source:(generator)







In this chapter we will deal with direct current.

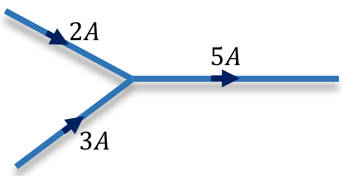
On the basis of nature

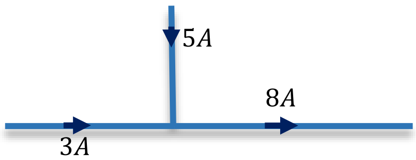
Average electric current	Instantaneous current
If Δq charge flows for Δt time interval.	The value of current at particular instant
$i_{avg} = \dfrac{\Delta q}{\Delta t}$	$i = \lim\limits_{\Delta t \to 0}\left(\dfrac{\Delta q}{\Delta t}\right) = \dfrac{dq}{dt}$ $dq = idt$ or $q = \int idt$

Some important points about current

(1) Current is a fundamental quantity with dimensions $[M^0L^0T^0A^1]$

(2) Current is a scalar quantity with its SI unit being ampere and it does not follow vector law of addition.





(3) Direction of current is in the direction of flow of positive charge or we can say opposite to the direction of flow of negative charge.

(4) Slope of q – t graph gives us current.

(5) Area under i-t curve gives us total charge flown.

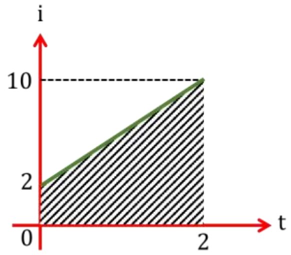
Ampere : The current through any conductor is said to be one ampere if one coulomb of charge is flowing per second through any cross-section of the wire.

Illustration 1:

Current I = (2 + 4 t) A where t is in seconds, is flowing through a wire for 2 seconds. Find out amount of charge flown through the wire.

Solution:

$q = \int_0^t idt = \int_0^2 (2+4t)dt = 12C$



Alternate method

At t = 0, I = 2A and at t = 2, I = 10A

Area under i-t curve = total charge $= \frac{1}{2}(2+10) \times 2 = 12C$

Illustration 2:

20 A current flows through a cross-section of 20 cm² for 20 msec. Find out number of free e^- crossed through that area?

Solution:

$$i = \frac{Q}{t} = \frac{ne}{t}$$

$$n = \frac{i \times t}{e} = \frac{20 \times 20 \times 10^{-3}}{1.6 \times 10^{-19}} = 2.5 \times 10^{18} \text{ electrons.}$$

Illustration 3:

In a discharge tube 3×10^{18} electrons/sec are moving from right to left and 2×10^{18} (+ve) ions per second are moving from left to right. Then find current through the discharge tube.

Solution:

As current due to both type of charges will be in same direction

$$i = \frac{(N_e q_e) + (N_\oplus q_\oplus)}{t}$$

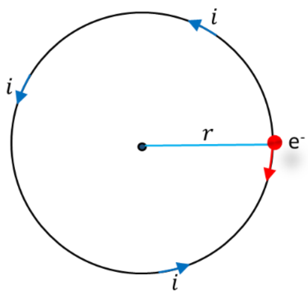
$$i = \frac{3 \times 10^{18}(e) + 2 \times 10^{18} \times (e)}{1}$$

$$i = 3 \times 10^{18} e + 2 \times 10^{18} e = 5 \times 10^{18} \times e = 5 \times 10^{18} \times 1.6 \times 10^{-19} = 0.8\text{A}$$

Illustration 4:

An e^- revolving on a circular track of radius 1.6 cm with a constant speed of 3.14×10^6 m/s. Find out the current flowing in the circle.

Solution:



$$T = \text{time period} = \frac{2\pi r}{v}$$

$$i = \frac{e}{T} = \frac{e}{\frac{2\pi r}{V}} = e\frac{V}{2\pi r}$$

$$i = 1.6 \times 10^{-19} \frac{(3.14 \times 10^6)}{2 \times 3.14 \times 1.6 \times 10^{-2}} = 5 \times 10^{-12}\text{A}$$

Flow of Electric charge in a Metallic Conductor

Classification of Materials according to Conductivity

(i) **Conductors :** In some materials, the outer electrons of each atoms or molecules are loosely bound to it. These electrons are almost free to move throughout the body of the material and are called free electrons. They are also known as conduction electrons. When such a material is placed in an electric field, the free electrons move in a direction opposite to the field. Such materials are called conductors.

(ii) **Insulators :** Materials of another class are called insulators in which all the electrons are tightly bound to their respective atoms or molecules. Effectively, there are no free electrons. When such a material is placed in an electric field, the electrons may slightly shift opposite to the field but they cannot leave their parent atoms or molecules and hence cannot move through long distances. Such materials are also called dielectrics.

(iii) **Semiconductors :** Semiconductors behave like insulators at low temperatures. But at higher temperatures, a small number of electrons are able to free themselves and they respond to the applied electric field. As the number of free electrons in a semiconductor is much smaller than that in a conductor, its response is intermediator to a conductor and an insulator and hence, it is named as semiconductor. A free electron in a semiconductor leaves a vacancy in its normal bound position. These vacancies also help in conduction and are called holes.

Assumptions about conductor

It is to be noted that we have to assume a conductor always of cylinder(solid) shape, and every conductor is associated with numerous free e^-, so we have to assume a term free e^- density for every conductor.

radius = r & cross-sectional Area = πr^2

$$n = \frac{\text{number of free } e^-}{\text{volume of conductor}} = \frac{\text{number}}{m^3}$$

Where 'n' =free e^- density or free charge density

- This free e^- density is independent of shape and size of conductor and it depends on nature of material.
- Free e^- density in conductor is 10^{28} to 10^{29} free electrons per unit volume

Illustration 5:

A conducting wire having material density 20g/cc and atomic mass number 200. Calculate the number of free e^- per unit volume (each atom of conductor gives $1e^-$).

Solution:

Let, V = volume, m = mass, ρ = density

$n = \frac{m}{M}$ n = no. of moles, M = molar mass

Number of atoms = nN_A

$$\text{Number of atoms} = \frac{m}{M} \times N_A = \frac{m}{200 \times 10^{-3}} \times N_A$$

$\therefore$ 200 g material having number of atom = N_A

As each atom gives one electron $\frac{\text{free } e^-}{\text{volume}} = \frac{\text{number of atoms}}{\text{volume}}$

$$\Rightarrow \quad \frac{m \times N_A}{200 \times 10^{-3}} \times \frac{1}{v} = \frac{\rho \times N_A}{200 \times 10^{-3}} \qquad \left[\because \ \rho = \frac{m}{v}\right]$$

$$= \frac{20 \times 10^{-3}}{(10^{-2})^3} \times \frac{6.023 \times 10^{23}}{200 \times 10^{-3}} = 6.023 \times 10^{28}$$

Discussion on conductor at room temperature: (without potential diff.)

At room temperature motion of the free e^- in metal can be compared with random motion of gaseous molecule in a container.

Thermal velocity (V_{th})

It is the velocity of free e^- inside the conductor due to the surrounding temperature (room temperature).

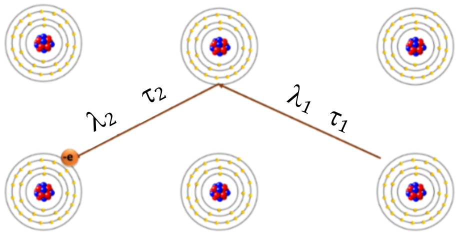
$$V_{th} = \sqrt{\frac{3KT}{M_e}} = \sqrt{\frac{3 \times 1.38 \times 10^{-23} \times 300}{9.1 \times 10^{-31}}} \approx 10^5 \, m/s \text{ or } 100 \frac{km}{s} \quad [K = \text{Boltzmann's constant}, M_e = \text{Mass of } e^-]$$

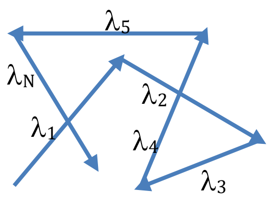
Due to random motion net velocity of all the free e^- is zero, so no current flows through the conductor at room temperature. $\vec{V}_{th} = \vec{V}_1 + \vec{V}_2 + \vec{V}_3 + \cdots = \vec{0}$.

Current Density, Drift Velocity and Relaxation Time

Mean Free Path (λ)

The distance travelled by free electron between two successive collision is called free path. The average of all these free paths is called mean free path.





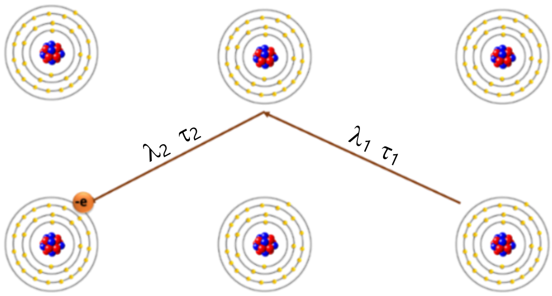
$$\lambda = \frac{\lambda_1 + \lambda_2 ..... + \lambda_N}{N}$$

Order of λ = 10Å

The average time taken by an electron between two successive collisions is the relaxation time (τ).

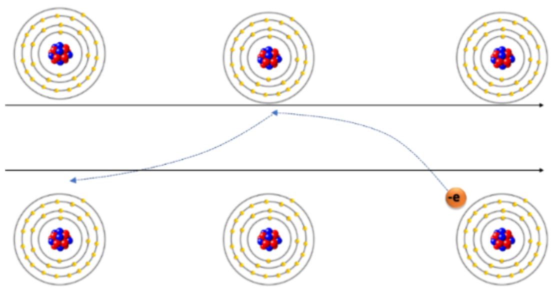
Behavior of conductor on applying potential difference across it at room temperature

➢ Before application of electric field between ends of conductor



Path is straight line between two successive collisions

➢ After application of electric field between ends of conductor



Path is curved between two successive collisions

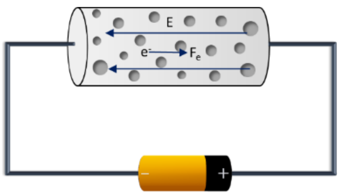
Relaxation time and their relation with electric current

Mean Relaxation Time (τ)

The time taken by free electrons between two successive collision is called relaxation time (t) and average of all these times is called mean relaxation time.

$$\tau = \frac{\tau_1 + \tau_2 + \tau_3 + \cdots ... \tau_N}{N}$$ (Order of $\tau = 10^{-14}$ sec.)

Drift Velocity and Mobility



- Due to application of potential difference/Voltage across conductor an electric field is produced inside the conductor.
- Due to electric field, free electron experience force opposite to the direction electric field.
- Due to this force these electrons start drifting in the opposite direction of electric field and during motion they face many collision with heavy positive ions in their path.
- So the velocity with which e^- effectively displaced is called **drift velocity.**

Let us imagine at any time (t = 0), any electron moving with initial velocity V_{th}, now due to external electric field (E) it experiences force opposite to direction of electric field (E)

$$\vec{F}_e = e\vec{E} \qquad ...(1)$$

$$\vec{a}_e = \frac{e\vec{E}}{m} \qquad ...(2)$$

Now after time t_1 (say) it will be having velocity V_1

$$\vec{V}_1 = \vec{V}_{th} + \frac{e\vec{E}}{m}t_1$$

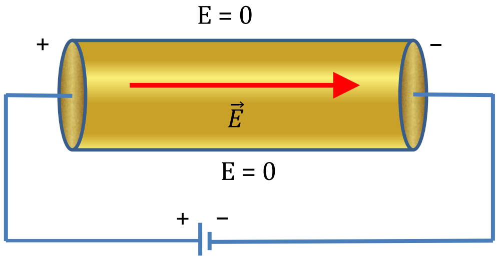
Averaging over N electrons, $<\vec{V}> \;=\; <\vec{V}_{th}> + \left\langle \frac{e\vec{E}}{m}t_1 \right\rangle$

$$\vec{V}_d = 0 + \frac{e\vec{E}\tau}{m} \quad \text{or} \quad \vec{V}_d = \frac{e\vec{E}\tau}{m}$$

Note: This drift velocity is very low of order 10^{-4} m/s but electrical appliances immediately starts working just after switch is on because electric field propagates with the speed of light in circuit and instantly exert the fore on free electron at their own location.

Important points about current carrying conductor

- All free e^- moves with V_d (drift velocity).
- There is no accumulation of charge in current carrying conductor. This implies charges entering at one end of conductor is equal to the charges leaving from the other end.
- Current carrying conductor is electrically neutral.



Relation between drift velocity and current

Radius r and area of Cross section A

Let 'n' represents number of free electron per unit volume or free electron density.

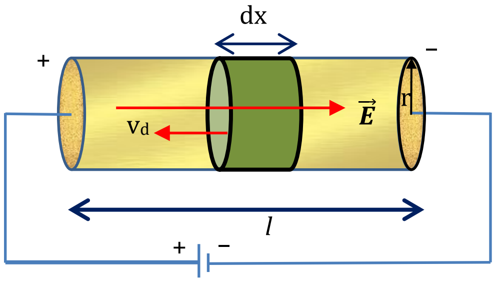
$V_d = \frac{dx}{dt}$ ($n = \frac{N}{V}$ = free electrons density)

$dV = Adx$

$dN = ndV = nAdx$

$dq = edN = enAdx$

$i = \frac{dq}{dt} = \frac{enAdx}{dt} = enA\frac{dx}{dt}$ or $I = neAV_d$



Golden Key Points

- If n particles, each having a charge q, pass per second per unit area then current associated with cross–sectional area A is $I = \frac{\Delta q}{\Delta t} = nqA$.
- If there are n particles per unit volume each having a charge q and moving with velocity v then current through cross–sectional area A is $I = \frac{\Delta q}{\Delta t} = nqvA$
- If a charge q is moving in a circle of radius r with speed v then its time period is $T = 2\pi r/v$. The equivalent current $I = \frac{q}{T} = \frac{qv}{2\pi r}$.

Different formulas of current

(1) If 1 atom releases only 1 e^-

No. of atom (N) = No. of free e^-

No. of atoms (N) = no. of moles × $N_A = \frac{m}{M_W} \times N_A$

$N = \frac{m}{M_W} \times N_A$

Free electron density (n)

$n = \frac{N}{v} = \frac{m}{M_W}\frac{N_A}{m}\rho = \frac{\rho N_A}{M_W}$

Where ρ = density of material, M_w = Molecular weight

$i = nAeV_d$

$i = \frac{\rho N_A}{M_W} eAV_d$

(2) $i = neAV_d = \frac{N}{V}eAV_d = \frac{N}{A \times \ell}eAV_d$

$i = \frac{NeV_d}{\ell} \times \frac{m_e}{m_e} = (N \times m_e V_d)\frac{e}{m_e \ell}$

$(N \times m_e V_d)$ = Net linear momentum of all free electrons

$i = \frac{p_e e}{m_e \ell}$ where me = mass of e– ; ℓ = length of wire

Current density (J)

- It is defined as current flowing per unit cross-sectional area of conductor.
- It gives us information about magnitude & direction of current passing through an area.
- Current density is a vector quantity having direction of electric field or direction of current flow.

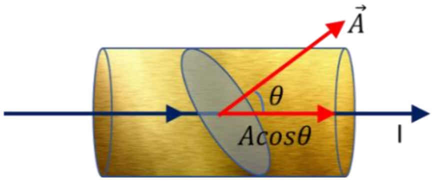
  from definition of current density (J) $= \frac{I}{A}$

- Remember that surface area is normal to the direction of charge flow (or current passing) through that point.

  Current density (J)= $\frac{I}{A\cos\theta}$

  I = (J)Acosθ

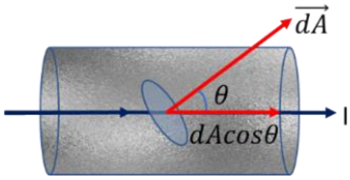
  $I = \vec{J}.\vec{A}$



  θ is angle between area vector and direction of current flow.

  For non-uniform distribution of current

  If dI is current flowing through small elementary area then



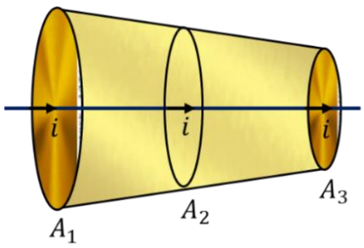
  current density (J) $= \frac{dI}{dA\cos\theta}$

  or $dI = J\,(dA\cos\theta)$ or $I = \int J\,(dA\cos\theta)$

Illustration 6:

For conductor of non-uniform cross-sectional area, compare current density at section 1, 2 and 3.

Solution:



Current through each area of cross section is same.

$J_1 = \frac{I}{A_1}$; $J_2 = \frac{I}{A_2}$; $J_3 = \frac{I}{A_3}$

$A_3 < A_2 < A_1 \Rightarrow J_1 < J_2 < J_3$

Relation between current density & electric field

From $J = \frac{i}{A} = \frac{neAV_d}{A} = neV_d$

$J = neV_d$

Also $V_d = \frac{eE\tau}{m}$

$$J = ne\frac{eE\tau}{m} = \frac{ne^2E\tau}{m} = \frac{ne^2E\tau}{m} = \left(\frac{ne^2\tau}{m}\right)E \quad \text{or} \quad J = \sigma E$$

($\sigma = \frac{ne^2\tau}{m}$ is conductivity of conductor)

- σ is constant for any conductor, is independent of shape, size, volume of conductor.
- In vector form $\vec{J} = \sigma\vec{E}$, this relation is known as microscopic Ohm's law.
- Conductivity is degree to which a material conducts electricity.

  Also $\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$ (ρ is resistivity of a material)

  σ & ρ are independent of conductors' shape and size but depends on nature of material and temperature.

Illustration 7:

A current I flows through a uniform wire of diameter d then the electrons drift velocity is v. The same current will flow through a wire of diameter d/2 made of the same material, the drift velocity of the electrons in second wire is

Solution:

$$j=\frac{I}{A}=nev_d$$

$\frac{4I}{\pi d^2}=nev_d$ ...(i) $\frac{16I}{\pi d^2}\, nev_{d_2}$ ... (ii)

From equation (i) and (ii)

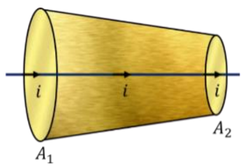
$$\frac{4I}{16I}=\frac{v_{d_1}}{v_{d_2}} \Rightarrow v_{d_2} = 4v_{d_1}$$

Illustration 8:

Figure shown above is representing a current carrying wire with having non-uniform cross sectional area find out the relation between physical quantities on their two ends.

Solution:

(i) Electric current is the rate flow of electric charge and that is completely independent from the cross section of given figure. Hence $i_1 = i_2 = i$



(ii) $i = ne\,Av_d$, $v_d = \frac{i}{neA}$, $\boxed{v_d \propto \frac{1}{A}}$

$A_1 > A_2$ $\boxed{v_{d_1} < v_{d_2}}$

(iii) $J = nev_d$, $J \propto v_d$ $\boxed{J_1 < J_2}$

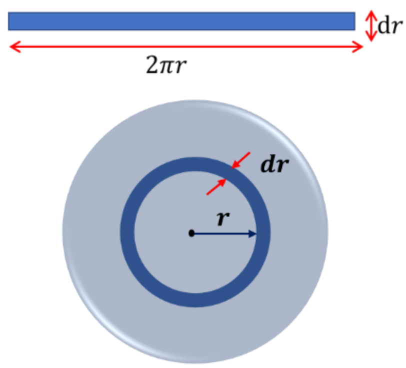
(iv) $J = \sigma E$, $J \propto E$ $\boxed{E_1 < E_2}$

Illustration 9:

The current density across a cylindrical conductor of radius R varies in magnitude according to the equation $J = J_0\left(1-\frac{r}{R}\right)$ where r is the distance from the central axis. Thus, the current density is maximum J_0 at the axis (r = 0) and decreases linearly to zero at the surface (r = R). The current in terms of J_0 and conductor's cross-sectional area A is.

