

PARABOLA

FOCUS, LATUS RECTUM, AXIS, FOCALCHORD

1. The length of the latus-rectum of the parabola

$$169\{(x-1)^2 + (y-3)^2\} = (5x-12y+17)^2 \text{ is}$$

1) $\frac{12}{13}$

2) $\frac{14}{13}$

3) $\frac{28}{13}$

4) $\frac{31}{13}$

Key (2)

Sol.

$$(x-1)^2 + (y-3)^2 = \left(\frac{5x-12y+17}{13}\right)^2$$

Length of latus rectum $= 4a$

Perpendicular distance from (1,3) to the line $5x-12y+17=0$ is

$$2a = \frac{|5 \times 1 - 12 \times 3 + 17|}{\sqrt{169}} = \frac{14}{13}$$

2. If parabola $y^2 = \lambda x$ and $25\{(x-3)^2 + (y+2)^2\} = (3x-4y-2)^2$ are equal, then value of λ is

Key (6)

Sol

Two parabolas are equal if the length of their latus rectum are equal.

Length of the latus rectum of is

Focus of the second parabola is (3,-2) and equation of the directrix is $3x-4y-2=0$

Length of the latus rectum is twice distance between focus and

$$\text{Directrix } 2 \left| \frac{3 \times 3 - 4 \times (-2) - 2}{\sqrt{3^2 + (-4)^2}} \right| = 6 = \lambda$$

3. The coordinates of the focus of the parabola $y^2 = 4(x+y)$, are

- 1) (-1, 1) 2) (0, 2) 3) (2, 1) 4) (2, -1)

Key (2)

Sol.

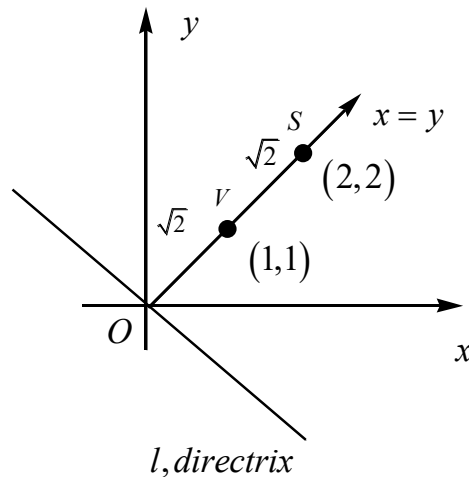
$$\begin{aligned} y^2 &= 4x + 4y \\ \Rightarrow (y - 2)^2 &= 4(x + 1) \\ \text{focus } &(0, 2) \end{aligned}$$

4. The axis of a parabola is along the line $x = y$ and the distance of its vertex from the origin is $\sqrt{2}$ and that of the focus from the origin is $2\sqrt{2}$. If the focus and vertex lie in the first quadrant, the equation of the parabola is

- 1) $(x + y)^2 = x - y - 2$ 2) $(x - y)^2 = x + y - 2$
 3) $(x - y)^2 = 4(x + y - 2)$ 4) $(x - y)^2 = 8(x + y - 2)$

Key. (4)

Sol. The vertex is $V(1,1)$ and focus is $S(2,2)$



The directrix is $x + y = 0$

The equation of the parabola is $(x - 2)^2 + (y - 2)^2 = \frac{(x + y)^2}{2}$

Or $(x - y)^2 = 8(x + y - 2)$

6. The equation of directrix of the parabola $x^2 + 4y - 6x + k = 0$ is $y + 1 = 0$ then

- 1) $k = 7$ 2) $k = -17$
 3) Focus is $(3, -3)$ 4) vertex is $(3, -3)$

Key. (3)

Sol.

$$\therefore (x-3)^2 = -4y - k + 9 = -4\left(y + \frac{k-9}{4}\right)$$

$$\text{Equation of the directrix is } y + \frac{k-9}{4} = 1 \Rightarrow y = \frac{13-k}{4}$$

$$\frac{13-k}{4} = -1 \Rightarrow k = 17$$

$$\text{Vertex is } \left(3, \frac{-k+9}{4}\right) = (3, -2)$$

$$\text{Focus } (3, -3)$$

7. If $Ay^2 + By + Cx + D = 0$, ($ABC \neq 0$) be the equation of parabola, then

1) the length of latusrectum is $\left|\frac{C}{A}\right|$

2) the axis of the parabola is a vertical line

3) Y- co-ordinate of the vertex is $-\frac{B}{2A}$

4) X- Co-ordinates of the vertex is $\left(\frac{B^2 - 4AD}{4AC}\right)$

Key. (1,3,4)

Sol.

$$-Cx = Ay^2 + By + D$$

$$-Cx = A\left(y^2 + \frac{B}{A}y + \frac{D}{A}\right)$$

$$x = -\frac{A}{C}\left(\left(y + \frac{B}{2A}\right)^2 + \frac{D}{A} - \frac{B^2}{4A^2}\right)$$

$$= -\frac{A}{C}\left[\left(y + \frac{B}{2A}\right)^2\right] - \frac{A}{C}\left[\frac{4AD - B^2}{4A^2}\right]$$

$$\Rightarrow \left(x + \frac{4AD - B^2}{4AC}\right) = -\frac{A}{C}\left(y + \frac{B}{2A}\right)^2$$

$$\Rightarrow \left(y + \frac{B}{2A}\right)^2 = -\frac{C}{A}\left(x + \frac{4AD - B^2}{4AC}\right)$$

