Circular Motion

SOLUTIONS

Exercise-I (Conceptual Questions)

1. $a_C = \frac{v^2}{r} = \frac{4}{r^2} \Rightarrow v^2 = \frac{4}{r}$

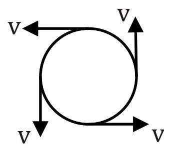
$\Rightarrow \quad v = \frac{2}{\sqrt{r}}$

Momentum $p = mv = \frac{2m}{\sqrt{r}}$

2. Uniform motion $a_t = 0$ and $a_C = \omega^2 r = v\omega$

So acceleration $a_C = v\omega$

3. $\omega = \frac{\theta}{t}$ = const.



– Uniform circular motion

– Magnitude of velocity is constant but the direction of velocity change

4. r = 12 cm ; $n = \frac{7}{100}$ = 0.07 rev./sec

$\because \quad \omega = 2\pi n \quad \therefore \quad v = \omega r = 2\pi nr$

$v = 2 \times \frac{22}{7} \times \frac{7}{100} \times 12 = 5.28$ cm/sec.

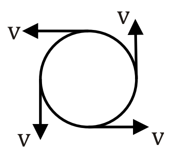
$\simeq$ 5.3 cm/sec.

5. $\omega = 2\pi n = 2\pi \times 2 = 4\pi$ rad/sec

$\because \quad a_t = 0$ (because ω = const.)

$\therefore \quad a_{net} = a_C = \omega^2 r = 16\pi^2 \times 0.25 = 4\pi^2$

6.



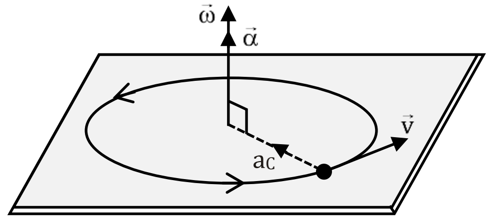
7. $T = m\omega^2 r$,

for constant $r = \frac{T_2}{T_1} = \frac{\omega_2^2}{\omega_1^2} = \frac{n_2^2}{n_1^2}$

$\frac{2T}{T} = \frac{n_2^2}{(5)^2} \quad \Rightarrow \quad n_2^2 = 25 \times 2$

$\Rightarrow \quad n_2 = 7$ r.p.m.

8.



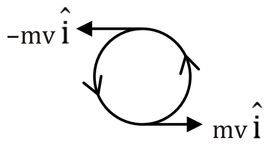
10. $\omega = \frac{d\theta}{dt} = \frac{d(2t^3 + 0.5)}{dt} \Rightarrow 6t^2 + 0$

At t = 2 $\quad \omega$ = 24 rad/s.

11. WD = F.d cos 90° = 0

12. At constant angular velocity, angular acceleration i.e., $\alpha = 0$

13.



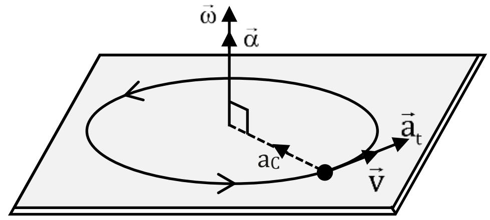
$|\Delta p| = \left|\left(\vec{p_f} - \vec{p_i}\right)\right| = 2mv$

14. Acceleration $= \frac{v^2}{r} = \frac{r^2\omega^2}{r} = r\omega^2$

r = 50 cm $\quad \omega = 2\pi n = 2\pi \times \frac{1}{2}$

acceleration $= \frac{50 \times 4\pi^2}{4}$ = 493 cm/s²

15.



16. Angular velocity (ω) $= 2\pi n = 2\pi \times \frac{100}{60}$

= 10.47 rad/sec

17. $T = \frac{2\pi}{\omega}$, For particle of mass M, $T_1 = \frac{2\pi}{\omega_1}$

For particle of mass m, $T_2 = \frac{2\pi}{\omega_2}$

since $T_1 = T_2$

$\frac{2\pi}{\omega_1} = \frac{2\pi}{\omega_2} \quad$ i.e $\frac{\omega_1}{\omega_2} = 1$

18. $\omega = \dfrac{2\pi}{60\times60} = \dfrac{\pi}{1800}$ rad/sec.

