

ELLIPSE

FOCUI, AXIS, LATUS RECTUM AND DIRECTRICES

- 1). The distance between the directrices of the ellipse

$$(4x-8)^2 + 16y^2 = (x + \sqrt{3}y + 10)^2 \text{ is K then } \frac{K}{2} \text{ is}$$

Key (8)

Sol:
$$(x-2)^2 + y^2 = \left(\frac{1}{2}\right)^2 \frac{(x + \sqrt{3}y + 10)^2}{4}$$

$$(h, k) = (2, 0), e = \frac{1}{2}$$

Perpendicular distance from $(2, 0)$ to $x + \sqrt{3}y + 10 = 0$ is $\frac{a}{e} - ae$

$$2a - \frac{a}{2} = 6 \Rightarrow a = 4$$

$$\text{Distance between directrices} = \frac{2a}{e} = 16 = K$$

- 2). Consider the ellipse $(3x-6)^2 + (3y-9)^2 = \frac{4}{169}(5x+12y+6)^2$ then match the following

1) The length of major axis $= \frac{48}{5}$

2) The length of minor axis $= \frac{16}{\sqrt{5}}$

3) The length of latus rectum $= \frac{16}{3}$

4) Distance between the directrices $= \frac{72}{5}$

Key (1,2,3,4)

Sol:
$$(x-2)^2 + (y-3)^2 = \frac{4}{9} \left(\frac{5x+12y+6}{13} \right)^2$$

$$d = \frac{5(2) + 12(3) + 6}{\sqrt{5^2 + 12^2}} = 4$$

And $e = 2/3$

$$\text{Length of major axis} = \frac{2e}{1-e^2} \cdot d = \frac{2(2/3)}{1-\frac{4}{9}} \times 4 = \frac{48}{5}$$

$$\text{Length of minor axis} = \frac{48}{5} \sqrt{1-\frac{4}{9}} = \frac{16}{\sqrt{5}}$$

$$\text{Distance between the directrices} = 2(a/e) = \frac{72}{5}$$

- 3). If the major axis is “n” ($n > 1$) times the minor axis of the ellipse, then eccentricity is

1. $\frac{\sqrt{n-1}}{n}$ 2. $\frac{\sqrt{n-1}}{n^2}$ 3. $\frac{\sqrt{n^2-1}}{n^2}$ 4. $\frac{\sqrt{n^2-1}}{n}$

Key (4)

Sol:

$$2a = n(2b)$$

$$\Rightarrow n = \frac{a}{b}$$

$$\therefore e = \sqrt{\frac{a^2 - b^2}{a^2}} = \sqrt{1 - \frac{b^2}{a^2}} =$$

$$\sqrt{1 - \frac{1}{n^2}} = \frac{\sqrt{n^2 - 1}}{n}$$

- 4). The equation $\frac{x^2}{10-a} + \frac{y^2}{4-a} = 1$ represents an ellipse if

1. $a < 4$ 2. $a > 4$ 3. $4 < a < 10$ 4. $a > 10$

Key (3)

Sol

$$10 - a > 0, 4 - a > 0$$

- 5). If A and B are foci of ellipse $(x - 2y + 3)^2 + (8x + 4y + 4)^2 = 20$ and P is any point on it, then $PA + PB = \underline{\hspace{2cm}}$.

1) 2

2) 4

3) $\sqrt{2}$ 4) $2\sqrt{2}$

Key (2)

Sol:

$$\frac{\left(\frac{x-2y+3}{\sqrt{5}}\right)}{4} + \frac{\left(\frac{2x-y+1}{\sqrt{5}}\right)}{1/4} = 1 \Rightarrow PA + PB = 2a = 4$$

- 6). If $\frac{x^2}{f(4a)} + \frac{y^2}{f(a^2-5)} = 1$ represents an ellipse with major axis as y-axis and f is a decreasing function positive for all 'a' then a belongs to

1) (0,6)

2) (-1,1)

3) (-1,5)

4) $(5, \infty)$

Key (3)

Sol:

$$f(a^2-5) > f(4a) \Rightarrow a^2-5 < 4a \Rightarrow a \in (-1, 5)$$

- 7). In the ellipse $25x^2 + 9y^2 - 150x - 90y + 225 = 0$

1) foci are at (3, 1), (3, 9)

2) $e = 4/5$

3) centre is (3, 5)

4) length of major axis is 10

Key (1,2,3,4)

Sol:

$$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1 \Rightarrow \text{centre (3, 5), } e = \frac{4}{5}$$

Length of major axis 10

Foci are (3, 1), (3, 9)

- 8). If latus rectum of the ellipse $x^2 \tan^2 \alpha + y^2 \sec^2 \alpha = 1$ is $\frac{1}{2}$, then

 $\alpha (0 < \alpha < \pi)$ is equal to1) $\pi/12$ 2) $\pi/6$ 3) $5\pi/12$

4) none of these

Key (1,3)

Sol:

$$x^2 \tan^2 \alpha + y^2 \sec^2 \alpha = 1$$

$$\Rightarrow \frac{x^2}{\cot^2 \alpha} + \frac{y^2}{\cos^2 \alpha} = 1 \quad \because \cos^2 \alpha = \cot^2 \alpha (1 - e^2)$$

$$\Rightarrow \sin^2 \alpha = (1 - e^2) \therefore e^2 = \cos^2 \alpha (\alpha \neq 90^\circ)$$

$$e = \cos \alpha \quad \therefore \text{Latustrectum} = 1/2 = 2b^2/a$$

$$\Rightarrow a = 4b^2$$

$$\Rightarrow \cot \alpha = 4 \cos^2 \alpha$$

$$\Rightarrow \frac{1}{\sin \alpha} = 4 \cos \alpha$$

$$\Rightarrow \sin 2\alpha = \frac{1}{2}$$

$$2\alpha = n\pi + (-1)^n \frac{\pi}{6} \quad \alpha = \frac{n\pi}{2} + (-1)^n \frac{\pi}{12}$$

$$\text{for } n = 0 \quad \alpha = \frac{\pi}{12} \quad \text{and} \quad \text{for } n = 1 \quad \alpha = \frac{\pi}{2} - \frac{\pi}{12} = \frac{5\pi}{12}$$

9). If the ellipse $\frac{x^2}{4} + y^2 = 1$ meets the ellipse $x^2 + \frac{y^2}{a^2} = 1$ in four points and

$$a = b^2 - 5b + 7 \text{ then } b \in \underline{\hspace{1cm}}.$$

1) $(-\infty, 0)$

2) $[4, 5]$

3) $[2, 3]$

4) $[0, \infty]$

Key (1,2)

Sol:

(a,b) Both the ellipse have their centre at (0,0). The major axis of first is along x-axis and in case the two ellipse meet in 4 distinct points, then the major axis of second ellipse should be along the y-axis.

$$\Rightarrow a^2 > 1 \& b^2 - 5b + 7 > 1 \Rightarrow b < 2 \text{ or } b > 3$$

10). If P is any point on ellipse $\frac{x^2}{4} + 4y^2 = 1$ whose foci are at A and B then,
max. Value of $|PA.PB|$ is

Key (4)

Sol:

$$PA + PB = 4$$

$$\frac{PA+PB}{2} \geq (PA.PB)^{1/2} \Rightarrow PA.PB \leq 4.$$

- 11). If the ellipse $\frac{x^2}{a^2-3} + \frac{y^2}{a+4} = 1$ is inscribed in a square of side length $a\sqrt{2}$ then a is
- 1) 4 2) 2 3) 1 4) None of these

Key (4)

Sol:

Sides of the square will be perpendicular tangents to the ellipse so, vertices of the square will lie on director circle.

