Gravitation

SOLUTIONS

Exercise-I (Conceptual Questions)

1. Control the rotational motion of satellites and planets. Explained in Kepler's law.

2. $$F_g = GMm\left(\frac{1}{r_1^2}+\frac{1}{r_2^2}+...\infty\right)$$

$$F_g = G(1)(3)\left(\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{4^2}+...\infty\right)$$

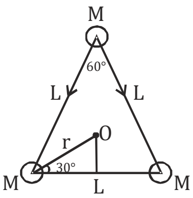
$$F_g = 3G\left(1+\frac{1}{4}+\frac{1}{16}+...\infty\right)$$

$$F_g = 3G\left(\frac{1}{1-\frac{1}{4}}\right) = 4G$$

3. Gravitational force is medium independent

4. It is a universal constant.

5. Net force on any particle :



$$= \sqrt{F^2+F^2+2F^2\cos 60°}$$

$$= \sqrt{3F^2} = F\sqrt{3}$$

$\left(F = \frac{GM^2}{L^2} \text{ is force of any one particle on another}\right)$

$$F\sqrt{3} = \frac{Mv^2}{r} \text{ (by centripetal force)}$$

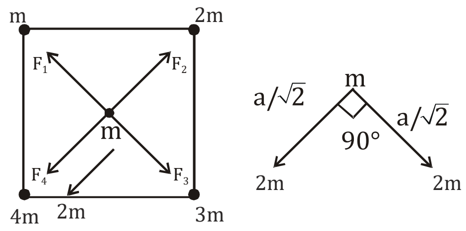
Where $\frac{L}{2r} = \cos 30° \Rightarrow r = \frac{L}{\sqrt{3}}$

$$\sqrt{3}\frac{GM^2}{L^2} = \frac{Mv^2}{r} \Rightarrow \sqrt{3}\frac{GM}{L^2} = \sqrt{3}\frac{v^2}{L}$$

$$\Rightarrow v = \sqrt{\frac{GM}{L}} \qquad \because v = \omega r$$

$$\therefore \quad \omega = \frac{v}{r} = \sqrt{\frac{3GM}{L^3}}$$

6.



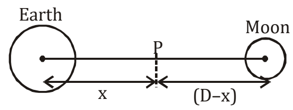
$$F_{net} = \sqrt{2}F$$

$$= \sqrt{2}\frac{G\times 2m\times m}{(a/\sqrt{2})^2} = \frac{4\sqrt{2}Gm^2}{a^2}$$

7. The tidal waves in seas are primarily due to gravitational force of the moon.

8. More work is done against the gravitational pull of the earth.

9.



At point 'p' for gravitational field to be zero field due to earth = field due to moon

$$\Rightarrow \frac{GM_e}{x^2} = \frac{GM_m}{(D-x)^2} \Rightarrow \frac{81M_m}{x^2} = \frac{M_m}{(D-x)^2}$$

$$\Rightarrow \frac{x}{D-x} = 9 \Rightarrow 9(D-x) = x \Rightarrow x = \frac{9D}{10}$$

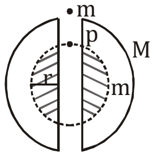
10. By first law of motion i.e. law of inertia, satellite will fly off tangentially from the orbit with speed V.

11. $$I_g = \frac{-GMr}{R^3}; \quad r < R$$

$$I_g = \frac{-GM}{r^2}; \quad r \geq R$$

12. Weight is due to gravitational pull of earth which varies with distance as $\frac{1}{r^2}$. Weight becomes zero when gravitational pull of both planets become equal & opposite. After that its weight again increase to 240 N due to mars.

13. The gravitational field at P is due to the mass enclosed. (m')



the force acting = $\frac{Gm'm}{r^2}$,

m = mass of the particle released.

$$F = G\rho\frac{4}{3}\pi r^3 \cdot \frac{m}{r^2} \qquad \left\{m' = \frac{4}{3}\pi\rho r^3\right\}$$

$$F = \left[\frac{4}{3}\pi G\rho m\right]r$$

14. $g = \frac{GM_e}{R_e^2}$

$$g_{mass} = \frac{G(0.1)M_e}{(0.5)^2 R_e^2} = \frac{0.4GM_e}{R_e^2} = 0.4\,g$$

15. By symmetrically all forces will cancel each other and resultant will be zero.

16. $g = \frac{GM_E}{R_E^2}$

where M_E and R_E is the mass and radius of the earth respectively.

$$M_E = \frac{g}{G}R_E^2$$

17. $g_d = g_s\left(1 - \frac{d}{Re}\right)$ $\qquad$ $d = R_e$; $g_d = 0$

18. The value of 'g' on earth surface depends on both structure and rotational motion.

$$g = \frac{GM_e}{R_e^2}$$

$g' = g - R_e\omega^2\cos^2\lambda$

19. As $\quad g' = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

$$\frac{g}{2} = \frac{g}{\left(1+\frac{h}{R}\right)^2} \Rightarrow \left(1+\frac{h}{R}\right)^2 = 2$$

$$\Rightarrow \frac{h}{R} = \sqrt{2} - 1 \Rightarrow h = R\left(\sqrt{2}-1\right)$$

20. Use $mg_1h_1 = mg_2h_2$

$\Rightarrow \quad (1.96)(h_1) = (9.8)(2)$

$h_1 = 10$ m

21. $g' = g - R_e\omega^2\cos^2\lambda \Rightarrow g' < g$

except for $\lambda = 90°$ i.e. for poles.