19. a = $\sqrt{a_T^2 + a_c^2} = \sqrt{(2)^2 + \dfrac{(30)^2}{500}}$ = 2.7 m/s²

20. $\theta = 90°$, WD = 0

21. v = $\omega r = 2\pi \times 3 \times 0.1 = 1.88$ m/s

$a_c = \dfrac{v^2}{r} = \dfrac{(1.88)^2}{0.1}$ = 35.5 m/s²

$T = \dfrac{mv^2}{r} = 1\times35.5 = 35.5N$

22. Distance s = $2(2\pi r)$ = 80m

by $v^2 = u^2 + 2a_T s$

$a_T = \dfrac{v^2-u^2}{2s} = \dfrac{(80)^2-0}{2(80)}$

$\boxed{a_T = 40 m/s^2}$

23. v = $r\omega \Rightarrow$ v = 0.5 × 70 = 35 m/s

24. $\omega = \dfrac{2\pi}{T} = 2\times\pi\times\dfrac{22}{44} = \pi$

centripetal acc. $a_C = \omega^2 r = \pi^2 \times 1 = \pi^2$ m/s²

(towards the center)

25. $\omega_0 = 2\pi n = 2\pi \times 10 = 20\pi$ rad/sec

$\omega = 0$

t = 120 sec

$\omega = \omega_0 + \alpha t$

$0 = 20\pi + \alpha \times 120$

$\alpha = -\dfrac{\pi}{6}$ rad/s²

26. a = αR

27. In uniform speed angular acceleration (α) = 0

28. $F_1 = \dfrac{mv^2}{r}$, $F_2 = \dfrac{m(3v)^2}{(3r)} = \dfrac{9mv^2}{3r} = 3F_1$

31. $T = m\omega^2 r$

$\omega_{max}^2 = \dfrac{T_{max}}{mr} = \dfrac{100}{100\times10^{-3}\times0.1}$

$\Rightarrow \quad \omega_{max}$ = 100 rad/sec

32. $F_C = m\omega^2 r = m(2\pi n)^2 r$

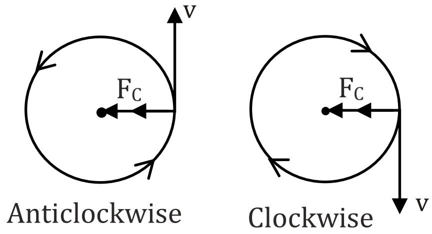
$\because$ Angular frequency n = constant

$\therefore$ $F_C \propto r$ so $\dfrac{F_{C_2}}{F_{C_1}} = \dfrac{2r}{r}$

$\Rightarrow \quad F_{C_2} = 2F_{C_1} = 2F$

33. $F_C = \dfrac{mv^2}{r} = \dfrac{0.5\times4\times4}{0.4}$ = 20N

34.



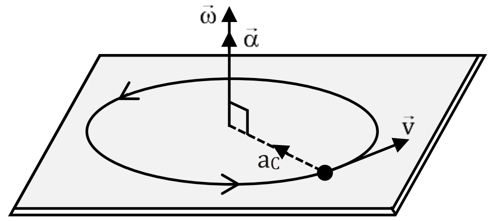
Centripetal force always toward centre.

35. In uniform circular motion.

magnitude of v = constant

so $a_t = 0$; $a_r \neq 0$

36.



37. $F_C = \dfrac{mv^2}{r} = \dfrac{mr^2\omega^2}{r} = mr\omega^2$ $\quad T_{max}$ = 10 N

At Balancing point $T_{max} = F_{cp} \Rightarrow 10 = mr\omega^2$

$\omega^2 = 400$

$\Rightarrow \quad \omega$ = 20 rad/sec.