So diameter of director circle is

$$2\sqrt{(a^2-3)+(a+4)} = \sqrt{2a^2+2a^2}$$

$$2\sqrt{a^2+a+1} = 2a \Rightarrow a = -1$$

But for ellipse $a^2 > 3$ & $a > -4$

So a cannot take the value '-1'

- 12). The perimeter of a triangle is 20 and the points (-2, -3) and (-2, 3) are two of the vertices of it. Then the locus of third vertex is :

$$1. \frac{(x-2)^2}{49} + \frac{y^2}{40} = 1$$

$$2. \frac{(x+2)^2}{49} + \frac{y^2}{40} = 1$$

$$3. \frac{(x+2)^2}{40} + \frac{y^2}{49} = 1$$

$$4. \frac{(x-2)^2}{40} + \frac{y^2}{49} = 1$$

Key (3)

Sol:

PA + PB + AB = 20 where A & B are foci

- 13). Let 'O' be the centre of ellipse for which A,B are end points of major axis and C,D are end points of minor axis, F is focus of the ellipse. If in radius of $\triangle OCF$ is '1' then $|AB| \times |CD| =$

1) 65

2) 52

3) 78

4) 47

Key (1)

Sol:

$$r = \frac{\Delta}{S} \Rightarrow \Delta = S$$

$$\frac{1}{2}(ae)b = \frac{ae + b + \sqrt{a^2e^2 + b^2}}{2}$$

$$ae = 6 \Rightarrow 6b = 6 + b + \sqrt{36 + b^2} \Rightarrow b = \frac{5}{2}$$

$$\Rightarrow a^2(1 - e^2) = \frac{25}{4} \Rightarrow a^2 - 36 = \frac{25}{4} \Rightarrow a = \frac{13}{2} \quad a > 1$$

- 14). There are exactly two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose distance from the center of the ellipse are equal to $\sqrt{\frac{a^2 + 2b^2}{2}}$. Eccentricity of this ellipse is equal to

1) $\frac{\sqrt{3}}{2}$

2) $\frac{1}{\sqrt{3}}$

3) $\frac{1}{\sqrt{2}}$

4) $\sqrt{\frac{2}{3}}$

Key (3)

Sol:

The given distance is clearly the length of semi major axis.

$$\text{Thus, } \sqrt{\frac{a^2 + 2b^2}{2}} = a$$

$$\Rightarrow 2b^2 = a^2$$

$$\Rightarrow 2a^2(1 - e^2) = a^2$$

$$\Rightarrow e^2 = \frac{1}{2} \Rightarrow e = \frac{1}{\sqrt{2}}$$

- 15). F is one of the foci of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ where AB is the major axis and CD is the minor axis and O is the centre of the ellipse. If $OF = 6$ and the in-radius of $\triangle OCF$ is 1, then the product of the lengths of AB and CD is

1) 60

2) 65

3) 70

4) 75

Key (2)

Sol:

$$ae = 6 \quad CF = a$$

$$r = \frac{\Delta}{s}, r = 1 \Rightarrow \Delta = S$$

$$\frac{1}{2} \times 6 \times b = \frac{b + 6 + a}{2}$$

$$5b = a + 6$$

$$25b^2 = a^2 + 36 + 12a$$

$$25(a^2 - a^2e^2) = a^2 + 36 + 12a$$

$$25(a^2 - 36) = a^2 + 36 + 12a$$

$$\Rightarrow a = \frac{13}{2}, b = \frac{5}{2}$$

$$AB \times CD = 13 \times 5 = 65$$

- 16). From any point P lying in first quadrant on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$, PN is drawn perpendicular to the major axis such that N lies on major axis. Now PN is produced to the point Q such that NQ equals to PS , where S is a focus. The point Q lies on which of the following lines

(1) $2y - 3x - 25 = 0$

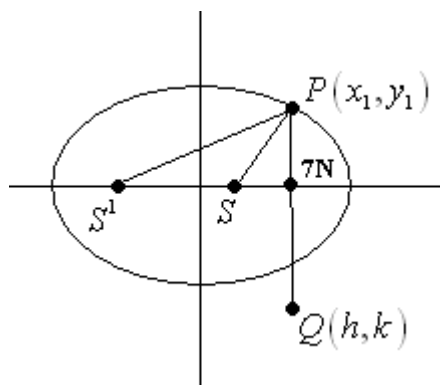
(2) $3x + 5y + 25 = 0$

(3) $2x - 5y - 25 = 0$

(4) $2x - 5y + 25 = 0$

Key (2)

Sol:



$$a^2 = 25$$

$$b^2 = 16$$

$$e = \sqrt{\frac{25-16}{25}} = \frac{3}{5}$$

Let point Q be (h, k), where $K < 0$

Given that $|K| = a + eh$ (as $x_1 = h$)

$$-y = a + ex$$

$$-y = 5 + \frac{3}{5}x$$

$$3x + 5y + 25 = 0$$

- 17). The line passing through the extremity A of the major axis and extremity B of the minor axis of the ellipse $x^2 + 9y^2 = 9$, meets its auxiliary circle at the point M. Then the area of the triangle with vertices at A, M and the origin 'O' is

1) $\frac{31}{10}$

2) $\frac{29}{10}$

3) $\frac{21}{10}$

4) $\frac{27}{10}$

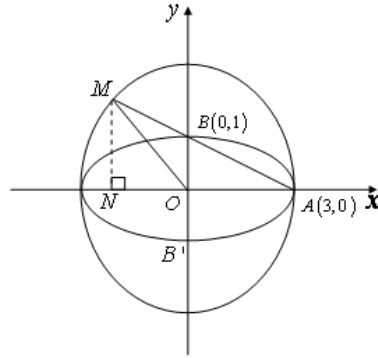
Key (2)

Sol:

Equation of given ellipse is $\frac{x^2}{9} + \frac{y^2}{1} = 1$

Equation of auxiliary circle is $x^2 + y^2 = 9$(1)

Equation of line AB is $\frac{x}{3} + \frac{y}{1} = 1 \Rightarrow x = 3(1 - y)$



Putting this in (1), we get $9(1-y)^2 + y^2 = 9 \Rightarrow 10y^2 - 18y = 0 \Rightarrow y = 0, \frac{9}{5}$

Thus, y coordinate of 'M' is $\frac{9}{5}$

$$\Delta OAM = \left(\frac{1}{2}\right)(OA)(MN) = \frac{1}{2}(3)\frac{9}{5} = \frac{27}{10}$$

18). The ratio of the area enclosed by the locus of the midpoint of PS and area of the ellipse is ____ (P-be any point on the ellipse and S, its focus)

- 1) $\frac{1}{2}$ 2) $\frac{1}{3}$ 3) $\frac{1}{5}$ 4) $\frac{1}{4}$

Key (4)

Sol:

$$\text{mid point of PS is (h,k) \& } h = \frac{a \cos \theta + ae}{2} \Rightarrow \cos \theta = \frac{2h - ae}{a}; k = \frac{b \sin \theta}{2}$$

$$\frac{(2h - ae)^2}{a^2} + \frac{4k^2}{b^2} = 1 \Rightarrow \frac{\left(h - \frac{ae}{2}\right)^2}{\frac{a^2}{4}} + \frac{k^2}{\frac{b^2}{4}} = 1 \text{ its area}$$

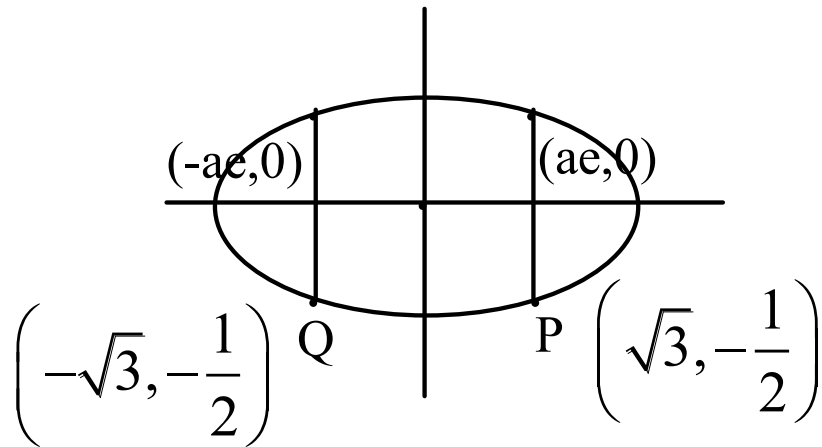
19). Let P (x_1, y_1) and Q (x_2, y_2) , $y_1 < 0, y_2 < 0$, be the end points of the latus rectum of the ellipse $x^2 + 4y^2 = 4$. The equations of parabolas with latus rectum PQ are

