03 Kinematics

Motion in a Straight Line

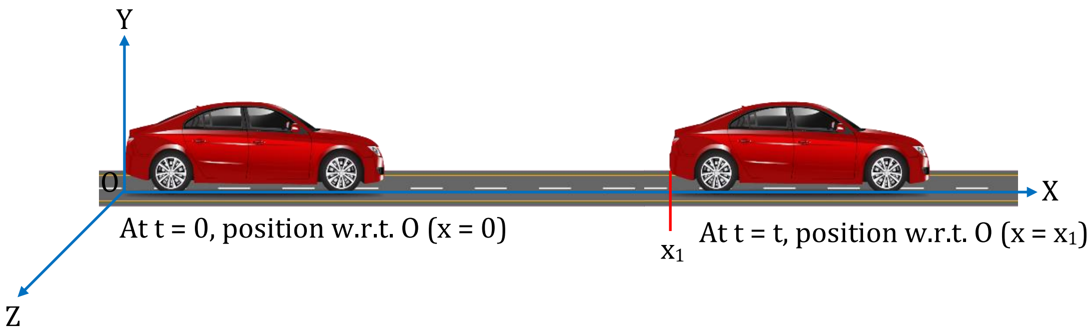
Introduction of Kinematics

Rest

If position does not change with time, then it is at rest.

Motion

If position of particle changes with time, then it is called in motion.



Frame of Reference

1. It is the reference with respect to which position or motion of particle is defined.
2. Motion and Rest are relative terms which depends on Frame of reference.
3. If not specified, we consider ground as reference frame.

Conclusion (With Respect to a Reference point)

1. If position of particle changes with time, then it is in Motion.
2. If position of particle does not change with time, then it is in Rest.

Illustration 1:

Give an example where object is at rest and also in motion at the same time

Solution:

Whether a object is in rest or motion depends upon the reference frame by which you are observing, Different reference frame may give different answers. Example: If you are travelling in a train and a woman is sitting near you. you will see the woman near you as stationary but if someone is observing the woman from outside the train he will see as if the woman is moving with the speed of train. Hence the two different reference point has given two results.

Types of Motion

1-D Motion:

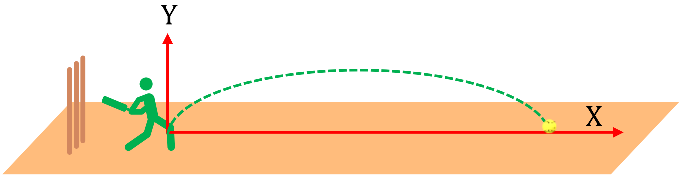
If position changes with time with respect to frame of reference along a straight line, motion is one dimensional (1-D) or straight-line motion or rectilinear motion.

Examples: Motion of Train Along Straight Track , A Ball Projected Upward.

2-D Motion:

If position changes in a plane with time with respect to frame of reference then motion is 2-D.

Examples: Projectile Motion



Examples: Circular Motion

3-D Motion:

If position changes in space with time with respect to frame of reference, motion is 3-D or motion in space.

Examples: Motions of Kite

Illustration 2:

Short following examples according to type of motion.

(1) Motion of an aeroplane

(2) Earth revolving around the sun

(3) Motion of the wheels of moving train

(4) Train running on a straight track.

(5) Lizard moving on the wall

(6) A sprinter running a 100 m race on straight track.

(7) A ball dropped from a building

(8) An out swinging delivery in cricket

Solution:

1D: - Train running on a straight track, A sprinter running a 100 m race on straight track, A ball dropped from a building.

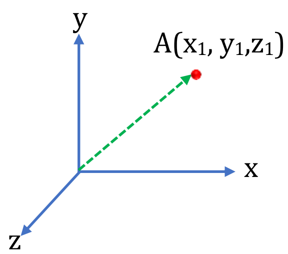
2D: - Earth revolving around the sun, Motion of the wheels of moving train, Lizard moving on the wall.

3D: - Motion of an aeroplane, An out swinging delivery in cricket.

Distance, Displacement and Position Vector

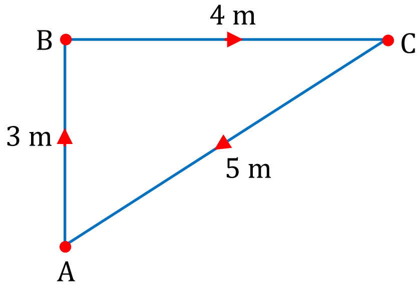
Position Vector

- It is used to specify the position of a certain particle.
- A vector that symbolizes the position of any given point with respect to any reference point like the origin.
- **Direction:** Always points from the Reference point (origin) of that vector towards a given point.



Distance

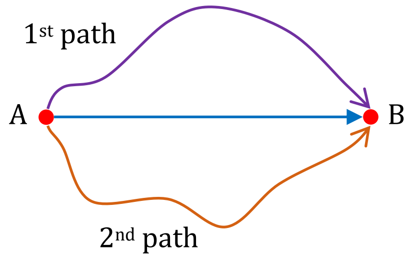
Actual length of the path traversed by the particle between initial and final position is called distance.



Distance travelled by man through path ABCA = 3 + 4 + 5 = 12m

Key Point:

- Distance always depends on path which is followed by the particle. Does not depend on initial and final position of the particle.

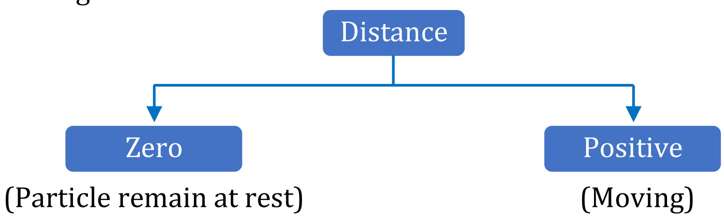


$(\text{Distance})_1 \neq (\text{Distance})_2$

- Infinite distances are possible between two fixed points because infinite paths are possible between two fixed points.
- It is a Scalar Quantity.
- Unit

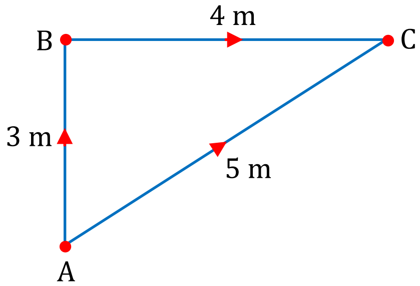
  S.I. $\rightarrow$ Metre (m)

  C.G.S. $\rightarrow$ Centimetre (cm)
- Dimension: $[M^0L^1T^0]$
- Distance cannot be negative.



Displacement

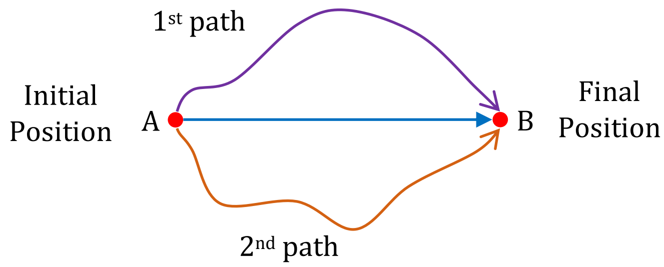
Shortest distance between initial & final position of the particle is called displacement.



Displacement of man when moving from A to B and B to C = 5m

Key Point:

- Displacement is a vector quantity, and its direction is always from initial position to final position.
- Displacement does not depend on path followed by particle. It depends only on initial & final position of the particle.



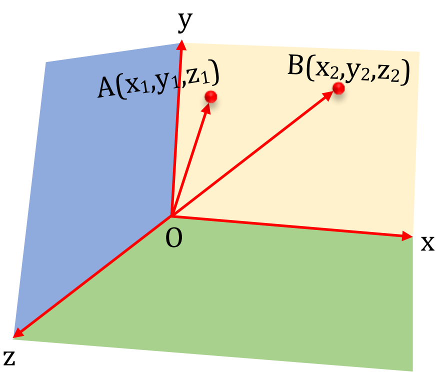
$(\text{Displacement})_1 = (\text{Displacement})_2$

- Only single value of displacement is possible between two fixed points.
- Displacement may be positive, negative or zero.
- If motion is in straight line without change in direction then distance = |displacement| = magnitude of displacement.
- Magnitude of displacement may be equal or less than distance but never greater than distance. i.e., distance $\geq$ |displacement|

Displacement

Zero	Positive/Negative
(Particle may be or may not be at rest)	(Moving)

Displacement in Vector Form



Initial position vector $\left(\overrightarrow{OA}\right)$

$\vec{r}_1 = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

Final position vector $\left(\overrightarrow{OB}\right)$

$\vec{r}_2 = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$

Displacement vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$

Illustration 3:

A particle starts from point A (1, –1, 0) m and reaches at point B (5, 3, –2) m, reference point is origin. Find:

(i) Initial position vector (ii) Final position vector
(iii) Displacement vector (iv) Length of shortest path

Solution:

(i) Initial position vector $\left(\overrightarrow{OA}\right) \Rightarrow \overrightarrow{OA} = \hat{i} - \hat{j}$

(ii) Final position vector $\overrightarrow{OB} \Rightarrow \overrightarrow{OB} = 5\hat{i} + 3\hat{j} - 2\hat{k}$

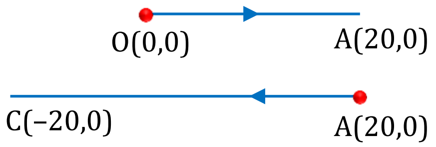
(iii) Displacement vector $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \overrightarrow{OB} - \overrightarrow{OA} = (5-1)\hat{i} + (3-(-1))\hat{j} + (-2-0)\hat{k} = \left(4\hat{i} + 4\hat{j} - 2\hat{k}\right)$

(iv) Length of shortest path $|\Delta\vec{r}| = \sqrt{4^2 + 4^2 + 4} = 6\text{m}$

Illustration 4:

A particle starts from the origin, goes along the X-axis upto the point (20m, 0) and then returns along the same line to the point (–20m, 0). Find the distance and displacement of the particle during the trip.

Solution:

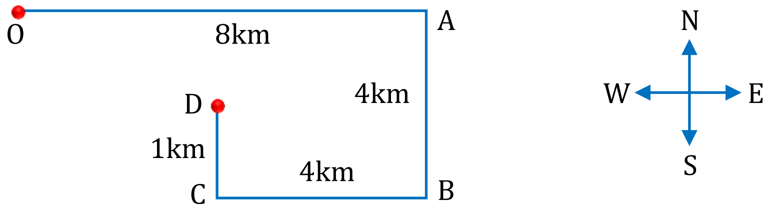


Distance = $|\overrightarrow{OA}|+|\overrightarrow{AC}|$ = 20 + 40 = 60m

Displacement = $\overrightarrow{OA}+\overrightarrow{AC} = 20\hat{i}+(-40\hat{i})=(-20\hat{i})$m

Illustration 5:

A car moves from O to D along the path OABCD shown in fig. What is distance travelled and its net displacement?



Solution:

$$\text{Distance} = |\overrightarrow{OA}|+|\overrightarrow{AB}|+|\overrightarrow{BC}|+|\overrightarrow{CD}| = 8 + 4 + 4 + 1 = 17 \text{ km}$$

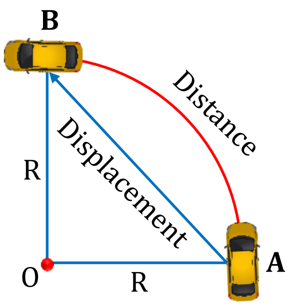
$$\text{Displacement } = \overrightarrow{OA}+\overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CD} = 8\hat{i}+(-4\hat{j})+(-4\hat{i})+\hat{j}=4\hat{i}-3\hat{j} \Rightarrow |\text{displacement}|=\sqrt{(4)^2+(3)^2}=5$$

So, Displacement = 5km, 37° S of E

Distance and Displacement in circular path

A car starts moving from point A and reaches at point B along a circular path then find out distance and displacement in following cases.

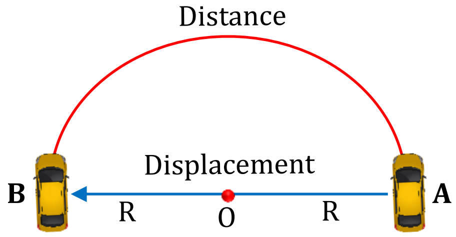
(1) One-fourth of the complete circle



Distance = $\frac{\pi R}{2}$

Displacement = $\sqrt{2}R$

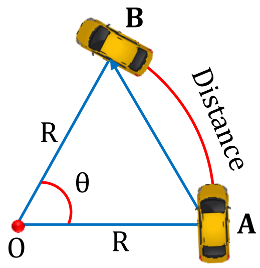
(2) Half of the complete circle



Distance = πR

Displacement = 2R

(3) At any arbitrary arc



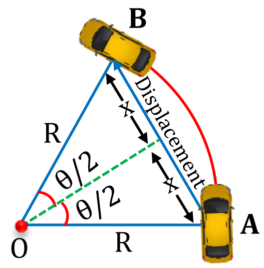


Illustration 6:

A cyclist moving on a circular track, completes one revolution in 10 sec. Then find out its displacement after 1 minute 5 s (radius = 1m)

Solution:

In 1 minute, 5s number of revolutions = 6.5

$\therefore$ Displacement = 2r = 2m

Useful Direction Conventions

Sense of Direction in terms of base vectors.

East : $\hat{i}$

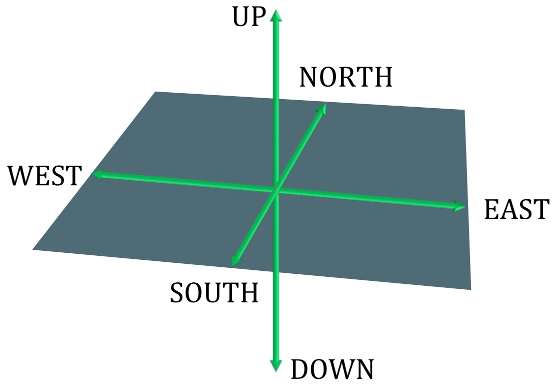
West : $-\hat{i}$

North : $\hat{j}$

South : $-\hat{j}$

Up : $\hat{k}$

Down : $-\hat{k}$



Sense of Direction in terms of base vectors.

Illustration 7:

A person moves 30 m due west, then 20 m due south then 20 m due east. Find out displacement.

Solution:

$$\vec{d}_1 = -30\hat{i}, \vec{d}_2 = -20\hat{j}, \vec{d}_3 = 20\hat{i}$$

$$\vec{d} = \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (-10\hat{i} - 20\hat{j})\text{m}$$

$$|\vec{d}| = 10\sqrt{5}\text{m}$$

$$\tan\theta = 2 \Rightarrow \theta = \tan^{-1}(2)$$

$$\vec{d} = 10\sqrt{5}\text{m}, \tan^{-1}(2)\text{S of W or } \vec{d} = 10\sqrt{5}\text{m. } \tan^{-1}(2) \text{ (w to s)}$$

Illustration 8:

A Body moves 6 m north 8 m east and 10m vertically upwards, what is its resultant displacement from initial position.

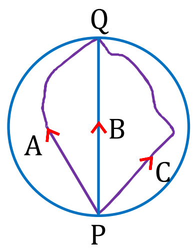
Solution:

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} \therefore r = \sqrt{x^2 + y^2 + z^2} = \sqrt{6^2 + 8^2 + 10^2} = 10\sqrt{2}\text{ m}$$

BEGINNER'S BOX-1

1. A particle moves on a circular path of radius 'r', It completes one revolution in 40 s. Calculate distance and displacement in 2 min 20 s. **PMP180**

2. Three girls skating on a circular ice ground of radius 200 m start from a point P on the edge of the ground and reach a point Q diametrically opposite to P following different paths as shown in figure. What is the magnitude of the displacement for each? For which girl is this equal to the actual length of path skate?

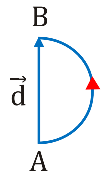


PMP181

3. A man moves 4 m along east direction, then 3m along north direction, after that he climbs up a pole to a height 12m. Find the distance covered by him and his displacement.

PMP182

4. A person moves on a semicircular track of radius 40 m. If he starts at one end of the track and reaches the other end, find the distance covered and magnitude of displacement of the person.



PMP183

5. A man has to go 50m due north, 40m due east and 20m due south to reach a cafe from his home.
(A) What distance he has to walk to reach the cafe?
(B) What is his displacement from his home to the cafe? **PMP184**

Speed

The Rate at which distance is covered with respect to time is called speed.
Scalar Quantity

Unit : S.I. $\rightarrow$ Metre per second (m/s)
C.G.S. $\rightarrow$ Centimetre per second (cm/s)

Conversion : km/hr $\underset{\times\frac{18}{5}}{\overset{\times\frac{5}{18}}{\rightleftharpoons}}$ m/s

Dimension : $[M^0L^1T^{-1}]$

For a moving particle speed can never be zero or negative, it is always positive.

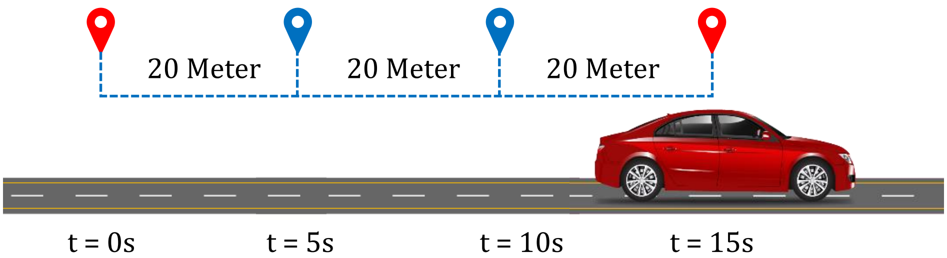
Speed

Zero (Particle at rest) | **Positive** (Moving)

Types of Speed

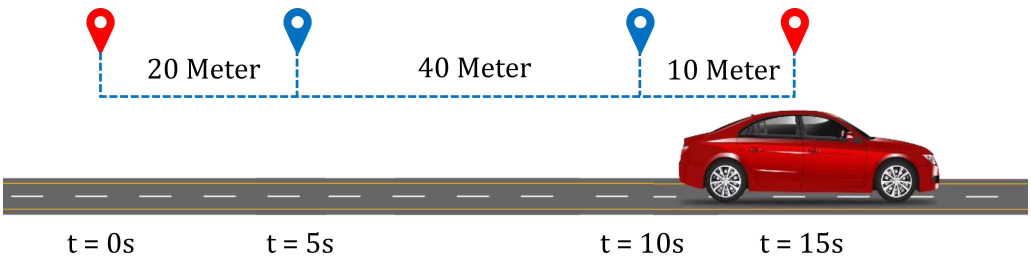
(1) Uniform Speed

Particle covers equal distances in equal interval of time. It is said to be moving with uniform speed.



(2) Non-uniform Speed (Variable Speed)

Particle covers unequal distances in equal intervals of time.



(3) Average Speed

For a given time interval is defined as the ratio of total distance travelled to total time taken.

$$\text{Average speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

(4) Instantaneous speed

It is the speed of a particle at a particular instant of time.

Average speed

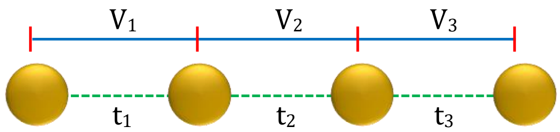
Cases of average speed to Remember

Case I : Particle moves with different uniform speeds v_1, v_2, v_3,.........v_n in different time intervals its average speed over the total time of journey is given as –

$$V_{avg} = \frac{\text{Total Distance covered}}{\text{Total time}} = \frac{s_1 + s_2 + s_3 + ... + s_n}{t_1 + t_2 + t_3 + ... + t_n} = \frac{v_1t_1 + v_2t_2 + v_3t_3 + ...}{t_1 + t_2 + t_3 + ...}$$

If $t_1 = t_2 = t_3 = ...= t_n$ then

$$V_{avg} = \frac{v_1 + v_2 + ... + v_n}{n}$$



For 2 equal intervals of times

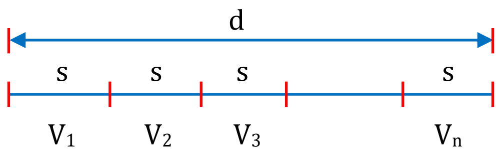
$$V_{avg} = \frac{v_1 + v_2}{2}$$

Case II : Particle describes equal distances with different speeds then the average speed of particle over the total distance will be given as –

If a person moves a certain distance in n equal parts with different speeds.

$$V_{avg} = \frac{d}{\frac{d}{nv_1} + \frac{d}{nv_2} + ... \frac{d}{nv_n}}$$

$$V_{avg} = \frac{n}{\frac{1}{v_1} + \frac{1}{v_2} + ... \frac{1}{v_n}}$$



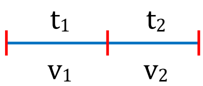
$$s = \frac{d}{n}$$

For 2 equal distances.

$$V_{avg} = \frac{2v_1v_2}{v_1 + v_2}$$

Illustration 9:

A particle moves for 20 sec such that first 10 sec it moves with 36 km/hr and for next 10 sec moves with 54 km/hr. Find average speed.



