

INFINITE INTEGRATION

ALGEBRA BASED QUESTIONS

1). If $I = \int \frac{5x^8 + 7x^6}{(x^2 + 1 + 2x^7)^2} dx$ then I is equal to

1) $\frac{x^7}{2x^7 + x^2 + 1} + C$

2) $\frac{x^5}{x^2 + 1 + 2x^7} + C$

3) $\frac{-1}{2x^7 + x^2 + 1} + C$

4) $\frac{p(x)}{q(x)}, \deg p(x) = \deg q(x) = 7$

Key (1)

Sol.
$$I = \int \frac{5x^8 + 7x^6}{x^{14} \left(\frac{1}{x^5} + \frac{1}{x^7} + 2 \right)^2} dx = \int \frac{\frac{5}{x^6} + \frac{7}{x^8}}{\left(\frac{1}{x^5} + \frac{1}{x^7} + 2 \right)^2} dx$$

Put $t = \frac{1}{x^5} + \frac{1}{x^7} + 2$, so that

$$I = \int \frac{-dt}{t^2} = \frac{1}{t} + C$$

$$= \frac{x^7}{x^2 + 1 + 2x^7} + C.$$

2) $\int (x^{3m} + x^{2m} + x^m) (2x^{2m} + 3x^m + 6)^{\frac{1}{m}} dx = \underline{\hspace{2cm}}; m > 0$

1) $\frac{1}{6m} 2x^{3m} + 3x^{2m} + 6x^m \frac{1}{m} + c$

2) $\frac{1}{6m+1} 2x^{3m} + 3x^{2m} + 6x^m \frac{m+1}{m} + c$

3) $\frac{1}{6m} 2x^{3m} + 3x^{2m} + 6x^m \frac{m+1}{m} + c$

4)

$\frac{1}{6m+1} 2x^{3m} + 3x^{2m} + 6x^m \frac{1}{m} + c$

Key (1)

Sol.

$$\int x^{3m-1} + x^{2m-1} + x^{m-1} (2x^{3m} + 3x^{2m} + 6x^m)^{\frac{1}{m}} dx$$

$$= \frac{1}{6m} \int t^{\frac{1}{m}} dt \quad \text{Where } t = 2x^{3m} + 3x^{2m} + 6x^m$$

$$= \frac{1}{6m+1} (2x^{3m} + 3x^{2m} + 6x^m)^{\frac{m+1}{m}} + c$$

3). A curve $g(x) = \int x^{27} (1+x+x^2)^6 (6x^2+5x+4) dx$ is passing through (0,0) then

$$1) g(1) = \frac{3^7}{7} \quad 2) g(1) = \frac{2^7}{7} \quad 3) g(-1) = 2/7 \quad 4) g(-1) = \frac{3^7}{14}$$

Key (1)

Sol.

$$g(x) = \int x^{24} \cdot (1+x+x^2)^6 \cdot (6x^5+5x^4+4x^3) dx$$

$$= \int (x^4+x^5+x^6)^6 \cdot (6x^5+5x^4+4x^3) dx$$

$$\text{Put } t = x^6+x^5+x^4$$

$$= \int t^6 dt = \frac{t^7}{7} + C$$

$$= \frac{(x^6+x^5+x^4)^7}{7} + C$$

But it passes through (0,0) $\Rightarrow C = 0$

$$\therefore g(1) = \frac{3^7}{7}$$

4) The value of the $\int (x^2+x)(x^{-8}+2x^{-9})^{\frac{1}{10}} dx$ is

$$1) \frac{5}{11} (x^2+2x)^{\frac{11}{10}} + C$$

$$2) \frac{5}{6} (x+1)^{\frac{11}{10}} + C$$

$$3) \frac{6}{7} (x+1)^{\frac{11}{10}} + C$$

$$4) \frac{11}{5} (x^2+2x)^{\frac{10}{11}} + C$$

Key (1)

Sol.

Let $I = \int (x^2+x)(x^{-8}+2x^{-9})^{\frac{1}{10}} dx = \int (x+1)(x^2+2x)^{\frac{1}{10}} dx$. Put $x^2+2x = t$ and solve

5). If $\int x^6 + x^3 \sqrt[3]{x^3 + 2} dx = \frac{1}{6} f x^{a/b} + C, a, b, c \in \text{real constants}$ then $f(1) =$

1) 0

2) 1

3) 2

4) 3

Key (4)

Sol.

Write the integral as $\int x^5 + x^2 \sqrt[3]{x^6 + 2x^3}$ and then substitute $x^6 + 2x^3$ as t .

6). If $\int \frac{x^2 - 1}{x^4 + 3x^2 + 1 \tan^{-1}\left(\frac{x^2 + 1}{x}\right)} dx = k \log \left| \tan^{-1} \frac{x^2 + 1}{x} \right| + c$, then k is equal to

1) 1

2) 2

3) 3

4) 5

Key (1)

Sol.

Dividing the numerator and denominator by x^2 , the given integral becomes

$$\int \frac{\left(1 - \left(\frac{1}{x^2}\right)\right) dx}{\left[\left(x + \left(\frac{1}{x}\right)\right)^2 + 1\right] \tan^{-1}\left(x + \left(\frac{1}{x}\right)\right)}$$

$$\text{Let } x + \frac{1}{x} = \tan v = \int \frac{dv}{v} = \log |v| + c$$

$$= \log \left| \tan^{-1} \left(\frac{x^2 + 1}{x} \right) \right| + c. \text{ Hence } k = 1.$$

7). $\int \frac{x(x-1)}{(x^2+1)(x+1)\sqrt{x^3+x^2+x}} dx = \frac{1}{2} \log \left| \frac{\sqrt{x+\frac{1}{x}+1}-1}{\sqrt{x+\frac{1}{x}+1}+1} \right| - A + c$ Then the value of A is equal to

1) $\cos^{-1} \sqrt{1 + \frac{1}{x}}$

2) $\tan^{-1} \sqrt{x + \frac{1}{x} + 1}$

$$3) \cot^{-1} \sqrt{x + \frac{1}{x}}$$

$$4) \sin^{-1} \sqrt{x + \frac{1}{x} + 1}$$

Key (2)

Sol.

We have, $\int \frac{x(x-1)}{(x^2+1)(x+1)\sqrt{x^3+x^2+x}} dx$

$$= \int \frac{x(x^2-1)}{(x^2+1)(x+1)^2 \sqrt{x^3+x^2+x}} dx$$

$$= \int \frac{x^3 \left(1 - \frac{1}{x^2}\right)}{x^3 \left(x + \frac{1}{x}\right) \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right)^2 \sqrt{x + \frac{1}{x} + 1}} dx$$

$$= \int \frac{\left(1 - \frac{1}{x^2}\right)}{\left(x + \frac{1}{x}\right) \left(x + \frac{1}{x} + 2\right) \sqrt{x + \frac{1}{x} + 1}} dx$$

$$I = \int \frac{2t}{(t^2-1)(t^2+1)\sqrt{t^2}} dt,$$

where $x + \frac{1}{x} + 1 = t^2$

$$= \int \frac{2}{(t^2-1)(t^2+1)} dt$$

$$= \int \left(\frac{1}{t^2-1} - \frac{1}{t^2+1} \right) dt$$

$$= \int \frac{1}{t^2-1} dt - \int \frac{1}{t^2+1} dt$$

$$= \frac{1}{2} \log \left| \frac{t-1}{t+1} \right| - \tan^{-1} t + c$$

$$= \frac{1}{2} \log \left| \frac{\sqrt{x + \frac{1}{x} + 1} - 1}{\sqrt{x + \frac{1}{x} + 1} + 1} \right| - \tan^{-1} \sqrt{x + \frac{1}{x} + 1} + c$$

8). $\int \left(\frac{x-1}{x+1} \right) \frac{dx}{\sqrt{x^3+x^2+x}} = 2 \tan^{-1} \sqrt{f(x)} + C$ then find $f(x)$.

$$1) x + \frac{1}{x} + 1$$

$$2) x + \frac{1}{x} + 2$$

$$3) x - \frac{1}{x} + 1$$

$$4) x - \frac{1}{x} - 2$$

Key (1)

Sol.

$$I = \int \left(\frac{x-1}{x+1} \right) \frac{dx}{x\sqrt{x+1+\frac{1}{x}}}$$

$$\text{so, } I = \int \frac{(x-1)dx}{(x+1)x\sqrt{x+1+\frac{1}{x}}} = \int \frac{\left(1-\frac{1}{x}\right)\left(1+\frac{1}{x}\right)dx}{(x+1)\left(1+\frac{1}{x}\right)\sqrt{x+1+\frac{1}{x}}} = \int \frac{\left(1-\frac{1}{x^2}\right)dx}{\left(x+\frac{1}{x}+2\right)\sqrt{x+\frac{1}{x}+1}}$$

$$\text{Put } x+1+\frac{1}{x} = t^2$$

$$\left(1-\frac{1}{x^2}\right)dx = 2t dt$$

$$= \int \frac{2t dt}{(t^2+1)t} = 2 \tan^{-1} t + c = 2 \tan^{-1} \left(\sqrt{x+\frac{1}{x}+1} \right) + c$$

$$x+\frac{1}{x}+1$$

9). $\int \frac{(x^3-1)dx}{x^4\sqrt{6x^6-4x^3+2}}$ is equal to

1) $\frac{\sqrt{6x^6-4x^3+2}}{6x^3} + C$

2) $x^2\sqrt{x^6-4x^3+2} + C$

3) $\sqrt{6x^6-4x^3+2} + C$

4) $\frac{1}{2} \ln (6x^6-4x^3+2) + C$

Key (1)**Sol.**

$$I = \frac{\int \left(\frac{1}{x^4} - \frac{1}{x^7} \right) dx}{\sqrt{6 - \frac{4}{x^3} + \frac{2}{x^6}}}$$

$$\text{Put } 6 - \frac{4}{x^3} + \frac{2}{x^6} = t$$

$$\Rightarrow I = \frac{1}{12} \int \frac{dt}{\sqrt{t}} = \frac{\sqrt{6x^6-4x^3+2}}{6x^3} + C$$

10). $\int \frac{dx}{(1+x)\sqrt{1+x-x^2}} =$

$$1) \cos^{-1} \frac{2\sqrt{1+x-x^2}}{\sqrt{5}(x+1)} + C$$

$$2) 2 \tan^{-1} \left(\frac{\sqrt{1+x-x^2}+1}{2} \right) + C$$

$$3) \sin^{-1} \left(\frac{3x+1}{5(x+1)} \right) + C$$

$$4) \sin^{-1} \left(\frac{3x+1}{\sqrt{5}(x+1)} \right) + C$$

Key (1,4)

Sol

Put $1+x = \frac{1}{t}$. Then

$$I = \int \frac{-\frac{1}{t^2} dt}{\frac{1}{t} \sqrt{1 + \left(\frac{1-t}{t}\right) - \left(\frac{1-t}{t}\right)^2}} = \int \frac{-dt}{\sqrt{t^2 + t - t^2 - (1 - 2t + t^2)}} = -\int \frac{dt}{\sqrt{-1 + 3t - t^2}}$$

$$= -\int \frac{dt}{\sqrt{\frac{5}{4} - \left(t - \frac{3}{2}\right)^2}} = -\sin^{-1} \left(\frac{2t-3}{\sqrt{5}} \right) + c = -\sin^{-1} \left(\frac{-3x-1}{\sqrt{5}(x+1)} \right) + c$$

$$= \sin^{-1} \left(\frac{3x+1}{\sqrt{5}(x+1)} \right) + c \Rightarrow \cos^{-1} \left(\frac{2\sqrt{1+x-x^2}}{\sqrt{5}(x+1)} \right) + c$$

$$11). \int \frac{x^2-1}{(x^2+1)\sqrt{x^4+1}} dx =$$

$$1) \sec^{-1} \left(\frac{x^2+1}{\sqrt{2}x} \right) + c$$

$$2) \frac{1}{\sqrt{2}} \sec^{-1} \left(\frac{x^2+1}{\sqrt{2}x} \right) + c$$

$$3) \frac{1}{\sqrt{2}} \sec^{-1} \left(\frac{x^2+1}{\sqrt{2}} \right) + c$$

$$4) \text{None}$$

Key (2)

Sol.

$$\begin{aligned}
 \int \frac{x^2-1}{x^2\left(x+\frac{1}{x}\right)\sqrt{x^2+\frac{1}{x^2}}}dx &= \int \frac{1-\frac{1}{x^2}}{\left(x+\frac{1}{x}\right)\sqrt{\left(x+\frac{1}{x}\right)^2-2}}dx \quad \text{put } x+\frac{1}{x}=t \\
 &= \int \frac{dt}{t\sqrt{t^2-2}} \quad t=\sqrt{2}\sec\theta \\
 &= \int \frac{\sqrt{2}\sec\theta\tan\theta d\theta}{\sqrt{2}\sec\theta\sqrt{2}\tan\theta} = \frac{1}{\sqrt{2}}\theta + C
 \end{aligned}$$

12). $\int \frac{x^2+1}{x^4-x^2+1}dx =$

1) $\tan^{-1}\left(\frac{1+x^2}{x}\right) + c$

2) $\cot^{-1}\left(\frac{1+x^2}{x}\right) + c$

3) $\tan^{-1}\left(\frac{x^2-1}{x}\right) + c$

4) $\cot^{-1}\left(\frac{x^2-1}{x}\right) + c$

Key (3)

Sol. $\int \frac{\left(1+\frac{1}{x^2}\right)dx}{\left(x-\frac{1}{x}\right)^2+1}$ Put $x-\frac{1}{x}=t$

$$\int \frac{dt}{t^2+1} = \tan^{-1}t + c = \tan^{-1}\left(\frac{x^2-1}{x}\right) + c$$

13). $\int \frac{x^8+4}{x^4-2x^2+2}dx =$

1) $\frac{x^5}{5} - \frac{2x^3}{3} + 2x + C$

2) $\frac{x^5}{5} - \frac{2x^3}{3} - 2x + C$

3) $\frac{x^5}{5} + \frac{2x^3}{3} - 2x + C$

4) $\frac{x^5}{5} + \frac{2x^3}{3} + 2x + C$

Key (4)

Sol.

$$\int \frac{(x^8 + 4 + 4x^4) - 4x^4}{x^4 - 2x^2 + 2} dx = \int \frac{(x^4 + 2)^2 - (2x^2)^2}{(x^4 - 2x^2 + 2)} dx$$

$$= \int \frac{(x^4 + 2 - 2x^2)(x^4 + 2 + 2x^2)}{(x^4 - 2x^2 + 2)} dx = \frac{x^5}{5} + \frac{2x^3}{3} + 2x + C$$

14). $\int \frac{dx}{x^{20}(1+x^{20})^{\frac{1}{20}}} =$

1) $-\frac{1}{19} \left(1 + \frac{1}{x^{20}}\right)^{\frac{19}{20}} + C$

2) $\frac{1}{21} \left(1 - \frac{1}{x^{20}}\right)^{\frac{19}{20}} + C$

3) $\frac{1}{19} \left(1 - \frac{1}{x^{20}}\right)^{\frac{19}{20}} + C$

4) $-\frac{1}{21} \left(1 + \frac{1}{x^{20}}\right)^{\frac{19}{20}} + C$

Key (1)

Sol.

$$\int \frac{dx}{x^{21} \left(\frac{1}{x^{20}} + 1\right)^{\frac{1}{20}}}, \text{ Put } \frac{1}{x^{20}} + 1 = t$$

15). Let $f(x) = \frac{x}{(1+x^n)^{1/n}}$ for $n \geq 2$ and $g(x) = \underbrace{f \circ f \circ f \dots \circ f}_{n \text{ times}}(x)$. Then $\int x^{n-2} g(x) dx$ equals

1) $\frac{1}{n(n-1)} (1+nx^n)^{1-\frac{1}{n}} + K$

2) $\frac{1}{n-1} (1+nx^n)^{1-\frac{1}{n}} + K$

3) $\frac{1}{n(n-1)} (1+nx^n)^{1+\frac{1}{n}} + K$

4) $\frac{1}{n-1} (1+nx^n)^{1+\frac{1}{n}} + K$

Key (1)

Sol.

$$f \circ f(x) = \frac{f(x)}{f(x)(1+(f(x))^n)^{1/n}} = \frac{x}{(1+2x^n)^{1/n}}$$

Similarly $f \circ f \circ f(x) = \frac{x}{(1+3x^n)^{1/n}}$ and likewise $f \circ f \dots \circ f(n \text{ times})(x) = \frac{x}{(1+nx^n)^{1/n}}$

$$\int x^{n-2} g(x) dx = \int \frac{x^{n-1}}{(1+nx^n)^{1/n}} dx. \text{ Put } (1+nx^n)^{1/n} = t$$

We get the above integral as

$$\begin{aligned}\frac{1}{n} \int \frac{t^{n-1}}{t} dt &= \frac{1}{n(n-1)} t^{n-1} + K \\ &= \frac{1}{n(n-1)} (1+nx^n)^{1-\frac{1}{n}} + K\end{aligned}$$

16). $\int \frac{x^2-1}{x^3\sqrt{2x^4-2x^2+1}} dx$ is equal to

$$\begin{aligned}1) & \frac{\sqrt{2x^4-2x^2+1}}{x^2} + C & 2) & \frac{\sqrt{2x^4-2x^2+1}}{x^3} + C \\ 3) & \frac{\sqrt{2x^4-2x^2+1}}{x} + C & 4) & \frac{\sqrt{2x^4-2x^2+1}}{2x^2} + C\end{aligned}$$

Key (4)

Sol.

$$I = \int \frac{\frac{1}{x^3} - \frac{1}{x^5}}{\sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}}} dx$$

$$\text{Put } 2 - \frac{2}{x^2} + \frac{1}{x^4} = t$$

$$= \frac{1}{4} \int \frac{dt}{\sqrt{t}}$$

$$= \frac{1}{2} \sqrt{t} + c$$

$$= \frac{1}{2} \sqrt{2 - \frac{2}{x^2} + \frac{1}{x^4}} + c$$

17). $\int \frac{dx}{x\sqrt{1-x^3}} dx =$

$$1) \frac{1}{3} \log \left| \frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1} \right| + c$$

$$2) \frac{1}{3} \log \left| \frac{\sqrt{1-x^2}+1}{\sqrt{1-x^2}-1} \right| + c$$

$$3) \frac{1}{3} \log \left| \frac{1}{\sqrt{1-x^3}} \right| + c$$

$$4) \frac{1}{3} \log |1-x^3| + c$$

Key (1)

Sol. $\int \frac{dx}{x\sqrt{1-x^3}}$ Put $1-x^3 = t^2$

$$-3x^2 dx = 2tdt$$

$$= -\frac{2}{3} \int \frac{dt}{1-t^2} = \frac{1}{3} \log \left| \frac{\sqrt{1-x^3}-1}{\sqrt{1-x^3}+1} \right| + c$$

18). $\int \frac{dx}{x^2(1+x^4)^{3/4}}$ is equal to

1) $\frac{-(1+x^4)^{1/4}}{x} + c$

2) $\frac{(1+x^4)^{1/4}}{x} + c$

3) $\frac{-(1+x^4)^{3/4}}{x} + c$

4) none of these

Key (1)

Sol.

$$\int \frac{dx}{x^2(1+x^4)^{3/4}} = \int \frac{dx}{x^5 \left(1 + \frac{1}{x^4}\right)^{3/4}} \quad \text{Put } 1 + \frac{1}{x^4} = t$$

$$\therefore -\frac{1}{4} \int t^{-3/4} dt = -\frac{(1+x^4)^{1/4}}{x} + c$$

19). If $\int \frac{x+2}{\sqrt{4x-x^2}} dx$ equals to $-\sqrt{4x-x^2} + k \sin^{-1} \frac{x-2}{2} + c$ then the value of k is

1) 1

2) 2

3) 3

4) 4

Key (4)

Sol.

$$\begin{aligned} I &= \int \frac{x+2}{\sqrt{4x-x^2}} dx = \int \frac{-\frac{1}{2}(4-2x) + 4}{\sqrt{4x-x^2}} dx \\ &= -\frac{1}{2} \int \frac{(4-2x)}{\sqrt{4x-x^2}} dx + 4 \int \frac{dx}{\sqrt{4-(x-2)^2}} \quad (\text{put } 4x-x^2 = t) \\ &= -\frac{1}{2} \int \frac{dt}{\sqrt{t}} + 4 \int \frac{dx}{\sqrt{2^2-(x-2)^2}} = -\frac{1}{2} \times 2\sqrt{t} + 4 \sin^{-1} \left(\frac{x-2}{2} \right) \end{aligned}$$

20). $\int \frac{dx}{(x-3)^{4/5} (x+1)^{6/5}} =$

1) $((x-3)(x+1))^{1/5} + c$

2) $\frac{5}{4} \left(\frac{x-3}{x+1} \right)^{1/5} + c$

$$3) \left(\frac{x+1}{x-3} \right)^{1/5} + c$$

$$4) (x-3)^{6/5} (x+1)^{4/5} + c$$

Key (2)

Sol.

$$\text{Put } t = \frac{x-3}{x+1}$$

$$\Rightarrow dx = \frac{(x+1)^2 dt}{4}$$

21). If $\int \frac{x dx}{\sqrt[2012]{(1+x^2)^{1012} (2+x^2)^{3012}}} = \frac{\alpha}{\beta} (1-f(x))^{\frac{\beta}{2\alpha}} + k$ then which is true

$$1) \alpha = 503; \beta = 500, f(\sqrt{2}) = \frac{1}{\beta - \alpha}$$

$$2) \alpha = 503; \beta = 250, f(\sqrt{2}) = \frac{1}{\alpha - \beta}$$

$$3) \alpha = 503; \beta = 500, f(1) = \frac{1}{\alpha - \beta}$$

$$4) \alpha = 503; \beta = 225, f(\sqrt{3}) = \frac{1}{\alpha - \beta}$$

Key (3)

Sol.

$$\frac{2+x^2}{1+x^2} = t$$

22). The value of $\int x \cdot \frac{\ln(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$ equals

$$1) \sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) - x + C$$

$$3) \frac{x}{2} \cdot \ln^2(x + \sqrt{1+x^2}) - \frac{x}{\sqrt{1+x^2}} + C$$

$$c) \frac{x}{2} \cdot \ln^2(x + \sqrt{1+x^2}) + \frac{x}{\sqrt{1+x^2}} + C$$

$$4) \sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) + x + C$$

Key (1)

