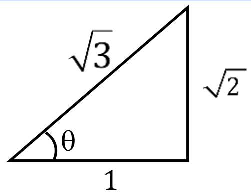
Basic Mathematics Used in Physics and Vectors

SOLUTIONS

Exercise-I (Conceptual Questions)

2.



$\sin\theta = \dfrac{\sqrt{2}}{\sqrt{3}}$

$\tan\theta = \dfrac{P}{B} = \dfrac{\sqrt{2}}{1}$

3. If $\sin\theta = \cos\theta$

$\tan\theta = 1$

For $180° < \theta < 360° \Rightarrow \theta = 225°$

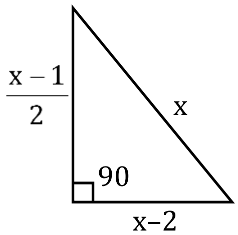
4. If $\sin\theta_1 + \sin\theta_2 + \sin\theta_3 = 3$

then $\theta_1 = \theta_2 = \theta_3 = 90°$

so $\cos\theta_1 + \cos\theta_2 + \cos\theta_3 = 0$

5. Max. value = $\sqrt{7^2 + (-24)^2} = \sqrt{49 + 576} = 25$

6.



Hypotenuse (H) = x

Base (B) = x – 2

Perpendicular (P) = $\dfrac{x-1}{2}$

$x^2 = (x-2)^2 + \left(\dfrac{x-1}{2}\right)^2$

$4x^2 = 4(x-2)^2 + (x-1)^2$

$x^2 - 18x + 17 = 0$

x = 17 cm and x = 1 cm (not considerable)

So, H = 17 cm

B = 17 – 2 = 15 cm

$P = \dfrac{17-1}{2} = 8$ cm

7. $x = at^4$; $y = bt^3$

$\dfrac{dx}{dt} = 4at^3$ & $\dfrac{dy}{dt} = 3bt^2$

So $\dfrac{dy}{dt} = \dfrac{3bt^2}{4at^3} = \dfrac{3b}{4at}$

8. $A = 5t^2 + 4t + 8$

$\dfrac{dA}{dt} = 10t + 4$

At t = 3 sec

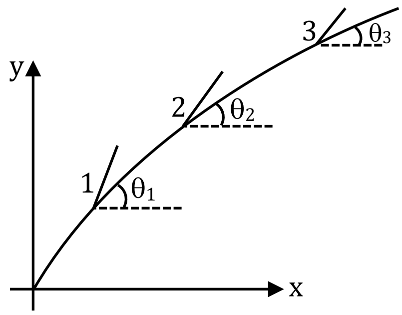
$\dfrac{dA}{dt} = 34\ m^2/s$

9. If side = a, then rate of increase of perimeter w.r.t. time = $4\left(\dfrac{da}{dt}\right) = 4(0.1) = 0.4\ cm/s$

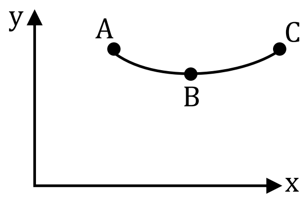
10. $3y = x + 5$

$\dfrac{3dy}{dt} = \dfrac{dx}{dt}$ (x changes at a faster rate)

11. $\theta_1 > \theta_2 > \theta_3 \Rightarrow m_1 > m_2 > m_3$



12.



From A to B : θ is obtuse and increases, so slope (tan θ) will decreases.

From B to C : θ is acute and increases

So, B to C magnitude will increases.

13. Area under the curve $\int_2^3 f(x)dx = \int_2^3 x^2 dx$

$$= \frac{1}{3}[x^3]_2^3 = \frac{19}{3}$$

14. $\int_0^4 |1-x|dx = \int_0^1 (1-x)dx + \int_1^4 -(1-x)dx$

$$= [x]_0^1 - \left[\frac{x^2}{2}\right]_0^1 - [x]_1^4 + \left[\frac{x^2}{2}\right]_1^4$$

