Vectors

Representation of Vectors - Symbolic and Geometrical

Physical Quantities

Those physical quantities which are used to define law of physics is known as physical quantities.

On the basis of directions, there are mainly two types of physical quantities.

(1) Scalar (2) Vector

(1) Scalar Quantities

A physical quantity which can be described completely by its magnitude only and does not require a direction is known as a scalar quantity.

It obeys the ordinary rules of algebra.

Ex : Distance, mass, time, speed, density, volume, temperature, electric current etc.

(2) Vector Quantities

A physical quantity which requires magnitude and a particular direction, when it is expressed.

(a) has a specified direction.

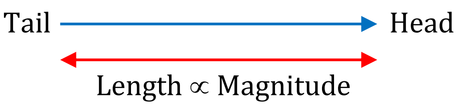
(b) obeys parallelogram law of vector addition, then only it is said to be a vector.

If any of the above conditions is not satisfied the physical quantity cannot be a vector.

If a physical quantity is a vector it has a direction, but the converse may or may not be true, i.e. if a physical quantity has a direction, it may or may not be a vector. e.g. time, pressure, Electric Current etc. have directions but are not vectors because they do not obey parallelogram law of vector addition. Example of vector quantity : Displacement, velocity, acceleration, force etc.

Representation of vector

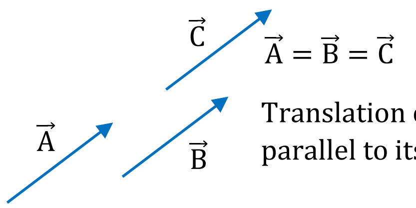
Geometrically, the vector is represented by a line with an arrow indicating the direction of vector as



Mathematically, vector is represented by $\vec{A}$

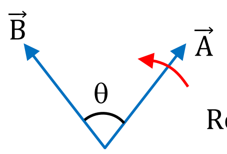
Important points

If a vector is displaced parallel to itself it does not change (see Figure)



Translation of a vector parallel to itself

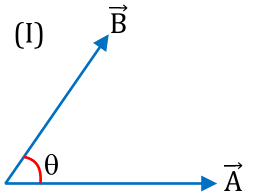
If a vector is rotated through an angle other than multiple of 2π (or 360°) it changes (see Figure)



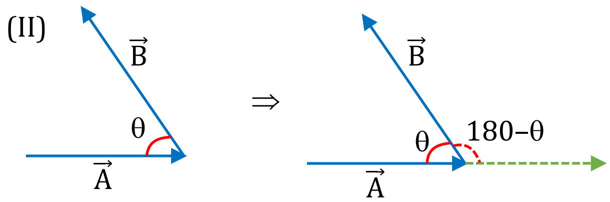
Rotation of a vector

Angle between the vectors

Angle between two vectors means smaller of the two angles between the vectors when they are placed tail to tail by displacing either of the vectors parallel to itself (i.e. $0 \le \theta \le \pi$).



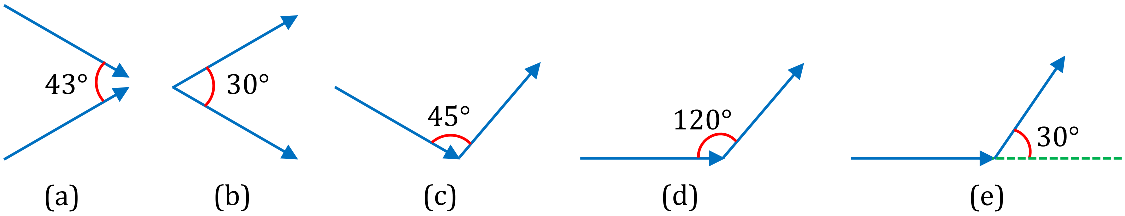
Angle between vectors is θ



Angle between vectors is 180-θ

Illustration 1:

Find angle between the vector.



Solution:

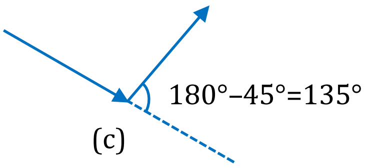
(a) $\theta = 43°$

Head to head are matching

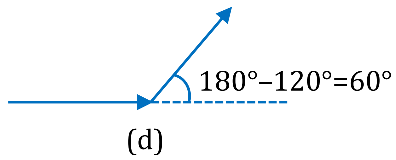
(b) $\theta = 30°$

tail to tail are matching

(c) $\theta = 135°$



(d) $\theta = 60°$

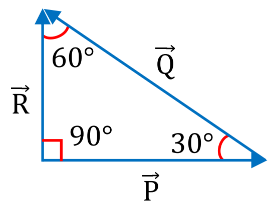


(e) $\theta = 30°$

tail to tail are matching

Illustration 2.

Find angle between all the vectors :



Solution:

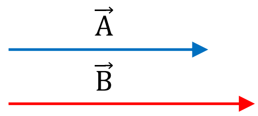
(a) angle between $\vec{P}$ and $\vec{Q} = 150°$

(b) angle between $\vec{Q}$ and $\vec{R} = 60°$

(c) angle between $\vec{P}$ and $\vec{R} = 90°$

Types of vector

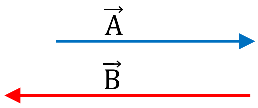
- **Parallel vectors :** Those vectors which have same direction are called parallel vectors. Angle between two parallel vectors is always 0°



- **Equal Vectors :** Vectors which have equal magnitude and same direction are called equal vectors.

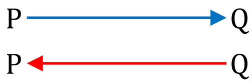
$$\vec{A} = \vec{B}$$

- **Anti–parallel Vectors :** Those vectors which have opposite direction are called anti–parallel vector.



Angle between two anti–parallel vectors is always 180°

Negative (or Opposite) Vectors : Vectors which have equal magnitude but opposite direction are called negative vectors of each other.

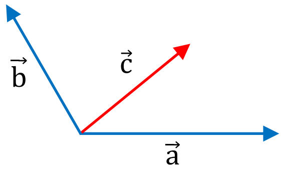


$\overrightarrow{PQ}$ and $\overrightarrow{QP}$ are negative vectors ; $\overrightarrow{PQ} = -\overrightarrow{QP}$

Co-initial vector :

Co-initial vectors are those vectors which have the same initial point.

In figure $\vec{a}, \vec{b}$ and $\vec{c}$ are co-initial vectors.



Collinear Vectors :

The vectors lying in the same line are known as collinear vectors.

Angle between collinear vectors is either 0° or 180°

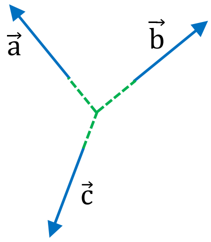
Example. (i) $\leftarrow \leftarrow$ (θ = 0°) (ii) $\rightarrow \rightarrow$ (θ = 0°) (iii) $\leftarrow \rightarrow$ (θ = 180°) (iv) $\rightarrow \leftarrow$ (θ = 180°)

Coplanar Vectors : Vectors located in the same plane are called coplanar vectors.

Note : Two vectors are always coplanar.

Concurrent vectors : Those vectors which pass through a common point are called concurrent vectors.

In figure $\vec{a}, \vec{b}$ and $\vec{c}$ are concurrent vectors



Null or Zero Vector : A vector having zero magnitude is called null vector.

Note : Sum of two vectors is always a vector so, $(\vec{A}) + (-\vec{A}) = \vec{0}$

$\vec{0}$ is a zero vector or null vector.

Unit Vector : A vector having unit magnitude is called unit vector. It is used to specify direction. A unit vector is represented by $\hat{A}$ (Read as A cap or A hat or A caret).

Unit vector in the direction of $\vec{A}$ is $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$ $\left(\text{unit vector} = \dfrac{\text{Vector}}{\text{Magnitude of the vector}}\right)$

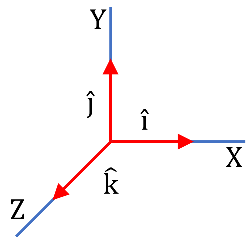
$\vec{A} = A\hat{A} = |\vec{A}|\hat{A}$

Note : A unit vector is used to specify the direction of a vector.

Base vectors

In an XYZ co-ordinate frame there are three unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$, these are used to indicate X, Y and Z direction respectively.

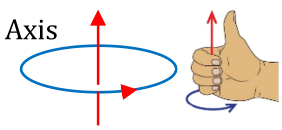
These three unit vectors are mutually perpendicular to each other.



