Properties of Matter and Fluid Mechanics

SOLUTIONS

Exercise-I (Conceptual Questions)

2. Total weight at height $\frac{L}{4}$ from its lower end = W_T

W_T = weight suspended + weight of $\frac{L}{4}$ of wire

$W_T = W_1 + \frac{W}{4}$

Stress = $\frac{F}{Area} = \frac{W_T}{S}$

Stress = $\frac{\left(W_1 + \frac{W}{4}\right)}{S}$

3. Stress = $\frac{F}{Area} = \frac{Mg}{A} \Rightarrow \frac{\rho LAg}{A} = \rho Lg$

$L_{max} = \frac{(Stress)_{max}}{\rho g}$; L_{max} = 75 m

4. From graph

Slope = $\frac{F}{\Delta\ell}$

also Y = $\frac{FL}{A\Delta\ell}$

Slope = $\frac{F}{\Delta\ell} = Y\frac{A}{L}$

$\Rightarrow$ more slope more Y

So $Y_A > Y_B$

5. When density increase, mass of material increase, for same strain more stress required. So Young's modulus increases.

6. Volume = constant

$A \times \ell$ = constant $\Rightarrow A \propto \frac{1}{\ell}$

$\Delta\ell = \frac{F\ell}{AY} \Rightarrow \Delta\ell \propto \frac{\ell}{A} \Rightarrow \Delta\ell \propto \ell^2 \quad \left(\text{as } A \propto \frac{1}{\ell}\right)$

7. In both case tension in wire are same and $\Delta\ell \propto T$

8. $\Delta L = \frac{\rho g L^2}{2Y} = \frac{1.5\times10\times64\times10^{-4}}{2\times5\times10^8}$

$= 9.6 \times 10^{-11}$ m.

9. Bulk modulus = $\frac{\Delta P}{\Delta V/V} = \frac{h\rho g}{0.1/100}$

$= \frac{200\times10^3\times9.8\times100}{0.1} = 19.6 \times 10^8 \text{ N/m}^2$

10. (Bulk Modulus) K = $\frac{\Delta P}{\Delta V/V}$

$= \frac{0.155\times10^5}{10/100} = 1.55 \times 10^5$ Pa

11. Potential energy per unit volume is

$= \frac{1}{2}\times\frac{(Stress)^2}{Y} \Rightarrow \frac{U_1}{U_2} = \frac{F_1^2}{A_1^2}\times\frac{A_2^2}{F_2^2} = \frac{r_2^4}{r_1^4} = 16:1$

12. If wire suddenly breaks then energy released on bond breaking appears in the form of heat.

13. $W_{ext} = \vec{F}.\vec{d}$ using COM

$= \frac{F\Delta\ell}{2}$

$= \frac{mg\Delta\ell}{2} = \frac{0.5\times9.8\times3\times10^{-3}}{2}$

$= 7.3\times10^{-3}$ Joule

14. $F = \frac{YA}{L}\Delta\ell = \pi\times\left(\frac{1}{2}\times10^{-3}\right)^2\times2\times10^{11}\times\frac{1}{1000}$

= 157 N

15. $\frac{F}{A} = \frac{Y\Delta L}{L}$

$\Delta L = \frac{FL}{YA}$ (as given ΔL = e)

$e = \frac{FL}{YA} = \frac{F\ell}{Y\pi r^2}$

For second wire

$e' = \frac{(2F)(2\ell)}{Y\pi(2r)^2} = \frac{F\ell}{Y\pi r^2}$

e' = e

16. $F = \frac{YA}{L}\Delta\ell$

$= 0.9 \times 10^{11} \times \pi \times (0.3 \times 10^{-3})^2 \times \frac{0.2}{100} = 51\text{N}$

17. $\frac{F}{A} = \frac{Y\Delta L}{L}$ (Load = F)

$F = \frac{YA}{L} \cdot \Delta L$

$\boxed{\frac{F}{\Delta L} \propto A}$

more the slope, more will be area.

18. Here $\Delta\ell = \ell_0$

ℓ_0 = initial length

ℓ_f = final length = $2\ell_0$

$\frac{F}{A} = \frac{Y\Delta\ell}{\ell_0}$

$F = \frac{YA(\ell_0)}{\ell_0}$

$\boxed{F = YA}$

$= 2.0 \times 10^{11} \times 0.1 \times 10^{-4} = 2 \times 10^6$ N

19. $\frac{\Delta\ell}{L} = \frac{0.1}{100}$; Area = $\pi r^2 = \pi \times (2 \times 10^{-3})^2$

$F = YA. \frac{\Delta\ell}{L}$

$= 9 \times 10^{10} \times (\pi \times 4 \times 10^{-6}) \times \frac{0.1}{100} = 360\ \pi$ N

21. Stress = $\frac{F}{A}$

$\Rightarrow \text{Stress} \propto \frac{1}{\text{Area}}$

$\frac{\text{Stress}_2}{\text{Stress}_1} = \frac{r_1^2}{r_2^2}$

22. $\left(-\frac{\Delta V}{V}\right) = \frac{0.004}{100}$

$\Delta P = \left(-\frac{\Delta V}{V}\right) K \times = 2100 \times 10^6 \times \frac{0.004}{100}$

