

Quadratic Equations

1. For $p > 0$ and $3x^2 + px + 3 = 0$ one root of above equation is square of the other then p is

(A) -6 (B) 10 (C) 2 (D) 3

KEY: D

Sol: $\alpha + \alpha^2 = \frac{-1}{3}; \alpha^3 = 1$

$$\alpha = 1, \omega, \omega^2$$

If $\alpha = 1$

$P = -6$ as $P > 0$ neglected

if $\alpha = \omega; P = 3$

2. If α, β are roots of $3x^2 + 2bx + c = 0$ whose discriminant is Δ_1 ; $\alpha + \delta, \beta + \delta$ are roots of $9x^2 + 2Bx + C = 0$ whose discriminant is Δ_2 then $\frac{\Delta_1}{\Delta_2}$ is

(A) $\frac{1}{9}$ (B) 9 (C) 3 (D) $\frac{1}{3}$

KEY: A

Sol: $\alpha - \beta = \frac{\sqrt{\Delta_1}}{3}$

$$(\alpha + \delta) - (\beta + \delta) = \frac{\sqrt{\Delta_2}}{9}$$

$$\frac{\Delta_1}{9} = \frac{\Delta_2}{81}; \frac{\Delta_1}{\Delta_2} = \frac{1}{9}$$

3. If α is a root of $ax^2 + bx + c = 0$; β is a root of $-ax^2 + bx + c = 0$ and γ is a root of $ax^2 + 2bx + 2c = 0$ then

A) $\gamma < \alpha < \beta$ B) $\alpha < \beta < \gamma$ C) $\alpha < \gamma < \beta$ D) $\frac{\alpha}{\beta} < \gamma < \frac{\beta}{\alpha}$

KEY: C

Sol: Let $f(x) = ax^2 + 2bx + 2c$

Then, we have $f(\alpha) = a\alpha^2 + 2b\alpha + 2c = -a\alpha^2 + 2(a\alpha^2 + b\alpha + c)$

$= -a\alpha^2$ [$\because \alpha$ is a root of $ax^2 + bx + c = 0 \therefore a\alpha^2 + b\alpha + c = 0$]

Also we have, $f(\beta) = a\beta^2 + 2b\beta + 2c = 3a\beta^2 + 2(-a\beta^2 + b\beta + c)$

$= 3a\beta^2$ [$\because \beta$ is a root of $-ax^2 + bx + c = 0 \therefore a^2\beta - b\beta - c = 0$]

Now, $f(\alpha)f(\beta) = -3a^2\alpha^2\beta^2 < 0$ which implies that $f(\alpha), f(\beta)$ are of opposite signs and hence, proves that the curve represented by $y = f(x)$ cuts the X-axis somewhere between α and β .

In other words $f(x) = 0$ has a root lying between α and β .

4. If α and β are the roots of the equation $(x-a)(x-b) = c \neq 0$, then the roots of the equation $(x-\alpha)(x-\beta) + c = 0$ are

a) a, b b) a, c c) b, c d) $a+c, b+c$

Sol: (A)

$$(x-\alpha)(x-\beta) = (x-a)(x-b) - c$$

$$\Rightarrow (x-\alpha)(x-\beta) + c = (x-a)(x-b) \Rightarrow a, b \text{ are roots}$$

5. If the equation $|x^2 + bx + c| = k$ has four distinct real roots, then

- a) $b^2 - 4c > 0$ and $0 < k < \frac{b^2 - 4c}{4}$ b) $b^2 - 4c < 0$ and $0 < k < \frac{4c - b^2}{4}$
 c) $b^2 - 4c > 0$ and $k > \frac{b^2 - 4c}{4}$ d) $b^2 - 4c < 0, k > \frac{4c - b^2}{4}$

Sol: (A)

For the equation to have four distinct real roots, the line $y = k$ must intersect $y = |x^2 + bx + c|$ at four distinct points.

$$\therefore b^2 - 4c > 0 \text{ and } k \in \left(0, \frac{b^2 - 4c}{4}\right)$$

6. The quadratic equation $x^2 - 6x + a = 0$ and $x^2 - cx + 6 = 0$ have one root in common, the other roots of the first and second equations are integers in the ratio 4:3 then the common root is

- a) 2 b) 1 c) 4 d) 3

Sol: (A)

Let the roots of $x^2 - 6x + a = 0$ are $\alpha, 4\beta$ and $x^2 - cx + 6 = 0$ are $\alpha, 3\beta$

$$\alpha + 4\beta = 6, 4\alpha\beta = a$$

$$\alpha + 3\beta = c, 3\alpha\beta = 6$$

$$\Rightarrow \alpha\beta = 2, a = 8$$

The roots of $x^2 - 6x + 8 = 0$ are 2, 4

Common root is 2

7. If $p(q-r)x^2 + q(r-p)x + r(p-q) = 0$ has equal roots then $\frac{2}{q} =$

- a) $\frac{p+r}{p-r}$ b) $\frac{p-r}{pr}$ c) $\frac{p+r}{pr}$ d) $\frac{p-r}{p+r}$

Sol: (C)

$$1 = \frac{r(p-1)}{p(q-r)}$$

$$\frac{2}{q} = \frac{1}{p} + \frac{1}{r} = \frac{p+r}{pr}$$

8. The ratio of the roots of $ax^2 + 2bx + c = 0$ is same as the ratio of the roots of $px^2 + 2qx + r = 0$ then

- a) $\frac{b^2}{ac} = \frac{p^2}{q^2}$ b) $\frac{b}{ac} = \frac{q}{pr}$ c) $\frac{b^2}{ac} = \frac{q^2}{pr}$ d) $\frac{b}{ac} = \frac{q^2}{pr}$

Sol: (C)

$$\frac{(m+n)^2}{mn} = \frac{4b^2}{ac} = \frac{4q^2}{pr} \Rightarrow \frac{b^2}{ac} = \frac{q^2}{pr}$$

9. The set of values of 'a' for which in-equation $(a-1)x^2 - (a+1)x + (a-1) \geq 0$ is true for all $x \geq 2$ is

- A) $\left[\frac{7}{3}, \infty\right)$ B) $\left[1, \frac{7}{3}\right]$ C) $(-\infty, 1)$ D) $(-3, -2)$

Sol: (B)

$$(a-1)x^2 - (a+1)x + (a-1) \geq 0$$

$$a(x^2 - x + 1) - (x^2 + x + 1) \geq 0$$

$$\alpha \geq \frac{x^2+x+1}{x^2-x+1} = 1 + \frac{2x}{x^2-x+1} = 1 + \frac{2}{x+\frac{1}{x}-1} \Rightarrow \alpha \in \left(1, \frac{7}{3}\right]$$

10. Solution set of $2\{x\}^2 - 3\{x\} + 1 \geq 0$ where $\{.\}$ denotes the fractional part in $[-1, 1]$ is

- A) $[-1, 1] \sim \left(\frac{1}{2}, 1\right)$ B) $\left[-1, \frac{-1}{2}\right] \cup \left[0, \frac{1}{2}\right] \cup \{1\}$
 C) $\left(-1, \frac{1}{2}\right)$ D) $\left(-\frac{1}{2}, 1\right)$

Sol: (B)

$$\{x\} \geq 1 \text{ or } \{x\} \leq \frac{1}{2}$$

$$\therefore \{x\} \leq \frac{1}{2} \Rightarrow 0 \leq x - 1 \leq \frac{1}{2}$$

$$0 \leq x - 2 \leq \frac{1}{2}$$

$$0 \leq x - n \leq \frac{1}{2} \Rightarrow x \in \left[n, n + \frac{1}{2}\right]$$

11. If α, β are the roots of the equation $ax^2 + bx + c = 0$ and $S_n = \alpha^n + \beta^n$ then

$$aS_{n+1} + bS_n + cS_{n-1} = (n \geq 2)$$

- 1) 0 2) $a + b + c$
 3) $(a + b + c)n$ 4) $n^2 abc$

KEY.1

Sol: $S_{n+1} = \alpha^{n+1} + \beta^{n+1}$

$$= (\alpha + \beta)(\alpha^n + \beta^n) - \alpha\beta(\alpha^{n-1} + \beta^{n-1})$$

$$= -\frac{b}{a}S_n - \frac{c}{a}S_{n-1}$$

12. The number of roots of equation $x^2 + x + 3 + 2 \sin x = 0$, where $x \in [-\pi, \pi]$ is

- (1) 0 (2) 2 (3) 3 (4) 4

Sol: (1)

