

HYPERBOLA

(FOCI, ECCENTRICITY, LATUSRECTUM, DIRECTRICES)

1). The length of latus-rectum of the conic $\left| \sqrt{x^2 + y^2} - \sqrt{(x-1)^2 + (y-3)^2} \right| = \sqrt{5}$ is

- 1) $\sqrt{10}$ 2) $\sqrt{5}$ 3) $5\sqrt{2}$ 4) cannot be evaluated.

Key (2)

Sol The given conic is hyperbola

$$2a = \sqrt{5} \quad \text{and} \quad 2ae = \sqrt{10}$$

$$\Rightarrow e = \sqrt{2}$$

$$\text{Length of latus-rectum} = 2a = \sqrt{5}.$$

2). If point P(x, y) is such that it moves on a hyperbola

$$\left| \sqrt{(x-3)^2 + (y-4)^2} - \sqrt{x^2 + y^2} \right| = k^2 + 1,$$

then number of possible integral values of k is

Key (3)

Sol $|PS_1 - PS_2| = \text{const (A)}$

for hyperbola

$$k^2 + 1 < 5, \quad k^2 < 4$$

$$-2 < k < 2 \quad \text{No. of integral values is 3.}$$

3). If e is the eccentricity of the hyperbola $(5x - 10)^2 + (5y + 15)^2 = (12x - 5y + 1)^2$ then $\frac{25e}{13}$ is equal to

Key (5)

Sol Equation can be rewritten as $\sqrt{(x-2)^2 + (y+3)^2} = \frac{13}{5} \left| \frac{12x-5y+1}{13} \right|$

So, $e = \frac{13}{5}$.

4). Length of the latus rectum of the hyperbola

$$(10x-5)^2 + (10y-2)^2 = 9(3x+4y-7)^2 \text{ is}$$

(1) $\frac{282}{100}$ (2) $\frac{141}{100}$ (3) $\frac{228}{100}$ (4) $\frac{822}{100}$

Key (1)

Sol

$$\begin{aligned} (10x-5)^2 + (10y-2)^2 &= 9(3x+4y-7)^2 \\ \text{Given, hyperbola } \Rightarrow \left[x - \frac{1}{2}\right]^2 + \left[y - \frac{1}{5}\right]^2 &= \frac{9}{4} \left[\frac{3x+4y-7}{5}\right]^2 \end{aligned}$$

$\Rightarrow$ Given curve is a hyperbola where focus is $\left[\frac{1}{2}, \frac{1}{5}\right]$ and directrix is $3x+4y-7=0$. Latus rectum is a line passing through the focus and parallel to the directrix.

$$\Rightarrow \text{Eq. of the latus rectum is } y - \frac{1}{5} = -\frac{3}{4} \left[x - \frac{1}{2}\right]$$

5). The eccentricity of the conic defined by

$$\left| \sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2} \right| = 3$$

1) $5/2$ 2) $5/3$ 3) $\sqrt{2}$ 4) $\sqrt{11}/3$

Key (2)

Sol Hyperbola for which (1, 2) and (5, 5) are foci and length of transverse axis 3.
 $2ae = 5$ and $2a = 3 \quad \therefore e = 5/3$

6). The equation of the transverse axis of the hyperbola

$$(x-3)^2 + (y+1)^2 = (4x+3y)^2 \text{ is}$$

$$1) x + 3y = 0 \quad 2) 4x + 3y = 9 \quad 3) 3x - 4y = 13 \quad 4) 4x + 3y =$$

Key (3)

Sol $(x-3)^2 + (y+1)^2 = (4x+3y)^2$

$$(x-3)^2 + (y+1)^2 = 25 \left(\frac{4x+3y}{5} \right)^2$$

$$PS = 5PM$$

$\therefore$ directrix is $4x + 3y = 0$ and focus $(3, -1)$

So transverse axis has slope $= \frac{3}{4}$ and equation of transverse axis $y+1 = \frac{3}{4}(x-3)$

7). The eccentricity of the hyperbola $9x^2 - 16y^2 - 72x + 96y - 144 = 0$ is

$$1) \frac{3}{2} \quad 2) \frac{4}{3} \quad 3) \frac{6}{5} \quad 4) \frac{5}{4}$$

Key (4)

Sol The given equation can be written as

$$\frac{(x-4)^2}{16} - \frac{(y+3)^2}{9} = 1$$

Therefore, the eccentricity is given by $e = \sqrt{\frac{16+9}{16}} = \frac{5}{4}$

8). The eccentricity of hyperbola given by the locus of point of intersection of lines $\sqrt{3}x - y = 4\sqrt{3}t$ and $\sqrt{3}tx + ty = 4\sqrt{3}$ (where t is a parameter) is

$$1) \sqrt{3} \quad 2) 2 \quad 3) \frac{2}{\sqrt{3}} \quad 4) \frac{4}{3}$$

Key (2)

Sol By eliminating t from the given equations we get

The eccentricity of the conic defined

$$\text{by } \left| \sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2} \right| = 3$$

9). Consider a branch of the hyperbola $x^2 - 2y^2 - 2\sqrt{2}x - 4\sqrt{2}y - 6 = 0$ with vertex at the point A. Let B be one of the end points of its latus rectum. If C is the

focus of the hyperbola nearest to the point A, then the area of the triangle ABC is

1) $1 - \sqrt{\frac{2}{3}}$

2) $\sqrt{\frac{3}{2}} - 1$

3) $1 + \sqrt{\frac{2}{3}}$

4) $\sqrt{\frac{3}{2}} + 1$

Key (2)

Sol

$$x^2 - 2\sqrt{2}x - 2(y^2 + 2\sqrt{2}y) = 6$$

$$\Rightarrow (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 - 2 + 4 = 6$$

$$\Rightarrow (x - \sqrt{2})^2 - 2(y + \sqrt{2})^2 = 4$$

$$\Rightarrow \frac{(x - \sqrt{2})^2}{4} - \frac{(y + \sqrt{2})^2}{2} = 1$$

$$b^2 = a^2 (e^2 - 1)$$

$$\Rightarrow 2 = 4 (e^2 - 1)$$

$$\Rightarrow e^2 - 1 = 1/2$$

$$e = \sqrt{3}/2$$

$$\text{area} = \frac{1}{2} (ae - a) \times b^2/a$$

$$= (e - 1) = \left(\sqrt{\frac{3}{2}} - 1 \right)$$

10). For the hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1$ where the value of α changes,

which of the following remain constant?