9. The ends of the latus rectum of the conic $x^2 + 10x - 16y + 25 = 0$ are

(1) $(3, -4), (13, 4)$

(2) $(-3, -4), (13, -4)$

(3) $(3, 4), (-13, 4)$

(4) $(5, -8), (-5, 8)$

Key. (3)**Sol.** $(x+5)^2 = 16y$ comparing it with $x^2 = 4ay$,

2. If parabola $y^2 = \lambda x$ and $25((x-3)^2 + (y+2)^2) = (3x-4y-2)^2$ are equal, then value of λ is

Key (6)**Sol**

Two parabolas are equal if the length of their latus rectum are equal.

Length of the latus rectum of is

Focus of the second parabola is $(3, -2)$ and equation of the directrix is $3x - 4y - 2 = 0$

Length of the latus rectum is twice distance between focus and

$$\text{Directrix } 2 \left| \frac{3 \times 3 - 4 \times (-2) - 2}{\sqrt{3^2 + (-4)^2}} \right| = 6 = \lambda$$

10. A parabola which have directrix $x+y+2=0$ and touches a line $2x+y-5=0$ at $(2,1)$. If m is the latus rectum of the parabola, then the value of $\frac{m}{\sqrt{2}}$ is _____

Key. (6)

Sol. $\frac{\alpha-2}{1} = \frac{\beta-1}{1} = \frac{-(2+1+2)}{2} \Rightarrow \alpha = -\frac{1}{2}, \beta = -\frac{3}{2}$

$$\frac{\gamma + \frac{1}{2}}{2} = \frac{\delta + \frac{3}{2}}{1} = \frac{-2\left(-1 - \frac{3}{2} - 5\right)}{5} = 3$$

$$\Rightarrow S = (\gamma, \delta) = \left(\frac{11}{2}, \frac{3}{2} \right)$$

$$\text{Semi-latus rectum} = \frac{\left| \frac{11}{2} + \frac{3}{2} + 2 \right|}{\sqrt{2}} = \frac{9}{\sqrt{2}}$$

12. Consider the parabola $y^2 - 5y + 3x + k = 0$ then

1) Its latusrectum is 3

2) its latus rectum is independent of k

3) The line $y = 2x + 1$ will touch the parabola if $k = \frac{73}{16}$

4) $y = \frac{5}{2}$ is only normal to the parabola whose slope is zero.

Key. (1,2,3,4)

Sol.
$$\left(y - \frac{5}{2} \right)^2 = -3 \left(x - \left(\frac{25 - 4k}{12} \right) \right)$$

The line $y = 2x + 1$ will touch the parabola $y^2 - 5y + 3x + k = 0$

$$\text{When } \Delta = 0 \text{ for } (2x + 1)^2 - 5(2x + 1) + 3x + k = 0 \Rightarrow k = \frac{73}{16}$$

$y = \frac{5}{2}$ is axis of the parabola

13. Coordinates of any point on the parabola, whose focus is $\left(\frac{-3}{2}, -3 \right)$

and the directrix is $2x + 5 = 0$ is given by

1) $(2t^2 + 2, 2t - 3)$

2) $(2t^2 - 2, 2t - 3)$

3) $(2t^2 - 2, 2t + 3)$

4) $(2t^2 + 2, 2t + 3)$

Key. (2)

Sol Equation of parabola

$$\Rightarrow \left(x + \frac{3}{2} \right)^2 + (y + 3)^2 = \frac{(2x + 5)^2}{4}$$

$$(y+3)^2 = 2(x+2)$$

$$x+2 = \frac{1}{2}t^2$$

$$= 2\left(\frac{t}{2}\right)^2$$

$$x = 2\left(\frac{t}{2}\right)^2 - 2 \left(\text{let } \frac{t}{2} \text{ is some } t \right)$$

$$\therefore x = 2t^2 - 2$$

$$y+3 = 2\frac{1}{2}t$$

$$y = 2\left(\frac{t}{2}\right) - 3$$

$$\text{Let } \frac{t}{2} \text{ is some } t. y = 2t - 3$$

14. If the focus of a parabola is $(-2,1)$ and the directrix has the equation $x+y=3$, then the vertex is:

1) $(0,3)$

2) $\left(-1, \frac{1}{2}\right)$

3) $(-1,2)$

4) $(2,-1)$

Key. (3)

Sol.

The vertex is the middle point of the perpendicular dropped from the focus to the directrix.

