

LIMITS 2026

STANDARD LIMITS

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1.$$

$$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1.$$

$$\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = n a^{n-1}$$

$$\lim_{x \rightarrow 0} \frac{e^x - 1}{x} = 1.$$

$$\lim_{x \rightarrow 0} (1 + x)^{\frac{1}{x}} = e$$

$$\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\lim_{x \rightarrow 0} \frac{\ln(1 + x)}{x} = 1.$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x^2} = \frac{1}{2}$$

EXPANSIONS:

$$1) (1 + x)^n = 1 + nx$$

$$2) e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$$

$$3) \ln(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3}$$

$$4) \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \dots$$

$$5) \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \dots$$

$$6) \tan x = x + \frac{x^3}{3} + \frac{2}{15}x^5$$

$$7) \sin^{-1}x = x + \frac{1^2}{3!}x^3 + \frac{1^2 \cdot 3^2}{5!}x^5 + \dots$$

$$8) \tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5}$$

$$9) (1 + x)^{\frac{1}{x}} = e \left[1 - \frac{x}{2} + \frac{11}{24}x^2 \dots \right]$$

TYPES OF INDETERMINATES

If the value of a function at a point acquires one of the form

$$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 1^\infty, 0^0, \infty^0$$

LIMITS IN THE FORM OF

$$\lim_{x \rightarrow a} (f(x))^{g(x)} = L$$



$$1^\infty$$

$$L = e^{\lim_{x \rightarrow a} g(x)(f(x)-1)}$$



$$0^0$$

$$L = e^{\lim_{x \rightarrow a} \frac{\ln f(x)}{\frac{1}{g(x)}}}$$

$$\infty^0$$

$$L = e^{\lim_{x \rightarrow a} \frac{\ln f(x)}{\frac{1}{g(x)}}}$$

'L' HOSPITAL RULE

→ Before applying L.H rule one must try to use standard limits by applying limit theorem .

$$\text{Let } \lim_{x \rightarrow a} \frac{f(x)}{g(x)}$$

$$\text{If } f(a) = g(a) = 0 \text{ or } f(a) = g(a) = \infty$$

Now $\lim_{x \rightarrow a} \frac{f(x)}{g(x)}$ has an indeterminacy at $x = a$

$$\frac{0}{0} \text{ or } \frac{\infty}{\infty} \text{ then}$$

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \lim_{x \rightarrow a} \frac{f'(x)}{g'(x)}$$

If $\frac{f'(x)}{g'(x)}$ to has an indeterminacy at $x = a$

Of the type $\frac{0}{0}$ or $\frac{\infty}{\infty}$ then

$$\lim_{x \rightarrow a} \frac{f'(x)}{g'(x)} = \lim_{x \rightarrow a} \frac{f''(x)}{g''(x)}$$

this can be continued till we finally arrive at a determinate result .

SANDWICH THEOREM :

If $g(x) < f(x) \leq h(x)$ for all x in the neighbourhood of $x = a$ and

$$\lim_{x \rightarrow a} g(x) = \lim_{x \rightarrow a} h(x) = L \quad \text{Then } \lim_{x \rightarrow a} f(x) = L$$

1. Let $f : (1, \infty) \rightarrow (0, \infty)$ be a continuous decreasing function

with $\lim_{x \rightarrow \infty} \frac{f(4x)}{f(8x)} = 1$ then $\lim_{x \rightarrow \infty} \frac{f(6x)}{f(8x)}$ is equal to :

Answer : 1

2. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a positive decreasing function with $\lim_{x \rightarrow \infty} \frac{f\left(\frac{x - x^3}{6}\right)}{f(x)} = 1$

then $\lim_{x \rightarrow \infty} \frac{f(\sin x)}{f(x)}$ is equal to

Answer : 1

$$\frac{1}{x} \rightarrow 0^+ \text{ if } x \rightarrow +\infty$$

$$\frac{1}{x} \rightarrow 0^- \text{ if } x \rightarrow -\infty$$

Consider the behavior of the function $\frac{1}{x}$ of x

It is quite clear $\frac{1}{x}$ increases in values as x approaches the least positive

Values, $\frac{1}{x}$ increases without limit this fact is written as

$$\frac{1}{x} \rightarrow \infty \text{ as } x \rightarrow 0^+$$

$$\frac{1}{x} \rightarrow -\infty \text{ as } \frac{1}{x} \rightarrow 0^-$$

∞ (infinity) $\rightarrow$ is a symbol and not a number. It is a symbol for the behaviour of a variable which continuously increases and passes all limits.

Thus the $x = \infty$ Meaningless, we may write $x \rightarrow \infty$

Similarly continuously decreases and passes all limits $x = -\infty$ meaningless.