1) Abscissae of the vertices

2) Abscissae of the foci

3) Eccentricity

4) Directrices

Key (2)

Sol

$$\text{Foci} = (\pm ae, 0) = (\pm \sqrt{a^2 + b^2}, 0)$$

$$= (\pm \sqrt{\cos^2 \alpha + \sin^2 \alpha}, 0) = (\pm 1, 0)$$

11). The equations of the transverse and conjugate axes of a hyperbola respectively

are $x + 2y - 3 = 0$ and $2x - y + 4 = 0$ and their respective lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$ The equation of the hyperbola is

$$1) \frac{2}{5} (x + 2y - 3)^2 - \frac{3}{5} (2x - y + 4)^2 = 1$$

$$2) \frac{2}{5} (2x - y + 4)^2 - \frac{3}{5} (x + 2y - 3)^2 = 1$$

$$3) 2(2x - y + 4)^2 - 3(x + 2y - 3)^2 = 1$$

$$4) 2(x + 2y - 3)^2 - 3(2x - y + 4)^2 = 1$$

Key (2)

Sol

The equation of the hyperbola is

$$\frac{\left(\frac{|2x - y + 4|}{\sqrt{5}}\right)^2}{\left(\frac{\sqrt{2}}{2}\right)^2} - \frac{\left(\frac{|x + 2y - 3|}{\sqrt{5}}\right)^2}{\left(\frac{2}{\sqrt{3}} \cdot \frac{1}{2}\right)^2} = 1$$

12). If the equation to the hyperbola is $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$ then equation to the conjugate hyperbola is

$$1) 3x^2 - 5xy - 2y^2 + 5x + 11y - 16 = 0$$

$$2) 3x^2 - 5xy - 2y^2 + 5x + 11y - 12 = 0$$

$$3) 3x^2 - 5xy - 2y^2 + 5x + 11y - 4 = 0$$

$$4) 3x^2 - 5xy - 2y^2 + 5x + 11y - 20 = 0$$

Key (1)

Sol

$3x^2 - 5xy - 2y^2 + 5x + 11y + c = 0$ be the equation to the pair of asymptotes then $c = -12$. And hence equation to the conjugate hyperbola is $3x^2 - 5xy - 2y^2 + 5x + 11y - 16 = 0$

13). A hyperbola has centre 'C' and one focus at $P(6,8)$. If its two directrices are $3x + 4y + 10 = 0$ and $3x + 4y - 10 = 0$ then $CP =$

$$1) 14$$

$$2) 13$$

$$3) 10$$

$$4) 11$$

Key (1)

Sol

$$\begin{aligned}\frac{2a}{e} = 4 &\Rightarrow a = 2e, P \text{ is nearest to } 3x + 4y - 10 = 0 \\ \Rightarrow ae - \frac{a}{e} &= 8 \Rightarrow e = \sqrt{5}, a = 2\sqrt{5} \\ CP &= ae = 10\end{aligned}$$

- 14). PQ is a double ordinate of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ such that OPQ is an equilateral triangle, O being the centre of the hyperbola, then the eccentricity e of the hyperbola satisfies

1) $1 < e < \frac{2}{\sqrt{3}}$ 2) $e = \frac{2}{\sqrt{3}}$ 3) $e = \frac{\sqrt{3}}{2}$ 4) $e > \frac{2}{\sqrt{3}}$

Key (4)

Sol

Let P be $(\sqrt{3}l, l)$ then $\left[\frac{3}{a^2} - \frac{1}{b^2} \right] = \frac{1}{l^2}$

$$\Rightarrow \frac{3}{a^2} - \frac{1}{b^2} > 0$$

$$\Rightarrow \frac{b^2}{a^2} > \frac{1}{3} \Rightarrow e^2 > \frac{4}{3}$$

- 15). The transverse axis of a hyperbola is of length 2a and a vertex divides the segment of the axis between the centre and the corresponding focus in the ratio 2: 1, the equation of the hyperbola is

1) $4x^2 - 5y^2 = 4a^2$ 2) $4x^2 - 5y^2 = 5a^2$ 3) $5x^2 - 4y^2 = 4a^2$ 4) $5x^2 - 4y^2 = 5a^2$

Key (4)

Sol

Clearly $\frac{2ae}{3} = a \Rightarrow e = \frac{3}{2}$

$$\therefore S = \left(\frac{3a}{2}, 0 \right)$$

Directrix is $x = \frac{2a}{3}$

$\therefore$ equation of hyperbola will be $\left(x - \frac{3a}{2}\right)^2 + y^2 = \frac{9}{4}\left(x - \frac{2a}{3}\right)^2$

Which reduces to $5x^2 - 4y^2 = 5a^2$

- 16). If the latusrectum of a hyperbola forms an equilateral triangle with the centre of the hyperbola, then the eccentricity is

1) $\frac{\sqrt{3}-1}{2\sqrt{3}}$ 2) $\frac{\sqrt{3}+1}{2\sqrt{3}}$ 3) $\frac{\sqrt{13}+1}{2\sqrt{3}}$ 4) $\frac{\sqrt{3}-1}{\sqrt{3}}$