= 84 kPa

23. Energy density = $\frac{1}{2} \times \frac{(\text{Stress})^2}{Y} = \frac{S^2}{2Y}$

24. For volume

Stress = ΔP

Strain = $-\frac{\Delta V}{V}$ ('−'represent decrease in volume)

$\frac{\text{Stress}}{\text{Strain}} = \frac{-\Delta P}{\frac{\Delta V}{V}} = B$

$\Rightarrow \quad \frac{\Delta V}{V} \propto \frac{1}{B}$

25. For a wire of length L stretched by a length x, the restoring elastic force is :

$F = \left[\frac{x}{L}\right] YA$

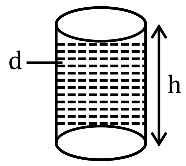
The work required to be done against the elastic restoring forces to elongate it further by a length dx is,

$dW = F.dx = \frac{YA}{L} x\,.\,dx$

The total work done in stretching the wire from x = 0 to x is,

$W = \int_0^x \frac{YA}{L} x.dx = \frac{YA}{L}\left[\frac{x^2}{2}\right]$

26. $\frac{F_A}{F_B} = \frac{P_A A_A}{P_B A_B} = \frac{\rho_A g h_A A_A}{\rho_B g h_B A_B} = 1$



27. d = Density

h = Height of water

A = Base cross sectional area

P_{Bottom} (pressure due to liquid) = hdg

P_{Top}(pressure due to liquid) = dg(0) = 0

av. pressure = $\frac{P_{Bottom} + P_{Top}}{2} = \frac{hdg}{2}$

Alternate : Use concept of COM for a uniform solid cylinder.

28. Due to upthrust on the block, its apparent wt. will reduce & hence the reading of spring balance A will be less then 2 kg. But since the liquid in the beaker exerts buoyant force on the block, the block will also exert reaction force on the liquid in the downward direction and the reading of balance B will increase due to this additional force.

29. Liquid with low density float over high density.

30. Weight of man = upthrust $\Rightarrow$ $Mg = V_{in}\,\rho_w\,g$

$M = (3 \times 2 \times 1 \times 10^{-2}) \times 10^3 = 60$ kg

31. Water density = 1000 kg/m^3

block density = 981 kg/m^3

so it will float with some part immersed in water.

32. When coin is removed the downward force an block decrease, which will reduce the volume submerged also the removing volume will be filled by water so h $\downarrow$.

33. Same density

34. RD of metal = $\dfrac{W_A}{W_A - W_W} = \dfrac{210}{210-180} = 7$

$\therefore$ density of metal = $7 \times \rho_w = 7$ g/cm^3

35. Torr is the unit of pressure.

36. In water for floating

W = Th

$V\rho_B g = V_{in}\rho_w g$

$V\rho_B g = \dfrac{2}{3} V \times 1 \times g \Rightarrow \rho_B = \dfrac{2}{3}$ g/cm^3

in liquid

W = Th (Buoyant force)

$V\rho_B g = \dfrac{V}{4} \times \rho_L g$

$\rho_L = 4\rho_B = \dfrac{8}{3}$ g/cm^3

37. Hydraulic lift, hydraulic jack, hydraulic piston works on Pascal's law.

38. $W_{APP} = W - F_B$ (F_B = force of buoyant)

$W_{APP} = V\rho g - V\rho_1 g$

$= V\rho g\left[1 - \dfrac{\rho_1}{\rho}\right]$

$W_{APP} = W\left[1 - \dfrac{\rho_1}{\rho}\right]$

39. Pascal's law states that the magnitude of pressure within fluid is equal in all parts.

40. $Th = W_A - W_W = 5 - 2 = 3N$

41. Hydraulic lift, hydraulic jack, hydraulic press works on Pascal's law.

42. For wooden block $\rho_{Block} < \rho_{Water}$

So $F_B > Mg$, hence it will rise up with an constant acceleration.

43. $A_P v_P = A_Q v_Q$

or $\dfrac{v_P}{v_Q} = \dfrac{A_Q}{A_P} = \dfrac{\pi \times (4\times10^{-2})^2/4}{\pi\times(2\times10^{-2})^2/4} = 4$

or $v_P = 4\, v_Q$

44. Writing equation of continuity for the tube & the holes

$\Rightarrow$ $A_{tube}\, v_{tube} = nA_{hole}\, v_{hole}$

$\pi R^2 \times v = n\pi r^2 v'$ $\Rightarrow$ $v' = \dfrac{vR^2}{nr^2}$

45. From Bernoulli equation

$\dfrac{1}{2}\rho v^2 = \rho g h$ $\Rightarrow v = \sqrt{2gh} = 2$ m/s

from equation of continuity $A_1v_1 = A_2v_2$

$A_2 = \left(\dfrac{v_1}{v_2}\right)\times A_1 = \left(\dfrac{1}{2}\right)\times 10^{-4} = 5 \times 10^{-5}$ m^2