We may write $x \rightarrow -\infty$

FOLLOWING FORMS ARE NOT INDETERMINATE FORMS

$$1. (1)^{\rightarrow\infty} = 1$$

$$2. \frac{\rightarrow 0}{\rightarrow \infty} = \rightarrow 0$$

$$3. 0 \times \rightarrow (\infty) = 0$$

$$4. \frac{\rightarrow 0}{0} = \text{Not defined}$$

$$5. \frac{0}{\rightarrow 0} = 0$$

1. $\lim_{x \rightarrow 0} \sin^{-1}(\sec x)$ is equal to :

1) $\frac{\pi}{2}$

2) 0

3) 1

4) undefined

key: 4

2. $\lim_{x \rightarrow 0} \sin^{-1}[\sec x]$ is equal to :

1) $\frac{\pi}{2}$

2) 0

3) 1

4) undefined

key: 1

3. $\lim_{x \rightarrow 0^-} \sin^{-1}[\tan x] = L$, then $\{L\}$ is equal to :

$[] \rightarrow \text{G. I. F}$ and $\{ \} \rightarrow \text{fractional part function}$

1) $2 - \frac{\pi}{2}$

2) $1 - \frac{\pi}{2}$

3) 0

4) $\frac{\pi}{2} - 1$

key: 1

Modal 1

Using standard results

1.
$$\lim_{x \rightarrow 0} \frac{(4^x - 1)^3}{\sin\left(\frac{x}{\alpha}\right) \ln\left(1 + \frac{x^2}{3}\right)} =$$

Answer : $3\alpha (\log 4)^3$

2. The value of $\lim_{x \rightarrow 0} \frac{\sin(\ln(1+x))}{\ln(1+\sin x)}$ is :

1) 0

2) $\frac{1}{2}$

3) $\frac{1}{4}$

4) 1

Key : 4

3. $\lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{2(x - \sin x)} =$

1) $-\frac{1}{2}$

2) $\frac{1}{2}$

3) 1

4) $\frac{3}{2}$

Key : 2

4. $\lim_{x \rightarrow 0} \frac{e^{\sin ax} - e^{\sin bx}}{\tan 2x} =$

1) $a - b$

2) $\frac{a-b}{2}$

3) $2(a - b)$

4) $a + b$

Key : 2

5. $\lim_{x \rightarrow 0} \frac{\ln(1+2x-3x^2)}{\ln(1-x+5x^5)} =$

Answer : -2

6. $\lim_{x \rightarrow 0} \frac{\ln(2-\cos 2x)}{\ln^2(\sin 3x+1)} =$

Answer : $\frac{2}{9}$

1. $\lim_{x \rightarrow 0^+} \frac{\tan\left(5(x)^{\frac{1}{3}}\right) \log_e(1+3x^2)}{(\tan^{-1} 3\sqrt{x})^2 (e^{5(x)^{\frac{4}{3}}} - 1)}$ is equal to _____.

1) $\frac{5}{3}$

2) 1

3) $\frac{1}{3}$

4) $\frac{1}{15}$

Key: 3

2. $\lim_{x \rightarrow 0} \left(\left(\frac{1 - \cos^2(3x)}{\cos^3(4x)} \right) \left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5} \right) \right)$ is equal to _____.

1) 15

2) 18

3) 9

4) 24

Key2

3. $\lim_{x \rightarrow 0} \frac{\sin^2(\pi \cos^4 x)}{x^4}$ is equal to :

1) π^2

2) $2\pi^2$

3) $4\pi^2$

4) 4π

Key : 3

4. $\lim_{x \rightarrow 0} \left(\frac{x}{\sqrt[8]{1 - \sin x} - \sqrt[8]{1 + \sin x}} \right)$ is equal to :

1) 0

2) 4

3) -4

4) -1

Key : 3

5. $\lim_{\theta \rightarrow 0} \frac{\tan(\pi \cos^2 \theta)}{\sin(2\pi \sin^2 \theta)}$ is equal to :

1) 0

2) $-\frac{1}{2}$

3) $\frac{1}{4}$

4) $-\frac{1}{4}$

6. $\lim_{x \rightarrow 0} \frac{x \cot(4x)}{\sin^2 x \cot^2(2x)}$ is equal to :

1) 0

2) 4

3) 1

4) 2

Key : 3

7. $\lim_{x \rightarrow 0} \left(\frac{\sin^2 x}{\sqrt{2} - \sqrt{1 + \cos x}} \right)$ equals:

A) $\sqrt{2}$

B) $2\sqrt{2}$

C) 4

D) $4\sqrt{2}$

Key : 4

JEE Main 2015

8. $\lim_{x \rightarrow 0} \frac{(1 - \cos 2x)(3 + \cos x)}{x \tan 4x}$ is equal to:

1) 2

2) $\frac{1}{2}$

3) 4

4) 3

Key : 1

10. $\lim_{x \rightarrow 0} \frac{8^x - 3.4^x + 3.2^x - 1}{\sin \frac{x}{4} \ln \left(1 + \frac{x^2}{2} \right)} = \underline{\hspace{2cm}}.$

