04 Laws of Motion and Friction

Force

According to Aristotle, a constant continuous force is required to keep a body in uniform motion. This is called Aristotle's Fallacy.

This cause was first understood in the late 1600s by Sir Isaac Newton.

Any push or pull which either changes or tends to change the state of rest or of uniform motion (constant velocity) of a body is known as force.

Effects of force: -

A non-zero resultant force may produce the following effects on a body:

(i) It may change the speed of the body.

(ii) It may change the direction of motion.

(iii) It may change both the speed and direction of motion.

(iv) It may change the size and the shape of the body.

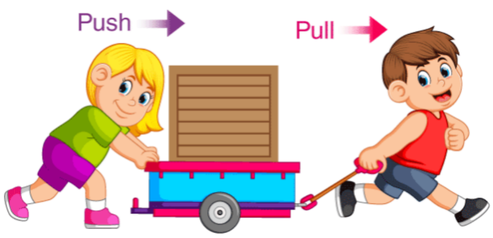
Units for measurement of force: -

Absolute units: (i) S.I $\Rightarrow$ Newton (N) (ii) C.G.S $\Rightarrow$ Dyne

Conversion: $1N = 10^5$ Dyne

Force is a vector quantity, with magnitude and direction.

Force: Push and Pull: -



For instance, the force has been defined as an interaction that changes the motion of an object if unopposed. When this statement is examined closely, we see the role of push-pull in this. A force that changes the direction of an object towards you, that would be a pull. On the other hand, if it moves away, it is a push. Sometimes, force is simply defined as a push or pull upon an object resulting from the object's interaction with another object. Hence, any kind of force is basically a push or a pull. Spring and elastic are also types of forces. The moment you push against it, it tends to resist and react or springs back with the same magnitude.

Push and Pull Examples

Push is defined as the force that is responsible for an object to move from the state of rest.

Examples of push:

- Pushing the trolley.
- Pushing of the car when it breaks down.
- Pushing the table from one place to another.

The pull is defined as the force that is responsible for an object to move from the state of rest but in the opposite direction when compared to the push.

Examples of pull:

- Pulling the curtain.
- Dragging the box.
- Opening of the door.

Whenever we consider a force in a given scenario, it can act as an internal as well as an external force depending on the system we have considered. This is how we have introduced this topic from the basic and it emphasizes on the fact that we have to be careful about the system chosen whenever we are labelling a force as an internal or an external force.

Balanced and Unbalanced Forces: -

Balanced Forces

If the resultant force of all the forces acting on a body sums up to zero, then the forces acting on the body are known as balanced forces. Let us look at some examples of balanced forces and understand how the body's state of motion remains unaffected.

Unbalanced Forces

If force or forces acting on an object change the state of rest or uniform motion then it is called unbalanced forces.

Practically anything that moves is a result of the exertion of unbalanced forces on it. If you kick a football and it moves from one place to another, it means that the unbalanced troops are acting upon it.

The ball moves from one place to another after it's kicked.

Laws of Motion

Newton formulated three laws, which we call Newton's laws of motion.

1. Newton's First Law of Motion or Law of Inertia
2. Newton's Second Law of Motion
3. Newton's Third Law of Motion

Newton's First law of motion

If a body is at rest, then it remains at rest and if it is moving with constant velocity then it continues to move with constant velocity until or unless it is acted upon by an external force.

Inertia

Inertia is property of a body by virtue of which it opposes any change in its state of rest or state of motion. Mass of a body is quantitative or numerical measure of a body's inertia.

Inertia $\propto$ Mass

- Mass of a body is quantitative or numerical measure of a body's inertia.
- Larger the inertia of a body, more will be its mass.

Inertia of Rest

It is the inability of a body to change its state of rest by itself.

Example:

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly, the passengers sitting inside tends to fall backwards.
- When a horse starts off suddenly, its rider falls backwards.
- A coin is placed on cardboard and this cardboard is placed over a tumbler such that coin is above the mouth of tumbler. Now if the cardboard is removed with a sudden jerk, then the coin falls into the tumbler.
- The dust particles in a blanket fall off when it is beaten with a stick.

Inertia of Motion

It is the inability of a body to change its state of uniform motion by itself.

Example:

- When a bus or train stops suddenly, the passengers sitting inside lean forward.
- A person who jumps out of a moving train may fall in the forward.
- A bowler runs with the ball before throwing it, so that his speed of running gets added to the speed of the ball at the time of throw.
- An athlete runs through a certain distance before taking a long jump because the velocity acquired during the running gets added to the velocity of athlete at the time of jump and hence he can jump over a longer distance.
- A ball is thrown in the upward direction by a passenger sitting inside a moving train then the ball will fall: -
- back to the hands of the passenger, if the train is moving with constant velocity.
- ahead of the passenger, if the train is retarding (slowing down)
- behind of the passenger, if the train is accelerating (speeding up)

Inertia of Direction

It is the inability of a body to change its direction of motion by itself.

Example:

- When a straight running car turns sharply, the person sitting inside feels a force radially outwards.
- Rotating wheels of vehicle throw out mud, mudguard fitted over the wheels prevent this mud from spreading.
- When a knife is pressed against a grinding stone, the sparks produced move in the tangential direction.

Linear Momentum

The total amount of motion possessed by a moving body is known as the momentum of the body.

- It is the product of the mass and velocity of a body.
- It is a vector quantity whose direction is along instantaneous velocity.

  Momentum $\vec{p} = m\vec{v}$ Momentum = Mass × Velocity
- The equation illustrates that momentum is directly proportional to an object's mass and directly proportional to the object's velocity.
- The direction of the momentum vector is the same as the direction of the velocity vector.

  SI unit : kg-m/s, N-s ; Dimension : $[MLT^{-1}]$

Illustration 1:

A block having mass 2 kg moving with velocity $\vec{v} = 2\hat{i} + 3\hat{j} + \hat{k}$ m/s. Find momentum of the block.

Solution:

Linear momentum is $\vec{P} = m \times \vec{v} = 2 \times (2\hat{i} + 3\hat{j} + \hat{k}\ \ m/s)$

The magnitude of momentum is $|P| = \sqrt{4^2 + 6^2 + 2^2}\ kg - m/s = 2\sqrt{14}\ kg - m/s$

Change in Momentum

Change in any physical quantity can be calculated by

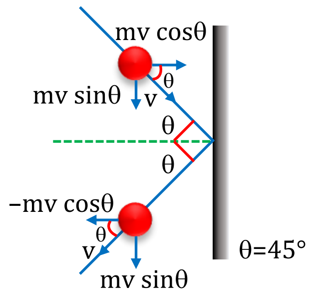
Change = Final – Initial

Similarly change in momentum can be calculated by

$\Delta\vec{p} = \vec{p}_f - \vec{p}_i = m\vec{v}_f - m\vec{v}_i = m\Delta\vec{v}$

Illustration 2:

A ball of 0.20 kg hits a wall with a velocity of 25 m/s at an angle of 45° with horizontal. if the ball rebounds at 90° to the direction of incidence with same speed, calculate the magnitude of change in momentum of the ball.



Solution:

Initial momentum $\vec{P}_i = m\times\vec{v}_i = m\times\vec{v}(\cos 45°)\hat{i} + mv\sin 45°\left(-\hat{j}\right)$

Final momentum $\vec{P}_f = m\times\vec{v}_f = m\times\vec{v}(\cos 45°)\left(-\hat{i}\right) + mv\sin 45°\left(-\hat{j}\right)$

Change in momentum = (–mv cos45°) – (mv cos45°) = –2mv cos45°

$$\left|\Delta\vec{p}\right| = 2mv\cos 45° = 2\times0.2\times25\times\frac{1}{\sqrt{2}} = 5\sqrt{2}\,\text{N}-\text{s}$$

Newton's Second Law of Motion

According to this law, "rate of change of momentum of any system is directly proportional to the applied external force".

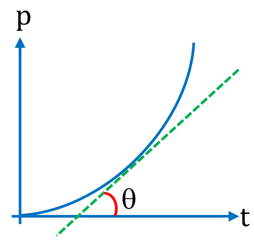
$$\vec{F}_{net} \propto \frac{d\vec{p}}{dt} \Rightarrow \vec{F} = k\frac{d\vec{p}}{dt} \quad \text{[In S.I. units k = 1]}$$

$$\vec{F}_{net} = \frac{d\vec{p}}{dt}$$

Change in momentum takes place in the direction of the applied force.

$$\vec{F}_{net} = \frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = \frac{md(\vec{v})}{dt} + \frac{\vec{v}dm}{dt}$$

If m is constant $\left(\frac{dm}{dt} = 0\right) \Rightarrow \vec{F}_{net} = \frac{md(\vec{v})}{dt} = m\vec{a}$



$$\vec{F}_{net} = \frac{d(m\vec{v})}{dt} = \frac{md(\vec{v})}{dt} + \frac{\vec{v}dm}{dt}$$

If $\vec{v}$ is constant $\left(\frac{d\vec{v}}{dt} = 0\right)$

$\vec{F}_{net} = \frac{\vec{v}dm}{dt}$ (variable mass system e.g. conveyor belt, rocket)

The slope of momentum-time graph is equal to the force on the particle at that instant.

$$F = \frac{dp}{dt} = \tan\theta = \text{slope}$$

Illustration 3:

A cricket ball of mass 150 g is moving with a velocity of 12 m/s and is hit by a bat so that the ball gets turned back with a velocity of 20 m/s. If the duration of contact between the ball and bat is 0.01 s, find the change in momentum and required force?

Solution:

According to given problem change in momentum of the ball

$\Delta p = P_f - p_i = m(v - u) = 150 \times 10^{-3}\,[20 - (-12)] = 4.8$ N-s

force in case of change in momentum = $\frac{\Delta p}{\Delta t} = \frac{4.80}{0.01} = 480\,\text{N}$

Illustration 4:

Momentum of a particle is given by p = $at^2 - 2bt$.

Find the time at which force on particle becomes zero (a & b are constants)

Solution:

$$F = \frac{dp}{dt} = 0 = 2at - 2b = 0 \Rightarrow t = \frac{b}{a} \text{ units}$$

Basic problems based on Newton's Second Law of Motion

Illustration 5:

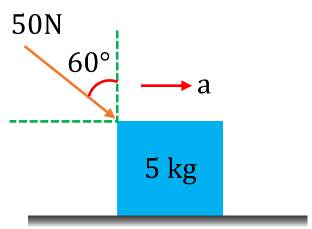
A force $\vec{F} = (6\hat{i} - 8\hat{j} + 10\hat{k})N$ produces an acceleration of $\sqrt{2}\ m/s^2$ in a body. Calculate the mass of the body.

Solution:

From Newton's II^{nd} law $|\vec{F}| = |m\vec{a}| = ma \Rightarrow$ Acceleration $a = \frac{|\vec{F}|}{m} \Rightarrow m = \frac{|\vec{F}|}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{\sqrt{2}} = 10\,kg$

Illustration 6:

A force of 50 N acts on a block in the direction as shown in figure. The block is of mass 5kg, resting on a smooth horizontal surface. Find out the acceleration of the block.



Solution:

horizontal component of the force $= 50\sin 60° = \frac{50\sqrt{3}}{2}$

acceleration of the block,

$$a = \frac{\text{component of force in the direction of acceleration}}{\text{mass}} = \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3}\ m/s^2$$

Illustration 7:

If force F = 40 – 30t, then impulse in interval [0, t]: -

Solution:

$$I = \int F dt = \left[40t - \frac{30t^2}{2}\right]_0^t = 40t - 15t^2$$

Illustration 8:

A force of 40 N acts on a body of mass 10 kg for 10 seconds. The change in its momentum is

Solution:

$$a = \frac{F}{m} = 4\ m/s^2 \text{ and } v = u + at$$

$v - u = 4(10) \Rightarrow v - u = 40 \Rightarrow \Delta P = m(v-u) = 10(40) = 400$ kg-m/s

Illustration 9:

A player catches a ball of 200 g moving with a speed of 10 m/s. If the time taken to complete the catch is 0.20 s, the force exerted on the player's hand is: -

Solution:

$$F_{avg} = \frac{\Delta p}{\Delta t} = \frac{200 \times 10^{-3} \times 10}{0.20} = 10N$$

Impulse and Average Force

Impulse

When a large force is applied on a body for a very short interval of time, then the product of force and time interval is known as impulse.

$$d\vec{I} = \vec{F}dt = d\vec{p} \qquad \left[\because \frac{d\vec{p}}{dt} = \vec{F}\right]$$

Unit of impulse = N-s or kg-m/s.

Dimension of impulse [F] [t] = [m] [a] [t] = $[M^1L^1T^{-2}T^1] = [M^1L^1T^{-1}]$

Case-I : If this force is acting from time t_1 to t_2 , then integrating the above equation, we get –

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p}$$

Impulse = Change in momentum

Case-II : If constant or average force acts on a body, then :-

$$\vec{I} = \int_{t_1}^{t_2} \vec{F}_{avg}\ dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p} \Rightarrow \vec{I} = \vec{F}_{avg}\int_{t_1}^{t_2} dt = \int_{\vec{p}_1}^{\vec{p}_2} dt = \int_{\vec{p}_1}^{\vec{p}_2} d\vec{p}$$

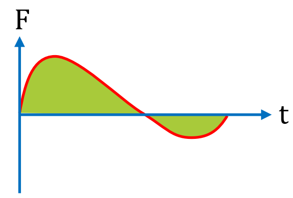
$$\vec{I} = \vec{F}_{avg}[t]_{t_1}^{t_2} = [\vec{p}]_{\vec{p}_1}^{\vec{p}_2} \Rightarrow \vec{I} = \vec{F}_{avg}(t_2 - t_1) = (\vec{p}_2 - \vec{p}_1) \Rightarrow \vec{I} = \vec{F}_{avg}\Delta t = \Delta\vec{p}$$

Case-III : Impulse from force time graph :

Area under the force time graph represents impulse or change in momentum.

e.g.

$$I = \Delta p = \int F dt = \text{Area under } F - t \text{ graph}$$



Average Force

Average force acting on a body in a given time interval can be calculated by

$$\frac{d\vec{p}}{dt} = \vec{F} \Rightarrow \vec{F}_{avg} = \frac{\Delta\vec{p}}{dt}$$

As $F_{avg} = \frac{\Delta p}{\Delta t}$ therefore, for a certain momentum change if the time interval is increased, then the average force exerted on body will decrease.

Examples :

(i) A cricketer lowers his hands while catching a ball so as to increase the time interval of momentum change consequently the average reaction force on his hands decreases. So he can save himself from getting hurt.

