Kinematics

SOLUTIONS

Exercise-I (Conceptual Questions)

4. Displacement ≤ distance

5. Total time = 100 sec.

distance = $2\pi R + 2\pi R + \pi R = 5\pi R$

displacement = 2R

6. displacement (diagonal of cube)

$=\sqrt{(10)^2+(10)^2+(10)^2}=10\sqrt{3}\,m$

7. distance = 5 × 4 + 12 × 4 = 68 cm

8. displacement

$S=\sqrt{(12)^2+(5)^2}$

S = 13 cm

9. $\dfrac{s_P}{s_R}=\dfrac{\frac{3}{2}\pi R}{\frac{\pi R}{2}}=\dfrac{3}{1}$

10. Distance = πR

Displacement = 2R

$\dfrac{\text{Distance}}{\text{Displacement}}=\dfrac{11}{7}$

11. Distance = $\dfrac{2\pi R}{3600}\times 100$

$=\dfrac{2\pi\times 10}{3600\times 100}\times 100=\dfrac{\pi}{180}m$

12. $\vec{S}=(3-2)\hat{i}+(4-3)\hat{j}+(5-5)\hat{k}$

$\vec{S}=\hat{i}+\hat{j}$

13. $\vec{d}=\vec{d}_1+\vec{d}_2+\vec{d}_3$

$\vec{d}=30\hat{j}+20\hat{i}+30\sqrt{2}\left(\dfrac{\hat{i}-\hat{j}}{\sqrt{2}}\right)$

$\vec{d}=30\hat{j}+20\hat{i}+30\hat{i}-30\hat{j}$

$\vec{d}=+50\hat{i}$

$\vec{d}=50\text{m towards east}$

14. $\vec{d}_1=80\hat{i}$

$\vec{d}_2=-30\hat{j}-40\hat{i}$

Position from starting point

$\vec{d}=\vec{d}_1+\vec{d}_2$

$\vec{d}=40\hat{i}-30\hat{j}$

$|\vec{d}|=\sqrt{40^2+30^2}=50\text{ m}$

$\tan\theta=-\left(\dfrac{40}{30}\right)=\left(-\dfrac{4}{3}\right)$

θ = 53° east of south

15. 1 cycle = 5 step forward & 3 step backward then net displacement in 1 cycle = (+5) + (–3) = +2m

Net displacement in 4 cycle = 8 m

So, he will fall down in the pit after completing 4 cycles & 5 forward steps.

Total time = 4(5 + 3) + 5(1) = 37 sec.

16. $v_{avg}=\dfrac{2v_1v_2}{v_1+v_1}=\dfrac{2\times 30\times 70}{30+70}=42\text{km/h}$

17. Average velocity $=\dfrac{\text{Displacement}}{\text{Time interval}}$

A particle moving in a given direction with non-zero velocity cannot have zero speed. In general, average speed is not equal to magnitude of average velocity. However, it can be so if the motion is along a straight line without change in direction.

19. $v_{avg}=\dfrac{d_1+d_2}{2t}=\dfrac{vt+vt}{2t}$

v_{avg} = v

20. Average speed = $\dfrac{\text{total distance}}{\text{total time}}=\dfrac{2d}{2+3}=\dfrac{2d}{5}$

21. Average velocity = $\dfrac{(15\hat{i}\times 2)+5\hat{j}\times 8}{2+8}=3\hat{i}+4\hat{j}$

$|\vec{v}|=5\text{ m/s}$

22. Let the constant speed is V, then

average speed = V

average velocity = $\dfrac{a}{\frac{2a}{V}}=\dfrac{V}{2}$

$\dfrac{\text{average speed}}{\text{average velocity}}=\dfrac{V}{V/2}=\dfrac{2}{1}$

23. Average speed

$$= \frac{\text{Total distance}}{\text{Total time}} = \frac{\pi R}{3.14} = 10\ m/s$$

Average velocity

$$= \frac{\text{Total displacement}}{\text{Total time}} = \frac{2R}{3.14} = \frac{20}{\pi}\ m/s$$

24. Average velocity $= \dfrac{3\times20+4\times20+5\times20}{20+20+20}$

$$= \frac{240}{60} = 4\ m/s$$

25. $t_1 = \dfrac{10\times1000}{100}$ = 100 sec.

$t_2 = \dfrac{10\times1000}{50}$ = 200 sec.

Avg. speed $= \dfrac{\text{Total distance}}{\text{Total time}}$

$= \dfrac{10000+10000}{300}$

= 66.66

$\simeq$ 66.7 m/s

26. Average speed

$$= \frac{3V_1V_2V_3}{V_1V_2+V_2V_3+V_1V_3}$$

$$= \frac{3\times V\times 2V\times 3V}{2V^2+6V^2+3V^2} = \frac{18}{11}V$$

27. $v_{avg} = \dfrac{v_1t+v_2t+v_3t}{3t}$

$v_{avg} = \dfrac{v_1+v_2+v_3}{3}$

28. Average speed $= \dfrac{S}{\dfrac{\frac{2S}{5}}{V_1}+\dfrac{\frac{3S}{5}}{V_2}}$

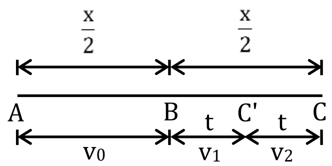
$$= \frac{5}{\dfrac{2}{V_1}+\dfrac{3}{V_2}} = \frac{5V_1V_2}{3V_1+2V_2}$$

29. Average speed in last half $= \dfrac{4.5+7.5}{2}$ = 6 m/s

Average speed in total journey $= \dfrac{2\times10\times6}{10+6}$

= 7.5 m/s

30.



for second half $v_{avg} = \dfrac{v_1+v_2}{2}$

for total $v_{avg} = \dfrac{2(v_0)\left(\dfrac{v_1+v_2}{2}\right)}{v_0+\dfrac{v_1+v_2}{2}}$

$$v_{avg} = \frac{2v_0(v_1+v_2)}{2v_0+v_1+v_2}$$

31. $\Delta\vec{V} = \vec{V}_f - \vec{V}_i$

$\Delta\vec{V} = (-10\hat{j} - 10\hat{i})\ m/s$

$\Delta\vec{V} = (-10\hat{i} - 10\hat{j})\ m/s$

$|\Delta\vec{V}| = 10\sqrt{2}\ m/s$, in south-west direction

$|\Delta\vec{V}| = 14.14\ m/s$, in south-west direction

32. $\Delta V = 2V\sin\dfrac{\theta}{2}$

$\Delta V = 2V\sin\dfrac{40°}{2}$

$\Delta V = 2V\sin 20°$

33. $\vec{V}_{avg} = \dfrac{4\hat{j}+3\hat{i}}{10+5}\ m/s$

$\vec{V}_{avg} = \left(\dfrac{4}{15}\hat{j}+\dfrac{3}{15}\hat{i}\right) m/s$

$V_{avg} = \dfrac{1}{3}\ m/s$

34. $x = at^2 - bt^3$

$v = \dfrac{dx}{dt} = 2at - 3bt^2 = 0 \Rightarrow t = \dfrac{2a}{3b}$

35. $x = a\sin t$

$v = \dfrac{dx}{dt} = a\cos t \Rightarrow a = \dfrac{dv}{dt} = -a\sin t$

36. $x = a + bt^2,\ v = \dfrac{dx}{dt} = 2bt$

$a = \dfrac{dv}{dt} = 2b = 2 \times 3\ cm/s^2 = 6\ cm/s^2$

37. $s = 3t^3 + 7t^2 + 14t + 8$

$v = \frac{ds}{dt} = 9t^2 + 14t + 14$

$a = \frac{dv}{dt} = 18t + 14$

at $t = 2$ $a = 50\ m/s^2$

38. $v = \frac{dx}{dt} = 2at + b$

39. $t = \sqrt{x} + 3 \Rightarrow \sqrt{x} = t - 3$

$x = (t-3)^2 = t^2 - 6t + 9$

$v = \frac{dx}{dt} = 2t - 6 \Rightarrow a = \frac{dv}{dt} = 2\ m/s^2$

40. $y = a + bt + ct^3$

$v = \frac{dy}{dt} = b + 3ct^2$

at $t = 0$ $v = b$

$a = \frac{dv}{dt} = 6ct$

at $t = 0$ $a = 0$

41. $x = u\ (t - 2) + a(t - 2)2$

$v = \frac{dx}{dt} = u + 2a\ (t - 2)$

acceleration $= \frac{dv}{dt} = 2a$

42. $x = kt$

$v = \frac{dx}{dt} = k\ (\text{constant}) \Rightarrow a = \frac{dv}{dt} = 0$

43. $v = 20 + 0.1\ t$

$a = \frac{dv}{dt} = 0.1$

it is uniform acceleration.

44. Acceleration $= \frac{dv}{dt}$, It is the first derivative where t terms can only yield a constant value.

$v = 6 - 7t$

$a = \frac{dv}{dt} = -7$

45. Velocity $= \frac{dy}{dt}$

It is the first derivative where t terms can only yield a constant value.

46. $s = 6t^2 - t^3$

$v = 12t - 3t^2$

$a = 12 - 6t = 0 \Rightarrow t = 2$ sec.

47. $v = \frac{dx}{dt}$

$v = \frac{d}{dt}\left[\frac{k}{b} - \frac{ke^{-bt}}{b}\right] = \frac{d}{dt}\left(\frac{k}{b}\right) - \frac{d}{dt}\left[\frac{k}{b}e^{-bt}\right]$

$v = -\frac{d}{dt}\left[\frac{k}{b} \cdot e^{-bt}\right]$

$v = k(e^{-bt})$

48. $s = t^3 - 6t^2 + 3t + 4$

$v = \frac{ds}{dt} = 3t^2 - 12t + 3$

$a = \frac{dv}{dt} = 6t - 12$ If $a = 0$ Þ $t = 2$

$s = 2^3 - 6(2)^2 + 3(2) + 4 = 8 - 24 + 6 + 4 = -6m$

49. $x = at + bt^2 - ct^3$

$v = \frac{dx}{dt} = a + 2bt - 3ct^2$

acceleration $a' = 2b - 6ct$

if $a' = 0$ then $t = \frac{b}{3c}$

So at $t = \frac{b}{3c}$

$V = a + 2b \times \frac{b}{3c} - 3c \times \frac{b^2}{9c^2}$

$V = a + \frac{2b^2}{3c} - \frac{b^2}{3c}$

$V = a + \frac{b^2}{3c}$

50. $v_x = \frac{dx}{dt} = t$

At $t = 2$ sec. $v_x = 2\ m/s$

$v_y = \frac{dy}{dt} = \frac{d}{dt}\left(\frac{t^4}{8}\right) = \frac{t^3}{2}$

At $t = 2$ sec. $v_y = 4\ m/s$

$\boxed{\vec{v} = (2\hat{i} + 4\hat{j})\ m/s}$

51. $u = kt \Rightarrow \frac{ds}{dt} = kt$ ($\because k = 2\ m/s^2$)

$$\Rightarrow \int_0^s ds = 2\int_0^4 tdt \quad \Rightarrow s = 2\left|\frac{t^2}{2}\right|_0^4 \Rightarrow s = 16m$$

52. $f = at^2 = \frac{dv}{dt}$

$$\Rightarrow \int_u^v dv = \int_0^t at^2\, dt \Rightarrow v - u = \frac{at^3}{3} \Rightarrow v = u + \frac{at^3}{3}$$

53. Acceleration $\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{10\hat{j} - 10\hat{i}}{20}\ m/s^2$

$$= \frac{-\hat{i} + \hat{j}}{2} \text{ (west-north)}$$

$$|\vec{a}| = \sqrt{\frac{1}{4} + \frac{1}{4}} = \frac{1}{\sqrt{2}}\ m/s^2 \text{ (north-west)}$$