Solution:

Here we consider elementary ring of radius **r** and thickness **dr**



Area of elementary ring

$$dA = (2\pi r)dr$$

From $I = \int \vec{J}.\overrightarrow{dA} = \int JdA\cos\theta$

$$I = \int_0^R J_0\left(1-\frac{r}{R}\right)(2\pi r dr)$$

$$I = \int_0^R J_0\left(1-\frac{r}{R}\right)(2\pi r dr)$$

$$I = 2\pi J_0\int_0^R\left(r-\frac{r^2}{R}\right)dr = 2\pi J_0\left[\frac{r^2}{2}-\frac{r^3}{3R}\right]_0^R = 2\pi J_0\left[\frac{R^2}{2}-\frac{R^3}{3R}\right] = 2\pi J_0\left[\frac{R^2}{2}-\frac{R^2}{3}\right]$$

$$I = 2\pi J_0\left[\frac{R^2}{6}\right] I = 2\pi J_0\left[\frac{R^2}{6}\right]$$

As $A = \pi R^2$

$$I = \frac{J_0 A}{3}$$

Mobility (μ)

Ease of movement of charge carriers within a conductor known as mobility of charge carriers.

- It is defined as drift velocity per unit electric field

$$\mu = \frac{V_d}{E} = \frac{1}{E}\frac{eE\tau}{m} \Rightarrow m = \frac{e\tau}{m}$$

- μ is constant for a given material (If temperature is constant)
- Electrons have more mobility than proton $\left(\mu \propto \frac{1}{\text{mass}}\right)$

Relation between relaxation time and current

$I = neAv_d$

$$V_d = \frac{eE\tau}{m}$$

$$I = neA\frac{eE\tau}{m} \Rightarrow I = \frac{ne^2AE\tau}{m}$$

Note: On increasing temperature of conductor, frequency of collision increases and hence average relaxation time decreases.

temperature $\uparrow, \tau \downarrow$

$$V_d = \frac{eE\tau}{m}, \; V_d \downarrow$$

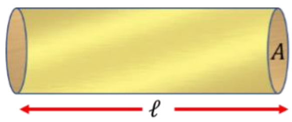
$$\sigma = \frac{ne^2\tau}{m}, \sigma \downarrow, \rho \uparrow$$

$$m = \frac{e\tau}{m}, \mu \downarrow$$

Illustration 10:

The number of free e^- per 10 mm of an ordinary copper wire is 2×10^{21}. The avg. drift speed of e^- is 0.25 mm/sec. Calculate the current flow in the wire.

Solution:



N = Total no of free e^-

$i = neAV_d$

$$\Rightarrow \quad i = \left(\frac{N}{A\ell}\right)eAV_d = \left(\frac{N}{\ell}\right)eV_d$$

$$\Rightarrow \quad i = \left(\frac{2 \times 10^{21}}{10}\right) \times 1.6 \times 10^{-19}(0.25) = 8A$$

Illustration 11:

Two wires each of radius of cross-section r but of different materials are connected together end to end (in series). If the densities of charge carriers in the two wires are in the ratio 1 : 4, the drift velocity of electrons in the two wires will be in the ratio :

Solution:

$I = neAv_d$

$$v_d \propto \frac{1}{n} \Rightarrow \frac{v_{d_1}}{v_{d_2}} = \frac{n_2}{n_1} = \frac{4n_1}{n_1} = 4:1$$

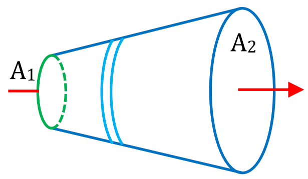
Golden Key Points

Terms	Thermal speed V_T	Mean free path λ	Relaxation time τ	Drift speed V_d
Order of magnitude	10^5 m/s	10 Å	10^{-14} S	10^{-4} m/s

- If the temperature of a conductor increases, the amplitude of vibrations of positive ions in the conductor also increase. Consequently, the free electrons collide more frequently with the vibrating ions and as a result, the average relaxation time decreases.
- Mean free path of conduction electrons = Drift velocity × Relaxation time.
- The conductor remains uncharged when current flows through it because the charge entering at one end per second is equal to charge leaving the other end per second.
- Order of free e^- density in conductors = 10^{28} electrons/m^3, while in semiconductors = 10^{16} electrons/m^3
- Electric field outside a current carrying conductor is zero but inside a conductor it is $\frac{V}{\ell}$.

System	Current carriers
A bar made of silver or any metal	Free electrons
Hydrogen discharge tube	Electrons and (+ve) Ions
Voltaic cell	H^+ and SO_4^{-12} (Ions)
Lead Accumulator	H^+, SO_4^{-2} Ions
Semiconductor	Electrons and holes
Super conductor	Electrons

- For a given conductor current does not change with change in its cross–section because current is simply the rate of flow of charge. If a steady current flows in a metallic conductor of non-uniform cross-section :



(i) I is same along the wire.

(ii) Current density, electric field strength, drift velocity are inversely proportional to area.

Here $I_1 = I_2$ but $A_1 < A_2$ so $J_1 > J_2$, $E_1 > E_2$, $v_{d_1} > v_{d_2}$

BEGINNER'S BOX-1

1. 10,000 alpha particles pass per minute through a straight tube of radius r. What is the resulting electric current? **PCE235**
2. The potential difference applied to an X-ray tube is 5 kV and the current through it is 3.2 mA. What is the number of electrons striking the target per second ? **PCE236**
3. The diameter of a copper wire is 2 mm, a steady current of 6.25 A is generated by 8.5×10^{28}/m^3 electrons flowing through it. Calculate the drift velocity of conduction electrons. **PCE237**
4. A silver wire of 1mm diameter has a charge of 90 coulombs flowing in 1 hours and 15 minutes. Silver contains 5.8×10^{28} free electrons per cm^3. Find the current in the wire and drift velocity of the electrons. **PCE238**

Ohm's Law

If physical quantities like temperature, pressure, volume, length, cross-section or nature of the material kept constant then current through a conductor is directly proportional to potential difference applied across it. This is called Ohm's law.

$V \propto I$ [Ohm's law]

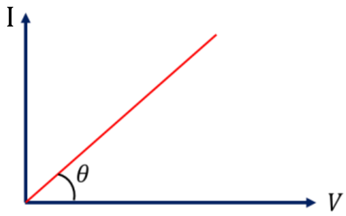
$\frac{V}{I}$ = constant = R (Resistance)

$I \propto V$

$I = GV$

G is constant called conductance of conductor

S.I. unit of G $\Rightarrow$ mho (Ω^{-1})/Siemens



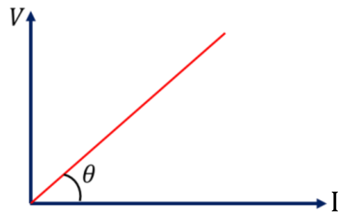
Slope = $\frac{I}{V}$ = tan θ = G

$V \propto I$

$V = IR$

R is called resistance of conductor

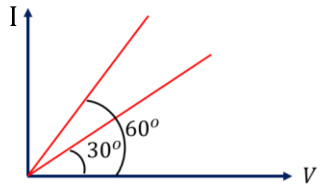
S.I. unit of R $\Rightarrow$ ohm(Ω)



Slope = $\frac{V}{I}$ = tan θ = R

Illustration 12:

In the shown I-V curve $\frac{R_1}{R_2}$ will be?



Solution:

Slope of given graph gives conductance (G)

$\tan 60° = G_1 = \sqrt{3} = \frac{1}{R_1}, R_1 = \frac{1}{\sqrt{3}}$

$\tan 30° = G_2 = \frac{1}{\sqrt{3}} = \frac{1}{R_2}, R_2 = \sqrt{3}$

$\frac{R_1}{R_2} = \frac{1/\sqrt{3}}{\sqrt{3}} = \frac{1}{3}$

Proof of Ohm's Law

From microscopic form of Ohm's law

$J = \sigma E$

$\frac{I}{A} = \sigma\left(\frac{V}{\ell}\right)$

$\frac{V}{I} = \left(\frac{1}{\sigma}\right) \times \frac{\ell}{A} = \rho \times \frac{\ell}{A}$ $\left[\text{where } \sigma = \frac{1}{\sigma} = \frac{m}{ne^2\tau}\right]$

$\frac{V}{I} = R$ Ohm's law, where $R = \frac{\rho\ell}{A}$ = Resistance [S.I. unit of ρ is ohm-m(Wm)]

Where ℓ is length of the conductor & A is area of cross-sectional, ρ is called resistivity or specific resistance.

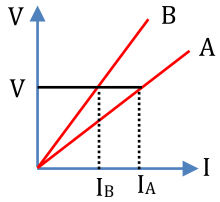
Illustration 13:

The voltage-current graphs for two resistors of the same material and the same radius with lengths, L_1 and L_2 are shown in figure. If $L_1 > L_2$, state with reason which of these graphs represents voltage current change for L_1.

Solution:

As $R_A = V/I_A$ and $R_B = V/I_B$,

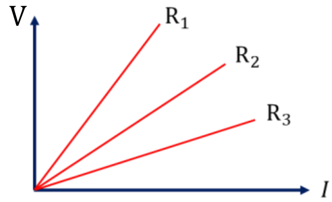
$$\frac{I_A}{I_B} = \frac{R_B}{R_A}$$



From figure, as $I_A > I_B$, $R_B > R_A$, Since, $R \propto L$, graph B represents the voltage current change for L_1.

Illustration 14:

In the shown V-I curve will be?



(1) $R_3 > R_2 > R_1$

(2) $R_1 > R_2 > R_3$

(3) $R_2 > R_3 > R_1$

(4) $R_1 > R_3 > R_2$

Solution: (2)

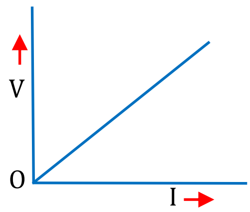
Slope of given graph gives resistance (R)

$R_1 > R_2 > R_3$

V – I Characteristics (Linear and Non-linear)

(A) Linear V-I characteristics

At constant temperature, current is directly proportional to the applied potential difference. This law is called ohm's law and substances which obey it are called ohmic or linear conductors.



(B) Non- Linear V-I characteristics

The relation between V and I depends on the sign of V. In other words, if I is the current for a certain V, then reversing the direction of V keeping its magnitude fixed, does not produce a current of the same magnitude as I in the opposite direction (Figure 1).

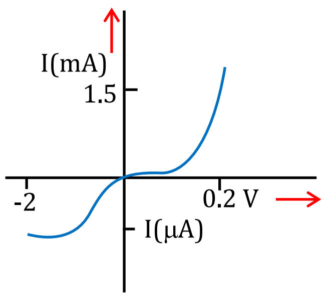


Figure 1 : Characteristic curve of a diode, Note the different scales for negative and positive values of the voltage and current

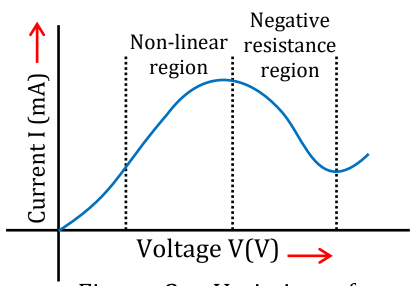
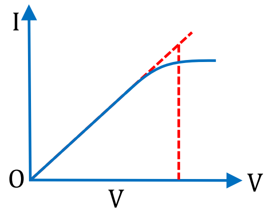


Figure 2 : Variation of current versus voltage for GaAs

The relation between V and I is not unique, i.e., there is more than one value of V for the same current I (Figure 2). A material exhibiting such a behaviour is GaAs.

Illustration 15:

The I-V characteristics of a resistor is observed to deviate from a straight line for higher value of current as shown in figure Why?

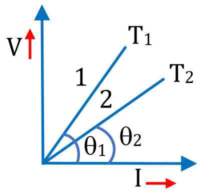


Solution:

For higher value of current, the resistor gets heated and consequently its resistance increases. The resistor becomes non-ohmic due to which the I-V characteristic deviates from straight line thereby showing lesser current for the same voltage.

Golden Key Points

- Ohm's law is not a fundamental law of nature. As it is possible that for an element :-
  (i) V depends on I nonlinearly (e.g. vacuum tubes)
  (ii) Relation between V and I depends on the sign of V for the same value [forward and reverse bias in diode]
  (ii) The relation between V and I is not unique. That is for the same value of I there is more than one value of V.
- Thus, Ohm's law is not followed in the following cases :
  Materials :
  (i) Vacuum tubes; (ii) crystal rectifiers; (iii) transistors;
  (iv) thermistors, thyristors; (v) superconductors,
  Conditions:
  (i) At very high temperatures (ii) At very low temperatures (Superconductivity)
  (iii) At very high potential differences
- At different temperatures V–I curves are different.
  Here $\tan\theta_1 > \tan\theta_2$ So $R_1 > R_2$ i.e. $T_1 > T_2$



Electrical Resistance, Resistivity and Conductivity

Electrical Resistance

The resistance of a conductor is the opposition to the flow of charge which the conductor offers. When a potential difference is applied across a conductor, free electrons get accelerated and collide with positive ions and their motion is thus opposed. This opposition offered by the ions is called the resistance of the conductor.

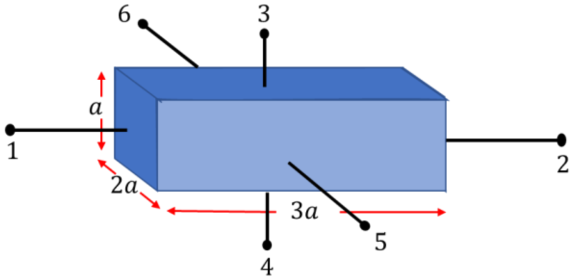
Unit : ohm, volt/ampere,

Dimensions = $[M L^2 T^{-3} A^{-2}]$

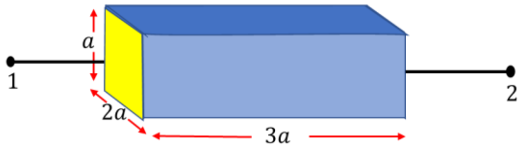
Resistance depends on :

- Length of the conductor $R \propto \ell$
- Area of cross-section of the conductor $R \propto \frac{1}{A}$
- Nature of material of the conductor $R = \frac{\rho\ell}{A}$
- Temperature
  Resistance of conductor also depends on direction of flow of current.
  $R = \frac{\rho\ell}{A}$
  Where ℓ is the length of conductor along which current is flowing.
  And A is area of cross-sectional of conductor perpendicular to the direction of current flow.

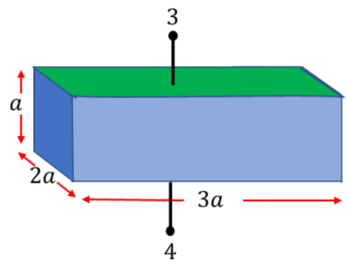
Illustration 16:
Find R_{12}, R_{34}, R_{56} on the basis direction of current flow through different paths.



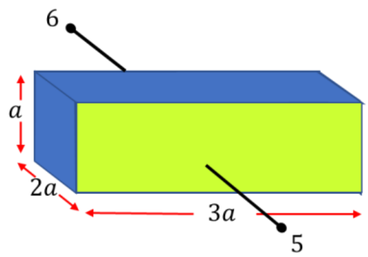
Solution:



$$R_{12} = \frac{\rho(3a)}{2a^2} = \frac{3\rho}{2a}$$



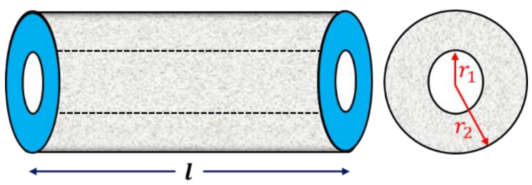
$$R_{34} = \frac{\rho(a)}{6a^2} = \frac{\rho}{6a}$$



$$R_{56} = \frac{\rho(2a)}{3a^2} = \frac{2\rho}{3a}$$

$R_{12} > R_{56} > R_{34}$

Illustration 17:
Find out the resistance of the current flowing through hollow cylindrical conductor.



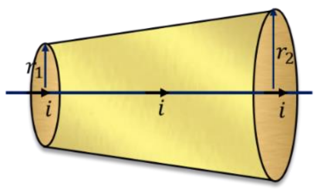
Solution:
Electric current is flowing along the length of conductor through area of cross section

$$A = \pi(r_2^2 - r_1^2)$$

If ℓ is length of conductor along current flow

$$\text{From} \quad R = \frac{\rho l}{A} \Rightarrow \quad R = \frac{\rho l}{\pi(r_2^2 - r_1^2)}$$

Note: Similarly we need to remember some standard value of resistances of standard conductors.



$$R = \frac{\rho l}{\pi r_1 r_2}$$

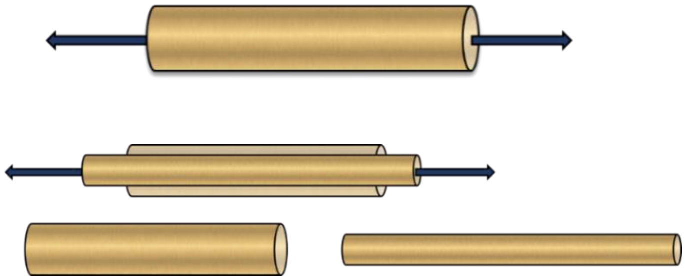
EFFECT OF STRETCHING ON RESISTANCE OF WIRE

Before stretching $V_1 = A_1 l_1$ After stretching $V_2 = A_2 l_2$

Volume of wire remains same in stretching

$A_1 l_1 = A_2 l_2$

Initial resistance of wire : $R_1 = \frac{\rho \ell_1}{A_1}$



Final resistance of wire : $R_2 = \frac{\rho \ell_2}{A_2}$

$$\frac{R_1}{R_2} = \frac{\rho \ell_1}{A_1} \times \frac{A_2}{\rho \ell_2} = \frac{\ell_1}{\ell_2} \times \frac{A_2}{A_1} = \frac{\ell_1^2}{\ell_2^2}$$

$R \propto \ell^2$

If $\ell_2 = n\ell_1$

$$\frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{n\ell_1}{l_1}\right)^2 = n^2 R_2 = n^2 R_1$$

Also

$$\frac{R_2}{R_1} = \left(\frac{l_2}{l_1}\right)^2 = \left(\frac{A_1}{A_2}\right)^2 = \left(\frac{r_1}{r_2}\right)^4$$

$$R \propto \frac{1}{A^2} \propto \frac{1}{r^4} \propto \frac{1}{D^4}$$

Illustration 18:

Three copper wires have their lengths in the ratio 5 : 3 : 1 and their masses are in the ratio 1 : 3 : 5. Their electrical resistance will be in the ratio

(1) 5 : 3 : 1 (2) 1 : 3 : 5

(3) 125 : 15 : 1 (4) 1 : 15 : 125

Solution: (3)

$l_1 : l_2 : l_3 :: 5 : 3 : 1$ And $m_1 : m_2 : m_3 :: 1 : 3 : 5$

$$R = \frac{\rho \ell}{A} = \frac{\rho \ell}{V}\ell = \frac{\rho \ell^2}{m}$$

$R \propto \frac{\ell^2}{m}$ [density = Constant]

$$R_1 : R_2 : R_3 = \frac{25}{1} : \frac{9}{3} : \frac{1}{5} = 125 : 15 : 1$$

Note:

(1) On stretching: $\ell\uparrow$, $A\downarrow$, $R\uparrow$

(2) If percentage change in length is less than 5% than percentage fractional change in resistance of wire will be

$$\frac{\Delta R}{R} \times 100\% = 2\left(\frac{\Delta \ell}{\ell}\right) \times 100\% \quad \left(\text{use } \frac{\Delta \ell}{\ell} \text{ with sign}\right)$$

(3) Similarly for area

$$\frac{\Delta R}{R} \times 100\% = -2\left(\frac{\Delta A}{A}\right) \times 100\% \text{ (use } \frac{\Delta A}{A} \text{ with sign)}$$

(4) Similarly for radius

$$\frac{\Delta R}{R} \times 100\% = -4\left(\frac{\Delta r}{r}\right) \times 100\% \text{ (use } \frac{\Delta r}{r} \text{ with sign)}$$

(5) If percentage increase in dimensions is greater than 5% than percentage fractional change in resistance will be

$$\frac{\Delta R}{R} \times 100\% = \frac{R_f - R_i}{R_i} \times 100\%$$

Illustration 19:

A wire is stretched so that its radius is decreased by 3%. Find % change in its resistance.

