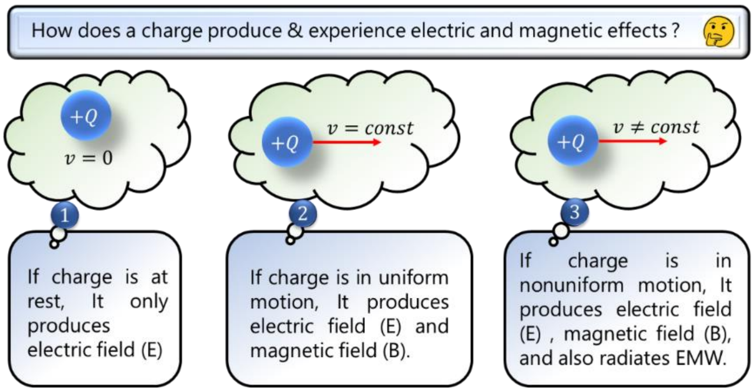
01 Electrostatics

Electric Charge and Field

Electro (charge) + statics (rest) = Electrostatics : It is a branch of physics which deals with the study of forces, fields and potential arising from static charges.

Electric Charges and their Properties

Electric Charge : Charge is an intrinsic property associated with matter due to which it produces and experiences, electric and magnetic effects.



❖ A body is said to be electrified or charged whenever it gains or loses electrons.

Types of charge :

There exist two types of charges in nature.

(i) Positive charge : Due to deficiency of electrons.

(ii) Negative charge : Due to excess of electrons.

Units of charge :

SI $\rightarrow$ coulomb (C)

CGS $\rightarrow$ stat coulomb (stat C) or e.s.u. of charge or franklin

Another CGS $\rightarrow$ e.m.u. of charge or absolute coulomb (abC)

practical unit $\rightarrow$ faraday (F)

1C = 3×10^{9} stat C 1F = 96500 C	Smallest unit of charge $\Rightarrow$ franklin Largest unit of charge $\Rightarrow$ faraday

Dimensional formula of charge : [AT] or $[M^0L^0T^1A^1]$

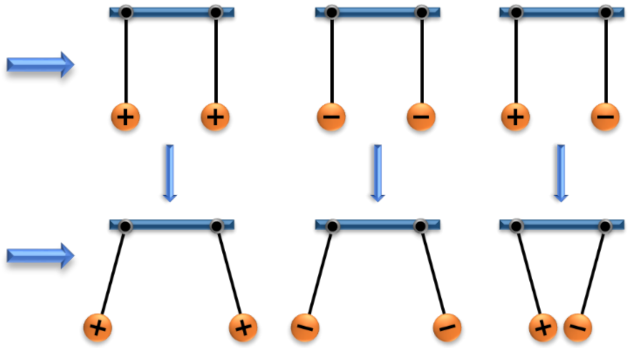
Specific Properties of Charge

(i) **Charge is a scalar quantity :** It has no direction.

(ii) Like point charges always repel each other while unlike point charges always attract each other.

As two charged bodies or balls are brought close to each other as shown

After interaction with each other they finally stay as shown.



If the sizes of charged bodies are very small as compared to the distance between them, we treat them as point charges.

(iii) **Charge is transferable:** If a charged body is kept in contact with another body, then charge can be transferred to another body.

(iv) **Charge is always associated with mass:** Charge cannot exist without mass though mass can exist without charge.

❖ The presence of charge itself is a convincing proof of existence of mass.

Ex. (i) Photons don't have any charge as they have zero rest mass.

Ex. (ii) Rest mass of the charged body can never be zero.

❖ The mass of a body changes after being charged.

When a body is given a positive charge, its mass decreases and

When a body is given a negative charge, its mass increases.

(v) **Charge is realistically invariant, whereas mass is variant :** Charge is independent of frame of reference, i.e. charge on a body does not change whatever be its speed.

Concept

$$\underbrace{m}_{\text{Dynamic mass}} = \frac{\overbrace{m_0}^{\text{Rest mass}}}{\sqrt{1-\underbrace{\frac{v^2}{c^2}}_{\text{Speed of particle}}}}$$

If $v\uparrow$ $\quad v^2$ *also* $\uparrow$ $\quad$ *that means* $\left(\frac{v^2}{c^2}\right)$ *also* $\uparrow$

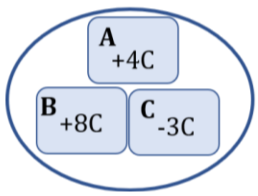
the value of $\left(1-\frac{v^2}{c^2}\right)\downarrow$

That means the particle moving with larger speed will have larger mass.

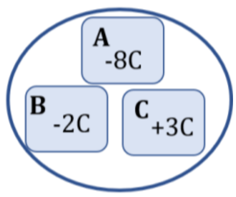
Note : If (v<<c) then m is taken as constant parameter.

(vi) **Charge is additive :** If a system contains n charges q_1, q_2, $-q_3$ .... q_n, then the total charge of the system is obtained simply by adding them algebraically (with their respective signs).

$$Q_t = \sum q = q_1 + q_2 - q_3 + ...q_n$$



Q_{total} = 4 + 8 – 3 = +9C



Q_{total} = –8 – 2 + 3 = –7C

(vii) Charge is conserved : Total charge of an isolated system always remains constant. This is also called as "Law of conservation of charge."

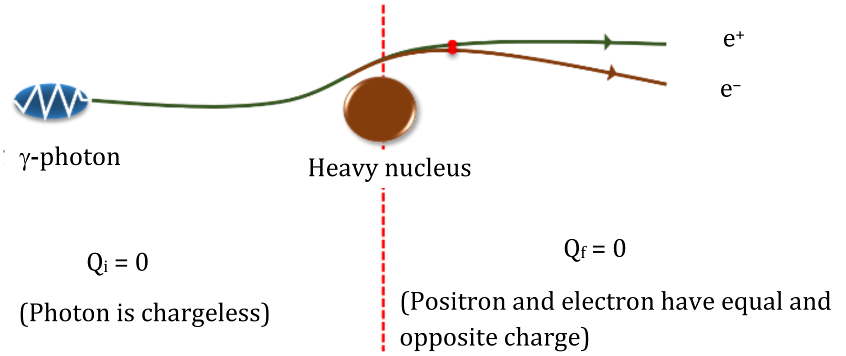
Conservation of charge holds good in all type of reactions, for example

(i) Ionization : $H(atom) \rightarrow H^+ + e^-$

(ii) Radioactive decay : $n \rightarrow p + e^- + \bar{\upsilon}$

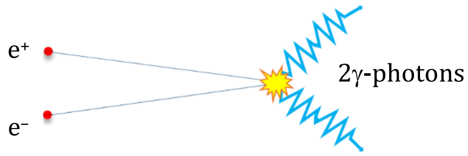
(iii) Pair production phenomenon : conversion of energy into mass.

Pair production phenomenon



Note: For this phenomenon energy of photon must be greater than 1.02 MeV.

(iv) Pair annihilation phenomenon : conversion of mass into energy

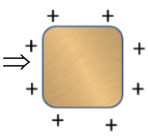


(viii) Charge is quantized :

Charge can have only discrete values, rather than any value.

Charge on any body must be an integral multiple of a basic unit of charge represented by e.

e = basic unit or quanta of charge or minimum transferable value of charge = 1.6×10^{-19} C.

In general we can write $\boxed{Q = \pm ne}$ $\Rightarrow$ 

where n = an integer

n = 0,1,2,3.......

Illustration 1:

Which of the following charges is/are not possible?

(1) $\sqrt{3}$e (2) 48×10^{-19} C (3) 3.2×10^{-20} C (4) 1C

Solution:

(1) $\because$ q = ne $\Rightarrow$ $\sqrt{3}$ e = ne $\Rightarrow$ n = 1.732(fraction) $\rightarrow$ not possible

(2) 48×10^{-19}C = n(1.6×10^{-19})C $\Rightarrow$ n = 30(integer) $\rightarrow$ possible

(3) 3.2×10^{-20}C = n(1.6×10^{-19})C $\Rightarrow$ n = 0.2(fraction) $\rightarrow$ not possible

(4) 1 = n(1.6×10^{-19}) $\Rightarrow$ n = 6.25×10^{18} (integer) $\rightarrow$ possible

Illustration 2:

How many electrons must be removed from a body to make it electrified by 3.2C of charge?

Solution:

From Q = ne $\Rightarrow$ $n = \frac{Q}{e} = \frac{3.2}{1.6\times10^{-19}} \Rightarrow n = 2\times10^{19}$

Illustration 3:

10^{10} alpha particles are ejected per second from a body, then after how much time, the body will acquire a charge of 8 μC?

Solution:

Charge appear per second on body $Q = \left(\frac{N}{t}\right) q_\alpha$

$Q = 10^{10} \times (2 \times 1.6 \times 10^{-19}) = 3.2 \times 10^{-9}\,C/sec$

Let t be the time taken to acquire charge of 8 μC then

$t = \frac{8 \times 10^{-6}}{3.2 \times 10^{-9}} = 2.5 \times 10^{3}\,sec$

Note: For macroscopic charges (big charges) quantisation rule can be ignored.

Conductors & Insulators

Conductors : Those materials which allow electricity to pass through them easily are called conductors. They have electric charge (electrons) that are comparatively free to move inside the material. Ex. Metals, human and animal bodies, earth etc.

Insulators : Those materials which offer high resistance to the passage of electricity through them, are called insulators. Ex. Glass, rubber, plastic, nylon, wood etc.

Methods of Charging

(1) Charging by friction or rubbing :

If we rub one body with another body, electrons get transferred from one body to the another. Both insulators and conductors can be charged by this method.

When an object in column-1 is rubbed against the object of column-2 then they acquire charges specified in the following table.

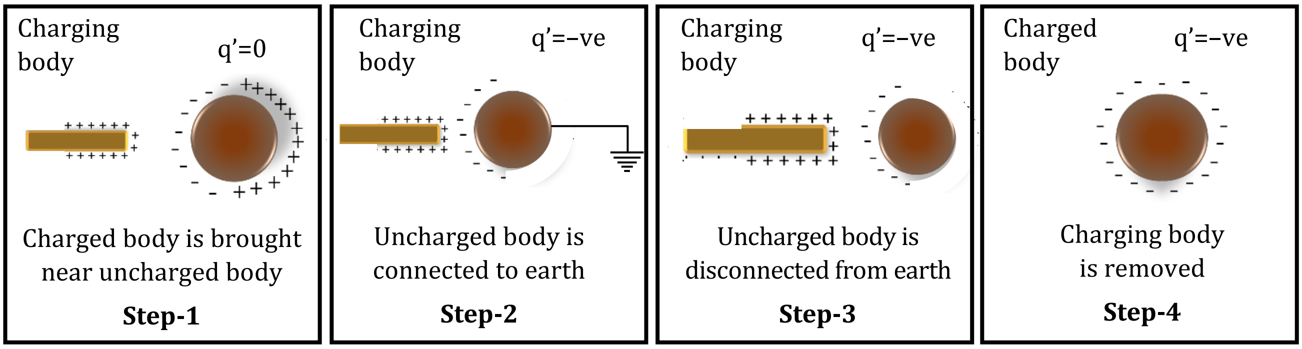
Column-1	Column-2
Positive Charge	**Negative Charge**
Glass rod	Silk cloth
Woolen cloth	Rubber shoe, Amber, Plastic objects
Dry hair	Comb

Note: Clouds get charged due to friction.

(2) Charging by induction :

If a charged body is brought near a neutral body, the charged body will attract opposite charges and repel similar charges present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'. Hence, "Induction is a phenomenon of redistribution of charges on a body in the influence of other charged object or external field."

Charging a body by induction (in four successive steps)



In case of induction it is worth noting that :

(i) Inducing body neither gains nor loses charge.

(ii) The nature of induced charge is always opposite to that of inducing charge.

(iii) Induced charge can be lesser or equal to inducing charge (but never greater).

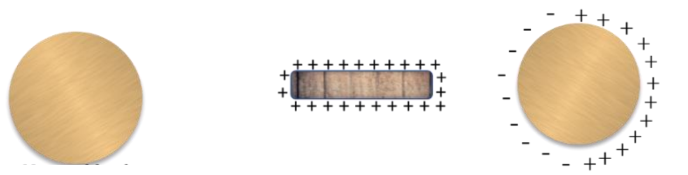
(iv) Induction takes place only in bodies (either conducting or non conducting) and not in particles.

Illustration 4:

If a charged body is put near a neutral conductor, will it attract the conductor or repel it?

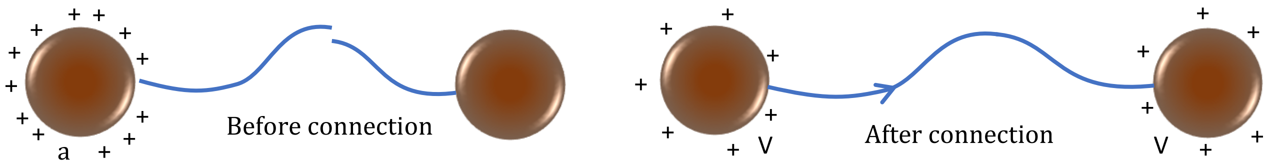
Solution:

They attract each other due to induction effect



Neutral Body   Body is still Neutral

(3) Charging by conduction: Whenever a charged conductor brought in contact with other conductor, charge continuously flows from one to another until their potential becomes equal. This is also called sharing of charges.



In case of conduction it is worth noting that:

(i) Contact is necessary

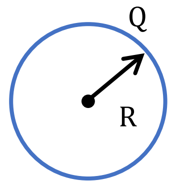
(ii) Only conductors can be charged by this method

(iii) In this method both conductors will acquire same nature of charge.

(iv) Total charge of the system distributes in the ratio of their radii (only for spherical objects)

Consider two charged spheres (q_1, R_1) & (q_2, R_2) are connected by a wire or touched with each other.

For sphere



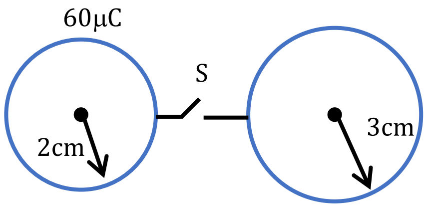
$V = \frac{kQ}{R}$

After conduction let their final charges become Q_1 & Q_2 respectively then $q_1 + q_2 = Q_1 + Q_2$ & $\boxed{\frac{Q_1}{Q_2} = \frac{R_1}{R_2}}$

Illustration 5:

Find final charges on the spheres when switch S is closed.

Solution:



Total charge of the system = Q = 60μC +0= 60μC

Then after conduction

$$\frac{Q_1'}{Q_2'} = \frac{R_1}{R_2} \Rightarrow \frac{2}{3} \quad \Rightarrow \quad Q_1' = \left(\frac{2}{2+3}\right)60\mu C = 24\mu C$$

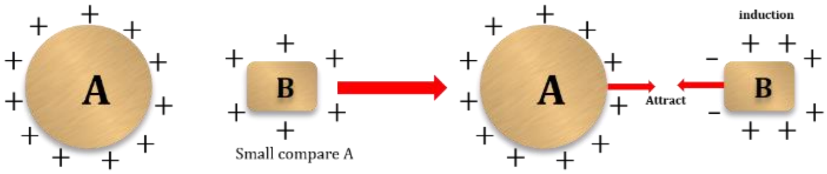
$$Q_2' = \left(\frac{3}{2+3}\right)60\mu C = 36\mu C$$

Illustration 6:
A positively charged body 'A' attracts a body 'B' then charge on body 'B' may be:

Solution:

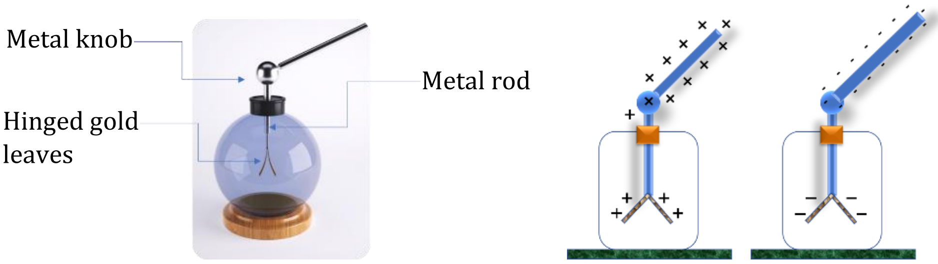
If B is –ve

If B is +ve

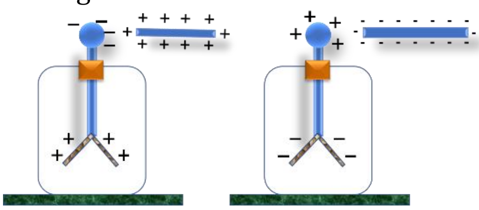


Gold leaf electroscope
It is used to detect the presence of charge.

When a conducting body touches the metal knob of an electroscope, then knob & leaves both wire acquire same nature of charge.



When a charged body is placed near the electroscope then knob will acquire opposite nature of charge & leaves will acquire same nature of charge.



Golden Key Points

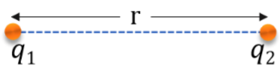
- Charge differs from mass in the following aspects :
  (i) In SI system of units, charge is a derived physical quantity while mass is a fundamental quantity.
  (ii) Charge is always conserved but mass not.
  (iii) Charge cannot exist without mass but mass can exist without charge.
  (iv) Charges are of two type (positive and negative) but mass is of only one type.
  (v) For a moving charged body, mass increases while charge remains constant.
- True test of electrification is repulsion, not attraction
- As attraction can take place between a charged and an uncharged body or between two similarly charged bodies
- For a non relativistic (i.e. v << c) charged particle, specific charge $\frac{q}{m}$ = constant.
- Charge can be detected and/or measured with the help of gold-leaf electroscope, electrometer, voltmeter etc.

BEGINNER'S BOX-1

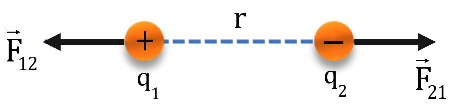
1. In a neutral sphere, 5×10^{21} electrons are present. If 10 percent electrons are removed, then calculate the charge on the sphere. **PEL180**
2. Calculate the number of electrons in 100 grams of CO_2. **PEL181**
3. Can a body have a charge of (a) 0.32×10^{-18} C (b) 0.64×10^{-20} C (c) 4.8×10^{-21} C ? **PEL182**
4. A glass tumbler contains 1 billion amoeba and two sodium ions are there on the body of each amoeba. Find out the charge contained in the glass. **PEL183**
5. How many electrons should be removed from a conductor so that it acquires a positive charge of 3.5 nC ? **PEL184**
6. It is now believed that protons and neutrons (which constitute nuclei of ordinary matter) are themselves made up of more elementary units called "quarks". A proton and a neutron consist of three quarks each. Two types of quarks, the so called 'up' quark (denoted by u) of charge +(2/3)e, and the 'down' quark (denoted by d) of charge (–1/3)e, together with electrons build up ordinary matter. (Quarks of other types have also been found which give rise to different unusual varieties of matter.) Suggest a possible quark composition of a proton and neutron. **PEL185**
7. A polythene piece rubbed with wool is found to have a negative charge of 3.2×10^{-7} C. Find the
   (a) number of electrons transferred.
   (b) mass gained by the polythene. **PEL186**
8. Two identical metal spheres A and B placed in contact are supported on insulating stand. What kinds of charge will A and B develop when a negatively charged ebonite rod is brought near A ? **PEL187**

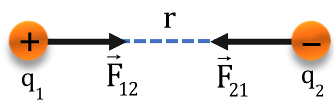
Coulomb's Law

"The electrostatic force of interaction between two point charges is directly proportional to magnitude of the product of charges and inversely proportional to the square of distance between them."



This force always works along the line joining the two charges.





$$\left|\vec{F}_{12}\right| = \left|\vec{F}_{21}\right| = F$$

i.e. $F \propto \dfrac{|q_1 q_2|}{r^2} \Rightarrow \boxed{F = \dfrac{k|q_1 q_2|}{r^2}}$

Where k = proportionality constant or coulomb's constant or electrostatic constant

❖ Value of k depends upon the choice of system of units.

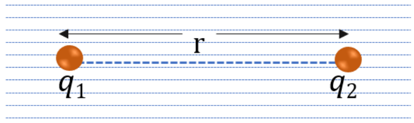
❖ In SI system, $k = 9 \times 10^9 \dfrac{N \times m^2}{C^2}$

❖ This constant is also written as, $k = \dfrac{1}{4\pi\varepsilon_0}$

ϵ_0 = permittivity of free space or vacuum = $8.85 \times 10^{-12} \dfrac{C^2}{N-m^2}$

Then $\boxed{F = \dfrac{kq_1q_2}{r^2} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q_1q_2}{r^2}}$

❖ When the point charge system is placed in a homogeneous dielectric medium.



$$F_m = \frac{1}{4\pi \in} \frac{q_1 q_2}{r_2}$$

F_m = net force on any charge

$\in$ = permittivity of medium = $\in_0 \in_r$

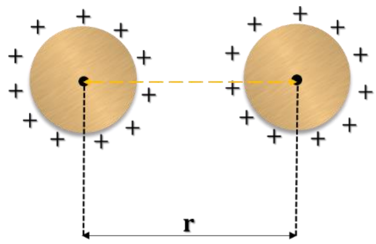
$\in_r$ = relative permittivity of medium or Dielectric constant of medium (K)

$$F_m = \frac{1}{4\pi \in_0 \in_r} \frac{q_1 q_2}{r^2} = \frac{1}{\in_r}\left(\frac{1}{4\pi \in_0} \frac{q_1 q_2}{r^2}\right) \Rightarrow \boxed{F_m = \frac{F}{\in_r}}$$

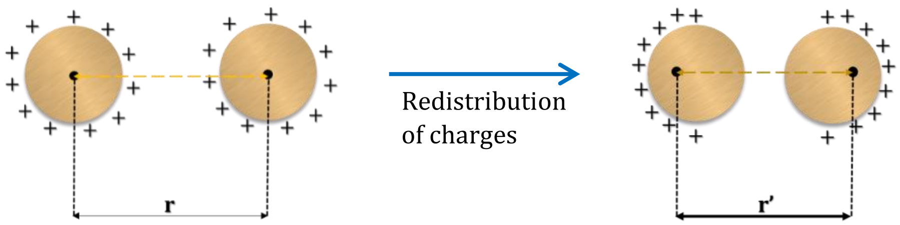
The value of $\in_r$ or $K \geq 1$, i.e. $F_m \leq F$

Illustration 7:

Why we can not use Coulomb's law for large size bodies ?



Solution:

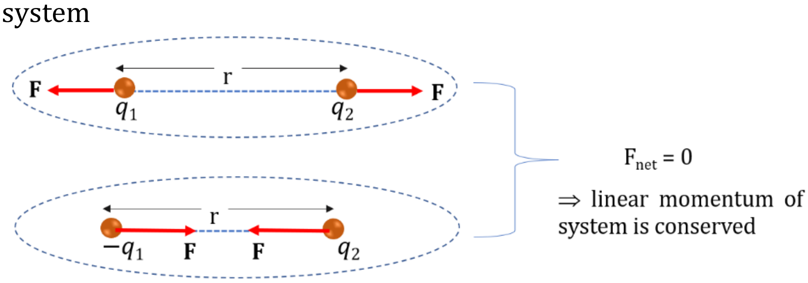


When large size charged conducting spheres brought close to each other, there charges moves away due to repulsion hence effective distance between their centers increases (r' > r)

$$F_{actual} < F_{calculated} = \frac{kq_1 q_2}{r^2}$$

Note:

❖ For two charge system



In both the cases shown -

Net external force on the system is zero so linear momentum of system remains conserved.

❖ For the system shown, line of action of force for both forces is same. So, net torque about any point for the system is zero.

$\Rightarrow$ Angular momentum = Constant

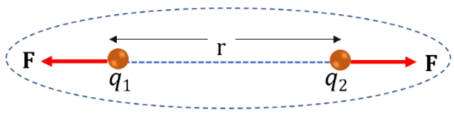


Illustration 8:

Two point charges $+q_1$ and $+q_2$ are placed r meter apart in vacuum. Force on q_1 due to q_2 is F. If now a third charge $+q_3$ is placed at mid point of line joining the charges, then

(1) force on q_1 due to q_2

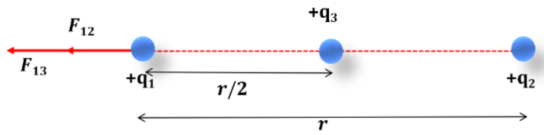
(a) Increases (b) Decreases (c) Remains same (d) Can't say

(2) net force on q_1

(a) Increases (b) Decreases (c) Remains same (d) Can't say

Solution:

When q_3 is placed at mid point,



Now net force on q_1 will definitely increase but force due to q_2 will remain F_{12} on q_1.

$$F_{12} = \frac{kq_1q_2}{r^2}$$

Illustration 9:

Calculate the force between two charges, each of 1μC separated in air by 1cm.

Solution:

$q_1 = q_2 = 1\mu C$ & $r = 1cm$

$$F = \frac{kq_1q_2}{r^2} = 9\times10^9 \frac{(10^{-6})^2}{(10^{-2})^2} = 90N$$

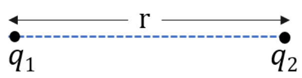
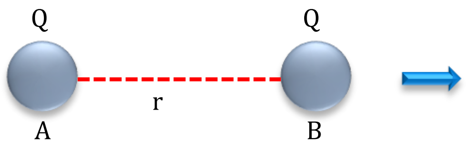


Illustration 10:

Force between two identical spheres A and B carrying same charge and separated by distance r in vacuum is F. A third identical sphere C first brought in contact with A then B and finally, C is taken away. Now force between A and B becomes.

(1) F/4 (2) F/8 (3) 3F/4 (4) 3F/8

Solution:

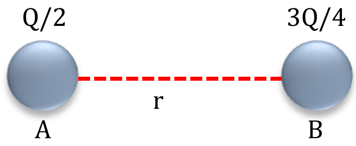


$$F = \frac{kQ^2}{r^2} \quad ...(1)$$

Now C (uncharged sphere) is touched with A first

Then C is touched with B

New force between A & B now is



$$F' = \frac{k\left(\frac{Q}{2}\right)\left(\frac{3Q}{4}\right)}{r^2} = \frac{3kQ^2}{8r^2} = \frac{3F}{8}$$

Illustration 11:

Two spheres having equal charges exert F force on each other. Now 20% charge of one sphere is transferred to another sphere. Then find new force between them in terms of F.

Solution:

Let us assume initially both the sphere were having equal charge q so, Force between them can be written as -

$$F = \frac{kqq}{r^2}$$

When 20% charge is transferred from one the another then new force F' -

$$F' = \frac{k(0.8q)(1.2q)}{r^2} = \frac{kqq}{r^2}\ (0.96) = 0.96F$$

Illustration 12:

Two spheres having equal charges are kept at long separation. If the gravitational force between two spheres is equal to electrostatic force between them. Find the ratio of specific charge $\frac{q}{m}$.

Solution:

Equating the gravitational and electrostatics force -

$$\frac{kq^2}{r^2} = \frac{Gm^2}{r^2}$$

$$\frac{q}{m} = \sqrt{\frac{G}{k}} = \sqrt{\frac{6.67\times10^{-11}}{9\times10^{9}}} = \sqrt{\frac{0.74}{10^{20}}} = \frac{0.86}{10^{10}} = 0.86\times10^{-10}$$

Illustration 13:

12C charge is to be distributed among the two spheres placed at a separation of 2m then find the charge on each sphere so that coulomb force between them is maximum also find the maximum force.

Solution:

For maximum force divide the charge as half - half

$$F = \frac{k(6)(6)}{2^2} = \frac{9\times10^{9}\times36}{4} = 81\times10^{9}\ \text{N}$$

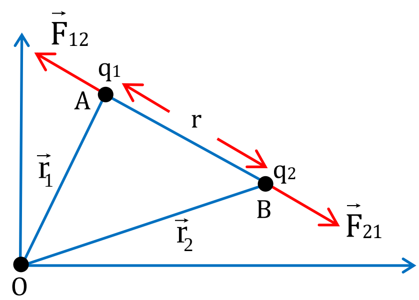
Vector form of Coulomb's Law –

$\vec{F}_{12}$ = force on q_1 due to $q_2 = \frac{kq_1q_2}{r^2}\hat{r}_{21} = \frac{kq_1q_2}{r^3}\vec{r}_{21}$

$\vec{F}_{21}$ = force on q_2 due to $q_1 = \frac{kq_1q_2}{r^2}\vec{r}_{12} = \frac{kq_1q_2}{r^3}\vec{r}_{12}$

$\because r = |\overrightarrow{AB}| = |\overrightarrow{BA}| \Rightarrow r = |\vec{r}_2 - \vec{r}_1| = |\vec{r}_1 - \vec{r}_2|$

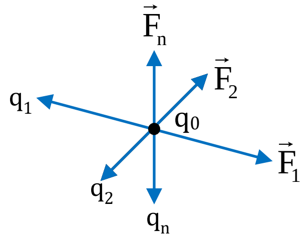
or $\vec{F}_{12} = \frac{kq_1q_2}{|\vec{r}_1-\vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$ and $\vec{F}_{21} = \frac{kq_1q_2}{|\vec{r}_2-\vec{r}_1|^3}(\vec{r}_2 - \vec{r}_1)$



Forces between Multiple Charges

Superposition Principle

When a number of charges are interacting then, total force on a given charge is vector sum of all the individual forces exerted on it by all other charges.

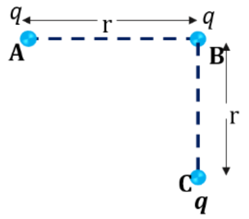


$\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + ...\vec{F}_{1N}$ or

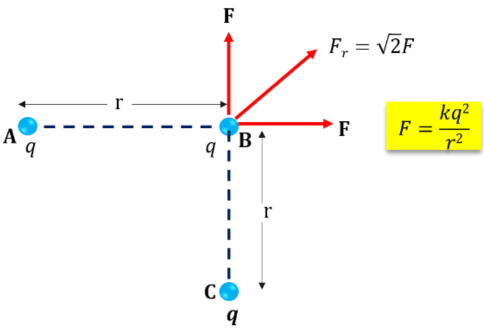
$$\vec{F} = \frac{kq_0q_1}{r_1^2}\hat{r}_1 + \frac{kq_0q_2}{r_2^2}\hat{r}_2 + ... + \frac{kq_0q_i}{r_i^2}\hat{r}_i + ... \frac{kq_0q_n}{r_n^2}\hat{r}_n$$

Illustration 14:

Find net force on charge at B.



Solution:



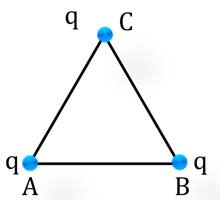
So, net force on B will be

$$F_r = \sqrt{2}\frac{kq^2}{r^2}$$

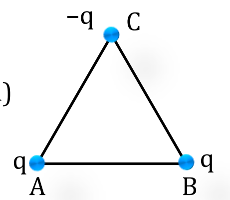
Illustration 15:

Find net force on the charge, placed at B, in the given arrangements

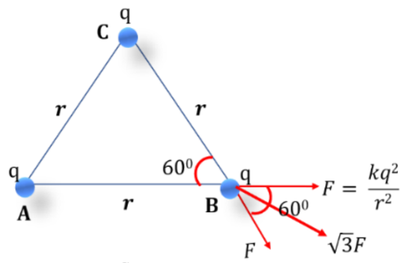
(i)



(ii)

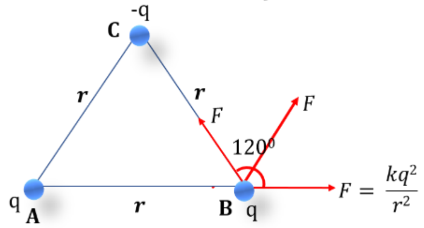


Solution:



(i) Required force on charge at B will be -

$$F_B = \sqrt{3}\frac{kq^2}{r^2}$$

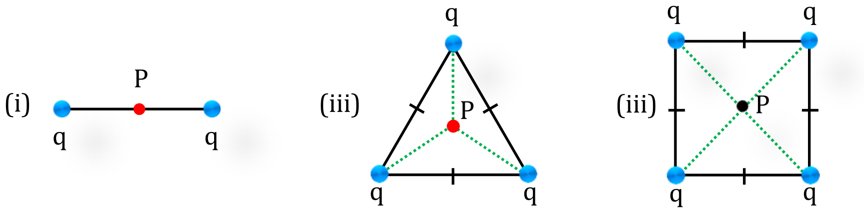


(ii) Required force on charge at B will be -

$$F_B = \frac{kq^2}{r^2}$$

Illustration 16:

Find net force on the charge Q, placed at centre point P as shown :



Solution:

If n identical charges placed at corners of a regular polygon then net force on charge placed at center of polygon is always zero.