Sol.

$$\int x \cdot \frac{\ln(x + \sqrt{1+x^2})}{\sqrt{1+x^2}} dx$$

$$I = \int \ln(x + \sqrt{1+x^2}) \cdot \frac{x}{\sqrt{1+x^2}} dx$$

$$= \sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) - \int \frac{1}{\sqrt{1+x^2}} \cdot \sqrt{1+x^2} dx \quad (\text{using integration by parts})$$

$$= \sqrt{1+x^2} \ln(x + \sqrt{1+x^2}) - x + C$$

23). If $y^2 = x^2 - x + 1$ and $I_n = \int \frac{x^n}{y} dx$ and $AI_3 + BI_2 + CI_1 = x^2 y$, for all $x \in \mathbb{R}$ then the value of $A + B + C$ is

- 1) 0 2) $3/2$ 3) 2 4) $5/2$

Key (4)

Sol.

$$y^2 = x^2 - x + 1 \Rightarrow 2y \frac{dy}{dx} = 2x - 1$$

$$AI_3 + BI_2 + CI_1 = x^2 y$$

$$\Rightarrow \int \frac{Ax^3 + Bx^2 + Cx}{y} dx = x^2 y$$

$$\Rightarrow \frac{Ax^3 + Bx^2 + Cx}{y} = x^2 \frac{dy}{dx} + 2xy$$

$$\Rightarrow Ax^3 + Bx^2 + Cx = x^2 \frac{(2x-1)}{2} + 2x(x^2 - x + 1)$$

$$= 3x^3 - \frac{5x^2}{2} + 2x$$

$$\Rightarrow A=3, B=-\frac{5}{2}, C=2$$

$$\Rightarrow A + B + C = \frac{5}{2}$$

24). If $x f(x) = 3f^2(x) + 2$, then $\int \frac{2x^2 - 12xf(x) + f(x)}{(6f(x) - x)(x^2 - f(x))^2} dx$ equals

- 1) $\frac{1}{x^2 - f(x)} + c$ 2) $\frac{1}{x^2 + f(x)} + c$ 3) $\frac{1}{x - f(x)} + c$ 4) $\frac{1}{x + f(x)} + c$

Key (1)

Sol.

$$f'(x) = \frac{f(x)}{6f(x) - x}$$

$$\text{Now, } I = \int \frac{2x(x - 6f(x)) + f(x)}{(6f(x) - x)(x^2 - f(x))^2} dx$$

$$\Rightarrow I = -\int \frac{2x - f(x)}{(x^2 - f(x))^2} dx = \frac{1}{x^2 - f(x)} + C$$

25). $\int \frac{(x^3 - 1)}{(x^4 + 1)(x + 1)} dx$, is

1) $\frac{1}{4} \ln(1 + x^4) + \frac{1}{3} \ln(1 + x^3) + c$ 2) $\frac{1}{4} \ln(1 + x^4) - \frac{1}{3} \ln(1 + x^3) + c$

3) $\frac{1}{4} \ln(1 + x^4) - \ln(1 + x) + c$ 4) $\frac{1}{4} \ln(1 + x^4) + \ln(1 + x) + c$

Key (3)

Sol

$$\int \frac{x^3 - 1}{(x^4 + 1)(x + 1)} dx = \int \frac{(x^4 + x^3) - (x^4 + 1)}{(x^4 + 1)(x + 1)} dx = \int \frac{x^3}{x^4 + 1} dx - \int \frac{1}{x + 1} dx = \frac{1}{4} \ln(x^4 + 1) - \ln(x + 1) + c$$

26). $\int \frac{2xe^{x^2}(x^2 - 1)}{(x^2 + 1)^3} dx =$

1) $\frac{xe^{x^2}}{(x^2 + 1)^2} + c$ 2) $\frac{e^{x^2}}{(x^2 + 1)^2} + c$ 3) $\frac{e^{x^2}}{x(x^2 + 1)^2} + c$ 4) $\frac{e^x}{(x^2 + 1)^2} + c$

Key (2)

Sol.

$$-x^2 = t.$$

27). The evaluation of $\int \frac{px^{p+2q-1} - qx^{q-1}}{x^{2p+2q} + 2x^{p+q} + 1} dx$ is

1) $-\frac{x^p}{x^{p+q} + 1} + C$ 2) $\frac{x^q}{x^{p+q} + 1} + C$ 3) $-\frac{x^q}{x^{p+q} + 1} + C$ 4) $\frac{x^p}{x^{p+q} + 1} + C$

Key (3)

Sol.

$$\int \frac{px^{p+2q-1} - qx^{q-1}}{(x^{p+q} + 1)^2} dx = \int \frac{px^{p-1} - qx^{-q-1}}{(x^p + x^{-q})^2} dx$$

taking x^q as x^{2q} common from Denominator and take it in N^r

28). Let $S(x) = \int \frac{dx}{e^x + 8e^{-x} + 4e^{-3x}}$, $R(x) = \int \frac{dx}{e^{3x} + 8e^x + 4e^{-x}}$ and $M(x) = S(x) - 2R(x)$. If

$M(x) = \frac{1}{2} \tan^{-1}(f(x)) + c$ where c is an arbitrary constant then $f(\log_e^2) =$

- 1) $\frac{3}{2}$ 2) $\frac{1}{2}$ 3) $\frac{5}{2}$ 4) $\frac{7}{2}$

Key. (1)

Sol.

$$M(x) = \int \frac{e^x(e^{2x} - 2)}{e^{4x} + 8e^{2x} + 4} dx \quad e^x = t \Rightarrow \int \frac{(t^2 - 2) dt}{t^4 + 8t^2 + 4} = \frac{1}{2} \tan^{-1}\left(\frac{t + 2/t}{2}\right) + c$$

$$= \frac{1}{2} \tan^{-1}\left(\frac{e^x + 2e^{-x}}{2}\right) + c$$

29). $\int \frac{\sqrt[3]{x^2} + \sqrt[6]{x}}{x(1 + \sqrt[3]{x})} dx =$

- 1) $\frac{1}{2} \tan^{-1}\left(\frac{x^6 + x^{-6}}{2}\right) + C$ 2) $\frac{x^{24}}{2} - \log(1 + x^{24}) + \tan^{-1}(x^3) + C$
- 3) $\frac{3}{2} x^{12} + 6 \tan^{-1}(x^6) + C$ 4) $6 \tan^{-1}(x^6) + 3x^{12} - 6 \log_e \sqrt{1 + x^{12}} + C$

Key (4)

$$x = t^6$$

Sol

30). If $f\left(\frac{3x-4}{3x+4}\right) = x+2$, then $\int f(x) dx = \frac{2}{3}x + \frac{k}{3} \log|x-1| + c$, where k is equal to

- 1) -4 2) 4 3) -8 4) 8

Key. (3)

Sol

$$\frac{3x-4}{3x+4} = y$$

$$\frac{6x}{-8} = \frac{y+1}{y-1}$$

$$x = \frac{-4}{3} \left(\frac{y+1}{y-1} \right)$$

$$f(y) = \frac{-4}{3} \left(\frac{y+1}{y-1} \right) + 2$$

$$= \frac{2y-10}{3(y-1)}$$

$$\int f(x)dx = \frac{2(x-1)}{3(x-1)} - \frac{8}{3(x-1)}$$

$$\frac{2}{3}x - \frac{8}{3}\log(x-1) + c$$

31). If y is implicit differentiable function of x such that $y(x+y)^2 = x$

then $\int \frac{dx}{x+3y} =$

1) $\ln((x+y)^2+1)+c$

2) $\frac{1}{2}\ln((x-y)^2+1)+c$

3) $\ln((x-y)^2+1)+c$

*4) $\frac{1}{2}\ln((x+y)^2+1)+c$

Key (4)

$$y(x+y)^2 = x$$

Sol

$$\Rightarrow \frac{dy}{dx} = \frac{1-2y(x+y)}{(x+y)(x+3y)}$$

$$\Rightarrow 1 + \frac{dy}{dx} = \frac{(x+y)^2+1}{(x+y)(x+3y)}$$

$$\frac{(x+y)(dx+dy)}{(x+y)^2+1} = \frac{dx}{x+3y}$$

$$\therefore \int \frac{dx}{x+3y} = \frac{1}{2} \ln((x+y)^2+1) + c$$

32). The value of $\int \frac{dx}{\sqrt{x-x^2}}$ is equal to

1) $2\sin^{-1}\sqrt{x} + C$

2) $\sin^{-1}(2x-1) + C$

3) $C - 2\cos^{-1}(2x-1)$ 4) $\cos^{-1} 2\sqrt{x-x^2} + C$

Key. (1,2,4)

Sol $I = \int \frac{dx}{\sqrt{x-x^2}} = \int \frac{dx}{\sqrt{\frac{1}{4} - \left(x - \frac{1}{2}\right)^2}}$

$$= \sin^{-1} \frac{\left(x - \frac{1}{2}\right)}{1/2} = \sin^{-1}(2x-1) + C \quad \dots\dots(1)$$

Also $I = \int \frac{dx}{\sqrt{x}\sqrt{1-x}}$ put $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$$\Rightarrow I = \int \frac{2dt}{\sqrt{1-t^2}} = 2 \sin^{-1} \sqrt{x} + C \quad \dots\dots(2)$$

Now let $\theta = \sin^{-1}(2x-1)$

$$\Rightarrow \sin \theta = 2x-1 \Rightarrow \cos \theta = \sqrt{1-(2x-1)^2} = 2\sqrt{x-x^2}$$

$$\Rightarrow \theta = \cos^{-1} 2\sqrt{x-x^2}$$

33). If $\int \frac{(x-1)dx}{x^2\sqrt{2x^2-2x+1}} = \frac{\sqrt{f(x)}}{g(x)} + C$, then ($x > 0$ and $f(0) = 1$)

1) $f(x) = 2x^2 - 2x + 1$

2) $g(x) = x + 1$

3) $g(x) = x$

4) $f(x) = \sqrt{2x^2 - 2x}$

Key (1,3)

Sol

$$I = \int \frac{(x-1)}{x^2\sqrt{2x^2-2x+1}} = \int \frac{(x-1)}{x^3\sqrt{2-\frac{2}{x}+\frac{1}{x^2}}} \quad \text{Put } 2-\frac{2}{x}+\frac{1}{x^2} = t^2, \text{ then } \left(\frac{x-1}{x^3}\right)dx = t dt$$

$$\Rightarrow I = \int \frac{t dt}{t} = t + C = \sqrt{2-\frac{2}{x}+\frac{1}{x^2}} + C = \frac{\sqrt{2x^2-2x+1}}{x} + C$$

So, $f(x) = \sqrt{2x^2-2x+1}$ and $g(x) = x$

34). If

$$\int \frac{dx}{1+x^{1/2}-1+x^{1/3}} = A x + 1^{1/2} + B 1+x^{1/3} + C 1+x^{1/6} + 6 \log 1+x^{1/6} - 1 + K$$

then

1) $A + B = 5$

2) $A + B + C = 11$

3) $C+B=9$

4) None of these

Key. (1,2,3)**Sol**

$$\text{Put, } 1+x=t^6 \Rightarrow dx=6t^5$$

$$\int \frac{dx}{1+x^{\frac{1}{2}} 1+x^{\frac{1}{3}}} = \int \frac{6t^5}{t^3-t^2} dt$$

$$= \int \frac{6t^3}{t-1} dt$$

$$= 6 \int \left(t^2 + t + 1 + \frac{1}{t-1} \right) dt$$

$$= 6 \left(\frac{t^3}{3} + \frac{t^2}{2} + t + \frac{\ln}{t} - 1 \right) + c$$

$$= 2 \left(1+x \right)^{\frac{1}{2}} + 3 \left(1+x \right)^{\frac{1}{3}} + 6 \left(1+x \right)^{\frac{1}{6}} + 6 \ln \left(\left(1+x \right)^{\frac{1}{6}} - 1 \right) + c$$

35). If $f'(x) = \frac{1}{-x + \sqrt{x^2+1}}$ and $f(0) = -\left(\frac{1+\sqrt{2}}{2}\right)$ then $f(1)$ is equal to

1) $-\frac{1}{2} \log(\sqrt{2}-1)$

2) 1

3) $1+\sqrt{2}$

4) $\frac{1}{2} \log(1+\sqrt{2})$

Key. (1,4)

So **l** $f(x) = \int (x + \sqrt{x^2+1}) dx$

$$= \frac{x^2}{2} + \frac{x}{2} \sqrt{x^2+1} + \frac{1}{2} \log|x + \sqrt{x^2+1}| + c$$

36) $\int \frac{dx}{\sqrt[4]{(x-1)^3(x+2)^5}} = \lambda \sqrt[4]{\frac{x-1}{x+2}} + C$, then $3\lambda =$ _____

Key. (4)**Sol**

$$\int \frac{dx}{(x-1)^{3/4}(x-1+3)^{5/4}} = \int \frac{dx}{(x-1)^2 \left(1 + \frac{3}{x-1}\right)^{5/4}} \quad \text{put } 1 + \frac{3}{x-1} = t, \quad \frac{-3}{(x-1)^2} dx = dt$$

$$= \frac{-1}{3} \int \frac{dt}{t^{5/4}} = \frac{-1}{3} \times \frac{t^{-\frac{5}{4}+1}}{-\frac{5}{4}+1} = \frac{4}{3} \frac{1}{t^{1/4}} + c$$

$$= \frac{4}{3} \sqrt[4]{\frac{x-1}{x+2}} + c$$

$$\therefore \lambda = \frac{4}{3} \Rightarrow 3\lambda = 4$$

37). If $I = \int \frac{dx}{1 + \sqrt{x^2 + 2x + 2}} = \frac{A}{7} \log \left| x + 1 + \sqrt{x^2 + 2x + 2} \right| - \frac{\sqrt{x^2 + 2x + 2} - 1}{x + 1} + C$, then A is equal to

Key. (7)

Sol

$$I = \int \frac{dx}{\sqrt{(x+1)^2 + 1} + 1}$$

$$= \int \frac{\sqrt{(x+1)^2 + 1} - 1}{(x+1)^2} dx$$

put $(x+1)^2 + 1 = t^2$ so that

$$I = \int \frac{t-1}{t^2-1} \frac{t}{\sqrt{t^2-1}} dt$$

$$= \int \frac{t}{t+1} \frac{dt}{\sqrt{t^2-1}} = \int \frac{dt}{\sqrt{t^2-1}} - \int \frac{dt}{(t+1)\sqrt{t^2-1}}$$

$$\log \left| t + \sqrt{t^2 - 1} \right| - I_1$$

In I_1 put $t+1 = 1/u$ to, so that

$$I_1 = \int \frac{\left(-\frac{1}{u^2}\right) du}{\left(\frac{1}{u}\right) \sqrt{\left(\frac{1}{u}-1\right)^2 - 1}} = -\int \frac{du}{\sqrt{1-2u}}$$

$$\sqrt{1-2u} = \sqrt{\frac{t-1}{t+1}}$$

$$\text{Thus, } I = \log \left| t + \sqrt{t^2 - 1} \right| - \frac{t-1}{\sqrt{t^2 - 1}} + C$$

$$= \log \left| x+1 + \sqrt{x^2 + 2x + 2} \right| - \frac{\sqrt{x^2 + 2x + 2} - 1}{x+1} + C$$

38). If $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx = A\sqrt{1-x} + B\sin^{-1} \sqrt{x} + C\sqrt{x-x^2} + D$, then $A + B + C =$

1) -1

2) -2

3) -3

4) -4

Key. (2)

Sol $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}} dx$, put $x = \sin^2 \theta$, $dx = 2 \sin \theta \cos \theta d\theta$

$$= \int \frac{\sqrt{1-\sin \theta}}{\sqrt{1+\sin \theta}} \cdot 2 \sin \theta \cos \theta d\theta$$

$$= 2 \int (1-\sin \theta) \sin \theta d\theta$$

$$= \int 2 \sin \theta - (1-\cos 2\theta) d\theta$$

$$= -2 \cos \theta - \theta + \frac{\sin 2\theta}{2}$$

$$= -2\sqrt{1-x} - \sin^2 \sqrt{x} + \sqrt{x-x^2} + D$$

$$\therefore A+B+C = -2-1+1 = -2$$

39). The integration of $\int 2^x (g'(x) + g(x) \log 2) dx$ is equal to

1) $2^x g(x) + C$

2) $2^x \log 2 g(x) + C$

3) $\frac{2^x}{\log 2} g(x) + C$

4) $2^x + C$

Key. (1)

Sol

$$\begin{aligned}
& \int 2^x g'(x) dx + \log 2 \int 2^x g(x) dx \\
&= 2^x g(x) - \int 2^x \log 2 g(x) dx + \int \log 2 \cdot 2^x g(x) dx \\
&= 2^x g(x) + C
\end{aligned}$$

40). The integration of $\int x^2 \frac{d\left(x - \frac{1}{x}\right)}{x^4 + 1}$ is equal to

- 1) $\left(x - \frac{1}{x}\right)^2 + C$ 2) $\sin^{-1}\left(x - \frac{1}{x}\right)^2 + C$
- 3) $\frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x - \frac{1}{x}}{\sqrt{2}}\right) + C$ 4) $\frac{1}{\sqrt{2}} \sin^{-1}\left(\frac{x - \frac{1}{x}}{\sqrt{2}}\right) + C$

Key. (3)

Sol

$$\begin{aligned}
&= \int \frac{x^2}{x^4 + 1} d\left(x - \frac{1}{x}\right) = \int \frac{1}{x^2 + \frac{1}{x^2}} d\left(x - \frac{1}{x}\right) \\
&= \int \frac{1}{\left(x - \frac{1}{x}\right)^2 + 2} d\left(x - \frac{1}{x}\right) = \frac{1}{\sqrt{2}} \tan^{-1}\left(\frac{x - \frac{1}{x}}{\sqrt{2}}\right) + C
\end{aligned}$$

41). If $n \in N$ and $\int (x^{7n} + x^{2n} + x^n)(2x^{6n} + 7x^n + 14)^{\frac{1}{n}} dx$

$$= \frac{(2x^{7n} + 7x^{2n} + 14x^n)^{\frac{n+1}{n}}}{K(n+1)} + C \text{ then K is equal to}$$

- 1) 4 2) 7 3) 10 4) 14

Key. (4)

Sol

$$= \int (2x^{7n-1} + x^{2n-1} + x^{n-1})(2x^{7n} + 7x^{2n} + 14x^n)^{1/n} dx$$

$$\text{Let } t = 2x^{7n} + 7x^{2n} + 14x^n$$

$$\therefore I = \frac{1}{14n} \int t^{1/n} dt = \frac{1}{14n} \left(\frac{t^{\frac{1}{n}+1}}{\frac{1}{n}+1} \right) + C$$

$$= \frac{(2x^{7n} + 7x^{2n} + 14x^n)^{\frac{n+1}{n}}}{14(n+1)} + C$$

42). The integration of $\int \frac{x f'(x) - 2f(x)}{\sqrt{x^4 f(x)}} dx$ is equal to

1) $x^2 f(x) + C$

2) $|x| f(x) + C$

3) $\frac{2f(x)}{|x|}$

4) $\frac{2\sqrt{f(x)}}{|x|} + C$

Key. (4)

Sol $\int \frac{x f'(x) - 2f(x)}{x^2 \sqrt{f(x)}} dx = \int \frac{x^2 f'(x) - 2f(x) / x^4}{\sqrt{\frac{f(x)}{x^2}}} dx =$

Let $t = \frac{f(x)}{x^2} \Rightarrow \frac{dt}{dx} = \frac{x^2 f'(x) - 2xf(x)}{x^4}$
 $= \int \frac{1}{\sqrt{t}} dt = \frac{2\sqrt{f(x)}}{|x|} + C$

43). If $\int \frac{dx}{x(x^{4/5} + 1)} = \frac{1}{\lambda} \log\left(\frac{x^K}{x^K + 1}\right) + C$ then λK is equal

1) 0

2) 1

3) 2

4) 4

Key. (2)

Sol

$= \int \frac{1}{x^{\frac{9}{5}} \left(1 + \frac{1}{x^{4/5}}\right)} dx$ Let $1 + \frac{1}{x^{4/5}} = t$
 $I = \frac{5}{4} \int \frac{1}{t} dt = -\frac{5}{4} \log(t) + C = \frac{-5}{4} \log\left(\frac{1 + x^{4/5}}{x^{4/5}}\right) + C$
 $= \frac{5}{4} \log\left(\frac{x^{4/5}}{1 + x^{4/5}}\right) + C$

44). If $\int \frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} dx = Ax + B \log(4e^x + 5e^{-x}) + C$

1) $A = \frac{-1}{8}, B = \frac{7}{8}$

2) $A = \frac{1}{8}, B = \frac{7}{8}$

3) $A = \frac{-1}{8}, B = \frac{-7}{8}$

4) $A = \frac{1}{8}, B = \frac{-7}{8}$

Key. (1)

Sol

Differentiation both sides

$$\frac{3e^x - 5e^{-x}}{4e^x + 5e^{-x}} = A + B \frac{(4e^x - 5e^{-x})}{4e^x + 5e^{-x}}$$

On solving, $A = -1/8$, $B = 7/8$

45). If $\int \sqrt{\frac{x}{a^3 - x^3}} dx = \lambda \sin^{-1} \left(\frac{x^\mu}{a^K} \right) + C$ then which of the following is correct

- 1) $\lambda = \mu = K$ 2) $\mu \neq K \neq \lambda$ 3) $\frac{1}{\lambda} = \mu = K$ 4) $\lambda = \mu \neq K$

Key. (3)

Sol

$$I = \int \frac{x^{1/2} dx}{\sqrt{(a^{3/2})^2 - \left(x^{\frac{3}{2}}\right)^2}} \quad \text{Let } x^{3/2} = t \Rightarrow x^{1/2} dx = \frac{2}{3} dt$$

$$\therefore I = \frac{2}{3} \int \frac{dt}{\sqrt{(a^{3/2})^2 - t^2}} = \frac{2}{3} \sin^{-1} \left(\frac{t}{a^{\frac{3}{2}}} \right) + C = \frac{2}{3} \sin^{-1} \left(\frac{x^{\frac{3}{2}}}{a^{\frac{3}{2}}} \right) + C$$