(ii) Shockers are provided in vehicles to avoid jerks.

(iii) Buffers are provided in bogies of train to avoid jerks.

(iv) A person jumping on a hard cement floor receives more injury than a person jumping on muddy or sandy road.

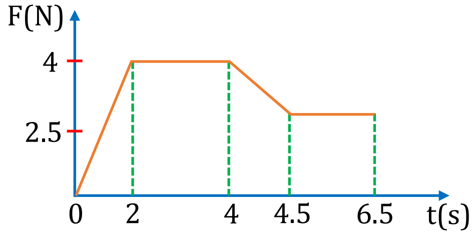
Illustration 10:

A football of mass 0.8 kg coming towards player with a velocity of 5 m/s. Player hit straight back the football with velocity 10 m/s. If the contact time is 0.02 second then the average force involved is: -

Solution:

$$F_{avg} = \frac{\Delta p}{\Delta t} = \frac{0.8(10-(-5))}{0.02} = 600\text{N}$$

Illustration 11:



A body of mass 3 kg has an initial speed 4 ms^{-1}. A force acts on it for some time in the direction of motion. The force-time graph is shown in figure. The final speed of the body is : -

Solution:

Δp = Area under F-t graph

$m(v-u) = 4 + 8 + 1.625 + 5 \Rightarrow 3(v-4) = 18.625 \Rightarrow v = 10.208$ m/s

Impulse Momentum Theorem

$$d\vec{I} = \vec{F}dt = d\vec{p}\left[\because \frac{d\vec{p}}{dt} = \vec{F}\right]$$

Impulse-momentum theorem states that the impulse applied to an object will be equal to the change in its momentum.

This theorem can be proven from newton's law. Signifies instantaneous rate of change of momentum hence the impulse is equal to change in momentum.

Illustration 12:

A hammer of mass 1 kg moving with a speed of 6 m/s strikes a wall and comes to rest in 0.1 s. Calculate

(a) Impulse of the force

(b) Average retarding force that stops the hammer

(c) Average retardation of the hammer

Solution:

(a) Impulse = F × Δt = m(v – u) = 1 (0 – 6) = – 6 N-s

(b) Average retarding force that stops the hammer $F = \frac{\text{Impulse}}{\text{time}} = \frac{6}{0.1} = 60\,N$

(c) Average retardation $a = \frac{F}{m} = \frac{60}{1} = 60 m/s^2$

Conservation of Linear Momentum

According to Newton's Second law, if net external force on the system is zero, then linear momentum of the system remains conserved. According to Newton's Second Law.

$$\vec{F}_{ext} = \frac{d\vec{p}}{dt}$$

If $\vec{F}_{ext} = \vec{0} \Rightarrow \frac{d\vec{p}}{dt} = \vec{0} \Rightarrow \vec{p} = \text{constant} \Rightarrow \vec{p}_{in} = \vec{p}_{final}$

for two particles system $\vec{p}_1 + \vec{p}_2 = \text{constant}$

$$\Delta\vec{p}_1 + \Delta\vec{p}_2 = 0$$

$\boxed{\Delta\vec{p}_1 = -\Delta\vec{p}_2}$ (change in momentum of I^{st} particle = – change in momentum of II^{nd} particle)

Examples of Law of Conservation of Momentum

Following are the examples of law of conservation of momentum:

1. Air-filled balloons 2. System of gun and bullet 3. Motion of rockets

Illustration 13:

Two cars with masses 5 kg and 10 kg respectively are moving towards each other collides and comes to rest. Initially a car having the mass 10 kg moves towards the east with a velocity of 5 ms^{-1}. Find initial velocity of the car with mass 5 kg with respect to ground.

Solution:

Given, $m_1 = 5kg;\ m_2 = 10kg;\ v_2 = 5\ m/s$

$v_1 = ?$

We know from the law of conservation of momentum that,

$\vec{P}_{final} = 0$, as the cars comes to rest after collision

$\vec{P}_{final} = \vec{P}_{initial}$ (Considering both cars as single system)

$\vec{P}_{final} = \vec{p}_1 + \vec{p}_2 = m_1.\vec{v}_1 + m_2.\vec{v}_2$

$0 = 5\ kg.v_1 + 10\ kg.\ 5\ m.s^{-1} \Rightarrow 0 = 5\ kg.v_1 + 50 kg.m.s^{-1} \Rightarrow v_1 = -10\ m.s^{-1}$

BEGINNER'S BOX-1

1. A force of 72 dynes is inclined at an angle of 60° to the horizontal, find the acceleration in a mass of 9 g which moves under the effect of this force in the horizontal direction. **PLM208**

2. A constant retarding force of 50N is applied to a body of mass 20 kg moving initially with a speed of 15 m/s. What time does the body take to stop? **PLM209**

3. A constant force acting on a body of mass 3 kg changes its speed from 2 m/s to 3.5 m/s in 25 s in the direction of the motion of the body. What is the magnitude and direction of the force? **PLM210**

4. A body of mass 5kg is acted upon by two perpendicular forces of 8N and 6N, find the magnitude and direction of the acceleration. **PLM211**

5. A ball of mass 1 kg dropped from 9.8 m height, strikes the ground and rebounds to a height of 4.9 m. If the time of contact between ball and ground is 0.1 s, then find impulse and average force acting on ball. **PLM212**

6. A machine gun fires a bullet of mass 50 gm with velocity 1000 m/s. If average force acting on gun is 200 N then find out the maximum number of bullets fired per minute. **PLM213**

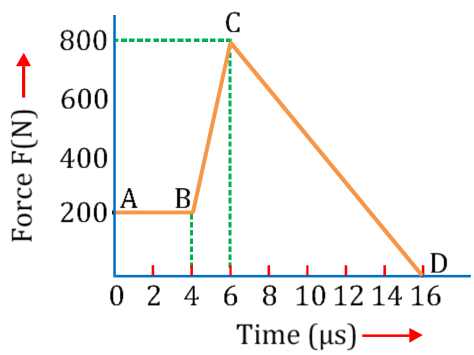
7. A machine gun has a mass of 20 kg. It fires 35 g bullets at the rate of 400 bullets per minute with a speed of 400m/s. What average force must be applied to the gun to keep it in position? **PLM214**

8. A force of 10N acts on a body for 3 μs. If mass of the body is 5 g, calculate the impulse and the change in velocity. **PLM215**

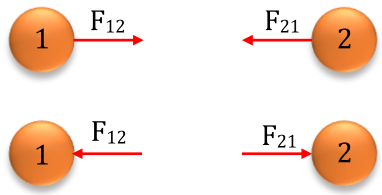
9. A body of mass 0.25 kg moving with velocity 12 m/s is stopped by applying a force of 0.6 N. Calculate the time taken to stop the body. Also calculate the impulse of this force. **PLM216**

10. A batsman deflects a ball by an angle of 90° without changing its initial speed, which is equal to 54 km/hr. What is the impulse imparted to the ball? (Mass of the ball is 0.15 kg) **PLM217**

11. The magnitude of the force (in newton) acting on a body varies with time t (in microsecond) as shown in fig. AB, BC and CD are straight line segments. Find the magnitude of the total impulse of the force (in N–s) on the body from t = 4 μs to t = 16 μs. **PLM218**



Newton's Third Law of Motion



According to Newton's third law, to every action, there is always an equal (in magnitude) and opposite (in direction) reaction. This law is also known as action-reaction law.

Here, $\vec{F}_{12}$ (force on first body due to second body) is equal in magnitude and opposite in direction to (force on second body due to first body).

$$\boxed{\vec{F}_{12} = -\vec{F}_{21}}$$

The force between two objects 1 and 2 are equal and opposite, whether they are attractive or repulsive.

Important points about Newton's III law

(i) We cannot produce a single isolated force in nature. Forces are always produced in action-reaction pair.

(ii) There is no time gap in between action and reaction. So we cannot say that action is the cause and reaction is its effect. Any one force can be action and the other reaction.

(iii) Action - reaction law is applicable on both the states either at rest or in motion.

(iv) Action and reaction is also applicable between bodies which are not in physical contact.

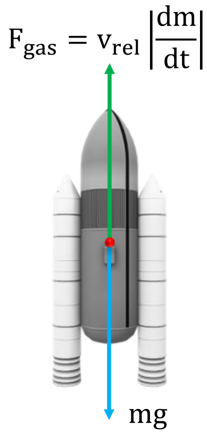
(v) Action - reaction law is applicable to all the interaction forces eg. gravitational force, electrostatic force, electromagnetic force, tension, friction, viscous force, etc.

(vi) Action and reaction never cancel each other because they act on two different bodies.

Examples : Walking, Swimming, recoiling of gun when a bullet is fired from it, Rocket propulsion.

Variable Mass Problems

Rocket Propulsion



m = Mass of rocket

v_{rel} = Velocity of exhaust gases w.r.t rocket

$\frac{dm}{dt}$ = Rate of burning of fuel

Case-I : If rocket is accelerating upwards, then -

Net upwards force on rocket = ma

$$v_{rel}\left|\frac{dm}{dt}\right| - mg = ma$$

Where v_{rel} is the relative velocity of the ejected mass w.r.t rocket

The acceleration of the rocket is given as:

$$a = \frac{v_{rel}}{m}\left|\frac{dm}{dt}\right| - g$$

Case-II : If rocket is moving with constant velocity, then a = 0

$$v_{rel}\left|\frac{dm}{dt}\right| = mg$$

Illustration 14:
A 800 kg rocket is set for a vertical firing. If the exhaust speed of gases is 2000 m/s w.r.t rocket, then calculate the mass of gas ejected per second to supply the thrust needed to overcome the weight of rocket.

Solution:

Force required to overcome the weight of rocket F = mg and thrust needed = $v_{rel}\dfrac{dm}{dt}$

so $v_{rel}\dfrac{dm}{dt}=mg \Rightarrow \dfrac{dm}{dt}=\dfrac{mg}{v_{rel}}=\dfrac{800\times 9.8}{2000}=3.92\ kg/s$

Illustration 15:
2.80×10^6 kg is the mass of rocket. The fuel is burnt at the rate of 1.40×10^4 kg/s and the exhaust velocity is 2.40×10^3 m/s w.r.t rocket. What is the initial acceleration?

Solution:

Given: Exhaust velocity, $v_{rel}=2.40\times10^3\ m/s$; Mass of the rocket, $m=2.80\times10^6$ kg

Rate of fuel burnt, $\dfrac{dm}{dt}=1.40\times10^4$ kg/s ; Acceleration due to gravity, $g=9.80\ m/s^2$

Substituting the above, in the formula, we get

$$a=\frac{v_{rel}}{m}\frac{dm}{dt}-g=\frac{2.40\times10^3\,m/s}{2.80\times10^6\,kg}\left(1.40\times10^4\,kg/s\right)-9.80\ m/s^2=2.20\ m/s^2$$

Illustration 16:
If the force on a rocket moving in force free space with an exhaust velocity of gases 600 m/sec w.r.t rocket is 500 N, then the rate of combustion of the fuel, is :-

Solution:

$F=v\cdot\dfrac{dm}{dt}$ (g = 0, force free space) $\Rightarrow \dfrac{dm}{dt}=\dfrac{F}{v}=\dfrac{500}{600}=0.833\,kg/s$

Illustration 17:
For a Rocket propulsion velocity of exhaust gases relative to rocket is 1.8 km/s. If mass of rocket system is 600 kg, then the rate of fuel consumption for a rocket to rise up with an acceleration 2.4 m/s² will be :-

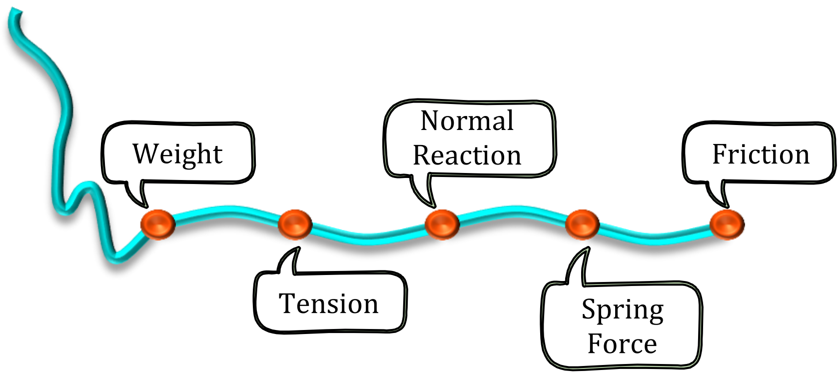
Solution:

$$v\frac{dm}{dt}-mg=ma$$

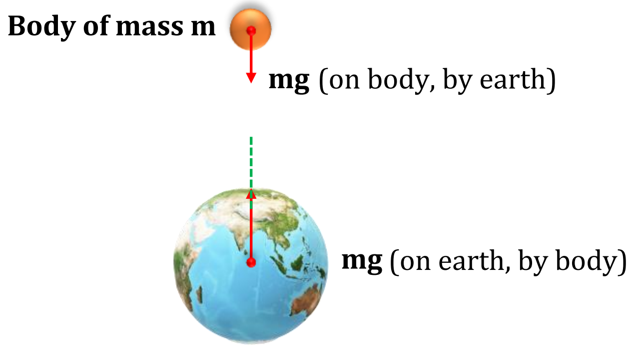
$$600\times9.8+600\times2.4=\frac{dm}{dt}\times1800\Rightarrow\frac{dm}{dt}=\frac{7320}{1800}=4.067\,kg/s$$

Common Forces in Mechanics and Free body diagram

Common Forces in Mechanics



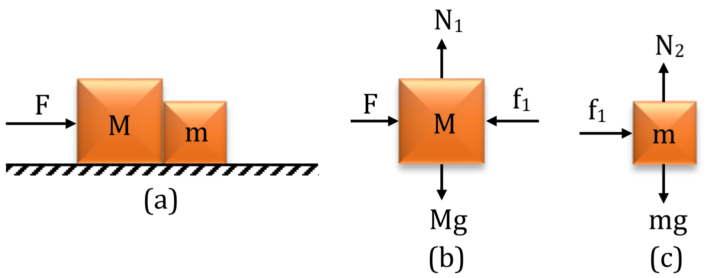
Weight : Weight is the gravitational force with which the earth pulls an object. The weight of an object is mg, where g is acceleration due to gravity.



Normal Reaction : The contact force by which two surfaces in contact push each other perpendicular to the contacting surfaces, is known as normal reaction.
Tension : A pulling force exerted on an object by a rope or string. This is a contact force.