Solution:

We know that here radius is decreased by 3% then % change in resistance is given by the formula.

$$\frac{\Delta R}{R} \times 100\% = -4\left(\frac{\Delta r}{r}\right) \times 100\% \text{ (use } \frac{\Delta r}{r} \text{ with sign)}$$

$$\frac{\Delta R}{R} \times 100\% = [-4(-3)]\% = +12\%$$

Electric Resistivity and Conductivity

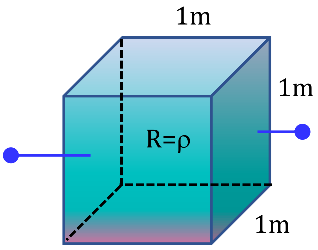
Resistivity :

The resistivity (specific resistance) of a material is equal to the resistance of a wire of that material with unit cross sectional area and unit length.

$\rho = RA/\ell$ if ℓ = 1 units, A = 1 units then ρ = R units

From Ohm's law,

$$R = \frac{V}{I} = \frac{E\ell}{neAV_d} = \frac{E\ell}{neA\left(\frac{eE\tau}{m_e}\right)} = \frac{m_e\ell}{ne^2A\tau}$$



$$R = \left(\frac{m_e}{ne^2\tau}\right) \times \frac{\ell}{A} = \frac{\rho\ell}{A}$$

Where $\rho = \left(\frac{m_e}{ne^2\tau}\right)$, Resistivity

S.I. unit of resistivity is = Ωm.

Conductivity is the inverse of resistivity $\boxed{\sigma = \frac{1}{\rho}}$

Where $\sigma = \frac{ne^2\tau}{m}$ is Conductivity of a material

Conductivity is degree to which a material conducts electricity.

Resistivity (ρ) and conductivity (σ) depends on :

(i) Nature of material

(ii) Temperature of material

Resistivity and conductivity does not depend on the size and shape of the material because it is a characteristic property of the conducting material.

Golden Key Points

- If a wire is stretched to n times of its original length, its new resistance will be n^2 times.
- If a wire is stretched such that its radius is reduced to $\frac{1}{n}$th of its original value, then its new resistance will increase to n^4 times; similarly resistance will decrease to n^4 times if radius is increased to n times by contraction.
- If x % change is brought in length of a wire, its resistance will change by 2x %. (if change is very small)
- $\rho_{alloy} > \rho_{semiconductor} > \rho_{conductor}$
- Temperature coefficients of alloys are lower than pure metals.
- Resistance of most of non-metals decrease with increase in temperature. (e.g. carbon)
  The resistivity of an insulator (e.g. amber) is greater than a metal by a factor of 10^{22}.

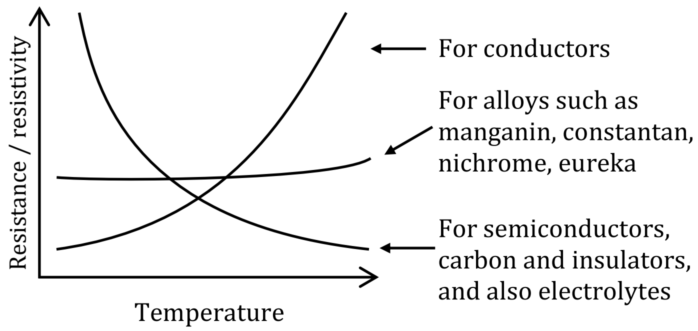
Temperature Dependence of Resistance

Temperature Dependence of Resistivity and Resistance

The resistivity of a material is found to be dependent on the temperature over a limited range of temperatures (not too large). The resistivity of a metallic conductor is approximately given by

$$\rho_T = \rho_0[1 + \alpha(T - T_0)]$$

Where ρ_T is the resistivity at a temperature T and ρ_0 is the resistivity at a reference temperature T_0. Here, α is called the temperature co-efficient of resistivity.



Unit of α is $\frac{1}{°C}$

Resistance corresponding to temperature difference (ΔT) is given as

$R_T = R_0 (1 + \alpha\Delta T)$

Where, R_T = Resistance at T° C,

R_0 = Resistance at 0° C

ΔT = Change in temperature,

α = Temperature coefficient of resistance

For metals : α is positive

For semiconductors and insulators : α is negative

- Resistance of the conductor decreases linearly with decrease in temperature and becomes zero at a specific temperature. This temperature is called critical temperature. Below this temperature a conductor becomes a superconductor.

Illustration 20:

The temperature coefficient of resistance for a wire is 0.00125/°C. At 300K its resistance is 1 ohm. The temperature at which the resistance becomes 2 ohm is

(1) 1154 K (2) 1100 K (3) 1400 K (4) 1127 K

Solution: (4)

$R = R_0 (1+\alpha t)$

$t = 300 - 273 = 27°C$

$$\frac{R_1}{R_2} = \frac{(1+\alpha t_1)}{(1+\alpha t_2)} \Rightarrow \frac{1}{2} = \frac{(1+0.00125 \times 27)}{(1+0.00125 \times t)} \Rightarrow t = 854°C \Rightarrow T = 1127K$$

Golden Key Points

- Temperature coefficient (α) of semiconductor including carbon (graphite), insulator and electrolytes is negative.
- The **heating element** devices like heater, geyser, electric iron (usually known as press) etc are made of **nichrome** because it has high resistivity and high melting point. It does not react with air and acquires steady state when red hot at 800°C.
- **Fuse wire** is made of **tin lead alloy** because it has low melting point and high resistivity. The fuse is used in series, and melts to result in an open circuit when current exceeds the safety limit.
- **Resistances** of resistance box are made of **manganin** or **constantan** because they have moderate resistivity and very small temperature coefficient of resistance. The resistivity is nearly independent of temperature.
- The **filament of bulb** is made up of **tungsten** because it has low resistivity, high melting point of 3300 K and emits light at 2400 K. The bulb is filled with inert gas because at high temperatures tungsten reacts with air forming oxide.

  The **connection wires** are made of **copper** because it has low resistivity.

BEGINNER'S BOX-2

1. The dimensions of a block is 100 m× 1 m× 1 m. The specific resistance of the material of the block is 4×10^{-6} Ω-m. Calculate its resistance across its square faces.

PCE239

2. If a copper wire is stretched to make its cross-sectional radius 0.1% thinner, then what is the percentage increase in its resistance ?

PCE240

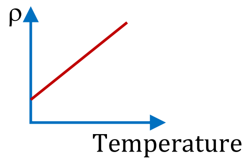
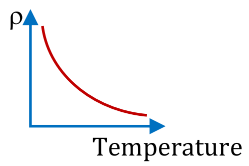
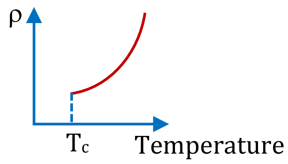
3. A wire of resistance 5 Ω is drawn out so that its length is increased to twice its original length. Calculate its new resistance and resistivity.

PCE241

4. At what temperature, would the resistance of a copper wire be double its resistance at 0°C ? If $\alpha = \frac{1}{273}\ °C^{-1}$.

PCE242

5. Match the following items –

Graph	Solid
(A) 	(p) Semiconductor
(B) 	(q) Conductor
(C) 	(r) Conductor which exhibits super conductivity

PCE243

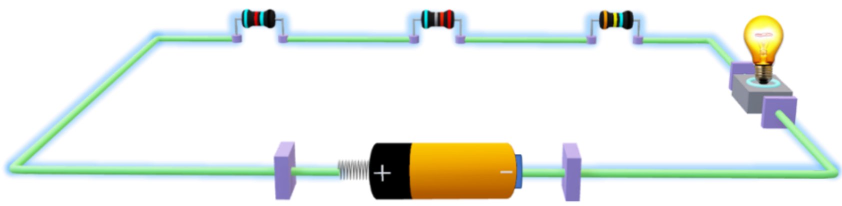
6. Match the column I with column II –

	Column I		Column II
(A)	Filament of electric bulb	(p)	tin-lead alloy
(B)	Fuse wire	(q)	Manganin
(C)	Resistance of resistance box	(r)	Nichrome
(D)	Filament of heating devices (heater, geyser, press etc)	(s)	Tungsten
		(t)	Constanton

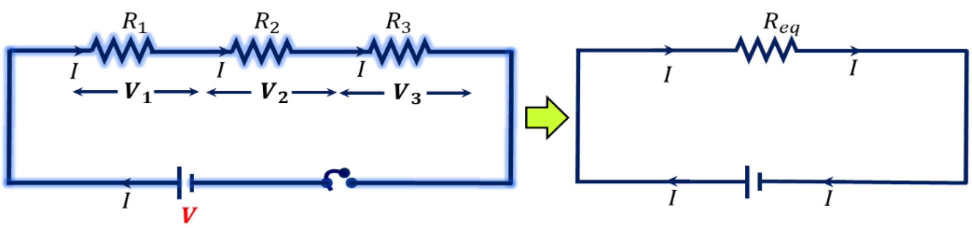
PCE244

Combination of Resistors & Kirchhoff's laws

Series Combination



Let's see circuit diagram and formulas used in series combination



1. **In series, $I \rightarrow$ same**

If V is P.D. of source and V_1, V_2 and V_3 is potential drop across each resister then

$V = V_1 + V_2 + V_3$

$IR_{eq} = IR_1 + IR_2 + IR_3 \Rightarrow R_{eq} = R_1 + R_2 + R_3$

2. **Voltage divider rule**

Since I is same; $V \propto R$ (As $V = IR$)

$V_1 : V_2 : V_3 \Rightarrow R_1 : R_2 : R_3$

In series combination voltage drop across resistors is in the ratio of resistance.

Also $V_1 = IR_1$ and $I = V_{battery}/R_{eq}$

$$V_1 = \frac{R_1}{R_{eq}} \times V_{battery}$$

3. **For n identical resistances in series:**

$R_{eq} = nR$

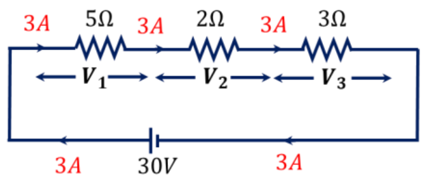
$$V_1 = V_2 = V_3 = V_n = \frac{V_{battery}}{n}$$

4. $\mathbf{R_{eq} > (R_{max})_{combination}}$

Illustration 21:

In given network find out equivalent resistance of network. Also find P.D. across each resistor.

Solution:



All resistances are in series so we use

$R_{eq} = R_1 + R_2 + R_3$

$R_{eq} = 5 + 2 + 3 = 10\Omega$

Current supplying by source will be

$$I = \frac{V_{battery}}{R_{eq}}, \quad I = \frac{30}{10} = 3A$$

$V_1 = 3 \times 5 = 15$ Volt, $V_2 = 3 \times 2 = 6$ Volt, $V_3 = 3 \times 3 = 9$ volt

Illustration 22:

Two wires of equal diameters, of resistivities ρ_1 and ρ_2 and lengths l_1 and l_2, respectively, are joined in series. The equivalent resistivity of the combination is

Solution:

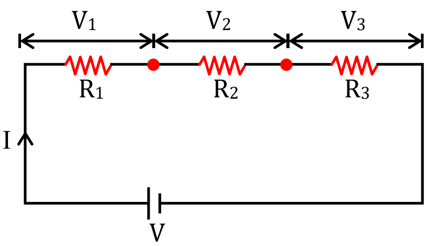
$$R_1 = \frac{\rho_1 l_1}{A} \text{ and } R_2 = \frac{\rho_2 l_2}{A}.$$

In series $R_{eq} = R_1 + R_2$

$$\frac{\rho_{eq.}(l_1 + l_2)}{A} = \frac{\rho_1 l_1}{A} + \frac{\rho_2 l_2}{A} \Rightarrow \rho_{eq} = \frac{\rho_1 l_1 + \rho_2 l_2}{l_1 + l_2}$$

Golden Key Points

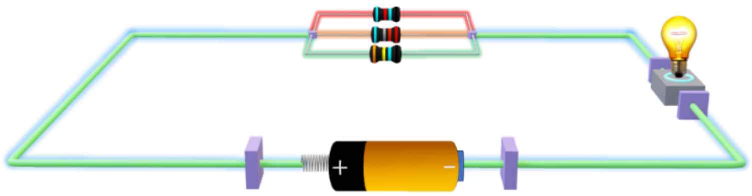
- If n resistance (each R) are connected in series their effective resistance will be nR.
- When resistors are connected in series in a battery circuit then the ratio of potential differences across the resistors is equal to ratio of their respective resistances.



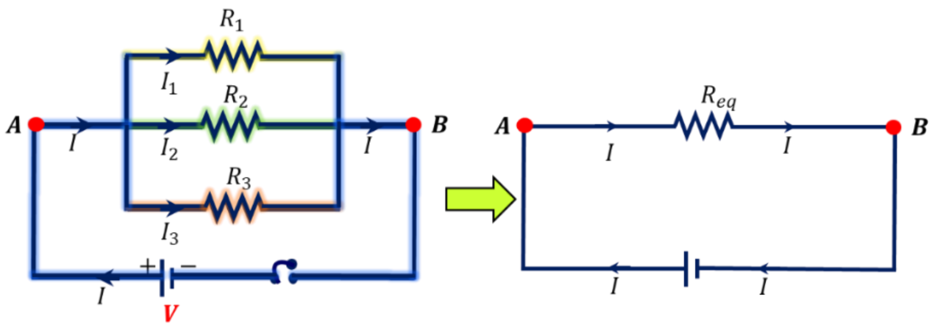
$V_1 : V_2 : V_3 = R_1 : R_2 : R_3$

$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} V \,;\; V_2 = \frac{R_2}{R_1 + R_2 + R_3} V;\; V_3 = \frac{R_3}{R_1 + R_2 + R_3} V$$

Parallel Combination



Let's see circuit diagram and formulas used in parallel combination



1. In parallel V is same across all resistors but current will be different if all resistors are different

If I_1, I_2 and I_3 are the currents through resistors R_1, R_2 and R_3 then

$I = I_1 + I_2 + I_3$

or $\dfrac{v}{R_{eq}} = \dfrac{V}{R_1} + \dfrac{V}{R_2} + \dfrac{V}{R_3}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

For n resistors in parallel, $\dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3} + ... + \dfrac{1}{R_n}$

2. **Current divider rule :**

Since V is same, $I \propto \dfrac{1}{R}$

$$I_1 : I_2 : I_3 \Rightarrow \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3}$$

3. **In case of 2 resistance**

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow R = \frac{R_1 R_2}{R_1 + R_2}$$

4. **If n equal resistance are connected in parallel**

$R_{eq} = R/n$

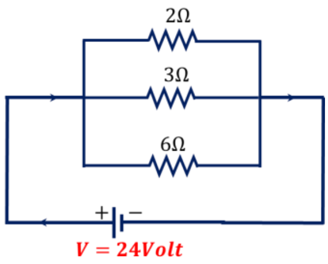
$I_1 = I_2 = I_3 = I_n = \dfrac{I_{total}}{n}$

5. $\mathbf{R_{eq} < (R_{min})_{combination}}$

Illustration 23:

In given network find out equivalent resistance of network. Also find current passing through each resistor.

Solution:



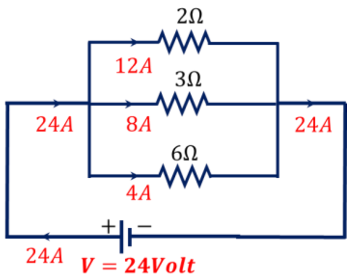
All resistances are in parallel so $\frac{1}{R_{eq}}=\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}$

$$\frac{1}{R_{eq}}=\frac{1}{6}+\frac{1}{3}+\frac{1}{2}=\frac{1+2+3}{6}$$

$$\Rightarrow \quad R_{eq}=1\Omega$$

$$I=\frac{V}{R_{eq}}=\frac{24}{1} \Rightarrow I = 24\text{ A}$$

P.D. across each resistor will be 24 V



Current through 2Ω resistor will be $I_{2\Omega}=\frac{24}{2}=12\text{A}$

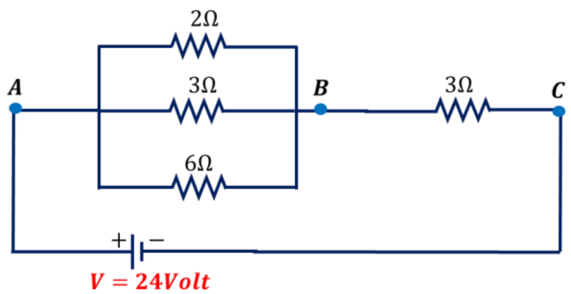
Current through 3Ω resistor will be $I_{3\Omega}=\frac{24}{3}=8\text{A}$

Current through 6Ω resistor will be $I_{6\Omega}=\frac{24}{6}=4\text{A}$

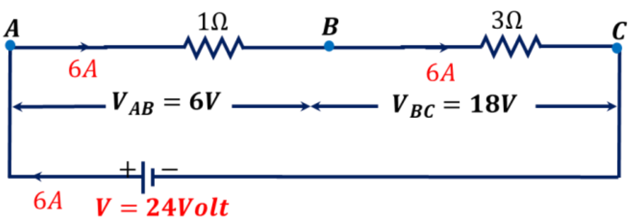
Problems of combination of resistors

Illustration 24:

In given network find out equivalent resistance of network. Also P.D. across each resistor.