$$= 1 - \frac{1}{2} - 3 + \frac{15}{2} = 5$$

15. $y = x^2 + 2 - 3x$

on y-axis $x = 0$

$\Rightarrow \quad y = 2$

16. at $t = 2$ $\quad x_A = 6, \quad y_A = 2$

$x_B = 6, \quad y_B = 2 + 3(2)^2 = 14$

distance $= \sqrt{(6-6)^2 + (14-2)^2} = 12$

17. distance d $= \sqrt{\{a-b-a-b\}^2 + \{c-b-b-c\}^2}$

$d = \sqrt{4b^2 + 4b^2} = 2\sqrt{2}\,b$

18. Minimum distance

$= \sqrt{(5-0)^2 + (3-3)^2 + (-8-4)^2}$

$= \sqrt{5^2 + 12^2} = 13$

19. $\because y = x^2 + 2x - 3$

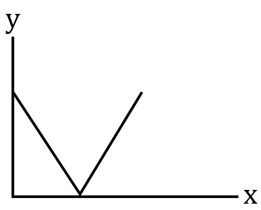
$\therefore y = (x + 3)(x - 1)$

For $y = 0 \Rightarrow x = -3$ and $x = 1$

For $x = 0 \Rightarrow y = -3$

20. For $0 \le x \le 1$, $\quad y = -(x - 1) = -x + 1$

For $x \ge 1$, $\quad y = x - 1$



22. $f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$

$f^2 = \frac{1}{4\pi^2}\frac{g}{\ell}$

$\Rightarrow f^2 \propto \frac{1}{\ell}$ graph between f & $\frac{1}{\ell}$ is parabolic

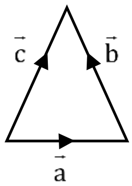
23. $1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + ...\infty$

$$S_\infty = \frac{a}{1-r} = \frac{1}{1-1/3} = \frac{3}{2}$$

29. Angular velocity is an axial vector

30. $\hat{n} = \frac{\vec{A}}{|\vec{A}|}$

31.



Resultant of $\vec{a}$ & $\vec{b}$ is $\vec{c}$

$(\vec{a} + \vec{b} = \vec{c})$

so $\quad \vec{a} + \vec{b} + \vec{c} = \vec{c} + \vec{c} = 2\vec{c}$

32. $10 - 5 \le R \le 10 + 5$

$5N \le R \le 15N$

33. $10 - 8 \le R \le 10 + 8$

$2N \le R \le 18N$, So 20N cannot be the resultant of these forces.

34. $(A-B) \le R \le (A+B)$

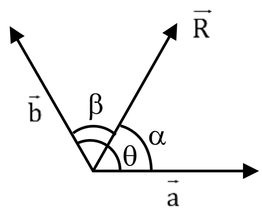
35. $R = 2a\cos(\theta/2) \Rightarrow a = 2a\cos(\theta/2)$

$\Rightarrow \cos(\theta/2) = 1/2 \Rightarrow \theta/2 = 60° \Rightarrow \theta = 120°$

36. If $R = a = b$

then angle between a & b will be 120° ($\theta = 120°$) & resultant will be bisector

$$\alpha = \beta = \frac{\theta}{2} = \frac{\pi}{3}$$



37. $\sqrt{a^2 + b^2 + 2ab\cos 60°} = \sqrt{7}\,b$

$\Rightarrow \quad a^2 + b^2 + ab = 7b^2$

$\Rightarrow \quad a^2 + ab - 6b^2 = 0$

$\Rightarrow \quad (a - 2b)(a + 3b) = 0$

$\Rightarrow \quad \boxed{\frac{b}{a} = \frac{1}{2}}$ or $\boxed{\frac{b}{a} = -\frac{1}{3}}$ (not considerable)

38. $\vec{A}+\vec{B}=\vec{C} \Rightarrow \sqrt{A^2+B^2+2AB\cos\theta}=C$

and A + B = C $\Rightarrow$ $\sqrt{A^2+B^2+2AB\cos\theta}$ = A + B

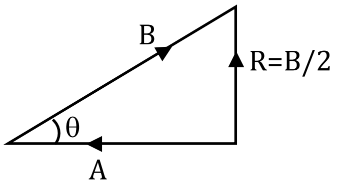
$\Rightarrow A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 + 2AB$

$\Rightarrow \cos\theta = 1 \Rightarrow \theta = 0°$

39. $\vec{A}+\vec{B}=\vec{C}$ and $|\vec{C}|=A-B$

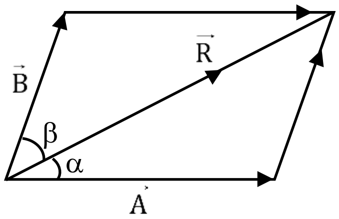
$\Rightarrow$ angle between $\vec{A}$ & $\vec{B}$ is 180°

40.