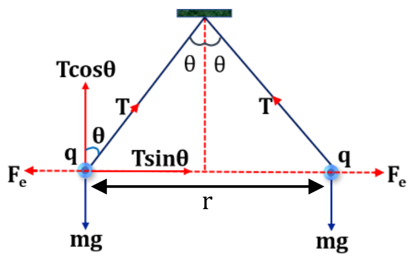
Equilibrium of suspended charge system (Pith Ball problems)

Two identical charged pith balls are suspended by insulated strings of equal length, by a common point of suspension, if each ball has mass m then the problem related to it can be treated as,

$T \sin\theta = F_e$ ...(i)

$T \cos\theta = mg$ ...(ii)

$\tan\theta = \dfrac{F_e}{mg}$ ...(iii)

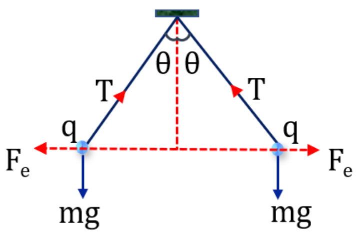


$r = 2\ell \sin\theta$ (distance between charges, where ℓ is length of string)

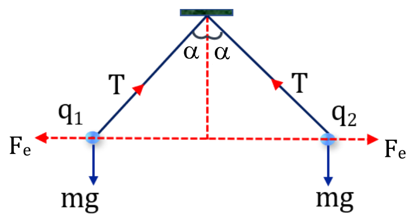
$$F_e = \frac{kq^2}{r^2} = \frac{kq^2}{4\ell^2 \sin^2\theta}$$

$$T = \sqrt{(F_e)^2 + (mg)^2}$$

Case -1 : If $q_1 = q_2$, $m_1 = m_2$, then $\theta_1 = \theta_2$ (say θ)

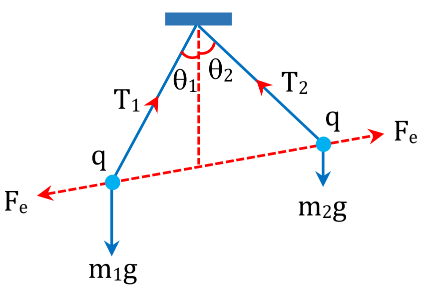


Case -2 : If $q_1 \neq q_2$, $m_1 = m_2$, then $\theta_1 = \theta_2$ (say α)

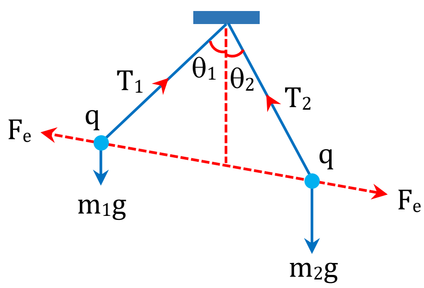


Case -3 : If $q_1 = q_2$, $m_1 \neq m_2$, then $\theta_1 \neq \theta_2$

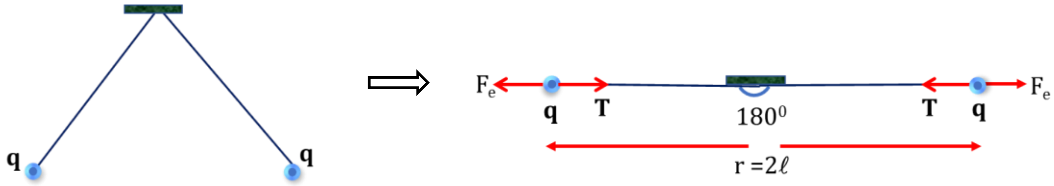
If $m_1 > m_2$ then $\theta_1 < \theta_2$



If $m_1 < m_2$ then $\theta_1 > \theta_2$



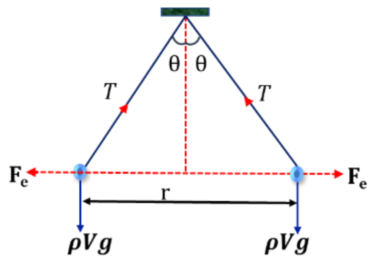
Case -4 : If whole system placed in gravity free space (g = 0)



$$F_e = \frac{kq^2}{r^2} = \frac{kq^2}{4l^2}$$

Tension in thread $T = \frac{kq^2}{4l^2}$

Case -5 : If density of material is ρ and volume is V then -

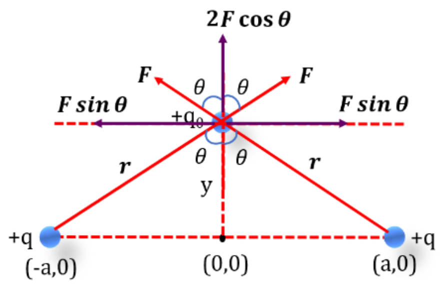


In air, $\tan\theta = \frac{F_e}{mg} = \frac{F_e}{V\rho g}$

Illustration 17:

Two point charges each of +q are fixed at (a,0) & (-a,0). Another point charge $+q_0$ is free to move along y-axis is placed at (0,y). For what value of y force on $+q_0$ is maximum.

Solution:



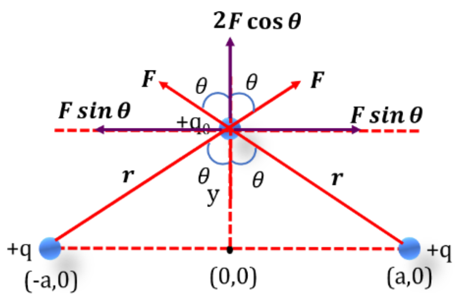
F_r on $+q_0$

$$F_r = 2F\cos\theta$$

$$r = \sqrt{(a^2 + y^2)}$$

$$F_r = 2\left(\frac{kqq_0}{r^2}\right)\left(\frac{y}{r}\right)$$

$$F_r = 2\left(\frac{kqq_0}{r^3}\right)y = 2\left(\frac{kqq_0}{\left(a^2+y^2\right)^{3/2}}\right)y$$



$$F_r = 2\left(\frac{kqq_0}{r^3}\right)y = 2\left(\frac{kqq_0}{\left(a^2+y^2\right)^{3/2}}\right)y$$

For maximum value of Force

$\frac{dF}{dy} = 0$ (concept of maxima)

By doing this we obtained $y = \pm \frac{a}{\sqrt{2}}$

Golden Key Points

- Coulomb's law is based on physical observations and is not logically derivable from any other concept. Experiments reveal its universal nature till today.
- The law is analogous to Newton's law of gravitation : $F = G\dfrac{m_1m_2}{r^2}$
- Electric force between charged particles is much stronger than gravitational force, i.e., $F_E >> F_G$. Consequently, F_G is neglected when both F_E and F_G are present.
- Electric force can be attractive or repulsive while gravitational force is always attractive.
- Electric force depends on the nature of medium between the charges while gravitational force does not.
- The force is conservative, i.e., work done in moving a point charge round a closed path under the action of Coulomb's force is zero.
- The law expresses the force between two point charges at rest. In applying it to the case of extended bodies of finite size care should be taken in assuming the whole charge of a body to be concentrated at its 'centre' as this is true only for spherically charged bodies, that too for external points.
- Electric force between two charges does not depend on neighbouring charges.
- Although the net electric force on both particles change in the presence of dielectric but force due to one charged particle on another charged particle does not depend on the medium between them.

Electrostatic Equilibrium

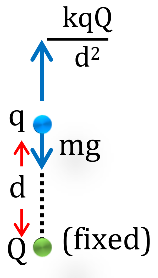
In physics if a particle or system is in equilibrium means net force on particle or system is zero.

Equilibrium
- Static equilibrium (System is at rest)
- Dynamic equilibrium (System is in uniform motion)

Illustration 18:

A small charge q of mass m is in equilibrium at a distance d above another charge Q. What should be the value of q in terms of other given quantities?

Solution:



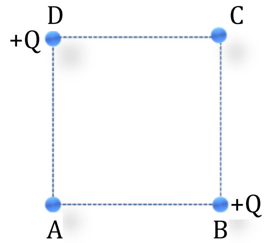
In equilibrium : $mg = F_e$

$$mg = \frac{kqQ}{d^2}$$

$$\frac{mgd^2}{kQ} = q$$

Illustration 19:

What equal charges should be placed at remaining two vertices so that net force on +Q charge becomes zero (Side of square is a)



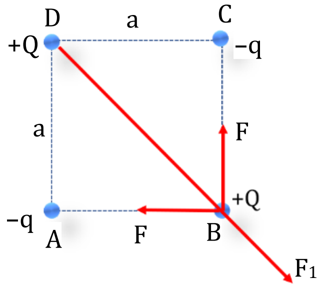
(1) $\frac{Q}{\sqrt{2}}$ (2) $-\frac{Q}{\sqrt{2}}$

(3) $\frac{Q}{2\sqrt{2}}$ (4) $-\frac{Q}{2\sqrt{2}}$

Solution:

To make net force on +Q charge zero, Field due to charges on vertices A & C should be opposite to that of field due to other +Q charge so there must be -q charge placed at A and C.

For net force to be zero on +Q



$\sqrt{2}F = F_1$

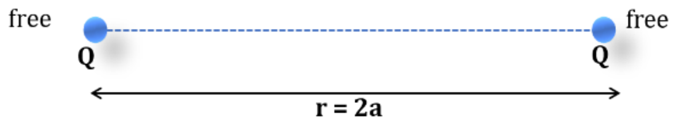
Here $F = \frac{kqQ}{a^2}$ & $F_1 = \frac{kQ^2}{(\sqrt{2}a)^2} = \frac{kQ^2}{2a^2}$

$\sqrt{2}\frac{kqQ}{a^2} = \frac{kQ^2}{2a^2}$

$q = \frac{Q}{2\sqrt{2}}$ (magnitude) $\therefore q = -\frac{Q}{2\sqrt{2}}$ option (4) is correct.

Illustration 20:

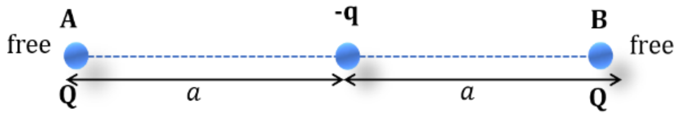
Where and what value of charge should be placed in between the given two charges so that whole system may remain in equilibrium.



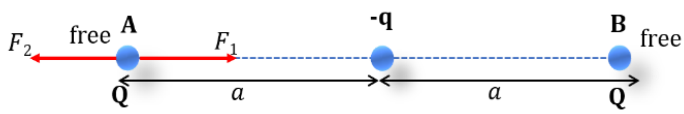
Solution:

As both free charges are identical so third charge will remain in equilibrium at mid point.

Third charge will be opposite in sign otherwise system's equilibrium is not possible



For equilibrium of charge placed at A, force of attraction of –q must be balanced by force of repulsion of charge placed at B



For equilibrium of Q, $F_1 = F_2$

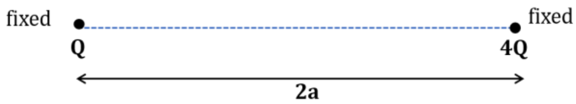
$\frac{kQq}{a^2} = \frac{kQ^2}{4a^2}$

$\frac{q}{1} = \frac{Q}{4}$ or $q = \frac{Q}{4}$ (in magnitude)

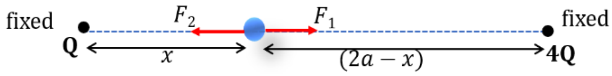
$\therefore q = -\frac{Q}{4}$

Illustration 21:

In the system of two charges Q and 4Q where should be third charge must be placed so that net force on third charge will be zero.



Solution:



F_1 : Force exerted by Q on q

F_2 : Force exerted by 4Q on q

Suppose q is placed at distance x from Q. For net force to be zero on third charge q force due to Q and 4Q must be equal and opposite.

For net force to be zero on q, $F_1 = F_2$

$$\frac{kQq}{x^2}=\frac{4kQq}{(2a-x)^2} \Rightarrow \frac{(2a-x)^2}{x^2}=\frac{4}{1}$$

$$\frac{(2a-x)}{x}=\frac{2}{1}$$

$$x=\frac{2a}{3}$$

Short Trick: -

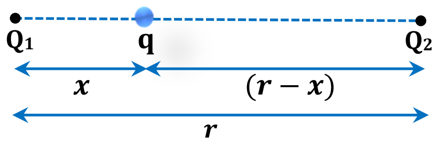
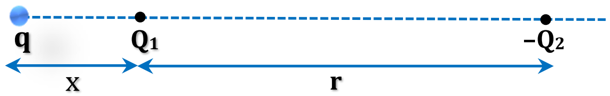
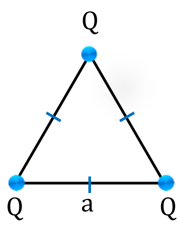
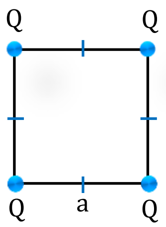
Case-1: -	Case-2: -
 In this case we can directly calculate the value of x. $x=\left(\dfrac{r}{\sqrt{\dfrac{Q_2}{Q_1}}+1}\right)$ Here $Q_1Q_2 > 0$ and $\lvert Q_1\rvert < \lvert Q_2\rvert$ Equilibrium position will be near to smaller charge.	 In this case we can directly calculate the value of x $x=\left(\dfrac{r}{\sqrt{\left\lvert\dfrac{Q_2}{Q_1}\right\rvert}-1}\right)$ Here $Q_1Q_2 < 0$ and $\lvert Q_1\rvert < \lvert Q_2\rvert$ Equilibrium position will be near to smaller charge.

Illustration 22:

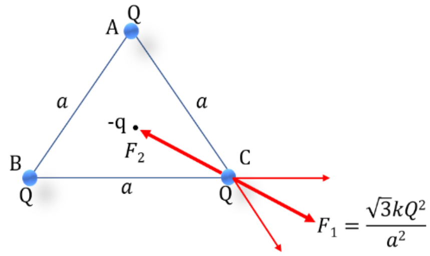
Find the value of charge, placed at the centre of given system, so that the systems will remain in equilibrium :

(i) 

(ii) 

Solution:

Concept - As all charges are positive so they will repel each other.
Definitely other charge must be opposite in sign.

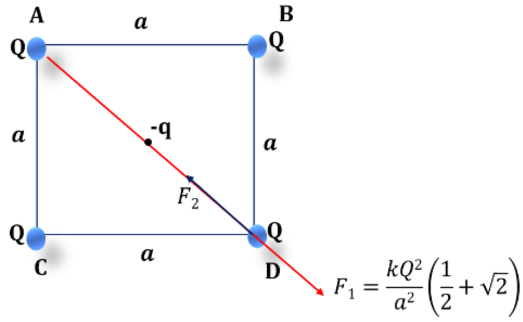


(i) Distance between center to corner of triangle is $x = \frac{a}{\sqrt{3}}$

For equilibrium of Q, $F_1 = F_2$

or $\frac{\sqrt{3}kQ^2}{a^2} = \frac{kQq}{x^2}$ or $\frac{\sqrt{3}Q}{a^2} = \frac{3q}{a^2}$ or $q = \frac{Q}{\sqrt{3}}$ (magnitude)

or $q = -\frac{Q}{\sqrt{3}}$



(ii) Distance between center to corner of square is $x = \frac{a}{\sqrt{2}}$

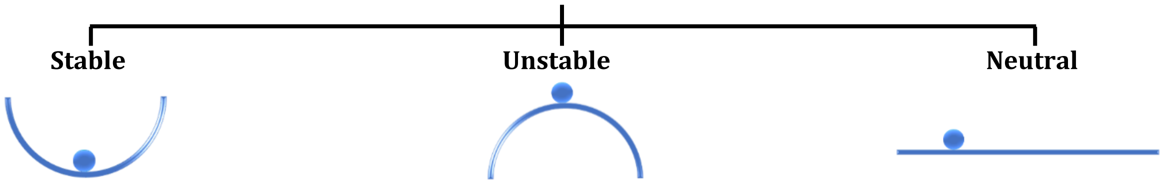
For equilibrium of Q, $F_1 = F_2$

or $\frac{kQq}{x^2} = \frac{kQ^2}{a^2}\left(\frac{1}{2}+\sqrt{2}\right)$ or $\frac{2kQq}{a^2} = \frac{kQ^2}{a^2}\left(\frac{1}{2}+\sqrt{2}\right)$

or $q = \frac{Q}{4}\left(1+2\sqrt{2}\right)$ (magnitude) or $q = -\frac{Q}{4}\left(1+2\sqrt{2}\right)$

Nature of Equilibrium

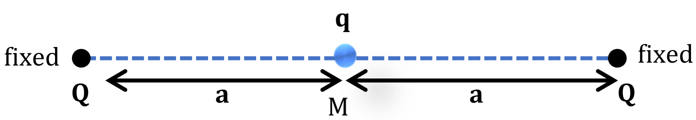
Types of equilibrium



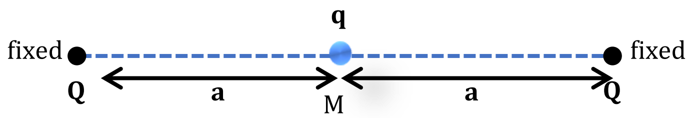
A charge is initially in equilibrium position and is displaced by a small distance. If the charge tries to return back to the same equilibrium position then this equilibrium is called position of **stable equilibrium.**	If charge is displaced by a small distance from its equilibrium position and the charge has no tendency to return to the same equilibrium position. Instead it goes away from the equilibrium position, then this is called position of **unstable equilibrium.**	If charge is displaced by a small distance and it is still in equilibrium condition then it is called **neutral equilibrium**.

Illustration 23:

What will be net force on charge q placed at mid-point as shown

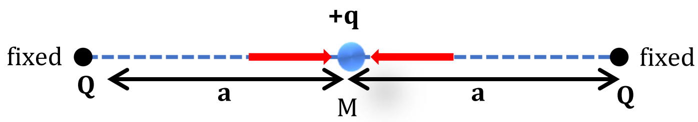


Solution:



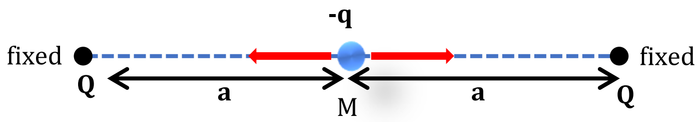
net force on q is zero whatever be sign and magnitude of q

Case-1 when positive charge is at the mid point



F_{net} on +q will be zero because it will be repel by equal and opposite force

Case-2 when negative charge is at the mid point



F_{net} on +q will be zero because it will be attract by equal and opposite force

Illustration 24:

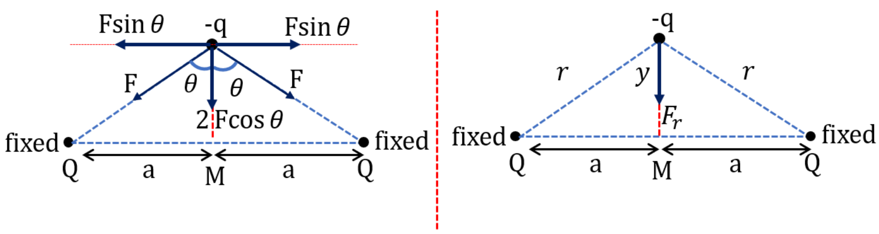
Two charges each of magnitude Q are fixed at 2a distance apart. A third charge (–q of mass 'm') is placed at the mid point of the two charges. When –q charge is slightly displaced perpendicular to the line joining the charges then find its time period.

Solution:

First of all we must know necessary condition for SHM.

When restoring force acting on particle changes linearly w.r.t. displacement from mean position, particle will perform SHM. $F_r \propto -y$

$$F_r = -2F\cos\theta$$



$$F_r = -2\left(\frac{kQq}{r^2}\right)\frac{y}{r}$$

$$F_r = -2\left(\frac{kQq}{r^3}\right)\frac{y}{1}$$

$$F_r = -2\left(\frac{kQq}{(a^2+y^2)^{3/2}}\right)y$$

$$F_r = -2\left(\frac{kQq}{(a^2+y^2)^{3/2}}\right)y$$ For small value of y, y^2 can be neglected w.r.t. a^2

$$F_r = -\left(\frac{2kQq}{a^3}\right)y$$

$$F_r = -ky$$

For calculation of time period.

$T = -2\pi\sqrt{\frac{m}{k}}$ (where k is coefficient of force, i.e. the value of all parameters except displacement in formula of restoring force)

So, $T = -2\pi\sqrt{\frac{ma^3}{2kqQ}}$

BEGINNER'S BOX-2

1. Two identical metal spheres carry charges of + q and – 2q respectively. When the spheres are separated by a large distance r, the force between them is F. Now the spheres are allowed to touch and then moved back to the same separation. Find the new force of repulsion between them. **PEL188**

2. The electrostatic force of repulsion between two positive ions carrying equal charges is 4×10^{-9} N, when their separation is 5 Å. How many electrons are missing from each ? **PEL189**

3. Two identical particles each of mass M and charge Q are placed a certain distance apart. If they are in equilibrium under mutual gravitational and electric force then calculate the order of $\frac{Q}{M}$ in SI units. **PEL190**

4. The force between two point charges is 100 N in air. Calculate the force if the distance between them is increased by 50%. **PEL191**

5. Two neutral insulating small spheres are rubbed against each other and are then kept 4 m apart. If they attract each other with a force of 3.6 N, then

(i) calculate the charge on each sphere, and

(ii) calculate the number of electrons transferred from one sphere to the other during rubbing. **PEL192**

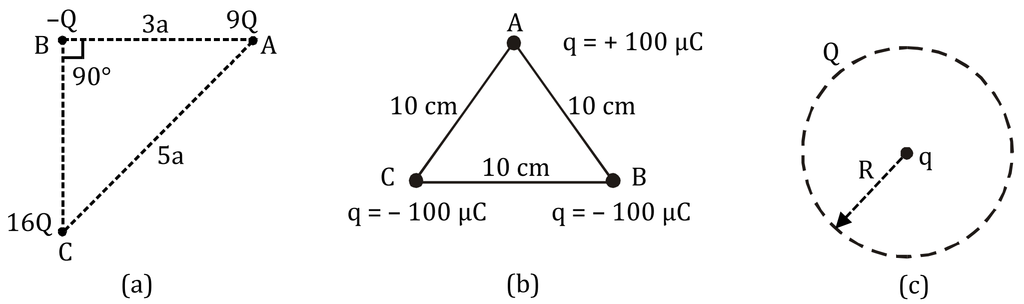
6. Two equal point charges $Q = +\sqrt{2}$ μC are placed at each of the two opposite corners of a square and equal point charges q at each of the other two corners. What must be the value of q so that the resultant force on Q is zero ? **PEL193**

7. In the given figure (b) three point charges are situated at the corners of an equilateral triangle of side 10 cm. Calculate the resultant force on the charge at B. What is its direction? **PEL194**

8. Two positively charged particles, each of mass 1.7×10^{-27} kg and carrying a charge of 1.6×10^{-19} C are placed at a distance d apart. If each experiences a repulsive force equal to its weight, find the value of d. **PEL195**

9. In figure (a) ABC is a right angled triangle. Calculate the magnitude of force on charge –Q. **PEL196**

10. In figure (c) Charge Q of mass m revolves around a point charge q due to electrostatic attraction. Show that its period of revolution is given by $T^2 = \frac{16\pi^3 \in_0 mR^3}{Qq}$.



PEL197

Electric Field Intensity

- The space around a charge or charge distribution, in which another charge experiences an electric force, is called electric field.
- Every charge has its own electric field.
- Electric field at a point can be characterized by

  (i) A vector function of position, called intensity of electric field ($\vec{E}$)

  (ii) A scalar function of position, called electrostatic potential(V).

  (iii) Graphically, with help of electric field lines (EFL)
- It is defined as the net force experienced by a unit positive test charge.

  Mathematically, $\vec{E}=\dfrac{\vec{F}}{q_0}$ SI unit : N/C or V/m.
- Dimensional formula $[M^1L^1T^{-3}A^{-1}]$.
- **Test Charge :** It is a charge of very small magnitude which do not produce significant electric field.

  So more precise definition of electric field intensity is $\vec{E}=\lim\limits_{q_0\to 0}\left(\dfrac{\vec{F}}{q_0}\right)$.
- It is a vector quantity whose direction is always in the direction of force experienced by a positive test charge, and opposite to that of force experienced by a negative test charge.
- Electric field due to a positive charge is always away from it while due to a negative charge, this is always towards it.
- Force on a point charge is in the same direction as that of electric field on positive charge and in opposite direction as that of electric field on a negative charge.

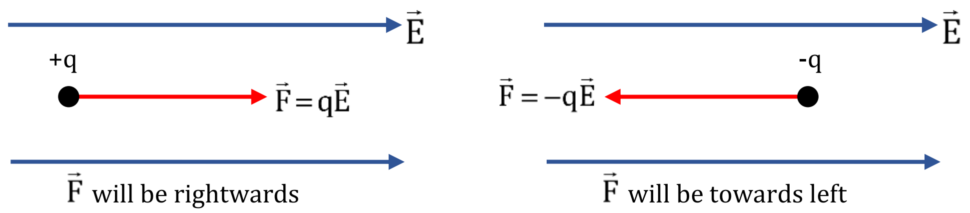
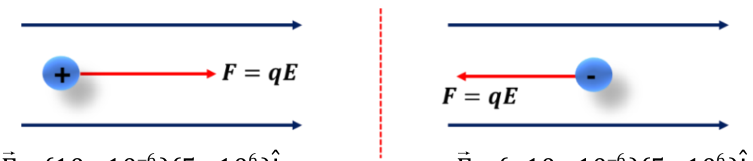


Illustration 25:

A charge of 10μC and -10 μC is placed in uniform electric field of 5×10^6 N/C directed along positive x axis, find out force acting on positive and negative charge?

Solution:



$\vec{F}=(10\times10^{-6})(5\times10^{6})\hat{i}$

$\vec{F}=50\hat{i}N$

$\vec{F}=(-10\times10^{-6})(5\times10^{6})\hat{i}$

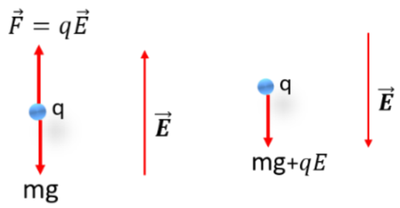
$\vec{F}=-50\hat{i}N$

Illustration 26:

A positively charged oil drop is in equilibrium in a uniform electric field. If suddenly direction of electric field is reversed, then acceleration of drop becomes?

(1) g (2) 2g (3) $\frac{g}{2}$ (4) None of these

Solution:

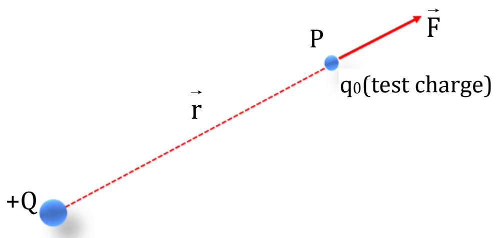


For equilibrium of particle qE = mg

When electric field is reversed : net force on particle = mg + qE = 2mg

Acceleration will be 2g (downwards)

Electric Field due to a Point Charge

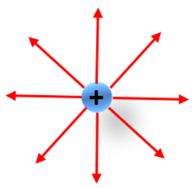


Force on test charge $\vec{F} = \frac{kQq_0}{r^2}\hat{r}$

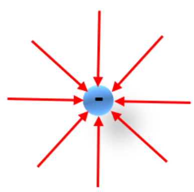
According to definition of electric field intensity

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{kQ}{r^2}\hat{r} = \frac{kQ}{r^3}\vec{r}$$

Electric Field due to Positive and Negative Charge



Electric field due to positive charge is radially outward

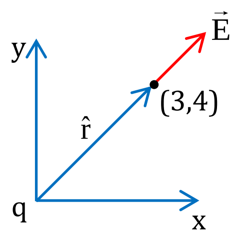


Electric field due to negative charge is radially inward

Illustration 27:

A charge particle of 1μC is placed at origin. Find intensity of electric field due to the charge at position (3,4)m.

Solution:



Position of particle is $\vec{r} = 3\hat{i} + 4\hat{j}$

Magnitude of r is $r = \sqrt{3^2 + 4^2} = 5\text{m}$

Electric field due to point charge $\vec{E} = \frac{kQ}{r^2}\hat{r}$

Unit vector along E, $\hat{r} = \frac{\vec{r}}{r}$ or $\hat{r} = \frac{3\hat{i} + 4\hat{j}}{5}$

$$\vec{E} = \frac{kQ}{r^2}\hat{r} = \frac{(9\times10^9)10^{-6}}{(5)^2}\left(\frac{3\hat{i}+4\hat{j}}{5}\right) = 72(3\hat{i}+4\hat{j})\ \text{N|C}$$

Illustration 28:

Calculate the electric field at origin due to infinite number of charges as shown in figures (a) and (b) below.

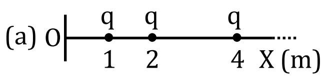


figure (a)

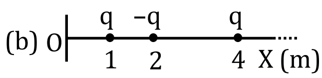


figure (b)

Solution:

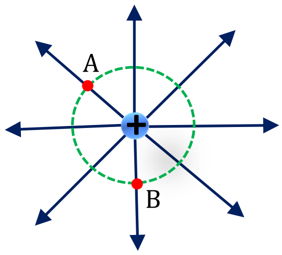
(a) $E_0 = kq\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+----\right] = \frac{kq.1}{(1-1/4)} = \frac{4kq}{3}$

(b) $E_0 = kq\left[\frac{1}{1}-\frac{1}{4}+\frac{1}{16}-----\right] = \frac{kq.1}{(1+1/4)} = \frac{4kq}{5}$

Illustration 29:

There are two different points in the surrounding of point charge at same distance, electric field at those point will be same

(i) True (ii) False



Solution:

If distance of A and B points from point charge is r, the magnitude of electric field will be $E = \frac{kQ}{r^2}$. As electric field is vector quantity direction its direction at A and B will be different. So above statement is false

Electric Field due to System of Charges

- **Electric field follows superposition principle :**

  When two or more than two charges are present in space than electric field at a particular point will be equal to vector sum of electric fields due to individual charges.

  According to superposition principle, net electric field at the given point will be

  $\vec{E}_{net} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 .... + \vec{E}_n$

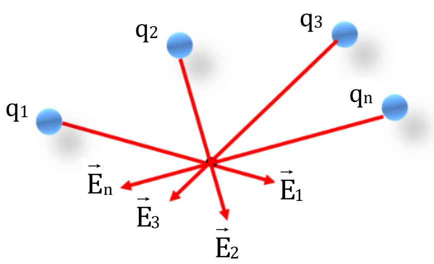
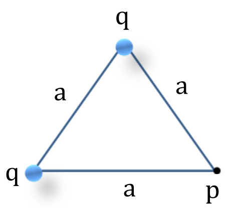
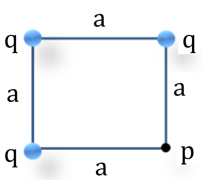
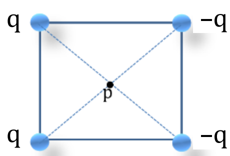


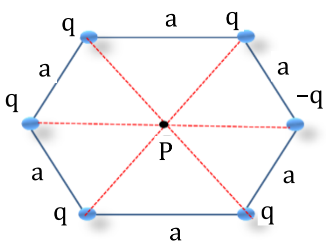
Illustration 30:

Find intensity of electric field at point P (as shown) in the given point charge distributions.

(i)


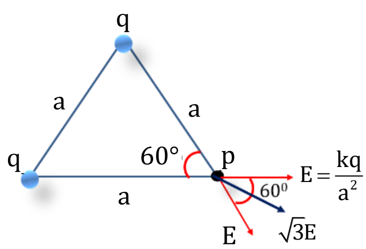
(ii)


(iii)


(iv)


Solution:

(i) Resultant electric field at p will be $E_r = \sqrt{3}E = \sqrt{3}\frac{kq}{a^2}$

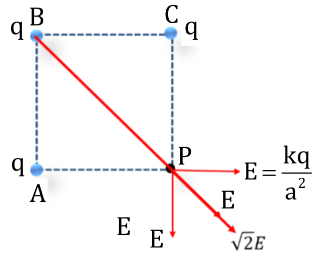


(ii) E_{net} at p will be

$$\vec{E}_r = \vec{E}' + \sqrt{2}\vec{E}$$

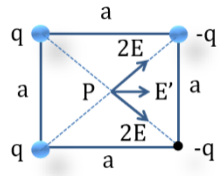
$$E_r = \frac{kq}{(\sqrt{2}a)^2} + \sqrt{2}\frac{kq}{a^2}$$

$$E_r = \frac{kq}{a^2}\left(\frac{1}{2} + \sqrt{2}\right)$$



(iii) Center to corner distance is $\frac{a}{\sqrt{2}}$ and here field $E = \frac{kq}{\left(a/\sqrt{2}\right)^2} = \frac{2kq}{a^2}$

So, net electric field $E' = 2(2E)\cos 45° = 4\sqrt{2}\frac{kq}{a^2}$



(iv) Now, if charge at point A becomes –q then we can assume it as q-2q so field at centre due to all +ve q charges will be zero and net field will be due to –2q charge at A.

$$E_{net} = \frac{2kq}{a^2} \text{ (along point A)}$$

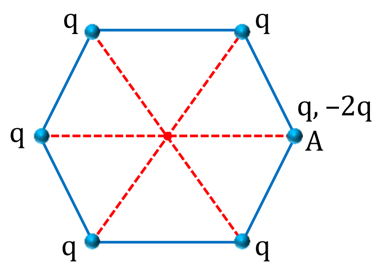
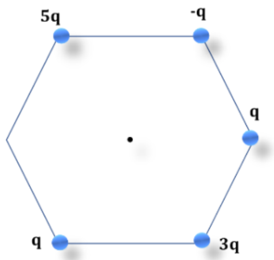
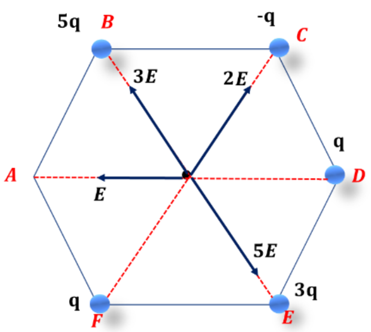


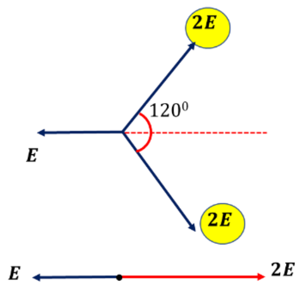
Illustration 31:

Five point charges placed on the vertices of a regular hexagon (as shown). Find net electric field at center of hexagon?