22. $g \propto \frac{M}{R^2}$

$$\frac{g'}{g} = \left(\frac{M'}{M}\right)\left(\frac{R}{R'}\right)^2 = (2)\left(\frac{1}{2}\right)^2 = \frac{1}{2}$$

As $\quad T \propto \frac{1}{\sqrt{g}}$

Hence $\frac{T'}{T} = \sqrt{\frac{g}{g'}} = \sqrt{2}$

then $\quad T' = \sqrt{2}T$

23. There is no atmosphere on the moon, means no buoyant force in upward direction so it will fall down with acceleration g/6.

24. $g = \frac{GM}{R^2}$

$$\therefore \quad M = \rho \times \frac{4}{3}\pi R^3 \Rightarrow g = \frac{G \times 4\pi R^3 \times \rho}{3 \times R^2}$$

$$\Rightarrow \rho = \frac{3g}{4\pi GR}$$

25. $g' = g - \omega^2 R_e \cos^2\lambda$

at poles $\lambda = 90°$, $\cos 90° = 0$

at equator $\lambda = 0°$, $\cos 0° = 1$

$\therefore g_{poles} > g_{equator}$

Given $m_p g_p = m_{eq} g_{eq}$

26. $g_{equator} = g - \omega^2 R_e$

$g_{pole} > g_{equator}$

27. As $\quad g = \frac{GM}{R^2}$

$$\frac{\Delta g}{g}\% = \frac{-2\Delta R}{R}\%$$

$$\frac{\Delta g}{g}\% = -2(-1)\% = +2\%$$

28. g decreased by 1% for h height

$$g_h = g\left[1+\frac{h}{R}\right]^{-2}$$

$$\frac{\Delta g_h}{g} = \frac{-2h}{R}$$

for depth.

$$g_d = g\left[1-\frac{d}{R}\right]$$

$$\frac{\Delta g_d}{g} = \frac{-d}{R} \text{ (here d = h)}$$

$$\frac{\Delta g_h}{g} = \frac{2\Delta g_d}{g}, \ \frac{\Delta g_d}{g}\times 100 = \frac{1}{2}\left[\frac{(\Delta g)_h}{g}\times 100\right]$$

$$\Delta g_d = \frac{1}{2}\Delta g_h$$

29. $g_{eff} = g\left(1-\frac{2h}{R}\right) = g\left(1-\frac{32\times 2}{6400}\right) = g(1-0.01)$

$g_{eff} = 0.99g \text{ m/s}^2$

30. Action and reaction force are equal in magnitude.

31. As $g = \frac{4}{3}\pi\rho RG$

$g \propto R$ (As ρ = constant)

$$\frac{g'}{g} = \left(\frac{R'}{R}\right)$$

$$\frac{g'}{g} = 3 \quad \Rightarrow \quad g' = 3g$$

32. $g_h = g\left[1-\frac{2h}{R_e}\right]$

$$g_d = g\left[1-\frac{d}{R_e}\right]$$

put $g_h = g_d$

$$\frac{2h}{R_e} = \frac{d}{R_e} \Rightarrow d = 2h$$

33. The acceleration due to gravity at equator is given by

$g' = g - R\omega^2$

When rotational speed of earth i.e. ω increased, the acceleration due to gravity at equator (g') will decrease, it means weight of body will decrease.

34. At surface of earth,

$$g = \frac{GM}{R^2} \quad ....(i)$$

At a height, h = R/2, the value of acceleration due to gravity is

$$g' = \frac{GM}{(h+R)^2} = \frac{GM}{\left(\frac{R}{2}+R\right)^2} = \frac{GM}{\left(\frac{3R}{2}\right)^2} = \frac{GM}{\left(\frac{3R}{2}\right)^2},$$

$$g' = \frac{4}{9}\frac{GM}{R^2} = \frac{4}{9}g$$

$\therefore$ weight at a height h = R/2 is

$$W' = mg' = \frac{4}{9}mg = \frac{4}{9}W.$$

35. At equator $g' = g - \omega^2 R_e$

For the case of weightlessness g' = 0

$\omega^2 R_e = g$

$$\omega = \sqrt{\frac{g}{R_e}} = \sqrt{\frac{10}{6.4\times 10^6}} = \frac{1}{8}\times 10^{-2}$$

$\omega = 0.125 \times 10^{-2}$ rad/sec

$\omega = 1.25 \times 10^{-3}$ rad/sec

36. Acceleration due to gravity is independent of the mass of object $a = \frac{GM_e}{r^2}$ [M_e = mass of earth]

37. Gravitational potential

$$V = \frac{-GM}{r} \text{ for } r > R$$

$$V = \frac{-GM}{R} \text{ for } r \le R$$

Potential remains constant inside the hollow sphere.