15. The co-ordinates of a point on the parabola $y^2 = 8x$ whose focal distance is 4 is

1) $(2,4)$

2) $(4,2)$

3) $(2,-6)$

4) $(4,-2)$

Key. (1)

Sol. $a+x=4 \Rightarrow 2+x=4 \Rightarrow x=2, y=4$

16. Co-ordinate of the focus of the parabola $x^2 - 4x - 8y - 4 = 0$ are

1) $(0,2)$

2) $(2,1)$

3) $\left(-3, \frac{-71}{10}\right)$

4) $(2,-1)$

Key. (2)

Sol. $(x-2)^2 = 8(y+1)$

Focus $x-2=0, y+1=2 \Rightarrow x=2, y=1$

Focus (2,1)

17. Length of latus rectum of the parabola whose parametric equations are $x=t^2+t+1, y=t^2-t+1$ where $t \in R$, is equal to

1) 4

2) +1

3) $\sqrt{2}$

4) 3

Key. (3)

Sol. $x+y=2(t^2+1) \& x-y=2t$

$$\therefore (x+y-2) = 2\left(\frac{x-y}{2}\right)^2 \Rightarrow \left(\frac{x-y}{\sqrt{2}}\right)^2 = \sqrt{2}\left(\frac{x+y-2}{\sqrt{2}}\right)$$

Length of latusrectum = $\sqrt{2}$

18. Length of the latus rectum of the parabola $\sqrt{x} + \sqrt{y} = \sqrt{a}$

1. $a\sqrt{2}$

2. $\frac{a}{\sqrt{2}}$

3. a

4. 2a

Key. (1)

Sol $\sqrt{x} = \sqrt{a} - \sqrt{y}$

$$x = a + y - 2\sqrt{ay}$$

$$(x - y - a)^2 = 4ay$$

$$x^2 + (y+a)^2 - 2x(a+y) = 4ay$$

$$x^2 + y^2 - 2xy + 2ay + a^2 - 2ax = 4ay$$

$$x^2 + y^2 - 2xy = 2ax + 2ay - a^2$$

$$(x-y)^2 = 2a\left(x+y-\frac{a}{2}\right)$$

Axis is $x-y=0$

$$\left(\frac{x-y}{\sqrt{2}}\right)^2 = \frac{2a}{2} \left(\frac{x+y-\frac{a}{2}}{\sqrt{2}}\right) \times \sqrt{2}$$

$$\left(\frac{x-y}{\sqrt{2}}\right)^2 = a\sqrt{2} \left(\frac{x+y-\frac{a}{2}}{\sqrt{2}}\right) \therefore \text{lengthy } L.R = a\sqrt{2}$$

19. Axis of a parabola is $y = x$ and vertex and focus are at a distance $\sqrt{2}$ and $2\sqrt{2}$ respectively from the origin. Then, equation of the parabola is

1) $(x-y)^2 = 8(x+y-2)$

2) $(x+y)^2 = 2(x+y-2)$

3) $(x-y)^2 = 4(x+y-2)$

4) $(x+y)^2 = 2(x-y+2)$

Key. (1)

Sol. Since, distance of vertex from origin is $\sqrt{2}$ and focus is $2\sqrt{2}$.

$\therefore V(1,1)$ and $F(2,2)$ (i.e. lying on $y = x$)

Where, length of latusrectum $= 4a = 4\sqrt{2}$ [$a = \sqrt{2}$]

$\therefore$ By definition of parabola

Where PN is length of perpendicular upon $x+y-2=0$, i.e. tangent at vertex

$$\Rightarrow \frac{(x-y)^2}{2} = 4\sqrt{2} \left(\frac{x+y-2}{\sqrt{2}} \right)$$

$$\Rightarrow (x-y)^2 = 8(x+y-2)$$

$$x - h^2 = 4a \quad y - k$$

$$\Rightarrow x^2 = -4y - 10$$

PARABOLA

TANGENTS

- 1). Let $(2, 3)$ be the focus of a parabola and $x + y = 0$ and $x - y = 0$ be its two tangents, then equation of its directrix will be

1) $2x - 3y = 0$

2) $3x + 4y = 0$

3) $x + y = 5$

4) $12x - 5y + 1 = 0$

Key (3)

Sol.

Mirror image of focus in the tangent of parabola lie on its directrix.

- 2). The locus of the Orthocentre of the triangle formed by three tangents of the parabola $(4x-3)^2 = -64(2y+1)$ is

1) $y = \frac{-5}{2}$ 2) $y = 1$ 3) $x = \frac{7}{4}$ 4) $y = \frac{3}{2}$

Key (4)

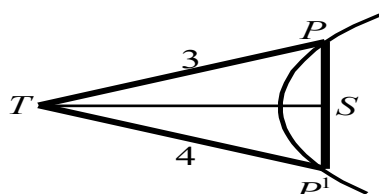
Sol.

The locus is directrix of the parabola

- 3). If the lengths of two perpendicular tangents from a point to a parabola are of 3 and 4 units and the length of latus rectum is K units, then $[K]$ = -----
 ([.] represent greatest integer function)

1)2 2)4 3)8 4)10

Key (3)



Sol.

The foot of perpendicular are dropped from T onto the line PP' lies on the focus.

$$\angle TPS = \tan^{-1} 4/3$$

$$\cos \angle TPS = \frac{SP}{PT}$$

$$SP = 3 \times \frac{3}{5} = 9/5$$

$$\text{Similarly } SP' = 16/5 \quad \frac{1}{SP} + \frac{1}{SP'} = \frac{1}{a}$$

- 5). If $a_1x + by + c = 0$ and $a_2x + by + c = 0$ are two tangents to the parabola

$$y^2 = 8a(x - 2a), \text{ then the value of } \frac{a_1a_2 + b^2 + 4}{2} =$$

Key (2)

Sol.