$$\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + 3 + 2 \sin x = 0$$

$$\left(x + \frac{1}{2}\right)^2 = -2 \sin x - \frac{11}{4}$$

$$\text{LHS} \geq 0 \quad \therefore -2 \sin x - \frac{11}{4} \geq 0$$

$$\Rightarrow 2 \sin x + \frac{11}{4} \leq 0$$

$$\Rightarrow \sin x \leq -\frac{11}{8}$$

not possible

no solution

13. Number of rational roots of the equation $|x^2 - 2x - 3| + 4x = 0$ is

- a) 0 b) 1 c) 2 d) 4

KEY.B

Sol: $x^2 - 2x - 3 \geq 0 \Rightarrow x^2 - 2x - 3 = 0 \Rightarrow x = -3$

$$x^2 - 2x - 3 < 0 \Rightarrow x^2 - 6x - 3 = 0 \text{ no rational root}$$

14. If the equations $2x^2 - 7x + 1 = 0$ and $ax^2 + bx + 2 = 0$ have a common root, then

- a) $a=2, b=-7$ b) $a=\frac{-7}{2}, b=1$ c) $a=4, b=-14$ d) $a=-4, b=1$

KEY.C

Sol: First equation has irrational roots.∴ both roots common

15. If $p, q, r \in \mathbb{R}$ and the quadratic equation $px^2 + qx + r = 0$ has no real root, then

- a) $p(p+q+r) > 0$ b) $p(p+q+r) < 0$
 c) $q(p+q+r) > 0$ d) $q(p+q+r) < 0$

KEY.A Sol: $p(px^2 + qx + r) > 0$ for $x \in \mathbb{R}$. Take $x=1$

16. Total number of distinct integral values of 'a' such that $x^2 + ax + a + 1 = 0$ has integral roots is equal to

- (a) one (b) two (c) three (d) four

Sol: (B)

$$D = a^2 - 4(a+1) = a^2 - 4a - 4$$

$$= (a-2)^2 - 8 = \lambda^2 \text{ (say)}$$

$$(a-2)^2 - \lambda^2 = 8$$

$$(a-2+\lambda)(a-2-\lambda) = 8$$

Clearly $a-2+\lambda$ and $a-2-\lambda$ are even

$$a-2+\lambda = 4, a-2-\lambda = 2$$

$$a = 5, \lambda = 1$$

$$\text{or } a-2+\lambda = 2, a-2-\lambda = 4$$

$$a = 5, \lambda = -1$$

$$\text{or } a-2+\lambda = -4, a-2-\lambda = -2$$

$$a = -1, \lambda = -1$$

$$\text{or } a-2+\lambda = -2, a-2-\lambda = -4$$

$$a = -1, \lambda = 1$$

Thus $a = -1$ or 5

17. The condition that the equation $\frac{1}{x} + \frac{1}{x+b} = \frac{1}{m} + \frac{1}{m+b}$ has real roots that are equal in magnitude but opposite in sign is

- a) $b^2 = m^2$ b) $b^2 = 2m^2$ c) $2b^2 = m^2$ d) $b^2 = 3m^2$

Sol: (B)

$x = m$ is one root, other root is $x = -m$

$$-\frac{1}{m} + \frac{1}{-m+b} = \frac{1}{m} + \frac{1}{m+b}$$

$$\Rightarrow b^2 = 2m^2$$

18. If the difference between the roots of equation $x^2 + ax + 1 = 0$ is less than $\sqrt{5}$ then the set of values of a is

- a) $(-3, \infty)$ b) $(3, \infty)$ c) $(-\infty, -3)$ d) $(-3, 3)$

Sol: (D)

$$|\alpha - \beta| < \sqrt{5}$$

$$\sqrt{a^2 - 4} < \sqrt{5} \Rightarrow a^2 < 9 \Rightarrow -3 < a < 3$$

19. If $0 < c < b < a$ and the roots α, β of the equation $cx^2 + bx + a = 0$ are imaginary then

- A) $\frac{|\alpha| + |\beta|}{2} = |\alpha||\beta|$ B) $|\alpha| + |\beta| > 2$ C) $|\alpha| + |\beta| < 2$ D) $|\alpha| + |\beta| = 2$

Sol: (B)

$$\therefore \alpha = \frac{-b+i\sqrt{4ac-b^2}}{2c} \text{ and } \beta = \frac{-b-i\sqrt{4ac-b^2}}{2c} \text{ since disc. } < 0.$$

$$|\alpha| = |\beta| = \sqrt{\frac{b^2+4ac-b^2}{4c^2}} = \sqrt{\frac{a}{c}} > 1$$

20. If α, β are the roots of $x^2 + px + 1 = 0$ and γ, δ are the two roots of $x^2 + qx + 1 = 0$ then $(\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta) =$
- a) $q^2 - p^2$ b) $p^2 - q^2$ c) $p^2 + q^2$ d) $p^2 q^2$

Sol: (A)

$$\alpha + \beta = -p, \quad \alpha\beta = 1$$

$$\gamma + \delta = -q, \quad \gamma\delta = 1$$

$$\text{Required value} = (\alpha - \gamma)(\beta - \gamma)(\alpha + \delta)(\beta + \delta)$$

$$= (\alpha\beta - \gamma(\alpha + \beta) + \gamma^2)(\alpha\beta + (\alpha + \beta)\delta + \delta^2)$$

$$= (\gamma^2 + p\gamma + 1)(\delta^2 - p\delta + 1)$$

$$= (p\gamma - q\gamma)(-p\delta - q\delta)$$

$$= (q^2 - p^2)\gamma\delta = q^2 - p^2$$

21. Three distinct positive real numbers a, b, c are in H.P. then for the quadratic equation

$$x^2 - kx + 2b^{101} - a^{101} - c^{101} = 0, k \in \mathbb{R} \text{ has}$$

- (a) roots of same sign (b) roots of opposite sign
(c) roots of imaginary (d) roots are real and equal

KEY.B

Sol: if α, β are roots

$$\text{then } \alpha\beta = 2b^{101} - a^{101} - c^{101}$$

$$\text{now } \frac{a^{101} + c^{101}}{2} \geq (\sqrt{ac})^{101} \geq b^{101}$$

$$\Rightarrow 2b^{101} - a^{101} - c^{101} < 0$$

$$\Rightarrow \alpha\beta < 0$$

$\Rightarrow$ roots are opposite in sign.

22. If α and β , α and γ , α and δ are the roots of the equations $ax^2 + 2bx + c = 0$, $2bx^2 + cx + a = 0$ and $cx^2 + ax + 2b = 0$ respectively where a, b, c are positive real numbers, then $\alpha + \alpha^2 =$
- a) -1 b) 1 c) 0 d) abc

KEY.A

$$\text{Sol:} \therefore a\alpha^2 + 2b\alpha + c = 0$$

$$a + 2b\alpha^2 + c\alpha = 0 \quad \text{then } (a + 2b + c)(1 + \alpha + \alpha^2) = 0$$

$$a\alpha + 2b + c\alpha^2 = 0$$

$$\therefore a, b, c \in \mathbb{R}^+ \text{ then } \alpha + \alpha^2 = -1$$

23. If $p, q, r, s \in \mathbb{R}$ then the equation $(x^2 + px + 3q)(-x^2 + rx + q)(-x^2 + sx + 2q) = 0$

- A) 6 real roots B) at least 2 real roots
C) 2 real and 4 imaginary roots D) 4 real and 2 imaginary roots

Sol: (B)

$$\Delta_1 + \Delta_2 + \Delta_3 = p^2 - 12q + r^2 + 4q + s^2 + 8q = p^2 + r^2 + s^2 \geq 0$$

$\therefore$ At least one equation will have real roots

COMMON ROOTS

24. The values of 'a' for which the equation $x^3 + ax + 1 = 0$ and $x^4 + ax^2 + 1 = 0$ have a common root

A) 2 B) -2 C) 0 D) 1

KEY: B

Sol:. Let α be a common root

$$\text{Then } \alpha^3 + a\alpha + 1 = 0 \text{ --- (1)}$$

$$\text{And } \alpha^4 = a\alpha^2 + 1 = 0 \text{ --- (2)}$$

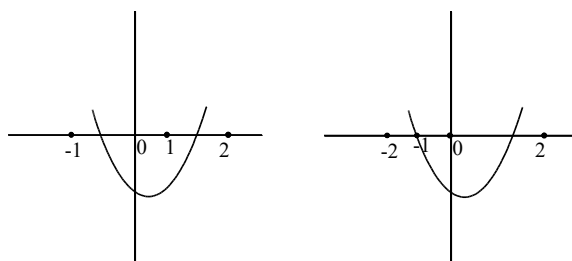
$$\alpha \times (1) - (2) \Rightarrow \alpha - 1 = 0 \Rightarrow \alpha = 1$$

$$\text{So, from } x^3 + ax + 1 = 0 \Rightarrow 1 + a + 1 = 0 \Rightarrow a = -2$$

LOCATION OF ROOTS

25. If the equation $x^2 + (a-6)x - 1 = 0$ is such that there are exactly two integers between the roots of it, then the number of integral values of 'a' is

A) 1 B) 2 C) 3 D) 4



$$\begin{aligned} f(-2) &\geq 0, f(-1) < 0, f(2) \geq 0 \\ -2a + 15 &\geq 0, 6 - a < 0, 2a - 9 \geq 0 \\ a &\leq \frac{15}{2}, a > 6, a \geq \frac{9}{2} \\ a &\in \left[6, \frac{15}{2}\right] \end{aligned}$$

Sol: (B)

$$f(0) = -1 < 0$$

$$f(1) < 0, f(2) \geq 0, f(-1) \geq 0$$

$$a - 6 < 0, 2a - 9 \geq 0, 6 - a \geq 0$$

$$a < 6, a \geq \frac{9}{2}, a \leq 6$$

$$a \in \left[\frac{9}{2}, 6\right]$$

$$\therefore a \in \left[\frac{9}{2}, \frac{15}{2}\right] - \{6\}$$

$$\text{Integers: } a = 5, a = 7$$

$$\text{No. of integers} = 2$$

26. If both roots of the quadratic equation $x^2 - 2kx + k^2 + k - 5 = 0$ are less than 5, then k lies in the interval

a) (5,6) b) (6,∞) c) (−∞,4) d) [4,5]