38. Gravitational field intensity

$$E_g = -\frac{dV}{dr} = \frac{2}{20} = 0.1$$

Hence gravitational force = mE_g

= 5(0.1) = 0.5N

Work = Force × displacement = (0.5)(4) = 2J

39. $$= -\frac{GM_e m}{R} - \frac{GM_m m}{r} = -\frac{G(81M_m)m}{R} - \frac{GM_m m}{r}$$

$$= -GmM_m\left(\frac{81}{R}+\frac{1}{r}\right)$$

40. $E_p = -\frac{GMm}{r}$, when $r \to \infty$, $E_p = 0$ (max.)

41. To escape from earth total energy would be zero. If body is unable to escape then total energy
(PE + KE) is negative, i.e. less than zero.

42. Apply energy conservation
Energy at surface = Energy at height

$$-\frac{GMm}{R}+\frac{1}{2}mv^2=\frac{-GMm}{2R}$$

$$\Rightarrow \quad \frac{1}{2}v^2=\frac{GM}{2R} \quad \Rightarrow \quad v=\sqrt{\frac{GM}{R}}$$

43. GPE of body is $|U|=\frac{GMm}{r}$

Force at that point is $F=\frac{GMm}{r^2}$

$$\frac{U}{F}=r \quad \Rightarrow \quad F=\frac{U}{r}$$

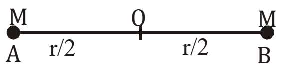
44. Apply energy conservation

$$\frac{-GMm}{R}+\frac{1}{2}mv^2=0 \quad \Rightarrow \quad v^2=\frac{2GM}{R}$$

$$v=\sqrt{\frac{2GM}{R}} \quad \Rightarrow \quad v=\sqrt{2gR}$$

45. $\Delta U = U_f - U_i = -\frac{GMm}{R+h}-\left(-\frac{GMm}{R}\right)$

$$=GmM\left[\frac{1}{R}-\frac{1}{R+h}\right]=\frac{GmMh}{R(R+h)}$$

46. 

Gravitational potential of A at

$$O=-\frac{GM}{r/2}=-\frac{2GM}{r}$$

due to B, potential at $O=-\frac{GM}{r/2}=-\frac{2GM}{r}$

$\therefore$ Total potential = $-\frac{4GM}{r}$

47. The potential energy of a satellite is equal to twice its total energy.

$\therefore$ Potential energy = $2E_0$

48. $\frac{mv^2}{R}=\frac{A}{R^2}$

$KE=\frac{1}{2}mv^2$ $\qquad$ $\boxed{KE=\frac{1}{2}\frac{A}{R}}$

$F=-\frac{dU}{dr}$

$-\frac{A}{r^2}=-\frac{dU}{dr}$

$\int_0^R \frac{A}{r^2}dr=\int dU=\frac{-A}{R}$

TE = PE + KE = $-\frac{A}{R}+\frac{A}{2R}=-\frac{A}{2R}$

49. $PE=\frac{-GMm}{R}$

$-54=\frac{-GM\times 3}{R}$

Hence $\frac{GM}{R}=18$

but $\quad v_{esc}=\sqrt{\frac{2GM}{R}}=\sqrt{36}$ = 6 m/sec

50. $v_{esc}=\sqrt{\frac{2GM}{R}}$

$100=\sqrt{\frac{2GM}{R}} \Rightarrow \frac{GM}{R}=5000$

Gravitational Potential energy = $\frac{-GMm}{R}$

= –5000 × 1 = –5000J

51. V_{escape} = $\sqrt{2gR}$

$\frac{V_1}{V_2}=\sqrt{\frac{g_1R_1}{g_2R_2}}=\sqrt{pq}$

52. Escape velocity $v=\sqrt{\frac{2GM_e}{R}}$ is independent of angle at projection

53. As $v_{esc}\propto\sqrt{\frac{M}{R}}$

Hence $\qquad \frac{v_2}{v_1}=\sqrt{\frac{M_2}{M_1}}\sqrt{\frac{R_1}{R_2}}$

$$\frac{v_2}{v_1}=\sqrt{100}\sqrt{\frac{1}{4}}=10\times\frac{1}{2}=5$$

v_2 = 11.2 × 5 = 56 km/sec

54. Let a body is projected from the surface of the earth with the velocity V and reaches maximum height h.

According to law of conservation of mechanical energy, we get

$$\frac{1}{2}mV^2-\frac{GMm}{R}=0-\frac{GMm}{(R+h)}$$

or $\frac{1}{2}V^2-\frac{GM}{R}=-\frac{GM}{(R+h)}$

As per question,

$$V=\frac{V_e}{2}=\frac{1}{2}\sqrt{\frac{2GM}{R}} \qquad \left[\because\ V_e=\sqrt{\frac{2GM}{R}}\right]$$