- 1) $x^2 + 2\sqrt{3} y = 3 + \sqrt{3}$ 2) $x^2 - 2\sqrt{3} y = 3 + \sqrt{3}$
 3) $x^2 + 2\sqrt{3} y = 3 - \sqrt{3}$ 4) $x^2 - 2\sqrt{3} y = 3 - \sqrt{3}$

Key (2,3)

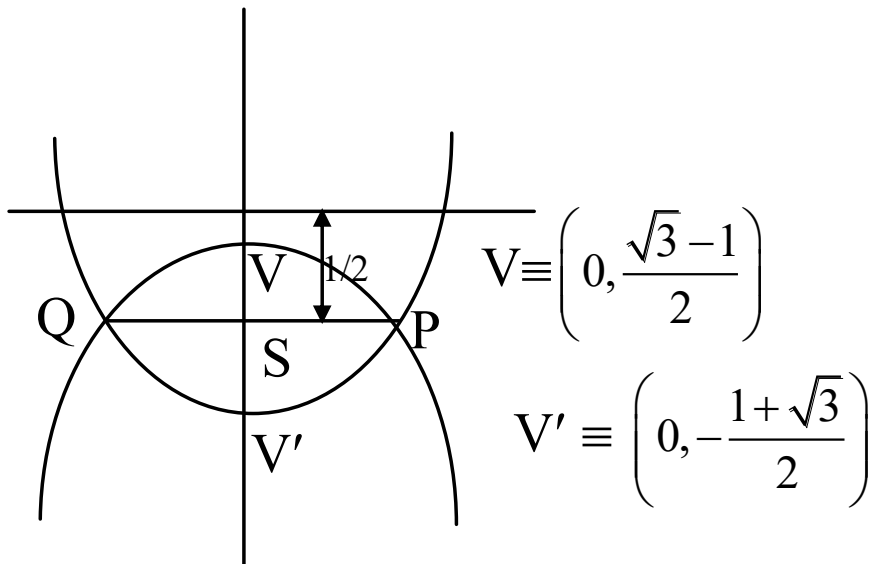
Sol:

$$\text{Given ellipse is } \frac{x^2}{4} + \frac{y^2}{1} = 1 \Rightarrow e = \sqrt{1 - \frac{1}{4}} = \frac{\sqrt{3}}{2}$$



$$P \equiv (ae, -b^2/a) = \left(\sqrt{3}, -\frac{1}{2}\right), Q \equiv (-ae, -b^2/a) = \left(-\sqrt{3}, -\frac{1}{2}\right)$$

$$\text{length of PQ} = 2\sqrt{3}$$



$$VS = SV' = \frac{PQ}{4} = \frac{\sqrt{3}}{2}$$

$\therefore$ Equations of parabolas

are

$$x^2 = -2\sqrt{3}\left(y - \frac{\sqrt{3}-1}{2}\right) \Rightarrow x^2 + 2\sqrt{3}y = 3 - \sqrt{3}$$

$$\text{and } x^2 = 2\sqrt{3}\left(y + \frac{1+\sqrt{3}}{2}\right) \Rightarrow x^2 - 2\sqrt{3}y = 3 + \sqrt{3}$$

- 20). A circle concentric to an ellipse $\frac{4x^2}{289} + \frac{4y^2}{\lambda^2} = 1 \left(\lambda < \frac{17}{2} \right)$ passes through foci F_1 and F_2 cuts the ellipse at 'P' such that area of triangle $P F_1 F_2$ is 30 sq.units. If $F_1 F_2 = 13K$ where $K \in \mathbb{Z}$ then $K =$

Key (1)

Sol:

Since F_1 & F_2 are the ends of the diameter

$$\text{Area of } \Delta P F_1 F_2 = \frac{1}{2}(F_1 P)(F_2 P) = \frac{1}{2}x(17-x) = 30 \Rightarrow x = 5 \text{ or } 12 \Rightarrow F_1 F_2 = 13$$

- 21). The graph of the equation $x + y = x^3 + y^3$ is the union of

- 1) line and an ellipse 2) line and a parabola
3) line and a hyperbola 4) pair of lines

Key (1)

Sol: Given, $(x+y) = (x+y)(x^2 - xy + y^2)$

$$\Rightarrow (x+y)(x^2 - xy + y^2 - 1) = 0$$

$$\therefore \text{ Either } x + y = 0 \text{ or } x^2 - xy + y^2 - 1 = 0$$

Now, $x + y = 0$ represents a line.

$$\text{Also, } \Delta = \begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = \begin{vmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & 0 \\ 0 & 0 & -1 \end{vmatrix} = -\frac{3}{4} \neq 0$$

$$\text{and } h = -\frac{1}{2}, a = 1 = b, \text{ so } h^2 < ab$$

$$\therefore x^2 - xy + y^2 - 1 = 0 \text{ represent an ellipse.}$$

- 22). An ellipse is inscribed in a rectangle whose axes are parallel to the sides of the rectangle and if the angle between the diagonals is $\tan^{-1} 2\sqrt{2}$ then $e =$

1) $\frac{1}{\sqrt{3}}$

2) $\frac{1}{3}$

3) $\frac{1}{\sqrt{2}}$

4) $\frac{1}{2}$

Key (3)

Sol: Slopes of diagonals are $\frac{b}{a}, -\frac{b}{a}$ $\therefore 2\sqrt{2} = \left| \frac{\frac{b}{a} + \frac{b}{a}}{1 - \frac{b^2}{a^2}} \right| \Rightarrow e = \frac{1}{\sqrt{2}}$

23). P and Q are points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose center is C . The eccentric angles of P and Q differ by a right angle. If $\angle PCQ$ minimum, the eccentric angle of P can be

1) $\frac{\pi}{6}$

2) $\frac{\pi}{4}$

3) $\frac{\pi}{3}$

4) $\frac{\pi}{12}$

Key (2)

Sol Since the eccentric angles of P and Q differ by a right angle, we can take P as $(a \cos \theta, b \sin \theta)$ and Q as $(-a \sin \theta, b \cos \theta)$

$$\text{slope of } CP = \frac{b \sin \theta}{a \cos \theta} \quad \text{slope of } CQ = \frac{b \cos \theta}{-a \sin \theta}$$

If A is the angle between CP and CQ , then

$$A = \tan^{-1} \left| \frac{\frac{b \sin \theta}{a \cos \theta} + \frac{b \cos \theta}{a \sin \theta}}{1 - \frac{b^2 \sin \theta \cos \theta}{a^2 \cos \theta \sin \theta}} \right| = \tan^{-1} \left| \frac{2ab}{a^2 - b^2} \frac{1}{\sin 2\theta} \right|$$

A is minimum, if $\sin 2\theta$ is maximum, i.e, if $2\theta = \pi/2, \theta = \pi/4$

24). Consider an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 (a > b)$. Let a hyperbola is having its vertices at the extremities of minor axis of an ellipse and length of major axis of an ellipse is

equal to the distance between the foci of hyperbola. Let e_1 and e_2 be the

eccentricities of an ellipse and hyperbola respectively.