Solution:

$$v_1 = 36 \times \frac{5}{18} = 10 m/s \quad \text{and} \quad v_2 = 54 \times \frac{5}{18} = 15 m/s$$

$$v_{avg} = \frac{\text{total distance}}{\text{total time}} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{10 \times 10 + 15 \times 10}{20} = \frac{250}{20} = 12.5 m/s = 12.5 \times \frac{18}{5} = 45 km/hr$$

Illustration 10:

A train travels from city A to city B with a constant speed of 10 m/s and return back to city A with a constant speed of 20 m/s. Find its average speed during its entire journey.

Solution:

Let the distance between the two cities A and B = x m.

Time taken by the train to travel from A to B = $\frac{x}{10} = t_1$

Time taken to come back from B to A = $\frac{x}{20} = t_2$

Average speed = $\frac{\text{total distance}}{\text{total time}} = \frac{x+x}{t_1+t_2} = \frac{2x}{\frac{x}{10}+\frac{x}{20}} = \frac{40}{3} m/s$

Illustration 11:

A particle covers first one third distance with speed 10m/s and remaining distance with speed 20m/s. Find average speed.

Solution:

$$v_{avg} = \frac{d}{t_1+t_2} = \frac{d}{\frac{d}{3\times10}+\frac{2d}{3\times20}} = \frac{3\times10\times20}{2\times10+20} = \frac{600}{40} = 15 m/s$$

Velocity

The rate of change of position with time is called velocity.

It is a Vector Quantity.

Dimension : $[M^0L^1T^{-1}]$

Unit: S.I. $\rightarrow$ m/s

C.G.S. $\rightarrow$ cm/s

Velocity can be positive, Negative or Zero.

Types of Velocity

(1) Uniform velocity (Constant velocity)

If magnitude as well as direction of its velocity remains same.

This is possible only when it moves in a straight line without reversing its direction.

(2) Non-uniform velocity (variable)

If either magnitude or direction or both varies then velocity changes.

(3) Average velocity

It is defined as the ratio of displacement to time taken by body.

$$\text{Average velocity} = \frac{\text{Net Displacement}}{\text{Time taken}} \Rightarrow \vec{V}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{t_f - t_i}$$

Its direction is along the displacement.

Average velocity and Average Speed

Illustration 12:

A particle moves 20 m in 2 sec due east, then it moves 40 m in 3 sec due south. Then find out.
(i) Distance travelled (ii) Displacement (iii) Average speed (iv) Average velocity

Solution:

(i) Distance = 20 + 40 = 60 m

(ii) Displacement = $20\hat{i}-40\hat{j}$

(iii) Average speed = $\dfrac{\text{Total distance}}{\text{Total time}}=\dfrac{60}{5}=12\text{m}/\text{s}$

(iv) Average velocity = $\dfrac{\text{Net displacement}}{\text{Total time}}=\dfrac{20\hat{i}-40\hat{j}}{5}=(4\hat{i}-8\hat{j})\text{m}/\text{s}$

$\vec{v}_{avg}=4\sqrt{5}$ m/s, E $\tan^{-1}(2)$ S (E to S)

Illustration 13:

A bird flies due north at velocity 20 ms^{-1} for 15 s it rests for 5 s and then flies due south at velocity 24ms^{-1} for 10 s. For the whole trip find the average speed and magnitude of average velocity.

Solution:

Average speed = $\dfrac{\text{Total Distance}}{\text{Total Time}}=\dfrac{20\times15+24\times10}{15+5+10}=\dfrac{540}{30}=18\text{ms}^{-1}$

Average velocity = $\dfrac{\text{Displacement}}{\text{Total Time}}=\dfrac{(20\times15)\hat{j}+(24\times10)(-\hat{j})}{15+5+10}=\dfrac{60\hat{j}}{30}=2\hat{j}$

Magnitude of average velocity = $|2\hat{j}|$ = 2 m/s

Illustration 14:

A car moves with a velocity 2.24 km/h in first minute, with 3.60 km/h in the second minute and with 5.18 km/h in the third minute. Calculate the average velocity in these three minutes.

Solution:

Distance travelled in first minute $s_1=v_1\times t_1=2.24\times\dfrac{1}{60}\text{km}$

Distance travelled in second minute $s_2=v_2\times t_2=3.60\times\dfrac{1}{60}\text{km}$

Distance travelled in third minute $s_3=v_3\times t_3=5.18\times\dfrac{1}{60}\text{km}$

Total distance travelled $s=s_1+s_2+s_3=\dfrac{2.24}{60}+\dfrac{3.60}{60}+\dfrac{5.18}{60}=\dfrac{11.02}{60}\text{km}$

Total time taken, t = 1 + 1 + 1 = 3 min = $\dfrac{1}{20}$ h

$\therefore$ average velocity = $\dfrac{s}{t}=\dfrac{11.02}{60}\times\dfrac{20}{1}=3.67$ km/h

Note:-

(i) Time average velocity

If v = f (t) $\Rightarrow\langle v\rangle=\dfrac{\int_{t_1}^{t_2}v\,dt}{\int_{t_1}^{t_2}dt}$

(ii) Space average velocity

If v = f (x) $\Rightarrow\langle v\rangle=\dfrac{\int_{x_1}^{x_2}v\,dx}{\int_{x_1}^{x_2}dx}$

Illustration 15:

If velocity of particle is given by v = $(2t^2+3t+1)$, where 't' is time. Find average velocity for interval $0 \le t \le 2$ sec.

Solution:

$$<v> = \frac{\int_0^2 v\,dt}{\int_0^2 dt} \rightarrow \frac{\left[\frac{2t^3}{3}+\frac{3}{2}t^2+t\right]_0^2}{2-0} = \frac{20}{3}\,m/s$$

Illustration 16:

If v = sinx, find average velocity for interval $0 \le x \le \frac{\pi}{2}$.

Solution:

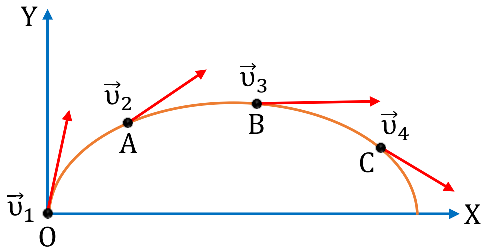
$$<v> = \frac{\int_0^{\frac{\pi}{2}} v\,dx}{\int_0^{\frac{\pi}{2}} dx} = \frac{\int_0^{\pi/2} \sin x\,dx}{\frac{\pi}{2}-0} = \frac{[-\cos]_0^{\frac{\pi}{2}}}{\frac{\pi}{2}} = \frac{2}{\pi}\,m/s$$

Instantaneous Velocity

It is the velocity of a particle at a particular instant of time.

$$\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t}$$

$$\vec{v}_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{r}}{\Delta t}\right) \Rightarrow \vec{v}_{inst} = \frac{d\vec{r}}{dt}$$



The direction of instantaneous velocity is always tangential to the path followed by the particle.

Important Points

Average speed $\ge$ |Average velocity |

- When particle moves with constant velocity then magnitude of displacement and distance covered by particle is same.
- A particle may have constant speed but variable velocity.
- A particle may have constant speed but variable velocity. Example: When a particle is performing uniform circular motion then for every instant of its circular motion its speed remains constant but velocity changes at every instant.
- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.
- When particle moves with uniform velocity then its instantaneous speed, magnitude of instantaneous velocity, average speed and magnitude of average velocity are all equal.
- When a particle is moving on any path, the magnitude of instantaneous velocity is equal to the instantaneous speed.

Illustration 17:

Position of particle as function of time is given by $\vec{r} = t^2\hat{i} + 2t\hat{j} + (\sin t)\hat{k}$.

Find velocity as function of time.

Solution:

velocity $\vec{v} = \frac{d\vec{r}}{dt} = 2t\,\hat{i} + 2\,\hat{j} + \cos t\,\hat{k}$

Illustration 18:

A particle is moving along a straight-line OX. At a time t (in seconds) the distance x (in metres) of particle from point O is given by $x = 10 + 6t - 3t^2$. How long would the particle travel before coming to rest?

Solution:

Initial value of x, at t = 0, $x_1 = 10m$

Velocity $v = \frac{dx}{dt} = 6 - 6t$ When v = 0, t = 1s

Final value of x, at t = 1s, $x_2 = 10 + 6 \times 1 - 3(1^2) = 13$ m

Distance travelled = $x_2 - x_1 = 13 - 10 = 3m$

Change in velocity

$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$

If $|\vec{v}_f| = |\vec{v}_i| = v$, then

$$|\Delta\vec{v}| = 2v\sin\left(\frac{\theta}{2}\right)$$

where θ is the angle between initial and final velocity

Illustration 19:

A car is moving with speed 10 m/s along north, then it takes 60° left turn and moves with same speed. Find out change in velocity.

Solution:

$\vec{u} = 10\,(\hat{j})\ m/s$

$\vec{v} = 5\sqrt{3}\,(-\hat{i}) + 5\,(+\hat{j})\ m/s$

$\Delta\vec{v} = \vec{v} - \vec{u} = \left(5\sqrt{3}\,(-\hat{i}) + 5\,(+\hat{j})\right) - 10\,(\hat{j}) = -5\sqrt{3}\,\hat{i} - 5\,\hat{j}$

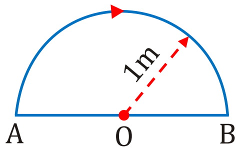
$|\Delta\vec{v}| = \sqrt{\left(5\sqrt{3}\right)^2 + 5^2} = 10\,m/s$

BEGINNER'S BOX-2

1. Air distance between Kota to Jaipur is 260 km and road distance is 320 km. A deluxe bus which moves from Jaipur to Kota takes 8 h while an aeroplane reaches in just 15 min. Find
(i) average speed of bus in km/h
(ii) average velocity of bus in km/h
(iii) average speed of aeroplane in km/h
(iv) average velocity of aeroplane in km/h **PMP185**
2. A particle moves on a straight line in such way that it covers 1st half distance with speed 3 m/s and next half distance in 2 equal time intervals with speeds 4.5 m/s and 7.5 m/s respectively. Find average speed of the particle. **PMP186**
3. Length of a minute hand of a clock is 4.5 cm. Find the average velocity of the tip of minute's hand between 6 A.M. to 6.30 A.M. & 6 A.M. to 6.30 P.M. **PMP187**
4. A particle of mass 2 kg moves on a circular path with constant speed 10 m/s. Find change in speed and magnitude of change in velocity. When particle completes half revolution. **PMP188**

5. The distance travelled by a particle in time t is given by $s = (2.5\ t^2)$ m. Find (a) the average speed of the particle during time 0 to 5.0s and (b) the instantaneous speed at t = 5.0 s. **PMP189**

6. A particle goes from point A to point B, moving in a semicircle of radius 1m in 1 second. Find the magnitude of its average velocity. **PMP190**



7. Straight distance between a hotel and a railway station is 10 km, but circular route is followed by a taxi covering 23 km in 28 minute. What is average speed and magnitude of average velocity? Are they equal? **PMP191**

Acceleration

Rate of change of velocity is called acceleration.

It is a vector quantity.

Its direction is same as that of change in velocity (not in the direction of the velocity).

Dimension: $[M^0L^1T^{-2}]$

Unit : (S.I.) $\rightarrow$ m/s^2 (C.G.S.) $\rightarrow$ cm/s^2

There are 3 ways to change a velocity (vector)

- Only magnitude change
- Only direction change
- Both direction+ magnitude change

Types of Acceleration

(1) Uniform Acceleration (2) Non-Uniform Acceleration

(3) Average Acceleration (4) Instantaneous acceleration

(1) Uniform Acceleration

A body is said to have uniform acceleration if magnitude and direction of the acceleration remains constant during motion of particle.

(2) Non-Uniform Acceleration

A body is said to have non-uniform acceleration, if either magnitude or direction or both change during motion.

(3) Average Acceleration

It is the ratio of total change in velocity to the total time taken by the particle

$$\vec{a}_{avg} = \frac{\text{change in velocity}}{\text{time interval}} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Direction of average acceleration is along the change in velocity

(4) Instantaneous Acceleration

It is the acceleration of a particle at a particular instant of time.

$$\vec{a}_{inst} = \lim_{\Delta t \to 0}\left(\frac{\Delta\vec{v}}{\Delta t}\right) \Rightarrow \vec{a}_{inst} = \frac{d\vec{v}}{dt}$$ When $\vec{v}$ in function of time (t).

i.e. first derivative of velocity is called instantaneous acceleration

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d^2\vec{r}}{dt^2} \quad \left[\text{As } \vec{v} = \frac{d\vec{r}}{dt}\right]$$

i.e. second derivative of position vector is called instantaneous acceleration

$$\vec{a}_{inst} = \frac{d\vec{v}}{dt} \times \frac{dx}{dx} \Rightarrow \vec{a}_{inst} = v\frac{dv}{dx}$$ When $\vec{v}$ in function of position (x).

Application of Calculus

$$x \xrightarrow[\text{Differentiate w.r.t.}]{\frac{dx}{dt}} v \xrightarrow[\text{Differentiate w.r.t.}]{\frac{dv}{dt}} a$$

$$a \xrightarrow[\text{Integrate w.r.t.}]{\int a\,dt} \Delta v \xrightarrow[\text{Integrate w.r.t.}]{\int v\,dt} \Delta x$$

For v-t relation $\Rightarrow a = \frac{dv}{dt}$ For v-x relation $\Rightarrow a = v\frac{dv}{dx}$

Illustration 20:

The velocity of a particle is given by v = $(2t^2 - 4t + 3)$ m/s where t is time in seconds. Find its acceleration at t = 2 second.

Solution:

Acceleration (a) = $\frac{dv}{dt} = \frac{d}{dt}(2t^2 - 4t + 3) = 4t - 4$

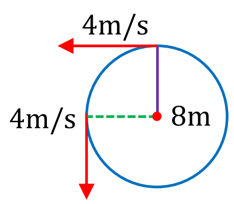
Therefore acceleration at t = 2s is equal to, a = $(4 \times 2) - 4 = 4$ m/s^2

Illustration 21:

A particle is moving along a circular path of radius 8m with uniform speed 4m/s. Find out magnitude of average acceleration when particle completes ¼ half revolution.

Solution:

Average acceleration = $\frac{\text{change in velocity}}{\text{Time}}$



$$\text{time} = \frac{\frac{1}{4}(2\pi r)}{V}$$

$t = \frac{2\pi \times 8}{4 \times 4} = \pi$ s so, $a_{avg} = \frac{-4\hat{j} - (-4\hat{i})}{\pi} = \frac{4\sqrt{2}}{\pi}$ m/s^2

Illustration 22:

The velocity of a body depends on time according to the equation $v = 20 + 0.1t^2$. The body is undergoing what kind of acceleration?

Solution:

Acceleration $a = \frac{dv}{dt} = 0.1 \times 2t = 0.2t$

Which is time dependent i.e. non-uniform acceleration.

Illustration 23:

One particle have velocity $v = \sqrt{x+1}$ then find acceleration of particle?

Solution:

$$\frac{dv}{dx} = \frac{1}{2\sqrt{x+1}} \Rightarrow v\frac{dv}{dx} = \sqrt{x+1}\frac{1}{2\sqrt{x+1}} = \frac{1}{2}$$

Illustration 24:

Position of a particle is given by $x = (6t^2 - t^3)$m, then find out its position when velocity becomes maximum.

Solution:

$x = 6t^2 - t^3 \qquad \Rightarrow \qquad v = \frac{dx}{dt} = 12t - 3t^2$

For maximum velocity:

$\frac{dv}{dt} = 0 \Rightarrow 12 - 6t = 0 \qquad t = 2$ sec

$\therefore$ Position at t = 2 sec.

$x = 6(2)^2 - (2)^3 = 16$ m

Key points

- When particle moves with constant velocity then its acceleration will be zero.

  If particle moves with constant velocity then $v_{avg} = |\vec{v}_{avg}| = v_{inst} = |\vec{v}_{inst}|$

- Acceleration which opposes the motion of body is called retardation.

  Increment or decrement in speed depends on direction of $\vec{v}$ and $\vec{a}$.

Case- I

If $\vec{v}$ and $\vec{a}$ are in same direction then speed increases

(i) $\xrightarrow{\vec{v}=+ve}$ $\xrightarrow{\vec{a}=+ve}$ (ii) $\xleftarrow{\vec{v}=-ve}$ $\xleftarrow{\vec{a}=-ve}$

Case- II

If $\vec{v}$ and $\vec{a}$ are in opposite direction then speed decreases.

(i) $\xrightarrow{\vec{v}=+ve}$ $\xleftarrow{\vec{a}=-ve}$ (ii) $\xleftarrow{\vec{v}=-ve}$ $\xrightarrow{\vec{a}=+ve}$

Velocity and Acceleration

Case-I

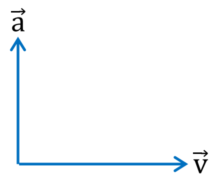
Angle between $\vec{v}$ & $\vec{a}$ is zero or they are parallel, speed increases

$\longrightarrow \vec{a}$

$\longrightarrow \vec{v}$

Case-II

Angle between $\vec{v}$ & $\vec{a}$ is 90° or they are orthogonal, speed remains constant



Case-III

Angle between $\vec{v}$ & $\vec{a}$ is 180° or they are anti-parallel, speed decreases

$\longleftarrow \vec{a}$

$\longrightarrow \vec{v}$

Path of Motion Based on Acceleration (To Remember)

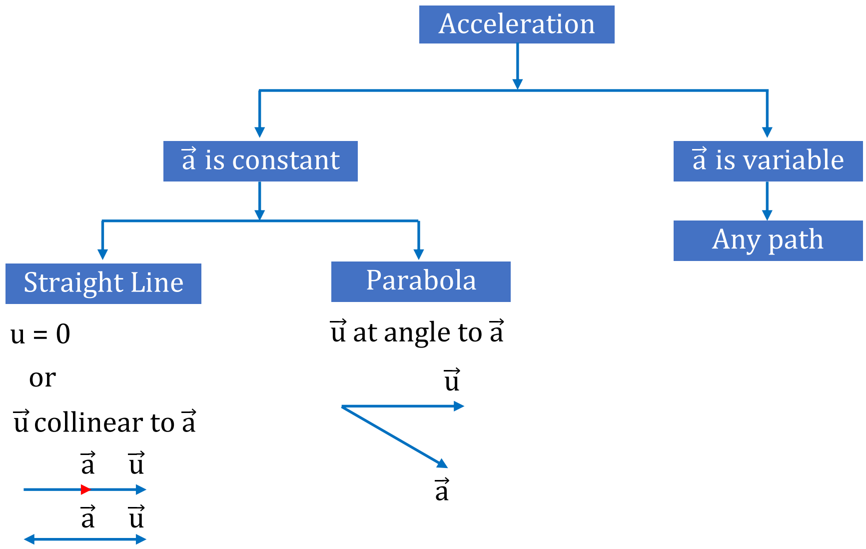


Illustration 25:

The path of a particle moving under the influence of a force fixed in magnitude and direction is (force and velocity at angle $\theta = 0°$ or $\theta = 180°$)

Solution:

$\vec{u}$ collinear with $\vec{a}$ i.e. straight line

BEGINNER'S BOX-3

1. A particle moves on circular path of radius 5 m with constant speed 5 m/s. Find the magnitude of its average acceleration when it completes half revolution. **PMP192**

2. The position of a particle moving on X-axis is given by $x = At^2 + Bt + C$ The numerical values of A, B and C are 7, –2 and 5 respectively and SI units are used. Find

(a) The velocity of the particle at t = 5

(b) The acceleration of the particle at t = 5

(c) The average velocity during the interval t = 0 to t = 5

(d) The average acceleration during the interval t = 0 to t = 5 **PMP193**

Equations of Motion

S.No.	Scalar Form	Vector Form
first equation	$v = u + at$	$\vec{v} = \vec{u} + \vec{a}t$
second equation	$s = ut + \frac{1}{2}at^2$	$\vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2$
third equation	$v^2 = u^2 + 2as$	$\vec{v}\cdot\vec{v} = \vec{u}\cdot\vec{u} + 2\vec{a}\cdot\vec{s}$

Here :

v = final velocity s = displacement

u = initial velocity t = time

a = acceleration = constant

These are valid only when acceleration is constant.

(i) $a = \frac{dv}{dt}$	(ii) $v = \frac{ds}{dt}$	(iii) $a = v\frac{dv}{ds}$
$\int_{v}^{v} dv = \int_{0}^{t} a dt$ $v - u = at$ $v = u + at$	$\int_{0}^{s} ds = \int_{0}^{t} v dt$ $s = ut + \frac{1}{2}at^2$	$\int_{u}^{v} v dv = \int_{0}^{s} a ds$ $\left[\frac{v^2}{2}\right]_u^v = a[s]_0^s$ $v^2 = u^2 + 2as$

Some other useful equations

(i) Displacement in the n^{th} second, $s_{n^{th}} = u + \frac{1}{2}a(2n-1)$

(ii) $s = v_{av}t = \frac{(u+v)}{2}t$

Vector form of following equations

(i) $\vec{s}_{n^{th}} = \vec{u} + \frac{1}{2}\vec{a}(2n-1)$ (ii) $\vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t$

Steps to follow for solving numerical:

(1) Select origin. Generally, we take starting point as origin if not given.
(2) Select any one direction as positive.
(3) As per direction selected in step 2, write all known variable with sign.
(4) Apply the suitable equations and put variables values with sign.