Solution:





Net electric field will be rightwards and it will be $E_{net} = \frac{kq}{r^2}$

(r is side of hexagon which is also distance b/w center to vertex)

Important Note:

When identical charges are placed at the corners of a regular polygon (symmetric arrangement) then resultant field at centre of the polygon is always zero.

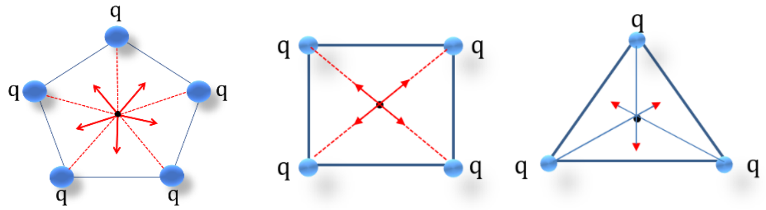
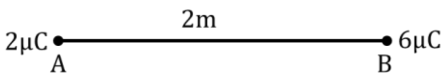
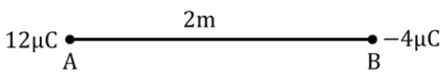


Illustration 32:

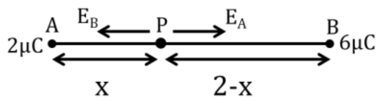
Find the point at which resultant $\vec{E}$ of the systems will become zero.

(i) 

(ii) 

Solution:

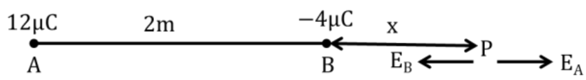
(i) Let us assume at point P resultant $\vec{E}$ is zero due to A and B so



$$|\vec{E}_A| = |\vec{E}_B| \qquad \frac{k(2\mu)}{x^2} = \frac{k(6\mu)}{(2-x)^2} \Rightarrow x = \frac{2}{1+\sqrt{3}}\,m$$

(ii) Let us assume at point P resultant $\vec{E}$ is zero due to A and B so

$$|\vec{E}_A| = |\vec{E}_B|$$



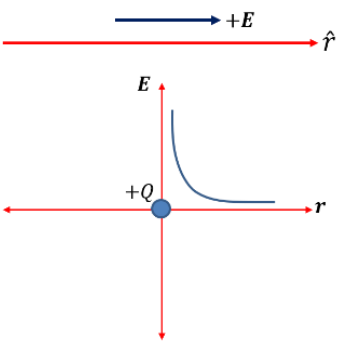
$$\frac{k(12\mu)}{(2+x)^2} = \frac{k(4\mu)}{x^2} \Rightarrow x = \frac{2}{\sqrt{3}-1}\,m$$

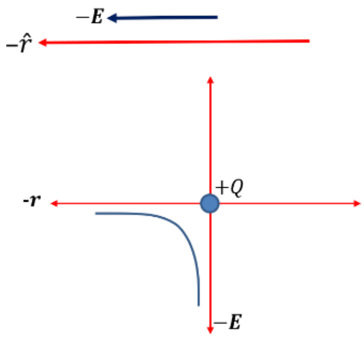
Graphical problems

Electric field v/s distance

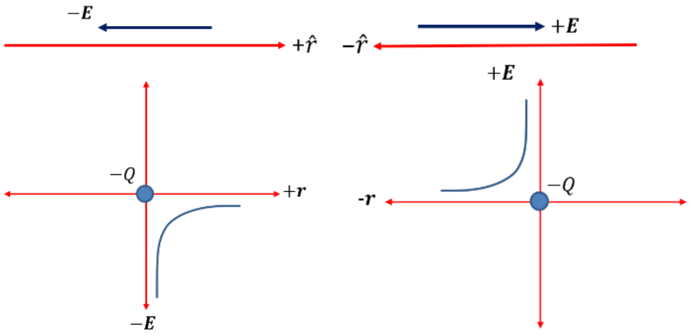
For point charge $E = \frac{kQ}{r^2}$ or $E \propto \frac{1}{r^2}$

(i) For positive charge :

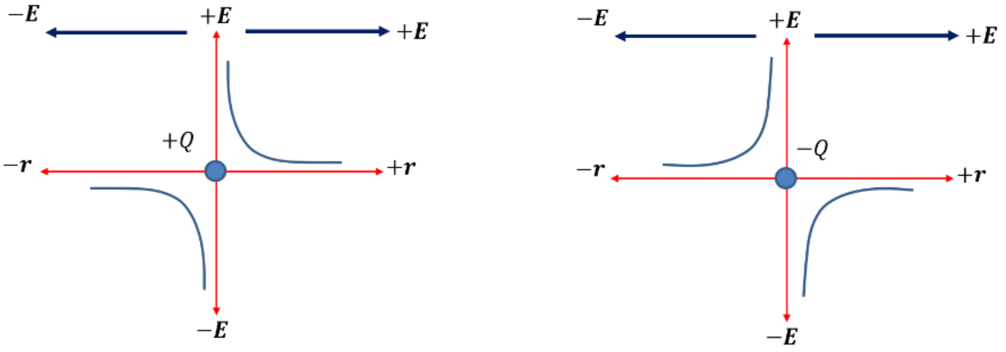




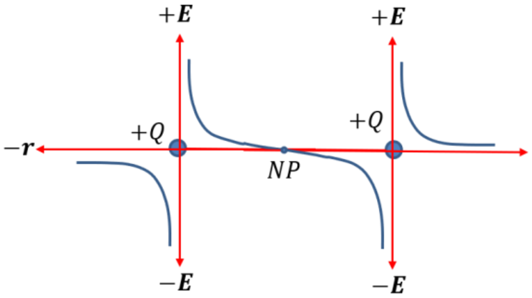
(ii) For negative charge :



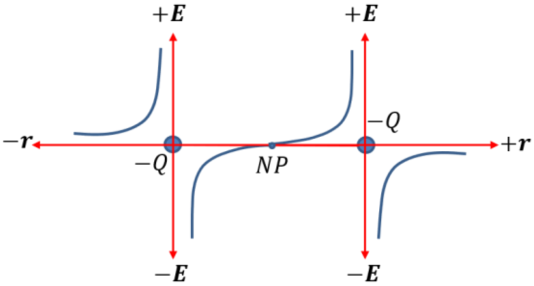
(iii) For positive and negative charge (combined analysis) :



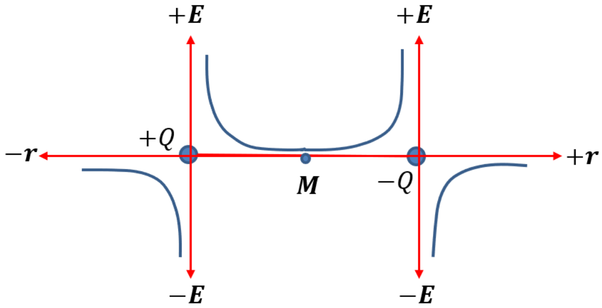
(iv) Graph for pair of positive charges :



(v) Graph for pair of negative charges :



(vi) Graph for pair of positive and negative charges :



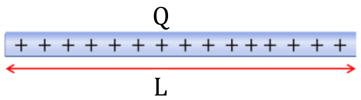
Here M is the mid point of line joining the two charges at x distance from any charge.

Here electric field will be $E = \dfrac{2kQ}{x^2}$ (rightward minimum electric field)

Continuous Charge Distributions

Types of continuous charge distributions

Linear Charge distribution

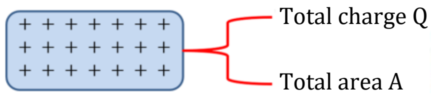


Used for linear objects (1-D) such as wire, thin rod, ring etc.
Linear charge density is,

$$\lambda = \frac{Q}{L}\,C/m$$

For non-uniform distribution of charge, charge of small element can be find by, $\boldsymbol{dQ=\lambda dl}$

Surface Charge distribution

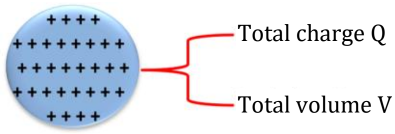


Used for flat objects (2-D) such as plate, disc, etc.
Surface charge density,

$$\sigma = \left(\frac{Q}{A}\right)C/m^2$$

For non-uniform distribution of charge, charge of small element can be find by, $dQ = \sigma dA$

Volume Charge distribution



Used for 3-D objects such as sphere, cube, cylinder etc.
Volume charge density,

$$\rho = \left(\frac{Q}{V}\right)C/m^3$$

For non-uniform distribution of charge, charge of small element can be find by, $dQ = \rho dV$

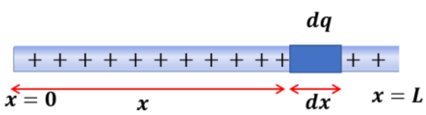
Illustration 33:

If linear charge density of rod of length L varies as $\lambda = \lambda_0 x$, find total charge on the rod.

Solution:

We consider a small element of length dx at distance x

Then $dq = \lambda dx = (\lambda_0 x)dx$



$$\int_0^Q dq = \int_0^L \lambda_0 x\,dx$$

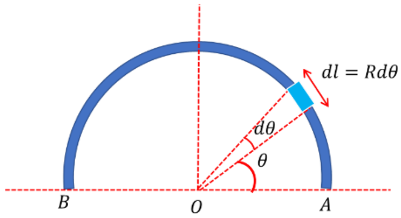
$$Q = \frac{\lambda_0 L^2}{2}$$

Illustration 34:

If linear charge density of semi-circular ring of radius R is $\lambda = \lambda_0 \sin\theta$, find total charge on semi-circular ring.

Solution:

For finding total charge on semi-circular ring -



Charge on small element $dq = \lambda dx$

$$dq = (\lambda_0 \sin\theta)dl = (\lambda_0 \sin\theta)Rd\theta$$

$$Q = \int_0^{\pi} (\lambda_0 R)\sin\theta d\theta = (\lambda_0 R)[-\cos\theta]_0^{\pi}$$

$$Q = (-\lambda_0 R)[\cos\pi - \cos 0]$$

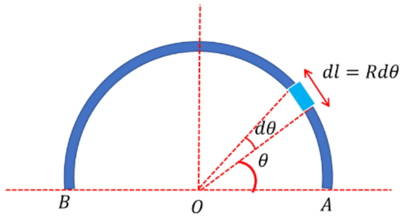
$$= 2\lambda_0 R$$

Illustration 35:

If linear charge density of a semi-circular ring is $\lambda = \lambda_0 \cos\theta$, find total charge on the semi-circular ring.

Solution:

For finding total charge on semi-circular ring



$dq = \lambda dx$ or $dq = (\lambda_0 \cos\theta)dl$

or $dq = (\lambda_0 \cos\theta)\, Rd\theta$ or $Q = \int_0^{\pi} (\lambda_0 R)\cos\theta d\theta$

or $Q = (\lambda_0 R)\ [\sin\theta]_0^{\pi}$ or $Q = (\lambda_0 R)\ [\sin\pi - \sin 0]$

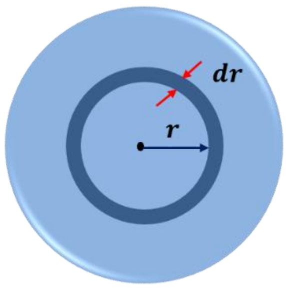
or $Q = 0$ (i.e. total charge on ring is zero)

Illustration 36:

Find total charge on a disc of radius R, Whose surface charge density varies according to $\sigma = \sigma_0 \left(1 - \frac{r}{R}\right)$ (r is radial distance from the centre of disc).

Solution:

Here we consider elementary ring of radius **r** and thickness **dr**



Area of elementary ring $\Rightarrow dA = (2\pi r)dr$

so Charge on elementary ring

$$dq = \sigma dA = \sigma(2\pi r)dr = \sigma_0 \left(1 - \frac{r}{R}\right)(2\pi r)dr$$

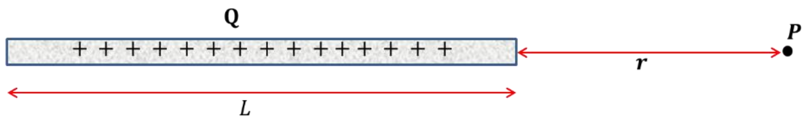
$\therefore$ Total charge -

$$Q = \int_0^R \sigma_0 \left(1 - \frac{r}{R}\right)(2\pi r)dr$$

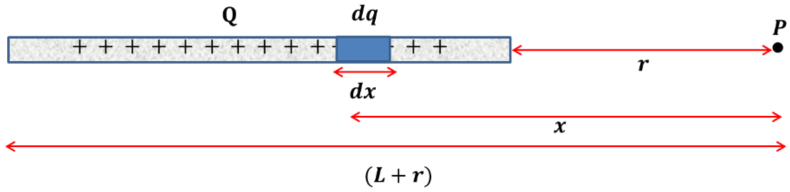
$$Q = 2\pi\sigma_0 \int_0^R \left(r - \frac{r^2}{R}\right)dr = 2\pi\sigma_0 \left(\frac{R^2}{2} - \frac{R^3}{3R}\right) = 2\pi\sigma_0 \left(\frac{R^2}{2} - \frac{R^2}{3}\right) = \frac{2\pi\sigma_0 R^2}{6} = \frac{\pi\sigma_0 R^2}{3}$$

Illustration 37:

Find electric field at point P as shown, due to a uniformly charged thin rod (Q, L).



Solution:



Now consider an element as shown -

Linear charge density, $\lambda = \frac{Q}{L} C/m$

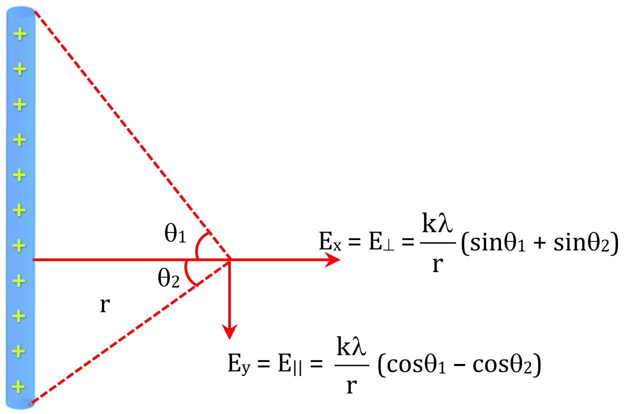
Field at point P due to the small element

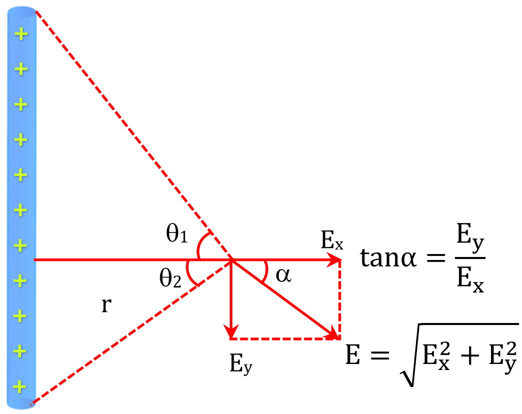
$dE = \frac{k(dq)}{x^2}$ here $dq = \lambda dx = \frac{Q}{L} dx$

Total field at P, due to the whole rod

$$E = \int_r^{(r+L)} \frac{k(dq)}{x^2} = \int_r^{(r+L)} \frac{k}{x^2}\frac{Q}{L}dx = \frac{kQ}{L}\int_r^{(r+L)} \frac{1}{x^2}dx = \frac{kQ}{L}\left[-\frac{1}{x}\right]_r^{(r+L)} = \frac{kQ}{L}\left[\frac{1}{r} - \frac{1}{r+L}\right] = \frac{kQ}{r(r+L)}$$

Electric field intensity due to charged wire





Special Cases:

(i) **For infinite wire,** (both ends goes to infinite)

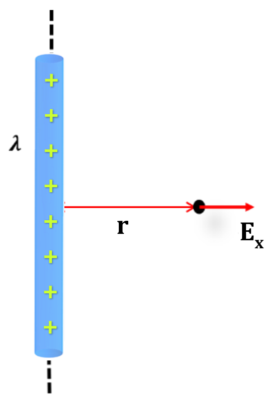
$\theta_1 \to 90°, \theta_2 \to 90°$

$$E_x = E_\perp = \frac{k\lambda}{r}(\sin 90° + \sin 90°)$$

$$E_x = E_\perp = \frac{2k\lambda}{r}$$

$$E_y = E_{||} = \frac{k\lambda}{r}(\cos 90° - \cos 90°) = 0$$

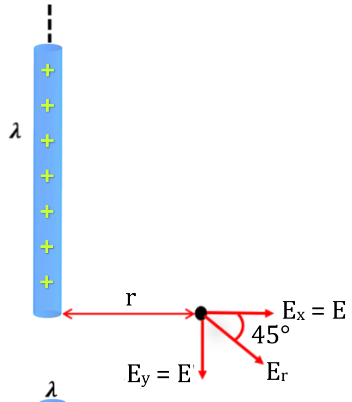
Net electric field will be $E_x = E_\perp = \frac{2k\lambda}{r}$ (perpendicular to wire)



(ii) For semi – infinite wire -

(one end goes to infinite and we find electric field at a point which is at perpendicular distance r from finite end)

For the given point $\theta_1 \to 0,\ \theta_2 \to 90°$



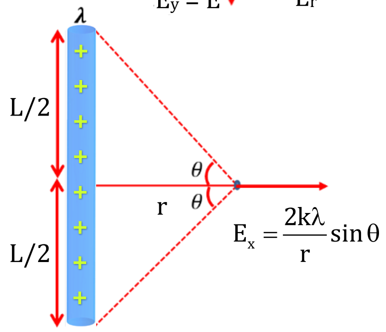
$$E_x = E_\perp = \frac{k\lambda}{r}(\sin 0° + \sin 90°) = \frac{k\lambda}{r}$$

$$E_y = E_{\parallel} = \frac{k\lambda}{r}(\cos 0° - \cos 90°) = \frac{k\lambda}{r}$$

$$E_r = \sqrt{E_x^2 + E_y^2} = \sqrt{E^2 + E^2} = \sqrt{2}E$$

$$E_r = \frac{\sqrt{2}k\lambda}{r}$$

(iii) Electric field due to finite wire at symmetric point :

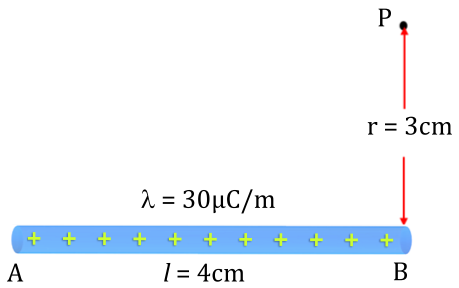


$$E_x = E_\perp = \frac{k\lambda}{r}(\sin\theta + \sin\theta) = \frac{2k\lambda}{r}\sin\theta$$

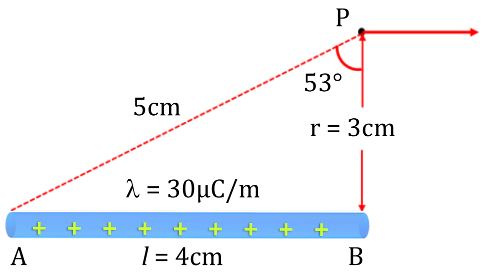
$$E_y = E_{\parallel} = \frac{k\lambda}{r}(\cos\theta - \cos\theta) = 0$$

Illustration 38:

Find x component of electric field at point P (see figure).



Solution:



$$E = \frac{k\lambda}{r}(\cos 0° - \cos 53°)$$

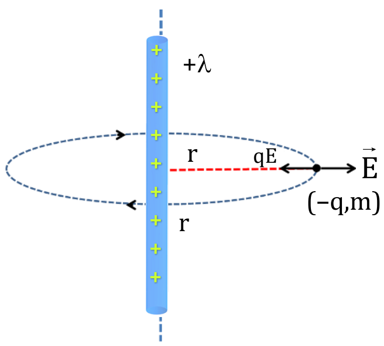
$$E_x = \frac{k\lambda}{r}\left(1 - \frac{3}{5}\right) = \frac{k\lambda}{r}\left(\frac{2}{5}\right)$$

$$E_x = \frac{(9\times10^9)(30\times10^{-6})}{3\times10^{-2}}\left(\frac{2}{5}\right) = 3.6\times10^6\ \text{N/C}$$

Illustration 39:

A negative charge particle (-q, m) is revolving around a uniformly charged long wire (+λ). Find speed of the particle.

Solution:



E will be radially away from wire, so force on charge will be towards the wire

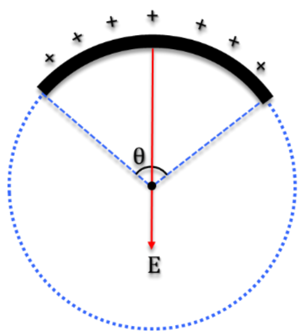
Force of attraction of wire on -q will provide required centripetal force

$$qE = \frac{mv^2}{r} \qquad \text{where, } E = \frac{2k\lambda}{r}$$

$$\frac{2k\lambda q}{r} = \frac{mv^2}{r},\ v = \sqrt{\frac{2k\lambda q}{m}}\ \text{m/s}$$

Electric Field due to a Uniformly Charged Arc

Consider a uniformly charged circular arc of radius R whose ends subtends an angle θ at centre.



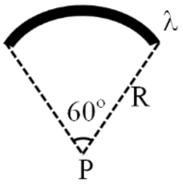
Electric field at centre of arc

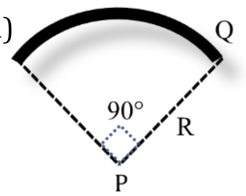
$$E=\frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right) \qquad \theta \to \text{angle of arc}$$

Here λ is linear charge density

Illustration 40:

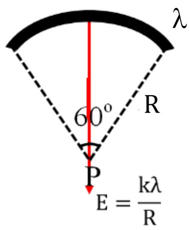
Find electric field at point P in the following cases.

(i)


(ii)


Solution:

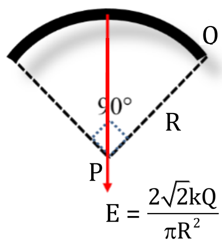
(i) From the formula of electric field due to arc



$$E=\frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right) \qquad \theta \to \text{angle of arc}$$

$$E=\frac{2k\lambda}{R}\sin\left(\frac{60^\circ}{2}\right)=\frac{2k\lambda}{R}\times\left(\frac{1}{2}\right)=\frac{k\lambda}{R}$$

(ii) From the formula of electric field due to arc



$$E=\frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right) \qquad \theta \to \text{angle of arc}$$

$$E=\frac{2kQ}{R}\sin\left(\frac{90^\circ}{2}\right)=\frac{2kQ}{R}\times\frac{1}{\sqrt{2}}=\frac{\sqrt{2}k\lambda}{R} \quad (\text{here } \lambda = 2Q/\pi R)$$

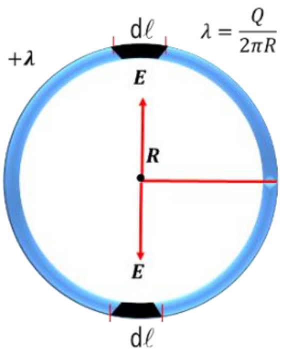
$$E=\frac{2\sqrt{2}kQ}{\pi R^2}$$

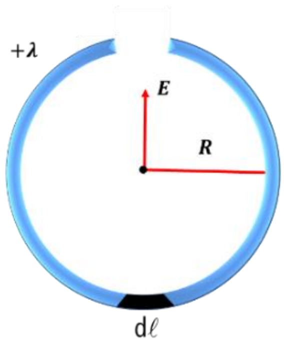
Illustration 41:

Charge Q is uniformly distributed over a ring of radius R. If a small portion of length $d\ell$ is removed from ring, then find electric field at the centre of ring.

Solution:

If dq is charge of small element then, $E=\frac{kdq}{R^2}=\frac{k\lambda d\ell}{R^2}=\frac{kd\ell}{R^2}\left(\frac{Q}{2\pi R}\right)$





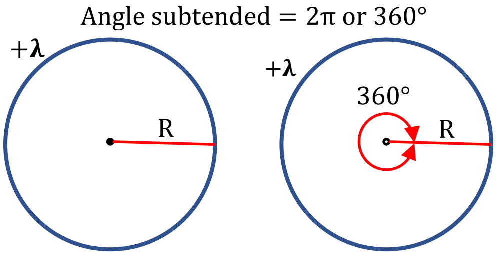
Electric Field at Centre of Uniformly Charged Ring

Electric field due to uniformly charged ring at its centre.

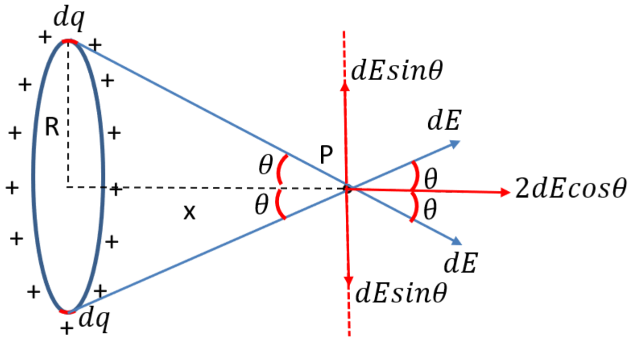
From the formula of electric field due to arc

$$E = \frac{2k\lambda}{R}\sin\left(\frac{\theta}{2}\right) \qquad \theta \to \text{angle of arc}$$

$$E = \frac{2k\lambda}{R}\sin\left(\frac{360°}{2}\right) = 0$$

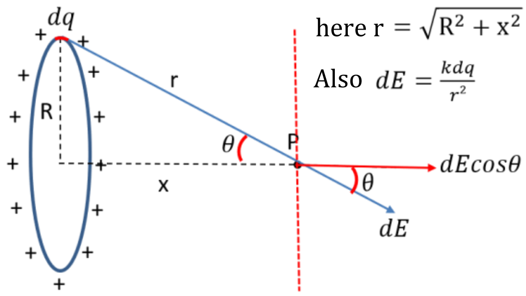


Electric Field due to a Uniformly Charged Ring at its Axis



Here it is clear that dEsinθ, of all the elements will cancel out each other and sum of all dEcosθ , will give net electric field.

∴ Net electric field along x axis



here $r = \sqrt{R^2 + x^2}$

Also $dE = \frac{kdq}{r^2}$

$$E = \int dE\cos\theta = \int \frac{kdq}{r^2}\left(\frac{x}{r}\right)$$

Here $\int dq = Q$

So $$\boxed{E = \frac{kQx}{(R^2 + x^2)^{3/2}}}$$

Special Cases:

(1) Electric field on the axis for small values of x.

If x is very small then x^2 can be neglected w.r.t. R^2

$$E = \frac{kQ}{(R^2 + \underbrace{x^2}_{\text{(neglected)}})^{3/2}}.x \approx \frac{kQ}{R^3}.x \quad \text{(E changes linearly w.r.t. x)}$$

(2) Electric field at the centre of ring.

From $E = \frac{kQx}{(x^2 + R^2)^{3/2}}$ ⇒ E will be zero at x=0

(3) Electric field at the axis for larger values of x.

If x is very large, then R^2 can be neglected w.r.t. x^2

$$E = \frac{kQ}{(\underbrace{R^2}_{\text{(neglected)}} + x^2)^{3/2}}.x \approx \frac{kQ}{x^2} \quad \text{(Ring behaves like a point charge)}$$

(4) Maximum value of electric field

For maximum value of electric field $\frac{dE}{dx} = 0 \Rightarrow \frac{d\left(\frac{kQ}{(R^2+x^2)^{3/2}}.x\right)}{dx} = 0$

From here we obtained $x = \pm\frac{R}{\sqrt{2}}$

So, E_{max} = E at $x = \frac{R}{\sqrt{2}}$, substituting values, we have $E_{max} = \frac{2kQ}{3\sqrt{3}R^2}$

Variation of Electric Field for Ring

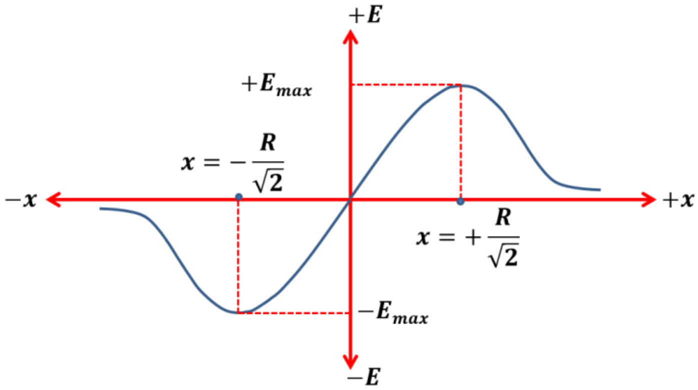


Illustration 42:

Find electric field on the axis of a uniformly charged ring (Q,R) at a distance $x = \sqrt{3}R$.

Solution:

From $E = \frac{kQ}{(R^2+x^2)^{3/2}}.x$

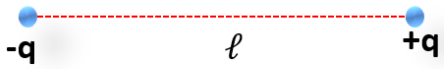
Here we put $x = \sqrt{3}R$

$$E = \frac{kQ}{(R^2+(\sqrt{3}R)^2)^{3/2}}.(\sqrt{3}R)$$

$$E = \frac{kQ}{(4R^2)^{3/2}}.(\sqrt{3}R) = \frac{kQ}{(8R^3)}.(\sqrt{3}R) \Rightarrow E = \frac{\sqrt{3}kQ}{8R^2}$$

Electric Dipole

A system of two equal and opposite charges placed at very small separation is known as electric dipole.



Every dipole has a characteristic property called dipole moment.

Dipole Moment:

The dipole moment of a dipole is equal to product of magnitude of either charge and separation between the charges.

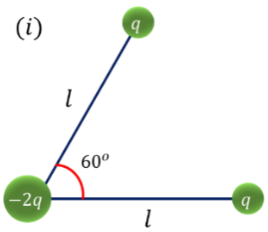
$$\boxed{\vec{p} = q\vec{\ell}}$$

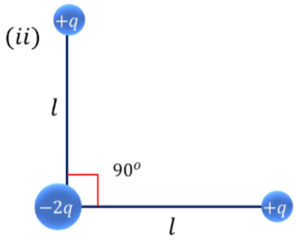
It is a vector quantity whose direction is from (-q) to (+q)

SI unit $\rightarrow$ C-m, Practical unit $\rightarrow$ debye (3.33×10^{-30} C-m)

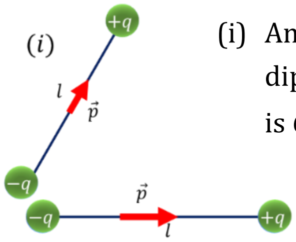
Illustration 43:

Find electric dipole moment of the given arrangement.