Answer : $8 (\ln 2)^3$

11. $\lim_{x \rightarrow 0} \frac{27^x - 9^x - 3^x + 1}{\sqrt{2} - \sqrt{1 + \cos x}} = \underline{\hspace{2cm}}.$

Answer : $8\sqrt{2}(\ln 3)^2$

13. The value of $\lim_{x \rightarrow 0} \frac{(140)^x - (35)^x - (28)^x - (20)^x + 7^x + 5^x + 4^x - 1}{x \sin^2 x}$
= $2 \log 2 \log \log 7$. Then $k = \text{-----}$

Answer : 5

14. $\lim_{x \rightarrow 0} \frac{\cos^2(1 - \cos^2(1 - \cos^2(1 - \cos^2(1 - \cos^2(x))))))}{\sin\left\{\pi\left(\frac{\sqrt{x+4}-2}{x}\right)\right\}}$ is equal to

1) 4π

2) $\frac{4}{\pi}$

3) $\frac{2}{\pi}$

4) 2π

Key : 2

14. $\lim_{x \rightarrow 0} \frac{\cos^2(1 - \cos^2(1 - \cos^2(1 - \cos^2(x))))}{\sin\left\{\pi\left(\frac{\sqrt{x+4}-2}{x}\right)\right\}}$ is equal to

1) 4π

2) $\frac{4}{\pi}$

3) $\frac{2}{\pi}$

4) 2π

Key : 2

15. $\lim_{n \rightarrow 0} \frac{\sin^3 x - 3x \sin^2 x + 3x^2 \sin x - x^3}{(\sin x)^9} = \underline{\hspace{2cm}}.$

1) $-\frac{1}{216}$

2) $\frac{1}{113}$

3) $\frac{2}{211}$

4) $\frac{-1}{213}$

Key -1

$$16) \lim_{x \rightarrow 0} \frac{(\sin x - x)^2 + 1 - \cos x^3}{x^5 \sin x + (1 - \cos x)^2 (2x^2 - \sin x^2)} = \underline{\hspace{2cm}}.$$

Answer: $\frac{19}{45}$

$$17) \lim_{x \rightarrow 0} \frac{1 - \cos(\sqrt{x} - \sin x)}{\sqrt{1 + x^3} - 1} = \underline{\hspace{2cm}}.$$

Answer : 1/6

L. H . RULE

$$1. \lim_{x \rightarrow 0} \frac{x \cos x - \ln(1+x)}{x^2} =$$

Answer : $\frac{1}{2}$

$$2. \lim_{x \rightarrow \frac{3\pi}{4}} \frac{1 + \sqrt[3]{\tan x}}{1 - 2\cos^2 x} =$$

Answer : $\frac{1}{3}$

$$3. \lim_{\theta \rightarrow \frac{\pi}{4}} \frac{\sqrt{2} - \cos\theta - \sin\theta}{(40 - \pi)^2} =$$

Answer : $\frac{1}{16\sqrt{2}}$

$$4. \lim_{x \rightarrow 0} \frac{1 + \sin x - \cos x + \ln(1 - x)}{x \tan^2 x} =$$

Answer : $-\frac{1}{2}$

Model : 2

$\frac{\infty}{\infty}$ form

1. $\lim_{x \rightarrow 0} \frac{\ln \sin 2x}{\ln \sin x} =$

Answer : 1

$$2. \lim_{x \rightarrow 1} \sec \frac{\pi}{2^x} \cdot \ln x =$$

$$\text{Answer : } \frac{2}{\pi \ln 2}$$

Model : 3

$\infty - \infty$ form

Such problems are simplified (generally rationalized) first, thereafter they generally acquire $\left(\frac{\infty}{\infty}\right)$ form.

1. Evaluate $\lim_{x \rightarrow \infty} (x - \sqrt{x^2 + x})$.

Answer : $-\frac{1}{2}$

2. Evaluate $\lim_{x \rightarrow \infty} (\sqrt{x^2 + x + 1} - \sqrt{x^2 + 1})$.

Answer : $-\frac{1}{2}$

3. $\lim_{x \rightarrow 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right) =$

Answer : 0

4) $\lim_{x \rightarrow \infty} \left(\frac{x^3}{3x^2 - 4} - \frac{x^2}{3x + 2} \right)$

Answer : $\frac{2}{9}$

5) $\lim_{x \rightarrow 0} \left(\frac{1}{x^2} - \frac{1}{\sin^2 x} \right)$

Answer : $\frac{-1}{3}$

Model : 4

$$\lim_{x \rightarrow \infty} a^x = \begin{cases} \infty, & \text{if } a > 1 \\ 1, & \text{if } a = 1 \\ 0, & \text{if } 0 \leq a < 1 \end{cases}$$

1. $\lim_{x \rightarrow \infty} (2^x + 3^x + 5^{-x} + 7^x)^{\frac{1}{x}}$ is equal to _____.