Free Body Diagram

A diagram showing all external forces acting on an object is called "Free Body Diagram" (F.B.D.)
In a specific problem, we are required to choose a body first and then we access the different forces acting on it, and all the forces are drawn on the body, The resulting diagram is known as free body diagram (FBD).
For example, if two bodies of masses m and M resting on a smooth floor are in contact and a force F is applied on M from the left as shown in figure (a), the free body diagrams of M and m will be as shown in figures (b) and (c).



Important Point:

Two forces in Newton's third law never occur in the same free–body diagram. This is because a free–body diagram shows forces acting on a single object, and the action–reaction pair in Newton's third law always act on different objects.

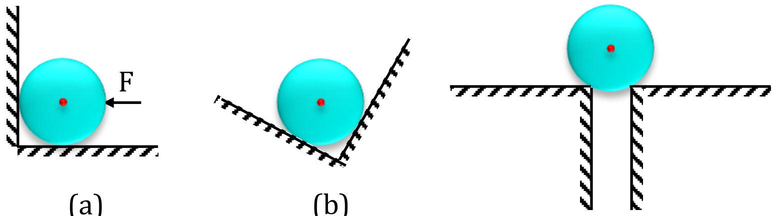
Normal Reaction Force

The contact force by which two surfaces in contact push each other perpendicular to the contacting surfaces, is known as normal reaction.

- Electromagnetic in nature
- The pushing force acting towards the body.
- Contact force in a direction perpendicular to surfaces in contact
- Always produced in action reaction pair
- Normal Reaction cannot pull an object.
- Self-adjusting in nature
- Normal Reaction can never be negative, its minimum value is ZERO
- N = 0 implies two conditions
- Bodies are not in physical contact
- Bodies are in physical contact but not pushing each other

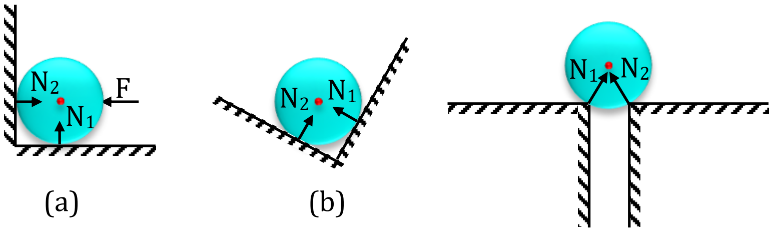
Illustration 18:

Show the normal reaction forces in following figures?



(a) (b) (c)

Solution:



(a) (b) (c)

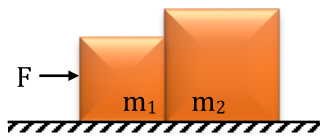
Here N_1 and N_2 normal reaction forces perpendicular to the well define surface towards the centre of the body.

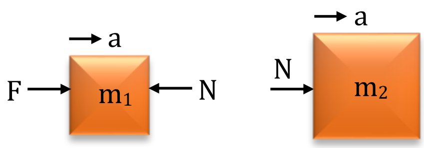
Problems on Normal Reaction

Using free body diagrams :

$F - N = m_1 a$ ...(1)

$N = m_2 a$ ...(2)





On adding the above equations

$F = m_1 a + m_2 a$

$F = a(m_1 + m_2)$

$\Rightarrow a = \dfrac{F}{m_1 + m_2}$ ...(3) OR $a = \dfrac{F_{net}}{m_{total}}$

putting the value of 'a' from equation (3) in (2), we get

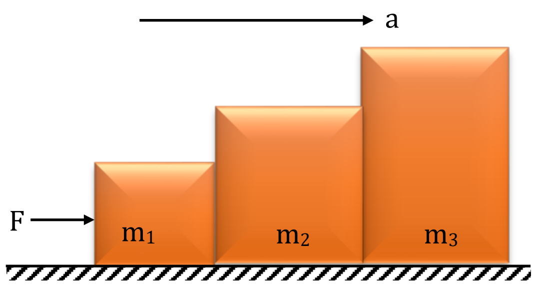
$N = \dfrac{m_2 F}{m_1 + m_2}$ (here N = contact force)

Three bodies in contact :

$F - N_1 = m_1 a$ ...(1)

$N_1 - N_2 = m_2 a$ ...(2)

$N_2 = m_3 a$ ...(3)



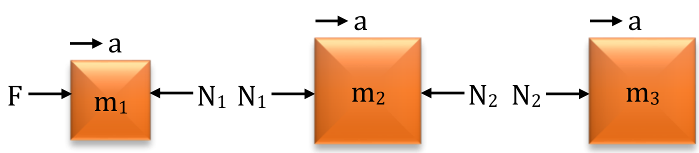
On adding the above equations

$F = m_1 a + m_2 a + m_3 a$

$F = a(m_1 + m_2 + m_3)$

$\Rightarrow a = \dfrac{F}{m_1 + m_2 + m_3}$ ...(4)

OR $a = \dfrac{F_{Net}}{m_{total}}$



Putting the value of 'a' from equation (4) in (3), we get

$$N_2 = \frac{Fm_3}{m_1 + m_2 + m_3}$$

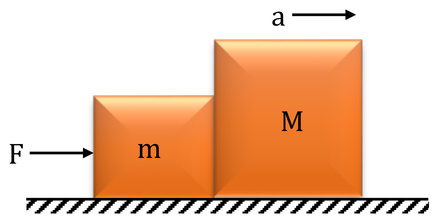
Putting the value of 'a' from (4) in (1);

$$F - m_1 a = N_1$$

$$N_1 = F - \frac{Fm_1}{m_1 + m_2 + m_3} \Rightarrow N_1 = \frac{(m_2 + m_3)F}{m_1 + m_2 + m_3}$$

Illustration 19:

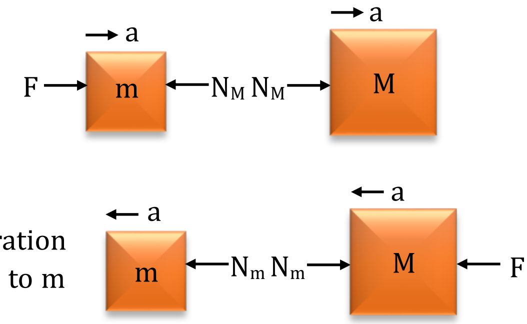
Two blocks of masses m = 2 kg and M = 5 kg are in contact on a frictionless table. A horizontal force F (= 35 N) is applied to m. Find the force of contact between the blocks. Will the force of contact remain same if F is applied to M?



Solution:

As the blocks are rigid both will move with same acceleration under the action of a force F

$$a = \frac{F}{m+M} = \frac{35}{2+5} = 5\ m/s^2$$



Force of contact $N_M = Ma = 5 \times 5 = 25N$

If the force is applied to M then its action on m will be

$$N_M = ma = 2 \times 5 = 10N$$

Note: - From this problem it is clear that magnitude of acceleration does not depend on the fact that whether the force is applied to m or M, but force of contact does.

BEGINNER'S BOX-2

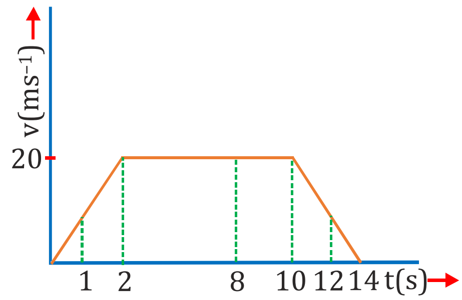
1. A person of mass M kg is standing on a lift. If the lift moves vertically upwards according to given v-t graph then find out the weight of man at the following instants :

($g = 10\ m/s^2$)

(i) t = 1 second

(ii) t = 8 seconds

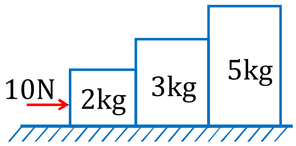
(iii) t = 12 seconds



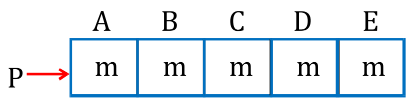
PLM219

2. Find the acceleration of the system and the contact force between

(i) 2kg and 3 kg blocks (ii) 3kg and 5 kg blocks **PLM220**



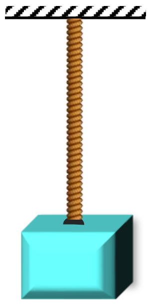
3. Calculate : (i) a_{system} (ii) F_{DE} (iii) F_{CD} (iv) F_{BC} (v) F_{AB} corresponding to the following diagram.



PLM221

Tension Force

A pulling force exerted on an object by a rope or cord. This is a contact force.



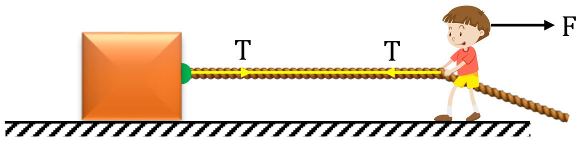
Tension in String

Tension is the intermolecular force between the molecules of a string, which becomes active when the string is stretched.

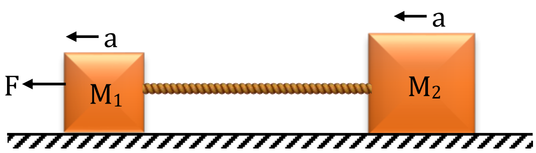
Important points about tension:

- Electromagnetic in nature
- A pull type force, acting on a body in the direction away from the point of contact or tied end of the string.
- Tension in slack string is always zero
- Tension in a string can never be negative, its minimum value is zero

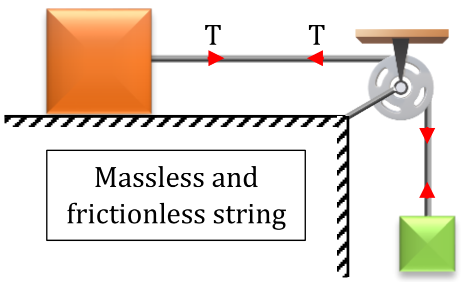
(a) Force of tension act on a body in the direction away from the point of contact or tied end of the string.



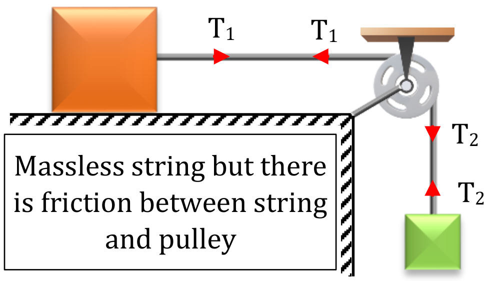
(b) String is assumed to be inextensible so that the magnitude of accelerations of the blocks tied to the strings are always same.



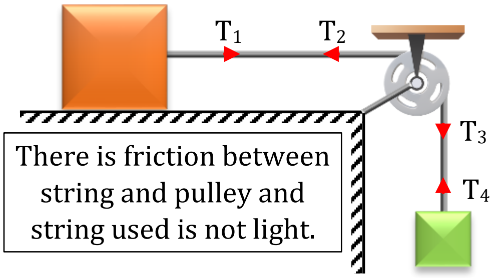
(c) (i) If the string is massless and frictionless, tension throughout the string remains same.



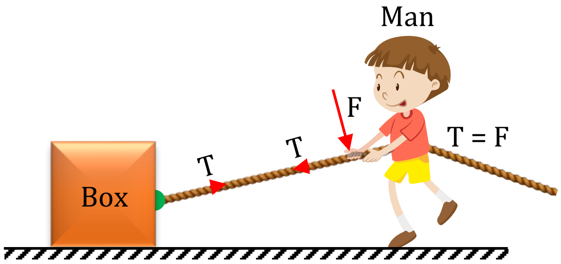
(ii) If the string is massless but not frictionless, at every contact tension changes.



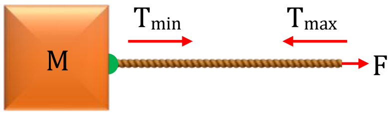
(iii) If the string is not light, tension at each point of the string will be different depending on the acceleration.



(d) If a force is directly applied to a string, say a man is pulling a string from the other end with some force, then tension will be equal to the applied force irrespective of the motion of the pulling agent, irrespective of whether the box moves or not, man moves or not.



(e) String is assumed to be massless unless stated, hence tension in it remains the same every where and equal to the applied force. However, if a string has a mass, tension at different points will be different being maximum (= applied force) at the end through which force is applied and minimum at the other end connected to a body.

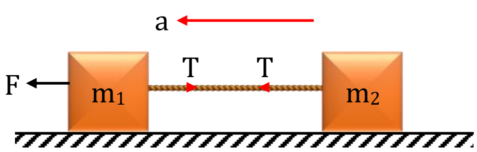


(f) In order to produce tension in a string two equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e., T = F) and is directed inwards opposite to F.

(g) Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break beyond a certain limit. The maximum tension which a string can bear without breaking is called its "breaking strength". It is finite for a string and depends on its material and dimensions.