Solution:



Equivalent resistance between A and B is R_{AB}

$$\frac{1}{R_{AB}}=\frac{1}{2}+\frac{1}{3}+\frac{1}{6}$$

$$\frac{1}{R_{AB}}=\frac{3+2+1}{6} \quad \Rightarrow \quad R_{AB}=1\Omega$$

Given network can be reduced as

Equivalent resistance of network is R_{eq} = (1+3) = 4 Ω

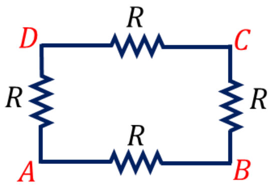
Current supplying by source will be $I = V_{battery}/R_{eq}$, $I=\frac{24}{4}=6\text{ A}$

P.D. between A and B will be $V_{AB}=6\times 1=6\text{ V}$

P.D. between B and C will be $V_{BC}=6\times 3=18\text{ V}$

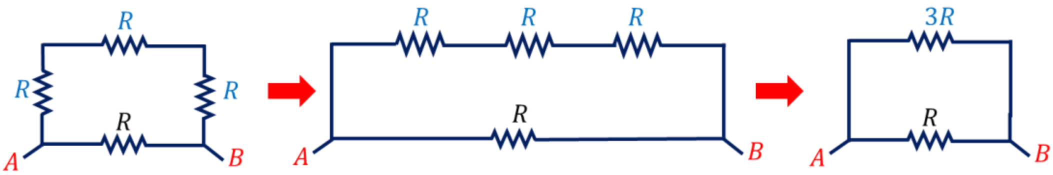
Illustration 25:

Find equivalent resistance across (i) R_{AB} & (ii) R_{AC}



Solution:

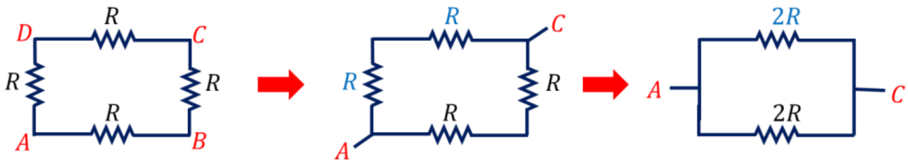
(i) R_{AB}



$$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{3R \times R}{3R \times R}$$

$$R_{AB} = \frac{3R}{4}$$

(ii) R_{AC}

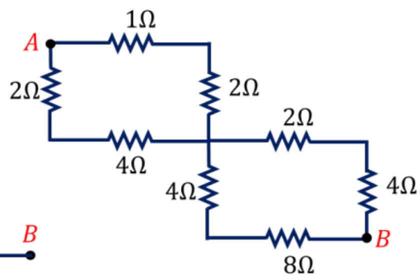


$$R_{eq} = \frac{R_1R_2}{R_1 + R_2} = \frac{2R \times 2R}{2R + 2R}$$

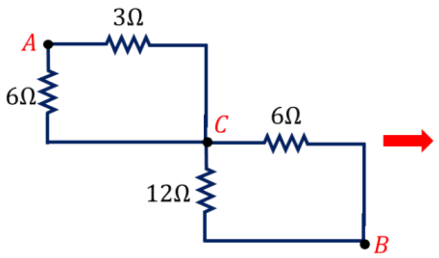
$R_{AC} = R$

Illustration 26:

Find equivalent resistance across A & B -



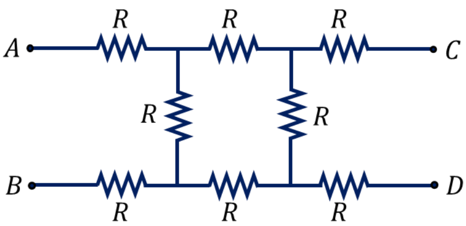
Solution:



$R_{AB} = 2 + 4 = 6\ \Omega$

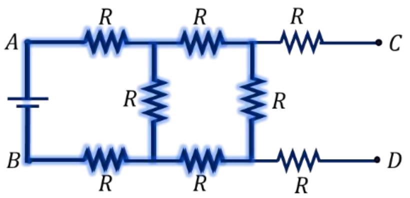
Illustration 27:

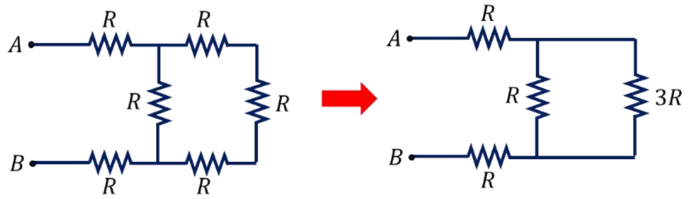
Find equivalent resistance across (i) R_{AB} & (ii) R_{AD}

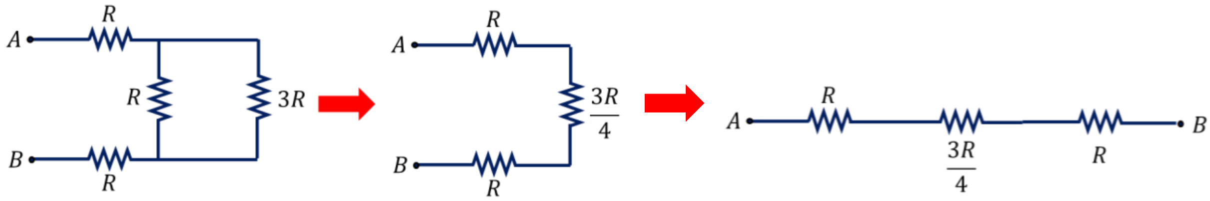


Solution:

(i) for R_{AB}

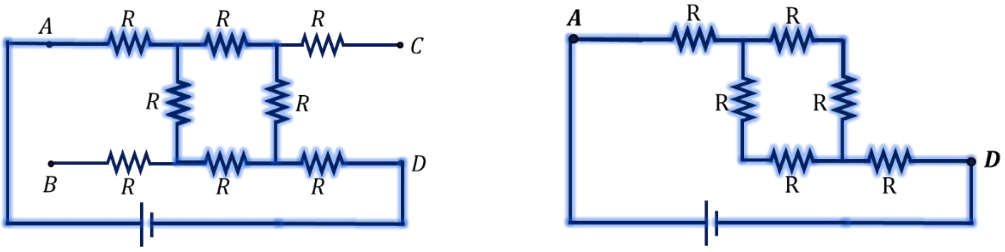


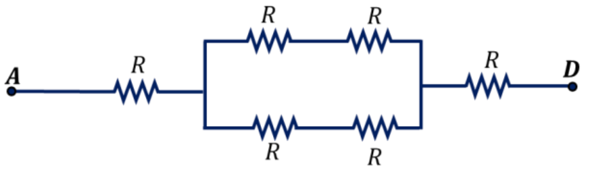




$$R_{AB} = R + \frac{3R}{4} + R = \frac{11\ R}{4}$$

(ii) for R_{AD}

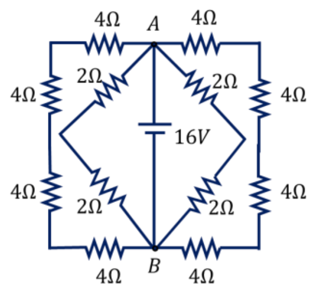




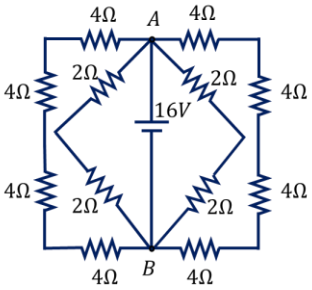
$R_{AD} = 3R$

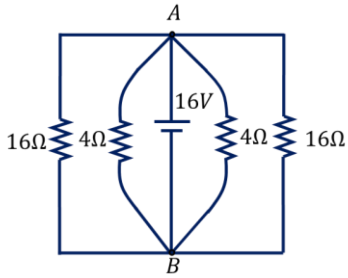
Illustration 28:

Find current supplied by battery in given network?



Solution:





Now all resistances are in parallel so

$$\frac{1}{R_{eq}}=\frac{1}{16}+\frac{1}{4}+\frac{1}{4}+\frac{1}{16}=\frac{1+4+4+1}{16}=\frac{10}{16}$$

$$R_{eq}=\frac{16}{10}=1.6\Omega \qquad I=\frac{16}{1.6}=10A$$

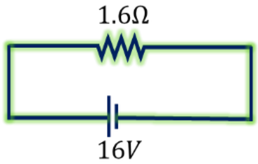
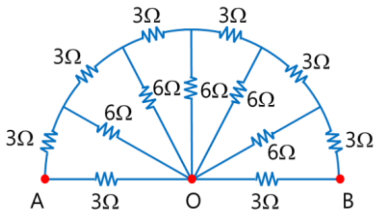


Illustration 29:

Find equivalent resistance across O and A (R_{OA}) ?



Solution:

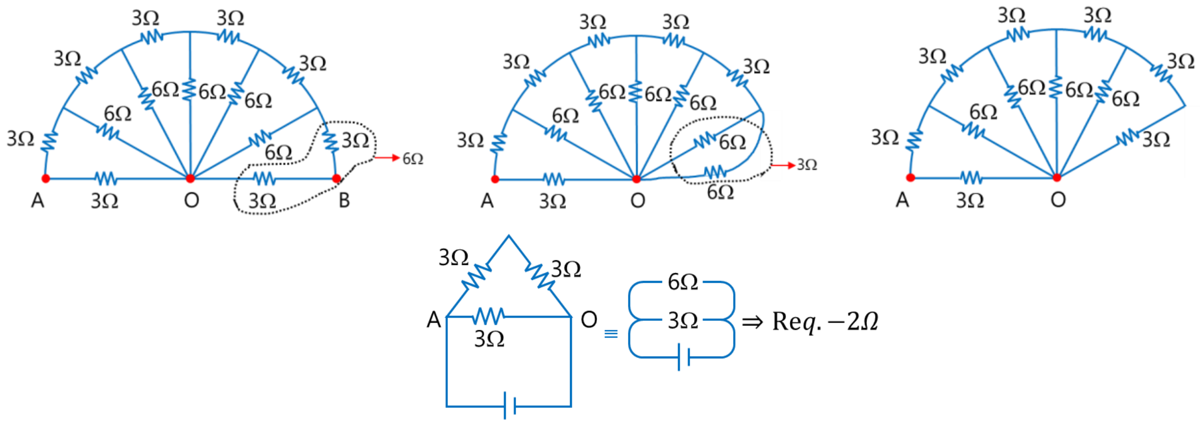
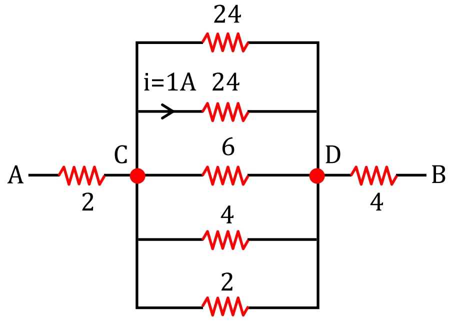


Illustration 30:

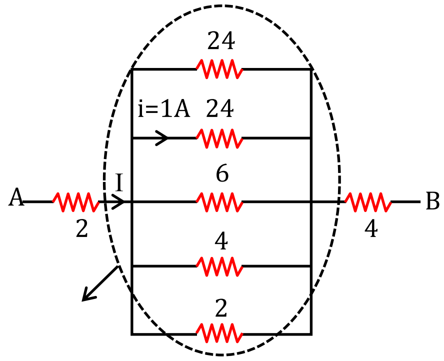
Find $V_A - V_B$?



Solution:

$$\frac{1}{R}=\frac{1}{24}+\frac{1}{24}+\frac{1}{6}+\frac{1}{4}+\frac{1}{2}$$

$$\frac{1}{R}=\frac{1}{24}+\frac{1}{24}+\frac{4+6+12}{24} \quad \text{or} \quad R = 1$$



$R_{net} = 2 + 1 + 4 = 7\ \Omega$

The ratio of resistance between C and D is

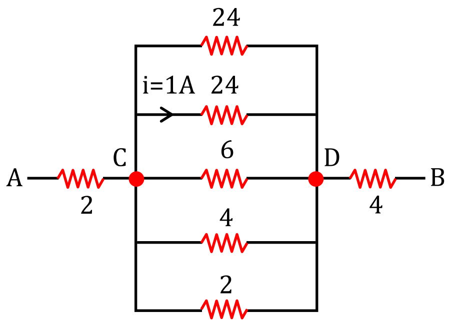
$24 : 24 : 6 : 4 : 2 = 12 : 12 : 3 : 2 : 1$

The ratio of current between C and D will be

$\frac{1}{12}:\frac{1}{12}:\frac{1}{3}:\frac{1}{2}:1 = 1 : 1 : 4 : 6 : 12$

$I = 24$ A

$V = 24 \times 7 = 168$ V



Alternate method

$V = 1 \times 24 = 24VR = 1$

$V = IR$

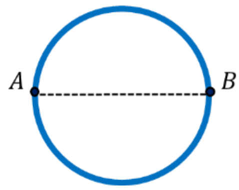
$I = 24$ A

$R_{net} = 2 + 4 + 1 = 7\ \Omega$

$V = 24 \times 7 = 168$ V

Illustration 31:

A wire of resistance $\frac{6}{\pi}\Omega/m$ is bend to form a circular loop of radius (r = 1m). Find equivalent resistance of the loop between two diametrically opposite points.

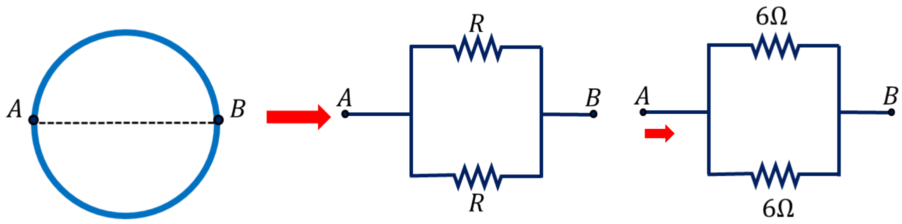


Solution:

Both upper and lower part will be in parallel so it can be treated as

Resistance of 1m length is $\frac{6}{\pi}\Omega$

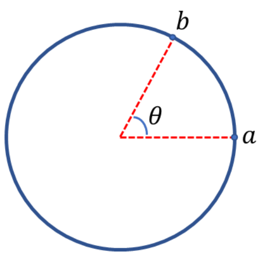
Resistance of $2\pi r$ length will be $\frac{6}{\pi}\Omega\times 2\pi r=\frac{6}{\pi}\Omega\times 2\pi(1) = 12\ \Omega = 2\ R \Rightarrow R = 6\Omega$



$R_{AB} = 3\Omega$

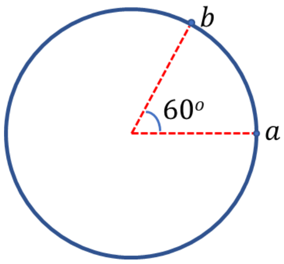
Illustration 32:

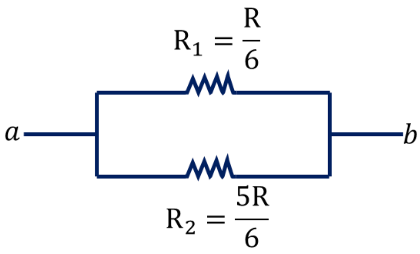
A uniform wire of resistance 18 Ω is bent in the form of a circle. Calculate effective resistance across the points a & b.



$\theta = 60^o$

Solution:

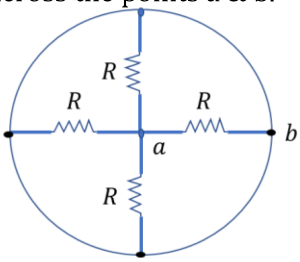




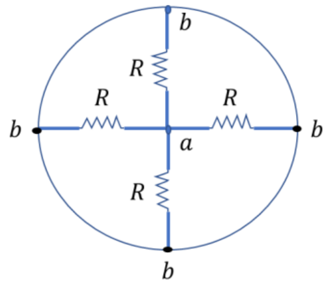
$$R_{ab} = \frac{R_1 \cdot R_2}{R_1 + R_2} = \frac{\frac{5R}{6} \cdot \frac{R}{6}}{\frac{5R}{6} + \frac{R}{6}} = \frac{5R}{6 \times 6} = \frac{5R}{36} = \frac{5}{36}(18) = 2.5\Omega$$

Illustration 33:

Calculate effective resistance across the points a & b.



Solution:



$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{R}$$

$$R_{eq} = \frac{R}{4}$$

Illustration 34:

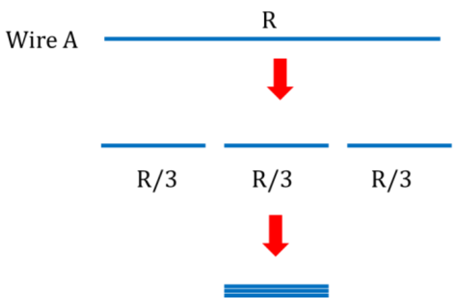
Two wires of same material and length are having cross-section in the ratio 1 : 2. Now, thin wire is cut into 3 equal parts and arranged to form a bundle and connected in series with thick wire. If resistance of the thick wire is 18Ω. Then find resistance of the given combination.

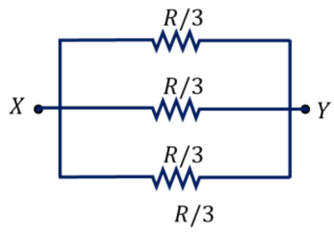
Solution:

As resistance of wire is = $\frac{\rho l}{A}$

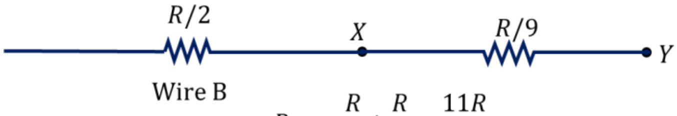
Let us assume resistance of wire A (thin) is R then resistance of wire B (thick) will be R/2

When thin wire (A) cut in three equal parts resistance of each part will be R/3





$$R_{XY} = \frac{\frac{R}{3}}{3} = \frac{R}{9}$$



$$R_{eq} = \frac{R}{2} + \frac{R}{9} = \frac{11R}{18}$$

$$R_{eq} = \frac{R}{2} + \frac{R}{9} = \frac{11R}{18} = \frac{11}{9}\left(\frac{R}{2}\right) = \frac{11}{9} \times 18 = 22\Omega$$

Concept of Short Circuit

It both terminals of a resistance are connected with a conducting wire then potential of both terminals will become same. In this condition no current will flow through the resistance & we can remove them from the circuit, this process is known as short circuit and that resistor is called short circuited or by pass resistor.

- ➢ Both terminals of a resistance are connected with a conducting wire.
- ➢ No current will flow through the resistance.

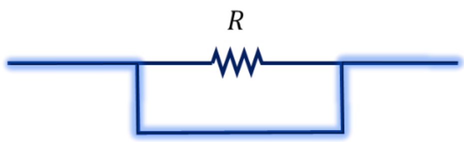
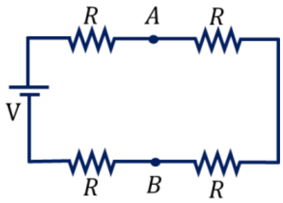
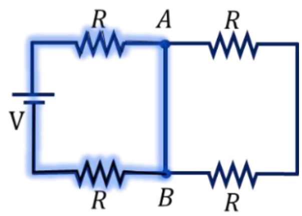


Illustration 35:

If circuit is short circuited between A and B then what will be current supply by source.



Solution:



After short circuiting equivalent resistance will be

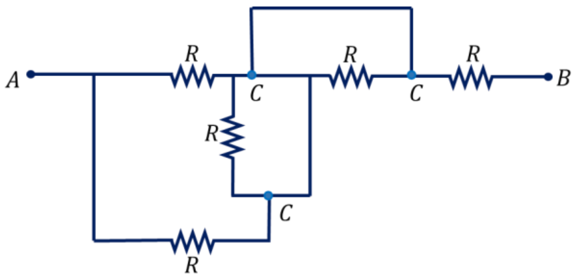
$R_{eq} = 2\ R$

Current supplying by source will be

$$I = \frac{V}{2R}$$

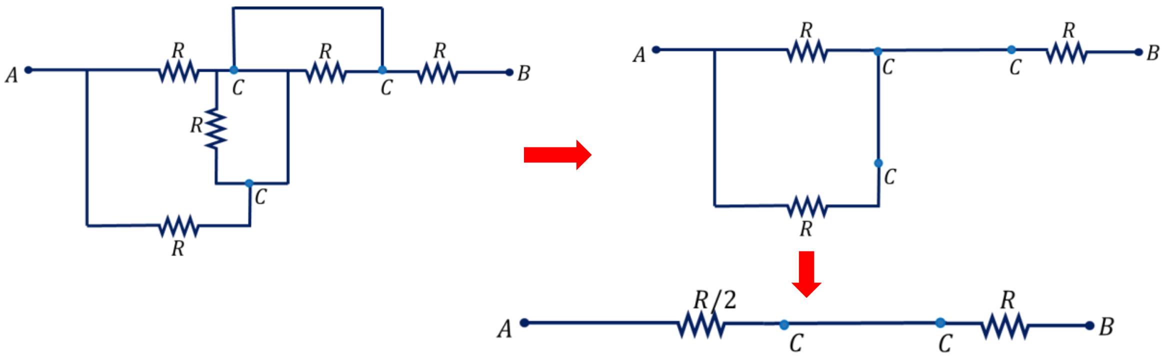
Illustration 36:

Find out equivalent resistance between A and B.



Solution:

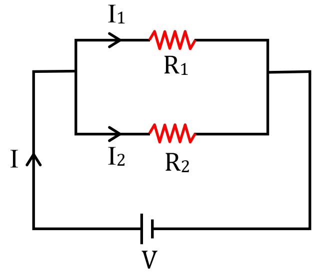
In given circuit all the points connected by straight conducting wire are at same potential.



$$R_{AB} = \frac{R}{2} + R = \frac{3R}{2}$$

Golden Key Points

- If n resistances (each R) are connected in parallel then their effective resistance will be $\frac{R}{n}$.
- The equivalent resistance of parallel combination is lower than the value of the lowest resistance in the combination.
- When resistors are connected in parallel in a battery circuit then the ratio of currents through them will be the ratio of reciprocals of their respective resistances.



$$\frac{I_1}{I_2}=\frac{\frac{1}{R_1}}{\frac{1}{R_2}}=\frac{R_2}{R_1}$$

$$I_1=\frac{R_2}{R_1+R_2}I\ ;\ I_2=\frac{R_1}{R_1+R_2}I \qquad [I = I_1 + I_2]$$

Kirchhoff's Laws

There are two laws given by Kirchhoff for determination of potential difference and current in different branches of any complicated network.