54. $a = 2t - 2$

$$\int_0^v dv = \int_0^5 (2t - 2)dt$$

$$V = \left[t^2\right]_0^5 - 2[t]_0^5$$

$V = 25 - 10 = 15\ m/s$

55. $V = (10 + 2t^2)\ m/s$

$$a_{avg} = \frac{V_2 - V_1}{t_2 - t_1}$$

at $t = 2$ sec, $v_1 = 18\ m/s$

at $t = 5$ sec, $v_2 = 60\ m/s$

$$a_{avg} = \frac{60 - 18}{5 - 2} = 14\ m/s^2$$

56. $V = (3t^2 + 2)\ m/s$

$S = (t^3 + 2t)\ m$

at $t = 0 \Rightarrow S_1 = 0$

at $t = 2 \Rightarrow S_2 = 12\ m$

$$v_{avg} = \frac{S_2 - S_1}{t_2 - t_1} = \frac{12 - 0}{2 - 0} = 6\ m/s$$

57. $v = \alpha\sqrt{x} \Rightarrow \frac{dx}{dt} = \alpha\sqrt{x} \Rightarrow \frac{dx}{\sqrt{x}} = \alpha dt$

Integrating, $\int_{x=0}^{x} x^{-1/2}dx = \int_{t=0}^{t} \alpha\, dt$

$$2\sqrt{x} = \alpha t \Rightarrow \sqrt{x} = \alpha t/2$$

Put this value of $\sqrt{x}$ in the original given eq^n.

$$v = \alpha\sqrt{x} = \alpha(\alpha.t/2) = \alpha^2 t/2$$

$\therefore v \propto t$

58. $v = (180 - 16x)^{1/2}$

$$\because\ a = \frac{(v)dv}{dx}$$

$$a = (180 - 16x)^{1/2} \cdot \frac{d}{dx}(180 - 16x)^{1/2}$$

$$a = \frac{1}{2}\left[\frac{(180 - 16x)^{1/2}}{(180 - 16x)^{1/2}}\right][0 - 16]$$

$= -8\ m/s^2$

59. $a = 2s + 1$

$$\frac{(v)dv}{ds} = 2s + 1$$

$v(dv) = (2s + 1)ds$

$$\int_0^v vdv = \int_0^s (2s + 1)ds$$

$$\Rightarrow \frac{v^2}{2} = \left(\frac{2s^2}{2}\right) + s$$

$$\Rightarrow v = \sqrt{2s(s+1)}$$

60. $\vec{r} = (t^2 - 4t + 6)\hat{i} + (t^2)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = (2t - 4)\hat{i} + (2t)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = 2\hat{i} + 2\hat{j}$$

For $\vec{v} \perp \vec{a}$

$\vec{v} \cdot \vec{a} = 0$

$4t - 8 + 4t = 0$

$t = 1$ sec.

61. Rate of change of speed = component of acceleration along velocity

$$= \left(\frac{\vec{a} \cdot \vec{v}}{|\vec{v}|}\right)(\hat{v})$$

$$= \left(\frac{6 + 8}{10}\right)\left(\frac{6\hat{i} + 8\hat{j}}{10}\right)$$

$$= \frac{7}{25}(3\hat{i} + 4\hat{j})$$

62. $s = ut + \frac{1}{2}at^2$

$u = 0$

$s = \frac{1}{2}at^2$

$t^2 = \frac{2s}{a}$

$t \propto \sqrt{s}$

63. As force is opposite to velocity so body will be retarded and slow down.

64. $v = 144 \times \frac{5}{18}$ = 40 m/s

$a = \frac{40-0}{40}$ = 1m/s^2

$s = \frac{1}{2}at^2 = \frac{1}{2} \times 1 \times 40 \times 40$ = 800 m

65. $s = \left(\frac{u+v}{2}\right) \times t = \left(\frac{0+15}{2}\right) \times 10 = 75\,m$

66. u = 72 km/h = 20 m/s;

v = 0 ; S = 100 m

a = $\frac{v^2 - u^2}{2s}$ = –2 m/s^2

67. $S = 0 + \frac{5}{2}(2 \times 5 - 1) = \frac{45}{2} = 22.5m$

68. $S_{n^{th}} = U + \frac{a}{2}(2n-1)$

$19 = 0 + \frac{a}{2}(2 \times 10 - 1)$

a = 2 m/s^2

69. $\frac{S_{n^{th}}}{S_n} = \frac{0 + \frac{a}{2}(2n-1)}{0 + \frac{1}{2}an^2}$

$\frac{S_{n^{th}}}{S_n} = \frac{2}{n} - \frac{1}{n^2}$

70. $\frac{S_{4^{th}}}{S_{5^{rd}}} = \frac{2 \times 4 - 1}{2 \times 5 - 1} = \frac{7}{9}$

71. $S \propto u^2 \Rightarrow \frac{S_1}{S_2} = \frac{u_1^2}{u_2^2}$

$\Rightarrow \frac{20}{s_2} = \frac{10^2}{40^2} = s_2 = 320m$

72. $s = \frac{u^2}{2a}$ so $s \propto u^2$

So if speed is tripled, stopping distance will be nine times i.e. 9 × 2 m = 18 m

73. u + 2a = 30

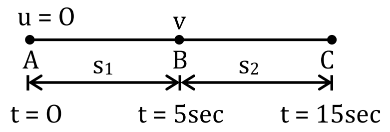
u + 4a = 50

$\Rightarrow$ a = 10 m/s^2, u = 10 m/s

74. $s_1 = \frac{1}{2} \times 5 \times (5)^2 = \frac{125}{2}m$

v = 0 + 5(5) = 25 m/s

s_2 = 25 × 10 = 250 m



Total distance travelled = $s_1 + s_2$ = 312.5 m

75. $a = \frac{20-10}{5} = 2m/s^2$

for 2 seconds before t_0

10 = v + 2(2)

v = 10 – (2)(2) = 6 m/s

76. Retardation remains same in both situations

$\frac{0-(200)^2}{2 \times 4} = \frac{0-V^2}{2 \times 9}$

V = 300 cm/s

78. Time of free fall does not depend on mass.

79. $v^2 = 2as$

$v = \sqrt{2gh}$ [does not depend upon mass]

80. $h = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 5^2 = 125$ m

81. Let total time of fall = n, u = 0

$S_{last} = S_3$

$\frac{g}{2}(2n-1) = \frac{1}{2}g(3)^2$

2n – 1 = 9 $\Rightarrow$ t = 5 sec

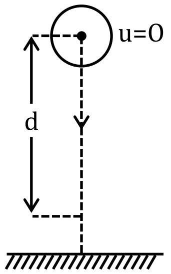
82. $x = \frac{1}{2}g(1)^2 = \frac{1}{2}g$

$5x = \frac{g}{2}(2n-1)$

$5 \times \frac{g}{2} = \frac{g}{2}(2n-1)$

n = 3 sec.

83.



$v^2 = u^2 + 2as$

$v^2 = 2g(2d)$

$v = 2\sqrt{gd}$

84. use $v^2 = u^2 - 2as$

$\Rightarrow 0 = u^2 - 2as$

$\Rightarrow a = \dfrac{u^2}{2s}$

85. Distance covered in 1st sec is $x = \dfrac{1}{2} g (1)^2 = \dfrac{g}{2}$

Let it falls for n sec then last sec is n^{th} sec.

$\therefore \quad 9x = 0 + \dfrac{g}{2}(2n - 1)$

$\Rightarrow \quad 9 \times \dfrac{g}{2} = \dfrac{g}{2}(2n - 1)$

$\Rightarrow \quad 2n = 10$

$\Rightarrow \quad n = 5$ sec

86. Time taken by stone to go down $t_1 = \sqrt{\dfrac{2h}{g}}$

Time taken by sound to come up $t_2 = \dfrac{h}{v}$

$\therefore \quad T = t_1 + t_2 = \sqrt{\dfrac{2h}{g}} + \dfrac{h}{v}$

87. $H = \dfrac{1}{2}gt^2$

$h = \dfrac{1}{2}g\left(\dfrac{t}{3}\right)^2 = \dfrac{H}{9}$ from releasing point

so distance from the ground

$= H - H/9 = 8H/9$

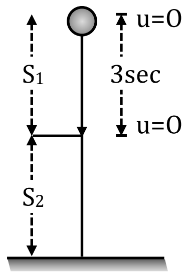
88. $h = 0 + \dfrac{1}{2}gt_1^2$

$\Rightarrow \quad t_1 = \sqrt{\dfrac{2h}{g}}$

$\dfrac{h}{4} = 0 + \dfrac{1}{2}gt_2^2 \Rightarrow t_2^2 = \dfrac{2}{g} \times \dfrac{h}{4}$

$\Rightarrow \quad t_2 = \sqrt{\dfrac{2}{g} \times \dfrac{h}{4}} = \dfrac{1}{2}\sqrt{\dfrac{2h}{g}} = \dfrac{t_1}{2}$

89.



Total height to fall

$h = \dfrac{1}{2}gt^2 = \dfrac{1}{2} \times 10 \times (5)^2 = 125\,m$

In first 3 sec.

$s_1 = \dfrac{1}{2}gt_1^2 = \dfrac{1}{2} \times 10 \times (3)^2 = 45$ m

Now remaining height to fall

$\therefore \quad s_2 = h - s_1 = 125 - 45 = 80$ m

$t_2 = \sqrt{\dfrac{2s_2}{g}} = \sqrt{\dfrac{2 \times 80}{10}} = 4$ sec

$t = t_1 + t_2 = 3 + 4 = 7$ sec

90. Separation $= S_1 - S_2$

$= \dfrac{1}{2}g(4)^2 - \dfrac{1}{2}g(2)^2$

$= \dfrac{12}{2}g = 58.8$ m

91. $h_1 - h_2 = \dfrac{1}{2}g(t_1^2 - t_2^2)$

$= \dfrac{1}{2} \times 9.8\,[(5)^2 - (3)^2]$

$= 8 \times 9.8 = 78.4$ m

92. In first 2 sec the distance travelled is

$h_1 = 0 + \dfrac{1}{2}(10)(2)^2 = 20$ m

After stopped & released the distance travelled in next 2 sec $h_2 = 0 + \dfrac{1}{2}10(2)^2 = 20$ m

So height from ground = (H – 40)m

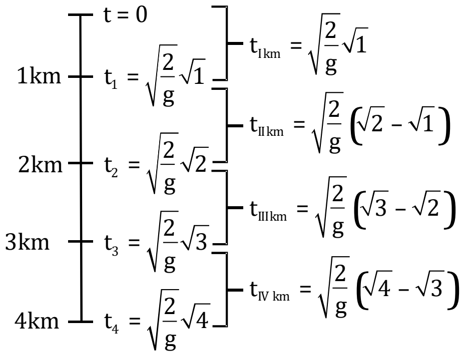
93. $S_{n^{th}} = u + \frac{a}{2}(2n-1)$

$u = 0$ and $a = g$ (const.)

$S_{n^{th}} \propto (2n-1)$ (here n = 1,2,3.....)

$D_1 : D_2 : D_3 : \text{.........} = 1 : 3 : 5 : \text{.........}$

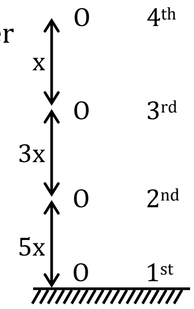
94. $\because \ h = \frac{1}{2}gt^2$ $\therefore \ t = \sqrt{\frac{2h}{g}}$



$\therefore t_{I\,km} ; t_{II\,km} ; t_{III\,km} ;....$

$= \sqrt{1} : \left(\sqrt{2}-\sqrt{1}\right) : \left(\sqrt{3}-\sqrt{2}\right) : ...$

95. By galileo's law of odd number



$x + 3x + 5x = 27m$

$x = 3$ m

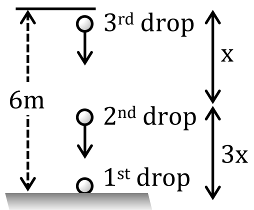
Position of 2nd drop from top

$x + 3x = 4x = 12m$

Position of 3rd drop from top

$x = 3$ m

96. By galileo's law of odd number



$x + 3x = 6m$

$4x = 6$

$x = \frac{6}{4} = 1.5m$

Height of 2nd drop from ground

$= 3x = 3 \times \frac{6}{4} = 4.5m$

97. $S_1 = \frac{1}{2}g(4)^2$

$S_1 = 80$ m

$S_2 = \frac{1}{2}g(3)^2 = 45$ m

$S_3 = \frac{1}{2}g(2)^2 = 20$ m

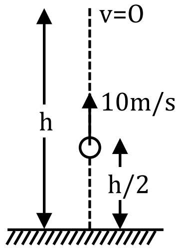
$S_4 = \frac{1}{2}g(1)^2 = 5$ m

$S_1 - S_2 = 80 - 45 = 35$ m

$S_2 - S_3 = 45 - 20 = 25$ m

$S_3 - S_4 = 20 - 5 = 15$ m

98.