46). If $x f(x) = 3(f(x))^2 + 2$ then $\int \frac{2x^2 - 12xf(x) + f(x)}{(6f(x) - x)(x^2 - f(x))^2} dx$ is equal to

- 1) $\frac{1}{x^2 - f(x)} + C$ 2) $\frac{1}{x^2 + f(x)} + C$
 3) $\frac{1}{x - f(x)} + C$ 4) $\frac{1}{x + f(x)} + C$

Key. (1)

Sol

$$f'(x) = \frac{f(x)}{6f(x) - x}$$

$$\therefore I = \int \frac{2x(x - 6f(x) + f(x))}{(6f(x) - x)(x^2 - f(x))^2} dx$$

$$= - \int \frac{2x - f'(x)}{(x^2 - f(x))^2} dx = \frac{1}{x^2 - f(x)} + C$$

47). If $\int \frac{dx}{x^2(x^n + 1)^{\frac{n-1}{n}}} = -[f(x)]^{\frac{1}{n}} + C$ then $f(x)$ is

1) $(1+x^n)$

2) $1+x^{-n}$

3) $x^n + x^{-n}$

4) None

Key. (4)**Sol**

$$I = \int \frac{dx}{x^2 (x^n + 1)^{\frac{n-1}{n}}} = \int \frac{dx}{x^2 x^{n-1} \left(1 + \frac{1}{x^n}\right)^{\frac{n-1}{n}}}$$

$$= \int \frac{dx}{x^{n+1} (1 + x^{-n})^{\frac{n-1}{n}}}$$

$$\text{Let } 1 + x^{-n} = t \Rightarrow \frac{dx}{x^{n+1}} = -\frac{dt}{n}$$

$$I = -\frac{1}{n} \int \frac{dt}{t^{\frac{n-1}{n}}} = -\frac{1}{n} \int t^{\frac{1}{n}-1} dt = -\frac{1}{n} \frac{t^{\frac{1}{n}}}{\frac{1}{n}} + C$$

$$= -t^{\frac{1}{n}} + C = -(1 + x^{-n})^{\frac{1}{n}} + C$$

$$48). \int \frac{3x^2 + 2x}{x^6 + 2x^5 + x^4 + 2x^3 + 2x^2 + 5} dx =$$

1) $\sqrt{\frac{x^3 + x^2 + 1}{2}} + c$

2) $\sqrt{x^2 + 1 + \frac{2}{x^2}} + c$

3) $\sqrt{x + \frac{1}{x} + 1} + c$

4) $\frac{1}{2} \tan^{-1} \left(\frac{x^3 + x^2 + 1}{2} \right) + c$

Key. (4)**Sol****Key**

$$\text{Sol: Req} = \int \frac{3x^2 + 2x}{(x^3 + x^2)^2 + 2(x^3 + x^2) + 5} dx$$

$$\text{Put } x^3 + x^2 = t$$

$$49). \int (x^{3m} + x^{2m} + x^m)(2x^{2m} + 3x^m + 6)^{\frac{1}{m}} dx =$$

1) $\frac{1}{6m} (2x^{3m} + 3x^{2m} + 6x^m)^{\frac{1}{m}} + c$

2) $\frac{1}{6(m+1)} (2x^{3m} + 3x^{2m} + 6x^m)^{\frac{(m+1)}{m}} + c$

$$3) \frac{1}{6m} (2x^{3m} + 3x^{2m} + 6x^m)^{(m+1)/m} + c \quad 4) \frac{1}{6(m+1)} (2x^{3m} + 3x^{2m} + 6x^m)^{1/m} + c$$

Key. (2)

Sol
$$I = \int (x^{3m} + x^{2m} + x^m) \frac{(2x^{3m} + 3x^{2m} + 6x^m)^{\frac{1}{m}}}{x} dx$$

$$= \int (x^{3m-1} + x^{2m-1} + x^{m-1}) (2x^{3m} + 3x^{2m} + 6x^m)^{\frac{1}{m}} dx$$

Put $2x^{3m} + 3x^{2m} + 6x^m = t$

$$\Rightarrow I = \frac{1}{6m} \int t^{\frac{1}{m}} dt = \frac{1}{6m} \frac{t^{1+\frac{1}{m}}}{1+\frac{1}{m}} + c = \frac{1}{6(m+1)} (2x^{3m} + 3x^{2m} + 6x^m)^{(m+1)/m} + c$$

50).
$$\int \frac{(x-1)dx}{(x+1)\sqrt{x+x^2+x^3}} =$$

1) $\tan^{-1} \sqrt{\left(x + \frac{1}{x}\right)} + c$

2) $2 \tan^{-1} \sqrt{\left(x - \frac{1}{x}\right)} + c$

3) $2 \tan^{-1} \sqrt{x + \frac{1}{x} + 1} + c$

4) $\tan^{-1} \sqrt{x + \frac{1}{x} + 1} + c$

Key (3)

Sol

$$I = \int \frac{(x^2 - 1)/dx}{(x+1)^2 \sqrt{x+x^2+x^3}}$$

$$= \int \frac{x \left(1 - \frac{1}{x^2}\right) dx}{(x+1)^2 \sqrt{\frac{1}{x} + 1 + x}}, \text{ put } t^2 = x + \frac{1}{x} + 1$$

$$= 2 \int \frac{dt}{t^2 + 1} = 2 \tan^{-1} t + c = 2 \tan^{-1} \sqrt{x + \frac{1}{x} + 1} + c.$$

51). If $\int \frac{x^2 - 1}{x^3 \sqrt{3x^4 - 2x^2 + 1}} dx = \frac{\sqrt{3x^4 - 2x^2 + 1}}{\phi(x)} + c$ then $\phi^1(1), \phi^1(2), \phi^1(3)$ are in ____

1) AP

2) GP

3) HP

4) AGP

Key (1)

Sol $G.I = \int \frac{\frac{1}{x^3} - \frac{1}{x^5}}{\sqrt{3 - \frac{2}{x^2} + \frac{1}{x^4}}} dx = \frac{1}{4} \int \frac{dt}{\sqrt{t}} = \frac{1}{2} \sqrt{t} + c = \frac{\sqrt{3x^4 - 2x^2 + 1}}{2x^2} + c$

$\phi(x) = 2x^2 \Rightarrow \phi'(x) = 4x \Rightarrow \phi'(1), \phi'(2), \phi'(3)$ are in AP

52). If $\int \ln(\sqrt{1-x} + \sqrt{1+x}) dx = x \ln(\sqrt{1-x} + \sqrt{1+x}) + \frac{1}{2}(f(x) - x) + c$ then the value of $f'(0)$ equals

- 1) 0 2) 1 3) 2 4) $\frac{1}{2}$

Key (2)

Using by parts, $f(x) = \sin^{-1} x \Rightarrow f'(0) = 1$

Sol

53). If $\int \frac{(1 + \ln x)^2}{1 + \ln x^{x+1} + (\ln(x^{\sqrt{x}}))^2} dx = \text{_____}$

- 1) $\ln(x + \ln x) + c$ 2) $\ln(1 + x \ln x) + c$ 3) $\ln(x - \ln x) + c$ 4) $\ln(1 - x \ln x) + c$

Key (2)

Sol $G.I = \int \frac{1 + \ln x}{1 + x \ln x} dx = \ln(1 + x \ln x) + c$

Trigonometry based questions

54). $\int \frac{(1 + \sqrt{\tan x})(1 + \tan^2 x)}{2 \tan x} dx$ equal to

- 1) $\log \tan^2 x + \sqrt{\tan x} + c$ 2) $\log \tan^2 x + \frac{1}{2\sqrt{\tan x}} + c$
 3) $\log |\tan x| + 2\sqrt{\tan x} + c$ 3) $\log |\tan| + \sqrt{\tan x} + c$

Key (1)

Sol.

$$\int \frac{1}{2 \sin x \cos x} dx + \frac{1}{2} \int \frac{\sqrt{\tan x}}{\sin x \cos x} dx$$

$$= \frac{1}{2} \int \frac{\sin^2 x + \cos^2 x}{\sin x \cos x} dx + \frac{1}{2} \int \frac{\sec^2 x}{\sqrt{\tan x}} dx = \log(\tan^2 x) + \sqrt{\tan x} + c$$

55). $\int \frac{(2 + \sec x) \sec x}{(1 + 2 \sec x)^2} dx =$

1) $\frac{1}{2 \operatorname{cosec} x + \cot x} + C$

2) $2 \operatorname{cosec} x + \cot x + C$

3) $\frac{1}{2 \operatorname{cosec} x - \cot x} + C$

4) $2 \operatorname{cosec} x - \cot x + C$

Key (1)**Sol.**

$$I = \int \frac{(2 \cos x + 1)}{(2 + \cos x)^2} dx = \int \frac{(2 + \cos x) \cos x + \sin^2 x}{(2 + \cos x)^2} dx$$

$$= \int \frac{\cos x}{2 + \cos x} dx - \int \frac{-\sin^2 x}{(2 + \cos x)^2} dx = \frac{\sin x}{2 + \cos x} + c$$

56). If $\int (2 - 3 \sin^2 x) \sqrt{\sec x} dx = 2f(x) \sqrt{g(x)} + c$ and $f(x)$ is non constant function then

1) $f^2(x) + g^2(x) = 1$

2) $f^2(x) - g^2(x) = 1$

3) $f(x)g(x) = 1$

4) $f(x) = g(x)$

Key (1)**Key****(1)****Sol.**

$$\int \frac{2 - 3 \sin^2 x}{\sqrt{\cos x}} dx = \int \frac{2 \cos^2 x - \sin^2 x}{\sqrt{\cos x}} dx = 2 \int (\cos x) \sqrt{\cos x} dx - \int \frac{\sin^2 x}{\sqrt{\cos x}} dx = 2 \sin x \sqrt{\cos x} + c$$

$$\Rightarrow f(x) = \sin x, g(x) = \cos x$$

57). The value of $2 \int \sin x \cdot \operatorname{cosec} 4x dx$ is equal to

$$1) \frac{1}{2\sqrt{2}} \ln \left| \frac{1+\sqrt{2} \sin x}{1-\sqrt{2} \sin x} \right| - \frac{1}{4} \ln \left| \frac{1+\sin x}{1-\sin x} \right| + C$$

$$2) \frac{1}{2\sqrt{2}} \ln \left| \frac{1+\sqrt{2} \sin x}{1-\sqrt{2} \sin x} \right| + \frac{1}{4} \ln \left| \frac{1+\sin x}{1-\sin x} \right| + C$$

$$3) \frac{1}{2\sqrt{2}} \ln \left| \frac{1-\sqrt{2} \sin x}{1+\sqrt{2} \sin x} \right| - \frac{1}{4} \ln \left| \frac{1+\sin x}{1-\sin x} \right| + C$$

$$4) \frac{1}{2\sqrt{2}} \ln \left| \frac{1-\sqrt{2} \sin x}{1+\sqrt{2} \sin x} \right| - \frac{1}{4} \ln \left| \frac{1+\cos x}{1-\cos x} \right| + C$$

Key (1)

Sol.

$$\begin{aligned} I &= 2 \int \sin x \cdot \operatorname{cosec} 4x \, dx = 2 \int \frac{\sin x \, dx}{4 \sin x \cos x \cos 2x} \\ &= \frac{1}{2} \int \frac{\cos x \, dx}{(1-\sin^2 x)(1-2\sin^2 x)} \quad \text{Put } \sin x = t, \text{ then } \cos x \, dx = dt \\ \Rightarrow I &= \frac{1}{2} \int \frac{dt}{(1-t^2)(1-2t^2)} \\ &= \frac{1}{2} \left[-\int \frac{1}{1-t^2} \, dt + \int \frac{2}{1-2t^2} \, dt \right] \quad [\text{By partial fraction}] \\ &= \frac{1}{2} \left[-\frac{1}{2} \ln \left| \frac{1+t}{1-t} \right| + \frac{2}{2\sqrt{2}} \ln \left| \frac{1+\sqrt{2}t}{1-\sqrt{2}t} \right| \right] \\ &= \frac{1}{2\sqrt{2}} \ln \left| \left(\frac{1+\sqrt{2} \sin x}{1-\sqrt{2} \sin x} \right) \right| - \frac{1}{4} \ln \left| \left(\frac{1+\sin x}{1-\sin x} \right) \right| + C \end{aligned}$$

58). If $y = 2 \cot^{-1} \left(\frac{x}{2} \right) + x$, then $\int \frac{x^2 \, dx}{(x^2-4) \sin x + 4x \cos x} =$

$$1) \log |\operatorname{cosec} 2y + \cot y| + c$$

$$2) \log |\operatorname{cosec} y + \cot 2y| + c$$

$$3) \log |\operatorname{cosec} y + \cot y| + c$$

$$4) \log |\operatorname{cosec} y - \cot y| + c$$

Key (4)

Sol.

$$y = 2 \tan^{-1} \left(\frac{2}{x} \right) + x = \tan^{-1} \left(\frac{4x}{x^2 - 4} \right) + x \Rightarrow \tan(y - x) = \frac{4x}{x^2 - 4}$$

$$\Rightarrow \cos(y - x) = \frac{x^2 - 4}{x^2 + 4}$$

$$\int \frac{\frac{x^2}{x^2 - 4} dx}{\sin x + \frac{4x}{x^2 + 4} \cos x} = \int \frac{\frac{x^2}{x^2 - 4} \frac{x^2 - 4}{x^2 + 4} dx}{\sin y}$$

$$\frac{dy}{dx} = \frac{2}{1 + \frac{4}{x^2}} - \frac{2}{x^2} + 1 = \frac{x^2}{x^2 + 4} = \int \frac{dy}{\sin y}$$

59). $\int \frac{dx}{\sin 2x \tan^9 x} = \underline{\hspace{2cm}}$

1) $\frac{1}{9} \tan^9 x + c$ 2) $\frac{1}{18} \tan^9 x + c$ 3) $\frac{1}{18} \cot^9 x + c$ 4) $\frac{-1}{18} \cot^9 x + c$

Key (4)

Sol. $\frac{1}{2} \int \cot^8 x \operatorname{cosec}^2 x dx = \frac{-\cot^9 x}{18} + c$

60). $\int \frac{\cos 14x - \cos 13x}{1 + 2 \cos 9x} dx = \underline{\hspace{2cm}}$

1) $\frac{\sin 6x}{6} - \frac{\sin 5x}{5} + C$

2) $-\frac{\sin 6x}{6} - \frac{\sin 5x}{5} + C$

3) $\frac{\sin 5x}{5} - \frac{\sin 4x}{4} + C$

4) $-\frac{\sin 5x}{5} - \frac{\sin 4x}{4} + C$

Key (3)

Sol

$$\begin{aligned}\int \frac{-2 \sin\left(\frac{27x}{2}\right) \sin\left(\frac{x}{2}\right)}{3 - 4 \sin^2\left(\frac{9x}{2}\right)} dx &= \int -2 \sin\left(\frac{9x}{2}\right) \sin\left(\frac{x}{2}\right) \\ &= \int \cos 5x - \cos 4x \, dx \\ &= \frac{\sin 5x}{5} - \frac{\sin 4x}{4} + C\end{aligned}$$

61). $\frac{\sin^3 x}{(\cos^4 x + 3 \cos^2 x + 1) \tan^{-1}(\sec x + \cos x)} dx =$

1) $\tan^{-1}(\sec x + \cos x) + c$

2) $\log |\tan^{-1}(\sec x + \cos x)| + c$

3) $\frac{1}{(\sec x + \cos^2 x)^2} + c$

4) $\log |\sec x + \cos x| + c$

Key (2)

Sol.

Put $\tan^{-1}(\sec x + \cos x) = f(x)$

$$f^1(x) = \frac{\sin^3 x}{\cos^4 x + 3 \cos^2 x + 1}$$

$$\therefore \int \frac{f^1(x)}{f(x)} = \log |f(x)| + c$$

62). If $\int \frac{2 \cos x - \sin x + \lambda}{\cos x + \sin x - 2} dx = A \ln |\cos x + \sin x - 2| + Bx + C$ Then the ordered triplet A, B, λ is

1) $\left(\frac{1}{2}, \frac{3}{2}, -1\right)$

2) $\left(\frac{3}{2}, \frac{1}{2}, -1\right)$

3) $\left(\frac{1}{2}, -1, -\frac{3}{2}\right)$

4) $\left(\frac{3}{2}, -1, \frac{1}{2}\right)$

Key (2)

Sol.

$$\begin{aligned} & \frac{d}{dx} (A/n |\cos x + \sin x - 2| + Bx + C) \\ &= A \frac{\cos x - \sin x}{\cos x + \sin x - 2} + B = \frac{A \cos x - A \sin x + B \cos x + B \sin x - 2B}{\cos x + \sin x - 2} \\ \therefore & \quad 2 = A + B, -1 = -A + B, \lambda = -2B \\ \therefore & \quad A = 3/2, B = 1/2, \lambda = -1 \end{aligned}$$

63). $\int \frac{\cos^4 x \, dx}{\sin^3 x (\sin^5 x + \cos^5 x)^{3/5}} = -\frac{1}{2} (1 + \cot^4 x)^B + C$

- 1) 5 2) $\frac{2}{5}$ 3) 2 4) 1

Key (3)

Sol.

$$\begin{aligned} I &= \int \frac{\cos^4 x \, dx}{\sin^3 x (\sin^5 x + \cos^5 x)^{3/5}} \\ &= \int \frac{\cos^4 x \, dx}{\sin^6 x (1 + \cot^5 x)^{3/5}} \\ &= \int \frac{\cot^4 x \operatorname{cosec}^2 x \, dx}{(1 + \cot^5 x)^{3/5}} \text{ put } 1 + \cot^5 x = t \quad 5 \cot^4 x \operatorname{cosec}^2 x \, dx = -dt \end{aligned}$$

64). $\int 2 \cos 2x \ln(\tan x) \, dx$, is equal to

- 1) $\sin 2x \ln(\tan x) - 2x + C$ 2) $\sin 2x \ln(\tan x) + 2x + C$
 3) $\sin x \ln(\tan x) - x + C$ 4) none of these

Key (1)

Sol.

$$\begin{aligned} \text{We have } I &= \int 2 \cos 2x \ln(\tan x) \, dx = \sin 2x \ln(\tan x) - \int \sin 2x \cdot \frac{\sec^2 x}{\tan x} \, dx \\ &= \sin 2x \ln(\tan x) - \int 2 \, dx = \sin 2x \ln(\tan x) - 2x + C \end{aligned}$$

65). $\int \frac{\cos x + x \sin x}{x(x + \cos x)} dx =$

1) $\log \left| \frac{x}{x + \cos x} \right| + c$

2) $-\log \left| \frac{x}{x + \cos x} \right| + c$

3) $\log \left| \frac{x + \cos x}{2x} \right| + c$

4) $-\log \left| \frac{x + \cos x}{2x} \right| + c$

Key (1)

Sol.

$$\int \frac{x + \cos x + x \sin x - x}{x(x + \cos x)} dx$$

66) $\int [1 + \tan x \tan(x + \alpha)] dx$, is equal to

1) $\tan \alpha \ln \left| \frac{\sin(x + \alpha)}{\sin x} \right| + C$

2) $\cot \alpha \ln \left| \frac{\sin(x + \alpha)}{\sin x} \right| + C$

3) $\cot \alpha \ln \left| \frac{\sin x}{\sin(x + \alpha)} \right| + C$

4) $\cot \alpha \ln \left| \frac{\cos x}{\cos(x + \alpha)} \right| + C$

Key (4)

Sol.