Sol: (C)

Both the roots are less than 5

$$\Rightarrow \frac{2k \pm \sqrt{4k^2 - 4(k^2 + k - 5)}}{2} < 5$$

$$\Rightarrow k \pm \sqrt{5 - k} < 5 \Rightarrow \pm 1 < 5 - k$$

$$\Rightarrow 1 < 5 - k \Rightarrow k < 4 \Rightarrow k \in (-\infty, 4)$$

27. Let $k \in \mathbb{R}$ and the equation $(k-2)\{x-[x]\}^2 + 2\{x-[x]\} + k^2 = 0$ where $[]$ stands for the greatest integer function, has exactly one solution in $(3,4)$, then k lies in the interval

A) $(3,4)$ B) $(1,3)$ C) $(0,1)$ D) $(-1,0)$

Sol:

(D)

$$\text{Let } y = x - [x]$$

Then the given equation is

$$(k-2)y^2 + 2y + k^2 = 0 \text{ that is } f(y) = 0$$

x is not an integer $\Rightarrow k \neq 0$

$$3 < x < 4 \Rightarrow [x] = 3 \Rightarrow 0 < x - [x] < 1, \quad 0 < y < 1$$

When the equation in x has exactly one solution in $(3,4)$

Then the equation in y has exactly one solution in $(0,1)$

$$\Rightarrow f(0)f(1) < 0$$

$$k^2 \{(k-2) + 2 + k^2\} < 0$$

$$k^2 \{k^2 + k\} < 0 \Rightarrow k(k+1) < 0$$

$$k \in (-1,0)$$

28. If a, b, c are positive numbers such that $a > b > c$ and the equation $(a+b-2c)x^2 + (b+c-2a)x + (c+a-2b) = 0$ has a root in the interval $(-1,0)$, then

A) b cannot be the G.M. of a, c B) b may be the G.M. of a, c
C) b is the G.M. of a, c D) none of these

KEY.A

Sol.: Let $f(x) = (a+b-2c)x^2 + (b+c-2a)x + (c+a-2b)$

According to the given condition, we have

$$f(0)f(-1) < 0$$

$$\text{i.e. } (c+a-2b)(2a-b-c) < 0$$

$$\text{i.e. } (c+a-2b)(a-b+a-c) < 0$$

$$\text{i.e. } c+a-2b < 0 \quad [a > b > c, \text{ given } \Rightarrow a-b > 0, a-c > 0]$$

$$\text{i.e. } b > \frac{a+c}{2}$$

$\Rightarrow b$ cannot be the G.M. of a, c , since $\text{G.M} < \text{A.M}$ always.

29. If the equation $x^4 - 4x^3 + ax^2 + bx + 1 = 0$ has four positive roots then (a, b) is given by

(A) $(4, 6)$ (B) $(6, -4)$
(C) $(-4, -6)$ (C) $(2, 3)$

KEY.B

Sol.: Let the roots of the equation be x_1, x_2, x_3, x_4 then $x_1 + x_2 + x_3 + x_4 = 4$
and $x_1 x_2 x_3 x_4 = 1$

As $\text{A.M} \geq \text{G.M}$ and equality sign holds only when numbers are equal.

We have $1 = \frac{x_1 + x_2 + x_3 + x_4}{4} \geq (x_1 x_2 x_3 x_4)^{\frac{1}{4}} = 1$

$$\Rightarrow x_1 = x_2 = x_3 = x_4 = 1$$

$$\Rightarrow x^4 - 4x^3 + ax^2 + bx + 1 = (x-1)^4 \Rightarrow a = 6, b = -4.$$

30. For $x \in \mathbb{R}$, the equation $x^4 + 4x^3 - 8x^2 + k = 0$ has

A) Only two real roots when $k \in [0, 3]$ B) no real roots when $k \in (3, 4]$

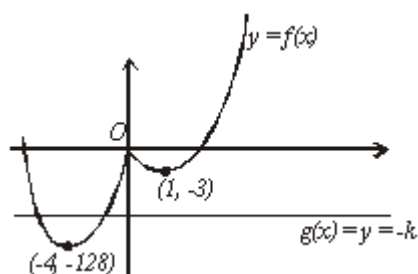
C) Two real roots when $k \in (4, \infty)$ D) Two real roots when $k \in (-\infty, 0)$

Sol: (D)

We have $x^4 + 4x^3 - 8x^2 + k = 0$

$$\Rightarrow x^4 + 4x^3 - 8x^2 = -k$$

$$\Rightarrow f(x) = x^4 + 4x^3 - 8x^2, g(x) = -k$$



When $k \in [0, 3]$, $y = -k$ intersects at 4 points

$k \in (3, 4]$, $y = -k$ intersects at 2 points

$k \in (4, \infty)$, $y = -k$ does not cut curve at all

$k \in (-\infty, 0)$, $y = -k$ meets at 2 points

31. The values of a for which the roots of the equation $(a+1)x^2 - 3ax + 4a = 0$ ($a \neq -1$) are real and greater than 1

a) $\left[-\frac{10}{7}, 1\right]$ b) $\left[-\frac{12}{7}, 0\right]$ c) $\left[-\frac{16}{7}, -1\right)$ d) $\left(-\frac{16}{7}, 0\right)$

KEY: C

Sol.: $D = 9a^2 - 16a(a+1) \geq 0, x_1 > 1, x_2 > 1$

Where $x_1 + x_2 = \frac{3a}{a+1}, x_1 x_2 = \frac{4a}{a+1} \Rightarrow x_1 + x_2 - 1 > 0 \& (x_1 - 1)(x_2 - 1) > 0$

$$\Rightarrow a(7a+16) \leq 0 \quad (1)$$

$$\frac{a-2}{a+1} > 0 \quad (2)$$

$$\frac{2a+1}{a+1} > 0 \quad (3)$$

Solving $-\frac{16}{7} \leq a < -1$.

32 If both the roots of $(2a-4)9^x - (2a-3)3^x + 1 = 0$ are non-negative, then

A) $0 < a < 2$ B) $2 < a < \frac{5}{2}$ C) $a < \frac{5}{4}$ D) $a > 3$

KEY: B

Sol.: Putting $3^x = y$, we have

$$(2a-4)y^2 - (2a-3)y + 1 = 0$$

This equation must have real solution

$$\Rightarrow (2a-3)^2 - 4(2a-4) \geq 0$$

$$\Rightarrow 4a^2 - 20a + 25 \geq 0$$

$$\Rightarrow (2a-5)^2 \geq 0. \text{ This is true.}$$

$y=1$ satisfies the equation

Since 3^x is positive and $3^x \geq 3^0$, $y \geq 1$

Product of the roots $= 1 \times y > 1$

$$\Rightarrow \frac{1}{2a-4} > 1$$

$$\Rightarrow 2a-4 < 1 \Rightarrow a < \frac{5}{2}$$

$$\text{Sum of the roots} = \frac{2a-3}{2a-4} > 1$$

$$\Rightarrow \frac{(2a-3)-(2a-4)}{2a-4} > 0$$

$$\Rightarrow \frac{1}{2a-4} > 0 \Rightarrow a > 2$$

$$\Rightarrow 2 < a < \frac{5}{2}$$

33. The value of a , for which one root of the equation $(a-5)x^2 - 2ax + (a-4) = 0$ is smaller than 1 and the other greater than 2 is _____

- A) $a \in (5, 24)$ B) $a \in \left(\frac{20}{3}, \infty\right)$ C) $a \in (5, \infty)$ D) $a \in (-\infty, \infty)$

Sol:A

(i) $D > 0$

$$4a^2 - 4(a-5)(a-4) > 0$$

$$9a - 20 > 0 \Rightarrow a > \frac{20}{9} \Rightarrow a \in \left(\frac{20}{9}, \infty\right) \text{ --- (1)}$$

(ii) $(a-5)f(1) < 0$; $(a-5)f(2) < 0$

$$\Rightarrow (a-5)(a-5-2a+a-4) < 0$$

$$\Rightarrow a > 5 \Rightarrow a \in (5, \infty) \text{ --- (2)}$$

$$\text{and } (a-5)\{(a-5).4 - 4a + a - 4\} < 0$$

$$\Rightarrow (a-5)(a-24) < 0 \Rightarrow 5 < a < 24$$

$$\Rightarrow a \in (5, 24) \text{ --- (3)}$$

Using (1), (2) & (3)

The common condition is $a \in (5, 24)$

34. If both the roots of the equation $x^2 + 2bx + \log_3(b^2 - 4b + 4) = 0$ are of opposite sign then 'b' belongs to

- (A) (1, 3) (B) $(-\infty, 1) \cup (3, \infty)$
(C) [1, 3] (D) $(1, 2) \cup (2, 3)$