1) 7

2) 3

3) 5

4) 8

key : 1

2. $\lim_{x \rightarrow 0} \left(1^{\frac{1}{\sin^2 x}} + 2^{\frac{1}{\sin^2 x}} + \dots + n^{\frac{1}{\sin^2 x}} \right)^{\sin^2 x}$ is equal to _____.

1) n

2) ∞

3) 0

4) $\frac{n(n+1)}{2}x$

key : 1

3. The value of $\lim_{x \rightarrow \pi} \frac{2^{\cot x} + 3^{\cot x} - 5^{1+\cot x} + 2}{(2^{\cot x})^2 + (9^{\cot x})^{\frac{1}{2}} - 5^{\cot x} + 1}$ is :

1) 5

2) 2

3) non – existent

4) -2

key : 3

An important result :

If m, n are positive integers and $a_0, b_0 \neq 0$ and non – zero real numbers, then

$$\lim_{x \rightarrow \infty} \frac{a_0 x^m + a_1 x^{m-1} + \dots + a_{m-1} x + a_m}{b_0 x^n + b_1 x^{n-1} + \dots + b_{n-1} x + b_n} = \begin{cases} 0, & m < n \\ \frac{a_0}{b_0}, & m = n \\ \infty, & m > n \text{ when } a_0 b_0 > 0 \\ -\infty, & m > n \text{ when } a_0 b_0 < 0 \end{cases}$$

1. $\lim_{x \rightarrow -\infty} \frac{x^2 \sin\left(\frac{1}{x}\right)}{\sqrt{9x^2 + x + 1}} = \underline{\hspace{2cm}}.$

Answer : -1/3

2. $\lim_{x \rightarrow -\infty} \frac{x^5 \tan\left(\frac{1}{\pi x^2}\right) + 3|x|^2 + 7}{|x|^3 + 7|x| + 8} = \underline{\hspace{2cm}}.$

key : $-\frac{1}{\pi}$

2. find the following limits :

$$i) \lim_{x \rightarrow \infty} \frac{x^3 - 4x^2 + 5}{x(x-1)(x-2)} =$$

Answer : 1

$$ii) \lim_{x \rightarrow \infty} \frac{x^2 - 4x + 5}{x(x-1)(x-2)} =$$

Answer : 0

$$iii) \lim_{x \rightarrow \infty} \frac{x^3 - 4x^2 + 5}{(x-1)(x-2)} =$$

Answer : ∞

$$iv) \lim_{x \rightarrow +\infty} \frac{\sqrt{x} + \sqrt[3]{x} - \sqrt[5]{x}}{\sqrt{4x-1} - \sqrt[4]{2+3x}} =$$

Answer : $\frac{1}{2}$

$$v) \lim_{x \rightarrow +\infty} (\sqrt{3x^2 - 1} + 2x) =$$

Answer : ∞

vi) $\lim_{x \rightarrow -\infty} (\sqrt{3x^2 - 1} - 2x) =$

Answer : ∞

vii) $\lim_{x \rightarrow +\infty} (\sqrt{4x^2 + 9} - 2x) =$

Answer : 0

viii) $\lim_{x \rightarrow \infty} x (\sqrt{4x^2 + 9} - 2x) =$

Answer : limit does not exist

ix) $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 1}}{x - 1} =$

Answer : limit does not exist

x) $\lim_{x \rightarrow \infty} \left(\frac{2x^2}{2x - 3} - \frac{3x^2}{3x - 4} \right) =$

Answer : $\frac{1}{6}$

3. $\lim_{x \rightarrow \infty} \frac{\sum_{k=1}^{1000} (x+k)^m}{x^m + 10^{1000}}$, ($m > 101$) is equal to :

1) 10^3

2) 10^{100}

3) 10

4) 10^{1000}

key : 1

4. $\lim_{x \rightarrow \infty} \frac{\sum_{k=1}^{10} (x + r)^{2010}}{(x^{1006} + 1)(2x^{1004} + 1)}$, is equal to :

1) 10

2) 5

3) 25

4) 2

key : 2

$$10) \text{ i) } \lim_{n \rightarrow \infty} \left(\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \text{upto } n \text{ terms} \right) = \frac{1}{6}$$

$$\text{ii) } \lim_{n \rightarrow \infty} \left(\frac{1}{1.4} + \frac{1}{4.7} + \frac{1}{7.10} + \text{upto } n \text{ terms} \right) = \frac{1}{3}$$

$$\text{iii) } \lim_{n \rightarrow \infty} \left(\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{(2n-1)(2n+1)} \right) = \frac{1}{2}$$

Modal 5

1^∞ form

To evaluate exponential limits of the form 1^∞ , we use the following results.