38. $F_{gravitation} = F_{centripetal}$

$F_{cen} = mr\omega^2$

$= 6 \times 10^{24} \times 1.5 \times 10^8 \times 10^3 \times 4 \times 10^{-14}$

$= 36 \times 10^{21}$ N

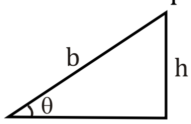
39. $F = \dfrac{mv^2}{r} = \dfrac{500\times(10)^2}{50}$ = 1000 N

40. $F_C = \dfrac{mv^2}{r}$

$F_C' = \dfrac{mv'^2}{r'} = \dfrac{m(2v)^2}{r} = 4F_C \qquad (r' = r)$

42. Lack of centripetal forces causes overturning.

43.

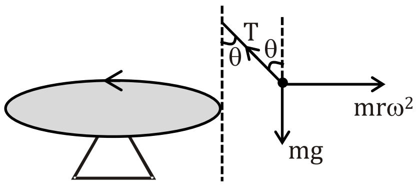


$\tan\theta \simeq \sin\theta = \dfrac{h}{b}$

$\therefore \quad \dfrac{h}{b} = \dfrac{v^2}{Rg}$

$\therefore \quad h = \dfrac{v^2 b}{Rg}$

44.

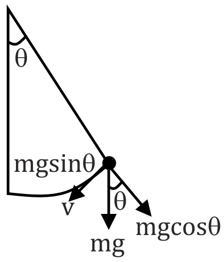


$T\cos\theta = mg$

$T\sin\theta = mr\omega^2$

$\therefore \quad \tan\theta = \dfrac{r\omega^2}{g}$

45. Tension at θ displacement



$T = mg\cos\theta + \dfrac{mv^2}{\ell}$

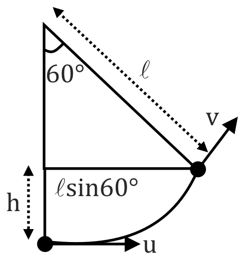
at extreme position v = 0

$\therefore \quad T = mg\cos\theta$

(at extreme position)

46. $h = \ell - \ell\cos 60° = \ell - \ell \times \dfrac{1}{2}$

$= \ell/2$



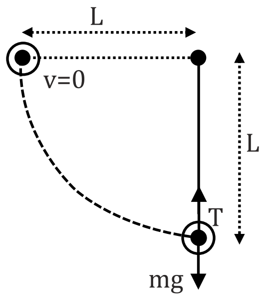
by law of conservation of mechanical energy

$\dfrac{1}{2}mu^2 = \dfrac{1}{2}mv^2 + mgh$

$\therefore \quad v^2 = u^2 - 2g\dfrac{\ell}{2}$

$v = \sqrt{(3)^2 - 10 \times \dfrac{1}{2}} = 2\ m/s$

47. $0 + mgh = \dfrac{1}{2}mv^2 + 0$



$v^2 = 2gL$

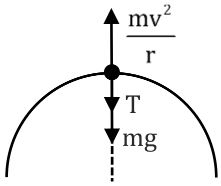
$\Rightarrow \quad v = \sqrt{2gL}$

$T = mg + \dfrac{mv^2}{L}$

$T = mg + \dfrac{m}{L} \times 2gL$

$T = 3mg$

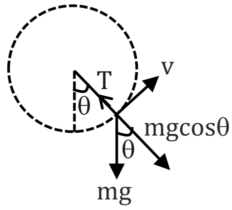
48. $T = \dfrac{mv^2}{\ell} - mg$



$= \dfrac{1 \times (4)^2}{1} - 1 \times 10 = 6\,N$

49. At highest point minimum possible value of tension is zero.

50. $T = \dfrac{mv^2}{r} + mg\cos\theta$



as θ increases cosθ and v both decreases

$\because$ v maximum at lowest point

$\therefore$ T maximum at lowest point

51. $\omega = \sqrt{\dfrac{T - mg}{mr}} = \sqrt{\dfrac{30 - 0.5 \times 10}{0.5 \times 2}} = 5\ rad/sec$

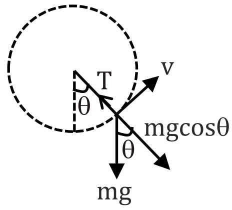
52. For looping the loop minimum velocity at top point $v = \sqrt{gL}$

time taken by particle

$t = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2 \times 2L}{g}} = 2\sqrt{\dfrac{L}{g}}$