The tangents are drawn from $\left(0, -\frac{c}{b}\right)$ which is on Y-axis, the directrix of the given parabola.

$$\Rightarrow \left(-\frac{a_1}{b}\right)\left(-\frac{a_2}{b}\right) = -1 \Rightarrow a_1a_2 + b^2 = 0$$

- 6). The triangle formed by the tangent to the parabola $y^2 = 4x$ at the point whose abscissa lies in the interval $[a^2, 4a^2]$, the ordinate and the x -axis has the greatest area equal to

- 1) $12a^3$ 2) $8a^3$ 3) $16a^3$ 4) None

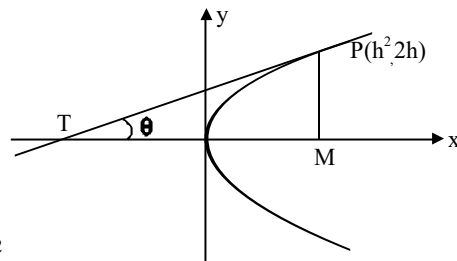
Key (3)

Sol

$$P \equiv (h^2, 2h)$$

$$\tan \theta = \frac{1}{h}$$

$$\text{And } \Delta PTM = \frac{1}{2} \times 2h \times 2h \cot \theta = 2h^3$$



$$a^2 \leq h^2 \leq 4a^2$$

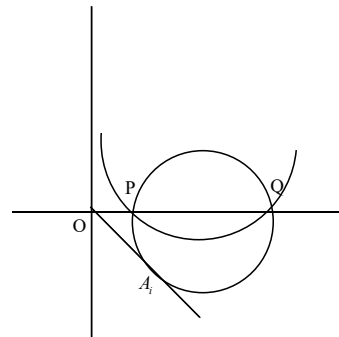
$$\therefore \text{maximum area} = 2(2a)^3 = 16a^3$$

- 7). The parabola $y = 4x^2 - 72x + 49$ cuts x -axis at P, Q. If S_i be the circles drawn through P, Q and if OA_i be the length of tangent drawn from origin to the circle S_i then $\sum_{i=1}^{10} OA_i$

- 1) 180 2) $6\sqrt{5}$ 3) 35 4) $7\sqrt{5}$

Key (3)

Sol.



$$OA_i = \sqrt{OP \cdot OQ} = \sqrt{\frac{49}{4}} = \frac{7}{2}$$

$$\sum OA_i = 10 \times \frac{7}{2} = 35$$

- 10). The locus of point of intersection of any tangent to the parabola $y^2 = 4a(x-2)$ with a line perpendicular to it and passing through the focus, is
 1) $x = 2$ 2) $y = 0$ 3) $x = a$ 4) $x = a + 2$

Key (1)

Sol. It is well known property of a parabola that a tangent and normal to it from focus intersect at tangent at vertex

- 12). If a line $y = 3x + 1$ cuts the parabola $x^2 - 4x - 4y + 20 = 0$ at A and B, then the tangent of the angle subtended by line segment AB at origin is
 1) $\frac{8\sqrt{3}}{205}$ 2) $\frac{8\sqrt{3}}{209}$ 4) $\frac{8\sqrt{3}}{215}$ D) 1

Key (2)

Sol.

The pair of lines joining origin to A and B is given by

$$x^2 - (4x + 4y)(y - 3x) + 20(y - 3x)^2 = 0$$

Angle between pair of lines $ax^2 + 2hxy + by^2 = 0$ is given by $\tan^{-1} \left(\frac{2\sqrt{h^2 - ab}}{a + b} \right)$

- 13). $\overline{PQ}$ is a focal chord of a parabola which is along the straight line $2x - y = 1$ and the tangent at P to the parabola is $y = x$, if its axis passes through the origin then
 1) The focus is $\left(\frac{1}{3}, \frac{-1}{3}\right)$ 2) The focus is $\left(\frac{2}{3}, \frac{1}{3}\right)$
 3) The directrix is $6x + 3y - 4 = 0$ 4) The directrix is $3x + 6y - 4 = 0$

Key (2,3)

Sol.
$$\left. \begin{array}{l} 2x - y = 1 \\ y = x \end{array} \right\} \Rightarrow p(1,1)$$

L is a line passing p and parallel to axis

$\Rightarrow$ The tangent at p bisects the angle between L and focalchord

$$\Rightarrow L \text{ is } x - 2y + 1 = 0; \Rightarrow \text{axis is } x - 2y = 0 \Rightarrow \text{focus } s\left(\frac{2}{3}, \frac{1}{3}\right)$$

- 14). Let $3x - y - 8 = 0$ be tangent to a parabola at the point $P(7,13)$. If the focus of the parabola be at the point $(-1, -1)$, the area of the triangle formed by the directrix and co-ordinate axis is K^2 then $[K] =$ (Where $[.]$ is greatest integer function, $K > 0$)

Key (4)

Sol.