Key (3)

Sol $\tan^{30} = \frac{(b^2/a)}{ae} \Rightarrow \frac{1}{\sqrt{3}} = \frac{a^2(e^2-1)}{a^2e} \Rightarrow \sqrt{3}e^2 - e - \sqrt{3} = 0$

$$e = \frac{1 \pm \sqrt{1+12}}{2\sqrt{3}} = \frac{1 + \sqrt{13}}{2\sqrt{3}}$$

- 17). A hyperbola has its axes along the coordinate axes, length of latus rectum is 8 and conjugate axis is half of the distance between the foci. Then the eccentricity is

(1) $\frac{2}{\sqrt{3}}$ 2) $\frac{3}{\sqrt{2}}$ 3) $\frac{4}{\sqrt{3}}$ 4) $\frac{4}{3}$

Key (1)

Sol $\frac{2b^2}{a} = 8$

$$\Rightarrow b^2 = 4a \quad \text{----- (1)}$$

and $2b = ae$

$$\Rightarrow b = \frac{ae}{2} \quad \text{----- (2)}$$

From eq (1), we have $a^2(e^2 - 1) = 4a$

$$\Rightarrow a = \frac{4}{e^2 - 1} \quad \text{----- (3)}$$

From eq (1) and (3), we have

$$\frac{a^2 e^2}{4} = b^2 = 4a$$

$$\Rightarrow a = \frac{16}{e^2} \quad \text{----- (4)}$$

From eq (3) and (4), we have

$$\frac{4}{e^2 - 1} = \frac{16}{e^2}$$

$$\Rightarrow e^2 = 4e^2 - 4$$

$$\Rightarrow e = \frac{2}{\sqrt{3}}$$

- 18). A hyperbola has center C and one focus at P(6, 8). If it's two directrices are $3x + 4y + 10 = 0$ and $3x + 4y - 10 = 0$, then

1) CP = 10 2) eccentricity = $\sqrt{5}$ 3) CP = 1) eccentricity = $\frac{3}{2}$

Key (4)

Sol A.M of distances of focus from two directrices is CP i.e. 10

$$\text{CP/distance between two directrices} = \frac{(\text{eccentricity})^2}{2}$$

$$\Rightarrow \frac{10}{4} = \frac{e^2}{2} \Rightarrow e = \sqrt{5}$$

- 19). The foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{b^2} = 1$ and the hyperbola $\frac{x^2}{144} - \frac{y^2}{81} = \frac{1}{25}$ coincide then the value of b^2 is

1) 1 2) 5 3) 7 4) 9

Key (3)

Sol $16 - b^2 = \frac{144 \times 81}{25} \Rightarrow b^2 = 7$

- 20). The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at (0, 3) is

1) 3 2) 4) 5 4) $2\sqrt{3}$

Key (2)

Sol $e = \sqrt{\frac{a^2 - b^2}{a^2}} \Rightarrow ae = \sqrt{16 - 9} = \sqrt{7}$

$\therefore$ Foci $\pm\sqrt{7}, 0$ lies on a circle with centre $0, 3$

$\therefore r = \sqrt{7 + 9} = 4$

- 21). If the focal distance of an end of the minor axis of any ellipse (referred to its axes as the axes of x and y respectively) is k and the distance between the foci is $2h$, then its equation is

1) $\frac{x^2}{k^2} + \frac{y^2}{k^2 + h^2} = 1$ 2) $\frac{x^2}{k^2} + \frac{y^2}{h^2 - k^2} = 1$ 3) $\frac{x^2}{k^2} + \frac{y^2}{k^2 - h^2} = 1$ 4) $\frac{x^2}{k^2} + \frac{y^2}{h^2} = 1$

Key (3)

Sol $B(0, b)$

Focal distance $a + e(o) = k$

$a = k$

$2ae = 2h$

$ae = h$

$b^2 = a^2(1 - e^2)$

$b^2 = k^2 - h^2$

- 22). F is one of the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where AB is the major axis and CD is the minor axis and O is the centre of the ellipse. If $OF = 6$ and the in-radius of $\triangle OCF$ is 1, then the product of the lengths of AB and CD is

1) 60

2) 65

3) 70

4) 75

Key (2)

Sol $ae = 6$

$CF = a$

$r = \frac{\Delta}{s}, r = 1$

$\Rightarrow \Delta = S$

$\frac{1}{2} \times 6 \times b = \frac{b + 6 + a}{2}$

$$5b = a + 6$$

$$25b^2 = a^2 + 36 + 12a$$

$$25(a^2 - a^2e^2) = a^2 + 36 + 12a$$

$$25(a^2 - 36) = a^2 + 36 + 12a$$

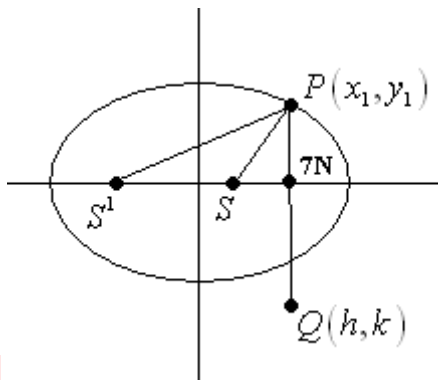
$$\Rightarrow a = \frac{13}{2}, b = \frac{5}{2}$$

$$AB \times CD = 13 \times 5 = 65$$

- 23). From any point P lying in first quadrant on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, PN is drawn perpendicular to the major axis such that N lies on major axis. Now PN is produced to the point Q such that NQ equals to PS, where S is a focus. The point Q lies on which of the following lines