$\therefore$ horizontal range $x = vt = \sqrt{gL} \times 2\sqrt{\dfrac{L}{g}} = 2L$

53. $T = \dfrac{mv^2}{r} + mg\cos\theta$

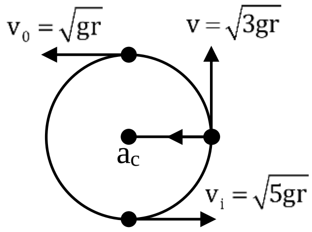


as θ increases cosθ and v both decreases hence for θ = 60°,

T will be less i.e., $T_1 > T_2$

54. $a_C = \dfrac{v^2}{r} = \dfrac{3gr}{r}$

$a_C = 3g$



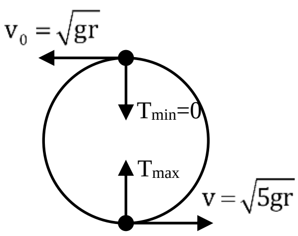
55. $T_{max} = mg + \dfrac{mv^2}{r} = mg + \dfrac{m}{r} \times 5gr = 6mg$

$T_{min} = 0$

$T_{max} - T_{min} = 6mg$

$\therefore \quad 6mg = 2g$

$m = \dfrac{1}{3}kg$



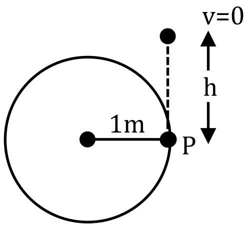
56. Acc to COME at point P

KE of stone = PE

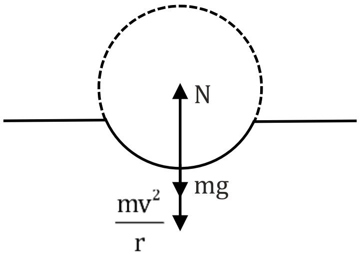
gained by stone.

$\frac{1}{2}m(5)^2 = m \times 10 \times h$

$h = \frac{25}{20} = 1.25m$



57. $N = mg + \frac{mv^2}{r}$



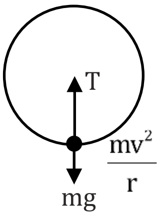
58. $\frac{1}{2}m(5Rg) = mgh$

$h = \frac{5R}{2}$

$R = \frac{2}{5} \times h = 2$ cm

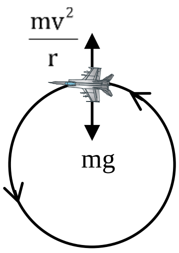
59. At position B tension is maximum ;

$T = mg + \frac{mv^2}{r}$



60. $\frac{mv^2}{r} = mg$

$\rightarrow v = \sqrt{gr}$

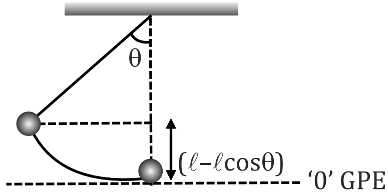


61. $T - mg = \frac{mv^2}{r}$

$\rightarrow \quad 52 - 2 = \frac{0.2v^2}{0.1}$

$\rightarrow \quad v = 5$ m/s

62.



$\therefore \quad mg\ell(1-\cos\theta) = \frac{1}{2}mv^2$

$v = \sqrt{2g\ell(1-\cos\theta)}$

Exercise-II (Previous Years Questions)

1. $(F_C)_{heavier} = (F_C)_{lighter}$

$\Rightarrow \frac{2mv^2}{(r/2)} = \frac{m(nv)^2}{r}$

$\Rightarrow n^2 = 4 \Rightarrow n = 2$

3. By using work-energy theorem, W=ΔKE

$\Rightarrow (ma_t)(4\pi R) = \frac{1}{2}mv^2 \Rightarrow a_t = \frac{\left(\frac{1}{2}mv^2\right)}{4\pi mR}$

