05 Properties of Matter and Fluid Mechanics

Mechanical Properties of Solids

Introduction to types of forces and types of bodies

Types of forces

Deforming Forces

External force which tends to bring about a change in the length, volume or shape of a body is called deforming force.

Restoring Forces

When an external force acts at any object then an internal resistance produced in the material due to the intermolecular forces which is called restoring force.

Elasticity

Elasticity is that property of a material of a body by virtue of which it opposes any change in its shape or size when deforming forces are applied on it, and recover its original state as soon as the deforming force are removed

Types of Bodies

Rigid Body

A body is said to be rigid if the relative positions of its constituent particles remain unchanged when external deforming forces are applied to it. The nearest approach to a rigid body is diamond or carborundum.

Perfectly Elastic Body

A body which perfectly regains its original form on removing the external deforming force, is defined as a perfectly elastic body. Example : Quartz – It is quite close to a perfect elastic body.

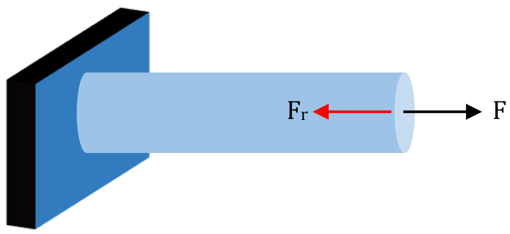
Plastic Body

A body which does not have the property of opposing the deforming forces, is known as a plastic body. All bodies which remain in the deformed state even after the removal of the deforming forces are known as plastic bodies. Example : clay, wax, putty.

Stress and its types, breaking stress

Stress

The restoring force acting per unit area of the deforming body is called **stress**.



$$\text{stress} = \frac{\text{internal restoring forces}}{\text{area of cross section}} \ (F_r)$$

$$\text{stress} = \frac{F_{\text{external}} \text{(at equilibrium)}}{A}$$

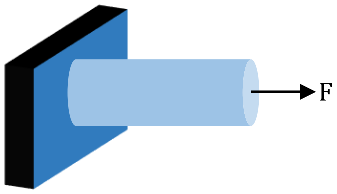
SI Unit: N/m^2 Dimensions: $[M^1L^{-1}T^{-2}]$

Types of Stress

- Longitudinal Stress
  - Tensile Stress
  - Compressive Stress
- Volumetric Stress
- Shear Stress

Types of stress

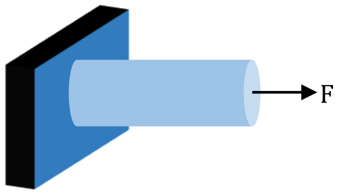
(i) Longitudinal Stress



When the stress is normal to the cross sectional area, then it is known as **longitudinal stress.**

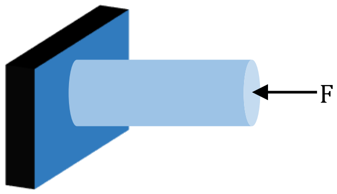
Longitudinal Stress = $\frac{F_{\perp}}{A}$

There are two types of longitudinal stress:



(a) Tensile Stress

The longitudinal stress, produced due to increase in the length of a body, is defined as tensile stress.



(b) Compressive Stress

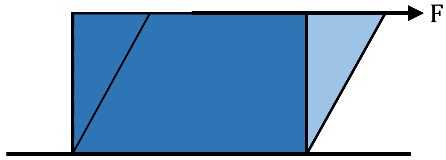
The longitudinal stress, produced due to the decrease in the length of a body, is defined as compressive stress.

(ii) Volume Stress/Hydraulic Stress

If equal normal forces are applied over every unit surface of a body, then it undergoes a certain change in volume. The force opposing this change in volume per unit area is defined as volume stress.

Volume Stess/Hydraulic Stress = $\frac{F_{\perp}}{A} = P$

(iii) Shear Stress/Tangential Stress



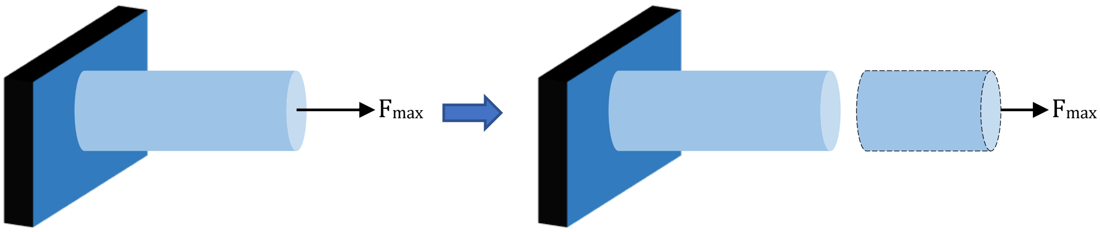
When the stress is tangential or parallel to the surface of a body then it is known as shear stress. Due to this stress, the shape of the body changes or it gets twisted but not its volume.

Shear stress = $\frac{F_t}{A} = \frac{F_{Tangential}}{A}$

(iv) Breaking Stress

The minimum stress required to cause the actual fracture of a material is called the **breaking stress** or ultimate strength. **or**

The maximum stress that a material can bear is called breaking stress.



Breaking stress = $\frac{F_{max}}{A}$ $\qquad$ F_{max} = force required to break the body.

Dependence of breaking stress:

(i) Nature of material $\qquad$ (ii) Temperature $\qquad$ (iii) Impurities.

Independence of breaking stress :

(i) Cross sectional area or thickness $\qquad$ (ii) Applied force (F_{max} = B.S. × Area).

Maximum load (force) which can applied on the wire depends on

(i) Cross sectional area or thickness $\qquad$ (ii) Nature of material

(iii) Temperature $\qquad$ (iv) Impurities

Illustration 1

The ratio of radii of two wires of same materials is 2 : 1. Find if they are stretched by the same force, the ratio of stress:

Solution:

$$\text{Stress} = \frac{\text{force}}{\text{area}} = \frac{F}{\pi r^2} \Rightarrow \frac{(\text{stress})_1}{(\text{stress})_2} = \frac{F}{\pi r_1^2} \times \frac{\pi r_2^2}{F} = \left[\frac{r_2}{r_1}\right]^2 = \left[\frac{1}{2}\right]^2 = \frac{1}{4}$$

Illustration 2

A body of mass 10 kg is attached to a 30 cm long wire whose breaking stress is 4.8×10^7 N/m^2. The area of cross section of the wire is 10^{-6} m^2. What is the maximum angular velocity with which it can be rotated in a horizontal circle?

Solution:

$$\frac{m\omega^2 L}{A} = \text{breaking stress (BS)} \Rightarrow \omega = \sqrt{\frac{(BS)A}{mL}} = \sqrt{\frac{4.8\times10^7\times10^{-6}}{10\times0.3}} = 4 \text{ rad/s}$$

Illustration 3

The breaking stress of aluminium is 7.5×10^8 dyne/cm^2. Find the maximum length of aluminium wire that can hang vertically without getting broken under its own weight. Density of aluminium is 2.7 g/cm^3. Given : g = 980 cm/s^2.

Solution:

Let L be the maximum length of the wire that can hang vertically without getting broken.

Mass of the wire, m = cross-sectional area (A) × length (L) × density (ρ)

Weight of the wire = mg = ALρg

This is equal to the maximum force that the wire can withstand.

$$\therefore \text{ Breaking stress} = \frac{LA\rho g}{A} = L\rho g$$

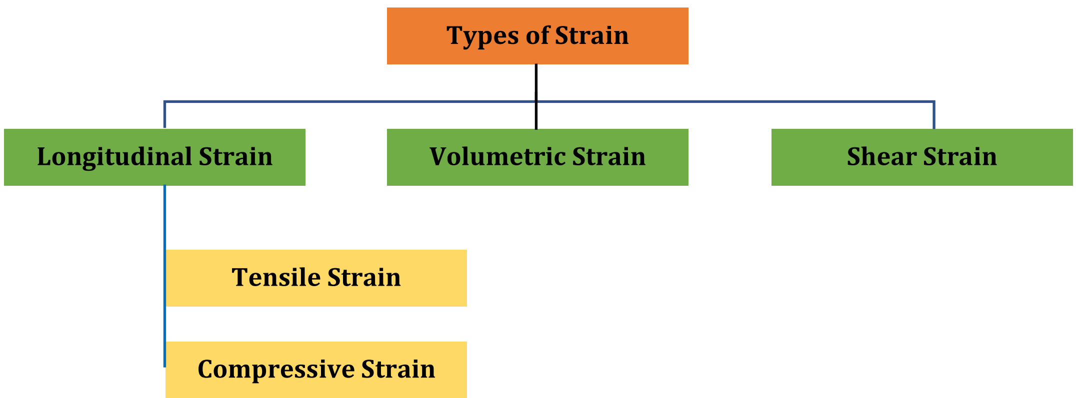
$$\text{Or } 7.5 \times 10^8 = L \times 2.7 \times 980 \Rightarrow L = \frac{7.5\times10^8}{2.7\times980} = 2.834 \times 10^5 \text{ cm} = 2.3 \text{ km}$$

Strain and its types

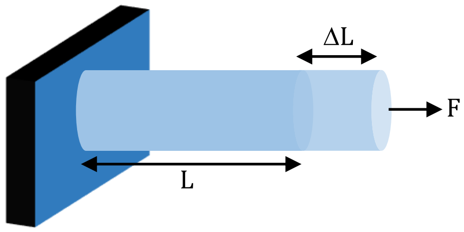
Strain

$$\text{Strain} = \frac{\text{change in the dimension of the body}}{\text{original dimension of the body}}$$

It is a unitless and dimensionless quantity.



(i) Longitudinal strain:



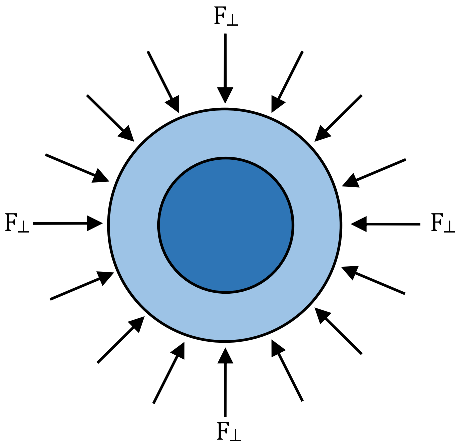
When applied force is $\perp$ to cross-section, length changes

$$\text{Longitudinal strain} = \frac{\text{change in length of the body}}{\text{initial length of the body}} = \frac{\Delta L}{L}$$

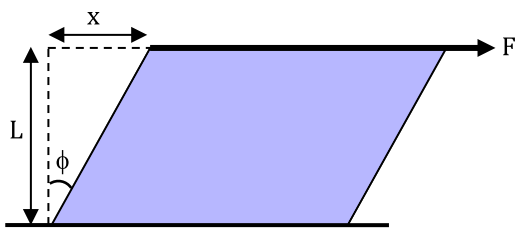
(ii) Volumetric strain:

When pressure is applied on body, volume changes

$$\text{Volume strain} = \frac{\text{change in volume of the body}}{\text{original volume of the body}} = \frac{\Delta V}{V}$$



(iii) Shear strain:



When applied force is ∥ to cross-section, shape changes

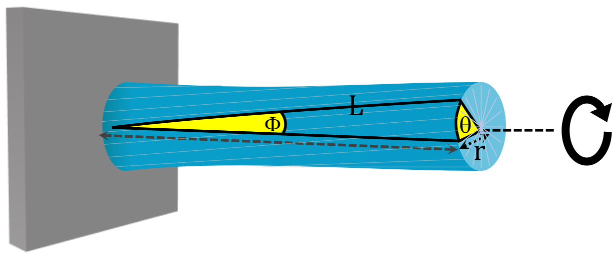
$\tan\phi = \frac{x}{L}$ (Here ϕ is very small)

$$\phi = \frac{x}{L} = \frac{\text{displacement of upper face relative to the lower face}}{\text{distance between two faces}}$$

ϕ = shear strain OR angle of shear

Relation between angle of twist and angle of shear

When a cylinder of length L and radius r is fixed at one end and a tangential force is applied at the other end, then the cylinder gets twisted. Figure shows the angle of shear ϕ and angle of twist θ.



From diagram

$r\theta = L\phi$

$\phi = \frac{r\theta}{L}$

θ = angle of twist

ϕ = angle of shear

Illustration 4

A wire of length 1m and radius 2mm is clamped from it's upper end. The lower end is twisted through an angle of 45°. The angle of shear is?

Solution:

$$\phi = \frac{r}{L}\theta = \frac{2\times10^{-3}}{1} \times 45° = 0.09°$$

Hooke's law

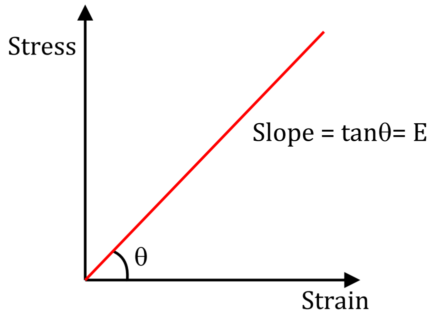
According to this law within the elastic limit the stress produced in a body is directly proportional to the corresponding strain.

Stress α Strain Or **Stress = E × strain**

Here, E = coefficient of elasticity or modulus of elasticity

$$E = \frac{\text{Stress}}{\text{Strain}}$$

The slope of stress & strain graph gives coefficient of elasticity.

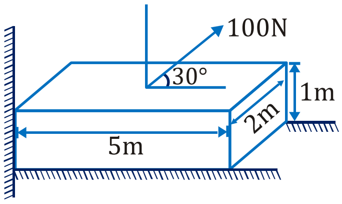


(i) E depends on :
  1. Nature of material
  2. Impurities
  3. Temperature

(ii) E independent from:-
  1. Stress
  2. Strain

BEGINNER'S BOX-1

1. Find out the longitudinal stress and tangential stress on the fixed block shown in figure.



PPF162

2. A 2 m long rod of radius 1 cm which is fixed at one end is given a twist of 0.8 radians at the other end. Find the shear strain developed.

PPF163

3. The maximum stress that can be applied to the material of a wire employed to suspend an elevator is $\frac{3}{\pi} \times 10^8$ N/m^2. If the mass of the elevator is 900 kg and it moves up with an acceleration of 2.2 m/s^2 then calculate the minimum radius of the wire.

PPF164

4. A human bone is subjected to a compressive force of 5.0×10^5 N. The bone is 25 cm long and has an approximate cross sectional area of 4.0 cm^2. If the ultimate compressive strength of the bone is 1.70×10^8 N/m^2, will the bone be compressed or will it break under this force?

PPF165

5. The breaking stress of steel is 7.9×10^8 N/m^2 and density is 7.9×10^3 kg/m^3. What should be the maximum length of a steel wire so that it may not break under its own weight?

PPF166

6. A wire can bear a weight of 20 kg before it breaks. If the wire is divided into two equal parts, then each part will support a maximum weight .....

PPF167

Significance of Modulus of Elasticity

More is the value of Modulus of Elasticity, more is the Elasticity of material.

It means more elastic material will have more tendency to regain its shape under elastic limit deformation (not permanent deformation).

Young's Modulus of Elasticity

Within elastic limit, the ratio of longitudinal stress to longitudinal strain is called Young's modulus of elasticity.

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{\Delta L/L}$$

$$Y = \frac{FL}{A\Delta L}$$

Unit of Y: N/m^2 Dimensions of Y: $[M^1L^{-1}T^{-2}]$

$$Y = \frac{\text{longitudinal stress}}{\text{longitudinal strain}} = \frac{F/A}{\Delta L/L} \Rightarrow F = \frac{YA}{L}\Delta L$$

Note : For spring : F = kx ...(i)

For wire : $F = \frac{YA}{L}\Delta L$ ...(ii)

From (i) and (ii), $k = \frac{YA}{L}$

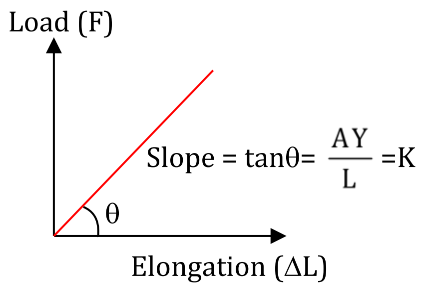


Illustration 5

A copper wire of negligible mass, length 1 m and cross-sectional area 10^{-6} m^2 is kept on a smooth horizontal table with one end fixed. A ball of mass 1kg is attached to the other end. The wire and the ball are rotating with an angular velocity of 20 rad/s. If the elongation in the wire is 10^{-3} m, obtain the Young's modulus of copper.