System of masses tied by strings

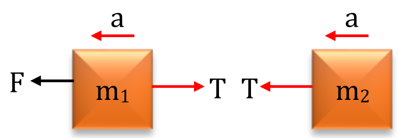
- **Two Connected Bodies :**



- **Free body diagram :**

$F-T=m_1a$ ...(1)

$T=m_2a$ ...(2)



On adding the above equations

$F=m_1a+m_2a$

$F=a(m_1+m_2)$

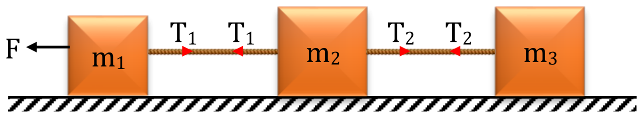
$\boxed{a=\dfrac{F}{m_1+m_2}}$ ...(3) Or $\boxed{a=\dfrac{F_{net}}{m_{total}}}$

Putting the value of 'a' from equation (3) in (2), we get –

$\boxed{T=\dfrac{m_2F}{m_1+m_2}}$

- **Three Connected Bodies :**

Free body diagram: -



$F-T_1=m_1a$ ...(1)

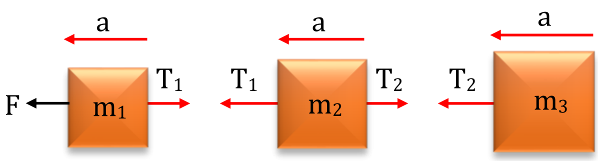
$T_1-T_2=m_2a$ ...(2)

$T_2=m_3a$ ...(3)

On adding above equations

$F=(m_1+m_2+m_3)a$

$\therefore$ $\boxed{a=\dfrac{F}{m_1+m_2+m_3}}$ or $\boxed{a=\dfrac{F_{net}}{m_{total}}}$



Put the value of 'a' in equation (1)

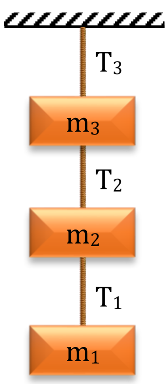
$T_1=F-m_1a$

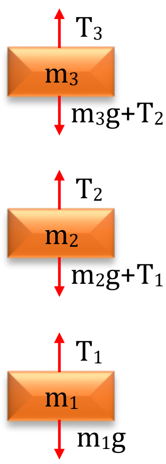
$\Rightarrow T_1=F-\dfrac{m_1F}{m_1+m_2+m_3}=\dfrac{m_1F+m_2F+m_3F-m_1F}{m_1+m_2+m_3}$ $\boxed{T_1=\dfrac{(m_2+m_3)F}{m_1+m_2+m_3}}$

Similarly, putting the value of 'a' in equation (3) $\boxed{T_2=\dfrac{m_3F}{m_1+m_2+m_3}}$

Bodies Hanging Vertically

Since all the bodies are in equilibrium, therefore net force on each body is zero





$T_1=m_1g$ ...(1)

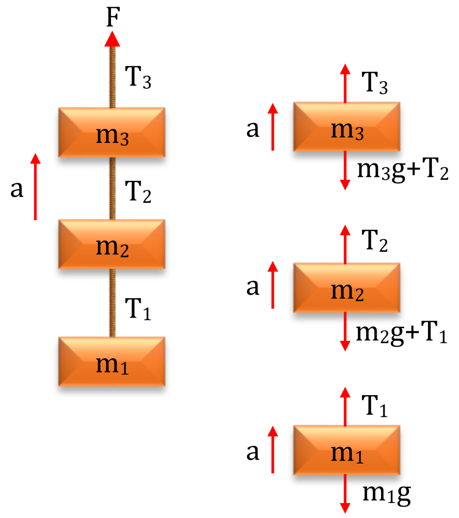
$T_2=T_1+m_2g$

$\Rightarrow$ $T_2=(m_1+m_2)g$ ...(2)

$T_3=T_2+m_3g$

$\Rightarrow$ $T_3=(m_1+m_2+m_3)g$

Bodies Accelerating Vertically Upwards



For m_1 $T_1 - m_1 g = m_1 a$

$\Rightarrow \quad T_1 = m_1(g+a)$ ...(1)

For m_2 $T_2 - m_2 g - T_1 = m_2 a$

$\Rightarrow \quad T_2 = m_2 a + m_2 g + T_1$

$\Rightarrow \quad T_2 = m_2(g+a) + m_1(g+a)$ (from equation 1)

$\Rightarrow \quad T_2 = (m_1 + m_2)(g+a)$ ...(2)

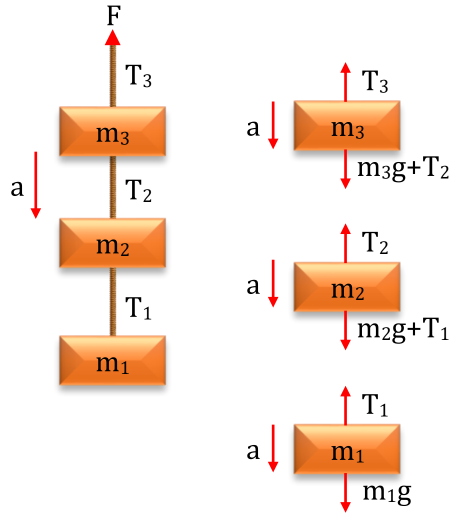
For m_3 $F = T_3$ and $T_3 - m_3 g - T_2 = m_3 a$

$\Rightarrow \quad T_3 = T_2 + m_3 g + m_3 a$

$\Rightarrow \quad T_3 = (m_1 + m_2)(g+a) + m_3(g+a)$ (from equation 2)

$\Rightarrow \quad T_3 = (m_1 + m_2 + m_3)(g+a)$ ...(3)

Bodies Accelerating Vertically Downwards



For m_1 $m_1 g - T_1 = m_1 a$

$\Rightarrow \quad T_1 = m_1(g-a)$ ...(1)

For m_2 $m_2 g + T_1 - T_2 = m_2 a$

$\Rightarrow \quad T_2 = m_2 g - m_2 a + T_1$

$\Rightarrow \quad T_2 = m_2(g-a) + m_1(g-a)$ (from equation 1)

$\Rightarrow \quad T_2 = (m_1 + m_2)(g-a)$ ...(2)

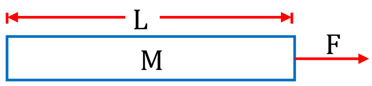
For m_3 $F = T_3$ and $m_3 g + T_2 - T_3 = m_3 a$

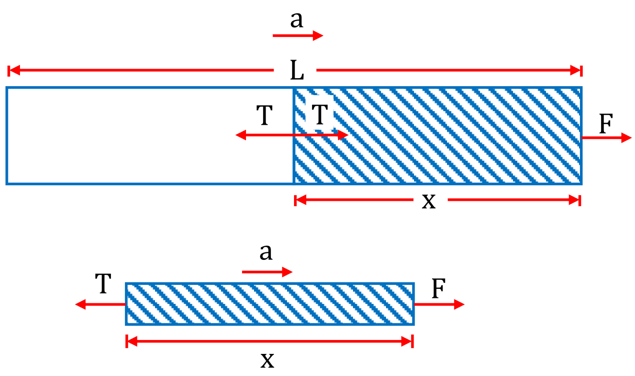
$\Rightarrow \quad T_3 = m_3(g-a) + (m_1 + m_2)(g-a)$ (from equation 2)

$\Rightarrow \quad T_3 = (m_1 + m_2 + m_3)(g-a)$ ...(3)

Tension in rod and heavy strings

Given





Mass of Rod = M

Length of Rod = L

F – T = ma

$\Rightarrow$ T = F – ma

$\Rightarrow \quad T = F - m\dfrac{F}{M} \qquad \left(\because a = \dfrac{F}{M}\right)$

$\because$ Mass of length 'L' = M $\qquad \therefore$ Mass of unit length $= \dfrac{M}{L}$

$\therefore$ Mass (m) of length 'x' = $\dfrac{M}{L}x$

Put this value of 'm' in equation (1) $T = F - \left(\dfrac{M}{L}x\right)\dfrac{F}{M} \Rightarrow \boxed{T = F\left(1 - \dfrac{x}{L}\right)}$

Illustration 20:

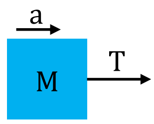
A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find the (i) Acceleration of the rope and the block (ii) Force that the rope exerts on the block. (iii) Tension in the rope at its mid-point.

Solution:

(i) Acceleration $a = \dfrac{F}{(m+M)}$

(ii) Force exerted by the rope on the block is

$$T = Ma = \frac{M.F}{(m+M)}$$

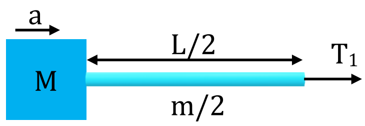


FBD of block

FBD of rope

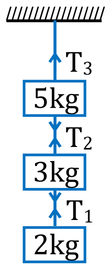
(iii) $T_1 = \left(\dfrac{m}{2} + M\right) a = \left(\dfrac{m+2M}{2}\right)\left(\dfrac{F}{m+M}\right)$

Tension in rope at midpoint is $T_1 = \dfrac{(m+2M)F}{2(m+M)}$

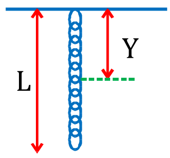


BEGINNER'S BOX-3

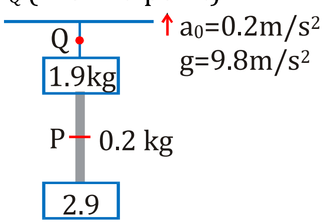
1. In the given figure determine $T_1 : T_2 : T_3$ **PLM222**



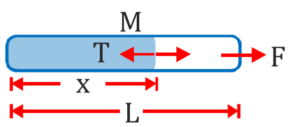
2. Find the tension in the chain at a distance Y from the support. Mass of chain is M. **PLM223**



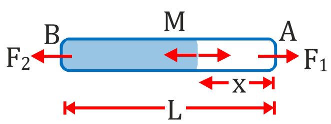
3. Calculate T_P and T_Q (P is mid point) **PLM224**



4. A uniform rope of mass M and length L is placed on a smooth horizontal surface. A horizontal force F is acting at one end of rope. Calculate the tension in the rope at a distance x as shown. **PLM225**

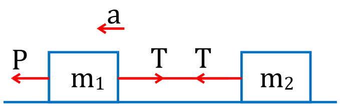


5. Calculate the tension T in the rope at a distance x from end A in the following diagram assuming $F_1 > F_2$. **PLM226**

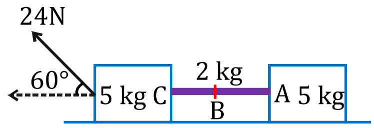


6. A man in a lift carrying a 5 kg bag. If the lift moves vertically downwards with g/2 acceleration. Find the tension in the handle of the bag. **PLM227**

7. Calculate T for the following diagram : **PLM228**



8. Calculate T_A, T_B, T_C for the following diagram : (B is the mid point of rope) **PLM229**



9. A dynamometer is attached to two blocks of masses 6 kg and 4 kg. Forces of 20 N and 10 N are applied on the blocks as shown in figure. Find the dynamometer reading in the steady state.

PLM230

Pulley Block Systems

- Ideal pulley is considered massless and frictionless.
- Ideal string is massless and inextensible.
- A pulley may change the direction of force in the string but not the tension.

The only function of pulley (which has no friction on its axle to retard rotation) is to change the direction of force through the cord that joins the two blocks.

Some Cases of Pulley

Case-I : $m_1 = m_2 = m$

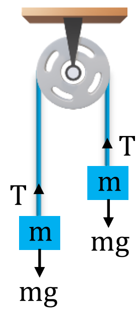
Tension in the string

$T = mg$

Acceleration 'a' = zero

Reaction at the point of suspension of the pulley or thrust on pulley.

$R = 2T = 2mg$.



Case-II : $m_1 > m_2$

now for mass m_1,

$m_1 g - T = m_1 a$ ...(i)

for mass m_2

$T - m_2 g = m_2 a$ ...(ii)

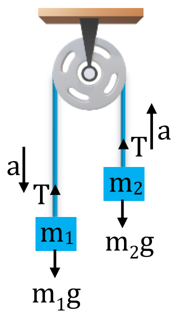
adding (i) and (ii)

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g \text{ and } T = \frac{2m_1 m_2}{(m_1 + m_2)} g = \frac{2W_1 W_2}{W_1 + W_2}$$

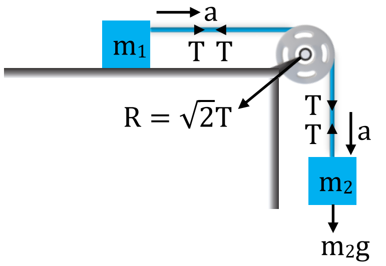
$$\boxed{\text{acceleration} = \frac{\text{net pulling force}}{\text{total mass to be pulled}}}$$

$$\boxed{\text{Tension} = \frac{2 \times \text{Product of masses}}{\text{Sum of masses}} g}$$

$$\boxed{\begin{array}{l}\text{Reaction at the suspension point of pulley (Thrust on pulley)} \\ R = 2T = \dfrac{4m_1 m_2 g}{(m_1 + m_2)} = \dfrac{4W_1 W_2}{W_1 + W_2}\end{array}}$$



Case-III :



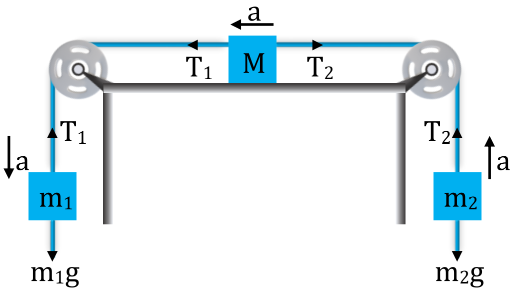
For mass m_1 : $T = m_1a$

For mass m_2 : $m_2g - T = m_2a$

acceleration $a = \dfrac{m_2g}{(m_1+m_2)}$ and $T = \dfrac{m_1m_2}{(m_1+m_2)}g$

Reaction at suspension point of pulley $R = \sqrt{2}\,T$

Case-IV : $(m_1 > m_2)$



$m_1g - T_1 = m_1a$ ...(i)

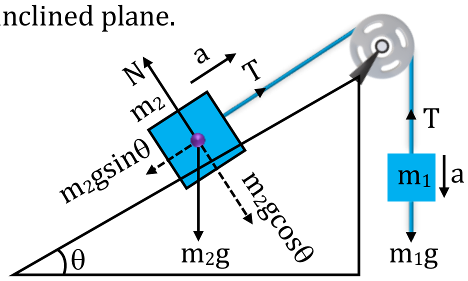
$T_2 - m_2g = m_2a$ ...(ii)

$T_1 - T_2 = Ma$ ...(iii)

adding (i), (ii) and (iii)

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)}g$$

Case-V : Mass suspended over a pulley along with another on an inclined plane.



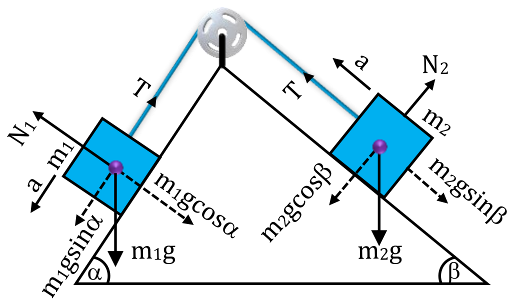
For mass m_1 : $m_1g - T = m_1a$

For mass m_2 : $T - m_2g\sin\theta = m_2a$

acceleration $a = \dfrac{(m_1 - m_2\sin\theta)}{(m_1+m_2)}g$

$$T = \frac{m_1m_2(1+\sin\theta)}{(m_1+m_2)}g$$

Case-VI : Masses m_1 and m_2 are connected by a string passing over a pulley $m_1\sin\alpha > m_2\sin\beta$



$m_1g\sin\alpha - T = m_1a$ ...(i)

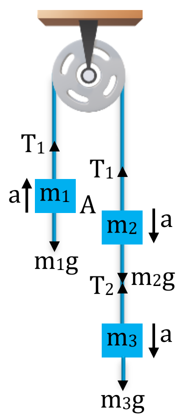
$T - m_2g\sin\beta = m_2a$ ...(ii)

After solving equation (i) and (ii)

Acceleration $a = \dfrac{(m_1\sin\alpha - m_2\sin\beta)}{(m_1+m_2)}g$

Tension $T = \dfrac{m_1m_2(\sin\alpha + \sin\beta)}{(m_1+m_2)}g$

Case-VII : For mass m_1 : $T_1 - m_1g = m_1a$



For mass m_2 : $m_2g + T_2 - T_1 = m_2a$

For mass m_3 : $m_3g - T_2 = m_3a$

$$\Rightarrow \qquad a = \frac{(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)}g$$

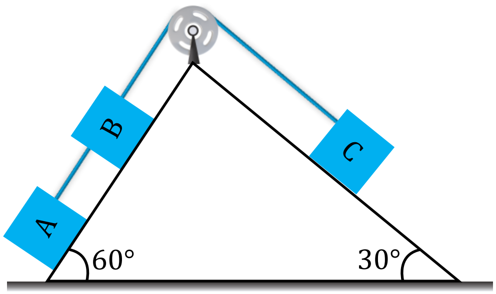
we can calculate tension T_1 and T_2 from above equations

Illustration 21:

In the adjacent figure, masses of A, B and C are 1 kg, 3 kg and 2 kg respectively.