Then the relation between e_1 and e_2 is given by

$$1) e_1 e_2 = 1 \quad 2) e_2^2 (1 - e_1^2) = 1 \quad 3) e_1^2 (e_2^2 - 1) = 1 \quad 4) e_1 e_2 (1 - e_1^2) = 1$$

Key (2)

Sol: $b^2 = a^2 (1 - e_1^2)$ And $2be_2 = 2a$

$$e_2 = \frac{a}{b}; \frac{1}{e_2^2} = 1 - e_1^2; \Rightarrow e_2^2 (1 - e_1^2) = 1$$

- 25). An ellipse with major and minor axes are of lengths $10\sqrt{3}$ and 10 respectively, sides along the coordinate axes and always remains confined in the first quadrant. The locus of the center of the ellipse will be the arc of a circle. The length of this arc will be equal to

$$1) 10\pi \text{ units} \quad 2) 5\pi \text{ units} \quad 3) \frac{5}{4}\pi \text{ units} \quad 4) \frac{5}{3}\pi \text{ units}$$

Key (4)

Sol:

Clearly origin will be a point lying on the director circle of the ellipse. Thus, equation of locus of center of ellipse will be

$$x^2 + y^2 = a^2 + b^2 \\ \Rightarrow x^2 + y^2 = 100$$

The adjacent figure shows the position of center of ellipse in two extreme positions.

$$\theta = \angle C_1OC_2 = \frac{\pi}{2} - 2 \tan^{-1} \left(\frac{5}{5\sqrt{3}} \right) = \frac{\pi}{6}$$

Thus, arc length $C_1C_2 = 10 \cdot \frac{\pi}{6} = \frac{5\pi}{3}$ units.

- 26). A triangle ABC is inscribed inside the ellipse $x^2 + 2y^2 = 9$. If $A \equiv (1, 2)$, $B \equiv (3, 0)$, then the co-ordinate of the vertex C for which the area of the triangle is maximum is

$$1) \left(-\sqrt{6}, \sqrt{\frac{3}{2}}\right) \quad 2) \left(\sqrt{6}, -\sqrt{\frac{3}{2}}\right) \quad 3) \left(\sqrt{6}, \sqrt{\frac{3}{2}}\right) \quad 4) \left(-\sqrt{6}, -\sqrt{\frac{3}{2}}\right)$$

Key (4)

Sol:

Tangent at C (h, k) will be parallel to AB.

Comparing the slopes of $hx + 2ky = 9$ and $x + y = 3$, we get $h = 2k$.

On solving with ellipse, we get h, k = $\left(-\sqrt{6}, -\sqrt{\frac{3}{2}}\right)$.

27). A variable point P on the ellipse of eccentricity e is joined to foci S and S'. The eccentricity of the locus of the in centre of triangle PSS' is

$$1) \sqrt{\frac{2e}{1+e}} \quad 2) \sqrt{\frac{e}{1+e}} \quad 3) \sqrt{\frac{1-e}{1+e}} \quad 4) \frac{e}{2(1+e)}$$

Key (1)

Sol: Let any point be $P(a \cos \theta, b \sin \theta)$.

$$SP = a[1 - e \cos \theta] \quad S'P = a[1 + e \cos \theta] \quad SS' = 2ae$$

(h, k) be in centre of $\Delta PSS'$ upon solving we get

$$\Rightarrow h = ae \cos \theta$$

$$k = \frac{b \sin \theta e}{1+e}$$

Eliminating ' θ '

$$\frac{h^2}{a^2 e^2} + \frac{k^2}{\frac{e^2 b^2}{(1+e)^2}} = 1$$

$$\text{Locus of } (h, k) \quad \frac{x^2}{a^2 e^2} + \frac{y^2}{\frac{e^2 b^2}{(1+e)^2}} = 1$$

$$\Rightarrow e_1^2 = 1 - \frac{e^2 b^2}{(1+e)^2 e^2 a^2} = \frac{2e}{1+e}$$

$$e_1 = \sqrt{\frac{2e}{1+e}}$$

- 28). An ellipse is drawn with major and minor axis of lengths 10 and 8 respectively. Using one focus as centre, a circle is drawn that is tangent to the ellipse, with no part of circle being outside the ellipse. The radius of circle is

- 1) 4 2) 5 3) 2 4) 1

Key (3)

Sol:

The circle must touch the end of major axis

$$\therefore \text{radius} = a - ae = a - \sqrt{a^2 - b^2} = 5 - \sqrt{5^2 - 4^2} = 2$$

- 29). The eccentric angle of a point p lying in the first quadrant on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is θ . If OP makes an angle ϕ with x-axis, then $\theta - \phi$ will be maximum when $\theta =$

- 1) $\tan^{-1} \sqrt{\frac{a}{b}}$ 2) $\tan^{-1} \sqrt{\frac{b}{a}}$ 3) $\frac{\pi}{4}$ 4) $\frac{\pi}{3}$

Key (1)

Sol:

$$\tan \theta = \frac{b}{a} \tan \theta$$

$$y = \frac{a - b}{a \cot \theta + b \tan \theta} \text{ if will be maximum}$$

$$\text{If } \tan^2 \theta = \frac{a}{b} \Rightarrow \tan \theta = \sqrt{\frac{a}{b}}$$

- 30). The ellipse $4x^2 + 9y^2 = 36$ and the hyperbola $a^2x^2 - y^2 = 4$ intersect at right angles then the equation of the circle through the points of intersection of two conics is:

- 1) $x^2 + y^2 = 5$ 2) $\sqrt{5}(x^2 + y^2) - 3x - 4y = 0$
 3) $\sqrt{5}(x^2 + y^2) + 3x + 4y = 0$ 4) $x^2 + y^2 = 25$

Key (2)

Sol:

Since ellipse and hyperbola intersect orthogonally, they are confocal.

Hence, $a = 2$ (equating foci).

Let point of intersection in the first quadrant be $P(x_1, y_1)$.

P lies on both the curves. Therefore $4x_1^2 + 9y_1^2 = 36$ and $4x_1^2 - y_1^2 = 4$.

Adding these two results, we get

$$8(x_1^2 + y_1^2) = 40$$

$$\Rightarrow x_1^2 + y_1^2 = 5 \Rightarrow r = \sqrt{5}.$$

Hence, equation of the circle is

$$x^2 + y^2 = 5.$$

- 31). An ellipse slides between two lines at right angles to one another. The locus of its centre is

1) a parabola 2) an ellipse 3) a circle 4) straight line

Key (3)

Sol:

Let Ellipse be $\frac{(x - \alpha)^2}{a^2} + \frac{(y - \beta)^2}{b^2} = 1$

Director circle is $(x - \alpha)^2 + (y - \beta)^2 = a^2 + b^2$ -----(1)

Now (0,0) lies on (1)

$$\Rightarrow \alpha^2 + \beta^2 = a^2 + b^2$$

Locus of $c(\alpha, \beta)$ is $x^2 + y^2 = a^2 + b^2$

- 32). If the curve $x^2 + 3y^2 = 9$ subtends an obtuse angle at the point $(2\alpha, \alpha)$, then a possible value of α^2 is

1) 1 2) 2 3) 3 4) 4

Key (2)

Sol:

The given curve is $\frac{x^2}{9} + \frac{y^2}{3} = 1$, whose director circle is $x^2 + y^2 = 12$. For the required condition $(2\alpha, \alpha)$ should lie inside the circle and outside the ellipse i.e., $(2\alpha)^2 + 3\alpha^2 - 9 > 0$ and $(2\alpha)^2 + \alpha^2 - 12 < 0$ i.e., $\frac{9}{7} < \alpha^2 < \frac{12}{5}$.