Solution:

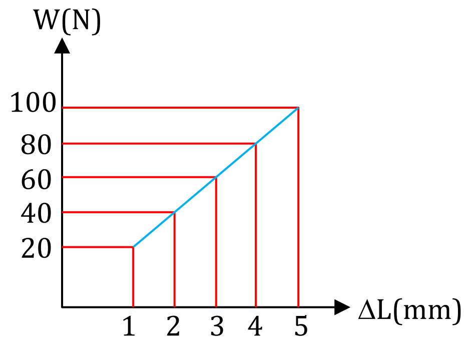
Centripetal force $F = m\omega^2 r$

$\therefore$ Stress in the wire = $\frac{F}{A} = \frac{m\omega^2 r}{A}$ and strain in the wire = $\frac{\Delta L}{L}$

Young's modulus Y = $\frac{\text{Stress}}{\text{Strain}} = \frac{m\omega^2 L \cdot L}{A.\Delta L} = \frac{1\times(20)^2\times(1)^2}{10^{-6}\times10^{-3}}$ = 4×10^{11} N/m^2

Illustration 6

The graph shows the extension of a wire of length 1m suspended from a roof at one end and with a load W connected to the other end. If the cross-sectional area of the wire is 1 mm^2, then the Young's modulus of the material of the wire is



Solution:

$$Y = \frac{F/A}{\Delta L/L} = \frac{WL}{A\Delta L} \Rightarrow \frac{W}{\Delta L} = \frac{YA}{L} = \text{slope}$$

$$\Rightarrow Y = \frac{L}{A} \times (\text{slope}) = \frac{1}{10^{-6}}\left(\frac{40-20}{(2-1)\times10^{-3}}\right) = 2 \times 10^{10}\ N/m^2$$

Illustration 7

Two wires are made of the same metal. The length of the first wire is half that of the second wire and its diameter is double that of the second wire. If equal loads are applied on both the wires, find the ratio of increase in their lengths.

Solution:

$$Y = \frac{\frac{F}{A}}{\frac{\Delta L}{L}} \Rightarrow \Delta L = \frac{FL}{AY} = \frac{FL}{\pi r^2 Y} \Rightarrow \frac{\Delta L_1}{\Delta L_2} = \frac{4FL_1}{\pi d_1^2 Y} \times \frac{\pi d_2^2 Y}{4FL_2} = \frac{L_1}{L_2} \times \frac{d_2^2}{d_1^2} = \frac{1}{2} \times \frac{1}{(2)^2} = \frac{1}{8}$$

Illustration 8

Calculate the force required to increase the length of a steel wire of cross-sectional area $10^{-6}m^2$ by 0.5%. given: Y(for steel) = 2×10^{11} N/m^2.

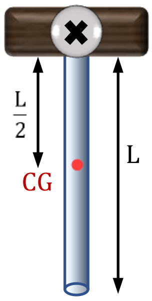
Solution:

$$\frac{\Delta L}{L} \times 100 = 0.5\% \Rightarrow \frac{\Delta L}{L} = 5 \times 10^{-3}$$

So, $F = YA\frac{\Delta L}{L} = 2 \times 10^{11} \times 10^{-6} \times 5 \times 10^{-3} = 10^3$ N

Elongation in hanging wire due to its own weight

Since the weight can assumed to be act at the centre of gravity, therefore



$\therefore$ The original length will be taken as $\frac{L}{2}$ $\therefore$ $Y = \frac{Mg \times \frac{L}{2}}{A \times \Delta L} \Rightarrow \Delta L = \frac{MgL}{2AY}$

But, M = (LA) ρ $\qquad \therefore \Delta L = \frac{LA\rho gL}{2AY}$ or $\Delta L = \frac{\rho g L^2}{2Y}$

Illustration 9

A rubber rope of length 8 m is hung from the ceiling of a room. What is the increase in length of the rope due to its own weight? (Given : Young's modulus of elasticity of rubber = 5×10^6 N m^{-2} and density of rubber = 1.5×10^3 kg/m^3 and g = 10 m/s^2).

Solution:

$$\Delta L = \frac{\rho g \ell^2}{2Y} = \frac{1.5 \times 10^3 \times 10 \times 8 \times 8}{2 \times 5 \times 10^6} = 9.6 \times 10^{-2} \text{ m} = 9.6 \times 10^{-2} \times 10^3 \text{ mm} = 96 \text{ mm.}$$

Bulk modulus of elasticity 'K' or 'B'

Within elastic limit, the ratio of the volumetric stress (i.e., change in pressure) to the volumetric strain is called Bulk modulus of elasticity.

$$K \text{ or } B = \frac{\text{volumetric stress}}{\text{volumetric strain}} = \frac{F/A}{\frac{-\Delta V}{V}} = \frac{\Delta P}{\frac{-\Delta V}{V}}$$

The negative sign indicates a decrease in volume with an increase in pressure and vice-versa.

Unit of K: N/m^2 or Pascal

Compressibility 'C'

The reciprocal of Bulk modulus of elasticity is defined as compressibility.

$C = \frac{1}{K}$ $\qquad$ SI unit of C: m^2/N or pascal^{-1}

Illustration 10

A sphere contracts in volume by 0.01% when taken to the bottom of sea 1 km deep. Find the Bulk modulus of the material of the sphere. Given: density of sea water is 1 g/cm^3, g = 980 cm/s^2.

Solution:

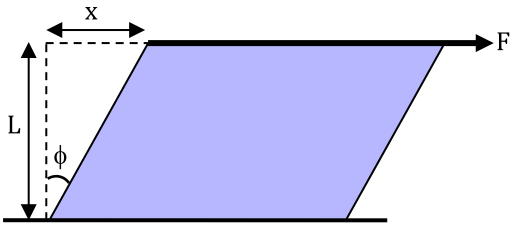
$\frac{|\Delta V|}{V} = \frac{0.01}{100}$, h = 1 km = 10^5 cm, ρ = 1 g/cm^3 ; ΔP = hρg = $10^5 \times 1 \times 980$ dyne/cm^2, K = ?

$$K = \frac{\Delta P}{|\Delta V|/V} = \frac{\Delta P \times V}{|\Delta V|} = \frac{10^5 \times 980 \times 100}{0.01} \text{ dyne/cm}^2 = 9.8 \times 10^{11} \text{ dyne/cm}^2.$$

Modulus of Rigidity 'η'

Within elastic limit, the ratio of shearing stress to shearing strain is called modulus of rigidity of a material.

$$\eta=\frac{\text{shearing stress}}{\text{shearing strain}}=\left(\frac{\frac{F_{\text{tangential}}}{A}}{\phi}\right)=\frac{F_{\text{tangential}}}{A\phi}$$



Note: Angle of shear 'Φ' is always taken in radians

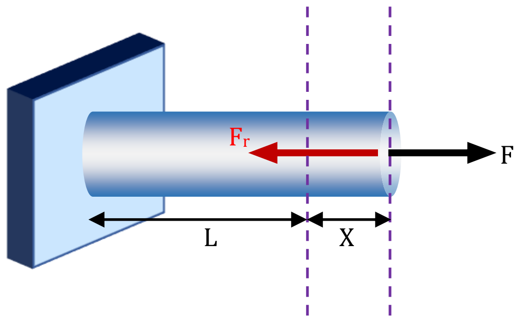
Material / E	Solid	Liquid	Gas	Ideal rigid body
Young's modulus	defined	Not defined	Not defined	Infinite
Bulk modulus	defined	Defined	Defined	infinite
Shear modulus	defined	Not defined	Not defined	infinite

Work done in stretching a wire (Potential energy of a stretched wire)

When a wire of length 'L' is stretched by an external force F, then work has to be done against the restoring force this work is stored as potential energy of wire.

For spring : W.D.= Elastic Potential energy = $\frac{1}{2}kx^2$

For wire : W = PE = $\frac{1}{2}\left(\frac{YA}{L}\right)(\Delta L)^2$



EPE = $\frac{1}{2}\left(\frac{YA}{L}\Delta L\right)(\Delta L)=\frac{1}{2}F(\Delta L)$

EPE = $\frac{1}{2}\frac{F}{A}\left(\frac{\Delta L}{L}\right)(AL)$

$$EPE=\frac{1}{2}(\text{stress})\ (\text{strain})\ (\text{volume})$$

$$EPE=\frac{1}{2}Y(\text{strain})^2(\text{volume})$$

$$EPE=\frac{1}{2}\left(\frac{(\text{stress})^2}{Y}\right)(\text{volume})$$

Illustration 11

A rubber cord has a cross–sectional area 1 mm² and total unstretched length 10 cm. It is stretched to 12 cm and then released to project a mass of 5 g. Young's modulus for rubber is 5 × 10⁸ N/m². Find the tension in the cord and velocity of the mass.

Solution:

Tension in the cord = $\frac{YA}{L}\Delta L=\frac{5\times10^{8}\times1\times10^{-6}\times2\times10^{-2}}{10\times10^{-2}}$ = 100 N.

When the mass is released, elastic energy stored = Kinetic energy of mass

$\frac{1}{2}F\times\Delta L=\frac{1}{2}mv^2 \Rightarrow v=\sqrt{\frac{F\times\Delta L}{m}}=\sqrt{\frac{100\times2\times10^{-2}}{5\times10^{-3}}}$ = 20 m/s.

Illustration 12

Young modulus of elasticity of brass is 10^{11} N/m². The increase in its energy on pressing a rod of length 0.1 m and cross–sectional area 1 cm² made of brass with a force of 10 kg along its length, will be ....

Solution:

Increase in energy = $\frac{1}{2}F \times \Delta L = \frac{1}{2} \times F \times \frac{FL}{AY} = \frac{1}{2}\times(10\times10)^2\times\frac{0.1}{10^{-4}\times10^{11}} = 5 \times 10^{-5}$ J

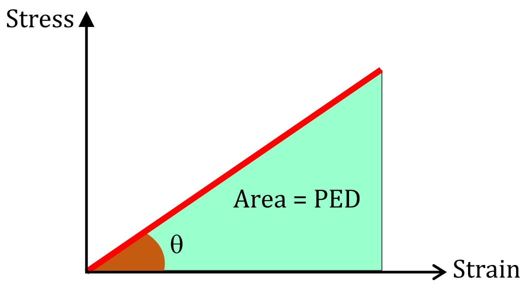
Potential energy density

Potential energy per unit volume stored in the wire

$$PED = \frac{PE}{vol}$$

SI unit : J/m³ or N/m²

$$PED = \frac{1}{2}(stress)(strain) = \frac{1}{2}Y(strain)^2 = \frac{1}{2}\left(\frac{(stress)^2}{Y}\right)$$



Note: Slope = tanθ = E = coefficient of elasticity

$$\text{Area} = \frac{1}{2}\times\text{stress}\times\text{strain} = \text{Potential Energy Density}$$

Poisson's Ratio (σ)

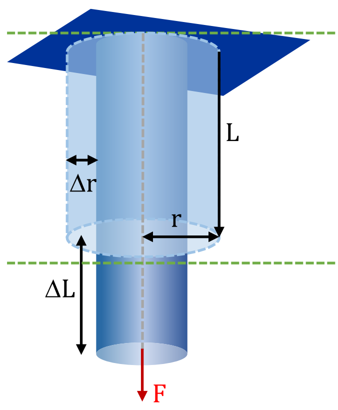
longitudinal strain $= \frac{\Delta L}{L}$; lateral strain $= \frac{\Delta r}{r}$

Within elastic limit, the ratio of lateral strain to the longitudinal strain is called poisson's ratio.

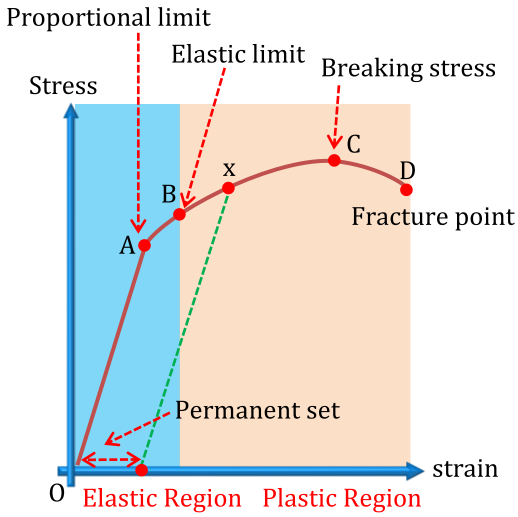
$$\sigma = \frac{\text{lateral strain}}{\text{longitudinal strain}} \Rightarrow \sigma = -\frac{\Delta r / r}{\Delta L / L}$$

$-1 \le \sigma \le 0.5$ (theoretical limit)

$\sigma \approx 0.2 - 0.4$ (experimental limit)



Stress Strain relationship and Graphical analysis



1. In the region between O to A the curve is linear. Hooke's law is obeyed. In this region the solid behaves as an elastic body.
2. In the region A to B stress and strain are not proportional but body regains its original shape and size when the load is removed. Point B is known as the elastic limit.
3. If the load is increased further the strain developed increases rapidly than stress.
4. In the region from B to D, when load is removed the body does not regain its original dimensions. Even when stress is zero the strain is not zero. The material is said to have a permanent set. The region beyond point B is known as the plastic region.
5. Point C corresponds to the ultimate tensile strength; beyond this point additional strain is produced by even a reduced applied force and fracture occurs at point D.
6. If plastic region is large then material will be ductile.
7. If plastic region is small then material will be brittle.

Note: For some materials elastic region is very large and the material does not obey Hooke's law over most of the region. These are called elastomers e.g. Tissue of Aorta, rubber, etc.

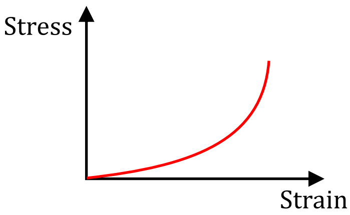
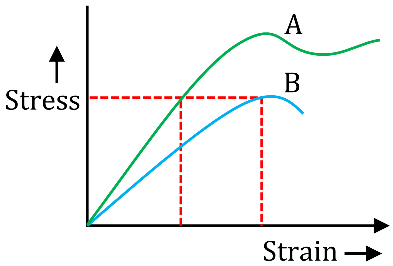


Illustration 13

The stress versus strain graphs for two materials A and B are shown below. Explain the following



(a) Which material has greater Young's modulus?

(b) Which material is more ductile?

(c) Which material is more brittle?

(d) Which of the two is stronger material?

Solution:

(a) Material A has greater value of Young's modulus, because slope of A is greater than that of B.

(b) Material A is more ductile because there is a large plastic deformation range between the elastic limit and the breaking point.

(c) Material B is more brittle because the plastic region between the elastic limit and breaking point is small.

(d) Strength of a material is determined by the stress required to cause fracture. Material A is stronger than material B.

BEGINNER'S BOX-2

1. A stress of 20×10^8 N/m² is developed when the length of a wire is doubled. Its Young's modulus of elasticity in N/m² will be ....

PPF168

2. The ratio of lengths of two wires made up of the same material is 3 : 1 and the ratio of their radii is 1 : 3. The ratio of increments of lengths on account of suspending the same weight will be ....

PPF169

3. The following four wires are made of same material. Which one will have the largest elongation when subjected to the same tension?

(1) Length 500 cm, diameter 0.05 mm.
(2) Length 200 cm, diameter 0.02 mm.
(3) Length 300 cm, diameter 0.03 mm.
(4) Length 400 cm, diameter 0.01 mm.

PPF170

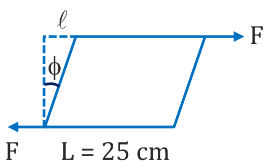
4. The bulk's modulus of copper is 138×10^9 Pa. The additional pressure generated in an explosion chamber is 345×10^6 Pa. Then the percentage change in the volume of a piece of copper placed in this chamber will be ....