$$\text{if } \lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0,$$

$$\text{Then } \lim_{x \rightarrow a} \{1 + f(x)\}^{1/g(x)} = e^{\lim_{x \rightarrow a} \frac{f(x)}{g(x)}}$$

$$\text{Or when } \lim_{x \rightarrow a} f(x) = 1 \text{ and } \lim_{x \rightarrow a} g(x) = \infty$$

$$\text{Then, } \lim_{x \rightarrow a} \{f(x)\}^{g(x)} = \lim_{x \rightarrow a} [1 + f(x) - 1]^{g(x)}$$

$$= \lim_{x \rightarrow a} (f(x) - 1)g(x)$$

Particular cases

$$\text{i) } \lim_{x \rightarrow 0} (1 + x)^{1/x} = e$$

$$\text{ii) } \lim_{x \rightarrow \infty} \left(1 + \frac{1}{x}\right)^x = e$$

$$\text{iii) } \lim_{x \rightarrow 0} (1 + \lambda x)^{1/x} = e^\lambda$$

$$\text{iv) } \lim_{x \rightarrow \infty} \left(1 + \frac{\lambda}{x}\right)^x = e^\lambda$$

$$\lim_{x \rightarrow \infty} \left(1 + \frac{2}{x}\right)^x = \underline{\hspace{2cm}}$$

Answer : e^2

$$\lim_{x \rightarrow 1} (\log_3 3x)^{\log_x 3} = \underline{\hspace{2cm}}$$

Answer : e^1

$$\lim_{x \rightarrow a} \left(2 - \frac{a}{x}\right)^{\tan \frac{\pi x}{2a}} = \underline{\hspace{2cm}}$$

Answer : $e^{-2/\pi}$

$$\lim_{x \rightarrow \infty} \left(\frac{x+6}{x+1} \right)^{x+4} = \underline{\hspace{2cm}}$$

Answer : e^5

$$\lim_{x \rightarrow 0} (1 + \tan^2 \sqrt{x})^{1/2x} = \underline{\hspace{2cm}}$$

Answer : $e^{1/2}$

$$\lim_{x \rightarrow 0} \left(\sin \frac{x}{m} + \cos \frac{3x}{m} \right)^{2m/x} = \underline{\hspace{2cm}}$$

1) e

2) 1

3) e^{-1}

4) e^2

Key : (4)

46. $\lim_{x \rightarrow 0} \left(\frac{1 + 5x^2}{1 + 3x^2} \right)^{\frac{1}{x^2}} = \underline{\hspace{2cm}}$

Answer : e^2

48. $\lim_{x \rightarrow 0} \left\{ \sin^2 \left(\frac{\pi}{2-ax} \right) \right\}^{\sec^2 \frac{\pi}{2-bx}} = \underline{\hspace{2cm}}$

1) $e^{-\frac{a}{b}}$

2) $e^{\frac{-a^2}{b^2}}$

3) $a^{\frac{2a}{b}}$

4) $e^{\frac{4a}{b}}$

Key : 1

50. $\lim_{x \rightarrow 1} \left(\tan \left(\frac{\pi}{4} + \log x \right) \right)^{\frac{1}{\log x}} = \underline{\hspace{2cm}}$

1) e

2) e^{-1}

3) e^2

4) e^{-2}

Key : (3)

$$11) \text{ i) } \lim_{x \rightarrow 0} \left(\frac{a^x + b^x + c^x}{3} \right)^{\frac{2}{x}} = (\mathbf{abc})^{\frac{2}{3}}$$

$$\text{ii) } \lim_{x \rightarrow 0} \left(\frac{1^x + 2^x + 3^x + \dots + n^x}{n} \right)^{\frac{a}{x}} = (\mathbf{n!})^{\frac{a}{n}}$$

$$\text{iii) } \lim_{x \rightarrow \frac{\pi}{2}} \left(\frac{1^{\cos^2 x} + 2^{\cos^2 x} + \dots + n^{\cos^2 x}}{n} \right)^{\frac{1}{\cos^2 x}} = \mathbf{n\sqrt{n!}}$$

$$\text{iv) } \lim_{x \rightarrow \infty} \left(\frac{a_1^{\frac{1}{x}} + a_2^{\frac{1}{x}} + \dots + a_n^{\frac{1}{x}}}{n} \right)^{mx} = (\mathbf{a_1, a_2 \dots a_n})^{\frac{m}{n}}$$

$$\text{v) } \lim_{x \rightarrow 0} \left(\frac{2^x + 2^{2x} + 2^{7x} + 2^{4x}}{4} \right)^{\frac{1}{x}} = \mathbf{2^{\frac{7}{2}}}$$

8) $\lim_{x \rightarrow 0^-} \left((\tan x)^{\frac{1}{\sin^2 x}} + \frac{\sin 2x + 2 \tan^{-1} 3x + 3x^2}{\ln(1 + 3x + \sin^2 x) + x e^x} \right)$ is =

1) -1

2) 2

3) + 2

4) - 2

Key -2

24) For $x > 0$ $\lim_{x \rightarrow 0} \left((\sin x)^{\frac{1}{x}} + \left(\frac{1}{x}\right)^{\sin x} \right) = \underline{\hspace{2cm}}.$