$$\frac{1}{2}\left(\frac{1}{2}\frac{GM}{R}\right)-\frac{GM}{R}=-\frac{GM}{(R+h)}$$

$$\frac{GM}{4R}-\frac{GM}{R}=-\frac{GM}{R+h}$$

$$-\frac{3}{4R}=-\frac{1}{(R+h)}$$

3 (R + h) = 4R or 3R + 3h = 4R

3h = R or h = $\frac{R}{3}$

55. Binding energy of moon and earth is $\frac{GM_eM_m}{2r_{em}}$

56. $v\propto\frac{1}{\sqrt{R+h}}$

Hence $\frac{v_A}{v_B}=\sqrt{\frac{r_B+R}{r_A+R}}$

57. $\frac{v_1}{v_2}=\sqrt{\frac{R_2}{R_1}}=\sqrt{\frac{1.5\times10^8}{6\times10^7}}=\frac{\sqrt{5}}{\sqrt{2}}$

58. For a body in geo-stationary orbit gravitational force is equal to the centripetal force so it will remain stationary with respect to the earth like GSS.

59. According to kepler law angular momentum is conserved for planetary motion or satellite motion.

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61. As $T^2\propto r^3$

$$\frac{T_2}{T_1}=\left(\frac{r_2}{r_1}\right)^{3/2}=\left(\frac{9R}{R}\right)^{3/2}=27$$

$$T_2=27\times2\pi\sqrt{\frac{R}{g}}$$

62. Using conservation of angular momentum

$\Rightarrow m v_{max} r_{min} = m v_{min} r_{max}$

$\Rightarrow v_{max} r_{min} = v_{min} r_{max}$

63. For system to be bounded

Net energy i.e. (KE + PE) must be negative

TE = KE + PE = -ve at points A,B & C.

64. $W = E_2 - E_1 = \frac{GMm}{2}\left(\frac{1}{2R}-\frac{1}{3R}\right)=\frac{GMm}{12R}$

65. $T=\frac{2\pi r}{v}=\frac{2\pi r}{\sqrt{\frac{GM}{r}}}=\frac{2\pi r^{3/2}}{\sqrt{GM}}$

Hence $T^2=\frac{4\pi^2r^3}{GM}$

$$T^2=\left(\frac{4\pi^2}{GM}\right)r^3$$

Hence slope of T^2 Vs r^3 curve is = $\frac{4\pi^2}{GM}$

66. $T^2=\frac{4\pi^2}{GM}a^3$

$$T^2=\frac{4\pi^2}{GM}\left(\frac{r_p+r_A}{2}\right)^3=\frac{\pi^2}{2GM}\left(r_p+r_A\right)^3$$

mvr = constant $\Rightarrow$ vr = constant

$\therefore$ $v_Ar_A = v_pr_p$

$\therefore$ $r_A > r_p$

$\therefore$ $v_A < v_p$

67. Near equator 'g' is minimum due to rotation and shape of earth which help to minimize fuel consumption.

68. For escape from earths gravitational field T.E. must be zero.

KE + PE = 0

$$v_0=\sqrt{\frac{GM}{R}}, \qquad v_e=\sqrt{\frac{2GM}{R}}$$

additional velocity = $v_e - v_0$

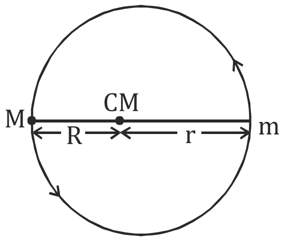
= 11.2 – 8 ≃ 3.2 km/s

69. TE = – KE = $-\frac{1}{2}mv^2$

70. $T = 2\pi\sqrt{\frac{r^3}{GM}} \Rightarrow T = 2\pi\sqrt{\frac{(R+h)^3}{GM}}$

$$\Rightarrow h = \left[\frac{GMT^2}{4\pi^2}\right]^{1/3} - R$$

71.