Axial Vector

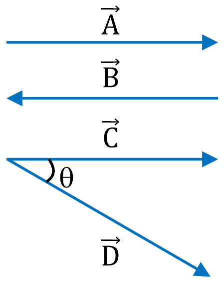
These vectors are used in rotational motion to define rotational effects.

Direction of these vectors is always along the axis of rotation in accordance with right hand screw rule or right hand thumb rule.



Ex. : Infinitesimal angular displacement $(\overrightarrow{d\theta})$, Angular velocity, $(\vec{\omega})$, Angular momentum $(\vec{J})$, Angular acceleration $(\vec{\alpha})$ and Torque $(\vec{\tau})$

Illustration 3:



(1) $\vec{A}$ & $\vec{C}$ are parallel

(2) $\vec{A}$ & $\vec{B}$ and $\vec{B}$ & $\vec{C}$ are antiparallel

(3) $\vec{C}$ & $\vec{D}$ are coinitial vectors

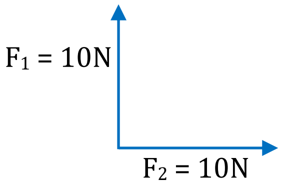
(4) $\vec{A}, \vec{B}, \vec{C}, \vec{D}$ are coplanar

Solution:

All are correct

Illustration 4.

Which one is true or false :



(a) $|\vec{F}_2| = |\vec{F}_1|$ (b) $\vec{F}_1 = \vec{F}_2$ (c) $\vec{F}_1 \perp \vec{F}_2$

(d) $\hat{i} = \hat{j}$ (e) $|\hat{i}| = |\hat{j}|$ (f) $\vec{F}_2 = \vec{F}_1$

(g) $|\vec{F}_1| = |\vec{F}_2|$

Solution:

Here $\vec{F}_1 = 10\hat{j}$ N and $\vec{F}_2 = 10\hat{i}$ N $\Rightarrow \left|\vec{F}_1\right| = \left|\vec{F}_2\right| = 10$ N $\Rightarrow \left|\hat{i}\right| = \left|\hat{j}\right| = 1$

(a) True (b) False (c) True (d) False (e) True (f) False (g) True

Addition of Vectors-Graphical Method

Addition of two vectors

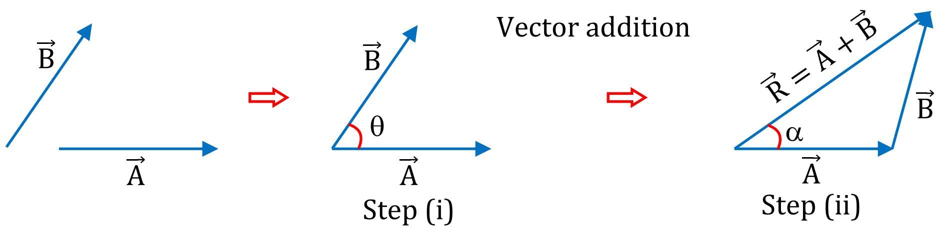
Vector addition can be performed by using following methods

(i) Graphical methods (ii) Analytical methods

Addition of two vectors is quite different from simple algebraic sum of two numbers.

Triangle law of addition of two vectors

If two vectors are represented by two sides of a triangle in same order then their sum or 'resultant vector' is given by the third side of the triangle taken in opposite order of the first two vectors.

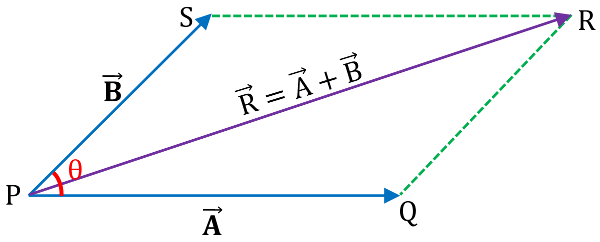


Shift one vector $\vec{B}$, without changing its direction, such that its tail coincide with head of the other vector $\vec{A}$. Now complete the triangle by drawing third side, directed from tail of $\vec{A}$ to head of $\vec{B}$ (it is in opposite order of $\vec{A}$ and $\vec{B}$ vectors).

Sum of two vectors is also called resultant vector of these two vectors. Resultant $\vec{R} = \vec{A} + \vec{B}$

Parallelogram Law

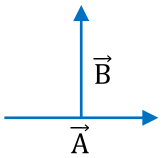
If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the parallelogram passing away through that common point.

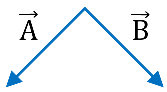


$\overrightarrow{PQ}+\overrightarrow{PS}=\overrightarrow{PR} \Rightarrow \vec{A}+\vec{B}=\vec{R}$

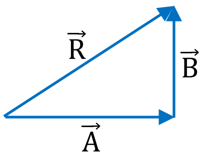
Illustration 5:

Add vectors by triangle law :

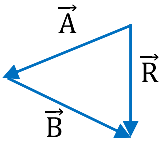
(i) 

(ii) 

Solution:

(i) 

By using Triangle law

(ii) 

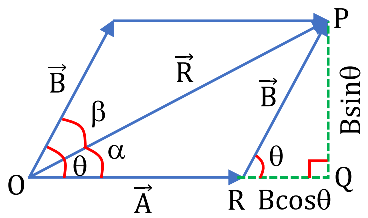
By using Triangle law

Analytical method :

In ΔPQR

$\sin\theta = \dfrac{PQ}{PR} \Longrightarrow PQ = B\sin\theta$

$\cos\theta = \dfrac{RQ}{PR} \Longrightarrow RQ = B\cos\theta$



In ΔOPQ

$OP^2 = OQ^2 + PQ^2$

$R^2 = (A + B\cos\theta)^2 + (B\sin\theta)^2$

$R^2 = A^2 + B^2\cos^2\theta + 2AB\cos\theta + B^2\sin^2\theta$

$R^2 = A^2 + 2AB\cos\theta + B^2(\cos^2\theta + \sin^2\theta)$

$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$|\vec{A}+\vec{B}| = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

In ΔOPQ : $\tan\alpha = \dfrac{QP}{OQ} = \dfrac{B\sin\theta}{A+B\cos\theta}$ Similarly, $\tan\beta = \dfrac{A\sin\theta}{B+A\cos\theta}$

Illustration 6:

If magnitude of resultant of 2N and 3N is 4N find angle between 2N and 3N:

Solution:

$4=\sqrt{2^2+3^2+2(2)(3)\cos\theta} \Rightarrow 16 = 4 + 9 + 12\cos\theta \Rightarrow 3 = 12\cos\theta \Rightarrow \theta=\cos^{-1}\left(\dfrac{1}{4}\right)$

Illustration 7:

Two vectors of magnitude 4N and 6N are acting at an angle 60° then find :

(i) Magnitude of their resultant vectors (ii) Angle between resultant vector and 4N

(iii) Angle between resultant vector and 6N

Solution:

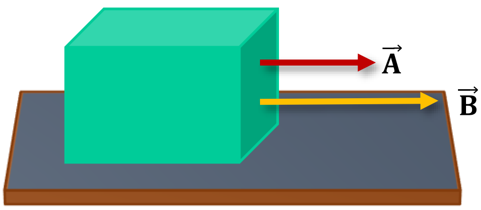
(i) $|\vec{R}| = \sqrt{4^2 + 6^2 + 2(4)(6)\cos 60^\circ} = \sqrt{16 + 36 + 48\left(\frac{1}{2}\right)} = \sqrt{76} = 2\sqrt{19}\text{N}$

(ii) $\tan\alpha = \frac{6\sin(60^\circ)}{4 + 6\cos(60^\circ)} = \frac{6\left(\frac{\sqrt{3}}{2}\right)}{4 + 6\left(\frac{1}{2}\right)} \Rightarrow \alpha = \tan^{-1}\left(\frac{3\sqrt{3}}{7}\right)$

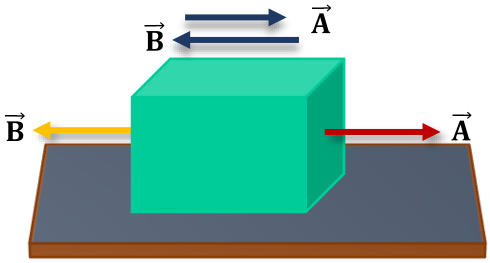
(iii) $\tan\beta = \frac{4\sin 60^\circ}{6 + 4\cos 60^\circ} = \frac{4\left(\frac{\sqrt{3}}{2}\right)}{6 + 4\left(\frac{1}{2}\right)} \Rightarrow \beta = \tan^{-1}\frac{\sqrt{3}}{4}$