$\sin\theta = \frac{R}{B} = \frac{B/2}{B} = \frac{1}{2} \Rightarrow \theta = 30°$

41.



Resultant is inclined towards vector of large magnitude thus. If A > B then α < β.

42. If two vectors have equal magnitude then resultant will be bi-sector of angle between them.

43. $\tan\beta = \frac{A\sin\theta}{B+A\cos\theta} = \tan 90° = \infty$

thus, $B + A\cos\theta = 0 \Rightarrow \cos\theta = -B/A$

44. $\tan 30° = \frac{(Q)\sin(90°)}{P+Q\cos(90°)}$ $(\because 30° + 60° = 90°)$

$\frac{1}{\sqrt{3}} = \frac{Q}{P} \Rightarrow \boxed{P=\sqrt{3}(Q)}$

As $\sqrt{(\sqrt{3}Q)^2+Q^2+2(\sqrt{3}Q)(Q)\cos 90°}=40$

$\Rightarrow$ Q = 20N and $P=20\sqrt{3}N$

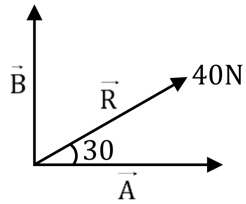
OR

A = R cos30°

$A = 40\times\frac{\sqrt{3}}{2}$

$A = 20\sqrt{3}N$

$B = R\cos 60° = 40\times\frac{1}{2}$



B = 20N

46. $R_1^2 = A^2 + B^2 + 2AB\cos\theta$

$|\vec{R}_2| = |\vec{A}-\vec{B}| \Rightarrow R_2^2 = A^2 + B^2 - 2AB\cos\theta$

thus $R_1^2 + R_2^2 = 2(A^2 + B^2)$

$\Rightarrow 2\left(R_1^2+R_2^2\right) = 4(A^2+B^2)$

47. If $|\vec{A}+\vec{B}|=|\vec{A}|=|\vec{B}|$

then angle between them θ = 120°

Let $|\vec{A}|=|\vec{B}|$ = a

So $|\vec{A}-\vec{B}|=\sqrt{A^2+B^2-2AB\cos\theta}$

$=\sqrt{a^2+a^2-2a^2\left(-\frac{1}{2}\right)}=\sqrt{3}|\vec{A}|$

48. $|\hat{n}_1-\hat{n}_2|=1$

$\sqrt{1^2+1^2-2\cos\theta}=1 \Rightarrow 2-2\cos\theta=1$

$\Rightarrow \cos\theta=\frac{1}{2} \Rightarrow \theta=60°$

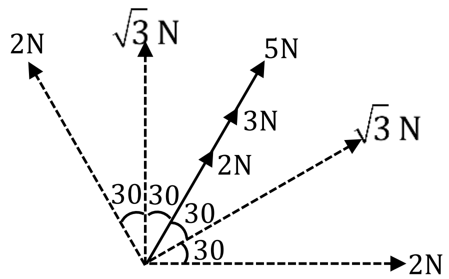
$|\hat{n}_1+\hat{n}_2|=\sqrt{1^2+1^2+2\times1\times1\cos 60°}=\sqrt{3}$