- 33). The radius of the circle passing through the foci of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ and having centre at $(0, 3)$ is

- 1) 3 2) 4 3) 5 4) $2\sqrt{3}$

Key (2)

Sol: $e = \sqrt{\frac{a^2 - b^2}{a^2}} \Rightarrow ae = \sqrt{16 - 9} = \sqrt{7}$

$\therefore$ Foci $\pm\sqrt{7}, 0$ lies on a circle with centre $(0, 3)$

$\therefore r = \sqrt{7 + 9} = 4$

- 34). If the focal distance of an end of the minor axis of any ellipse (referred to its axes as the axes of x and y respectively) is k and the distance between the foci is $2h$, then its equation is

- 1) $\frac{x^2}{k^2} + \frac{y^2}{k^2 + h^2} = 1$ 2) $\frac{x^2}{k^2} + \frac{y^2}{h^2 - k^2} = 1$ 3) $\frac{x^2}{k^2} + \frac{y^2}{k^2 - h^2} = 1$ 4) $\frac{x^2}{k^2} + \frac{y^2}{h^2} = 1$

Key (3)

Sol:

$B(0, b)$

Focal distance $a + e(0) = k$

$a = k$

$2ae = 2h$

$ae = h$

$b^2 = a^2(1 - e^2)$

$b^2 = k^2 - h^2$

- 35). The graphs of $x^2 + y^2 + 6x - 24y + 72 = 0$ and $x^2 - y^2 + 6x + 16y - 46 = 0$ intersect at four points. Then the sum of the distances from the four points to $(-3, 2)$ is

1) 20

2) 30

3) 40

4) 50

Key (3)**Sol:**

Adding two equations we get parabola $(x + 3)^2 = 4(y - 1)$. Sum of distances from given points on the parabola to $(-3, 2)$, the focus of parabola is equal to sum of their distances from directrix $y = 0$ which is equal to the sum of the ordinates of points of intersection. These points lie on $y^2 - 20y + 59 = 0$

(subtracting two equations) hence sum = $2 \times 20 = 40$

CHORDS

36). If PSQ is a focal chord of the ellipse $16x^2 + 25y^2 = 400$ such that $SP = 8$ then the length of SQ =

1) 2

2) $\frac{11}{3}$

3) 16

4) 25

Key (1)**Sol**

$$\frac{1}{SP} + \frac{1}{SQ} = \frac{2a}{b^2}$$

37). If PSQ and $PS'R$ are two chords of an ellipse through its foci S and S' , then

$$\frac{PS}{SQ} + \frac{PS'}{S'R} =$$

1) $2\left(\frac{1+e^2}{1-e^2}\right)$

2) $\frac{1+e^2}{1-e^2}$

3) $2\left(\frac{1-e^2}{1+e^2}\right)$

4) $\frac{1-e^2}{1+e^2}$

Key (1)

Sol $SP + S'P = 2a, SS' = 2ae, \frac{1}{SP} + \frac{1}{SQ} = \frac{2}{\text{length of semi latus rectum}}.$

38). If SP^1 is a focal chord of $\frac{x^2}{25} + \frac{y^2}{9} = 1$ and $SP=3$ then $6l(SP^1) - 56 =$

Where S, S^1 are foci of ellipse.

Key (7)

Sol $\because \frac{1}{SP} + \frac{1}{S^1P^1} = \frac{2}{1} = \frac{2}{\left(\frac{b}{a}\right)^2} \Rightarrow \frac{1}{S^1P^1} = \frac{10}{9} - \frac{1}{7} = \frac{61}{63}$

$\therefore 6l(SP^1) - 56 = 7$

39). A man running round a race course notes that the sum of the distances of two flag posts from him is 8 meters. The area of the path he encloses in square meters if the distance between flag posts is 4 is

1. $15\sqrt{3}\pi$ 2. $12\sqrt{3}\pi$ 3. $18\sqrt{3}\pi$ 4. $8\sqrt{3}\pi$

Key (4)

Sol Area = πab

40). If the variable lines $l_1(x-a)+y=0$ and $l_2(x+a)+y=0$ are conjugate

lines with respect to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then the locus of their

point of intersection is

1. $\frac{2x^2}{a^2} + \frac{y^2}{b^2} = 1$ 2. $\frac{x^2}{a^2} + \frac{2y^2}{b^2} = 1$ 3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ 4. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$

Key (2)

Sol

$$\ell_1 \ell_2 = \frac{-b^2}{2a^2}, \ell_1 \ell_2 (x^2 - a^2) = -y^2$$

41). If the mid -point of a chord of the ellipse $\frac{x^2}{16} + \frac{y^2}{25} = 1$ is (0,3) length of the

chord is $\frac{4K}{5}$ then K is

Key (8)

Sol

Equation of the chord is

$$\frac{3y}{25} - 1 = \frac{9}{25} - 1 \quad \text{i.e. } y = 3 \text{ intersect}$$

$$\text{The ellipse } \frac{x^2}{16} + \frac{y^2}{25} = 1$$

$$\Rightarrow x = \frac{16}{5}$$

$$\text{Length of the chord } \frac{32}{5} \Rightarrow \frac{4k}{5} = \frac{32}{5} \Rightarrow k = 8$$

42). The locus of the mid-point of the portion of a tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ included between the coordinate axes is } \frac{a^2}{x^2} + \frac{b^2}{y^2} = k$$

where $k =$

1) $\frac{1}{2}$

2) 2

3) $\frac{1}{4}$

4) 4

Key (4)

Sol Let $M = (\alpha, \beta)$ be the mid-point of portion of a tangent included between the axes

$$\text{Equation of that line is } \frac{x}{2\alpha} + \frac{y}{2\beta} = 1 \Rightarrow y = \left(\frac{-\beta}{\alpha} \right)x + 2\beta$$

$$\text{Now } c^2 = am^2 + b^2 \Rightarrow 4\beta^2 = a^2 \left(\frac{\beta^2}{\alpha^2} \right) + b^2$$

$$\therefore \frac{a^2}{\alpha^2} + \frac{b^2}{\beta^2} = 4$$

$$\text{Locus of M is given by } \frac{a^2}{x^2} + \frac{b^2}{y^2} = 4$$

43). The locus of mid point of chords of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

which passes through the foot of the directrix from focus is

1. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{ex}{a^2}$

2. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x}{ae}$

3. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x}{a^2e}$

4. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{x}{ae^2}$

Key (2)

Sol $S_1 = S_{11}$ passes through $\left(\frac{a}{e}, 0\right)$

44). Locus of the vertices of an equilateral triangle circumscribing the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

1) $3(x^2 + y^2 - a^2 - b^2)^2 = 4(b^2x^2 + a^2y^2 - a^2b^2)$

2) $(x^2 + y^2 - a^2 - b^2)^2 = (b^2x^2 + a^2y^2 - a^2b^2)$

3) $(x^2 + y^2 - a^2 - b^2)^2 = 4(b^2x^2 + a^2y^2 - a^2b^2)$

4) $4(x^2 + y^2 - a^2 - b^2)^2 = 3(b^2x^2 + a^2y^2 - a^2b^2)$

Key (1)

Sol

Put $\theta = 60^\circ$ in

$$(x^2 + y^2 - (a^2 + b^2))^2 \tan^2 \theta = 4(b^2x^2 + a^2y^2 - a^2b^2)$$

45). A ray of light along the line $x=3$ is reflected at a point on the

ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$. The slope of the reflected ray is

1) $\frac{4}{15}$

2) $\frac{8}{15}$

3) $\frac{15}{8}$

4) $\frac{15}{4}$

Key (2)

Sol

Reflection property of ellipse

46). AB and CD are two equal and parallel chords of their ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Tangents to the ellipse at A and B intersect at P and at C and D at Q, then line PQ

1) passes through the origin

2) is bisected at origin

3) cannot pass through the origin

4) is not bisected at the origin

Key (1,2)

Sol

(ab) let $P(\alpha, \beta), Q(\alpha_1, \beta_1)$. Equations of AB and Cd are

$$\frac{x}{a}\alpha + \frac{y}{b}\beta = 1 \quad \& \quad \frac{x}{a}\alpha_1 + \frac{y}{b}\beta_1 = 1$$

$$\frac{\alpha}{\alpha_1} = \frac{\beta}{\beta_1} = k \Rightarrow \frac{\alpha^2}{a^2} + \frac{\beta^2}{b^2} = \frac{\alpha_1^2}{a^2} + \frac{\beta_1^2}{b^2} \Rightarrow \frac{\alpha}{\alpha_1} = \frac{\beta}{\beta_1} = -1 ;$$

‘PQ’ passes through origin and is bisected at origin.