Key points

When particle starts from rest and moves with constant acceleration then ratio of distance travelled by it in successive equal intervals of time is

1 : 3 : 5 : 7........ (2n – 1)

Known as **Galileo's law of odd numbers**.

Illustration 26:

The velocity of a body moving with a uniform acceleration of $2\ m/sec^2$ is 10 m/sec. Its velocity after an interval of 4 sec is

Solution:

$v = u + at = 10 + 2 \times 4 = 18\ m/sec$

Illustration 27:

The initial velocity of the particle is 10 m/sec and its retardation is $2 m/sec^2$. The distance moved by the particle in 5^{th} second of its motion is

Solution:

$S_n = u - \frac{a}{2}(2n-1) = 10 - \frac{2}{2}(2 \times 5 - 1) = 1\ \text{meter}$

Illustration 28:

If a car at rest accelerates uniformly to a speed of 144 km/h in 20 s. Then it covers a distance of

Solution:

Here

v = 144 km/h = 40 m/s

$v = u + at \Rightarrow 40 = 0 + 20 \times a \Rightarrow a = 2\ m/s^2$

$\therefore s = \frac{1}{2}at^2 = \frac{1}{2} \times 2 \times (20)^2 = 400\ m$

Illustration 29:

A body starts from rest. What is the ratio of the distance travelled by the body during the 4th and 3rd second

Solution:

$$S_n = u + \frac{a}{2}(2n-1) = \frac{a}{2}(2n-1)$$

because u = 0

Hence $\frac{S_4}{S_3} = \frac{7}{5}$

Illustration 30:

A particle starting from rest travels a distance x in first 2 seconds and a distance y in next two seconds, then find ratio of distances.

Solution:

If particle starts from rest and moves with constant acceleration then in successive equal interval of time the ratio of distance covered by it will be

1:3:5:7 ....(2n – 1)

i.e. ratio of x and y will be a : 3 i.e. $\frac{x}{y} = \frac{1}{3} \Rightarrow y = 3x$

Illustration 31:

Two cars start off a race with velocities 2m/s and 4m/s travel in straight line with uniform accelerations $2 m/s^2$ and $1 m/s^2$ respectively. What is the length of the path if they reach the final point at the same time?

Solution:

Let both particles reach at same position in same time t then from s = ut + $\frac{1}{2}$ at^2

For 1st particle : $s = 4(t) + \frac{1}{2}(1)t^2 = 4t + \frac{t^2}{2}$

For 2nd particle : $s = 2(t) + \frac{1}{2}(2)t^2 = 2t + t^2$

Equating above equation we get $4t + \frac{t^2}{2} = 2t + t^2 \Rightarrow t = 4s$

Substituting value of t in above equation s = 4(4)+ $\frac{1}{2}$ (1) $(4)^2$ = 16 + 8 = 24 m

Problems based on Uniformly Accelerated Motion

Illustration 32:

A particle having initial velocity 10 m/s, moves with uniform acceleration. After 2 seconds its velocity becomes 32 m/s. Find distance covered in this duration.

Solution:

u = 10 m/s $\qquad s = \left(\frac{u+v}{2}\right)t$

a = constant $\qquad s = \left(\frac{10+32}{2}\right) \times 2 = 42m$

t = 2s

v = 32 m/s

Average velocity for uniformly accelerated motion

Illustration 33:

A car is moving along a straight road with a uniform acceleration. It passes through two points P and Q separated by a distance with velocity 30 km/h and 40 km/h respectively. The velocity of the car midway between P and Q is?

Solution:

Here, u = 30 km/hr, v = 40 km/hr

Between points P and M

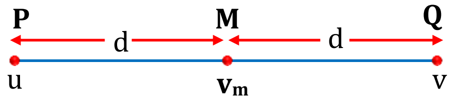
$v_m^2 = 30^2 + 2a(d)$

$2ad = v_m^2 - 900$ ...(i)

Between points M and Q

$40^2 = v_m^2 + 2a(d)$

$2ad = 1600 - v_m^2$ ...(ii)



From eq. (1) and (2)

$v_m^2 - 900 = 1600 - v_m^2$

$2v_m^2 = 2500$

$v_m = \sqrt{\dfrac{2500}{2}}$ km/hr

Stopping Distance

For Vehicles,

When brakes are applied to a moving vehicle, the distance it travels before stopping is called stopping distance.

$a = -a_0$ $u = u_0$

$v = 0$ $s = ?$

$v^2 = u^2 + 2as$

$\Rightarrow 0 = u_0^2 - 2a_0 s \Rightarrow 2a_0 s = u_0^2 \Rightarrow s = \dfrac{u_0^2}{2a_0}$ [since a is constant] $\Rightarrow s \propto u_0^2$

So, we can say that if u becomes n times then s becomes n^2 times that of previous value.

Stopping Time

$a = -a_0$ $u = u_0$

$v = 0$ $t = ?$

$\Rightarrow v = u + at \Rightarrow 0 = u_0 - a_0 t \Rightarrow a_0 t = u_0 \Rightarrow t = \dfrac{u_0}{a_0}$ [since a is constant] $\Rightarrow t \propto u_0$

So, we can say that if u becomes n times then t becomes n times that of previous value.

Reaction Time

When a situation demands our immediate action, it takes some time before we really respond. Reaction time is the time a person takes to observe, think and act.

Illustration 34:

A motor car moving with a uniform speed of 20m/sec comes to stop on the application of brakes after travelling a distance of 10m Its acceleration is

Solution:

From $v^2 = u^2 + 2aS \Rightarrow 0 = u^2 + 2aS \Rightarrow a = \dfrac{-u^2}{2S} = \dfrac{-(20)^2}{2 \times 10} = -20 m/s^2$

Illustration 35:
A car moving with a velocity of 10 m/s can be stopped by the application of a constant force F in a distance of 20 m. If the velocity of the car is 30 m/s, it can be stopped by this force in

Solution:

$S \propto u^2$ If u becomes 3 times then S will become 9 times

i.e. $9 \times 20 = 180$ m

Illustration 36:
Two trains travelling on the same track are approaching each other with equal speeds of 40 m/s. The drivers of the trains begin to decelerate simultaneously when they are just 2.0 km apart. Assuming the decelerations to be uniform and equal, the value of the deceleration to barely avoid collision should be

Solution:

Both trains will travel a distance of 1 km before to come in rest. In this case by using

$v^2 = u^2 + 2as \Rightarrow 0 = (40)^2 + 2a \times 1000 \Rightarrow a = -0.8\ m/s^2$

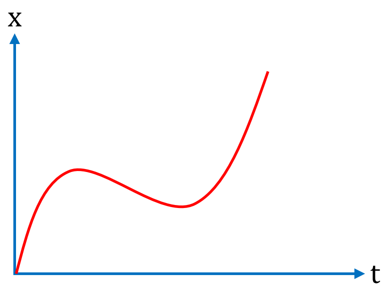
BEGINNER'S BOX-4

1. A particle starts from rest, moves with constant acceleration for 15s. If it covers s_1 distance in first 5s then distance s_2 in next 10s, then find the relation between s_1 & s_2. **PMP194**
2. The engine of a train passes an electric pole with a velocity 'u' and the last compartment of the train crosses the same pole with a velocity v. Then find the velocity with which the mid-point of the train passes the pole. Assume acceleration to be uniform. **PMP195**
3. A bullet losses 1/n of its velocity in passing through a plank. What is the least number of planks required to stop the bullet? (Assuming constant retardation) **PMP196**
4. A car moving along a straight highway with speed 126 km h^{-1} is brought to a halt within a distance of 200m. What is the retardation of the car (assumed uniform) and how long does it take for the car to stop? **PMP197**
5. A car is moving with speed u. Driver of the car sees red traffic light. His reaction time is t, then find out the distance travelled by the car before coming at rest. Assume uniform retardation 'a' after applying brakes. **PMP198**
6. If a body starts from rest and travels 120cm in the 6^{th} second then what is the acceleration? **PMP199**

Basic Graphs and their Conversions
Mathematical + Graphical approach

$$x \xrightarrow[\text{Slope of x-t graph}]{\frac{dx}{dt}} v \xrightarrow[\text{Slope of v-t graph}]{\frac{dv}{dt}} a \ : \ a \xrightarrow[\text{Area of a-t curve with time axis}]{\int a\,dt} \Delta v \ : \ v \xrightarrow[\text{Area of v-t curve with time axis}]{\int v dt} \Delta x$$

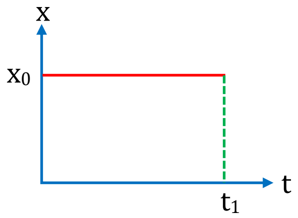
Position-time graph



x-t Graph
- Slope of this graph represents instantaneous velocity.
- Area = Defines No physical Quantity
- Displacement = final position – initial position
- Distance = sum of magnitude of all length one by one.

$\because \tan\theta = \dfrac{\text{displacement}}{\text{time}} = \text{velocity}$

Case-I



Slope = instantaneous velocity

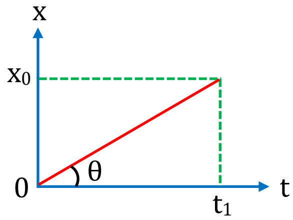
$\theta = 0$

$\tan\theta = \tan 0 = 0$

velocity = 0

i.e. body is at rest.

Case-II



Slope = instantaneous velocity

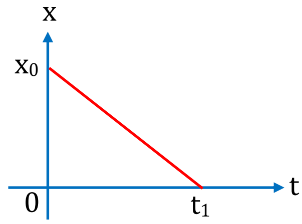
θ = constant

$\tan\theta$ = constant

velocity = constant

i.e. the body is in uniform motion

Case-III



Slope = instantaneous velocity

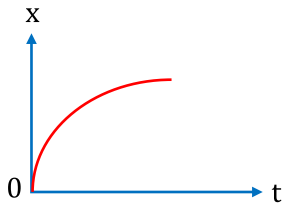
$\theta > 90$

$\tan\theta = -ve$

velocity = –ve but constant

i.e. uniform motion

Case-IV



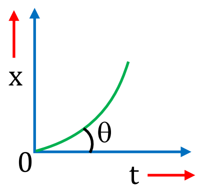
When particle starts from rest and moves with constant acceleration

θ is decreasing with time

$\therefore \tan\theta$ is decreasing with time

$\therefore$ velocity is decreasing with time

i.e. non uniform motion



θ is increasing with time

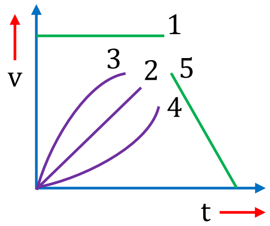
$\therefore \tan\theta$ is increasing with time

$\therefore$ velocity is increasing with time

i.e. non-uniform motion

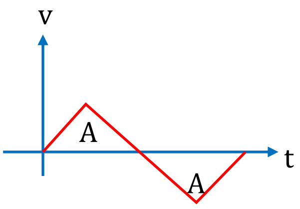
- Area of x-t graph = $\int x\,dt$ = No physical significance

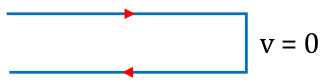
Velocity-time graph



Slope of this graph represents acceleration.

$\because \tan\theta = \frac{\text{velocity}}{\text{time}} = \text{acceleration}$





Area under the v-t curve gives

|displacement| = $A_1 - A_2$

Distance = $A_1 + A_2$

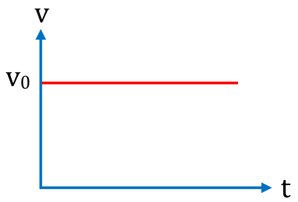
Case-I

$\theta = 0°$

$\tan\theta = \tan 0° = 0$

acceleration = 0

i.e. v = constant or uniform motion



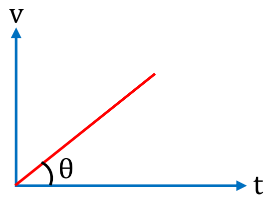
Case-II

θ = constant

tan θ = constant

acceleration = constant

i.e. uniformly accelerated motion



Case-III

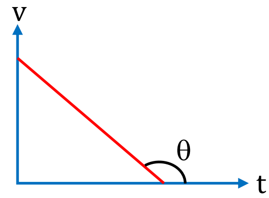
$\theta > 90°$

$\tan\theta$ = –ve

acceleration = –ve but constant

i.e. constant or uniform retardation

is acting on the body



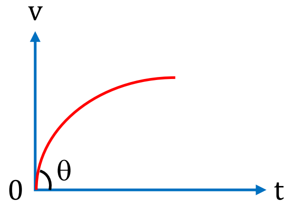
Case-IV

θ is decreasing with time

$\therefore \tan\theta$ is decreasing with time

$\therefore$ acceleration is decreasing with time

i.e. acceleration goes on decreasing with time but it is not retardation

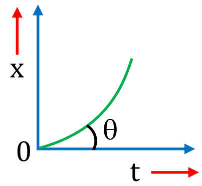


θ is increasing with time

$\therefore \tan\theta$ is increasing with time

$\therefore$ acceleration is increasing with time

i.e. acceleration goes on increasing with time



- Area of v-t graph = $\int vdt$ = displacement = change in position

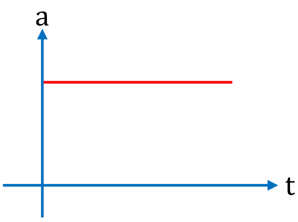
Acceleration-time graph

Slope of a-t graph defines nothing

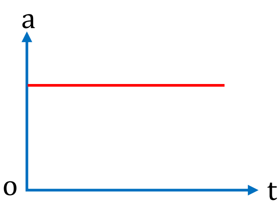
But Area of a-t graph gives change in velocity

$\therefore$ Area of a–t graph = $\int a\,dt = \int dv = v_2 - v_1$ = change in velocity

- Area ≠ final velocity



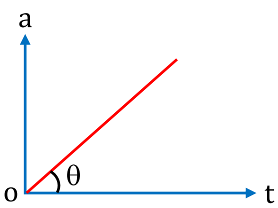
Case-I



$a \propto t^0$

i.e. uniform or constant acceleration

Case-II

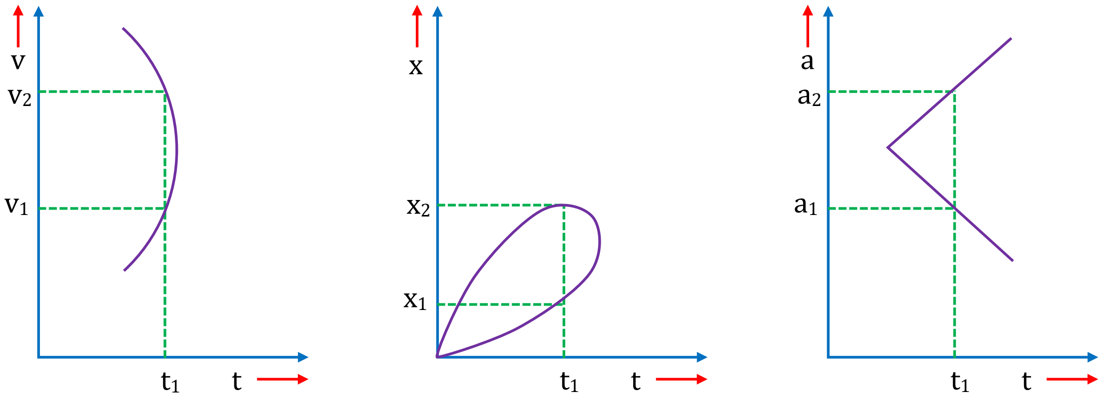


i.e. uniformly increasing acceleration.

Key Points

Following graphs do not exist in practice :

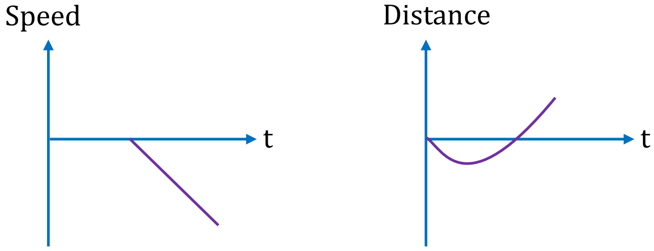
Case-I



Explanation:

In practice, at any instant body can not have two velocities or displacements or accelerations simultaneously.

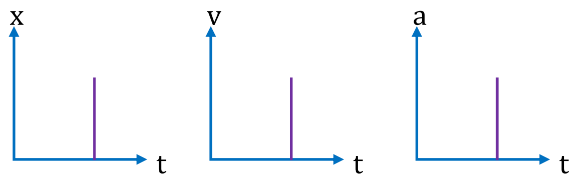
Case-II



Explanation:

Speed or distance can never be negative.

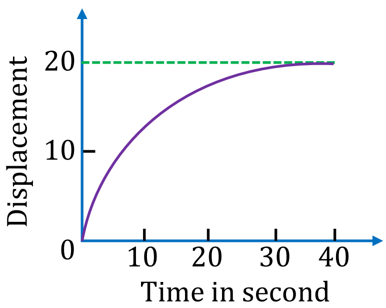
Case-III



Explanation : It is not possible to change any quantity without consuming time i.e. time can't be constant.

Illustration 37:

The displacement of a particle as a function of time is shown in the figure. The figure shows that velocity is

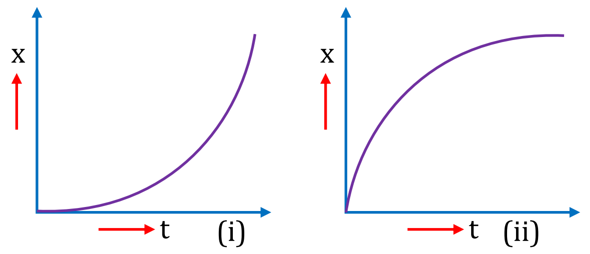


Solution:

The slope of displacement-time graph goes on decreasing, it means the velocity is decreasing i.e. It's motion is retarded and finally slope becomes zero i.e. particle stops.

Illustration 38:

Figures (i) and (ii) below show the displacement-time graphs of two particles moving along the x-axis. We can say that

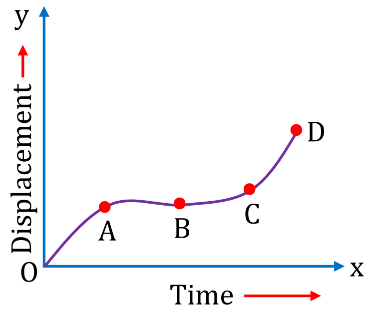


Solution:

Particle (i) is having a uniformly accelerated motion while particle (ii) is having a uniformly retarded motion.

Illustration 39:

The graph between the displacement x and time t for a particle moving in a straight line is shown in figure. During the interval OA, AB, BC and CD, the acceleration of the particle is

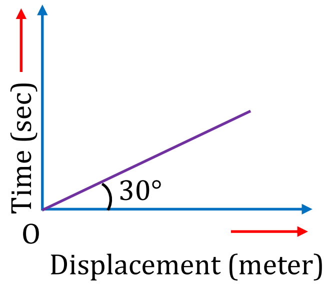


Solution:

Region OA shows that graph bending toward time axis i.e. acceleration is negative.
Region AB shows that graph is parallel to time axis i.e. velocity is zero. Hence acceleration is zero.
Region BC shows that graph is bending towards displacement axis i.e. acceleration is positive.
Region CD shows that graph having constant slope i.e. velocity is constant. Hence acceleration is zero.

Illustration 40:

From the following displacement-time graph find out the velocity of a moving body



Solution:

Slope of the x-t graph gives instantaneous velocity but here t-x graph,

$$\upsilon = \tan 30^\circ = \frac{1}{\sqrt{3}} \text{ m/s.}$$

But it is wrong because formula $v = \tan\theta$ is valid when angle is measured with time axis.
Here angle is taken from displacement axis. So angle from time axis = 90° – 30° = 60°
Now $\upsilon = \tan 60^\circ = \sqrt{3}$

Graphical Problems in Horizontal Motion

Shortcut to Remember

If a car, starts from rest moves with constant acceleration α for some time retards uniformly at rate β and finally comes at rest total time of motion is T

Then:

$$v_{max} = \left(\frac{\alpha\beta}{\alpha+\beta}\right)T$$

$$s = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)T^2$$

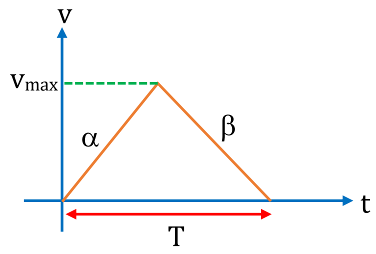


Illustration 41:

A particle is moving along x-axis. Its position is given by $x = -2t + 3t^2$. Draw its :-

(i) v-t graph

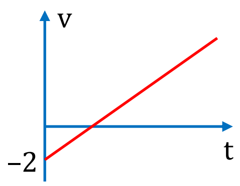
(ii) a-t graph

Solution:

$x = -2t + 3t^2$

$v = -2 + 6t$

$a = 6$



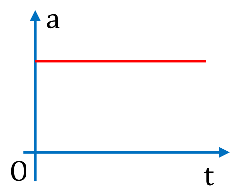
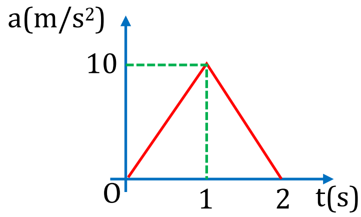


Illustration 42:

A particle have initial velocity 10 m/s. Its acceleration time graph is shown. Find out its velocity at t = 2 sec.