Special cases

(1) If two vectors are parallel



(2) If two vectors are anti-parallel



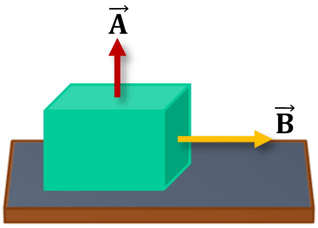
$\theta = 0^\circ, \theta = 180^\circ$

$|\vec{R}| = \sqrt{A^2 + B^2 + 2AB\cos\theta} \Rightarrow R = \sqrt{A^2 + B^2 + AB\cos 180^\circ}$

$|\vec{R}|_{max} = A + B \qquad |\vec{R}|_{min} = A - B$

Note : $\mathbf{A - B \le |\vec{A} + \vec{B}| \le A + B}$

(3) If two vectors are perpendicular



$\theta = 90^\circ \quad |\vec{R}| = \sqrt{A^2 + B^2}$

Illustration 8:

What can be resultant of two vectors $\vec{A}$ & $\vec{B}$ of magnitude 3 N and 5 N

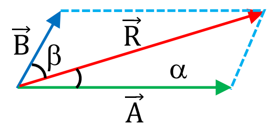
(1) 10 N (2) 1N (3) 12 N (4) 2 N (5) 6 N (6) 5 N (7) 0 N

Solution:

$|A - B| \le R \le |A + B|$ So, $2 \le R \le 8$

Hence, option (4) 2N (5) 6N (6) 5N are correct options

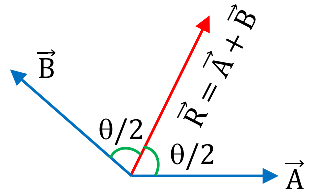
Properties of vector addition



Vector addition is commutative $\vec{A}+\vec{B}=\vec{B}+\vec{A}$

Vector addition is associative $\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$

Resultant of two vectors $\vec{A}$ and $\vec{B}$ situated in common plane.



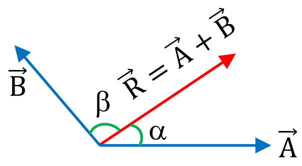
If $|\vec{A}|>|\vec{B}|$

then $\alpha < \beta$

$\vec{A},\vec{B}$ and $\vec{R}$ are co-planar

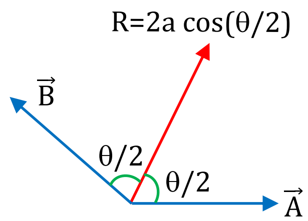
Resultant of two vector of equal magnitude will be at their bisector.

If $|\vec{A}|=|\vec{B}|$



But if $|\vec{A}|>|\vec{B}|$ then angle $\beta > \alpha$

If two vectors have equal magnitude i.e. $|\vec{A}|=|\vec{B}|$ = a and angle between them is θ then resultant will be along the bisector of $\vec{A}$ and $\vec{B}$ and its magnitude is equal to $2a\cos\left(\frac{\theta}{2}\right)$



$$|\vec{R}|=|\vec{A}+\vec{B}|=2a\cos\left(\frac{\theta}{2}\right)$$

Special Case : If = θ = 120° then R = $2a\cos\left(\frac{120°}{2}\right)=a$

i.e. If θ = 120° then $|R|=|\vec{A}+\vec{B}|=|\vec{A}|=|\vec{B}|=a$

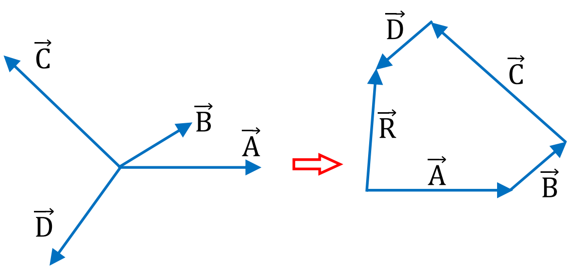
If resultant of two unit vectors is another unit vector then the angle between them (θ) = 120°.

OR

If the angle between two unit vectors (θ) = 120°, then their resultant is another unit vector.

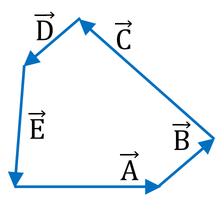
Addition of more than two vectors (law of polygon)

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order. $\vec{R}=\vec{A}+\vec{B}+\vec{C}+\vec{D}$



In a polygon if all the vectors taken in same order are such that the head of the last vector coincides with the tail of the first vector then their resultant is a null vector.

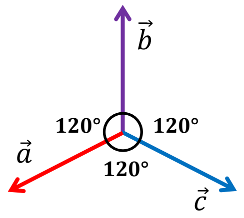
$$\vec{A}+\vec{B}+\vec{C}+\vec{D}+\vec{E}=\vec{0}$$



2. If n coplanar vectors of equal magnitude are at equal angular separation $\left(\frac{360°}{n}\right)$, then their resultant is always ZERO

If n = 3 & $|\vec{a}| = |\vec{b}| = |\vec{c}| = a$

If n=4 & $|\vec{a}| = |\vec{b}| = |\vec{c}| = |\vec{d}| = a$



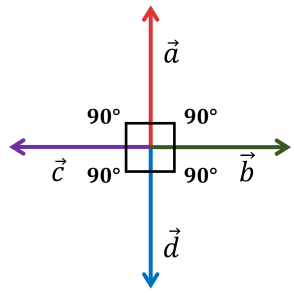


Illustration 9:

If 100 coplanar vectors each having magnitude 10 units are equally inclined with each other then find the magnitude of their resultant?

(1) 0 (2) 10 (3) 100 (4) 1000

Solution:

Because all the coplanar vectors of equal magnitude are at equal angular separation then their resultant is always ZERO

Note : The resultant of 3 vectors can be zero if they satisfy following conditions :

$$|A - B| \leq C \leq |A + B|$$

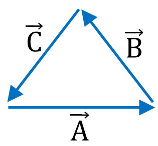


Illustration 10:

Which of the following groups can give zero resultant (Equilibrium)

Vectors	$\vec{P}$	$\vec{Q}$	$\vec{R}$
(1)	2N	4N	8N
(2)	12N	10N	23N
(3)	1N	2N	3N
(4)	8N	1N	2N

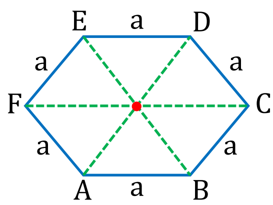
Solution:

$|P - Q| \leq R \leq |P + Q|$

(1) $2 \leq R \leq 6$ (Not Possible) (2) $2 \leq R \leq 22$ (Not Possible)

(3) $1 \leq R \leq 3$ (Possible) (4) $7 \leq R \leq 9$ (Not Possible)

Illustration 11:



It is regular hexagon and O is centre then find $\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}=?$

Solution:

$|\overrightarrow{OA}|=|\overrightarrow{OB}|=|\overrightarrow{OC}|=|\overrightarrow{OD}|=|\overrightarrow{OE}|=|\overrightarrow{OF}|$

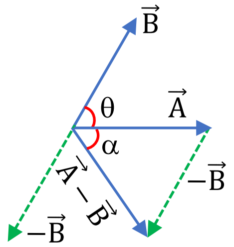
All the vectors are at equal angle so their resultant will be

$\overrightarrow{OA}+\overrightarrow{OB}+\overrightarrow{OC}+\overrightarrow{OD}+\overrightarrow{OE}+\overrightarrow{OF}=\vec{0}$

Vector Subtraction, Miscellaneous problems on Vector addition and Subtraction

Subtraction of two vectors

Let $\vec{A}$ and $\vec{B}$ are two vectors. Their difference i.e. $\vec{A}-\vec{B}$ can be treated as sum of the vector $\vec{A}$ and vector $(-\vec{B})$.



$$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$$

To subtract $\vec{B}$ from $\vec{A}$, reverse the direction of $\vec{B}$ and add to vector $\vec{A}$ according to law of triangle.

$$|\vec{A}-\vec{B}|=\sqrt{A^2+B^2+2AB\cos(\pi-\theta)}=\sqrt{A^2+B^2-2AB\cos\theta}\ \ \&\ \ \tan\alpha=\frac{B\sin\theta}{A-B\cos\theta}$$

Where θ is the angle between $\vec{A}$ and $\vec{B}$.