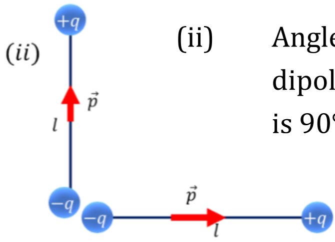


Solution:



(i) Angle between both the dipole moment vectors is 60°.

$$P_{res} = \sqrt{3}p = \sqrt{3}(ql)$$

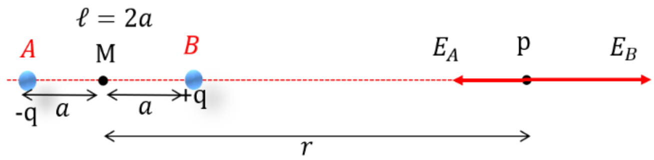


(ii) Angle between both the dipole moment vectors is 90°.

$$P_{net} = \sqrt{2}p = \sqrt{2}(ql)$$

Electric Field Due to a Dipole

(1) At axial / End on position :



Here $E_A = \dfrac{kq}{(r+a)^2}$ & $E_B = \dfrac{kq}{(r-a)^2}$

Net electric field at point P on axis is

$$\text{Eaxis} = E_B - E_A = \frac{kq}{(r-a)^2} - \frac{kq}{(r+a)^2} = kq\left(\frac{4ar}{(r^2-a^2)^2}\right)$$

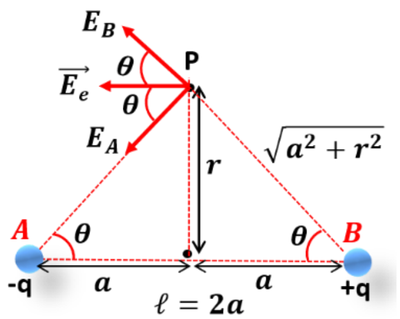
For r>>a ∴ $r^2 - a^2 \approx r^2$

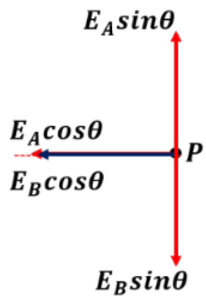
$$E_{axis} = kq\left(\frac{4ar}{r^4}\right) = \left(\frac{2k(q2a)}{r^3}\right) = \left(\frac{2kp}{r^3}\right)$$

For axial point electric field vector and dipole moment vector both are parallel to each other or angle between both vectors is zero degree.

In vector form $\vec{E}_{axis} = \left(\dfrac{2kp}{r^3}\right)\hat{r} = \dfrac{2k\vec{p}}{r^3}$

(2) At equator / Broad side on position :





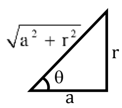
$E_A = E_B$ (in magnitude)

$$E_A = E_B = \frac{kq}{\left(\sqrt{a^2+r^2}\right)^2} = \frac{kq}{(a^2+r^2)}$$

Net electric field at equator

$$E_P = E_B\cos\theta + E_A\cos\theta$$

$$\because \cos\theta = \frac{a}{\sqrt{a^2+r^2}}$$



$$E_P = 2E_B\cos\theta = 2E_A\cos\theta = 2\left(\frac{kq}{(a^2+r^2)}\right)\frac{a}{\sqrt{a^2+r^2}}$$

$$E_P = \left(\frac{kq(2a)}{(a^2+r^2)}\right)\frac{1}{\sqrt{a^2+r^2}} = \frac{kp}{(a^2+r^2)^{3/2}}$$

For r>>a ∴ $r^2 + a^2 \approx r^2$

$$E_P = \frac{kp}{r^3} \qquad \text{in vector from } \vec{E}_P = \frac{k\vec{p}}{r^3}(-\hat{i})$$

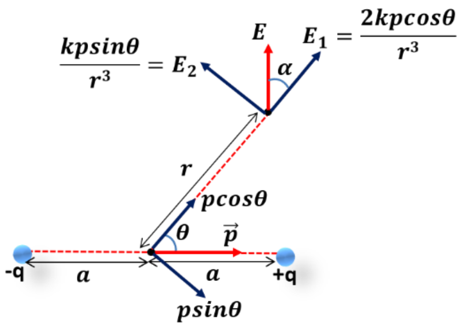
Negative sign is used because electric field vector and dipole moment vector both are opposite in direction.

(3) At general position :

Net electric field at that point will be $E = \sqrt{E_1^2 + E_2^2}$

$$E = \frac{kp}{r^3}\sqrt{(2\cos\theta)^2 + (\sin\theta)^2} = \frac{kp}{r^3}\sqrt{3\cos^2\theta + 1}$$

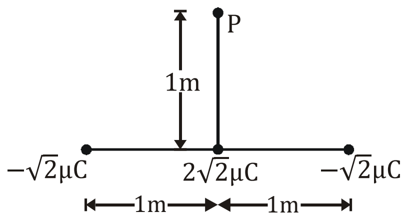
Net electric field is making angle $(\alpha+\theta)$ from the direction of dipole moment vector.



$$\tan\alpha = \frac{E_2}{E_1} = \frac{\frac{kp\sin\theta}{r^3}}{\frac{2kp\cos\theta}{r^3}} = \frac{1}{2}\tan\theta \Rightarrow \boxed{\tan\alpha = \frac{1}{2}\tan\theta}$$

BEGINNER'S BOX-3

1. Two charges of value 2 μC and – 50 μC are placed 80 cm apart. Calculate the distance of the point from the smaller charge where the intensity is zero. **PEL198**
2. A charged particle of mass 2 mili gram remains freely in air in an electric field of strength 4 N/C directed upward. Calculate the charge and determine its nature (g = 10 m/s²). **PEL199**
3. How many electrons should be added or removed from a neutral body of mass 10 mili gram so that it may remain stationary in air in an electric field of strength 100 N/C directed upwards (g = 10 m/s²) ? **PEL200**
4. Work out the magnitude and direction of field at point P, when a charge of 2 μC experiences an electrical force of $5 \times 10^{-2}\,\hat{j}$ N at point P. **PEL201**
5. Two charges 4 μC and 36 μC are placed 60 cm apart. At what distance from the larger charge is the electric field intensity is zero ? **PEL202**
6. Three charges of respective values $-\sqrt{2}\,\mu C$, $2\sqrt{2}\,\mu C$ and $-\sqrt{2}\,\mu C$ are arranged along a straight line as shown in the figure. Calculate the total electric field intensity due to all three charges at the point P.



PEL203

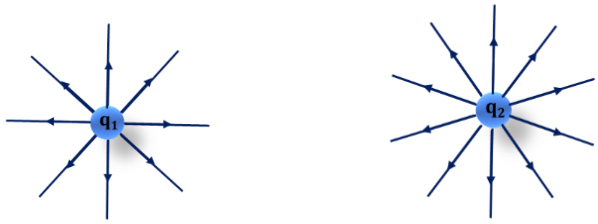
Electric Field Lines

- ❖ These are imaginary lines used to represent electric field pictorially.
- ❖ These are such a smooth continuous curve that tangent drawn at any point gives direction of field at that point.

Properties of Electric Field Lines:

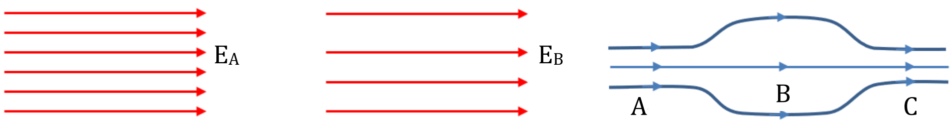
(i) They originate from (+) charge and terminate at (-)charge.

(ii) It gives an idea about the magnitude of charge.
$|q| \propto$ number of field lines
Here $|q_1| < |q_2|$



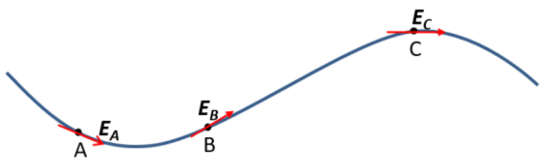
(iii) It gives an idea about the strength of electric field.
Density of electric field lines (EFL) $\propto$ strength of intensity of electric field.

- ❖ If electric field lines are denser electric field intensity is high.
- ❖ If electric field lines are rarer electric field intensity is low.



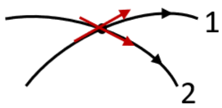
$E_A > E_B$ $\quad$ $E_A = E_C > E_B$

(iv) Tangent drawn at any point of the electric field line, gives the direction of net force on a charge particle placed at that given point.



Note: It gives the direction of force not the direction of motion.

(v) Two electric field lines can never intersect each other.
Reason: If they do so, then at the point of intersection, there will be two tangents at same point representing two different directions of electric field at the point which is not possible.

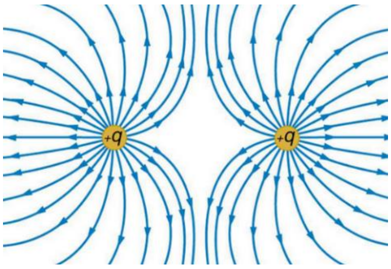


(vi) Electrostatic field lines can never form any closed loop, but induced electric field lines can form a closed loop.

Electric Field

Electrostatic field	Induced electric field
Produced due to static charges	Produced due to change in magnetic field
(no close loops)	(forms close loops)

(vii) Electric field lines due to a pair of like nature of charges.



(viii) Electric field lines due to a pair of equal and opposite charges.

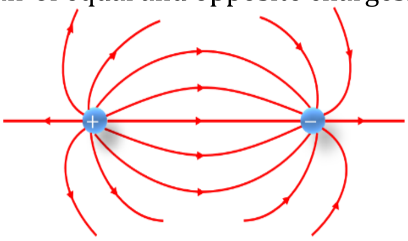


Illustration 44:

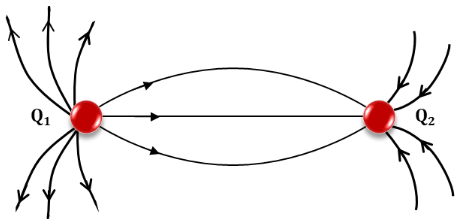
Identify the correct option(s)

(1) $|Q_1| > |Q_2|$

(2) $|Q_1| < |Q_2|$

(3) $Q_1 \rightarrow$ +ve ; $Q_2 \rightarrow$ –ve

(4) $Q_1 \rightarrow$ –ve ; $Q_2 \rightarrow$ +ve



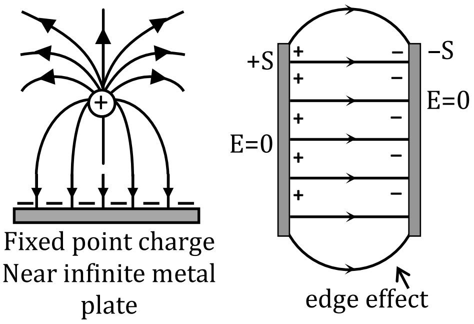
Solution:

(3) and (1) both correct

field lines starts from +ve and ends at -ve, and no. of field lines $\propto$ q

Golden Key Points

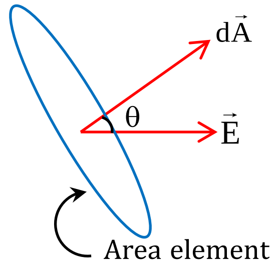
- Lines of force starts from (+ve) charge and end on (–ve) charge.
- Lines of force start and end normally on the surface of a conductor.



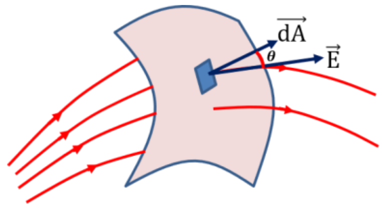
Fixed point charge Near infinite metal plate

- Lines of force never intersect because, the field at a point (point of intersection) cannot have two distinct directions.
- Electric field inside a solid conductor is always zero.
- Electric field inside a hollow conductor may or may not be zero ($E \neq 0$ if net charge is present inside the sphere).
- The electric field due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius i.e. $x >> R$.

Electric Flux



- This physical quantity is used to measure strength of electric field (also called flux density).
- The electric flux can be understood graphically by the total number of electric field lines passing through an area.
- For an area element $d\vec{A}$, placed in an electric field $\vec{E}$ as shown, electric flux is defined as $d\phi = \vec{E}.d\vec{A}$
- Flux is a scalar quantity.
- Unit of flux : Nm^2/C or Vm.
- Dimensional formula of ϕ is $[M^1L^3T^{-3}A^{-1}]$.
- Electric flux through a large surface.



$$\phi = \int d\phi = \int \vec{E}.d\vec{A}$$

❖ If field is uniform $\Rightarrow \vec{E}$ = constant

$\phi = \vec{E}.\int d\vec{A}$ $\qquad$ $\int d\vec{A}$ = total area vector of a plane surface

$$\Rightarrow \boxed{\phi = \vec{E}\,.\,\vec{A}}$$

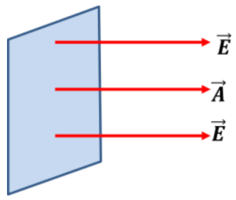
$$\Rightarrow \boxed{\phi = EA\cos\theta}$$

Different Cases:

(i) When surface is perpendicular to the field and $\vec{A}$ is parallel to $\vec{E}$

Here θ is 0°.

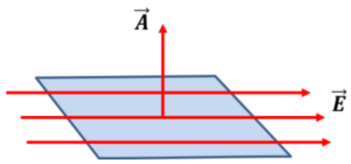
$\phi = EA\cos 0° = EA$ (positive flux means outgoing or leaving)



(ii) When surface is along the field and $\vec{A}$ is perpendicular to $\vec{E}$

Here θ is 90°

$\phi = EA\cos 90^0 = 0$



(iii) When surface is perpendicular to the field and $\vec{A}$ is anti parallel to $\vec{E}$

Here θ is 180^0

$\phi = EA\cos 180^0 = -EA$

Negative flux means incoming or entering

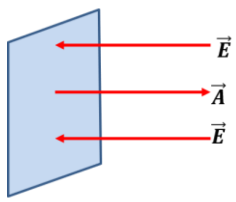


Illustration 45:

If $\vec{E} = \left(2\hat{i} + 3\hat{j} - 5\hat{k}\right)$ N/C and $\vec{A} = \left(2\hat{i} - \hat{j} - \hat{k}\right)$ cm^2 then find electric flux.

Solution:

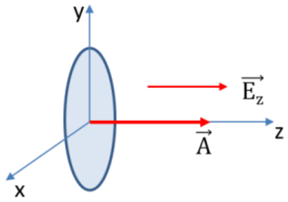
$\phi = \vec{E}\,.\,\vec{A}$

$\phi = \left(2\hat{i} + 3\hat{j} - 5\hat{k}\right).\left(2\hat{i} - \hat{j} - \hat{k}\right)$

$\phi = 4 - 3 + 5 = 6Ncm^2/C$

Illustration 46:

A thin disc of area 2m² is placed in x-y plane in a uniform field of strength $\vec{E}=\left(2\hat{i}-\hat{j}+4\hat{k}\right)$N/C, find flux of the field passing through the disc.



Solution:

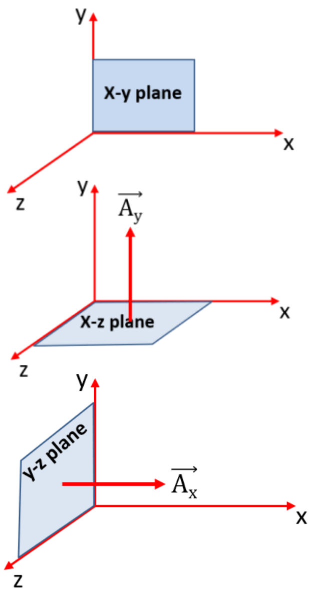
$\vec{E}=\left(2\hat{i}-3\hat{j}+4\hat{k}\right)$N/C and $\vec{A}=\left(2\hat{k}\right)m^2$

$\phi=\vec{E}.\vec{A}=(2\hat{i}-3\hat{j}+4\hat{k}).2\hat{k}=8\dfrac{Nm^2}{C}$

Illustration 47:

An uniform electric field $\vec{E}=\left(10\hat{i}+5\hat{j}\right)$ V/m is present in space. Find out flux passing through the area of 10m², if it lies in,

(1) x-y plane (2) x-z plane (3) y-z plane



Solution:

(1) Only E_z field will be passed through the x-y plane

$\phi_{(x\text{-}y\ plane)}$ =0 (because E_z is zero)

$\phi_{(x\text{-}y\ plane)}=\vec{E}.\vec{A_z}=\left(10\hat{i}+5\hat{j}\right).\left(10\hat{k}\right)=0$

(2) Flux passing through x-z plane

Only E_y field will be passed through the x-z plane

$\phi_{(x-z\,plane)}=\vec{E}.\vec{A_y}=\left(10\hat{i}+5\hat{j}\right).\left(10\hat{j}\right)$, $\phi_{(x-z\,plane)}$ =50 V-m

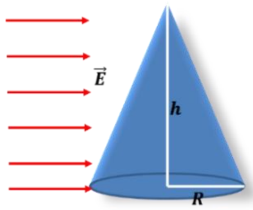
(3) Flux passing through y-z plane

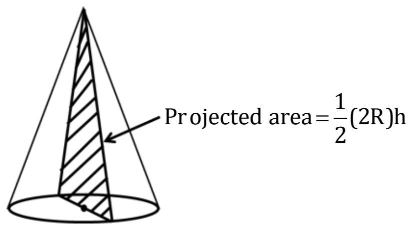
Only E_x field will be passed through the y-z plane

$\phi_{(y\text{-}z\ plane)}=\vec{E}.\vec{A_x}=\left(10\hat{i}+5\hat{j}\right).\left(10\hat{i}\right)$ =100V-m

Illustration 48:

A cone (radius R & height h) is placed in a uniform field E as shown. Find flux passing through the cone.





Solution:

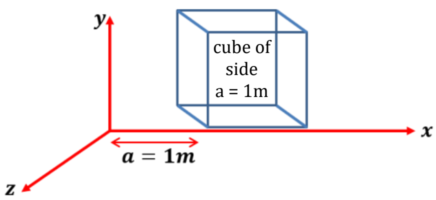
$\phi_{entering}=\phi_{leaving}$ = E × projected area

$\phi_{entering}=\phi_{leaving}$ = ERh

So, net flux through the cone is zero

Illustration 49:

If electric field $\vec{E}=x^2\hat{i}$, find net flux passing through cube

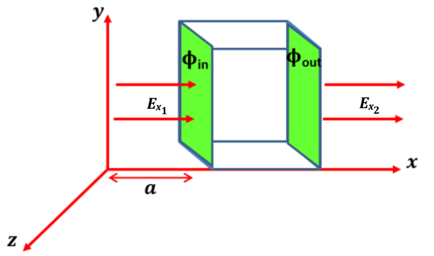


Solution:

$\phi_{in} = -E_{x_1}\left(a^2\right) = -(1^2)(1^2) = -1$ unit

$\phi_{out} = E_{x_2}\left(a^2\right) = 2^2 \times 1^2 = 4$ unit

$\phi_{net} = 4 - 1 = 3$ unit

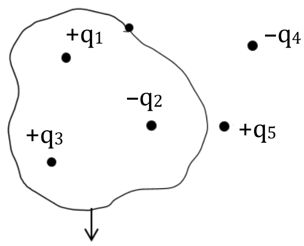


Note: Electric flux through a large surface (open): $\phi = \int_S \vec{E}.d\vec{A}$

Gauss's Law & Applications of Gauss's Law

Gauss's Law: Net electric flux passing through any closed imaginary surface (Gaussian surface) is $\frac{1}{\varepsilon_0}$ times of the total charge enclosed by it.

$$\boxed{\phi_{net} = \frac{Q_{enclosed}}{\varepsilon_0}}$$



Gaussian Surface

To understand this concept, consider a closed surface as shown -

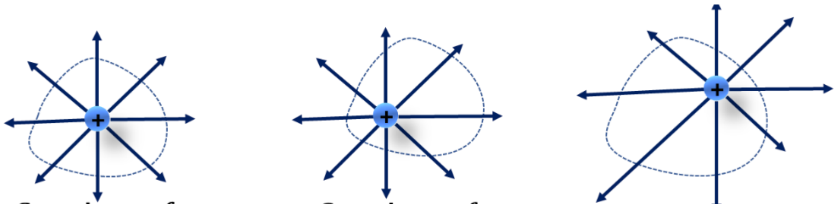
$$\phi_{net} = \frac{(q_1 - q_2 + q_3)}{\varepsilon_0}$$

Charges present outside the closed surface $-q_4$ and $+q_5$ do not play any role in net flux passing through the surface (also called gaussian surface).

$$\boxed{\phi_{net} = \oint \vec{E}.\vec{ds} = \frac{Q_{enclosed}}{\epsilon_0}}$$

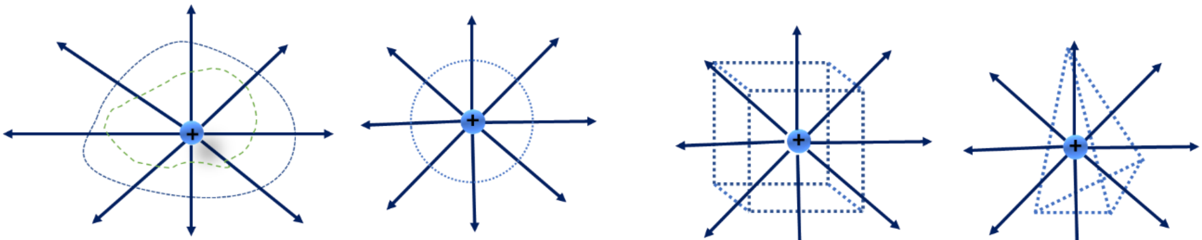
Note:

(i) Flux through Gaussian surface is independent of location of charge within Gaussian surface.



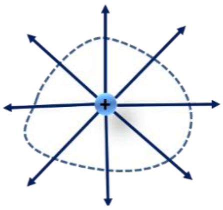
Gaussian surface Gaussian surface Gaussian surface

(ii) Flux through Gaussian surface is independent of shape and size of Gaussian surface provided the net charge enclosed remains same.



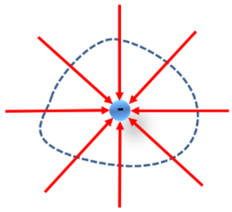
Gaussian surface Gaussian surface Gaussian surface Gaussian surface

(iii) When net flux is going out then positive charge is enclosed by Gaussian surface



Gaussian surface

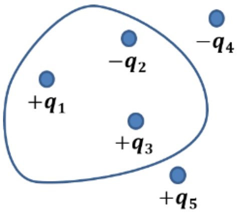
When net flux is coming in then negative charge is enclosed by Gaussian surface



Gaussian surface

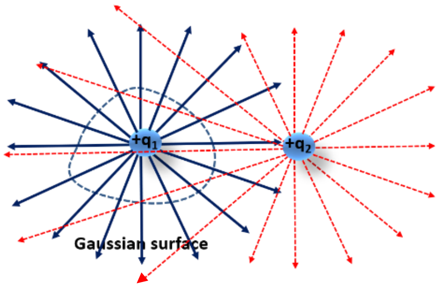
(iv) In Gauss's theorem

$E \rightarrow$ due to all the charges, but $\phi_{closed} \rightarrow$ due to enclosed charges only.



Net flux passing out is only due to $+q_1, -q_2, +q_3$ charges but field at surface is due to $+q_1, -q_2, +q_3, -q_4$ and $+q_5$.

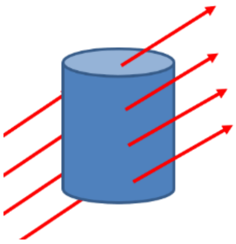
Given diagram shows that why external charges don't contribute in flux through the gaussian surface.



(v) When an imaginary closed surface placed in an external electric field, net flux passing through it, will be zero.

(vi) Gauss's theorem is applicable for all the forces following inverse square law.

(vii) For a Gaussian surface $\phi = 0$ does not imply $E = 0$, but $E = 0$ implies $\phi = 0$.



ϕ(entering) = ϕ (leaving)

$\therefore$ Net flux is zero

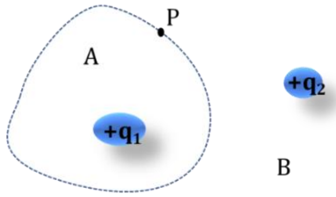
Illustration 50:

What effect will be observed on ϕ_{closed} and electric field at point P, if

(i) charge q_1 is shifted to A

(ii) charge q_1 is shifted to B and

(iii) charge q_2 is shifted to A

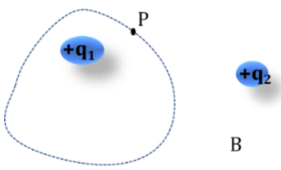


Solution:

(i) when q_1 is shifted to A

$\phi_{closed} \Rightarrow$ same

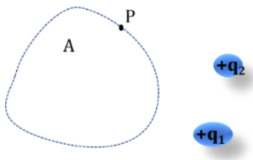
$\vec{E}_P \Rightarrow$ change



(ii) when q_1 is shifted to B

$\phi_{closed} \Rightarrow$ change

$\vec{E}_P \Rightarrow$ change



(iii) charge q_2 is shifted to A

$\phi_{closed} \Rightarrow$ change

$\vec{E}_P \Rightarrow$ change

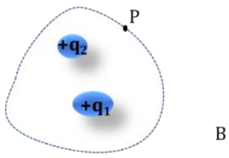
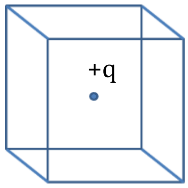


Illustration 51:
Find total flux through cube and flux through each face of cube.



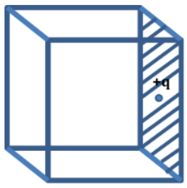
Solution:

Total flux passing through cube will be

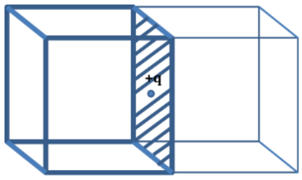
$\phi_{net} = \frac{Q_{enclosed}}{\varepsilon_0}$ (From Gauss's theorem) $\Rightarrow$ $\phi_T = \frac{q}{\varepsilon_0}$

Due to symmetry, flux passing through each face will be same $\Rightarrow$ $\phi_{face} = \frac{q}{6\varepsilon_0}$

Illustration 52:
Find total flux through give cube.



Solution:

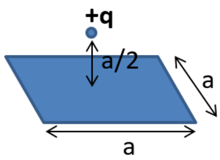


Total flux passing through both the cubes will be $\phi_T = \frac{q}{\varepsilon_0}$

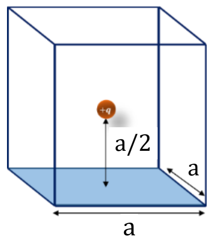
Flux passing through each cube will be $\phi = \frac{q}{2\varepsilon_0}$

Note: Flux through the surface containing charge is zero.

Illustration 53:
Find flux passing through sheet.



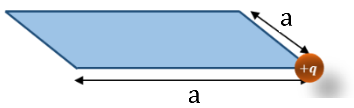
Solution:



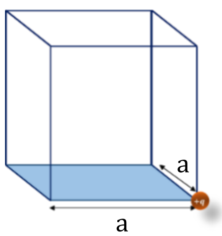
If we assume 6 similar sheets and arranged them in form of a cube in such a way that charge +q comes at centre

Now from gauss's theorem flux $\phi = \frac{q}{6\varepsilon_0}$

Illustration 54:
Find flux passing through sheet.



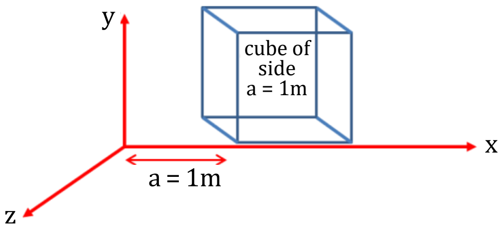
Solution:

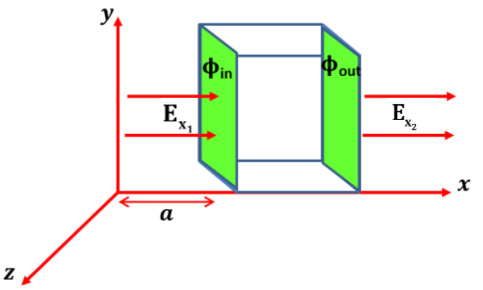


Here, electric field and area vector of the sheet will be perpendicular so flux through it will be zero.

Illustration 55:

If E_x is $5\sqrt{x}$, and E_y & E_z is zero then find,
(1) Net flux passing through cube
(2) Net charge enclosed by the cube





Solution:

$$\phi_{in} = -E_{x_1}\left(a^2\right) = -5\sqrt{1}\left(1\right)^2 = -5 \text{ unit}$$

$$\phi_{out} = E_{x_2}\left(a^2\right) = +5\sqrt{2}\left(1\right)^2 = 5\sqrt{2} \text{ unit}$$

$$\phi_{net} = 5\sqrt{2} - 5 = 5\left(\sqrt{2}-1\right) \text{ unit}$$

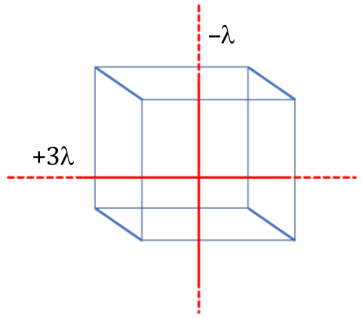
According to Gauss's theorem $\phi_{net} = \dfrac{q}{\varepsilon_0}$

Or $\quad 5\left(\sqrt{2}-1\right) = \dfrac{Q_{in}}{\varepsilon_0} \quad \Rightarrow \quad Q_{in} = 5\left(\sqrt{2}-1\right)\varepsilon_0$

Charge enclosed by the cube is $Q_{in} = 5\left(\sqrt{2}-1\right)\varepsilon_0$

Illustration 56:

Find flux passing out from cube of side L where linear charge densities of line charges kept in cube are λ and $-\lambda$ as shown in the figure.



Solution:

Charge enclosed by cube

$$Q_{in} = +3\lambda L - \lambda L$$

$$Q_{in} = +2\lambda L$$

According to Gauss's theorem

$$Q_{in} = \frac{Q_{in}}{\varepsilon_0} = \frac{2\lambda L}{\varepsilon_0}$$

Applications of Gauss's Law

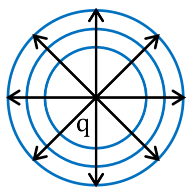
Gauss's Law is the most useful tool to find E, in situations where the charge distributions has some symmetry. Alternatively, if we are given the field, we can use Gauss's Law to determine about the charge distributions.

To determine E for symmetric charge distributions, we select a symmetric Gaussian surface as follows :

Charge Distributions	Direction of EF	Gaussian surface
Point charge	Radial	Concentric surface
Spherical charge	Radial	Concentric surface
Line charge	Radial	Coaxial cylinder
Planar charge or Charged sheets	Normal to surface	Parallel planes like cylindrical

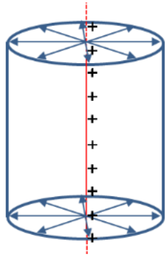
Different shapes of Gaussian surfaces

Point charge and charged sphere



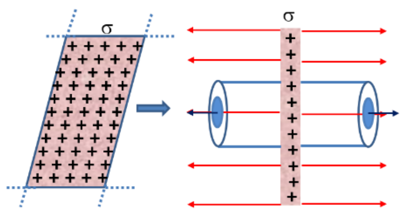
Concentric spherical Gaussian surface

Infinite line charge



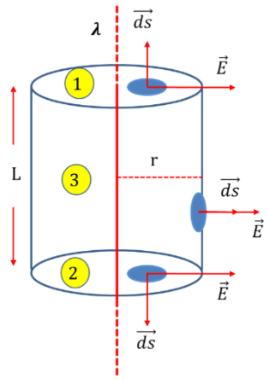
Coaxial Cylindrical Gaussian surface

Infinite sheet



Gaussian surface

(1) Electric field due to a uniformly charged long wire :



Considering a coaxial gaussian cylinder of radius r and length L as shown. Then applying Gauss's theorem

$$\oint \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0} \qquad \text{...(i)}$$

Here, Gaussian Cylinder has three surfaces as :

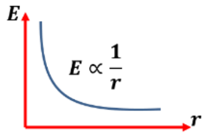
Surfaces (i) and (ii) are flat, and for these surfaces area vector is perpendicular to E

Surfaces (iii) is curved and for this surfaces area vector is along E

$$\oint \vec{E}.d\vec{s} = \int_{S_1} Eds\cos 90° + \int_{S_2} Eds\cos 0° + \int_{S_2} Eds\cos 90° = 0 + \int_{S_2} Eds + 0 = \int_{S2} Eds = E\int_{S2} ds = E(2\pi rL)$$

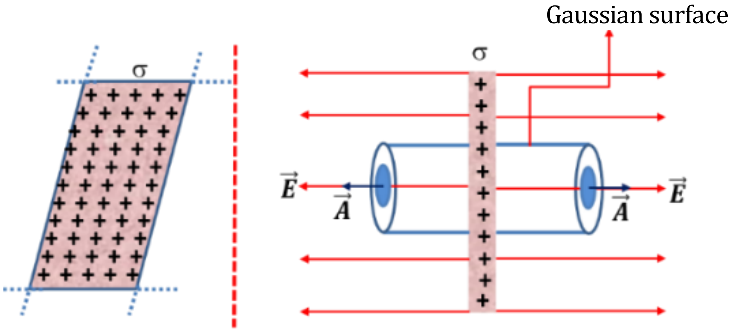
From equation No. (i)

$$E(2\pi rL) = \frac{\lambda L}{\varepsilon_0} = \frac{\lambda}{2\pi\varepsilon_0 r} \text{ or } E = \frac{2k\lambda}{r}$$



Variation of E with r

(2) Electric Field due to Uniformly Charged Infinite Sheet (Non conducting)

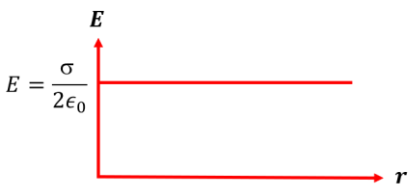


According to Gauss's theorem

$$\phi_{closed} = EA + EA + 0 = \frac{Q_{in}}{\varepsilon_0}$$

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

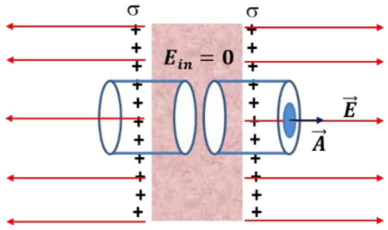


Variation of E with r

Electric field due to non conducting sheet is uniform

(3) Electric Field due to Uniformly Charged Infinite Sheet (Conducting)

Conducting sheet or metal plate -



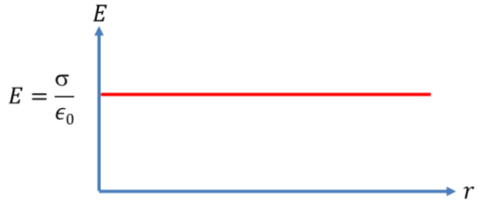
According to Gauss's theorem

$$\phi_{closed} = EA + 0 + 0 = \frac{q_{in}}{\varepsilon_0}$$

$$EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{\varepsilon_0}$$

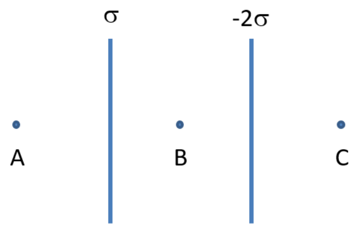
Electric field due to conducting sheet is uniform



Variation of E with r

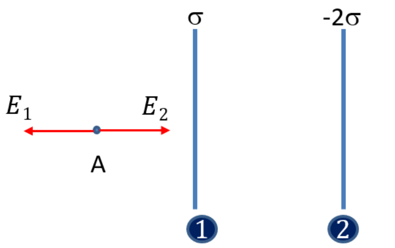
Illustration 57:

Find E_A, E_B and E_C. Consider non conducting sheets.