Solution:

$$\Delta V = V_f - V_i = \text{Area} = \frac{1}{2}\times 2\times 10$$

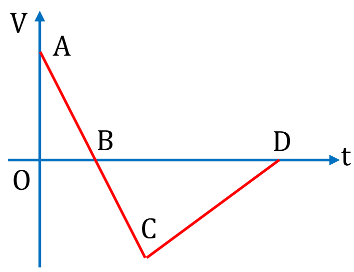
$$V_f - 10 = 10$$

$$V_f = 20\text{m/s}$$

Illustration 43:

For a given velocity time graph, find out :

(i) In which duration acceleration is Positive, Negative and Zero.



Solution:

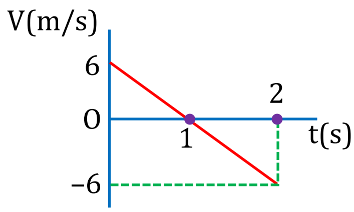
AB $\rightarrow$ Negative

BC $\rightarrow$ Negative

CD $\rightarrow$ Positive

Illustration 44:

Velocity-time graph for a particle moving in a straight line is given. Calculate the displacement of the particle and distance travelled in first two sec.



Solution:

(i) Displacement = area $= \left(\frac{1}{2}\times 6\times 1\right) + \left[\frac{1}{2}\times 1\times(-6)\right] = 0$

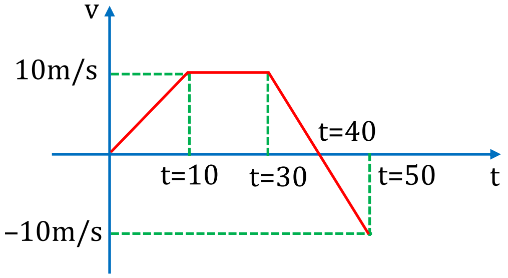
(ii) Distance $= \left(\frac{1}{2}\times 6\times 1\right) + \left(\frac{1}{2}\times 6\times 1\right) = 6\text{m}$

Illustration 45:

Find :

(i) average velocity from t = 0 to t = 30

(ii) average speed from t = 10 to t = 40



Solution:

Average velocity from t = 0 to t = 30

$$< v >= \frac{\Delta x}{\Delta t} = \frac{50+200}{30} = \frac{25}{3} m/s$$

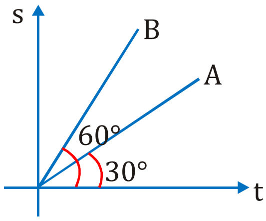
Average speed from t = 10 to t = 40

$$\frac{\text{distance}}{\text{time}} = \frac{200+50}{30} = \frac{25}{3} m/s$$

because there is no turning point.

BEGINNER'S BOX-5

1. s-t graph of two particles A and B are shown in fig. Find the ratio of velocity of A to velocity of B.



PMP200

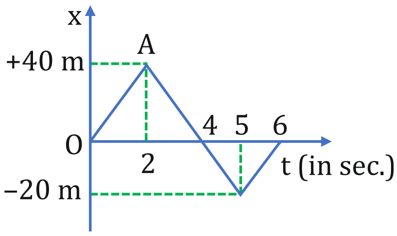
2. Position-time graph of a particle in motion is shown in fig. Calculate –

(i) Total distance covered

(ii) Displacement

(iii) Average speed

(iv) Average velocity.



PMP201

3. The position-time (x-t) graphs for two children A and B returning from their school O to their homes P and Q respectively are shown in fig. Choose correct entries in the brackets below :

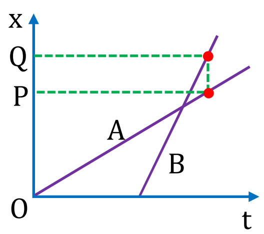
(a) (A/B) lives closer to the school than (B/A)

(b) (A/B) starts from the school earlier than (B/A)

(c) (A/B) walks faster than (B/A)

(d) A and B reach home at the (same / different) time

(e) (A/B) overtakes (B/A) on the road (once/ twice).



PMP202

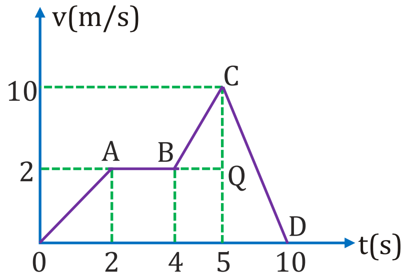
4. A particle moves on straight line according to the velocity–time graph shown in fig. Calculate –

(i) Total distance covered

(ii) Average speed

(iii) In which part of the graph the acceleration is maximum and also find its value.

(iv) Retardation



PMP203

5. A body starts from rest and moves with a uniform acceleration of 10 ms^{-2} for 5 seconds. During the next 10 seconds it moves with uniform velocity. Find the total distance travelled by the body (Using graphical analysis).

PMP204

Motion under Gravity-Vertical Projection from Ground

Motion Under Gravity

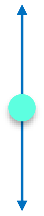
Acceleration produced in the body by the force of gravity, is called acceleration due to gravity. It is represented by the symbol g.

Value of g = 9.8 m/s^2 or g = 980 cm/s^2 or g=32 ft/s^2

In the absence of air, it is found that all bodies (irrespective of the size, weight or composition) fall with the same acceleration near the surface of the earth. This motion of a body falling towards the earth from a small altitude (h << earth's radius) is called motion under gravity. Free fall means acceleration of body is equal to acceleration due to gravity.

Sign Conventions

Negative and positive sign are matter of our choice, so we can select any direction as positive and opposite side as negative.



Vertical Projection from ground (Body is Projected Vertically Upward)

Equations of motion : Taking initial position as origin and direction of motion (i.e. vertically up) as positive and (vertical downward) as negative.

u (initial velocity) = +u

a (acceleration) = –g

at time t,

Final velocity = v

height above ground = h

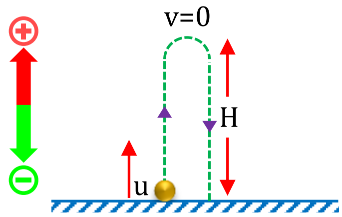
Positive / Negative directions are a matter of choice.

$\therefore\ v = u - gt$ ...(i)

$h = ut - \frac{1}{2}gt^2$ ...(ii)

$v^2 = u^2 - 2gh$ ...(iii)

$h_{n^{th}} = u - \frac{g}{2}(2n-1)$



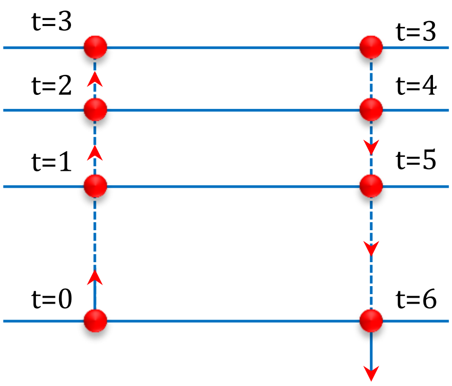
Maximum height (H)

Final velocity (v) = 0 so,

From above equation (iii)

$0 = u^2 - 2gH \quad \Rightarrow \quad H = \frac{u^2}{2g}$

Total time of flight(T)



at maximum height v = 0

So from above equation (i) u = g t

it is called time of ascent (t_1) = u/g

In case of motion under gravity, time taken to go up is equal to the time taken to fall down through the same distance. Time of descent (t_2) = time of ascent (t_1) = u/g

$\therefore$ Total time of flight $T = t_1 + t_2$

$T = \frac{2u}{g}$

Illustration 46:

A ball is projected vertically upward with a speed of 50 m/s.

Find

(a) the maximum height,

(b) the time to reach the maximum height,

(c) the speed at half the maximum height.

(d) time of flight

(e) velocity after t = 1 and t = 6 s

(f) height from ground at t = 1 and t = 6 sec

Take $g = 10 m/s^2$

Solution:

(a) Initial velocity of ball (u)=50m/s, acceleration of ball =–g,

final velocity at the highest point (v)=0

So applying the 3rd equation of motion we get:

$v^2 = u^2 - 2gh_{max} \Rightarrow 0 = 50^2 - 2\times10\times h_{max} \Rightarrow h_{max} = \frac{2500}{20} = 125\text{ m}$

(b) Let the time required to reach max height be t. Then applying 1st equation of motion we get:

$v = u - gt \Rightarrow 0 = 50 - 10t \Rightarrow t = 5s$

(c) Let speed at half of max height be V then:

$V^2 = 50^2 - 2g\frac{125}{2} \Rightarrow V^2 = 2500 - 1250 = 1250 \Rightarrow V = \sqrt{1250} = 35.35 m/s$

(d) $T = \frac{2u}{g} = \frac{50\times2}{10} = 10\,\text{sec}$

(e) velocity after one second

$v = u - gt$

$v = 50 - 10 \times 1 = 40$ m/s

velocity after 6 sec

$v = 50 - 10 \times 6 = -10$m/s or 10 m/s downward

(f) Height after one second from ground

$h_1 = 50 - \frac{1}{2}\times10 = 45m$

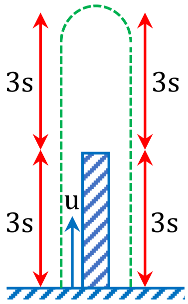
height after 6 sec from ground

$h_6 = 50\times6 - \frac{1}{2}\times10\times36 = 120m$

Illustration 47:

A ball is projected upwards from the foot of a tower. The ball crosses the top of the tower twice after an interval of 6s and the ball reaches the ground after 12s. The height of the tower is (g=10m/s²):

Solution:



$T = \frac{2u}{g} = 12$

$2u = 120$

$u = 60$ m/s

$h = 60 \times 3 - \frac{1}{2} \times 10 \times 9$

$= 180 - 45 = 135$ m

Illustration 48:

A body A is projected upwards with a velocity of 98m/s. The second body B is projected upwards with the same initial velocity but after 4 sec. Both the bodies will meet after

Solution:

Let t be the time of flight of the first body after meeting, then (t-4)sec will be the time of flight of the second body. Since $h_1 = h_2$

$\therefore 98t - \frac{1}{2}gt^2 = 98(t-4) - \frac{1}{2}g(t-4)^2$

On solving, we get t = 12seconds

Illustration 49:

When a ball is thrown up vertically with velocity V_0, it reaches a maximum height of 'h'. If one wishes to triple the maximum height then the ball should be thrown with velocity

Solution:

$H_{max} \propto u^2 \qquad \therefore u \propto \sqrt{H_{max}}$

i.e. to triple the maximum height, ball should be thrown with velocity $\sqrt{3}u$

Illustration 50:

With what velocity a ball be projected vertically so that the distance covered by it in 5^{th} second is twice the distance it covers in its 6^{th} second (g=10m/s²)

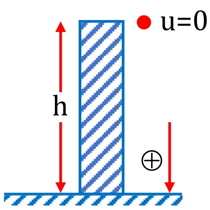
Solution:

$h_{n^{th}} = u - \frac{g}{2}(2n-1)$; $h_{5^{th}} = u - \frac{10}{2}(2\times5-1) = u - 45$; $h_{6^{th}} = u - \frac{10}{2}(2\times6-1) = u - 55$

Given $h_{5^{th}} = 2\times h_{6^{th}}$. By solving we get u = 65 m/s

Motion under Gravity-Dropping from height

Body dropped from some height (initial velocity zero)



Equations of Motion

Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, here we have

$u = 0$ [As body starts from rest]

$a = +g$ [As acceleration is in the direction of motion]

at time t,

final velocity = v

height below dropping point = h

$v = gt$ ...(i) $h = \frac{1}{2}gt^2$ ...(ii) $v^2 = 2gh$ ...(iii)

Results

From equations (i) (ii) and (iii)

- Time to reach bottom(T)

  $T = \sqrt{\frac{2H}{g}}$

- Maximum speed (at bottom)

  $v = \sqrt{2gH}$

Illustration 51:

A ball is released from a height and it reaches the ground in 3 s. If $g = 9.8\ ms^{-2}$, find

(a) the height from where the ball was released

(b) the velocity with which the ball will strike the ground.

(c) at t = 2s, velocity and height from the surface

Solution:

(a) $s = ut + \frac{1}{2}at^2 = \frac{1}{2}\times 9.8\times 3^2 = 44.1m$

(b) $v = u + at = 0 + (9.8)(3) = 29.4\ m/s$

(c) $v = gt = 9.8\times 2 = 19.6 m/s$; $h = \frac{1}{2}\times 9.8\times 2^2 = 19.6m$

So, height from surface H = 44.1 – 19.6 = 12.4 meter

Key Points

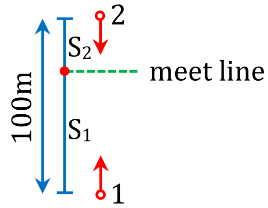
- As $h = (1/2)gt^2$, i.e., $h \propto t^2$, distance covered in time t, 2t, 3t, etc., will be in the ratio of $1^2 : 2^2 : 3^2$, i.e., square of consecutive integers. (in case of free fall, from rest)
- A particle at rest, is dropped vertically from a height. The time taken by it to fall through successive distance of 1m each will then be in the ratio of the difference in the square roots of the integers i.e. $\sqrt{1}, (\sqrt{2}-\sqrt{1}), (\sqrt{3}-\sqrt{2}), (\sqrt{4}-\sqrt{3}), \ldots$
- The motion is independent of the mass of body, as mass is not involved in any equation of motion. It is due to this reason that a heavy and light body when released from the same height, reach the ground simultaneously and with same velocity i.e., $t = \sqrt{(2h/g)}$ and $v = \sqrt{2gh}$.
- The distance covered in the n^{th} second, $h_n = \frac{1}{2}g(2n-1)$

  So distance covered in 1^{st}, 2^{nd}, 3^{rd} second, etc., will be in the ratio of 1 : 3 : 5, i.e., odd integers only.

Illustration 52:

A particle is dropped from height 100 m and another particle is projected vertically up with velocity 50 m/s from the ground along the same line. Find out the height where two particle will meet? (take g = 10 m/s^2)

Solution:



$S_1 = ut - \frac{1}{2}gt^2$ ...(i)

$S_2 = \frac{1}{2}gt^2$ ...(ii) $S_1 + S_2 = 100$ m

From eq. (i) and (ii) $S_2 = \frac{1}{2}gt^2 = \frac{1}{2}\times 9.8 \times 2^2 = 19.62\text{m}$ (distance from top)

We get $S_1 + S_2 = ut$ Height (s_1) = 100-19.62=80.38m

ut = 100 $\Rightarrow$ 50t = 100 $\Rightarrow$ t = 2s

Illustration 53:

A body falls freely from rest. It covers as much distance in the last second of its motion as covered in the first three seconds. The body has fallen for a time of

Solution:

$\frac{1}{2}g(3)^2 = \frac{g}{2}(2n-1) \Rightarrow n = 5\,s$

Illustration 54:

A body is released from the top of a tower of height h. It takes t sec to reach the ground. Where will be the ball after time t/2 sec

Solution:

Let the body after time t/2 be at x from the top, then

$x = \frac{1}{2}g\frac{t^2}{4} = \frac{gt^2}{8}$ ...(i) $h = \frac{1}{2}gt^2$ ...(ii)

Eliminate t from (i) and (ii), we get $x = \frac{h}{4}$

$\therefore$ Height of the body from the ground $= h - \frac{h}{4} = \frac{3h}{4}$

Illustration 55:

A body is released from a great height and falls freely towards the earth. Another body is released from the same height exactly one second later. The separation between the two bodies, two seconds after the release of the second body is

Solution:

The separation between the two bodies, two seconds after the release of second body

$$= \frac{1}{2}\times 9.8[(3)^2 - (2)^2] = 24.5\,\text{m}$$

Illustration 56:

A body falls from a height h = 200m. The ratio of distance travelled in each 2 sec during t = 0 to t = 6 second of the journey is

Solution:

$S_n \propto (2n-1)$ In equal time interval of 2 seconds

Ratio of distance = 1 : 3 : 5

Motion under Gravity-Vertical Projection from Height

Vertical Projection from Height

CASE 1 : Downward Projection

Equations of motion:

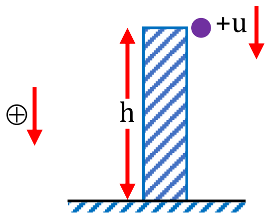
Taking initial position as origin and direction of motion (i.e., downward direction) as a positive, we have

$v = u + gt$ ...(i)

$h = ut + \frac{1}{2}gt^2$ ...(ii)

$v^2 = u^2 + 2gh$ ...(iii)

$h_n = u + \frac{g}{2}(2n-1) \Rightarrow v_{max} = \sqrt{u^2 + 2gh}$



CASE 2: Upward Projection

Equations of motion :

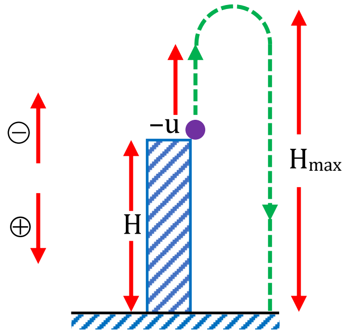
Taking initial position as origin and direction of motion (i.e., upward direction) as negative, here we have

$v = -u + gt$ ...(i)

$h = -ut + \frac{1}{2}gt^2$ ...(ii)

$v^2 = u^2 + 2gh$ ...(iii)

$h_{n^{th}} = -u + \frac{g}{2}(2n-1)$



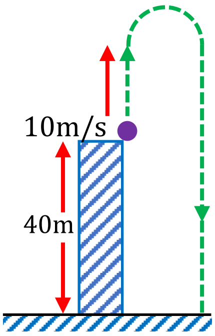
- Maximum height attained by the body $H_{max} = H + \frac{u^2}{2g}$
- Distance travelled by the body = $H + \frac{u^2}{g}$
- Time taken by the body to reach the ground

  $H = -ut + \frac{1}{2}gt^2 \Rightarrow \frac{1}{2}gt^2 - ut - H = 0 \Rightarrow gt^2 - 2ut - 2H = 0$

  After solving this equation, we get the result.

Illustration 57:

A ball is thrown upwards from the top of a tower 40m high with a velocity of 10 m/s, find the time when it strikes the ground ($g = 10\ m/s^2$)



Solution:

In the problem $u = +10\ m/s$, $a = -10\ m/s^2$ and $s = -40m$

Substituting in $s = ut + \frac{1}{2}at^2$

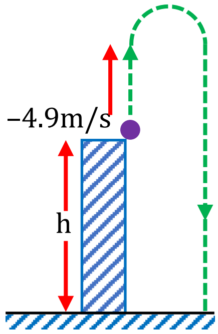
$-40 = 10t - 5t^2$ or $5t^2 - 10t - 40 = 0$ or $t^2 - 2t - 8 = 0$

Solving this we have t = 4 s and – 2s. Taking the positive value t = 4s

Illustration 58:

A pebble is thrown vertically upwards from a bridge with an initial velocity of 4.9 m/s. It strikes the water after 2s. If acceleration due to gravity is $9.8 m/s^2$ (a) what is the height of the bridge? (b) with what velocity does the pebble strike the water?

Solution:



(a) Let height of the bridge be h then

$$h = -4.9 \times 2 + \frac{1}{2} \times 9.8 \times (2)^2$$

$$h = 9.8 \text{ m}$$

(b) velocity with which the ball strikes the water surface

$$v = u + at$$

$$v = -4.9 + 9.8 \times 2 = 14.7 \text{ m/s}$$

Illustration 59:

A man in a balloon rising vertically with an acceleration of $4.9 m/sec^2$ releases a ball 2 sec after the balloon is let go from the ground. The greatest height above the ground reached by the ball is $(g = 9.8 m/sec^2)$

Solution:

Height travelled by ball (with balloon) in 2 sec

$$h_1 = \frac{1}{2}at^2 = \frac{1}{2} \times 4.9 \times 2^2 = 9.8 \text{ m}$$

Velocity of the balloon after 2 sec

$$v = at = 4.9 \times 2 = 9.8 \text{ m/s}$$

Now if the ball is released from the balloon then it acquire same velocity in upward direction.

Let it move up to maximum height h_2

$$v^2 = u^2 - 2gh_2 \Rightarrow 0 = (9.8)^2 - 2 \times (9.8) \times h_2$$

$$\therefore h_2 = 4.9 \text{m}$$

Greatest height above the ground reached by the ball $= h_1 + h_2 = 9.8 + 4.9 = 14.7 \text{ m}$

Illustration 60:

Three particles A, B and C are thrown from the top of a tower with the same speed. A is thrown up, B is thrown down and C is horizontally. They hit the ground with speeds V_A, V_B and V_C respectively.

Solution:

$$v^2 = u^2 + 2gh \Rightarrow v = \sqrt{u^2 + 2gh}$$

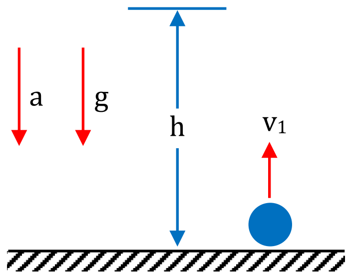
So, for all the cases velocity will be equal.