Foot of $\perp$ ler from focus $(-1, -1)$ to tangent $= (2, -2)$ image of $(7, 13)$ w.r.t $(2, -2)$ will lie on axis and image $(-1, 1)$ w.r.t $(2, -2)$ will lie on directrix

So, $(-3, -17)$ is the point on axis and $(5, -3)$ is the point on directrix

Now slope of axes $= 8$

And slope of directrix $= -1/8$

Eq of directrix is $x + 8y + 19 = 0$

$$\text{And } k^2 = \frac{-381}{16} \Rightarrow [k] = 4$$

- 15). If foot of perpendicular from focus upon any tangent of parabola $y^2 - 4y + 4x + 8 = 0$ lies on the line $x + k = 0$ then k is

Key (1)**Sol.**

$$(y - 2)^2 = -4(x + 1)$$

foot of $\perp$ will lie on tangent at vertex. $x + 1 = 0$

$$\Rightarrow k = 1.$$

- 16). The parabola $y = x^2 - 5x + 4$ cuts the x -axis at P and Q . A circle is drawn through P and Q so that the origin lies outside it. The length of tangent to the circle from the origin is equal to

Key (2)

$$P \equiv (1, 0), Q \equiv (4, 0)$$

Sol.

$$(x - 1)(x - 4) + y^2 + \lambda y = 0$$

the length of tangent from $(0, 0)$ is $\sqrt{4} = 2$

- 17). The value of a for which two curves $y = ax^2 + ax + \frac{1}{24}$ and $x = ay^2 + ay + \frac{1}{24}$ touch each other is

$$1) \frac{2}{3}$$

$$2) \frac{3}{2}$$

$$3) \frac{-2}{3}$$

$$4) -\frac{3}{2}$$

Key (1,2)**Sol.**

Two curves are symmetric about $y = x$

Let $p(\alpha, \alpha)$ be the point of contact

$$\Rightarrow 24a\alpha^2 + 24\alpha(a - 1) + 1 = 0 \text{ has equal roots ; } \Rightarrow a = \frac{3}{2}, \frac{2}{3}$$

TAGNENTS IN (Point form)

- 18). Let $3x - y - 8 = 0$ be the equation of tangent to a parabola at the point $(7, 13)$. If the focus of the parabola is at $(-1, -1)$. Its directrix is
- 1) $x - 8y + 19 = 0$
 - 2) $8x + y + 19 = 0$
 - 3) $8x - y + 19 = 0$
 - 4) $x + 8y + 19 = 0$

Key (4)

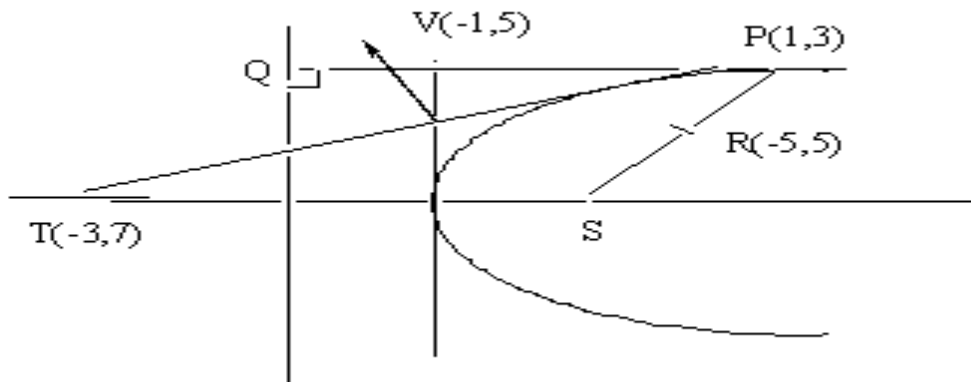
Sol

A point on the axis is $(-3, -17)$ point on the directrix i.e image of the focus w. R. t the tangent is $(5, -3)$ equation of directrix is $x + 8y + 19 = 0$

- 19). Tangent drawn at point $P(1, 3)$ to a parabola intersects its tangent at vertex at $V(-1, 5)$. If $R(-5, 5)$ is a point on SP ; where S is focus of the parabola then
- 1) Slope of directrix is $1/3$
 - 2) radius of circumcircle of ΔSVP is 2 units
 - 3) tangent cuts the axis of parabola at $(-3, 7)$
 - 4) focus is $(-2, 4)$

Key (2,3)

Sol



Equation of tangent at p is $x + y = 4$

Clearly mirror image of $R(-5, 5)$ lies on line PQ

Now mirror image R' of R

$$\Rightarrow \frac{\alpha' + 5}{1} = \frac{\beta' - 5}{1} = \frac{-2(-5 + 5 - 4)}{2} = 4$$

$$\Rightarrow (\alpha', \beta') = (-1, 9)$$

Let PV cuts the axis at T; As V is midpoint of PT

$\Rightarrow T$ is $(-3, 7)$

$$\text{Equation SP} \equiv y - 3 = -\frac{1}{3}(x - 1)$$

$$\Rightarrow x + 3y - 10 = 0$$

$$\text{Let } S = (10 - 3\beta, \beta)$$

$$\text{Again } TS \parallel PQ \Rightarrow \frac{\beta - 7}{13 - 3\beta} = \frac{9 - 3}{-1 - 1} = -3$$

$$\Rightarrow 8\beta = 32 \Rightarrow \beta = 4 \quad \therefore \text{focus is } (-2, 4)$$