PPF171

5. The compressibility of water per unit atmospheric pressure is 4×10^{-5}. Decrease in the volume of 100 cm³ water of volume at 100 atmospheric pressure will be ....

PPF172

6. Two parallel and opposite forces, each of magnitude 4000 N, are applied tangentially to the upper and lower faces of a cubical metal block of side 25 cm. If the shear modulus for the metal is 8×10^{10} Pa, then the displacement of the upper surface relative to the lower surface will be ....



PPF173

7. Young modulus of elasticity of brass is 10^{11} N/m². The increase in its energy on pressing a rod of length 0.1 m and cross–sectional area 1 cm² made of brass with a force of 10 kg along its length, will be ....

PPF174

Mechanical Properties of Fluids

Fluid

Fluids are the substances that can flow or deforms. Therefore, liquids and gases both are fluids.
Study of fluids at rest is called fluid statics or hydrostatics and the study of fluid in motion is called fluid dynamics or hydrodynamics. Fluid statics and fluid dynamics collectively known as fluid mechanics.
The intermolecular forces in liquids are comparatively weaker than in solids. Therefore, their shapes can be changed easily. When external force (shear stress) is present, liquid can flow until it conforms to the boundaries of its container. Most liquids resist compression. Unlike a gas, a liquid does not disperse to fill every space of a container and it forms a free surface.
The intermolecular forces are weakest in gases, so their shapes and sizes can be changed much easily. Gases are highly compressible and occupy the entire space of the container quite rapidly. Unlike liquid, gases can't form free surface.

Fluid Mechanics

Fluid Statics | **Fluid dynamics**

(i) Density

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} \Rightarrow \rho = \frac{M}{V}$$

Units - (i) $\frac{kg}{m^3}$ (SI system) (ii) $\frac{gm}{cm^3}$ (CGS system)

Note: (i) ρ_{water} = 1000 kg/m³ = 1 gm/cc (ii) ρ_{Hg} = 13600 kg/m³ = 13.6 gm/cc

Density of mixture

If two immiscible liquids of mass m_1 and m_2 and volume V_1 and V_2 are mixed together then density of mixture is given by : $\rho_{mix} = \frac{\text{Total mass of mixture}}{\text{Total volume of mixture}} = \frac{m_1 + m_2}{v_1 + v_2} = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_1}{\rho_2}}$

Case (1) :

If liquid with same masses are mixed i.e. $m_1 = m_2 = m$ then -

$\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$ (Harmonic mean of individual densities)

Case (2) :

If liquid with same volumes are mixed i.e. $V_1 = V_2 = V$ then -

$\rho_{mix} = \frac{\rho_1 + \rho_2}{2}$ (Arithmetic mean of individual densities)

Illustration 14

Two immiscible liquids having density 2 gm/cc and 4 gm/cc are mixed then find density of mixture if -
(1) Same volumes are taken (2) Same masses are taken

Solution:

(1) $\rho_{mix} = \frac{\rho_1 + \rho_2}{2} = \frac{2+4}{2} = 3 gm/cc$

(2) $\rho_{mix} = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} = \frac{2(2)(4)}{2+4} = \frac{8}{3} gm/cc$

Illustration 15

Density of mixture is 4 gm/cc when equal volumes are taken and 3 gm/cc when equal masses are taken, then find density of individual liquid.

Solution:

Let the density of liquids are ρ_1 and ρ_2

If equal volumes are taken, $\frac{\rho_1+\rho_2}{2}=4 \Rightarrow \rho_1+\rho_2=8$ ...(i)

If equal masses are taken, $\frac{2\rho_1\rho_2}{\rho_1+\rho_2}=3 \Rightarrow \rho_1\rho_2=12$ ...(ii)

By solving both equations, ρ_1 = 2 'or' 6 gm/cc ; ρ_2 = 6 'or' 2 gm/cc

(ii) Relative density:

$$R.D. = \frac{\text{Density of substance}}{\text{Density of pure water at } 4^\circ C}$$

R.D. is an unitless and dimensionless quantity

(iii) Specific weight:

$$\text{Specific weight} = \frac{\text{Weight of substance}}{\text{Volume of substance}} = \frac{mg}{V} = \rho g$$ unit $\rightarrow$ N/m^3

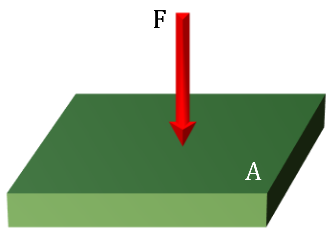
(iv) Specific Gravity:

$$S.G. = \frac{\text{Specific weight of substance}}{\text{Specific weight of pure water at } 4^\circ C} = \frac{\rho g}{\rho_w g} = \frac{\rho}{\rho_w} = R.D.$$

Note: The numerical value of specific gravity and relative density are same.

Pressure and Pressure due to Liquid Column

Pressure P is defined as the magnitude of the normal force acting per unit surface area.



$P=\frac{F_\perp}{A}$, here $F_\perp$ = normal force on a surface of area A

SI UNIT : Pascal (Pa) ; 1 Pa = 1 N/m^2 **Dimensions :** $[ML^{-1}T^{-2}]$

Practical units : atmospheric pressure (atm), bar and torr .

1 atm = 1.01325 × 10^5 Pa = 1.01325 bar = 760 torr = 760 mm of Hg = 10.33 m of water

1 bar = 10^5 Pa ; 1 torr = 1 mm of mercury column = 133 Pa.

Pressure is a scalar quantity.

Illustration 16

A 50 kg girl wearing heel shoes balances on a single heel. The heel is circular with a diameter 1 cm. The pressure exerted by the heel on the horizontal floor is (Take g = 10 m s^{-2})

(1) 6.4 × 10^4 Pa (2) 6.4 × 10^5 Pa (3) 6.4 × 10^6 Pa (4) 6.4 × 10^7 Pa

Solution: (1)

Here, m = 50 kg, D = 1 cm = 10^{-2} m, g = 10 ms^{-1}

$\therefore$ Pressure exerted by the heel on the horizontal floor is

$$p=\frac{F}{A}=\frac{mg}{\pi(D/2)^2}=\frac{4mg}{\pi D^2}=\frac{4\times 50kg\times 10ms^{-2}}{3.14\times(10^{-2}m)^2} = 6.4 \times 10^6 \text{ Pa}$$

Types of Pressures

Pressure is of three types :

(i) Atmospheric pressure (P_o/P_{atm}) (ii) Gauge pressure (P_{gauge}) (iii) Absolute pressure ($P_{abs.}$)

(1) Atmospheric Pressure

Force exerted by atmospheric column on unit cross-sectional area at mean sea level is called atmospheric pressure (P_0).

$$P_o = 101.3 \text{ kN/m}^2 \Rightarrow P_o = 1.013 \times 10^5 \text{ N/m}^2$$

Note: Barometer is used to measure the atmospheric pressure.

(2) Gauge Pressure

Excess Pressure over the atmospheric pressure ($P - P_{atm}$) measured with the help of pressure measuring instruments is called gauge pressure.

Note: Gauge pressure is always measured with the help of a **"manometer".**

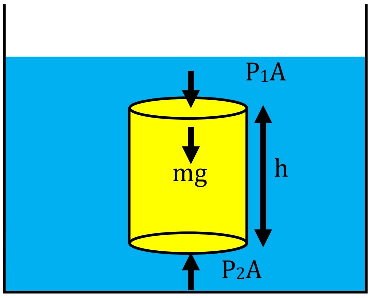
(3) Absolute Pressure :-

Sum of the atmospheric and gauge pressure is called absolute pressure.

$P_{abs} = P_{atm} + P_{gauge}$

$P_{abs} = P_o + h\rho g$

Pressure due to liquid at depth 'h'



m = mass of liquid element

A = Area of cross - section of liquid element

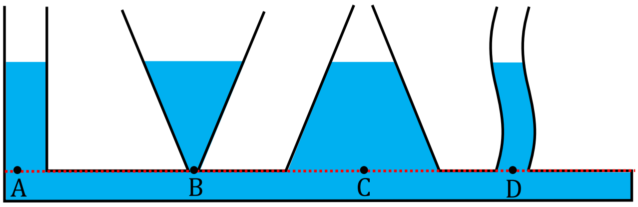
Since fluid is at rest

$\Rightarrow$ $P_1A + mg = P_2A$

$\Rightarrow$ $P_1A + (Ah\rho)g = P_2A$

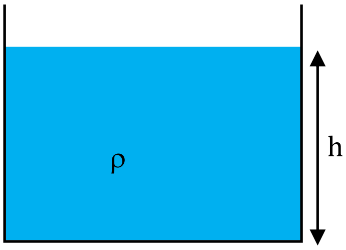
$\Rightarrow$ $P_2 = P_1 + \rho gh$

Note: Pressure exerted by a same liquid at any point does not depend on shape and size of the container (it means quantity of liquid). It depends only on the height of liquid column.



Pressure due to liquid, $P_A = P_B = P_C = P_D$

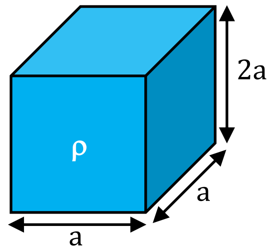
Pressure due to liquid on a vertical wall of container



Pressure due to liquid on a vertical wall is different at different depths, so average fluid pressure on side wall of the container is equal to mean pressure = $\frac{\rho gh}{2}$ (h = height of wall)

Problems based on Pressure due to Liquid Column and Barometer

Illustration 17



A cuboid (a × a × 2a) is filled with a liquid of density 'ρ' as shown in figure. Neglecting atmospheric pressure, find out

(a) Force on base wall of the cuboid (b) Force on side wall of the cuboid

Solution:

(a) Force on base wall = $P_{\text{base wall}} \times A_{\text{base wall}} = \rho g(2a) \times a^2 = 2\rho g a^3$

(b) Force on side wall = $P_{\text{side wall}} \times A_{\text{side wall}} = \left[\dfrac{0+\rho g(2a)}{2}\right] \times 2a^2 = 2\rho g a^3$

Illustration 18

A tank 5 m high is half filled with water and then is filled to the top with oil density 0.85 g-wt/cm³. The pressure at the bottom of the tank, due to these liquids is.

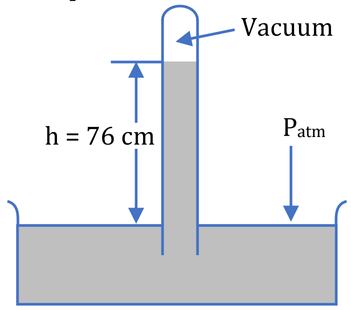
Solution:

Pressure at the bottom,

$$P = (h_1 d_1 + h_2 d_2)\frac{\text{g-wt}}{\text{cm}^2} = [250 \times 1 + 250 \times 0.85] = 250[1.85]\frac{\text{g-wt}}{\text{cm}^2} = 462.5\frac{\text{g-wt}}{\text{cm}^2}$$

Barometer

A tube of length 1 m and uniform cross section is taken. It is filled with mercury and inverted into a mercury tray. The height of the mercury column in equilibrium inside the tube is 76 cm.



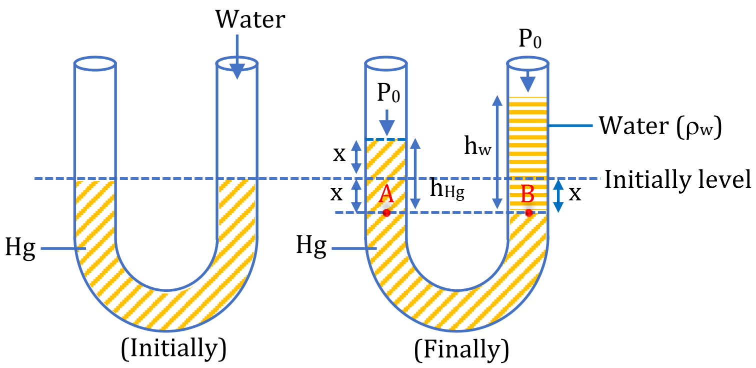
∴ Atmospheric pressure $P_0 = \rho g h = 13.6 \times 10^3 \times 9.81 \times 76 \times 10^{-2} = 1.013 \times 10^5\ \text{N/m}^2$

Barometer in lift

Case (1) : Lift accelerating upwards with acceleration 'a' ; $P_{atm} = \rho(g+a)h$

Case (2) : Lift accelerating downwards with acceleration 'a' ; $P_{atm} = \rho(g-a)h$

U- tube concept

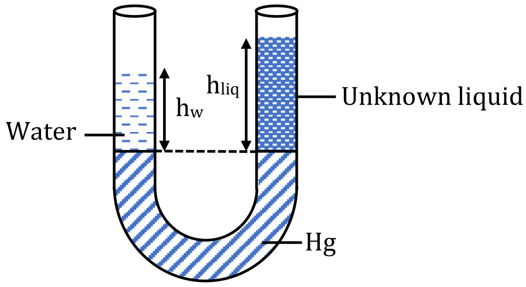


For the same stationary liquid, pressure at same horizontal level remains same.

$P_A = P_B$

$P_0 + h_{Hg}\rho_{Hg}g = P_0 + h_w\rho_w g \Rightarrow h_{Hg}\rho_{Hg} = h_w\rho_w$

Illustration 19
An open U-tube contains water and unknown liquid separated by mercury. The mercury columns in two arms are in level with 8 cm of water in one arm and 10 cm of unknown liquid in the other. Find the specific gravity of unknown liquid.



Solution:

$h_w\rho_w = h_{liq}\rho_{liq}$

$(S.G.)_{liq.} = \dfrac{\rho_{liq}}{\rho_w} = \dfrac{h_w}{h_{liq}}$

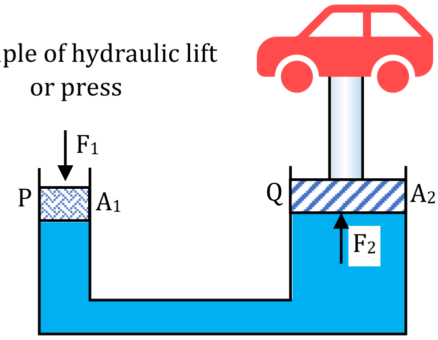
$(S.G.)_{liq.} = \dfrac{8}{10} = 0.8$

Pascal's Law
Pascal's law is stated in following ways.
- A liquid exerts equal pressures in all directions at point.
- If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude.

Applications of pascal's law
hydraulic lifts, hydraulic press, hydraulic brakes, etc

Hydraulic lift

Principle of hydraulic lift or press



$(A_2 > A_1)$

Pressure applied $= \dfrac{F_1}{A_1}$

$\therefore$ Pressure transmitted $= \dfrac{F_2}{A_2}$

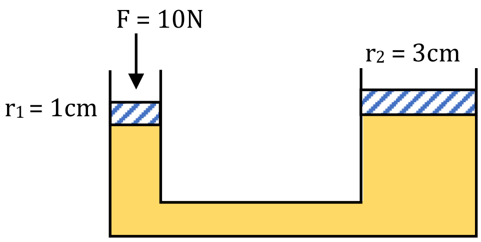
$\because$ Pressure is equally transmitted

$\because$ $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2}$

$\therefore$ Upwards force on A_2 is $F_2 = \dfrac{F_1}{A_1} \times A_2 = \dfrac{A_2}{A_1} \times F_1$

Illustration 20
(1) Find the force transmitted on larger piston.

(2) If smaller piston is pushed down through 6cm, then how much the larger piston will move up.