Properties of vector subtraction :

Vector subtraction dos not follow commutative law i.e. $\vec{A}-\vec{B}\neq\vec{B}-\vec{A}$

Vector subtraction does not follow associative law i.e. $(\vec{A}-\vec{B})-\vec{C}\neq\vec{A}-(\vec{B}-\vec{C})$

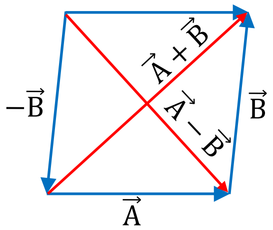
If two vectors have equal magnitude i.e. $|\vec{A}|=|\vec{B}|=a$ and θ is the angle between them, then

$$|\vec{A}-\vec{B}|=\sqrt{a^2+a^2-2a^2\cos\theta}=2a\sin\left(\frac{\theta}{2}\right)$$

Special case : If θ = 60° then 2a sin$\left(\frac{\theta}{2}\right)$=a i.e. $|\vec{A}-\vec{B}|=|\vec{A}|=|\vec{B}|=a$ at θ = 60°

If difference of two unit vectors is another unit vector then the angle between them is 60° or If two unit vectors are at angle of 60°, then their difference is also a unit vector.
In physics whenever we want to calculate change in a vector quantity, we have to use vector subtraction.
For example, change in velocity, $\Delta\vec{v}=\vec{v}_2-\vec{v}_1$

Note: In parallelogram one diagonal represent vector addition other diagonal represent vector subtraction.



Use of vector subtraction

Vector subtraction is used to find change in vector quantities.

Change in velocity $\Delta\vec{V}=\vec{V}_f-\vec{V}_i$

Change in momentum $\Delta\vec{P}=\vec{P}_f-\vec{P}_i$

Change in position vector $\Delta\vec{r}=\vec{r}_f-\vec{r}_i$

In θ is angle then the major diagonal will represent the vector sum and minor diagonal represent vector subtraction.

Illustration 12:

Angle between 2 vectors $\vec{A}$ and $\vec{B}$ having magnitude 5N and 10N is 37°, then find out $|\vec{A}-\vec{B}|$

Solution:

$$|\vec{A}-\vec{B}| = \sqrt{A^2+B^2-2AB\cos\theta} = \sqrt{(5)^2+(10)^2-(2)(5)(10)\cos37°} = \sqrt{25+100-100\times\frac{4}{5}} = 3\sqrt{5}$$

Illustration 13:

Find angle between $\vec{A}$ and $\vec{B}$ if $|\vec{A}-\vec{B}| = |\vec{A}+\vec{B}|$

Solution:

$|\vec{A}-\vec{B}| = |\vec{A}+\vec{B}| \Rightarrow \sqrt{a^2+b^2-2ab\cos\theta} = \sqrt{a^2+b^2+2ab\cos\theta}$

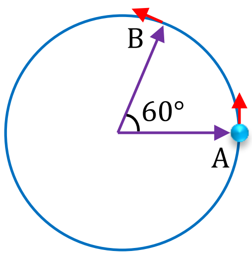
On squaring both sides

$a^2+b^2-2ab\cos\theta = a^2+b^2+2ab\cos\theta$

$4ab\cos\theta = 0 \Rightarrow \theta = \frac{\pi}{2}$

Illustration 14:

If a particle moves with constant speed v on circumference, find magnitude of change in velocity from A to B.



Solution:

$|\vec{V}_A - \vec{V}_B| = 2V\sin\left(\frac{\theta}{2}\right) = 2\times V\times \sin\left(\frac{60°}{2}\right) = V$

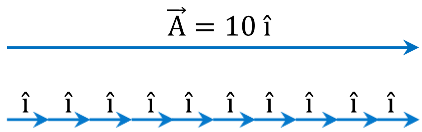
BEGINNER'S BOX-8

1.	If two forces act in opposite direction then their resultant is 10N and if they act mutually perpendicular then their resultant is 50N. Find the magnitudes of both the forces. **PBM168**
2.	If $\vec{A}=3\hat{i}+2\hat{j}$ and $\vec{B}=2\hat{i}+3\hat{j}-\hat{k}$, then find a unit vector along $(\vec{A}-\vec{B})$. **PBM169**
3.	If magnitude of sum of two unit vectors is $\sqrt{2}$ then find the magnitude of subtraction of these unit vectors. **PBM170**

Resolution of a Vector

To break or split a vector in components is called resolution of vector

Any vector can be split in two or more than two components.



Note : Maximum number of components of a vector can be infinite.

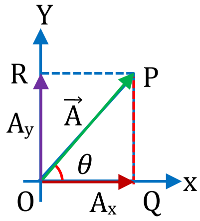
Vector Resolution in A Plane (2D)

If the angle between components is 90°, then they are known as perpendicular/orthogonal /rectangular components.

A_x : Component of $\vec{A}$ along x axis

A_y : Component of $\vec{A}$ along y axis

In ΔOQP,



$\cos\theta = \frac{OQ}{OP} = \frac{OQ}{A}$

$\Rightarrow OQ = A_x = A\cos\theta$

$\sin\theta = \frac{PQ}{OP} = \frac{PQ}{A}$

$\Rightarrow PQ = A_y = A\sin\theta$

According to law of vector addition,

$\vec{A} = \overrightarrow{OP} = \overrightarrow{OQ} + \overrightarrow{OR}$

$\vec{A} = A_x\hat{i} + A_y\hat{j} \Rightarrow \vec{A} = A\cos\theta\,\hat{i} + A\sin\theta\,\hat{j}$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2}$ and $\tan\theta = \frac{A_y}{A_x}$ and $\tan\phi = \frac{A_x}{A_y}$

Here θ is angle between $\vec{A}$ and its x component Here ϕ is angle between $\vec{A}$ and its y component

Note : If $\vec{A}$ vector makes an angle θ from a given direction, then

- its component along that direction will be $A\cos\theta$ and
- the remaining perpendicular component is $A\sin\theta$
- The component of a vector along its perpendicular direction is always **ZERO**

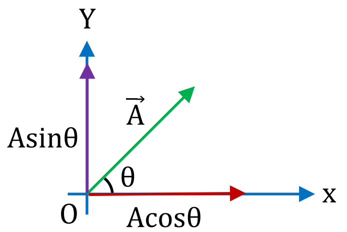


Illustration 15:

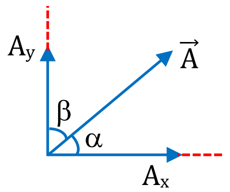
$\vec{A} = 3\hat{i} + 4\hat{j}$

Find (i) A_x (ii) A_y (iii) $|\vec{A}|$ (iv) Angle of $\vec{A}$ from x-axis

(v) Angle of $\vec{A}$ from y-axis (vi) Unit vector along $\vec{A}$

Solution:

(i) $A_x = 3$ (ii) $A_y = 4$ (iii) $|\vec{A}| = 5$



(iv) Angle of $\vec{A}$ from x-axis

$$\tan\alpha = \frac{A_y}{A_x} = \frac{4}{3} \Rightarrow \alpha = 53°$$

(v) Angle of $\vec{A}$ from y-axis

$$\tan\beta = \frac{A_x}{A_y} = \frac{3}{4} \Rightarrow \beta = 37°$$

(vi) Unit vector along $\vec{A}$

$$\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{3\hat{i} + 4\hat{j}}{5}$$

Illustration 16:

If $\vec{A} = 0.6\hat{i} + b\hat{j}$ is a unit vector, find value of b.

Solution:

$|\vec{A}| = 1 \Rightarrow \sqrt{(0.6)^2 + b^2} = 1 \Rightarrow b^2 = 1 - .36 \Rightarrow b^2 = 0.64 \Rightarrow b = 0.8$

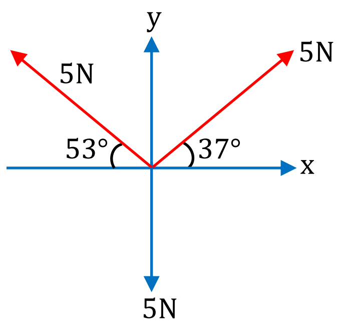
Illustration 17:

If Force F=10N is acting along positive x-axis. Find its y-component.

Solution:

The component of a vector along its perpendicular direction is always **ZERO**

Illustration 18:



Solution:

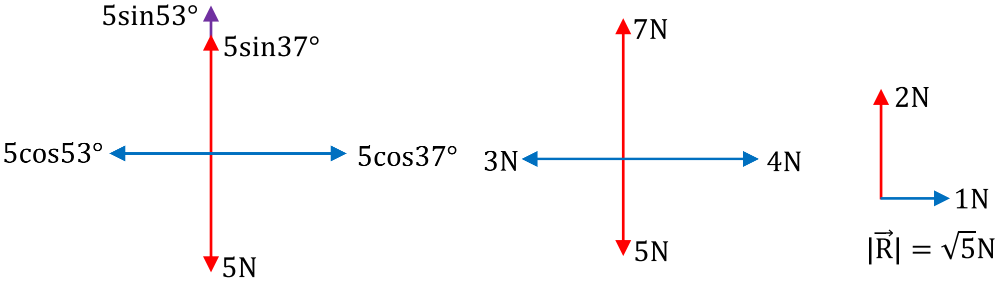
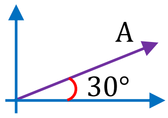


Illustration 19:

A vector makes an angle of 30° with the horizontal. If the horizontal component of the vector is 250 N, find the magnitude of vector and its vertical component.