Effect of Air Resistance

If 'a' is retardation due to air resistance,

Case-1

For ascending motion(upward motion)



net acceleration $a_{Net} = g + a$ (downwards)

If maximum height attained by the particle is 'h' then

Time to go up,

$$t_1 = \sqrt{\frac{2h}{g + a}}$$

Velocity of projection,

$$v_1 = \sqrt{2(g + a)h}$$

Case-2

For descending motion (downward motion)

net acceleration $a_{Net} = g - a$ (downwards)

If body is released from maximum height 'h' then

Time to go up,

$$t_2 = \sqrt{\frac{2h}{g-a}}$$

Velocity on reaching ground,

$$v_2 = \sqrt{2(g-a)h}$$

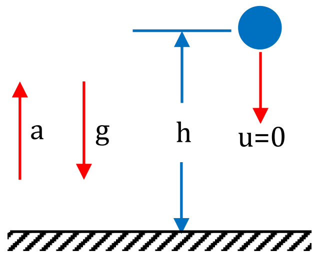


Illustration 61:

A body is thrown vertically upwards. If air resistance is to be taken into account, then the time during which the body rises then the time of fall

Solution:

For ascending motion $t_1 = \sqrt{\frac{2h}{g+a}}$

For descending motion $t_2 = \sqrt{\frac{2h}{g-a}}$

$t_2 > t_1$

$\therefore$ time during the body rises is less than the time of fall.

BEGINNER'S BOX-6

1. A particle is projected vertically upwards from ground with velocity 10 m/s. Find the time taken by it to reach at the highest point?

PMP205

2. A particle is projected vertically up from the top of a tower with velocity 10 m/s. It reaches the ground in 5s. Find–

(a) Height of tower. (b) Striking velocity of particle at ground

(c) Distance traversed by particle. (d) Average speed & average velocity of particle.

PMP206

3. A balloon starts rising from the ground with an acceleration of 1.25 m/s^2. A stone is released from the balloon after 10s. Determine

(1) maximum height of stone from ground (2) time taken by stone to reach the ground

PMP207

4. A rocket is fired vertically up from the ground with an acceleration 10 m/s^2. If its fuel is finished after 1 minute then calculate –

(a) Maximum velocity attained by rocket in ascending motion.

(b) Height attained by rocket before fuel is finished.

(c) Time taken by the rocket in the whole motion.

(d) Maximum height attained by rocket.

PMP208

5. A particle is dropped from the top of a tower. During its motion it covers $\frac{9}{25}$ part of height of tower in the last 1 second Then find the height of tower.

PMP209

6. A particle is dropped from the top of a tower. It covers 40 m in last 2s. Find the height of the tower.

PMP210

7. A player throws a ball upwards with an initial speed of 29.4 m/s.

(a) What is the direction of acceleration during the upward motion of the ball?

(b) What are the velocity and acceleration of the ball at the highest point of its motion?

(c) Choose x = 0 m and t = 0 s to be the location and time of the ball at its highest point, vertically downward direction to be the positive direction of x-axis and give the signs of position, velocity and acceleration of the ball during its upward and downward motion.

(d) To what height does the ball rise and how long does the ball take to return to the player's hands? (Take g = 9.8 m/s^2 and neglect air resistance).

PMP211

8. A particle is dropped from the top of a tower. The distance covered by it in the last one second is equal to that covered by it in the first three seconds. Find the height of the tower.

PMP212

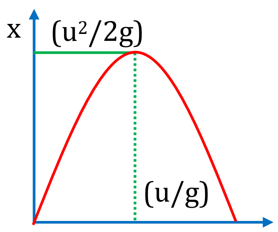
9. Water drops are falling in regular intervals of time from top of a tower of height 9 m. If 4th drop begins to fall when 1st drop reaches the ground, find the positions of 2nd & 3rd drops from the top of the tower.

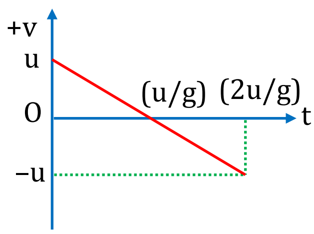
PMP213

Graphical Problems in Vertical Motion

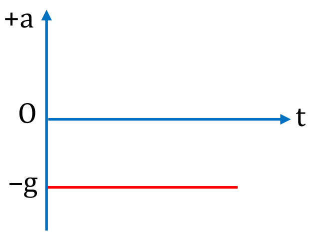
1. A body is projected vertically upwards then

(a) Graphs of displacement with respect to time : (b) Graph of velocity with respect to time



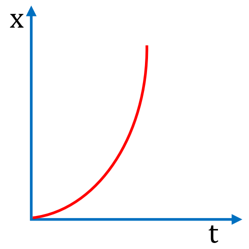


(c) Graph of acceleration with respect to time

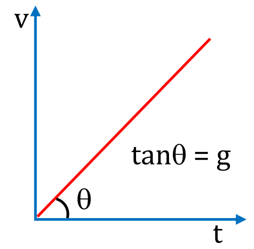


2. A body dropped from some height

(a) Graphs of displacement with respect to time:



(b) Graph of velocity with respect to time



(c) Graph of acceleration with respect to time

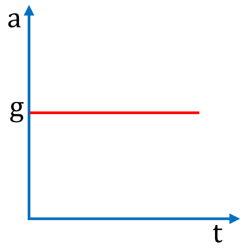


Illustration 62:

A particle is thrown vertically upward with 30 m/s, reaches on maximum height and come on ground after some time. Consider vertically upward direction positive and point of projection as origin. (if $g=10\ m/sec^2$)

(i) Acceleration time graph

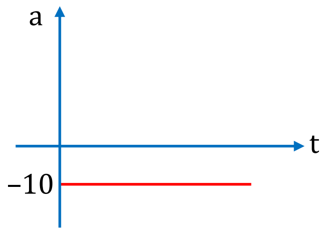
(ii) Velocity time graph

(iii) Speed time graph

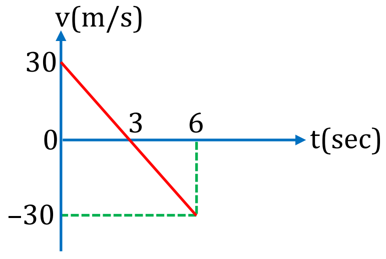
(iv) Displacement time graph

Solution:

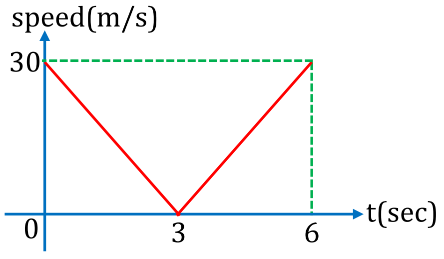
(i) Acceleration time graph



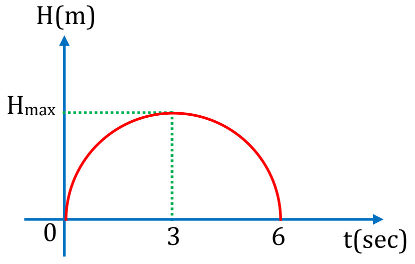
(ii) Velocity time graph



(iii) Speed time graph



(iv) Displacement time graph



Motion in a Plane

Ground to Ground Projectile Motion

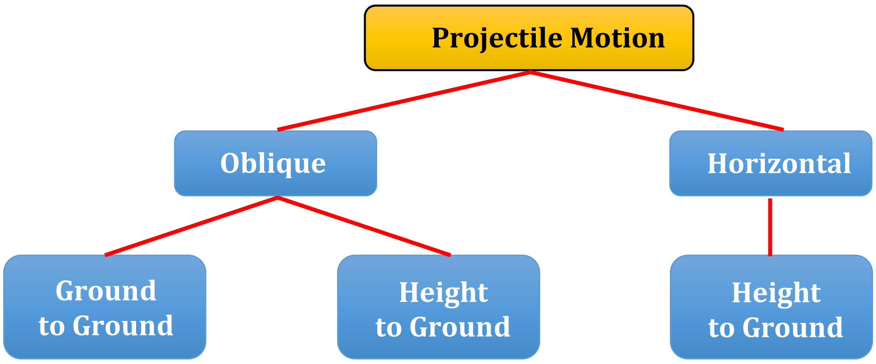
2D Motion:

- When the motion of an object is restricted within a plane, it is said to undergo a motion in 2D.
- 2D motion can be studied as two independent 1D motions. (One along x-axis and the other along y-axis).

  Example: Motion of a carrom coin, Projectile Motion, Circular Motion.

Projectile Motion

When a body moves with constant acceleration such that its initial velocity and acceleration are non-collinear then its path is parabola and motion is known as projectile motion.



Projectile Motion = Horizontal Motion + Vertical Motion

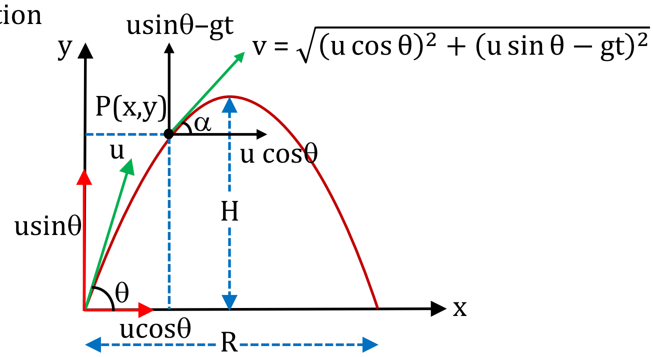
$u_x = u\cos\theta$; $u_y = u\sin\theta$

$a_x = 0$ $a_y = -g$

$v_x = u_x = u\cos\theta$ $v_y = u_y - gt$

$x = (u\cos\theta)t$

$y = (u\sin\theta)t - \frac{1}{2}gt^2$



Horizontal Motion

- Initial velocity in horizontal direction = $u\cos\theta = u_x$
- Acceleration along horizontal direction = $a_x = 0$. (Neglect air resistance)
- Therefore, Horizontal velocity remains unchanged.
- At any instant horizontal velocity $v_x = u\cos\theta$
- In time t, displacement along horizontal direction or x co-ordinate is, $x = u_x t$ or $x = (u\cos\theta)t$

Vertical Motion

- It is motion under the effect of gravity so that as particle moves upwards the magnitude of its vertical velocity decreases.
- Initial velocity in vertical direction = $u\sin\theta = u_y$
- Acceleration along vertical direction = $a_y = -g$ (Neglect air resistance)
- At any instant, vertical speed $v_y = u_y - gt = u\sin\theta - gt$
- In time t, displacement in vertical direction or "height" of the particle above the ground

$$y = u_y t - \frac{1}{2}gt^2 = u\sin\theta t - \frac{1}{2}gt^2$$

- Equations of motion should be applied separately for x and y directions.
- Velocity and position can be dealt separately in x and y directions as shown in the table given below:

x-axis	y-axis	Resultant
$v_x = u_x + a_x t$	$v_y = u_y + a_y t$	$\vec{v} = v_x\hat{i} + v_y\hat{j};\ \lvert\vec{v}\rvert = \sqrt{v_x^2 + v_y^2}$
$\Delta x = u_x t + \frac{1}{2}a_x t^2$	$\Delta y = u_y t + \frac{1}{2}a_y t^2$	$\vec{r} = x\hat{i} + y\hat{j};\ \lvert\vec{r}\rvert = \sqrt{x^2 + y^2}$

Illustration 1:

A particle has initial velocity, $\vec{u} = 2\hat{i} + 3\hat{j}$ and acceleration, $\vec{a} = 4\hat{i} + 2\hat{j}$ then, Find

(a) $\vec{v}$ at t = 2sec. (b) $\vec{s}$ from 0 to 2 sec.

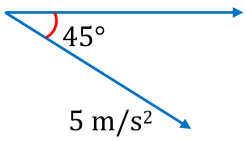
Solution:

x-direction	y-direction
$u_x = 2$	$u_y = 3$
$a_x = 4$	$a_y = 2$
$t = 2$	$t = 2$
$v_x = u_x + a_x t$	$v_y = u_y + a_y t$
$v_x = 2 + 4 \times 2 = 10$	$v_y = 3 + 2 \times 2 = 7$
$s_x = \left(\frac{u_x + v_x}{2}\right)t$	$s_y = \left(\frac{u_y + v_y}{2}\right)t$
$s_x = \left(\frac{2+10}{2}\right)2 = 12$	$s_y = \left(\frac{3+7}{2}\right)2 = 10$

$\vec{v} = 10\hat{i} + 7\hat{j}$ and $\vec{s} = 12\hat{i} + 10\hat{j}$

Illustration 2:

A roller coaster goes down along 45° incline with an acceleration of 5.0 ms^{-2}. (Starts from rest)



(a) How far will the roller coaster travel in 10 seconds horizontally?

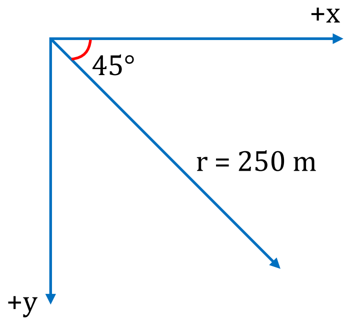
(b) How far will the roller coaster travel in 10 seconds vertically?

Solution:

The motion of the roller coaster can be considered as a 1D motion along the direction of acceleration. Taking the direction of acceleration as r-direction.

$$x = x_0 + ut + \frac{1}{2}at^2$$

In r-direction, $r = r_0 + ut + \frac{1}{2}at^2 = 0 + 0 \times t + \frac{1}{2}(5)(10)^2$ = 250 m



Taking right direction as positive x axis and downwards as positive y axis,

Displacement along y-direction

$$\Delta y = r\sin\theta = 250 \sin 45° = \frac{250}{\sqrt{2}}\text{m}$$

Displacement along x-direction

$$\Delta x = r\cos\theta = 250 \cos 45° = \frac{250}{\sqrt{2}}\text{m}$$

Illustration 3:
A body is projected with a velocity u at an angle θ with the horizontal. The velocity of the body will become perpendicular to the velocity of projection after a time t given by.

Solution:

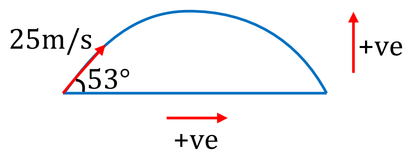
$v_y = u\sin\theta - gt$

$v_x = u\cos\theta \Rightarrow \left(u\cos\theta\hat{i} + u\sin\theta\hat{j}\right)\cdot\left(u\cos\theta\hat{i} + \left(u\sin\theta - gt\right)\hat{j}\right) = 0$

$\Rightarrow u^2\cos^2\theta + u^2\sin^2\theta - u\sin\theta gt = 0 \Rightarrow u^2 = u\sin\theta gt \;\; \Rightarrow t = \dfrac{u}{g\sin\theta}$

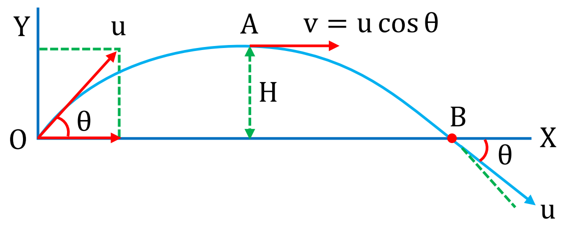
Illustration 4:
A particle is projected with initial velocity u = 25m/s and θ = 53° with horizontal? Find height of projectile when horizontal distance is 45 m?

Solution:



x-direction	y-direction
$u_x = 25\cos 53°$	$u_y = 25\sin 53°$
$= 15$ m/s	20 m/s
$a_x = 0$	$a_y = -10$
$s_x = 45$	t = 3 sec.
$45 = 15 \times t$	$s_y = 20\times3 - \frac{1}{2}\times10\times3^2$
$t = \frac{45}{15} = 3s$	$s_y = 60 - 45 = 15$ m

Standard Results of Ground to Ground Projectile Motion
Time of flight (T)
Time of flight is the time for which projectile remains in air.
At time T particle will be at ground again, i.e. displacement along Y-axis becomes zero.



$\because\; y = u_y t - \frac{1}{2}gt^2 \qquad \therefore\; 0 = u_y T - \frac{1}{2}gT^2$

or $T = \dfrac{2u_y}{g} = \dfrac{2u\sin\theta}{g}$

Time of ascent = Time of descent = $\dfrac{T}{2} = \dfrac{u_y}{g} = \dfrac{u\sin\theta}{g}$

At time $\dfrac{T}{2}$ particle attains maximum height of its trajectory.

Maximum height (H)
At maximum height vertical component of velocity becomes zero. At this instant y coordinate is, its maximum height.

$\because\; v_y^2 = u_y^2 - 2gy \qquad \therefore\; 0 = u_y^2 - 2gH \;\{\because v_y = 0, y = H\}$ so, $H = \dfrac{u_y^2}{2g} = \dfrac{u^2\sin^2\theta}{2g}$

Horizontal range or Range (R)
It is the displacement of particle along X-direction during its complete flight.

$\because x = u_x t \qquad \therefore R = u_x T = u_x\dfrac{2u_y}{g}; \qquad R = \dfrac{2u_x u_y}{g}$

$R = \dfrac{2(u\cos\theta)(u\sin\theta)}{g} \Rightarrow R = \dfrac{u^2\sin 2\theta}{g} \qquad (\because 2\sin\theta\cos\theta = \sin 2\theta)$

Maximum horizontal range (R_{max})

If value of θ is increased from θ = 0° to 90°, then range increases from θ = 0° to 45° but it decreases beyond 45°. Thus range is maximum at θ = 45°

For maximum range, $\theta = 45°$ and $R_{max} = \frac{u^2 \sin 2(45°)}{g} = \frac{u^2 \sin 90°}{g} \Rightarrow R_{max} = \frac{u^2}{g}$

Comparison of two projectiles of equal range

When two projectiles are thrown with equal speeds at angles θ and (90° – θ) then their ranges are equal but maximum heights attained are different and time of flights are also different.

At angle θ, $R = \frac{u^2 \sin 2\theta}{g}$

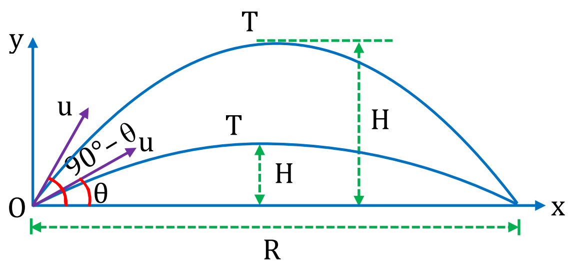
At angle $(90° - \theta), R' = \frac{u^2 \sin 2(90° - \theta)}{g} = \frac{u^2 \sin(180° - 2\theta)}{g} = \frac{u^2 \sin 2\theta}{g}$

Thus, $R' = R$

Maximum height of projectiles

$$H = \frac{u^2 \sin^2 \theta}{2g} \quad \text{and} \quad H' = \frac{u^2 \sin^2(90° - \theta)}{2g} = \frac{u^2 \cos^2 \theta}{2g}$$

- $\frac{H}{H'} = \frac{\sin^2 \theta}{\cos^2 \theta} = \tan^2 \theta$
- $HH' = \frac{u^4 \sin^2 \theta \cos^2 \theta}{4g^2} = \frac{R^2}{16} \Rightarrow R = 4\sqrt{HH'}$
- $H + H' = \frac{u^2 \sin^2 \theta}{2g} + \frac{u^2 \cos^2 \theta}{2g} \Rightarrow H + H' = \frac{u^2}{2g}$



Time of flight of projectiles

$$T = \frac{2u \sin \theta}{g}; T' = \frac{2u \sin(90 - \theta)}{g} = \frac{2u \cos \theta}{g}$$

- $\frac{T}{T'} = \frac{\sin \theta}{\cos \theta} = \tan \theta$
- $TT' = \frac{4u^2 \sin \theta \cos \theta}{g^2} = \frac{2R}{g} \quad \Rightarrow \quad TT' \propto R$

Equation of Trajectory

Along horizontal direction $x = u_x t$ or $x = u \cos \theta t$

Along vertical direction $y = u_y t - \frac{1}{2} g t^2$ or $y = u \sin \theta t - \frac{1}{2} g t^2$

On eliminating t from these two equations

$$y = (u \sin \theta)\left(\frac{x}{u \cos \theta}\right) - \frac{1}{2} g \left(\frac{x}{u \cos \theta}\right)^2 \Rightarrow y = x \tan \theta - \frac{1}{2} g \frac{x^2}{u^2 \cos^2 \theta}$$

This is an equation of a parabola so it can be stated that projectile follows a parabolic path.

Again $y = x \tan \theta \left[1 - \frac{gx}{2u^2 \sin \theta \cos \theta}\right] = x \tan \theta \left[1 - \frac{x}{R}\right]$

Kinetic Energy of a Projectile

Kinetic energy = $\frac{1}{2}\times$ Mass × (Speed)^2

Let a body is projected with velocity u at an angle θ .

Thus, initial kinetic energy of projectile, $K_0 = \frac{1}{2}mu^2$

Since velocity of projectile at maximum height is $u\cos\theta$.

Kinetic energy at highest point, $K = \frac{1}{2}m(u\cos\theta)^2 = K_0\cos^2\theta$

Which is the minimum kinetic energy during whole motion.