49. If vectors of equal magnitude are acting at equal angle of separation then resultant will be zero.

51. Sum of all small forces should be greater than largest force then resultant can be zero.

52. Resultant of 2N & 2N

= (2) (2) cos $\left(\frac{120°}{2}\right)$ = 2N

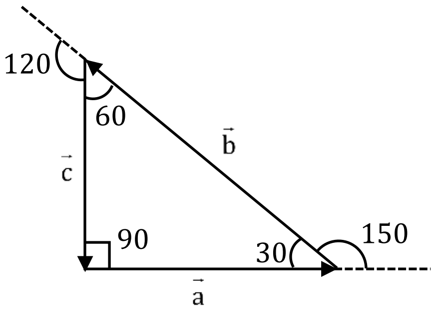


Resultant of $\sqrt{3}N$ & $\sqrt{3}N$

$= (2)(\sqrt{3})\cos\left(\frac{60°}{2}\right) = 3N$

Net resultant = 5 + 3 + 2 = 10 N

53.

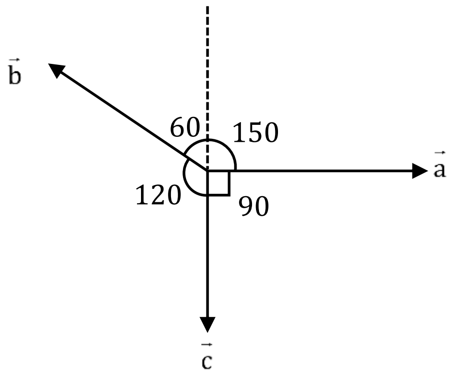


According to this

$\boxed{b^2 = a^2 + c^2}$

a : b : c = $\sqrt{3}$: 2 : 1

OR



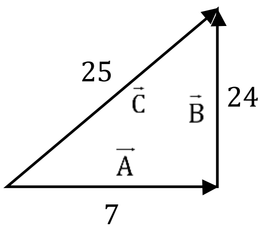
bcos60° = c

$\frac{b}{c} = \frac{2}{1}$

bsin60° = a

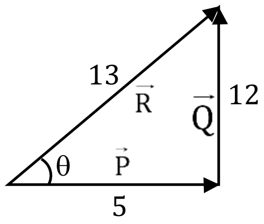
$\frac{b}{a} = \frac{2}{\sqrt{3}}$ so, ratio of a, b & c is $\sqrt{3}$: 2 : 1

54. $\vec{A} + \vec{B} = \vec{C}$ & $A^2 + B^2 = C^2$



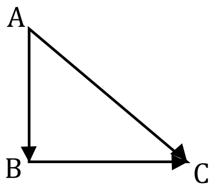
Angle between $\vec{A}$ & $\vec{B}$ is 90°

55. $\vec{P} + \vec{Q} = \vec{R}$ and $P^2 + Q^2 = R^2$



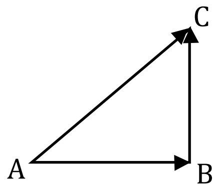
$\cos\theta = \frac{5}{13} \Rightarrow \theta = \cos^{-1}\left(\frac{5}{13}\right)$

56.



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC} = (\hat{i} + \hat{j} + \hat{k}) + 3\hat{i}$

$= 4\hat{i} + \hat{j} + \hat{k}$



$= \sqrt{16 + 1 + 1} = \sqrt{18}$ unit

$\overrightarrow{BC} = \overrightarrow{AC} - \overrightarrow{AB} = 3\hat{i} - (\hat{i} + \hat{j} + \hat{k})$

$= 2\hat{i} + \hat{j} + \hat{k}$

$|\overrightarrow{BC}| = \sqrt{4 + 1 + 1} = \sqrt{6}$ unit

60. The direction of a vector remains unchanged on multiplying the vector by a positive scalar.

64. $\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{i} - 2\hat{j}}{\sqrt{5}}$

65. $\vec{A} + \vec{B} = \hat{j}$

$\Rightarrow \vec{B} = \hat{j} - \vec{A} = \hat{j} - (\hat{i} - \hat{j} + \hat{k}) = -\hat{i} + 2\hat{j} - \hat{k}$