KEY:D

Sol.: Let $f(x) = x^2 + 2bx + \log_3(b^2 - 4b + 4)$

For both roots to be of opposite sign

$$f(0) < 0 \Rightarrow \log_3(b^2 - 4b + 4) < 0$$

$$\Rightarrow b^2 - 4b + 4 < 1$$

$$\Rightarrow b^2 - 4b + 3 < 0$$

$$\Rightarrow (b-1)(b-3) < 0 \Rightarrow 1 < b < 3$$

But $b \neq 2$

$$\therefore b \in (1, 2) \cup (2, 3).$$

35. The values of 'a' for which the quadratic expression $ax^2 + (a-2)x - 2$ is negative for exactly two integral values of x, belongs to
- (A) $[-1, 1]$ (B) $[1, 2]$
 (C) $[3, 4]$ (D) $[-2, -1]$

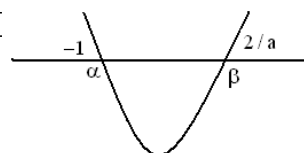
KEY: B

Sol: Let $f(x) = ax^2 + (a-2)x - 2$

$f(x)$ is negative for two integral values of x, so graph should be vertically upward parabola i.e., $a > 0$

Let two roots of $f(x) = 0$ are α and β then $\alpha, \beta = \frac{-(a-2) \pm (a+2)}{2a}$

$$\Rightarrow \alpha = -1, \beta = \frac{2}{a} \Rightarrow 1 < \beta \leq 2 \Rightarrow 1 < \frac{2}{a} \leq 2 \Rightarrow a \in [1, 2]$$



36. If α, β are the roots of $x^2 - 3x + \lambda = 0$ ($\lambda \in \mathbb{R}$) and $\alpha < 1 < \beta$, then the true set of values of λ equals

a) $\lambda \in \left(2, \frac{9}{4}\right]$

b) $\lambda \in \left(-\infty, \frac{9}{4}\right]$

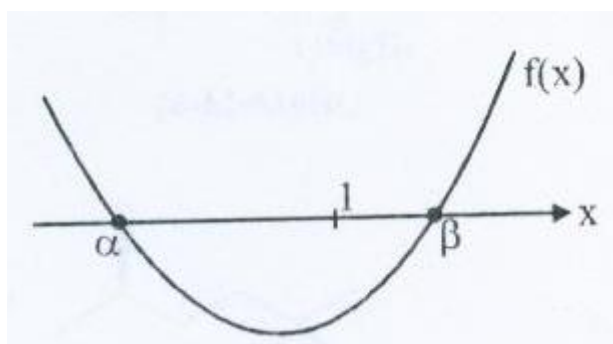
c) $\lambda \in (2, \infty)$

d) $\lambda \in (-\infty, 2)$

KEY: :d

Sol: Let $f(x) = x^2 - 3x + \lambda$

Clearly $f(1) < 0$



$$\Rightarrow 1 - 3 + \lambda < 0 \Rightarrow \lambda < 2 \Rightarrow \lambda \in (-\infty, 2)$$

37. If the roots of the quadratic equation $(4p - p^2 - 5)x^2 - (2p - 1)x + 3p = 0$ lie on either side of unity, then the number of integral values of p is
- 1) 1 2) 2 3) 3 4) 4

Sol: (2)

$$4p - p^2 - 5 < 0$$

$$\therefore f(1) < 0$$

$$(p - 1)(p - 4) < 0$$

No. of integral values of $p = 2$

38. If the roots of the equation $x^2 - 2ax + a^2 + a - 3 = 0$ are real and less than 3 then

1) $a < 2$

2) $a < 3$

3) $a < \frac{3}{2}$

4) $a < 4$

Sol: (1)

$$\text{Let } f(x) = x^2 - 2ax + a^2 + a - 3 = 0$$

Then (i) $\Delta \geq 0$ (ii) sum of root < 6 (iii) $f(3) > 0$

39. If $b > a$, then the equation $(x-a)(x-b) - 1 = 0$ has
 (A) both roots in (a, b) (B) both roots in $(-\infty, a)$
 (C) both roots in (b, ∞) (D) one root in $(-\infty, a)$ and other in (b, ∞)

KEY.D

Sol.: The given equation can be written as

$$x^2 - (a+b)x + ab - 1 = 0$$

Let α, β be its roots.

$$\therefore \alpha + \beta = a + b \quad \alpha\beta = ab - 1$$

$\therefore$ if one root is less than a , then the other root is greater than b ,

$\therefore$ we have one root in $(-\infty, a)$ and other in (b, ∞) .

CUBIC AND BIQUADARATIC EQUATIONS

40. The equation $(x^2 + 6x + 7)^2 + 6(x^2 + 6x + 7) + 7 = x$ has
 A) no real solutions B) exactly two real and equal solutions
 C) exactly two distinct real solutions D) all its solutions are real

Sol: (A)

$$\text{let } x^2 + 6x + 7 = f(x)$$

$$f(f(x)) = x \Rightarrow f(x) = f^{-1}(x)$$

$$\Rightarrow f(x) = f^{-1}(x)$$

$$\Rightarrow x^2 + 6x + 7 = x \Rightarrow x^2 + 5x + 7 = 0 \text{ has no real solutions}$$

The given equation has no real solution.

41. If the function $f(x) = x^3 + 3(a-7)x^2 + 3(a^2-9)x - 1$ has a point of maximum at positive values of x then

- (a) $a \in \left(-\infty, \frac{29}{7}\right)$ (b) $a \in (-\infty, 7)$
 (c) $a \in (-\infty, -3) \cup \left(3, \frac{29}{7}\right)$ (d) $a \in (3, \infty) \cup (-\infty, -3)$

KEY.C

Sol: $f(x) = x^3 + 3(a-7)x^2 + 3(a^2-9)x - 1$

$$f'(x) = 3x^2 + 6(a-7)x + 3(a^2-9)$$

The roots of $f'(x) = 0$ positive and distinct which is possible if

(i) $b^2 - 4ac > 0 \Rightarrow 6(a-7)^2 - 4(3)(3)(a^2-9) > 0$

$$\Rightarrow a < \frac{29}{7}$$

(ii) Product of Roots > 0 $a^2 - 9 > 0$

(iii) Sum of Roots > 0 $a - 7 < 0$

$$a < 7$$

$$\Rightarrow \text{From i, ii, iii } a \in (-\infty, -3) \cup \left(3, \frac{29}{7}\right)$$

43. If $f(x) = (x^2 + 3x + 2)(x^2 - 7x + a)$ and $g(x) = (x^2 - x - 12)(x^2 + 5x + b)$ then the values of a and b , If $(x+1)(x-4)$ is HCF of $f(x)$ and $g(x)$

- a) $a=10:b=6$ b) $a=4:b=12$
 c) $a=12:b=4$ d) $a=6:b=10$

KEY.C

Sol.: $x^2 - 7x + a$ is divisible by $x - 4$ & $x^2 + 5x + b$ is divisible by $x + 1$
 $\therefore a = 12; b = 4$

44. The equation $(x^2 + 3x + 4)^2 + 3(x^2 + 3x + 4) + 4 = x$ has

- a) all its solutions real but not all positive b) only two of its solutions real
 c) two of its solutions positive and two negative d) none of solutions real.

KEY.D

Sol.: $f(x) = ax^2 + bx + c$: If $f(x) = x$ has no real solution then $f(f(x)) = x$ also has no real solution:

45. If $p, q, r, s \in R$ then the equation $(x^2 + px + 3q)(-x^2 + rx + q)(-x^2 + sx + 2q) = 0$

- A) 6 real roots B) at least 2 real roots
 C) 2 real and 4 imaginary roots D) 4 real and 2 imaginary roots

Sol: (B)

$$\Delta_1 + \Delta_2 + \Delta_3 = p^2 - 12q + r^2 + 4q + s^2 + 8q = p^2 + r^2 + s^2 \geq 0$$

$\therefore$ At least one equation will have real roots

46. If every root of a polynomial equation (of degree 'n') $f(x) = 0$ with leading coefficient "1" is real and distinct, then the equation $f''(x)f(x) - \{f'(x)\}^2 = 0$ has.

- (A) at least one real root (B) no real root
 (C) at most one real root (D) exactly two real roots

KEY.B

Sol.: Let $f(x) = (x - a_1)(x - a_2) \dots (x - a_n)$ where $a_1, a_2, \dots, a_n \in R$ take log both sides and differentiate. Then

$$\frac{f'(x)}{f(x)} = \frac{1}{x - a_1} + \frac{1}{x - a_2} + \dots + \frac{1}{x - a_n}$$

Again diff w.r.t. 'x'

$$\frac{f f'' - (f')^2}{f^2} = - \left[\frac{1}{(x - a_1)^2} + \frac{1}{(x - a_2)^2} + \dots + \frac{1}{(x - a_n)^2} \right]$$

$$< 0 \forall x \in R$$

$\Rightarrow f f'' - (f')^2 = 0$ has no real root

47. Consider the equation $x^4 - ax^3 - bx^2 - cx - d = 0, a, b, c, d \in I^+, a \geq b \geq c \geq d$, then the number of integral solution(s) will be

- A) 0 b) 2 C) 4 D) None of These

Sol: (A)

Suppose m is an integral root of $x^4 - ax^3 - bx^2 - cx - d = 0$

As $d \neq 0 \Rightarrow m \neq 0$

$$\begin{aligned}
 \text{(I)} \quad m &> 0 \\
 m^4 - am^3 - bm^2 - cm &= d \\
 \Rightarrow d &= m(m^3 - am^2 - bm - c) \\
 \Rightarrow d &\geq m \\
 \text{Also, } m^4 - am^3 &= bm^2 + cm + d \\
 m^3(m - a) &= bm^2 + cm + d \\
 \Rightarrow m &> a \Rightarrow \text{contradiction} \\
 \text{(II)} \quad m &< 0 \\
 m &= -n \\
 \Rightarrow n^4 + an^3 - bn^2 + cn - d &= 0
 \end{aligned}$$

$$n^4 + n^2(an - b) + (cn - d) > 0$$

48. The number of values of k for which $(x^2 - (k-2)x - 2k)(x^2 + kx + 2k - 4)$ is a perfect square is
- a) 1 b) 2 c) 0 d) none of these

Sol: (A)

Let the expression is of the form $(x - \alpha)(x - \beta)(x - \gamma)(x - \delta)$.