20). If a parabola touches the lines $y = x$ and $y = -x$ at $A(3, 3)$ and $B(1, -1)$ respectively, then

1) equation of directrix is $2x + y = 0$

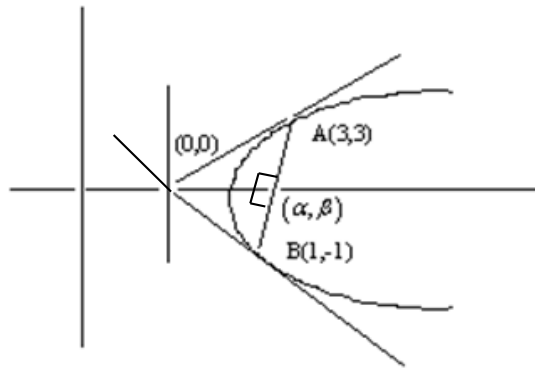
2) equation of line through origin and focus is $x + 2y = 0$

3) focus is $\left(-\frac{3}{5}, \frac{6}{5}\right)$

4) directrix passes through $(1, -2)$

Key (1, 2, 4)

Sol.



Clearly tangents are perpendicular

$\Rightarrow$ their point of intersection i.e $(0, 0)$ lies at directrix

$\Rightarrow AB$ is focal chord of the parabola

$\Rightarrow$ focus is foot of perpendicular from $(0, 0)$ on focal chord AB whose equation is $2x - y - 3 = 0$

$$\Rightarrow \frac{\alpha - 0}{2} = \frac{\beta - 0}{-1} = \frac{-(0 - 0 - 3)}{2^2 + 1^2}$$

$$\Rightarrow (\alpha, \beta) \text{ is } \left(\frac{6}{5}, -\frac{3}{5}\right)$$

$$\therefore \text{Focus is } \left(\frac{6}{5}, -\frac{3}{5}\right)$$

$$\Rightarrow \text{Slope of axis} = \text{slope of line joining } (0, 0) \text{ and midpoint of } AB = \frac{1 - 0}{2 - 0} = \frac{1}{2}$$

$$\text{Equation of directrix } y - 0 = -2(x - 0)$$

$$\text{Equation of line through origin and focus is } x + 2y = 0$$

- 21). If area of Triangle formed by tangents from the point (x_1, y_1) to the parabola $y^2 = 4ax$ and their chord of contact is

1) $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a^2}$ 2) $\frac{(y_1^2 - 4ax_1)^{3/3}}{a^2}$ 3) $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$ 4) none of these

Key (3)

Sol.

Let $A(x_1, y_1)$ be any point outside the parabola and $B(\alpha, \beta), C(\alpha', \beta')$ be the points of contact of tangents from point A eq of chord BC, $YY_1 = 2a(x+x_1)$

Lengths of $\perp$ from A to BC

$$= \frac{2a(x_1+x) - y_1y}{\sqrt{y^2+4a^2}} = \frac{y_1^2 - 4ax}{\sqrt{y_1^2+4a^2}}$$

Area of $\Delta ABC = \frac{1}{2}AL \times BC$ We get $\frac{(y_1^2 - 4ax_1)^{3/2}}{2a}$

- 22). The locus of mid-point of the chords of the parabola $y^2 = 4ax$ such that the tangents at the extremities of the chords are perpendicular is

1) $y^2 = 2a(x-a)$ 2) $y^2 = 2a(x+a)$ 3) $y^2 = ax$ 4) $y^2 = 2ax$

Key (1)

Sol. Let (x_1, y_1) be the mid point of the chord $y^2 = 4ax$

Its equation is $S_1 = S_{11} \Rightarrow yy_1 - 2ax = y_1^2 - 2ax_1$

Tangents the extremities of the chord are perpendicular to each other.

Hence the chord is a focal chord and it passes through $S(a, 0)$

$$\therefore -2a^2 = y_1^2 - 2ax$$

Locus of (x_1, y_1) is $y^2 = 2a(x-a)$

- 23). Tangents are drawn from the point $(-1, 2)$ to the parabola $y^2 = 4x$. The length of the intercept made by the line $x = 2$ on these tangents is

1) 6 2) $6\sqrt{2}$ 3) $2\sqrt{6}$ 4) none

Key (2)

Sol.