Solution:

At point A -

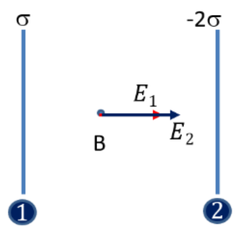


Electric field due to sheet 1 will be leftwards $E_1 = \frac{\sigma}{2\varepsilon_0}$

Electric field due to sheet 2 will be rightwards $E_2 = \frac{2\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$

Net electric field will be rightwards $E_A = E_2 - E_1 = \frac{\sigma}{\epsilon_0} - \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0}$

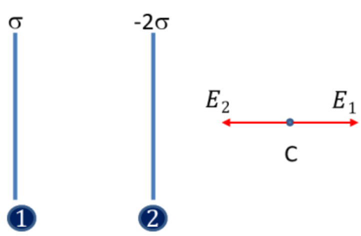
At point B –



Electric field due to both sheets will be rightwards

Net electric field will be rightwards $E_B = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{2\sigma}{2\epsilon_0} = \frac{3\sigma}{2\epsilon_0}$

At point C –



Electric field due to sheet 1 will be rightwards = $E_1 = \frac{\sigma}{2\epsilon_0}$

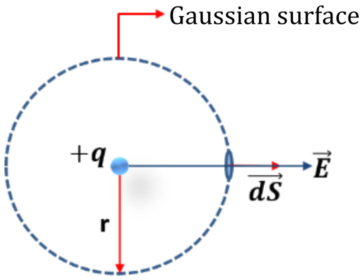
Electric field due to sheet 2 will be leftwards $E_2 = \frac{2\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0}$

Net electric field will be leftwards $E_A = E_2 - E_1 = \frac{2\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{2\epsilon_0}$

Electric field due to the charged conducting sphere and non conducting sphere

(1) Electric field due to point charge.

Considering a concentric spherical gaussian surface of radius r, as shown,

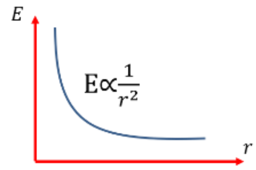


We now apply gauss's theorem as :

$$\oint \vec{E}.d\vec{s} = \frac{q_{in}}{\varepsilon_0} \quad \text{...(i)}$$

Angle between $\vec{E}$ and $d\vec{s}$ at every point of the Gaussian surface is 0° $\Rightarrow \oint \vec{E}.d\vec{s} = \oint Eds\cos\theta = \oint Eds$

Since all points on spherical Gaussian surface are equidistant from the point charge :



Variation of E with r

$\therefore \oint Eds = E\oint ds = E(4\pi r^2)$

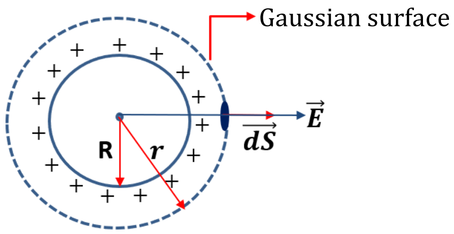
From $E\left(4\pi r^2\right) = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{q}{4\pi\varepsilon_0 r^2}$ or $\frac{kq}{r^2}$

(2) Electric field due to uniformly charged thin shell or conducting sphere.

(i) EF at any point outside the sphere (r>R)

Considering a Gaussian surface and applying Gauss's theorem

$$\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0}$$



For spherical surface, we can directly write

$$\oint \vec{E}.\vec{ds} = E(4\pi r^2)$$

$\therefore E(4\pi r^2) = \frac{q}{\varepsilon_0}$ or $E = \frac{q}{4\pi\varepsilon_0\left(r^2\right)}$ or $E = \frac{kq}{r^2}$

Thus for point lying outside the sphere, sphere behaves as point charge centered at centre.

(ii) EF at any point lying on the surface of sphere (r = R)

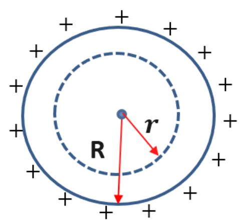
For the point on surface, we putting r = R in $E = \frac{kq}{r^2}$, we have

$\boxed{E_S = \frac{kq}{R^2}}$ (max. value of electric field)

$$E_S = \frac{q}{4\pi\epsilon_0 R^2} = \frac{\sigma}{\epsilon_0} \quad \left(\because \sigma = \frac{q}{4\pi R^2}\right)$$

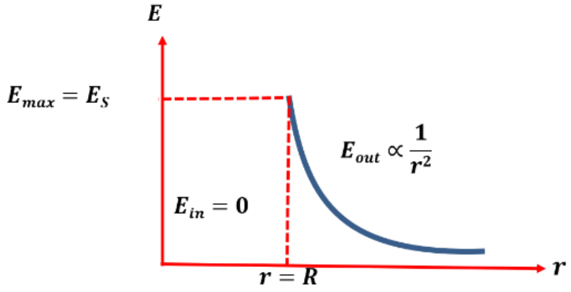
(iii) EF at any point inside the sphere (r < R)

In this case, charge enclosed by the gaussian surface is zero, so using Gauss's theorem



$$\oint \vec{E_{in}}.\vec{ds} = \frac{0}{\varepsilon_0} \Rightarrow E_{in} = 0$$

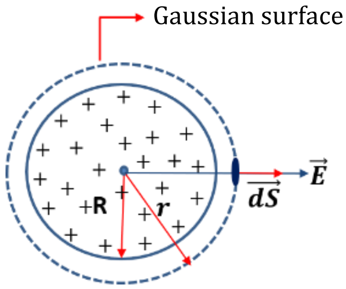
(iv) Variation of E with r



(3) Electric field due to uniformly charged nonconducting sphere.

(i) EF at any point outside the sphere (r > R)

According to Gauss's theorem



$$\oint \vec{E}\,\vec{ds} = \frac{q_{in}}{\varepsilon_0}$$

$$\oint E ds = \frac{q}{\varepsilon_0} \Rightarrow E\oint ds = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E(4\pi r^2) = \frac{q}{\varepsilon_0} \Rightarrow E = \frac{kq}{r^2}$$

Thus for point lying outside the sphere, sphere behaves as point charge centered at centre.

(ii) EF at any point lying on the surface of sphere (r=R)

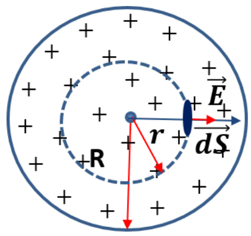
$$E_S = \frac{kq}{R^2} \text{ (max. value of electric field)}$$

$$E_S = \frac{q}{4\pi\varepsilon_0 R^2} = \frac{qR}{3\left(\frac{4}{3}\pi R^3\right)\varepsilon_0} = \frac{\rho R}{3\epsilon_0}$$

[$\therefore$ ρ is volume charge density]

(iii) EF at any point inside the sphere (r<R)

If q_{in} is the charge enclosed by G.S., then



$$\Rightarrow q_{in} = \rho\left(\frac{4}{3}\pi r^3\right) = \frac{q}{\frac{4}{3}\pi R^3}\left(\frac{4}{3}\pi r^3\right)$$

According to Gauss's theorem

$$\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\varepsilon_0}$$

$$\oint E ds = \frac{q_{in}}{\varepsilon_0} \Rightarrow E\oint ds = \frac{q_{in}}{\varepsilon_0} \Rightarrow E\left(4\pi r^2\right) = \frac{q}{R^3}\left(r^3\right)\frac{1}{\varepsilon_0} \Rightarrow E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}(r) = \frac{kq}{R^3}(r)$$

So, inside the nonconducting sphere electric field is directly proportional to r.

$$E_{in} = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^3}(r) = \frac{q}{3\left(\frac{4}{3}\pi R^3\right)}(r) = \frac{\rho}{3\varepsilon_0}(r)$$

For r = 0, E = 0 $\Rightarrow$ electric field intensity at the centre of the sphere is zero.

(iv) Variation of E with r

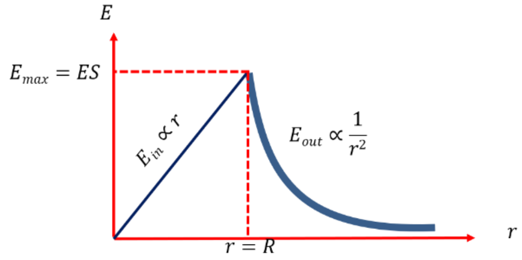


Illustration 58:

Electric field at a distance 20 cm from a charged metallic sphere of radius 10 cm is E. Find E at a distance :

(1) r = 5cm (2) r = 10cm (3) r =30cm, from centre.

Solution:

(1) 5cm (r <R), so E = 0 (for conductor E= 0 at inside point)

(2) 10cm (r = R), $E_s = \frac{kQ}{R^2}$ As $E = \frac{kQ}{(20)^2} = \frac{kQ}{4R^2}$ so $E_s = \frac{kQ}{R^2} = 4E$

(3) 30cm (r > R), $E_{out} = \frac{kQ}{9R^2}$ so $E_{out} = \frac{4E}{9}$

Illustration 59:

Electric field at a distance 30 cm from a uniformly charged non conducting sphere of radius 15 cm is E. Find E at a distance :

(1) r = 5 cm (2) r = 15 cm (3) r =20 cm, from centre.

Solution:

(1) 5 cm (r <R),

$E_{out} = \frac{kQ}{r^2} = E$ (given) or kQ = $(30)^2$E

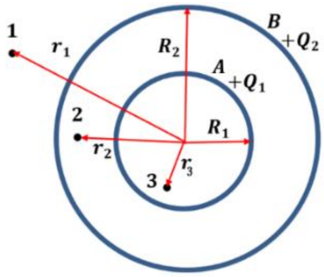
$$E_{in} = \frac{kQ}{R^3}(r) = \frac{(30)^2 E}{(15)^3}(5) = \frac{4E}{3}$$

(2) 15cm (r = R), $E_S = \frac{kQ}{R^2} \Rightarrow E_s = \frac{(30)^2 E}{(15)^2} = 4E$

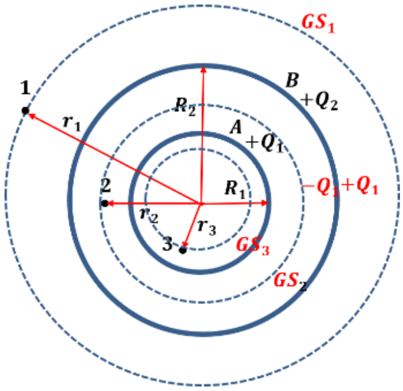
(3) 20cm (r > R), $E_{out} = \frac{kQ}{r^2} = \frac{(30)^2 E}{(20)^2} = \frac{9E}{4}$

Illustration 60:

Two concentric conducting spherical shells are shown in figure. Find electric field at points 1,2 & 3.



Solution:



For GS_1 charge enclosed is $+Q_1$ and $+Q_2$ i.e.

$$E_1 = \frac{k(Q_1+Q_2)}{r_1^2}$$

For GS_2 charge enclosed is $+Q_1$, i.e.

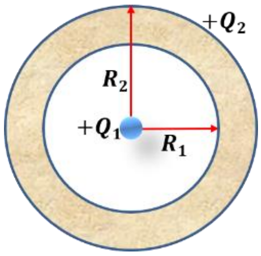
$$E_2 = \frac{kQ_1}{r_1^2}$$

For GS_3 charge enclosed is zero, i.e.

$E_3 = 0$

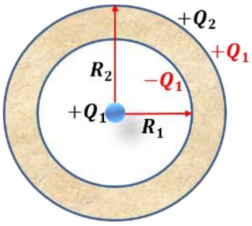
Illustration 61:

A point charge $+Q_1$ is placed at the centre of a thick conducting shell of inner radius R_1 and outer radius R_2 as shown then find. (1) $\sigma_{\text{inner surface}}$ (2) $\sigma_{\text{outer surface}}$



Solution:

due to charge at centre $-Q_1$ & Q_2 will be induced charge on inner & outer surface respectively of shell.



(1) $\sigma_{\text{inner surface}} = \frac{-Q_1}{4\pi R_1^2}$

(2) $\sigma_{\text{outer surface}} = \frac{(Q_1+Q_2)}{4\pi R_2^2}$

BEGINNER'S BOX-4

1. Two point charges Q and 4Q are 12 cm apart. Sketch the lines of force and calculate the distance of neutral point from 4Q charge. **PEL204**
2. A charge Q is uniformly distributed over a large plastic (non-conducting) sheet. The electric field at a point close to the centre of the plate near the surface is 20 V/m. If the plate is replaced by a copper plate of the same geometrical dimensions and carrying the same charge Q, then what is the electric field at that point? **PEL205**
3. What is the net flux of a uniform electric field $\vec{E} = 3\times 10\,\hat{i}\ NC^{-1}$ through a cube of side 20 cm oriented such that its faces are parallel to the coordinate planes ? **PEL206**
4. Charges Q_1 and Q_2 lie inside and outside a closed surface S respectively. Let E be the field at any point on S and ϕ be the flux of E over S. Which statement is wrong?
   (A) If Q_1 changes, both E and ϕ will change.
   (B) If Q_2 changes, E will change but ϕ will not change.
   (C) If Q_1=0 and $Q_2 \neq 0$ then $E \neq 0$ but ϕ =0
   (D) If $Q_1 \neq 0$ and Q_2=0 then E=0 but $\phi \neq 0$ **PEL207**

5. A charge 'q' is placed at the centre of a cube whose top face is open (it has only 5 faces). Calculate the total electric flux passing through the cube. **PEL208**

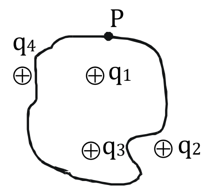
6. A point charge of 2.0 μC is at the centre of a cubic Gaussian surface of edge 9.0 cm. What is the net electric flux through the surface ? **PEL209**

7. An electric flux of -6×10^{-3} Nm^2/C passes normally through a spherical Gaussian surface of radius 10 cm, due to a point charge placed at its centre.

(a) What is the charge enclosed by the Gaussian surface ?

(b) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface ? **PEL210**

8. A Gaussian surface encloses two of the four positively charged particles as shown in figure. Which of the particles contribute to the electric field at a point P on the surface ? **PEL211**



9. Is it possible to have flux associated with an imaginary closed surface to be zero even when electric field on this surface is non–zero. If yes, then give one example. **PEL212**

10. Two large, thin metal plates are parallel and close to each other. The plates have surface charge densities of opposite signs and of magnitude 17.0×10^{-12} C/m^2 on their inner faces. What is electric field,

(a) in the outer region of the first plate ?

(b) between the plates ? **PEL213**

11. Plot the following graphs –

(a) Electric field inside a conducting sphere with distance from its centre.

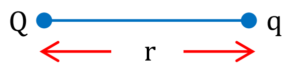
(b) E versus (1/r) where E is electric field due to a point charge and r is the distance from the charge. **PEL214**

Electrostatic Potential Energy & Electric Potential

Electrostatic Potential Energy (EPE)

- ❖ EPE of a point charge, is the amount of work done in bringing the charge from infinity to the present position without changing its kinetic energy.
- ❖ EPE of a system of charges, is the minimum amount of work done in assembling the given charge system.
- ❖ An object may have EPE by virtue of
  (i) Its own charge called self-energy and
  (ii) Its relative position in the system of charges, called interaction energy.
  Therefore, total EPE = self-energy + interaction energy
- ❖ EPE is defined only for conservative field like gravitational field, electrostatic field etc.
- ❖ From Work Energy Theorem
  $W_{\text{external agent}} + W_{\text{internal conservative field}} + W_{\text{internal non-conservative field}} = \Delta KE$
  If we assume $W_{\text{internal non-conservative field}} = 0$, then $W_{\text{external agent}} + W_{\text{internal conservative field}} = \Delta KE$
  For $\Delta KE = 0$
  $W_{\text{external agent}} = -W_{\text{internal conservative field}} = U_f - U_i$ ($\because W_{\text{internal conservative field}} = U_i - U_f$)
  $W_{\text{external agent}} = U_f - U_i$

Electrostatic Potential Energy (EPE) of two point charge system:



To find this, consider Q is at rest and q is brought from ∞ to r. (Here reference is at infinity)
At any instant suppose charge **q** is at distance **x** from **Q.**

At this instant electric force on q is

$F_{elec} = \dfrac{kQq}{x^2}$ (rightwards)

Then, $\Delta U = (W_{\infty\to r})_{ext.} = \int_{\infty}^{r} \vec{F}_{ext}.d\vec{r}$

$$\Rightarrow U_r - U_\infty = \Delta U = -\left(W_{\infty\to r}\right)_{elec} = -\int_{\infty}^{r} \vec{F}_{elec} d\vec{x}$$

$$\Rightarrow U_r - U_\infty = -\int_{\infty}^{r} \vec{F}_{elec} d\vec{x} \qquad \left[\because \vec{F}_{elec} \,||\, d\vec{x}\right]$$

$$\Rightarrow U_r - U_\infty = -\int_{\infty}^{r} \frac{kQq}{x^2} dr = kQq\left[+\frac{1}{x}\right]_{\infty}^{r} \Rightarrow U_r - U_\infty = kQq\left[\frac{1}{r} - \frac{1}{\infty}\right] = \frac{kQq}{r}$$

As we consider infinity as reference & $U_\infty = 0$, so we have

$U\text{-}0 = \dfrac{kQq}{r} \Rightarrow U = \dfrac{kQq}{r}$

Note: Always put sign of the charge in this formula.

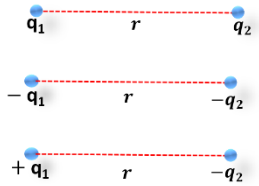
❖ Potential energy under repulsive forces is positive.

❖ Potential energy under attractive forces is negative.

(1) $U = \frac{kq_1q_2}{r}$

(2) $U = \frac{kq_1q_2}{r}$

(3) $U = -\frac{kq_1q_2}{r}$



❖ **Amount of work done in changing the configuration of charge particles.**

$W = \Delta U = U_f - U_i$

Where U_f = EPE of final configuration of charge particles

U_i = EPE of initial configuration of charge particles

❖ **Variation of U with r for two-point charge system**

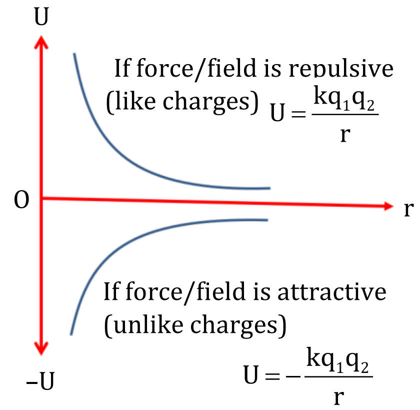
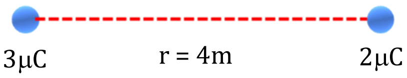


Illustration 62:

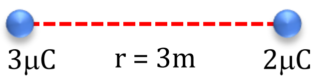
Find potential energy of the system shown. Also find work done in decreasing the distance by 1m.



Solution:

$$U_i = \frac{kq_1q_2}{r} = \frac{9\times10^9\left(3\times10^{-6}\right)\left(2\times10^{-6}\right)}{4} = 13.5\times10^{-3}\text{ J}$$

When distance decreases by 1m, separation between charges becomes 3m



$$U_f = \frac{9\times10^9\left(3\times10^{-6}\right)\left(2\times10^{-6}\right)}{3} = 18\times10^{-3}\text{ J}$$

Work done in this process

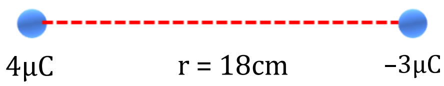
$W = U_f - U_i = 18 \times 10^{-3} - 13.5 \times 10^{-3} = 4.5 \times 10^{-3}$ J

Illustration 63:

For the given charge system, find –

(1) PE of system

(2) Work done in carrying the charges to infinity



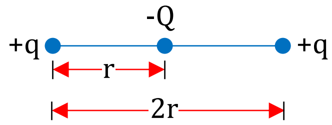
Solution:

(1) $$U = \frac{kq_1q_2}{r} = \frac{(9\times10^9)(4\times10^{-6})(-3\times10^{-6})}{(18\times10^{-2})} = -0.6\text{J}$$

(2) $W = U_f - U_i = 0-(-0.6\text{J}) = +0.6\text{J}$

Illustration 64:

Figure shows an arrangement of three-point charges. The total potential energy of this arrangement is zero. Calculate the ratio $\frac{q}{Q}$.



Solution:

$\because U_{sys} = 0$

$$\Rightarrow \frac{1}{4\pi\varepsilon_0}\left[\frac{-qQ}{r}+\frac{(+q)(+q)}{2r}+\frac{Q(-q)}{r}\right] = 0 \Rightarrow -Q + \frac{q}{2} - Q = 0 \Rightarrow 2Q = \frac{q}{2} \Rightarrow \frac{q}{Q} = \frac{4}{1}$$

Electrostatic Potential Energy of a System of Charges

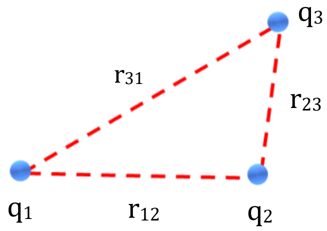
It is defined as work done by external agent or work done against electric field in bringing charges from infinity to their respective positions, in forming system of charges.

Example –

(i) For 3-point charge system -

$U_S = U_{12} + U_{23} + U_{31}$

$$U_s = \frac{kq_1q_2}{r_{12}} + \frac{kq_2q_3}{r_{23}} + \frac{kq_3q_1}{r_{31}}$$



(ii) For 4-point charge system –

$U_S = U_{12} + U_{13} + U_{14} + U_{23} + U_{24} + U_{34}$

Similarly, for n point charge system

$$\text{No. of pairs } = \frac{n(n-1)}{2}$$

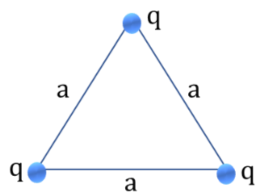
Illustration 65:

Consider the charge system shown, find

(i) Potential energy of any one of the charges.

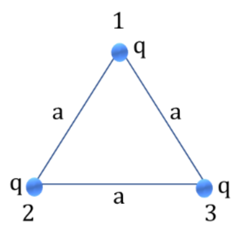
(ii) Potential energy of the system.

(iii) Work done in associating the system.



Solution:

(i) $$U_1 = U_{12} + U_{13} = \frac{kq^2}{a} + \frac{kq^2}{a} = \frac{2kq^2}{a}$$



(ii) $$U_S = U_{12} + U_{13} + U_{23} = \frac{kq^2}{a} + \frac{kq^2}{a} + \frac{kq^2}{a} = \frac{3kq^2}{a}$$

(iii) $$W = U_f - U_i = U_s - U_\infty = \frac{3kq^2}{a} - 0 = \frac{3kq^2}{a}$$

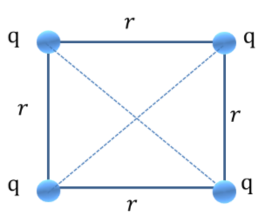
Illustration 66:

Consider the point charge system shown, find

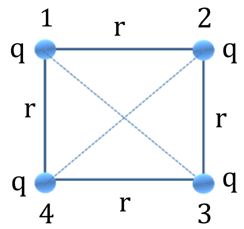
(i) Potential energy of any one of the charges.

(ii) Potential energy of the system.

(iii) Work done in dissociating the system.



Solution:



(i) $U_1 = U_{12} + U_{13} + U_{14} = \frac{2kq^2}{r} + \frac{kq^2}{r\sqrt{2}} = \frac{kq^2}{r}\left(2 + \frac{1}{\sqrt{2}}\right)$

(ii) $U_s = U_{12} + U_{23} + U_{34} + U_{41} + U_{13} + U_{42}$

$$U_S = \frac{kq_1q_2}{r} + \frac{kq_2q_3}{r} + \frac{kq_3q_4}{r} + \frac{kq_4q_1}{r} + \left\{\frac{kq_1q_3}{\sqrt{2}r} + \frac{kq_4q_2}{\sqrt{2}r}\right\}$$

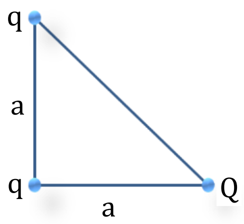
$\because$ If all charges are identical,

$$U_S = \frac{4kq^2}{r} + \frac{2kq^2}{\sqrt{2}r} = \frac{kq^2}{r}(4 + \sqrt{2})$$

(iii) $W = U_f - U_i = U_\infty - U_s = -\frac{kq^2}{r}\left(\sqrt{2} + 4\right)$

Illustration 67:

For what value of Q, EPE of the system shown is zero.



Solution:

$$U_s = \left(\frac{k(q\times q)}{a} + \frac{k(q\times Q)}{a} + \frac{k(q\times Q)}{\sqrt{2}a}\right)$$

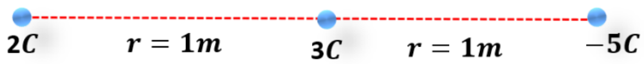
According to the given condition

$$\left(\frac{k(q\times q)}{a} + \frac{k(q\times Q)}{a} + \frac{k(q\times Q)}{\sqrt{2}a}\right) = 0 \Rightarrow \left(q^2 + (q\times Q) + \frac{(q\times Q)}{\sqrt{2}}\right) = 0 \Rightarrow \left(q + (Q) + \frac{(Q)}{\sqrt{2}}\right) = 0$$

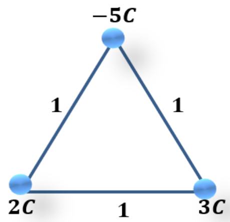
$$\Rightarrow (Q) + \frac{(Q)}{\sqrt{2}} = -q \Rightarrow Q = \frac{-q\sqrt{2}}{1+\sqrt{2}}$$

Illustration 68:

Find work done in shifting -5C charge to a point 1m equidistant from both the remaining charges.



Solution:



$$W = \Delta U = (U_f) - (U_i)$$

$$\Delta U = \left[\frac{k(2)(-5)}{1} + \frac{k(2)\times(3)}{1} + \frac{k(3)\times(-5)}{1}\right] - \left[\frac{k(2)\times(3)}{1} + \frac{k(3)\times(-5)}{1} + \frac{k(2)\times(-5)}{2}\right]$$

$= (-10k + 6k - 15k) - (6k - 15k - 5k) = -5k = -45\times10^9 J$

Illustration 69:

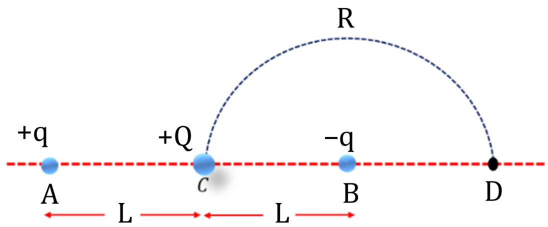
If separation between two charges is increased, whether EPE of the system increases or decreases.

Solution:

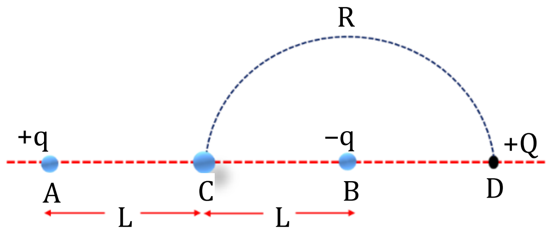
$\therefore$ It may increase or decrease depending upon sign of charges of system.

Illustration 70:

Find work done in transferring the charge +Q via semicircular path CRD centered at B.



Solution:

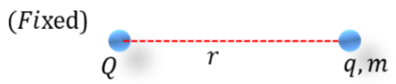


$$W = \Delta U = (U_f) - (U_i)$$

$$W = \left[\frac{k(q)(-q)}{2L} + \frac{k(-q)(Q)}{L} + \frac{k(q)(Q)}{3L}\right] - \left[\frac{k(q)(Q)}{L} + \frac{k(Q)(-q)}{L} + \frac{k(q)(-q)}{2L}\right] = -\frac{2kqQ}{3L}$$

Illustration 71:

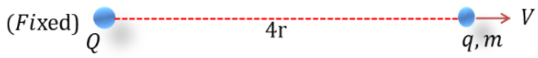
Charge (q, m) is released from rest position as shown in figure. Find the speed of the particle when its separation from charge Q becomes 4r.



Solution:

On releasing q, let its speed becomes v at separation of 4r then,

Using COME, $K_i + U_i = K_f + U_f$



$$\Rightarrow 0 + \frac{k(Q)(q)}{r} = \frac{1}{2}mv^2 + \frac{k(Q)(q)}{4r}$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{kQq}{r} - \frac{kQq}{4r} \Rightarrow \frac{1}{2}mv^2 = \frac{3kQ}{4r} \Rightarrow v = \sqrt{\frac{3kQq}{2mr}}$$

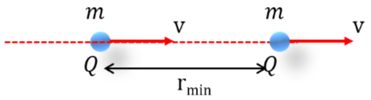
Illustration 72:

Find distance of closest approach for the given system of two charge particles, shown below.

Solution:

Initial situation is -

Finally, at minimum distance both will acquire same speed (v) –



As there is no external force acting on system, so we use Conservation of Linear Momentum (COLM) to find final velocities of charge particles

$$\Rightarrow mv_0 = mv + mv \Rightarrow v = \frac{v_0}{2}$$

Now we use COME to find minimum distance of approach

$\Rightarrow K_i + U_i = K_f + U_f$

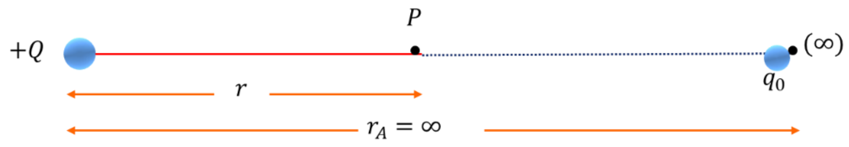
$$\Rightarrow \frac{1}{2}mv_0^2 + 0 = \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{kQ^2}{r}$$

$$\Rightarrow \frac{1}{2}mv_0^2 = 2\left[\frac{1}{2}m\left(\frac{v_0}{2}\right)^2\right] + \frac{kQ^2}{r}$$

$$\frac{1}{4}mv_0^2 = \frac{kQ^2}{r} \Rightarrow r_{min} = \frac{4kQ^2}{mv_0^2}$$

Electric Potential

In electrostatic field, the electric potential (due to some source charges) at a point P is defined as the work done by external agent in taking a unit point positive charge from a reference point (generally taken at infinity) to that point P without changing its kinetic energy.



If $(W_{\infty \to P})_{ext}$ is the work required in moving a point charge q from infinity to a point P, the electric potential of the point P is

$$V_p = \left.\frac{\left(W_{\infty\to p}\right)_{ext}}{q}\right]_{\Delta K=0}$$

❖ It is scalar quantity

❖ S.I. Unit of potential is volt = $\frac{\text{joule}}{\text{coulomb}}$ and its dimensional formula is $[M^1L^2T^{-3}I^{-1}]$.