Solution:

(1) $\dfrac{F_1}{A_1} = \dfrac{F_2}{A_2} \Rightarrow \dfrac{10}{\pi(1)^2} = \dfrac{F_2}{\pi(3)^2} \Rightarrow F_2 = 90N$

(2) $V_1 = V_2$

$A_1h_1 = A_2h_2 \quad \Rightarrow \dfrac{6}{9} = 0.67cm$

Illustration 21
The neck and bottom of a bottle are 4 cm and 12 cm in radius respectively. If the cork is pressed with a force 10 N in the neck of the bottle, then find out the force exerted on the bottom of the bottle:

Solution:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_2 = \left(\frac{A_2}{A_1}\right)F_1 = \left(\frac{R_2}{R_1}\right)^2 F_1 = \left(\frac{12}{4}\right)^2 \times 10 = 90N$$

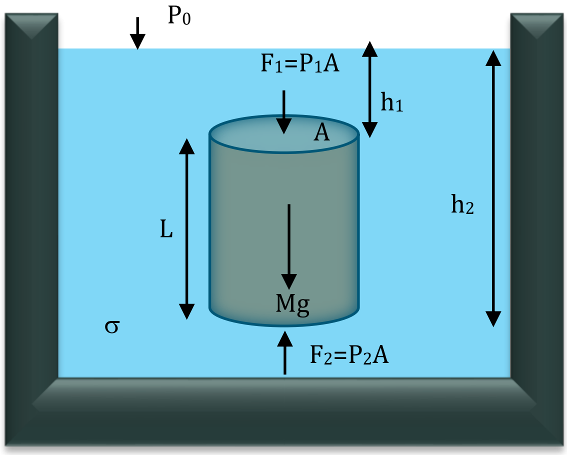
Buoyancy, Archimedes' Principle and Laws of Floatation

Buoyant Force

If a body is partially or fully immersed in a fluid, it experiences an upward force due to the fluid surrounding it. This phenomenon of force exerted by fluid on the body is called buoyancy and force is called buoyant force or force of upthrust.

Archimedes' Principle

It states that the upward buoyant force on a body that is partially or totally immersed in a fluid is equal to the weight of the fluid displaced by it.



Consider a body immersed in a liquid of density σ.

Top surface of the body experiences a downward force

$F_1 = AP_1 = A[h_1.\sigma.g + P_0]$ ...(i)

Lower face of the body will experience an upward force

$F_2 = AP_2 = A[h_2.\sigma.g + P_0]$ ...(ii)

As $h_2 > h_1$ so F_2 is greater than F_1

so net upward force $F = F_2 - F_1 = A\sigma g[h_2 - h_1]$

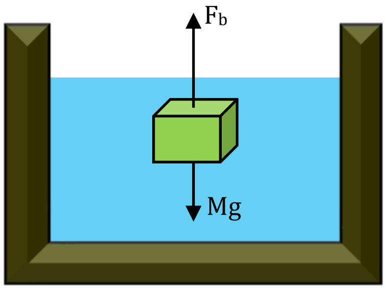
$\therefore$ $F = A.\sigma.g.L. = V_{in}.\sigma.g$

[$\because$ V_{in} = volume of the body submerged in the fluid

AL = volume of displaced liquid

mg = weight of displaced liquid]

Apparent weight



Relative density of body = $\dfrac{W}{W - W_{app}}$

$$= \frac{\rho_s}{\rho_L} = \frac{W}{W - W_{app}}$$

We consider that a body is completely submerged inside fluid :-

$W_{app} = W - F_b$

$W_{app} = mg - V\sigma g$

$W_{app} = V\rho g - V\sigma g = V(\rho - \sigma)g$

Principle of Floatation

When a body of density (ρ) and volume (V) is completely immersed in a liquid of density (σ), the forces acting on the body are :

(i) Weight of the body $W = Mg = V\rho g$ directed vertically downwards

(ii) Buoyant force or Upthrust $F_b = V\sigma g$ directed vertically upwards

The following three cases are possible :

Case I : Density of the body is greater than that of liquid ($\rho > \sigma$)

In this case $W > F_b$

So the body will sink to the bottom of the liquid.

$W_{App} = W - F_b = V\rho g - V\sigma g = V\rho g\,(1 - \sigma/\rho) = W\,(1 - \sigma/\rho)$.

Case II : Density of the body is equal to the density of liquid ($\rho = \sigma$)

In this case $W = F_b$

So the body will float fully submerged in the liquid. It will be in neutral equilibrium.

$W_{App} = W - F_b = 0$

Case III : Density of the body is lesser than that of liquid ($\rho < \sigma$)

In this case $W < F_b$

So the body will float partially submerged in the liquid. In this case the volume of liquid displaced by the body (V_{in}) will be less than the volume of body (V). This ensures that upthrust equals to W.

$\therefore W_{App} = W - F_b = 0$

The above three cases constitute the ***laws of floatation*** which states that a body will float in a liquid if weight of the liquid displaced by the immersed part of the body is at least equal to the weight of the body.

Illustration 22

If a block of wood floats in water with $\frac{3}{5}$ of its volume inside the water then what is the density of wood?

Solution:

In floating condition $W = F_b$

$\Rightarrow \rho_b Vg = \rho_w V_{in} g \Rightarrow \rho_b V = 1000 \times \frac{3}{5} V \Rightarrow \rho_b = 0.6 \times 10^3 \text{ kg/m}^3$

Illustration 23

A cube with an edge of 10cm is immersed in a vessel containing water. A layer of liquid immiscible with water and having a density of 0.8×10^3 kg/m³ is poured above water. The interface between the liquid is at the middle of the cube height. Find the mass of the cube :

Solution:

$W = F_b(\text{liq}) + F_b(\text{water})$

$\Rightarrow \quad mg = \rho_\ell V_{in_{liq}} g + \rho_w V_{in_{water}} g \Rightarrow m = (\rho_\ell + \rho_w)V_{in}$

$= (0.8 \times 10^3 + 10^3) \times (10 \times 10 \times 5 \times 10^{-6}) = 0.90 \text{ kg}$

$[\because V_{in} = \frac{V}{2} = 5 \times 10^{-4} \text{ m}^3]$

BEGINNER'S BOX-3

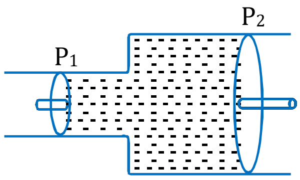
1. When two metals with equal volumes are mixed together, the density of the mixture is 4 kg/m³. When equal masses of the same two metals are mixed together, the mixture density is 3 kg/m³. Calculate the densities of each metal.

PPF175

2. A mercury barometer reads 75 cm in a stationary lift. What reading does it show when the lift is moving downwards, with an acceleration of 1 m/s²?

PPF176

3. The diameter of a piston P_2 is 50 cm and that of a piston P_1 is 10 cm. What is the force exerted on P_2 when a force of 1 N is applied on P_1?



PPF177

4. An open U-tube of uniform cross-section contains mercury. If 27.2 cm of water column is poured into one limb of the tube, how high does the mercury surface rise in the other limb from its initial level? [ρ_w = 1 g/cm³ and ρ_{Hg} = 13.6 g/cm³]

PPF178

5. A certain block weighs 15 N in air, 12 N in water. When immersed in another liquid, it weighs 13 N. Calculate the relative density of (i) the block (ii) the other liquid.

PPF179

6. A block of wood floats in water with two-third of its volume submerged. The block floats in oil with 0.90 of its volume submerged. Find the density of (i) wood and (ii) oil. Density of water is 10^3 kg/m^3.

PPF180

7. A 700 g solid cube having an edge of length 10 cm floats in water. What volume of the cube is outside water?

PPF181

8. If a block of iron of density 5 g/cm^3 and size 5 cm × 5 cm × 5 cm was weighed whilst completely submerged in water, what would be the apparent weight in g – f (gram-force)?

PPF182

Types of Fluid flow

Steady flow

Steady flow is defined as that type of flow in which the fluid velocity at a point does not change with time. The fluid particle may have a different velocity at some other point.

In steady flow all the particles passing through a given point follow the same path and hence a unique line of flow.

This line or path is called a streamline. Streamlines do not intersect each other, if they do so any particle at the point of intersection can move in either directions and consequently the flow cannot be steady.

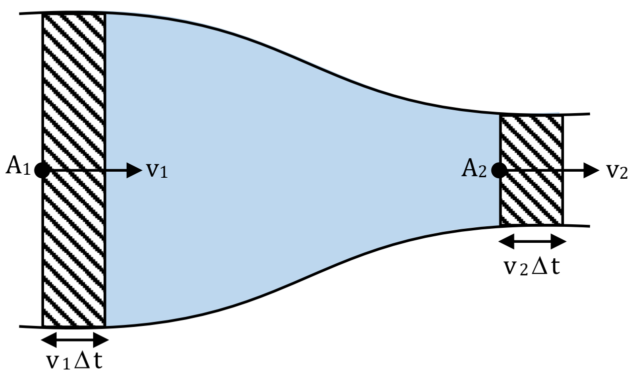
Laminar and Turbulent flow

Laminar flow is the flow in which fluid particles move along well-defined streamlines which are straight and parallel. In laminar flow the velocities at different points in the fluid may have different magnitudes, but their directions are parallel. Thus the particles move in laminae or layers sliding smoothly over the adjacent layer. Turbulent flow is an irregular flow in which the particles move in zig-zag way due to which eddy formation take place which are responsible for high energy losses.

Rotational and Irrotational flow

Rotational flow is the flow in which the fluid particles while flowing along different path-lines also rotate about their own axes. In irrotational flow the particles do not rotate about their axes.

Equation of Continuity



(i) The continuity equation is the mathematical expression of the law of conservation of mass in fluid dynamics.

(ii) In the steady flow mass of the fluid entering into a tube of flow in a particular time interval is equal to the mass of fluid leaving the tube.

$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$ or $\rho_1 A_1 v_1 = \rho_2 A_2 v_2$

for incompressible fluid $\rho_1 = \rho_2$

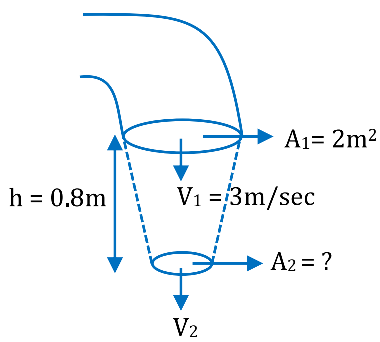
or $A_1v_1 = A_2v_2$ or Av = constant

Volume flux = Rate of flow = Volume of liquid flowing per second $Q = \frac{dV}{dt} = Av$

Unit of (Q); m^3/s or $\frac{\text{Litre}}{\text{sec}}$ or $\frac{m\ell}{s}$

Illustration 24

Water from a tap emerges vertically downwards with an initial speed of 3 m/sec. The cross sectional area of tap is 2m². Assume that the pressure is constant throughout the stream of water and that the flow is steady. Find the cross sectional area of stream 80 cm below the tap. (g =10 m/s²)



Solution:

By 3rd equation of motion

$v^2 = u^2 + 2as$

$v_2^2 = (3)^2 + 2(10)(0.8)$

$v_2 = 5$ m/sec

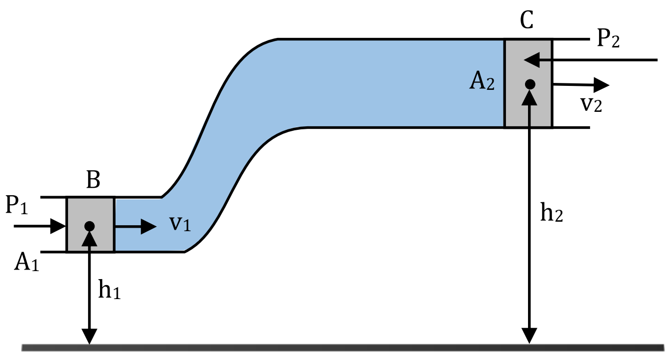
By equation of continuity $\rightarrow$

$A_1v_1 = A_2v_2$

$(2)(3) = (A_2)(5) \Rightarrow A_2 = \frac{6}{5} = 1.2\text{m}^2$

Bernoulli's Theorem

According to Bernoulli's Theorem, in case of steady flow of incompressible and non-viscous fluid through a tube of non-uniform cross section then the sum of the pressure energy per unit volume, the potential energy per unit volume and the kinetic energy per unit volume is same at every point in the tube, i.e., $P + \rho gh + \frac{1}{2}\rho v^2$ = constant.



This equation is the Bernoulli's equation and expresses principle of conservation of mechanical energy in case of moving fluids.

The sum of pressure energy, kinetic energy and potential energy per unit volume remains constant along a streamline in an ideal fluid flow i.e.,

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant} \qquad \text{(Energy per unit volume)}$$

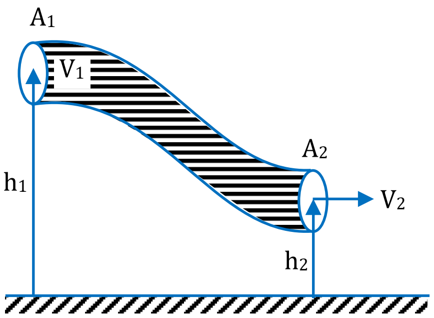
Or $$\frac{P}{\rho} + \frac{v^2}{2} + gh = \text{constant} \qquad \text{(Energy per unit mass)}$$

Or $$\frac{P}{\rho g} + \frac{v^2}{2g} + h = \text{constant} \qquad \text{(Energy per unit weight)}$$

In the above equation $\frac{P}{\rho g}$ is called the pressure head, $\frac{v^2}{2g}$ is called the velocity head and h is called the gravitational/potential head.

Illustration 25

Water is moving with a speed of 5 m/sec through a pipe with cross sectional area of 4cm^2. The water gradually descends 10m as the pipe increase in area to 5cm^2. If the pressure at the upper level is 2 atm then find the pressure at lower level. (g =10 m/s^2)



Solution:

Given - A_1 = 4cm^2, A_2 = 5cm^2

$h_1 - h_2$ = 10 m

v_1 = 5 m/sec

From equation of continuity $\rightarrow$

$A_1v_1 = A_2v_2$

(4) (5) = (5) $v_2 \Rightarrow v_2$ = 4 m/sec

Applying Bernoulli's theorem –

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2$$

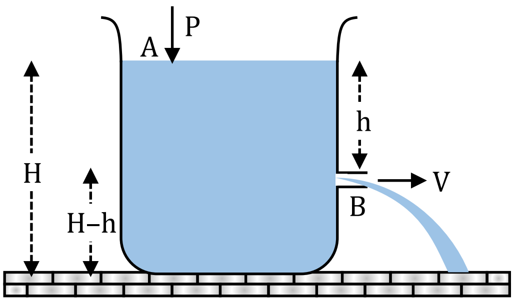
$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) + \rho g(h_1 - h_2)$$

$$P_2 = (2\times10^5) + \frac{1}{2}(10^3)(5^2 - 4^2) + (10^3)(10)(10) = 2 \times 10^5 + 4.5 \times 10^3 + 1 \times 10^5 = 3.045 \text{ atm}$$

Torricelli's law

Speed of Efflux

As shown in the figure the area of cross-section of the vessel A is very large as compared to that orifice B, therefore speed of liquid flow at A is zero i.e. $v_A \approx 0$. The fluid at sections A and B are at the same pressure P (atmospheric pressure). Applying Bernoulli's theorem at A and B.



$$P_0 + \rho gH + \frac{1}{2}\rho v_A^2 = P_0 + \rho g(H-h) + \frac{1}{2}\rho v_B^2$$

or $\frac{1}{2}\rho v_B^2 = \rho gh$ or $v_B = \sqrt{2gh}$

This equation is same as that of the velocity acquired by a freely falling body after falling through h height and is known as **Torricelli's law**.

Writing the equation of uniformly accelerated motion in the vertical direction

$$H - h = 0 + \frac{1}{2}gt^2 \left(\text{from } s_y = u_y t + \frac{1}{2}a_y t^2\right)$$

$\Rightarrow t = \sqrt{\frac{2(H-h)}{g}}$, t = time of flight as in case of horizontal projection from the top of a tower.

Horizontal range $R = v_x t = \sqrt{2gh} \times \sqrt{\frac{2(H-h)}{g}}$ or $R = 2\sqrt{h(H-h)}$

Range will be maximum when h = H – h or $h = \frac{H}{2}$

$$\therefore \quad R_{max} = 2\sqrt{\frac{H}{2}\left[H - \frac{H}{2}\right]} = H$$

Reaction force on vessel

By Newton's third law –

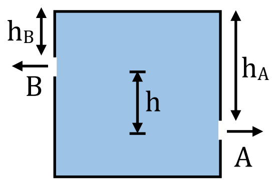
$$F = v\left(\frac{dm}{dt}\right) = v\frac{d(\rho V)}{dt} = v\rho\frac{dV}{dt} = v\rho(Av)$$

$$F = \rho Av^2$$

Illustration 26

There are two identical small holes on the opposite sides of a tank containing a liquid. The tank is open at the top. The difference in heights between the two holes is h. Find the resultant force of reaction of liquid flowing out of vessel.