It can be a perfect square if either both expressions are simultaneously perfect squares or both roots common. i.e., $(k-2)^2 + 8k = 0$ and $k^2 - 4(2k-4) = 0$ or

$$\frac{2-k}{k} = \frac{-2k}{2k-4}$$

Solving we get k can take only one value.

50. The number of real roots of the equation $x^3 - 3x + 1 = 0$ is
- 1) 0 2) 1 3) 2 4) 3

Sol: (4)

$$\text{Let } f(x) = x^3 - 3x + 1$$

$$\text{Then } f'(x) = 3(x^2 - 1)$$

$$\text{Let } f'(x) = 0 \Rightarrow x = \pm 1 \text{ and } f(1) f(-1) < 0$$

51. If one root of equation $x^2 - 4ax + a + f(a) = 0$ is three times of the other, then minimum value of $f(a)$ is
- (A) $\frac{-1}{6}$ (B) $\frac{-1}{10}$ (C) $\frac{-1}{5}$ (D) $\frac{-1}{12}$

Sol (D)

Let roots are α and 3α , then $4\alpha = 4a$ $\alpha = a$ and

$$a^2 - 4a^2 + a + f(a) = 0 \Rightarrow f(a) = 3a^2 - a$$

$$\text{minimum value of } f(a) = \frac{-(-1)}{4 \times 3} = \frac{-1}{12}$$

52. The equation $a_8x^8 + a_7x^7 + a_6x^6 + \dots + a_0 = 0$ has all its roots positive and real (where $a_8 = 1, a_7 = -4, a_0 = 1/2^8$), then
- A) $a_1 = \frac{1}{2^8}$ B) $a_1 = -\frac{1}{2^4}$ C) $a_2 = \frac{7}{2^5}$ D) $a_2 = \frac{7}{2^8}$

Sol (B)

Let the roots be $\alpha_1, \alpha_2, \dots, \alpha_8$

$$\Rightarrow \alpha_1 + \alpha_2 + \dots + \alpha_8 = 4$$

$$\alpha_1 \alpha_2 \dots \alpha_8 = \frac{1}{2^8}$$

$$\Rightarrow (\alpha_1 \alpha_2 \dots \alpha_8)^{1/8} = \frac{1}{2} = \frac{\alpha_1 + \alpha_2 + \dots + \alpha_8}{8}$$

$$\Rightarrow \text{AM} = \text{GM} \Rightarrow \text{all the roots are equal to } \frac{1}{2}.$$

$$\Rightarrow a_1 = -{}^8C_7 \left(\frac{1}{2}\right)^7 = -\frac{1}{2^4}$$

$$a_2 = {}^8C_6 \left(\frac{1}{2}\right)^6 = -\frac{7}{2^4}$$

$$a_3 = -{}^8C_5 \left(\frac{1}{2}\right)^5$$

53. If $p_1 p = 2(q_1 + q)$ then which of following statement about the quadratic equations

$x^2 + px + q = 0; x^2 + p_1 x + q_1 = 0$ is always true (Here $p, p_1, q, q_1 \in R$)

- (a) Both the equations have real roots
- (b) Both the equations have imaginary roots
- (c) at least one of the equations has real roots
- (c) exactly one of them has equal roots

Sol (C)

$$\Delta_1 = p^2 - 4q; \Delta_2 = p_1^2 - 4q_1$$

$$\Delta_1 + \Delta_2 = p^2 + p_1^2 - 4(q + q_1) = p^2 + p_1^2 - 2p_1 p = (p - p_1)^2 \geq 0$$

Both Δ_1, Δ_2 can be negative.

Hence 3.

54. If the roots of the equation $ax^2 + bx + c = 0$ are of the form $\frac{\alpha}{\alpha-1}$ and $\frac{\alpha+1}{\alpha}$, then

value of $(a+b+c)^2$ is

- A) $2b^2 - ac$ B) $b^2 - 2ac$ C) $b^2 - 4ac$ D) $4b^2 - 2ac$

KEY: C

Sol: By hypothesis $\frac{\alpha}{\alpha-1} + \frac{\alpha+1}{\alpha} = -\frac{b}{a}$ and $\frac{\alpha}{\alpha-1} \cdot \frac{\alpha+1}{\alpha} = \frac{c}{a}$

$$\Rightarrow \frac{2\alpha^2 - 1}{\alpha^2 - \alpha} = -\frac{b}{a} \text{ and } \alpha = \frac{c+a}{c-a}$$

$$\Rightarrow (c+a)^2 + 2b(c+a) + b^2 = b^2 - 4ac \Rightarrow (a+b+c)^2 = b^2 - 4ac$$

55. If for any real x , we have $-1 \leq \frac{x^2 + nx - 2}{x^2 - 3x + 4} \leq 2$ then the value of n is

- A) $n \in [-1, \sqrt{40} - 6]$ B) $n \in [-1, 3)$ C) $n \in [-\sqrt{40} - 6, -1]$ D) $n \in [1, \sqrt{40} + 6]$

KEY: A

Sol: $\frac{x^2 + nx - 2}{x^2 - 3x + 4} - 2 \leq 0$

$$\Rightarrow x^2 - (n+6)x + 10 \geq 0, \text{ true } \forall x \in R \text{ then}$$

$$D \leq 0 \Rightarrow (n+6)^2 - 40 \leq 0 \Rightarrow -\sqrt{40} - 6 \leq n \leq \sqrt{40} - 6 \text{ --- (1)}$$

$$\text{Similarly } \frac{x^2 + nx - 2}{x^2 - 3x + 4} + 1 \geq 0 \Rightarrow 2x^2 + (x-3)x + 2 \geq 0$$

$$\Rightarrow D \leq 0 \Rightarrow (n-3)^2 - 16 \leq 0 \Rightarrow -1 \leq n \leq 7 \text{ --- (2)}$$

Combined (1) & (2) we get $n \in [-1, \sqrt{40} - 6]$

56. Let $f(x) = 2x^3 + ax^2 + bx$, ($a, b \in \mathbb{N}$), if graph of $f(x)$ cuts the x -axis at 3 distinct points, then minimum value of $a^2 + b^2$ is
- (A) 10 (B) 2
(C) 9 (D) 8

Sol: (A)

$$f'(x) = 6x^2 + 2ax + b$$

If $f(x) = 0$ has 3 distinct

real roots, then $f'(x) = 0$

must have 2 distinct real roots for it

$$4a^2 - 24b > 0 \Rightarrow a^2 > 6b$$

$$\Rightarrow a \geq 3, b \geq 1 \text{ and for } a = 3, b = 1, f(x) = 0$$

has 3 distinct real roots.

57. For the equation $3x^2 + px + 3 = 0$, $p > 0$, if one root is the square of the other then value of P is

- 1) $\frac{1}{3}$ 2) 1
3) 3 4) $\frac{2}{3}$

KEY.3

Sol: $\alpha + \alpha^2 = -\frac{p}{3}$

$$\alpha^3 = 1$$

58. If the equations $2x^2 + kx - 5 = 0$ and $x^2 - 3x - 4 = 0$ have a common root, then the value of k is

- 1) -2 2) -3
3) $\frac{27}{4}$ 4) $-\frac{1}{4}$

KEY.2

Sol: If ' α ' is the common root then $2\alpha^2 + k\alpha - 5 = 0$, $\alpha^2 - 3\alpha - 4 = 0$ solve the equations.

59. If α and β are the roots of the equation $x^2 - x + 1 = 0$ then $\alpha^{2009} + \beta^{2009} =$

- 1) 1 2) 2
3) -1 4) -2

KEY.1

Sol: $x = \frac{1 \pm i\sqrt{3}}{2}$

$$\therefore \alpha = -\omega, \beta = -\omega^2$$

60. If $P(Q-r)x^2 + Q(r-P)x + r(P-Q) = 0$ has equal roots then $\frac{2}{Q} =$ (where $P, Q, r \in \mathbb{R}$)

- 1) $\frac{1}{P} + \frac{1}{r}$ 2) $\frac{1}{P} - \frac{1}{r}$
3) $P+r$ 4) Pr

KEY.1

Sol: Product of the roots = 1

61. If the equations $x^2 + 2\lambda x + \lambda^2 + 1 = 0$, $\lambda \in R$ and $ax^2 + bx + c = 0$ where a, b, c are lengths of sides of triangle have a common root, then the possible range of values of λ is

1) $(0, 2)$ 2) $(\sqrt{3}, 3)$ 3) $(2\sqrt{2}, 3\sqrt{2})$ 4) $(0, \infty)$

KEY.1

Sol: $(x + \lambda)^2 + 1 = 0$ has clearly imaginary roots

So, both roots of the equations are common

$$\therefore \frac{a}{1} = \frac{b}{2\lambda} = \frac{c}{\lambda^2 + 1} = k(\text{say})$$

Then $a = k$, $b = 2\lambda k$, $c = (\lambda^2 + 1)k$

As a, b, c are sides of triangle

$$a + b > c \Rightarrow 2\lambda + 1 > \lambda^2 + 1 \Rightarrow \lambda^2 - 2\lambda < 0$$

$$\Rightarrow \lambda \in (0, 2)$$

The other conditions also imply same relation.