❖ Potential can be positive, negative and even zero. Hence charges must be taken with sign.

❖ Electric potential at a point is also equal to the negative of the work done by the electric field in taking the point charge from reference point (i.e. infinity) to that point.

$$V_p = -\left.\frac{\left(W_{\infty\to p}\right)_{elec}}{q}\right]_{\Delta K=0}$$

Illustration 73:

A point charge of 20μC is brought from infinity to a point in an electric field. If -40 μJ of work had to be done against the field then find potential at that point.

Solution:

$$V_P = \left(\frac{W_{agent}}{q_0}\right)_{\Delta K=0} = \left(\frac{-(40\times10^{-6})}{(20\times10^{-6})}\right)_{\Delta K=0}$$

So electric potential at that point is -2V.

Illustration 74:

When a point charge of 20μC is brought from infinity to a point in an electric field, -20μJ of work is done by the electric field. Then what amount of work would have to be done by the external agent, if a 40μC charge brought from infinity to the same point ?

Solution:

$$V_P = -\left(\frac{W_{(\infty\to P)ele}}{q_0}\right) \Rightarrow V_P = -\left(\frac{-\left(20\times10^{-6}\right)}{\left(20\times10^{-6}\right)}\right)$$

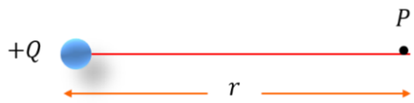
$V_P = 1V$

Now from, $V_P = \left(\frac{W_{(\infty\to P)}ext}{q}\right) \Rightarrow W_{(\infty\to P)}ext = q\,V_P = \left(40\times10^{-6}\right)\times1 \Rightarrow W_{(\infty\to P)ext} = 40\mu J$

Electrostatic Potential due to a point charge

Electric potential at a point in electric field is defined as amount of work done in bringing a unit positive charge from infinity to given point against the electric field. (without change in KE)

If a test charge q_0 is brought from ∞ to point P then

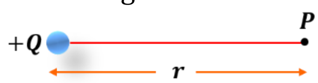


Work done in this process $W_{\infty\to P} = \frac{kQq_0}{r}$

According to definition, $V_P = \frac{W_{\infty\to P}}{q_0} = \frac{kQq_0}{q_0 r} = \frac{kQ}{r} \Rightarrow \boxed{V_P = \frac{kQ}{r}}$

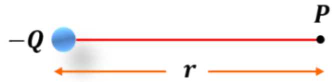
Conclusion:

- Potential due to a positive point charge



$V_P = +\frac{kQ}{r}$

➢ Potential due to a negative point charge



$V_P = -\frac{kQ}{r}$

Where *reference potential is 0 at infinity.*

Illustration 75:

A charge particle of 2C is released from a point where electric potential is 10V. Find kinetic energy of the particle when charge particle reaches to infinity.

Solution:

Initially — At infinity

$KE_i = 0$ — $KE_f = ?$

$U_i = 20J$ — $U_f = 0$

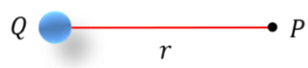
Using COME, $KE_i + U_i = KE_f + U_f$

$0 + 20 = KE_f + 0 \quad \Rightarrow \quad KE_f = 20J$

Illustration 76:

Calculate potential due to a charge 5×10^{-7} C at a point P located 9 cm away. Also find work done in bringing an another charge of 3×10^{-8}C from infinity to the point.

Solution:

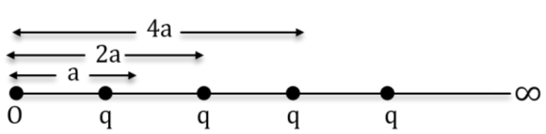


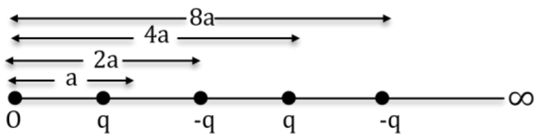
$$V_p = \frac{kQ}{r} = \frac{9\times10^9\times5\times10^{-7}}{9\times10^{-2}} = 5\times10^4\,V$$

$W = q(\Delta V) = q\,(V_p - V_\infty) = 3\times10^{-8}\,(5\times10^4 - 0) = 15\times10^{-4}\,J = 1.5\text{ mJ}$

Illustration 77:

Find potential at origin O due to the given charge distributions.

(i)


(ii)


Solution:

$$\text{(i)}\ V_0 = \frac{kq}{a} + \frac{kq}{2a} + \frac{kq}{4a} + \frac{kq}{8a} + ...\infty = \frac{kq}{a}\left[1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + ...\infty\right] = \frac{kq}{a}\left[\frac{1}{1-\frac{1}{2}}\right] \Rightarrow V_0 = \frac{2kq}{a}$$

$$\text{(ii)}\ V_0 = \frac{kq}{a} - \frac{kq}{2a} + \frac{kq}{4a} - \frac{kq}{8} + ...\infty = \frac{kq}{a}\left[1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + ...\infty\right] = \frac{kq}{a}\left[\frac{1}{1+\frac{1}{2}}\right] \Rightarrow V_0 = \frac{2kq}{3a}$$

Electrostatic Potential Due to a System of Charges

Consider a system of charges q_1, q_2,..., q_n with position vectors r_1, r_2,...,r_n relative to some origin as shown in Fig.

By the superposition principle, the potential V at P due to the total charge configuration is the algebraic sum of the potentials due to the individual charges

$V = V_1 + V_2 + ... + V_n$

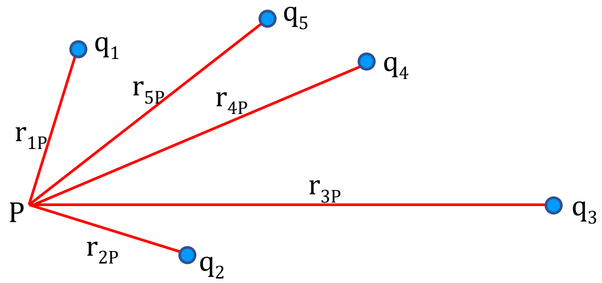
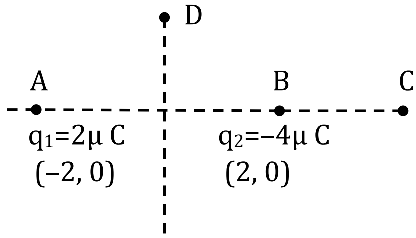


Illustration 78:

Two-point charges $2\mu C$ and $-4\mu C$ are situated at points (–2m, 0m) and (2 m, 0 m) respectively. Find out potential at point C (4 m, 0 m) and D (0 m, $\sqrt{5}$ m).



Solution:

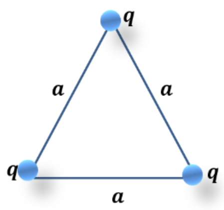
Potential at point C

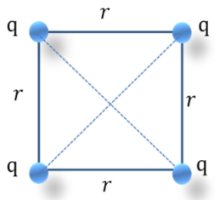
$$V_C = V_{q_1} + V_{q_2} = \frac{k(2\mu C)}{6} + \frac{k(-4\mu C)}{2} = \frac{9\times10^9\times2\times10^{-6}}{6} - \frac{9\times10^9\times4\times10^{-6}}{2} = -15000\text{ V}.$$

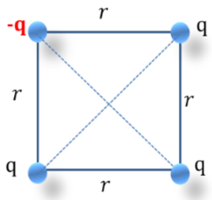
Similarly, $$V_D = V_{q_1} + V_{q_2} = \frac{k(2\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} + \frac{k(-4\mu C)}{\sqrt{(\sqrt{5})^2+2^2}} = \frac{k(2\mu C)}{3} + \frac{k(-4\mu C)}{3} = -6000\text{ V}.$$

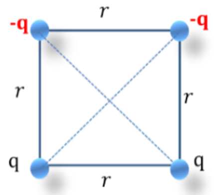
Illustration 79:

Find potential at the centre of the charge systems shown-

(i) 

(ii) 

(iii) 

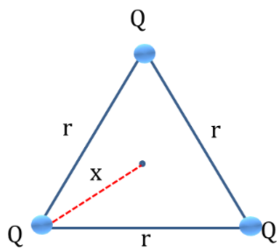
(iv) 

Solution:

(i) Here value of x is $x = \frac{r}{\sqrt{3}}$

Potential at centre due to all three charges is

$$V = \frac{3kQ}{x} = \frac{3\sqrt{3}kQ}{r}$$

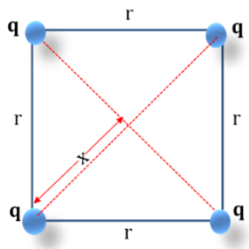


(ii) Here value of $x = \frac{r}{\sqrt{2}}$

Potential at centre due to all four charges is

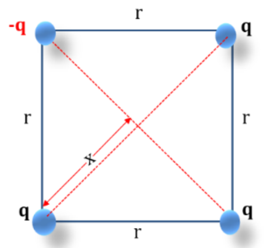
$$V = \frac{4kq}{x}$$

$$V = \frac{4\sqrt{2}kq}{r}$$



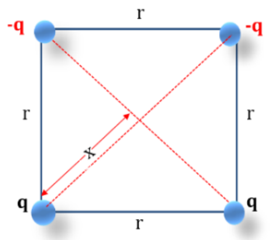
(iii) Potential at centre due to all four charges is

$$V=\frac{3kq}{x}-\frac{kq}{x}=\frac{2kq}{x}=\frac{2\sqrt{2}kq}{r}$$



(iv) Potential at centre due to all four charges is

$$V=\frac{2kq}{x}-\frac{2kq}{x}=0$$

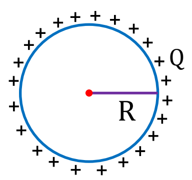


Electrostatic Potential due to a Uniformly Charged Ring:

(i) At the centre -

Consider a small element on the ring having charge dq then potential due to this at centre will be

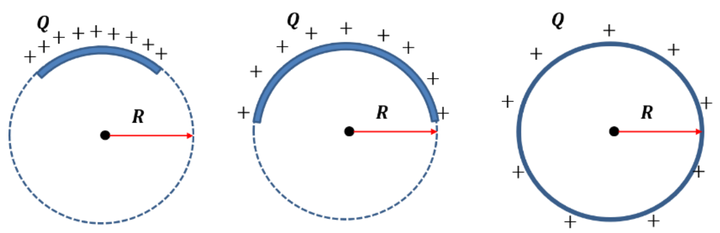
$$dV=\frac{kdq}{R}$$



so potential due to complete ring

$$V=\int dV \Rightarrow V=\int_0^Q \frac{kdq}{R} \Rightarrow V=\frac{kQ}{R}$$

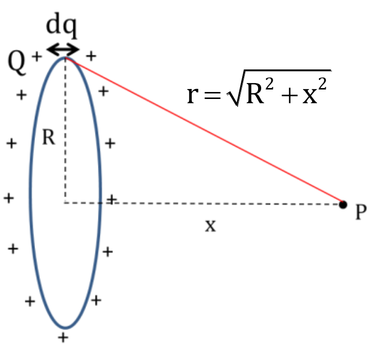
Note: Consider following charge distributions:



Here, distance of total charge from centre of ring is R in all cases so potential at centre in every case will be $V=\frac{kQ}{R}$

(ii) On its axis -

Let electric potential at point P due to a small element of length $d\ell$ be dV, then



$$dV=\frac{dq}{4\pi\varepsilon_0 r}$$

$\therefore$ Total potential due to the ring on its axis:

$$V=\int dV=\int\frac{dq}{4\pi\varepsilon_0 r}=\frac{1}{4\pi\varepsilon_0 r}\int dq$$

$$V=\frac{Q}{4\pi\varepsilon_0(R^2+x^2)^{1/2}} \quad \Rightarrow \quad \boxed{V=\frac{kQ}{\sqrt{R^2+x^2}}=\frac{kQ}{r}}$$

Illustration 80:

Find potential on the axis of a ring (Q, R) where its field is maximum?

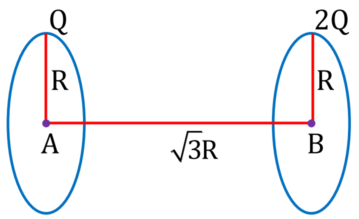
Solution:

As we know that field due to a charged ring is maximum at x = $\pm\frac{R}{\sqrt{2}}$ from the centre on its axis, so potential at these points will be –

$$V=\frac{kQ}{\sqrt{R^2+\left(\frac{R}{\sqrt{2}}\right)^2}}=\frac{kQ}{\sqrt{R^2+\frac{R^2}{2}}}=\frac{kQ}{\sqrt{\frac{3R^2}{2}}}=\frac{\sqrt{2}kQ}{\sqrt{3}R}$$

Illustration 81:

Find $V_A - V_B$ for the system of charged rings placed coaxially as shown -



Solution:

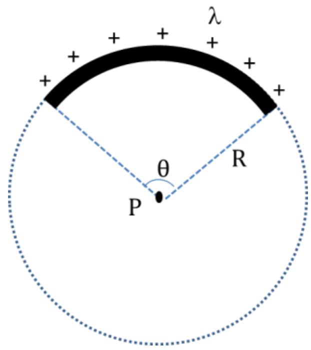
Using formula we can write -

$$V_A=\frac{kQ}{R}+\frac{2kQ}{2R}=\frac{2kQ}{R}$$

$$V_B=\frac{2kQ}{R}+\frac{kQ}{2R}=\frac{5kQ}{2R}$$

$$\therefore\ V_A-V_B=\frac{2kQ}{R}-\frac{5kQ}{2R}=-\frac{kQ}{2R}$$

Electrostatic Potential due to a Uniformly Charged Circular Arc –

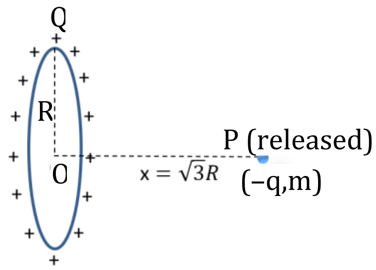


Here, linear charge density of arc is λ so total charge of the area can be written as $Q=\lambda R\theta$

Hence, potential will be $V=\frac{kQ}{R}=\frac{k(\theta R\lambda)}{R}=k\theta\lambda$

Illustration 82:

Find speed of the charge particle, when it reaches at centre of the ring from point P on the axis as shown.



Solution:

Using COME between points P and O -

$(KE)_i + (PE)_i = (KE)_f + (PE)_f$

$$\Rightarrow\ 0-q\frac{kQ}{2R}=\frac{1}{2}mv^2-q\frac{kQ}{R}$$

$$\Rightarrow\ \frac{kQq}{2R}=\frac{1}{2}mv^2\Rightarrow\ v=\sqrt{\frac{kQq}{mR}}$$

Illustration 83:

A ball of mass 4g has 2μC of charge. It is released from the position where potential is 60 Volt then find its speed when reaches the position where potential is 20 Volt.

Solution:

Using COME between initial point and final position

$(KE)_i + (PE)_i = (KE)_f + (PE)_f$

$$\Rightarrow 0 + qV_i = \frac{1}{2}mv^2 + qV_f$$

$$\Rightarrow 0 + (2\mu C)60 = \frac{1}{2}mv^2 + (2\mu C)20$$

$$\Rightarrow 80\times10^{-6} = \frac{1}{2}(4\times10^{-3})v^2 \Rightarrow v^2 = 4\times10^{-2} \Rightarrow v = 0.2\ m/s$$

Equipotential Surfaces, Relation between Electric Field and Potential

Equipotential Surfaces (E.P.S.)

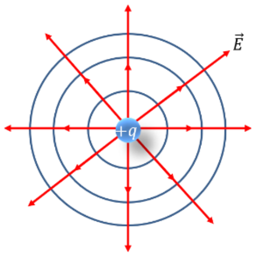
Locus of all such points which are at same potential or the surface at which potential of all the points are equal is known as equipotential surface (EPS).

Properties of equipotential surface

- Electric field intensity is always perpendicular to *EPS*.
- Work done in moving a charge from one point of EPS to other point of same EPS is always zero.
- Two EPS can never intersect each other.

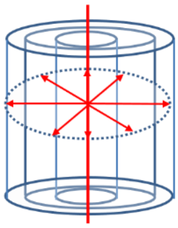
(1) EPS due to point charge -

EPS of point charge is spherical in shape.



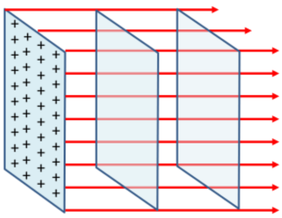
(2) EPS due to line charge –

EPS of line charge is cylindrical in shape.

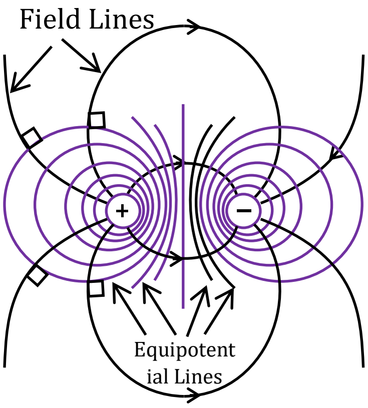


(3) EPS due to large plane charged sheet –

EPS of large plane sheet is plane in shape.



(4) EPS due to unlike charges -



(5) EPS due to like charges -

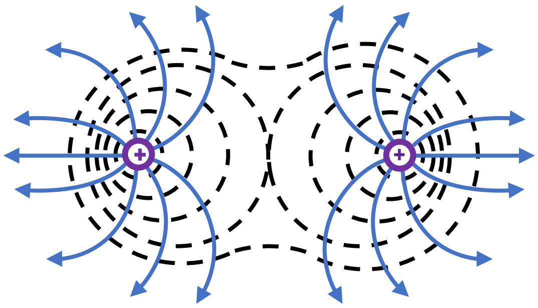
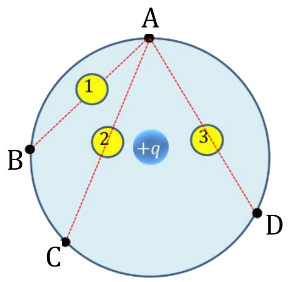


Illustration 84:

A charge in moving via three paths, 1, 2, and 3. as shown in figure. Find work done in each case.



Solution:

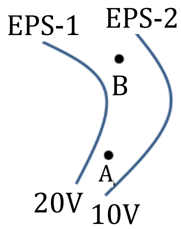
Potential at points A, B, C and D is

$$V = \frac{kq}{R}$$

All the points are equipotential points, so work done in each case is zero.

Illustration 85:

Find relation between E_A and E_B.

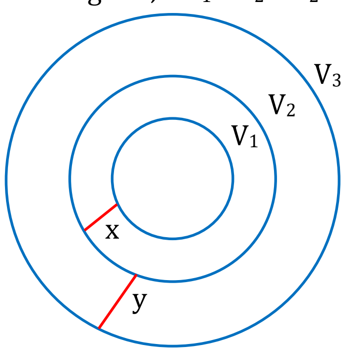


Solution:

$\because \ |E| = \frac{\Delta V}{d}$ and here ΔV is same, but d is less near point A so E_A will be greater than E_B.

Illustration 86:

Concentric spherical equipotential surfaces due to a point charge are shown in the diagram, if $V_1 - V_2 = V_2 - V_3$ then compare x and y -



Solution:

If electric field between V_1 and V_2 be the E_1 and between V_2 and V_3 be the E_2 then we can write (using approximation)

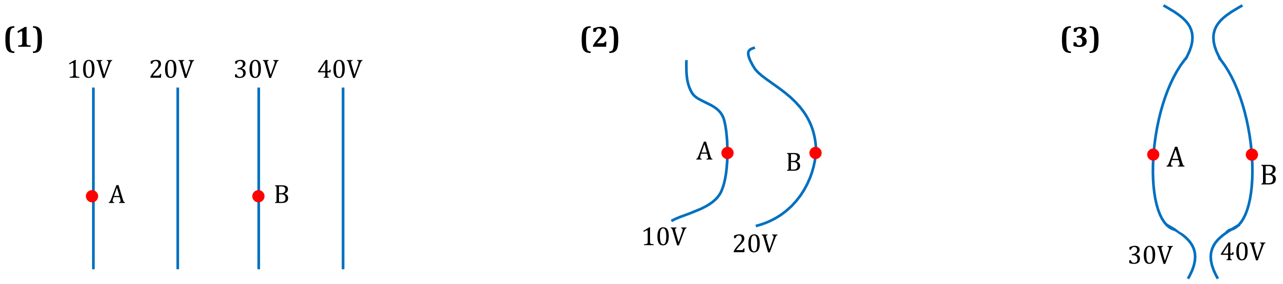
$V_1 - V_2 = E_1(x)$

$V_2 - V_3 = E_2(y)$

We can see here $E_1 > E_2$ so according to given condition that $V_1 - V_2 = V_2 - V_3$ we can conclude that x<y.

Illustration 87:

Find which has more work done in moving a charge from A to B?



Solution:

Use W = q(ΔV)

$W_1 > W_2 = W_3$

Relation between electrostatic field (E) and potential (V):

We know that $\quad F = -\dfrac{dU}{dr}$

Dividing both sides by q

$\Rightarrow \dfrac{F}{q} = -\dfrac{d}{dr}\left(\dfrac{U}{q}\right) \Rightarrow E = -\dfrac{dV}{dr}$ (As we know $\vec{F} = q\vec{E}$ & U = qV)

$\vec{E} = -\vec{\nabla}V$

So, negative gradient of electric potential is equal to electric field.

Since electric field is negative gradient of potential, thus potential decreases in the direction of $\vec{E}$.

Finding E when V is given:

1. Gradient $\rightarrow$ Rate of change with respect to distance
2. In cartesian system

As $\vec{E} = -\vec{\nabla}V$ and $\nabla = \dfrac{\partial}{\partial x}\hat{i} + \dfrac{\partial}{\partial y}\hat{j} + \dfrac{\partial}{\partial z}\hat{k}$

$\vec{E} = -\dfrac{\partial V}{\partial x}\hat{i} - \dfrac{\partial V}{\partial y}\hat{j} - \dfrac{\partial V}{\partial z}\hat{k}$

$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k}$

Here, $E_x = -\dfrac{\partial V}{\partial x}$; $\quad E_y = -\dfrac{\partial V}{\partial y}$; $\quad E_z = -\dfrac{\partial V}{\partial z}$ and

$\dfrac{\partial}{dx} \rightarrow$ partial differentiation with respect to x, keeping y & z constant.

$\dfrac{\partial}{dy} \rightarrow$ partial differentiation with respect to y, keeping x & z constant.

$\dfrac{\partial}{dz} \rightarrow$ partial differentiation with respect to z, keeping x & y constant.

Finding V when E is given:

As we know

$\because E = -\dfrac{dV}{dr} \Rightarrow dV = -Edr \Rightarrow dV = -\vec{E}\,.\,d\vec{r}$ $\quad$ where $d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$

On integration

$$\Delta V = V_P - V_{ref} = -\int_{ref}^{P} \vec{E}.\vec{dr} \quad \text{or} \quad V_P - V_{ref} = -\int_{X_{ref}}^{X_P} E_x dx - \int_{Y_{ref}}^{Y_P} E_y dy - \int_{Z_{ref}}^{Z_P} E_z dz.$$

Illustration 88:
If V = 8x² volt, find electric field in the region.

Solution:

$$E_x = -\frac{\partial V}{\partial x} = -16x$$

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} \qquad (E_y \text{ and } E_z \text{ is zero}) \Rightarrow \vec{E} = -16x\hat{i} \Rightarrow \vec{E} = 16x(-\hat{i})\ V/m$$

Illustration 89:
If $V = (x^2y + y^2xz + 5)$ volt then find $\vec{E}$ at (1,1,0).

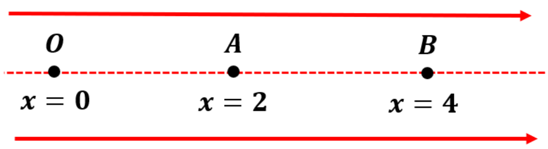
Solution:

$$E_x = -\frac{\partial V}{\partial x} = -[2xy + y^2z] \quad , \quad E_y = -\frac{\partial V}{\partial y} = -[x^2 + 2yxz] \quad , \quad E_z = -\frac{\partial V}{\partial Z} = -[0 + y^2x]$$

At (1,1,0) $\Rightarrow E_x = -2\hat{i}; \quad E_y = -\hat{j}; \quad E_z = -\hat{k} \Rightarrow \vec{E} = (-2\hat{i} - \hat{j} - \hat{k})\ V/m$

Illustration 90:
If $\vec{E} = \frac{40}{x^2}(\hat{i})\ N/m$, then find potential difference between the points x = 2m and x = 4m.

Solution:



$$\int dV = -\int E dx \cos\theta$$

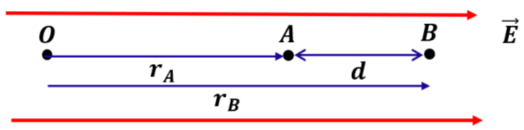
$$\int_{V_A}^{V_B} dV = -\int_{x=2}^{x=4} E dx \cos 0°$$

$$(V_B - V_A) = -\int_{x=2}^{x=4} \frac{40}{x^2} dx$$

$$(V_B - V_A) = -40\left[-\frac{1}{x}\right]_2^4$$

$$(V_B - V_A) = +40\left[\frac{1}{4} - \frac{1}{2}\right] = -\frac{40}{4} = -10V \Rightarrow (V_A - V_B) = +10V$$

Note: In uniform electric field -



$$\int dV = -\int E dr \cos\theta$$

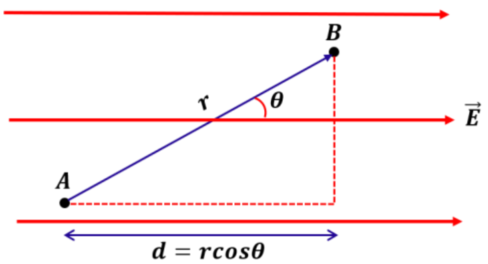
$$\int_{V_A}^{V_B} dV = -E\int_{r_A}^{r_B} dr$$

$(V_B - V_A) = -E(r_B - r_A) \Rightarrow (V_A - V_B) = +E(r_B - r_A)$

$\Rightarrow \Delta V = Ed$

P.D. = E (distance between two points along electric field)

Illustration 91:



Solution:

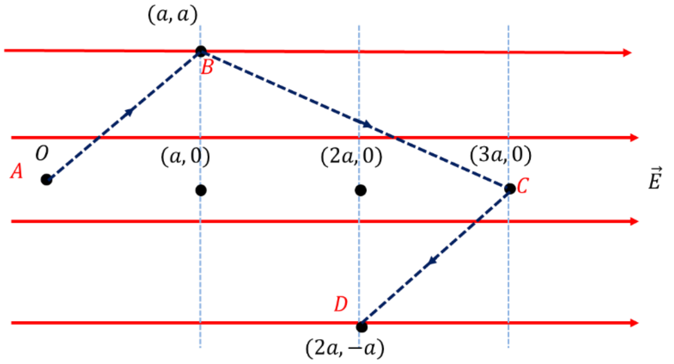
P.D.= E(distance between two points along electric field)

$\Rightarrow \Delta V = Ed = Er\cos\theta$

Illustration 92:

There is a uniform electric field parallel to x axis as shown. Find work done by the electric field in moving a charge +q from

(a) A to B (b) B to C (c) A to D



Solution:

$W_{(A\to B)} = +qE[0 - (-a)] = +qEa$

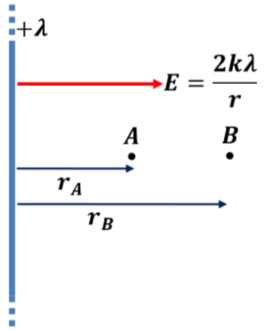
$W_{(B\to C)} = +qE[(-a) - (-3a)] = +2qEa$

$W_{(A\to D)} = +qE[0 - (-2a)] = +2qEa$

Note: In conservative field work done depends only on initial and final positions, does not depend on path followed.

Electrostatic Potential difference for line charge and sheet -

Net potential at a point due to infinite geometry cannot be obtained but potential difference between two nearby points can be obtained.



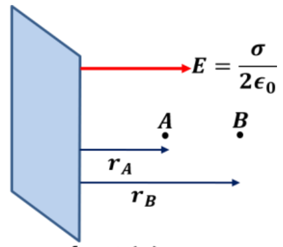
(i) For infinite line charge -

For nonuniform electric field:

$$\int_{V_B}^{V_A} dV = -\int_{r_B}^{r_A} \vec{E}.\vec{dr}$$

$$\Rightarrow V_A - V_B = -\int_{r_B}^{r_A} \frac{2k\lambda dr}{r}$$

$$\Rightarrow (V_A - V_B) = 2k\lambda \ln\left(\frac{r_B}{r_A}\right)$$



(ii) For large plane sheet -

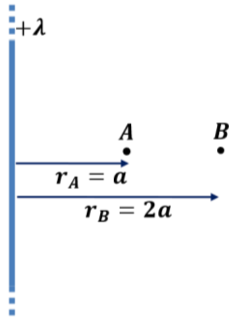
Here electric field is uniform so,

$(V_A - V_B) = Ed$

$$(V_A - V_B) = \frac{\sigma}{2\epsilon_0}(r_B - r_A)$$

Illustration 93:

Find p.d. between points A and B as shown in figure.



Solution:

$$(V_A - V_B) = 2k\lambda \ln\left(\frac{r_B}{r_A}\right)$$

$$(V_A - V_B) = 2k\lambda \ln\left(\frac{2a}{a}\right)$$

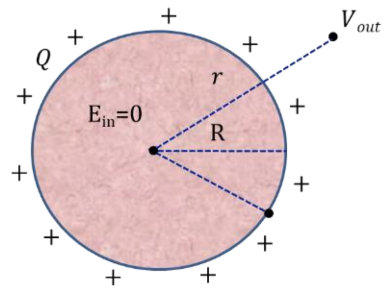
$(V_A - V_B) = 2k\lambda \ln(2)$

Electrostatic Potential due to a Charged Conducting Sphere or Spherical Shell

Surface charge density $\sigma = \frac{Q}{4\pi R^2}$

(i) For outside points (r > R)

$\because$ Whole charge is supposed to be at centre so



$$\int_0^V dV = -\int_\infty^r E.dr$$

$$V = \int_\infty^r \frac{kQ}{r^2}dr = \frac{kQ}{r} = \frac{Q}{4\pi\varepsilon_0 r}$$

$$V = \underbrace{\boxed{\frac{Q}{4\pi R^2}}}_{\sigma}\frac{R^2}{\varepsilon_0 r} \Rightarrow V = \frac{kQ}{r} = \frac{\sigma R^2}{\varepsilon_0 r}$$

(ii) For points on the surface (r = R)

$$V = \frac{kQ}{R} = \frac{Q}{4\pi\varepsilon_0 R} \quad \Rightarrow \quad V = \frac{Q}{4\pi R^2} \times \frac{R}{\varepsilon_0} \quad \Rightarrow \quad V = \frac{\sigma R}{\varepsilon_0} = \frac{kQ}{R}$$

(iii) For Inside points (r < R)

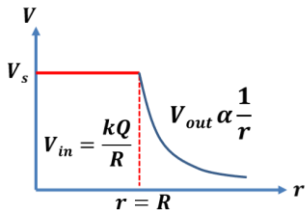
$\because$ For all points inside the sphere E_{in} is same so

$$V = \int_0^v dV = -\int E.dr$$

$$\int_0^v dV = -\int_\infty^R \vec{E}.d\vec{r} - \int_R^r \vec{E}.d\vec{r} \quad \Rightarrow \quad V = -\int_\infty^R \frac{kQdr}{r^2} - \int_R^r Edr \quad \Rightarrow \quad V = \frac{kQ}{R}$$

Variation of potential with r

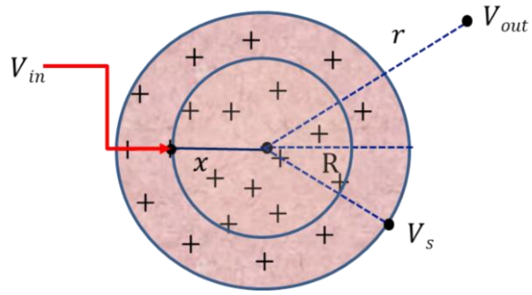
$V_{out} = \frac{kQ}{r}$ and $V_S = V_{in} = \frac{kQ}{R}$



Electric Potential due to Uniformly Charged Non-conducting Sphere

(i) Outside (r > R)

Because sphere will act like charge Q kept at centre.



$$\int_0^V dV = -\int_\infty^r \vec{E}.d\vec{r}$$

$$V = -\int_\infty^r \frac{KQ}{r^2}dr \qquad V = \frac{KQ}{r}$$

(ii) On the surface (r = R)

Putting r = R in above equation we get -

V surface = $\frac{KQ}{R}$

(iii) Inside (r < R) (Here, x = r for this case in above diagram)

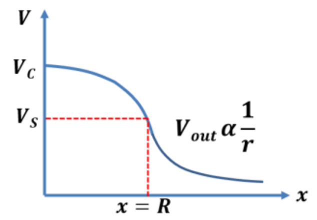
$$\int dV = -\int_{\infty}^{R} \vec{E}.\vec{dr} - \int_{R}^{r} \vec{E}.\vec{dr}$$

$$\Rightarrow V = -\int_{\infty}^{R} \frac{KQ}{r^2}dr - \int_{R}^{r} \frac{KQr}{R^3}dr \quad \Rightarrow V = \frac{KQ}{R} - \frac{KQ}{R^3}\left(\frac{r^2}{2} - \frac{R^2}{2}\right) \quad \Rightarrow \frac{KQ}{R} - \frac{KQ}{2R^3}(r^2 - R^2)$$

$$\Rightarrow V = \frac{KQ}{2R^3}(2R^2 - (r^2 - R^2)] \Rightarrow Vin = \frac{KQ}{2R^3}\left[3R^2 - r^2\right]$$

At center, r = 0

$$\Rightarrow V_{center} = \frac{3}{2}\frac{KQ}{R} = \frac{3}{2}V_{surface}$$



Variation of potential with r

$$V_{in} = \frac{kQ}{2R^3}\left(3R^2 - x^2\right); \quad V_s = \frac{kQ}{R}; \quad V_{out} = \frac{kQ}{r}$$

Illustration 94:

A conducting sphere of radius R has charge Q. Find potential at distance of (a) $\frac{R}{2}$ (b) $\frac{3R}{2}$ from centre.