Solution:



Net reaction force

$F = F_A - F_B$

$$= \rho a\left(v_A^2 - v_B^2\right)$$

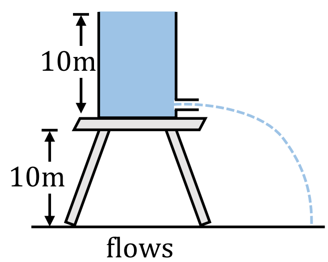
$$= \rho a(2gh_A - 2gh_B) = 2\rho agh$$

Illustration 27

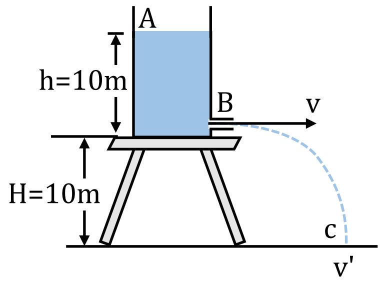
A cylindrical tank 1m in radius rests on a platform 10 m high. Initially, the tank is filled with water to a height of 10 m. A small plug whose area is 10^{-4} m^2 is removed from an orifice located on the side of the tank at the bottom. Calculate the :

(i) initial speed with which water flows out from the orifice

(ii) initial speed with which the water strikes the ground.



Solution:



(i) Applying Bernoulli's theorem between the water surface and the orifice,

$$P_0+\frac{1}{2}\rho(0)^2+\rho gh=P_0+\frac{1}{2}\rho v^2+\rho g(0)$$

$$\rho gh=\frac{1}{2}\rho v^2; v=\sqrt{2gh}=\sqrt{2\times10\times10}=10\sqrt{2}\ m/s$$

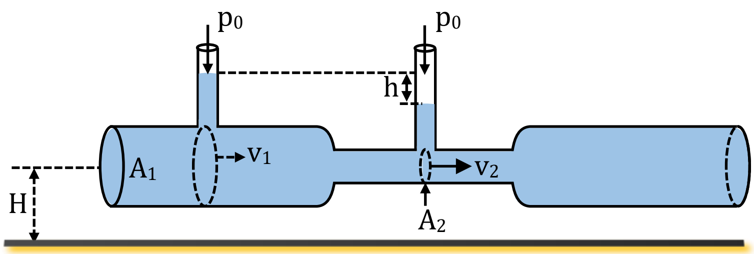
(ii) Let v' be the initial velocity with which the water strikes the ground

Then, applying Bernoulli's theorem between the top of the tank and the ground level, we get

$$v'=\sqrt{2g(H+h)}=\sqrt{2\times10\times20}=20\,m/s$$

Venturimeter

Venturimeter is used to measure the flow velocities in an incompressible fluid. As shown in figure if P_1 and P_2 are the pressures and v_1 and v_2 are the velocities of the fluid of density ρ at points 1 and 2 on the same horizontal level and A_1 and A_2 be the respective areas, then from equation of continuity



$A_1v_1 = A_2v_2$ or $v_2=\left[\frac{A_1}{A_2}\right]v_1$ ...(i)

From Bernoulli's equation for horizontal flow, $P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho v_2^2$

Or $P_1+\frac{1}{2}\rho v_1^2=P_2+\frac{1}{2}\rho\left[\frac{A_1^2}{A_2^2}\right]v_1^2$ [from equation (i)]

Or $P_1-P_2=\frac{1}{2}\rho v_1^2\left[\frac{A_1^2}{A_2^2}-1\right]$

But $P_1 - P_2 = \rho gh$ [$\because$ difference in heights between the liquid surfaces in the two arms is h]

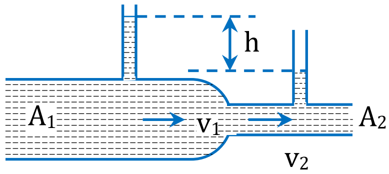
$\therefore$ $\rho gh=\frac{1}{2}\rho v_1^2\left[\frac{A_1^2}{A_2^2}-1\right]$

$\therefore$ $v_1=\sqrt{2gh}\left[\frac{A_1^2}{A_2^2}-1\right]^{-\frac{1}{2}}=A_2\sqrt{\frac{2gh}{A_1^2-A_2^2}}$

If Q be the volume of liquid flowing per unit time then $Q=A_1v_1=A_2v_2=A_1A_2\sqrt{\frac{2gh}{A_1^2-A_2^2}}$

Thus at a point where the cross-sectional area is smaller velocity is greater and pressure is lower and vice versa.

Illustration 28



A liquid flows through a horizontal tube. The velocities of the liquid in the two sections, which have areas of cross-section A_1 and A_2, are v_1 and v_2 respectively. The difference in the levels of the liquid in the two vertical tubes is *h*

(1) The volume of the liquid flowing through the tube in unit time is A_1v_1

(2) $v_2 - v_1 = \sqrt{2gh}$

(3) $v_2^2 - v_1^2 = 2gh$

(4) The energy per unit mass of the liquid is the same in both sections of the tube

Solution: (1,3,4)

According to equation of continuity the volume of liquid flowing through the tube in unit time remains constant *i.e.* $A_1v_1 = A_2v_2$, hence option (1) is correct

According to Bernoulli's theorem,

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \Rightarrow h\rho g = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right)$$

$$\therefore\ v_2^2 - v_1^2 = 2gh$$

Hence option (3) is correct.

Also, according to Bernoulli's theorem option (4) is correct

Dynamic Lift

Aerofoil

This is a structure which is shaped in such a way so that its motion relative to a fluid produces a force perpendicular to the flow. As shown in the figure the shape of the aerofoil section causes the fluid to flow faster over the top surface than below the bottom i.e. the streamlines are closer above than below the aerofoil. By Bernoulli's theorem the pressure at above reduced whereas that underneath it gets increased. Thus, a resultant upward thrust is generated normal to the flow and it is this force which provides most of the upward lift for an aeroplane.

Examples of aerofoils are aircraft wings, turbine blades and propellers.

Applying Bernoulli's theorem between upper and bottom surface of aerofoil.

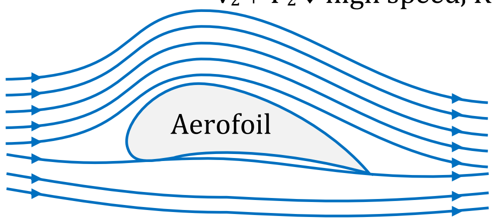
$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

$$\Rightarrow \quad P_1 - P_2 = \frac{1}{2}\rho_{air}\left(v_2^2 - v_1^2\right)$$

Net upward lifting force

$$F = (P_1 - P_2)A = \frac{1}{2}\rho_{air}A\left(v_2^2 - v_1^2\right)$$

$v_2\uparrow$ $P_2\downarrow$ high speed, Reduced Pressure

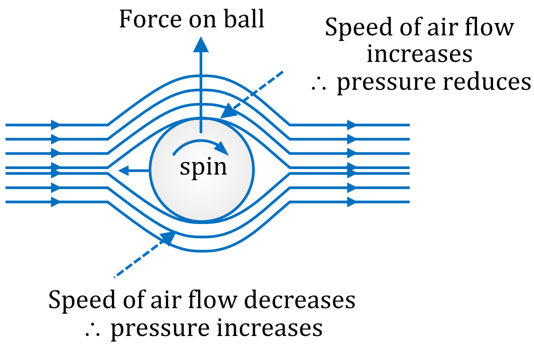


$V_1\downarrow$ $P_1\uparrow$ low speed, Pressure increased

Conceptual Applications of Bernoulli's Theory

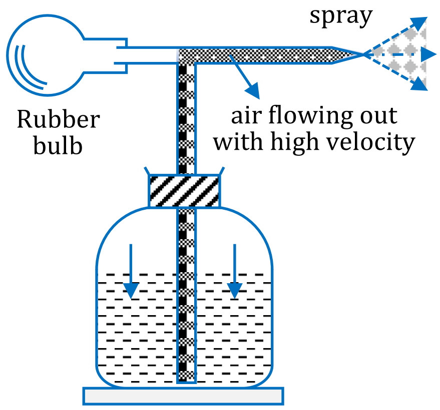
Magnus Effect (Observed in a Spinning Ball)

Tennis and cricket players usually experience that when a ball is thrown spinning, it moves along a curved path. This is called swing of the ball. This is due to the air which is being dragged round by the spinning ball. When the ball spins, the layer of the air around it also moves with the ball. So, as shown in figure the resultant velocity of air increases on the upper side and reduces on the lower side. Hence according to Bernoulli's theorem, the pressure on the upper side becomes lower than that on the lower side. This pressure difference exerts a force on the ball due to which it moves along a curved path. This effect is known as Magnus-effect.



Sprayer or Atomizer

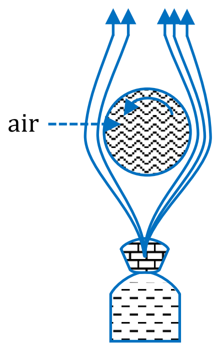
This is an instrument used to spray a liquid in the form of small droplets (fine spray). It consists of a vertical tube whose lower end is dipped in the liquid to be sprayed, filled in a vessel. The upper end opens in a horizontal tube. At one end of the horizontal tube there is a rubber bulb and the other end has is a fine bore (hole). When the rubber bulb is squeezed, air rushes out through the horizontal tube with very high velocity and thus the pressure reduces (according to Bernoulli's theorem). Consequently, the liquid in the vessel rises up and mixes with air in the form of small droplets which gets ejected in the form of a fine spray.



Example: paint guns, perfume or deodorant sprayer, etc.

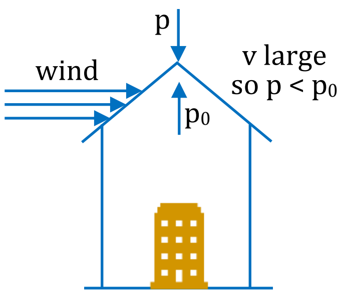
Motion of the Ping-Pong Ball

When a ping-pong ball is placed on a vertical stream of water-fountain, it rises upto a certain height above the nozzle of the fountain and spins about its axis. The reason for this is that the air streams of water rise up from the fountain with large velocity so that the air-pressure decreases. Therefore, whenever the ball tends to fall out from the stream, the outer air which is at atmospheric pressure pushes it back into the stream (in the region of low pressure). Thus, the ball remains more or less stable in the fountain.



Blowing-off of Tin Roof Tops in Wind Storm

When wind blows with a high velocity above a tin roof, it causes lowering of pressure above the roof, while the pressure below the roof is still atmospheric. Due to this pressure-difference the roof is lifted up and is blown away during storms.

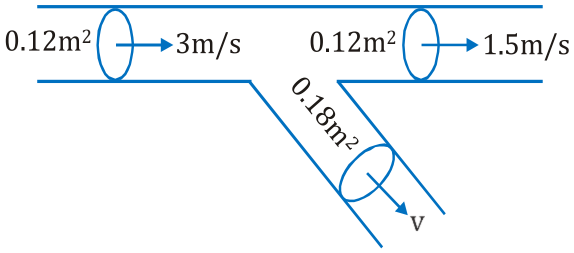


Pull-in or Attraction Force by Fast Moving Trains

If we are standing on a platform and a train passes through the platform with very high speed we are pulled towards the train. This is because as the train moves at high speed, the pressure close to the train decreases. Thus, the air away from the train which is still at atmospheric pressure pushes us towards the train. The reason behind flying-off of small papers, straws and other light objects towards the train is also the same.

BEGINNER'S BOX-4

1. An incompressible liquid flows as shown in the figure. Calculate the speed v of the fluid in the lower branch.

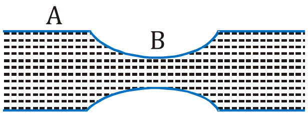


PPF183

2. A hole is made in a vessel containing water at a depth of 3.2 m below the free surface. What would be the velocity of efflux?

PPF184

3. Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m^2 between A and B where the area of cross-section are 30 cm^2 and 15 cm^2 respectively. Find the rate of flow of water through the tube.



PPF185

Molecular Theory of Surface Tension

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum possible area. This property of liquid is called surface tension.

Intermolecular forces

Forces of attraction or repulsion acting among the molecules are known as intermolecular forces. The nature of intermolecular force is electromagnetic.

There are two types of intermolecular attractive forces.

Cohesive Force

The force of attraction acting between the molecules of same material is defined as cohesive force.

Ex. : force acting between water molecules, Hg molecules.

Adhesive Force

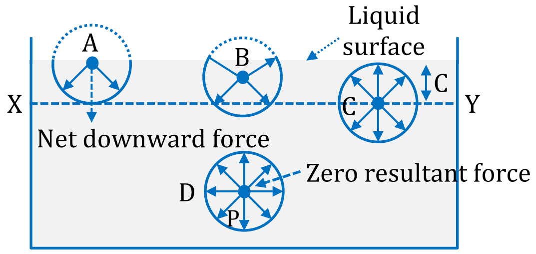
The force of attraction acting between molecules of two different materials is defined as adhesive force.

Ex. : Force acting between the molecules of paper and ink, black board and chalk etc.

- Intermolecular forces are different from gravitational forces in the sense that the former does not obey inverse- square law.
- The distance upto which these forces remains effective, is called molecular range. This distance is nearly 10^{-9} m. Within this limit the forces increases very rapidly as the distance decreases.
- Molecular range depends on the nature of the substance.

Explanation of Surface Tension

Laplace explained the phenomenon of surface tension on the basis of intermolecular forces. According to him surface tension is a molecular phenomenon and its root cause is electromagnetic force. He explained the cause of surface tension as described below. If the distance between two molecules is less than the molecular range C ($\approx 10^{-9}$ m) then they attract each other, but if the distance is more than this, the attraction becomes negligible. If a sphere of radius C with a molecule at centre is drawn, then only those molecules which are enclosed within this sphere can attract or be attracted by the molecule at the centre of the sphere. This sphere is called sphere of molecular activity or sphere of influence. In order to understand the tension acting at the free surface of liquid, let us consider four liquid molecules A, B, C and D along with their spheres of molecular activity.



(a) According to figure sphere D is completely inside liquid. So, molecule is attracted equally in all directions and hence resultant cohesive force is equal to zero.

(b) According to figure, sphere of molecule C is just below the liquid surface. So resultant cohesive force is equal to zero.

(c) The molecule B which is a little below the liquid surface is attracted downwards due to excess of molecules present below. Hence the resultant cohesive force is acting downwards.

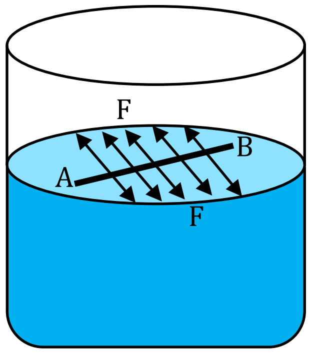
(d) Molecule A is situated at the surface so that its sphere of molecular activity is half outside the liquid and half inside. Only lower portion has liquid molecules. Hence it experiences a maximum downward force. Thus, all the molecules situated between the surface and a plane XY, distant C below the surface, experience a resultant downward cohesive force.

When the surface area of liquid is increased molecules from the interior of the liquid rise to the surface. As these molecules reach close to the surface, work is done against the downward cohesive force. This work is stored in the molecules in the form of potential energy. Thus, the potential energy of the molecules lying close to the surface is greater than that of the molecules in the interior of the liquid. A system is in stable equilibrium when its potential energy is minimum. Hence in order to have minimum potential energy the liquid surface tends to have minimum number of molecules. In other words, any surface tends to contract to a minimum possible area. This tendency is exhibited as surface tension.

Mathematical Formula of Surface Tension

The force acting per unit length on one side of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is defined as surface tension.