Equation of pair of tangent is

$$\begin{aligned}
SS_1 &= T^2 \\
\Rightarrow (y^2 - 4x)(8) &= 4(y - x + 1)^2 \\
\Rightarrow y^2 - 2y(1 - x) - (x^2 + 6x + 1) &= 0 \\
\text{Put } x &= 2 \\
\Rightarrow y^2 + 2y - 17 &= 0 \\
\Rightarrow |y_1 - y_2| &= 6\sqrt{2}
\end{aligned}$$

- 24). The radius of the circle passing through the origin and touches the parabola $y^2 = 8x$ at $(2, 4)$ is _____

1) $5\sqrt{2}$ 2) $\frac{5}{\sqrt{2}}$ 3) $10\sqrt{2}$ 4) $\frac{15}{\sqrt{2}}$

Key (1)

Sol

Eq. of tangent at $(2, 4)$ to $y^2 = 8x$ is $x - y + 2 = 0$, equation of the required circle is $(x - 2)^2 + (y - 4)^2 + k(x - y + 2) = 0$, but it passes through $(0, 0) \Rightarrow k = -10$

Therefore the radius of the circle is $5\sqrt{2}$

- 25). From a point T a tangent is drawn and it touches the parabola $y^2 = 16x$ at $P = (16, 16)$. If S be the focus of the parabola, then $\angle TPS$ can be equal to

1) $\tan^{-1}\left(\frac{3}{4}\right)$ 2) $\frac{1}{2}\tan^{-1}\left(\frac{1}{4}\right)$
 3) $\tan^{-1}\left(\frac{1}{2}\right)$ 4) $\frac{\pi}{4}$

Key (3)

Sol.

Slope of tangent = $1/2$ and slope of PS = $4/3$

$$\begin{aligned}
\therefore \angle TPS &= \tan^{-1} \frac{\left(\frac{4}{3} - \frac{1}{2}\right)}{1 + \frac{4}{3} \cdot \frac{1}{2}} \\
&= \tan^{-1} \left(\frac{1}{2}\right)
\end{aligned}$$

TANGENTS In Slope form

- 32). If m_1, m_2 are slopes of tangents drawn from $(1, 4)$ to the parabola $y^2 = 4x$, then

1) $m_1 + m_2 = 4$ 2) $|m_1 - m_2| = 2\sqrt{3}$
 3) $m_1 \cdot m_2 = -1$ 4) $m_1 = m_2$

Key (1)

ANY TANGENT OF THE PARABOLA $y = mx + \frac{a}{m}$

Sol.

$$\begin{aligned}\Rightarrow 4 &= m + \frac{1}{m} \Rightarrow 4m = m^2 + 1 \\ \Rightarrow m^2 - 4m + 1 &= 0 \\ \Rightarrow m_1 + m_2 &= 4 \text{ and } m_1 m_2 = 1\end{aligned}$$

33). If two tangents from the point α, β to the parabola $y^2 = 4x$ be such that the slope of one tangent is double of the other then

$$1) \beta = \frac{2}{9}\alpha^2 \quad 2) \alpha = \frac{2}{9}\beta^2 \quad 3) 2\alpha = 9\beta^2 \quad 4) \alpha = 2\beta^2$$

Key (2)

Sol.

$$\begin{aligned}y &= mx + \frac{1}{m} & \beta &= m\alpha + \frac{1}{m} \\ \alpha m^2 - \beta m + 1 &= 0 & \text{Roots } m_1, 2m_1 \\ m_1 + 2m_1 &= \frac{\beta}{\alpha}, m_1 \cdot 2m_1 = \frac{1}{\alpha} & 2 \cdot \left(\frac{\beta}{3\alpha}\right)^2 &= \frac{1}{\alpha} \quad 2\beta^2 = 9\alpha\end{aligned}$$

34). If the lines $(y-b) = m_1(x+a)$ and $(y-b) = m_2(x+a)$ are the tangents of $y^2 = 4ax$ then

$$1) m_1 + m_2 = 0 \quad 2) m_1 m_2 = 1 \quad 3) m_1 m_2 = -1 \quad 4) m_1 + m_2 = 1$$

Key (3)

Sol

$$\begin{aligned}y &= mx + \frac{a}{m} \\ \Rightarrow m^2 x - 3y + a &= 0, m_1 m_2 = -1\end{aligned}$$

35). Two tangents are drawn from point $(1, 4)$ to the parabola $y^2 = 4x$. Angle between these tangents is $\frac{\pi}{K}$ then $K = \dots\dots\dots$

Key (3)

Sol. $y = mx + \frac{1}{m}$ is tangent to the parabola $y^2 = 4x$

$$\Rightarrow y^2 = m + \frac{1}{m} \Rightarrow m^2 - 4m + 1 = 0$$

$$m_1 + m_2 = 4, m_1 m_2 = 1$$

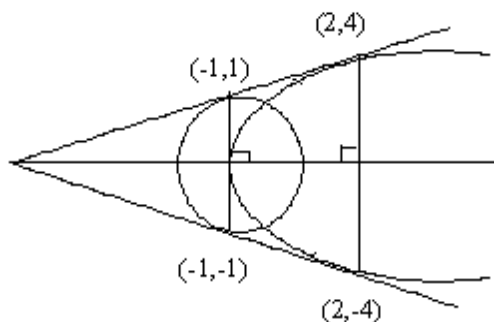
$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \sqrt{3}$$

$$\theta = \frac{\pi}{3}$$

- 39). The common tangents to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ touch the circle at the points P, Q and the parabola at the points R, S. Then the area of the quadrilateral PQRS is
- 1) 3 2) 6 3) 9 4) 15

Key (4)

Sol.



$$\text{Area} = \left(\frac{1}{2}(1+4)3 \right) \times 2 = 15$$

- 40). An equation of common tangent to the parabola $y^2 = 8x$ and the hyperbola $3x^2 - y^2 = 3$ is
- 1) $2x - y + 1 = 0$ 2) $x - y + 2 = 0$ 3) $x + y + 2 = 0$ 4) $2x + y - 1 = 0$