1) $2y - 3x - 25 = 0$ 2) $3x + 5y + 25 = 0$ 3) $2x - 5y - 25 = 0$ 4) $2x - 5y + 25 = 0$

Key (2)



Sol

$$a^2 = 25$$

$$b^2 = 16$$

$$e = \sqrt{\frac{25-16}{25}} = \frac{3}{5}$$

Let point Q be (h, k), where K < 0

Given that $|K| = a + eh$ (as $x_1 = h$)

$$-y = a + ex$$

$$-y = 5 + \frac{3}{5}x$$

$$3x + 5y + 25 = 0$$

24). The locus of the point of intersection of the line $\sqrt{3}x - y - 4\sqrt{3}K = 0$ and $\sqrt{3}Kx + Ky - 4\sqrt{3} = 0$ is a hyperbola of eccentricity is

1) 1

2) 2

3) 2.5

4) $\sqrt{3}$

Key (2)

Sol

$$K = \frac{\sqrt{3}x - y}{4\sqrt{3}} = \frac{4\sqrt{3}}{\sqrt{3}x - y}$$

$$\Rightarrow 3x^2 - y^2 = 48 \Rightarrow \frac{x^2}{16} - \frac{y^2}{48} = 1$$

$$48 = 16(e^2 - 1) \Rightarrow e = 2$$

25). If (5,12) and (24,7) are the foci of a hyperbola passing through origin, then its

1) Eccentricity = $\frac{\sqrt{386}}{13}$

2) Eccentricity = $\frac{\sqrt{386}}{12}$

3) Length of latus rectum = $\frac{121}{6}$ units 4) Length of conjugate axis = $\sqrt{242}$

Key (2,3)

Sol

$$|SP - S^1P| = 12$$

$$2ae = \sqrt{386} \quad e = \frac{\sqrt{386}}{12} \quad 2a = 12 \quad a = 6$$

$$\frac{2b^2}{a} = \frac{121}{6}$$

26). A hyperbola has center C and one focus at P(6,8). If it's two directrices are

$$3x + 4y + 10 = 0 \text{ and } 3x + 4y - 10 = 0, \text{ then}$$

1) CP = 10

2) eccentricity = $\sqrt{5}$

3) CP = 8

4) eccentricity = $\sqrt{5}/2$

Key (1,2)

Sol

A.M. of distances of focus from two directrices is CP i.e. 10

$$\text{CP/distance between two directrices} = \frac{(\text{eccentricity})^2}{2}$$

$$\Rightarrow \frac{10}{4} = \frac{e^2}{2} \Rightarrow e = \sqrt{5}$$

27). A hyperbola having the transverse axis of length $\frac{1}{2}$ unit is confocal with the ellipse $3x^2 + 4y^2 = 12$, then

- 1) Equation of the hyperbola is $\frac{x^2}{15} - \frac{y^2}{1} = \frac{1}{16}$
- 2) Eccentricity of the hyperbola is 4
- 3) Distance between the directrices of the hyperbola is $\frac{1}{8}$ units
- 4) Length of latus rectum of the hyperbola is $\frac{15}{2}$ units

Key (2,3,4)

Sol

$$\text{Ellipse is } \frac{x^2}{4} + \frac{y^2}{3} = 1$$

$$\text{Here } \frac{3}{4} = 1 - e^2 \Rightarrow e = \frac{1}{2}$$

Foci are $(\pm 1, 0)$

Now the hyperbola is having same focus i.e. $(\pm 1, 0)$. Let e_1 be the eccentricity of hyperbola

$$2ae_1 = 2$$

$$\text{But } 2a = \frac{1}{2} \text{ So, } e_1 = 4$$

$$b^2 = a^2(e_1^2 - 1) = \frac{1}{16}(16 - 1) = \frac{15}{16}$$

So, the equation of the hyperbola is

$$\frac{x^2}{\frac{1}{16}} - \frac{y^2}{\frac{15}{16}} = 1 \Rightarrow \frac{x^2}{1} - \frac{y^2}{15} = \frac{1}{16}$$

$$\text{Its distance between the directrices} = \frac{2a}{e_1} = \frac{1}{2 \times 4} = \frac{1}{8} \text{ units}$$

$$\text{Length of latus-rectum} = \frac{2b^2}{a}$$

$$= \frac{2 \times 15 \times 4}{16 \times 1} = \frac{15}{2} \text{ units}$$

28). The equation $16x^2 - 3y^2 - 32x - 12y - 44 = 0$ represents a hyperbola with

1) length of transverse axis $= 2\sqrt{3}$ 2) length of conjugate axis $= 8$

3) centre at $(1, -2)$

4) eccentricity $= \sqrt{19}$

Key (1,2,3)

Sol
$$\frac{(x-1)^2}{(\sqrt{3})^2} - \frac{(y+2)^2}{4^2} = 1$$

$$2a = 2\sqrt{3}, 2b = 8$$

Centre $(1, -2)$

$$e = \sqrt{19/3}$$

29). For the hyperbola $9x^2 - 16y^2 - 18x + 32y - 151 = 0$

1) one of the directrix is $x = \frac{21}{5}$ 2) Length of latus rectum $= \frac{9}{2}$

3) Foci are $(6, 1)$ and $(-4, 1)$ 4) eccentricity is $\frac{5}{4}$

Key (1,2,3,4)