Solution:

(a) r < R i.e. inside the surface $V_{in} = V_S = \frac{kQ}{R}$

(b) r > R i.e. outside the surface $V_{out} = \frac{kQ}{r} = \frac{2kQ}{3R}$

Illustration 95:

A nonconducting uniformly charged sphere of radius R has charge Q. Find potential at distance of

(a) $\frac{R}{2}$

(b) $\frac{3R}{2}$ from centre.

Solution:

(a) r < R i.e. inside the surface $V_{in} = \frac{kQ}{2R^3}\left(3R^2 - r^2\right)$ Put r = $\frac{R}{2}$ $\Rightarrow V_{in} = \frac{11kQ}{8R}$

(b) r > R i.e. outside the surface $V_{out} = \frac{kQ}{r} = \frac{kQ}{r} = \frac{2kQ}{3R}$

Illustration 96:

A conducting sphere of radius R has electric field E at distance 3R from centre. Find potential difference between centre of the sphere & the point where electric field is E.

Solution:

$$E = \frac{kQ}{(3R)^2} = \frac{kQ}{9R^2} \Rightarrow kQ = 9ER^2$$

$$V_{centre} = \frac{kQ}{R} = \frac{9ER^2}{R} = 9ER$$

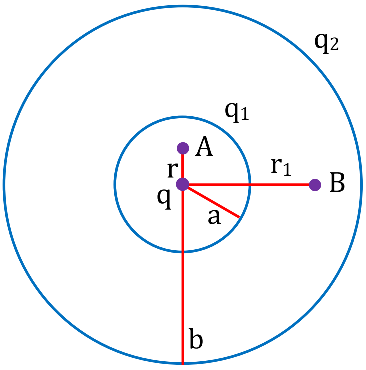
$$V_P = \frac{kQ}{r} = \frac{9ER^2}{3R} = 3ER$$

$$V_{centre} - V_P = 9ER - 3ER = 6ER$$

Problems on potential due to concentric spherical shells -

Illustration 97:

Find potential at points A and B due to given arrangement of shells.



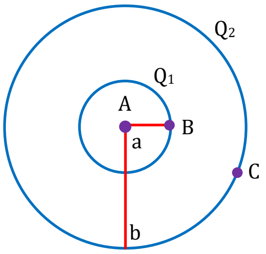
Solution:

$$V_A = \frac{kq}{r} + \frac{kq_1}{a} + \frac{kq_2}{b}$$

$$V_B = \frac{kq}{r_1} + \frac{kq_1}{r_1} + \frac{kq_2}{b}$$

Illustration 98:

Find potential at points A,B and C due to given arrangement of shells.



Solution:

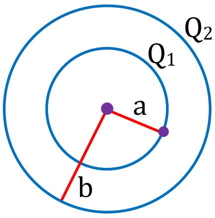
$$V_A = \frac{kQ_1}{a} + \frac{kQ_2}{b}$$

$$V_B = \frac{kQ_1}{a} + \frac{kQ_2}{b}$$

$$V_C = \frac{kQ_1}{b} + \frac{kQ_2}{b}$$

Illustration 99:

Find V_{in} - V_{out} for the given arrangement of shells.



Solution:

$$V_{in} = \frac{kQ_1}{a} + \frac{kQ_2}{b}$$

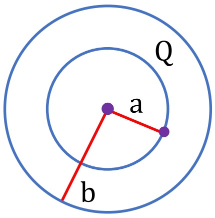
$$V_{out} = \frac{kQ_1}{b} + \frac{kQ_2}{b}$$

$$V_{in} - V_{out} = \frac{kq_1}{a} - \frac{kQ_2}{b} = kQ_1\left[\frac{1}{a} - \frac{1}{b}\right]$$

Note: P.D. between 2 concentric spherical shalls depends only on the charge of inner sphere.

Illustration 100:

Initially V_{in} - V_{out} = V now if outer sphere is given charge 4Q then V_{in} - V_{out} will be?

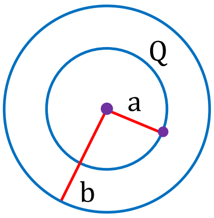


Solution:

Potential difference between them will not depend upon charge on outer surface so V_{in} - V_{out} will be V only.

Illustration 101:

Initially V_{in} - V_{out} = V now if charge on inner sphere is increased to 4Q then V_{in} - V_{out} will be?



Solution:

Potential difference between them is directly proportional to charge on inner sphere so V_{in} - V_{out} will be 4V.

Golden Key Points

- If potential energy at infinity is zero, external work done in changing the configuration of a system from 'i' to 'f' is $W = U_f - U_i$.
- In assembling a given charge system ($U_i = 0$) $\Rightarrow W = U_f$ and in disassembling ($U_f = 0$) $\Rightarrow W = -U_i$
- eV is the smallest practical unit of energy used in atomic and nuclear physics. ($1 \text{ eV} = 1.6 \times 10^{-19}$ J)
- Potential is theoretically zero at infinity; practically we consider it zero on the earth surface.
- Potential on earth is assumed to be zero as it is a large conductor and its potential is more or less constant in relation to finite charge given to or taken from it.
- The direction of electric field is from regions of higher electric potential to regions of lower potential irrespective of the charge configuration.
- If one moves in the direction of $\vec{E}$, electric potential V decreases; If one moves in the direction opposite to $\vec{E}$ potential increases.
- Electric potential is the same at all points in a region where E = 0.
- Equipotential surfaces need not be physical surfaces. Any imaginary surface over which the electric field is perpendicular to it everywhere is called an equipotential surface.
- Equipotential surfaces are closely spaced where electric field intensity is large and widely spaced where electric field intensity is small.
- Electric field must be zero within an equipotential volume.
- Different equipotential surfaces due to a point charge whose potentials differ by a constant amount (say 1 volt) are not equispaced.
- Two equipotential surfaces never intersect.
- When we place a charged conductor inside a hollow conductor and connect them by a conducting wire, the charge will be completely transferred from the inner conductor to the other conductor.

BEGINNER'S BOX-5

1. Three charges –q, Q and –q are placed at equal distances on a straight line. If the potential energy of the system of three charges is zero, then what is the ratio Q: q ? **PEL215**

2. Work done in moving a charge q coulombs on the surface of a given charged conductor of potential V volts is **PEL216**

3. Three point charges q, –2q and –2q are placed at the vertices of an equilateral triangle of side a. Find the work done by external forces to increase their separation to 2a ? **PEL217**

4. Two electrons lying 10 cm apart are released. What will be their speed when they are 20 cm apart ? **PEL218**

5. (i) Determine the electrostatic potential energy of a system consisting of two charges 7 μC and –2μC (and with no external field) placed at (–9 cm, 0, 0) and (9 cm, 0, 0) respectively.
(ii) How much work is required to separate them to an infinitely large distance ? **PEL219**

6. (i) Calculate the potential at point P due to a charge of 4×10^{-7} C located 9 cm away.
(ii) Hence obtain the value of work done in bringing a charge of 2×10^{-9} C from infinity to the point P. Does the answer depend on the path along which the charge is brought ? **PEL220**

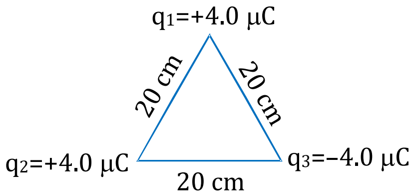
7. A regular hexagon of 10 cm side has charges of 5 μC at each of its vertices. Calculate the potential at its centre. **PEL221**

8. (i) What is the difference in potentials between two points, one at 10 cm, and the other at 20 cm distance from a charge of –5.5 μC ?
(ii) Which point is at a higher potential ? **PEL222**

9. (i) How far would a point be from a $+0.60\ \mu C$ charge if it had a value of electric potential equal to 10 kV ?

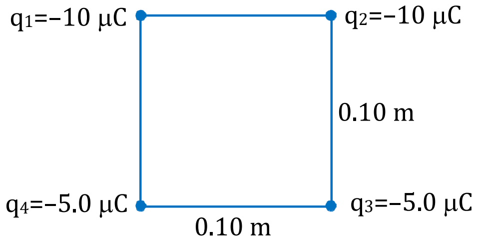
(ii) If the point were moved to four times of its initial distance from the charge then what potential change would occur ? Is it an increase or a decrease ? **PEL223**

10. What is the electric potential at the center of the triangle shown in figure ?



PEL224

11. Calculate the electric potential at the center of the square shown in figure.



PEL225

12. If potential at a point P (1,1,1) is given by the relation $V_P = x^2 - y^2 + 2z$, then calculate the electric field at P. **PEL226**

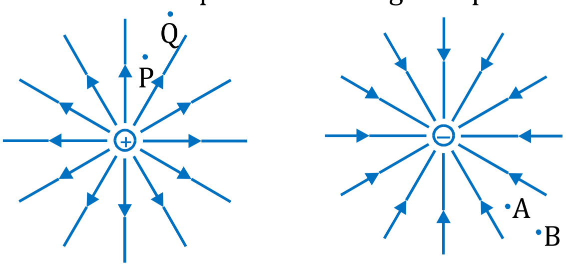
13. For a point charge of $+3.50\ \mu C$ what is the radius of the equipotential surface which is at a potential of 2.50 kV ? **PEL227**

14. A proton is moved 15 cm on a path parallel to the field lines of a uniform electric field of 2.0×10^5 V/m.

(i) What are the possible changes in potential ? Consider both cases of moving the proton along and against the field.

(ii) How much work would be done if the proton were moved perpendicular to the electric field ? **PEL228**

15. Figure (a) and (b) show the field lines of positive and negative point charge respectively.



(a) **(b)**

(a) Give the signs of the potential difference: $V_P - V_Q$; $V_B - V_A$

(b) Give the sign of the potential energy difference of a small negative charge between the points Q and P ; A and B.

(c) Give the sign of the work done by the field in moving a small positive charge from Q to P.

(d) Give the sign of the work done by an external agency in moving a small negative charged from B to A.

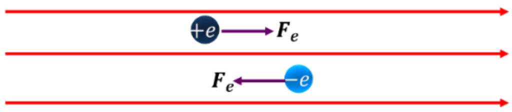
(e) Does the kinetic energy of a small negative charge increase or decrease in going from B to A ? **PEL229**

Motion of a Charge Particle in Uniform Electric Field

Case – I: When a charged particle is released from rest (i.e. u = 0)

Consider an electron and a proton both are released in uniform electric field, they travelled along straight line in opposite direction with constant acceleration.

$a = \dfrac{eE}{m} =$ constant, hence equations of motion can be applied.



Also, $F_{proton} = F_{electron}$ but, $\vec{F}_{proton} = -\vec{F}_{electron}$ and

$\vec{a}_{proton} \neq -\vec{a}_{electron}$

$a_e = \dfrac{eE}{m_e}$ and $a_p = \dfrac{eE}{m_p}$

$\dfrac{a_e}{a_p} = \dfrac{m_p}{m_e} = 1837$ {As $m_p = 1837m_e$}

Distance travelled by charge particle in uniform electric field.

Using $s = ut + \dfrac{1}{2}at^2 \Rightarrow s = \dfrac{1}{2}at^2$ or $t = \sqrt{\dfrac{2s}{a}} = \sqrt{\dfrac{2sm}{eE}}$

Here t is time taken by charge particle to move distance s in uniform electric field.

Illustration 102:

An electron is allowed to fall through some distance in uniform electric field in time t_1. Now the direction of electric field is reversed and proton is allowed to fall through same distance in time t_2. Then relation between t_1 and t_2.

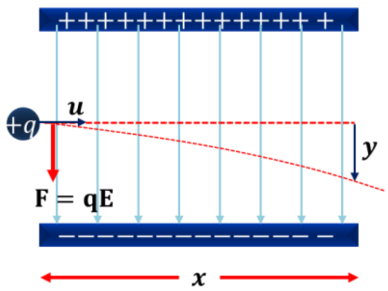
Solution:

In **Case (1)**, $s = \dfrac{1}{2}a_e t_1^2$ and in **case (2)**, $s = \dfrac{1}{2}a_p t_2^2$

$$\frac{t_1}{t_2} = \sqrt{\frac{a_p}{a_e}} = \sqrt{\frac{m_e}{m_p}} = \frac{1}{\sqrt{1837}} \Rightarrow t_1 < t_2$$

Charge Particle Projected Perpendicular to Uniform Electric Field

When the charge particle is projected perpendicular to a uniform electric field $\left(\vec{u} \perp \vec{E}\right)$ as shown



$u_x = u$; but $a_x = 0$

$u_y = 0$; but $a_y = \dfrac{qE}{m}$ = constant

Along x-direction	Along y-direction
$u_x = u$	$u_y = 0$
$a_x = 0$	$a_y = \dfrac{qE}{m}$ =constant
$v_x = u_x + a_x t$ $v_x = u$	$v_y = u_y + a_y t$ $v_y = \dfrac{qEt}{m}$
$x = u_x t + \dfrac{1}{2}a_x t^2$ $x = ut$	$y = a_y t + \dfrac{1}{2}a_y t^2$ $y = \dfrac{1}{2}\dfrac{qE}{m}t^2$

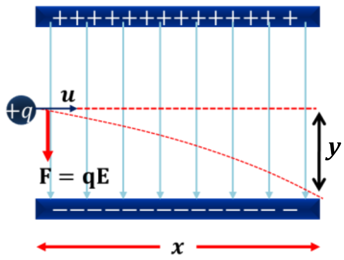
Deviation of the particle from its path

$$y=\frac{1}{2}\frac{qE}{m}t^2$$

Trajectory equation (x & y relation)

$$y=\frac{1}{2}\frac{qE}{m}t^2=\frac{1}{2}\frac{qE}{m}\left(\frac{x}{u}\right)^2 \qquad \left[\text{from } x=ut\Rightarrow t=\frac{x}{u}\right]$$

$$y=\left(\frac{1}{2}\frac{qE}{mu^2}\right)x^2 \quad\Rightarrow\quad y = kx^2 \text{ (parabolic path)}$$



Also, deviation –

$$y=\frac{1}{2}\left(\frac{qE}{mu^2}\right)x^2=\left(\frac{qE}{4K_i}\right)x^2$$

(Where $K_i=\frac{1}{2}mu^2$ & x = length of the plates)

Velocity of particle at time 't'

$$\vec{v}=v_x\hat{i}+v_y\hat{j}$$

$$\vec{v}=u\hat{i}+\left(\frac{qE}{m}t\right)\hat{j}$$

Speed, $v=\sqrt{v_x^2+v_y^2}$ and in diagram $\tan\alpha=\frac{v_y}{v_x}$

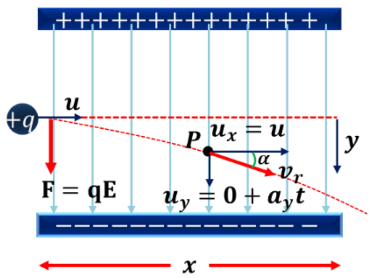


Illustration 103:

A particle of mass 1g and charge 10μC is projected with velocity 30 m/s in direction perpendicular to a uniform electric field of 1000 V/m. Find (a) its speed after 4 second (b) deviation after 4 second.

Solution:

(a) $v_x = u = 30$ m/s ; $v_y=\left(\frac{qE}{m}\right)t$

$$v_y=\left(\frac{10\times10^{-6}\times10^{3}}{10^{-3}}\right)\times4\Rightarrow v_y=40\text{m/s}$$

Speed after 4 second

$$c=\sqrt{v_x^2+v_y^2}=\sqrt{30^2+40^2}=50\text{m/s}$$

(b) Deviation $\rightarrow y$

$$y=\frac{1}{2}\left(\frac{qE}{m}\right)t^2=\frac{1}{2}\times10\times4^2\Rightarrow y=80\text{m}$$

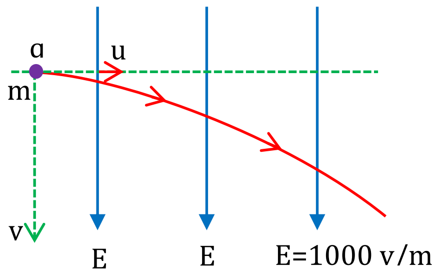


Illustration 104:

An electron enters into a uniform electric field of 364 V/m with a velocity 100 m/s in a direction perpendicular to the electric field. If it travels a distance of 0.01 mm in the direction of electric field, find deviation in its path:

Solution:

Here $y=\frac{qEx^2}{2mu^2}$

$q = e = 1.6\times10^{-19}$ C ; $E = 364$ V/m

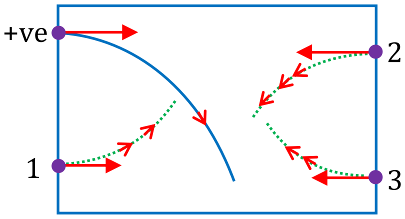
$m = 9.1\times10^{-31}$ kg ; $u = 100$ m/s

and $x = 0.01$ mm $= 10^{-5}$ m

$$y=\frac{1.6\times10^{-19}\times364\times(10^{-5})^2}{2\times9.1\times10^{-31}\times(10^2)^2}=\frac{1.6\times364}{18.2}\times10^{-2}=32\times10^{-2}\text{ m or } y = 0.32\text{ m}$$

Illustration 105:

Find nature of charge of particles 1, 2 and 3



Solution:

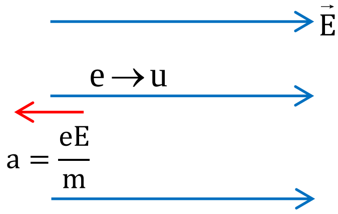
Since $\vec{E}$ is in downward direction

So 1 $\rightarrow$ -ve; 2$\rightarrow$ +ve ; 3$\rightarrow$ -ve

Illustration 106:

An electron is projected with speed u along a uniform electric field E, then find distance travelled before coming to rest.

Solution:



For electron $a = \frac{-eE}{m}$ and $v_{final} = 0$

If it comes to rest after t time,

Putting values in $v = u + at$

$$0 = u - \frac{Ee}{m}t \Rightarrow t = \frac{um}{Ee}$$

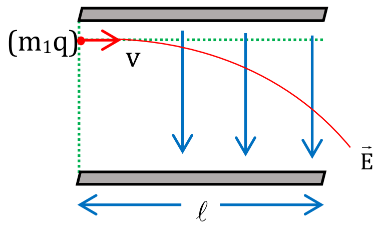
Distance covered $S = ut + \frac{1}{2}at^2$

$$\Rightarrow S = u\left(\frac{mu}{eE}\right) - \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{mu}{eE}\right)^2 \Rightarrow S = \frac{mu}{eE}\left[u - \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{mu}{Ee}\right)\right]$$

$$\Rightarrow S = \frac{mu^2}{2eE} = \frac{K_i}{eE}$$

Illustration 107:

Charge (q,m) is projected with speed v in uniform transverse $\vec{E}$ confined in a region of length ℓ as shown in figure.



(1) Time for which particle remains in $\vec{E}$

(2) v of particle when it leaves $\vec{E}$

(3) Deviation of particle

Solution:

(1) Till the time particle covers horizontal distance, ℓ it will remain in $\vec{E}$.

For particle $u_x = v$ (initial speed), $a_x = 0$, $S_x = \ell$

$$S = ut + \frac{1}{2}at^2$$

$$\ell = vt + 0 \Rightarrow t = \frac{\ell}{V}$$

(2) Speed of particle where it leaves $v_{final} = u + at$

$v_{x(final)} = v$ & $v_{y(final)} = u_y + a_y t$

$$v_y = 0 + \left(\frac{qE}{m}\right)\frac{\ell}{v}$$

So final speed = $\sqrt{v_x^2 + v_y^2}$ Final speed $v_{(final)} = \sqrt{v^2 + \left(\frac{qE\ell}{mv}\right)^2}$

(3) Deviation (vertically) in time $t = \frac{\ell}{v}$

$$S_y = u_y t + \frac{1}{2}at^2 = 0 - \frac{1}{2}\left(\frac{qE}{m}\right)\left(\frac{\ell}{v}\right)^2$$

$$|S_y| = \frac{qE\ell^2}{2mv^2}$$

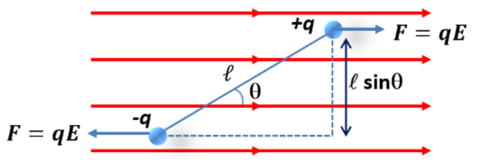
Force & Torque on Dipole in a Uniform Electric Field

In uniform electric field both charges experience equal and opposite force, so net force on dipole is zero.

Torque/Couple of force = magnitude of either force × perpendicular distance between lines of force.

$\vec{\tau} = (qE)l\sin\theta$ or $\vec{\tau} = (ql)E\sin\theta$

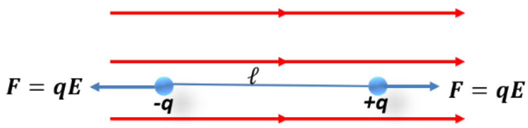
$\vec{\tau} = pE\sin\theta$ or $\vec{\tau} = \left(\vec{p}\times\vec{E}\right)$



- **When dipole is aligned along the field**

  Here angle between EF and dipole moment is zero.

  So, $\tau = pE\sin 0 = 0$ (torque is zero)



- **When dipole is placed perpendicular to the field.**

  Here angle between $\vec{E}$ & $\vec{p}$ = 90°.

  So, $\tau = pE\sin 90 = pE(\max)$ (Torque on dipole is maximum)

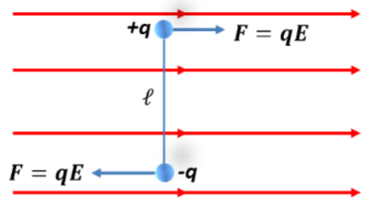
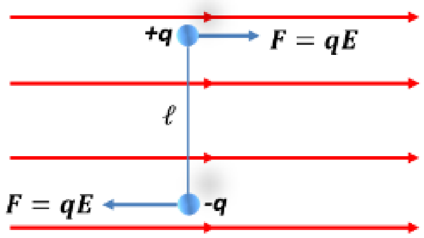


Illustration 108:

If dipole is released from given situation, in which direction it will start rotating.



(1) Clockwise (2) Anticlockwise

Solution:

It will start rotating clockwise.

Illustration 109:

When a dipole is placed in a uniform electric field, net force and net torque on it, is always zero. This statement is -

(1) true (2) false

Solution:

In uniform electric field net force is always zero but net torque may or may not be zero always, so statement is false.

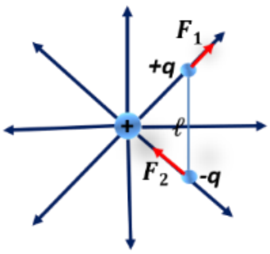
Illustration 110:

When a dipole is placed in a nonuniform electric field, net force and net torque on it is always nonzero. This statement is -

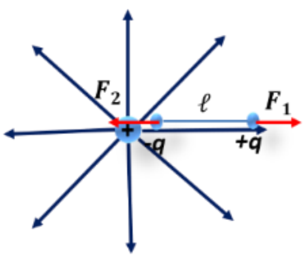
(1) true (2) false

Solution:

This statement is false.



Net force and net torque both are non zero



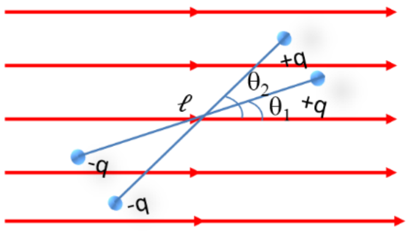
Net force is non zero but net torque is zero

Work done in rotating a dipole in uniform electric field

Work done in rotating a dipole by small angle $d\theta$ is

$$dW = \tau d\theta$$

Work done in rotating the dipole from initial angular position θ_1 to θ_2 is



$$\int dW = \int_{\theta_1}^{\theta_2} \tau d\theta = pE\int_{\theta_1}^{\theta_2} \sin\theta d\theta \qquad [\because \tau = PE\sin\theta]$$

$$W = pE[-\cos\theta]_{\theta_1}^{\theta_2} = pE(-\cos\theta_2 + \cos\theta_1) = pE(\cos\theta_1 - \cos\theta_2)$$

Illustration 111:

A dipole of dipole moment 2×10^{-14} Cm is placed in a uniform electric field of $5\times10^{4}\,N/C$. Find work done in rotating the dipole from $60°$ to $90°$.

Solution:

$$W = pE(\cos\theta_1 - \cos\theta_2) = 2\times10^{-14}\times5\times10^{4}(\cos60° - \cos90°) = 5\times10^{-10}\,J$$

Illustration 112:

Work done in rotating dipole from 0° to 60° is W. Find torque required to hold the dipole at this position.

Solution:

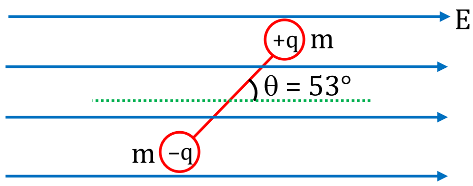
$$W = pE\,(\cos0° - \cos60°) = \left(\frac{pE}{2}\right)$$

Torque required to hold the dipole at 60° position

$$\tau = pE\sin60° = pE\left(\frac{\sqrt{3}}{2}\right) = \sqrt{3}W$$

Illustration 113:

Two-point masses of mass m and equal and opposite charge of magnitude q are attached on the corners of a non-conducting uniform rod of mass m and the system is released from rest in uniform electric field E as shown in the figure from $\theta = 53°$



(i) Find angular acceleration of the rod just after releasing

(ii) What will be angular velocity of the rod when it passes through stable equilibrium.

(iii) Find work required to rotate the system by 180°.

Solution:

(i) $\tau_{net} = pE\sin 53° = I\alpha$

$$\left(\text{Where } I = \frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2 = \frac{7m\ell^2}{12}\right)$$

$$\therefore \qquad \alpha = \frac{(q\ell)\ E\left(\frac{4}{5}\right)}{\frac{m\ell^2}{12} + m\left(\frac{\ell}{2}\right)^2 + m\left(\frac{\ell}{2}\right)^2} = \frac{48qE}{35\ m\ell}$$

(ii) From energy conservation: $K_i + U_i = K_f + U_f$

$$\therefore \qquad 0 + (-pE\cos 53°) = \frac{1}{2}I\omega^2 + (-pE\cos 0°)$$

$$\therefore \qquad \frac{1}{2}I\omega^2 = pE\,(1 - 3/5) = \frac{2}{5}pE$$

$$\therefore \qquad \frac{1}{2} \times \frac{7m\ell^2}{12} \times \omega^2 = \frac{2}{5}q\ell E \qquad \text{or} \quad \omega = \sqrt{\frac{48qE}{35\ m\ell}}$$

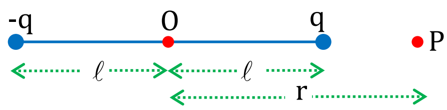
(iii) $W_{ext} = U_f - U_i$

$$\therefore W_{ext} = (-pE\cos(180° + 53°)) - (-pE\cos 53°) \quad \text{or} \quad W_{ext} = (q\ell)E\left(\frac{3}{5}\right) + (q\ell)E\left(\frac{3}{5}\right)$$

$$\Rightarrow W_{ext} = \left(\frac{6}{5}\right)q\ell E$$

Electric Potential due to Dipole

- **At axial point**



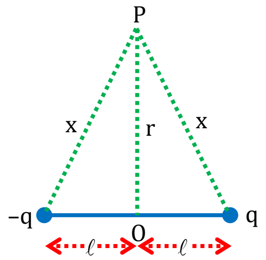
Electric potential due to +q charge $\quad V_1 = \frac{kq}{(r-\ell)}$

Electric potential due to –q charge $\quad V_2 = \frac{-kq}{(r+\ell)}$

Net electric potential $\quad V = V_1 + V_2 = \frac{kq}{(r-\ell)} + \frac{-kq}{(r+\ell)} = \frac{kq \times 2\ell}{(r^2 - \ell^2)} = \frac{kp}{r^2 - \ell^2}$

If $r >> \ell$ then $V = \frac{kp}{r^2}$

- **At equatorial point**



Electric potential at P due to +q charge, $V_1 = \frac{kq}{x}$

Electric potential at P due to -q charge, $V_2 = \frac{-kq}{x}$

Net potential $\quad V = V_1 + V_2 = \frac{kq}{x} - \frac{kq}{x} = 0 \quad (\therefore V = 0)$

- **Potential at general point (r, θ):**

Let's resolve the dipole moment $\vec{p}$ into components: pcosθ along radial line and p sinθ ⊥ to the radial line

For the p cosθ component, the point A is an axial point,

So, potential at A due to p cosθ = $\frac{k(p\cos\theta)}{r^2}$

And for P sinθ component, the point A is an equatorial point,

So potential at A due to p sinθ = 0

So, net potential at point A, V = $\frac{k(p\cos\theta)}{r^2}$

$\therefore \quad V = \frac{k(\vec{p}.\vec{r})}{r^3} \qquad (\text{here } \vec{r} = \overrightarrow{OA})$

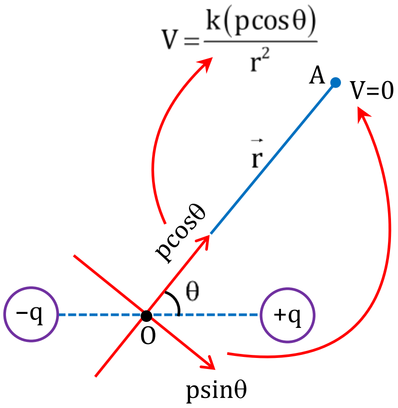
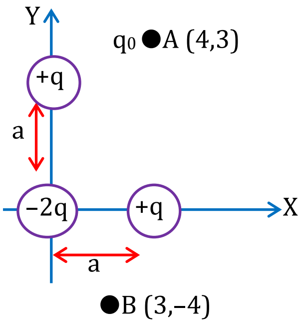


Illustration 114:

(i) Find potential at point A and B due to the small charge system fixed near origin. (Distance between the charges is negligible).

(ii) Find work done to bring a test charge q_0 from point A to point B, slowly. All parameters are in S.I. units.



Solution:

(i) Dipole moment of the system is $\vec{P} = (qa)\,\hat{i} + (qa)\,\hat{j}$

Potential at point A due to the dipole

$$V_A = k\frac{(\vec{P}\cdot\vec{r})}{r^3} = \frac{k[(qa)\hat{i}+(qa)\hat{j}]\cdot(4\hat{i}+3\hat{j})}{5^3} = \frac{(7)k(qa)}{125}$$

$$V_B = \frac{k\left[(qa)\hat{i}+(qa)\hat{j}\right]\cdot\left(3\hat{i}-4\hat{j}\right)}{(5)^3} = \frac{-k(qa)}{125}$$

(ii) $W_{A\to B} = U_B - U_A = q_0\,(V_B - V_A) = q_0\left[-\frac{k(qa)}{125} - \left(\frac{k(qa)(7)}{125}\right)\right]$

$$\Rightarrow \qquad W_{A\to B} = \frac{-kqq_0a}{125}(8)$$

Potential Energy of Dipole in Electric Field

$\vec{E}$ is a conservative field so whatever work is done in rotating a dipole from θ_1 to θ_2 it is just equal to change in the electrostatic potential energy.