Key Points

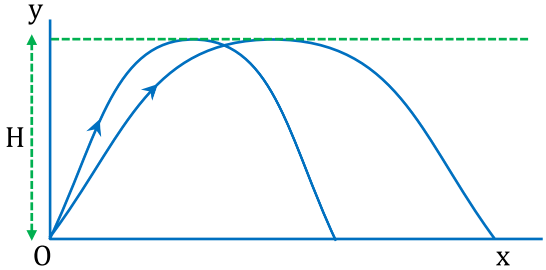
- At maximum height, v_y = 0 and $v_x = u_x = u\cos\theta$ so that at maximum height $v = \sqrt{v_x^2 + v_y^2} = u\cos\theta$
- At maximum height angle between velocity and acceleration is 90°.
- Magnitude of velocity at height 'h'.

  $v_y^2 = u_y^2 - 2gh$

  $v_y^2 = (u\sin\theta)^2 - 2gh$ and $v_x = u\cos\theta$

  $|\vec{v}| = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2\cos^2\theta + (u\sin\theta)^2 - 2gh}$

  $|\vec{v}| = \sqrt{u^2 - 2gh}$



- $T = \frac{2u_y}{g}$, $H = \frac{u_y^2}{2g}$, $R = \frac{2u_x u_y}{g}$

  T and H depend only upon initial vertical speed u_y

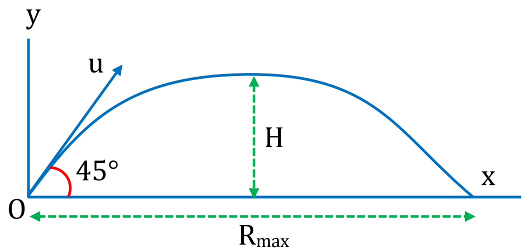
  If two projectiles thrown in different directions, have equal times of flight then their initial vertical speeds are same so that their maximum height are is also same.

  If $H_A = H_B$ then $(u_y)_A = (u_y)_B$ and $T_A = T_B$

- For situation shown in figure for $\theta = 45°$

  Here $R_{max} = \frac{u^2}{g}$ and $H = \frac{u^2\sin^2 45°}{2g} = \frac{u^2}{4g}$

  $\therefore$ R_{max} = 4H = 4 (maximum height attained)



- When R = H $R = \frac{u^2(2\sin\theta\cos\theta)}{g}$ and $H = \frac{u^2\sin^2\theta}{2g}$ $\Rightarrow$ $\frac{R}{H} = 4\cot\theta = 1$

  $\Rightarrow 4\cot\theta = 1$ $\Rightarrow$ $\tan\theta = 4$ $\Rightarrow$ $\theta = \tan^{-1}(4) \approx 76°$

Illustration 5:

A projectile is thrown with speed u making angle θ with horizontal at t = 0. It just crosses two points of equal height, at time t = 1s and t = 3s respectively. Calculate the maximum height attained by it? (g=10m/s^2)

Solution:

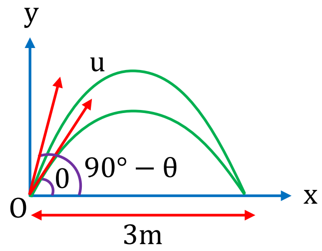
Displacement in y direction y = $u_y \times 1 - \frac{1}{2}g \times (1)^2 = u_y \times 3 - \frac{1}{2}g(3)^2 \Rightarrow u_y = 2g = 20$ m/s

Maximum height attained $h_{max} = \frac{u_y^2}{2g}$ = 20m

Illustration 6:
A stone is to be thrown so as to cover a horizontal distance of 3m. If the velocity of the projectile is 7 m/s, find: (a)the angle at which is must be thrown.

(b) the largest horizontal displacement that is possible with the projection speed of 7 m/s.

Solution:



(a) Range $R=\frac{u^2}{g}\sin 2\theta$

$\Rightarrow \quad \sin 2\theta=\frac{gR}{u^2}=\frac{9.8\times 3}{(7)^2}=0.6=\sin 37°\Rightarrow 2\theta=37°\Rightarrow\theta=18.5°$

angle of projection may also be = 90° – θ = 90° – 18.5° = 71.5°

(b) For largest horizontal displacement θ = 45°

Maximum range $R_{max.}=\frac{u^2}{g}=\frac{(7)^2}{9.8}=\frac{49}{98}\times 10$ = 5 m

Illustration 7:
Two projectiles are projected at angles (θ) and $\left(\frac{\pi}{2}-\theta\right)$ to the horizontal respectively with same speed 20 m/s. One of them rises 10 m higher than the other. Find the angles of projection. (Take g=10 m/s^2)

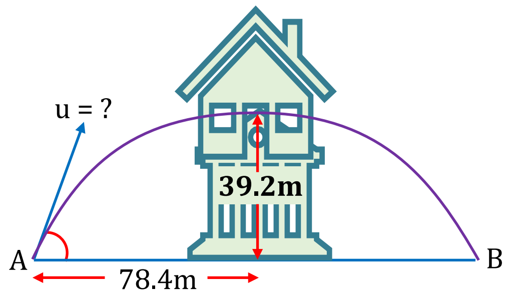
Solution:

Maximum height H = $\frac{u^2\sin^2\theta}{2g}\Rightarrow h_1=\frac{(20)^2\sin^2\theta}{2g}=20\sin^2\theta$ & $h_2=\frac{(20)^2\sin^2(\pi/2-\theta)}{2g}=20\cos^2\theta$

$h_2-h_1=20\left[\cos^2\theta-\sin^2\theta\right]=10\Rightarrow 20\cos 2\theta=10\Rightarrow\cos 2\theta=\frac{1}{2}\Rightarrow 2\theta=60°\Rightarrow\theta=30°$

$\{\because \cos 2\theta=\cos^2\theta-\sin^2\theta\}$ and $\theta'=90°-\theta=60°$

Illustration 8:
A boy stands 78.4 m away from a building and throws a ball which just enters a window at maximum height 39.2m above the ground. Calculate the velocity of projection of the ball.



Solution:

Maximum height = $\frac{u^2\sin^2\theta}{2g}$ = 39.2 m ...(i)

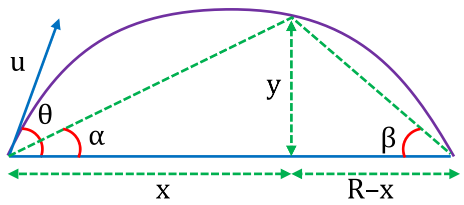
Range = $\frac{u^2\sin 2\theta}{g}=\frac{2u^2\sin\theta\cos\theta}{g}=2\times 78.4$ ...(ii)

From equation (i) divided by equation (ii) $\tan\theta=1\Rightarrow\theta=45°$

From equation (ii) range = $\frac{u^2\sin 90°}{g}=2\times 78.4\Rightarrow u=\sqrt{2\times 78.4\times 9.8}=39.2$ m/s

Illustration 9:
A particle thrown over a triangle from one end of a horizontal base falls on the other end of the base after grazing the vertex. If α and β are the base angles of triangle and angle of projection is θ, then prove that $\tan\theta=\tan\alpha+\tan\beta$.

Solution:



From triangle y = x tan α and y = (R – x) tan β

$$\tan\alpha+\tan\beta=\frac{y}{x}+\frac{y}{R-x}=\frac{yR}{x(R-x)} \quad \text{...(i)}$$

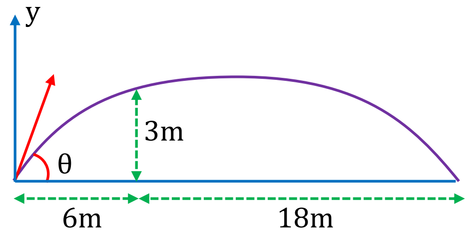
Equation of trajectory

$$\because\ y=x\tan\theta\left[1-\frac{x}{R}\right]\Rightarrow \tan\theta=\frac{yR}{x(R-x)} \quad \text{...(ii)}$$

From equations (i) and (ii) $\therefore \tan\theta=\tan\alpha+\tan\beta$

Illustration 10:

A ball is thrown from the ground to clear a wall 3 m high at a distance of 6 m and falls 18 m away from the wall, the angle of projection of ball is: -



Solution:

Given : R = 18 + 6 = 24m

x = 6m, y = 3m

From equation of trajectory, $y=x\tan\theta\left[1-\frac{x}{R}\right]\Rightarrow 3=6\tan\theta\left[1-\frac{1}{4}\right]\Rightarrow \tan\theta=\frac{2}{3}$ so, $\theta=\tan^{-1}\left(\frac{2}{3}\right)$

Problems based on Ground to Ground Projectile Motion

Illustration 11:

A body is projected from ground with velocity $\left(4\hat{i}+3\hat{j}\right)$ m/s. Find out:

1. Time of flight
2. Maximum height
3. Horizontal range
4. Velocity when body reaches ground

Solution:

1. Time of flight (T) = $\frac{2u_y}{g}=\frac{2\times3}{10}=0.6$ s

2. Maximum height $H_{max}=\frac{u_y^2}{2g}=\frac{9\times100}{2\times10}=45$ cm

3. Horizontal range

   R = u_x × T = 4 × 0.6 = 2.4 m

4. Velocity when body reaches ground....

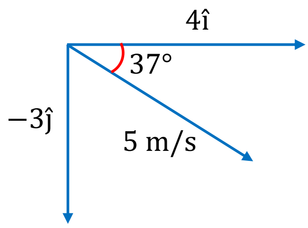


Illustration 12:

A body is projected from ground with speed 40 m/s for maximum range. Then find out maximum height attained by the body.

Solution:

When $\theta=45°$,

Horizontal range is maximum,

$$R_{max}=\frac{u^2}{g}$$

And height is,

$$H=\frac{u^2\sin^2 45°}{2g}=\frac{u^2}{4g}=\frac{R_{max}}{4}$$

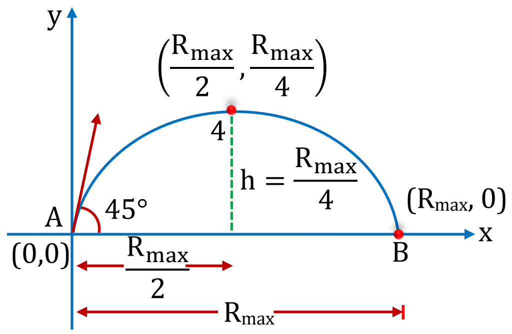


Illustration 13:

A ground to ground projectile has equation of trajectory, $y = x - \frac{x^2}{40}$. Find angle of projection and speed of projection.

Solution:

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$ (General equation) ...(i)

$y = x - \frac{x^2}{40}$ (Given equation) ...(ii)

By comparing equation (i) and (ii) $\tan\theta = 1; \theta = 45°$

$\frac{g}{2u^2\cos^2\theta} = \frac{1}{40}$ $\therefore u = 20\ m/s$

$\left(y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}\right)$ is valid when we take starting point as origin and $\xrightarrow{+ve} \uparrow +ve$

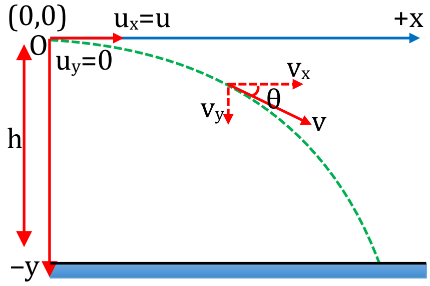
BEGINNER'S BOX-7

1. A football player kicks a ball at an angle of 30° to the horizontal with an initial speed of 20 m/s. Assuming that the ball travels in a vertical plane, calculate (a) the time at which the ball reaches the highest point (b) the maximum height reached (c) the horizontal range of the ball (d) the time for which the ball is in the air. ($g = 10\ m/s^2$) **PMP214**
2. A cricketer can throw a ball to a maximum horizontal distance of 100 m. How high above the ground can the cricketer throw the ball, with the same speed? **PMP215**
3. Two bodies are thrown with the same initial speed at angles α and $(90° - \alpha)$ with the horizontal. What will be the ratio of (a) maximum heights attained by them and (b) horizontal ranges ? **PMP216**
4. A ball is thrown at angle θ and another ball is thrown at angle $(90° - \theta)$ with the horizontal direction from the same point each with speeds of 40 m/s. The second ball reaches 50m higher than the first ball. Find their individual heights. $g = 10\ m/s^2$. **PMP217**
5. The range of a particle when launched at an angle of 15° with the horizontal is 1.5 km. What is the range of the projectile when launched at an angle 45° to the horizontal. **PMP218**
6. Show that the projection angle θ_0 for a projectile launched from the origin is given by :

   $\theta_0 = \tan^{-1}\left[\frac{4H_m}{R}\right]$ Where H_m = Maximum height, R = Horizontal range **PMP219**
7. A ball of mass m is thrown vertically up. Another ball of mass 2m is thrown at an angle θ with the vertical. Both of them stay in air for the same periods of time. What is the ratio of the height attained by the two balls? **PMP220**
8. The ceiling of a long hall is 25 m high. What is the maximum horizontal distance that a ball thrown with a speed of 40 m/s can go without hitting the ceiling of the hall? ($g = 10\ m/s^2$) **PMP221**

Horizontal Projection from Height

Horizontal Projection



Consider a projectile thrown from point O at some height h from the ground with a velocity u in horizontal direction.

Now we shall deal the characteristics of projectile motion separately along horizontal and vertical directions i.e.

Horizontal Direction

(i) Initial velocity $u_x = u$ (ii) Acceleration $a_x = 0$

Vertical Direction

(i) Initial velocity $u_y = 0$ (ii) Acceleration $a_y = -g$ (downward)

Equation of Trajectory

The path traced by projectile is called its trajectory. After time t,

x = ut

$y = -\frac{1}{2}gt^2$ {negative sign indicates that the direction of vertical displacement is downward}

So $y = -\frac{1}{2}g\frac{x^2}{u^2}$ $(\because t = \frac{x}{u})$

This is an equation of a parabola so it can be stated that projectile follows a parabolic path.

Velocity at a General Point P(x,y)

$v = \sqrt{v_x^2 + v_y^2}$

Velocity of the projectile in horizontal direction after time t is

$v_x = u$ (remains constant)

Velocity of projectile in vertical direction after time t is

$v_y = 0 - (g)t = -gt$ (downward) $\therefore v = \sqrt{u^2 + g^2t^2}$

and $\tan\theta = \frac{v_y}{v_x}$ or $\tan\theta = -\frac{gt}{u}$ {negative sign indicates clockwise direction}

Displacement

The displacement of the particle is expressed by; $\vec{s} = x\hat{i} + y\hat{j}$ Where $|\vec{s}| = \sqrt{x^2 + y^2} = \left|(ut)\hat{i} - (\frac{1}{2}gt^2)\hat{j}\right|$

Time of Flight

From equation of motion for vertical direction; $h = u_y t + \frac{1}{2}gt^2$

At highest point $u_y = 0$ $\Rightarrow h = \frac{1}{2}gT^2 \Rightarrow T = \sqrt{\frac{2h}{g}}$

Horizontal Range

Distance covered by the projectile along the horizontal direction between the point of projection to the point on the ground; $R = u_x t = u\sqrt{\frac{2h}{g}}$

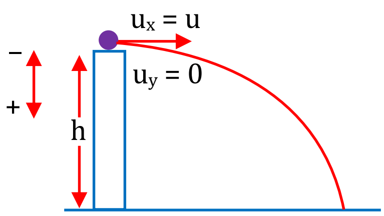
Velocity After Falling a Height h_1

Along vertical direction $v_y^2 = 0^2 + 2(h_1)(g)$ so, $v_y = \sqrt{2gh_1}$

Along horizontal direction $v_x = u_x = u$

So net velocity, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh_1}$

Important Parameters



Time of flight

$$h = u_y t + \frac{1}{2} g T^2$$

$$h = \frac{1}{2} g T^2; \quad R = u\sqrt{\frac{2h}{g}}; \quad T = \sqrt{\frac{2h}{g}}$$

Horizontal Range

R = uT

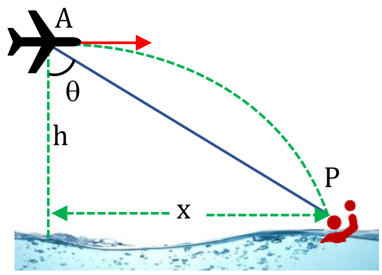
Illustration 14:

A relief aeroplane is flying at a constant height of 1960 m with 600 km/hr speed above the ground towards a point directly over a person struggling in flood water. At what angle of sight with the vertical should the pilot release a survival kit if it is to reach the person in water? (g = 9.8m/s²)

Solution:

Plane is flying at a speed = $600 \times \frac{5}{18} = \frac{500}{3}$ m/s horizontally

(at a height 1960m)



Time taken by the kit to reach the ground $t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 1960}{9.8}} = 20s$

In this time the kit will move horizontally by $x = ut = \frac{500}{3} \times 20 = \frac{10,000}{3} m$

So, $\tan\theta = \frac{x}{h} = \frac{10,000}{3 \times 1960} = \frac{10}{5.88} = 1.7 \simeq \sqrt{3}$ or $\theta = 60°$

Illustration 15:

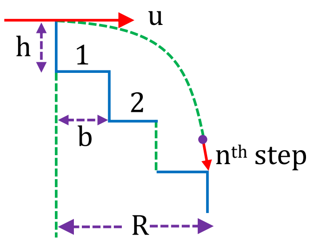
A ball rolls off the top of a stair way with a horizontal velocity u. If each step has height h and width b the ball will just hit the edge of nth step. Find the value of n.

Solution:

If the ball hits the n^{th} step, the horizontal and vertical distances traversed are nb and nh respectively. Let t be the time taken by the ball for these horizontal and vertical displacements.

Velocity along horizontal direction = u (remains constant)

Initial vertical velocity = zero.



$\therefore \quad nb = ut$ and $\quad nh = 0 + \frac{1}{2} g t^2$

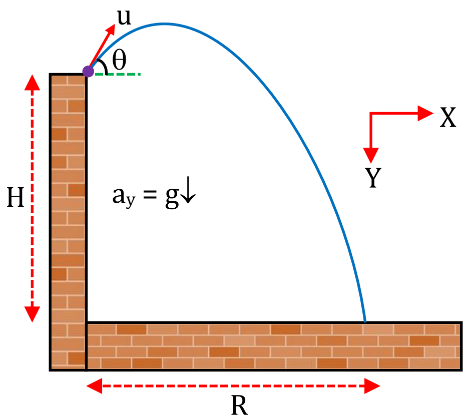
Eliminating t from the equation

$$nh = \frac{1}{2} g \left(\frac{nb}{u}\right)^2 \Rightarrow n = \frac{2hu^2}{gb^2}$$

Oblique Projection from Height

Oblique Projection from a Certain Height

CASE 1: Projection from a height at an angle θ above horizontal:



$u_x = u\cos\theta \qquad u_y = -u\sin\theta$

$x = (u\cos\theta)t \qquad a_y = g$

$$H = (-u\sin\theta)t + \frac{1}{2} g t^2$$

$$gt^2 - (2u\sin\theta)t - 2H = 0$$

After solving the above equation, we get the result i.e. value of 't'

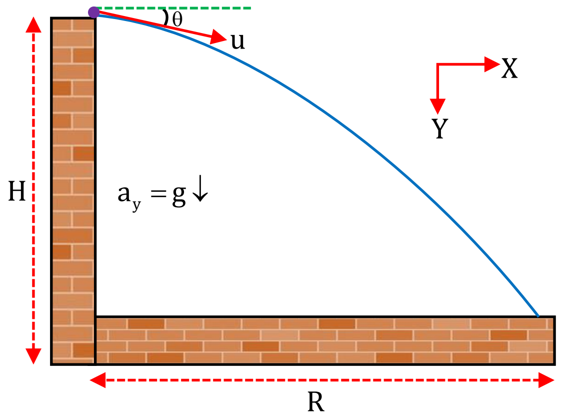
Velocity after falling height h:

Along vertical direction; $v_y^2 = (-u\sin\theta)^2 + 2(h)(g)$

Along horizontal direction, $v_x = u_x = u\cos\theta$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$$

CASE 2: Projection from a height at an angle θ Below horizontal



$u_x = u\cos\theta$ $\qquad$ $u_y = u\sin\theta$

$x = (u\cos\theta)t$ $\qquad$ $a_y = g$

$$H = (u\sin\theta)t + \frac{1}{2}gt^2$$

$$gt^2 + (2u\sin\theta)t - 2H = 0$$

After solving the above equation, we get the result.