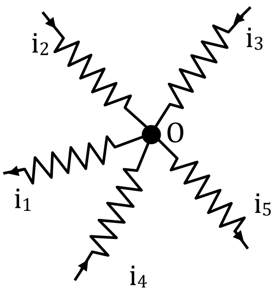
Kirchhoff's First law: This law is also known as junction law or current law (KCL). According to it-

In an electric circuit, the algebraic sum of the currents meeting at any junction in the circuit is zero.

OR

Sum of the currents entering the junction is equal to sum of the currents leaving the junction.

$\sum i=0$

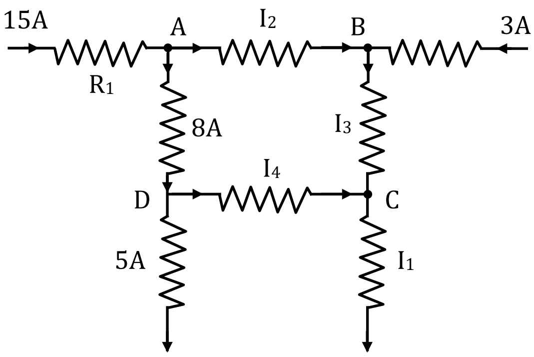


$i_1 - i_2 - i_3 - i_4 + i_5 = 0 \quad \Rightarrow \quad i_1 + i_5 = i_2 + i_3 + i_4$

This is based on law of conservation of charge.

Illustration 37:

Calculate the values of currents I_1, I_2, I_3, and I_4 in the section of networks shown in figure.



Solution:

The distribution of currents in different resistances can be calculated using Kirchhoff's junction rule. We will apply Kirchhoff's junction rule at junctions A, B, C and D as shown in figure.

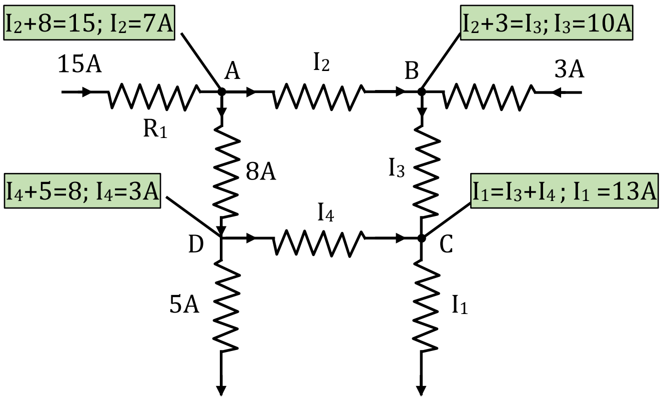
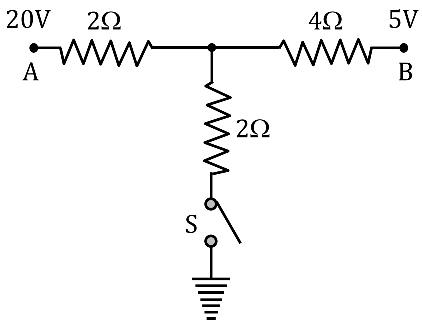


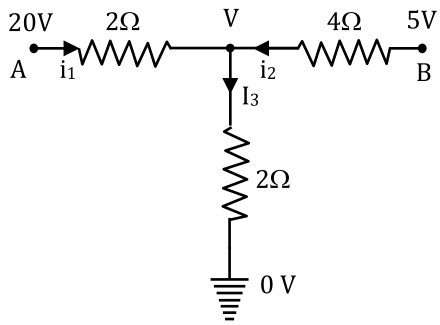
Illustration 38:

Figure shows three resistances are connected with switch S. Initially the switch is open. When the switch S is closed find the current passing through it.



Solution:

Let V be the potential of the junction as shown in figure.



Applying junction law, we have

$$i_1 + i_2 = i_3$$

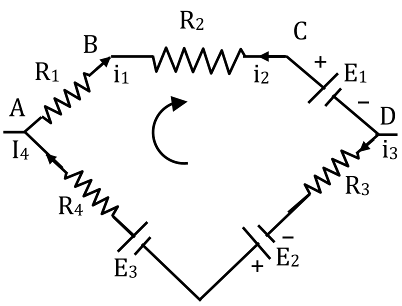
$\Rightarrow$ $$\frac{20-V}{2} + \frac{5-V}{4} = \frac{V-0}{2}$$

$\Rightarrow$ 40-2V+5-V=2V

$\Rightarrow$ 5V = 45 or V = 9V

$\therefore$ $$i_3 = \frac{V}{2} = 4.5A$$

Kirchhoff's Second law: This law is also known as loop rule or voltage law (KVL) and according to it in any closed circuit the algebraic sum of e.m.f. and algebraic sum of potential drops is zero.



$\Sigma IR + \Sigma E = 0$.

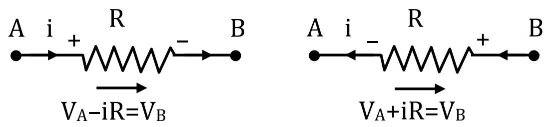
In the following closed loop,

$-i_1R_1 + i_2R_2 - E_1 - i_3R_3 + E_2 + E_3 - i_4R_4 = 0$

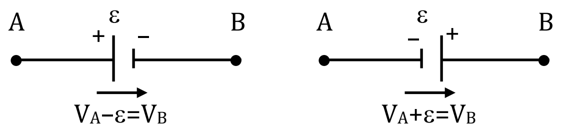
Sign convention for the application of Kirchhoff's law :

For the application of Kirchhoff's laws, the following sign conventions are to be considered.

❖ The change in potential in traversing a resistance in the direction of the current is –iR, while in the opposite direction it is +iR.



❖ The change in potential in traversing an emf source from negative to positive terminal is +e, while in the opposite direction it is -e irrespective of the direction of current in the circuit.



❖ The change in potential in traversing a capacitor from the negative terminal to the positive terminal is +q/C, while in the opposite direction it is -q/C.

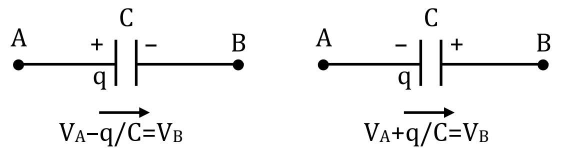
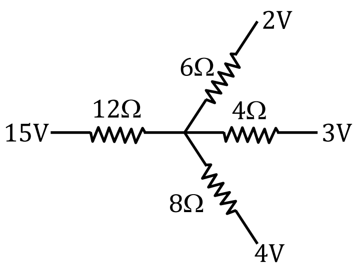


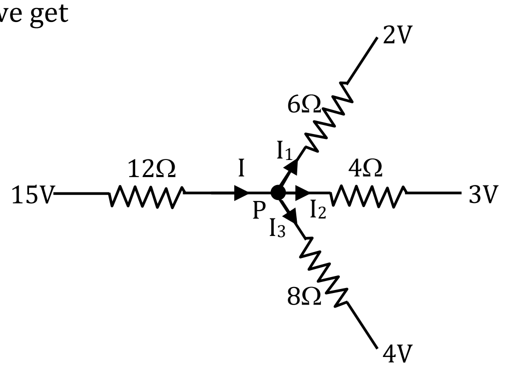
Illustration 39:

Find the current through 12Ω resistor in figure.



Solution:

Let V be the potential at P; then applying KCL at junction P, we get



$I = I_1 + I_2 + I_3$

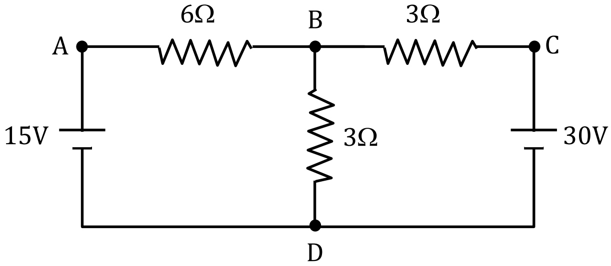
$\Rightarrow \quad \dfrac{15-V}{12} = \dfrac{V-2}{6} + \dfrac{V-3}{4} + \dfrac{V-4}{8}$

$\Rightarrow \quad V = \dfrac{68}{15} V$

and $I = \dfrac{15-(68/15)}{12} = \dfrac{157}{180} A$

Illustration 40:

In the circuit shown in figure, find the current through the branch BD.



Solution:

Applying KVL along the loop ABDA and moving in the clockwise direction, we get,

$-6I_1 - 3I_2 + 15 = 0$

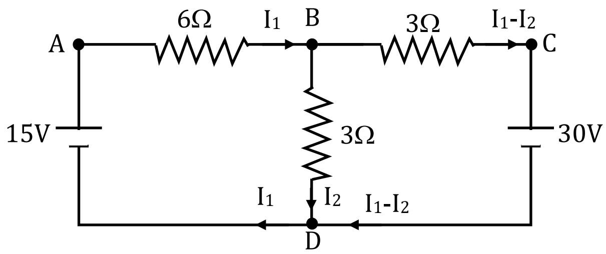
$\Rightarrow \quad 2I_1 + I_2 = 5 \quad$ ...(i)

Applying KVL along the loop BCDB, we get

$-3(I_1 - I_2) - 30 + 3I_2 = 0$

$\Rightarrow \quad -I_1 + 2I_2 = 10 \quad$ ...(ii)

Solving Eqs. (i) and (ii) for I_2, we get $I_2 = 5A$.



Symmetric Circuits and Infinite Circuits

Symmetric Circuits

Mirror Symmetry

(1) It is applied along perpendicular bisector of line AB (A and B are the points about which Req. is to be calculated).

(2) We can remove resistances and junctions which are on the line of symmetry.

(3) Points on line of symmetry are equipotential points.

Folding symmetry:

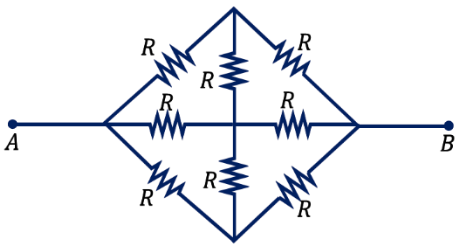
(1) It is applied along line AB.

(2) We can fold the circuit about line of symmetry.

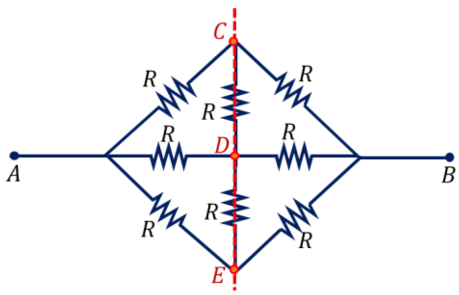
(3) When we fold the circuit, the parallel combination of resistance can be solved easily.

Illustration 41:

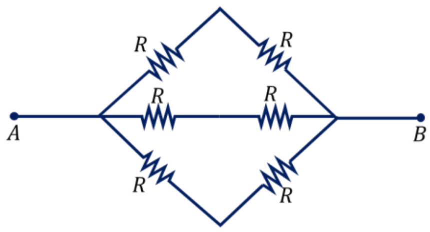
Find equivalent resistance across point A and B ?



Solution:



Points C, D and E will be equipotential points so there is no current flowing from resistances connected between these points.



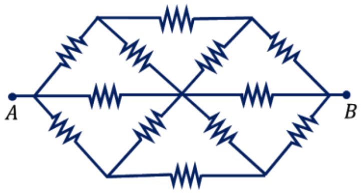
$$\frac{1}{R_{AB}} = \frac{1}{2R} + \frac{1}{2R} + \frac{1}{2R}$$

$R_{AB} = 2R/3$

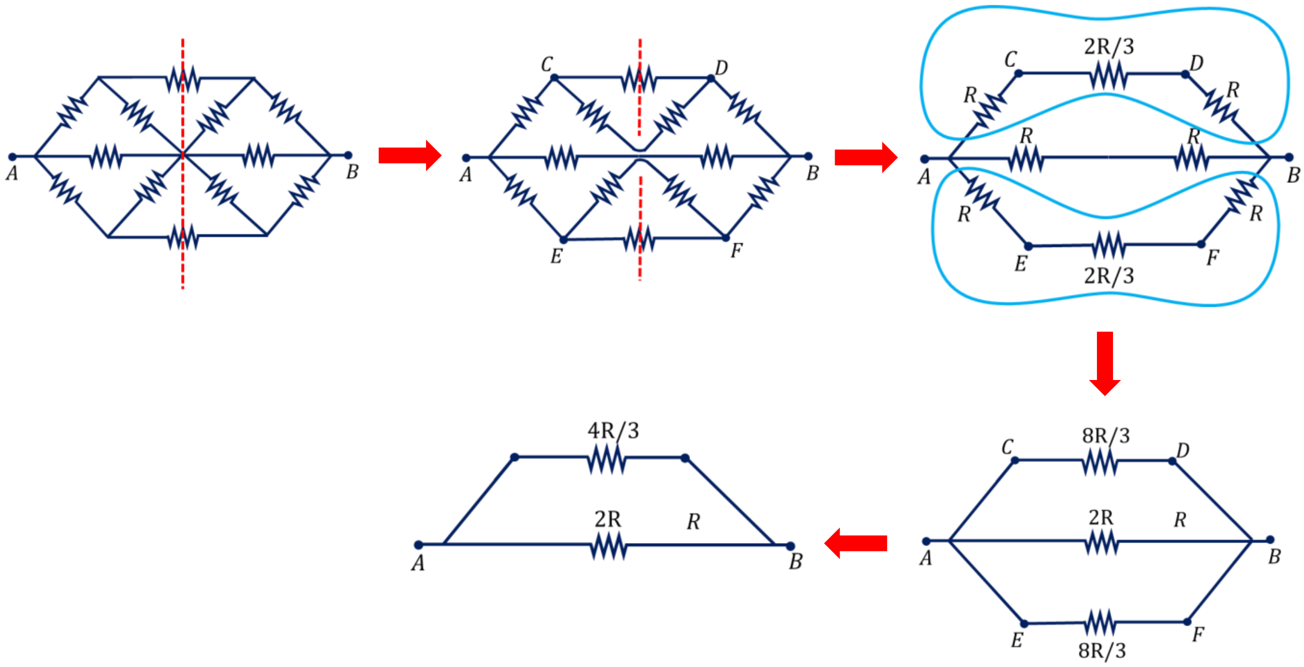
Illustration 42:

Find equivalent resistance across point A and B ?

(all resistances are R)

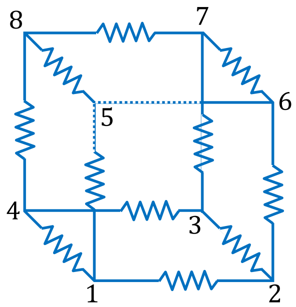


Solution:



$$\frac{1}{R_{AB}} = \frac{1}{2R} + \frac{3}{4R} = \frac{2}{4R} + \frac{3}{4R} \Rightarrow R_{AB} = \frac{4R}{5}$$

Illustration 43:



A frame of cube is made by wires each of resistance r then find -

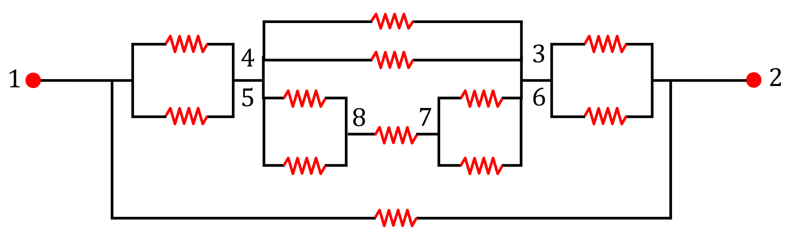
(a) Resistance between two nearer corners (R_{12})

(b) Resistance across face diagonal (R_{13})

(c) Resistance across main diagonal (R_{17})

Solution:

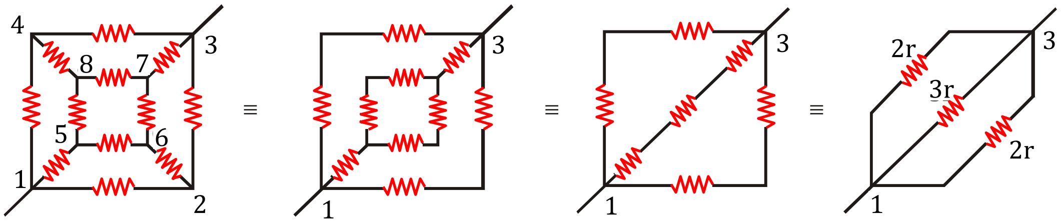
(a) Resistance between two nearer corners



Equivalent resistance between 1 and 2 $\Rightarrow R_{12} = \frac{7}{12}r$

(b) Resistance across face diagonal

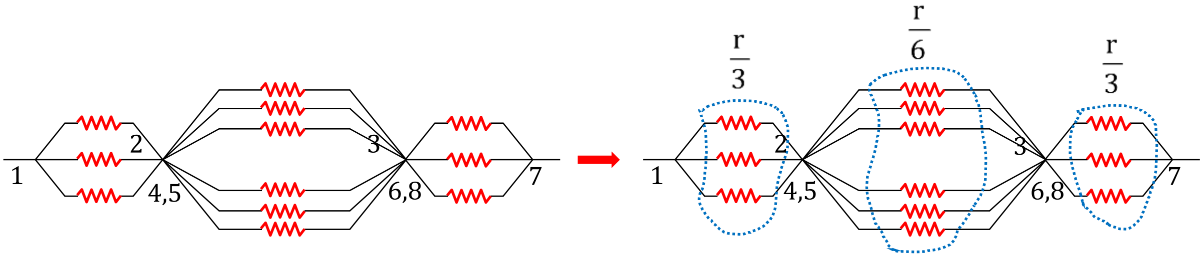
Given circuit can be drawn as



Equivalent resistance between 1 and 3 $\Rightarrow R_{13} = \frac{3}{4}r$

(c) Resistance across main diagonal

The given circuit can be simplified as follows

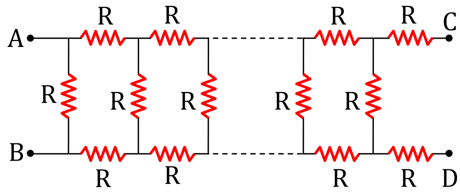


$$R_{17} = \frac{r}{3} + \frac{r}{6} + \frac{r}{3} \quad \therefore R_{17} = \frac{5r}{6}$$

Infinite Circuits

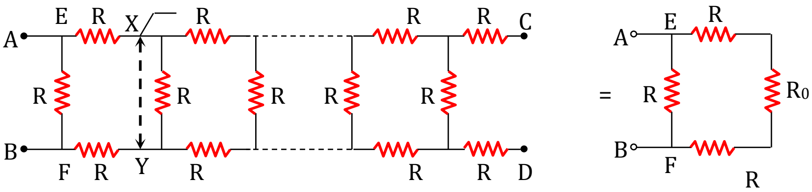
Illustration 44:

In the figure, the value of resistors to be connected between C and D so that the resistance of the entire circuit between A and B does not change with the number of elementary sets used is



Solution:

Cut the series from XY and let the resistance towards right of XY be R_0 whose value should be such that when connected across AB does not change the entire resistance. The combination is reduced to as shown below.