Sol Given hyperbola can be written as
$$\frac{(x-1)^2}{16} - \frac{(y-1)^2}{9} = 1$$

$$\Rightarrow \frac{X^2}{16} - \frac{Y^2}{9} = 1 \quad \{\text{where } X = x - 1, Y = y - 1\}$$

$$e = \sqrt{1 + \frac{b^2}{a^2}} = \sqrt{1 + \frac{9}{16}} = \frac{5}{4}$$

Directrices are $X = \pm \frac{a}{e}$

$$\Rightarrow x - 1 = \pm \frac{16}{5} \Rightarrow x = \frac{21}{5} \text{ and } x = -\frac{11}{5}$$

$$\text{Length of Latus rectum} = \frac{2b^2}{a} = \frac{9}{2}$$

and foci are

$$\Rightarrow X = \pm ae, Y = 0 \Rightarrow (6, 1) \text{ and } (-4, 1)$$

$$\Rightarrow 3x - 4y = 13$$

- 30). If the latusrectum of a hyperbola forms an equilateral triangle with the centre of the hyperbola, then the eccentricity is

1) $\frac{\sqrt{3}-1}{2\sqrt{3}}$ 2) $\frac{\sqrt{3}+1}{2\sqrt{3}}$ 3) $\frac{\sqrt{13}+1}{2\sqrt{3}}$ 4) $\frac{\sqrt{3}-1}{\sqrt{3}}$

Key (3)

Sol $\tan^{30} = \frac{(b^2/a)}{ae} \Rightarrow \frac{1}{\sqrt{3}} = \frac{a^2(e^2-1)}{a^2e} \Rightarrow \sqrt{3}e^2 - e - \sqrt{3} = 0$

$$e = \frac{1 \pm \sqrt{1+12}}{2\sqrt{3}} = \frac{1+\sqrt{13}}{2\sqrt{3}}$$

- 31). Let the length of major axis of an ellipse equal to the length of transverse axis of a hyperbola and their director circle have radius equal to $2R$ and R respectively. If e_1 and e_2 are the eccentricities of the ellipse and hyperbola then correct relation is

1) $4e_1^2 - e_2^2 = 6$ 2) $e_1^2 - 4e_2^2 = 6$
 3) $2e_1^3 - e_2^2 = 4$ 4) $4e_2^2 - e_1^2 = 6$

Key (4)

Sol $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{B^2} = 1$

$$a^2 + b^2 = 4R^2 = a^2(2 - e_1^2) \text{ and } a^2 - B^2 = R^2 = a^2(2 - e_2^2)$$

$$4 = \frac{2 - e_1^2}{2 - e_2^2} \Rightarrow 4e_2^2 - e_1^2 = 6$$

32). If radii of director circles of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{a^2} - \frac{y^2}{(b')^2} = 1$ are $2r$ and r respectively and e_e and e_h be the eccentricities of the ellipse and the hyperbola respectively then

- 1) $2e_n^2 - e_e^2 = 6$ 2) $e_e^2 - 4e_n^2 = 6$ 3) $4e_n^2 - e_e^2 = 6$ 4) none of these

Key (3)

Sol Equation of director circles of ellipse and hyperbola are respectively.

$$x^2 + y^2 = a^2 + b^2 \quad \text{and} \quad x^2 + y^2 = a^2 - b^2$$

$$a^2 + b^2 = 4r^2 \quad \dots(1)$$

$$a^2 - b^2 = r^2 \quad \dots(2)$$

$$\text{So } 2a^2 = 5r^2$$

$$a^2 = \frac{5r^2}{2}$$

$$b^2 = 4r^2 - \frac{5r^2}{2}$$

$$b^2 = \frac{3r^2}{2}$$

$$e_n^2 = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow e_n^2 = 1 - \frac{3r^2}{2} \times \frac{2}{5r^2} = 1 - \frac{3}{5} = \frac{2}{5}$$

$$e_n^2 = 1 + \frac{b^2}{a^2}$$

$$\Rightarrow e_n^2 = 1 + \frac{3}{5} = \frac{8}{5}$$

$$\text{So } 4e_n^2 - e_e^2 = 4 \times \frac{8}{5} - \frac{2}{5} = \frac{30}{5} = 6$$

33). The equations of the transverse and conjugate axes of a hyperbola are respectively $x + 2y - 3 = 0$, $2x - y + 4 = 0$, and their

respective lengths are $\sqrt{2}$ and $\frac{2}{\sqrt{3}}$. The equation of the hyperbola is

$$(1) \frac{2}{5}(x+2y-3)^2 - \frac{3}{5}(2x-y+4)^2 = 1$$

$$(2) \frac{2}{5}(2x-y+4)^2 - \frac{3}{5}(x+2y-3)^2 = 1$$

$$(3) 2(2x-y+4)^2 - 3(x+2y-3)^2 = 1$$

$$(4) 2(x+2y-3)^2 - 3(2x-y+4)^2 = 1$$

Key (2)

Sol The equation of the hyperbola is

$$\frac{\left(\frac{2x-y+4}{\sqrt{5}}\right)^2}{\frac{1}{2}} - \frac{\left(\frac{x+2y-3}{\sqrt{5}}\right)^2}{\frac{1}{3}} = 1$$

$$\text{Or } \frac{2}{5}(2x-y+4)^2 - \frac{3}{5}(x+2y-3)^2 = 1$$

34). Let F_1, F_2 are the foci of the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$ and F_3, F_4 are the foci of its conjugate hyperbola. If e_H and e_C are their eccentricities respectively then statement which holds true

1) Their equation of asymptotes are different 2) $e_H > e_C$

3) Area of the quadrilateral formed by their foci is 50squnits

4) Their auxiliary circle will have the same equation

Key (3)

Sol $e_H = \frac{5}{4}$, $F_1 = (5,0)$ and $F_2 = (-5,0)$

$$e_C = \frac{5}{3}, F_3 = (0,5) \text{ and } F_4 = (0,-5)$$

$$\text{Area of } F_1F_2F_3F_4 = 50$$

35). For each positive integer n , consider the point P with abscissa n on the curve $y^2 - x^2 = 1$. If d_n represents the shortest distance from point P to the line $y = x$ then $\lim_{n \rightarrow \infty} (n.d_n)$ has the value equal to