Work done $W = pE(\cos\theta_1 - \cos\theta_2) = U_2 - U_1$

$\Rightarrow pE\cos\theta_1 - pE\cos\theta_2 = U_2 - U_1$

$\Rightarrow (-pE\cos\theta_2) - (-pE\cos\theta_1) = U_2 - U_1$

$\Rightarrow U_2 = -pE\cos\theta_2$ or $U_1 = -pE\cos\theta_1$

$\Rightarrow U = -pE\cos\theta$ or $U = -\vec{p}.\vec{E}.$

Case-(i): When $\vec{E} || \vec{p} \Rightarrow \theta = 0° \quad \Rightarrow U = -pE = U_{min}$

($\because$ Here F = 0, τ = 0 $\Rightarrow$ **Stable equilibrium**)

Case-(ii): When $\vec{E} \perp \vec{p} \Rightarrow \theta = 90° \Rightarrow U = 0$ (reference position)

($\because$ Here F = 0, τ = maximum) $\Rightarrow$ **Not in equilibrium**

Case-(iii): When angle between $\vec{E}$ & $\vec{p}$ is 180° $\Rightarrow U = +pE = U_{max}$

($\because$ Here F = 0, τ = 0) $\Rightarrow$ **Unstable equilibrium**

θ	$\tau = pE\sin\theta$	$U = -pE\cos\theta$	Nature of equilibrium
0°	0	–pE	Stable equilibrium
90°	max	0	Not in equilibrium
180°	0	+pE	Unstable equilibrium

Force on electric dipole in non-uniform electric field:

From relation between force and potential energy

$$F = -\frac{dU}{dr} = -\frac{d(-pE\cos\theta)}{dr} = p\cos\theta\left(\frac{dE}{dr}\right)$$

$$F = p\cos\theta\left(\frac{dE}{dr}\right)$$

In uniform EF

$\left(\frac{dE}{dr} = 0\right)$

F = 0

$\tau = pE\sin\theta$

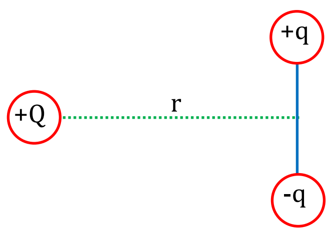
In non-uniform EF

$\left(\frac{dE}{dr} \neq 0\right)$

τ and F may or may not be zero

Illustration 115:

A short dipole of dipole moment P is placed near a point charge Q as shown in figure. Find force on the dipole due to the point charge.



Solution:

Force on the point charge due to dipole $F = (Q)\ (E_{dipole})$

$F = (Q)\left(\frac{KP}{r^3}\right)(\downarrow)$

So, force on the dipole due to the point charge will also be

$F = \left(\frac{KPQ}{r^3}\right)(\uparrow)$ (but in opposite direction) as shown

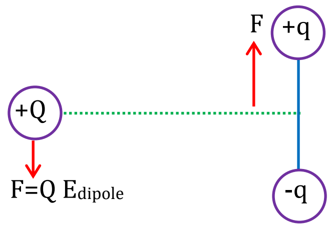


Illustration 116:

Find force on short dipole P_2 due to short dipole P_1 if they are placed at a distance r a part as shown in figure.

Solution:

Force on P_2 due to P_1

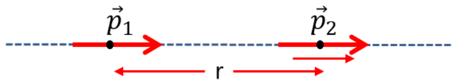
$$F_2 = (P_2)\left(\frac{dE_1}{dr}\right)$$

$$\therefore \quad F_2 = (P_2)\left(\frac{d}{dr}\left(\frac{2\ kP_1}{r^3}\right)\right) = -\ \frac{6kP_1P_2}{r^4}$$

Here – sign indicates that this force will be attractive (opposite to r).

Dipole-Dipole Interaction:

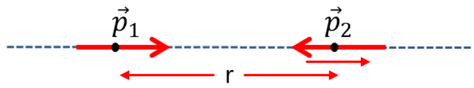
Case-(i)



Force on p_2 due to p_1 will be

$$F_2 = \left(-\frac{6kp_1p_2}{r^4}\right)$$ (Negative sign shows force of attraction between both dipoles.)

Case-(ii)



Force on p_2 due to p_1 will be

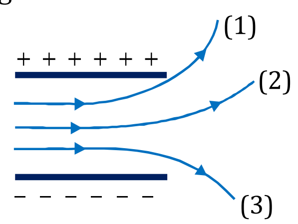
$$F_2 = \left(+\frac{6kp_1p_2}{r^4}\right)$$ (Positive sign shows force of repulsion between both dipoles.)

Golden Key Points

- For a dipole, potential is zero at equatorial point, while at any finite point $E \neq 0$
- In a uniform $\vec{E}$, dipole may experience a torque but may not experience a force.
- If a dipole is placed in a field due a point charge, then torque on dipole may be zero.

BEGINNER'S BOX-6

1. A sample of HCl gas is placed in an electric field of strength 2.5×10^4 NC^{-1}. The dipole moment of each HCl molecule is 3.4×10^{-30} Cm. Find the maximum torque that can act on a molecule. **PEL230**
2. An electric dipole when placed in a uniform electric field E will have minimum potential energy, when the angle made by the dipole moment with the field E is **PEL231**
3. An electric dipole is placed making at an angle 60° with an electric field of strength 4×10^5 N/C. It experiences a torque equal to $8\sqrt{3}$ N-m. Calculate the charge on the dipole, if it is of length 4 cm. **PEL232**
4. Figure shows the tracks of three charged particles in a uniform electric field.
   Give the signs of the three charges.
   Which particle has the largest charge to mass ratio ?



PEL233

5. An electron is projected into a uniform transverse electric field of strength 200 NC^{-1} with initial velocity 10^6 ms^{-1}. By what length is the electron deflected after it has travelled 1 cm ?

PEL234

6. A system has two charges $q_A = 2.5 \times 10^{-7}$ C and $q_B = -2.5 \times 10^{-7}$C located at points A ≡ (0,0,–15) cm and B ≡ (0, 0, 15) cm respectively. What are the total charge and electric dipole moment vector of the system ? **PEL235**

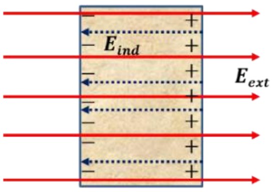
Conductors and their Properties

(i) Conductors are materials which contain large number of free electrons which can move freely inside the conductor.

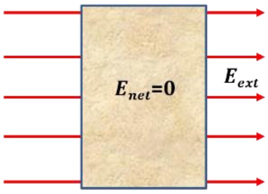
(ii) Electrostatic field lines never exist inside a conductor.

Consider a conductor (or a metal body) placed in uniform electric field :

Metal Body
When just placed in external electric field.



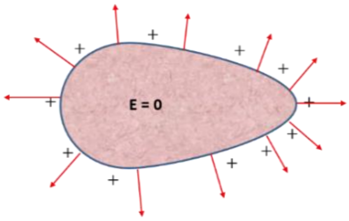
Metal Body
An electrical field gets develop inside the metal due to induced charges.



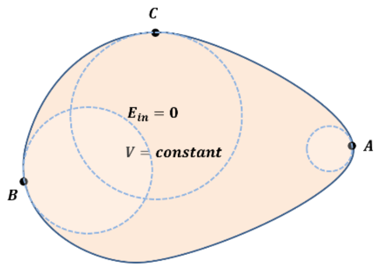
Metal Body
E_{ext} = E_{ind} So net electric field inside metal body will be zero.

Because there is no electric field inside the conductor so we can say electrostatic field lines never exist within conducting materials.

(iii) Electric field lines terminates & originates always perpendicular to the surface of conductor because the surface of conductor is an equipotential surface.



(iv) In electrostatics, conductors are always equipotential surfaces.



For any conductor at its surface, V= constant

$V_A = V_B = V_C$

$R_A < R_B < R_C$

∵ For conductors σR = constant

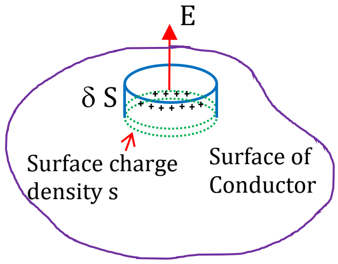
So $\sigma_A > \sigma_B > \sigma_C$

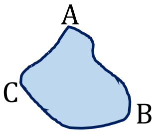
Charge density on sharper edges will be more.

(v) Charge always resides on the outer surface of a conductor.

(vi) If there is a cavity inside a charged conductor with the cavity devoid of any charge then charge will always reside only on the outer surface of the conductor.

(vii) Electric field intensity near a conducting surface is given by the formula $\vec{E}=\frac{\sigma}{\varepsilon_0}\hat{n}$



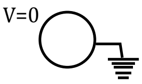


$\vec{E}_A=\frac{\sigma_A}{\varepsilon_0}\hat{n}$; $\vec{E}_B=\frac{\sigma_B}{\varepsilon_0}\hat{n}$; $\vec{E}_C=\frac{\sigma_C}{\varepsilon_0}\hat{n}$

Because conductors are always equipotential surfaces so we can say $V_A=V_B=V_C$

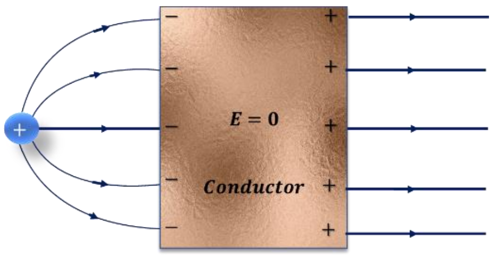
It means q $\propto$ R and $\sigma \propto \frac{1}{R}$ so we can say $\frac{q_1}{q_2}=\frac{R_1}{R_2}$ and $\frac{\sigma_1}{\sigma_2}=\frac{R_2}{R_1}$

(viii) When a conductor is grounded its potential becomes zero.



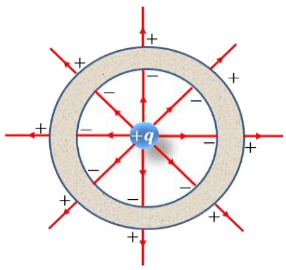
This concept is known as earthing, after which potential of body becomes zero but charge may or may not be zero. If isolated body is earthed then only we can say that its charge is zero.

(ix) EFL pattern when a point charge is placed near a metallic conductor.

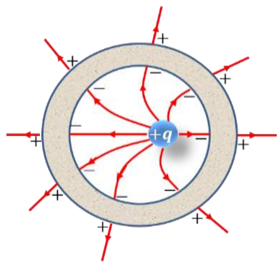


If all the EFL originate from point charge entering into metal body then magnitude of induced charge is equal to point charge.

(x) EFL Pattern when a point charge placed inside spherical metal shell -



(1) Charge placed at centre



(2) Charge not placed at centre

Illustration 117:

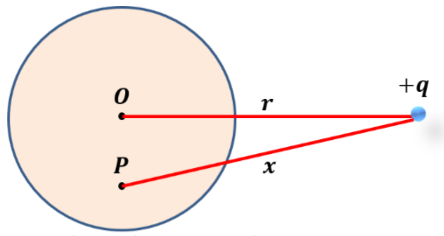
For the below neutral conducting sphere find -

(1) E due to q at P

(2) E due to sphere at P

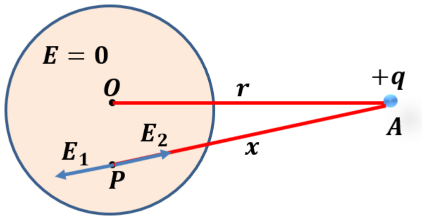
(3) Electric potential at P due to q

(4) Electric potential at P due to sphere



Neutral conducting sphere

Solution:



In every conductor net electric field is always zero, so electric field at P will be equal and opposite due to point charge and induced charges on conductor.

(1) E due to $+q$ charge at P will be $E_1 = \frac{kq}{x^2}\left(\text{along } \overrightarrow{AP}\right)$

(2) E due induce charges charge at P will be

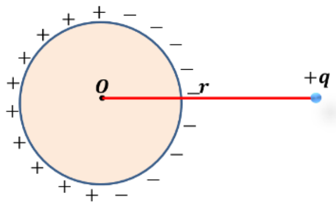
$E_2 = \frac{kq}{x^2}\left(\text{opposite to } \overrightarrow{AP}\right)$

At Point P

$\vec{E} = \vec{E}_1 + \vec{E}_2$

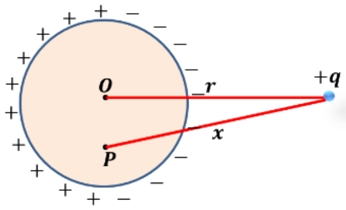
$\vec{E} = 0 \Rightarrow \vec{E}_1 = -\vec{E}_2$ (i.e. both EF are equal and opposite).

(3) Potential at centre due to induced charges is zero, and due to $+q$ is $V_q = \frac{kq}{r}$



As potential at every point inside the conductor is same so potential at P is also $V = \frac{kq}{r}$

(4) At point P

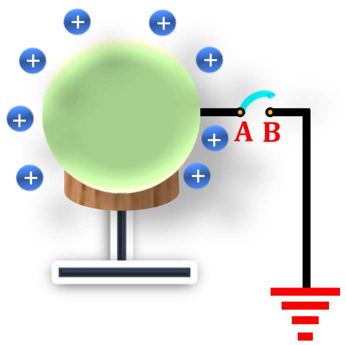


$V_{ind} + V_q = \frac{kq}{r}$

$V_{ind} + \frac{kq}{x} = \frac{kq}{r} = \frac{kq}{r} - \frac{kq}{x}$

Concept of Earthing

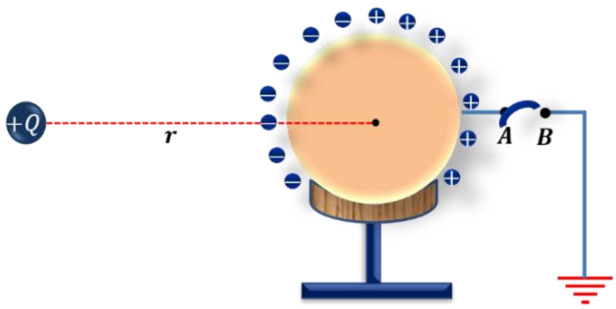
Earth is conducting body with zero potential (assumed). When a charged body is connected to earth, charge flows continuously from higher potential to lower potential till potential of body becomes zero.



As soon as switch is closed whole charge of sphere goes to earth to make potential of sphere zero

Illustration 118:

What final charge will be appearing on conducting sphere when switch is closed.



Solution:

If x is charge (which would be appearing) on sphere after closing the switch then,

$$\frac{kQ}{r}+\frac{kx}{R}=0$$

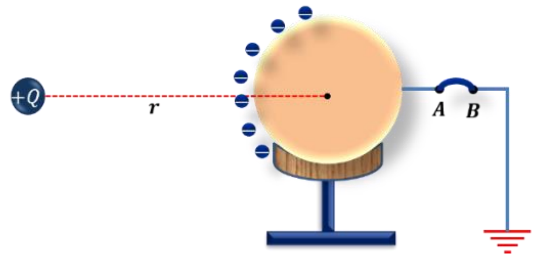
(Potential due to charge appears on sphere)

(Potential due to point charge)

From $\frac{kQ}{r}+\frac{kx}{R}=0$

$\Rightarrow \frac{kQ}{r}=-\frac{kx}{R}$

$\Rightarrow x=-Q\left(\frac{R}{r}\right)$



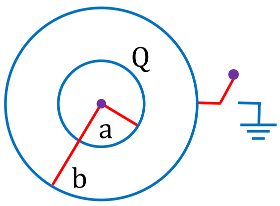
As $R>r$ magnitude of x will be smaller than Q .

When earthing is removed charge on sphere remains at same position due to force of attraction of +Q charge.

When point charge +Q is removed whole negative charge of sphere is distributed uniformly on surface of conductor.

Illustration 119:

Find the charge of outer sphere after closing the switch



Solution:

After closing the switch outer sphere will get earthed so its potential will become zero. Let us assume Q' charge appears on it finally after earthing -

$\Rightarrow \quad V_{out} = 0$

$\Rightarrow \quad \frac{kQ}{b}+\frac{kQ'}{b}=0$

$\Rightarrow \quad Q' = -Q$

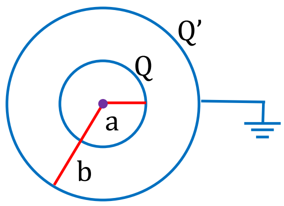
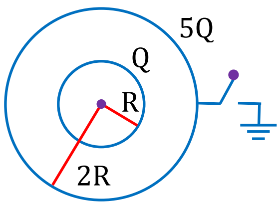
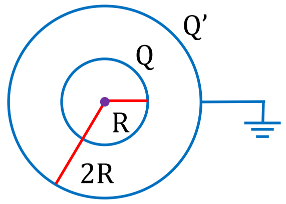


Illustration 120:

Find the charge transferred from body to earth after closing the switch



Solution:

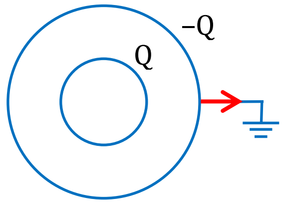


After closing the switch outer sphere will get earthed so its potential will become zero, Let us assume Q' charge appears on it finally after earthing.

$\Rightarrow V_{out} = 0$

$$\Rightarrow \frac{kQ}{2R}+\frac{kQ'}{2R}=0$$

$\Rightarrow Q' = -Q$

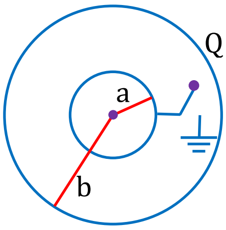


Initially outer sphere was having 5Q charge and finally it becomes -Q that means 6Q charged transferred from body to earth. (or -6Q from earth to body)

$\Rightarrow Q_{Transfer}$ = 6Q from Body to earth -6Q from earth to Body.

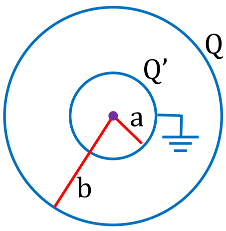
Illustration 121:

Find the charge on inner sphere after closing the switch



Solution:

After closing the switch inner sphere will get earthed so its potential will become zero. Let us assume Q' charge appears on it finally after earthing.



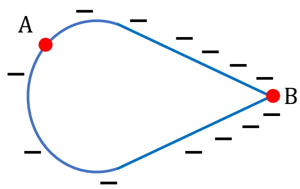
$\Rightarrow V_{in} = 0$

$$\Rightarrow \frac{kQ'}{a}+\frac{kQ}{b}=0$$

$$\Rightarrow Q'=-\frac{Qa}{b}$$

Leakage of Charge, Electrostatic Shielding and Sharing of Charges

Leakage of Charge



Leakage of charge from sharp points of a conductor

Here $V_A = V_B$ but $\sigma_B > \sigma_A$,

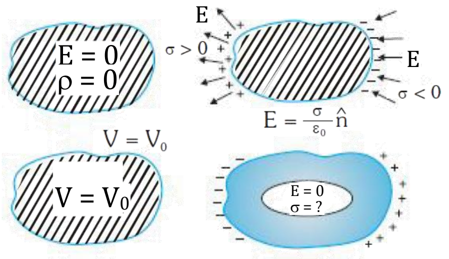
hence $E_B > E_A \left\{\because E=\frac{\sigma}{\varepsilon_0}\right\}$

Thus charge density & hence electric field is larger at sharp corners as compared to flat surfaces due to their less radius of curvature.

$$\sigma_1=\frac{Q_1'}{4\pi R_1^2} \ \& \ \sigma_2=\frac{Q_2'}{4\pi R_2^2} \qquad \because \frac{Q_1'}{Q_2'}=\frac{R_1}{R_2}\Rightarrow\frac{\sigma_1}{\sigma_2}=\frac{R_2}{R_1}\Rightarrow\sigma\propto\frac{1}{R}$$

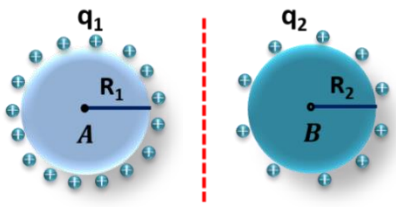
Electrostatic Shielding

- Electrostatic shielding is the method of protecting a certain region from the effect of electric field.
- A cavity surrounded by conducting walls is a field free region as long as there are no charges inside the cavity.
- Whatever be the charge and field configuration the field inside the cavity is always zero (Provided no charge present inside the cavity) This is known as electrostatic shielding.
- Electrostatic shielding can be achieved by enclosing sensitive instruments in a hollow conductor.
- Figure gives a summary of the important electrostatic properties of a conductor.



Sharing of charges

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of both conductors become equal.

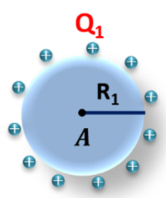


A and B both are isolated conducting spheres having initial charges q_1 and q_2. (total charge of system $Q = q_1 + q_2$). Charge will flow continuously until potential of both spheres becomes equal. Let the amounts of charges after the conductors are connected be Q_1 and Q_2 respectively and their common potential be V then

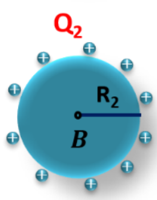
Finally

$$V = \frac{kQ_1}{R_1} = \frac{kQ_2}{R_2} \Rightarrow \frac{Q_1}{Q_2} = \frac{R_1}{R_2} \quad \text{or} \quad Q_1 = \left(\frac{R_1}{R_1 + R_2}\right)Q \text{ or } Q_2 = \left(\frac{R_2}{R_1 + R_2}\right)Q \qquad \{Q_1 + Q_2 = Q\}$$

Final charge density of spheres



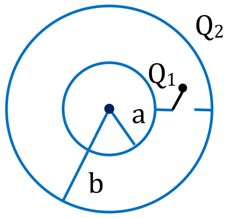
$$\sigma_1 = \frac{Q_1}{A_1} = \frac{\left(\frac{R_1}{R_1 + R_2}\right)Q}{4\pi R_1^2}$$



$$\sigma_2 = \frac{Q_2}{A_2} = \frac{\left(\frac{R_2}{R_1 + R_2}\right)Q}{4\pi R_2^2}$$

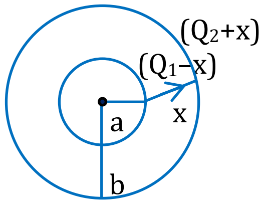
Illustration 122:

Find charge on inner sphere after closing the switch



Solution:

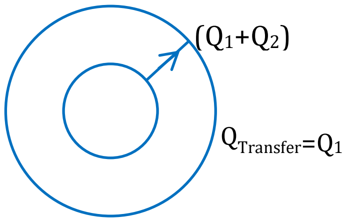
$\because$ After closing the switch potential of both the spheres will become equal so $V_{in} = V_{out}$. Now if x charge is flown from inner to outer sphere then equating their potential, we get the below equation



$$\Rightarrow \frac{k(Q_1 - x)}{a} + \frac{k(Q_2 + x)}{b} = \frac{k(Q_1 - x)}{b} + \frac{k(Q_2 + x)}{b}$$

$$\Rightarrow k(Q_1 - x)\left(\frac{1}{a} - \frac{1}{b}\right) = 0 \Rightarrow Q_1 - x = 0$$

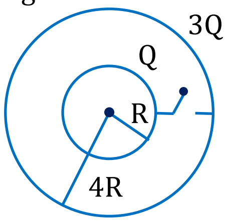
$\Rightarrow x = Q_1 = Q_{transfer}$ (that means complete charge of inner sphere will get transferred to outer sphere)



Charge on inner sphere = 0 and on outer sphere = $(Q_1 + Q_2)$

Illustration 123:

Find the charge of each sphere after closing the switch



Solution:

$\because$ Complete charge of inner sphere will get transferred to outer sphere so Q transfer will be Q

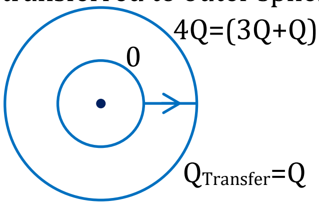
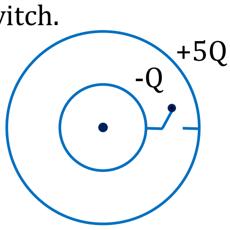


Illustration 124:

Find the charge transfer after closing the switch.



Solution:

$\because$ Complete charge of inner sphere will get transferred to outer sphere so $Q_{transfer}$ will be -Q

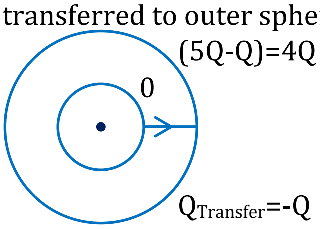


Illustration 125:

Two thin spherical shells made of metal are at a large distance apart. One, of radius 10 cm, carries a charge of +0.5 mC and the other, of radius 20 cm, carries a charge of +0.7 mC. The charge on each, when they are connected by a suitable conducting wire is respectively -

Solution:

$$Q_1 = \left(\frac{R_1}{R_1+R_2}\right)Q = \left(\frac{10cm}{30cm}\right)1.2mc \Rightarrow Q_1 = 0.4mc$$

$$Q_2 = \left(\frac{R_2}{R_1+R_2}\right)Q = \left(\frac{20cm}{30cm}\right)1.2mc \Rightarrow Q_2 = 0.8mc$$

Illustration 126:

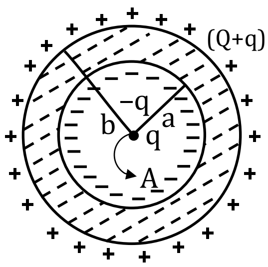
Two spherical conductors of radii 4 m and 5m, are charged to the same potential. If $\sigma 1$ and σ_2 be the respective values of the surface charge density on the two conductors, then the ratio $\frac{\sigma_1}{\sigma_2}$.

Solution:

For spherical conductors

$V = \sigma R$ so $\sigma \propto \frac{1}{R}$

$$\frac{\sigma_1}{\sigma_2} = \frac{R_2}{R_1} = \frac{5}{4}$$



Note:

Only inner charge will be responsible for charge distribution over inner surface.

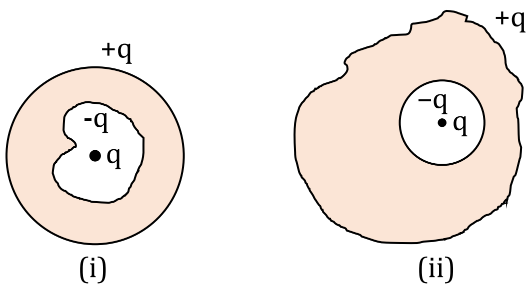
Only outer charge will be responsible for charge distribution over outer surface. In the above figure

Surface charge density over inner surface $\sigma_{inner} = \frac{-q}{4\pi a^2}$ and outer surface $\sigma_{outer} = \frac{Q+q}{4\pi b^2}$.

If charge q is shifted to A then charge over inner surface will get distributed non-uniformly and charge distribution over outer surface will be same as before (It would not get effected by inner charge).

Illustration 127:

Find charge distribution on each surface.



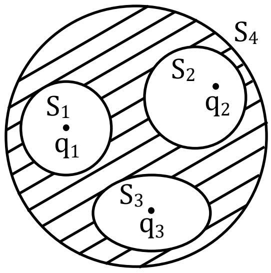
Solution:

(i) Inner $\rightarrow -q$ (Non-uniform distribution) Outer $\rightarrow$ q (Uniform distribution)

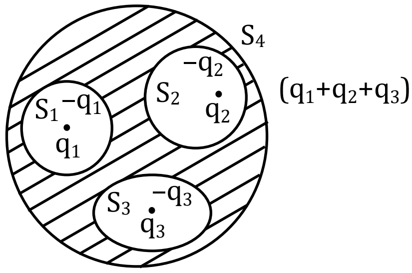
(ii) Inner $\rightarrow -q$ (Uniform distribution) Outer $\rightarrow$ q (Non-uniform distribution)

Illustration 128:

Find charge distribution on each surface of the body.



Solution:



$S_1 \rightarrow -q_1$ (Uniform)

$S_2 \rightarrow -q_2$ (Non-uniform)

$S_3 \rightarrow -q_3$ (Non-uniform)

$S_4 \rightarrow q_1+q_2+q_3$ (Uniform)

BEGINNER'S BOX-7

1. A cube of metal is given a charge (+ Q); which of the following statements is true ?

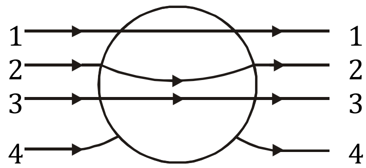
(A) Potential on the surface of cube is zero

(B) Potential within the cube is zero

(C) Electric field is normal to the surface of the cube

(D) Electric field varies within the cube **PEL236**

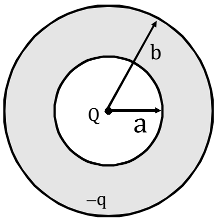
2. A solid metallic sphere is placed in a uniform electric field. The lines of force follow the path(s) (shown in figure):-



(A) 1 (B) 2 (C) 3 (D) 4

PEL237

3. Shown in the figure is a spherical shell with inner radius 'a' and outer radius 'b' which is made of conducting material. A point charge +Q is placed at the centre of the shell and a total charge −q is placed on the shell. Charge −q is distributed on the surfaces as:-



(A) −Q on the inner surface, −q on the outer surface

(B) −Q on the inner surface, −q+Q on the outer surface

(C) +Q on the inner surface, −q−Q on the outer surface

(D) The charge −q is spread uniformly between the inner and outer surfaces. **PEL238**

BEGINNER'S BOX — ANSWER KEY

BEGINNER'S BOX-1

1. 80 C **2.** 3×10^{25} electrons. **3.** (a) Yes, (b) No, (c) No

4. 3.2×10^{-10} C **5.** 2.18×10^{10}

6. For proton $\Rightarrow$ u, u, d and For neutron $\Rightarrow$ d, d, u

7. (a) 2×10^{12} ; (b) 18.2×10^{-19} kg

8. A will have positive charge and B will have negative charge.

BEGINNER'S BOX-2

1. $\frac{F}{8}$ **2.** 2 **3.** 10^{-10} **4.** 44.4 N

5. (i) 8×10^{-5} C (ii) 5×10^{14} **6.** $q = -\frac{1}{2}\,\mu C$ **7.** $F_{net} = 9 \times 10^{3}$ N along to $\overrightarrow{CA}$

8. d = 0.118 m **9.** $F = \frac{kQ^2}{a^2}\sqrt{2}$

BEGINNER'S BOX-3

1. 20 cm **2.** 5×10^{-6} C ; +ve charge

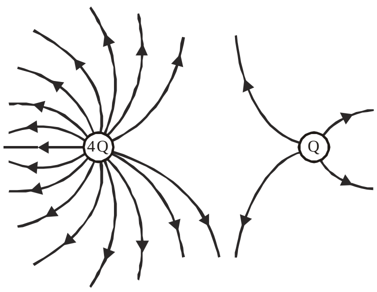
3. 6.25×10^{12} electrons should be removed

4. $|\vec{E}| = 2.5 \times 10^{4}$ N/C electric field along y-direction **5.** 45 cm

6. $E_{net} = 16.46 \times 10^{3}$ N/C Direction of net electric field is perpendicular and away from the line AB

BEGINNER'S BOX-4

1.



x = 8 cm

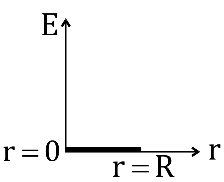
2. 20 V/m **3.** Zero **4.** D **5.** $\frac{5}{6}\frac{q}{\epsilon_0}$

6. 2.25×10^{5} Nm^2/C

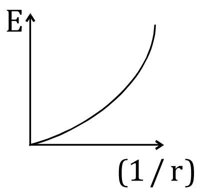
7. (a) -5.31×10^{-14} C (b) Flux remains the same **8.** All i.e. q_1, q_2, q_3 & q_4

9. Yes, Ex : A dipole placed in a closed surface **10.** (a) Zero (b) 1.92 V/m

11. (a)



(b)



BEGINNER'S BOX-5

1.	1: 4	**2.**	Zero	**3.**	Zero	**4.**	36 m/s each

5. (i) –0.7 J, (ii) 0.7 J **6.** (i) 4×10^4 V (ii) 8×10^{-5} J, No

7. 2.7×10^6 V

8. (i) 2.5×10^5 V, (ii) The more distant point from the charge

9. (i) 0.54 m, (ii) 7.5 kV, a decrease

10.	3.1×10^5 V	**11.**	$-27\sqrt{2}\times10^5\text{V}$	**12.**	$-2\hat{i}+2\hat{j}-2\hat{k}$	**13.**	12.6 m

14. (i) against the field 3.0×10^4 V; in the field direction -3.0×10^4 V, (ii) Zero

15. (a) $V_P - V_Q$ is positive

$V_B - V_A$ is positive

(b) $U_Q - U_P$ is positive

$U_A - U_B$ is positive

(c) negative

(d) positive

(e) decreases

BEGINNER'S BOX-6

1.	8.5×10^{-26} Nm	**2.**	0	**3.**	10^{-3} C or 1 mC

4. (1) –ve, (2) –ve (3) +ve

Particle (1) has largest charge to mass ratio.

5. 0.176 cm

6. Total charge = 0;

Dipole moment = -7.5×10^{-8} C-m

BEGINNER'S BOX-7

1.	(C)	**2.**	(D)	**3.**	(B)