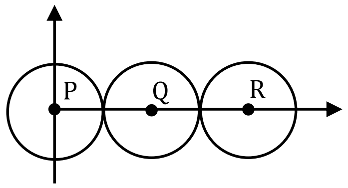
Collisions and Centre of Mass

SOLUTIONS

Exercise-I (Conceptual Questions)

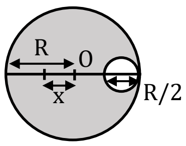
1.



Distance of C.O.M. of system from P is

$$=\frac{1\times PQ+1\times PR}{3}=\frac{PQ+PR}{3}$$

2. Mass of disc = M



Mass per unit area $=\frac{M}{\pi R^2}$

Mass of area of cutting part

$$=\frac{M}{\pi R^2}\times\pi\left(\frac{R}{4}\right)^2=\frac{M}{16}$$

Remaining mass $=\frac{15M}{16}$

$$\frac{15M}{16}\times x=\frac{M}{16}\times\frac{3R}{4}$$

$\Rightarrow\ x=\frac{R}{20}$ from centre O

3. $m_1 = M$ (Solid sphere)

$r_{x_1}=0$ $\qquad r_{y_1}=a$

Hollow sphere $\qquad m_2 = M$

$r_{x_2}=0$ $\qquad r_{y_2}=0$

Disc. $m_3 = M$

$r_{x_3}=a$ $\qquad r_{y_3}=0$

$$r_{x_{cm}}=\frac{m_1r_{x_1}+m_2r_{x_2}+m_3r_{x_3}}{m_1+m_2+m_3}=\frac{0+0+aM}{3M}=\frac{a}{3}$$

$$r_{y_{cm}}=\frac{m_1r_{y_1}+m_2r_{y_2}+m_3r_{y_3}}{m_1+m_2+m_3}=\frac{Ma+0+0}{3M}=\frac{a}{3}$$

So co-ordinate of centre of mass of system will be

$$\left(\frac{a}{3},\frac{a}{3}\right)$$

4. The centre of mass of the system of particles does not depend on internal forces on the particles.

If $\vec{F}_{ext.}=\vec{0}$ then $\vec{v}_{CM}$ = constant

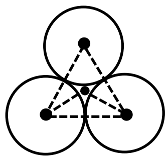
If no external force acts on a system the velocity of its centre of mass remains constant, i.e., velocity of centre of mass is unaffected by internal forces.

5. $m_1r_1 = m_2r_2$

$$\frac{r_1}{r_2}=\frac{m_2}{m_1}\qquad\Rightarrow\qquad r\propto\frac{1}{m}$$

6. COM may lie within (i.e. in solid sphere), outside of the surface of the body (L shaped Lamina)

7.



The COM of the system is located at the point of intersection of their median.

8. Centre of mass is towards heavier mass and

$m << M$.

So centre of mass of the system in nearer to M.

9.

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3}{m_1+m_2+m_3}$$

$$\vec{r}_{cm}=0$$

$$\Rightarrow\ m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3=0$$

$$\vec{r}_{cm}=\frac{m_1\vec{r}_1+m_2\vec{r}_2+m_3\vec{r}_3+m_4\vec{r}_4}{m_1+m_2+m_3+m_4}$$

$$\left(\hat{i}+2\hat{j}+3\hat{k}\right)=\frac{0+4\alpha(\hat{i}+2\hat{j}+3\hat{k})}{1+2+3+4}$$

$$\Rightarrow\qquad\alpha=\frac{5}{2}$$

10. By third law for a system

$\vec{F}_{12} = -\vec{F}_{21}$ or $d\vec{P}_1 = -d\vec{P}_2$

11. Let plank shifted by x then

$\Delta x_{CM} = 0$, as there is no external force on the system.

$m(L - x_{plank}) - Mx_{plank} = 0$

$x_{plank} = \dfrac{mL}{M+m}$

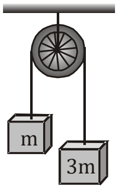
12. $200 = n \times \dfrac{40}{1000} \times 10^3$

n = 5 bullets/sec

$\therefore$ n = 300 bullets/min

13. $a = \dfrac{(3m-m)}{3m+m} g = \dfrac{g}{2}$

$\vec{a}_{cm} = \dfrac{3m\vec{a}_1 + m\vec{a}_2}{3m+m}$



Both mass have same magnitude of accleration but in opposite direction $\vec{a}_1 = -\vec{a}_2 = a$ Let)

$a_{cm} = \left(\dfrac{3m-m}{4m}\right) \times \dfrac{g}{2} = \dfrac{g}{4}$

14. Since in this system their is absence of external force which can move the centre of mass.

Here particles are moving due to mutual interaction between them that's why velocity of COM of the system will be zero.