47). Let PQ be a chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, which subtends an angle of $\pi/2$ radians at the centre. If L is the foot of perpendicular from (0, 0) to PQ, then

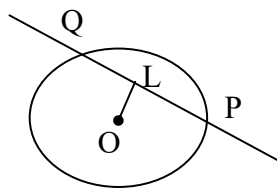
- 1) locus of L is an ellipse
- 2) locus of L is circle concentric with given ellipse
- 3) locus of L is a hyperbola concentric with given ellipse
- (4) a square concentric with given ellipse

Key (2)

Sol PQ : $x\cos\alpha + y\sin\alpha - p = 0 \quad \dots (A)$

Homogenising $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with (A)

$$\frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} = \text{constant}$$



48) If the chords of contact of tangents from two points (x_1, y_1) and (x_2, y_2) to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are at right angles then $\left(\frac{x_1 x_2}{y_1 y_2}\right)$ is equal to

- 1) $\frac{a^2}{b^2}$
- 2) $-\frac{b^2}{a^2}$
- 3) $-\frac{a^4}{b^4}$
- 4) $-\frac{b^4}{a^4}$

Key (3)

Sol Chord of contact from (x_1, y_1) is

$$\frac{xx_1}{a^2} + \frac{yy_1}{b^2} = 1$$

Whose slope is $-\frac{b^2 x_1}{a^2 y_1}$

Similarly slope of another chord of contact is $-\frac{b^2 x_2}{a^2 y_2}$

We have $\left(-\frac{b^2}{a^2} \frac{x_1}{y_1}\right) \times \left(-\frac{b^2}{a^2} \frac{x_2}{y_2}\right) = -1 \Rightarrow \frac{x_1 x_2}{y_1 y_2} = -\frac{a^4}{b^4}$

- 49). If a line through P(a, 2) meets the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at A and D and meets the axes at B and C, so that PA, PB, PC, PD are in G.P., then the minimum value of |a|, is....

Key (6)

Sol

$$\frac{x-a}{\cos \theta} = \frac{y-2}{\sin \theta} = r \quad (a + r \cos \theta, 2 + r \sin \theta) \text{ lies on ellipse for A and D.}$$

$$\frac{(a + r \cos \theta)^2}{9} + \frac{(2 + r \sin \theta)^2}{4} = 1 \Rightarrow r_1 r_2 = \text{PA} \cdot \text{PD}$$

PA, PB, PC, PD are in G.P. $\text{PA} \cdot \text{PD} = \text{PB} \cdot \text{PC}$. etc.....

- 50). Maximum length of the chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

such that eccentric angles of the extremities of the chord are differ by $\frac{\pi}{2}$,
is (a>b)

1) $a\sqrt{2}$ 2) $b\sqrt{2}$ 3) $ab\sqrt{2}$ 4) ab

Key (1)

Sol $P = (a \cos \theta, b \sin \theta) \quad Q = \left(a \cos \frac{\pi}{2} + \theta, b \sin \left(\frac{\pi}{2} + \theta\right)\right)$

- 51). The distance of a point P on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ from centre C is 2, then

eccentric angle of P may be

1) $\frac{\pi}{4}$ 2) $\frac{\pi}{6}$ 3) $\frac{3\pi}{4}$ 4) $\frac{7\pi}{4}$

Key (1,3,4)

Sol

$$\cos^2 \theta = \frac{d^2 - b^2}{a^2 - b^2} = \frac{4 - 2}{6 - 2} = \frac{1}{2} \Rightarrow \cos \theta = \pm \frac{1}{\sqrt{2}} \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{7\pi}{4}$$

52). If the equation of the chord joining the points $P(\theta)$ and

$$D\left(\theta + \frac{\pi}{2}\right) \text{ on } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \text{ is } x \cos \alpha + y \sin \alpha = p \text{ then } a^2 \cos^2 \alpha + b^2 \sin^2 \alpha =$$

1) $4p^2$

2) p^2

3) $\frac{p^2}{2}$

4) $2p^2$

Key (4)

$$\begin{aligned} \frac{x}{a} \cos \left(\frac{\theta + \theta + \frac{\pi}{2}}{2} \right) + \frac{y}{b} \sin \left(\frac{\theta + \theta + \frac{\pi}{2}}{2} \right) \\ = \cos \left(\frac{\theta - \theta - \frac{\pi}{2}}{2} \right) \rightarrow 1 \text{ eq} \end{aligned}$$

$$x \cos \alpha + y \sin \alpha = P \rightarrow 2 \text{ eq}$$

$$(1) = (2)$$

53). PQ is variable chord of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ If PQ subtends a right

angle at the centre ' O ' of the ellipse then $\frac{1}{OP^2} + \frac{1}{OQ^2}$ is equal to

1) $\frac{1}{a^2} + \frac{1}{b^2}$

2) $\frac{2}{a^2} + \frac{1}{b^2}$

3) $\frac{1}{a^2} + \frac{2}{b^2}$

4) $2\left(\frac{1}{a^2} + \frac{2}{b^2}\right)$

Key (1)

Sol

$$\angle POQ = 90^\circ, P = (r_1 \cos \theta, r_1 \sin \theta) \text{ where } r_1 = OP$$

$$Q = (r_2 \cos(90^\circ + \theta), r_2 \sin(90^\circ + \theta)) \text{ where } r_2 = OQ$$

54). The locus of the point of intersection of tangents at the extremities of a chord whose eccentric angles are differ by $\frac{\pi}{2}$, of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity 'e' is another ellipse of the eccentricity.

- 1) e 2) 2e 3) 3e 4) $\frac{e}{2}$

Key (2)

Sol

$$p(\theta) = (a \cos \theta, b \sin \theta) \text{ and } Q\left(\frac{\pi}{2} + \theta\right) = (-a \sin \theta, b \cos \theta)$$

Eliminate θ from equation of tangents at P and CQ.

$$\text{We get the locus } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

55). The length of the longest chord of the ellipse $4x^2 + 9y^2 = 1$ drawn through

$$\left(0, -\frac{1}{3}\right) \text{ is } \frac{3}{\sqrt{\lambda}} \text{ where } \lambda \text{ is equal to } \underline{\hspace{2cm}}$$

- 1) 15 2) 25 3) 20 4) 36

Key (3)

Sol

$$P\left(\frac{1}{2} \cos t, \frac{1}{3} \sin t\right) \& Q\left(0, -\frac{1}{3}\right)$$

$$PQ^2 = \frac{5}{36} \sin^2 t + \frac{2}{9} \sin t + \frac{13}{36}$$

$$(PQ^2)' = 0 \Rightarrow \sin t = \frac{4}{5}$$

$$\max \text{ of } (PQ^2) = \frac{9}{20} \Rightarrow \max(PQ) = \frac{3}{\sqrt{20}}$$

56) .If α, β are the eccentric angles of the ends of a focal chord of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ then the eccentricity of the ellipse is}$$

- 1) $\frac{\sin \alpha - \sin \beta}{\sin(\alpha + \beta)}$ 2) $\frac{\sin \alpha + \sin \beta}{\sin(\alpha + \beta)}$ 3) $\frac{\cos \alpha - \cos \beta}{\cos(\alpha + \beta)}$ 4) $\frac{\cos \alpha + \cos \beta}{\cos(\alpha - \beta)}$

Key (2)

Sol

$$\frac{x}{a} \cos\left(\frac{\alpha + \beta}{2}\right) + \frac{y}{b} \sin\left(\frac{\alpha + \beta}{2}\right) = \cos\left(\frac{\alpha - \beta}{2}\right) \text{ Passing through } (ae, 0)$$

57). P and Q are points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose center is C .

The eccentric angles of P and Q differ by a right angle.