$$\frac{GMm}{(R+r)^2} = m\omega^2 r$$

72. $\frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v^2 = \frac{GM}{r}$

Now T.E. = $\frac{1}{2}mv^2 - \frac{GMm}{r}$

$= \frac{1}{2}\frac{mGM}{r} - \frac{GMm}{r} \Rightarrow -\frac{GMm}{2r}$

So total energy of system

$= -\frac{GMm}{2r} - \frac{GMm}{2r} = -\frac{GMm}{r}$

73. $T^2 \propto a^3$

$a = \frac{r_{max} + r_{min}}{2} = \frac{r_1 + r_2}{2}$

$T = \left(\frac{r_1 + r_2}{2}\right)^{3/2}$

$T \propto (r_1 + r_2)^{3/2}$

74. By conservation of mechanical energy

$-\frac{GMm}{2R} + 0 = \frac{1}{2}mv^2 - \frac{GMm}{R}$

$v = \sqrt{\frac{GM}{R}}$ = 7.919 km/s

75. COME in conservative field is valid & total energy of bounded system is negative.

76. $F_{cp} = F_g$

$\frac{mv^2}{R} = \frac{K}{R} \Rightarrow v^2 = \frac{K/m}{R^0}$ So proportional to R^0

77. $v = \sqrt{\frac{GM}{R+h}} = \sqrt{\frac{gR^2}{R+h}}$

$v = R\sqrt{\frac{g}{R+h}}$

78. By $V = \sqrt{\frac{GM}{r}}$

$v \propto \frac{1}{\sqrt{r}}$

$\Rightarrow V_1 r_1 = V_2 r_2$

so, if $r_1 > r_2$, then $v_1 < v_2$

79. $T^2 \propto R^3$

$\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{R}{4R}\right)^3$

$\frac{T_1}{T_2} = \sqrt{\frac{1}{64}}$

$T_2 = 8T_1$

T_1 = 1 day for communication satellite

T_2 = 8 days

80. $v_0 = \sqrt{\frac{GM}{R}}$ where M = mass of earth

81. $TE_1 = -\frac{GMm}{2(R+R)} = -\frac{GMm}{4R}$

$TE_2 = -\frac{GMm}{2(R+7R)} = -\frac{GMm}{16R}$

$\frac{TE_1}{TE_2} = \frac{4}{1}$

$\frac{PE_1}{PE_2} = \frac{\frac{-GMm}{2R}}{-\frac{GMm}{8R}} = \frac{4}{1}$

$\frac{KE_1}{KE_2} = \frac{+\frac{GMm}{4R}}{+\frac{GMm}{16R}} = \frac{4}{1}$

82. For satellite net pulling force of gravity must be equal to centripetal force

$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$v = \sqrt{\frac{GM}{r}}$$

if $r \to R$

$$v = \sqrt{\frac{GM}{R}} \approx 8 \text{ km/s}$$

$= 8 \times 10^3$ m/s

83. Apply angular momentum conservation law

$mv_1r_1 = mv_2r_2$

$\Rightarrow \quad 60 \times 1.6 \times 10^{12} = v_2 \times 8 \times 10^{12}$

$v_2 = 12$ m/sec

84. For satellite,

Centripetal force = gravitational force

$$\frac{mv^2}{r} = \frac{Gmm_E}{r^2}$$

$$\text{or } v^2 = \frac{Gm_E}{r}$$

Hence v depends m_E and r.

85. As $\quad T^2 \propto r^3$

$$\left(\frac{T_2}{T_1}\right)^2 = \left(\frac{r_2}{r_1}\right)^3$$

$$\frac{T_2}{1.4} = \left(\frac{4R}{R}\right)^{3/2} = 8$$

$T_2 = 1.4 \times 8 = 8\sqrt{2}$ hr.

86. $$T^2 = \frac{4\pi^2}{GM} r^3$$

$$\left(\frac{T_1}{T_2}\right)^{2/3} = \frac{r_1}{r_2} \qquad [\therefore\ T_1 = T_2]$$

$$\Rightarrow \frac{r_1}{r_2} = 1$$

87. Apply energy conservation

$$\frac{-GM_em}{R_e} + \frac{1}{2}m(2v_e)^2 = \frac{1}{2}mv'^2 = 0$$

$$\Rightarrow \quad \frac{-GM_em}{R_e} + \frac{1}{2}m\left(2\sqrt{\frac{2GM_e}{R_e}}\right)^2 = \frac{1}{2}mv'^2$$

$$\Rightarrow \quad \frac{3GM_em}{R_e} = \frac{1}{2}mv'^2$$

$$v'^2 = \frac{6GM_e}{R_e} = 3 \times \frac{2GM_e}{R_e}$$

$$v' = \sqrt{3}\sqrt{\frac{2GM_e}{R_e}} = \sqrt{3}v_e$$

88. for a satellite

$$\text{K.E.} = \frac{GMm}{2r}$$

$$\text{PE} = \frac{-GMm}{r}$$

89. Orbital velocity of satellite, $v = \sqrt{\dfrac{GM}{R}} = v_0$

$$\therefore\ v' = \sqrt{\frac{GM}{R + \frac{R}{2}}} = \sqrt{\frac{2GM}{3R}} = \sqrt{\frac{2}{3}}v_0.$$

90. According to Kepler's law, $T^2 \propto r^3$

$$\text{or, } \frac{T_1}{T_2} = \left(\frac{r_1}{r_2}\right)^{3/2}$$

$$\text{or, } T_2 = T_1\left(\frac{r_2}{r_1}\right)^{3/2} = T_1\left(\frac{2r_1}{r_1}\right)^{3/2} = T_1 2\sqrt{2}$$

$$T_2 = 2\sqrt{2}\,(1)$$

$$= 2\sqrt{2} \text{ year}$$

91. The satellite revolving around a planet has two types of energies.

(i) Gravitational potential energy U due to position of the satellite is given by $U = -\dfrac{GMm}{R}$.