Let an imaginary line AB be drawn in any direction on a liquid surface. The surface on either side of this line exerts a pulling force, which is perpendicular to line AB. If force is F and length of AB is L then surface tension $T = \frac{F}{L}$



SI UNITS : N/m and J/m^2

CGS UNITS : dyne/cm and erg/cm^2

Dimensions : [$M^1L^0T^{-2}$]

Illustration 29

A stick of length 5 cm is floating on the surface of water. If one side of it has surface tension of 70 dyne/cm and on the other side by keeping a piece of camphor the surface tension is reduced to 50 dyne/cm, then find the resultant force (in dyne) acting on the stick.

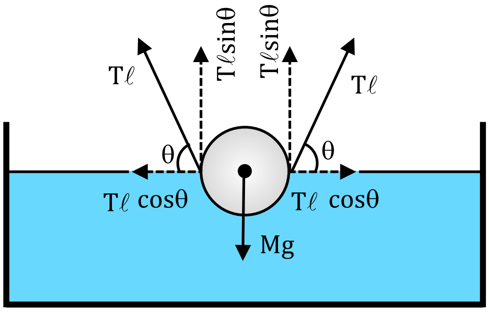
Solution:

$F_{net} = F_1 - F_2 = (T_1 - T_2)\ell = (70 - 50)\,5 = 100$ dyne

Bodies on Liquid Surface

Floatation of bodies due to surface tension

(i) When a needle floats on the liquid surface then $2T\ell \sin\theta = Mg$



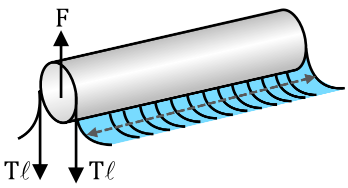
Special Case : -

Maximum mass of needle which can float on the liquid surface

In this case –

$$\theta = 90°$$
$$\therefore\ 2T\ell = M_{max}g$$
$$\therefore\ M_{max} = \frac{2T\ell}{g}$$

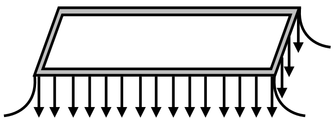
(ii) If the needle is lifted from the liquid surface then required excess force will be $F_{excess} = 2T\ell$



Minimum force required $F_{min} = Mg + 2T\ell$

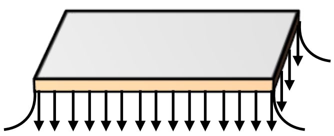
(iii) Required excess force for a square frame of side ℓ

$F_{excess} = 8\,T \times \ell$



(iv) Required excess force for a square lamina of side ℓ

$F_{excess} = 4\,T \times \ell$



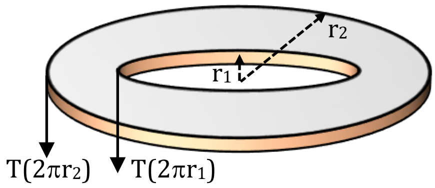
(v) Required excess force for a rectangular frame of length ℓ and width b

$F_{excess} = 4\,T\,(\ell + b)$

(vi) Required excess force for a rectangular lamina of length ℓ and width b

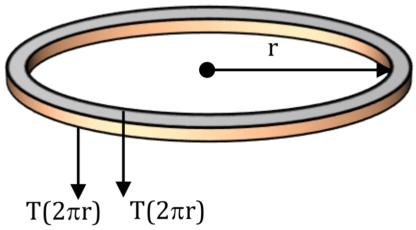
$F_{excess} = 2\,T\,(\ell + b)$

(vii) Required excess force for a circular thick ring (or annular ring) having internal and external radii r_1 and r_2 is dipped in and taken out from liquid. $F_{excess} = F_1 + F_2 = T(2\pi\, r_1) + T(2\pi\, r_2) = 2\pi T(r_1 + r_2)$.



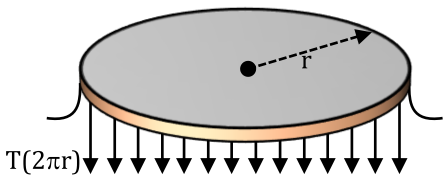
(viii) Required excess force for a circular ring ($r_1 = r_2 = r$)

$F_{excess} = 2\pi\, T(r + r) = 4\pi rT$.



(ix) Required excess force for a circular disc ($r_1 = 0$, $r_2 = r$)

$F_{excess} = 2\pi rT$



Dependency of Surface Tension

- **On Cohesive Force**

  Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.

- **On Impurities**

  If the impurity is completely soluble then on dissolving it in the liquid, its surface tension increases. e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively, e.g. On mixing detergent in water its surface tension decreases.

- **On Temperature**

  On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero.

  Note : Surface tension of water is maximum at 4°C.

- **On Contamination**

  Dust particles or lubricating materials on the liquid surface decrease its surface tension.

Effects of surface tension

(i) Small liquid drops and soap bubbles are spherical.

(ii) The hairs of the brush remain separated from each other inside water, but when the brush is taken out, the hairs stick together.

(iii) Floatation of needle on water.

(iv) Formation of lead shots.

(v) Dirty clothes become clean in hot detergent solution in comparison to pure water at room temperature.

Illustration 30

The length of a needle floating on water is 2.5 cm. Calculate the additional force required to pull the needle out of water. [$T = 7.2 \times 10^{-2}$ N/m]

Solution:

$F = T(2\ell) = (7.2 \times 10^{-2})(2 \times 2.5 \times 10^{-2}) = 3.6 \times 10^{-3}$ N

Illustration 31

Find the maximum weight of needle which can float on water having surface tension 0.05 N/m. Length of needle is 1 cm.

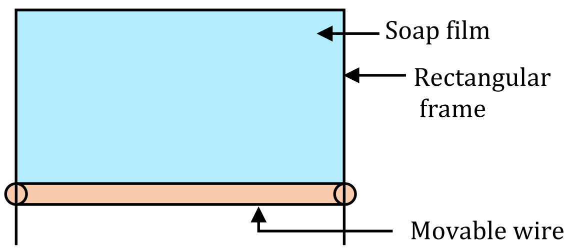
Solution:

$W_{max} = 2T\ell = 2(0.05)(1\times10^{-2}) = 1 \times 10^{-3}$ N

Illustration 32

If a soap film is formed on the frame then find the radius of the wire to maintain equilibrium.

Surface tension of soap solution $= 5\times\frac{10^{-3}N}{m}$ Density of wire $= \frac{\frac{5}{\pi}kg}{m^3}, g = \frac{10m}{s^2}$



Solution:

At equilibrium $\rightarrow Mg = 2T\ell$

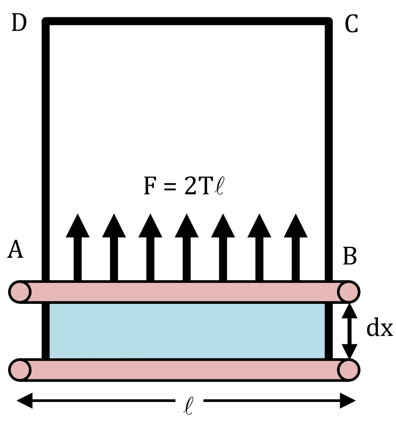
$\rho(\pi r^2\ell)g = 2T\ell$

$r = \sqrt{\frac{2T}{\pi\rho g}} = 1.4 \times 10^{-2}$ m $= 1.4$ cm

Surface Energy

According to molecular theory of surface tension the molecules on the surface have certain additional energy due to their position. This additional energy of the surface is called "Surface energy".

Let a liquid film be formed on a wire frame and a straight wire of length ℓ can slide on this wire frame as shown in figure. The film has two surfaces and both the surfaces are in contact with the sliding wire and hence, exert forces of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force $T\ell$. Thus, the net force on the wire due to both the surfaces is $2T\ell$.



Apply an external force F equal and opposite to it to keep the wire in equilibrium. Thus, $F=2T\ell$

Now, suppose the wire is moved through a small distance dx, then work done by the force is

$dW = F\,dx = (2T\ell)\,dx$

But (2ℓ) (dx) is the total increase in area of both the surfaces of the film. Let it be dA

then $dW = T\,dA \Rightarrow T = dW/dA$

Thus, the surface tension T can also be defined as the work done in increasing the surface area by unity. Further, since there is no change in kinetic energy, work done by the external force is stored as potential energy of the new surface. $T = \frac{dU}{dA}$ [as dW = dU]

Special Cases

Work done in blowing of small drop of radius r_1 to large drop of radius r_2.

$W = T \times \Delta A$ or $W = T \times (4\pi r_2^2 - 4\pi r_1^2) = 4\pi T(r_2^2 - r_1^2)$

Work done in blowing of small bubble of radius r_1 to large bubble of radius r_2.

$W = T \times \Delta A$ or $W = T \times 2 \times (4\pi r_2^2 - 4\pi r_1^2) = 8\pi T(r_2^2 - r_1^2)$

Work done (surface energy) in formation of a drop of radius r :-

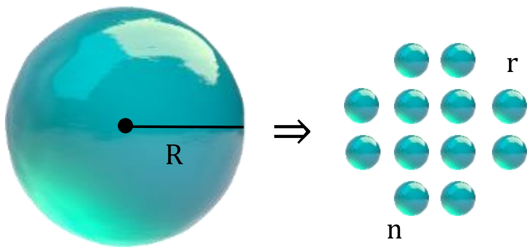
$W = T \times \Delta A = T \times 4\pi r^2 = 4\pi r^2 T$

Work done (surface energy) in blowing of a soap bubble of radius r :-

$W = T \times \Delta A = T \times 2 \times 4\pi r^2 = 8\pi r^2 T$

Splitting of big drop into smaller droplets

If a big a drop is split into smaller droplets then in this process volume of liquid always remain conserved. Let the big drop have a radius R. It is splitted into n smaller drops of radius r then by conservation of volume



(i) $\frac{4}{3}\pi R^3 = n\left[\frac{4}{3}\pi r^3\right] \Rightarrow n = \left[\frac{R}{r}\right]^3 \Rightarrow r = \frac{R}{n^{\frac{1}{3}}}$

(ii) Initial surface area = $4\pi R^2$ and final surface area = $n(4\pi r^2)$

Change in area $\Delta A = n4\pi r^2 - 4\pi R^2 = 4\pi\,(nr^2 - R^2)$.

Therefore the amount of surface energy absorbed i.e. $\Delta E = E_f - E_i = 4\pi T(nr^2 - R^2)$

$\therefore$ Magnitude of work done against surface tension i.e. $W = 4\pi(nr^2 - R^2)T$

$$W = 4\pi T\,(nr^2 - R^2) = 4\pi R^2 T\,(n^{1/3} - 1) = 4\pi R^2 T\left[\frac{R}{r} - 1\right] \Rightarrow W = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

Initial surface energy $E_i = 4\pi R^2 T$ and final surface energy $E_f = n(4\pi r^2 T)$

$$\frac{(S.E)_f}{(S.E)_i} = \frac{TA_f}{TA_i} = \frac{n(4\pi r^2)}{4\pi R^2} = n\left(\frac{r}{R}\right)^2$$

$$\frac{(S.E)_f}{(S.E)_i} = n\left(\frac{1}{n^{2/3}}\right) = n^{1/3}$$

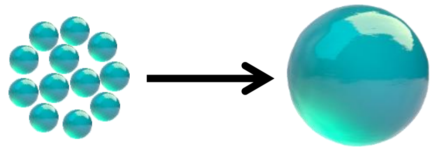
In this process area increases, surface energy increases, internal energy decreases, temperature decreases, and energy is absorbed.

S. A. $\uparrow\rightarrow$ S. E. $\uparrow\rightarrow$ I. E. $\downarrow\rightarrow$ Temp $\downarrow\rightarrow$ Energy Absorbed

$$W = Jms\Delta\theta = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right] \Rightarrow \Delta\theta = \frac{4\pi R^3 T}{\frac{4}{3}\pi R^3 J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right] = \frac{3T}{J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Where ρ = liquid density, s = specific heat of liquid, J = Joule's constant

Droplets are coalesce to form a big drop



$|W| = 4\pi R^2 T (n^{1/3} - 1)$

$$\frac{(S.E)_f}{(S.E)_i} = \frac{1}{n^{1/3}}$$

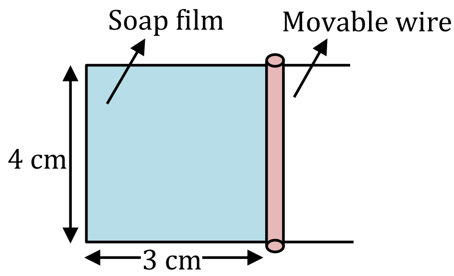
S. A.↓→ S. E.↓→ I. E.↑→ Temp ↑→ Energy Released

Thus, in this process surface area decreases, surface energy decreases, internal energy increases, temperature increases, and energy is released.

Illustration 33

If work done required to displace a movable wire by 2cm is 6×10^{-4}J. Then find out surface tension of the soap

Solution:



$W = T(2\Delta A)$

$$\Rightarrow T = \frac{W}{2\Delta A} = \frac{6\times10^{-4}}{2[20\times10^{-4} - 12\times10^{-4}]}$$

$$= \frac{6}{2\times8} = \frac{3}{8} = 0.375 \text{ N/m}$$

Illustration 34

Calculate work done required to increase the area of a rectangular film of liquid from (4cm × 5cm) to (5cm × 6cm). (surface tension of the liquid is 0.3 N/m)

Solution:

$W = T(2\Delta A) = 0.3 \times 2 \times (30 \times 10^{-4} - 20 \times 10^{-4}) = 6\times10^{-4}$ J

Illustration 35

If a big drop of radius R spitted into 27 identical small droplets. Then find out work done in this process.

Solution:

$W = 4\pi R^2T [n^{1/3} - 1] = 4\pi R^2T [(27)^{1/3} - 1] = 8\pi R^2T$

Illustration 36

If a drop of mercury having radius 1 cm is broken into 10^6 identical small droplets. If surface tension of mercury is 35×10^{-3} N/m, then find out work done in this process.

Solution:

$$W = 4\pi R^2T [n^{1/3} - 1] = 4 \times \frac{22}{7} \times (10^{-2})^2 \times 35 \times 10^{-3} (100 - 1) = 4.35 \times 10^{-3}\text{J}$$

Illustration 37

A big drop of radius 1 cm is converted into 1000 small identical droplets. If surface tension of liquid is 70 dyne/ cm. Then find out :

(1) radius of small drop

(2) work done required

(3) ratio of final surface energy to initial surface energy.

Solution:

(1) $$\frac{R}{r} = n^{\frac{1}{3}} \Rightarrow \frac{1\text{cm}}{r} = 10 \Rightarrow r = 0.1 \text{ cm}$$

(2) $$\text{W.D.} = 4\pi R^2T (n^{1/3} - 1) = 4 \times \frac{22}{7} (1)^2 \times 70 \times (10 - 1) = 7920 \text{ erg.}$$

(3) $$\frac{SF_f}{SE_i} = \frac{n\times4\pi r^2T}{4\pi R^2T} = n^{\frac{1}{3}} = 10$$

BEGINNER'S BOX-5

1. A stick of length 4 cm is floating on the surface of water. If one side of it has surface tension of 80 dyne/cm and on the other side by keeping a piece of camphor the surface tension is reduced to 60 dyne/cm, then find the resultant force (in dyne) acting on the stick.
PPF186

2. A thin wire ring of radius 1m is situated on the surface of a liquid. If the excess force required to lift it upwards (before the liquid film breaks) from the liquid surface is 8 N. calculate the surface tension of liquid?
PPF187

3. A circular frame made of 20 cm long thin wire is floating on the surface of water. The surface tension of water is 70 dyne/cm. Calculate the required excess force to separate this frame from water?
PPF188

4. A metallic wire of density d floats horizontally in water. Find out the maximum radius of the wire so that the wire may not sink. (surface tension of water = T)
PPF189

5. A rectangular film of liquid is 5 cm long and 3 cm wide. If the work done in increasing its area to 9 cm × 5 cm is 6×10^{-4} joule. find the surface tension of the liquid?
PPF190

6. Calculate the surface energy of ring floating on the liquid surface? (surface tension of liquid is 75 N/m and area of ring is 0.04 m^2)
PPF191

7. Find the work done in increasing the volume of a soap bubble by 700% if its radius is R and surface tension is T.
PPF192

8. A bubble of radius 2 cm is blown inside a cold drink using a straw. If the surface tension of the liquid is 60 dyne/cm find the work done (in ergs) in blowing the bubble.
PPF193

9. 10^6 tiny drops coalesce to form a big drop. The surface tension of liquid is T. Calculate the percentage fractional energy loss.
PPF194

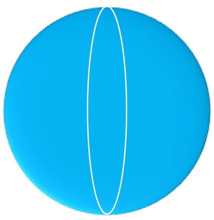
10. A liquid drop of radius 'r' is divided into 64 similar tiny droplets. If the surface tension of liquid is T, calculate the increase in energy?
PPF195

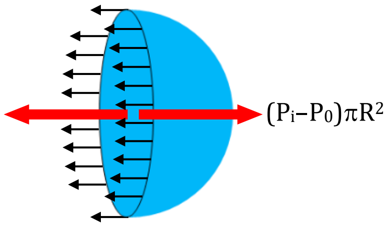
Excess Pressure

Excess pressure inside a curved liquid surface

The pressure on the concave side of curved liquid surface is greater than that on the convex side. Therefore, a pressure difference exists across two sides of a curved surface. This pressure difference is called excess pressure.