Key (1)

Sol. Let m be the slope of the common tangent

$$\therefore \frac{2}{m} = \sqrt{m^2 - 3} \Rightarrow m = \pm 2$$

Equation of common tangents are $y = 2x + 1$ or $y = -2x - 1$

- 41). The equation of a common tangent to the parabola $y^2 = 2x$ and the circle $x^2 + y^2 + 4x = 0$ is
- 1) $x - 2\sqrt{6}y - 2 = 0$ 2) $x + 2\sqrt{6}y + 12 = 0$ 3) $x - 2\sqrt{6}y + 12 = 0$ 4) $x = 0$

Key (2,3,4)

Sol.

Equation of tangent to $y^2 = 2x$ with slope 'm' is $y = mx + \frac{1}{2m}$

$$\Rightarrow 2m^2x - 2my + 1 = 0$$

It touches the circle $x^2 + y^2 + 4x = 0$

$$\therefore \frac{|2m^2(-2)+1|}{\sqrt{4m^4+4m^2}} = 2 \Rightarrow 16m^4 + 1 - 8m^2 = 16m^4 + 16m^2$$

$$24m^2 = 1$$

$$m = \pm \frac{1}{2\sqrt{6}}$$

Also y -axis is a common tangent

Hence the common tangents are $x - 2\sqrt{6}y + 12 = 0$, $x + 2\sqrt{6}y + 12 = 0$ and $x = 0$

42). Equation of common tangent to $x^2 = 32y$ and $y^2 = 32x$

1. $x + y = 8$ 2. $x + y + 8 = 0$ 3. $x - y = 8$ 4. $x - y + 8 = 0$

Key (2)

Sol.

Common tangents $y^2 = 4ax$ and $x^2 = 4ay$ is $xa^{\frac{1}{3}} + yb^{\frac{1}{3}} + a^{\frac{2}{3}}b^{\frac{2}{3}} = 0$

Here $a=8$, $b=8$

43). Equation of common tangent of $y = x^2$, $y = -x^2 + 4x - 4$ is

- 1) $y = 4(x-1)$ 2) $y = 0$
3) $y = -4(x-1)$ 4) $y = -30x - 50$

Key (1,2)

Sol.

The equation of tangent to $y = x^2$, be $y = mx - \frac{m^2}{4}$

Putting in $y = -x^2 + 4x - 4$, we should only get one value of x i.e. Discriminant must be zero

$$\therefore mx - \frac{m^2}{4} = -x^2 + 4x - 4$$

$$\Rightarrow x^2 + x(m-4) + 4 - \frac{m^2}{4} = 0$$

$$D = 0$$

$$\text{Now, } (m-4)^2 - (16 - m^2) = 0$$

$$\Rightarrow 2m(m-4) = 0$$

$$\Rightarrow m = 0, 4$$

$\therefore y = 0$ and $y = 4(x-1)$ are the required tangents.

Hence, (a) and (b) are correct answers

44). Equation of a common tangent to the curves $y^2 = 4ax$ and $x^2 + 3y^2 = 37$ is

- 1) $3y = 9x + 2$ 2) $y = 2x + 1$ 3) $2y = x + 8$ 4) $y = x + 2$

Key (4)

Sol.

$$y^2 = 8k, xy = -1$$

Let $P\left(t, \frac{-1}{t}\right)$ be any point on $xy = -1$

Equation of the tangent to $xy = -1$ at $P\left(t, \frac{-1}{t}\right)$ is

$$\frac{xy_1 + yx_1}{2} = -1$$

$$\frac{-x}{t} + yt = -2$$

$$y = \frac{x}{t^2} + \left(\frac{-2}{t}\right) \dots \dots \dots (1)$$

If (1) is tangent to the parabola $y^2 = 8x$ then

$$\frac{-2}{t} = \frac{2}{1/t^2} \Rightarrow t^3 = -1$$

$$t = -1$$

$\therefore$ Common tangent is $y = x + 2$

46). The equations of the common tangents of the curves $x^2 + 4y^2 = 8$ and $y^2 = 4x$ are

$$1) x + 2y + 4 = 0 \quad 2) x - 2y + 4 = 0 \quad 3) 2x + y = 4 \quad 4) 2x - y + 4 = 0$$

Key (1,2)

Sol.

$$\frac{x^2}{8} + \frac{y^2}{2} = 1, \quad y^2 = 4x$$

Any tangent to parabola is $y = mx + \frac{1}{m}$

If this line is tangent to ellipse then $\frac{1}{m^2} = 8m^2 + 2 \Rightarrow 8m^4 + 2m^2 - 1 = 0$

$$m^2 = \frac{-2 \pm \sqrt{4 + 32}}{16} = \frac{-2 \pm 6}{16}$$

$$\Rightarrow m^2 = \frac{1}{4} \Rightarrow m = \pm \frac{1}{2}$$

$$y = \frac{x}{2} + 2 \text{ or } y = -\frac{x}{2} - 2$$

$$x - 2y + 4 = 0 \text{ or } x + 2y + 4 = 0$$