$v^2 = u^2 - 2g\frac{h}{2}$

$0 = (10)^2 - 10h$

$h = 10$ m

99. $T = \frac{2u}{g} = \frac{2 \times 50}{10} = 10$ sec

100. $\because$ Height $H = \frac{u^2}{2g}$

$H \propto u^2$

$\frac{H_1}{H_2} = \frac{(2u)^2}{(3u)^2} = \frac{4}{9}$

$\because$ Time $T = \frac{2u}{g}$

$T \propto u$

$\frac{T_1}{T_2} = \left(\frac{2u}{3u}\right) = \frac{2}{3}$

101. $H_{max.} = \frac{u^2}{2g} = \frac{40 \times 40}{2 \times 10}$

$H_{max.} = 80$ m

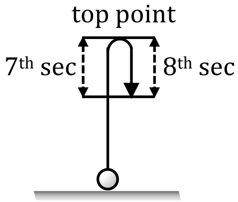
102. $v = u + at$,

$v = 40 - 10(3) = 10$ m/s.

103. $\frac{h_2}{h_1} = \frac{\frac{u_2^2}{2g}}{\frac{u_1^2}{2g}} \Rightarrow \frac{2h_1}{h_1} = \frac{u_2^2}{v_0^2}$

$u_2^2 = 2v_0^2 \Rightarrow u_2 = \sqrt{2}v_0$

104. It is only possible when 7^{th} sec is the last second for its ascending motion and 8^{th} sec is first second of its descending motion, which is clear from figure.



$\therefore \quad 0 = u - gt$

so $\quad u = gt = 9.8 \times 7 = 68.6$ m/sec

105. Distance covered in last t second of ascent

= distance covered in first t second of descent

$= 0 + \frac{1}{2}gt^2 = \frac{1}{2}gt^2$

106. Distance travelled in 1^{st} second during upward motion = distance travelled in last second during downward motion.

$$S_{1^{st}} = \frac{g}{2}(2\times6-1) = \frac{11g}{2}$$

$$S_{7^{th}} = \frac{1}{2}g(1)^2 = \frac{g}{2}$$

$$\frac{S_{1^{st}}}{S_{7^{th}}} = \frac{11}{1}$$

107. $S = \frac{1}{2}gt^2$

$$S = \frac{1}{2}g(2)^2 = 20\,m$$

108. $S_1 = S_2$

$$98(t) - \frac{1}{2}(9.8)t^2 = 98(t-4) - \frac{1}{2}(9.8)(t-4)^2$$

$$10t - \frac{t^2}{2} = 10(t-4) - \frac{1}{2}(t-4)^2$$

$\Rightarrow \quad 4t - 48 = 0$

$\Rightarrow \quad t = \frac{48}{4} = 12s$

109. For distance

'S' time should be $\Delta t/2$

$$S = \frac{1}{2}g\left(\frac{\Delta t}{2}\right)^2$$

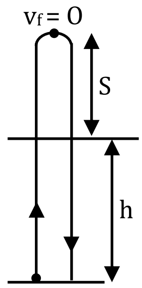
For total height

$$(h + S) = \frac{u^2}{2g}$$

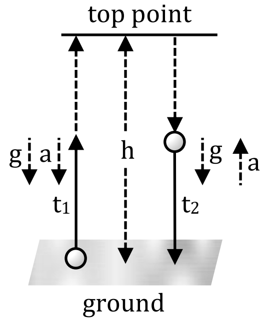
$$u = \sqrt{2g(h+S)}$$

$$= \sqrt{2g\left(h + \frac{1}{2}g\left(\frac{\Delta t}{2}\right)^2\right)}$$

$$= \sqrt{2gh + \left(\frac{g\Delta t}{2}\right)^2}$$



110.



Let retardation due to air resistance = a

then for upward journey

net retardation = (g + a)

$\therefore \quad t_{asc} = \sqrt{\frac{2h}{g+a}}$

and for downward journey

net acceleration = (g – a)

$\therefore \quad t_{desc} = \sqrt{\frac{2h}{g-a}}$

$\therefore \quad t_{asc} < t_{desc}$

111. There is loss of kinetic energy during the motion so it is clear that $v_1 > v_2$

112. $\frac{t_{ascent}}{t_{descent}} = \sqrt{\frac{g-a}{g+a}} = \sqrt{\frac{10-2}{10+2}} = \sqrt{\frac{2}{3}}$

113. $\because v^2 = u^2 + 2g(H)$

$v^2 = (20)^2 + 2(9.8)(200)$

$v = \sqrt{4320}$

$\boxed{v \simeq 65\ m/s}$

114. h = 81 m

$h = -ut + \frac{1}{2}gt^2$

$81 = -12(t) + \frac{1}{2}(10)t^2$

$5t^2 - 12t - 81 = 0$

t = 5.4 sec

115. $S = ut + \frac{1}{2}at^2$

$-h = 4.9 \times 3 - \frac{1}{2} \times 9.8 \times (3)^2$

$h = 29.4$ m

116. $V^2 = u^2 + 2as$

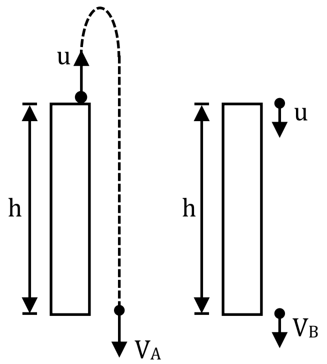
$(-3u)^2 = u^2 + 2(-g)(-h)$

$h = \frac{4u^2}{g}$

117. Because acceleration remains same in both cases, then :

$$\frac{(3)^2-(0)^2}{2h} = \frac{V^2-(-4)^2}{2h}$$

V = 5 m/s

118. 

Taking upward direction as negative and downward direction positive.

For A : $V_A^2 = (-u)^2 + 2gh$

$V_A = \sqrt{u^2+2gh}$

For B : $V_B^2 = u^2 + 2gh$

$V_B = \sqrt{u^2+2gh}$

119. Initial velocity of stone u = 10 m/sec downwards

$\therefore$ displacement $h = ut + \frac{1}{2}gt^2$

$= 10 \times 10 + \frac{1}{2} \times 10 \times (10)^2$

$= 100 + 500 = 600$ m

120. Speed after fall of 50 m

$V = \sqrt{2g(50)} = \sqrt{980}$

After parachute opens

$(3)^2 = \left(\sqrt{980}\right)^2 - 2(2)(S)$

$S = \frac{971}{4} = 242.75$ m

So total height

$242.75 + 50 \simeq 293$ m

121. Velocity at the time when fuel is finished :-

$V = U + at = 0 + 20 \times 30$

V = 600 m/s

Now, the time taken by the rocket to reach at max. height after this instant :-

$0 = 600 - 10 \times t$

t = 60 sec.

The total time taken by the rocket to reach at max. height = 30 + 60 = 90 sec.

124. Slope of s-t curve is equal to velocity.

When slope of s-t curve is zero, velocity must be zero. So it is at point B.

125. Slope of displacement-time graph gives instantaneous velocity.

126. Avg. speed $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{40+40+40+40}{20} = 8$ m/s

127. Speed at t = 4 sec. is zero and it is the minimum value.

128. $\frac{V_A}{V_B} = \frac{\tan 30°}{\tan 60°} = \frac{\frac{1}{\sqrt{3}}}{\sqrt{3}} = \frac{1}{3}$

130. Particle cannot have different displacements at same time

131. Uniformly accelerated motion $\Rightarrow$ constant acceleration

132. Avg. speed $= \frac{\text{Total distance}}{\text{Total time}}$

$= \frac{(60-10)+(40-10)}{10} = 8$ m/s

133. distance

$= \frac{1}{2}(40+20) \times 2 + (20 \times 1) + \frac{1}{2}(20 \times 1) + \frac{1}{2}$ (40×1)

$= 110$ m

displacement

$= \frac{1}{2}(40 + 20) \times 2 + (20 \times 1) + \frac{1}{2}(20 \times 1)$

$- \frac{1}{2}(40 \times 1)$

$= 70$ m

134. Let particle is moving along x-axis then, distance travelled in first 4 sec. $= \frac{1}{2}(4+2) \times 4 = 12$ m

distance travelled in next 4 sec. (along –x axis)

$= \frac{1}{2}(4+2) \times 2 = 6$ m

distance of particle from origin = 12 – 6 = 6 m

135. $\frac{\text{distance in last 4 sec.}}{\text{distance in 9 sec.}} = \frac{\frac{1}{2} \times 4 \times 10}{\frac{1}{2}(8+2) \times 10} = \frac{2}{5}$

136. displacement = 2 × 4 – 2 × 2 + 2 × 2 = 8 m

distance = 2 × 4 + 2 × 2 + 2 × 2 = 16 m

137. Distance travelled in 3 sec

= area under the curve

= area of ΔAOH + area of rectangle ABGH

+ area of ΔBCC' + area of rectangle C' CFG

$= \frac{20 \times 1}{2} + 20 \times 1 + \frac{10 \times 1}{2} + 10 \times 1 = 45\text{m}$

138. Distance = area under v-t curve

= 20 × 1 + (60 – 40) × 1 = 40 m

139. Acceleration will be maximum when slope will be maximum

$a_{max} = \frac{60-20}{40-30} = \frac{40}{10} = 4 \text{ m/s}^2$

140. $a_{avg} = \frac{V_2 - V_1}{t_2 - t_1} = \frac{0-10}{4-2}$

$a_{avg} = -5 \text{ m/s}^2$

141. Area under a-t graph gives the change in velocity during given time interval.

$\therefore \Delta V = \frac{1}{2} \times 5 \times 8 = 20$ m/s

Since initial velocity = 0

∴ Maximum speed of the particle = 20 m/s

142. Area under the acceleration-time graph = change in velocity

$\frac{1}{2}(4+2) \times 10 = v - 0$

v = 30 m/s

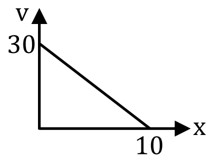
143. because the displacement is zero, hence the average velocity is also zero.

144. from 0 – 2s a = +ve ⇒ slope of v-t curve =+ve

from 2 – 4s a = – ve ⇒ Slope of v-t curve =–ve

Hence correct option (1)

146. $v = -3x + 30$



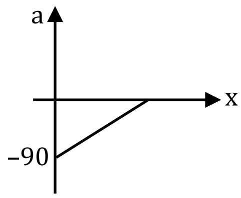
$\frac{dv}{dx} = -3$

$\because a = \frac{vdv}{dx} = (-3x+30)(-3)$

a = 9x – 90 {equation of straight line}

when x = 0 ; a = –90

& when a = 0 ; x = 10



148. $v^2 = 2as$

hence graph will be parabola

so correct option (2)

151. The slope of s-t curve gives velocity.

152. Area of (v–t) curve = displacement (height)

$= \frac{1}{2} \times 120 \times 1000 = 60{,}000\text{m} = 60\text{km}$

153. Height before retardation

$= \frac{1}{2} \times (20) \times 1000 = 10{,}000 = 10$ km

154. Mean velocity $= \frac{\text{Displacement}}{\text{Total time}} = \frac{\text{Total height}}{\text{Total time}}$

$= \frac{60{,}000\text{m}}{120\text{sec}} = 500 \text{ m/s}$

155. Retardation $= \frac{1000}{(120-20)} = \frac{1000}{100} = 10 \text{ m/s}^2$

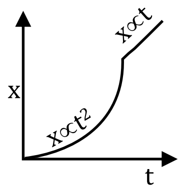
156. Acceleration = slope of OA line

$\tan\theta = \frac{1000}{20}$ = 50 m/s^2

158. Acceleration is always downward i.e. positive so slope of v – t curve will always be positive Hence correct option (3)

159. Direction of motion is changed so velocity will be first positive & then negative and $v^2 \propto h$

161. a = constant, $\Rightarrow x \propto t^2$



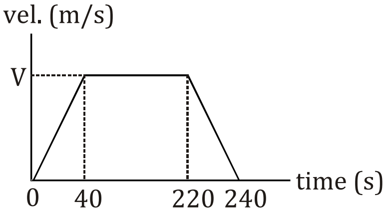
v = constant, $\Rightarrow x \propto t$

163. $v^2 = u^2 + 2as$

$a = \frac{v^2 - u^2}{2s} = \frac{(900)-(4600)}{2\times0.6} = -\frac{3700}{1.2}$

= –3084 km/hr^2

164.