46. According to Bernoulli's theorem.

$\dfrac{1}{2}\rho v^2 + \rho g h + P = \text{const.}$

So if A$\downarrow$ v$\uparrow$ and P$\downarrow$

and if A$\uparrow$ v$\downarrow$ and P$\uparrow$

47. Volume flowing per second = Av

$= A\sqrt{2gh} = 10^{-4}\sqrt{2\times10\times5} = 10^{-3}$ m^3/s

48. $P_1 + \dfrac{1}{2}\rho v_1^2 = P_2 + \dfrac{1}{2}\rho v_2^2$

$(P_1 - P_2) = \dfrac{1}{2}\rho v_2^2$

since tap is closed, $v_1 = 0$

$(3.5 \times 10^5 - 3 \times 10^5) = \dfrac{1}{2} \times 10^3 \times v^2$

v = 10 m/s

49. Dynamic lift = Pressure diff. × Area

$= (P_{lower} - P_{upper})\,A$

(According to Bernoullis Theorem)

$= \left(\dfrac{1}{2}\rho v_{upper}^2 - \dfrac{1}{2}\rho v_{lower}^2\right)A$

$= \dfrac{1}{2}\rho A\left[(\sqrt{2}v)^2 - v^2\right] = \dfrac{1}{2}\rho A v^2$

50. $A_Av_A = A_Bv_B$

$v_B = \frac{A_A}{A_B} \times v_A = \frac{\pi(2R)^2}{\pi R^2} \times v = 4v$

51. Equation of continuity $A_1v_1 = A_2v_2$

Av = constant

$\pi r^2 v$ = constant

$\boxed{v \propto \frac{1}{r^2}}$

52. Level flight means net vertical force is zero

$\therefore \quad mg = \Delta P \times A$

$\Delta P = \frac{mg}{A} = \frac{3\times10^4\times10}{120} = 2.5 \times 10^3$ Pa = 2.5 kPa

53. Scent sprayer is based on Bernoulli's theorem.

54. Bernoulli's equation for steady, non-viscous, in-compressible flow expresses the conservation of energy.

55. Bernoulli's theorem can be seen in dynamic lift to aeroplane.

56. By equation of continuity

$A_1v_1 = A_2v_2$

$v_2 = \left(\frac{A_1}{A_2}\right) v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{0.2}{0.1}\right)^2 \times 20$

= 80 cm/s

57. Final velocity is terminal velocity which is independent from 'h'

$v_T = \frac{2r^2}{9}\frac{(\rho-\sigma)}{\eta}g$

58. $v_T \propto r^2 \therefore \frac{v_T'}{v_T} = \left(\frac{R}{r}\right)^2$ or $v_T' = \left(\frac{R}{r}\right)^2 v_T$ ...(1)

when 2 droplets of radius r coalesce the new radius R of the big drop will be given by

$\frac{4}{3}\pi R^3 = 2\times\frac{4}{3}\pi r^3 \Rightarrow \frac{R}{r} = 2^{1/3}$ ...(2)

from equation (1) and (2)

$v'_T = (2^{1/3})^2 \times 5 = 4^{1/3} \times 5$ cm/sec.

59. $v_T = \frac{2r^2}{9}\frac{(\rho-\sigma)}{\eta}g$

v_T = 0.2 m/s for gold

ρ = 19.5 kg/m^3 (gold)

σ = 1.5 kg /m^3

From above data we can find value of $\frac{r^2}{n}$

putting in 2nd case for silver

ρ_2 = 10.5 kg/m^3

$\boxed{v_T = 0.1\,m/s}$

60. $v_T = \frac{2r^2}{9}\frac{(\rho-\sigma)}{\eta}g$

$v_T \propto r^2 \qquad \therefore \frac{v_{T_2}}{v_{T_1}} = \frac{r_2^2}{r_1^2}$

$\therefore v_{T_2} = v_{T_1}\left(\frac{r_2}{r_1}\right)^2 = 20\times\left(\frac{1}{2}\right)^2$ = 5 cm/s.

61. The velocity of falling rain drop attain limited value because of viscous force exerted by air.

62. Poise is the unit of viscosity.

63. $v_T = \frac{2r^2}{9}\frac{(\rho-\sigma)}{\eta}g$

$v_T \propto r^2 \qquad \Rightarrow \frac{v_{T_1}}{v_{T_2}} = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{1}{2}\right)^2 = 1:4$

64. $v_T = \frac{2R^2}{9}\frac{(\rho-\sigma)}{\eta}g$

$v_T \propto R^2$ but $M \propto R^3$

$\Rightarrow v_T \propto \frac{M}{R}$

65. $v_T = \frac{2}{9}r^2\frac{(\rho_w - \rho_{air})g}{\eta}$

$\because \ \rho_{air} \approx 0$

$\Rightarrow v_T = \frac{2}{9}\times\frac{(1.50\times10^{-6})^2\times10^3\times10}{2\times10^{-5}}$

= 2.5 × 10^{-4} m/s

66. Elastic membrane is formed on the surface of water due to surface tension. This help's spider & insects to move and run on the surface of water.