We have $\tan \alpha = \tan(x + \alpha - x) = \frac{\tan(x + \alpha) - \tan x}{1 + \tan x \tan(x + \alpha)}$

Then, we have

$$\int [1 + \tan x \tan(x + \alpha)] dx = \int \cot \alpha [\tan(x + \alpha) - \tan x] dx$$

$$= \cot \alpha [-\ln |\cos(x + \alpha)| + \ln |\cos x|] + C$$

$$= \cot \alpha \ln \left| \frac{\cos x}{\cos(x + \alpha)} \right| + C$$

67). Let $x^2 \neq n\pi - 1, n \in \mathbb{N}$. Then, the value of $\int x \sqrt{\frac{2 \sin(x^2 + 1) - \sin 2(x^2 + 1)}{2 \sin(x^2 + 1) + \sin 2(x^2 + 1)}} dx$ is equal to

1) $\log \left| \frac{1}{2} \sec(x^2 + 1) \right| + C$

2) $\log \left| \sec \left(\frac{x^2 + 1}{2} \right) \right| + C$

3) $\frac{1}{2} \log |\sec(x^2 + 1)| + C$

4) None of these

Key (2)

Sol.

$$\begin{aligned} \text{We have, } & \int x \sqrt{\frac{2 \sin(x^2 + 1) - \sin 2(x^2 + 1)}{2 \sin(x^2 + 1) + \sin 2(x^2 + 1)}} dx \\ &= \int x \sqrt{\frac{2 \sin(x^2 + 1) - 2 \sin(x^2 + 1) \cos(x^2 + 1)}{2 \sin(x^2 + 1) + 2 \sin(x^2 + 1) \cos(x^2 + 1)}} dx \\ &= \int x \sqrt{\frac{1 - \cos(x^2 + 1)}{1 + \cos(x^2 + 1)}} dx \\ &= \int x \tan \left(\frac{x^2 + 1}{2} \right) dx \\ &= \int \tan \left(\frac{x^2 + 1}{2} \right) d \left(\frac{x^2 + 1}{2} \right) \\ &= \log \left| \sec \left(\frac{x^2 + 1}{2} \right) \right| + C \end{aligned}$$

68). $\int \frac{dx}{\cos(2x) \cos(4x)}$ is equal to

1) $\frac{1}{2\sqrt{2}} \log \left| \frac{1 + \sqrt{2} \sin 2x}{1 - \sqrt{2} \sin 2x} \right| - \frac{1}{2} (\log |\sec 2x - \tan 2x|) + C$

2) $\frac{1}{2\sqrt{2}} \log \left| \frac{1 + \sqrt{2} \sin 2x}{1 + \sqrt{2} \sin x} \right| - \frac{1}{2} (\log |\sec 2x - \tan 2x|) + C$

$$3) \frac{1}{\sqrt{2}} \log \left| \frac{1 + \sqrt{2} \sin 2x}{1 - \sqrt{2} \sin 2x} \right| - \frac{1}{2} (\log |\sec 2x - \tan 2x|) + C$$

4) None of these

Key (1)

Sol.

$$\int \frac{\sin(4x - 2x) dx}{\sin(2x) \cos(2x) \cos(4x)}$$

$$= \int \frac{\sin(4x) dx}{\sin(2x) \cos(4x)} - \int \sec 2x dx$$

$$= 2 \int \frac{\cos 2x dx}{\cos 4x} - \frac{1}{2} (\log |\sec 2x - \tan 2x|)$$

69). $\int \frac{1 - 7 \cos^2 x}{\sin^7 x \cos^2 x} dx = \frac{f(x)}{(\sin x)^7} + C$, then f(x) is equal to

1) Sin x

2) Cos x

3) Tan x

4) Cot x

Key (3)

Sol.

$$\int \frac{1 - 7 \cos^2 x}{\sin^7 x \cos^2 x} dx = \int \left(\frac{\sec^2 x}{\sin^7 x} - \frac{7}{\sin^7 x} \right) dx$$

$$= \int \frac{\sec^2 x}{\sin^7 x} dx - \int \frac{7}{\sin^7 x} dx = I_1 + I_2$$

$$\text{Now, } I_1 = \int \frac{\sec^2 x}{\sin^7 x} dx$$

$$= \frac{\tan x}{\sin^7 x} + 7 \int \frac{\tan x \cdot \cos x}{\sin^8 x} dx$$

$$= \frac{\tan x}{\sin^7 x} - I_2$$

$$\therefore I_1 + I_2 = \frac{\tan x}{\sin^7 x} + C$$

$$\Rightarrow f(x) = \tan x$$

70). Integral of $\sqrt{1 + 2 \cot x (\cot x + \operatorname{cosec} x)}$ with respect to x is:

1) $2 \ln \cos \frac{x}{2} + c$

2) $2 \ln \sin \frac{x}{2} + c$

3) $\frac{1}{2} \ln \cos \frac{x}{2} + c$

4) $\ln \sin x - \ln(\operatorname{cosec} x - \cot x) + c$

Key (2)

Sol.

$$\begin{aligned} I &= \int \sqrt{1 + 2 \operatorname{cosec} x \cot x + 2 \cot^2 x} \\ &= \int \sqrt{\operatorname{cosec}^2 x + 2 \operatorname{cosec} x \cot x + \cot^2 x} dx \\ &= \int (\operatorname{cosec} x + \cot x) dx \end{aligned}$$

71). $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx$ equals

1) $\sin x - 6 \tan^{-1}(\sin x) + c$

2) $\sin x - 2 \sin^{-1} x + c$

3) $\sin x - 2(\sin x)^{-1} - 6 \tan^{-1}(\sin x) + c$

4) $\sin x - 2(\sin x)^{-1} + 5 \tan^{-1}(\sin x) + c$

Key (3)

Sol.

$$\begin{aligned} \sin x = t ; \quad I &= \int \frac{(1-t^2)(2-t^2)}{t^2(1+t^2)} dt = \int \frac{(y-1)(y-2)}{y(1+y)} dy = 1 + \frac{2(1-2y)}{y(y+1)} ; \quad y = t^2 \\ &= 1 + 6 \left[\frac{1}{3y} - \frac{1}{y+1} \right] = \left(1 + \frac{2}{t^2} - \frac{6}{1+t^2} \right) dt \end{aligned}$$

72). If $I = \int \frac{dx}{\sin\left(x - \frac{\pi}{3}\right) \cos x}$, then I equals

- 1) $2 \log \left| \sin x + \sin\left(x - \frac{\pi}{3}\right) \right| + C$ 2) $2 \log \left| \sin\left(x - \frac{\pi}{3}\right) \sec x \right| + C$
 3) $2 \log \left| \sin x - \sin\left(x - \frac{\pi}{3}\right) \right| + C$ 4) None of these

Key (2)

Sol.

$$I = \frac{1}{\cos\left(\frac{\pi}{3}\right)} \int \frac{\cos\left(x - \left(x - \frac{\pi}{3}\right)\right)}{\sin\left(x - \frac{\pi}{3}\right) \cos x} dx$$

73). $\int \frac{\sin\left(\frac{\pi}{4} - x\right) dx}{2 + \sin 2x} = A \tan^{-1}(f(x)) + B$, where A, B are constants then the range of $Af(x)$ is

- 1) $[-1, 1]$ 2) $[-\sqrt{2}, \sqrt{2}]$ 3) $[0, 1]$ 4) $[-1, 0]$

Key(1)

Sol.

$$\frac{1}{\sqrt{2}} \int \frac{d(\sin x + \cos x)}{(\sin x + \cos x)^2 + 1}$$

74). $\int \left(1 + 2 \tan x (\tan x + \sec x)\right)^{1/2} dx$

- 1) $\log \sec x (\sec x - \tan x) + C$ 2) $\log \csc x (\sec x + \tan x) + C$
 3) $\log \sec x (\sec x + \tan x) + C$ 4) $\log (\sec x + \tan x) + C$

Key (3)

Sol.

$$\int \left(1 + 2 \tan^2 x + 2 \tan x \sec x\right)^{1/2} dx = \int \left(\sec^2 x + \tan^2 x + 2 \sec x \tan x\right)^{1/2} dx$$

$$= \log(\sec x + \tan x) \sec x + c$$

75). $\int \frac{1}{1 + \cos x + \sin x} dx = \log_e |f(x)| + c$ then $f(x) =$

1) $2 + \tan \frac{x}{2}$

2) $1 - \tan \frac{x}{2}$

3) $2 - \tan \frac{x}{2}$

4) $1 + \tan \frac{x}{2}$

Key (4)

Sol.

Put $\tan \frac{x}{2} = t$

76). If $\int \frac{(\cos x - \sin x) dx}{2 - \sin x \cos x - \sin^2 \cos^2 x} = A \tan^{-1}(f(x)) + B \log \left| \frac{1+f(x)}{1-f(x)} \right| + C$, then

1) $A = 3B$

2) $f(x) = \frac{\sin x + \cos x}{\sqrt{3}} + 1$

3) $9(A^2 - B^2) = 1$

4) $A = -2B$

Key (3)

Sol.

Put $\sin x + \cos x = t$ and evaluate

77). $\int \frac{(\cos x + x \sin x)}{x(\cos x - x)} dx = \dots$

1) $\ln \left(\left| \frac{cx}{\cos x - x} \right| \right)$

2) $\ln \left(\left| \frac{\cos x - x}{cx} \right| \right)$

3) $\ln(|\cos x + x|) + c$

4) $\ln x + \ln(|\cos x + x|) - 2 \ln(|\cos x|) + c$

Key (1)

Sol.

$$I = \int \frac{\cos x + x \sin x}{x(\cos x - x)} dx = \int \frac{\cos x + x \sin x - x + x}{x(\cos x - x)} dx$$

$$= \int \frac{1}{x} dx - \int \frac{-(\sin x + 1)}{\cos x - x} dx = \log x - \log(\cos x - x) + \log c$$

$$= \log cx - \log(\cos x - x)$$

$$= \log \left| \frac{Cx}{\cos x - x} \right|$$

78). $\int \frac{(9 \tan^5 x + 7 \tan^3 x)(1 + \tan^2 x) dx}{(\tan^3 x + \tan x)^{\frac{1}{2}}} = \dots$

- 1) $2(\tan x)^{\frac{5}{2}} + c$ 2) $2(\tan x)^{\frac{7}{2}} + c$ 3) $2(\tan x)^{\frac{7}{2}} |\sec x| + c$ 4) None

Key (3)

Sol.

$$t = \tan x, dt = \sec^2 x \, dx$$

$$\int \frac{(9t^5 + 7t^3) dt}{(t^3 + t)^{1/2}}$$

$$\int \frac{9t^8 + 7t^6}{(t^9 + t^7)^{1/2}} dt$$

$$y = t^9 + t^7$$

$$\Rightarrow \int \frac{dy}{\sqrt{y}} \Rightarrow 2\sqrt{y} + c$$

79). The values of a and b which satisfy

$$\int \frac{2 \sin 2x - \cos x}{4 - \cos^2 x - 4 \sin x} dx = a \log |\sin x - 1| + b \log |\sin x - 3| + c, \text{ are respectively}$$

- 1) $-\frac{3}{2}, \frac{11}{2}$ 2) $\frac{3}{2}, \frac{11}{2}$ 3) $\frac{3}{2}, -\frac{11}{2}$ 4)

Key(1)

Sol. $-\frac{3}{2}, -\frac{11}{2}$

$$\int \frac{4 \sin x - 1 \cos x}{3 + \sin^2 x - 4 \sin x} dx$$

Put $\sin x = t$

$$= \int \frac{4t-1}{t^2-4t+3} dt$$

$$= \int \frac{4t-1}{t-1} \frac{1}{t-3} dt$$

$$= \int \left(\frac{a}{t-1} + \frac{b}{t-3} \right) dt$$

$$\text{Then } a = \left[\frac{4t-1}{t-3} \right]_{t=1} = -\frac{3}{2} \text{ and } b \left[\frac{4t-1}{t-1} \right]_{t=3} = \frac{11}{2}$$

80). The value of $\int \frac{x \cot x \operatorname{cosec} x}{(x + \cot x)^2} dx$ is equal to

1) $\frac{-1}{(x \tan x + 1)} + C$

2) $\frac{-1}{(x \sin x + \cos x)} + C$

3) $\frac{-1}{(x + \cot x)} + C$

4) $\frac{1}{(x \sin x + \cos x)} + C$

Key (2)

Sol.

$$\begin{aligned} \text{Let } I &= \int \frac{x \cot x \operatorname{cosec} x}{(x + \cot x)^2} dx = \int \frac{x \cot x \operatorname{cosec} x}{\cot^2 x (x \tan x + 1)^2} dx = \int \frac{x \cos x}{(x \sin x + \cos x)^2} dx \\ &= \frac{-1}{(x \sin x + \cos x)} + C. \end{aligned}$$

81). For $x \in \left(0, \frac{\pi}{2}\right)$, If $\left(\int \frac{\tan x \sqrt{\sec x} (1 + \cos^2 x)}{\sqrt{\cos x + \sin^2 x}} dx = 2\sqrt{f(x)+1} + c \right)$ and $f\left(\frac{\pi}{3}\right) = \frac{3}{2}$ then

$$\left[f\left(\frac{\pi}{4}\right) \right] = \text{(where } [.] \text{ denotes G.I.F)}$$

1) $1/2$

2) 1

3) 2

4) 0

Key (4)

Sol.

$$\int \frac{\tan x \sqrt{\sec x} (1 + \cos^2 x)}{\sqrt{\cos x + \sin^2 x}} dx = \int \frac{\tan x \sqrt{\sec x} (1 + \cos^2 x)}{\sqrt{1 - \cos^2 x + \cos x}} dx$$

$$\begin{aligned}
&= \int \frac{\sec x \tan x (1 + \cos^2 x)}{\sqrt{\sec x - \cos x + 1}} dx \\
&= \int \frac{\sec x \tan x + \sin x}{\sqrt{\sec x - \cos x + 1}} dx \\
&= 2\sqrt{\sec x - \cos x + 1} + c \\
\Rightarrow f(x) &= \sec x - \cos x \therefore \left[f\left(\frac{\pi}{4}\right) \right] = 0
\end{aligned}$$

82). If $f'(x) = 3x^2 \sin \frac{1}{x} - x \cos \frac{1}{x}$, $x \neq 0$, $f(0) = 0$ then the value of $f\left(\frac{1}{\pi}\right)$ is

1) 0

2) 1

3) 2

4) 3

Key (1)

Sol.

$$f'(x) = 3x^2 \sin \frac{1}{x} - x \cos \frac{1}{x}$$

$$\Rightarrow f(x) = \int (3x^2 \sin \frac{1}{x} - x \cos \frac{1}{x}) dx$$

$$f(x) = \sin \frac{1}{x} \cdot x^3 - \int \cos \frac{1}{x} \left(-\frac{1}{x^2} x^3 \right) dx - \int x \cos \frac{1}{x} dx$$

$$f(x) = x^3 \sin \frac{1}{x} + c$$

$$f(0) = 0 + c = 0 \Rightarrow c = 0$$

$$\therefore f(1/\pi) = \frac{1}{\pi^3} \cdot \sin \pi + 0 = 0.$$

83). If $\int \frac{dx}{\sin^6 x + \cos^6 x} = \cot^{-1}(f(x)) + c$ then $\left[f\left(\frac{\pi}{6}\right) \right] =$ (Where $[.]$ denotes G.I.F).

1) $\frac{1}{2}$

2) 2

3) 0

4) 1

Key (4)

Sol.

$$\int \frac{dx}{\sin^6 x + \cos^6 x} = \int \frac{dx}{1 - 3\sin^2 x \cos^2 x} = 4 \int \frac{\operatorname{cosec}^2 2x}{1 + 4 \cot^2 2x} dx = \cot^{-1}(2 \cot 2x) + c$$

$$\therefore f(x) = 2 \cot 2x \Rightarrow \left[f\left(\frac{\pi}{6}\right) \right] = 1$$

85). If $\int \tan^5 x dx = A \tan^4 x + B \tan^2 x + g(x) + c$ then

1) $A = \frac{1}{4}, B = -\frac{1}{2}$

2) $g(x) = -\log_e |\sec x|$

3) $g(x) = x$

4) $A = \frac{-1}{4}, B = \frac{1}{3}$

Key (1)

Sol.

$$\int \tan^2 x (\tan^3 x) dx = \int \tan^3 x \sec^2 x dx - \int \tan^2 x (\tan x) dx$$

$$= \frac{\tan^4 x}{4} - \int \sec^2 x \cdot \tan x dx + \int \tan x dx$$

$$= \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log_e |\sec x| + c$$

86). The value of $\int \frac{x^2 + \cos^2 x}{1 + x^2} \operatorname{cosec}^2 x dx$ is equal to

1) $\cot x - \cot^{-1} x + C$

2) $C - \cot x + \cot^{-1} x$

3) $-\tan^{-1} x - \frac{\operatorname{cosec} x}{\sec x} + C$

4) $-e^{\ln \tan^{-1} x} - \cot x + C$

Key (2,3,4)

Sol.

Add 1 & sub tract 1 in numerator and then apply by parts

87). The value of $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx$ is equal to

1) $\cot^{-1}(\cot^2 x) + C$

2) $-\cot^{-1}(\tan^2 x) + C$

$$3) \tan^{-1}(\tan^2 x) + C$$

$$4) -\tan^{-1}(\cos 2x) + C$$

Key (1,2,3,4)

Sol.

$$I = \int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx = \int \frac{2 \tan x \sec^2 x}{\tan^4 x + 1} dx$$

$$\text{Put } \tan^2 x = t \Rightarrow 2 \tan x \sec^2 x dx = dt$$

$$= \int \frac{dt}{t^2 + 1} = \tan^{-1}(t) + C = \frac{\pi}{2} - \cot^{-1}(\tan^2 x) + C$$

$$= -\cot^{-1}(\tan^2 x) + C_1 = -\cot^{-1}\left(\frac{1}{\cot^2 x}\right) + C_1 = \cot^{-1}(\cot^2 x) + C_1$$

$$\text{Also } \cos 2x = \frac{1 - \tan^2 x}{1 + \tan^2 x} \Rightarrow \left(\frac{1 - \cos 2x}{1 + \cos 2x}\right) = \tan^2 x, \text{ using these values in given integral}$$

$$I = \int \frac{\sin 2x}{(\cos^2 x - \sin^2 x)^2 + 2 \sin^2 x \cos^2 x} dx = \int \frac{2 \sin 2x}{(\cos 2x)^2 + 1} dx$$

$$\text{Put } \cos 2x = t \Rightarrow -2 \sin 2x dx = dt$$

$$\Rightarrow I = \int \frac{-dt}{t^2 + 1} = -\tan^{-1} t + C_2 = -\tan^{-1}(\cos 2x) + C_2$$

88). If $\int \frac{3 \cot 3x - \cot x}{\tan x - 3 \tan 3x} dx = p f(x) + q g(x) + C$, where, 'C' is a constant of integration and $f(2014) = 2014$, then

$$1) p=1; q=\frac{1}{\sqrt{3}}; f(x)=x; g(x)=\ln \left| \frac{\sqrt{3}-\tan x}{\sqrt{3}+\tan x} \right|$$

$$2) p=1; q=-\frac{1}{\sqrt{3}}; f(x)=x; g(x)=\ln \left| \frac{\sqrt{3}-\tan x}{\sqrt{3}+\tan x} \right|$$

$$3) p=1; q=-\frac{2}{\sqrt{3}}; f(x)=x; g(x)=\ln \left| \frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x} \right|$$

$$4) p=1; q=-\frac{1}{\sqrt{3}}; f(x)=x; g(x)=\ln \left| \frac{\sqrt{3}+\tan x}{\sqrt{3}-\tan x} \right|$$

Key (1,4)**Sol.**

$$I = \int \frac{3 \cot 3x - \cot x}{\tan x - 3 \tan x} dx = p f(x) + g(x) + C \dots (i)$$

$$\text{Using } \tan 3x = \frac{3 \tan x - \tan^3 x}{1 - 3 \tan^2 x} \text{ and } \cot 3x = \frac{\cot^3 x - 3 \cot x}{3 \cot^2 x - 1}$$

$$I = \int \frac{1 - 3 \tan^2 x + 2 - 2}{3 - \tan^2 x}$$

$$I = \int 1 \cdot dx - 2 \int \frac{1 + \tan^2 x}{3 - \tan^2 x} dx = x - 2 \int \frac{\sec^2 x}{3 - \tan^2 x} dx + C$$

$$= x - \frac{2}{2\sqrt{3}} \ln \left| \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} \right| + C$$

$$= 1 \cdot x + \frac{1}{\sqrt{3}} \ln \left| \frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x} \right| + C$$

Comparing with R.H.S. of equation (i), we get result

89). If $I = \int \sec^2 x \operatorname{cosec}^4 x dx = A \cot^3 x + B \tan x + C \cot x + D$, then

1) $A = -1/3$

2) $B = 2$

3) $C = -2$

4) $B = 1/3$

Key (1,3)**Sol.**

$$I = \int \sec^2 x \operatorname{cosec}^4 x dx$$

$$= \int \frac{(\sin^2 x + \cos^2 x)^2}{\cos^2 x \sin^4 x} dx$$

$$= \int \frac{\sin^4 x + \cos^4 x + 2 \sin^2 x \cos^2 x}{\cos^2 x \sin^4 x} dx$$

$$= \int \left(\sec^2 x + 2 \operatorname{cosec}^2 x + \frac{\cos^2 x}{\sin^4 x} \right) dx$$

$$= \tan x - 2 \cot x + \int \cot^2 x \operatorname{cosec}^2 x dx$$

$$= \tan x - 2 \cot x - \frac{\cot^3 x}{3} + D$$

90). $\int \frac{1}{(\sin x + 2 \cos x)(\cos x + 2 \sin x)} dx =$

1) $\frac{1}{3} \ln \left| \frac{2 \tan x + 1}{2(\tan x + 2)} \right| + c$

2) $\frac{1}{3} \ln \left| \frac{\tan x + 2}{\tan x + 1} \right| + c$

3) $\frac{1}{3} \ln \left| \frac{\sin x + 2 \cos x}{\sin x + \cos x} \right| + c$

4) $\frac{1}{3} \ln \left| \frac{2 \sin x + \cos x}{\sin x + 2 \cos x} \right| + c$

Key (1,4)

Sol.

$$I = \int \frac{1}{3 \sin x} \frac{2(2 \sin x + \cos x) - (\sin x + 2 \cos x)}{(\sin x + 2 \cos x)(\cos x + 2 \sin x)} dx$$

$$= \int \left(\frac{2 \sec^2 x}{3 \tan^2 x + 6 \tan x} - \frac{\sec^2 x}{3 \tan x + 6 \tan^2 x} \right) dx = \frac{2}{3} \int \frac{dt}{t^2 + 2t} - \frac{1}{3} \int \frac{dt}{t + 2t^2}$$

$$(\because \tan x = t)$$

91). $\int \frac{\cos 9x + \cos 6x}{2 \cos 5x - 1} dx = A \sin 4x + B \sin x + c$ Then

1) $4A + 4B = 15$ 2) $4A + 4B = 5$ 3) $4A + 2B = 3$ 4) $A - B = 1$

Key (2,3)

Sol.

$$\int \frac{2 \cos \frac{15x}{2} \cos \frac{3x}{2}}{2 \left(2 \cos^2 \frac{5x}{2} - 1 \right) - 1} dx$$

$$= \int \frac{2 \left(4 \cos^3 \frac{5x}{2} - 3 \cos \frac{5x}{2} \right)}{4 \cos^2 \frac{5x}{2} - 3} \cos \frac{3x}{2} dx$$

$$= \int 2 \cos \frac{5x}{2} \cos \frac{3x}{2} dx$$

$$= \int \cos 4x + \cos x \, dx$$

$$= \frac{1}{4} \sin 4x + \sin x + c$$

$$\therefore A = \frac{1}{4}, B = 1$$

91). If $\int \frac{\sin 2x + 2 \tan x}{\cos^6 x + 6 \cos^2 x + 4} dx = \frac{1}{\alpha} \beta + \gamma \sin^{4x} + \delta \sin^6 x + C$

(α, β are co-prime)

1) $\alpha + \beta = 13$ 2) $\beta + \gamma = 7$ 3) $\alpha - \beta + \gamma = 17$ 4) $\alpha + \beta - \delta = 14$

Key (2,3)

Sol

$$\int \frac{\frac{2 \sin x}{\cos^5 x} + \frac{2 \sin x}{\cos^7 x}}{1 + \frac{6}{\cos^4 x} + \frac{4}{\cos^6 x}} dx = \frac{1}{12} \ln \left(1 + \frac{6}{\cos^4 x} + \frac{4}{\cos^6 x} \right) + C$$

92). If $\int \frac{dx}{\cos^3 x - \sin^3 x} = A \tan^{-1}(\sin x + \cos x) + B \ln \left| \frac{\cos x + \sin x - \sqrt{2}}{\cos x + \sin x + \sqrt{2}} \right| + C$, then