(i) Excess pressure inside the drop

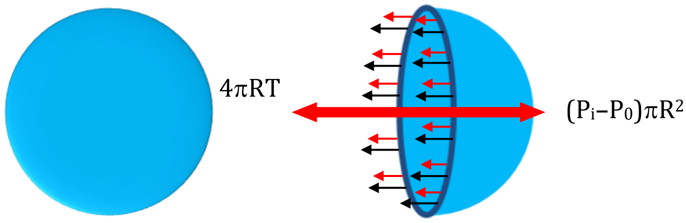




For equilibrium of hemispherical drop

$$2\pi RT = (P_i - P_0)\pi R^2 \quad \text{or } (P_i - P_0) = \frac{2T}{R} \qquad P_{ex} = \frac{2T}{R}$$

(ii) Excess pressure inside a soap bubble :

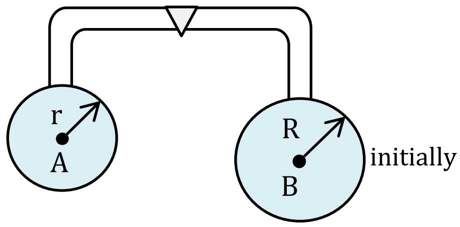


For equilibrium of hemispherical bubble

$4\pi RT = (P_i - P_0)\pi R^2$ or $(P_i - P_0) = \dfrac{4T}{R}$ $P_{ex} = \dfrac{4T}{R}$

Special Cases :

(i) Two unequal soap bubbles are formed one on each side of a tube closed in the middle by a tap. When the tap is opened, then:-



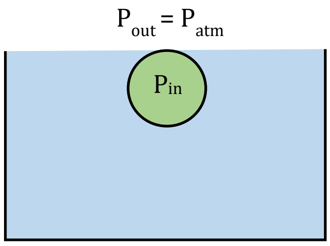
$P_A = P_0 + \dfrac{4T}{r}$;

$P_B = P_0 + \dfrac{4T}{R}$ {P_0 = atmospheric pressure}.

Clearly $P_A > P_B$; so air will flow from A to B.

Ultimately bubble A becomes smaller and bubble B becomes bigger in size.

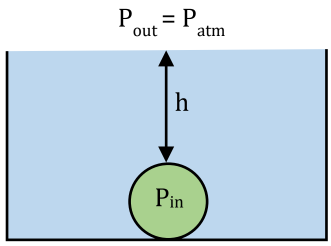
(ii) Total pressure inside an air bubble in liquid



$P_{in} = P_{out} + P_{ex}$

$P_{in} = P_{out} + \dfrac{2T}{R}$

$P_{in} = P_o + \dfrac{2T}{R}$



$P_{in} = P_{out} + P_{ex}$

$P_{in} = P_{out} + \dfrac{2T}{R}$

$P_{in} = P_o + \rho gh + \dfrac{2T}{R}$

(iii) Total pressure inside a cavity or air bubble in liquid

$P_{excess} = P_{in} - P_{out} = \rho gh + \dfrac{2T}{R}$ (ρ density of liquid)

$P_{inside} = P_{atm} + \rho gh + \dfrac{2T}{R}$

(iv) If two bubbles of radii r_1 and r_2 coalesce isothermally in vacuum then the radius of the new bubble will be :

When two bubbles coalesce then total number of molecules of air will remain same and temperature will also remain constant

So $n_1 + n_2 = n \Rightarrow P_1 V_1 + P_2V_2 = PV \Rightarrow \dfrac{4T}{r_1}\left(\dfrac{4}{3}\pi r_1^3\right) + \dfrac{4T}{r_2^2}\left(\dfrac{4}{3}\pi r_2^3\right) = \dfrac{4T}{r}\left(\dfrac{4}{3}\pi r^3\right) \Rightarrow r = \sqrt{r_1^2 + r_2^2}$

(v) If two bubbles of radii r_1 and r_2 ($r_1 < r_2$) come in contact with each other then the radius of curvature of the common surface:

$\because$ $r_1 < r_2$

$\therefore$ $P_1 > P_2$ Small portion of bubbles is in contact and in equilibrium

$\Rightarrow$ $P_1 - P_2 = \frac{4T}{r} \Rightarrow \frac{4T}{r_1} - \frac{4T}{r_2} = \frac{4T}{r} \Rightarrow r = \frac{r_1 r_2}{r_2 - r_1}$

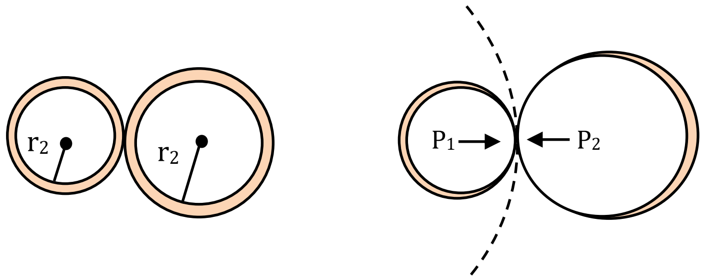


Illustration 38

If the excess pressure inside two soap bubbles are 1.02 atm and 1.03 atm respectively. Find ratio of their volumes.

Solution:

$$\frac{(P_{ex})_1}{(P_{ex})_2} = \frac{(P_{in} - P_{out})_1}{(P_{in} - P_{out})_2} = \frac{1.02 - 1}{1.03 - 1} = \frac{2}{3} \Rightarrow \frac{\frac{4T}{r_1}}{\frac{4T}{r_2}} = \frac{2}{3} \Rightarrow \frac{r_1}{r_2} = \frac{3}{2} \Rightarrow \frac{V_1}{V_2} = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

Illustration 39

The excess pressure inside an air bubble of radius r just below the surface of water is P_1. The excess pressure inside a drop of the same radius just outside the surface is P_2. If T is surface tension, then find the relation between P_1 and P_2.

Solution:

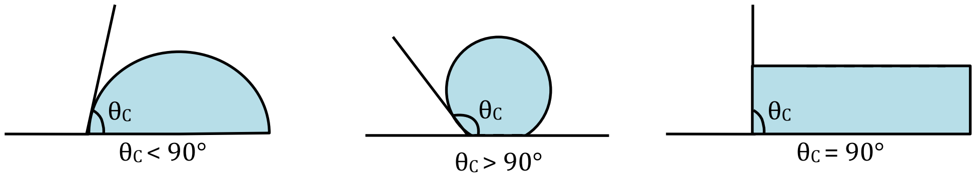
Excess pressure in air bubble just below the water surface. $P_1 = \frac{2T}{r}$

Excess pressure inside a drop $P_2 = \frac{2T}{r}$

So, $P_1 = P_2$

Angle of Contact (θ_C)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the angle of contact. The angle of contact depends on the nature of the solid and liquid in contact



Some important angle of contact-

(i) Pure water and glass : $\rightarrow 0°$

(ii) Ordinary water and glass : $\rightarrow$ 8° to 18°

(iii) Water and silver : $\rightarrow 90°$

(iv) Mercury and glass : $\rightarrow 138°$

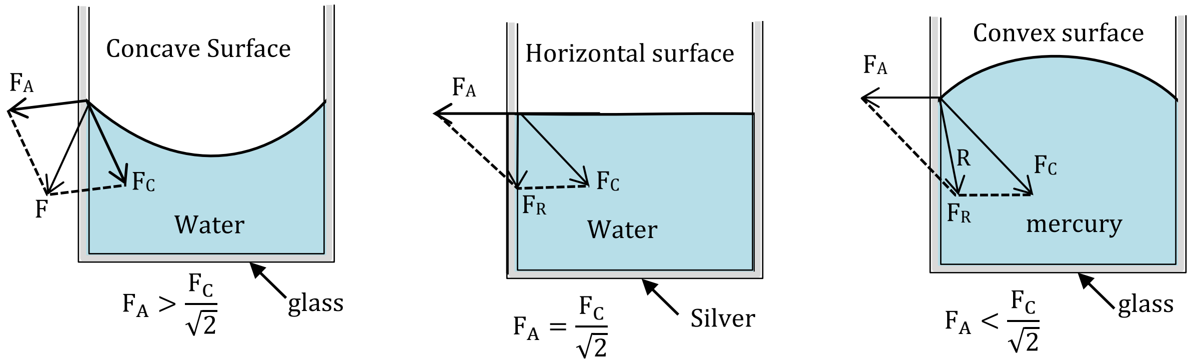
Shape of Liquid Surface (Shape of meniscus)

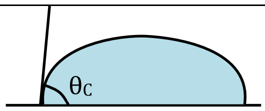
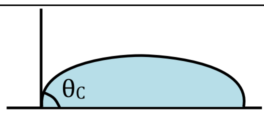
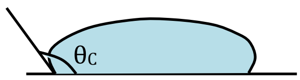
When a liquid is brought in contact with a solid surface, the surface of the liquid becomes curved near the place of contact. The shape of the surface (concave or convex) depends upon the relative magnitude of cohesive force between the liquid molecules and the adhesive force between the molecules of the liquid and the solid.

The free surface of a liquid which is near the walls of a vessel and which is curved because of surface tension is known as meniscus. The cohesive force acts at an angle of 45° from liquid surface whereas the adhesive force acts at right angles to the solid surface.

The relation between the shape of liquid surface, cohesive/adhesive forces, angle of contact, etc. are summarized in the table below :

Relation between cohesive and adhesive force



Shape of meniscus	Concave	Plane	Convex
Angle of contact	$\theta_C < 90°$(Acute angle)	$\theta_C = 90°$ (Right angle)	$\theta_C > 90°$ (Obtuse angle)
Shape of liquid drop			
Level of liquid	Liquid rises up	Liquid neither rises nor falls	Liquid falls
Wetting property	Liquid wets the solid surface	Liquid does not wet the solid surface	Liquid does not wet the solid surface
Example	Glass – Water	Silver – Water	Glass – Mercury

Dependency of Angle of Contact

(i) Effect of Temperature on angle of contact

On increasing temperature surface tension decreases, thus $\cos\theta_C$ $\left[\because \cos\theta_C \propto \frac{1}{T}\right]$ decreases and θ_c increases.

So, on increasing temperature, θ_c decreases.

(ii) Effect of Impurities on angle of contact

(a) Soluble impurities increase surface tension, so $\cos\theta_c$ decreases and angle of contact θ_c increases.

(b) Partially soluble impurities decrease the surface tension, so angle of contact θ_c decreases.

(iii) Effect of Water Proofing Agent

Angle of contact increases due to the presence of water proofing agent. It changes from acute to obtuse angle.

Capillary Tube and Capillarity

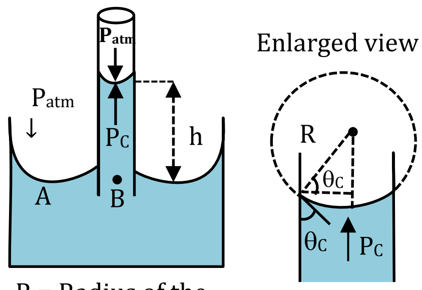
A glass tube with a fine bore and open at both ends is known as a capillary tube. The property by virtue of which a liquid rises or gets depressed in a capillary tube is known as capillarity. Rise or fall of liquid in tubes of narrow bore (capillary tube) is called capillary action.

Ex. : (i) Kerosene oil in lanterns rise upward due to capillarity.
(ii) Working of fountain pen's nib split due to capillarity.

Calculation of Capillary Rise

(i) Pressure Balance Method :

When a capillary tube is first dipped in a liquid as shown in the figure, the liquid climbs up the walls curving the surface. Let the radius of the meniscus be R and the radius of the capillary tube be r. Angle of contact is θ_C, surface tension is T, density of liquid is ρ and the liquid rises to a height h.



R = Radius of the meniscus

$R = \dfrac{r}{\cos\theta}$

Now let us consider two points A and B at the same horizontal level as shown. By Pascal's law

$P_A = P_B \qquad \Rightarrow \qquad P_A = P_C + \rho g h$

Now, point C is on the curved meniscus which has P_{atm} and P_C as the pressures on its concave and convex sides respectively.

$$\therefore \quad P_{atm} = \left(P_{atm} - \frac{2T}{R}\right) + h\rho g \Rightarrow h = \frac{2T}{R\rho g} = \frac{2T\cos\theta_C}{r\rho g}$$

(ii) Force Balance Method :

The liquid continues to rise in the capillary tube until the weight of the liquid column becomes equal to force due to surface tension.

On liquid force due to surface tension = $(2\pi r)\, T\cos\theta_C$

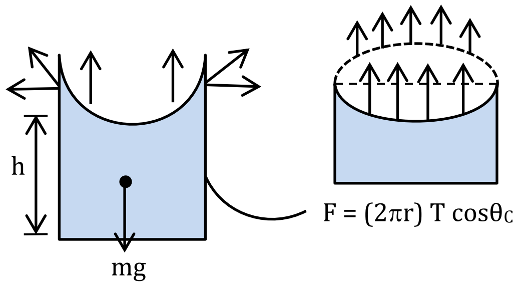
In equilibrium :

force due to S.T = weight of rise liquid

$(2\pi r)T\cos\theta_C = mg$

$(2\pi r)T\cos\theta_C = (\pi r^2 h\rho)g$

$$h = \frac{2T\cos\theta_C}{r\rho g}$$



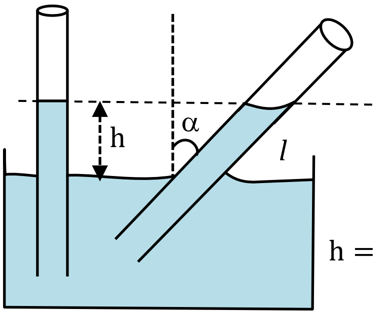
Special Points

(i) Zurin's Law :

The height of rise of liquid in a capillary tube is inversely proportional to the radius of the capillary tube, if T, θ_C, r and g are constant $h \propto \frac{1}{r}$ or rh = constant. It implies that liquid will rise more in capillary tube of less radius and vice versa.

(ii) Inside a satellite, water will rise upto the top level but will not overflow and shape of meniscus will be plane.

(iii) If a capillary tube is dipped into a liquid and tilted at an angle α from vertical then the vertical height of the liquid column remains same whereas the length of liquid column in the capillary tube increases.



$h = \ell\cos\alpha \Rightarrow \ell = \dfrac{h}{\cos\alpha}$

Illustration 40

Water rise to a height of 10mm in a capillary. If the radius of the capillary is made half of its previous value then what is the new value of the capillary rise?

Solution:

$$\frac{h_2}{h_2} = \frac{r_1}{r_2} = \frac{r}{(r/2)} = 2$$

Hence $h_2 = 2h_1 = 2 \times 10$ mm $= 20$ mm

Illustration 41

Water rises in a vertical capillary tube upto a length of 10cm. If the tube is inclined at 45° with the vertical then find the length of water rises in the tube.

Solution:

$\ell\cos\theta = h$

$$\therefore \quad \ell = \frac{h}{\cos 45^\circ} = \frac{10}{(1/\sqrt{2})} = 10\sqrt{2}\,\text{cm}$$

Illustration 42

In a capillary tube experiment, a vertical 30 cm long capillary is dipped in water. The water rises upto a height of 10cm due to capillary action. If this experiment is conducted in a freely falling elevator, what will be the length of water column.