66. $\hat{n} = 0.3\hat{i} - 0.4\hat{j} + c\hat{k}$

$\sqrt{(0.3)^2 + (-0.4)^2 + c^2} = 1$

$\Rightarrow$ $c^2 = 1 - 0.09 - 0.16$

$\Rightarrow$ $c^2 = 0.75$

$\Rightarrow$ $c = \sqrt{0.75}$

67. $\hat{e}_r = |\hat{e}_r|\cos\theta\hat{i} + |\hat{e}_r|\sin\theta\hat{j}$

$\because |\hat{e}_r| = 1$

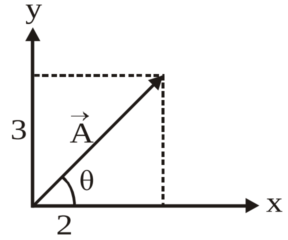
$\Rightarrow \hat{e}_r = (\cos\theta)\hat{i} + (\sin\theta)\hat{j}$

68. $\vec{R} = 7\hat{i} + 24\hat{j} + 25\hat{k}$

$|\vec{R}| = \sqrt{7^2 + 24^2 + 25^2} = \sqrt{49 + 576 + 625}$

$= 25\sqrt{2}\,N$

69.



$\vec{A} = 2\hat{i} + 3\hat{j}$

$\tan\theta = \frac{3}{2}$

$\theta = \tan^{-1}\left(\frac{3}{2}\right)$

70. Let $\vec{A}$ has to the added

$(\hat{i} - 2\hat{j} + 2\hat{k}) + (2\hat{i} + \hat{j} - \hat{k}) + (\vec{A}) = \hat{j}$

$\vec{A} = -3\hat{i} + 2\hat{j} - \hat{k}$

71. $\vec{R} = \vec{A} + \vec{B} = 4\hat{i} + 3\hat{j} + 6\hat{k} + 2\hat{i} - 3\hat{j} + 2\hat{k}$

$\vec{R} = 6\hat{i} + 8\hat{k}$

$\hat{R} = \frac{6\hat{i} + 8\hat{k}}{\sqrt{6^2 + 8^2}} = \frac{6\hat{i} + 8\hat{k}}{10}$

72. $\vec{c} = 3|\vec{b}|\hat{a}$

$|\vec{b}| = \sqrt{3}; \hat{a} = \frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}$

so $\vec{c} = (3)(\sqrt{3})\left(\frac{2\hat{i} + 2\hat{j} - \hat{k}}{3}\right) = 2\sqrt{3}\hat{i} + 2\sqrt{3}\hat{j} - \sqrt{3}\hat{k}$

73. Unit vector $\hat{v} = \frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}$

Force in the direction of velocity

$\vec{F} = |\vec{F}|\hat{v}$

$\vec{F} = (10\sqrt{3})\left(\frac{\hat{i} + \hat{j} - \hat{k}}{\sqrt{3}}\right) = (10\hat{i} + 10\hat{j} - 10\hat{k})N$

74. $\vec{A} = \sqrt{2}\,\hat{i} + \sqrt{2}\,\hat{j} + \hat{k}$

$\cos\alpha = \frac{A_x}{A}, \ \cos\beta = \frac{A_y}{A}, \ \cos\gamma = \frac{A_z}{A}$

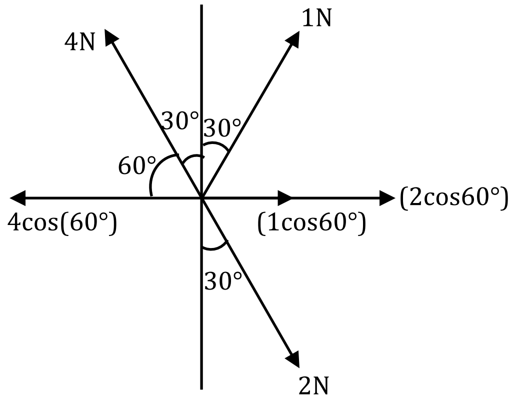
$\cos\alpha = \frac{\sqrt{2}}{\sqrt{5}}, \ \cos\beta = \frac{\sqrt{2}}{\sqrt{5}}, \ \cos\gamma = \frac{1}{\sqrt{5}}$