$2250 = \frac{1}{2}(240+180)\times V$

V = 10.71 m/s

165. $v_{max} = \frac{abt}{a+b} = \frac{2\times4\times3}{6} = 4\ m/s$

166. Horizontal component of velocity remain same.

168. If air resistance is zero then for horizontal motion

$F_x = 0$; $a_x = 0$

So u_x = constant

169. At highest point velocity is in horizontal direction only & acceleration (g) is vertically downwards.

170. If air resistance is not considered but acceleration 'g' is considered, which is constant, $|\vec{a}| = \left|\frac{d\vec{v}}{dt}\right| = g$

171. For projectile motion acceleration is constant

$\frac{d\vec{v}}{dt}$ = constant & $\frac{d^2\vec{v}}{dt^2}$ = 0

172. $t = \frac{U\sin\theta}{g} = \frac{1000\sin30°}{10}$

t = 50 sec.

173. $\because \frac{2U\sin\theta}{g} = 10$

$\therefore U\sin\theta = 50$

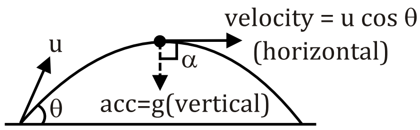
$\because H = \frac{U^2\sin^2\theta}{2g} = \frac{(50)^2}{2\times10}$

$\therefore$ H = 125 m

174. $T = \frac{2u\sin\theta}{g} = \frac{2\times19.6\sin30°}{9.8} = 2$ sec

175. $H = \frac{u_y^2}{2g} = \frac{u^2\cos^2\theta}{2g}$ (θ from vertical)

176. $\therefore$ $\angle\alpha = 90°$



177. $\because$ Range R = (u_x) (T)

Here $(v_A)_x = (v_B)_x$

but $(T)_A > (T_B)$

So $R_A > R_B$ is true

178. $\frac{2U\sin\theta}{g} = 4$

$U\sin\theta = 2g$

$H = \frac{U^2\sin^2\theta}{2g} = 2g = 19.6$ m

Height of the ball from ground = 19.6 + 1.5

= 21.1 m

179. After time 't'

velocity $\vec{v} = (v_x)\hat{i} + (v_y)\hat{j}$

$\vec{v} = (v\cos\theta)\hat{i} + (v\sin\theta - gt)\hat{j}$

$|\vec{v}| = \sqrt{(v\cos\theta)^2 + (v\sin\theta - gt)^2}$

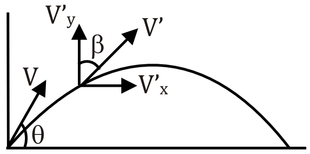
$= \sqrt{v^2 + g^2t^2 - (2v\sin\theta)gt}$

180. $\Delta v_x = 0$ (as $\vec{v}_x$ = const)

$\Delta\vec{v}_y$ = – u sin θ – (u sin θ) = –2 u sin θ

magnitude of change in velocity $\Delta v_y = 2u\sin\theta$

182. $\tan\beta = \dfrac{v'_x}{v'_y} = \dfrac{v\cos\theta}{v\sin\theta - gt}$



183. For vertical motion

$v_y^2 = u_y^2 - 2g(h)$

$(2)^2 = u_y^2 - 2(10)(0.4)$

$u_y = 2\sqrt{3}$

$\because\ \vec{u} = (u_x)\hat{i} + (u_y)\hat{j}$

$\therefore\ \vec{u} = 6\hat{i} + 2\sqrt{3}\hat{j}$

$\tan\theta = \dfrac{2\sqrt{3}}{6} = \dfrac{1}{\sqrt{3}} \Rightarrow \boxed{\theta = 30^\circ}$

184. $\theta = 60^\circ$; $R = \dfrac{u^2\sin(2\times60^\circ)}{g} = \dfrac{\sqrt{3}u^2}{2g}$

185. $\vec{v} = a\hat{i} + (b - ct)\hat{j}$

$v_x = a$; $v_y = (b - ct)$

$\because$ At maximum height

$v_y = 0$

$b - ct = 0$

$t = \dfrac{b}{c}$

So time of Flight $T = \dfrac{2b}{c}$

Range $R = (v_x)(T)$

$\boxed{R = \dfrac{2ab}{c}}$

186. $R = \dfrac{2u_xu_y}{g} = \dfrac{2\times9.8\times4.9}{9.8} = 9.8\text{m}$

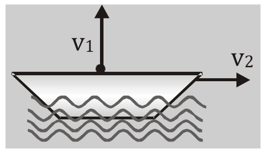
187. $\vec{v} = 5\hat{i} + 12\hat{j}$

$\Rightarrow$ $u\cos\theta = 5$ and $u\sin\theta = 12$

$R = \dfrac{2\,(u\sin\theta)(u\cos\theta)}{g} = \dfrac{2\times5\times12}{10} = 12\text{m}$

188. For vertical motion, time of flight $t = \dfrac{2v_1}{g}$

for horizontal motion $R = v_2 \times t = \dfrac{2v_1v_2}{g}$



189. $0.5 = \dfrac{u^2\sin(2\times60^\circ)}{g} \Rightarrow 0.5 = \dfrac{u^2}{g}\times\dfrac{\sqrt{3}}{2}$

$\dfrac{u^2}{g} = \dfrac{1}{\sqrt{3}} \Rightarrow R' = \dfrac{u^2\sin(2\times45^\circ)}{g} = \dfrac{u^2}{g} = \dfrac{1}{\sqrt{3}}$ km

190. $u\cos\theta = \dfrac{1}{2}u \quad\Rightarrow\quad \theta = 60^\circ$

$\Rightarrow R = \dfrac{u^2\sin(2\times60^\circ)}{g} = \dfrac{u^2\sin120^\circ}{g} = \dfrac{\sqrt{3}u^2}{2g}$

191. $R = \dfrac{u^2\sin(2\theta)}{g}$

$R = \dfrac{500\times500\times\sin(30^\circ)}{10}$

$R = 12500$ m

192. Range of projectile $= \dfrac{u^2\sin2\theta}{g}$

$\because$ Range $\propto u^2$

$\therefore$ When u is tripled, range becomes 9 times.

193. For maximum range $\theta = 45^\circ$

So $\tan\theta = 1$

194. $R_{max} = \dfrac{u^2}{g} \Rightarrow u^2 = Rg = 25000 \times 10$

$\therefore\quad u = 500$ m/s

195. For maximum range

$R_{max} = \dfrac{u^2}{g} = 1.6$

$u = 4$ m/s

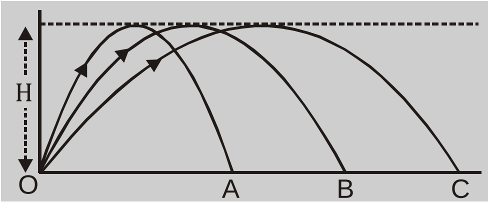
So horizontal distance in 10 sec. will be

$d = (u_x)\,(t)$

$d = (4)\cos(45^\circ)\,(10)$

$= 20\sqrt{2}$ m

196.



$H = \frac{u^2 \sin^2\theta}{2g}$ which is same for all

$\therefore$ u sinθ is same for all

$T = \frac{2u\sin\theta}{g}$ {same for all}

for large θ value of sinθ is large and hence u will be less

$\therefore$ $u_A < u_B < u_C$

i.e. launch speed is maximum for particle C

horizontal range R = $u_x \times T$

T is same for all and R_C is maximum for C

$\therefore$ u_x is maximum for C

197. $\vec{v} = a\hat{i} + b\hat{j} \Rightarrow$ u cos θ = a and u sin θ = b

R = $4H_{max}$

$\tan\theta = \frac{b}{a}$ also $\frac{u^2 \sin 2\theta}{g} = \frac{4u^2 \sin^2\theta}{2g}$

$\Rightarrow$ tanθ = 1

$\therefore$ $\frac{b}{a} = 1 \Rightarrow$ b = a

198. $\because$ $\theta_1 + \theta_2 = 90°$

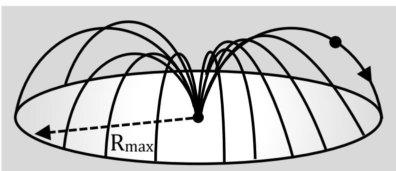
$\therefore$ $R_1 = R_2$

199. $\because$ $\theta_A + \theta_C = 90°$

$\therefore$ $R_A = R_C = \frac{u^2 \sin 2\times30°}{g} = \frac{u^2\sqrt{3}}{2g}$

$R_B = \frac{u^2 \sin 2\times 45°}{g} = \frac{u^2}{g}$ $\Rightarrow$ $R_B > R_A = R_C$

200. $R_{max} = \frac{u^2}{g}$



Area = $\pi R_{max}^2 = \pi\left[\frac{u^2}{g}\right]^2$

201. $R = \frac{U^2 \sin 2\theta}{g}$

For 'Q' and 'S' range will be same (complementary angles)

For 'R' it will be maximum

For 'P' it will be minimum

202. For maximum height

$\because$ $H_{max} = \frac{u^2}{2g}$

$(10) = \frac{u^2}{2g}$

$U = 10\sqrt{2}$ m/s

For maximum range

$R_{max} = \frac{u^2}{g} = \frac{(10\sqrt{2})^2}{10} = 20$ m

203. For maximum range

$R_{max} = \frac{u^2}{g}$

$100 = \frac{u^2}{g}$

For maximum height

$H_{max} = \frac{u^2}{2g} = \frac{100}{2}$

$\boxed{H_{max} = 50\text{ m}}$

204. $R = \frac{u^2 \sin 2\theta}{g}$ At θ = 45°

$R_{max} = \frac{u^2}{g} = 2000$

$\Rightarrow$ $h = \frac{u^2 \sin^2 45°}{2g} = \frac{2000}{2\times2} = 500\text{m}$

205. For θ < 45°, with increase in θ, Range also increases.

206. $\because$ Time of flight

$T = \frac{2(u)\sin\theta}{g}$

$\boxed{T \propto \sin\theta}$

For projectile '1' angle of projection is small so time of flight will be less.