If $\angle PCQ$ minimum, the eccentric angle of P can be

- 1) $\frac{\pi}{6}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{12}$

Key (2)

Sol

Since the eccentric angles of P and Q differ by a right angle, we can take P as $(a \cos \theta, b \sin \theta)$ and Q as $(-a \sin \theta, b \cos \theta)$

$$\text{slope of } CP = \frac{b \sin \theta}{a \cos \theta} \qquad \text{slope of } CQ = \frac{b \cos \theta}{-a \sin \theta}$$

If A is the angle between CP and CQ , then

$$\begin{aligned} A &= \tan^{-1} \left| \frac{\frac{b \sin \theta}{a \cos \theta} + \frac{b \cos \theta}{a \sin \theta}}{1 - \frac{b^2 \sin \theta \cos \theta}{a^2 \cos \theta \sin \theta}} \right| \\ &= \tan^{-1} \left| \frac{2ab}{a^2 - b^2} \frac{1}{\sin 2\theta} \right| \end{aligned}$$

A is minimum, if $\sin 2\theta$ is maximum, i.e, if 2θ

$$= \pi / 2, \theta = \pi / 4$$

58). The points of intersection of two ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

be at the extremities of conjugate diameters of $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then $\frac{a^2}{\alpha^2} + \frac{b^2}{\beta^2} =$

- 1) 1 2) 2 3) 3 4) 4

Key (2)

Sol

Clearly $P(a \cos \theta, b \sin \theta)$ $Q(-a \sin \theta, b \cos \theta)$ are extremities of conjugate

diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

P and Q lies on $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

$$\frac{a^2 \cos^2 \theta}{\alpha^2} + \frac{b^2 \sin^2 \theta}{\beta^2} = 1$$

$$\frac{a^2 \sin^2 \theta}{\alpha^2} + \frac{b^2 \cos^2 \theta}{\beta^2} = 1$$

$$(+)$$

$$\frac{a^2}{\alpha^2} + \frac{b^2}{\beta^2} = 2$$

59). If circumcentre of an equilateral triangle inscribed in $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with

vertices having eccentric angles α, β, γ respectively is

(x_1, y_1) then $\sum \cos \alpha \cdot \cos \beta + \sum \sin \alpha \cdot \sin \beta$ is

1) $\frac{9x_1^2}{2a^2} + \frac{9y_1^2}{2b^2} - \frac{3}{2}$

2) $\frac{x_1^2}{2a^2} + \frac{y_1^2}{2b^2} - \frac{5}{2}$

3) $\frac{x_1^2}{9a^2} + \frac{y_1^2}{9b^2} - \frac{5}{9}$

4) $\frac{x_1^2}{a^2} + \frac{y_1^2}{b^2} - \frac{1}{2}$

Key (1)

Sol

$A(a \cos \alpha, b \sin \alpha), B(a \cos \beta, b \sin \beta), C(a \cos \gamma, b \sin \gamma)$

Centroid = circumcentre $= (x_1, y_1) = \left[\frac{\sum a \cos \alpha}{3}, \frac{\sum b \sin \alpha}{3} \right]$

$$\frac{3x_1}{a} = \sum \cos \alpha, \frac{3y_1}{b} = \sum \sin \alpha$$

$$\left(\frac{9x_1^2}{a^2} + \frac{9y_1^2}{b^2} - 3 \right) = 2(\sum \cos \alpha \cos \beta + \sum \sin \alpha \sin \beta)$$

60). If $\phi_1, \phi_2, \phi_3, \phi_4$ are the eccentric angles of four concyclic points on the

ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then $\cos(\phi_1 + \phi_2 + \phi_3 + \phi_4)$ is equal to _____

Key (1)

Sol

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 = 2n\pi$$

$$\cos(\phi_1 + \phi_2 + \phi_3 + \phi_4) = 1$$

61). If P and Q are the ends of a pair of conjugate diameters and C is the centre of the ellipse $4x^2 + 9y^2 = 36$ then the area of ΔCPQ in square units.

Key (3)

Sol

$$\frac{x^2}{9} + \frac{y^2}{4} = 1,$$

$$\text{so } P = (3\cos\theta, 2\sin\theta) \text{ and } Q = \left(3\cos\left(\frac{\pi}{2} + \theta\right), 2\sin\left(\frac{\pi}{2} + \theta\right)\right)$$

$$\text{Area of } \Delta CPQ = \frac{1}{2} \begin{vmatrix} 0 & 0 & 1 \\ 3\cos\theta & 2\sin\theta & 1 \\ -3\sin\theta & 2\cos\theta & 1 \end{vmatrix} = 3.$$

62). The point 'P' on the ellipse $4x^2 + 9y^2 = 36$ is such that the area of the triangle PF_1F_2 is $\sqrt{5}$ where F_1, F_2 are foci. Then the co-ordinates of 'P' are

$$1) \left(\frac{3}{\sqrt{2}}, \sqrt{2}\right) \quad 2) (3/2, -2) \quad 3) \left(-\frac{3\sqrt{3}}{2}, 4\right) \quad 4) \left(\frac{3\sqrt{3}}{2}, 1\right)$$

Key (4)

Sol

$$\text{Let } P = (3\cos\theta, 2\sin\theta), F_1F_2 = 2ae = 2\sqrt{5}$$

$$\Delta PF_1F_2 = \frac{1}{2}(2\sqrt{5})(2\sin\theta) = \sqrt{5} \Rightarrow \sin\theta = \pm \frac{1}{2}$$

$$\cos\theta = \pm \frac{\sqrt{3}}{2}$$

63). If P, Q, R are three points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angles are $\theta, \theta + \alpha, \theta + 2\alpha$ then

- 1) The area of ΔPQR when $\theta = \frac{\pi}{6}$ is $\frac{\sqrt{3}}{2}$ times area of ΔPQR when $\theta = \frac{\pi}{3}$
- 2) The area of ΔPQR is maximum when $\alpha = \frac{2\pi}{3}$
- 3) The area of ΔPQR is independent of ' α '
- 4) When $\alpha = \frac{\pi}{2}, \theta = \frac{\pi}{4}$ then area of ΔPQR is 'ab'

Key (2,4)

Sol
$$\Delta PQR = 2ab \sin^2 \frac{\alpha}{2} \sin \alpha$$

It is maximum when $\alpha = \frac{2\pi}{3}$

64). The points P, Q, R with eccentric angles $\theta, \theta + \alpha, \theta + 2\alpha$ are taken on

the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then

- 1) area of ΔPQR is independent of θ only.
- 2) area of ΔPQR is independent of α only
- 3) area of ΔPQR is independent of both θ and α
- 4) area of ΔPQR is maximum when $\alpha = \frac{2\pi}{3}$

Key (1,4)

Sol
$$A = \text{area of } \Delta PQR = ab \sin \alpha (1 - \cos \alpha)$$

Which is independent of θ only.

$$\frac{dA}{d\alpha} = ab [\cos \alpha - \cos 2\alpha]$$

For maxima, $\frac{dA}{d\alpha} = 0$

$$\Rightarrow \cos \alpha = \cos 2\alpha \Rightarrow 2\alpha = 2\pi - \alpha \quad \Rightarrow \alpha = \frac{2\pi}{3}$$

65). If the chord through the points whose eccentric angles are θ and ϕ on the

ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ passes through a focus then $\tan \frac{\theta}{2} \cdot \tan \frac{\phi}{2}$ is

1) $\frac{1}{9}$

2) -9

3) $-\frac{1}{9}$

4) 9

Key (3,4)

Sol Equation of the chord $\frac{x}{5} \cos\left(\frac{\theta+\phi}{2}\right) + \frac{y}{3} \sin\left(\frac{\theta+\phi}{2}\right) = \cos\left(\frac{\theta-\phi}{2}\right)$

If it passes through $(4, 0)$, $\frac{\cos\left(\frac{\theta+\phi}{2}\right)}{\cos\left(\frac{\theta-\phi}{2}\right)} = \frac{5}{4} \Rightarrow \tan \frac{\theta}{2} \cdot \tan \frac{\phi}{2} = -\frac{1}{9}$