67. $2T\ell$ = mg (soap film has two free surface)

$T = \frac{mg}{2\ell} = \frac{1.5\times10^{-2}}{2\times0.3}$ = 0.025 N/m

68. $\Delta E = 4\pi R^2 T\left(n^{1/3}-1\right)$

$$= 4\pi \frac{D^2}{4} T\,[(27)^{1/3}-1] = 2\pi D^2 T$$

69. Excess pressure in air bubble just below the water surface. $P_1 = \frac{2T}{r}$

Excess pressure inside a drop $P_2 = \frac{2T}{r}$

So $P_1 = P_2$

70. By conservation of volume

$$\frac{4}{3}\pi R^3 = n\left[\frac{4}{3}\pi r^3\right] \Rightarrow n = \left[\frac{R}{r}\right]^3 \Rightarrow r = \frac{R}{n^{1/3}}$$

$R = n^{1/3} r$

$$P_{ex.} \propto \frac{1}{r} \Rightarrow \frac{(P_{ex})_{big}}{(P_{ex})_{small}} = \frac{r}{R} = \frac{1}{n^{1/3}} = \frac{1}{2}$$

71. $h = \frac{2T\cos\theta}{r\rho g}$

If the radius of capillary is reduced to half, the rise of liquid column becomes two times because $h \propto \frac{1}{r}$

72. $h = \frac{2T\cos\theta}{r\rho g}$

$$h \propto \frac{1}{r} \quad 0 \Rightarrow \frac{h_2}{h_1} = \frac{r_1}{r_2} \Rightarrow h_2 = \frac{h_1}{2}$$

Mass of water = V × ρ_{water}

$$\frac{M'}{M} = \frac{\pi(2r)^2 \times \frac{h}{2} \times \rho_w}{\pi r^2 \times h \times \rho_w} = 2$$

$\Rightarrow M' = 2M$

73. When water rise upto a height h then mass of liquid rise $m = (\pi r^2 h)\rho$

$\because$ total mass be located at centre of mass.

then potential energy $U = mg\frac{h}{2} = (\pi r^2 H\rho)g\frac{h}{2}$

by Zurin law r × h = constant

$\therefore$ U = constant

74. $\frac{2T}{r} = h\rho g$

$$h = \frac{2T}{r\rho g} = \frac{2\times 75}{\left(\frac{0.1}{2}\times 10^{-1}\right)\times 1\times 1000} = 30 \text{ cm}$$

75. Inside a satellite, water will rise upto the top level but sill not overflow. Radius of curvature (R') increases in such a way that final height h' is reduced and given by $h' = \frac{hR}{R'}$. (It is in accordance with Zurin's law).

76. When two bubbles coalesce then total number of molecules of air will remain same and temperature will also remain constant

so $n_1 + n_2 = n \Rightarrow P_1V_1 + P_2V_2 = PV$

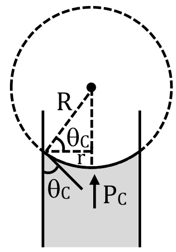
$$\Rightarrow \frac{4T}{r_1}\left(\frac{4}{3}\pi r_1^3\right) + \frac{4T}{r_2}\left(\frac{4}{3}\pi r_2^3\right) = \frac{4T}{r}\left(\frac{4}{3}\pi r^3\right)$$

$$\Rightarrow r = \sqrt{r_1^2 + r_2^2}$$

77. Spherical shape of rain-drop is due to surface tension which tries to minimize the surface area.

78. $h = \frac{2T}{r\rho g}$

$\Rightarrow h \propto \frac{1}{r}$



$R = \frac{r}{\cos\theta}$

79. $h = \frac{2T}{r\rho g}$

$\Rightarrow h \propto \frac{1}{g}$

$\Rightarrow$ at moon $g' = \frac{g}{6}$

80. For hemispherical

R = r

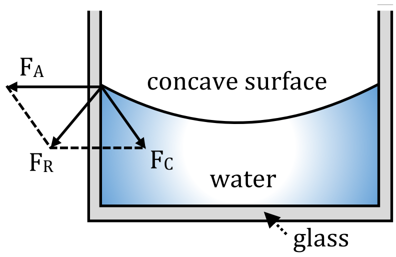
i.e. $\cos\theta = 1$

$\theta = 0°$

as $R\cos\theta = r$

in capillary tube

81.



Liquid rises up for acute angle of contact.

82. Splitting of big drop into smaller droplets.
In this process area increases, surface energy increases, internal energy decreases, temperature decreases, and energy is absorbed.
For reverse process energy is liberated.

83. $h = \dfrac{2T\cos\theta}{r\rho g}$

$h \propto \dfrac{T}{\rho} \Rightarrow \dfrac{h_1}{h_2} = \dfrac{T_1\rho_2}{T_2\rho_1} = \dfrac{60}{50} \times \dfrac{0.6}{0.8} = \dfrac{9}{10}$

84. Surface tension of liquid lead, which tries to minimize the surface area and gives a spherical shape.

85. S.E. = T × 2A
= 5 × 2 × 0.02
= 2×10^{-1} J

86. $h = \dfrac{2T\cos\theta}{r\rho g}$

$\Rightarrow r = \dfrac{2T\cos\theta}{h\rho g}$

$= \dfrac{2 \times 75 \times 10^{-3} \times 1}{3 \times 10^{-2} \times 10^{3} \times 10}$

= 0.5×10^{-3} m
diameter d = 2r = 10^{-3} m = 1 mm

87. W = TΔA
W = TA
as ΔA = A

88. $P_1 - P_2 = \dfrac{4T}{r}$

$\Rightarrow \dfrac{4T}{r_1} - \dfrac{4T}{r_2} = \dfrac{4T}{r}$

$\Rightarrow r = \dfrac{r_1 r_2}{r_2 - r_1}$

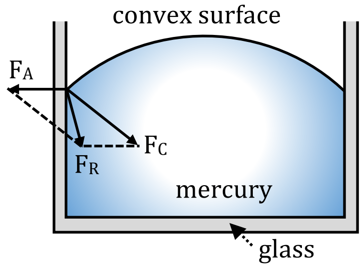
$r = \dfrac{r_1 r_2}{r_2 - r_1} = \dfrac{4 \times 5}{5 - 4}$ cm. = 20 cm.