(ii) Kinetic energy K due to orbital velocity of satellite is given by

$$K = \frac{1}{2}mv_0^2 = \frac{1}{2}m\left(\frac{GM}{R}\right) = \frac{1}{2}\frac{GMm}{R}$$

$\therefore$ Total energy = U + K

$$= -\frac{GMm}{R} + \frac{GMm}{2R} = -\frac{GMm}{2R}$$

92. Kinetic energy of a satellite $K = \frac{GMm}{2r}$

where G = Universal gravitational constant

M = Mass of a earth

m = Mass of a satellite

Potential energy of a satellite $U = -\frac{GMm}{r}$

Total energy of a satellite $E = -\frac{GMm}{2r}$

In a graph 1 represents kinetic energy, 2 total energy and 3 represent potential energy.

93. According to Kepler's third law

$T^2 \propto r^3$ or $T \propto r^{3/2}$

$$\frac{T_E}{T_M} = \left(\frac{r_E}{r_M}\right)^{3/2}$$

where T_E = time of revolution of the earth around the sun = 1 year

r_E = mean distance of earth from the sun

T_M = time of revolution of the mars around the sun

r_M = mean distance of mars from the sun.

$$T_M = T_E\left(\frac{r_M}{r_E}\right)^{3/2} = (1)\left[\frac{1.5r_E}{r_E}\right]^{3/2} = 1.84 \text{ years}$$

Exercise-II (Previous Year Questions)

1. Escape velocity = $\sqrt{\frac{2GM}{R}}$ = c = speed of light

$$R = \frac{2GM}{c^2} = \frac{2\times6.6\times10^{-11}\times5.98\times10^{24}}{(3\times10^8)^2} = 10^{-2}\text{ m}$$

2. Gravitational field or intensity of gravitational field. w.r.t. distance

$$I = \frac{-GM}{r^2} \quad \text{for } r > R$$

$$I = \frac{-GMr}{R^3} \; ; r \le R$$

3. $T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{GM}}\sqrt{r}$ $\quad \left(\text{as } v = \sqrt{\frac{GM}{r}}\right)$

$T = \frac{2\pi}{\sqrt{GM}} r^{3/2} \Rightarrow \quad T^2 = \frac{4\pi^2}{GM} r^3$

Comparing $K = \frac{4\pi^2}{GM}$

4. For satellite S moving elliptical orbit around the earth net force will be towards centre of the earth.

(like centripetal force in circular motion)

5. For the satellite revolving around earth

$$v_0 = \sqrt{\frac{GM_e}{(R_e+h)}} = \sqrt{\frac{GM_e}{R_e\left(1+\frac{h}{R_e}\right)}} = \sqrt{\frac{gR_e}{1+\frac{h}{R_e}}}$$

substituting the values

$v_0 = \sqrt{60\times10^6}$ m/s

$v_0 = 7.76 \times 10^3$ m/s = 7.76 km/s

6. $V = \frac{-GM}{R+h} = -5.4 \times 10^7$ ...(1)

and $g = \frac{GM}{(R+h)^2} = 6$ ...(2)

dividing (1) and (2)

$\Rightarrow \frac{5.4\times10^7}{(R+h)} = 6$

$\Rightarrow$ R + h = 9000 km so h = 2600 km

7. Escape Velocity

$$= \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2G}{R}.\left(\frac{4}{3}\pi R^3\rho\right)} \propto R\sqrt{\rho}$$

$\therefore$ Ratio, $\frac{v_e}{v_p} = 1 : 2\sqrt{2}$

8. $g = \left(\frac{GM_e}{R_e^3}\right) r$ for $0 < r \le R_e \Rightarrow \mathbf{g \propto r}$

$g = \frac{GM_e}{r^2}$ for $r \ge R_e \Rightarrow \mathbf{g \propto \frac{1}{r^2}}$

9. Total energy = $-\dfrac{GM_em}{2(R+h)}$

$\because \quad g_0 = \dfrac{GM_e}{R^2} \quad \Rightarrow M_e = \dfrac{g_0R^2}{G}$

$\therefore$ Energy = $-\dfrac{mg_0R^2}{2(R+h)}$

10. $g_h = g_d$

$g\left(1-\dfrac{2h}{R}\right) = g\left(1-\dfrac{d}{R}\right)$

$\dfrac{2h}{R} = \dfrac{d}{R}$

So $\boxed{d = 2\ \text{km}}$

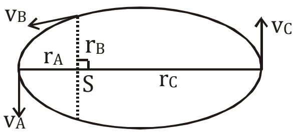
$2 \times 1 = d$

12. $m_s' = \dfrac{m_s}{10}$ & G' = 10 G

$g_E = \dfrac{G'M_E}{R^2} = \dfrac{10GM_E}{R^2} = 10g$

$\therefore$ Raindrops will fall faster, Walking on the ground would become more difficult, Time period of a simple pendulum on the earth would decrease.