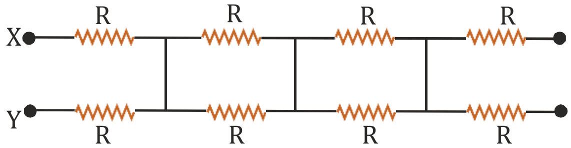
The resistance across $EF = R_{EF} = (R_0 + 2R)$

Thus $R_{AB} = \frac{(R_0 + 2R)R}{R_0 + 2R + R}$

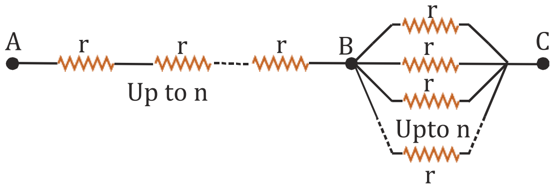
$$R_0 = \frac{R_0R + 2R^2}{R_0 + 3R} \Rightarrow R_0^2 + 2RR_0 - 2R^2 + 0 \Rightarrow R_0 = R(\sqrt{3} - 1)$$

BEGINNER'S BOX-3

1. For following circuit, what is the value of total resistance between X and Y ? **PCE245**



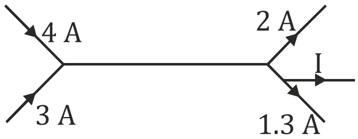
2. In the following figure what is the resultant resistance between A and C ? **PCE246**



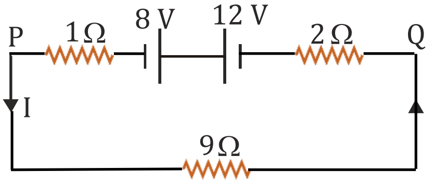
3. What is the smallest resistance that can be obtained by ten $\frac{1}{10}\,\Omega$ resistors? **PCE247**

4. Two wires of the same metal have the same length, but their cross sectional areas are in the ratio 3 :1. They are joined in series. The resistance of the thicker wire is 10 Ω. Then what will be the total resistance of the combination? **PCE248**

5. What is the value of current I in the following circuit? **PCE249**



6. In the given circuit, calculate the potential difference between the points P and Q. **PCE250**



Cells, Combinations of Cells, Electrical Heating and Power

Internal resistance of a cell, Potential difference and EMF of a cell

Electro motive force (E:M:F:)

The potential difference across the terminals of a cell when it is not delivering any current is called emf of the cell. The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called the emf of the cell.

- emf depends on :

  (i) nature of electrolyte (ii) metal of electrodes

- emf does not depend on :

  (i) area of plates (ii) distance between the electrodes

  (iii) quantity of electrolyte (iv) size of cell

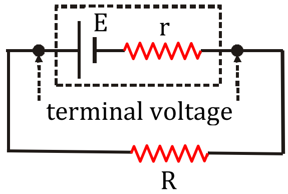
Internal resistance

Resistance offered by the electrolyte of the cell when an electric current flows through it is known as internal resistance.

Dependency of internal resistance r :

- Distance between two electrodes increases $\Rightarrow$ r increases
- Area dipped in electrolyte increases $\Rightarrow$ r decreases
- Concentration of electrolyte increases $\Rightarrow$ r increases
- Temperature increases $\Rightarrow$ r decreases

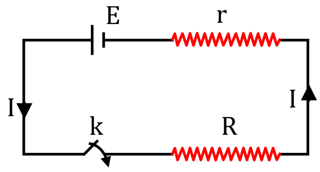
Terminal potential difference (V)



- When current is drawn through a cell or current is supplied to it then the potential difference across its terminals is called terminal voltage.
- When I current is drawn from cell then terminal voltage V is less than its e.m.f i.e., V = E – Ir

(a) When cell is discharging :

Current inside the cell is from cathode to anode.



Current $I = \dfrac{E}{r+R}$

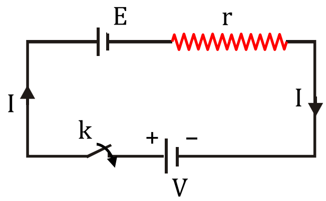
$\Rightarrow$ E = IR + Ir = V + Ir

$\Rightarrow$ V = E – Ir

When current is drawn from the cell, potential difference across the cell is less than the emf of the cell. Greater is the current drawn from the cell, smaller is the terminal potential difference. When a large current is drawn from a cell its terminal potential difference is reduced appreciably.

(b) When cell is getting charged :

Current inside the cell is from anode to cathode.



Current $I = \dfrac{V-E}{r}$

$\Rightarrow$ V = E + Ir

During charging terminal potential difference is greater than the emf of the cell.

(c) When cell is in the open circuit :

In open circuit R = ∞

$\therefore\ I = \dfrac{E}{R+r} = 0 \Rightarrow V = E$

In open circuit, terminal potential difference is equal to emf and is the maximum potential difference which a cell can provide.

(d) When cell is short circuited :

In short circuit R = 0

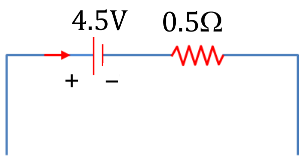
$I = \dfrac{E}{R+r} = \dfrac{E}{r}$ and V = IR = 0

Note: In short circuit, current from the cell is maximum and terminal potential difference is zero.

Illustration 45:

A current of 1A is flowing from (+)ve to (–)ve terminal inside the cell of emf 4.5 V and internal resistance 0.5Ω then find potential of the negative terminal if potential of positive terminal is 10V.

Solution:



Terminal Potential Difference V = E + ir (∵ cell is charging)

$\Rightarrow$ $V_+ - V_- = 4.5 + 0.5$

$\Rightarrow$ $10 - V_- = 5V \Rightarrow V_- = 5V$

Illustration 46:

When a resistance of 4Ω is connected to a cell then a current 2A flow through it. If the cell is connected to a resistance of 9Ω then current is decreased by 50%. Find current through the cell if its terminals are directly connected by a connecting wire.

Solution:

$$i = \frac{E}{R+r} = 2A$$

$$i = \frac{E}{4+r} \qquad ...(i)$$

$$i' = \frac{i}{2} = \frac{E}{9+r} \qquad ...(ii) \qquad \left[i' = \frac{50}{100}i = \frac{i}{2} = 1A\right]$$

By (i) / (ii)

$$\frac{2}{1} = \frac{E}{4+r} \times \frac{9+r}{E} \Rightarrow 8 + 2r = 9 + r \Rightarrow r = 1\Omega$$

From eq. (i) $2(4 + 1) = E \Rightarrow E = 10V$

Current through cell during Short circuit = $\frac{E}{r} = \frac{10}{1} = 10A$

★ Golden Key Points ★

- At the time of charging a cell, when current is being supplied to it, the terminal voltage is greater than the emf E.

  $$V = E + Ir \quad \text{So} \quad V > E$$
- Series combination is useful when internal resistance of the cell is less than external resistance.
- Parallel combination is useful when internal resistance of the cell is greater than external resistance.
- Internal resistance of an ideal cell = 0

  If external resistance is zero, then current delivered by the battery is maximum.

Combination of cells in Series and in Parallel

A. Series Combination

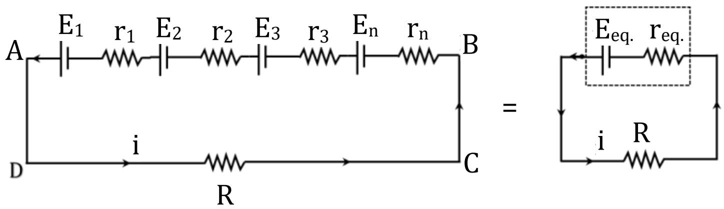
When cells are connected in series the total emf of the series combination is equal to the sum of the emf's of the individual cells and the internal resistance of the cells will also be in series.

$E_{eq.} = E_1 + E_2 + E_3 \ ...... \ E_n$

$r_{eq.} = r_1 + r_2 + r_3 \ ....r_n$

$R_{eq.} = r_1 + r_2 + r_3 \ ......r_n + R$

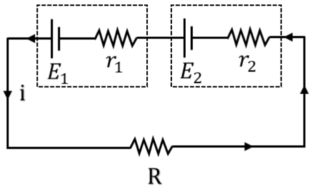
Current $i = \frac{E_{eq.}}{r_{eq.} + R}$



If all n cells are identical then $i = \frac{nE}{nr + R}$

- If nr >> R , $i = \frac{E}{r}$ = current from any cell when shot-circuited.
- If nr << R , $i = \frac{nE}{R}$ = n × current from any one cell, when connected with the external resistance.

When cell support each other



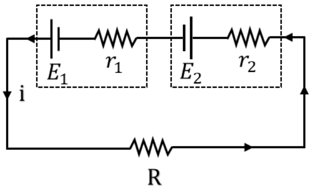
$$i = \frac{E_1 + E_2}{R + (r_1 + r_2)}$$

T.P.D. of first cell = $E_1 - ir_1$

T.P.D. of second cell = $E_2 - ir_2$

When cells are connected with opposite polarity

Let $E_1 > E_2$



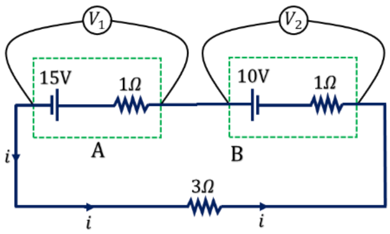
$$i = \frac{E_1 - E_2}{R + (r_1 + r_2)}$$

T.P.D. of first cell = $E_1 - ir_1$

T.P.D. of second cell = $E_2 + ir_2$

Illustration 47:

Find readings of voltmeter V_1 and V_2



Solution:

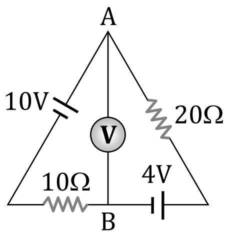
$$i = \frac{E_{eq}}{R + r_1 + r_2} = \frac{15 - 10}{5} = \frac{5}{5} = 1A$$

TPD of 15 V emf cell is = 15-1×1 = 14V

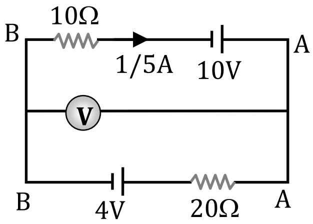
TPD of 10 V emf cell is = 10 + 1 × 1 = 11V

Illustration 48:

Find out the Reading of volt meter?



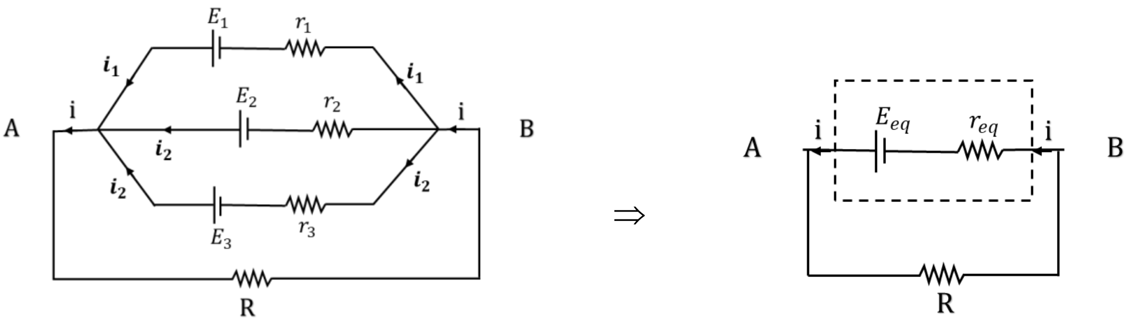
Solution:



Current flowing in the circuit $i = \frac{E}{R} = \frac{10 - 4}{20 + 10} = \frac{1}{5}A$

$\Rightarrow V_B - 10 \times \frac{1}{5} + 10 = V_A$ $\quad\quad \Rightarrow V_A - V_B = 8V$

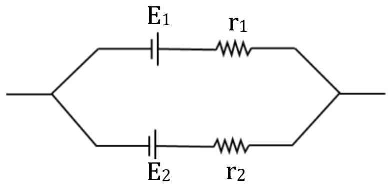
B. Parallel Combination



$$E_{eq} = \frac{\frac{E_1}{r_1} + \frac{E_2}{r_2} + \frac{E_3}{r_3} ...}{\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} ...} \qquad \frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} ...$$

TPD (V) = $E_{eq} - ir_{eq}$

When cell support each other

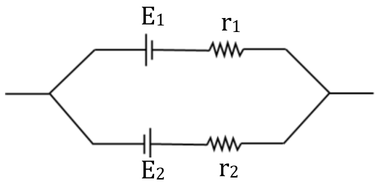


$$E_{eq} = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 + r_2}{r_1 + r_2}$$

When cells are connected with opposite polarity

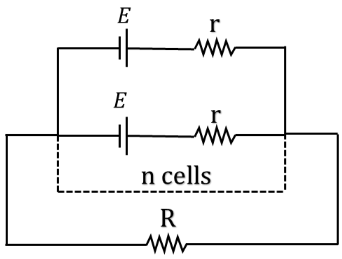
Let $E_1 > E_2$



$$E_{eq} = \frac{E_1 r_2 - E_2 r_1}{r_1 + r_2}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2}$$

Note: If n identical cells are connected in parallel and then :-



$$E_{eq} = n\frac{E/r}{n/r}$$

$$E_{eq} = E$$

$$r_{eq} = \frac{r}{n}$$

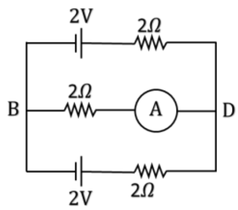
Current in the circuit $\quad i = \dfrac{E}{R + \frac{r}{n}} = \dfrac{nE}{nR + r}$

If r << nR $\quad$ i = E/R = Current from any one cell when connected with the external resistance

If r>> nR $\quad i = \frac{nE}{r}$ = n × current from any cell when short-circuited

Illustration 49:

Reading of Ammeter = ?

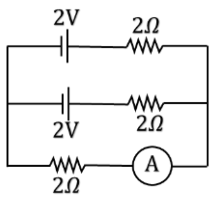


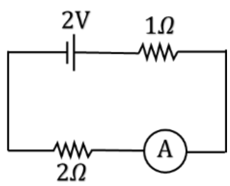
Solution:

$$E_{eq} = 2V$$

$$R_{net} = 2 + 1 = 3\Omega$$

$$I = \frac{2}{3}A$$

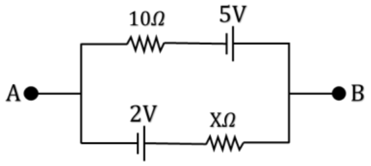




Hence reading of ammeter = $\frac{2}{3}$A

Illustration 50:

If $V_B - V_A = 4V$ then x will be ?



Solution:

$$V_B - V_A = \frac{E_2r_1 + E_1r_2}{r_1 + r_2}$$

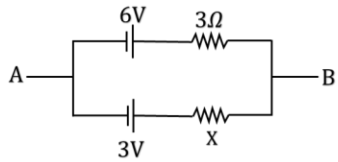
$$\Rightarrow 4 = \frac{5x+20}{10+x}$$

$$\Rightarrow 4(10 + x) = 5x + 20$$

$$\Rightarrow 40 + 4x = 20 + 5x \quad \Rightarrow x = 20$$

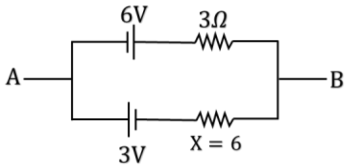
Illustration 51:

In the given diagram potential difference between A & B is 5V. Now terminals of the 3V cell are reversed then find new potential difference between A & B.



Solution:

$$E_{eq} = \frac{E_1r_2 + E_2r_1}{r_2 + r_1}$$



$$\Rightarrow 5 = \frac{6x+9}{3+x} \Rightarrow 15 + 5x = 6x + 9 \quad \Rightarrow x = 6$$

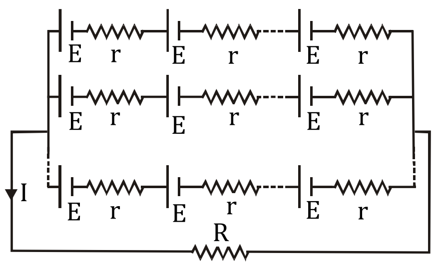
Reversing the terminals :-

$$V_A - V_B = E_{eq} = \frac{36-9}{6+3} = 3V$$

Circuit Analysis involving combination of cells

If n identical cells are connected in series and there are m such branches in the circuit then

Total number of cells in this circuit = nm



Internal resistance of the cells connected in a row = nr

total e.m.f. of the cells connected in a row = E_{net} = nE

Since there are m such rows,

Total internal resistance of the circuit $r_{net} = \frac{nr}{m}$

Total e.m.f. of the circuit = total e.m.f. of the cells connected in a row

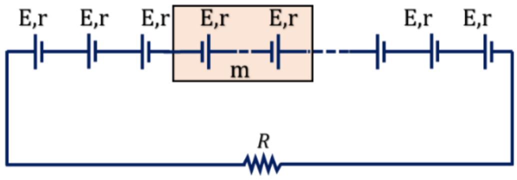
= E_{net} = nE

Current in the circuit $I = \frac{E_{net}}{R + r_{net}} = \frac{nE}{R + \frac{nr}{m}}$

Note: Current I in the circuit is maximum when external resistance in the circuit is equal to the total internal resistance of the cells.

Special case

If in a battery of n identical cells, m cells are wrongly connected.



Let m be the number of wrongly connected cells.

Number of cells helping one another = (n-m)

Total emf of such cells = (n-m)E

Total emf of cells opposing = mE

Resultant emf of battery= (n-m)E – mE = (n-2m) E

Total internal resistance of cells = nr

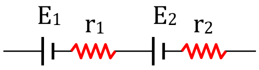
($\therefore$ resistance remains same irrespective of way of connections of cells.)

E_{net} = (n-2m)E (Condition n > 2m)

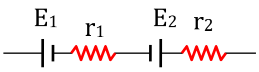
$$I_{net} = \frac{(n-2m)E}{R+nr}$$

★ Golden Key Points ★

- In series grouping of cells, their emfs are additive or subtractive, while their internal resistances are always additive.

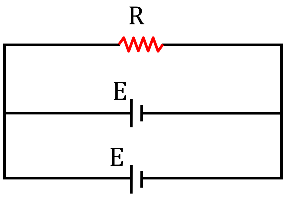


$E_{eq} = E_1 + E_2$ and $r_{eq} = r_1 + r_2$

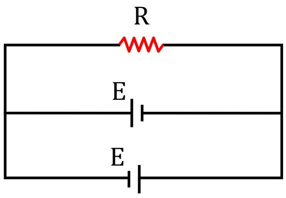


$E_{eq} = E_1 - E_2 (E_1 > E_2)$ and $r_{eq} = r_1 + r_2$

- In Parallel grouping of two identical cells having no internal resistance, we have the situation in figure.

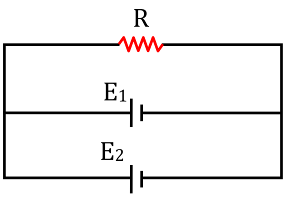


$E_{eq} = E$



$E_{eq} = 0$

- When two cells of different emf and no internal resistance are connected in parallel then equivalent emf is indeterminate.



Note that connecting a wire with a cell but with no resistance is equivalent to short-circuiting. Therefore, the total current that will be flowing will be infinite.

Wheatstone Bridge

It is used to find unknown resistance in circuit.

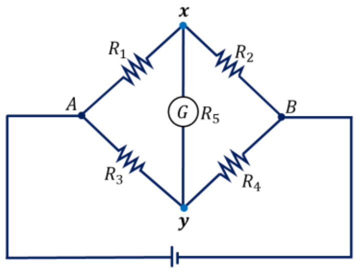
None of the shown resistance are in series and none of them are in a parallel.

Balancing condition

(a) If $\left[\frac{R_1}{R_2} = \frac{R_3}{R_4}\right]$ then bridge is said to be balanced, in this case:

$[V_x = V_y]$

Thus, no current flows in xy branch.