89. Work done (surface energy) in blowing of a soap bubble of radius r :
W = T × ΔA or W = T × 2 × $4\pi r^2$ = $8\pi r^2 T$
[$\because$ soap bubble has two surfaces]

$E = T \times 8\pi \left(r_2^2 - r_1^2\right)$

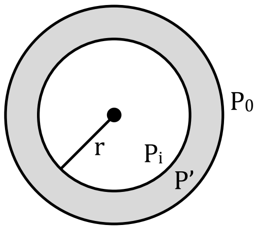
$= T \times 8\pi\,(4r^2 - r^2)$
$= 24\pi r^2 T$

90.



A liquid does not wet the sides of a solid, for obtuse (more than 90°) angle of contact.

91. Since a soap bubble has two surfaces, excess pressure will get doubled as compared to a drop



$P_i - P' = \dfrac{2T}{r}$, $P' - P_0 = \dfrac{2T}{r}$ $\Rightarrow$ excess pressure

$= P_i - P_0 = \dfrac{4T}{r}$

92. Inside a freely falling elevator, water will rise upto the top level but still not overflow. Radius of curvature (R') increases in such a way that final height h' is reduced and given by $h' = \dfrac{hR}{R'}$.
(It is in accordance with Zurin's law).

93. On liquid force due to surface tension = $(2\pi R)\,T\cos\theta_C$
In equilibrium : force due to S.T = weight of rise liquid
$(2\pi R)T\cos\theta_C = mg$

$T = \dfrac{\text{Weight}}{2\pi R} = \dfrac{6.2 \times 10^{-4}}{2 \times 3.14 \times 2 \times 10^{-3}} = 5 \times 10^{-2}$ N/m

94. On liquid force due to surface tension = $(2\pi r)\,T\cos\theta_C$
In equilibrium : force due to S.T = weight of rise liquid
$(2\pi r)T\cos\theta_C = mg$
$2\pi r.T$ = weight

$r = \dfrac{\text{weight}}{2\pi T}$

$= \dfrac{6.28 \times 10^{-4}}{2 \times 3.14 \times 5 \times 10^{-2}} = 2 \times 10^{-3}$ m

95. On liquid force due to surface tension = $(2\pi r) T\cos\theta_C$

In equilibrium : force due to S.T = weight of rise liquid

$(2\pi r)T\cos\theta_C = mg$

$2\pi r.T$ = weight

$2\pi r = \frac{\text{weight}}{T}$

$= \frac{75\times10^{-4}}{6\times10^{-2}}$

$= 12.5 \times 10^{-2}$ m

96. By conservation of volume

$$\frac{4}{3}\pi R^3 = n\left[\frac{4}{3}\pi r^3\right]$$

$$\Rightarrow n = \left[\frac{R}{r}\right]^3$$

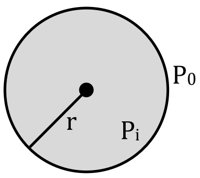
$$\Rightarrow r = \frac{R}{n^{1/3}}$$

Radius of big drop $R' = n^{1/3}.r = 2^{1/3}.R$

$$\frac{\text{Surface energy before change}}{\text{Surface energy after change}}$$

$$= \frac{n4\pi R^2 T}{4\pi R'^2 T} = \frac{2.R^2}{2^{2/3}.R^2} = 2^{1/3}$$

97.

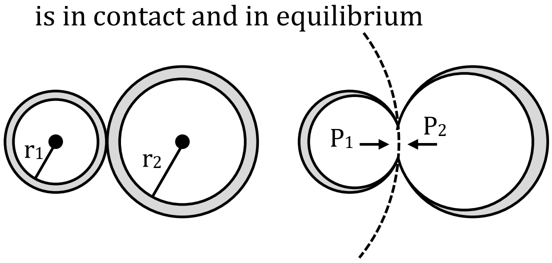


$\Rightarrow$ excess pressure $= P_i - P_0 = \frac{2T}{r}$

98. $W = 4\pi\left(\frac{D}{2}\right)^2\left(n^{1/3}-1\right)T = \pi D^2\left(n^{1/3}-1\right)T$

$= 3.14 \times (0.2 \times 10^{-2})^2(10 - 1) = 7.9 \times 10^{-6}$J

99. $\because r_1 < r_2 \quad \therefore P_1 > P_2$ Small portion of bubbles is in contact and in equilibrium



$\Rightarrow P_1 - P_2 = \frac{4T}{r}$

$\Rightarrow \frac{4T}{r_1} - \frac{4T}{r_2} = \frac{4T}{r}$

$\Rightarrow r = \boxed{\frac{r_1 r_2}{r_2 - r_1}}$

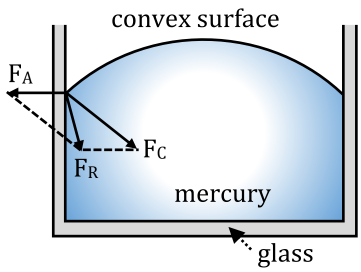
100. On liquid force due to surface tension = $(2\pi R) T\cos\theta_C$

In equilibrium : force due to S.T = weight of rise liquid

$(2\pi R)T\cos\theta_C = W$

$\frac{W_2}{W_1} = \frac{4\pi(2R)^2T}{4\pi R^2 T} = 4 \Rightarrow W_2 = 4W_1 = 4W$

101.