Find (a) the acceleration of the system and

(b) the tensions in the strings (Neglect friction).



Solution:

(a) In this case net pulling force

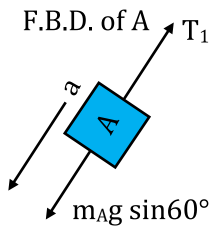
$= m_A g \sin 60° + m_B g \sin 60° - m_C g \sin 30°$

$= (m_A + m_B) g \sin 60° - m_C g \sin 30°$

$= (1 + 3) \times 10 \times \dfrac{\sqrt{3}}{2} - 2 \times 10 \times \dfrac{1}{2} = 20\sqrt{3} - 10 = 24.64$ N

Total mass being pulled = 1 + 3 + 2 = 6 kg

$\therefore$ Acceleration of the system $a = \dfrac{24.64}{6} = 4.1$ m/s^2

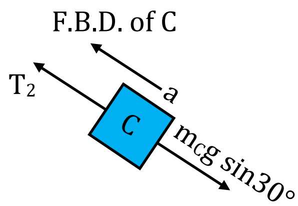


(b) For the tension in the string between A and B.

$m_A g \sin 60° - T_1 = (m_A)(a)$

$\therefore$ $T_1 = m_A g \sin 60° - m_A a = m_A(g \sin 60° - a)$

$\therefore$ $T_1 = (1)\left(10 \times \dfrac{\sqrt{3}}{2} - 4.1\right) = 4.56$ N



For the tension in the string between B and C.

$T_2 - m_C g \sin 30° = m_C a$

$\therefore$ $T_2 = m_C(a + g \sin 30°) = 2\left[4.1 + 10\left(\dfrac{1}{2}\right)\right] = 18.2\text{N}$

Illustration 22:

In the figure blocks A, B and C have acceleration a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitudes 2mg and mg respectively. Find the value of a_1, a_2 and a_3.

Solution:

$a_1 = \dfrac{2mg - mg}{m} = g$; $a_2 = \dfrac{2m - m}{2m + m} g = \dfrac{g}{3}$

$a_3 = \dfrac{mg + mg - mg}{2m} = \dfrac{g}{2}$

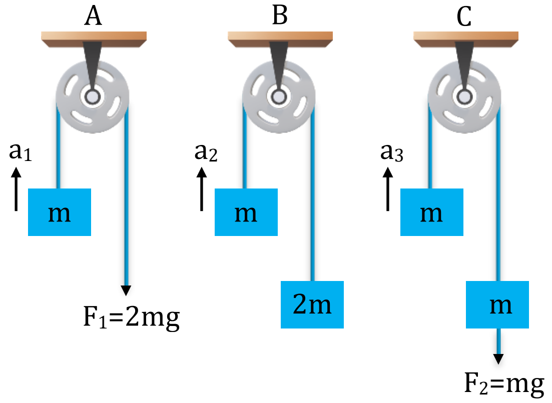
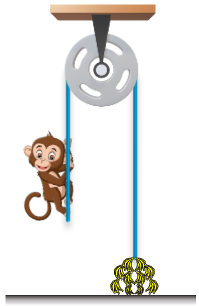


Illustration 23:

A 12 kg monkey climbs a light rope as shown in fig. The rope passes over a pulley and is attached to a 16 kg bunch of bananas. Mass and friction in the pulley are negligible so that the effect of pulley is only to reverse the direction of force of the rope. What maximum acceleration can the monkey have without lifting the bananas?

(Take $g = 10\ m/s^2$)



Solution:

For monkey

$T - 120 = 12 \times a$ ...(i)

For Bananas

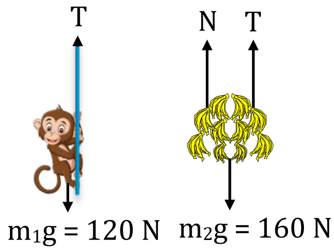
$160 - T = N$

Condition for just loosing the contact is N = 0

$160 - T = 0 \quad \Rightarrow \quad T = 160$ ...(ii)

from equation (i) & (ii)

$160 - 120 = 12 \times a \quad \Rightarrow \quad a = 3.33\ m/s^2$



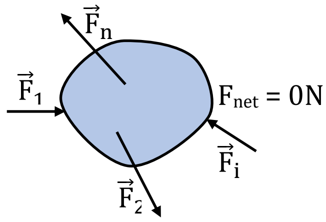
Translational Equilibrium

Conditions for translational equilibrium :

For a body to be in translational equilibrium, no net force must act on it i.e. vector sum of all the forces acting on it must be zero.

If several external forces $\vec{F}_1, \vec{F}_2 ......\vec{F}_i .... \text{and}\ \vec{F}_n$ act simultaneously on a body and the body is in translational equilibrium, the resultant of these forces must be zero.

$\sum \vec{F}_i = \vec{0}$



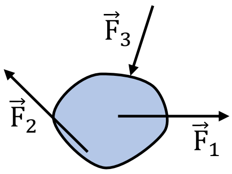
If the forces $\vec{F}_1, \vec{F}_2 ......\vec{F}_i .... \text{and}\ \vec{F}_n$ are expressed in Cartesian components, we have :

$\sum F_{ix} = 0 \qquad \sum F_{iy} = 0 \qquad \sum F_{iz} = 0$

If a body is acted upon by a single external force, it cannot be in equilibrium.

If a body is in equilibrium under the action of only two external forces, the forces must be equal and opposite.

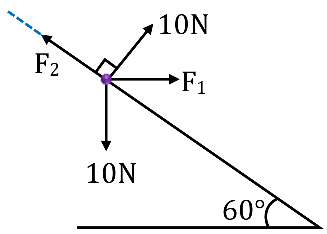
If a body is in equilibrium under the action of three forces, their resultant must be zero.



$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$

Illustration 24:

In the given figure if body is in equilibrium then calculate F_1 and F_2.



Solution:

Resolve the forces along x-direction & y-direction

In x-direction

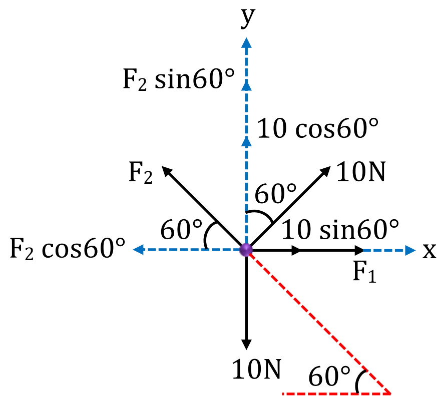
$\Sigma F_x = 0$ (for equilibrium)

$F_1 + 10 \sin 60° = F_2 \cos 60°$

$\Rightarrow \quad F_1 + 10\left(\frac{\sqrt{3}}{2}\right) = F_2\left(\frac{1}{2}\right)$

$\Rightarrow \quad 2F_1 - F_2 = -10\sqrt{3}$ ...(i)

In y-direction



$\Sigma F_y = 0$

$F_2 \sin 60° + 10 \cos 60° = 10$

$\Rightarrow \quad F_2 \times \left(\frac{\sqrt{3}}{2}\right) + 10\left(\frac{1}{2}\right) = 10 \qquad \Rightarrow F_2(\sqrt{3}) + 10 = 20$

$\Rightarrow \quad F_2 = \left(\frac{10}{\sqrt{3}} N\right)$

Now from equation (i)

$2F_1 - F_2 = -10\sqrt{3} \qquad \Rightarrow \qquad 2F_1 - \frac{10}{\sqrt{3}} = -10\sqrt{3}$

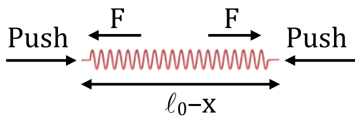
$\Rightarrow \quad 2\sqrt{3}F_1 - 10 = -30 \qquad \Rightarrow \qquad 2\sqrt{3}F_1 = -20 \qquad \Rightarrow \qquad \boxed{F_1 = \frac{-10}{\sqrt{3}} N}$

Spring Force

A linear spring is generally a helical metallic wire.

When the two free ends of the spring are pulled away or pushed towards each other, the length of the spring is changed.

The spring has a tendency to come back to its original length and it develops an opposition to change in its length.



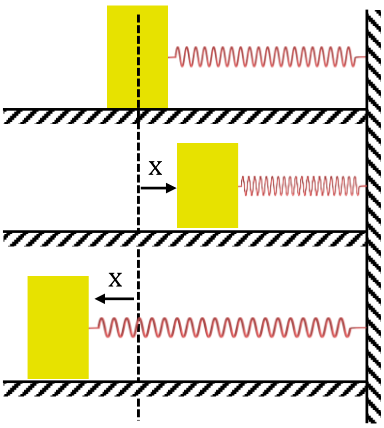
This opposing force (F) is the restoring force.

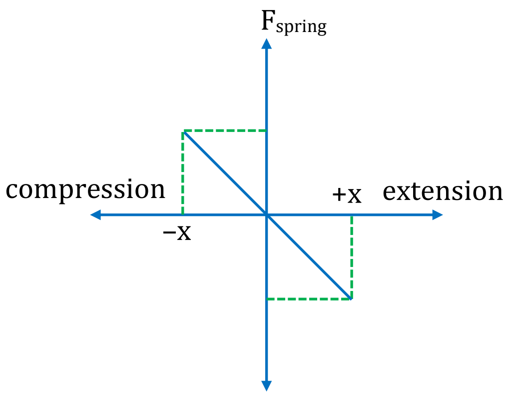
This force is electromagnetic in nature.

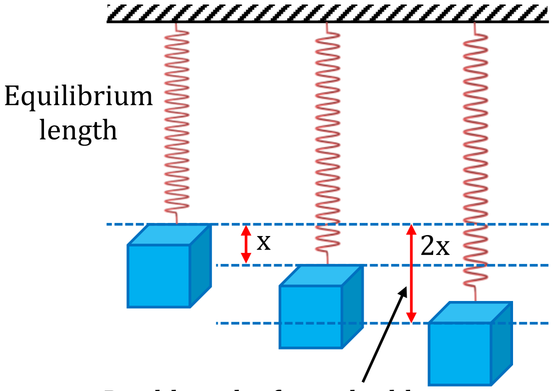
Hooke's Law

Under small extension/compression, The opposing force is directly proportional to the change in length and opposite to direction of pull/push.

$F_{spring} = -kx$







The force F varies linearly with x and acts in a direction opposite to x. Therefore, it is expressed by the following equation F = –kx

Here, the minus (–) sign represents the fact that force F is always opposite to x.

The constant of proportionality k is known as force constant of the spring or simply as spring constant. The modulus of slope of the graph equals the spring constant.

SI unit of spring constant is newton per meter of (N/m).

Dimensions of spring constant are $[MT^{-2}]$.

For a pulley – spring system (at steady state) :-

Acceleration $a = \dfrac{m_2 g}{m_1 + m_2}$

Tension $T = \dfrac{m_1 m_2}{(m_1 + m_2)} g$

If x is the extension in the spring,

then T = kx

$x = \dfrac{T}{k} = \dfrac{m_1 m_2 g}{k(m_1 + m_2)}$

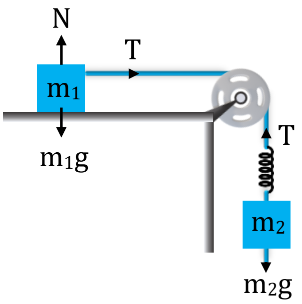
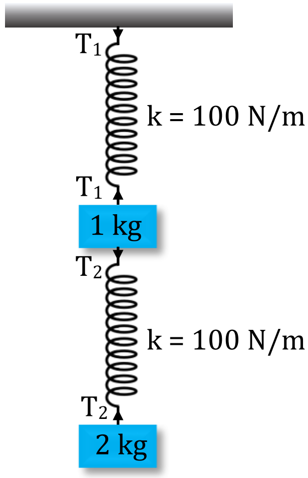


Illustration 25:

Find extension in both the springs if the system is in equilibrium.



Solution:

From figure (a)

$T_2 - 20 = 0 \Rightarrow T_2 = 20$ N ...(i)

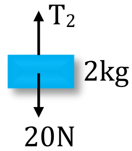


Figure (a)

From figure (b)

$T_1 = T_2 + 10$ ...(ii)

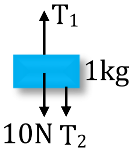


Figure (b)

From equation (i) & (ii)

$T_1 = 20 + 10 = 30$ N

For Ist Spring

$T_1 = kx_1 \quad \Rightarrow 30 = 100 \times x_1 \Rightarrow x_1 = 0.3$ m

For 2nd Spring

$T_2 = kx_2 \quad \Rightarrow 20 = 100 \times x_2 \Rightarrow x_2 = 0.2$ m

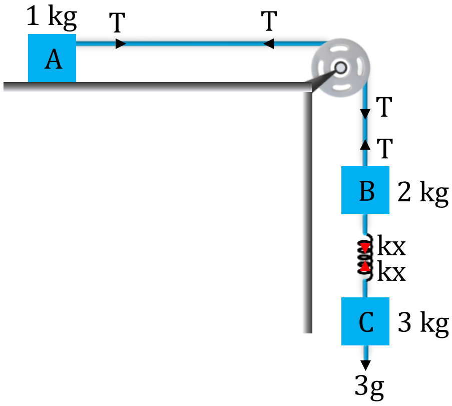
Illustration 26:

In the system shown in figure all surfaces are smooth, string is massless and inextensible. (in steady state) Find the

(a) acceleration of the system

(b) tension in the string and

(c) extension in the spring if force constant of spring is $k = 50$ N/m (Take $g = 10$ m/s²)



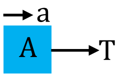
Solution:

(a) $3g - kx = 3a$ ...(i)

$2g + kx - T = 2a$ ...(ii)

$T = a$ ...(iii)

$\therefore$ Acceleration of the system is $a = \frac{50}{6} m/s^2$

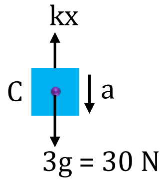
(b) 

Free body diagram of 1 kg block gives $T = ma = (1)\left(\frac{50}{6}\right)N = \frac{50}{6}N$

(c) Free body diagram of 3 kg block gives

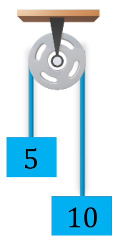
$30 - kx = ma$ but $ma = 3 \times \frac{50}{6} = 25$ N

$x = \frac{30-25}{k} = \frac{5}{50} = 0.1\,m = 10\,cm$



BEGINNER'S BOX-4

1. The respective masses of the blocks are shown in the diagram in appropriate units. Find acceleration of system, tension in the string and thrust on the pulley (in terms of g).