15. $\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$

$m_1 = 2kg$ $\quad$ $m_2 = 3kg$

$|\vec{v}_1| = 3m/s$ $\quad$ $|\vec{v}_2| = 2m/s$

$\vec{v}_{cm} = \dfrac{2\times3+3\times2}{2+3} = \dfrac{12}{5} m/s$

16. $m_1 = 200$ gm $\quad$ $m_2 = 500$ gm

$\vec{v}_1 = 10\hat{i}$ $\quad$ $\vec{v}_2 = 3\hat{i} + 5\hat{j}$

$\vec{v}_{cm} = \dfrac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \dfrac{200\times10\hat{i} + 500(3\hat{i}+5\hat{j})}{200+500}$

$\vec{v}_{cm} = \dfrac{20\hat{i} + 15\hat{i} + 25\hat{j}}{7} = 5\hat{i} + \dfrac{25\hat{j}}{7}$

17. 3 × 16 = 6 × v

v = 8 m/s $\quad \therefore \quad KE = \dfrac{1}{2} \times 6 \times 8 \times 8 = 192J$

18. $m\left(3\hat{i} + 2\hat{j}\right) + m\left(-\hat{i} - 4\hat{j}\right) + m\vec{v} = 0$

$\therefore \quad \vec{v} = -2\hat{i} + 2\hat{j}$

19. By COLM

$50 \times 600 = 10^3 \times v$

v = –30 m/s

negative sign show's opposite direction

20. For rubber ball $\Delta\vec{p}$ is more

as v_f will be in opp. direction

$\Delta\vec{p} = m\left(\vec{v}_f - \vec{v}_i\right)$

21. By $\vec{v} = \vec{u} + \vec{a}t$

$\vec{v} = 100\hat{j} - 10\hat{j} \times 5 = 50\hat{j}\, m/s$

using COLM $\vec{p}_f = \vec{p}_i$

$0.6\vec{v}_2 - 0.4 \times 25\hat{j} = 1 \times 50\hat{j}$

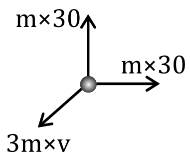
$0.6\vec{v}_2 = 60\hat{j}$

$\vec{v}_2 = 100\hat{j}$ m/s

22. Total momentum = 0

$\therefore \quad m \times 30 \times \sqrt{2} = 3m \times v$

$v = 10\sqrt{2}\, m/s$



23. $M_1 \times 8v = M_2 v$

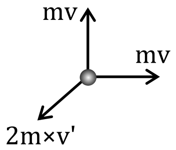
$M_2 = 8M_1$ $\quad \left(\because M = \dfrac{4}{3}\pi r^3 \rho, M \propto r^3\right)$

$\therefore \quad r_2 = 2r_1 \quad \Rightarrow \quad \dfrac{r_1}{r_2} = \dfrac{1}{2}$

24. $KE = \dfrac{p^2}{2m}$

$\therefore \quad \dfrac{KE_1}{KE_2} = \dfrac{m_2}{m_1} \quad (\because p = same)$

25.



$\therefore \quad 2mv' = mv\sqrt{2} \Rightarrow v' = \frac{v}{\sqrt{2}}$

$\therefore$ KE released

$$= \frac{1}{2}mv^2 \times 2 + \frac{1}{2}2m \times \frac{v^2}{2} = \frac{3}{2}mv^2$$

26. Total energy = $\frac{P^2}{2m_1} + \frac{P^2}{2m_2}$

$$= \frac{P^2}{2}\left[\frac{1}{m_1} + \frac{1}{m_2}\right]$$

$$= \frac{(1\times 80)^2}{2} \times \left[\frac{1}{1} + \frac{1}{2}\right] = 4.8 \text{ kJ}$$