A liquid does not wet the sides of a solid, for obtuse (more than 90°) angle of contact.

102. $h = \frac{2T\cos\theta}{r\rho g}$

$= \frac{2\times70\times1}{\frac{1}{28}\times1\times980} = 4$ cm

103. $W = T\times \Delta A$

$= 20 \times 2\,[120 - 60] = 2400$ erg

104. Work done $= 8\pi TR^2$

$= 8\pi\times60\times10^{-3}\times\left(\frac{2}{10}\right)^2 = 8\pi\times6\times10^{-2}\times\frac{4}{10^2}$

$W = 192\,\pi \times 10^{-4}$ J

105. Detergent decreases the oil-water surface tension and helps in removing dirty greasy stains.

106. $(P_{ex})_1 = 2(P_{ex})_2$

$\frac{4T}{r_1} = 2\left(\frac{4T}{r_2}\right) \Rightarrow \frac{r_1}{r_2} = \frac{1}{2}$

$\frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

107. For soap film we have two free surface

$W = T \times 2\Delta A$

$= 3\times10^{-2}\times 2[4 \times 5 - 4 \times 4] \times 10^{-4}$

$= 24 \times 10^{-6}$ J

Exercise-II (Previous Year Questions)

1. $Y = \dfrac{\frac{F}{A}}{\frac{\Delta\ell}{\ell}} \Rightarrow \Delta\ell = \dfrac{F\ell}{AY}$

But $V = A\ell$ so $A = \dfrac{V}{\ell}$ (V = volume)

Therefore $\Delta\ell = \dfrac{F\ell^2}{VY} \propto \ell^2$

2. As surface area decreases so energy is released.

Released energy

$= 4\pi R^2T[n^{1/3}-1]$ where $R = n^{1/3}r$

$= 4\pi R^3T\left[\dfrac{1}{r}-\dfrac{1}{R}\right] = 3VT\left[\dfrac{1}{r}-\dfrac{1}{R}\right]$

3. As we know

$B = \dfrac{P}{\frac{\Delta V}{V}}$

so $\dfrac{\Delta V}{V} = \dfrac{P}{B}$

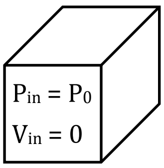
Now $P = \rho gh$ & compressibility 'C' $= \dfrac{1}{B}$

so $\dfrac{\Delta V}{V} = \rho gh(C)$

$= 10^3 \times 9.8 \times 2700 \times 45.4 \times 10^{-11}$

$= 1.201 \times 10^{-2}$

4. By Bernaulli's equation



$P + \dfrac{1}{2}\rho v^2 = P_0 + 0$

$P_0 - P = \dfrac{1}{2}\rho v^2$

$F = \dfrac{1}{2}\rho v^2 A$

$F = 2.4 \times 10^5$ N upward

5. Av = constant

$\pi R^2 V = n\pi r^2 v_1 \Rightarrow$

$v_1 = \dfrac{VR^2}{nr^2}$

6. $Y = \dfrac{F\ell}{A\Delta\ell} \Rightarrow \Delta\ell = \dfrac{F\ell}{AY}$

$(\Delta\ell)_{Steel} = (\Delta\ell)_{Brass}$

$\Rightarrow \dfrac{W_S\ell}{AY_S} = \dfrac{W_B\ell}{AY_B}$

$\Rightarrow \dfrac{W_S}{W_B} = \dfrac{Y_S}{Y_B} = \dfrac{2}{1}$

7. water will rise upto the top level but will not overflow. Radius of curvature (R') increases in such a way that final height h' is reduced and given by $h' = \dfrac{hR}{R'}$ (It is in accordance with Zurin's law)

8. Pressure = 150 mm Hg

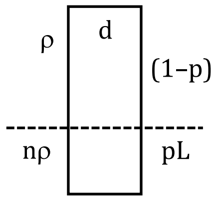
Pumping rate $= \dfrac{dV}{dt} = \dfrac{5\times10^{-3}}{60}$ m³/s

Power of heart $= P.\dfrac{dV}{dt} = \rho gh \times \dfrac{dV}{dt}$

$= (13.6 \times 10^3 \text{ kg/m}^3)(10)\times(0.15)\times\dfrac{5\times10^{-3}}{60}$

$= \dfrac{13.6\times5\times0.15}{6} = 1.70$ watt

9. Weight of cylinder = Upthurst by the two liquids



$L\,A\,d\,g = (pL)\,A\,(n\rho)g + (1-p)\,L\,A\,\rho\,g$

$\Rightarrow d = (1-p)\rho + pn\,\rho = [1 + (n-1)p]\rho$

10. $W = T(2\Delta A)$ $\{\Delta A = (20-8)\text{ cm}^2\}$

$\Rightarrow T = \dfrac{W}{2\Delta A} = \dfrac{3\times10^{-4}}{2\times12\times10^{-4}} = 0.125\text{ Nm}^{-1}$