Solution:

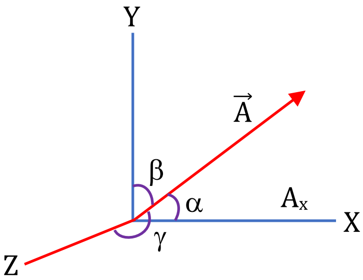


$A\cos 30° = 250$

$A = \dfrac{500}{\sqrt{3}}$

$A_y = A\sin 30° = \dfrac{250}{\sqrt{3}}$

Resolution of vectors in 3-D



If $\vec{A}$ makes

angle α with x axis

angle β with y axis

angle γ with z axis

and

Component of $\vec{A}$ along x axis is A_x

Component of $\vec{A}$ along y axis is A_y

Component of $\vec{A}$ along z axis is A_z

the

$\cos\alpha = \frac{A_x}{A} \Rightarrow A_x = A\cos\alpha$; $\cos\beta = \frac{A_y}{A} \Rightarrow A_y = A\cos\beta$; $\cos\gamma = \frac{A_z}{A} \Rightarrow A_z = A\cos\gamma$

Here, $\cos\alpha$, $\cos\beta$ and $\cos\gamma$ are known as direction cosines of $\vec{A}$

As, A_x, A_y and A_z are components of $\vec{A}$

so, $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \Rightarrow A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

Putting values of A_x, A_y and A_z

$A = \sqrt{A^2\cos^2\alpha + A^2\cos^2\beta + A^2\cos^2\gamma}$

$\Rightarrow A = A\sqrt{\cos^2\alpha + \cos^2\beta + \cos^2\gamma}$

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow (1 - \sin^2\alpha) + (1 - \sin^2\beta) + (1 - \sin^2\gamma) = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

Note : With help of angles made by a vector with x, y and z axes, we can write unit vector along the given vector.

As, A_x, A_y and A_z are components of $\vec{A}$

so, $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \Rightarrow \frac{\vec{A}}{A} = \frac{A_x}{A}\hat{i} + \frac{A_y}{A}\hat{j} + \frac{A_z}{A}\hat{k} \Rightarrow \hat{A} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$

(Unit vector in the direction of $\vec{A}$)

Points to remember

★ A vector can be resolved into infinite number of components.

For example $10\hat{i} = \hat{i} + \hat{i} + \hat{i}$ .... 10 times ; $= \frac{\hat{i}}{2} + \frac{\hat{i}}{2} + \frac{\hat{i}}{2}$ .... 20 times and so an.

★ Maximum number of rectangular components of a vector in a plane is two. But maximum number of rectangular components in space (3-dimensions) is three which are along X, Y and Z axes.

★ A vector is independent of the orientation of axes but the components of that vector depend upon the orientation of axes.

★ The component of a vector along its perpendicular direction is always zero.

Illustration 20:

$\vec{A} = 2\hat{i} + 3\hat{j} + 6\hat{k}$

(i) A_x (ii) A_y (iii) A_z (iv) $|\vec{A}|$ (v) Direction cosines of $\vec{A}$

(vi) Angle of $\vec{A}$ from x, y and z axis (vii) Unit vector along $\vec{A}$

Solution:

(i) $A_x = 2$ (ii) $A_y = 3$ (iii) $A_z = 6$ (iv) $|\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2 + (A_z)^2} = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{49} = 7$

(v) $\cos\alpha = \frac{2}{7}, \cos\beta = \frac{3}{7}, \cos\gamma = \frac{6}{7}$ (vi) $\alpha = \cos^{-1}\left(\frac{2}{7}\right), \beta = \cos^{-1}\left(\frac{3}{7}\right), \gamma = \cos^{-1}\left(\frac{6}{7}\right)$

(vii) $\hat{A} = \frac{\vec{A}}{|\vec{A}|} = \frac{2\hat{i} + 3\hat{j} + 6\hat{k}}{7}$

Illustration 21:

$\vec{A} = \hat{i} + 2\hat{j} + 2\hat{k}$; $\vec{B} = 3\hat{i} + \hat{j} + 10\hat{k}$ then find :

(i) $\vec{R}$ (ii) $|\vec{R}|$

(iii) Unit vector parallel to resultant vector : (iv) Direction cosine of $\vec{R}$

Solution:

(i) $\vec{R} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 12\hat{k}$ (ii) $|\vec{R}| = \sqrt{16 + 9 + 144} = \sqrt{169} = 13$

(iii) $\hat{R} = \frac{4\hat{i} + 3\hat{j} + 12\hat{k}}{13}$ (iv) $\cos\alpha = \frac{4}{13}$; $\cos\beta = \frac{3}{13}$; $\cos\gamma = \frac{12}{13}$

Illustration 22:

$\vec{A} = \hat{i} - 3\hat{j}$; $\vec{B} = 2\hat{j} - 3\hat{k}$ then find $|\vec{A} - 2\vec{B}|$

Solution:

$2\vec{B} = (4\hat{j} - 6\hat{k}) \Rightarrow |\vec{A} - 2\vec{B}| = \hat{i} - 3\hat{j} - 4\hat{j} + 6\hat{k} = \hat{i} - 7\hat{j} + 6\hat{k} = \sqrt{1 + 49 + 36} = \sqrt{86}$

Illustration 23:

Determine the vector which when added to the resultant of $\vec{P}$ and $\vec{Q}$, gives ZERO resultant (equilibrium).

$$\vec{P}=\hat{i}+2\hat{j}+\hat{k},\ \vec{Q}=2\hat{i}-\hat{j}+2\hat{k}$$

Solution:

Let's new vector = $\vec{x} \Rightarrow \vec{x}+(\vec{P}+\vec{Q})=0 \Rightarrow \vec{x}+(3\hat{i}+\hat{j}+3\hat{k})=0 \Rightarrow \vec{x}=-3\hat{i}-\hat{j}-3\hat{k}$

Illustration 24:

Determine the vector which when added to the resultant of $\vec{P}$ and $\vec{Q}$, gives unit vector along x-axis where $\vec{P}=\hat{i}+2\hat{j}+\hat{k},\ \vec{Q}=2\hat{i}-\hat{j}+2\hat{k}$.

Solution:

Let's new vector = $\vec{A} \Rightarrow \vec{A}+(\vec{P}+\vec{Q})=\hat{i}$

$\Rightarrow \vec{A}+(\hat{i}+2\hat{j}+\hat{k}+2\hat{i}-\hat{j}+2\hat{k})=\hat{i} \Rightarrow \vec{A}=\hat{i}-3\hat{i}-\hat{j}-3\hat{k} = -2\hat{i}-\hat{j}-3\hat{k}$

BEGINNER'S BOX-9

1. The x and y components of vector $\vec{A}$ are 4m and 6m respectively. The x and y components of vector $\vec{A}+\vec{B}$ are 10m and 9m respectively. For the vector $\vec{B}$ calculate the following -
(a) x and y components (b) length and (c) the angle it makes with x-axis **PBM171**
2. Find the directional cosines of vector $\left(5\hat{i}+2\hat{j}+6\hat{k}\right)$. Also write the value of sum of squares of directional cosines of this vector. **PBM172**
3. If $\vec{A}=6\hat{i}-6\hat{j}+5\hat{k}$ and $\vec{B}=\hat{i}+2\hat{j}-\hat{k}$, then find a unit vector parallel to the resultant of $\vec{A}$ & $\vec{B}$. **PBM173**

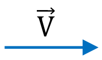
Multiplication of Vectors by Scalar and Dot product

Multiplication of a Vector by Scalar

To multiply a vector by a scalar, simply multiply the similar components, that is the vectors magnitude by the scalars magnitude. This will result in a new vector with the same direction but the product of two magnitudes.

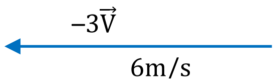
Note : Whenever a vector is multiplied by a positive number then the new vector has same direction but its magnitude changes depending on number.

Illustration 25:



If $\vec{V}=2\text{m/s}$ then $-3\vec{V}$ will be?