$\Rightarrow a_t = \frac{8\times10^{-4}}{4\times3.14\times10\times10^{-3}\times6.4\times10^{-2}} = 0.1\ m/s^2$

OR

$\frac{1}{2}mv^2 = KE \Rightarrow \frac{1}{2}\left(\frac{10}{1000}\right)v^2 = 8\times10^{-4}$

$\Rightarrow v^2 = 16\times10^{-2} \Rightarrow v = 4\times10^{-1} = 0.4$ m/s

Now,

$v^2 = u^2 + 2a_ts \qquad (s = 4\pi R)$

$\Rightarrow \frac{16}{100} = 0^2 + 2a_t\left(4\times\frac{22}{7}\times\frac{6.4}{100}\right)$

$\Rightarrow a_t = \frac{16}{100}\times\frac{7\times100}{8\times22\times6.4} = 0.1\ m/s^2$

4. When minimum speed of body is $\sqrt{5gR}$, then no matter from where it enters the loop, it will complete full vertical loop.

5. $\frac{v^2}{Rg} = \tan(\phi+\theta) = \frac{\tan\phi+\tan\theta}{1-\tan\phi\tan\theta} = \left(\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}\right)$

$\Rightarrow \quad v = \sqrt{Rg\left[\frac{\mu_s+\tan\theta}{1-\mu_s\tan\theta}\right]} \quad (\mu_s = \tan\phi)$

OR

Check by dimensions.

6. Centripetal acceleration $= \frac{v^2}{R} = a\cos30°$

$\Rightarrow v = \sqrt{aR\cos30°} = \sqrt{15\times2.5\times\frac{\sqrt{3}}{2}} = 5.7$m/s

7. $\vec{r}=\cos\omega t\,\hat{x}+\sin\omega t\,\hat{y}$

$\vec{v}=-\omega\sin\omega t\,\hat{x}+\omega\cos\omega t\,\hat{y}$

$\vec{a}=-\omega^2\cos\omega t\,\hat{x}-\omega^2\sin\omega t\,\hat{y}$

$\Rightarrow\ \vec{v}.\vec{r}=0$

velocity is perpendicular to $\vec{r}$.

$\Rightarrow\ \vec{a}=-\omega^2(\vec{r})$

acceleration is directed towards the origin.

8. Net force on the particle in uniform circular motion is centripetal force, which is provided by the tension in string.

9. To complete a vertical circle, speed at A should be $v_A=\sqrt{5gR}=\sqrt{\dfrac{5gD}{2}}$

using energy conservation mgh = $\dfrac{1}{2}mv_A^2$

$h=\dfrac{1}{2}\dfrac{v_A^2}{g}=\dfrac{1}{2}\dfrac{5g}{g}\dfrac{D}{2}=\dfrac{5D}{4}$

$\left(R=\dfrac{D}{2}\right)$

10. $f_L=\mu N=\mu mr\omega^2$

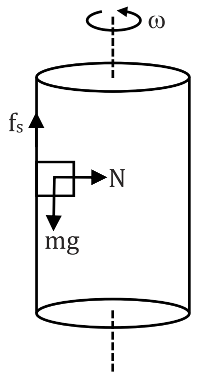
$f_s = mg$

As $f_s \le f_L$

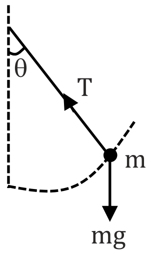
$\Rightarrow mg \le \mu mr\omega^2$

$\Rightarrow\ \omega \ge \sqrt{\dfrac{g}{\mu r}}$

$\Rightarrow \omega_{min} = 10$ rad/s



11. $T - mg\cos\theta = \dfrac{mv^2}{R}$



T will be maximum when θ = 0°,
When mass is at lowest point.