1) 1

2) -1

3) 2

4) $\frac{1}{2}$

Key - 1

23) $\lim_{x \rightarrow 0} \frac{2}{x^3} (\sin^{-1}x - \tan^{-1}x)^{\frac{2}{x^2}} = \underline{\hspace{2cm}}.$

1) e

2) $\sqrt{e}$

3) $\frac{1}{e}$

4) $\frac{1}{\sqrt{e}}$

Key - 4

JEE Main 2025

1. If $\lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{\frac{1}{x^2}} = p$, then $96 \log_e p$ is equal to_____.

Answer : 32

JEE Main 2024

6. $\lim_{x \rightarrow 0} \frac{e - (1+2x)^{\frac{1}{2x}}}{x}$ is equal to :

1) $\frac{-2}{e}$

2) $e - e^2$

3) 0

4) e

Key: 4

JEE Main 2022

13. If the value of $\lim_{x \rightarrow 0} (2 - \cos x \sqrt{\cos 2x})^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a , then a is equal to _____.

Answer : 3

15. $\lim_{n \rightarrow \infty} \left(1 + \frac{1 + \frac{1}{2} + \dots + \frac{1}{n}}{n^2}\right)^n$ is equal to:

1) $\frac{1}{2}$

2) 1

3) 0

4) $\frac{1}{e}$

Key : 2

JEE Main 2016

20. $\lim_{x \rightarrow 0} \left(1 + \frac{a}{x} - \frac{4}{x^2}\right)^{2x} = e^3$, then 'a' is equal to:

1) 2

2) $\frac{3}{2}$

3) $\frac{2}{3}$

4) $\frac{1}{2}$

Key : 2

16. $\lim_{x \rightarrow 0} \frac{x(e^{(\sqrt{1+x^2+x^4}-1)/x}-1)}{\sqrt{1+x^2+x^4}-1}$

Key: 3

1) is equal to 0

2) is equal to $\sqrt{e}$

3) is equal to 1

4) does not exist

17. $\lim_{x \rightarrow 0} (\tan(\frac{\pi}{4} + x))^{\frac{1}{x}}$ is equal to:

1) 2

2) 1

3) e

4) e^2

Key: 4

JEE Main 2020

18. $\lim_{x \rightarrow 0} \left(\frac{3x^2+2}{7x^2+2} \right)^{\frac{1}{x}}$ is equal to:

1) e

2) e^2

3) $\frac{1}{e^2}$

4) $\frac{1}{e}$

Key: 3

Modal 6 :

0^0 or ∞^0 form

$$\lim_{x \rightarrow 0} |x|^{\sin x} = \underline{\hspace{2cm}}$$

Answer : 1

$$\lim_{n \rightarrow \infty} (\pi n)^{\frac{2}{n}} = \underline{\hspace{2cm}}$$

Answer : 1

$$\lim_{n \rightarrow \infty} \left(\frac{e^n}{\pi} \right)^{1/n} = \underline{\hspace{2cm}}$$

Answer : e

$$\lim_{x \rightarrow 0} (\cos x)^x = \underline{\hspace{2cm}}$$

Answer : 1

$$\lim_{x \rightarrow 0} \frac{(1+x)^{1/x} - e}{x} = \underline{\hspace{2cm}}$$

Answer : $-\frac{e}{2}$

Modal 7 :

USING EXPANSIONS

$$\lim_{x \rightarrow 0} \frac{\sqrt{1 + \tan x} - \sqrt{1 + \sin x}}{x^3}$$

1) $\frac{1}{4}$

2) $\frac{1}{2}$

3) $\frac{1}{3}$

4) none of these

key : 1

$$\lim_{x \rightarrow 0} \frac{\tan x - \sin x}{x^3}$$

Answer : $\frac{1}{2}$

$$\lim_{x \rightarrow 0} \frac{\ln(1+x) - \sin x + \frac{x^2}{2}}{x \tan x \sin x}$$

Answer : $\frac{1}{2}$

$$\lim_{x \rightarrow 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5} =$$

1) $\frac{1}{120}$

2) $\frac{-1}{120}$

3) 0

4) $\frac{1}{2}$

Key : 1

13) $\lim_{x \rightarrow 0} \frac{\sin x^4 - x^4 \cos x^4 + x^{20}}{x^4 (e^{2x^4} - 1 - 2x^4)}$ is equal to :

1) 0

2) $1/6$

3) $1/12$

4) $1/3$

Key : $\frac{1}{6}$

1) if $\lim_{x \rightarrow 0} \frac{e^{x^3} - (1 - x^3)^{\frac{1}{3}} + \left((1 - x^2)^{\frac{1}{2}} - 1\right) \sin x}{x \sin^2 x} = p$, then the value of $6p = \underline{\hspace{1cm}}$.

Answer : 5

2) Let $\alpha, \beta \in \mathbb{R}$ be such that $\lim_{x \rightarrow 0} \frac{x^2 \sin x (\beta x)}{\alpha x - \sin x} = 1$. then $6(\alpha + \beta)$ ____.