1) $A = 2/3$

2) $B = -2/3\sqrt{2}$

3) $A = -2/3$

4) $B = -1/3\sqrt{2}$

Key (1,4)

Sol.

$$\int \frac{dx}{\cos^3 x - \sin^3 x} = \int \frac{dx}{(\cos x - \sin x) \left(1 + \frac{\sin 2x}{2}\right)} = \int \frac{(\cos x - \sin x) dx}{(1 - \sin 2x) \left(1 + \frac{\sin 2x}{2}\right)}$$

Let $\cos x + \sin x = t$

$$= \int \frac{2dt}{(2-t^2)(t^2+1)} = \frac{2}{3} \tan^{-1}(\sin x + \cos x) - \frac{1}{3\sqrt{2}} \ln \left| \frac{\cos x + \sin x - \sqrt{2}}{\cos x + \sin x + \sqrt{2}} \right| + C$$

93). If $\int \frac{\sin x}{\sin\left(x - \frac{\pi}{4}\right)} dx = A \left[f(x) + \log|\sin x - \cos x| \right] + c$ then

1) $A = \sqrt{2}$

2) $A = \frac{1}{\sqrt{2}}$

3) $f(x) = \sin x$

4) $f(x) = x$

Key (2,4)

Sol.

$$\int \frac{\sin x}{\sin\left(x - \frac{\pi}{4}\right)} dx = \int \frac{\sin\left(u + \frac{\pi}{4}\right)}{\sin u} du \quad \left(\text{put } u = x - \frac{\pi}{4} \right)$$

$$= \int \frac{\sin u \cos \frac{\pi}{4} + \cos u \sin \frac{\pi}{4}}{\sin u} du$$

$$= \frac{1}{\sqrt{2}} u + \frac{1}{\sqrt{2}} \log|\sin u| + c$$

$$= \frac{1}{\sqrt{2}} \left(x + \log|\sin x - \cos x| \right) + c$$

$$\therefore A = \frac{1}{\sqrt{2}}, f(x) = x$$

94). If $\int \frac{\sin x}{\sin 4x} dx = A \ln \left| \frac{1 - \sin x}{1 + \sin x} \right| + B \ln \left| \frac{1 - \sqrt{2} \sin x}{1 + \sqrt{2} \sin x} \right| + c$

1) $A = -\frac{1}{8}$

2) $A = \frac{1}{8}$

3) $B = \frac{1}{4\sqrt{2}}$

4) $B = -\frac{1}{4\sqrt{2}}$

Key (2,4)**Sol.**

$$\int \frac{\sin x}{4 \sin x \cdot \cos x \cdot \cos 2x} dx = \frac{1}{4} \int \frac{\cos x dx}{(1 - \sin^2 x)(1 - 2 \sin^2 x)} (t = \sin x)$$

$$= \frac{1}{4} \int \frac{dt}{(1 - t^2)(1 - 2t^2)} = \frac{1}{4} \int \left(\frac{2}{1 - 2t^2} - \frac{1}{1 - t^2} \right) dt$$

95). If $\int f(x) \sin x \cdot \cos x dx = \frac{1}{2(b^2 - a^2)} \ln |f(x)| + c$ then $\frac{1}{f(x)}$ can be

1) $a^2 \sin^2 x + b^2 \cos^2 x$

2) $-a^2 \cos^2 x - b^2 \sin^2 x$

3) $\frac{1}{2}(b^2 - a^2) \cos 2x$

4) $\frac{1}{2}(a^2 - b^2) \cos 2x$

Key (1,2,3)**Sol.**

$$f(x) \cdot \sin x \cdot \cos x = \frac{1}{2(b^2 - a^2)} \frac{f'(x)}{f(x)^2} \Rightarrow 2(b^2 - a^2) \sin x \cdot \cos x = \frac{f'(x)}{f(x)^2}$$

$$\int 2b^2 \sin x \cdot \cos x dx - \int 2a^2 \sin x \cos x dx = \int \frac{f'(x)}{f(x)} dx$$

$$\sin x = t, \text{ in Ist and } \cos x = t \text{ in IInd} \Rightarrow b^2 \sin^2 x + a^2 \cos^2 x = -\frac{1}{f(x)}$$

$$\text{Let } \cos x = t \text{ is Ist, } \sin x = t \text{ in IInd} \Rightarrow -b^2 \cos^2 x - a^2 \sin^2 x = -\frac{1}{f(x)}$$

$$\int (b^2 - a^2) \sin 2x dx = \int \frac{f'(x) dx}{f(x)^2} \Rightarrow -\frac{1}{2}(b^2 - a^2) \cos 2x = -\frac{1}{f(x)}$$

96). If $\int \frac{3 \cot 3x - \cot x}{\tan x - 3 \tan 3x} dx = px + q \log \left| \frac{\sqrt{3} - \tan x}{\sqrt{3} + \tan x} \right| + C$, then $\frac{p}{q^2} =$

Key (3)**Sol.**

$$3 \cot 3x - \cot x = \frac{3(1 - 3 \tan^2 x)}{3 \tan x - \tan^3 x} - \frac{1}{\tan x} = \frac{-8 \tan^2 x}{\tan x(3 - \tan^2 x)}$$

$$\tan x - 3 \tan 3x = \tan x - \frac{3(3 \tan x - \tan^3 x)}{1 - 3 \tan^2 x} = \frac{-8 \tan x}{1 - 3 \tan^2 x}$$

$$\therefore I = \int \frac{1 - 3 \tan^2 x}{3 - \tan^2 x} dx = \int 1 - 2 \frac{(1 + \tan^2 x)}{3 - \tan^2 x} dx = x - \frac{1}{\sqrt{3}} \ln \frac{\sqrt{3} + \tan x}{\sqrt{3} - \tan x} + c \therefore \frac{P}{Q^2} = 3$$

97). $\int \frac{dx}{\cos^3 x \sqrt{2 \sin 2x}} = \tan^{\frac{1}{2}} x \left(1 + \frac{\tan^2 x}{\lambda} \right) + C$ then $\lambda = \underline{\hspace{2cm}}$

Key (5)

Sol $I = \frac{1}{2} \int \frac{dx}{\cos^4 x \sqrt{\tan x}}$ & put $t = \tan x$

98). $\int \cos 2\theta \cdot \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) d\theta = K \ln(\cos 2\theta) + \sin 2\theta \cdot \ln \sqrt{\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta}} + c$ then K equals

- 1) 1 2) $\frac{1}{2}$ 3) 2 4) $\frac{1}{4}$

Key (2)**Sol.**

$$I = \frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) - \int \frac{\sin 2\theta}{\cos 2\theta} d\theta$$

$$I = \frac{\sin 2\theta}{2} \ln \left(\frac{\cos \theta + \sin \theta}{\cos \theta - \sin \theta} \right) + \frac{1}{2} \ln(\cos 2\theta) \Rightarrow k = \frac{1}{2}$$

99). If $\int x e^{2x} \cos 3x dx = e^{2x} [(px + q) \cos 3x + (rx + s) \sin 3x] + C$ then p:r=

- 1) 1:2 2) 2:3 3) 1:3 4) 2:1

Key (2)

Sol. $GI = x \cdot \frac{e^{2x}}{13} [2 \cos 3x + 3 \sin 3x] \cdot e^{2x} (2 \cos 3x + 3 \sin 3x) dx$

$$= x \cdot \frac{e^{2x}}{13} [2 \cos 3x + 3 \sin 3x] - \frac{1}{13} \cdot \frac{1}{13} e^{2x} (2 \cos 3x + 3 \sin 3x + 2 \sin 3x - 3 \cos 3x)$$

$$P = \frac{26}{169}, r = \frac{39}{169} \Rightarrow p:r = 2:3$$

100). If $\int \sin^4 x dx = Ax + B \sin 2x + C \sin 4x + D$ the value of $A + 2B + 4C$ equals

1) 0

2) 1

3) $\frac{1}{32}$

4) $\frac{1}{16}$

Key (1)

Sol.

Limit x and put x=0 $\Rightarrow$ G.E=0

101). The integration of $\int \frac{\sin x}{\sin\left(x - \frac{\pi}{6}\right) \sin\left(x + \frac{\pi}{6}\right)} dx$ is equal to

1) $\frac{1}{\sqrt{3}} \left[\log \tan\left(\frac{x}{2}\right) + \log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) \right] + C$

2) $\frac{1}{\sqrt{3}} \left[\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + \log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) \right] + C$

3) $\frac{1}{\sqrt{3}} \left[\log \cot\left(\frac{x}{2} + \frac{\pi}{12}\right) \right] + C$

4) None

Key (2)

Sol. $\int \frac{\sin x}{\sin\left(x - \frac{\pi}{6}\right) \sin\left(x + \frac{\pi}{6}\right)} dx = \frac{1}{\sqrt{3}} \int \frac{2 \sin x \cos \frac{\pi}{6}}{\sin\left(x - \frac{\pi}{6}\right) \sin\left(x + \frac{\pi}{6}\right)} dx$

$$\begin{aligned}
&= \frac{1}{\sqrt{3}} \int \frac{\sin\left(x + \frac{\pi}{6}\right) + \sin\left(x - \frac{\pi}{6}\right)}{\sin\left(x + \frac{\pi}{6}\right) \sin\left(x - \frac{\pi}{6}\right)} dx = \frac{1}{\sqrt{3}} \int \operatorname{cosec}\left(x - \frac{\pi}{6}\right) + \operatorname{cosec}\left(x + \frac{\pi}{6}\right) \\
&= \frac{1}{\sqrt{3}} \left[\log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + \log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) \right] + C
\end{aligned}$$

102) If $\int \tan^5 x \, dx = A \tan^4 x + B \tan^2 x + g(x) + C$

1) $A = \frac{1}{4}, B = \frac{1}{2}, g(x) = -\log|\sec x|$

2) $A = \frac{1}{4}, B = \frac{-1}{2}, g(x) = +\log|\sec x|$

3) $g(x) = \ln|\cos x|, A = \frac{1}{4}, B = \frac{1}{3}$

4) $A = -\frac{1}{4}, B = \frac{1}{3}, g(x) = \log|\cos x|$

Key (2)

Sol. $\int \tan^3 x (\sec^2 x - 1) dx = \int \tan^3 x \sec^2 x dx - \int \tan^3 x dx$

$$= \frac{\tan^4 x}{4} - \int \tan x (\sec^2 x - 1) dx = \frac{\tan^4 x}{4} - \frac{\tan^2 x}{2} + \log|\sec x| + C$$

103). The primitive of the function $x|\cos x|$ when $\frac{\pi}{2} < x < \pi$

1) $\cos x + x \sin x$

2) $-\cos x - x \sin x$

3) $x \sin x - \cos x$

4) None

Key (2)

Sol. If $\frac{\pi}{2} < x < \pi \Rightarrow |\cos x| = -\cos x$

$$= \int -x \cos x \, dx$$

104). Let $f(x) = \int \left(\frac{\cos x}{x} - \log x^{\sin x} \right) dx$ and $f(1) = 0$ then the value of $f\left(\frac{\pi}{2}\right)$ is

1) 1

2) 2

3) -1

4) 0

Key (4)

Sol. $\int \frac{\cos x}{x} dx - \int \sin x \log x dx = \cos x \log x + \int \sin x \log x dx - \int \sin x \log x dx$

$$= \cos x \log x + C$$

$$f(1) = 0 + C = 0 \Rightarrow C = 0$$

$$f(x) = \cos x \log x \Rightarrow f(\pi/2) = 0$$

105) If $\int \sqrt{\cos ec x + 1} dx = \lambda f(g(x)) + C$ where $\lambda \in R$ then

1) $\lambda = -2, f(x) = \cot^{-1} x, g(x) = \sqrt{\cos ec x + 1}$

2) $\lambda = -2, f(x) = \tan^{-1} x, g(x) = \sqrt{\cos ec x + 1}$

3) $\lambda = 2, f(x) = \tan^{-1} x, g(x) = \frac{\cot x}{\sqrt{\cos ec x - 1}}$

4) $\lambda = 2, f(x) = \cot^{-1} x, g(x) = \frac{\cot x}{\sqrt{\cos ec x + 1}}$

Key (4)

Sol. $I = \int \sqrt{\cos ec x + 1} dx = \int \frac{\cot x}{\sqrt{\cos ec x - 1}} dx = - \int \frac{-\cot x \cos ec x}{\cos ec x \sqrt{\cos ec x - 1}}$

Let $\cos ec x - 1 = t^2 \Rightarrow -\cos ec x \cot x dx = 2t dt$

$$I = - \int \frac{2dt}{1+t^2} = -2 \tan^{-1} t + c = -2 \tan^{-1} \sqrt{\cos ec x - 1} + C$$

$$= -2 \left[\frac{\pi}{2} - \cot^{-1} \sqrt{\cos ec x - 1} \right] + C = 2 \cot^{-1} \sqrt{\cos ec x - 1} + C'$$

$$= 2 \cot^{-1} \left(\frac{\cot x}{\sqrt{\cos ec x + 1}} \right) dx$$

106). If $\int \frac{3+2\cos x}{(2+3\cos x)^2} dx = \left(\frac{\sin x}{\mu + K \cos x} \right) + C$, then value of $1 + \mu + K$

1) 0

2) 1

3) 4

4) 6

Key (4)

Sol. $I = \int \frac{3+2\cos x}{(2+3\cos x)^2} dx$

Multiplying N^r & D^r by $\operatorname{cosec}^2 x$

$$I = -\int \frac{(3\operatorname{cosec}^2 x + 2\cot x \operatorname{cosec} x)}{(2\operatorname{cosec} x + 3\cot x)^2} dx = -\int \frac{-3\operatorname{cosec}^2 x - 2\cot x \operatorname{cosec} x}{(2\operatorname{cosec} x + 3\cot x)^2} dx$$

$$= \frac{1}{2\operatorname{cosec} x + 3\cot x} + C = \left(\frac{\sin x}{2+3\cos x} \right) + C$$

107). If $\int \frac{x^2+20}{(x\sin x+5\cos x)^2} dx = \frac{\lambda x}{\cos x(x^5\sin x+Kx^4\cos x)} + \mu \tan x + C$ then the value of $\lambda + k + \mu$ is

1) 5

2) 4

3) 3

4) 1

Key (1)

Sol. $I = \int \frac{x^8(x^2+20)}{x^8(x\sin x+5\cos x)^2} dx = \int \frac{(x^5+20x^3)\cos x}{(x^5\sin x+5x^4\cos x)^2} \times \frac{x^5}{\cos x} dx$

$$\therefore I = \frac{-x^5}{\cos x(x^5\sin x+5x^4\cos x)} + \int \frac{(5x^4\cos x+x^5\sin x)}{(x^5\sin x+5x^4\cos x)^2} \operatorname{sec}^2 x dx$$

$$\therefore I = \frac{-x^5}{\cos x(x^5\sin x+5x^4\cos x)} + \tan x + C$$

108). The value of $\int \frac{\cos 7x - \cos 8x}{1+2\cos 5x} dx = \frac{\sin 2x}{\lambda} + \frac{\sin 3x}{K} + C$ $\lambda + K$ is equal to

1) 0

2) 1

3) -1

4) 2

Key (3)

Sol. $I = \int \frac{\cos 7x - \cos 8x}{1 + 2 \cos 5x} dx = \int \frac{2 \sin \frac{15x}{2} \sin \frac{x}{2}}{1 + 2 \cos 5x} dx$

Multiplying & Dividing by $\sin \frac{5x}{2}$

$$\begin{aligned}
 &= \int \frac{2 \sin \frac{15x}{2} \sin \frac{x}{2} \sin \frac{5x}{2}}{\sin \frac{5x}{2} + 2 \sin \frac{5x}{2} \cos 5x} dx \\
 &= \int \frac{2 \sin \frac{15x}{2} \sin \frac{x}{2} \sin \frac{5x}{2}}{\sin \frac{5x}{2} + \sin \frac{15x}{2} - \sin \frac{5x}{2}} dx = \int 2 \sin \frac{x}{2} \sin \frac{5x}{2} dx \\
 &= \int (\cos 2x - \cos 3x) dx = \frac{\sin 2x}{2} - \frac{\sin 3x}{3} + C
 \end{aligned}$$

109). If $I = \int \frac{dx}{\sec x + \cos ecx}$ then I is equal to

1) $\frac{1}{2} \left(\cos x + \sin x - \frac{1}{\sqrt{2}} \log |\cos ecx - \cos x| \right) + C$ 2) $\frac{1}{2} \left(\sin x - \cos x - \frac{1}{\sqrt{2}} \log |\cos ecx + \cot x| \right) + C$

3) $\frac{1}{\sqrt{2}} \left(\sin x + \cos x + \frac{1}{2} \log |\cos ecx - \cos x| \right) + C$

4) $\frac{1}{2} \left(\sin x - \cos x - \frac{1}{\sqrt{2}} \log \left| \cos ec \left(x + \frac{\pi}{4} \right) - \cot \left(x + \frac{\pi}{4} \right) \right| \right) + C$

Key (4)

Sol. $I = \int \frac{\sin x \cos x}{\sin x + \cos x} dx = \frac{1}{2} \int \frac{(\sin x + \cos x)^2 - 1}{\sin x + \cos x} dx$

$$= \frac{1}{2} \int \left[\sin x + \cos x - \frac{1}{\sqrt{2} \sin \left(x + \frac{\pi}{4} \right)} \right] dx$$

$$= \frac{1}{2} \int \left[\sin x - \cos x - \frac{1}{\sqrt{2}} \log \left| \operatorname{cosec} \left(x + \frac{\pi}{4} \right) - \cot \left(x + \frac{\pi}{4} \right) \right| \right] dx + C$$

110). If $\int \frac{\left(\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x \right) dx}{\sqrt{\sin^3 x \cos^3 x \sin(x-\alpha)}} = a\sqrt{\cos \alpha \tan x - \sin \alpha} + b\sqrt{\cos \alpha - \sin \alpha \cot x} + c$, then $ab =$

- 1) $4 \operatorname{cosec} 2\alpha$ 2) $8 \operatorname{cosec} 2\alpha$ 3) $2 \sec \alpha$ 4) $4 \sec \alpha$

Key (2)

Sol. $I = \int \frac{dx}{\sqrt{\cos^3 x (\sin x \cos \alpha - \cos x \sin \alpha)}} + \int \frac{dx}{\sqrt{\sin^3 x (\sin x \cos \alpha - \cos x \sin \alpha)}}$

$$= \int \frac{\sec^2 x dx}{\sqrt{\cos \alpha \tan x - \sin \alpha}} + \int \frac{\operatorname{cosec}^2 x dx}{\sqrt{\cos \alpha - \sin \alpha \cot x}}$$

$$= \frac{2}{\cos \alpha} \sqrt{\cos \alpha \tan x - \sin \alpha} + \frac{2}{\sin \alpha} \sqrt{\cos \alpha - \sin \alpha \cot x} + c$$

$$ab = \frac{2}{\cos \alpha} \cdot \frac{2}{\sin \alpha} = 8 \operatorname{cosec} 2\alpha$$

111). If $\int \frac{\cos^3 x + \cos^5 x}{\sin^2 x + \sin^4 x} dx =$

- 1) $\sin x - 2 \operatorname{cosec} x - 6 \tan^{-1}(\sin x) + c$ 2) $\sin x - 2 \operatorname{cosec} x + 5 \tan^{-1}(\sin x) + c$
- 3) $\sin x + 2 \sec x + \tan^{-1}(\sin x) + c$ 4) $\sin x - 6 \tan^{-1}(\sin x) + c$

Key (1)

Sol. $\cos x = t \Rightarrow G.I. = \int \frac{(1-t^2) + (1-t^2)^2}{t^2 + t^4} dt = \int 1 + \frac{2-4t^2}{t^2(1+t^2)} dt = \int \left(1 + \frac{2}{t^2} - \frac{6}{t^2+1} \right) dt = t - \frac{2}{t} - 6 \tan^{-1} t + c$

112). If $\int \frac{dx}{\sin^2 x + \tan^2 x} = A \cot x + B \tan^{-1} \left(\frac{\tan x}{\sqrt{2}} \right) + c$ then $A + B\sqrt{2}$ equals

1) 1

2) -1

3) 2

4) -2

Key (2)

Sol. $\int \frac{dt}{t^2(2+t^2)} = \frac{1}{2} \int \left(\frac{1}{t^2} - \frac{1}{2+t^2} \right) dt = -\frac{1}{2t} - \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{t}{\sqrt{2}} \right) + c; \quad A + B\sqrt{2} = -1$

INTEGRATION BY PARTS

112). If $I = \int e^x \left(\frac{x+2}{x+4} \right)^2 dx$, then I equals

1) $\frac{x}{x+4} e^x + C$

2) $\frac{x-2}{x+4} e^x + C$

3) $\frac{x+1}{x+4} e^x + C$

4) $\frac{x-1}{x+4} e^x + C$

Key.1

sol. Put $x+4=t$, so that $I = \int e^{t-4} \left(\frac{t-2}{t} \right)^2 dt \cdot e^{-4} = \int e^t \left(\frac{t-4t+4}{t^2} \right) dt$

$$e^{-4} = \left[e^t - \frac{4}{t} e^t \right] + C = \frac{x}{x+4} e^x + C$$

113). $\int e^x \left(\tan^{-1} x + \frac{2x}{(1+x^2)^2} \right) dx$ is equal to

1) $e^x \left(\tan^{-1} x - \frac{1}{1+x^2} \right) + C$

2) $e^x \left(\tan^{-1} x + \frac{1}{1+x^2} \right) + C$

3) $e^x \left(\cot^{-1} x - \frac{1}{1+x^2} \right) + C$

4) $e^x \left(\tan^{-1} x + \frac{2}{1+x^2} \right) + C$

Key.1**sol.**

$$\int e^x \left(\tan^{-1} x + \frac{1}{1+x^2} - \frac{1}{1+x^2} + \frac{2x}{(1+x^2)^2} \right) dx = e^x \left(\tan^{-1} x - \frac{1}{1+x^2} \right) + C$$

114). $\int e^{\sin x} \left(\frac{x \cos^3 x - \sin x}{\cos^2 x} \right) dx$, is equal to