1) $\frac{1}{2\sqrt{2}}$ 2) $\frac{1}{2}$ 3) $\frac{1}{\sqrt{2}}$ 4) 0

Key (1)

Sol . Curve is rectangular hyperbola

$$\text{Perpendicular distance , } d_n = \left| \frac{n - \sqrt{n^2 + 1}}{\sqrt{2}} \right|$$

$$\lim_{n \rightarrow \infty} (n.d_n) = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{2}} \left| \sqrt{n^2 + 1} - n \right| = \lim_{n \rightarrow \infty} \frac{n}{\sqrt{2}} \left| \frac{1}{\sqrt{n^2 + 1} + n} \right| = \frac{1}{2\sqrt{2}}$$

36). If $H(x,y) = 0$ represents the equation of hyperbola and $A(x,y) = 0, C(x,y) = 0$ the equation of its asymptotes and conjugate hyperbola respectively the for any points (α, β) in the plane ; $H(\alpha, \beta), A(\alpha, \beta), C(\alpha, \beta)$ are in

1) Constant 2) H.P 3) G.P 4) A.P

Key (3)

Sol

Hyperbola + Conjugate Hyperbola = 2 Asymptotes

$$H(\alpha, \beta) + C(\alpha, \beta) = 2 A(\alpha, \beta)$$

37). If the eccentricity of the hyperbola $x^2 - y^2 \sec^2 \alpha = 5$ is $\sqrt{3}$ times the eccentricity of the

ellipse $x^2 \sec^2 \alpha + y^2 = 25$, then the value of α is:

- (1) $\pi/6$ (2) $\pi/4$ (3) $\pi/3$ (4) $\pi/2$

Key (2)

Sol

$$55. \frac{x^2}{1} - \frac{y^2}{\cos^2 \alpha} = 5;$$

$$\cos^2 \alpha + 1 = e_1^2$$

$$\frac{x^2}{\cos^2 \alpha} + \frac{y^2}{1} = 25$$

$$: 1(1 - e_2^2) \cos^2 \alpha \Rightarrow e_2^2 = \sin^2 \alpha$$

$$e_1^2 = 3e_2^2 \Rightarrow \cos^2 \alpha + 1$$

$$= 3 - 3\cos^2 \alpha \Rightarrow \cos^2 \alpha = \frac{1}{2}$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{2}}$$

CHORDS

38). Locus of the mid points of the chords of the hyperbola $x^2 - y^2 = a^2$ that touch

the parabola $y^2 = 4ax$ is

$$1) x^2(x - a) = y^3$$

$$2) y^2(x - a) = x^3$$

$$3) x^3(x - a) = y^2$$

$$4) y^3(x - a) = x^2$$

Key (2)

Sol

Let the mid point = (h, k)

$\therefore$ Equation of the chord $xh - yk = h^2 - k^2$

$$yk = xh + (k^2 - h^2)$$

$$y = \frac{xh}{k} + \frac{(k^2 - h^2)}{k} \quad \frac{k^2 - h^2}{k} = \frac{ak}{h}$$

$$\Rightarrow k^2h - h^3 = ak^2 \Rightarrow k^2(h - a) = h^3 \therefore x^3 = y^2(x - a)$$

39) The locus of the mid points of the chords passing through a fixed point

(α, β) of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ is

1) a circle with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

2) an ellipse with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

3) a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

4) straight line passing through $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

Key(3)

Sol Let (h, k) be the mid point

$$\therefore T = S_1 \Rightarrow \frac{xh}{a^2} - \frac{yk}{b^2} = \frac{h^2}{a^2} - \frac{k^2}{b^2} \quad \dots(i)$$

(1) passes through (α, β) so putting (α, β) in it

$$\Rightarrow \frac{\alpha x}{a^2} - \frac{\beta y}{b^2} = \frac{x^2}{a^2} - \frac{y^2}{b^2} \Rightarrow \left(\frac{x^2}{a^2} - \frac{\alpha x}{a^2}\right) - \left(\frac{y^2}{b^2} - \frac{\beta y}{b^2}\right) = 0$$

$$\Rightarrow \frac{\left(x - \frac{\alpha}{2}\right)^2}{a^2} - \frac{\left(y - \frac{\beta}{2}\right)^2}{b^2} + \frac{\alpha^2}{4a^2} - \frac{\beta^2}{4b^2} = 0$$

Which is a hyperbola with centre $\left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

40). If $P(\theta_1)$ and $Q(\theta_2)$ are the extremities of any focal chord of the

hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ then $\cos^2 \frac{\theta_1 + \theta_2}{2} = \lambda \cos^2 \frac{\theta_1 - \theta_2}{2}$, where λ is equal

to

$$1) \frac{a^2 + b^2}{a^2} \quad 2) \frac{a^2 + b^2}{b^2} \quad 3) \frac{a^2 + b^2}{ab} \quad 4) \frac{a^2 + b^2}{2ab}$$

Key (1)

Sol Equation of any chord joining the points $P(\theta_1)$ and $Q(\theta_2)$ is,

$$\frac{x}{a} \cos\left(\frac{\theta_1 - \theta_2}{2}\right) - \frac{y}{b} \sin\left(\frac{\theta_1 + \theta_2}{2}\right) = \cos\left(\frac{\theta_1 + \theta_2}{2}\right).$$