27. $18 \times 6 = 12 \times v$

$v = 9$ m/s

$\therefore \quad KE = \frac{1}{2} \times 12 \times 9 \times 9 = 486J$

28. $mv = MV$; $MV^2 = kd^2$

$\therefore V = \sqrt{\frac{kd^2}{M}} \quad \therefore v = \frac{M}{m}\sqrt{\frac{kd^2}{M}} = \frac{d}{m}\sqrt{kM}$

29. Momontum is conserved in all collision.

30. Consider ball and earth as a system.

31. By Conservation of linear momentum

$p_i = p_f$

$PQ + R(0) = (P + R)V$

$V = PQ/(P + R)$

32. By COLM

$mu + 0 = mv_1 + mv_2$ ...(1)

$e = \frac{v_2 - v_1}{u - 0}$ ...(2)

$\Rightarrow v_2 - v_1 = eu$ ...(3)

from (1)

$u = v_1 + v_2$ ...(4)

putting (4) in (1)

$v_2 - v_1 = e(v_1 + v_2)$

$\Rightarrow v_2 - ev_2 = v_1 + ev_1$

$\frac{v_1}{v_2} = \frac{1-e}{1+e}$

33. For same mass and e = 1 velocities exchange

34. 'e' is unit less

35. $m_1\vec{u}_1 + m_2\vec{u}_2 = 0 + 0$

$200 \times 0.3 + 400 \times \vec{u}_2 = 0$

$\vec{u}_2 = -0.15$ m/s

36. $\sqrt{2gh_f} = e\sqrt{2gh_i}$

$\Rightarrow e = \sqrt{\frac{h_f}{h_i}} = \sqrt{\frac{9}{25}} = \frac{3}{5} = 0.6$

37. $50 \times 10 = 1000 \times v \quad \therefore \quad v = \frac{1}{2}$ m/s

$E_i = \frac{1}{2} \times \frac{50}{1000} \times 10 \times 10 = 2.5J$

$E_f = \frac{1}{2} \times \frac{1000}{1000} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}J$

% loss = $\frac{2.5 - 1/8}{2.5} \times 100 = 95\%$

38. $mv = (M + m)v_f \quad \therefore \quad v_f = \frac{m}{M+m}v$

39. Let a ball fall from a height (h) and let it touch the ground with a velocity v taking time (t) to reach the ground.

Let v_1, v_2, v_3....... be the velocities immediately after first, second, third......collisions with the ground.

Height attained by the Ball After the 'n'th Rebound

$v_1 = ev \Rightarrow \sqrt{2gh_1} = e\sqrt{2gh} \Rightarrow h_1 = e^2h,$

$v_2 = e^2v \Rightarrow \sqrt{2gh_2} = e^2\sqrt{2gh} \Rightarrow h_2 = e^4h.$

Similarly $\boxed{h_n = e^{2n}h}$

40. By COLM

$mv - mv = 0$

41. Velocity interchange when mass are equal

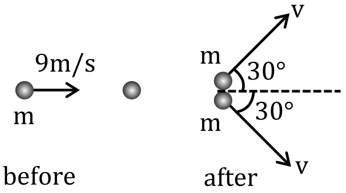
So KE = mgL

42. $mu_1 + mu_2 = 2mv$

$KE_{loss} = \frac{1}{2}(2m)v^2 - \frac{1}{2}mu_1^2 - \frac{1}{2}mu_2^2$

$$\frac{m}{2}\left[2\left(\frac{u_1+u_2}{2}\right)^2 - u_1^2 - u_2^2\right] = \frac{m}{4}(u_1 - u_2)^2$$

43.



$$2mv\times\frac{\sqrt{3}}{2}=m\times9$$

$$v=\frac{9}{\sqrt{3}}\ m/s = 3\sqrt{3} = 5.2\ m/s$$

44. Mass must be same

$$\therefore \quad \rho_1\times\frac{4}{3}\pi r^3=\rho_2\times\frac{4}{3}\pi\left(\frac{r}{2}\right)^3$$

$$\frac{\rho_2}{\rho_1}=8$$

45. $5\times v=(5+m)\frac{v}{3}$

$\Rightarrow \quad m = 10$ kg

46. $\frac{10}{1000}\times500=(10+0.01)\times v$

$\Rightarrow \quad v = 0.5$ m/s

47. $v_2 = -u_2 + 2u_1$ (using COLM & e = 1)