62. The number of solutions of $\sqrt{3x^2 + x + 5} = x - 3$ is

1) 0 2) 1 3) 2 4) 4

KEY.1

Sol: Note that we must have $3x^2 + x + 5 \geq 0$ and $x - 3 \geq 0$ or $x \geq 3$.

$$\sqrt{3x^2 + x + 5} = x - 3 \dots (1)$$

Squaring both sides of (1), we get

$$3x^2 + x + 5 = x^2 - 6x + 9$$

$$\Rightarrow 2x^2 + 7x - 4 = 0 \Rightarrow (2x - 1)(x + 4) = 0$$

$$\Rightarrow x = 1/2, -4$$

None of these satisfy the inequality $x \geq 3$. Thus, (1) has no solution.

63. If $x^2 + 2ax + 10 - 3a > 0$ for each $x \in R$, then

1) $a < -5$ 2) $-5 < a < 2$ 3) $a > 5$ 4) $2 < a < 5$

KEY.2

Sol: $x^2 + 2ax + 10 - 3a > 0 \forall x \in R$

$$\Rightarrow (x + a)^2 - (a^2 + 10 - 3a) > 0 \forall x \in R$$

$$\Rightarrow a^2 + 3a - 10 < 0$$

$$\Rightarrow (a + 5)(a - 2) < 0$$

$$\Rightarrow -5 < a < 2$$

64. The number of solutions of the equations of the equation $x^2 + [x] - 4x + 3 = 0$ is
Where $[]$ denotes G.I.F.

1) 0 2) 1 3) 2 4) 3

KEY.1

Sol: Given equation can be written as $(x^2 - 3x + 3) - f = 0$ where

$$f = x - [x] \text{ and } 0 \leq f < 1$$

$$\therefore 0 \leq x^2 - 3x + 3 < 1$$

solving $x^2 - 3x + 3 = 0$; roots are Imaginary

$$\therefore x^2 - 3x + 3 \geq 0 \forall x \in R$$

$$\text{solving } x^2 - 3x + 3 < 1 \Rightarrow 1 < x < 2$$

$$\text{if } 1 < x < 2; [x] = 1.$$

$$\Rightarrow \alpha + 2\alpha = \frac{1-3a}{a^2-5a+3} \text{ and } \alpha \cdot 2\alpha = \frac{2}{a^2-5a+3}$$

$$2 \left[\frac{1}{9} \frac{(1-3a)^2}{(a^2-5a+3)^2} \right] = \frac{2}{a^2-5a+3}$$

$$9a^2 - 6a + 1 = 9a^2 - 45a + 27$$

$$39a = 26$$

$$\frac{2}{3}$$

69. For $x^2 - (\alpha + 2)|x| + 9 = 0$ to have real solutions, the range of ' α ' is

(A) $[-\infty, 4]$

(B) $[4, \infty)$

(C) $(-\infty, 7] \cup [11, \infty)$

(D) $[-4, \infty)$

KEY.B

Sol: $\alpha = \frac{x^2+9}{|x|} - 2 = |x| + \frac{9}{|x|} - 2$

$$\Rightarrow \alpha \geq 4.$$

70. $0 < c < b < a$ and α, β are roots of equation $cx^2 + bx + a = 0$ if α, β are non real then

(A) $\frac{|\alpha|+|\beta|}{2} = |\alpha||\beta|$

(B) $\frac{2}{|\alpha|} = \frac{1}{|\beta|}$

(C) $\frac{1}{|\alpha|} + \frac{1}{|\beta|} < 2$

(D) $|\alpha| + \frac{1}{|\beta|} < 2$

KEY.C

Sol: $\alpha\beta = \frac{a}{c} > 1$

$$|\alpha||\beta| > 1 \Rightarrow |\alpha|^2 > 1 \Rightarrow |\alpha| > 1 \Rightarrow |\beta| > 1 \Rightarrow \frac{1}{|\alpha|} + \frac{1}{|\beta|} < 2$$

71. ω is a non real complex cube root of unity and $a, b \in R$. If ω, ω^2 are roots of

$$\frac{1}{a+x} + \frac{1}{b+x} = \frac{3}{x} \text{ then } a, b \text{ are roots of}$$

a) $3x^2 - 6x + 2 = 0$

b) $6x^2 - 3x + 2 = 0$

c) $2x^2 - 3x + 6 = 0$

d) $6x^2 - 2x + 3 = 0$

KEY.B

Sol: The given equation simplifies $x^2 + 2x(a+b) + 3ab = 0$, whose roots are given table ω, ω^2

$$\text{Hence } a+b = \frac{1}{2}, ab = \frac{1}{3}. \text{ So } a, b \text{ are roots of } x^2 - x\left(\frac{1}{2}\right) + \frac{1}{3} = 0$$

72. If a, b, c are positive numbers such that $a > b > c$ and the equation

$$(a+b-2c)x^2 + (b+c-2a)x + (c+a-2b) = 0 \text{ has a root in the interval } (-1, 0), \text{ then}$$

A) b cannot be the G.M. of a, c

B) b may be the G.M. of a, c

C) b is the G.M. of a, c

D) none of these

KEY.A

Sol: Let $f(x) = (a+b-2c)x^2 + (b+c-2a)x + (c+a-2b)$

According to the given condition, we have

$$f(0)f(-1) < 0$$

$$\text{i.e. } (c+a-2b)(2a-b-c) < 0$$

$$\text{i.e. } (c+a-2b)(a-b+a-c) < 0$$

$$\text{i.e. } c+a-2b < 0 \quad [a > b > c, \text{ given } \Rightarrow a-b > 0, a-c > 0]$$

$$\text{i.e. } b > \frac{a+c}{2}$$

$\Rightarrow b$ cannot be the G.M. of a, c , since $\text{G.M} < \text{A.M.}$ always.

73. If α, β are the real roots of $x^2 + (a-4)x + a^2 - 3a + 3 = 0$ and $\alpha^2 + \beta^2 = 6$ then $a =$

A) $-1-\sqrt{5}$

B) $1-\sqrt{5}$

C) $-1+\sqrt{5}$

D) $1+\sqrt{5}$

KEY.C

Sol: $\text{Disc} \geq 0 \Rightarrow (a-4)^2 - 4(a^2 - 3a + 3) \geq 0$

$$\Rightarrow a^2 + 16 - 8a - 4a^2 + 12a - 12 \geq 0$$

$$\Rightarrow 3a^2 - 4a - 4 \leq 0 \Rightarrow 3a^2 - 6a + 2a - 4 \leq 0$$

$$\Rightarrow (3a+2)(a-2) \leq 0 \Rightarrow -\frac{2}{3} \leq a \leq 2$$

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (4-a)^2 - 2(a^2 - 3a + 3) = 6$$

$$\Rightarrow a^2 + 16 - 8a - 2a^2 + 6a - 6 = 6$$

$$\Rightarrow a^2 + 2a - 4 = 0 \Rightarrow a = \frac{-2 \pm \sqrt{4+16}}{2} \Rightarrow a = -1 \pm \sqrt{5}$$

74. If $a \in \mathbb{R}$ and the equation $(a-2)(x-[x])^2 + 2(x-[x]) + a^2 = 0$ (where $[x]$ denotes the greatest integer $\leq x$) has no integral solution and has exactly one solution in $(2,3)$ then a lies in the interval

(A) $(-1,0)$

(B) $(0,1)$

(C) $(-1,3)$

(D) $(1,3)$

KEY.A

Sol: Let $y = x - [x]$

$$\text{Then equation } (a-2)(x-[x])^2 + 2(x-[x]) + a^2 = 0 \quad \dots(1)$$

can be written as

$$f(y) = (a-2)y^2 + 2y + a^2 = 0$$

since x cannot be an integer

$$\therefore y = x - [x] \neq 0. \text{ thus } a \neq 0$$

when $2 < x < 3$, $[x] = 2$.

$$\Rightarrow 0 < x - [x] < 1 \text{ i.e. } 0 < y < 1. \quad \dots(2)$$

since (1) has exactly one solution in the interval $(2,3)$, (2) has exactly one solution in the interval $(0,1)$.

This is possible if $f(0)f(1) < 0$ for otherwise the equation (2) has either no or two solution $(0,1)$.

$$\Rightarrow a^2(a-2+2+a^2) < 0$$

$$\Rightarrow a(a+1) < 0 \quad [\because a^2 > 0]$$

$$\Rightarrow -1 < a < 0 \text{ or } a \in (-1,0)$$

∴ (a) holds.