207. $R=\frac{2u^2\sin\theta\cos\theta}{g}=\frac{2u^2\sin 53°\cos 53°}{g}$

$R=\frac{24u^2}{25g}$

$H=\frac{u^2\sin^2\theta}{2g}=\frac{u^2(\sin 53°)^2}{2g}=\frac{16u^2}{2\times 25g}$

$H=\frac{8u^2}{25g}=\frac{R}{3}$

208. R is same for two angles θ & 90 – θ

$h_1=\frac{u^2\sin^2\theta}{2g}, h_2=\frac{u^2\sin^2(90-\theta)}{2g}$

$h_1h_2=\frac{u^4\sin^2\theta\cos^2\theta}{4g^2}$

$R^2=\frac{4u^4\sin^2\theta\cos^2\theta}{g^2} \Rightarrow h_1h_2 \propto R^2$

209. $h=\frac{u^2\sin^2\theta}{2g} \Rightarrow h_1=90=\frac{(u\sin 37°)^2}{2g}$

$90=\frac{u^2}{2g}\times\left[\frac{3}{5}\right]^2 \Rightarrow \frac{u^2}{2g}=250$

$h_2=\frac{(u\sin 53°)^2}{2g}=\frac{u^2}{2g}\times\left[\frac{4}{5}\right]^2=250\times\frac{16}{25}=160m$

210. For same range $\theta_1+\theta_2=90°$

$\theta_1=\frac{\pi}{6}rad=30°$

$\therefore \quad \theta_2=90°-\theta_1=60°$

$y_1=\frac{u^2\sin^2 30°}{2g}=\frac{u^2\times 1}{2g\times 4}=\frac{u^2}{8g}$

$y_2=\frac{u^2\sin^2 60°}{2g}=\frac{u^2\times 3}{2g\times 4}=\frac{3u^2}{8g}=3y_1$

211. Range(R) = same

then angles of projection are θ & (90 – θ)

$y_1=\frac{u^2\sin^2\theta}{2g}$ and

$y_2=\frac{u^2\sin^2(90°-\theta)}{2g}=\frac{u^2\cos^2\theta}{2g}$

$y_1+y_2=\frac{u^2}{2g}(\sin^2\theta+\cos^2\theta)=\frac{u^2}{2g}$

212. R = 4H

$\tan\theta=\frac{4H}{R}=\frac{4}{4}=1\Rightarrow\theta=45°$

213. $R=4\sqrt{3}H$

$\tan\theta=\frac{4}{4\sqrt{3}}=\frac{1}{\sqrt{3}}$

θ = 30°

214. R = 2H

$\tan\theta=\frac{4}{2}=2$

$\theta=\tan^{-1}(2)$

215. Given

$u_x=\left(\frac{\sqrt{3}}{2}\right)(u)$

$u\cos\theta=\frac{\sqrt{3}}{2}(u) \Rightarrow \theta=30°$ & R = (P) H

$\frac{u^2 2\sin(\theta)(\cos\theta)}{g}=(P)\frac{u^2\sin^2\theta}{2g}$

4 = (P) tan(θ)

$P=\frac{4}{\tan\theta}=\frac{4}{\tan(30°)}$

$P=\frac{4}{\frac{1}{\sqrt{3}}}=4\sqrt{3}$

216. Maximum range for a given velocity is attained at θ = 45°

$R=\frac{u^2\sin 2\theta}{g}\Rightarrow R_{max}=\frac{u^2\sin 90°}{g}=\frac{u^2}{g}$

$H=\frac{u^2\sin^2\theta}{2g}=\frac{u^2.\sin^2 45°}{2g}=\frac{u^2}{4g}$

$\therefore R_{max}=4H$

217. $y=x\tan\theta-\frac{1}{2}\frac{gx^2}{u^2\cos^2\theta}$ (standard equation)

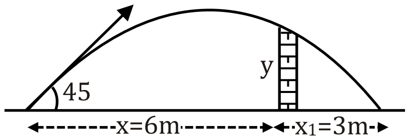
$y=x-\frac{1}{2}gx^2\Rightarrow\tan\theta=1\Rightarrow\theta=45°$

218. $y = 4x - \frac{x^2}{4}; y = 4x\left(1 - \frac{x}{16}\right)$

Comparing with y = x tanθ $\left(1 - \frac{x}{R}\right)$

R = 16m

219.



R = 6 + 3 = 9 m

$y = (x\tan\theta)\left[1 - \frac{x}{R}\right] = 6 \times 1 \times \left[1 - \frac{6}{9}\right] = 2\,m$

220. Time of flight $T = \frac{2u\sin\theta}{g}$

$T = \frac{(2)(50)\sin(30°)}{10} = 5\text{ sec}$

Horizontal range $R = \frac{u^2\sin 2\theta}{g}$

$R = \frac{(50)^2\sin(60°)}{10} = 216.5\text{ m}$

After 3 sec. the distance travelled is d = U_x × 3

= 50 cos (30°) (3) = 129.9m

So, remaining horizontal distance after wall is

R – d = 216.5 – 129.9

≃ 86.6 m

221. Range 'R' = 2(90) = 180 m

So $\frac{R}{H} = \frac{180}{45} = 4$

∴ R = 4H, $\tan\theta = \frac{4H}{R} = \frac{4}{4} = 1$

$\boxed{\theta = 45°}$

222. R = u_xT ⇒ $u_x = \frac{R}{T} = \frac{100}{5} = 20$

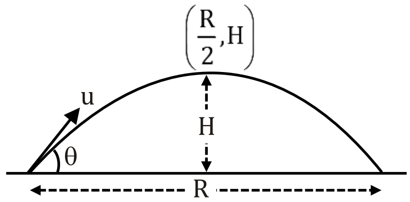
223. $H = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times \left[\frac{5}{2}\right]^2 = 31.25\,m$

224. $u_y = gt_{up}$ = 10 × 2.5 = 25 m/s

225. ∵ u_x = 20 and u_y = 25

∴ $\tan\theta = \frac{u_y}{u_x} = \frac{25}{20} = \frac{5}{4}$

226.



for maximum range θ = 45°

∵ $\tan\theta = \frac{4H}{R}$

$H = \frac{R}{4}\tan 45° = \frac{200}{4} = 50$

∴ required coordinates are $\left(\frac{R}{2}, H\right) = (100, 50)$

227. $R = \frac{v^2\sin 2\theta}{g} \quad \Rightarrow \quad \sin 2\theta = \frac{gR}{v^2}$

$\Rightarrow \quad 2\theta = \sin^{-1}\frac{gR}{v^2} \quad \Rightarrow \quad \theta = \frac{1}{2}\sin^{-1}\frac{gR}{v^2}$

228. R = const. ⇒ u² sin 2q = const.

$\Rightarrow u_1^2\sin 2\theta_1 = u_2^2\sin 2\theta_2$

$\Rightarrow \sin 2\theta_1 = \frac{u_2^2\sin 2\theta_2}{u_1^2} = \frac{u^2}{4}\frac{\sin 60°}{u^2} = \frac{\sqrt{3}}{8}$

$2\theta_1 = \sin^{-1}\frac{\sqrt{3}}{8} \quad \Rightarrow \quad \theta_1 = \frac{1}{2}\sin^{-1}\frac{\sqrt{3}}{8}$

229. $T_1 = T_2$

$\frac{2U_1}{g} = \frac{2U_2\sin 60°}{g}$

$\frac{U_1}{U_2} = \frac{\sqrt{3}}{2}$

230. $H_1 = \frac{U^2\sin^2\theta}{2g}$

$H_2 = \frac{U^2[\sin(90-\theta)]^2}{2g}$

$H_2 = \frac{U^2\cos^2\theta}{2g}$

$\frac{H_1}{H_2} = \frac{\tan^2\theta}{1}$

231. Horizontal component of velocity of projection of projectile should be equal to speed of person

$v_x = v\cos\theta = \frac{v}{2}$

$\cos\theta = \frac{1}{2}$ $\boxed{\theta = 60°}$

232. $\dfrac{H}{T^2} = \dfrac{\dfrac{U^2\sin^2\theta}{2g}}{\left(\dfrac{2U\sin\theta}{g}\right)^2}$

$= \dfrac{g}{8} = \dfrac{10}{8} = \dfrac{5}{4}$

233. $R = 5\sqrt{3}T^2$

$\Rightarrow \dfrac{2u^2\sin\theta\cos\theta}{g} = 5\left[\dfrac{2u\sin\theta}{g}\right]^2\sqrt{3}$

$\Rightarrow \dfrac{2u^2\sin\theta\cos\theta}{g} = 5\times\dfrac{4u^2\sin^2\theta}{g^2}\sqrt{3}$

$\Rightarrow \tan\theta = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \theta = 30°$

234. $E = \dfrac{1}{2}mu^2$

and $E' = \dfrac{1}{2}mu^2\cos^2 60° = E\times\dfrac{1}{4} = \dfrac{E}{4}$

235. Kinetic energy $K = \dfrac{1}{2}mu^2$

At highest point :

$v = u_x = u\cos 60° = \dfrac{u}{2}$

K.E. $= \dfrac{1}{2}m\left(\dfrac{u}{2}\right)^2$

$= \dfrac{1}{4}\left(\dfrac{1}{2}mu^2\right) = \dfrac{K}{4}$

236. In vertical upwards. PE = mgh

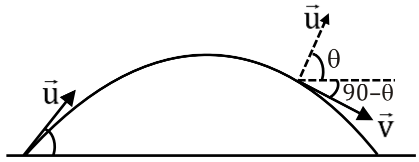
$= mg\left(\dfrac{v^2}{2g}\right) = \dfrac{mv^2}{2}$ ... (i)

at an angle of 30° with vertical PE = mgh

$= mg\left(\dfrac{v^2\sin^2 60°}{2g}\right) = 3\dfrac{mv^2}{8}$ ... (ii)

$\dfrac{(PE)_{up}}{(PE)_{at\ \theta=30°\ from\ vt.}} = \dfrac{\dfrac{mv^2}{2}}{3\dfrac{mv^2}{8}} = \dfrac{4}{3}$

237.



∵ horizontal component of velocity remains same

∴ u cosθ = v cos (90 – θ)

u cosθ = v sinθ ⇒ v = u cot θ

238. $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{(6t^2\hat{i}+4t\hat{j})}{1} = 6t^2\hat{i}+4t\hat{j}$

$\dfrac{d\vec{v}}{dt} = 6t^2\hat{i}+4t\hat{j}$

$\Rightarrow \int_0^{\vec{v}} d\vec{v} = \int_0^3 (6t^2\hat{i}+4t\hat{j})dt$

$\vec{v} = \left[\dfrac{6t^3}{3}\hat{i}+\dfrac{4t^2}{2}\hat{j}\right]_0^3 = 54\hat{i}+18\hat{j}$

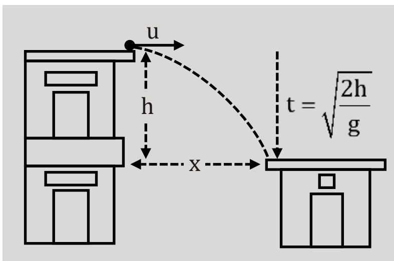
239. Time taken

$T = \sqrt{\dfrac{2h}{g}} = \sqrt{\dfrac{2\times 80}{10}} = 4\ \text{sec.}$

distance 'd' = (u_x) (T) = (10) (4) = 40 m

240. $x = t\times u = \sqrt{\dfrac{2h}{g}}\times u = \sqrt{\dfrac{2\times 490}{9.8}}\times 25 = 250\text{m}$

241.



$x = \sqrt{\dfrac{2h}{g}}\times u$

$\Rightarrow u = x\sqrt{\dfrac{g}{2h}} = 6.2\times\sqrt{\dfrac{9.8}{2\times 19.6}} = 3.1\ m/s$

242. Horizontal range

$R = (u_x)$ (T)

If projected horizontal from same height then time of flight will be same

$R \propto u_x$

$\dfrac{R_1}{R_2} = \dfrac{100}{40} = \dfrac{5}{2}$

243. Time of flight

$$T=\sqrt{\frac{2h}{g}}=\sqrt{\frac{2\times 20}{10}}=2\text{ sec.}$$

Range 'R' = (u_x) (T)

= (20) (2) = 40 m

244. $t=\sqrt{\frac{2h}{g}}\,;\,x=v_x.t=150\times\sqrt{\frac{2\times 19.6}{9.8}}$ = 300m

245. $t=\sqrt{\frac{2\times 20}{g}}=2\text{ sec}$

$$x=v_x t\Rightarrow v_x=\frac{8}{2}=4\text{ m/s}$$

246. Time taken

$$T=\sqrt{\frac{2(h)}{g}}$$

h = 39.2 – 19.6 = 19.6 m

$$T=\sqrt{\frac{2(19.6)}{9.8}}=2\text{sec.}$$

∵ Horizontal range

R = (u_x) (T)

20 = (u_x) (2)

u_x = 10 m/s

247. Difference in height

h = 150 – 27.5 = 122.5 m

time taken $T=\sqrt{\frac{2h}{g}}$

$$T=\sqrt{\frac{2(122.5)}{9.8}}=5\text{ sec.}$$

Range R = (u_x) (T)

$$u_x=\frac{30}{5}=6\text{ m/s}$$

248. For vertical motion

$$H=(u_yT)+\frac{1}{2}g(T)^2$$

Here 'H' & 'u_y' are same for both bodies so time will be same.

$$\boxed{\frac{T_1}{T_2}=\frac{1}{1}}$$

249. ∵ $h=u_y(t)+\frac{1}{2}gt^2$

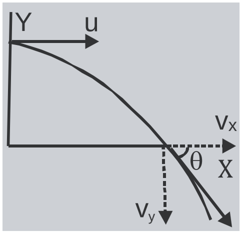
$$45=0+\frac{1}{2}(g)\,t^2$$

t = 3 sec.