$v_2 = 0 + 2(20) = 40$ m/s

48. $5 \times 10 - 2 \times 15 = 7 \times v$

$v=\frac{20}{7}$ = 2.8 cm/s in east

49. Momentum is conserved.

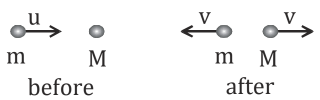
50. $20 \times 10 = 25 \times v$

$v=\frac{200}{25}$ = 8 m/s

$$\therefore \quad KE=\frac{1}{2}\times25\times8\times8=800J$$

51. Velocity interchange when m = M

52.



$$mu = (M - m)v \qquad ...(1)$$

$$e=\frac{2v}{u}=1$$

$$\therefore \quad v=\frac{u}{2} \qquad ...(2)$$

$$\therefore \quad m=(M-m)\times\frac{1}{2}$$

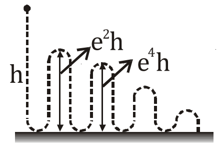
$$2m = M - m$$

$$3m = M$$

$$\frac{m}{M}=\frac{1}{3}$$

53. e = 0, for perfectly inelastic.

54.



Total distance = $h + 2e^2h + 2e^4h + ...$

$= h + 2e^2h(1 + e^2 + e^4+...)$

$$= h+2e^2h\times\frac{1}{1-e^2}$$

$$= h\left[1+\frac{2e^2}{1-e^2}\right]=\left(\frac{1+e^2}{1-e^2}\right)h$$

55. According to the law of conservation of momentum

$$m_1\vec{u}_1+m_2\vec{u}_2=m_1\vec{v}_1+m_2\vec{v}_2 \qquad ......(i)$$

Here $\vec{u}_1,\vec{u}_2$ = Initial velocities of m_1 and m_2

$\vec{v}_1,\vec{v}_2$ = Final velocities of m_1 and m_2

Change in velocity of m_1 = $\vec{v}_1-\vec{u}_1$

Change in velocity of m_2 = $\vec{v}_2-\vec{u}_2$

$$m_1\vec{v}_1-m_1\vec{u}_1=-m_2\vec{v}_2+m_2\vec{u}_2$$

or $m_1\left(\vec{v}_1-\vec{u}_1\right)=m_2\left(-\vec{v}_2+\vec{u}_2\right)$

or $\frac{\vec{v}_1-\vec{u}_1}{-\vec{v}_2+\vec{u}_2}=\frac{m_2}{m_1}$

$$\therefore \quad \frac{\text{changes in velocity of } m_1}{\text{changes in velocity of } m_2}=\frac{m_2}{m_1}$$

56. $\frac{M_A}{M_B}=1$

By COLM $M_Av_A = M_Bv_B$ = $\frac{v_A}{v_B}=\frac{M_B}{M_A}=1$

57. In option (3)

It is not possible that after collision one ball moves along the original line of motion while the other ball moves along some angle (α) with original line.The momuntum perpendicular to original line of motion cannot be conserved in this situation.

Initial momentum along perpendicular direction=zero

Final momentum along perpendicular direction = $m_2v_2 \sin\alpha$. Hence momentum is not conserved. Hence the situation is physically impossible. Rest option may be physically possible.

58. Velocity would interchange

59. For elastic oblique collision having object of same mass. After collision they move perpendicular to each other.

60. Initial energy = mgh = mg(10)

if 40% is lost

Remaining energy = 60% of mg(10)

$$\Rightarrow \frac{60}{100} \times mg(10) = mg(h)$$

h = 6m

61. $$e = \sqrt{\frac{h_f}{h_i}} = \sqrt{\frac{1.8}{5}} = 0.6 = \frac{3}{5}$$

loss in velocity = $1 - \frac{3}{5} = \frac{2}{5}$

62. $\vec{p}$ conserved in all collision

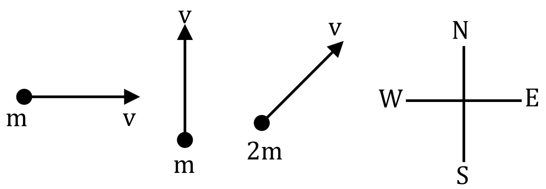
KE conserved only in elastic collision.