75. $\vec{a} - \vec{b} = \hat{j} - 2\hat{k}$

$|\vec{a} - \vec{b}| = \sqrt{1^2 + (-2)^2} = \sqrt{5}$

$\cos\alpha = 0 \ ; \cos\beta = \frac{1}{\sqrt{5}}; \cos\gamma = -\frac{2}{\sqrt{5}}$

76. To have the resultant force along the y-direction the force along x-axis must be zero.



Let F is the require force then,

$-4\cos(60°) + (1)\cos(60°) + 2\cos(60°) + F = 0$

F = 0.5N

77. If a vector rotates through any angle then its magnitude remain same.

so, $\sqrt{1^2 + b^2 + 3^2} = \sqrt{2^2 + (2b-1)^2 + 1^2}$

$\Rightarrow 10 + b^2 = 5 + 4b^2 + 1 - 4b$

$\Rightarrow 3b^2 - 4b - 4 = 0$

$\Rightarrow (3b + 2)(b - 2) = 0$

$\Rightarrow b = 2$ and $b = -\frac{2}{3}$

78. $\vec{P}\cdot\vec{Q} = -PQ$

$\vec{P}\vec{Q}\cos\theta = -PQ$

$\cos\theta = -1$

$\Rightarrow \theta = 180°$

79. $w=\vec{F}\cdot\vec{S}$

$w=(2\hat{i}+3\hat{j})\cdot(3\hat{i}+4\hat{j})$

$w = 6 + 12 = 18J$

80. $W=\vec{F}.\vec{S}=(2\hat{i}+2\hat{j}).(2\hat{i}-3\hat{j})$

$W = 4 - 6 = -2J$

81. $\vec{F}=14\left(\dfrac{3\hat{i}+2\hat{j}-6\hat{k}}{\sqrt{9+4+36}}\right)=6\hat{i}+4\hat{j}-12\hat{k}$

$\vec{S}=2\hat{i}+4\hat{j}-2\hat{k}$

$W=\vec{F}.\vec{S}=12+16+24=52J$

82. $\vec{A}=2\hat{i}+3\hat{j}+8\hat{k}$ & $\vec{B}=4\hat{i}-4\alpha\hat{j}+\alpha\hat{k}$

$\because \quad \vec{A}\perp\vec{B}\Rightarrow \vec{A}\cdot\vec{B}=0$

$\Rightarrow \quad 8 - 12\alpha + 8\alpha = 0$

$\Rightarrow \quad 4\alpha = 8$

$\Rightarrow \quad \alpha = 2$

83. $\vec{P}.\vec{Q}=0\Rightarrow\left(a\hat{i}+a\hat{j}+3k\right).\left(a\hat{i}-2\hat{j}-k\right)=0$

$a^2 - 2a - 3 = 0 \Rightarrow (a-3)(a+1) = 0$

$a = 3$ and $a = -1$

84. $\therefore \ \vec{A}\perp\vec{B}$

$\vec{A}.\vec{B}=0$

85. $\because \ \vec{A}$ is normal to $\vec{B} \Rightarrow \vec{A}\cdot\vec{B}=0$

86. $\hat{n}=a\hat{i}+b\hat{j}$ is perpendicular to $\hat{i}-\hat{j}$

$\Rightarrow(a\hat{i}+b\hat{j}).(\hat{i}-\hat{j})=0$

$\Rightarrow a - b = 0$

$\Rightarrow a = b$

88. $\left(\vec{A}+\vec{B}\right).\left(\vec{A}-\vec{B}\right)=0$

$\Rightarrow A^2-\vec{A}.\vec{B}+\vec{B}.\vec{A}-B^2=0$

$\Rightarrow A = B \ (\because \vec{A}.\vec{B}=\vec{B}.\vec{A})$

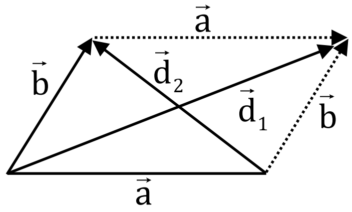
89. $(\vec{A}+\vec{B}).(\vec{A}-\vec{B})=A^2-\vec{A}.\vec{B}+\vec{B}.\vec{A}-B^2$