1) $e^{\sin x} (\tan x + x) + C$

2) $e^{\sin x} (x - \sec x) + C$

3) $e^{\sin x} (\sec x + \tan x) + C$ 4) none of these

Key.2

sol.

$$\begin{aligned} I &= \int e^{\sin x} \left(\frac{x \cos^3 x - \sin x}{\cos^2 x} \right) dx = \int x e^{\sin x} \cos x dx - \int e^{\sin x} (\sec x \tan x) dx \\ &= \left[x e^{\sin x} - \int e^{\sin x} dx \right] - \left[e^{\sin x} \sec x - \int e^{\sin x} dx \right] \\ &= e^{\sin x} (x - \sec x) + C \end{aligned}$$

115). $\int e^{\sec x} \cdot \sec^3 x \sin^2 x + \cos x + \sin x + \sin x \cos x \, dx =$

- 1) $e^{\sec x} \sec x + \tan x + c$ 2) $e^{\sec x} \cdot \tan x + c$
 3) $e^{\sec x} \sec x \tan x + c$ 4) $e^{\sec x} \cdot \sec x + c$

Key.1

Sol

$$\begin{aligned} &\int e^{\sec x} \cdot \sec x \tan x \sec x + 1 \, dx + \int e^{\sec x} \sec^2 x + \sec x \tan^2 x \, dx \\ &= e^{\sec x} \sec x + e^{\sec x} \cdot \tan x - \int e^{\sec x} \sec x \tan^2 x \, dx + \int e^{\sec x} \sec x \tan^2 x \, dx \quad 116). \\ &= e^{\sec x} \sec x + \tan x + C \end{aligned}$$

$$\int e^{x \sin x + \cos x} \left(\frac{x^4 \cos^3 x - x \sin x + \cos x}{x^2 \cos^2 x} \right) dx$$

- 1) $e^{x \sin x + \cos x} \left(x - \frac{\sec x}{x} \right) + c$
 2) $e^{x \sin x + \cos x} \left(x \sin x - \frac{\cos x}{x} \right) + c$
 3) $e^{x \sin x + \cos x} \left(\frac{x}{\tan x} - \frac{\sec x}{x} \right) + c$
 4) $x e^{x \sin x + \cos x} - \int e^{x \sin x + \cos x} \left(1 - \frac{\cos x - x \sin x}{x^2 \cos^2 x} \right) dx$

Key . 1

$$\int e^{x \sin x + \cos x} \left(x^2 \cos x - \frac{1}{x} \tan x \sec x + \frac{1}{x^2} \sec x \right) dx$$

$$= e^{x \sin x + \cos x} \cdot x - \int e^{x \sin x + \cos x} dx - \frac{\sec x \cdot e^{x \sin x + \cos x}}{x} -$$

Sol.

$$\int \sec x \left(\frac{e^{x \sin x + \cos x} - x^2 \cos x e^{x \sin x + \cos x}}{x^2} \right) dx + \int \frac{e^{x \sin x + \cos x} \cdot \sec x}{x^2} dx$$

$$= e^{x \sin x + \cos x} \left(x - \frac{\sec x}{x} \right) + C$$

118). $\int e^{x^x} [1 + x^x (x + \ln x^x)] dx =$

- 1) $e^{x^x} + c$ 2) $x e^{x^x} + c$ 3) $x^x \cdot e^{x^x} + c$ 4) $\frac{1}{x} e^{x^x} + c$

Key. 2

Sol $\int e^{x^x} (1 + x^x \cdot x (1 + \ln x^x)) dx = x \cdot e^{x^x} + c$

119). If $\int e^x \left(\frac{x^2 - 2}{x^3} \right) dx = e^x \left(\frac{ax^2 + bx + c}{x^2} \right) + d$ then the value of a+b+c equals

- 1) 1 2) 2 3) $\frac{1}{2}$ 4) 3

Key. 2

sol. $\int e^x \left(\frac{1}{x} - \frac{2}{x^3} \right) dx = \int e^x \left(\frac{1}{x} + \frac{1}{x^2} - \frac{1}{x^2} - \frac{2}{x^3} \right) dx$

$$= e^x \left(\frac{1}{x} + \frac{1}{x^2} \right) + c$$

$$\left. \begin{array}{l} a = 0 \\ \text{Clearly } b = 1 \\ c = 1 \end{array} \right\} a + b + c = 2$$

120). $\int e^x \left[\frac{1}{1 + \sin x} - \frac{\cos \frac{x}{2} - \sin \frac{x}{2}}{\cos \frac{x}{2} + \sin \frac{x}{2}} \right] dx = K e^x \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) + C$ then the value of K equals

- 1) 1 2) -1 3) 2 3) -2

Key. 2

sol.

$$G.I = \int e^x \left\{ \frac{1}{2} \sec^2 \left(\frac{\pi}{4} - \frac{x}{2} \right) - \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) \right\} dx$$

$$= -e^x \tan \left(\frac{\pi}{4} - \frac{x}{2} \right) + c \Rightarrow k = -1$$

121). $\int e^x \frac{(x^2 + 5x + 7)}{(x+3)^2} dx = e^x \phi(x) + c$ then number of solutions of $\phi(x) = x$ equals

1) 0

2) 1

3) 2

4) 3

Key. 3

sol. $G.I = e^x \left(\frac{x+2}{x+3} \right) + c \Rightarrow \frac{x+2}{x+3} = x$ posses solutions $x = -1 \pm \sqrt{3}$

No. of solutions = 2

122). $\int \frac{e^x (2-x^2)}{(1-x)\sqrt{1-x^2}} dx =$

1) $e^x \left(\frac{\sqrt{2}+x}{\sqrt{2}-x} \right) + c$ 2) $e^x \left(\frac{\sqrt{2}-x}{\sqrt{2}+x} \right) + c$ 3) $e^x \sqrt{\frac{1-x}{1+x}} + c$ 4) $e^x \sqrt{\frac{1+x}{1-x}} + c$

Key. 4

sol. $I = \int e^x \left(\frac{1+1-x^2}{(1-x)\sqrt{1-x^2}} \right) dx = \int \left(\frac{1}{(1-x)\sqrt{1-x^2}} + \sqrt{\frac{1+x}{1-x}} \right) dx$

$$= e^x \sqrt{\frac{1-x}{1+x}} + c$$

123). If $\int \frac{\operatorname{cosec}^2 x - 2013}{\cos^{2013} x} dx =$

1) $\cot x \sec^{2013} x + C$

2) $-\tan x \sec^{2013} x + C$

3) $-\cot x \sec^{2013} x + C$

4) $\tan x \sec^{2013} x + C$

Key.3**sol.**

$$\begin{aligned}
& \int \frac{\sec^2 x}{\cos^{2013} x} dx - \int \frac{2013}{\cos^{2013} x} dx \\
&= \frac{-\cot x}{\cos^{2013} x} - \int \cot x \frac{-2013}{\cos^{2014} x} - \sin x - \int \frac{2013}{\cos^{2013} x} dx \\
&= \frac{-\cot x}{\cos^{2013} x} + C
\end{aligned}$$

124). If $\int \frac{\sec^2 x - 2010}{\sin^{2010} x} dx = \frac{P(x)}{\sin^{2010} x} + C$, then value of $P\left(\frac{\pi}{3}\right)$ is

1) 0

2) $\frac{1}{\sqrt{3}}$

3) $\sqrt{3}$

3) None of these

Key.3

sol.

$$\begin{aligned}
& \int \frac{\sec^2 x - 2010}{\sin^{2010} x} dx \\
&= \int \sec^2 x (\sin x)^{-2010} - 2010 \int \frac{1}{(\sin x)^{2010}} dx = I_1 - I_2
\end{aligned}$$

Applying by parts on I_1 , we get

$$I_1 = \frac{\tan x}{(\sin x)^{2010}} + 2010 \int \frac{\tan x \cos x}{(\sin x)^{2011}} dx = \frac{\tan x}{(\sin x)^{2010}} + 2010 \int \frac{dx}{(\sin x)^{2010}}$$

$$\Rightarrow I = I_1 - I_2 = \frac{\tan x}{(\sin x)^{2010}} = \frac{P(x)}{(\sin x)^{2010}}$$

$$P\left(\frac{\pi}{3}\right) = \tan \frac{\pi}{3} = \sqrt{3}$$

125). $\int \frac{x^2}{x \sin x + \cos x} dx = \underline{\hspace{2cm}}$

1) $\cot x + \frac{x \sec x}{x \sin x + \cos x} + c$

2) $\tan x - \frac{x \sec x}{x \sin x + \cos x} + c$

3) $\cot x - \frac{x \sec x}{x \sin x + \cos x} + c$

4) $\tan x + \frac{x \sec x}{x \sin x + \cos x} + c$

Key.3

sol.

$$\int \underbrace{\frac{x}{\cos x}}_I \underbrace{\frac{x \cos x}{x \sin x + \cos x}}_{II} dx ; \text{ Integrate using byparts}$$

126). Let $f(x) = \frac{2\sin^2 x - 1}{\cos x} + \frac{\cos x(2\sin x + 1)}{1 + \sin x}$ then $\int e^x (f(x) + f'(x)) dx$ equals

(where c is the constant of integration)

- 1) $e^x \tan x + c$ 2) $e^x \cot x + c$ 3) $e^x \cos ec^2 x + c$ 4) None of these

Key (1)

Sol.

$$\begin{aligned} & \frac{\cos x(1 + 2\sin x)}{1 + \sin x} - \frac{\cos^2 x - \sin^2 x}{\cos x} \\ &= \frac{\cos^2 x(1 + 2\sin x) - (1 + \sin x)(\cos^2 x - \sin^2 x)}{\cos x(1 + \sin x)} = \frac{-\sin x \cos^2 x + \sin^3 x}{\cos x(1 + \sin x)} \\ &= \frac{\sin x \cos^2 x + \sin^2 x(1 + \sin x)}{\cos x(1 + \sin x)} = \frac{\sin x(1 - \sin x) + \sin^2 x}{\cos x} = \tan x \end{aligned}$$

127). $\int \frac{e^{\cot x}}{\sin^2 x} (2 \ln \cos ecx + \sin 2x) dx =$

- 1) $-2e^{\cot x} \ln(\cos ecx) + c$ 2) $e^{\cot x} \ln x + c$
 3) $e^{\cot x} \ln(\cos ecx) + c$ 4) $e^{\cot x} \ln(\sin x) + c$

Key (1)

Sol.

$$t = \cot x \Rightarrow -\cos ec^2 x dx = dt \Rightarrow dx = \frac{-1}{1+t^2} dt$$

$$I = -\int e^t \left(\ln(1+t^2) + \frac{2t}{1+t^2} \right) dt = -e^t \ln(1+t^2) + c = -2e^{\cot x} \ln \cos ecx + c$$

128). If $\int (\ln x)^4 dx = x \sum_{r=0}^4 A_r (\ln x)^r + c$ then $\sum_{r=0}^4 A_r =$

- 1) 4 2) 6 3) 9 4) None

Key.3

sol.

$$\text{G.I. } \int t^4 e^t dt = e^t (6^4 - 4t^3 + 12t^2 - 24t + 24) + c$$

$$\therefore GE = 1 - 4 + 12 - 24 + 24 = 9$$

129). $\int e^{\tan \theta} (\sec \theta - \sin \theta) d\theta$ equals :

1) $-e^{\tan \theta} \sin \theta + c$

2) $e^{\tan \theta} \sin \theta + c$

3) $e^{\tan \theta} \sec \theta + c$

4) $e^{\tan \theta} \cos \theta + c$

Key

Sol: integrate $3 \int e^{\tan \theta} \cdot \sin \theta d\theta$ by parts. One integral cancels.

130). If $\int x^4 \ln x dx = x^5 [A \ln x + B] + c$ then the value of $\sqrt{A+B} =$

1) $\frac{4}{5}$

2) $\frac{2}{5}$

3) $\frac{1}{5}$

4) $\frac{2}{3}$

Key.1**sol.**

$$\int x^n \ln x dx = \frac{x^{n+1}}{n+1} \left[\ln x - \frac{1}{n+1} \right] + c \quad A = \frac{1}{5} \quad B = \frac{1}{25}$$

131). $\int (1+x-x^{-1}) e^{x+x^{-1}} dx =$

1) $(x+1) e^{x+x^{-1}} + c$ 2) $(x-1) e^{x+x^{-1}} + c$ 3) $\frac{1}{2} x e^{x+x^{-1}} + c$ 4) $x e^{x+x^{-1}} + c$

Key.4**sol.**

$$\int e^{x+x^{-1}} dx + \int e^{x+x^{-1}} x \left(1 - \frac{1}{x^2} \right) dx = x e^{x+x^{-1}} + c$$

132). Let $F(x) = e^{\sin^{-1} x} \left(1 - \frac{x}{\sqrt{1-x^2}} \right)$ and $F(0) = 1$, If $F(1/2) = \frac{k\sqrt{3}e^{x/6}}{\pi}$, then $k =$

1) $\frac{\pi}{4}$

2) $\frac{\pi}{6}$

3) $\frac{\pi}{2}$

4) $\frac{\pi}{3}$

Key.3**sol.**

$$F(x) = \int e^{\sin^{-1}x} \left(1 - \frac{x}{\sqrt{1-x^2}} \right) dx = \int e^{\sin^{-1}x} \left(\frac{1}{\sqrt{1-x^2}} \sqrt{1-x^2} - \frac{x}{\sqrt{1-x^2}} \right) dx$$

$$F(x) = e^{\sin^{-1}x} \sqrt{1-x^2} + c$$

$$F(0) = 1 + c \quad \Rightarrow \quad c = 0 \quad (\because F(0) = 1)$$

$$F(1/2) = e^{\pi/6} \cdot \frac{\sqrt{3}}{2} = \frac{k\sqrt{3}}{\pi} e^{\pi/6}$$

$$\therefore k = \frac{\pi}{2}$$

133). $\int \frac{e^{\tan^{-1}x}}{(1+x^2)} \left[\left(\sec^{-1} \sqrt{1+x^2} \right)^2 + \cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) \right] dx \quad (x > 0)$

1) $e^{\tan^{-1}x} \cdot \tan^{-1}x + C$

2) $\frac{e^{\tan^{-1}x} \cdot (\tan^{-1}x)^2}{2} + C$

3) $e^{\tan^{-1}x} \cdot \left(\sec^{-1}(\sqrt{1+x^2}) \right)^2 + C$

4) $e^{\tan^{-1}x} \cdot \left(\cos^{-1}(\sqrt{1+x^2}) \right)^2 + C$

Key.3

sol.

note that $\sec^{-1} \sqrt{1+x^2} = \tan^{-1}x$; $\cos^{-1} \left(\frac{1-x^2}{1+x^2} \right) = 2 \tan^{-1}x$ for $x > 0$

$$\begin{aligned} I &= \int \frac{e^{\tan^{-1}x}}{1+x^2} \left((\tan^{-1}x)^2 + 2 \tan^{-1}x \right) dx \quad \text{put } \tan^{-1}x = t \\ &= \int e^t (t^2 + 2t) dt = e^t \cdot t^2 = e^{\tan^{-1}x} (\tan^{-1}x)^2 + C \end{aligned}$$

134). $\int \ln x^3 dx = x \sum_{r=0}^3 A_r (\ln x)^r + C$ Then

1) $\sum_{r=0}^3 A_r = -2$

2) $\sum_{r=0}^2 A_r = -3$

3) $\sum_{r=0}^3 A_r^2 = -4$

4) $\sum_{r=0}^2 A_r^2 = 2$

Key.1,2

sol.

Put $x = e^t$

$$\int t^3 e^t dt = x \ln x^3 - 3 \ln x^2 + 6 \ln x - 6 + c$$

$$\therefore A_0 = -6, A_1 = 6, A_2 = -3, A_3 = 1$$

135). If $\int e^{\tan \theta} (1 - \tan \theta)^2 \cos^2 \theta d\theta = A e^{\tan \theta} \cos^B \theta + K$, then the value of $A+B$ is

Key. 3

sol.

Put $\tan \theta = t$

$$\therefore I = \int e^t \frac{(1-t)^2}{(1+t^2)^2} dt$$

$$= \int e^t \left(\frac{1}{1+t^2} - \frac{2t}{(1+t^2)^2} \right) dt = e^t \frac{1}{1+t^2} + k = e^{\tan \theta} \cos^2 \theta + k$$

$$\therefore A+B=3$$

INVERSE FUNCTIONS

136). $\int \frac{1}{1-x^8} \left\{ \sin^{-1} \left(\frac{2x}{1+x^2} \right) + \tan^{-1} \left(\frac{2x}{1-x^2} \right) \right\} dx; x > 1$

1) C 2) $\frac{\pi}{8} \tan^{-1} x + c$ 3) $-\frac{\pi}{8} \sin^{-1} x + c$ 4) $\frac{\pi}{2} + x + c$

key (1)

sol:

$$\text{Let } I = \int \frac{1}{(1-x^8)} \left\{ \sin^{-1} \left(\frac{2x}{1+x^2} \right) + \tan^{-1} \left(\frac{2x}{1-x^2} \right) \right\} dx$$

$$\begin{aligned}
&= \int \frac{1}{(1-x^8)} \left\{ (\pi - 2 \tan^{-1} x) + (-\pi + 2 \tan^{-1} x) \right\} dx \\
&= \int 0 dx \\
&= 0 + c \\
&= c
\end{aligned}$$

137). If $\int \sin \left(2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right) dx = A \sin^{-1} x + Bx\sqrt{1-x^2} + c$ then A+B=

1) 1

2) -1

3) 0

4) -2

key. 1

Sol. $G.I. = \int \sqrt{1-x^2} dx = \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x + c \Rightarrow A + B = \frac{1}{2} + \frac{1}{2} = 1$

138). If $\int \sin^{-1} x \cos^{-1} x dx = \sin^{-1} x \left(Ax - x \sin^{-1} x - 2\sqrt{1-x^2} \right) + \frac{\pi}{2} \sqrt{1-x^2} + 2x + C$, Then $\frac{A}{\pi} =$

key (2)

sol:

$$\begin{aligned}
\sin^{-1} x \cos^{-1} x dx &= \sin^{-1} x \left(Ax - x \sin^{-1} x - 2\sqrt{1-x^2} \right) \\
&\quad + \frac{\pi}{2} \sqrt{1-x^2} + 2x + C,
\end{aligned}$$

139). If $\int \sin \left(2 \tan^{-1} \sqrt{\frac{1-x}{1+x}} \right) dx = A \sin^{-1} x + Bx\sqrt{1-x^2} + c$ then A+B=

1) 1

2) -1

3) 0

4) -2

key (1)

sol:

$$G.I. = \int \sqrt{1-x^2} dx = \frac{x}{2} \sqrt{1-x^2} + \frac{1}{2} \sin^{-1} x + c \Rightarrow A+B = \frac{1}{2} + \frac{1}{2} = 1$$

140). If $I = \int \sec^{-1} \sqrt{x} dx$, then I equals 1)

$$x \sec^{-1} \sqrt{x} - \log(1+x) + C \quad 2) \sec^{-1} \sqrt{x} - \tan^{-1} \sqrt{x} + C$$

$$3) x \sec^{-1} \sqrt{x} - \sqrt{x-1} + C \quad 4) \text{None of these}$$

key (3)

sol

Putting $\sqrt{x} = \sec \theta$ or $x = \sec^2 \theta$

$$\therefore I = \int \theta (2 \sec^2 \theta \tan \theta) d\theta = \int \theta \frac{d(\sec^2 \theta)}{d\theta} d\theta$$

$$= \theta \sec^2 \theta - \int \sec^2 \theta d\theta = \theta \sec^2 \theta - \tan \theta + C = \theta \sec^2 \theta - \sqrt{\sec^2 \theta - 1} + C$$

$$= x \sec^{-1} \sqrt{x} - \sqrt{x-1} + C$$

$$= -\frac{1}{x} \log(x^2 + a^2) + \frac{2}{a} \tan^{-1} \left(\frac{x}{a} \right) + C$$

141). If $I = \int \tan^{-1}(1+\sqrt{x}) dx$, then I equals

$$1) x^2 \tan^{-1}(1+\sqrt{x}) - \sqrt{x} + \log(x+2\sqrt{x}+2) + C$$

$$2) (2\sqrt{2}+1) \tan^{-1}(1+\sqrt{x}) + \sqrt{x} - \log(x+2\sqrt{x}+2) + C$$

$$3) 3x \tan^{-1} 1 + \sqrt{x} - \sqrt{x} + \log x + 2\sqrt{x} + 2 + C$$

4) None of these

key (3)

Put $1+\sqrt{x} = \tan \theta$, so that

$$I = \int \theta 2(\tan \theta - 1) \sec^2 \theta d\theta$$

$$\begin{aligned}
 &= (\tan \theta - 1)^2 \theta - \int (\sec^2 \theta - 2 \tan \theta) d\theta \\
 &= x^2 \tan^{-1} (1 + \sqrt{x})
 \end{aligned}$$

142). If $y = 2 \cot^{-1} \left(\frac{x}{2} \right) + x$, then $\int \frac{x^2 dx}{(x^2 - 4) \sin x + 4x \cos x} =$

- 1) $\log |\operatorname{cosec} 2y + \cot y| + c$ 2) $\log |\operatorname{cosec} y + \cot 2y| + c$
 3) $\log |\operatorname{cosec} y + \cot y| + c$ 4) $\log |\operatorname{cosec} y - \cot y| + c$

key (4)

sol:

$$\begin{aligned}
 y &= 2 \tan^{-1} \left(\frac{2}{x} \right) + x = \tan^{-1} \left(\frac{4x}{x^2 - 4} \right) + x \Rightarrow \tan(y - x) = \frac{4x}{x^2 - 4} \\
 \Rightarrow \cos(y - x) &= \frac{x^2 - 4}{x^2 + 4}
 \end{aligned}$$

$$\int \frac{\frac{x^2}{x^2 - 4} dx}{\frac{\sin x + \frac{4x}{x^2 + 4} \cos x}{\sin y}} = \int \frac{\frac{x^2}{x^2 - 4} \frac{x^2 - 4}{x^2 + 4} dx}{\sin y}$$

$$\frac{dy}{dx} = \frac{2}{1 + \frac{4}{x^2}} - \frac{2}{x^2} + 1 = \frac{x^2}{x^2 + 4} = \int \frac{dy}{\sin y}$$