63. $mv = (M + m)v_f$

$$v_f = \left(\frac{m}{m+M}\right)v$$

64. $Mu = mu + Mv \Rightarrow v = \frac{u(M-m)}{M}$

65.



According to the law of conservation of linear momentum along horizontal direction,

$mv + 0 = 2mv'_x$

$$v'_x = \frac{v}{2}$$

According to the law of conservation of linear momentum along vertical direction,

$0 + mv = 2\,mv'_y$

$$v'_y = \frac{v}{2}$$

$\therefore$ Speed of the new mass 2m is

$$v' = \sqrt{v'^2_x + v'^2_y} = \sqrt{\left(\frac{v}{2}\right)^2 + \left(\frac{v}{2}\right)^2} = \frac{v}{\sqrt{2}}$$

66. By Conservation of linear momentum

$\vec{p}_i = \vec{p}_f$

in x direction

$m_A\ v_A = (m_A + m_B)v_x$

in y direction

$m_B\ v_B = (m_A + m_B)v_y$

$$v_x = \frac{30}{50} m/s$$

$$v_y = \frac{40}{50} m/s$$

$$v_{net} = \sqrt{(v_x)^2 + (v_y)^2} = \sqrt{\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2}$$

= 1 m/s

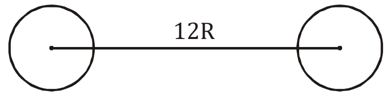
Exercise-II (Previous Year Questions)

1. Energy will always be conserved so

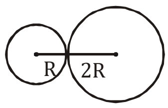
$K.E_{initial} = K.E_{final}$ + Excitation energy

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + \varepsilon$$

2.



Initial distance between their centers = 12 R



At time of collision the distance between their centers = 3R

So total distance travelled by both

=12R–3R = 9R

Since the bodies move under mutual forces, center of mass will remain stationary so

$m_1x_1 = m_2x_2$

mx = 5m(9R –x)

x = 45R – 5x

6x = 45R

$$x = \frac{45}{6}R \qquad \boxed{x = 7.5R}$$

3. Let ball rebounds with speed v so

$$v = \sqrt{2gh} = \sqrt{2\times10\times20} = 20\ m/s$$

Energy just after rebound

$$E = \frac{1}{2}\times m\times v^2 = 200\ m$$

50% energy lost in collision means just before collision energy is 400 m

By using energy conservation

$$\frac{1}{2}mv_0^2 + mgh = 400m$$

$$\Rightarrow \frac{1}{2}mv_0^2 + m\times10\times20 = 400m$$

$\Rightarrow v_0 = 20\ m/s$

4. In elastic collision energy of system remains same so.

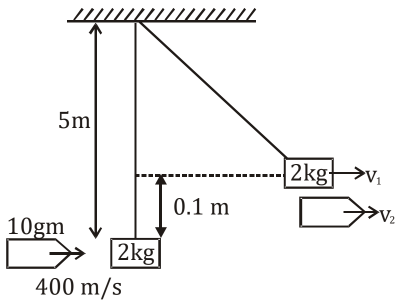
$(K.E)_{\text{before collision}} = (K.E)_{\text{After collision}}$

Let speed of second body after collision is v'

$$\frac{1}{2}mv^2 + 0 = \frac{1}{2}m\left(\frac{v}{3}\right)^2 + \frac{1}{2}m(v')^2$$

$$\Rightarrow \quad v' = \frac{2\sqrt{2}}{3}v$$

5.



Applying momentum conservation

$$\frac{10}{1000}\times400 + 0 = 2\times v_1 + \frac{10}{1000}\times v_2$$

$\Rightarrow\ 4 = 2v_1 + 0.01v_2 \qquad ......(1)$

Applying work energy theorem for block

$W = \Delta KE$

$$\Rightarrow\ 2\times10\times0.1 = \frac{1}{2}\times2\times v_1^2$$

$\Rightarrow\ v_1 = \sqrt{2} = 1.4\ m/s$

Putting the value of v_1 in equation (1)

$4 = 2\times1.4 + 0.01\ v_2 \Rightarrow v_2 = 120\ m/s$

6. Since both bodies are identical and collision is elastic.

Therefore velocities will be interchanged after collision.