$= A^2 - B^2$

$= 0 \ [\because |\vec{A}|=|\vec{B}|]$

Hence $\vec{A}+\vec{B}$ & $\vec{A}-\vec{B}$ are perpendicular to each other

90. According to triangle law



$\vec{a}+\vec{b}=\vec{d}_1$

$\Rightarrow \quad a^2+b^2+2ab\cos\theta=d_1^2 \quad$ ....(1)

& $\quad \vec{a}-\vec{b}=-\vec{d}_2$

$\Rightarrow \quad a^2+b^2-2ab\cos\theta=d_2^2 \quad$ ....(2)

By (1) + (2)

$a^2+b^2=\dfrac{d_1^2+d_2^2}{2}$

91. $\vec{A}=3\hat{i}+4\hat{j}+5\hat{k}, \quad \vec{B}=6\hat{i}+8\hat{j}+10\hat{k}$

$\vec{B}=2[3\hat{i}+4\hat{j}+5\hat{k}]$

$\vec{B}=2\vec{A}$

$\Rightarrow \quad \vec{B}$ is parallel to $\vec{A}$

92. $\vec{R}=\vec{A}+2\vec{B}+2\vec{A}-2\vec{B}$

$\vec{R}=3\vec{A} \Rightarrow \vec{R}$ is parallel to $\vec{A}$

93. $\cos\theta=\dfrac{\vec{A}.\vec{B}}{AB}=\dfrac{(\hat{i}+\hat{j}).(\hat{i}+\hat{k})}{\sqrt{2}\sqrt{2}}=\dfrac{1}{2}\Rightarrow\theta=60°$

94. $\because \ \cos\theta=\dfrac{\vec{A}.\vec{B}}{|\vec{A}||\vec{B}|}$

$\Rightarrow \ \cos\theta=\dfrac{(2\hat{i}+\hat{j}+2\hat{k}).(\hat{i}-\hat{j}+\hat{k})}{\left(\sqrt{2^2+1^2+2^2}\right)\sqrt{1^2+(-1)^2+(1)^2}}$

$\Rightarrow \cos\theta=\dfrac{1}{\sqrt{3}}\Rightarrow\boxed{\theta=\cos^{-1}\left(\dfrac{1}{\sqrt{3}}\right)}$

95. Projection $=\dfrac{(3\hat{i}+4\hat{k}).\hat{k}}{|\hat{k}|}=4$

96. Projection of $\vec{B}$ on $\vec{A}=B\cos\theta$

$=B\left[\dfrac{\vec{A}\cdot\vec{B}}{AB}\right]=\vec{B}\cdot\hat{A}$

100. $\vec{P}\times\vec{Q}=PQ\sin\theta\hat{n}\Rightarrow\hat{n}=\dfrac{\hat{P}\times\hat{Q}}{\sin\theta}=\dfrac{\vec{P}\times\vec{Q}}{|\vec{P}\times\vec{Q}|}$

102. $(\vec{A}+\vec{B})\times(\vec{A}-\vec{B})$

$=(\vec{A}\times\vec{A})-(\vec{A}\times\vec{B})+(\vec{B}\times\vec{A})-(\vec{B}\times\vec{B})$

$=2(\vec{B}\times\vec{A})=-2(\vec{A}\times\vec{B})$

103. $\vec{A}\times\vec{B}=-(\vec{B}\times\vec{A})$

$\therefore$ angle between these vectors is zero

104. Vertically upward $=\hat{k}$

South $=-\hat{j}$

$\vec{A}\times\vec{B}\Rightarrow\hat{k}\times(-\hat{j})=\hat{i} \qquad \hat{i} \Rightarrow$ east

105. $\frac{3}{4}AB\sin\theta = AB\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

$\frac{\sin\theta}{\cos\theta} = \frac{4}{3}$

$\theta = 53°$

106. $\vec{A}\times\vec{B} = (3\hat{i}+4\hat{j})\times(6\hat{i}+8\hat{j}) = \vec{0}$