Answer : 7

12) $\lim_{x \rightarrow 0} \frac{1 - \cos x \cos 2x \cos 3x}{\sin^2 2x}$ is equal to

1) $\frac{7}{2}$

2) $\frac{7}{4}$

3) $\frac{8}{3}$

4) $\frac{9}{4}$

Key : 2

26) $\lim_{n \rightarrow 0} \frac{(\cos x - 1)(\cos x - e^x)}{x^n}$ is finite non – zero number, ($n \in \mathbb{I}$),
then n is equal to _____.

1) 1

2) 2

3) 3

4) 4

Key : 3

25) $\lim_{n \rightarrow \infty} \left(\frac{x^3+1}{x+1} - ax^2 - bx - c \right) = 4$, then $a - b - c = \underline{\hspace{1cm}}$.

1) 2

2) 4

3) 5

4) 6

Key : 3

1. If $\lim_{x \rightarrow 1^+} \frac{(x-1)(6 + \lambda \cos(x-1)) + \mu \sin(1-x)}{(x-1)^3} = -1$, where $\lambda, \mu \in \mathbb{R}$,

then $\lambda + \mu$ is equal to

1) 20

2) 19

3) 18

4) 17

Key : 3

2. If $\lim_{x \rightarrow 0} \frac{\cos(2x) + a\cos(4x) - b}{x^4}$ is finite, then $(a + b)$ is equal to:

1) 0

2) $\frac{3}{4}$

3) -1

4) $\frac{1}{2}$

Key : 4

5. For $\alpha, \beta, \gamma \in \mathbb{R}$, if $\lim_{x \rightarrow 0} \frac{x^2 \sin \alpha x + (\gamma - 1)e^{x^2}}{\sin 2x - \beta x} = 3$, then $\beta + \gamma - \alpha$ is equal to:

1) -1

2) 4

3) 6

4) 7

JEE MAIN 2024

$\lim_{x \rightarrow 0} \frac{ax^2e^x - b \log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$ then $16(a^2 + b^2 + c^2)$ is equal to _____.

Answer : (81)

JEE MAIN 2024

15. If $\lim_{x \rightarrow 0} \frac{3 + \alpha \sin x + \beta \cos x + \log_e(1 - x)}{3 \tan^2 x} = \frac{1}{3}$, $2\alpha - \beta$ is equal to:

(1) 2

(2) 7

(3) 5

(4) 1

Key : 3

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34. If $\lim_{x \rightarrow 0} \frac{ae^x - b \cos x + ce^{-x}}{x \sin x} = 2$, then $a + b + c$ is equal to_____.

Answer : 4

16. If $\lim_{x \rightarrow 0} \frac{e^{ax} - \cos(bx) - \frac{cxe^{-cx}}{2}}{1 - \cos(2x)}$ 17, then $5a^2 + b^2$ is equal to:

1) 64

2) 68

3) 72

4) 76

Key: 2

26. $\lim_{x \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{r^2 + 3r + 3} \right) \right\}$ is equal to:

1) 1

2) 2

3) 3

4) 6

Key: 2

19. If $\lim_{x \rightarrow 0} \frac{e^{\alpha x} + \beta e^{-x} + \gamma \sin x}{x \sin^2 x} = \frac{2}{3}$, where $\alpha, \beta, \gamma \in \mathbb{R}$, then which of the following is NOT correct

1) $\alpha^2 + \beta^2 + \gamma^2 = 6$

2) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$

3) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$

4) $\alpha^2 - \beta^2 + \gamma^2 = 4$

Key : 3

11. Let $\beta = \lim_{x \rightarrow 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$ for some $\alpha \in \mathbb{R}$. Then the value of $\alpha + \beta$ is:

1) $\frac{14}{5}$

2) $\frac{3}{2}$

3) $\frac{5}{2}$

4) $\frac{7}{2}$

Model – 8 :

**L.H.S & R. H . S LIMITS
(Greatest integer function)**

1) $\lim_{x \rightarrow 0} \left[\frac{\sin x}{x} \right] = [.] \rightarrow \text{G. I. F}$

1) 0

2) 1

3) -1

4) Does not exist

key : 1

2) $\lim_{x \rightarrow 0} \left[\frac{\sin |x|}{x} \right] = [.] \rightarrow \text{G. I. F}$

1) 0

2) 1

3) -1

4) Does not exist

key : 4

3) $\lim_{x \rightarrow 0} \left[\frac{\sin |x|}{|x|} \right]$ is equal to :

1) 0

2) 1

3) -1

4) does not exist

key : 1

4) $\lim_{x \rightarrow 0} \left[\frac{\sin^{-1} x}{x} \right]$ is equal to :

Answer : 1

5) $\lim_{x \rightarrow 0} \left[\frac{\tan x}{x} \right]$ is equal to :