250. $x = ut$, $\quad y=\frac{1}{2}gt^2$

$$\text{Displacement}=\sqrt{x^2+y^2}=\sqrt{u^2t^2+\frac{g^2t^4}{4}}$$

251.



$v_x = 98\sqrt{3}$ m/s

v_y = 98 m/s

$v_y = u_y + a_yt$

v_y = 0 + 9.8 × 10 = 98 m/s

$$\tan\theta=\frac{v_y}{v_x}=\frac{98}{98\sqrt{3}}=\frac{1}{\sqrt{3}}\Rightarrow\theta=30^\circ$$

252. $\tan 45^\circ=\frac{H}{x}=1$

x = H = 78.4m

time of flight $t=\sqrt{\frac{2H}{g}}=\sqrt{\frac{2\times 78.4}{9.8}}$ = 4 sec

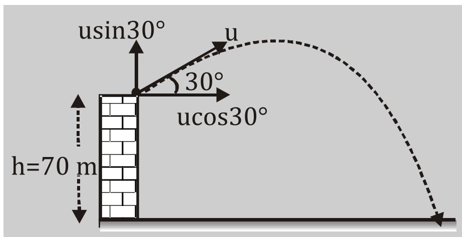
$$u=\frac{x}{t}=\frac{78.4}{4}=19.6\text{ m/s}$$

253. $\tan 30^\circ=\frac{v_y}{v_x}\Rightarrow v_y=\frac{v_x}{\sqrt{3}}=\frac{20}{3}$ m/s

$v_y = 0 + gt$ $\quad t=\frac{20/3}{g}=\frac{2}{3}$ sec.

254. $x=\frac{20}{\sqrt{3}}\times\frac{2}{3}=\frac{40}{3\sqrt{3}}$ m

255.

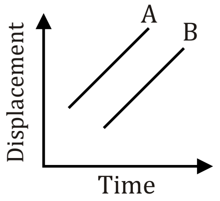


by $S_y=u_yt+\frac{1}{2}a_yt^2$

$$\Rightarrow\quad h=-u\sin 30^\circ t+\frac{1}{2}gt^2$$

$$\Rightarrow\quad 70=-50\times\frac{1}{2}t+\frac{1}{2}\times 10t^2\Rightarrow t=7\text{ sec}$$

256. Slope of displacement-time graph gives velocity



Slope of A & B is same

257. $\because$ Both velocities are perpendicular

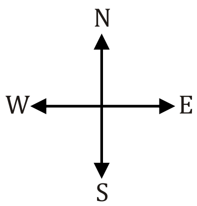
$$\vec{v}_{rel} = 4\hat{i} - 3\hat{j}$$

$$|\vec{v}_{rel}| = \sqrt{4^2 + 3^2} = 5\ m/s$$

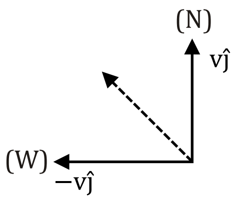
258. $\vec{v}_{Train} = v\hat{i}$

$$\vec{v}_{Car} = v\hat{j}$$

Velocity of car relative to train



$$\vec{v}_{rel} = v\hat{j} - v\hat{i}$$



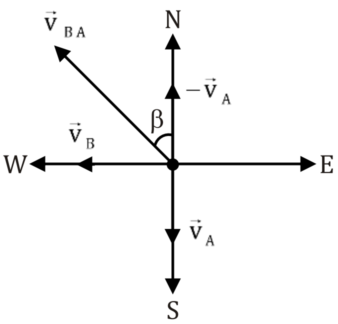
West-north direction

259. $a_A = 8\ m/s^2$ & $a_B = 4\ m/s^2$

$a_{BA} = a_B - a_A$

$= 4 - 8 = -4 m/s^2$

260. $v_{rel} = 80 + 65 = 145$ km/hr

261. 

The velocity of car A and car B are represented by the vectors in the direction as given in the question as shown in the figure.

Given : $v_A = v_B = v = 5$ km/min

The relative velocity of car B w.r. t to car A is given by : $\vec{v}_{BA} = \vec{v}_B - \vec{v}_A$

$$\vec{v}_{BA} = 5(-\hat{i}) - (-5\hat{j}) = -5\hat{i} + 5\hat{j}$$

$$|\vec{v}_{BA}| = \sqrt{5^2 + 5^2} = 5\sqrt{2}\ km/min$$

$$\tan\beta = \frac{5}{5} = 1$$

β = 45° in north-west.

262. $\vec{v}_{MT} = \vec{v}_M - \vec{v}_T$

$$\vec{v}_M = \vec{v}_{MT} + \vec{v}_T$$

$$\vec{v}_M = +3\hat{i} + 20\hat{i} = 23\hat{i}$$

23 m/s towards east

263. $v_{gas,\ plane} = v_{gas} + v_{plane}$

$v_{gas} = v_{gas,\ plane} - v_{plane}$

$= 1600 - 500 = 1100$ km/hr

264. $\vec{v}_{TG} = 54 \times \frac{5}{18}\hat{j} = 15\hat{j}\ m/s$

$$\vec{v}_{MT} = -36 \times \frac{5}{18}\hat{j} = -10\hat{j}\ m/s$$

$$\vec{v}_{MT} = \vec{v}_{MG} - \vec{v}_{TG}$$

$$\vec{v}_{MG} = \vec{v}_{MT} + \vec{v}_{TG} = -10\hat{j} + 15\hat{j} = 5\hat{j}\ m/s$$

= 5 m/s along north

267. Length of train L = 100 m

Let speed of train = v_t

and speed of man $v_m = 5$ km/hr

relative speed $v_{rel} = v_t + v_m$

$$v_{rel} = \frac{\text{Distance}}{\text{Time}} = \frac{100}{6}\ m/s$$

$$= \frac{100}{6} \times \frac{18}{5} = 60\ \text{km/hr}$$

$\therefore$ $v_t + 5 = 60$ $\Rightarrow$ $v_t = 55$ km/hr

268. Relative velocity of the bird $(v_{rel}) = 20 - (-5)$

= 25 m/s

Distance = 300 m

$\therefore$ Time $= \frac{\text{Distance}}{v_{rel}} = \frac{300}{25} = 12$ sec.

269. $t = \frac{100 + 100}{35 + 15} = 4s$

270. $\because$ $v^2 = u^2 + 2as$

$(50)^2 = (30)^2 + 2(a)(500)$

$a = 1.6\ m/s^2$

So, time $t = \dfrac{v-u}{a}$

$t = \dfrac{50-30}{1.6} = 12.5$ sec

271. For opposite direction

$v_1 + v_2 = 8$ ...(1)

For same direction

$v_1 - v_2 = 0.8$ ...(2)

From (1) & (2)

$v_1 = 4.4$ m/s & $v_2 = 3.6$ m/s

272. $v_M - v_R = \dfrac{30}{10}$

$5 - v_R = 3$

$\boxed{v_R = 2\ m/s}$

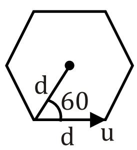
273. $\dfrac{5}{60} = \dfrac{10}{v+60} \Rightarrow v + 60 = 120$

$v = 60$ km/hr

274. $t = \dfrac{d}{u + u\cos 90} = \dfrac{d}{u}$

275. $t = \dfrac{d}{u - u\cos 60°}$

$= \dfrac{d}{u - \frac{u}{2}} = \dfrac{d}{\frac{u}{2}} = \dfrac{2d}{u}$



Alternative method: -

$t = \dfrac{d}{u\cos 60°}$

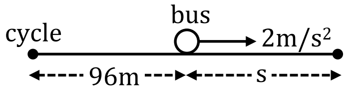
$t = \dfrac{2d}{u}$

276. speed of man $(v_m) = \dfrac{d}{45}$

speed of escalator $(v_s) = \dfrac{d}{60}$

$t = \dfrac{d}{v_m + v_s} = \dfrac{d}{\frac{d}{45} + \frac{d}{60}} = \dfrac{60 \times 45}{105} = 25.71$ sec

277.



For cycle $96 + s = 20 \times t$

for bus $s = \dfrac{1}{2}at^2 = \dfrac{1}{2} \times 2 \times t^2 = t^2$

$\therefore 96 + t^2 = 20t \Rightarrow t^2 - 20t + 96 = 0$

$(t-8)(t-12) = 0 \therefore t = 8$ s or 12 s

cyclist over takes bus in 8 s

Note : At t = 12s, bus overtakes cyclist.

278. $\dfrac{10}{v_{mr}} + \dfrac{10}{v_{mr}} = 2 \Rightarrow v_{mr} = 10$ km / hr.

$\Rightarrow t = \dfrac{10}{v_{mr} + v_r} + \dfrac{10}{v_{mr} - v_r}$

$= \dfrac{10}{10+5} + \dfrac{10}{10-5} = \dfrac{10}{15} + \dfrac{10}{5} = \dfrac{8}{3}$ hr

$\Rightarrow t = \dfrac{8}{3} \times 60 = 160$ min.

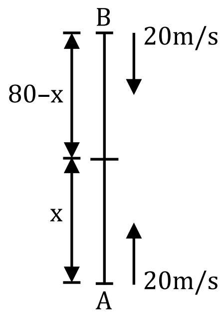
279. The velocity upstream is (6 – 2) km/hr and downstream is (6 + 2) km/hr.

$\therefore$ Total time taken $= \dfrac{2}{4} + \dfrac{2}{8} = \dfrac{6}{8} = \dfrac{3}{4}$ hr

= 45 minutes

280. Distance will remain the same as before because acceleration of both the bodies is same and initial velocity is zero.

281.



Let us assume they are meeting at x distance from ground

$80 - x = 20t + \dfrac{1}{2}gt^2$ ... (i)

$x = 20t - \dfrac{1}{2}gt^2$ ... (ii)

From eq. (i) and (ii)

$80 = 40t$

$t = 2$ s

$x = 20$ m from ground.

282. Total time taken by the first walnut to reach at ground :-

$h = \frac{1}{2} gt^2$

$20 = \frac{1}{2} \times 10 \times t^2$

t = 2 sec.

time taken by it to cover a distance of 5 m :

$5 = \frac{1}{2} gt^2 \Rightarrow t = 1$ sec.

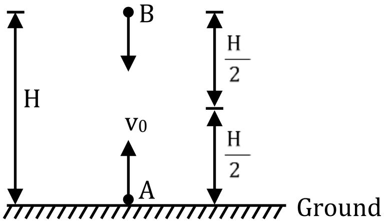
So, For 2nd walnut :

$S = Ut + \frac{1}{2} at^2$

$20 = U \times 1 + \frac{1}{2} \times 10 \times (1)^2$

U = 15 m/s

283. Let the two bodies A and B respectively meet at a time t, at a height $\frac{H}{2}$ from the ground.



Using $S = ut + \frac{1}{2} at^2$

For a body A, $u = v_0$, $a = -g$, $S = \frac{H}{2}$

$\therefore \frac{H}{2} = v_0 t - \frac{1}{2} gt^2$ ... (i)

For a body B, u = 0, a = +g, $S = \frac{H}{2}$

$\therefore \frac{H}{2} = \frac{1}{2} gt^2$

Equating equations (i) and (ii), we get

$v_0 t - \frac{1}{2} gt^2 = \frac{1}{2} gt^2$

$v_0 t = gt^2$ or $t = \frac{v_0}{g}$

Substituting the value of t is equation (i), we get

$$\frac{H}{2} = v_0 \times \left(\frac{v_0}{g}\right) - \frac{1}{2} g\left(\frac{v_0}{g}\right)^2 = \frac{v_0^2}{g} - \frac{1}{2}\frac{v_0^2}{g}$$

$\frac{H}{2} = \frac{1}{2}\frac{v_0^2}{g}$ or $v_0^2 = gH$

$v_0 = \sqrt{gH}$

Alternate solution: -

$U_{AB} = U_A - U_B = v_0 - 0 = v_0$

$a_{AB} = a_A - a_B = -g - (-g) = 0$, $S_{AB} = H$

$\therefore t = \frac{S_{AB}}{U_{AB}} = \frac{H}{v_0}$, for body B :- $S = 1/2\ gt^2$

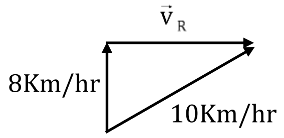
$\Rightarrow \frac{H}{2} = \frac{1}{2} g \times \frac{H^2}{v_0^2} \Rightarrow v_0 = \sqrt{gH}$

284. Effective acceleration in ascending lift = (g + a)

$t = \sqrt{\frac{2h}{g+a}} = \sqrt{\frac{2 \times 19}{32+6}}$

$t = \sqrt{\frac{2 \times 19}{38}}$ = 1 sec

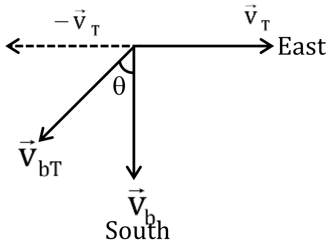
285.