Velocity after falling height h

Along vertical direction, $u_y^2 = (u\sin\theta)^2 + 2(h)(g)$

Along horizontal direction, $v_x = u_x = u\cos\theta$

so velocity, $\qquad v = \sqrt{v_x^2 + v_y^2} = \sqrt{u^2 + 2gh}$

Key Note

Four objects are projected with same speed 'u' from top of a tower of height 'h' as shown in figure, then their speed when they reach the ground is :

$v = \sqrt{u^2 + 2gh}$ v is independent of θ

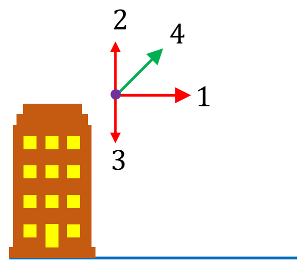
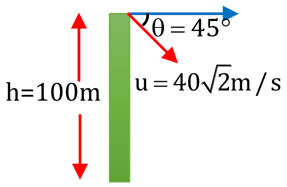
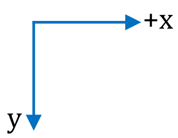


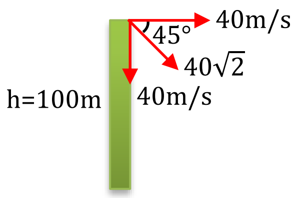
Illustration 16:

For the given body, find: (1) Time of flight (2) Horizontal range





Solution:



(1) Time of flight

$$100 = 40t + \frac{1}{2}\times 10t^2$$

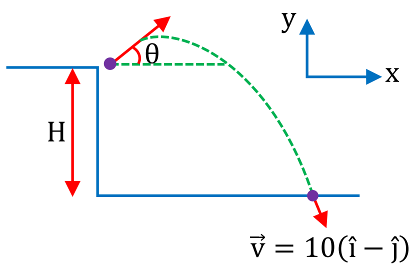
$100 = 40t + 5t^2$

$t^2 + 8t - 20 = 0 \Rightarrow t = 2$ sec

(2) Horizontal range; $\qquad R = u_x \times T = 40 \times 2 = 80$ m

Illustration 17:

A ball was thrown from height H and the ball hit the floor with velocity $10\hat{i} - 10\hat{j}$ after 1.5 sec of its projection. Find initial speed of ball.



Solution:

$\vec{a}_x = 0 \qquad \vec{a}_y = -10\hat{j}$

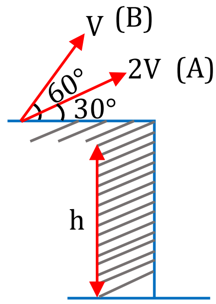
$t = 1.5$ sec.

$(\vec{u}_y) = -10\hat{j} - (-10\hat{j})\times(1.5)$

$\vec{u}_x = 10\hat{i} \qquad \vec{u}_y = 5\hat{j} \qquad$ so, $\qquad \vec{u} = 10\hat{i} + 5\hat{j} \Rightarrow |\vec{u}| = 5\sqrt{5}$ m/s

Illustration 18:

Two particles A & B are projected from a building. A is projected with speed 2v making an angle 30° with horizontal & B with speed v making an angle 60° with horizontal. Which particle will hit the ground earlier?



Solution:

$\left(v_y\right)_B = v\sin 60° = v\sqrt{3}/2$

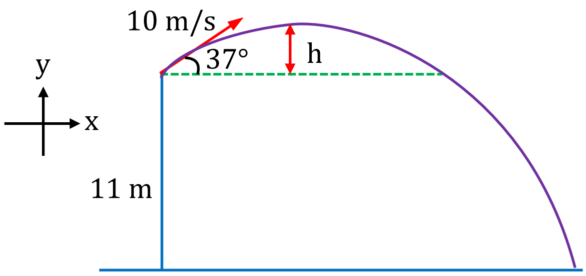
$\left(v_y\right)_A = 2v\sin 30° = v$

$\left(v_y\right)_A > \left(v_y\right)_B$

So, $T_A > T_B$

Illustration 19:

From the top of a 11 m high tower a stone is projected with speed 10 m/s, at an angle of 37° as shown in figure. Find



(a) Speed after 2s

(b) Time of flight.

(c) Horizontal range.

(d) The maximum height attained by the particle.

(e) Speed just before striking the ground.

Solution:

(a) Initial velocity in horizontal direction = 10 cos 37° = 8 m/s

Initial velocity in vertical direction = 10 sin 37° = 6 m/s

Speed after 2 seconds

$v = v_x\hat{i} + v_y\hat{j} = 8\hat{i} + (u_y + a_y t)\hat{j} = 8\hat{i} + (6 - 10 \times 2)\hat{j} = 8\hat{i} - 14\hat{j}$

(b) $S_y = u_y t + \frac{1}{2} a_y t^2 \Rightarrow -11 = 6 \times t + \frac{1}{2} \times (-10)\, t^2$

$5t^2 - 6t - 11 = 0 \Rightarrow (t + 1)(5t - 11) = 0 \Rightarrow t = \frac{11}{5}$ sec.

(c) Range $= u_x T = 8 \times \frac{11}{5} = 17.6$ cm

(d) Maximum height above the level of projection, $h = \frac{u_y^2}{2g} = \frac{6^2}{2\times10} = 1.8$ m

$\therefore$ Maximum height above ground = 11 + 1.8 = 12.8 m

(e) $v = \sqrt{u^2 + 2gh} = \sqrt{100 + 2\times10\times11} \Rightarrow v = 8\sqrt{5}$ m/s

BEGINNER'S BOX-8

1. A projectile is fired horizontally with a velocity of 98 ms^{-1} from the top of a hill 490m high. Find (i) the time taken to reach the ground (ii) the distance of the target from the hill and (iii) the angle at which the projectile hits the ground. ($g = 9.8\ m/s^2$)

PMP222

2. Two tall buildings face each other and are at a distance of 180m from each other. With what velocity must a ball be thrown horizontally from a window 55m above the ground in one building, so that it enters a window 10m above the ground in the second building ? ($g = 10\ m/s^2$)

PMP223

3. Two paper screens A and B are separated by a distance of 100m. A bullet pierces A and then B. The hole in B is 10 cm below the hole in A. If the bullet is travelling horizontally at the time of hitting the screen A, calculate the velocity of the bullet when it hits the screen A. ($g = 9.8\ m/s^2$)

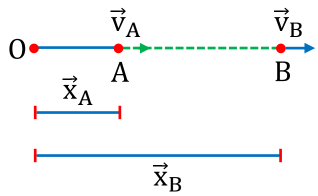
PMP224

4. A ball is thrown up from the top of a tower with an initial velocity of 10 m/s at an angle of 30° with the horizontal. It hits the ground at a distance of 17.3 m from the base of tower. Calculate the height of the tower. ($g = 10\ m/s^2$) **PMP225**

Concept of Relative Motion

Relative Displacement

Displacement of B with respect to A = Displacement of B as measured from A

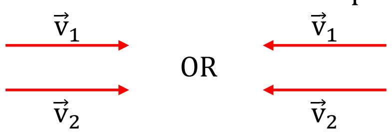


$$\Rightarrow \vec{x}_{BA} = \vec{x}_B - \vec{x}_A \Rightarrow \frac{d\vec{x}_{BA}}{dt} = \frac{d\vec{x}_B}{dt} - \frac{d\vec{x}_A}{dt} \Rightarrow \vec{v}_{BA} = \vec{v}_B - \vec{v}_A$$

Relative = Actual – Reference

For same direction

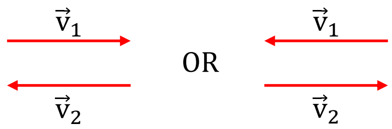
When two particles are moving in the same direction, then magnitude of their relative velocity is equal to the difference between their individual speeds.



$$|\vec{v}_{12}| \text{ or } |\vec{v}_{21}| = v_1 \sim v_2$$

For opposite directions

When two particles are moving in the opposite directions, then magnitude of their relative velocity is always equal to the sum of their individual speeds.



$$|\vec{v}_{12}| \text{ or } |\vec{v}_{21}| = v_1 + v_2$$

Note:- When two particles move simultaneously then the concept of relative motion becomes applicable conveniently.

Numerical Applications

(i) When two particles are moving along a straight line with constant speeds then their relative acceleration must be zero and, in this condition, relative velocity is the ratio of relative displacement to time.

v_1 = const. v_2 = const.

when $\vec{a}_{rel.} = 0$

$$v_{rel.} = \frac{S_{Relative}}{time}$$

(ii) When two particles move in such a way that their relative acceleration non-zero but constant then we apply equations of motion in the relative form.

$\vec{a}_{AB} = \vec{a}_A - \vec{a}_B$

$= 0 - a = -a \neq 0 = \text{constant}$

$\vec{v}_A$ = constant $\vec{v}_B \neq$ constant

$\vec{a}_A = 0$ $\vec{a}_B \neq 0$

Equations of Motion (Relative)

- $v_{rel.} = u_{rel.} + a_{rel.}t$
- $s_{rel.} = u_{rel.}t + \frac{1}{2}a_{rel.}t^2$
- $v_{rel.}^2 = u_{rel.}^2 + 2.a_{rel.}s_{rel.}$
- $s_{rel.} = \frac{1}{2}(u_{rel.} + v_{rel.})t$

Illustration 20:

Buses A and B are moving in the same direction with speeds 20 m/s and 15 m/s respectively. Find the relative velocity of A w.r.t. B and relative velocity of B w.r.t A.

Solution:

Let their direction of motion be along + x-axis then $\vec{v}_A = (20 \text{ m/s})\hat{i}$ and $\vec{v}_B = (15 \text{ m/s})\hat{i}$

(a) Relative velocity of A w.r.t. B is $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$ = (actual velocity of A) – (velocity of B)

$= (20\text{m/s})\hat{i} - (15 \text{ m/s})\hat{i} = 5 \text{ m/s } \hat{i}$

i.e. A is moving with speed 5 m/s w.r.t B in the same direction.

(b) Relative velocity of B w.r.t. A is $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$ = (actual velocity of B) – (velocity of A)

$= (15 \text{ m/s})\hat{i} - (20 \text{ m/s})\hat{i} = (-5 \text{ m/s})\hat{i} = (5 \text{ m/s})(-\hat{i})$

i.e. B is moving in opposite direction w.r.t. A, at a speed 5 m/s

Illustration 21:

A police van moving on a highway with a speed of 30 km/hr fires a bullet at a thief's car which is speeding away in the same direction with a speed of 190 km/hr. If the muzzle speed of the bullet is 150 m/s, find the speed of the bullet with respect to the thief's car.

Solution:

$v_b \rightarrow$ velocity of bullet ; $v_p \rightarrow$ velocity of police van ; $v_t \rightarrow$ velocity of thief's car

$v_{bp} = v_b - v_p$ so, $v_b = v_{bp} + v_p = 150 \times \frac{18}{5}$ km/hr + 30 km/hr =570 km/hr

$v_{bt} = v_b - v_t$ = 570 km/hr – 190 km/hr = 380 km/hr.

Illustration 22:

Three boys A, B and C are situated at the vertices of an equilateral triangle of side d at t = 0. Each of the boys move with constant speed v. A always moves towards B, B towards C and C towards A. when and where will they meet each other?

Solution:

By symmetry they will meet at the centroid of the triangle. Approaching velocity of A and B towards each other is v + v cos 60° and they cover distance d when they meet.

So that time taken, is given by

$$\therefore \ t = \frac{d}{v + v\cos 60^\circ} = \frac{d}{v + \frac{v}{2}} = \frac{2d}{3v}$$

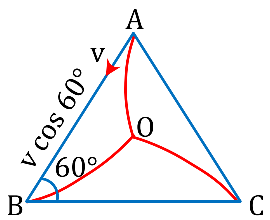


Illustration 23:

Two cars approach each other on a straight road with velocities 10 m/s and 12 m/s respectively. When they are 150 metres apart, both drivers apply their brakes and each car decelerates at 2 m/s^2 until they stop. How far apart will they be when both come to a halt?

Solution:

Let x_1 and x_2 be the distances travelled by the cars before they stop under deceleration.

From IIIrd equation of motion $v^2 = u^2 + 2as$, $\Rightarrow 0 = (10)^2 - 2(2)\,x_1 \Rightarrow x_1 = 25$ m

and $0 = (12)^2 - 2(2)\,x_2 \Rightarrow x_2 = 36$ m

Total distance covered by the two cars = $x_1 + x_2 = 25 + 36 = 61$ m

Distance between the two cars when they stop = 150 – 61 = 89 m.

Illustration 24:

A man 'A' moves in the north direction with a speed 10 m/s and another man B moves in E–30°–N with 10 m/s. Find the relative velocity of B w.r.t. A.

Solution:

$\vec{v}_A = 10\hat{j}$

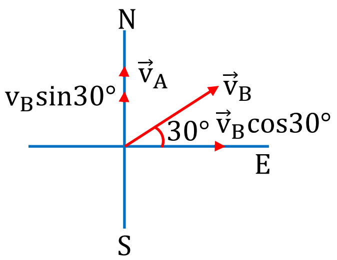
$\vec{v}_B = 5\sqrt{3}\,\hat{i} + 5\hat{j}$

$\vec{v}_{BA} = \vec{v}_B - \vec{v}_A = 5\sqrt{3}\,\hat{i} - 5\hat{j}$

Angle with east direction

$\tan\theta = \dfrac{5}{5\sqrt{3}} = \dfrac{1}{\sqrt{3}}$

$\theta = 30°$ i.e. E – 30° – S or 30°, S of E



Standard Questions of Relative Motion

When two particles are moving with constant velocities, Or

When two particles move with equal acceleration,

Then, $|\vec{a}_{rel}| = |\vec{a}_2 - \vec{a}_1| = 0$ and $s_{rel} = v_{rel} \times$ time

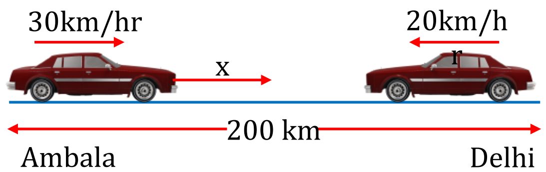
Illustration 25:

Delhi is at a distance of 200 km from Ambala. Car A sets out from Ambala at a speed of 30 km/hr. and car B set out at the same time from Delhi at a speed of 20 km/hr. When will they cross each other ? What is the distance of that crossing point from Ambala?

Solution:

$\vec{v}_{AB} = \vec{v}_A - \vec{v}_B = 30 - (-20) = 50$ km/hr

They will meet after time t given by $t = \dfrac{s}{v_{AB}} = \dfrac{200}{50} = 4$ hr



Distance from Ambala where they will meet is ;

x = 30 × 4 = 120 km

Illustration 26:

Two trains A and B which are 100 m and 60 m long are moving in opposite directions on parallel tracks. Velocity of the shorter train is 3 times that of the longer one. If the trains take 4 seconds to cross each other find the velocities of the trains?

Solution:

Given that $v_B = 3v_A$

Trains move in opposite directions then

Relative velocity $v_{rel} = \dfrac{d_{rel}}{\text{time}} \Rightarrow v_A + v_B = \dfrac{100+60}{4} \Rightarrow 4v_A = 40 \Rightarrow v_A = 10\ m/s, v_B = 3v_A = 30\ m/s$

Illustration 27:

A train is moving towards East with a speed 20 m/s. A person is running on the roof of the train with a speed 3 m/s in the direction of motion of train. Velocity of the person as seen by an observer on ground will be:

Solution:

$\vec{v}_{MT} = \vec{v}_M - \vec{v}_T$

$\vec{v}_M = \vec{v}_{MT} + \vec{v}_T = +3\hat{i} + 20\hat{i} = 23\hat{i}$ so velocity is 23 m/s towards east

Illustration 28:

A 300 metres long train is moving due north with a speed of 20 m/s. A small bird is flying due south a little above the train with 5 m/s speed. The time taken by the bird to cross the train is: -

Solution:

Relative velocity of the bird (v_{rel}) = 20 – (–5) = 25 m/s

Distance = 300 m $\quad\quad \therefore$ Time = $\dfrac{\text{Distance}}{v_{rel}} = \dfrac{300}{25} = 12$ sec.

Illustration 29:

Two trains each 100 m long, are travelling in opposite directions with respective velocities 35 m/s and 15 m/s. The time of crossing is :-

Solution:

$$t = \frac{100+100}{35+15} = 4s$$

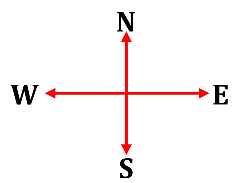
Illustration 30:

Two bodies A and B are 10 km apart such that B is to the south of A. A and B start moving with the same speed 20 km/hr eastward and northward respectively then find.

(a) relative velocity of A w.r.t. B.

(b) minimum separation attained during motion

(c) time elapsed from starting, to attain minimum separation.

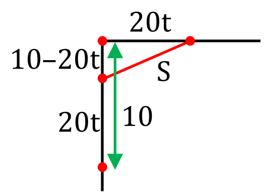


Solution:

(a) $\vec{V}_{rel} = 20\hat{i} - 20\hat{j}$

(b) & (c) Let they are at minimum separation at time 't'

From Figure



$$S = \sqrt{(20t)^2 + (10-20t)^2} \quad \text{...(i)}$$

$$S = 10\sqrt{(2t)^2 + (1-2t)^2}$$

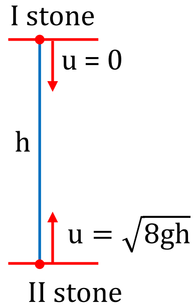
$$S = 10\sqrt{(4t^2 + 4t^2 + 1 - 4t)} = 10\sqrt{8t^2 - 4t + 1}$$

For S_{min} applying concept of maximum and minimum, $\dfrac{dS}{dt} = 10 \times \dfrac{1}{2} \times \dfrac{(16t-4)}{\sqrt{8t^2-4t+1}} = 0$

$t = \dfrac{1}{4}s$ put in (i) get $S_{min} = 5\sqrt{2}m$

Illustration 31:

A stone is dropped from height h. Another stone is thrown up simultaneously from the ground which reaches height 4h. The two stones will cross each other after time: -



Solution:

Initial velocity of 2nd stone = u.

Using $v^2 - u^2 = 2as$.

$$0 = u^2 - 2g(4h) \Rightarrow u = \sqrt{8gh}$$

Now, $d_{rel} = h, u_{rel} = \sqrt{8gh} + 0 = \sqrt{8gh}$

$a_{rel} = 0$ c then using $t = \frac{d_{rel}}{u_{rel}} = \frac{h}{\sqrt{8gh}} = \sqrt{\frac{h}{8g}}$

BEGINNER'S BOX-9

1. Two trains A and B each of length 50 m, are moving with constant speeds. If one train A overtakes the other in 40s, while crosses the other in 20s. Find the speeds of each train. **PMP226**

2. Two trains A and B of length 400 m each are moving on two parallel tracks with a uniform speed of 72 km/h in the same direction, with A ahead of B. The driver of B decides to overtake A and accelerates by 1 m/s^2. If after 50 s, the guard of B just brushes past the driver of A, what was the original distance between the guard of B & driver of A? **PMP227**

3. A boy standing on a stationary lift (open from above) throws a ball upwards with the maximum initial speed he can, equal to 49 m/s. How much time does the ball take to return to his hands? If the lift starts moving up with a uniform speed of 5 m/s and the boy again throws the ball up with the maximum speed he can, how long does the ball take to return to his hands? **PMP228**

Problems based on Relative Motion - Under Gravity

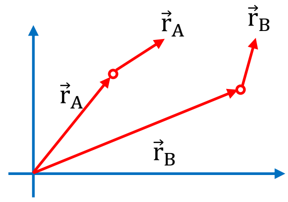
Relative velocity in a plane

- **For 2-dimensional motion:**

  Relative velocity of A with respect to 'B' can be calculated as

  $\vec{v}_{AB} = \vec{v}_A - \vec{v}_B$

  $|\vec{v}_{AB}| = \sqrt{v_A^2 + v_B^2 - 2v_A \cdot v_B \cos\theta}$



Note:

- **For two particles to collide:**
  (i) Their combined relative displacement becomes zero.
  (ii) Their combined vertical velocities will be same: if they are projected from same level (in case of projectiles)
  (iii) Their combined motion can be converted into two 1D motions.
- **Relative path of a projectiles w.r.t. another projectile**

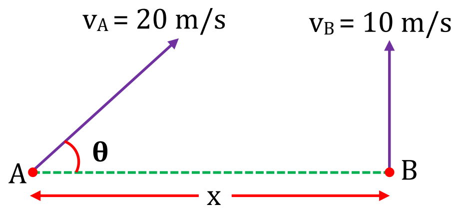
  Two projectiles are thrown from ground with different velocities at different angles. Since both projectiles have equal accelerations so their relative acceleration is zero. Thus, path of one projectile w.r.t. other is a straight line and motion of one projectile w.r.t. other is uniform.

  If $u_1 \cos\theta_1 = u_2 \cos\theta_2$ then relative path is a vertical line.

  If $u_1 \sin\theta_1 = u_2 \sin\theta_2$ then relative path is a horizontal line.

Illustration 32:

Two particles A and B are projected from the ground simultaneously in the directions shown in the figure with initial velocities v_A = 20 m/s and v_B =10 m/s respectively. They collide after 0.5 s. Find out the angle θ and the distance x.



Solution:

Both particles will collide if they are at same height in same time.

So, $y_A = y_B \Rightarrow (u_y)_A t - \frac{1}{2}gt^2 = (u_y)_B t - \frac{1}{2}gt^2 \Rightarrow (u_y)_A = (u_y)_B$

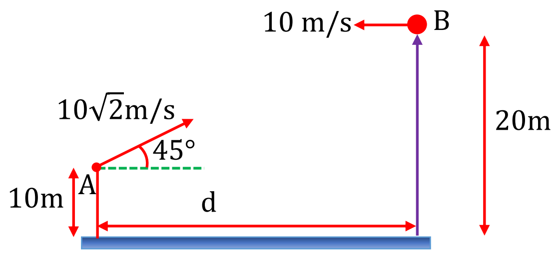
$\Rightarrow (v_A \sin\theta) = v_B \Rightarrow 20\sin\theta = 10 \Rightarrow \sin\theta = \frac{1}{2} \Rightarrow \theta = 30°$

In 0.5s horizontal distance covered by A is $x = (u_x)_A t = (20\cos 30°)0.5 = 10 \times \frac{\sqrt{3}}{2} = 5\sqrt{3}$m

Illustration 33:

Two particles are projected from the two towers simultaneously, as shown in the figure.