Answer : 1

6) $\lim_{x \rightarrow 0} \left[\frac{\tan^{-1} x}{x} \right]$ is equal to :

Answer : 0

7) $\lim_{x \rightarrow 0} \left[\frac{\sin[x - 3]}{[x - 3]} \right]$ is equal to :

1) 0

2) 1

3) -1

4) does not exist

key : 4

8) $\lim_{x \rightarrow 0} |x|^{\cos x}$ is equal to :

1) 0

2) 1

3) -1

4) does not exist

key : 2

9) $\lim_{x \rightarrow 1^-} [\sin(\sin^{-1} x)]$ is equal to :

Answer : 0

10) $\lim_{x \rightarrow \frac{\pi}{2}} [\sin^{-1}(\sin x)]$ is equal to :

Answer : 1

11) $\lim_{x \rightarrow \frac{5\pi}{4}} [\sin x + \cos x]$ is equal to :

1) 2

2) -2

3) 4

4) 3

key : 2

11) $\lim_{x \rightarrow 0} \sum_{k=1}^{15} \left[k^3 \frac{x}{\tan^{-1} x} \right]$ is equal to :

1) 14400

2) 1440

3) 120

4) 14425

key : 1

12) $\lim_{x \rightarrow 0} \left[\frac{\sin(\operatorname{sgn} x)}{(\operatorname{sgn} x)} \right] =$ [.] $\rightarrow$ G. I. F

1) 0

2) 1

3) -1

4) ∞

key : 1

13) $\lim_{x \rightarrow 0} \left[\frac{100x}{\sin x} \right] + \left[\frac{99 \sin x}{x} \right]$ is equal to : [.] $\rightarrow$ G. I. F

1) 198

2) 199

3) 197

4) 0

key : 1

14) $\lim_{x \rightarrow 0} \left[(1 - e^x) \frac{\sin x}{|x|} \right]$ is _____. [.] $\rightarrow$ G. I. F

1) 0

2) 1

3) -1

4) Does not exist

key : 3

15) $f(x) = x^{\frac{e^{[x]+|x|} - 2}{[x]+|x|}}$, then

1) $\lim_{x \rightarrow 0^+} + f(x) = 0$

2) $\lim_{x \rightarrow 0^-} - f(x) = -1$

3) $\lim_{x \rightarrow 0} + f(x) = -1$

4) $\lim_{x \rightarrow 0} f(x) = 0$

key : 1,2

17) If $\lim_{x \rightarrow 0} \left(\left[\frac{\sin^{-1} x}{x} \right] + \left[\frac{2^2 \sin^{-1} 2x}{x} \right] + \left[\frac{3^2 \sin^{-1} 3x}{x} \right] + \dots + \left[\frac{n^2 \sin^{-1} nx}{x} \right] \right) = 100,$

Then the value of n is _____.

1) 2

2) 3

3) 4

4) 5

key : 3

$$20) \lim_{n \rightarrow \infty} n^2 \sqrt{(1 - \cos \frac{1}{n}) \sqrt{(1 - \cos \frac{1}{n}) \sqrt{(1 - \cos \frac{1}{n}) \dots \dots \infty}} = \underline{\hspace{2cm}}.$$

1) $\frac{1}{2}$

2) $-\frac{1}{2}$

3) 2

4) -2

Key - 1

18) $\lim_{x \rightarrow 0} \frac{\tan^{-1}(1-\{x\}) \cos^{-1}(1-\{x\}) \sin^{-1}(1-\{x\})}{\sqrt{2\{x\}} (1-\{x\})^2}$

1) $\frac{\pi^2}{8}$

2) $\frac{\pi}{2}$

3) $\frac{\pi^2}{4}$

4) does not exist

Key : 4

21. $\lim_{x \rightarrow 0} \frac{e^{2|\sin x|} - 2|\sin x| - 1}{x^2}$

1) is equal to 1

2) does not exist

3) is equal to -1

4) is equal to 2

Key: 4

22. Let $\{x\}$ denote the fractional part of x and

$$f(x) = \frac{\cos^{-1}(1-\{x\}^2)\sin^{-1}(1-\{x\})}{\{x\}-\{x\}^3}, x \neq 0.$$

If L and R respectively denotes the left hand limit and the right hand

limit of $f(x)$ at $x = 0$, then $\frac{32}{\pi^2} (L^2 + R^2)$ is equal to

JEE Main 2023

23. $\lim_{x \rightarrow \infty} 6 \tan \left\{ \sum_{r=1}^n \tan^{-1} \left(\frac{1}{r^2 + 3r + 3} \right) \right\}$ is equal to:

1) 1

2) 2

3) 3

4) 6

Key : 2

JEE Main 2021

24. $\lim_{x \rightarrow \infty} \left(1 + \frac{1 + \frac{1}{2} + \dots + \frac{1}{n}}{n^2} \right)^n$ is equal to:

1) $\frac{1}{2}$

2) 1

3) 0

4) $\frac{1}{e}$

Key : 2