143). If $f(x) = \sqrt{x}$, $g(x) = e^{x-1}$, and $\int f \circ g(x) dx = A f \circ g(x) + B \tan^{-1}(f \circ g(x)) + C$, then $A + B$ is equal to

- 1) 1 2) 2 3) 3 4) 0

key (4)

sol:

$$f \circ g(x) = \sqrt{e^x - 1}$$

$$\therefore I = \int \sqrt{e^x - 1} dx = \int \frac{2t^2}{t^2 + 1} dt \quad \left\{ \text{where } \sqrt{e^x - 1} = t \right\}$$

$$= 2t - 2 \tan^{-1} t + C = 2\sqrt{e^x - 1} - 2 \tan^{-1}(\sqrt{e^x - 1}) + C = 2f \circ g(x) - 2 \tan^{-1}(f \circ g(x)) + C$$

$$\therefore A + B = 2 + (-2) = 0$$

144). If $\int \sin^{-1} x \cos^{-1} x dx = f^{-1} \left[\frac{\pi}{2} x - x f^{-1}(x) - 2\sqrt{1-x^2} \right] + \frac{\pi}{2} \sqrt{1-x^2} + 2x + C$, then $f(x)$ is equal to

1) $\sin 3x$

2) $\sin 2x$

3) $\sin x$

4) $\sin 4x$

key (3)

sol:

$$\int \sin^{-1} x \cos^{-1} x dx = \int \left[\frac{\pi}{2} \sin^{-1} x - (\sin^{-1} x)^2 \right] dx$$

$$\Rightarrow \frac{\pi}{2} \left(x \sin^{-1} x + \sqrt{1-x^2} \right) - \left(x (\sin^{-1} x)^2 + \sin^{-1} x \sqrt{1-x^2} - x \right) + c \text{ By parts}$$

$$\Rightarrow \sin^{-1} x \left[\frac{\pi}{2} x - x \sin^{-1} x - 2\sqrt{1-x^2} \right] + \frac{\pi}{2} \sqrt{1-x^2} + 2x + c$$

$$\therefore f^{-1}(x) = \sin^{-1} x, f(x) = \sin x$$

145). $\int \tan^{-1} \alpha \cdot \tan x d\alpha$ equals

1) $\tan^{-1} \alpha \cdot \sec x + c$

2) $\tan^{-1} \alpha \cdot \log(\sec x) + c$

3) $\tan x \left\{ \alpha \tan^{-1} \alpha - \frac{1}{2} \log(1 + \alpha^2) \right\} + c$

4)

$\tan \alpha \left\{ x \tan^{-1} x - \frac{1}{2} \log(1 + x^2) \right\} + c$

key (3)

sol:

$$\tan x \int \tan^{-1} \alpha d\alpha = \tan x \left[\alpha \tan^{-1} \alpha - \int \frac{\alpha}{1 + \alpha^2} d\alpha \right]$$

$$= \tan x \left[\alpha \tan^{-1} \alpha - \frac{1}{2} \ln(1 + \alpha^2) \right] + c$$

REDUCTION

146). If $I_n = \int \tan^n x dx$, ($n \in N$) then the value of $(n-1)(I_n + I_{n-2}) + (n-3)(I_{n-2} + I_{n-4})$ at $x = \pi/4$ equals

1) 0

2) 1

3) 2

4) 4

key. 3

Sol. $GE = \tan^{n-1} x + \tan^{n-3} x = 1 + 1 = 2$

147). If $I_n = \int \tan^n x dx$, then $I_0 + I_1 + 2(I_2 + \dots + I_8) + I_9 + I_{10}$, is equal to

1) $\sum_{n=1}^9 \frac{\tan^n x}{n}$

2) $1 + \sum_{n=1}^8 \frac{\tan^n x}{n}$

3) $\sum_{n=1}^9 \frac{\tan^n x}{n+1}$

4) $\sum_{n=2}^{10} \frac{\tan^n x}{n+1}$

Key 1

Sol. $I_n = \int \tan^n x dx = \int \tan^{n-2} x (\sec^2 x - 1) dx = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$

$$I_{n-2} + I_n = \frac{\tan^{n-1} x}{n-1} \quad (n \geq 2)$$

Thus, we have

$$I_0 + I_1 + 2(I_2 + \dots + I_8) + I_9 + I_{10}$$

$$= (I_0 + I_2) + (I_1 + I_3) + (I_2 + I_4) + (I_3 + I_5) + (I_4 + I_6) + (I_5 + I_7) + (I_6 + I_8) + (I_7 + I_9) + (I_8 + I_{10}).$$

$$= \sum_{n=2}^{10} \frac{\tan^{n-1} x}{n-1} = \sum_{n=1}^9 \frac{\tan^n x}{n}$$

148). If $I_n = \int \cot^n x dx$ and $I_0 + I_1 + 2(I_2 + \dots + I_8) + I_9 + I_{10} = A \left(u + \frac{u^2}{2} + \dots + \frac{u^9}{9} \right) + C$, where

$u = \cot x$ and C is an arbitrary constant, then

- 1) A is constant 2) $A = -1$ 3) $A = 1$ 4) A is dependent on x

Key (1,2)

Sol:

$$I_n = \int \cot^n x \, dx = \int \cot^{n-2} x (\sec^2 x - 1) \, dx = \frac{-\cot^{n-1} x}{n-1} - I_{n-2} + C_1$$

$$\Rightarrow I_n + I_{n-2} = \frac{-\cot^{n-1} x}{n-1} + C_1$$

Now, $I_0 + I_1 + 2(I_2 + \dots + I_8) + I_9 + I_{10}$

$$= (I_2 + I_0) + (I_3 + I_1) + (I_4 + I_2) + (I_5 + I_3) + (I_6 + I_4) + (I_7 + I_5) + (I_8 + I_6) + (I_9 + I_7) + (I_{10} + I_8)$$

$$= -\left(\frac{\cot x}{1} + \frac{\cot^2 x}{2} + \dots + \frac{\cot^9 x}{9} \right) + C$$

$$\therefore A = -1$$

149). Let $I_{m,n} = \int \frac{\sin^m x}{\cos^n x} dx (n \neq 1)$ and $I_{m,n} = f(n) \frac{\sin^{m-1} x}{\cos^{n-1} x} + g(m,n) I_{m-2,n-2}$ then

1) $f(n) = \frac{1}{n-1}$

2) $f(n) = \frac{1}{1-n}$

3) $g(m,n) = (m-1)f(n)$

4) $g(m,n) = (1-m)f(n)$

Key (1,4)

Sol.

$$I_{m,n} = \int \frac{\sin^m x}{\cos^n x} dx = \int \underbrace{\sin^{m-1} x}_u \underbrace{\frac{\sin x}{\cos^n x}}_v dx$$

Passage .1

Let $I_{n,m} = \int \sin^n x \cos^m x \, dx$. Then we can relate $I_{n,m}$ with each of the following

(i) $I_{n-2,m}$

ii) $I_{n+2,m}$

iii) $I_{n,m-2}$

iv) $I_{n,m+2}$

v) $I_{n-2,m+2}$

vi) $I_{n+2,m-2}$

Suppose we want to establish a relation between $I_{n,m}$ and $I_{n,m-2}$, then we set

$$P(x) = \sin^{n+1} x \cos^{m-1} x \quad \dots\dots(1)$$

In $I_{n,m}$ and $I_{n,m-2}$ the exponent of $\cos x$ is m and $m-2$ respectively, the minimum of the two is $m-2$ adding 1 to the minimum we get $m-2+1=m-1$. Now choose the exponent $m-1$ of $\cos x$ in $P(x)$. Similarly choose the exponent of $\sin x$ for $P(x)$

Now differentiating both sides of (1), we get

$$\begin{aligned} P'(x) &= (n+1)\sin^n x \cos^m x - (m-1)\sin^{n+2} x \cos^{m-2} x \\ &= (n+1)\sin^n x \cos^m x - (m-1)\sin^n x (1-\cos^2 x) \cos^{m-2} x \\ &= (n+1)\sin^n x \cos^m x - (m-1)\sin^n x \cos^{m-2} x + (m-1)\sin^n x \cos^m x \\ &= (n+m)\sin^n x \cos^m x - (m-1)\sin^n x \cos^{m-2} x \end{aligned}$$

Now integrating both sides, we get

$$\sin^{n+1} x \cos^{m-1} x = (n+m)I_{n,m} - (m-1)I_{n,m-2}$$

Similarly we can establish the other relations

1). The relation between $I_{4,2}$ and $I_{2,2}$ is

$$\begin{aligned} \text{a) } I_{4,2} &= \frac{1}{6}(-\sin^3 x \cos^3 x + 3I_{2,2}) & \text{b) } I_{4,2} &= \frac{1}{6}(\sin^3 x \cos^3 x + 3I_{2,2}) \\ \text{c) } I_{4,2} &= \frac{1}{6}(\sin^3 x \cos^3 x - 3I_{2,2}) & \text{d) } I_{4,2} &= \frac{1}{4}(-\sin^3 x \cos^3 x + 2I_{2,2}) \end{aligned}$$

Key (a)

2). The relation between $I_{4,2}$ and $I_{4,4}$ is

$$\begin{aligned} \text{a) } I_{4,2} &= \frac{1}{3}(\sin^5 x \cos^3 x + 8I_{4,4}) & \text{b) } I_{4,2} &= \frac{1}{3}(-\sin^5 x \cos^3 x + 8I_{4,4}) \\ \text{c) } I_{4,2} &= \frac{1}{3}(\sin^5 x \cos^3 x - 8I_{4,4}) & \text{d) } I_{4,2} &= \frac{1}{3}(\sin^5 x \cos^3 x + 6I_{4,4}) \end{aligned}$$

Key (B)

Follow the instructions in passage

Passage .2

$$\text{It is known that } \sqrt{\tan x} + \sqrt{\cot x} = \begin{cases} \frac{\sqrt{\sin x}}{\sqrt{\cos x}} + \frac{\sqrt{\cos x}}{\sqrt{\sin x}} & \text{if } 0 < x < \frac{\pi}{2} \\ \frac{\sqrt{-\sin x}}{\sqrt{-\cos x}} + \frac{\sqrt{-\cos x}}{\sqrt{-\sin x}} & \text{if } \pi < x < \frac{3\pi}{2} \end{cases},$$

$$\frac{d}{dx}(\sqrt{\tan x} - \sqrt{\cot x}) = \frac{1}{2}(\sqrt{\tan x} + \sqrt{\cot x})(\tan x + \cot x), \forall x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \text{ and}$$

$$\frac{d}{dx}(\sqrt{\tan x} + \sqrt{\cot x}) = \frac{1}{2}(\sqrt{\tan x} - \sqrt{\cot x})(\tan x + \cot x), \forall x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right).$$

3). Value of integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx$, where $x \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right)$ is

- a) $\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + C$ b) $\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}} \right) + C$
 c) $-\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + C$ d) $-\sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} + \sqrt{\cot x}}{\sqrt{2}} \right) + C$

Key (a)

Sol:

$$I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx = 2 \int \frac{(\sqrt{\tan x} + \sqrt{\cot x})(\tan x + \cot x)}{2(\tan x + \cot x)} dx$$

$$I = 2 \int \frac{d(\sqrt{\tan x} - \sqrt{\cot x})}{(\sqrt{\tan x} - \sqrt{\cot x})^2 + 2} = \sqrt{2} \tan^{-1} \left(\frac{\sqrt{\tan x} - \sqrt{\cot x}}{\sqrt{2}} \right) + C$$

4). Value of integral $I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx$, where $x \in \left(\pi, \frac{3\pi}{2}\right)$ is

- a) $\sqrt{2} \sin^{-1} (\cos x - \sin x) + C$ b) $\sqrt{2} \sin^{-1} (\sin x - \cos x) + C$
 c) $\sqrt{2} \sin^{-1} (\sin x + \cos x) + C$ d) $-\sqrt{2} \sin^{-1} (\sin x + \cos x) + C$

Key (a)

Sol:

$$\text{If } x \in \left(\pi, \frac{3\pi}{2}\right), \text{ then } I = \int (\sqrt{\tan x} + \sqrt{\cot x}) dx = \int \left(\frac{\sqrt{-\sin x}}{\sqrt{-\cos x}} + \frac{\sqrt{-\cos x}}{\sqrt{-\sin x}} \right) dx$$

$$= - \int \frac{\sin x + \cos x}{\sqrt{\sin x \cdot \cos x}} dx = - \int \frac{\sqrt{2} d(\sin x - \cos x)}{\sqrt{1 - (\sin x - \cos x)^2}} = \sqrt{2} \sin^{-1} (\cos x - \sin x) + C$$

Passage .3

Integrals of class of functions following a definite pattern can be found by the method of reduction and recursion. Reduction formulas make it possible to reduce an integral dependent on the index $n > 0$, called the order of the integral, to an integral of the same type with a smaller index. Integration by parts helps us to derive reduction formulas.

5). If $I_n = \int \frac{dx}{(x^2 + a^2)^n}$ then $I_{n+1} + \frac{1-2n}{2n} \cdot \frac{1}{a^2} I_n$ is equal to

(A) $\frac{x}{(x^2 + a^2)^n}$ (B) $\frac{1}{2na^2} \frac{1}{(x^2 + a^2)^{n-1}}$ (C) $\frac{1}{2na^2} \frac{1}{(x^2 + a^2)^n}$ (D) $\frac{1}{2na^2} \frac{x}{(x^2 + a^2)^n}$

Key (c)

Sol:

$$I_n = \frac{x}{(x^2 + a^2)^n} + 2n \int \frac{x^2}{(x^2 + a^2)^{n+1}} dx$$

$$= \frac{x}{(x^2 + a^2)^n} + 2n \int \frac{1}{(x^2 + a^2)^n} dx$$

$$2na^2 \int \frac{1}{(x^2 + a^2)^{n+1}} dx$$

Hence, $I_{n+1} + \frac{1-2n}{2n} \frac{1}{a^2} I_n = \frac{1}{2na^2} \cdot \frac{x}{(x^2 + a^2)^n}$

6). If $I_{n,-m} = \int \frac{\sin^n x}{\cos^m x} dx$, then $I_{n,-m} + \frac{n-1}{m-1} I_{n-2,2-m}$ is equal to

(A) $\frac{\sin^{n-1} x}{\cos^{m-1} x}$ (B) $\frac{1}{(m-1)} \frac{\sin^{n-1} x}{\cos^{m-1} x}$ (C) $\frac{1}{(n-1)} \frac{\sin^{n-1} x}{\cos^{m-1} x}$ (D) $\frac{n-1}{m-1} \frac{\sin^{n-1} x}{\cos^{m-1} x}$

Key (B)

Sol:

$$I_{n,-m} = \frac{\sin^{n-1} x}{(m-1) \cos^{m-1} x} - \frac{n-1}{m-1} \int \frac{\sin^{n-2} x}{\cos^{m-2} x} dx$$

$$= \frac{\sin^{n-1} x}{(m-1) \cos^{m-1} x} - \frac{n-1}{m-1} I_{n-2,2-m}$$

7). If $u_n = \int \frac{x^n}{\sqrt{ax^2 + 2bx + c}} dx$, then $(n+1)au_{n+1} + (2n+1)bu_n + nc u_{n-1}$ is equal to

- (A) $x^{n-1}\sqrt{ax^2 + 2bx + c}$ (B) $\frac{x^{n-2}}{\sqrt{ax^2 + 2bx + c}}$ (C) $\frac{x^n}{\sqrt{ax^2 + 2bx + c}}$ (D) $x^n\sqrt{ax^2 + 2bx + c}$

Key (d)

Sol:

$$u_{n+1} = \int \frac{x^{n+1}}{\sqrt{ax^2 + 2bx + c}} dx$$

$$u_{n+1} = \frac{1}{2a} \int \frac{x^n(2ax + 2b) - 2bx^n}{\sqrt{ax^2 + 2bx + c}} dx$$

$$u_{n+1} = \frac{1}{2a} \int \frac{x^n(2ax + 2b)}{\sqrt{ax^2 + 2bx + c}} dx - \frac{b}{a} u_n$$

$$= I_n - \frac{b}{a} u_n, \text{ where } \quad \quad \quad - \quad \quad \quad \mathbf{1}$$

$$I_n = \frac{1}{2a} \int \frac{x^n(2ax + 2b)}{\sqrt{ax^2 + 2bx + c}} dx$$

$$I_n = \frac{1}{2a} \left[x^n \sqrt{ax^2 + 2bx + c} - \int nx^{n-1} 2\sqrt{ax^2 + 2bx + c} \right] dx$$

$$= \frac{x^n}{a} \sqrt{ax^2 + 2bx + c}$$

$$- \frac{n}{a} \int x^{n-1} \frac{(ax^2 + 2bx + c)}{\sqrt{ax^2 + bx + c}} dx$$

$$aI_n = x^n \sqrt{ax^2 + 2bx + c} - na u_{n+1}$$

$$-2bn u_n - nc u_{n-1}$$

Putting this value in (1) we have

$$\begin{aligned}
 a u_{n+1} &= x^n \sqrt{ax^2 + 2bx + c} - nau_{n+1} - 2bnu_n - ncu_{n-1} - bu_n \\
 \Rightarrow (n+1)au_{n+1} + (2n+1)bu_n + ncu_{n-1} \\
 &= x^n \sqrt{ax^2 + 2bx + c}
 \end{aligned}$$

Passage .4

In calculating a number of integrals we had to use the method of integration by parts several times in succession. The result could be obtained more rapidly and in a more concise form by using the so-called generalized formula for integration by parts.

$$\int u(x)v(x)dx = u(x)v_1(x) - u'(x)v_2(x) + u''(x)v_3(x) + \dots + (-1)^{n-1}u^{(n-1)}(x)v_n(x) - (-1)^{n-1} \int u^{(n)}(x)v_n(x)dx$$

Where $v_1(x) = \int (x)dx = \int v_1(x)dx, \dots, v_n(x) = \int v_{n-1}(x)dx$ of

course, we assume that all derivatives and integrals appearing in this formula exist. The use of the generalized formula for integration by parts is especially useful when calculating $\int P_n(x)Q(x)dx$, where $P_n(x)$ is polynomial of degree n and the factor $Q(x)$ is such that it can be integrated successively $n+1$ times.

8). If $\int (x^3 - 2x^2 + 3x - 1)\cos 2x dx = \frac{\sin 2x}{4}u(x) + \frac{\cos 2x}{8}v(x) + C$, then

(A) $u(x) = x^3 - 4x^2 + 3x$

(B) $u(x) = 2x^3 - 4x^2 + 3x$

(C) $v(x) = 3x^2 - 4x + 3$

(D) $v(x) = 6x^2 - 8x$

Key (B)

Sol:

The given integral is equal to

$$(x^3 - 2x^2 + 3x - 1)\frac{\sin 2x}{2} - (3x^2 - 4x + 3)\left(-\frac{\cos 2x}{4}\right) +$$

$$(6x - 4)\left(-\frac{\sin 2x}{8}\right) - 6\frac{\cos 2x}{16} + C$$

$$= \frac{\sin 2x}{4} [2x^3 - 4x^2 + 3x] + \frac{\cos 2x}{8} (6x^2 - 8x + 3) + C$$

9). If $\int (2x^3 + 3x^2 - 8x + 1)\sqrt{2x+6} \, dx = \frac{\sqrt{2x+6}}{5.7.9} (2x+6)f(x) + C$ then $f(x)$ is equal to

(A) $x^3 - 6x^2 - 91x + 297$

(B) $7x^3 - 3x^2 - 132x + 597$

(C) $70x^3 - 45x^2 - 396x + 897$

(D)

$70x^3 - 45x^2 - 132x + 597$

Key (c)

Sol:

. applying the above formula in the comprehension, the given integral is equal to

$$\begin{aligned} & (2x^3 + 3x^2 - 8x + 1) \frac{(2x+6)^{3/2}}{3} - (6x^2 + 6x - 8) \\ & \frac{(2x+6)^{5/2}}{3.5} + (12x+6) \frac{(2x+6)^{7/2}}{3.5.7} - 12 \frac{(2x+6)^{9/2}}{3.5.7.9} + C \\ & \frac{\sqrt{(2x+6)}}{5.7.9} (2x+6) (70x^3 - 45x^2 - 396x + 897) + C \end{aligned}$$

Passage .5

Let $I_n = \int \tan^n x dx, I_n^1 = \int \cot^n x dx, n \in N$

$$f(x, n) = I_n + I_{n-2} (n \neq 1), g(x, n) = I_n^1 + I_{n-2}^1 (n \neq 1), f(0, n) = g\left(\frac{\pi}{2}, n\right) = 0$$

10). $\lim_{x \rightarrow 0} \frac{f(x, 2n+1)}{x^{2n} g(x, 2n+1)} =$

A) 0

B) 1

C) -1

D) 2

Key (a)

11). If $I_0 + I_1 + 2(I_2 + I_3 + \dots + I_{2006} + I_{2009}) + 3I_{2007} + I_{2008} = \sum_{r=1}^{2008} a_r \tan^r x$ then, $a_{2008} =$

- A) 1 / 2007 B) 2 / 2007 C) 1 / 1004 D) 1 / 2008

Key (c)

12). If $I_0^1 + I_1^1 + 2(I_2^1 + I_3^1 + \dots + I_{18}^1) + I_{19}^1 + I_{20}^1 = \sum_{r=1}^{19} a_r \cot^r x$ then $a_{10} =$

- A) 1/10 B) -1/10 C) 1/9 D) -2/9

Key (B)

Sol: $f(x, 2n+1) = \frac{\tan^{2n} x}{2n}, g(x, 2n+1) = -\frac{\cot^{2n} x}{2n}$

$$I_n + I_{n-2} = \frac{\tan^{n-1} x}{n-1}$$

$$I_n^1 + I_{n-2}^1 = -\frac{\cot^{n-1} x}{n-1}$$

Passage .6

Let $f(x) = \int \frac{dx}{e^x + 8e^{-x} + 4e^{-3x}}, g(x) = \int \frac{dx}{e^{3x} + 8e^x + 4e^{-x}}, h(x) = \int \frac{dx}{e^{2x} + 4e^{-2x} + 8}.$

13). $f(x) - 2g(x) =$

A) $\frac{1}{2} \log \left| \frac{e^x + 2e^{-x} - 2}{e^x + 2e^{-x} + 2} \right| + C$

B) $\frac{1}{4\sqrt{3}} \log \left| \frac{e^x - 2e^{-x} - 2\sqrt{3}}{e^x + 2e^{-x} + 2\sqrt{3}} \right| + C$

C) $\frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{e^x - 2e^{-x}}{2\sqrt{3}} \right) + C$

D) $\frac{1}{2} \tan^{-1} \left(\frac{e^x + 2e^{-x}}{2} \right) + C$

Key (d)

$$f(x) - 2g(x) = \int \frac{e^x(e^{2x} - 2)}{e^{4x} + 8e^{2x} + 4} dx \quad \text{put } e^x = t$$

$$= \int \frac{1 - \frac{2}{t^2}}{\left(t + \frac{2}{t}\right)^2 + 4} dt = \frac{1}{2} \tan^{-1} \left(\frac{t + \frac{2}{t}}{2} \right) + C$$

14). $f(x) + 2g(x) =$

A) $\frac{1}{4\sqrt{3}} \log \left| \frac{e^x - 2e^{-x} - 2\sqrt{3}}{e^x + 2e^{-x} + 2\sqrt{3}} \right| + C$

B) $\frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{e^x - 2e^{-x}}{2\sqrt{3}} \right) + C$

C) $\frac{1}{2} \tan^{-1} \left(\frac{e^x + 2e^{-x}}{2} \right) + C$

D) $\frac{1}{4} \log \left| \frac{e^x + 2e^{-x} - 2}{e^x + 2e^{-x} + 2} \right| + C$

Key (B)

$$f(x) + 2g(x) = \int \frac{e^x(e^{2x} + 2)}{e^{4x} + 8e^{2x} + 4} dx \quad \text{put } e^x = t$$

$$= \int \frac{1 + \frac{2}{t^2}}{\left(t - \frac{2}{t}\right)^2 + 12} dt = \frac{1}{2\sqrt{3}} \tan^{-1} \left(\frac{t - \frac{2}{t}}{2\sqrt{3}} \right) + C$$

15). Let $h(x) = \frac{1}{8\sqrt{3}} \log \left| \frac{f(x) - 2\sqrt{3}}{f(x) + 2\sqrt{3}} \right| + C$, then $f(\log_e^2) =$

A) 1

B) 8

C) 4

D) 10

Key (B)

$$h(x) = \frac{1}{2} \int \frac{dt}{t^2 + 8t + 4} \quad \text{where } t = e^{2x}$$

Passage .7

Let n be a non-negative integer and , Let $I_n = \int x^n \sqrt{a^2 - x^2} dx$ ($a > 0$) we can find relation among

I_n, I_{n-1}, I_{n-2} . It can be observed that I_1 is elementary integration whose value is $-\frac{1}{3}(a^2 - x^2)^{3/2}$.