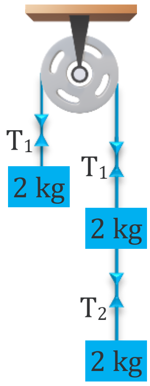
PLM231

2. Find the reading of the spring balance

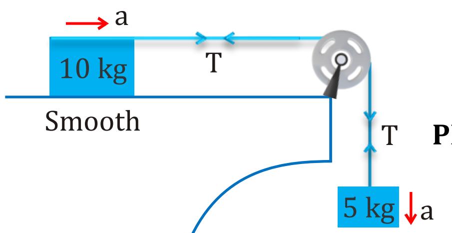
Note : Spring balance reads thrust on the pulley which is calibrated in kg-wt.

PLM232

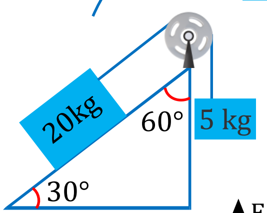
3. Calculate T_1 and T_2 for the following diagram. **PLM233**



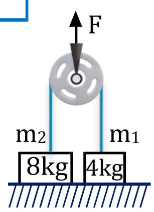
4. Calculate the acceleration of the system, tension in the string and thrust on the pulley in terms of g for the situation shown in following diagram. **PLM234**



5. Calculate the acceleration of the system and tension in the string for the situation shown in following diagram. **PLM235**



6. Two blocks of masses 8 kg and 4 kg respectively are connected by a string as shown.
Calculate their accelerations if they are initially at rest on the floor, after a force of 100N is applied on the pulley in the upward direction ($g = 10\ m/s^2$) **PLM236**

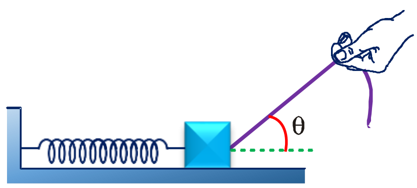


7. A block of mass m placed on a smooth floor is connected to a fixed support with the help of a spring of force constant k. It is pulled by a rope as shown in the figure. Tension T of the rope is increased gradually without changing its direction, until the block losses contact the floor. The increase in rope tension T is so gradual that acceleration in the block can be neglected.

(a) Draw its free body diagram, well before the block losses contact with the floor.

(b) What is the necessary tension in the rope so that the block looses contact with the floor?

(c) What is the extension in the spring, when the block looses contact with the floor? **PLM237**



Frame of Reference and Pseudo Force

Frame of Reference

A system with respect to which position or motion of a particle is described is known as frame of reference.
We can classify frame of reference in two categories: -

1. Inertial Frame of Reference
2. Non-inertial frame of Reference

Inertial Frame of Reference

The frame for which law of inertia is applicable is known as inertial frame of reference.
All the frames which are at rest or moving uniformly with respect to an inertial frame, are also inertial frame.

Non-Inertial Frame of Reference

The frame for which law of inertia is not applicable is known as non-inertial frame of reference.

All the frames which are accelerating or rotating with respect to an inertial frame will be non-inertial frames.

Pseudo Force

To apply Newton's law of motion in non-inertial reference frame we need to apply pseudo force.

It is an imaginary force which is used to explain the motion of objects from non-inertial reference frames.

Magnitude : $\boxed{F_{pseudo} = \text{Mass of body} \times \underset{(\text{w.r.t. Observer})}{\text{Acceleration of non-inertial frame}}}$

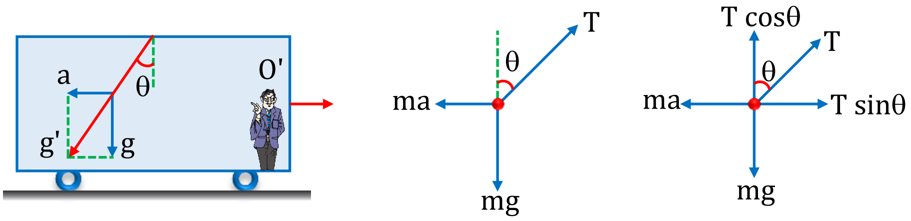
Direction : Opposite to the direction of acceleration of non-inertial frame(observer)

$\vec{F}_{pseudo} = -m \times \vec{a}_{obs}$

Pseudo force does not follow action reaction law.

Illustration 27:

A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle θ in which pendulum is in equilibrium w.r.t. train. Also calculate tension in the string.



F.B.D of bob
(for an observer in side the train)

Solution:

With respect to train, the bob is in equilibrium

$\therefore \quad \Sigma F_y = 0 \Rightarrow T\cos\theta = mg$ ...(i)

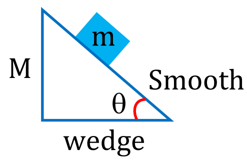
and $\quad \Sigma F_x = 0 \Rightarrow T\sin\theta = ma$ ...(ii)

Dividing equation (ii) by (i) $\tan\theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1}\left(\frac{a}{g}\right)$

Squaring and adding equation (i) and (ii) $T = m\sqrt{a^2 + g^2}$

Illustration 28:

What horizontal acceleration should be provided to the wedge so that the block of mass m kept on wedge remains at rest w.r.t. wedge?



Solution:

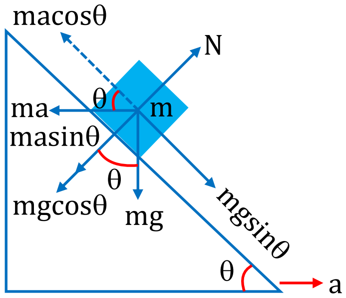
For equilibrium along wedge

(In reference of wedge frame)

macosθ = mgsinθ

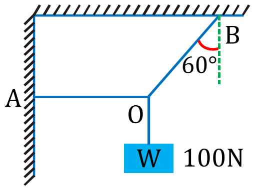
$\Rightarrow \quad a = g\frac{\sin\theta}{\cos\theta}$

$\Rightarrow \quad \boxed{a = g\tan\theta}$

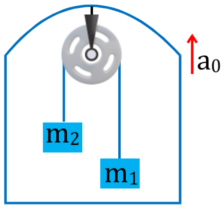


BEGINNER'S BOX-5

1. A block of weight 100 N is suspended (as shown) with the help of three strings. Find the tension in each of the three strings. **PLM238**



2. Determine the acceleration of the masses w.r.t. lift and tension in the string if the whole system is moving vertically upwards with uniform acceleration a_0. ($m_1 > m_2$) **PLM239**



Weighing Machine

Weighing machine measures the Normal Reaction applied on it

Reading of weighing machine is also called Apparent weight

W_{app} = Normal Reaction on machine

The Reading is expressed in two ways

in SI unit of force, N (newton)

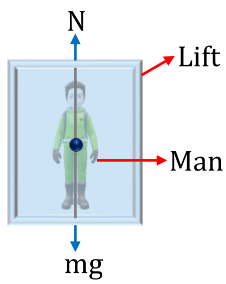
in practical/gravitational unit of force, kg-f or kg-wt

$$\text{Reading(kg} - \text{wt)} = \frac{\text{Normal Reaction(in N)}}{g}$$

$g = 9.8\text{ms}^{-2} \approx 10\text{ms}^{-2}$

Effective or Apparent weight of a man in lift

Case-I : If the lift is at rest or moving uniformly (a = 0), then



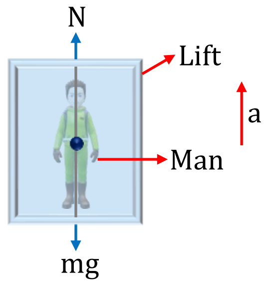
$N = mg$

So, $W_{app} = W_{actual}$

$W_{app} = N$

$W_{actual} = mg$

Case-II : If the lift is accelerated upwards, then –



Net upward force on man = ma

$N - mg = ma$

$N = mg + ma$

$\therefore N = m(g + a)$

$W_{(app)}$ or $N = m(g + a)$

So, $W_{app} > W_{actual}$

Case-III : If the lift is accelerated downwards, then –

$mg - N = ma$

$N = mg - ma$

W_{app} or $N = m(g - a)$

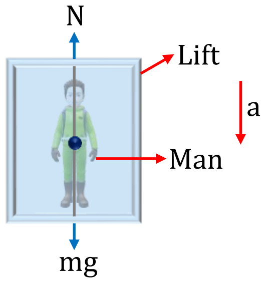
So, $W_{app} < W_{actual}$

If the lift is under free fall, it implies that its acceleration is equal to the acceleration due to gravity. i.e. a = g, then

$W_{app} = 0$

It means that person in lift will feel weightless

The apparent weight of any body falling freely is zero



Friction

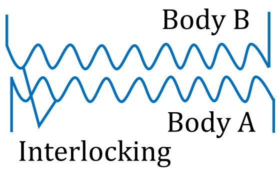
Friction is the opposing force that is set up between the surfaces of contact, when one body slides or tends to do so on the surface of another body.

Friction does not oppose motion.

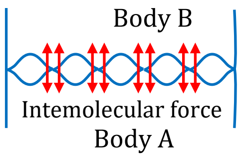
Rather it opposes relative motion between two bodies.

Cause of Friction

Old view: When two surfaces are in contact with each other, irregularities of one body get interlocked with the irregularities in the surface of the other. This interlocking opposes the tendency of relative motion.



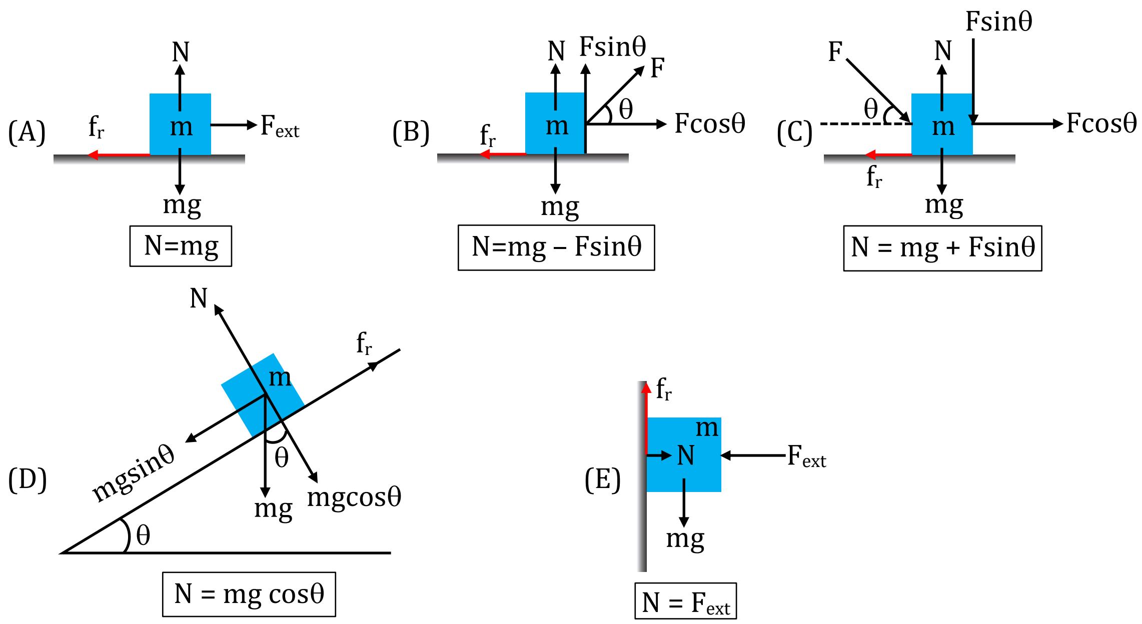
Modern view : Friction arises on account of Intermolecular forces of attraction between the two surfaces at the point of actual contact.



Friction depends on the following factors:

1. Friction force depends only on the area covered by contact particles of contact surfaces (actual contact area) it does not depend on the area covered by the body (apparent contact area)

2. Except static friction (f_s), friction force depends on normal reaction (N) $f \propto N$



Types of Friction

The frictional force developed before relative sliding initiates is defined as static friction. It opposes tendency of relative sliding. If the body is in relative motion, the friction opposing its relative motion is called dynamic or kinetic friction.

1. Static Friction

2. Kinetic Friction

Static Friction

When two bodies have tendency to move over each other (but they are not moving w.r.t. to each other) then their relative motion is opposed by the surfaces. This opposition force is called static friction.

- Direction is opposite to the tendency of motion
- Self-adjusting in nature
- Maximum value of static friction is called limiting friction.

$$f_s \le f_L$$

Limiting Friction

The maximum value of static friction is known as limiting friction

$$f_L \propto N \Rightarrow f_L = \mu_S N$$

μ_s : Coefficient of static friction

Limiting friction between two bodies depends on the nature of contact surfaces.

Its value is more than any other type of friction.

Kinetic Friction

When two contact surfaces slide on each other, friction acting between them is called Kinetic/Dynamic friction.

Its numerical value is $f_k \propto N \Rightarrow f_k = \mu_k N$ where μ_k = coefficient of kinetic friction.