13.



As L = mvr = contant

and $r_C > r_B > r_A$

so $v_A > v_B > v_C \Rightarrow K_A > K_B > K_C$

14. $g' = g\left(1-\dfrac{d}{R}\right)$

$g' = g\left(1-\dfrac{R/2}{R}\right)$

$mg' = mg\left(\dfrac{1}{2}\right)$

$W' = 200\left(\dfrac{1}{2}\right) = 100$ N

15. $W = \dfrac{mgh}{1+h/R}$

at h = R, W = $\dfrac{mgR}{2}$

16. Kepler's Third Law :-

$T \propto r^{3/2}$

$\dfrac{T_2}{T_1} = \left(\dfrac{r_2}{r_1}\right)^{3/2} = \left(\dfrac{R+2.5R}{R+6R}\right)^{3/2} = \dfrac{1}{2\sqrt{2}}$

$\Rightarrow T_2 = \dfrac{24}{2\sqrt{2}} = 6\sqrt{2}$ hours

17. $\Delta U = -GMm\left[\dfrac{1}{r_f}-\dfrac{1}{r_i}\right]$

$= -GMm\left[\dfrac{1}{R+h}-\dfrac{1}{R}\right]$

$= \dfrac{GMmh}{R(R+h)}$

18. $W_s = mg_s = 72$ N

$W_h = mg_h = \dfrac{mg_s}{\left(1+\dfrac{h}{R}\right)^2} = \dfrac{72N}{\left(1+\dfrac{R/2}{R}\right)^2} = \dfrac{72}{9/4}$

$W_h = 32$ N

19. At depth : $g_{eff} = g\left(1-\dfrac{d}{R}\right)$

$\Rightarrow \dfrac{g}{n} = g\left(1-\dfrac{d}{R}\right)$

$\Rightarrow d = (n-1)\, R/n$

20. $v_e = \sqrt{\dfrac{2GM}{R}} = \sqrt{\dfrac{2G}{R}\times\dfrac{4}{3}\pi R^3\rho}$

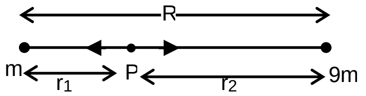
or $V_e = \sqrt{\dfrac{8\pi G\rho}{3}R^2}$

$\Rightarrow v_e \propto R$

$\Rightarrow \dfrac{v_e}{\upsilon} = \dfrac{4R}{R} \Rightarrow v_e = 4\upsilon$

21. $h = \dfrac{R}{\dfrac{2gR}{V^2}-1} = \dfrac{R}{\dfrac{V_e^2}{k^2V_e^2}-1} = \dfrac{Rk^2}{1-k^2}$

25.



Position of Neutral point (Zero Gravitational Field)

$$r_1 = \frac{\sqrt{m_1}\,R}{\sqrt{m_1}+\sqrt{m_2}} = \frac{\sqrt{m}\,R}{\sqrt{m}+\sqrt{9m}} = \frac{R}{4}$$

$r_2 = R - R/4 = 3R/4$

Now Gravitational potential at point P

$$V_P = -\frac{GM}{R/4} - \frac{9(GM)}{3R/4}$$

$$= \frac{-16GM}{R}$$

26. $$T = \frac{2\pi}{\sqrt{GM}} r^{3/2} \Rightarrow T^2 = \frac{4\pi^2R^3}{G\left(\frac{4}{3}\pi R^3 d\right)} \quad (r = R)$$