75 If the roots of the equation $ax^2 + bx + c = 0$ be α and β and those of the equation

$$Ax^2 + Bx + C = 0 \text{ be } \alpha + k \text{ and } \beta + k, \text{ then } \frac{b^2 - 4ac}{B^2 - 4AC} \text{ is}$$

(A) $\frac{a^2}{A^2}$

(B) 1

(C) $\frac{a}{A}$

(D) $\frac{A}{a}$

KEY.A

Sol: $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$

Let $\alpha^1 = \alpha + k \quad \dots(1)$

$\beta^1 = \beta + k \quad \dots(2)$

subtract (1) and (2)

$$\alpha^1 - \beta^1 = \alpha - \beta$$

$$(\alpha^1 - \beta^1)^2 = (\alpha - \beta)^2$$

$$(\alpha^1 + \beta^1) - 4\alpha^1\beta^1 = (\alpha + \beta)^2 - 4\alpha\beta$$

$$\left(-\frac{B}{A}\right)^2 - 4\left(\frac{C}{A}\right) = \left(-\frac{b}{a}\right)^2 - 4\left(\frac{c}{a}\right)$$

$$\Rightarrow \frac{b^2 - 4ac}{B^2 - 4Ac} = \left(\frac{a}{A}\right)^2.$$

76. If $p(x) = ax^2 + bx + c$ and $Q(x) = -ax^2 + dx + c$, $ac \neq 0$ then the equation $p(x).Q(x) = 0$ has

(A) Two real roots

(B) More than two real roots

(C) at least two real roots

(D) No real root

KEY.C

Sol: $P(x) = 0$ has real roots, if $b^2 - 4ac \geq 0$ and $Q(x) = 0$ has real roots, if $d^2 + 4ac \geq 0$

Now, $ac \neq 0$. if $ac < 0$, $b^2 - 4ac \geq 0$ and hence $P(x) = 0$ has real roots.

If $ac > 0$, then $d^2 + 4ac \geq 0$ and hence $Q(x) = 0$ has real roots..

Hence, atleast two roots of $P(x). Q(x) = 0$ are real.

77. Let α and β be the roots of $x^2 - 6x - 2 = 0$ with $\alpha > \beta$, If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{2a_9}$ is

Sol: (3)

$$\frac{\alpha^{10} - \beta^{10} - 2(\alpha^8 - \beta^8)}{2(\alpha^9 - \beta^9)}$$

$$\Rightarrow \frac{\alpha^8(\alpha^2 - 2) - \beta^8(\beta^2 - 2)}{2(\alpha^9 - \beta^9)}$$

$$\Rightarrow \frac{6(\alpha^9 - \beta^{10})}{2(\alpha^9 - \beta^9)} = 3 \begin{pmatrix} \because \alpha^2 - 2 = 6\alpha \\ \beta^2 - 2 = 6\beta \end{pmatrix}$$

78 The number of values of a for which $(a^2 - 3a + 2)x^2 + (a^2 - 5a + 6)x + a^2 - 4 = 0$ is identity in x is:

Sol: (1)

$$a^2 - 3a + 2 = 0 \text{ and } a^2 - 5a + 6 = 0 \text{ and } a^2 - 4 = 0$$

$$a = 1, 2 \text{ and } a = 2, 3 \text{ and } a = \pm 2$$

$$a = 2$$

79 If $a \neq 0$, $ax^2 + bx + c = 0$, $bx^2 + cx + a = 0$ have a common root then the value $\frac{a^2}{2bc} + \frac{b^2}{2ca} + \frac{c^2}{2ab}$ is equal to:

Sol: (8)

$$(ac - b^2)(ab - c^2) = (bc - a^2)^2$$

$$a^2bc - ac^3 - ab^3 + b^2c^2 = b^2c^2 + a^4 - 2a^2bc$$

$$a^3 + b^3 + c^3 = 3abc$$

$$\frac{a^2}{2bc} + \frac{b^2}{2ca} + \frac{c^2}{2ab} = 2 \frac{a^3 + b^3 + c^3}{abc} = 2^3 = 8$$

80. Number of real solutions of the equation $x^6 - x^5 + x^4 - x^3 + x^2 - x + \frac{3}{4} = 0$ is _____

Sol: (0)

$$x^4 \left(x - \frac{1}{2} \right)^2 + \frac{3}{4} x^2 \left(x - \frac{2}{3} \right)^2 + \frac{3}{8} = 0$$

$\Rightarrow$ No real solution.

81. If the biquadratic $x^4 + 4x^3 + 6px^2 + 4qx + r$ is divisible by $x^3 + 3x^2 + 9x + 3$, then the value of $\frac{(p+q)r}{5}$ is

Sol: (3)

$$\text{Let } x^4 + 4x^3 + 6px^2 + 4qx + r = (ax + b)(x^3 + 3x^2 + 9x + 3)$$

comparing coefficients of x^4 and x^3

$$a = 1; 3a + b = 4 \quad b = 1$$

comparing coefficient of x^2 , x and constant terms

$$9a + 3b = 3p \quad 6p = 12 \quad p = 2$$

$$3a + 9b = 4q \quad 4q = 12 \quad q = 3 \quad \text{and } r = 3$$

$$(p + q)r = 15$$

82. The integer value of 'K' such that $x^2 - 2(4k - 1)x + 15k^2 - 2k - 7 > 0 \quad \forall x \in R$ is

Sol: (3)

$$4(4k - 1)^2 - 4(15k^2 - 2k - 7) < 0$$

$$\Rightarrow k^2 - 6k + 8 < 0$$

$$\Rightarrow 2 < k < 4$$

83. The least positive integral value of real λ so that the equation

$(x - a)(x - c)(x - e) + \lambda(x - b)(x - d) = 0, (a > b > c > d > e)$ has distinct real roots is _____

Sol: (1)

$$f(x) = (x - a)(x - c)(x - e) + \lambda(x - b)(x - d)$$

$$\Rightarrow f(a) = \lambda(a - b)(a - d)$$

$$\Rightarrow f(b) = (b-a)(b-c)(b-e) < 0$$

$$f(c) = \lambda(c-b)(c-d)$$

$$f(d) = (d-a)(d-c)(d-e) > 0$$

$$f(e) = \lambda(e-b)(e-d)$$

If $\lambda > 0$ $f(a) > 0$, a root lies between b and a.

If $\lambda < 0$ $f(e) < 0$, a root lies between e and d.

Always a root lies between d and b $\Rightarrow$ all roots are real and distinct as exactly two can't be real.

If $\lambda = 0$ roots are a, c and e.

84. If $p(x) = ax^2 + bx$ and $q(x) = lx^2 + mx + n$ with $p(1) = q(1)$; $p(2) - q(2) = 1$ and $p(3) - q(3) = 4$ then $p(4) - q(4)$ is equal to ____.

Sol: (9)

$$p(1) - q(1) = 0 \Rightarrow (a+b) - (l+m+n) = 0 \quad \text{----- (1)}$$

$$p(2) - q(2) = 1 \Rightarrow 4a + 2b - (4l + 2m + n) = 1 \quad \text{----- (2)}$$

$$p(3) - q(3) = 4 \Rightarrow 9a + 3b - (9l + 3m + n) = 4 \quad \text{----- (3)}$$

$$3 \times (3) + 1(1) - 3 \times (2)$$

$$\text{We get } (16a + 4b) - (16l + 4m + n) = 9$$

$$p(4) - q(4) = 9$$

85. Let $P(x) = x^2 + bx + c$, where b and c are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of $P(1)$.

Key. 4

Sol: Since $P(x)$ divides into both of them

Hence $P(x)$ also divides

$$(3x^4 + 4x^2 + 28x + 5) - 3(x^4 + 6x^2 + 25)$$

$$= -14x^2 + 28x - 70 = -14(x^2 - 2x + 5)$$

Which is a quadratic, Hence $P(x) = x^2 - 2x + 5$

$$\therefore P(1) = 4$$

86. Largest integral value of m for which the quadratic expression

$$y = x^2 + (2m+6)x + 4m+12 \text{ is always positive, } \forall x \in R, \text{ is}$$

Key. 0

Sol: $D < 0 \Rightarrow -3 < m < 1 \Rightarrow m = 0$

87. Let α, β and γ be the roots of equation $f(x) = 0$, where $f(x) = x^3 + x^2 - 5x - 1$.

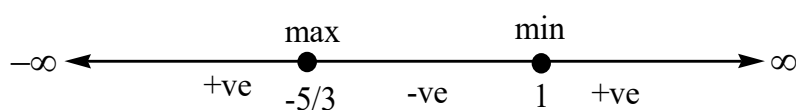
Then the value of $[\alpha] + [\beta] + [\gamma]$, where $[.]$ denotes the greatest integer function, is equal to

Key. 3

Sol: Given $f(x) = x^3 + x^2 - 5x - 1$

$$\therefore f'(x) = 3x^2 + 2x - 5. \text{ The roots of } f'(x) = 0 \text{ are } -\frac{5}{3} \text{ and } 1$$