11. $h = \dfrac{2T\cos\theta}{\rho gr}$

As r, h, T are same, $\dfrac{\cos\theta}{\rho} = \text{constant}$

$\Rightarrow \dfrac{\cos\theta_1}{\rho_1} = \dfrac{\cos\theta_2}{\rho_2} = \dfrac{\cos\theta_3}{\rho_3}$

As $\rho_1 > \rho_2 > \rho_3$

$\Rightarrow \cos\theta_1 > \cos\theta_2 > \cos\theta_3 \Rightarrow \theta_1 < \theta_2 < \theta_3$

As water rises so θ must be acute

So, $0 \le \theta_1 < \theta_2 < \theta_3 < \pi/2$

12. $B = \dfrac{\Delta P}{-\frac{\Delta V}{V}}$, $\dfrac{\Delta V}{V} = \dfrac{3\Delta R}{R}$

$B = \dfrac{\Delta P}{\frac{-3\Delta R}{R}} \Rightarrow -\dfrac{\Delta R}{R} = \dfrac{P}{3B}$ $(\Delta P = P)$

13. $\rho_0 g \times 140 \times 10^{-3} = \rho_w g \times 130 \times 10^{-3}$

$$\rho_0 = \frac{130}{140} \times 10^3 \approx 928\,kg/m^3$$

14. Rate of heat produced

$$\frac{dQ}{dt} = F_v \times v_T \quad \because\ v_T = \frac{2r^2}{9\eta}(\rho - \sigma)g \ \propto r^2$$

$= 6\pi\eta r v_T \times v_T$

$$\Rightarrow \frac{dQ}{dt} \propto r v_T^2 \ \propto r^5$$

15. $Y = \dfrac{F\ell}{A\Delta\ell} \quad \because V = A\ell \quad$ so $\ \ell = \dfrac{V}{A}$

$$\text{So } F = \frac{YA\Delta\ell}{\ell} = \frac{YA^2\Delta\ell}{V} \propto A^2$$

$$\frac{F_1}{F_2} = \left(\frac{A_1}{A_2}\right)^2 \Rightarrow \frac{F}{F_2} = \left(\frac{A}{3A}\right)^2 = \frac{1}{9} \Rightarrow F_2 = 9F$$

16. $P = P_0 + \rho g Z_0$ ....(i)

Also, $P = P_0 +$ ....(ii)

From (i) & (ii)

$$\rho g Z_0 = \frac{4T}{R}$$

$$\therefore \quad Z_0 = \frac{4T}{\rho g R} = \frac{4 \times 2.5 \times 10^{-2}}{10^3 \times 10 \times 10^{-3}}$$

$= 10^{-2}$ m = 1 cm

17. $U = \dfrac{1}{2}$ (force)(elongation)

$$= \frac{1}{2}(Mg)\ell = \frac{1}{2}Mg\ell$$

18. velocity of efflux $v = \sqrt{2gh}$

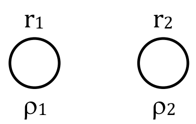
volume flow rate = Av = $A\sqrt{2gh}$

$= (2 \times 10^{-6})(2 \times 10 \times 2)^{1/2}$

$= 4\sqrt{10} \times 10^{-6}\ m^3/s$

$\cong 12.6 \times 10^{-6}\ m^3/s$

19.



$$v_T = \frac{2r^2(\sigma - \rho)g}{9\eta}$$

$$\frac{v_1}{v_2} = \left(\frac{r_1}{r_2}\right)^2 \frac{(\sigma_1 - \rho)}{(\sigma_2 - \rho)}$$

$$= \left(\frac{1}{2}\right)^2 \left(\frac{8\rho_2 - 0.1\rho_2}{\rho_2 - 0.1\rho_2}\right) = \frac{79}{36}$$

20. $\rho_{oil}\, h_{oil} = \rho_{water}\, h_{water}$

$$\Rightarrow \rho_{oil} = \frac{1000(15)}{20} = 750\ kg\ m^{-3}$$

22. $m \propto r \Rightarrow \dfrac{m_2}{m_1} = \dfrac{r_2}{r_1} \Rightarrow \dfrac{m_2}{5} = \dfrac{2r}{r} \Rightarrow m_2 = 10g$

23. $Y = \dfrac{FL}{A\Delta L} = \dfrac{MgL}{A(L_1 - L)}$

24. When angle of contact $\geq 90°$ then liquid doesn't wet solid.

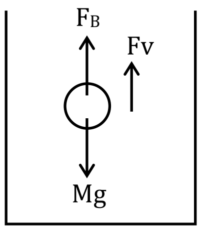
25. $76\ cm \times \rho_{Hg} \times g = h \times \rho_L \times g$

$$h = 76\,cm \times \frac{\rho_{Hg}}{\rho_L} = 76\ cm \times \frac{13600}{760} = 13.6\,m$$