μ_k : Coefficient of kinetic friction

It always opposes relative motion of a surface over other

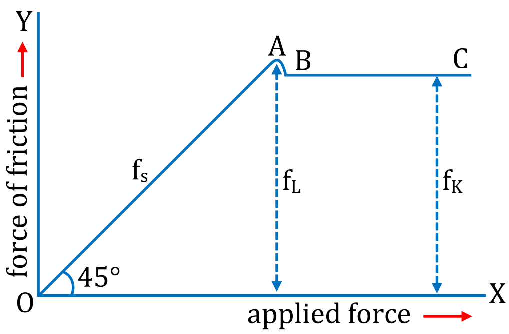
Its value does not depend on type of motion such as uniform or non-uniform motion, it has constant value

This is always slightly less than the limiting friction.

Its depends on N.

Graph between applied force and force of friction

The part OA of the curve represent static friction, (f_s) which goes on increasing, with the applied force. At A, the static friction is maximum. This represent the limiting friction. Beyond A, the force of friction is seen to decrease slightly. The portion BC of the curve, therefore, represents the kinetic friction (f_K).

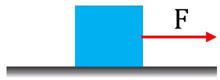


As the portion BC of the curve is parallel to OX, therefore, kinetic friction does not change with the applied force, It remains constant, whatever be the applied force.

Illustration 29:

A block of mass 1 kg is at rest on a rough horizontal surface having coefficient of static friction 0.2 and kinetic friction 0.15. Find the frictional force if a horizontal force,

(a) F = 1N (b) F = 1.96N (c) F = 2.5 N is applied on the block



Solution:

Maximum force of friction $f_{max} = 0.2 \times 1 \times 9.8$ N = 1.96 N

(a) For $F_{ext} = 1$ N ; $F_{ext} < f_{max}$

So, body is at rest which implies that static friction is present and hence $f_s = F_{ext} = 1$ N

(b) For $F_{ext} = 1.96$ N ; $F_{ext} = f_{max} = 1.96$ N so, f = 1.96 N

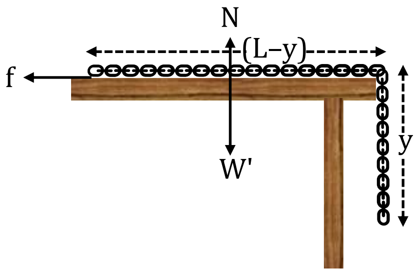
(c) For $F_{ext} = 2.5$ N so $F_{ext} > f_{max}$

Now the body is in motion $\therefore f_{max} = f_k = \mu_k N = \mu_k mg = 0.15 \times 1 \times 9.8 = 1.47$ N

Motion and Equilibrium on rough horizontal surface

Illustration 30:

Length of a chain is L and coefficient of static friction is μ. Calculate the maximum length of the chain which can hang from the table without sliding.



Solution:

Let y be the maximum length of the chain that can be held outside the table without sliding.

Length of chain on the table = (L – y)

Weight of the part of the chain on table $W' = \frac{M}{L}(L-y)g$

Weight of hanging part of the chain $W = \frac{M}{L}yg$

For equilibrium: limiting force of friction on (L–y) length = weight of hanging part of the chain of y length

$$\mu N = W \Rightarrow \mu W' = W \Rightarrow \mu\frac{M}{L}(L-y)g = \frac{M}{L}yg \Rightarrow \mu L - \mu y = y \Rightarrow y = \frac{\mu L}{1+\mu}$$

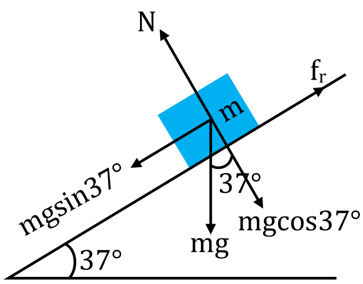
Illustration 31:

A block of mass 2 kg is placed on a plane inclined at an angle of 37° with the horizontal. The coefficient of friction between the block and the surface is 0.7.

(i) What will be the frictional force acting on the block?

(ii) What is the force applied by inclined plane on block?

Solution:



(i) Normal reaction of the surface N = mg cos 37° = 16 N

$\therefore$ limiting force of friction

$$f_L = \mu_s N = \mu_s mg\cos 37° = 0.7 \times 2 \times 10 \times \frac{4}{5} = 11.2 \text{ N}$$

$\because mg\sin 37° = 2 \times 10 \times \frac{3}{5} = 12$ N Because $f_L < mg\sin\theta$ so body is in motion $\therefore$ force of friction $(f_K) = 11.2$ N

(ii) $F = \sqrt{(f_K)^2 + (N)^2} = \sqrt{(11.2)^2 + (16)^2} = \sqrt{125.44 + 256} = \sqrt{381.44} = 19.53$ N

BEGINNER'S BOX-6

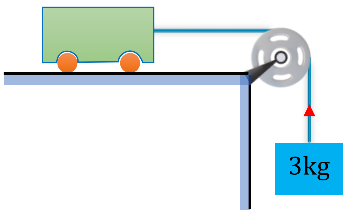
1. A body of mass 5 kg is placed on a rough horizontal surface. If coefficients of static and kinetic friction are 0.5 and 0.4 respectively, then find value of force of friction when external applied horizontal force is
(i) 15 N
(ii) 25 N and
(iii) 35 N. **PLM240**

2. A body of mass 5 kg is placed on a rough horizontal surface. If coefficient of friction is $\frac{1}{\sqrt{3}}$, find what pulling force should act on the body at an angle 30° to the horizontal so that the body just begins to move. **PLM241**

3. A body of mass 0.1 kg is pressed against a wall with a horizontal force 5 N. If coefficient of friction is 0.5 then find force of friction. **PLM242**

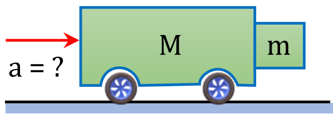
4. A body sliding on ice with a velocity of 8 m/s comes to rest after travelling 40 m. Find the coefficient of friction ($g = 9.8\ m/s^2$). **PLM243**

5. What is the acceleration of the block and trolley system shown in fig. if the coefficient of kinetic friction between the trolley and the surface is 0.04? What is the tension in the string? (Take $g = 10\ m/s^2$). Neglect the mass of the string.



PLM244

6. A cart of mass M has a block of mass m in contact with it as shown in fig. The coefficient of friction between the block and the cart is μ. What should be the minimum acceleration of the cart so that the block of mass m does not fall?



PLM245

7. Is it unreasonable to expect the coefficient of friction to exceed unity? **PLM246**

8. It is known that polishing a surface beyond a certain limit increases (rather than decreases) the frictional force. Explain. **PLM247**

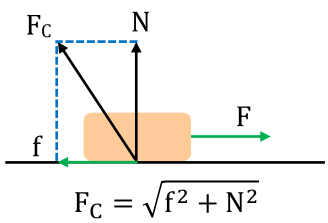
9. Why do we slip on a muddy road? **PLM248**

10. State whether the following statement is true or false :
When a person walks on a rough surface, the frictional force exerted by the surface on the person is opposite to the direction of his motion. **PLM249**

Contact Force and Angle of Friction

Contact Force

Let f be the force of friction and N the normal reaction, then the net contact force by the surface on the object is $F_{surface} = \sqrt{N^2 + f^2}$. Its minimum value (when f = 0) is N and maximum value (when f = μN) is $N\sqrt{1+\mu^2}$ Therefore $N \le F_{surface} \le N\sqrt{1+\mu^2}$



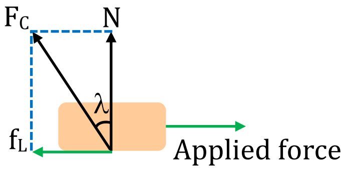
Angle of Friction (λ)

The angle which the resultant of the force of limiting friction f_L and normal reaction N makes with the direction of normal reaction N.

$$\tan\lambda = \frac{f_L}{N}$$

$$\tan\lambda = \frac{\mu N}{N}$$

$$\tan\lambda = \mu_S$$



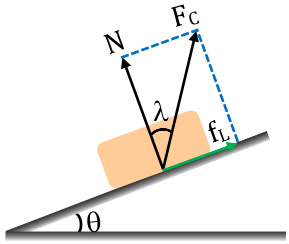


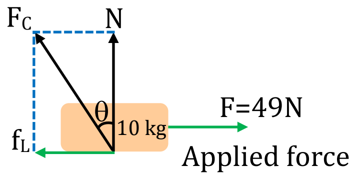
Illustration 32:

A horizontal force of 49 N is just able to move a block of wood weighing 10 kg on a rough horizontal surface. Calculate the coefficient of friction and angle of friction.

Solution:

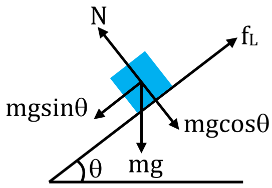
Here, F = 49 N, N = w = mg = 10 × 9.8N so $\mu = \frac{F}{N_s} = \frac{49}{10\times 9.8} = 0.5$

$\Rightarrow$ $\tan\theta = \mu = 0.5 \Rightarrow$ Angle of friction $\theta = \tan^{-1}(0.5)$



Angle of Repose or Angle of Sliding

It is defined as the minimum angle of inclination of a plane with the horizontal at which a body placed on it just begins to slide down or equivalently the maximum angle of inclination of plane with the horizontal at which a body placed on it down not slide.



$F_L = mg\sin\theta$ ...(i)

N = mg $\cos\theta$

$F_L = \mu N = \mu mg\cos\theta$ ...(ii)

From (i) and (ii)

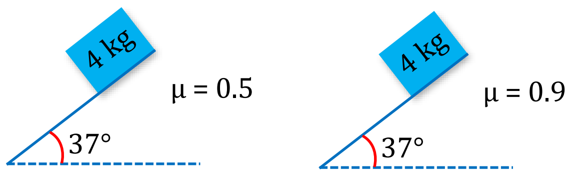
so $\mu_s = \frac{f_L}{N} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta \Rightarrow \theta = \tan^{-1}\mu_s$

This is fact is used for finding the coefficient of static friction in the laboratory.

Angle of repose (θ) = Angle of friction (λ)

Illustration 33:

Check in which case, extra force is needed for block to slide down.



Case-I **Case-II**

Solution:

Case-I : - Angle of repose = $\tan^{-1}(0.5) \Rightarrow \tan^{-1}(0.5) < 37°$

No extra force needed for block two slide down.

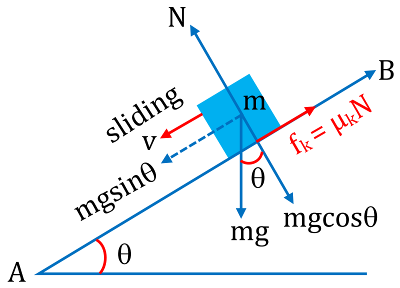
Case-II : - Angle of repose = $\tan^{-1}(0.9) \Rightarrow \tan^{-1}(0.9) > 37°$

Extra force needed for block two slide down.

Motion and Equilibrium on rough inclined surface

Downward sliding on rough incline plane

If angle of inclination is greater than the angle of repose, then the body accelerates down the incline. Net force on the body down the inclined plane



$F_{net} = mg\sin\theta - f_r$

Applying Newton's second laws of motion

$ma = mg\sin\theta - \mu_k N = mg\sin\theta - \mu_k\, mg\cos\theta$

$mg = mg[\sin\theta - \mu_k\cos\theta]$

$a = g[\sin\theta - \mu_k\cos\theta]$ hence $a < g$

Acceleration of a body down a rough inclined plane is always less than 'g'.

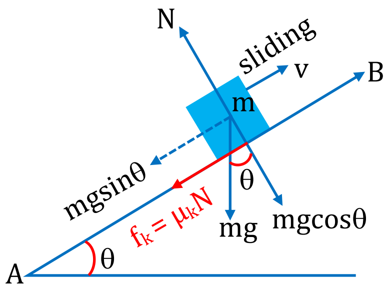
- For sliding down with constant velocity:

  $a = 0$

  $g\sin\theta = \mu_k g\cos\theta$

  $\tan\theta = \mu_k$

Upward sliding on rough incline



Body will suffer retardation

$$a = \frac{F_{total}}{mass}$$

$$a = \frac{mg\sin\theta + \mu_k mg\cos\theta}{m}$$

$$a = g\sin\theta + \mu_k g\cos\theta$$

Note :

(i) If we want to prevent the downward slipping of body then minimum upward force required is $= mg\sin\theta - \mu_k\, mg\cos\theta$

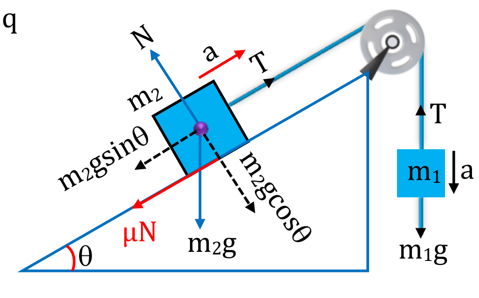
(ii) If a body is projected in upward direction along the inclined plane then retardation of body is $a = g[\sin\theta + \mu_k\cos\theta]$ retardation of a body up a rough inclined plane may be greater than 'g'

Pulley with friction between block and surface

For mass m_1 : $m_1g - T = m_1a$ and $N = m_2g\cos\theta \Rightarrow \mu N = \mu m_2 g\cos q$

For mass m_2 : $T - \mu m_2 g\cos\theta - m_2 g\sin\theta = m_2 a$

on solving,



$$\text{Acceleration } a = \left[\frac{m_1 - m_2(\sin\theta + \mu\cos\theta)}{(m_1 + m_2)}\right]g$$

$$\text{Tension } T = \frac{m_1 m_2(1 + \sin\theta + \mu\cos\theta)g}{(m_1 + m_2)}$$

Illustration 34:

A block of mass 2 kg slides down an inclined plane which makes an angle 30° with the horizontal. The coefficient of friction between the block and the surface is $\frac{\sqrt{3}}{2}$.

(i) What force must be applied to the block so that it moves down the plane without acceleration?

(ii) What force should be applied to the block so that it moves up without any acceleration?

Solution:

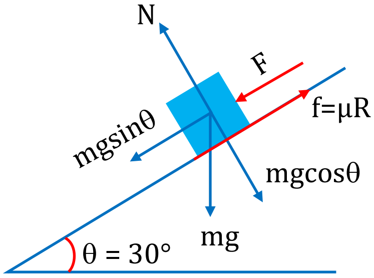
Make a 'free–body' diagram of the block. Take the force of friction opposite to the direction of motion.