Solution:

Liquid will rise upto top of the tube,

$\therefore \quad h' = 30$ cm

Illustration 43

Two capillary tubes of same material of a radius r_1 and r_2 are vertically immersed in the same liquid, then find: (1) Ratio of height of liquid rise (2) Ratio of mass of liquid rise (3) Ratio of potential energy of liquid rise

Solution:

(1) By Zurin's law : $h_1r_1 = h_2r_2 \Rightarrow \dfrac{h_1}{h_2} = \dfrac{r_2}{r_1}$

(2) $$\frac{m_1}{m_2} = \frac{\rho \times V_1}{\rho \times V_2} = \frac{\rho\left(\pi r_1^2\right)h_1}{\rho\left(\pi r_1^2\right)h_2} = \frac{r_1}{r_2}$$

(3) When water rises upto a height h then mass of liquid rise $m = \rho(\pi r^2 h)$

$\therefore$ Total mass be located at centre of mass, then potential energy $U = mg\dfrac{h}{2}$

$$\frac{(P.E.)_1}{(P.E.)_2} = \frac{m_1 g\left(\frac{h_1}{2}\right)}{m_2 g\left(\frac{h_2}{2}\right)} = \left(\frac{m_1}{m_2}\right)\left(\frac{h_1}{h_2}\right) = 1$$

Illustration 44

In a cylindrical vessel at its bottom a round hole of diameter 1mm is drilled and water is filled in it. Find the maximum height to which water can be filled in it without leakage. (Surface tension of water is 75×10^{-3} N/m, density of water is 1000 kg/m^3 and angle of contact is 0°)

Solution:

Water will start leaking out when ; $(h\rho g)\pi r^2 = T(2\pi r)$

$$h = \frac{2T}{\rho g r} = \frac{2 \times 75 \times 10^{-3}}{1000 \times 10 \times 0.5 \times 10^{-3}} = 3 \times 10^{-2} \text{ m} = 3 \text{ cm}$$

BEGINNER'S BOX-6

1. When a cylindrical tube is dipped vertically into a liquid the angle of contact is 80°. When the tube is dipped with an inclination of 40°, find the angle of contact?

PPF196

2. If A liquid rises in a capillary tube, then what can be the angle of contact?

PPF197

3. The excess pressure in a soap bubble is three times the excess pressure in another bubble. Find the ratio of their volumes?

PPF198

4. An air bubble of radius 0.02 mm is at a depth of 25 cm in oil of density 0.8 gm/cm^3. If the surface tension of oil is 25×10^{-3} N/m find the pressure inside the bubble (atmospheric pressure = 10^5 N/m^2)?

PPF199

5. The pressure inside two soap bubbles is 1.01 and 1.02 atmosphere respectively. Find the ratio of their volumes?

PPF200

6. On dipping one end of a capillary in a liquid and inclining the capillary at angles 30° and 60° with the vertical, the lengths of liquid columns in it are found to be ℓ_1 and ℓ_2 respectively. Find the ratio of ℓ_1 and ℓ_2?

PPF201

7. Water rises in two capillaries of same material up to heights of 40 and 60 mm. Find the ratio of their radii?

PPF202

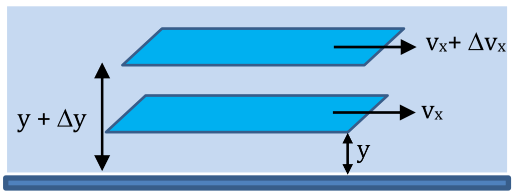
8. When a capillary tube is dipped inside water, water rises inside the capillary tube up to 0.015 m. If the surface tension of water is 75×10^{-3} N/m calculate the radius of the capillary tube?

PPF203

Newton's Law of Viscosity

Viscosity : Viscosity is the property of a fluid (liquid or gas) by virtue of which it opposes the relative motion between its adjacent layers. It is the fluid friction or internal friction.

The internal tangential force which tends to retard the relative motion between the adjacent layers is called viscous force.



The rate of change of velocity with distance perpendicular to the direction of flow i.e. $\frac{\Delta V_x}{\Delta y}$, is called velocity gradient.

According to Newton, the viscous force F acting between two adjacent layers of a liquid flowing in streamlined motion depends upon the following two factors :

(i) $F \propto$ contact area of the layers i.e $F \propto A$

(ii) $F \propto$ velocity gradient between the layers i.e $F \propto \frac{\Delta v_x}{\Delta y}$

Combining (i) and (ii) $F \propto A\frac{\Delta v_x}{\Delta y} \Rightarrow F = \eta A\frac{\Delta v_x}{\Delta y}$

where η is a constant called coefficient of viscosity of the liquid.

Coefficient of viscosity $\eta = \frac{F/A}{v/\ell} = \frac{\text{Shear stress}}{\text{Strain rate}}$

SI UNITS : $N - s\ m^{-2}$ = Pa-s = poiseuille (PI) = deca poise

CGS UNITS : dyne-s/cm^2 = poise; 1 decapoise = 10 poise.

Dimensions : $[M^1L^{-1}T^{-1}]$

Important points

(i) Viscosity of fluid depends only on the nature of fluid and is independent of area considered or velocity gradient.

(ii) Thin liquids like water, alcohol are less viscous than thick liquids like blood, glycerin, honey.

(iii) Viscosity of liquid is much greater (about 100 times more) than that of gases. Viscosity of water ≈ 0.01 poise, Viscosity of air $\approx 200\mu$poise

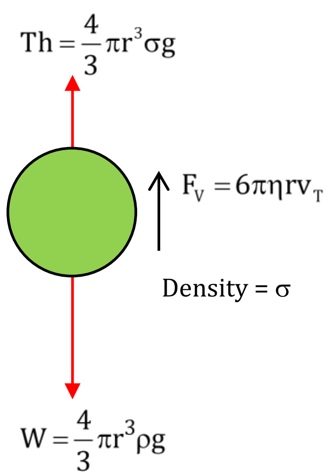
Stoke's Law and Terminal Velocity

Stoke's Law

Stoke showed that if a small sphere of radius r is moving with a velocity v through a homogeneous stationary medium (liquid or gas), of viscosity η then the viscous force acting on the sphere is $F_v = 6\pi\eta rv$.

Terminal Velocity

When a solid sphere falls in a liquid, its increasing velocity is controlled by the viscous force due to liquid and hence it attains a constant velocity which is known as the terminal velocity (v_T).



As shown in the figure when the body moves with constant velocity i.e. terminal velocity (with no acceleration) the net upward force (upthrust Th + viscous force F_v) balances the downward force (weight of the body W).

$$Th + F_V = W \Rightarrow \frac{4}{3}\pi r^3\sigma g + 6\pi\eta rv_T = \frac{4}{3}\pi r^3\rho g \Rightarrow v_T = \frac{2}{9}\frac{r^2(\rho-\sigma)}{\eta}g$$

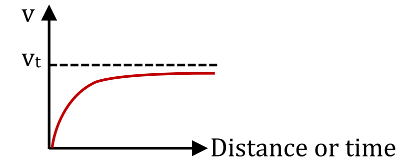
where, r = radius of body ρ = density of body

σ = density of medium η = coefficient of viscosity.

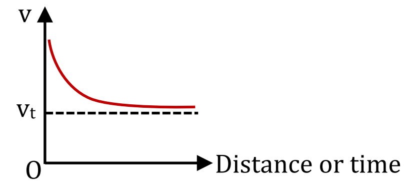
Graph

The variation of velocity with time (or distance) is shown in the adjacent graphs.

Case 1 : When initial velocity of body in medium is zero



Case 2 : When initial velocity of body in medium is greater than v_t



Some applications of terminal velocity:

(a) Actual velocity of rain drops is very small in comparison to the velocity which would have acquired by a body falling freely from the height of clouds.

(b) Descent of a parachute with moderate velocity.

(c) Determination of electronic charge in Milikan's oil drop experiment.

Illustration 45

Two balls made of same material having masses m and 8m respectively. Then find ratio of their terminal velocity in same liquid.

Solution:

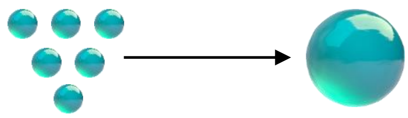
$m \propto r^3$ $\qquad$ $v_T \propto r^2$

$$\Rightarrow \frac{m_1}{m_2} = \left(\frac{r_1}{r_2}\right)^3 \qquad \Rightarrow \frac{v_{T_1}}{v_{T_2}} = \left(\frac{r_1}{r_2}\right)^2$$

$$\Rightarrow \frac{r_1}{r_2} = \left(\frac{m}{8m}\right)^{\frac{1}{3}} = \frac{1}{2} \qquad \Rightarrow \frac{v_{T_1}}{v_{T_2}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

Illustration 46

If terminal velocity of a drop is 10 m/sec in a fluid then find the terminal velocity of a big drop formed by 64 identical such drops:



Solution:

By volume conservation; $\frac{4}{3}\pi r^3 \times 64 = \frac{4}{3}\pi R^3 \Rightarrow R = 4r$

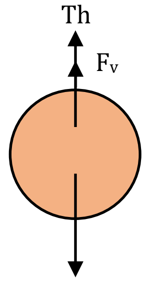
Terminal speed $v_T \propto r^2$

$$\Rightarrow \frac{v_{T_1}}{v_{T_2}} = \left(\frac{r_1}{r_2}\right)^2 \Rightarrow \frac{10}{v_{T_2}} = \left(\frac{r}{4r}\right)^2 \Rightarrow v_{T_2} = 160 \, m/s$$

Illustration 47

A tiny sphere of mass 2kg and density 5g/cc is dropped in a jar of glycerine of density 3g/cc. When the sphere acquires terminal velocity, then find the magnitude of the viscous force acting on the sphere.

Solution:



$$\text{Th} + F_v = W$$

$$\Rightarrow \quad F_v = W - \text{Th}$$

$$= \left(mg - \rho_\ell V_g g\right) = mg - \rho_\ell \frac{m}{\rho_b} g$$

$$= mg\left[1 - \frac{\rho_\ell}{\rho_b}\right] = 2\times10\left[1-\frac{3}{5}\right] = 8\text{N}$$

Illustration 48

A drop of water of radius 0.0015 mm is falling in air. The co-efficient of viscosity of air is 1.8×10^{-5} kg/m-s. What will be the terminal velocity of the drop? density of air can be neglected.

Solution:

$$v_T = \frac{2r^2(\rho-\sigma)g}{9\eta} = \frac{2\times\left[\dfrac{15\times10^{-4}}{1000}\right]^2 \times 10^3 \times 9.8}{9\times1.8\times10^{-5}} = 2.72\times10^{-4}\ \text{m/s}$$

Illustration 49

The velocity of a small ball of mass M and density d_1, when dropped in a container filled with glycerin becomes constant after some time. If the density of glycerin is d_2, the viscous force acting on the ball will be :

Solution:

At equilibrium, the viscous force acting upward = Effective force downward

Effective force = $Vd_1g - Vd_2g = V(d_1 - d_2)g = \frac{M}{d_1}(d_1 - d_2)g = Mg\left[1-\frac{d_2}{d_1}\right]$ $\quad \left[\because V = \frac{M}{d_1}\right]$

Illustration 50

A spherical ball of radius 1×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water. If the velocity of the ball does not change, after entering the water find h. Viscosity of water is 9.8×10^{-6} N-s/m^2.

Solution:

After falling a height h velocity of the ball will become $v=\sqrt{2gh}$. After entering into the water as this velocity does not change, this velocity is equal to the terminal velocity,

$$\sqrt{2gh} = \frac{2}{9}r^2\left[\frac{\rho-\sigma}{\eta}\right]g \Rightarrow 2gh = \left[\frac{2}{9}\times(10^{-4})^2\times\frac{(10^4-10^3)\times9.8}{9.8\times10^{-6}}\right]^2 \Rightarrow h = \frac{20\times20}{2\times9.8} = 20.41\text{m}$$

Dependency of viscosity

(i) On Temperature of Fluid

(a) Since cohesive forces decrease with increase in temperature. Viscosity of liquids decreases with a rise in temperature.

(b) Viscosity of gases is the result of diffusion of gas molecules from one moving layer to other. With an increase in temperature, the rate of diffusion increases. Consequently the viscosity increases. Thus, the viscosity of gases increases with the rise in temperature.

(ii) On Pressure of Fluid

(a) Viscosity is normally independent of pressure. However liquids under extreme pressure often undergo an increase in viscosity.

(b) Viscosity of gases is practically independent of pressure.

Steady flow in capillary tube

Poiseuille's Formula

In case of steady flow of liquid of viscosity (η) in a capillary tube of length (L) and radius (r) under a pressure difference (P) across it, the volume of liquid flowing per second is given by

$$Q=\frac{dV}{dt}=\frac{\pi P r^4}{8\eta L}$$

With the help of poiseuille's formula, coefficient of viscosity of a liquid can be determined.

BEGINNER'S BOX-7

1. A large wooden plate of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential force needed to keep the plate moving. Coefficient of viscosity of water at the temperature of the river = 10^{-2} poise.

PPF204

2. A square plate of 1m side moves parallel to a second plate with velocity 4 m/s. A thin layer of water exists between plates. If the viscous force is 2 N and the coefficient of viscosity is 0.01 poise then find the distance between the plates in mm.

PPF205

3. Find the terminal velocity of a rain drop of radius 0.01 mm. Coefficient of viscosity of air is 1.8×10^{-5} N-s/m^2 and its density is 1.2 kg/m^3. Density of water = 1000 kg/m^3. Take g = 10 m/s^2. (Force of buoyancy due to air is neglected)

PPF206

4. An air bubble of radius 1 mm is allowed to rise through a long cylindrical column of a viscous liquid of radius 5 cm and travels at a steady rate of 2.1 cm per second. If the density of the liquid is 1.47 g/cc, find its viscosity. Assume g = 980 cm/s^2 and neglect the density of air.

PPF207

5. A liquid flows through two capillary tubes connected in series. Their lengths are ℓ and 2ℓ and radii r and 2r respectively, then the pressure difference across the first and second tubes are in the ratio......

PPF208

BEGINNER'S BOX — ANSWER KEY

BEGINNER'S BOX-1

1.	$5\ N/m^2, 5\sqrt{3}\ N/m^2$	**2.**	0.004 radians	**3.**	6 mm	**4.**	Break
5.	10.2 km	**6.**	20 kg–wt				

BEGINNER'S BOX-2

1.	$2.0 \times 10^9\ N/m^2$	**2.**	27 : 1	**3.**	4	**4.**	0.25 %
5.	$0.4\ cm^3$	**6.**	2×10^{-7} m	**7.**	4.8×10^{-5} J		

BEGINNER'S BOX-3

1. $\rho_1 = 2\ kg/m^3, \rho_2 = 6\ kg/m^3$ or $\rho_1 = 6\ kg/m^3, \rho_2 = 2\ kg/m^3$

2.	83.33 cm	**3.**	25 N	**4.**	1 cm	**5.**	(i) 5 (ii) 0.67
6.	(i) $0.67 \times 10^3\ kg/m^3$		(ii) $0.74 \times 10^3\ kg/m^3$	**7.**	$300\ cm^3$	**8.**	500 g-f

BEGINNER'S BOX-4

1.	1 m/s	**2.**	8 m/s	**3.**	$\frac{6}{\sqrt{10}} \times 10^{-3}\ m^3/s$

BEGINNER'S BOX-5

1.	80 dynes	**2.**	$\frac{2}{\pi}$ N/m	**3.**	2800 dynes	**4.**	$\sqrt{\frac{2T}{\pi dg}}$
5.	0.1 N/m	**6.**	3 J	**7.**	$24\pi R^2 T$	**8.**	960π erg
9.	99%	**10.**	$12\ \pi r^2 T$				

BEGINNER'S BOX-6

1.	80°	**2.**	acute	**3.**	1 : 27	**4.**	1.045×10^5 Pa
5.	8 : 1	**6.**	$1 : \sqrt{3}$	**7.**	3 : 2	**8.**	1mm

BEGINNER'S BOX-7

1.	2×10^{-2} N	**2.**	2 mm	**3.**	0.012 m/s	**4.**	1.52 poise
5.	8 : 1						