26. Mass = M

Density of ball = d

Density of glycerine = $\dfrac{d}{2}$



$F_B = V_s \rho_\ell g = V\dfrac{d}{2}g$

$F_g = Mg = vdg$

for constant velocity, $F_{net} = 0$

$\therefore \quad F_B + F_v = M_g$

$$F_v = M_g - F_B = Vdg - \frac{Vdg}{2} = \frac{Vdg}{2} = \frac{Mg}{2}$$

31. $E = 2T(4\pi R^2)$

$= 2\,(0.03)\,(4)\,(3.14)\,(2 \times 10^{-2})^2$

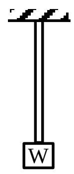
$= 3.01 \times 10^{-4}$J

32. Venturimeter works an Bernoulli's principle

33. Stress = $\dfrac{IRF}{A}$

Stress = $\dfrac{W}{A}$

(Here A Cross-sectional Area)



34. $F = 6\pi r\eta v$

$$= (6)(3.14)\left(\frac{1 \times 10^{-3}}{2}\right)(0.8 \times 10^{-1})(2)$$

$= 1.5 \times 10^{-3}$ N

35. Pressure is a scalar quantity.

36. $\frac{\text{E.P.E}}{\text{Volume}} = \frac{1}{2}(\text{stress})(\text{strain})$

$= \frac{1}{2}(Y)(\text{strain})^2$

$= \frac{1}{2}(Y)\left(\frac{\Delta L}{L}\right)^2$

$= \frac{1}{2}(2\times10^{11})\left(\frac{1\times10^{-3}}{100\times10^{-2}}\right)^2$

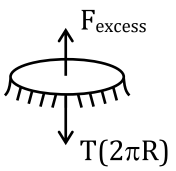
37. $Y = \frac{F\ell}{A\Delta\ell}$

$\Delta\ell = \frac{\left(\frac{F}{A}\right)\ell}{Y}$

$\Delta\ell = \frac{8\times10^{8}\times1}{2\times10^{11}}$

$\Delta\ell = 4\,\text{mm}$

38.



$F_{excess} = (T)(2\pi R)$

$= (0.07)\left[2\times\frac{22}{7}\times4.5\times10^{-2}\right]$

$= 44\times4.5\times10^{-4}$

$= 44\times4.5\times10^{-4}$

$= 19.8\times10^{-3}\,\text{N}$

$= 19.8\,\text{mN}$

39. $F = YA\alpha\Delta\theta$

$= 0.5 \times 10^{11} \times 10^{-3} \times 10^{-5} \times (100 - 0)$

$= 50 \times 10^{3}\text{N}$

40. From continuity equation,

Speed at Y is less than speed at X

$\therefore K_2 < K_1$

Exercise-III (Analytical Questions)

1. I. In water adhesive force is greater than cohesive force. $\therefore$ It wets the capillary, whereas in mercury, cohesive force is greater.

II. $\Delta P = \frac{4T}{R} \Rightarrow \Delta P \propto \frac{1}{R}$

3. In case of ice and water, the volume of water displaced by ice is equal to the volume of water formed by melting the ice, therefore level remains unchanged.

5. Velocity of efflux is given by

$$V = \sqrt{\frac{2gh}{1-\frac{a^2}{A^2}}}$$

$a \rightarrow$ area of orifice (hole)

$A \rightarrow$ Area of tank

7. $\Delta\ell = \frac{F\ell}{Ay}$

for coper wire :

$(\Delta\ell)_1 = \frac{500\times8}{0.5\times10^{-4}\times10^{11}}$

$(\Delta\ell)_1 = 0.8$ mm

for steel wire :

$(\Delta\ell)_2 = \frac{500\times4}{0.5\times10^{-4}\times2\times10^{11}}$

$(\Delta\ell)_2 = 0.2$ mm

Total elongation :

$\Delta\ell = (\Delta\ell)_1 + (\Delta\ell)_2$

= 1 mm

8. $\because \quad P_{e_x} = \frac{4T}{r}$

$\therefore \quad P_{e_x} \propto \frac{1}{r}$

The air pressure is greater inside the smaller bubble.

Hence air flows from the smaller to the larger bubble.

9. In given condition

$\therefore$ Tension in a rod is = F

$\therefore$ Stress = F/A

10. When a needle is placed carefully on the surface of water, it floats on the surface of water due to the surface tension of water, which does not allow the needle to sink. In case of a steel ball, the surface tension of the water is not sufficient to keep it floating, so it sinks down.

11. On spraying, surface area increases and hence surface energy; this increase in surface energy is on the expense of decrease in internal energy and hence temperature decreases.

12. Viscosity of liquids is greater than gases. Insoluable impurities decreases the cohesive force acting between the molecule of liquid. Therefore surface tension decreases.

13. For a substance:

$B = \frac{1}{C}$

$BC = 1$

for ideal rigid body

strain = 0

$\therefore \ \eta = \frac{\text{shear stress}}{\text{shear strain}}$

$\eta = \infty$

14. For concave meniscus

$F_A > \frac{F_C}{\sqrt{2}} \Rightarrow \theta_C < 90°$

For convex meniscus

$F_A > \frac{F_C}{\sqrt{2}} \Rightarrow \theta_C > 90°$

For plane meniscus

$F_A = \frac{F_C}{\sqrt{2}} \Rightarrow \theta_C = 90°$