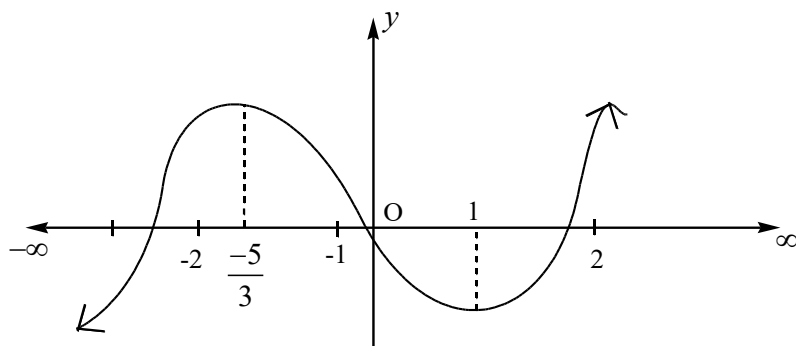
Writing the sign scheme for $f'(x)$,



Also, $f(-\infty) = -\infty < 0$, $f(\infty) = \infty > 0$

$$f(1) = -4, f\left(-\frac{5}{3}\right) = \frac{148}{27}$$

Now, graph of $y = f(x)$ is as follows



$$f(-3) = -27 + 9 + 15 - 1 = -4 < 0$$

$$f(-2) = -8 + 4 + 10 - 1 > 0$$

$$f(-1) = 4 > 0, f(0) = -1 < 0$$

$$f(2) = 1 > 0$$

$$\therefore -3 < \alpha < -2, -1 < \beta < 0, 1 < \gamma < 2$$

$$|[\alpha] + [\beta] + [\gamma]| = |-3 - 1 + 1| = 3$$

88. The set of real parameter 'a' for which the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ has all real solutions, is given by $\left[\frac{m}{n}, \infty\right)$ where m and n are relatively prime positive integers, then the value of $(m+n)$ is

Key. 7

Sol: We have $a^2 - (2x^2 + 1)a + x^4 + x = 0$

$$\therefore a = \frac{(2x^2 + 1) \pm \sqrt{(2x^2 + 1)^2 - 4(x^4 + x)}}{2}$$

$$2a = (2x^2 + 1) \pm (2x - 1)$$

On solving +ve & -ve sign we got

$$a \geq \frac{3}{4}$$

$$\therefore m + n = 7$$

89. Number of positive integer n for which $n^2 + 96$ is a perfect square is

Key. 4

Sol: Suppose m is positive integer such that $n^2 + 96 = m^2$ then

$$(m - n)(m + n) = 96$$

As $m - n < m + n$ and $m - n, m + n$ both must be even

So, the only possibilities are

$$m - n = 2, m + n = 48; m - n = 4, m + n = 24$$

$$m - n = 6, m + n = 16; m - n = 8, m + n = 12$$

So, the solutions of (m, n) are $(25, 23), (14, 10), (11, 5), (10, 2)$

90. Number of real roots of $2x^{99} + 3x^{98} + 2x^{97} + 3x^{96} + \dots + 2x + 3 = 0$ is

Key. 1

Sol: Given equation can be written as $(2x + 3)(x^{98} + x^{96} + \dots + 1) = (2x + 3) \frac{(x^{100} - 1)}{x^2 - 1}$

So, the real roots are $x = \pm 1, \frac{-3}{2}$, out of which ± 1 are not roots of given equation.

91. Let $P(x) = x^2 + bx + c$, where b and c are integer. If $P(x)$ is a factor of both $x^4 + 6x^2 + 25$ and $3x^4 + 4x^2 + 28x + 5$, find the value of $P(1)$.

Key. 4

Sol: Since $P(x)$ divides into both of them

Hence $P(x)$ also divides

$$(3x^4 + 4x^2 + 28x + 5) - 3(x^4 + 6x^2 + 25) \\ = -14x^2 + 28x - 70 = -14(x^2 - 2x + 5)$$

Which is a quadratic, Hence $P(x) = x^2 - 2x + 5$

$$\therefore P(1) = 4$$

92. Largest integral value of m for which the quadratic expression

$y = x^2 + (2m + 6)x + 4m + 12$ is always positive, $\forall x \in R$, is

Key. 0

Sol: $D < 0 \Rightarrow -3 < m < 1 \Rightarrow m = 0$

93. The least integral value of 'a' such that $(a - 3)x^2 + 12x + (a + 6) > 0 \forall x \in R$ is

Key: 7

Hint:

$$ax^2 + bx + c > 0 \forall x \in R \Rightarrow a > 0, D < 0$$

$$\Rightarrow (i) a - 3 > 0 (ii) (a + 9)(a - 6), a > 6$$

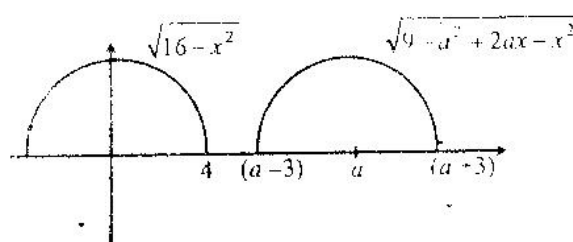
Least integral value of $a = 7$

94. Find the least integral value of a such that $\sqrt{9 - a^2 + 2ax - x^2} > \sqrt{16 - x^2}$ for at least one positive x .

Key : 6

$$\text{Sol : } y = \sqrt{9 - a^2 + 2ax - x^2}$$

$$(x - a)^2 + y^2 = 9$$



For given inequality to hold for positive x .

$$a - 3 < 4$$

$$\Rightarrow a < 7 \Rightarrow a = 6$$

95. If the equation $x^4 + px^3 + qx^2 + rx + 5 = 0$ has four positive real roots, then the minimum value of $pr/10$ is

Key. 8

Sol: Let $\alpha, \beta, \gamma, \delta$ be four positive real roots of given equation.

$$\text{Then } \alpha + \beta + \gamma + \delta = -p$$

$$\Sigma \alpha\beta = q$$

$$\Sigma \alpha\beta\gamma = -r$$

$$\alpha\beta\gamma\delta = 5$$

using A.M. $\geq$ G.M.

$$\frac{\alpha + \beta + \gamma + \delta}{4} \geq (\alpha\beta\gamma\delta)^{1/4}$$

$$\frac{\Sigma \alpha\beta\gamma}{4} \geq (\alpha^3\beta^3\gamma^3\delta^3)^{1/4}$$

$$\frac{(\Sigma \alpha) \cdot (\Sigma \alpha\beta\gamma)}{16} \geq (\alpha\beta\gamma\delta)$$

$$pr \geq 80$$

96. The set of real parameter 'a' for which the equation $x^4 - 2ax^2 + x + a^2 - a = 0$ has all real solutions, is given by $\left[\frac{m}{n}, \infty\right)$ where m and n are relatively prime positive integers, then the value of $(m+n)$ is

Key. 7

Sol: We have $a^2 - (2x^2 + 1)a + x^4 + x = 0$

$$\therefore a = \frac{(2x^2 + 1) \pm \sqrt{(2x^2 + 1)^2 - 4(x^4 + x)}}{2}$$

$$2a = (2x^2 + 1) \pm (2x - 1)$$

On solving +ve & -ve sign we got

$$a \geq \frac{3}{4}$$

$$\therefore m + n = 7$$

97. Number of positive integer n for which $n^2 + 96$ is a perfect square is

Key. 4

Sol: Suppose m is positive integer such that $n^2 + 96 = m^2$ then

$$(m - n)(m + n) = 96$$

As $m - n < m + n$ and $m - n, m + n$ both must be even

So, the only possibilities are

$$m - n = 2, m + n = 48; m - n = 4, m + n = 24$$

$$m - n = 6, m + n = 16; m - n = 8, m + n = 12$$

So, the solutions of (m, n) are $(25, 23), (14, 10), (11, 5), (10, 2)$

98. The sum of all the real roots of the equations $|x-2|^2 + |x-2| - 2 = 0$ is

Key. 4

Sol: $|x-2|^2 + |x-2| - 2 = 0$

$$\Rightarrow (|x-2| + 2)(|x-2| - 1) = 0 \Rightarrow |x-2| = -2, 1$$

$$\therefore |x-2| = 1 \text{ or } x = 3, 1 \Rightarrow \text{sum of the roots} = 4$$

99. If α, β are the roots of the equation $\lambda(x^2 - x) + x + 5 = 0$ and if λ_1 & λ_2 are two values of λ for which the roots α, β are related by $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{4}{5}$, then the value of $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} = (255 - k^2)$ then $|k|$ is

Key. 1

Sol: $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{(\alpha + \beta)^2 - 2\alpha\beta}{\alpha\beta} = \frac{4}{5} \Rightarrow \lambda^2 - 16\lambda + 1 = 0$

Now $\frac{\lambda_1}{\lambda_2} + \frac{\lambda_2}{\lambda_1} = \frac{\lambda_1^2 + \lambda_2^2}{\lambda_1\lambda_2} = 254$

100. The sum of squares of all integral values of $a > -5$ for which the inequality $x^2 - ax + 6a < 0$ is satisfied for all $x \in (-1, 1)$ must be equal to $6k$ then k is

Key. 5

Sol: $f(x) = x^2 - ax + 6a$

$D > 0, \quad f(1) < 0, f(-1) < 0$

value of a are $-4, -3, -2, -1$

$(-4)^2 + (-3)^2 + (-2)^2 + (-1)^2 = 30$

101. If $a, b \in \mathbb{R}$ and equations $ax^2 + 30x + b = 0$ and $x^2 + 3x + 4 = 0$ have a common root, then

$4a - b$ is

Key. 0

Sol: $x^2 + 3x + 4 = 0$ has imaginary roots so both roots are common

$\therefore \frac{a}{1} = \frac{30}{3} = \frac{b}{4}$

$a + b = 50$