Solution:

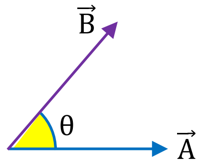


Note : whenever a vector is multiplied by a negative number then the new vector has opposite direction but its magnitude changes depending on number.

How to check parallel vectors ?

if $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ & $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k} \Rightarrow \frac{A_x}{B_x}=\frac{A_y}{B_y}=\frac{A_z}{B_z}$ = Positive constant

Scalar Product

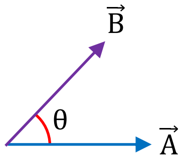
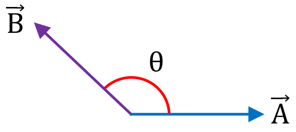


The scalar product of two vectors $\vec{A}$ and $\vec{B}$ equals to the product of their magnitudes and the cosine of the angle θ between them.

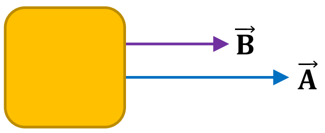
$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta$

Example : $W = \vec{F}\cdot\vec{S}$ Where $\vec{F}$ = force and $\vec{S}$ = displacement, Power (P) = $\vec{F}\cdot\vec{V}$ Where $\vec{F}$ = Force and $\vec{V}$ = velocity

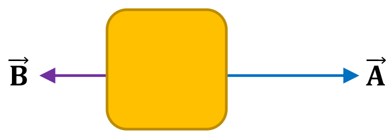
Angle dependence

	
$\theta < 90°$(Acute)	$\cos\theta \rightarrow$ Negative
$\cos\theta \rightarrow$ Positive	$\theta > 90°$(Obtuse)
$\vec{A}\cdot\vec{B} =$ positive	$\vec{A}\cdot\vec{B} =$ negative

Range of dot product



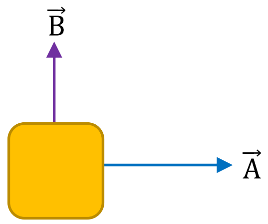
$\vec{A}||\vec{B}$



$\vec{A} \uparrow\downarrow \vec{B}$

$$-AB \le \vec{A}\cdot\vec{B} \le AB$$

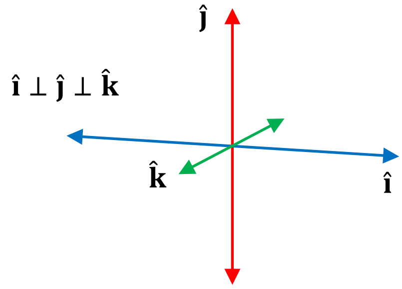
Key notes



When two vectors are perpendicular

Here, $\vec{A} \perp \vec{B}$

- Dot product of two perpendicular vectors is always zero
- If the dot product of two non-zero vectors is zero, then $\theta = 90°$



$$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$$

Self dot product

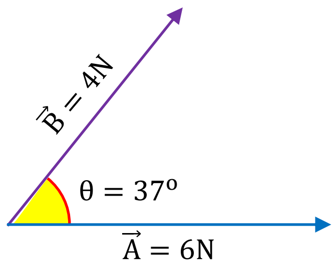
- $\vec{A}\cdot\vec{A} = A^2$
- $\hat{A}\cdot\hat{A} = 1$
- $\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = 1$

Dot product is commutative $\quad \vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$

Dot product is distributive $\quad \vec{C}\cdot(\vec{A}+\vec{B}) = \vec{C}\cdot\vec{A} + \vec{C}\cdot\vec{B}$

Illustration 26:

Find $\vec{A}\cdot\vec{B}$ & $\hat{A}\cdot\hat{B}$



Solution:

$$\vec{A}\cdot\vec{B} = |A|\,|B|\cos\theta = 4 \times 6 \times \cos 37° = 4 \times 6 \times \frac{4}{5} = \frac{96}{5}\,N$$

$$\hat{A}\cdot\hat{B} = |\hat{A}||\hat{B}|\cos\theta = 1 \times 1 \times \cos 37° = \frac{4}{5}\,N$$

Illustration 27:

Find angle between $\vec{A}$ & $\vec{B}$, if $\vec{A}$ (6N) and $\vec{B}$ (4N) and $\vec{A}\cdot\vec{B} = 12$

Solution:

$$\vec{A}\cdot\vec{B} = |\vec{A}||\vec{B}|\cos\theta \Rightarrow \cos\theta = \frac{\vec{A}\cdot\vec{B}}{|\vec{A}||\vec{B}|} = \frac{12}{6\times 4} = \frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\frac{1}{2}\right) \Rightarrow \theta = 60°$$

Dot product of vectors in component form

If two vectors are given as

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k} \;\&\; \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

Then, $\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$

Illustration 28:

Find Work done if Force $\vec{F} = 3\hat{i} + 2\hat{j} + \hat{k}$ & Displacement $\vec{s} = 2\hat{i} - 5\hat{j} + 3\hat{k}$.

Solution:

$$W = \vec{F}\cdot\vec{S} = (3\hat{i} + 2\hat{j} + \hat{k})\cdot(2\hat{i} - 5\hat{j} + 3\hat{k}) = 6 - 10 + 3 = -1J$$

Illustration 29:

Find Power if Force $\vec{F} = 2\hat{i} - 2\hat{j} + \hat{k}$ & velocity $\vec{v} = 3\hat{i} + 2\hat{j} - \hat{k}$.

Solution:

$$\vec{P} = \vec{F}\cdot\vec{V} = (2\hat{i} - 2\hat{j} + \hat{k})\cdot(3\hat{i} + 2\hat{j} - \hat{k}) = 6 - 4 - 1 = 1W$$

Illustration 30:

If angle between $\vec{a}$ and $\vec{b}$ is 60°, then find value of $(\hat{a} - 2\hat{b})\cdot(2\hat{a} + 4\hat{b})$.

Solution:

$$(\hat{a}\cdot 2\hat{a} + \hat{a}\cdot 4\hat{b} - 2\hat{b}\cdot 2\hat{a} - 8\hat{b}\cdot\hat{b}) = \left[(2 + 4(\hat{a}\cdot\hat{b}) - 4(\hat{b}\cdot\hat{a}) - 8)\right] = [-6 + 0] = -6$$

Illustration 31:

If $\vec{A} + \vec{B} + \vec{C} = \vec{0}$ and $|\vec{A}| = |\vec{B}| = |\vec{C}| = 1$, find $\vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{C} + \vec{C}\cdot\vec{A}$

Solution:

$$\vec{A} + \vec{B} + \vec{C} = 0 \Rightarrow \vec{A} + \vec{B} = -\vec{C} \Rightarrow |\vec{A} + \vec{B}| = |-\vec{C}|$$

$$\Rightarrow \sqrt{A^2 + B^2 + 2\vec{A}\cdot\vec{B}} = |-\vec{C}| \Rightarrow A^2 + B^2 + 2\vec{A}\cdot\vec{B} = |-\vec{C}|^2$$

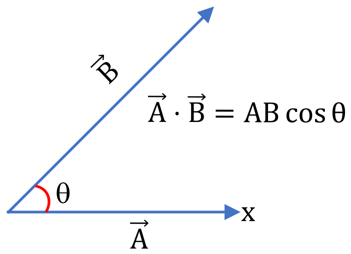
$$\because \quad (|A| = |B| = |C| = 1) \Rightarrow \vec{A}\cdot\vec{B} = \frac{1-1-1}{2} = -\frac{1}{2}$$

By same logic $\vec{B}\cdot\vec{C} = \frac{-1}{2}$ & $\vec{C}\cdot\vec{A} = \frac{-1}{2}$

So, $\vec{A}\cdot\vec{B} + \vec{B}\cdot\vec{C} + \vec{C}\cdot\vec{A} = \left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right) + \left(\frac{-1}{2}\right) = \frac{-3}{2}$

Application of DOT product

1. According to definition $\vec{A}\cdot\vec{B}=AB\cos\theta$ the angle between the vectors $\theta = \cos^{-1}\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)$



Note: To Check Orthogonal Vectors

for $\theta = 90°$

$\vec{A}\cdot\vec{B} = AB\cos\theta = 0$

If scalar product of two nonzero vectors is zero, then vectors are orthogonal or perpendicular

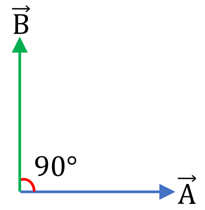


Illustration 32:

If the Vectors $\vec{P}=a\hat{i}+a\hat{j}+3\hat{k}$ and $\vec{Q}=a\hat{i}-2\hat{j}-\hat{k}$ are perpendicular to each other. Find the value of a?