$v_A = -0.3\ m/s$ and $v_B = 0.5\ m/s$

7. By conservation of linear momentum

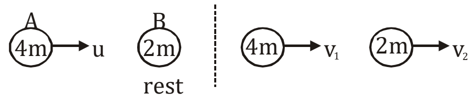
$$mv = 4mv' \Rightarrow v' = \frac{v}{4}$$

coefficient of restitution

$$(e) = \frac{\text{Velocity of separation}}{\text{Velocity of approach}}$$

$$= \frac{\frac{v}{4}-0}{v-0} = \frac{1}{4} = 0.25$$

8.



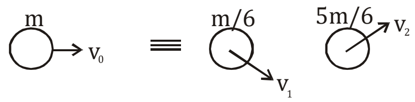
$$v_1 = \frac{4m-2m}{4m+2m}u = \frac{2mu}{6m} = \frac{u}{3}$$

Fraction of energy lost

$$= \frac{\frac{1}{2}(4m)u^2 - \frac{1}{2}(4m)\left(\frac{u}{3}\right)^2}{\frac{1}{2}(4m)u^2}$$

$$= 1 - \frac{1}{9} = \frac{8}{9}$$

9.



Conservation of linear momentum,

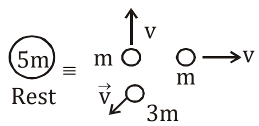
$$m\vec{v}_0 = \frac{m}{6}\vec{v}_1 + \frac{5m}{6}\vec{v}_2$$

$$\Rightarrow\ m(20\hat{i} + 25\hat{j} - 12\hat{k})$$

$$= \frac{m}{6}(100\hat{i} + 35\hat{j} + 8\hat{k}) + \frac{5m}{6}\vec{v}_2$$

$$\Rightarrow\ \vec{v}_2 = 4\hat{i} + 23\hat{j} - 16\hat{k}$$

10.



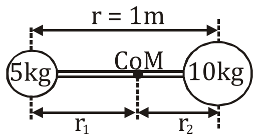
$$3m\vec{v} + mv\hat{i} + mv\hat{j} = 0$$

$$\Rightarrow\ \vec{v} = -\frac{v}{3}\hat{i} - \frac{v}{3}\hat{j} \qquad |\vec{v}| = \sqrt{2}\frac{v}{3}$$

Energy released

$$= \frac{1}{2}mv^2 + \frac{1}{2}mv^2 + \frac{1}{2}(3m)\left(\frac{2v^2}{9}\right) = \frac{4}{3}mv^2$$

11.

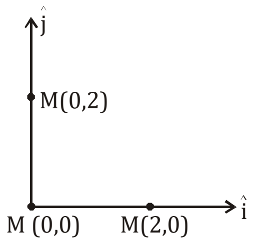


About CoM, mr = constant $\Rightarrow$ $r \propto \frac{1}{m}$

$\Rightarrow \frac{r_1}{r_2} = \frac{m_2}{m_1} = \frac{10}{5} = \frac{2}{1}$

$\Rightarrow r_1 = \frac{2}{3} r = \frac{2}{3} \times 1$ m = 67 cm

12.



$X_{CM} = \frac{M \times 0 + M \times 2 + M \times 0}{3M} = \frac{2}{3}$

$Y_{CM} = \frac{M \times 0 + M \times 2 + M \times 0}{3M} = \frac{2}{3}$

Position vector = $\vec{r} = X_{CM}\hat{i} + Y_{CM}\hat{j} = \frac{2}{3}\hat{i} + \frac{2}{3}\hat{j}$

16. In elastic collision maximum energy is transfer when M = m

17. Final velocity of centre of mass = Initial velocity of centre of mass = 0 because net external force on system is zero.

18. By Conservation of linear momentum :-

$mv_1 = (m + m) v_2$

$\Rightarrow mv_1 = 2mv_2$

$\Rightarrow \frac{v_1}{v_2} = 2:1$

Exercise-III (Analytical Questions)

1. Acceleration of centre of mass $\vec{a}_{cm} = \frac{\vec{F}_{ext.}}{M}$

If no external force acts on the system

$\vec{a}_{cm} = 0$ (Assertion is correct)

Reason is false because; $\vec{a}_{cm} = 0$ implies velocity will not change, if C.O.M is initially at rest, then it will remain at rest in absence of external force.

3. (A) Both particles will touch the ground together.

(B) C.O.M will follow same path.