Proof

In balancing condition

$i_g = 0$ i.e. $V_x = V_y$

$\therefore\ V - V_x = V - V_y$

$i_1R_1 = i_2R_3$ ...(i)

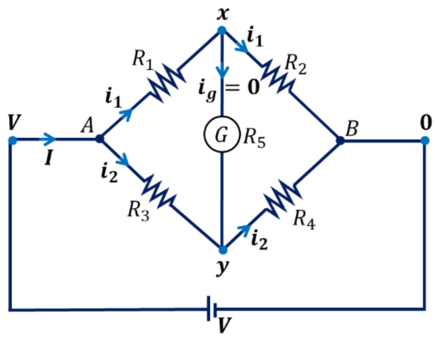
similarly

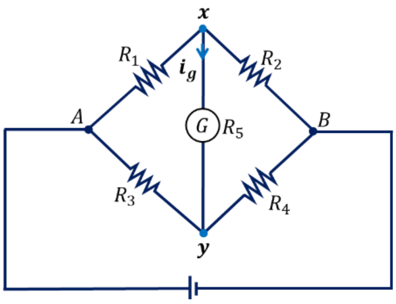
$V_x - 0 = V_y - 0$

$i_1R_2 = i_2R_4$ ...(ii)

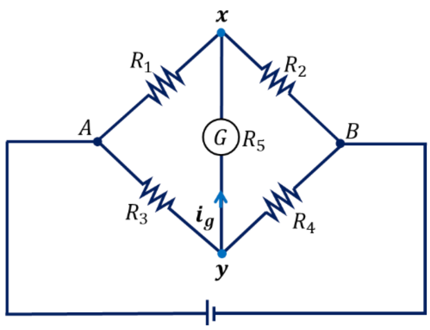
On dividing eq. (i) from (ii) we get

$\frac{R_1}{R_2} = \frac{R_3}{R_4}$ or $R_1R_4 = R_2R_3$





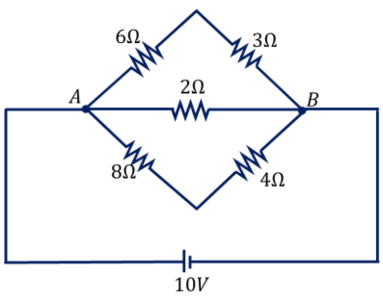
(b) If $\frac{R_1}{R_2} < \frac{R_3}{R_4}$; then $V_X > V_y$



(c) If $\frac{R_1}{R_2} > \frac{R_3}{R_4}$; then $V_X < V_y$

Illustration 52:

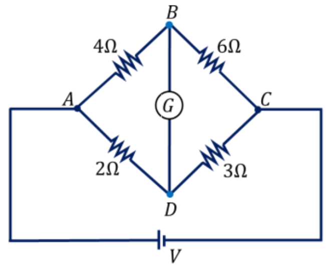
Find current in 2Ω resistance?



Solution:

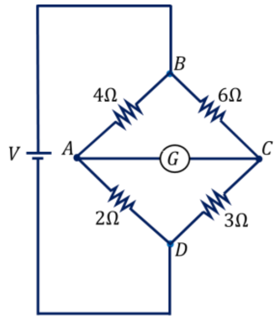
Potential difference across AB is 10 volt

Illustration 53:



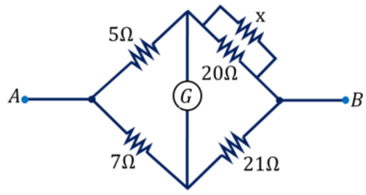
A balanced WSB is shown. If we interchange the position of battery and galvanometer then find current in galvanometer :

Solution:



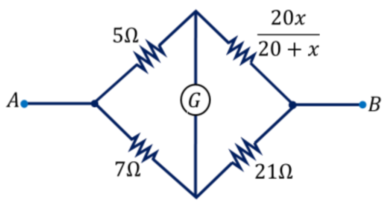
After interchange it will still remain balanced WSB so there will be no current flowing through G.

Illustration 54:



If reading of voltmeter of V is zero find equivalent resistance between A and B (R_{AB}).

Solution:



$$\frac{5}{7}=\frac{\frac{20x}{20+x}}{21}$$

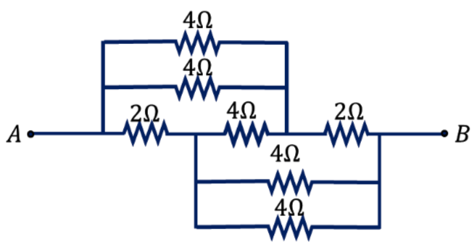
$$\Rightarrow \quad 3\times 5=\frac{20x}{20+x}$$

$$\Rightarrow \quad 300+15x=20x$$

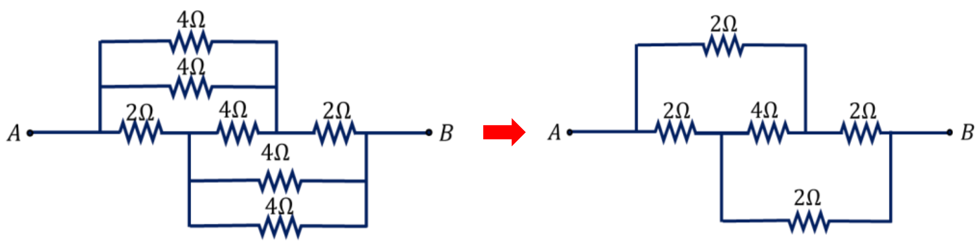
$$\Rightarrow \quad 300=5x \;\Rightarrow\; x=60\Omega$$

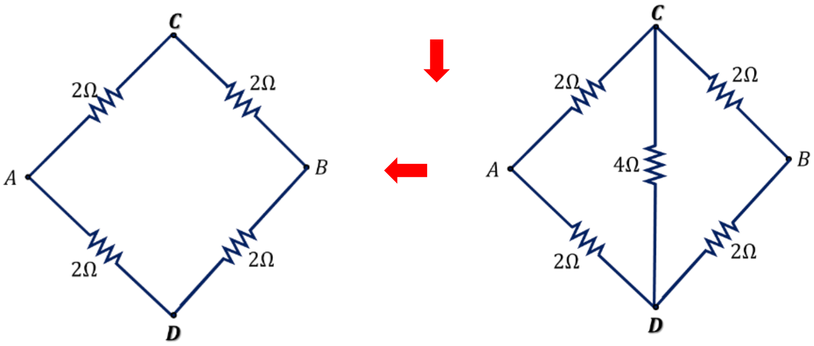
Illustration 55:

Find equivalent resistance across point A and B ?



Solution:





$$R_{AB} = \frac{4 \times 4}{4+4} = 2\Omega$$

Balanced wheatstone bridge

Golden Key Points

The wheatstone bridge is most sensitive when the resistances in all its four branches are almost equal.

Electrical Energy and Power

Cause of Heating

1. When current passes through the conductor then drifting electrons transfer their energy to atoms in collisions which increases thermal energy of the conductor, results in increase in temperature. Energy is lost in surroundings in form of heat.
2. Heat energy released from the conductor is equal to work done by the battery.
3. Let voltage 'V' is applied across the conductor of resistance R and a charge dq flows through it in time dt the work done by the battery in time dt is -

   $dW = (dq)V = (idt)V \ = (idt)(iR) \ = i^2Rdt$

   $\int dW = \int i^2Rdt$

   $H = \int i^2Rdt$ (as H = heat released = W)

   If I = constant

   $$H = i^2Rt = \frac{v^2}{R} = Vit$$

   SI unit : joule

   Practical Units : 1 kilowatt hour (kWh)

   $1kWh = 3.6 \times 10^6$ joules = 1 unit

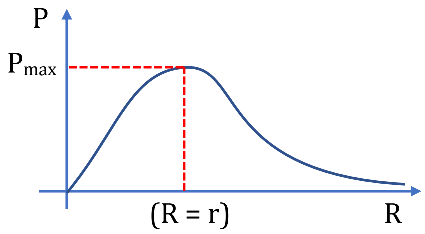
   1 BTU (British Thermal Unit) = 1055 J
4. Power delivered to the load resistance.

   $$P = \frac{dw}{dt} = \frac{i^2Rdt}{dt} = i^2R = \frac{V^2}{R} = Vi$$

   SI unit **:** Watt

   The watt–hour meter placed on the premises of every consumer records the electrical energy consumed.

5. Maximum power transfer theorem –



(a) $P = i^2R = \left[\frac{E}{R+r}\right]^2 R$

If P = max

Then $\frac{dP}{dR} = 0 \Rightarrow \frac{d}{dR}\left[\left[\frac{E}{R+r}\right]^2 R\right] = 0$

On solving R = r

So, to deliver maximum power to the load resistance. Load resistance should be equal to internal resistance of the cell.

(b) $i_{P(max)} = \frac{E}{R+R} = \frac{E}{2R} = \frac{E}{2r}$ where $i_{P(max)}$ = current in case of max. power delivered.

(c) $P_{(max)} = i^2R\left[\frac{E}{2R}\right]^2 R$

$$\boxed{P_{(max)} = \frac{E^2}{4R} = \frac{E^2}{4r}}$$

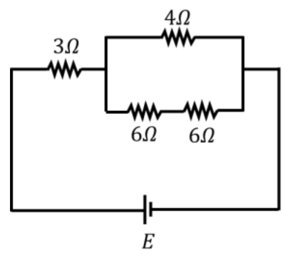
Illustration 56:

If power consumed by 4W resistance is 36 watt then find :

(i) Power consumed by 3W resistance

(ii) Value of E

(iii) Total power consumed in the circuit



Solution:

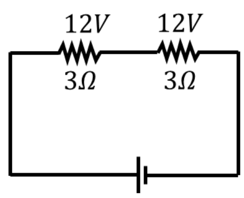
$P = \frac{v^2}{R}$

$\Rightarrow \quad 36 = \frac{v^2}{4}$

$\Rightarrow \quad V = 12$

$P_{3\Omega} = \frac{V_A^2}{R} = \frac{144}{3} = 48$

$P_{total} = \frac{V_{total}^2}{R_{total}} = \frac{(24)^2}{3+3} = 96W$

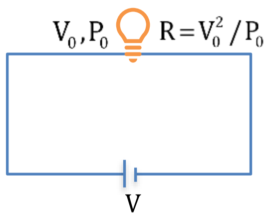


$E = 12 + 12 = 24$

Golden Key Points

- Resistance of a bulb (or other appliances) : Let a bulb be designed to operate on a voltage V_0 and its power indicated on it be P_0 (See figure) The resistance of the bulb is given by $R = V_0^2 / P_0$

  Now let a potential difference of V is applied across this bulb, then power consumed is given by $P = \frac{V^2}{R} = \left(\frac{V}{V_0}\right)^2 P_0$

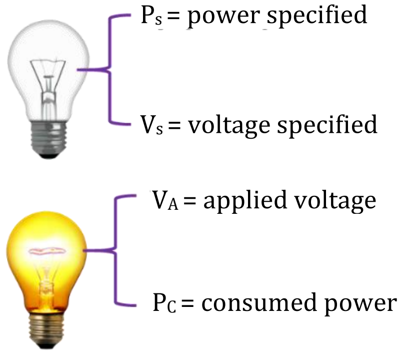


  An electric appliance consumes the specified power P_0 only if it runs at the specified voltage V_0. If the applied voltage V_A is greater than the specified voltage, the appliance may get damaged as in this situation, $I = V_A / R$ will exceed its current capacity $I_C = V_0 / R$. Further, if an appliance is made to run at a voltage lower than the specified then true power consumption will be less than the specified value.

Electric Bulbs and Heaters

Applications to Electric Bulbs and Heaters

If $V_A > V_S$, then electrical device will not work or will be fused.



- **Resistance of an electrical device**

$$R = \frac{V_s^2}{P_s} = \frac{V_A^2}{P_C} \Rightarrow \frac{V_S^2}{P_S} = \frac{V_A^2}{P_C}$$

Hence $P_C = \left(\frac{V_A}{V_S}\right) P_S$

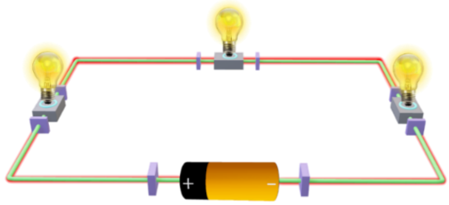
As V_s = constant $\Rightarrow R \propto \frac{1}{P_S}$

$P_S \uparrow$, $R \downarrow$, thickness of filament $\uparrow$

For example : $R_{40W} > R_{100w}$

- **Combination of bulbs**

(a) Series combination

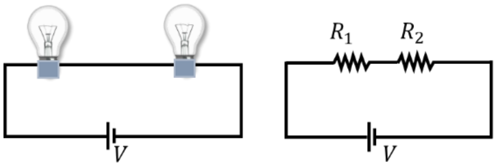


$R = R_1 + R_2$

$$\frac{V_1^2}{P} = \frac{V_2^2}{P_1} + \frac{V_3^2}{P_2}$$

(When rated voltage is not given, take it equal to applied voltage.)

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$



In case of n-identical bulbs

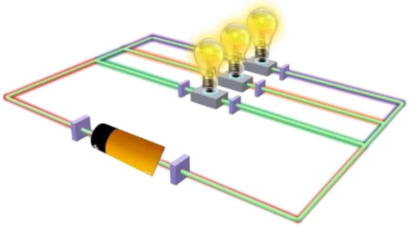
$$P_{series} = \frac{P}{n}$$

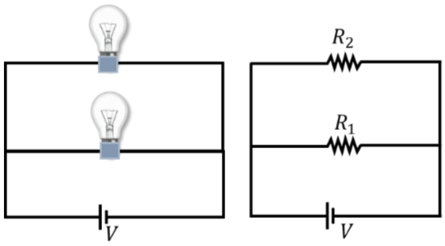
Note: Brightness $\propto$ Power consumed by bulb $\propto R \propto \frac{1}{P_{rated}}$.

Bulb of lesser wattage will shine more.

For same current $P = I^2R$ so $P \propto R \Rightarrow R\uparrow, P\uparrow$

(b) Parallel combination





$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{P}{V_S^2} = \frac{P_1}{V_S^2} + \frac{P_2}{V_S^2}$$

$P = P_1 + P_2$

In case of n-identical bulbs

$P_{parallel} = nP$

Note: Brightness $\propto$ power consumed by bulb $\propto \frac{1}{R} \propto P_s$

Bulb of greater wattage will shine more.

For same V, $P = \frac{V^2}{R}$ more power will be consumed in smaller resistance $P \propto \frac{1}{R}$

- Let two coil boil some amount of water in times t_1 and t_2 separately now these coils are connected in series and then parallel then time required to boil same water

In series combination	**In parallel combination**
$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$	$P = P_1 + P_2$
$\frac{t}{H} = \frac{t_1}{H} + \frac{t_2}{H}$	$\frac{H}{t} = \frac{H}{t_1} + \frac{H}{t_2}$
$t = t_1 + t_2$	$\frac{1}{t} = \frac{1}{t_1} + \frac{1}{t_2} \Rightarrow t = \frac{t_1 t_2}{t_1 + t_2}$

Illustration 57:

A coil boil some amount of H_2O in 20 mins. Now this coil is cut into two equal parts and these are connected in parallel then find time taken by this combination to boil same amount of water.

Solution:

$H = \frac{V^2 t}{R}$

As H and V are same

$\therefore t \propto R$

$\Rightarrow t' = \frac{t}{4} = 5\text{min}.$

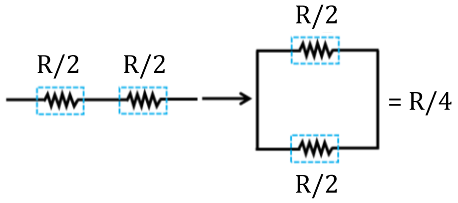


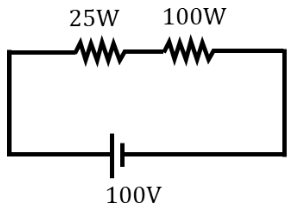
Illustration 58:

Two bulbs of specifications 25W, 100 V; 100W, 100 V are connected in series to a source of 100 V then find

(i) Resistance of each bulb (ii) Power consumed by each bulb

(iii) Which bulb will have more brightness.

Solution:



(i) $R = \frac{V_s^2}{P_s}$

$R_1 = \frac{100\times100}{25} = 400\Omega \quad , \quad R_2 = \frac{100\times100}{100} = 100\Omega$

(ii) $R \propto \frac{1}{P_C} \Rightarrow \frac{P_1}{P_2} = \frac{R_2}{R_1} = \frac{1}{4}$

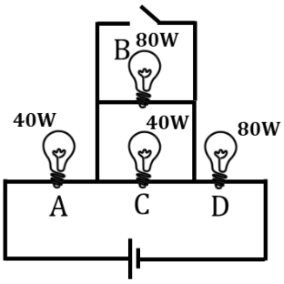
(iii) $V_{A_1} = \frac{4}{5} \times 100 = 80V, \quad V_{A_2} = \frac{1}{5} \times 100 = 20V$

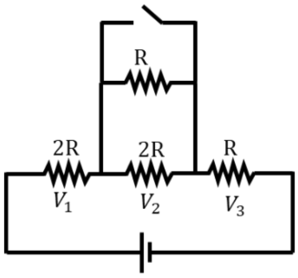
$P_{C_1} = \frac{{V_{A_1}}^2}{R_1} = \frac{80\times80}{400} = 16W, \quad P_{C_2} = \frac{{V_{A_2}}^2}{R_1} = \frac{20\times20}{100} = 4W$

As 25 watt bulb is consuming more power so it will have more brightness ($\because$ Brightness $\propto P_C$).

Illustration 59:

Compare brightness of the bulb before and after switch is closed.





Solution:

Let resistance of 80 W be R so resistance of 40 W be 2R.

As A and D are in series

So $P_A > P_D$

and B and C are in parallel $P_B > P_C$

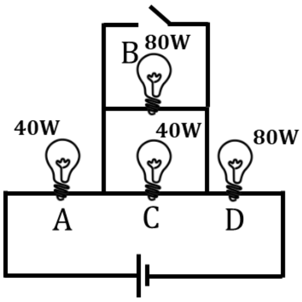
Here $V_A > V_D > V_C$

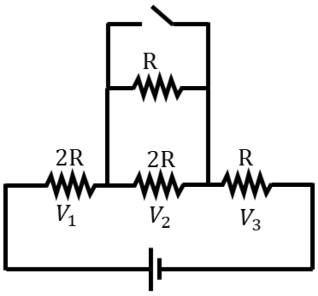
$$P_B = \frac{V_C^2}{R}$$

$$P_D = \frac{V_D^2}{R}$$

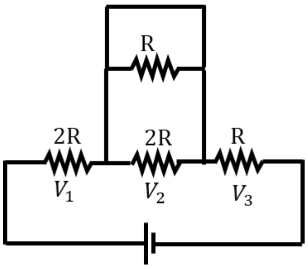
$P_D > P_B$

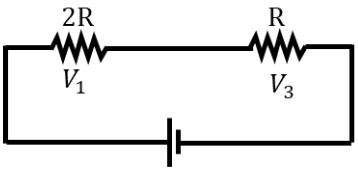
$P_A > P_D > P_B > P_C$





After switch is closed bulbs B and C are short-circuited and they will die out but brightness's of the bulb A & D increases due to increase in voltage.

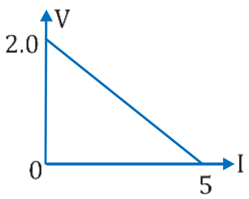




BEGINNER'S BOX-4

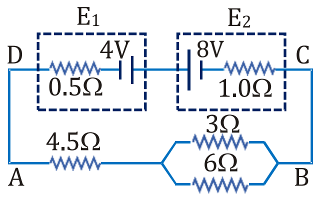
1. A battery of 20 cells (each having e.m.f. 1.8 V and internal resistance 0.1 Ω) is charged by 220 volts and the charging current is 15 A. Calculate the resistance to be put in the circuit. **PCE251**

2. For a cell, the graph between the potential difference (V) across the terminals of the cells and the current I drawn from the cell is as shown in figure. Calculate the e.m.f. and the internal resistance of the cell.

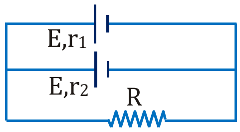


PCE252

3. In the given circuit, calculate the value of current in the 4.5 Ω resistor and indicate its direction. Also calculate the potential difference across each cell. **PCE253**



4. Two cells of equal e.m.f. E but of different internal resistances r_1 and r_2 are connected in parallel and the combination is connected with an external resistance R to obtain maximum power in R. Find the value of R. **PCE254**



5. What is the largest voltage you can safely apply across a resistor marked 196 Ω; 1W ? **PCE255**

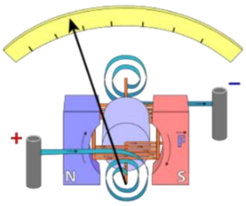
6. Two bulbs whose resistances are in the ratio 1 : 2 are connected in parallel to a source of constant voltage. What will be the ratio of the power dissipation in these? **PCE256**

Golden Key Points

Instruments based on heating effects of current, work with both A.C. and D.C. Equal values of A.C. (RMS) and D.C. produce equal heating effect. Due to this reason, the brightness of bulb is same whether it is operated by A.C. or an equivalent D.C.