Solution:

If vectors $\vec{P}$ and $\vec{Q}$ are perpendicular

$\Rightarrow \quad \vec{P}.\vec{Q}=0 \quad \Rightarrow \quad \left(a\hat{i}+a\hat{j}+3\hat{k}\right)\cdot\left(a\hat{i}-2\hat{j}-\hat{k}\right) = 0$

$\Rightarrow \quad a^2 - 2a - 3 = 0 \quad \Rightarrow \quad a^2 - 3a + a - 3 = 0$

$\Rightarrow \quad a(a-3) + 1(a-3) = 0 \quad \Rightarrow \quad a = -1, 3$

Illustration 33:

If $(\hat{a}+\hat{b})$ and $(2\hat{a}-3\hat{b})$ are perpendicular, then find angle between $\hat{a}$ and $\hat{b}$.

Solution:

$(\hat{a}+\hat{b})\cdot(2\hat{a}-3\hat{b})=0 \quad \Rightarrow \quad 2\hat{a}\cdot\hat{a}-3\hat{a}\cdot\hat{b}+2\hat{b}\cdot\hat{a}-3\hat{b}\cdot\hat{b}=0$

$\hat{a}\cdot\hat{a}=|\hat{a}|^2=1\ (\because|\hat{a}|=1) \quad \Rightarrow \quad \hat{b}\cdot\hat{b}=|\hat{b}|^2=1\ (\because|\hat{b}|=1)$

$(\hat{a}\cdot\hat{b}=\hat{b}\cdot\hat{a}) \quad \Rightarrow \quad 2(1) - 3\hat{a}\cdot\hat{b}+2\hat{a}\cdot\hat{b}-3(1)=0$

$\hat{a}\cdot\hat{b}=-1 \quad \Rightarrow \quad \cos\theta \times a \times b = -1$

$\cos\theta = -1 \quad \Rightarrow \quad \boxed{\theta=180°}$

3. Projection of $\vec{A}$ on $\vec{B}$

(i) In scalar form : Projection of $\vec{A}$ on $\vec{B}=A\cos\theta=A\left(\frac{\vec{A}\cdot\vec{B}}{AB}\right)=\frac{\vec{A}\cdot\vec{B}}{B}=\vec{A}\cdot\hat{B}$

(ii) In vector form : Projection of $\vec{A}$ on $\vec{B}$ = $(A\cos\theta)\hat{B}=\left(\frac{\vec{A}\cdot\vec{B}}{B}\right)\hat{B}=(\vec{A}\cdot\hat{B})\hat{B}$

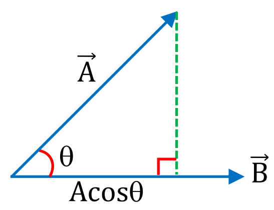


Illustration 34:

Find the component of $\vec{A}=3\hat{i}+4\hat{j}$ along $\vec{B}=\hat{i}+\hat{j}$?

1. Projection of $\vec{A}$ along $\vec{B}$ in scalar form
2. Projection of $\vec{A}$ along $\vec{B}$ in vector form
3. Projection of $(\vec{A}+\vec{B})$ along $\vec{A}$ in scalar form

Solution:

1. Component of $\vec{A}$ along $\vec{B}$ is given by $\frac{\vec{A}\cdot\vec{B}}{B}$ hence required component $=\frac{(3\hat{i}+4\hat{j}).(\hat{i}+\hat{j})}{\sqrt{2}}=\frac{7}{\sqrt{2}}$

2. $\frac{\vec{A}\cdot\vec{B}}{B}\hat{B}=\frac{\vec{A}\cdot\vec{B}}{B}\frac{\vec{B}}{(B)}=\frac{7}{\sqrt{2}}\frac{(\hat{i}+\hat{j})}{(\sqrt{2})}=\frac{7}{2}(\hat{i}+\hat{j})$

3. Let $\vec{A}+\vec{B}=\vec{R}=3\hat{i}+4\hat{j}+\hat{i}+\hat{j}=4\hat{i}+5\hat{j}$ So $\frac{\vec{R}\cdot\vec{A}}{A}=\frac{12+20}{5}=\frac{32}{5}$

BEGINNER'S BOX-10

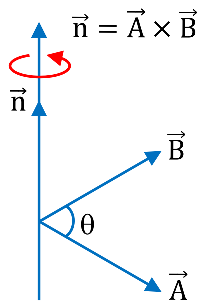
1. If $\vec{a}$ and $\vec{b}$ are two non collinear unit vectors and $|\vec{a}+\vec{b}|=\sqrt{3}$, then find the value of $(\vec{a}-\vec{b}).(2\vec{a}+\vec{b})$ **PBM174**
2. If $\vec{A}=4\hat{i}-2\hat{j}+4\hat{k}$ and $\vec{B}=-4\hat{i}+2\hat{j}+\alpha\hat{k}$ are perpendicular to each other then find value of α ? **PBM175**
3. If vector $(\hat{a}+2\hat{b})$ is perpendicular to vector $(5\hat{a}-4\hat{b})$, then find the angle between $\hat{a}$ and $\hat{b}$. **PBM176**
4. If $\vec{A}=2\hat{i}-2\hat{j}-\hat{k}$ and $\vec{B}=\hat{i}+\hat{j}$, then :
   (a) Find angle between $\vec{A}$ and $\vec{B}$.
   (b) Find the projection of resultant vector of $\vec{A}$ and $\vec{B}$ on x-axis. **PBM177**
5. Find the vector components of $\vec{a}=2\hat{i}+3\hat{j}$ along the directions of $\hat{i}+\hat{j}$. **PBM178**

Vector product

The vector product or cross product of any two vectors $\vec{A}$ and $\vec{B}$, denoted as $\vec{A}\times\vec{B}$ (read $\vec{A}$ cross $\vec{B}$) is defined as : $\vec{A}\times\vec{B}=AB\sin\theta\,\hat{n}$

Here θ is the angle between the vectors and the direction of $\hat{n}$ is given by the right-hand-thumb rule.

Right-Hand-Thumb Rule



To find the direction of $\hat{n}$, draw the two vectors $\vec{A}$ and $\vec{B}$ with both the tails coinciding. Now place your stretched right palm perpendicular to the plane of $\vec{A}$ and $\vec{B}$ in such a way that the fingers are along the vector $\vec{A}$ and when the fingers are closed they go towards $\vec{B}$. The direction of the thumb gives the direction of $\hat{n}$.

Examples of vector product

(i) Torque : $\vec{\tau}=\vec{r}\times\vec{F}$

(ii) Angular momentum : $\vec{J}=\vec{r}\times\vec{p}$

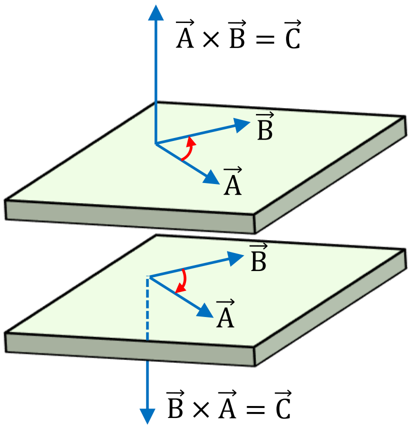
(iii) Velocity : $\vec{v}=\vec{\omega}\times\vec{r}$

(iv) Acceleration : $\vec{a}=\vec{\alpha}\times\vec{r}$

Here $\vec{r}$ is position vector and $\vec{F},\vec{p},\vec{\omega}$ and $\vec{\alpha}$ are force, linear momentum, angular velocity and angular acceleration respectively.