12. $T_A = T_B$

$\Rightarrow\ \dfrac{2\pi}{\omega_A}=\dfrac{2\pi}{\omega_B}$

$\Rightarrow\ \dfrac{\omega_A}{\omega_B}=1:1$

13. $\theta = (2\pi n)$, $\omega_0 = 0$, $\omega = V_0/r$

$\alpha=\dfrac{\omega^2-\omega_0^2}{2\theta}==\dfrac{(V_0/r)^2-0}{2(2\pi n)}=\dfrac{V_0^2}{4\pi nr^2}$

14. $\alpha=\dfrac{\Delta\omega}{\Delta t}=\dfrac{(1200-360)(2\pi)}{60\times14}$

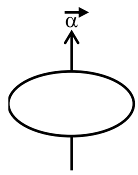
$\alpha = 2\pi$ rad/s²

15. $T-mg=\dfrac{mv^2}{r}$

$\Rightarrow\ T-mg=\dfrac{m(7gr)}{r}$

$\Rightarrow$ T = 8 mg

17.



Along the axis of rotation

18. $F_S = ma$

$f_L = ma_{max}$

μ mg = ma_{max}

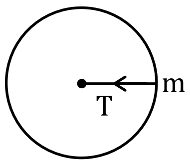
$a_{max} = \mu g$

= 0.15(10)

= 1.5 m/s²

19. Direction of velocity and centripetal acceleration changes continuously so $\vec{v}$ is not constant and $\vec{a}$ is not a constant.

20. $F_{cp} = ma_{cp}$



$F_{cp} = m\omega^2 r$

$T = m\omega^2 r$

Now speed becomes '2ω'

$T' = m(2\omega)^2 r$

$T' = 4m\omega^2 r$

T' = 4T

21. In uniform circular motion direction of velocity and acceleration keeps on changing

22. $T = m\omega^2 r$

$T \propto \omega^2$

$$\frac{T_2}{T_1} = \left(\frac{\omega_2}{\omega_1}\right)^2$$

$$\Rightarrow \frac{4T}{T} = \left(\frac{\omega_2}{10}\right)^2$$

$\Rightarrow \omega_2 = 20$ rpm

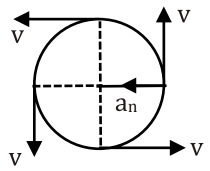
23. $\omega = \dfrac{2\pi}{T} \quad x = \dfrac{2\pi}{T}t$

$\dfrac{\omega}{x} = \dfrac{1}{t}$ = const.

$\therefore$ 1 minute

Exercise-III (Analytical Questions)

4.



a_t : tangential acceleration

a_n : Normal acceleration

$a_t = 0$

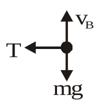
$a_n \neq 0$; $a_n = \dfrac{v^2}{r}$

For (A) $v_B^2 = v_A^2 - 2g\ell = 9g\ell - 2g\ell = 7g\ell$

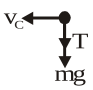
$\Rightarrow v_B = \sqrt{7g\ell}$

For (B) $v_C^2 = v_A^2 - 2g(2\ell) = 5g\ell$

$\Rightarrow v_C = \sqrt{5g\ell}$

For (C) 

$T_B = \dfrac{mv_B^2}{\ell} = 7mg$

For (D) 

$T_C + mg = \dfrac{mv_C^2}{\ell}$

$\Rightarrow T_C = 4mg$

5. As $V_H^2 = V_L^2 - 2gh$ where $h = 2r$

$\therefore V_H^2 = V_L^2 - 4gr$

Now, $V_L = 2V_H$ (given)

$\therefore V_H = \sqrt{\dfrac{4gr}{3}}$ & $V_L = 2V_H = 2\sqrt{\dfrac{4gr}{3}}$

$T_H = T_{min} = \dfrac{mv_H^2}{r} - mg = \dfrac{mg}{3}$

$T_L = T_{max} = \dfrac{mv_L^2}{r} + mg = \dfrac{19}{3}mg$

6. As $v = 2t$ and let radius of circular path is r then

$a_T = \dfrac{dv}{dt} = 2$

$a_r = \dfrac{v^2}{r} = \dfrac{4t^2}{r}$

Therefore,

$$a = \sqrt{a_T^2 + a_r^2}$$

$$a = \sqrt{4 + \frac{16t^4}{r^2}}$$

$\therefore$ (B) and (D) are correct.