If $I_n = -\frac{x^{n-1}(a^2 - x^2)^{3/2}}{A} + a^2 B I_{n-2}$ where A and B are constants. Then

16). A must be equal to

(A) $n + 1$

(B) $n - 1$

(C) $n + 2$

(D) n

Key (c)

17). B must be equal to

(A) $\frac{n+1}{n+2}$

(B) $\frac{n+2}{n+1}$

(C) $\frac{n}{n+2}$

(D) $\frac{n-1}{n+2}$

Key (d)

18). The value of $\int_0^a x^4 \sqrt{a^2 - x^2} dx$ must be

(A) $\frac{\pi a^6}{32}$

(B) $\frac{\pi a^4}{16}$

(B) $\frac{\pi a^4}{64}$

(D) $\frac{\pi a^6}{16}$

Key (a)

$$I_n = \int x^n \sqrt{a^2 - x^2} dx$$

$$= \int x^{n-1} (x \sqrt{a^2 - x^2}) dx$$

APPLY INTEGRATION BY PARTS

$$I_n = -\frac{1}{3} (a^2 - x^2)^{3/2} x^{n-1} + \frac{n-1}{3} \int x^{n-2} (a^2 - x^2)^{3/2} dx$$

$$= -\frac{1}{3} (a^2 - x^2)^{3/2} x^{n-1} + \frac{n-1}{3} \int (a^2 x^{n-2} \sqrt{a^2 - x^2} - x^n \sqrt{a^2 - x^2}) dx$$

$$I_n = -\frac{1}{3} (a^2 - x^2)^{3/2} x^{n-1} + \frac{n-1}{3} a^2 I_{n-2} - \frac{n-1}{3} I_n$$

$$\Rightarrow I_n = \frac{-(a^2 - x^2)^{3/2} x^{n-1}}{n+2} + \left(\frac{n-1}{n+2} \right) a^2 I_{n-2}$$

$$\Rightarrow A = N + 2, B = \frac{n-1}{n+2}$$

IF $I_n = \int_0^a x^n \sqrt{a^2 - x^2} dx$

$$\Rightarrow I_n = \left(\frac{n-1}{n+2} \right) a^n I_{n-2}$$

$$\Rightarrow I_4 = \frac{3}{6} a^2 I_2$$

$$\Rightarrow I_2 = \frac{1}{4} a^2 I_0$$

$$\Rightarrow I_0 = \frac{\lambda a^2}{4}$$

$$\Rightarrow I_4 = \frac{3}{6} \times \frac{1}{4} \times \frac{\pi}{4} a^6 = \frac{\pi a^6}{32}$$

Passage .8

Let $y^2 = 3x^2 + 2x + 1$, $I_n = \int \frac{x^n}{y} dx$, then $\alpha I_{10} + \beta I_9 + \gamma I_8 = x^9 y$.

- 19). $\alpha =$
 (A) 30 (B) 19 (C) 10
 (D) 9

Key (a)

- 20). $\beta =$
 (A) 30 (B) 19 (C) 10
 (D) 9

Key (B)

- 21). $\gamma =$
 (A) 30 (B) 19 (C) 10
 (D) 9

Key (d)

$$\alpha x^{10} + \beta x^9 + \gamma x^8 - y^2 9x^8 = x^9 y \frac{dy}{dx} \left(\because y \frac{dy}{dx} = 3x + 1 \right).$$

$$\alpha x^{10} + \beta x^9 + \gamma x^8 = (3x^2 + 2x + 1) 9x^8 + x^9 (3x + 1);$$

$$\alpha = 30; \beta = 19, \gamma = 9.$$

Passage .8

If the integrand is a rational function of x and fractional power of a linear fractional function of the form $\frac{ax+b}{cx+d}$ then rationalization of the integral is

affected by the substitution $\frac{ax+b}{cx+d} = t^m$, where m is l.c.m of fractional powers
of $\frac{ax+b}{cx+d}$

22). If $I = \int \frac{(2x-3)^{1/2}}{(2x-3)^{1/3} + 1} dx =$

$3 \left[\frac{1}{7}(2x-3)^{7/6} - \frac{1}{5}(2x-3)^{5/6} + \frac{1}{3}(2x-3)^{1/2} - (2x-3)^{1/6} + g(x) \right] + c$, then g(x) is equal to

a) $\tan^{-1}(2x-3)^{1/6}$

b) $\tan^{-1}(2x-3)^{1/6}$

Key (a)

c) $3 \tan^{-1}(2x-3)^{1/6}$

d) $4(2x-3)^{1/6}$

23). If $I = \int \frac{dx}{\sqrt[4]{(x-1)^3 + (x+2)^5}} = A \sqrt{\frac{x-1}{x+2}} + c$ then A is equal to

a) $\frac{1}{3}$

b) $\frac{2}{3}$

c) $\frac{3}{4}$

d) $\frac{4}{3}$

Key (d)

24). If $I = \int \frac{dx}{\sqrt[3]{(x+1)^2(x-1)^4}} = K \sqrt[3]{\frac{1+x}{1-x}} + c$ then K is equal to

a) $\frac{2}{3}$

b) $\frac{3}{2}$

c) $\frac{1}{3}$

d) $\frac{1}{2}$

Key (B)

Put $\frac{x+2}{x-1} = t$

$$\text{Put } \frac{x+1}{1-x} = t^3$$

Passage .9

We can derive reduction formulas for the integral of the form

$\int \sin^n x dx, \int \tan^n x dx, \int \sec^n x dx$ etc. using integration by parts. In turn these reduction formulas can be used to compute integrals of higher powers of $\sin x, \tan x$ etc.

25). If $\int \sin^5 x dx = -\frac{1}{5} \sin^4 x \cos x + A \sin^2 x \cos x - \frac{8}{15} \cos x + c$ then A is equal to

a) $-\frac{2}{15}$

b) $-\frac{3}{5}$

c) $-\frac{4}{15}$

d) $-\frac{1}{15}$

Key (c)

26). If $\int \tan^6 x dx = \frac{1}{5} \tan^5 x + A \tan^3 x + \tan x - x + c$ then A is equal to

a) $\frac{1}{3}$

b) $\frac{2}{3}$

c) $\frac{-2}{3}$

d) $-\frac{1}{3}$

Key (d)

27). If $\int \sec^6 x dx = \frac{1}{5} \tan^5 x + A \tan^3 x + \tan x + c$ then A is equal to

a) $\frac{1}{3}$

b) $\frac{2}{3}$

c) $-\frac{1}{3}$

d) $-\frac{2}{3}$

Key (B)

$$\sin^5 x = (1 - \cos^2 x)^2 \sin x$$

$$\tan^6 x = (\sec^2 x - 1) \tan^4 x$$

$$\sec^6 x = \sec^4 x \sec^2 x$$

Passage .10

If u and v are two functions in 'x', then $\int uv \, dx = u \int v \, dx - \int \left\{ \frac{du}{dx} \int v \, dx \right\} dx$.

In applying the above rule, care has to be taken in the selection of first function ' u ' and selection of second function ' v ' usually we use the following order for the first function Inverse, logarithmic, algebraic, trigonometric, exponent.

28). $\int \frac{x - \sin x}{1 - \cos x} \, dx =$

a) $-x \cot \frac{x}{2} + c$

b) $\cot \frac{x}{2} + c$

c) $-\cot \frac{x}{2} + c$

d) $x \tan \frac{x}{2} + c$

Key (a)

29). $\int \sec^{-1} \sqrt{x} \, dx =$

a) $x \sec^{-1} \sqrt{x} - \log(1+x) + c$

b) $\sec^{-1} \sqrt{x} - \tan^{-1} \sqrt{x} + c$

c) $x \sec^{-1} \sqrt{x} - \sqrt{x-1} + c$

d) $x \tan^{-1} \sqrt{x} - \sec^{-1} \sqrt{x} + c$

Key (c)

30). $\int \sin^{-1} \sqrt{\frac{x}{a+x}} \, dx =$

$$\begin{array}{ll} \text{a)} & a \left[\sqrt{\frac{x}{a}} \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1} \frac{x}{a} \right] + c \\ \text{b)} & a \left[\tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1} \frac{x}{a} \right] + c \\ \text{c)} & a \left[\frac{x}{a} \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + \tan^{-1} \sqrt{\frac{x}{a}} \right] + c \\ \text{d)} & \tan^{-1} \sqrt{\frac{x}{a}} - \sqrt{\frac{x}{a}} + c \end{array}$$

Key (c)

$$\int \frac{x - \sin x}{1 - \cos x} dx = \int \frac{x}{1 - \cos x} dx - \int \frac{\sin x}{1 - \cos x} dx = \frac{1}{2} \int x \csc^2 \frac{x}{2} - \int \cot \frac{x}{2} dx$$

Put $x = \sec^2 \theta$

Put $x = a \tan^2 \theta \Rightarrow dx = 2a \tan \theta \sec^2 \theta$

Passage .11

Some times it is useful to write the integral as a sum of two related integrals which can be evaluated by making suitable substitutions

Algebraic Twins: $\int \frac{2x^2}{x^4 + 1} dx = \int \frac{x^2 + 1}{x^4 + 1} dx + \int \frac{x^2 - 1}{x^4 + 1} dx$

$$\int \frac{2}{x^4 + 1} dx = \int \frac{x^2 + 1}{x^4 + 1} dx - \int \frac{x^2 - 1}{x^4 + 1} dx$$

$$\int \frac{2x^2}{x^4 + 1 + kx^2} dx, \quad \int \frac{2}{x^4 + 1 + kx^2} dx$$

Trigonometric Twins: $\int \sqrt{\tan x} dx, \int \sqrt{\cos x} dx, \int \frac{1}{\sin^4 x + \cos^4 x} dx, \int \frac{1}{\sin^6 x + \cos^6 x} dx$

31). $\int \frac{x^2 - 1}{x^4 + x^2 + 1} dx =$

$$\text{a) } \frac{1}{4} \log \left| \frac{x^2 - x - 1}{x^2 + x + 1} \right| + c$$

$$\text{b) } \frac{1}{2} \log \left| \frac{x^2 + x + 1}{x^2 - x + 1} \right| + c$$

$$\text{c) } \frac{1}{4} \log \left| \frac{x^2 + x + 1}{x^2 - x + 1} \right| + c$$

$$\text{d) } \frac{1}{2} \log \left| \frac{x^2 - x + 1}{x^2 + x + 1} \right| + c$$

Key (d)

$$32). \int \frac{1}{\cos^6 x + \sin^6 x} dx =$$

$$\text{a) } \tan^{-1}(\tan x - \cot x) + c$$

$$\text{b) } \log |\tan x - \cot x| + c$$

$$\text{c) } \sin^{-1}(\tan x - \cot x) + c$$

$$\text{d) } \cos^{-1}(\tan x - \cot x) + c$$

Key (a)

$$33). \int (\sqrt{\tan x} + \sqrt{\cot x}) dx =$$

$$\text{a) } \tan^{-1}(\sqrt{\tan x} + \sqrt{\cot x}) + c$$

$$\text{b) } \sin^{-1}(\sin x + \cos x) + c$$

$$\text{c) } \sqrt{2} \sin^{-1}(\sin x + \cos x) + c$$

$$\text{d) } \sqrt{2} \sin^{-1}(\sin x - \cos x) + c$$

Key (d)

$$\int \frac{x^2 \left(1 - \frac{1}{x^2}\right)}{x^2 \left(x^2 + 1 + \frac{1}{x^2}\right)} dx = \int \frac{1 - \frac{1}{x^2}}{\left(x + \frac{1}{x}\right)^2 - 1} dx \quad \text{Put } x + \frac{1}{x} = t$$

$$\int (\sqrt{\tan x} + \sqrt{\cot x}) dx = \int \frac{\sin x + \cos x}{\sqrt{\cos x \sin x}} dx \quad \text{Put } \sin x - \cos x = t$$

Passage .12

Integrals of the form $\int f(x), \sqrt{ax^2 + bx + c}$ are calculated with the help of Euler's substitution.

$$\sqrt{ax^2 + bx + c} = t \pm x\sqrt{a} \text{ if } a > 0$$

$$\sqrt{ax^2 + bx + c} = tx \pm \sqrt{c} \text{ if } c > 0$$

$$\sqrt{ax^2 + bx + c} = (x - \alpha)t \text{ if } ax^2 + bx + c = a(x - \alpha)(x - \beta)$$

$$34). \int \frac{(x + \sqrt{1+x^2})^{15}}{\sqrt{1+x^2}} dx =$$

$$(a) \frac{(x + \sqrt{1+x^2})^{16}}{16} + C$$

$$(b) \frac{(x - \sqrt{1+x^2})^{15}}{15} + C$$

$$(c) \frac{(x + \sqrt{1-x^2})^{15}}{15} + C$$

$$(d) \frac{(1 + \sqrt{1-x^2})^{15}}{15} + C$$

Key (c)

$$35). \int \frac{dx}{(x-1)\sqrt{-x^2+3x-2}} =$$

$$(a) -2\sqrt{\frac{x-2}{1-x}} + c$$

$$(b) -2\sqrt{\frac{x-2}{x-1}} + c$$

$$(c) 2\sqrt{\frac{1-x}{x-2}} + c$$

$$(d) \sqrt{\frac{x-1}{x-2}} + c$$

Key (a)

$$36). \int \frac{dx}{x\sqrt{x^2+2x-1}} =$$

$$(a) \frac{1}{\sqrt{x^2+2x-1}} + c$$

$$(b) 2 \tan^{-1} \left(x + \sqrt{x^2+2x-1} \right) + c$$

$$(c) \frac{1}{2\sqrt{3}} \ln \left| \frac{x^2+2x-1}{x^2-2x+1} \right| + c$$

$$(d) \sin^{-1} \left(x + \sqrt{x^2+2x-1} \right) + c$$

Key (B)

$$\sqrt{1+x^2}=t-x \Rightarrow x+\sqrt{1+x^2}=t \Rightarrow 1+\frac{x}{\sqrt{1+x^2}}dx=dt \Rightarrow \frac{dx}{\sqrt{1+x^2}}=\frac{dt}{t} \Rightarrow I=\int t^{14}dt=\frac{t^{15}}{15}+c$$

$$\text{Let } -x^2+3x-2=t(x-2) \Rightarrow t=\sqrt{\frac{1-x}{x-2}} \Rightarrow x=\frac{2t^2+1}{t^2+1} \Rightarrow dx=\frac{2t dt}{(t^2+1)^2} \Rightarrow I=\int \frac{2}{t^2} dt=-\frac{2}{t}+c$$

$$t=x+\sqrt{x^2+2x-1} \Rightarrow x=\frac{t^2+1}{2(t+1)} \Rightarrow \frac{dx}{dt}=\frac{1}{2} \frac{t^2+2t-1}{(1+t)^2} \Rightarrow I=2\int \frac{dt}{(1+t^2)}=2\tan^{-1}t$$

Passage .13

Repeated application of integration by parts give us, the reduction formula if the integrand is depend on a natural number n.

37). If $I_n = \int \cos nx \cdot \sec x \cdot dx$ then $I_n - I_{n-2} =$

- | | |
|--------------------------------|--------------------------------|
| (a) $\frac{2 \sin(n-1)x}{n-1}$ | (b) $\frac{2 \cos(n-1)x}{n-1}$ |
| (c) $\frac{2 \cos(n-1)x}{n}$ | (d) $\frac{2 \cos(n+1)x}{n}$ |

Key (B)

38). If $I_{m,n} = \int x^m (1-x)^{n-1} dx$ then $I_{m,n} - \frac{n-1}{m+n} I_{(m,n-1)} =$

- | | |
|---------------------------------------|---|
| (a) $\frac{x^m (1-x)^{n-2}}{m+n}$ | (b) $\frac{x^{m+1} (1-x)^{n-1}}{m+n}$ |
| (c) $\frac{x^{m-1} (1-x)^{n-1}}{m+n}$ | (d) $\frac{x^{m-1} (1-x)^{n-2}}{m+n-2}$ |

Key (B)

39). If $I_n = \int \frac{x^n}{\sqrt{ax^2+2bx+c}} dx$ then $(n+1)a I_{n+1} + (2n+1)b I_n + nc I_{n-1}$ is equal to

- | | |
|---------------------------------|---|
| (a) $x^{n-1} \sqrt{ax^2+2bx+c}$ | (b) $\frac{x^{n-2}}{\sqrt{ax^2+2bx+c}}$ |
|---------------------------------|---|

(c) $x^n \sqrt{ax^2 + 2bx + c}$

(d) $\frac{x^n}{\sqrt{ax^2 + 2bx + c}}$

Key (c)

Let $P = x^{m+1}(1-x)^{n-1}$

$$\frac{dP}{dx} = -(n-1) \cdot x^{m+1} \cdot (1-x)^{n-2} + (m+1)(1-x)^{n-1} \cdot x^m = (n-1) \cdot (1-x-1)x^m(1-x)^{n-2} + (m+1) \frac{d}{dx} I_{m,n}$$

$$= (n-1) \cdot (1-x)^{n-1} \cdot x^m - x^m (n-1)(1-x)^{n-2} + (m+1) \frac{d}{dx} I_{m,n}$$

$$\frac{dP}{dx} = (m+n) \frac{d}{dx} I_{m,n} - (n-1) \frac{d}{dx} I_{m,n-1}$$

Integrating

$$x^{m+1}(1-x)^{n-1} = (m+n)I_{m,n} - (n-1) \cdot I_{m,n-1}$$

$$I_{n+1} = \frac{1}{2a} \int \frac{x^n(2ax+2b)-2bx^n}{\sqrt{ax^2+2bx+c}} dx = \frac{1}{2a} \int \frac{x^n(2ax+2b)}{\sqrt{ax^2+2bx+c}} - \frac{b}{a} I_n = U_n - \frac{b}{a} I_n$$

$$U_n = \frac{1}{2a} \left(x^n 2\sqrt{ax^2+bx+c} - \int n x^{n-1} 2\sqrt{ax^2+2bx+c} dx \right)$$

$$aU_n = x^n \sqrt{ax^2+bx+c} - naI_{n+1} - 2bnI_n - ncI_{n-1}$$

Passage .14

Consider $\int \frac{x^3 + 3x^2 + 2x + 1}{\sqrt{x^2 + x + 1}} dx = (ax^2 + bx + c)\sqrt{x^2 + x + 1} + \lambda \int \frac{dx}{\sqrt{x^2 + x + 1}}$ then

40). Value of b =

(a) $\frac{13}{12}$

(b) $\frac{12}{5}$

(c) $\frac{1}{3}$

(d) 1

Key (a)

41). Value of c =

(a) $\frac{-13}{6}$

(b) $\frac{-7}{24}$

(c) $\frac{-10}{7}$

(d) 1

Key (B)42). Value of λ

(a) $\frac{1}{16}$

(b) $\frac{10}{3}$

(c) $\frac{10}{7}$

(d) 1

Key (a)

differentiating both side

$$\frac{x^3 + 3x^2 + 2x + 1}{\sqrt{x^2 + x + 1}} = \frac{(ax^2 + bx + c)(2x + 1)}{2\sqrt{x^2 + x + 1}} + (2ax + b)\sqrt{x^2 + x + 1} + \lambda \frac{1}{\sqrt{x^2 + x + 1}}$$

$$= \frac{6ax^3 + (5a + 4b)x^2 + (4a + 3b + 2c)x + (c + 2b + 2\lambda)}{2\sqrt{x^2 + x + 1}}$$

comparing coefficient of x on both sides.