If it passes through $(ae, 0)$, then

$$e^2 \cos^2\left(\frac{\theta_1 - \theta_2}{2}\right) = \cos^2\left(\frac{\theta_1 + \theta_2}{2}\right)$$

$$e^2 = 1 + \frac{b^2}{a^2} = \frac{a^2 + b^2}{a^2}$$

41). If α, β are the eccentric angles of the ends of a focal chord

of the ellipse $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then the eccentricity of the ellipse is

$$1) \frac{\sin \alpha - \sin \beta}{\sin(\alpha + \beta)}$$

$$2) \frac{\sin \alpha + \sin \beta}{\sin(\alpha + \beta)}$$

$$3) \frac{\cos \alpha - \cos \beta}{\cos(\alpha + \beta)}$$

$$4) \frac{\cos \alpha + \cos \beta}{\cos(\alpha - \beta)}$$

Key(2)

Sol

$$\frac{x}{a} \cos\left(\frac{\alpha + \beta}{2}\right) + \frac{y}{b} \sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha - \beta}{2}\right)$$

Passing through $(ae, 0)$

42). The area of the parallelogram formed by the lines $\frac{x}{a} + \frac{y}{b} = \pm 1$

and the directrices of the ellipse $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ with eccentricity ,

e and a, b being positive is

$$1) \frac{ab}{e^2}$$

$$2) \frac{4ab}{e^2}$$

$$3) \frac{2ab}{e^2}$$

$$4) \frac{9ab}{e^2}$$

Key

(2)

Sol

Solve directrix and lines

$$A = \left(\frac{a}{e}, b - b/e\right) B = \left(-\frac{a}{e}, b - b/e\right) C = \left(-\frac{a}{e}, \frac{b}{e} - b\right),$$

$$D = \left(\frac{a}{e}, -b - b/e\right) \text{ Area of ABCD} = 4ab/e^2$$

43). If a chord joining $P(a \sec \theta, a \tan \theta)$, $Q(a \sec \alpha, a \tan \alpha)$ on the hyperbola $x^2 - y^2 = a^2$ is the normal at P, then $\tan \alpha =$

- 1) $\tan \theta (4 \sec^2 \theta + 1)$ 2) $\tan \theta (4 \sec^2 \theta - 1)$
 3) $\tan \theta (2 \sec^2 \theta - 1)$ 4) $\tan \theta (1 - 2 \sec^2 \theta)$

Key(2)

Sol $\frac{\tan \alpha - \tan \theta}{\sec \alpha - \sec \theta} = -\frac{\tan \theta}{\sec \theta} \Rightarrow \tan \alpha - \tan \theta = -k \tan \theta$ and

$$\sec \alpha - \sec \theta = k \sec \theta$$

$$\therefore (1-k) \tan \theta = \tan \alpha \rightarrow 1. (1+k) \sec \theta = \sec \alpha \rightarrow 2.$$

$$[(1+k) \sec \theta]^2 - [(1-k) \tan \theta]^2 = \sec^2 \alpha - \tan^2 \alpha$$

$$\Rightarrow k = -2(\sec^2 \theta + \tan^2 \theta) = -4 \sec^2 \theta + 2$$

$$\text{From (1) } \tan \alpha = \tan \theta (1 + 4 \sec^2 \theta - 2) = \tan \theta (4 \sec^2 \theta - 1).$$

44). If two conics $a_1 x^2 + 2h_1 xy + b_1 y^2 = c_1$ and $a_2 x^2 + 2h_2 xy + b_2 y^2 = c_2$ intersect in four concyclic points, then

- 1) $(a_1 - b_1)h_2 = (a_2 - b_2)h_1$ 2) $(a_1 - b_1)h_1 = (a_2 - b_2)h_2$
 3) $(a_1 + b_1)h_2 = (a_2 + b_2)h_1$ 4) $(a_1 + b_1)h_1 = (a_2 + b_2)h_2$

Key (1)

Sol

On removing xy terms by multiplying $a_1 x^2 + 2h_1 xy + b_1 y^2 = C_1$ by h_2 and

$a_2 x^2 + 2h_2 xy + b_2 y^2 = C_2$ by h_1 and

subtracting we have

$$(a_1 h_2 - a_2 h_1) x^2 + (b_1 h_2 - b_2 h_1) y^2 = C_1 h_2 - C_2 h_1$$

Now this will represent a circle if coefficient of $x^2 =$ coefficient of y^2

$$\text{i.e. } a_1 h_2 - a_2 h_1 = b_1 h_2 - b_2 h_1$$

$$\text{i.e. } (a_1 - b_1) h_2 = (a_2 - b_2) h_1$$

45). The transverse axis of a hyperbola is of length $2a$ and a vertex divides the segment of the axis between the centre and the corresponding focus in the

ratio 2: 1, the equation of the hyperbola is

- 1) $4x^2 - 5y^2 = 4a^2$ 2) $4x^2 - 5y^2 = 5a^2$
 3) $5x^2 - 4y^2 = 4a^2$ 4) $5x^2 - 4y^2 = 5a^2$

Key (4)

Sol Clearly $\frac{2ae}{3} = a \Rightarrow e = \frac{3}{2}$

$\therefore S = \left(\frac{3a}{2}, 0\right)$

Directrix is $x = \frac{2a}{3}$

$\therefore$ equation of hyperbola will be $\left(x - \frac{3a}{2}\right)^2 + y^2 = \frac{9}{4}\left(x - \frac{2a}{3}\right)^2$

Which reduces to $5x^2 - 4y^2 = 5a^2$

- 46). A chord of the hyperbola $x^2 - 2y^2 = 1$ is bisected at the point $(-1, 1)$.
 Then the area of the triangle formed by the chord and the coordinate axes is

- 1) 1 2) 2 3) 1/2 4) 1/4

Key (4)

Sol Equation of the chord as $S_1 = S_{11} = \text{Req Area } \frac{1}{4}$

- 47). A tangent to the circle $x^2 + y^2 = 4$ intersects the hyperbola $x^2 - 2y^2 = 2$ at P and Q.