Galvanometer, Ammeter and Voltmeter:

GALVANOMETER



1. It is used to detect the current i.e. whether current is flowing or not and whether direction of current is reversed or not.
2. Its working is based on torque on current carrying coil. ($\theta \propto i$)
3. Measures small value of current.
4. Maximum value of current at which galvanometer shows maximum deflection or full scale deflection is known as 'i_g'.
5. Sensitivity = Response/input

   (a) Current sensitivity = $\frac{\theta}{I}$ (b) Voltage sensitivity = $\frac{\theta}{v}$

AMMETER

An ammeter is connected in series with a current carrying wire to measure current passing through it. Since it is connected to the wire and has a finite amount of resistance, there will be some potential difference across it. This will lead to the change in actual flow of current. In an ideal situation where the ammeter can measure the actual current passing through a wire, there should be no voltage drop across it and hence its resistance should be zero.

VOLTMETER

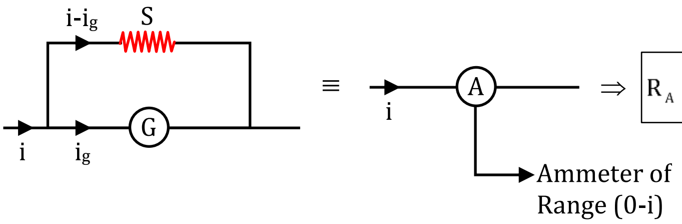
A voltmeter is connected in parallel to the current carrying wire, to measure the potential difference between two points. Since it is connected to the wire, a finite amount of current will pass through it. This will lead to the change in actual flow of current and hence change in the actual potential difference. In an ideal situation where the voltmeter can measure the actual potential difference across two points, there should be no current passing through it and hence its resistance should be infinite.

> **Golden Key Points**
>
> - The rate of variation of deflection depends upon the magnitude of deflection itself and hence on the accuracy of the instrument.
> - A suspended coil galvanometer can measure currents of the order of 10^{-9} amperes.

Conversion of Galvanometer into Ammeter and Voltmeter

AMMETER

1. To convert a galvanometer into an ammeter a very small resistance is connected in parallel to the galvanometer called SHUNT.
2. Resistance of an ammeter is very small and it is zero for ideal ammeter i.e. ideal ammeter behaves like conducting wire.
3. Value of shunt.



$$\Rightarrow R_A = \frac{SG}{S+G}$$

$$V_S = V_G \quad \Rightarrow \quad (i - i_g)S = i_g G$$

$$\Rightarrow S = \left[\frac{i_g}{i - i_g}\right]G \quad \Rightarrow S = \left[\frac{1}{\frac{i}{i_g} - 1}\right]G \quad \Rightarrow S = \left[\frac{G}{n-1}\right] \quad \text{Where } n = \frac{i}{i_g}$$

Where, G – resistance of galvanometer

i_g – range of galvanometer or current required to produce full deflection.

i – Range of ammeter [Max. current can be measured.]

Illustration 60:

A galvanometer of resistance 99Ω of range 10 mA is converted into an ammeter of range 10 A then find required resistance.

Solution:

Galvanometer $\rightarrow$ Ammeter

$G = 99\Omega$

$I_g = 10 \text{ mA} = 10^{-2}$

$I = 10 \text{ A} \Rightarrow n = \frac{10}{10^{-2}} = 1000$

$$S = \frac{G}{(n-1)} = \frac{99}{999} = 0.099\Omega$$

Illustration 61:

Resistance of a galvanometer is 45Ω. If only 10% of the main current passes through the galvanometer when it is connected to a shunt then find shunt resistance.

Solution:

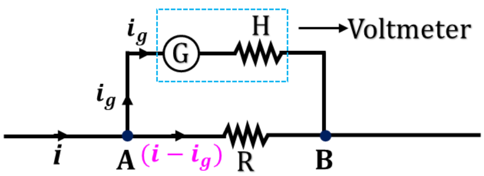
$$i_g = \frac{10}{100} i = 0.1i$$

$$n = \frac{i}{i_g} = 10 \Rightarrow S = \frac{G}{(n-1)} = \frac{45}{9} = 5\Omega$$

VOLTMETER

1. To convert galvanometer into voltmeter, a high resistance is connect in series with a galvanometer.
2. Resistance of voltmeter is very high and it is infinite for ideal voltmeter.

   so ideal voltmeter $\longrightarrow$ open circuit



Resistance R and (G+H) are in parallel between points A and B so we can write

$$i_g(G + H) = (i - i_g)R$$

$$i_gG + i_gH = (i - i_g)R$$

$$V_g + V_H = V \quad \Rightarrow \quad V_H = V - V_g$$

$$\Rightarrow i_gH = V - V_g \quad \Rightarrow \quad \frac{V_g}{G}H = V - V_g$$

$$\Rightarrow V_gH = (V - V_g)G \quad \Rightarrow \quad H = \left(\frac{V - V_g}{V_g}\right)G = \left(\frac{V}{V_g} - 1\right)G$$

$$H = (n-1)G \quad \text{where } n = \frac{V}{V_g}$$

V : Range of voltmeter , V_g : Range of galvanometer

Illustration 62:

Resistance of a galvanometer is 50 Ω and its current sensitivity is 1 mA/div and it has 20 division. It is to be converted into a voltmeter of 10V then find required resistance for the conversion.

Solution:

$I_g = 20 \times 1 = 2 \times 10^{-2}$ A

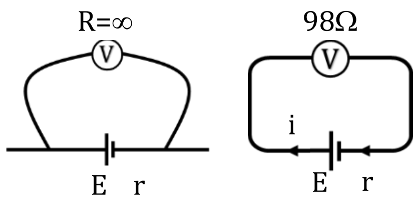
$V_g = i_g G = 2 \times 10^{-2} \times 50 = 1V$

$$H = \left(\frac{10}{1} - 1\right) \times 50 = 450\Omega$$

Illustration 63:

A voltmeter of 98 Ω is used to measure emf of a cell 2V, 2Ω. Find error in its reading.

Solution:



For ideal voltmeter :

When voltmeter of 98 Ω is used to measure emf:

Reading = E – ir ⇒ error = ir

$$\text{Error} = \left(\frac{E}{R+r}\right)r = \left(\frac{2}{98+2}\right)\times 2 = 0.04\text{V}$$

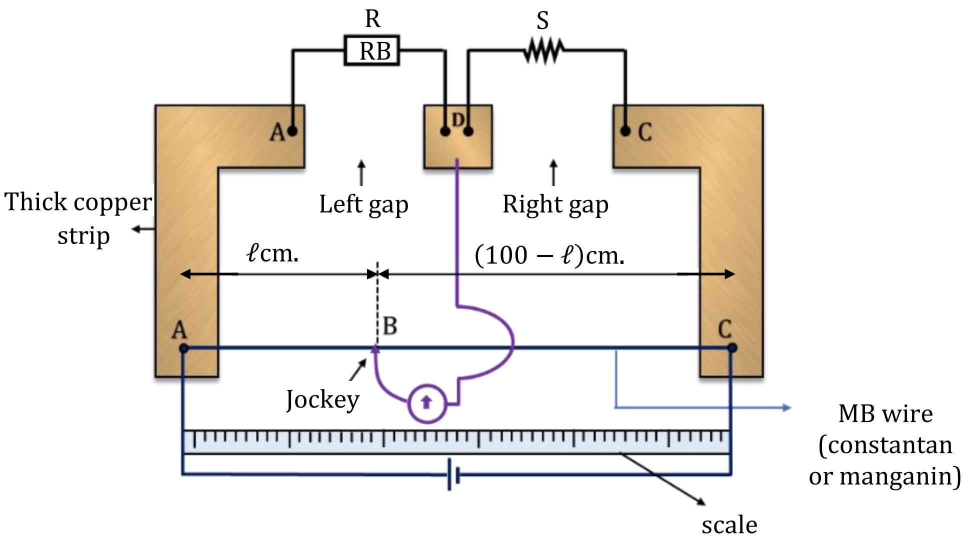
Golden Key Points

- I_g is the current for full scale deflection. If the current corresponding to a deflection of one division on the galvanometer scale is k and N is the total number of divisions on one side of the zero of galvanometer scale then $I_g = k \times N$.
- A ballastic galvanometer is a specially designed moving coil galvanometer used to measure charge flowing through a circuit for a small time interval.
- To increase the range of an ammeter, a shunt is connected in parallel with the galvanometer.
- To convert an ammeter of range I amperes and resistance $R_g\,\Omega$ into an ammeter of range nI amperes, the value of resistance to be connected in parallel will be $\frac{R_g}{n-1}\Omega$.
- To increase the range of a voltmeter a high resistance is connected in series with it.
- To convert a voltmeter of resistance $R_g\ \Omega$ and range V volts into a voltmeter of range nV volts, the value of resistance to be connected in series will be $(n-1)R_g\,\Omega$.
- Resistance of ideal ammeter is zero & resistance of ideal voltmeter is infinite.

Meter Bridge

(i) It is used to find unknown resistance in circuit.

(ii) Its working is based on WSB principle.



(iii) MB circuit :-

a. It has a wire of 1 m on which balancing point is to be search.

b. Resistance of copper-strips are negligible due to very low resistivity and greater cross-sectional area.

c. MB wire is taken of constantan or manganin due negligible temperature co-efficient of resistance α. So resistance of the wire doesn't change during the experiment.

(iv) Working

Initially a resistance R is taken out from a resistance box and jockey is tapped on the wire. Find balancing point B where deflection in galvanometer becomes zero. Distance of this point B from left end (A) is called balancing length 'ℓ'

In balancing condition

(a) $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{R}{\rho\ell / A} = \frac{S}{\frac{\rho(100-\ell)}{A}} \Rightarrow S = \left[\frac{100-\ell}{\ell}\right]R$

(b) Balancing length is independent of the radius of the cross-section of the wire.

(c) Jockey should not be rubbed against the wire because rubbing action changes the cross-section of the wire.

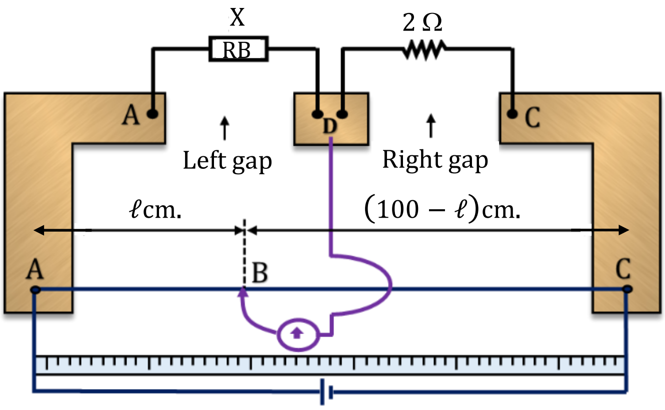
(d) END ERROR and END CORRECTION – this error occurs due to neglecting resistances at the connection point A & C. End corrections e_1 & e_2 are the lengths corresponding to resistances at connection points so equation becomes.

$$\frac{\ell + e_1}{(100-\ell)+e_2} = \frac{R}{S}$$

Illustration 64:

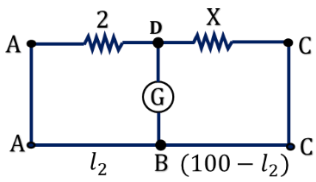
In meter bridge experiment, x Ω is connected in left side and 2 Ω (< x) is connected in right side. Now position of these two resistances interchanged and balancing length is shifted by 20 cm. Find value of x.

Solution:



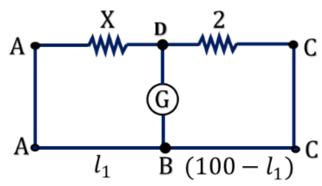
$\frac{x}{\ell_1} = \frac{2}{100-\ell_1}$

$\Rightarrow x = \frac{2\ell_1}{100-\ell_1}$ ...(1)



On inter changing

$$\frac{2}{\ell_1}=\frac{x}{100-\ell_2}$$



$$\Rightarrow \quad x=\frac{2(100-\ell_2)}{\ell_2} \quad ...(2)$$

$\therefore \ell_1-\ell_2=20$

Using (1) and (2) we get

$\ell_1+\ell_2=100$

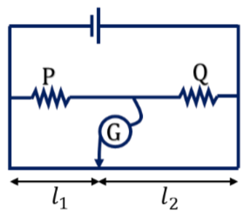
$\ell_1-\ell_2=20$ (given)

$2\ell_1=120$

$\Rightarrow \quad \ell_1=60$

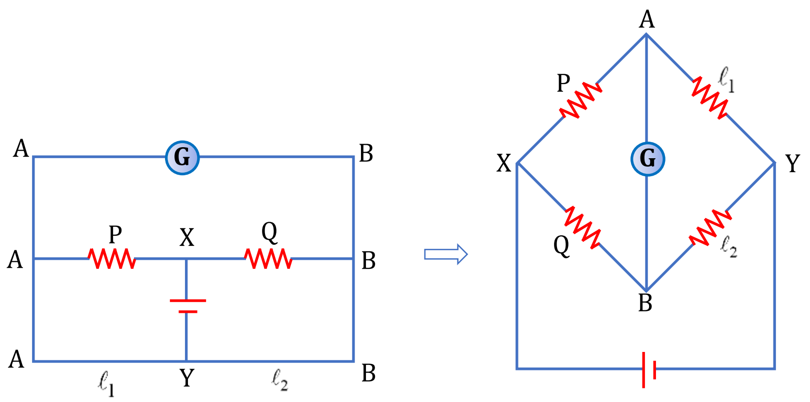
$$x=\frac{2\ell_1}{100-\ell_1}=\frac{2(60)}{100-60}=\frac{120}{40}=3\Omega$$

Illustration 65:



The metere bridge shown is in balanced position with $\frac{P}{Q}=\frac{\ell_1}{\ell_2}$. If we now interchange the positions of galvanometer and cell, will the bridge work? If yes, what will be balance condition?

Solution:



$$\frac{P}{\ell_1}=\frac{Q}{\ell_2}\Rightarrow\frac{P}{Q}=\frac{\ell_1}{\ell_2}$$

Yes, the bridge will work and the balance condition will $\frac{P}{Q}=\frac{\ell_1}{\ell_2}$

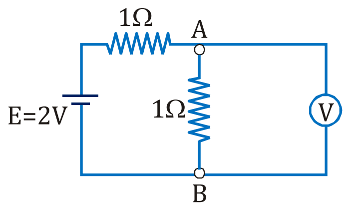
BEGINNER'S BOX-5

1. A galvanometer of resistance 100 Ω gives full scale deflection for 10 mA current. What should be the value of shunt, so that it can measure currents upto 100 mA ? **PCE257**
2. The resistance of a moving coil galvanometer is 20 Ω. It requires 0.01 A current for full scale deflection. Calculate the value of resistance required to convert it into a voltmeter of range 20 volts. **PCE258**
3. The resistance of an ammeter of range 5 A is 1.8 Ω. A shunt of 0.2 Ω is connected in parallel to it. When its indicator shows a current of 2 A then what will be the effective current? **PCE259**

4. A galvanometer of resistance 100 Ω gives full scale deflection for a current of 10^{-5} A. Calculate the shunt required to convert it into an ammeter of 1 ampere range. **PCE260**

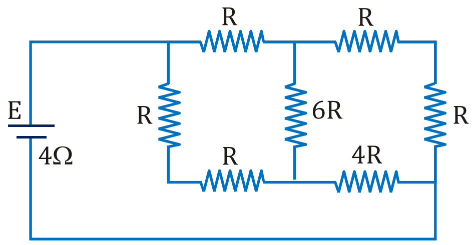
5. The shunt required for 10% of main current to be sent through a moving coil galvanometer of resistance 99Ω will be **PCE261**

6. In the given circuit, the voltmeter and the electric cell are ideal. Find the reading of the voltmeter (in volts)



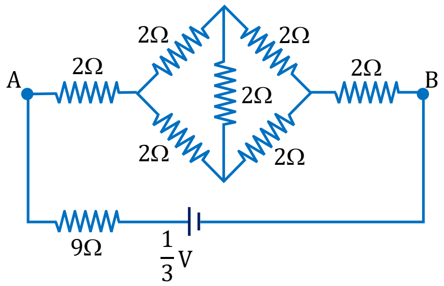
PCE262

7. A battery of internal resistance 4 Ω is connected to the network of resistances as shown in figure. In order that maximum power is delivered to the network, the value of R in Ω should be



PCE263

8. For the given circuit find out the –

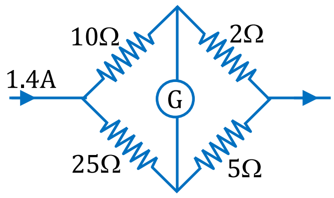


(a) total resistance of the circuit across the battery terminals.

(b) potential difference between A and B.

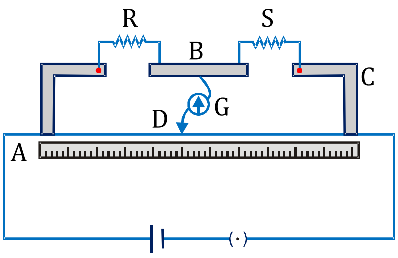
PCE264

9. Calculate the current flowing through the resistance 2 Ω.



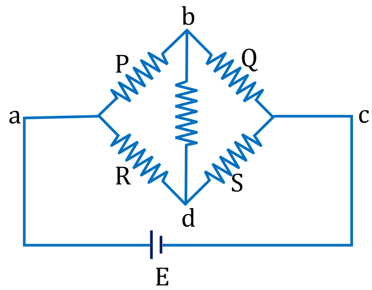
PCE265

10. Circuit diagram of metre bridge is shown in figure. The null point is found at a distance of 40 cm from A. If now a resistance of 12 Ω is connected in parallel with S, the null point shifts to 64 cm. Determine the values of R and S.



PCE266

11. What is the relation between P, Q, R and S if current in the branch bd is



(a) Zero?

(b) from b to d?

PCE267

Golden Key Points

- The meter bridge is also referred as the slide wire bridge.
- While performing the meter bridge experiment, do NOT slide the jockey continually along the wire. Just lift the jockey and TOUCH (not press) the constantan wire at specific points until you find the point of zero deflection. If you apply pressure (unintentionally even), the cross-sectional area of the wire will change and the experiment will be affected for the next student.
  The non-uniformity of the metal wire might lead to the generation of stray resistance and it will create an end error.

BEGINNER'S BOX — ANSWER KEY

BEGINNER'S BOX-1

1. 0.5×10^{-16} A **2.** 2×10^{16} **3.** 0.15 mm/s **4.** 0.02 A ; 2.74×10^{-12} m/s

BEGINNER'S BOX-2

1. $4 \times 10^{-4}\ \Omega$ **2.** 0.4%

3. R = 20 Ω, Resistivity of the wire remains unchanged as it does not change with change in dimensions of a material without change in its temperature.

4. 273°C **5.** (A) – q, (B) – p, (C) – r

6. (A) – s, (B) – p, (C) – q, t,(D) – r

BEGINNER'S BOX-3

1. 2R **2.** $R_{AC} = \left(\frac{n^2+1}{n}\right)r$ **3.** $\frac{1}{100}\ \Omega$

4. 40 Ω **5.** 3.7 A **6.** 3 V

BEGINNER'S BOX-4

1. 10.27 Ω **2.** 2 V ; 0.4 Ω

3. I = 0.5 A, anticlockwise CDABC $E_1 = 4.25$ V, $E_2 = 7.5$ V

4. $\frac{r_1 r_2}{r_1 + r_2}$ **5.** 14 volts **6.** $\frac{P_1}{P_2} = \frac{2}{1}$

BEGINNER'S BOX-5

1. 11.11 Ω **2.** 1980 Ω **3.** 20 A **4.** $10^{-3}\ \Omega$

5. 11 Ω **6.** 1 V **7.** 2 **8.** (a)15 Ω , (b) 0.133 V

9. 1A **10.** $\frac{40}{3}\ \Omega$, 20 Ω **11.** (a) $\frac{P}{Q} = \frac{R}{S}$, (b) $\frac{R}{S} > \frac{P}{Q}$