(i) Project forces along and perpendicular to the plane. Perpendicular to plane N = mg cosθ

Along the plane F + mg sinθ – f = 0

($\because$ there is no acceleration along the plane)

F + mg sinθ – μN = 0 ⇒ F + mg sinθ = μmg cosθ



$F = mg\,(\mu\cos\theta - \sin\theta) = 2 \times 9.8\left(\frac{\sqrt{3}}{2}\cos 30° - \sin 30°\right)$

$= 19.6\left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2}\right) = 19.6\left(\frac{3}{4} - \frac{1}{2}\right) = 4.9\ \text{N}$

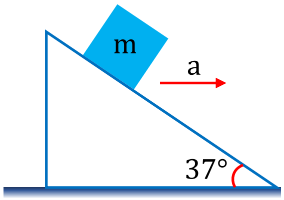
(ii) This time the direction of F is reversed and that of the frictional force is also reversed.

$\therefore$ N = mg cosθ ; F = mg sinθ + f

$\Rightarrow F = mg\,(\mu\cos\theta + \sin\theta) = 19.6\left(\frac{3}{4} + \frac{1}{2}\right) = 24.5\ \text{N}$

Illustration 35:

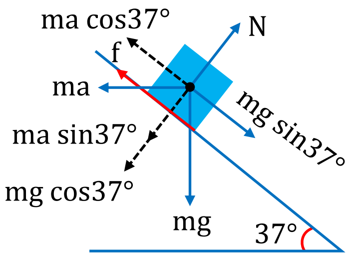
A block of mass 1 kg rests on an incline as shown in figure.



(a) What must be the frictional force between the block and the incline if the block is not to slide along the incline when the incline is accelerating to the right at 3 m/s^2?

(b) What is the least value of μ_s which can have for this to happen?

Solution:



N = m (g cos37° + a sin37°) = 1(9.8 × 0.8 + 3 × 0.6) = 9.64 N

mg sin 37° = ma cos 37° + f

(a) f = 1(9.8 × 0.6 – 3 × 0.8) = 3.48

(b) $\because$ f = μN ⇒ $\mu = \frac{f}{N} = \frac{3.48}{9.64} = 0.36$

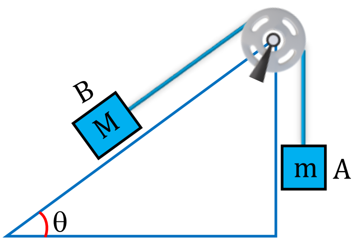
BEGINNER'S BOX-7

1. A cubical block rests on a plane of $\mu = \frac{1}{\sqrt{3}}$ determine the angle through which the plane be inclined to the horizontal, so that the block just slides down. **PLM250**

2. A body is in limiting equilibrium on a rough inclined plane at angle of 30° with horizontal. Calculate the acceleration with which the body will slide down, when inclination of the plane is changed to 60°. (Take g = 10 m/s^2) **PLM251**

3. A weight W rests on rough horizontal plane. If the angle of friction be θ, then calculate the least horizontal force that will move the body along the plane. **PLM252**

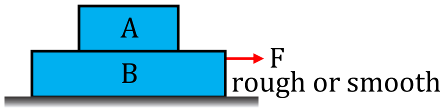
4. Two blocks A and B of masses m and M are connected to the two ends of a string passing over a pulley. B lies on plane inclined at an angle θ with the horizontal and A is hanging freely as shown. The coefficient of static friction between B and the plane is μ_S. Find the minimum and maximum values of m so that the system is at rest.



PLM253

Two Block System in Friction

Consider two blocks A and B placed one above the other, resting on a horizontal surface. A horizontal force F applied on either blocks, tend to move the system of blocks. Problems involving such situations can conveniently be solving by following mentioned step.



Step 1 : Draw the FBD of the combined block system. If friction appears in the FBD, then take its limiting value (maximum static friction)

If applied force > limiting friction then motion is possible, otherwise not.

If movement occurs then, either the blocks move together or separately depending on the fact that whether frictional forces are able to support the combined motion or not.

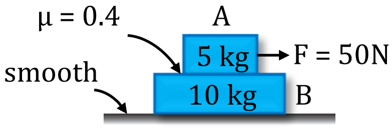
Step 2 : Assuming combined motion, find the common acceleration a_C. Draw the FBD of the body on which external force is not applied. Find the frictional force f required to make it move combinedly with the other block. Compare the above calculated force with the limiting value f_L (maximum static friction).

If $f \le f_L$, then both move together with common acceleration a_c. Otherwise, they move separately.

Step 3 : For separate motion, draw the individual FBD's of both blocks with kinetic friction forces acting wherever applicable. Find the individual accelerations of the two blocks using Newton's second law.

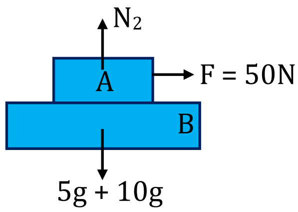
Illustration 36:

Calculate the accelerations of the blocks and the force of friction between them.



Solution:

Step 1 : Draw FBD of the combined block system.



Obviously, there is an unbalanced force (F = 50N) in the horizontal direction.

$\therefore$ Movement occurs.

Step 2 : Assuming that the blocks move together with common acceleration

$$a_c = \frac{50}{5+10} = 3.33 \text{ m/s}^2$$

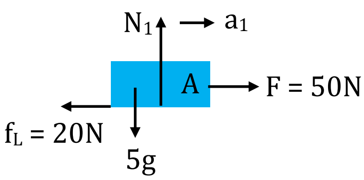
draw the FBD of block B (on which external force is not applied)

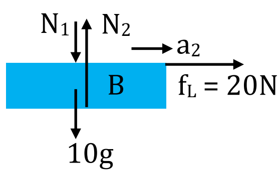
Required frictional force $f = m_B a_c = 10 \times 3.33 = 33.3$ N

Now limiting friction (maximum available static friction) $f_L = \mu N_1 = 0.4 \times 5 \times 10 = 20$ N

Obviously, f exceeds f_L. $\therefore$ The two blocks move separately.

Step 3 : Draw the individual FBD's of the two blocks



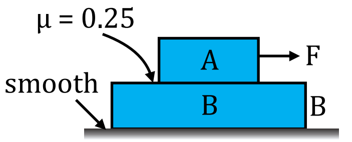


Applying Newton's II law, in the horizontal direction

$$a_1 = \frac{50-20}{5} = 6\text{m}/\text{s}^2 \text{ and } a_2 = \frac{20}{10} = 2\text{m}/\text{s}^2$$

Force of friction between the blocks= 20 N

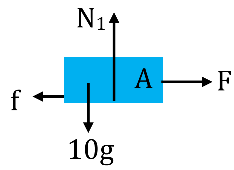
Illustration 37:



In the figure shown $m_A = 10$ kg, $m_B = 15$ kg. Find the maximum value of F, below which the block move together.

Solution:

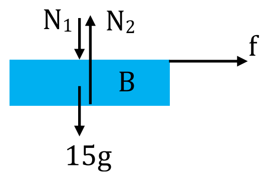
Assuming that both blocks move together, their common acceleration is



$$a_C = \frac{F}{10+15} = \frac{F}{25}$$

FBD of block B (on which no externally applied force acts).

The required friction force f is equal to $f = m_B a_C = \frac{15F}{25} = \frac{3F}{5}$

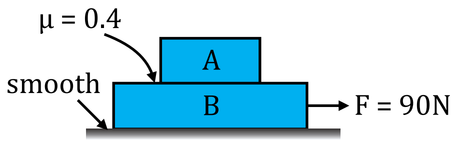


Now, maximum static friction available is

$f_L = \mu N_1 = 0.25(100) = 25$ N (here $N_1 = m_A g = 100$ N)

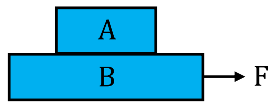
$$\therefore \qquad f \le f_L \Rightarrow \frac{3F}{5} \le 25 \Rightarrow F \le \frac{125}{3}\text{N} \Rightarrow F_{max} = \frac{125}{3}\text{N}$$

Illustration 38:



For the figure shown, $m_A = 10$ kg, $m_B = 20$ kg. F = 90 N. Find the acceleration of the two blocks and the frictional force between them.

Solution:



Step 1 : Draw the FBD of the two block system combinedly. Obviously, there is an unbalanced horizontal force

F = 90 N, so motion begins.

Step 2 : Assuming bot the blocks to move together, their common acceleration is

$$a_C = \frac{F}{m_A + m_B} = \frac{90}{10+20} = 3\text{m}/\text{s}^2$$

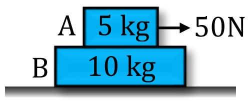
Draw the FBD of block A (the body on which no externally applied force acts).

The required frictional force $f = m_A a_C = 10 \times 3 = 30$ N

Now, maximum static friction (limiting friction) available is $f_L = \mu N_1 = 0.4 \times 10g = 40$ N

Obviously, $f < f_L$, so both move together with a common acceleration = 3m/s^2. The force of friction between them = 30 N

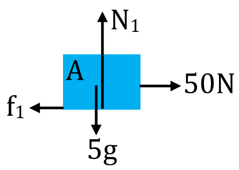
Illustration 39:

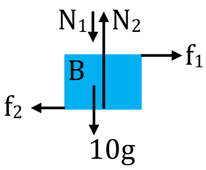


Blocks A and B of masses 5 kg and 10 kg are placed as shown in figure. If block A is pulled with 50 N. Find out the acceleration of A and B. If coefficient of friction between A and B is 0.5 and between B and ground is 0.4.

Solution:

FBDs of blocks:





Limiting friction between A and B, $f_{1L} = \mu_1 N_1 = 0.5 \times 5g = 24.5$ N

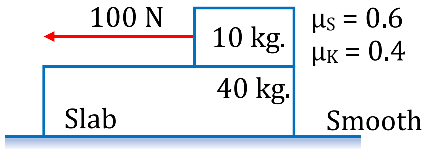
Limiting friction between B and ground $f_{2L} = \mu_2 N_2 = 0.4 \times 15g = 58.8$ N

For block A; $50 - f_1 = 5 \times a \Rightarrow 50 - 24.5 = 5 \times a \Rightarrow a = 5.1 \text{ m/s}^2$

So block B will remain at rest.

BEGINNER'S BOX-8

1. If 100 N force is applied to the 10 kg block as shown in diagram. What is the acceleration produced for slab and block ?

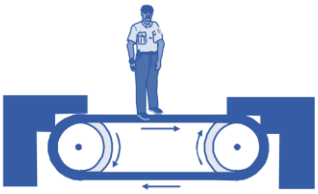


PLM254

2. A 3 kg block (A) is placed on a 6 kg block (B) which rests on a table. Coefficient of friction between (A) and (B) is 0.3 and between (B) and table is 0.6. A 30 N horizontal force is applied on the block (B), then calculate the frictional force between the blocks (A) and (B).

PLM255

3. Figure shows a man standing stationary with respect to a horizontal conveyor belt which is accelerating with 1 m/s^2. What is the net force on the man? If the coefficient of static friction between the man's shoes and the belt is 0.2, upto what acceleration of the belt can the man continue to remain stationary relative to the belt? (Mass of the man = 65 kg)



PLM256

BEGINNER'S BOX — ANSWER KEY

BEGINNER'S BOX-1

1. 4 cm/s^2 **2.** 6 s **3.** 0.18N, in the direction of motion

4. a = $2m/s^2$, 37° from 8N **5.** 23.89 Ns, 238.9 N

6. 240 **7.** 93.3N **8.** 3×10^{-5} Ns, 6×10^{-3} m/s

9. 5 s, 3Ns **10.** 3.18 Ns **11.** 5.0×10^{-3} N-s.

BEGINNER'S BOX-2

1. (i) 20M, (ii) 10M, (iii) 5M **2.** 1 ms^{-2}, (i) 8N (ii) 5 N

3. (i) (P/5m) (ii) P/5 (iii) 2P/5 (iv) 3P/5 (v) 4P/5

BEGINNER'S BOX-3

1. 2 : 5 : 10 **2.** $T = \frac{M(L-Y)}{L}g$ **3.** 30N, 50N **4.** T = F(x/L)

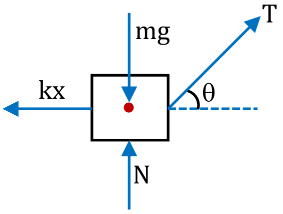
5. $T = F_1 + (F_2-F_1)(x/L)$ **6.** 25 N **7.** $T = \frac{Pm_2}{m_1+m_2}$ **8.** 5N, 6N, 7N

9. 14 N

BEGINNER'S BOX-4

1. $\frac{g}{3}, \frac{20g}{3}, \frac{40g}{3}$ **2.** $\frac{10}{3}$ kg – wt **3.** $T_1 = \frac{8g}{3}, T_2 = \frac{4g}{3}$

4. $\frac{g}{3}, \frac{10g}{3}, \frac{10\sqrt{2}g}{3}$ **5.** 2 m/s^2, 60N **6.** $a_{4\,kg} = 2.5\ m/s^2$, $a_{8\,kg} = 0$

7. (i) 

, (ii) mg cosec θ, (iii) $\frac{mg\cot\theta}{k}$

BEGINNER'S BOX-5

1. T_{OB} = 200N, $T_{OA} = 100\sqrt{3}$ N and T_{OW} = 100 N

2. $a = \left(\frac{m_1-m_2}{m_1+m_2}\right)(g + a_0)$, $T = \left(\frac{2m_1m_2}{m_1+m_2}\right)(g + a_0)$

BEGINNER'S BOX-6

1. 15N, 25N and 20N **2.** 25N **3.** 1 N **4.** 0.0816

5. $\frac{22}{23}$ m/s^2, 27.13 N **6.** $\frac{g}{\mu}$

7. For normal plane surfaces the coefficient of friction is less than unity. But when the surfaces are so irregular that sharp minute projections and cavities exist in the surfaces, the coefficient of friction may be greater than one.

8. When a surface is polished beyond a certain limit, the molecules of both surfaces come closer to each other to such an extent that inter molecular forces become appreciable, which exert strong attractive forces on each other. This is called surface adhesion. To overcome these forces, additional force is required. Hence the frictional force increases.

9. Water on a muddy road provides a thin layer in between our feet and road. This layer breakes the interlocking and decreases the friction.

10. False, when a person walks on a rough surface the man exerts a backward frictional force on the surface. As a result, the surface exerts a forward frictional force, according to Newton's III law.

BEGINNER'S BOX-7

1. 30°

2. $\frac{10}{\sqrt{3}}$ m/s^2

3. W tan θ

4. $m_{min} = M(\sin\theta - \mu\cos\theta)$ &
$m_{max} = M(\sin\theta + \mu\cos\theta)$

BEGINNER'S BOX-8

1. 0.98 m/s^2, 6.08 m/s^2

2. Zero

3. 65 N; 1.96 m/s^2