If locus of mid-point of PQ is $(x^2 - 2y^2)^2 = \lambda (x^2 + 4y^2)$; then λ equals

- 1) 2 2) 4 3) 6 4) 8

Key (2)

Sol Equation of chord of hyperbola $\frac{x^2}{2} - \frac{y^2}{1} = 1$, whose mid-point is (h, k) is

$$\frac{hx}{2} - ky = \frac{h^2}{2} - \frac{k^2}{1}$$

It is tangent to the circle $x^2 + y^2 = 4$, then $\left| \frac{\frac{h^2}{2} - k^2}{\sqrt{\frac{h^2}{4} + k^2}} \right| = 2$

$$\Rightarrow \left(\frac{h^2}{2} - k^2 \right)^2 = 4 \left(\frac{h^2}{4} + k^2 \right) \Rightarrow (x^2 - 2y^2)^2 = 4(x^2 + 4y^2) \Rightarrow \lambda = 4.$$

48). The locus of the middle points of chords of hyperbola

$$3x^2 - 2y^2 + 4x - 6y = 0 \quad \text{parallel to } y = 2x \text{ is}$$

1) $3x - 4y = 4$

2) $3y - 4x + 4 = 0$

3) $4x - 4y = 3$

4) $3x - 4y = 2$

Key (1)

Sol

Let locus be $P(h, k)$, $T = S_1$

$$3hx - 2ky + 2(x + h) - 3(k + y) = 3h^2 - 2k^2 + 4h - 6k$$

$$\text{Slope} = \frac{3h + 2}{2k + 3} = 2 \Rightarrow 3x - 4y = 4$$

49). If $\alpha + \beta = 3\pi$ then the chord joining the points α and β for the

Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ passes through

1) focus

2) centre

3) one of the end points of the transverse axis

4) one of the end points of the conjugates axis

Key (2)

Sol

(i) Equation of chord joining α and β is

$$\frac{x}{a} \cos\left(\frac{\alpha - \beta}{2}\right) - \frac{y}{b} \sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha + \beta}{2}\right)$$

$$\therefore \alpha + \beta = 3\pi$$

$$\frac{x}{a} \cos\left(\frac{\alpha - \beta}{2}\right) = \frac{y}{b} = 0$$

If passes through the centre $(0, 0)$

50) If a variable tangent of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$, cuts the circle $x^2 + y^2 = 4$

at point A, B and locus of midpoint of AB is $9x^2 - 4y^2 - \lambda(x^2 + y^2)^2 = 0$

then λ is

Key (1)

Sol

Equation of chord of circle with midpoint (h, k) is $xh + yk = h^2 + k^2$
or

$$y = \left(\frac{-h}{k} \right) x + \frac{h^2 + k^2}{k}, \text{ it touches the hyperbola}$$

51). If $(a \sec \theta, b \tan \theta)$ and $(a \sec \phi, b \tan \phi)$ are the ends of a focal chord

of $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, then $\tan \frac{\theta}{2} \tan \frac{\phi}{2}$ equal to

1) $\frac{e-1}{e+1}$ 2) $\frac{1-e}{1+e}$ 3) $\frac{1+e}{1-e}$ 4) $\frac{e+1}{e-1}$

Key (3)

Sol Equation of chord joining θ and ϕ

$$\frac{x}{a} \cos \frac{\theta - \phi}{2} - \frac{y}{b} \sin \frac{\theta + \phi}{2} = \cos \frac{\theta + \phi}{2}$$

It passes through $(ae, 0)$

$$\therefore e \cos \frac{\theta - \phi}{2} = \cos \frac{\theta + \phi}{2}$$

$$\therefore \frac{\cos \frac{\theta - \phi}{2}}{\cos \frac{\theta + \phi}{2}} = \frac{1}{e}$$

$$\frac{\cos \frac{\theta - \phi}{2} - \cos \frac{\theta + \phi}{2}}{\cos \frac{\theta - \phi}{2} + \cos \frac{\theta + \phi}{2}} = \frac{1 - e}{1 + e}$$

$$\frac{2 \sin \frac{\theta}{2} \sin \frac{\phi}{2}}{2 \cos \frac{\theta}{2} \cos \frac{\phi}{2}} = \frac{1 - e}{1 + e} \quad \Rightarrow \quad \tan \frac{\theta}{2} \tan \frac{\phi}{2} = \frac{1 - e}{1 + e}$$

Since the chord also passes thru $(-ae, 0)$

Similarly as above, we get $\tan \frac{\theta}{2} \tan \frac{\phi}{2} = \frac{1 + e}{1 - e}$

53). For a given non-zero value of m each of the lines $\frac{x}{a} - \frac{y}{b} = m$ and $\frac{x}{a} + \frac{y}{b} = m$

meets the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ at a point. Sum of the ordinate

of these points, as m varies is

1) $\frac{a(1+m^2)}{m}$

2) $\frac{b(1-m^2)}{m}$

3) 0

4) $\frac{a+b}{2m}$

Key (3)

Sol Ordinate of the point of intersection of the line $\frac{x}{a} - \frac{y}{b} = m$ and the hyperbola

is given by

$$\left(\frac{x}{a} - \frac{y}{b}\right)\left(\frac{x}{a} - \frac{y}{b} + \frac{2y}{b}\right) = 1 \quad \text{i.e.} \quad m\left(m + \frac{2y}{b}\right) = 1 \quad \text{i.e.} \quad y = \frac{b(1-m^2)}{2m}$$

Similarly ordinate of the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = m$ and the hyperbola is given by

$$y = \frac{b(m^2 - 1)}{2m}$$

$\therefore$ Sum of the ordinates is 0.