What should be the value of 'd' for their collision?



Solution:

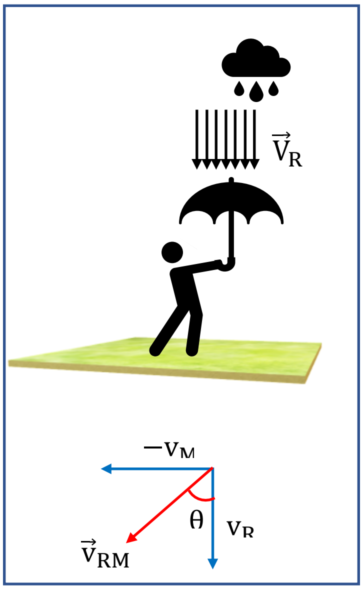
There is no relative acceleration of between A and B.

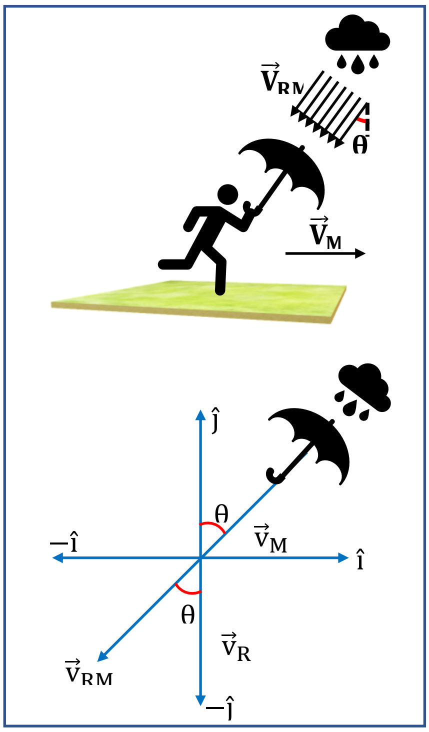
So time of collision t $= \frac{y_{BA}}{(v_y)_{BA}}$ where y_{BA} = vertical displacement of B w.r.t. A = 10m.

$(v_y)_{BA}$ = vertical velocity of B w.r.t. A = 0 – $\left(-10\sqrt{2}\sin 45°\right) = 10 \text{ m/s} \Rightarrow t = \frac{10}{10} = 1$s

d = horizontal distance travelled by B w.r.t. $A = (v_x)_{BA} \times t = \left(10 + 10\sqrt{2}\cos 45°\right) \times 1 = 20$ m.

Rain-Man Concept





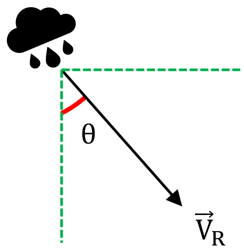
(i) If rain is falling vertically with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ then the velocity of rain relative to observer will be

$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M \Rightarrow \vec{v}_{RM} = -\vec{v}_R\hat{j} - \vec{v}_M\hat{i}$

Which by law of vector addition has magnitude; $v_{RM} = \sqrt{v_R^2 + v_M^2}$

The direction of $\vec{v}_{RM}$ is such that it makes an angle θ with the vertical given by $\theta = \tan^{-1}(v_m / v_R)$ as shown in figure.

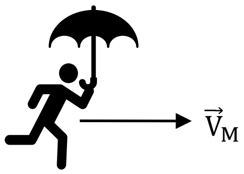
(ii) If rain is already falling at an angle θ with the vertical with a velocity $\vec{v}_R$ and an observer is moving horizontally with speed $\vec{v}_M$ finds that the rain drops are hitting on his head vertically downwards



Here $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

$\vec{v}_{RM} = (v_R \sin\theta - v_M)i - v_R \cos\theta\hat{j}$

Now for rain to appear falling vertically, the horizontal component of $\vec{v}_{RM}$ should be zero, i.e.



$\vec{v}_R \sin\theta - v_M = 0 \Rightarrow \sin\theta = \frac{v_M}{v_R}$

and $|\vec{v}_{RM}| = v_R \cos\theta = v_R\sqrt{1-\sin^2\theta} = v_R\sqrt{1-\frac{v_M^2}{v_R^2}} = v_{RM} = \sqrt{v_R^2 - v_M^2}$

Illustration 34:

A person moves due east at a speed 6m/s and feels the wind is blowing towards south at a speed 6m/s.

(a) Find actual velocity of wind blow.

(b) If person doubles his velocity then find the relative velocity of wind blow w.r.t. man.

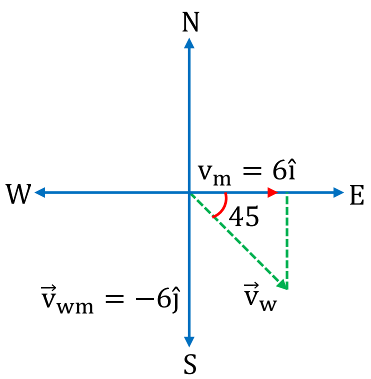
Solution:

(a) $\vec{v}_{wm} = \vec{v}_w - \vec{v}_m$

$\vec{v}_w = \vec{v}_{wm} + \vec{v}_m = -6j + 6\hat{i}$

$\vec{v}_w = 6\hat{i} - 6\hat{j}$

$|\vec{v}| = 6\sqrt{2}$ m/s and it is blowing along S–E



(b) Person doubles its velocity then $\vec{v}_m = 12\hat{i}$

But actual wind velocity remains unchanged.

$\vec{v}_{wm} = \vec{v}_w - \vec{v}_m = (6\hat{i} - 6\hat{j}) - 12\hat{i}$

$\vec{v}_{wm} = -6\hat{i} - 6\hat{j}$

Now relative velocity of wind is $6\sqrt{2}$m/s along S–W.

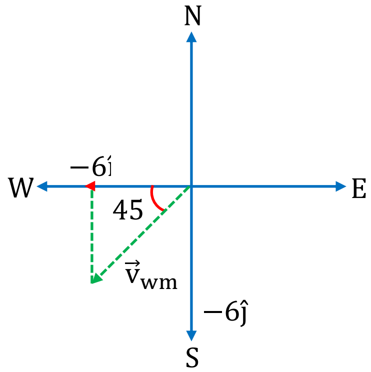
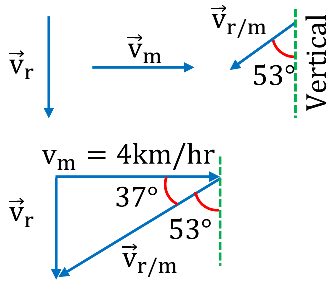


Illustration 35:

A man at rest observes the rain falling vertically. when he walks at 4 km/h, he has to hold his umbrella at an angle of 53° from the vertical. Find the velocity of raindrops.

Solution:

Assigning usual symbols $\vec{v}_m, \vec{v}_r$ and $\vec{v}_{r/m}$ to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following equation



$\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$

$\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$

The above equation suggests that a standstill man observes velocity $\vec{v}_r$ of rain relative to the ground and while he is moving with velocity $\vec{v}_m$, he observes

velocity of rain relative to himself $\vec{v}_{r/m}$. It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.

Therefore

$v_r = v_m \tan 37° = 3 \text{ km/h}$

Illustration 36:

A man is going east in a car with a velocity of 20 km/hr, a train appears to move towards north to him with a velocity of $20\sqrt{3}$ km/hr. What is the actual velocity and direction of motion of train?

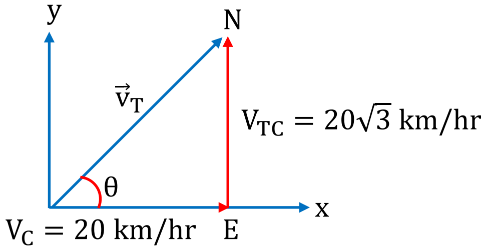
Solution:

$\vec{v}_{TC} = \vec{v}_T - \vec{v}_C$

$\vec{v}_T = \vec{v}_{TC} + \vec{v}_C = 20\sqrt{3}\,\hat{j} + 20\,\hat{i}$

$|\vec{v}_T| = \sqrt{(20\sqrt{3})^2 + (20)^2} = \sqrt{1600} = 40 \text{ km/hr}$

$\tan\theta = \dfrac{20\sqrt{3}}{20} \Rightarrow \theta = 60°$



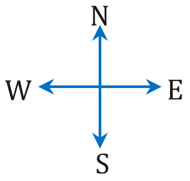
So, direction of motion of train is 60° N of E or E-60°-N

BEGINNER'S BOX-10

1. A man 'A' moves in the north direction with a speed 10 m/s and another man B moves in E–30°–N with 10 m/s. Find the relative velocity of B w.r.t. A. **PMP229**

2. A and B are moving with the same speed 10 m/s in the directions E–30°–N and E–30°–S respectively. Find the relative velocity of A w.r.t. B. **PMP230**

3. Two bodies A and B are 10 km apart such that B is to the south of A. A and B start moving with the same speed 20 km/hr eastward and northward respectively then find.



(a) relative velocity of A w.r.t. B.
(b) minimum separation attained during motion
(c) time elapsed from starting, to attain minimum separation. **PMP231**

4. Rain is falling vertically with a speed of 30 m/s. A woman rides a bicycle with a speed of 10 m/s in the north to south direction. What is the direction in which she should hold her umbrella? **PMP232**

5. A man is running up hill with a velocity $(2\hat{i}+3\hat{j})$ m/s w.r.t. ground. He feels that the rain drops are falling vertically with velocity 4 m/s. If he runs down hill with same speed, find v_{rm}. **PMP233**

6. A body is projected with velocity u_1 from point A as shown in figure. At the same time another body is projected vertically upwards with a velocity u_2 from point B. What should be the value of $\frac{u_1}{u_2}$ for both bodies to collide. **PMP234**

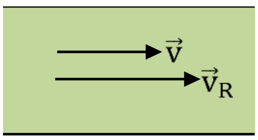
River-Boat (or Man) Problem

A man can swim with velocity $\vec{v}$, i.e. it is the velocity of man w.r.t. still water.

If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$

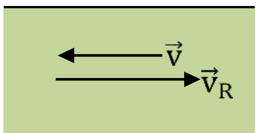
(i) If the swimming is in the direction of flow of water or along the downstream then

$$v_m = v + v_R$$



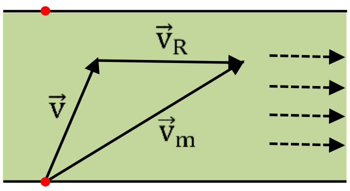
(ii) If the swimming is in the direction opposite to the flow of water or along the upstream then

$$v_m = v - v_R$$



(iii) If the man is crossing the river i.e. $\vec{v}$ and $\vec{v}_R$ are non colinear then use vector algebra.

$$\vec{v}_m = \vec{v} + \vec{v}_R$$



To Cross a River

Minimum distance of approach

OR

Cross the river along shortest possible path

OR

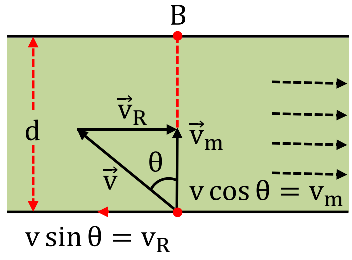
Cross the river and reach a point just opposite to the starting path.

Cross the river in shortest possible time

For shortest path :

If a man wants to cross the river such that his "displacement should be minimum", it means he intends to reach just opposite point across the river. He should start swimming at an angle θ with the perpendicular to the flow of river towards upstream.

Such that its resultant velocity $\vec{v}_m = (\vec{v} + \vec{v}_R)$. It is in the direction of displacement AB.



(For minimum displacement)

To reach at B $\quad v\sin\theta = v_R \quad \Rightarrow \quad \sin\theta = \dfrac{v_R}{v}$

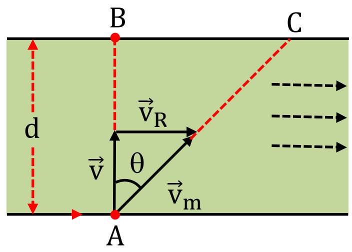
Component of velocity of man along AB is $v\cos\theta$ so, time taken $T = \dfrac{d}{v\cos\theta} = \dfrac{d}{\sqrt{v^2 - v_R^2}}$

For minimum time

To cross the river in minimum time, the velocity along $AB(v\cos\theta)$ should be maximum.

It is possible if $\theta = 0$, i.e. swimming should start perpendicular to water current.

Due to effect of river velocity man will reach at point C along resultant velocity, i.e. his displacement will not be minimum but time taken to cross the river will be minimum,



(For minimum time)

Minimum time = $t_{min} = \dfrac{d}{v}$

In time t_{min} swimmer travels distance BC along the river with speed of river v_R $\therefore BC = t_{min} v_R$

Distance travelled along river flow = drift of man = $t_{min} v_R = \dfrac{d}{v} v_R$

Illustration 37:

A boat moves along the flow of river between two fixed points A and B. It takes t_1 time when going downstream and takes t_2 time when going upstream between these two points. What time it will take in still water to cover the distance equal to AB

Solution:

$$t_1 = \frac{AB}{v_b + v_R}, t_2 = \frac{AB}{v_b - v_R} \quad \text{or} \quad v_b + v_R = \frac{AB}{t_1} \quad \text{and} \quad v_b - v_R = \frac{AB}{t_2}$$

$$\Rightarrow 2v_b = \frac{AB}{t_1} + \frac{AB}{t_2} = AB\left(\frac{t_1 + t_2}{t_1 t_2}\right) \quad \text{or} \quad \left(\frac{2t_1 t_2}{t_1 + t_2}\right) = \frac{AB}{v_b} = \text{time taken by the boat to cover AB}$$

Illustration 38:

A boat can be rowed at 5 m/s in still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.

(a) In which direction must it be steered to cross the river perpendicular to current?

(b) How long will it take to cross the river in a direction perpendicular to the river flow?

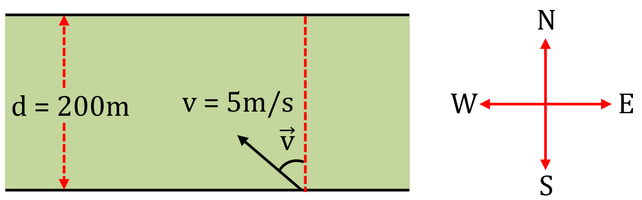
(c) In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

Solution:

(a) To cross the river perpendicular to current i.e. along shortest path

$$\sin\theta = \frac{u}{v} = \frac{3}{5} \Rightarrow \theta = 37°$$

(b) Time taken by boat, $t = \dfrac{d}{v\cos\theta} = \dfrac{200}{5 \times \frac{4}{5}} = 50s$



(c) To cross the river in minimum time, $\theta = 0$

Therefore, $t_{min} = \dfrac{d}{v} = \dfrac{200}{5} s = 40s$

$$\text{Drift} = u(t_{min}) = 3(40)m = 120m$$

Illustration 39:

A boat takes 2 hours to go 10 km and come back in still water lake. The time taken for going 10 km upstream and coming back with water velocity of 5 km/h is:

Solution:

$$\frac{10}{v_{mr}} + \frac{10}{v_{mr}} = 2 \Rightarrow v_{mr} = 10 \text{ km/hr} \Rightarrow t = \frac{10}{v_{mr} + v_r} + \frac{10}{v_{mr} - v_r} = \frac{10}{10+5} + \frac{10}{10-5} = \frac{10}{15} + \frac{10}{5} = \frac{8}{3}\text{hr}$$

Illustration 40:

A river 2 km wide flows at the rate of 2km/h. A boatman who can row a boat at a speed of 6 km/h in still water, goes a distance of 2 km upstream and then comes back. The time taken by him to complete his journey is

Solution:

The velocity upstream is (4 – 2) km/hr and downstream is (4 + 2) km/hr.

$\therefore$ Total time taken = $\dfrac{2}{4} + \dfrac{2}{8} = \dfrac{6}{8} = \dfrac{3}{4}$hr = 45 minutes

BEGINNER'S BOX-11

1. A man can swim at a speed 2 m/s in still water. He starts swimming in a river at an angle 150° to the direction of water flow and reaches the directly opposite point on the opposite bank.

(a) Find the speed of flowing water.

(b) If width of river is 1 km then calculate the time taken to cross the river.

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2. 2 km wide river flowing at the rate of 5 km/hr. A man can swim in still water with 10 km/hr. He wants to cross the river along the shortest path. Find –

(a) in which direction should the person swim.

(b) crossing time.

PMP236

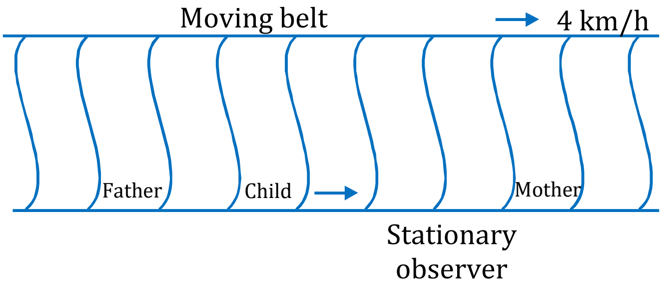
3. A child runs to and fro with a speed 9 km/h (with respect to the belt) on a long horizontally moving belt (fig.) between his father and mother located 50 m apart on the moving belt. The belt moves with a speed of 4 km/h. For an observer on stationary platform outside, what is the

(a) speed of the child while running in the direction of motion of the belt?

(b) speed of the child while running opposite to the direction of motion of the belt?

(c) time taken by the child in (a) and (b)?

Which of the answers will alter if motion is viewed by one of the parents?



PMP237

BEGINNER'S BOX — ANSWER KEY

BEGINNER'S BOX-1

1. $7\pi r$, 2r **2.** 400 m for each, B. **3.** 19m, 13m

4. (40π)m, 80m from A to B **5.** (A) 110m, (B) 50m, 37°N of E

BEGINNER'S BOX-2

1. (i) 40 km/hr (ii) 32.5 km/hr (iii) 1040 km/hr (iv) 1040 km/hr

2. 4 m/s **3.** 5×10^{-3} cm/s, 2×10^{-4} cm/s **4.** 0 m/s, 20 m/s

5. (a) 12.5 m/s (b) 25 m/s **6.** 2 m/s

7. 49.3 km/h, 21.4 km/h

BEGINNER'S BOX-3

1. $\frac{10}{\pi}$ m/s^2 **2.** (a) 68, (b) 14, (c) 33, (d) 14

BEGINNER'S BOX-4

1. $s_2 = 8s_1$ **2.** $\sqrt{\frac{u^2+v^2}{2}}$ **3.** $\frac{n^2}{2n-1}$

4. 3.06 ms^{-2} ; 11.4 s **5.** $ut+\frac{u^2}{2a}$ **6.** 0.218 m/s^2

BEGINNER'S BOX-5

1. 1 : 3

2. (i) 120m (ii) 0 (iii) 20m/s (iv) 0

3. (a) A, B, (b) A,B, (c) B, A, (d) Same, (e) B, A, once

4. (i) 37 m (ii) 3.7 m/s (iii) Part BC, a = 8 m/s^2 (iv) 2 m/s^2

5. 625 m

BEGINNER'S BOX-6

1. 1 s

2. (a) 75m, (b) 40m/s, (c) 85m, (d) 17m/s, 15m/s

3. (i) 70.3m (ii) 5 s

4. (a) 600 m/s, (b) 18 km, (c) $(2+\sqrt{2})$ min,(d) 36 km

5. 125 m

6. 45 m

7. (a) Vertically downwards;
(b) zero velocity, acceleration of 9.8 m/s^2 downwards.
(c) x > 0 (upward and downward motion) v < 0 (upward), v > 0 (downward), a > 0 throughout;
(d) 44.1 m, 6s.

8. 125m

9. 4m & 1m from top

BEGINNER'S BOX-7

1.	(a) 1s, (b) 5m, (c) 34.64 m, (d) 2s	**2.**	50 m.	**3.**	(a) $\tan^2\alpha$ (b)1
4.	15 m & 65 m	**5.**	3 km		
7.	1	**8.**	148.32 m		

BEGINNER'S BOX-8

1.	(i) 10s, (ii) 980m, (iii) $\beta = 45°$	**2.**	60 m/s	**3.**	700 m/s
4.	10 meter.				

BEGINNER'S BOX-9

1.	3.75 m/s & 1.25 m/s	**2.**	1250 m.	**3.**	10 sec, 10 sec

BEGINNER'S BOX-10

1.	$5\sqrt{3}\hat{i}-5\hat{j}$, E– 30° – S	**2.**	In north direction 10m/s
3.	$20\sqrt{2}$ km/hr S – E, $5\sqrt{2}$ km, 15 min.	**4.**	$\alpha = \tan^{-1}\left(\frac{1}{3}\right)$=18.4° from vertical towards south
5.	$\sqrt{20}$ m/s	**6.**	$\frac{2}{\sqrt{3}}$

BEGINNER'S BOX-11

1. (a) $\sqrt{3}$ m/s (b) 1000s.

2. (a) Direction from Down Stream = 120° (b) $\frac{2}{5\sqrt{3}}$ hr.

3. (a) 13 km/h (b) 5 km/h (c) 20s

If the motion is viewed by one of the parents answers to (a) and (b) are altered while answer to (c) remain unchanged.