$v_R^2 + 8^2 = 10^2$

v_R = 6 Km/hr

286.

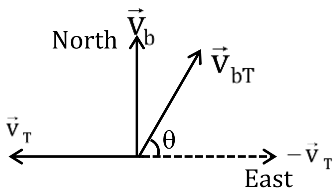


$\vec{v}_{bT} = -40\hat{j} - 40\hat{i}$

$|\vec{v}_{bT}| = \sqrt{40^2 + 40^2} = 40\sqrt{2}$ (south-west)

tanθ = 1 ; θ = 45°

287.



$\vec{v}_{bT} = \vec{v}_b - \vec{v}_T$

$\vec{v}_{bT} = 40\hat{j} - (-30\hat{i})$

$\vec{v}_{bT} = (30\hat{i} + 40\hat{j})$ km/h

$|\vec{v}_{bT}| = \sqrt{(30)^2 + (40)^2} = 50$ km/h

$$\tan\theta = \frac{40}{30} = \frac{4}{3}$$

$\theta = 53°$

53° North of east

288. $\vec{v}_{BW} = \vec{v}_{BG} - \vec{v}_{WG} = 9\hat{i} + 12\hat{j}$

289. Here,

$$\vec{r}_1(t) = 3t\,\hat{i} + 4t^2\,\hat{j}$$

$$\vec{r}_2(t) = 4t^2\,\hat{i} + 3t\,\hat{j}$$

Velocity,

$$\vec{v}_1(t) = \frac{d\vec{r}_1}{dt} = \frac{d}{dt}\left(3t\,\hat{i} + 4t^2\,\hat{j}\right) = 3\hat{i} + 8t\,\hat{j}$$

$$\vec{v}_2(t) = \frac{d\vec{r}_2}{dt} = \frac{d}{dt}\left(4t^2\,\hat{i} + 3t\,\hat{j}\right) = 8t\hat{i} + 3\hat{j}$$

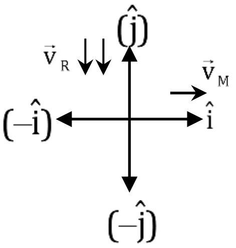
The relative speed of particle 1 with respect to particle 2 is : $\vec{v}_{12} = \vec{v}_1 - \vec{v}_2$

$$= \left(3\hat{i} + 8t\,\hat{j}\right) - \left(8t\,\hat{i} + 3\hat{j}\right) = (3 - 8t)\,\hat{i} + (8t - 3)\,\hat{j}$$

At t = 2 s,

$$\vec{v}_{12} = (3 - 16)\hat{i} + (16 - 3)\hat{j} = -13\hat{i} + 13\hat{j}$$

$$|\vec{v}_{12}| = \sqrt{(13)^2 + (13)^2} = 13\sqrt{2}\ \text{m/s}$$

290. 

$$\vec{v}_{RM} = \vec{v}_R - \vec{v}_M = -3\hat{j} - 3\hat{i}$$

$$\vec{v}_{RM} = \sqrt{3^2 + 3^2} = 4.2\ \text{m/s}$$

291. $\vec{V}_M = 5\hat{i}$

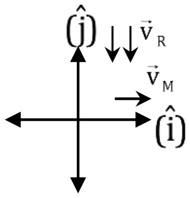
$$\vec{V}_R = -10\hat{j}$$

$$\vec{V}_{RM} = \vec{V}_R - \vec{V}_M = -10\hat{j} - 5\hat{i}$$

$$\left|\vec{V}_{RM}\right| = \sqrt{100 + 25} = 5\sqrt{5}\ \text{km/hr}$$

292. $\vec{v}_{RM} = \vec{v}_R - \vec{v}_M$

$$= -20\hat{j} - 30\hat{i}$$



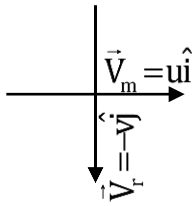
Angle with vertical

$$\tan\theta = \left(\frac{30}{20}\right)$$

$$\boxed{\theta = \tan^{-1}\left(\frac{3}{2}\right)}$$

293. If man has horizontal velocity equal to horizontal component of rain velocity, then relative velocity in horizontal direction becomes zero, so rain will appear vertically.

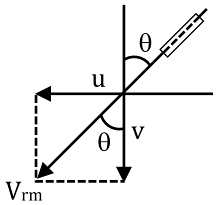
294. $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$



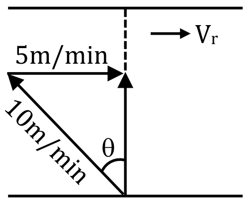
$$\vec{v}_{rm} = -v\hat{j} - u\hat{i}$$

$$\tan\theta = \frac{u}{v}$$

$$\theta = \tan^{-1}\left(\frac{u}{v}\right)$$



295. $v_{res} = \sqrt{v_P^2 + v_r^2} = \sqrt{164}\ \text{km/hr}$

296. 

$$\sin\theta = \frac{5}{10} = \frac{1}{2}$$

$\theta = 30°$

Angle with downstream = 90° + 30° = 120°

297. $t_{min} = \dfrac{D}{v} = \dfrac{4}{2} = 2\text{hr}$

298. For shortest time he has to cross the river with swimming perpendicular to flow i.e. due north.

299. $t=\dfrac{d}{\sqrt{v^2-v_R^2}}=\dfrac{4}{\sqrt{10^2-6^2}}=\dfrac{4}{8}=\dfrac{1}{2}\text{hr}$

300. $\sin\theta=\dfrac{v_R}{v}=\dfrac{1}{\sqrt{2}}\Rightarrow\theta=45°$

Angle with river flow = 90° + 45° = 135°

301. $u_{x_1}t+u_{x_2}t=x$

$\left[\sqrt{3}u\cos30°+u\cos60°\right]t=x$

$\left[\sqrt{3}u\times\dfrac{\sqrt{3}}{2}+\dfrac{u}{2}\right]t=x\Rightarrow t=\dfrac{x}{2u}$

Exercise-II (Previous Year Questions)

1. As Range = $\dfrac{u^2\sin2\theta}{g}$ so $g\propto u^2$

Therefore $g_{planet}=\left(\dfrac{3}{5}\right)^2(9.8\ m/s^2)=3.5\ m/s^2$

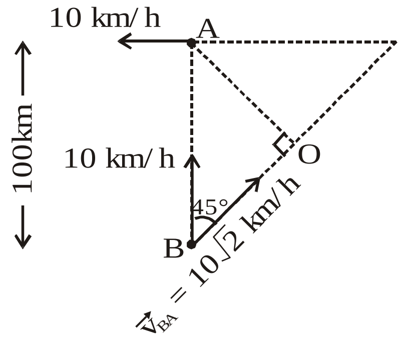
2. $\vec{v}_{avg}=\dfrac{\Delta\vec{r}}{\Delta t}=\dfrac{(13-2)\hat{i}+(14-3)\hat{j}}{5-0}=\dfrac{11}{5}(\hat{i}+\hat{j})$

3. $v=\beta x^{-2n}$ so $\dfrac{dv}{dx}=-2n\beta x^{-2n-1}$

Now $a=v\dfrac{dv}{dx}=(\beta x^{-2n})(-2n\beta x^{-2n-1})$

$\Rightarrow\ a=-2n\beta^2x^{-4n-1}$

4.



$|\vec{v}_{BA}|=\sqrt{10^2+10^2}=10\sqrt{2}\text{kmph}$

distance OB = 100 cos45° = $50\sqrt{2}$ km

Time taken to reach the shortest distance between A & B = $\dfrac{50\sqrt{2}}{|\vec{v}_{BA}|}=\dfrac{50\sqrt{2}}{10\sqrt{2}}$

t = 5 hrs.

5. For two particles to collide, the direction of the relative velocity of one with respect to other should be directed towards the relative position of the other particle

i.e. $\dfrac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|}\rightarrow$ direction of relative position of 1 w.r.t. 2.

& $\dfrac{\vec{v}_2-\vec{v}_1}{|\vec{v}_2-\vec{v}_1|}\rightarrow$ direction of velocity of 2 w.r.t. 1

so for collision of A & B $\dfrac{\vec{r}_1-\vec{r}_2}{|\vec{r}_1-\vec{r}_2|}=\dfrac{\vec{v}_2-\vec{v}_1}{|\vec{v}_2-\vec{v}_1|}$

6. $v=At+Bt^2\Rightarrow\dfrac{ds}{dt}=At+Bt^2$

$\int_0^s ds=\int_1^2(At+Bt^2)dt$

$s=\dfrac{A}{2}(2^2-1^2)+\dfrac{B}{3}(2^3-1^3)=\dfrac{3A}{2}+\dfrac{7B}{3}$

7. $x_P(t)=at+bt^2$ $\qquad x_Q(t)=ft-t^2$

$v_P=a+2bt$ $\qquad v_Q=f-2t$

as $v_P=v_Q$

$a+2bt=f-2t\Rightarrow\ t=\dfrac{f-a}{2(1+b)}$

8. $V_1\rightarrow$ velocity of Preeti

$V_2\rightarrow$ velocity of escalator

$\ell\rightarrow$ distance

$t=\dfrac{\ell}{V_1+V_2}=\dfrac{\ell}{\dfrac{\ell}{t_1}+\dfrac{\ell}{t_2}}=\dfrac{t_1t_2}{t_1+t_2}$

9. $v_x=5-4t,\ v_y=10$

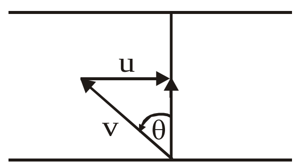
$a_x=-4,\ a_y=0$

$\vec{a}=a_x\hat{i}+a_y\hat{j}$

$\vec{a}=-4\hat{i}\ m/s^2$

10. v = 20 m/s

u = 10 m/s



$\sin\theta=\dfrac{u}{v}=\dfrac{10}{20}=\dfrac{1}{2}$

$\Rightarrow\theta$ = 30° west

11. $v^2 = u^2 - 2as$

$$\Rightarrow s = \frac{u^2}{2a} = \frac{u^2}{2g\sin\theta}$$

$$\frac{x_1}{x_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{\sin 30°}{\sin 60°} = \frac{1/2}{\sqrt{3}/2} \Rightarrow \frac{x_1}{x_2} = \frac{1}{\sqrt{3}}$$

12. $t = \sqrt{\frac{2h}{a_{rel}}}$

In both cases $a_{rel} = g - 0 = g$.

Hence $t_1 = t_2$

13. $t = \frac{d}{v_{rel}} = \frac{100}{50} = 2s$

distance travelled in downward direction

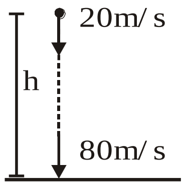
$$s_y = \frac{1}{2}gt^2 = \frac{1}{2} \times 10 \times 4 = 20$$

$\therefore$ Height = 200 – 20 = 180 m

14. Average velocity $= \frac{\text{Total distance}}{\text{Total time}} = \frac{x + x}{\frac{x}{v_1} + \frac{x}{v_2}}$

$$\Rightarrow v = \frac{2}{\frac{1}{v_1} + \frac{1}{v_2}} \Rightarrow \frac{2}{v} = \frac{1}{v_1} + \frac{1}{v_2}$$

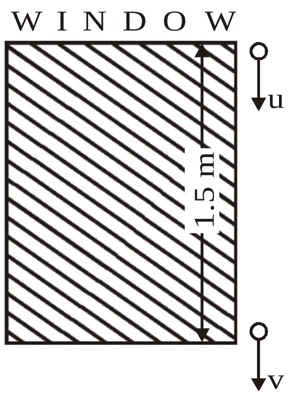
15.