107. $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

$\sqrt{A^2+B^2+2A\cos\theta} = \sqrt{A^2+B^2-2AB\cos\theta}$

$\Rightarrow$ $4AB\cos\theta = 0$

$\Rightarrow$ $\cos\theta = 0$

$\Rightarrow$ $\theta = 90°$

$\Rightarrow$ $\vec{A}.\vec{B} = 0$

108. Cross product of two || vector are zero.

$\therefore$ $\vec{F}_1$, $\vec{F}_2$ are || vectors

109. Cross product of two parallel or anti parallel vectors are zero.

110. $\vec{A}\times\vec{B} = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & 2 \\ 6 & 2 & S \end{vmatrix} = 0$$

$\hat{i}(S-4) - \hat{j}(3S-12) + \hat{k}(6-6)$

$(S-4)\hat{i} - \hat{j}(3S-12) = 0\hat{i} + 0\hat{j}$

S – 4 = 0 or 3S – 12 = 0

S = 4 or S = 4

112. $\vec{v} = \vec{\omega}\times\vec{r}$

$$\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 1 & 1 & 1 \end{vmatrix}$$

$= \hat{i}(-1-1) - \hat{j}(1-1) + \hat{k}(1+1)$

$= -2\hat{i} + 2\hat{k}$

113. $\vec{a}\times\vec{b} = \begin{vmatrix} i & j & k \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix}$

$= \hat{i}(-4-2) - \hat{j}(4-1) + \hat{k}(4+2)$

$\vec{a}\times\vec{b} = -6\hat{i} - 3\hat{j} + 6\hat{k}$

$\hat{n} = \pm\frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|} = \pm\left(\frac{-6\hat{i}-3\hat{j}+6\hat{k}}{3\times 3}\right)$

$= \pm\left(-\frac{2}{3}\hat{i} - \frac{1}{3}\hat{j} + \frac{2}{3}\hat{k}\right)$

Exercise-II (Previous Year Questions)

1. $\vec{A}\cdot\vec{B} = 0$

$\cos\omega t\cos\frac{\omega t}{2} + \sin\omega t\sin\frac{\omega t}{2} = 0$

$\cos\left(\omega t - \frac{\omega t}{2}\right) = 0 \Rightarrow \cos\frac{\omega t}{2} = 0$

2. $|\vec{A}+\vec{B}| = |\vec{A}-\vec{B}|$

$A^2 + B^2 + 2AB\cos\theta = A^2 + B^2 - 2AB\cos\theta$

$\cos\theta = 0 \Rightarrow \theta = 90°$.

3. $\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0} \Rightarrow \vec{a} = 0 \Rightarrow \vec{v} = \text{constant}$

4. $\vec{F} = 2\hat{i} + \hat{j} - \hat{k}$ $\quad$ $\vec{r} = 3\hat{i} + 2\hat{j} - 2\hat{k}$

$\vec{F}.\vec{r} = 6 + 2 + 2 = 10$

$\vec{F}\times\vec{r} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 3 & 2 & -2 \end{vmatrix} = \hat{i}(0) - \hat{j}(-1) + \hat{k}(1)$

$= \hat{j} + \hat{k}$

$|\vec{F}\times\vec{r}| = \sqrt{2}$

Exercise-III (Analytical Questions)

2. $|\vec{A}| = |\vec{B}| = Z, \theta = 60°$

$|\vec{A}+\vec{B}| = 2a\cos\left(\frac{\theta}{2}\right) = \sqrt{3}Z$

$|\vec{A}-\vec{B}| = 2a\sin\left(\frac{\theta}{2}\right) = Z$

$\vec{A}\cdot\vec{B} = AB\cos\theta = \frac{1}{2}\cdot Z^2$

$|\vec{A}\times\vec{B}| = AB\sin\theta = \frac{\sqrt{3}}{2}\cdot Z^2$

3. According to triangle law.

5. Direction of null vector is not defined and direction of $\vec{A} \times \vec{B}$ is given by right hand thumb rule which lies in $\perp$ plane of $\vec{A}, \vec{B}, (\vec{A}+\vec{B})$ and $(\vec{A}-\vec{B})$.

6. $\vec{A} \cdot \vec{B}$ in a scalar quantity that's why it is known as scalar product.