$$T^2 = \frac{3\pi}{Gd}$$

27. $$\boxed{V_\infty = \sqrt{V^2 - V_e^2}}$$

Given than

$V = 2V_e$

So,

$$V_\infty = \sqrt{(2V_e)^2 - V_e^2}$$

$$V_\infty = \sqrt{3}V_e = 11.2\sqrt{3}\ \text{km/s}$$

28. $$g = \frac{4}{3}\pi G R \rho$$

$$\rho = \frac{3g}{4\pi G R}$$

29. At Earth surface

$$g = \frac{GM}{R^2} = 9.8\ \text{m/s}^2$$

At given planet

$$m' = \frac{m}{10}$$

$$R' = \frac{R}{2}$$

$$g' = \frac{G\left(\frac{m}{10}\right)}{\left(\frac{R}{2}\right)^2} = 0.4g$$

$$\boxed{g' = 3.92\ \text{m/s}^2}$$

30. Final Energy of satellite

$$TE_f = -\frac{GMm}{2(R+h)} = -\frac{GMm}{2(3R)} = -\frac{GMm}{6R}$$

Initial energy

$$PE_i = -\frac{GMm}{R}$$

Now by COME

$KE_i + PE_i = (KE_f + PE_f)$

$$KE_i - \frac{GMm}{R} = -\frac{GMm}{6R}$$

$$KE_i = -\frac{GMm}{6R} + \frac{GMm}{R}$$

$$\boxed{KE_i = \frac{5}{6}\frac{GMm}{R}}$$

31. escape velocity

$$v_e = \sqrt{\frac{2GM}{R}}$$

$$\frac{v_e}{v_p} = \sqrt{\frac{M_e}{M_p}\cdot\frac{R_p}{R_e}}$$

$$\frac{v}{v_p} = \sqrt{\frac{M_e}{9M_e}\cdot\frac{16Re}{R_e}}$$

$$\frac{v}{v_p} = \frac{4}{3}$$

$$v_p = \frac{3v}{4}$$

32. $\Delta W = m(g_B - g_A)$

$$= 100\left(\frac{g}{\left(1+\frac{h_B}{R}\right)^2} - \frac{g}{\left(1+\frac{h_A}{R}\right)^2}\right)$$

$$= 1000\left(\frac{1}{\left(1+\frac{3}{2}\right)^2} - \frac{1}{(1+2)^2}\right)$$

$$= 1000\left(\frac{4}{25} - \frac{1}{9}\right)$$

$$= 1000\frac{11}{(225)}$$

$$= 40\frac{(11)}{9} = \frac{440}{9} = 48.88$$

$$\simeq 49\ \text{N}$$

Exercise-III (Analytical Questions)

1. After the force disappears, it continue to move in a straight line due to inertia of direction and orbital speed $V_0 = \sqrt{\dfrac{GM}{r}}$.

So, $V_0 \propto \dfrac{1}{\sqrt{r}} \Rightarrow r\uparrow \Rightarrow V_0\downarrow$

2. Work has to be done on a satellite by external agency to shift it in higher orbit. So total energy will not remain conserved.

3. (A) The magnitude of force between 1 Kg bodies is 6.67×10^{-11} N

(B) is correct, if escape velocity for a planet is high, the thermal speeds of lighter gases are less than the escape velocity.

$\therefore$ Lighter gases are not able to escape.

(C) is wrong; friction force is electromagnetic in nature.

(D) is correct.

4. Satellite in circular orbit has constant speed, but not constant velocity and it will have centripetal acceleration.

5. **I.** Gravitational force on satellite due to earth acts always towards the centre of earth.

II. Total mechanical energy remains conserve.

III. Since v changes in magnitude & direction.

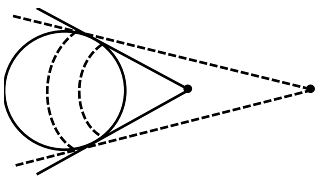
$\therefore$ Linear momentum is not conserved.

6. $r \rightarrow 2r$

(A) Angular Momentum : $L = mvr = m\sqrt{\dfrac{GM}{r}}.r$

$L \propto \sqrt{r} \quad r\uparrow \Rightarrow L\uparrow$

(B)



Area covered will $\uparrow$

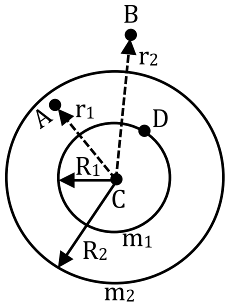
(C) $U = -\dfrac{GMm}{r} = -U_0$

$U' = -\dfrac{GMm}{2r} = -\left(\dfrac{U_0}{2}\right)$

P.E. will $\uparrow$

(D) $KE = \dfrac{GMm}{2r} \Rightarrow$ KE will $\downarrow$

7.



(A) $U_A = -\dfrac{Gm_1}{r_1} - \dfrac{Gm_2}{R_2}$; $U_B = -\dfrac{Gm_1}{r_2} - \dfrac{Gm_2}{r_2}$

$U_A < U_B$

(B) $I_A = \dfrac{Gm_1}{r_1^2}$; $I_B = \dfrac{G(m_1+m_2)}{r_2^2}$

Can't compare as data is insufficient.

(C) $U_C = -\dfrac{Gm_1}{R_1} - \dfrac{Gm_2}{R_2}$; $U_D = -\dfrac{Gm_1}{R_1} - \dfrac{Gm_2}{R_2}$

remains same

(D) $I_A = \dfrac{Gm_1}{r_1^2}$; $I_D = \dfrac{Gm_1}{R_1^2}$

$I_D > I_A$