$\because v^2 = u^2 + 2gh$

$\therefore 80^2 = 20^2 + 2 \times 10h \Rightarrow h = 300m$

16.



By using $s = ut + \frac{1}{2}at^2$

$$1.5 = u\,(0.1) + \frac{1}{2}(10)\,(0.1)^2$$

$\Rightarrow 15 = u + 0.5$

$\Rightarrow u = 14.5\ ms^{-1}$

17. S_n = Distance in n^{th} sec. i.e. t = n – 1 to t = n

S_{n+1} = Distance in $(n + 1)^{th}$ sec.

i.e. t = n to t = n + 1

So as we know

$S_n = \frac{a}{2}(2n - 1)$ a = acceleration

$$\frac{S_n}{S_{n+1}} = \frac{\frac{a}{2}(2n-1)}{\frac{a}{2}(2(n+1)-1)} = \frac{2n-1}{2n+1}$$

$$\frac{S_n}{S_{n+1}} = \frac{2n-1}{2n+1}$$

18. velocity of car at t = 4 sec is

v = u + at

v = 0 + 5 (4) = 20 m/s

At t = 6 sec

Acceleration due to gravity $\therefore$ a = g = 10 m/s²

v_x = 20 m/s (due to car)

v_y = u + a (t – 4)

= 0 + g (2) (downward)

= 20 m/s (downward)

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{20^2 + 20^2} = 20\sqrt{2}\ m/s$$

19. $T = \frac{2\pi R}{v} \Rightarrow v = \frac{2\pi R}{T}$ ... (1)

$$H_{max} = \frac{v^2\sin^2\theta}{2g} = \frac{2\pi^2R^2\sin^2\theta}{gT^2} = 4R$$

$$\sin\theta = \left(\frac{2gT^2}{\pi^2 R}\right)^{1/2}$$

$$\theta = \sin^{-1}\left[\frac{2gT^2}{\pi^2R}\right]^{1/2}$$

20. Velocity is slope of x–t graph

$$V = \frac{dx}{dt} = \tan\theta$$

$$\frac{V_1}{V_2} = \frac{\tan\theta_1}{\tan\theta_2} = \frac{\tan 30^o}{\tan 45^o} = \frac{1}{\sqrt{3}}$$

21. $S_{nth} = u + \frac{a}{2}(2n-1)$

$= 0 + \frac{a}{2}(2n-1)$

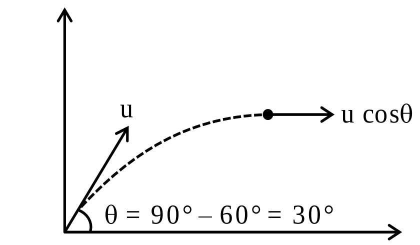
$S_{nth} \propto (2n-1)$

$\Rightarrow S_{1st}, S_{2nd}, S_{3rd}, S_{4th}$

$= [2(1) - 1] : [2(2) - 1] : [2(3) - 1] : [2(4) - 1]$

$= 1 : 3 : 5 : 7$

22. At highest point only horizontal component of velocity remains $\Rightarrow u_x = u\cos\theta$



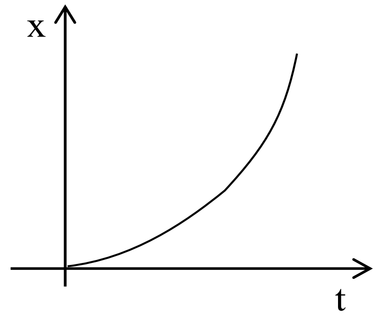
$u_x = u\cos\theta = 10\cos30°$

$= 5\sqrt{3}\,ms^{-1}$

23. $H = \frac{u^2\sin^2\theta}{2g} = \frac{(20)^2\sin^2 30°}{2(10)}$

$= 5m$

24.



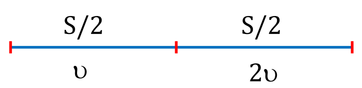
$x = \frac{1}{2}at^2$ (if coefficients of t^2 is positive ($a > 0$) upward opening parabola)

25. $H_{max} = \frac{u^2\sin^2\theta}{2g}$

$= \frac{(280)^2(\sin30°)^2}{2(9.8)}$

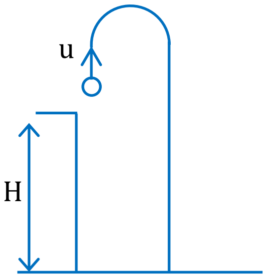
$= 1000$ m

26.



$V_{avg} = \frac{2\upsilon_1\upsilon_2}{\upsilon_1+\upsilon_2} = \frac{2(\upsilon)(2\upsilon)}{\upsilon+2\upsilon} = \frac{4\upsilon^2}{3\upsilon} = \frac{4\upsilon}{3}$

27. $S = ut + \frac{1}{2}at^2$



$-H = 4\times4 - \frac{1}{2}\times10\times4^2$

$-H = 16 - 80$

$-H = -64$

$H = 64$ m

28. By $v^2 = u^2 + 2as$

$\left(\frac{u}{3}\right)^2 = u^2 - 2ax$

$2ax = u^2 - \frac{u^2}{9}$

$\boxed{2ax = \frac{8u^2}{9}}$ ... (1)

Similarly from starting

$v^2 = u^2 + 2ax$

$0 = u^2 - 2ax_2$

$\boxed{2ax_2 = u^2}$ ... (2)

By (1) / (2)

$\frac{x}{x_2} = \frac{8}{9}$

$\frac{24}{x_2} = \frac{8}{9}$

$x_2 = 27$ cm

29. $\vec{V} = \frac{d\vec{r}}{dt} = 4\hat{i} + 4t\hat{j} + 0\hat{k}$

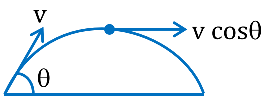
at t = 1sec

$\vec{V} = 4\hat{i} + 4(1)\hat{j}$

$|\vec{V}| = \sqrt{4^2 + 4^2} = 4\sqrt{2}$

$\tan\alpha = \frac{4}{4} = 1$

$\alpha = 45°$

30. 

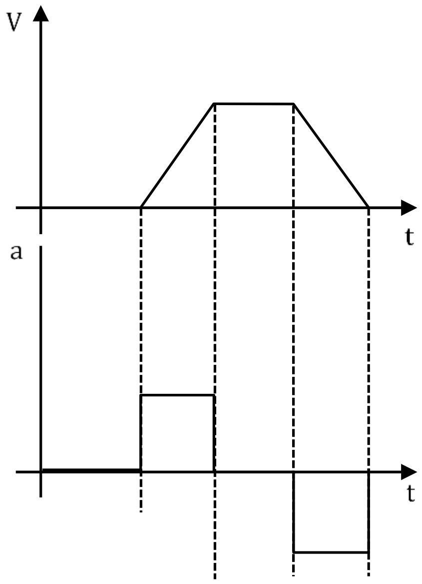
At the top most point of its trajectory particle will have only horizontal component of velocity

V at top = v cos θ

$= 20 \times \frac{1}{2}$

= 10 m/s

31.



32. $x = \alpha t^4 + \beta t^2 + \gamma t + \delta$

$V = \frac{dx}{dt} = 4\alpha t^3 + 2\beta t + \gamma$

$\Rightarrow$ at $t = 0,$ $v = \gamma$

$a = \frac{dv}{dt} = 12\alpha t^2 + 2\beta$

$\Rightarrow$ at $t = 0,$ $a = 2\beta$

$\therefore \frac{\text{Initial velocity}}{\text{Initial acceleration}} = \frac{\gamma}{2\beta}$

Exercise-III (Analytical Questions)

1. $\left(\vec{V}_i\right) = (u \cos \theta)\,\hat{i} + (u\sin\theta)\,\hat{j}$

$(\vec{V}_f) = (u\cos\theta)\hat{i} - (u\sin\theta)\hat{j}$

$(\vec{V}_{top}) = (u\cos\theta)\hat{i}$

$|\vec{V}_i|$ = Initial speed = u

$|\vec{V}_f|$ = final speed = u

$|\vec{V}_{top}|$ = speed at top = u cos θ (A)

(B) $\overrightarrow{\Delta V} = \vec{V}_f - \vec{V}_i \ \& \ \Delta|\vec{V}| = |\vec{V}_f| - |\vec{V}_i| = 0$

$|\overrightarrow{\Delta V}| = (2u\sin\theta)\hat{j}$

$|\overrightarrow{\Delta V}| = 2u\sin\theta$

(C) $|\vec{V}_i| - |\vec{V}_{top}| = u - u\cos\theta$

(D) $|\vec{V}_i - \vec{V}_{top}| = u\sin\theta$

2. $\vec{v}_i = 3\hat{i} + 4\hat{j} \quad \Rightarrow \quad |\vec{v}_i| = 5$

$\vec{v}_f = 3\hat{i} + 4\hat{j} \quad \Rightarrow \quad |\vec{v}_f| = 5$

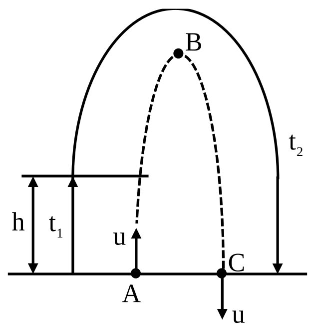
(A) $|\Delta\vec{v}| = |\vec{v}_f - \vec{v}_i| = |3\hat{i} + 4\hat{j} - 3\hat{i} + 4\hat{j}|$

$= |8\hat{j}| = 8$

(B) $\Delta|\vec{v}| = |\vec{v}_f| - |\vec{v}_i| = 5 - 5 = 0$

(C) $\vec{v}_f - \vec{v}_i = 8\hat{j}$

(D) $\vec{v}_f + \vec{v}_i = 6\hat{i}$

3. 

from the figure, it is clear that time of flight is $(t_1 + t_2)$

for maximum Ht :-

time taken from B to C is $\left(\frac{t_1 + t_2}{2}\right)$

Applying 2nd eqn of motion from B to C

$H_{max} = 0 + \frac{1}{2} g \left(\frac{t_1 + t_2}{2}\right)^2$

for (u) $\Rightarrow$ Speed at A = Speed at C

Apply 1st eqn of motion between B & C

$u = 0 + g\left(\frac{t_1 + t_2}{2}\right)$

for (h) $\Rightarrow h = ut_1 - \frac{1}{2}gt_1^2$

$$h = g\left(\frac{t_1+t_2}{2}\right)t_1 - \frac{1}{2}gt_1^2$$

$$\boxed{h = \frac{1}{2}gt_1t_2}$$

4. Two projectiles having same range may have different time of flight.

5. Projectile motion is an example of motion under constant acceleration.

9. When body moves with constant velocity then it moves with constant speed in same direction,

So |Inst. velocity| = |Avg. vel.|

= Inst. speed = Avg speed

10. Average speed for first 6 s

$$= \frac{\text{distance travelled in 6 s}}{6\text{ s}}$$

$$= \frac{\frac{1}{2}\times15\times4+\frac{1}{2}\times2\times10}{6} = \frac{20}{3}\ \text{ms}^{-1}$$

11. Average velocity for first 12 s

$$= \frac{\text{displacement in first 12 s}}{12\text{ s}}$$

$$= \frac{-\frac{1}{2}\times15\times4+\frac{1}{2}\times2\times10+2\times10+\left(\frac{20+10}{2}\right)(12-8)}{12}$$

$$= \frac{60}{12} = 5\ \text{ms}^{-1}$$

12. Average acceleration from t = 5 s to t = 15 s

$$= \frac{v(t=15)-v(t=5)}{15-5} = \frac{0-5}{10} = -\frac{1}{2}\text{ms}^{-2}$$