Properties :

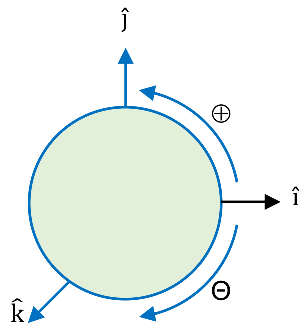
- Vector product of two vectors is always a vector perpendicular to the plane containing the two vectors, i.e. orthogonal (perpendicular) to both the vectors $\vec{A}$ and $\vec{B}$



- Vector product of two vectors is not commutative i.e.
  $\vec{A}\times\vec{B} \neq \vec{B}\times\vec{A}$ But $|\vec{A}\times\vec{B}|=|\vec{B}\times\vec{A}|=AB\sin\theta$

Note : $\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$ i.e., in case of vectors $\vec{A}\times\vec{B}$ and $\vec{B}\times\vec{A}$ magnitudes are equal but directions are opposite [See the figure]

- The vector product is distributive when the order of the vectors is strictly maintained, i.e. $\vec{A}\times(\vec{B}+\vec{C})=\vec{A}\times\vec{B}+\vec{A}\times\vec{C}$
- According to definition of vector product of two vectors $\vec{A}\times\vec{B}$ = AB $\sin\theta\,\hat{n} \Rightarrow \theta = \sin^{-1}\dfrac{|\vec{A}\times\vec{B}|}{AB}$
- The vector product of two vectors will be maximum when $\sin\theta$ = max. = 1, i.e., $\theta = 90°$
  $$|\vec{A}\times\vec{B}|_{max}=AB\sin 90°=AB$$
  i.e. vector product is maximum if the vectors are orthogonal (perpendicular)
- The vector product of two non-zero vectors will zero when $|\sin\theta| = 0$,
  i.e. when $\theta = 0°$ or $180°$, $|\vec{A}\times\vec{A}| = 0$ or $|\vec{A}\times(-\vec{A})|=0$
  Therefore if the vector product of two non-zero vectors is zero, then the vectors are collinear.
- The self cross product, i.e., product of a vector by itself is a zero vector or a null vector.
  i.e. $\vec{A}\times\vec{A}=(AA\sin 0°)\hat{n}=\vec{0}$
- In case of unit vector $\hat{n} : \hat{a}\times\hat{a} = 1 \times 1 \times \sin 0°\ \hat{n}=\vec{0}$ so that $\hat{i}\times\hat{i}=\vec{0}$, $\hat{j}\times\hat{j}=\vec{0}$, $\hat{k}\times\hat{k}=\vec{0}$
- In case of orthogonal unit vectors $\hat{i},\hat{j}$ and $\hat{k}$; according to right hand thumb rule
  $\hat{i}\times\hat{j}=\hat{k}$, $\hat{j}\times\hat{k}=\hat{i}$, $\hat{k}\times\hat{i}=\hat{j}$ and $\hat{j}\times\hat{i}=-\hat{k}$, $\hat{k}\times\hat{j}=-\hat{i}$, $\hat{i}\times\hat{k}=-\hat{j}$



- In terms of components, $\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}=\hat{i}(A_yB_z-A_zB_y)-\hat{j}(A_xB_z-A_zB_x)+\hat{k}(A_xB_y-A_yB_x)$,

  where $\vec{A}=A_x\hat{i}+A_y\hat{j}+A_z\hat{k}$ and $\vec{B}=B_x\hat{i}+B_y\hat{j}+B_z\hat{k}$
- If $\vec{A},\vec{B}$ and $\vec{C}$ are coplanar, then $\vec{A}\cdot(\vec{B}\times\vec{C})=0$. [$\because(\vec{B}\times\vec{C})$ is perpendicular to $\vec{A}$]
- Angle between $(\vec{A}+\vec{B})$ and $(\vec{A}\times\vec{B})$ is 90° as $\vec{A}\times\vec{B}$ is perpendicular to plane containing $\vec{A}\,\&\,\vec{B}$.
- A scalar or a vector, cannot be divided by a vector.
- Vectors of different types can be multiplied to generate new physical quantities which may be a scalar or a vector. If, in multiplication of two vectors, the generated physical quantity is a scalar, then their product is called scalar or dot product and if it is a vector, then their product is called vector or cross product.

Illustration 35:

$\vec{A}$ is East wards and $\vec{B}$ downwards. Find the direction of $\vec{A}\times\vec{B}$?

Solution:

Applying right hand thumb rule we find that $\vec{A}\times\vec{B}$ is along North.

Illustration 36:

If $\vec{A}\cdot\vec{B}=|\vec{A}\times\vec{B}|$, find angle between $\vec{A}$ and $\vec{B}$

Solution:

AB cosθ = AB sinθ

tan θ = 1 ⇒ θ = 45°

Illustration 37:

Find $\vec{A}\times\vec{B}$ if $\vec{A}=\hat{i}-2\hat{j}+4\hat{k}$ and $\vec{B}=2\hat{i}-\hat{j}+2\hat{k}$.

Solution:

$$\vec{A}\times\vec{B}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -2 & 4\\ 3 & -1 & 2\end{vmatrix}=i(-4-(-4))-\hat{j}(2-12)+\hat{k}(-1-(-6))=10\hat{j}+5\hat{k}$$

Illustration 38:

If $|\vec{A}|=2$; $|\vec{B}|=4$ and $|\vec{A}\times\vec{B}|=4$. Then find $\vec{A}\cdot\vec{B}$

Solution:

$$|\vec{A}\times\vec{B}|=AB\sin\theta\Rightarrow\sin\theta=\frac{4}{8}=\frac{1}{2}\Rightarrow\theta=30°$$

$$\vec{A}\cdot\vec{B}=AB\cos30°=2\times4\times\frac{\sqrt{3}}{2}=4\sqrt{3}$$

BEGINNER'S BOX-11

1.	If $\sqrt{3}\left|\vec{A}\times\vec{B}\right|=\vec{A}\cdot\vec{B}$, then find the angle between $\vec{A}$ and $\vec{B}$.	**PBM179**
2.	Find $\vec{A}\cdot\vec{B}$. if $\left|\vec{A}\right|=2$, $\left|\vec{B}\right|=5$, and $\left|\vec{A}\times\vec{B}\right|=8$	**PBM180**

BEGINNER'S BOX — ANSWER KEY

BEGINNER'S BOX-1

1. (i) $-\frac{1}{\sqrt{3}}$ (ii) $\frac{\sqrt{3}}{2}$ (iii) $\frac{1}{\sqrt{2}}$ (iv) $-\frac{\sqrt{3}}{2}$ (v) -1 (vi) 0

2. $\sin\theta = \frac{4}{5}$, $\cos\theta = \frac{3}{5}$, $\tan\theta = \frac{4}{3}$, $\cot\theta = \frac{3}{4}$, $\mathrm{cosec}\,\theta = \frac{5}{4}$

BEGINNER'S BOX-2

1. 2 **2.** 13 **3.** $-2/3$

BEGINNER'S BOX-3

1. (i) $\frac{7}{2}x^{5/2}$ (ii) $-3x^{-4}$ (iii) 1 (iv) $5x^4+3x^2+2x^{-1/2}$

(v) $20x^3 + 9x^{1/2} + 9$ (vi) $2ax+b$ (vii) $15x^4 - 3 + \frac{1}{x^2}$

2. $2t + 5$ **3.** $u + at$ **4.** $30\ cm^2\ s^{-1}$ **5.** $2\ \pi r$

6. (i) $4x + 3$ (ii) $-\frac{2}{(2x+1)^2}$ (iii) $-\frac{1}{(4x+5)^2}$ (iv) $\frac{2x-x^4}{(x^3+1)^2}$

BEGINNER'S BOX-4

1. (i) $\frac{x^{16}}{16} + c$ (ii) $-2x^{-1/2} + c$ (iii) $-\frac{x^{-6}}{2} + \log_e x + c$

(iv) $\frac{x^2}{2} + 2x + \log_e x + c$ (v) $\frac{x^2}{2} + \log_e x + c$ (vi) $-\frac{a}{x} + b\log_e x + c$

BEGINNER'S BOX-5

1. (i) $\frac{GMm}{R}$ (ii) $kq_1q_2\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$ (iii) $\frac{1}{2}M(v^2 - u^2)$

(iv) ∞ (v) 1 (vi) 1 (vii) 2

BEGINNER'S BOX-6

1. (i) $\frac{5}{2}, \frac{1}{5}$ (ii) $\frac{p}{q}, \frac{q}{p}$ **2.** (4)

BEGINNER'S BOX-7

1. 1275 **2.** 385

3. $\frac{2}{3}$ **4.** $F_{net} = \frac{2GMm}{r^2}$

BEGINNER'S BOX-8

1.	40N & 30N	**2.**	$\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$	**3.**	$\sqrt{2}$

BEGINNER'S BOX-9

1.	(a) 6m and 3m		(b) $\sqrt{45}$m		(c) $\tan^{-1}\left(\frac{1}{2}\right)$
2.	$\frac{5}{\sqrt{65}},\frac{2}{\sqrt{65}},\frac{6}{\sqrt{65}};1$			**3.**	$\frac{7\hat{i}-4\hat{j}+4\hat{k}}{9}$

BEGINNER'S BOX-10

1.	$\frac{1}{2}$	**2.**	5	**3.**	60°
4.	(a) 90° (b) 3	**5.**	$\frac{5}{2}(\hat{i}+\hat{j})$		

BEGINNER'S BOX-11

1.	30°	**2.**	$\vec{A}.\vec{B} = \pm 6$