If it passes through $(-4, 0)$ then $\tan \frac{\theta}{2} \cdot \tan \frac{\phi}{2} = 9$

66). P and Q are two points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose eccentric angles are differ by 90° then

1) Locus of point of intersection of tangents at P and Q is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

2) Locus of Mid point (P, Q) is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$

3) Product of slopes of OP and OQ where O is the centre is $-\frac{b^2}{a^2}$

4) Max area of $\Delta^{le}OPQ$ is $\frac{1}{2}ab$

Key (1,2,3,4)

Sol

$$P = (a \cos \theta, b \sin \theta) \quad Q = (-a \sin \theta, b \cos \theta)$$

Tangent at P $\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1 \rightarrow (1)$

$$\frac{-x \sin \theta}{a} + \frac{y \cos \theta}{b} = 1 \rightarrow (2)$$

Elimination θ $(1)^2 + (2)^2 \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \quad \therefore (1) \text{ is correct}$

Now mid(PQ) = $\left(\frac{a(\cos \theta - \sin \theta)}{2}, \frac{b(\sin \theta + \cos \theta)}{2} \right) = (x, y)$

$\cos \theta - \sin \theta = \frac{2x}{a} \rightarrow (3)$

$\cos \theta + \sin \theta = \frac{2y}{b} \rightarrow (4)$

$(3)^2 + (4)^2 \Rightarrow 2 = 4 \left(\frac{x^2}{a^2} + \frac{y^2}{b^2} \right) \Rightarrow \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2} \quad \therefore (2) \text{ is correct}$

Slope of OP = $\frac{b \sin \theta}{a \cos \theta} = m_1$

Slope of OQ = $\frac{-b \cos \theta}{a \sin \theta} = m_2$

Now $m_1 m_2 = \frac{-b^2}{a^2} \quad \therefore (1) \text{ is correct}$

Now area of triangle OPQ = $\frac{1}{2} \begin{vmatrix} a \cos \theta & b \sin \theta \\ -a \sin \theta & b \cos \theta \end{vmatrix}$

$\frac{1}{2} ab(\cos^2 \theta + \sin^2 \theta) = \frac{1}{2} ab \quad \therefore (4) \text{ is correct}$

67). The eccentric angle of a point on the ellipse $x^2 + 3y^2 = 6$ at a distance 2 unit from the origin is

1) $\frac{\pi}{4}$

2) $\frac{3\pi}{4}$

3) $\frac{5\pi}{4}$

4) None of these

Key (1,2,3)

Sol A point on the ellipse $\frac{x^2}{6} + \frac{y^2}{2} = 1$ is $(\sqrt{6} \cos \theta, \sqrt{2} \sin \theta)$

$6 \cos^2 \theta + 2 \sin^2 \theta = 4$

$\Rightarrow 4 \cos^2 \theta = 2$

$\cos^2 \theta = \frac{1}{2}$

$\cos^2 \theta = 0$

$2\theta = (2n+1)\frac{\pi}{2}, \theta = (2n+1)\frac{\pi}{4}, n \in I.$

TANGENTS IN SOLPE FORM

68). Tangents drawn to the ellipse $\frac{x^2}{36} + \frac{y^2}{9} = 1$ from the point P meet the co ordinate axes at con cyclic points . The locus of the point P is

1) $x^2 + y^2 = 7$ 2) $x^2 + y^2 = 25$ 3) $x^2 - y^2 = 7$ 4) $x^2 - y^2 = 25$

Key (3)

Sol Let $P = (h, k)$ Equation of any tangent is $y = mx \pm \sqrt{16m^2 + 9}$

$$\Rightarrow k = mh \pm \sqrt{16m^2 + 9} \Rightarrow m^2(h^2 - 16) - 2mhk + (k^2 - 9) = 0$$

Let m_1, m_2 are the slope of the tangents $m_1 m_2 = \frac{k^2 - 9}{h^2 - 16}$

For concyclic points $m_1 m_2 = 1 \Rightarrow h^2 - 16 = k^2 - 9$

$$\Rightarrow h^2 - k^2 = 7 \Rightarrow x^2 - y^2 = 7$$

69). The equation of common tangents of the curves

$x^2 + 4y^2 = 8$ and $y^2 = 4x$ are

1) $y - 2x - 4 = 0, y + 2x + 4 = 0$ 2) $y - 2x - 2 = 0, y + 2x + 2 = 0$
 3) $2y - x - 4 = 0, 2y + x + 4 = 0$ 4) $2y - x - 2 = 0, 2y + x + 2 = 0$

Key (3)

Sol Tangent in terms of 'm' for $y^2 = 4x$ is

$$y = mx + \frac{1}{m}$$

and for $\frac{x^2}{8} + \frac{y^2}{2} = 1$ tangent is,

$$y = mx \pm \sqrt{8m^2 + 2}$$

For common tangents we must have

$$\frac{1}{m} = \pm \sqrt{8m^2 + 2}$$

$$\Rightarrow m = \pm \frac{1}{2} \text{ Thus, common tangents are } y = \frac{x}{2} + 2 \text{ and } y = -\frac{x}{2} - 2$$

70). The angle between the tangents drawn to the ellipse $3x^2 + 2y^2 = 5$, from the point P(1,2) is equal to

$$1) \tan^{-1}(24\sqrt{5}) \quad 2) \tan^{-1}(12\sqrt{5}) \quad 3) \tan^{-1}\left(\frac{24}{\sqrt{5}}\right) \quad 4) \tan^{-1}\left(\frac{12}{\sqrt{5}}\right)$$

Key(4)

Sol

Tangent to the ellipse in terms of 'm' is,

$$y = mx \pm \sqrt{\frac{5m^2}{3} + \frac{5}{2}}.$$

If it passes through P(1,2), then

$$(2 - m)^2 = \frac{5m^2}{3} + \frac{5}{2}$$

$$\Rightarrow 4m^2 + 24m - 9 = 0$$

If it's roots are m_1 and m_2 then $m_1 + m_2 = -6, m_1 m_2 = -\frac{9}{4}$

$$\Rightarrow |m_1 - m_2| = \sqrt{36 + 9} = 3\sqrt{5}$$

If the required angle is ' θ ', then

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \frac{3\sqrt{5}}{\left| 1 - \frac{9}{4} \right|} = \frac{12}{\sqrt{5}}$$

71). A circle of radius 'r' is concentric with the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

Then inclination of common tangent with major axis is _____ ($b < r < a$)

$$1. \tan^{-1}\left(\frac{b}{a}\right) \quad 2. \tan^{-1}\left(\frac{rb}{a}\right) \quad 3. \tan^{-1}\sqrt{\frac{r^2 - b^2}{a^2 - r^2}} \quad 4. \frac{\pi}{2}$$

Key(3)

Sol

The tangent of Ellipse is $y = mx + \sqrt{a^2m^2 + b^2}$, this line touches $x^2 + y^2 = r^2$

$$\text{Condition is } \left| \frac{\sqrt{a^2m^2 + b^2}}{\sqrt{m^2 + 1}} \right| = r$$

$$a^2m^2 + b^2 = r^2m^2 + r^2$$

$$m^2(a^2 - r^2) = r^2 - b^2 \Rightarrow m^2 = \frac{r^2 - b^2}{a^2 - r^2}$$

$$m = \sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$$

$$\text{Inclination is } \tan^{-1} \sqrt{\frac{r^2 - b^2}{a^2 - r^2}}$$

72). If the tangents to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ make angles α, β with the major axis such that $\tan \alpha + \tan \beta = \lambda$, where λ is a non-zero constant then the possible locus of their point of intersection is

1) $x^2 + y^2 = a^2$

2) $x^2 + y^2 = b^2$

3) $x^2 - a^2 = 2\lambda xy$

4) $\lambda(x^2 - a^2) = 2xy$

Key(4)

Sol

Tangent to the ellipse is

$$y = mx + \sqrt{a^2m^2 + b^2}$$

It passes through the point $P(h, k)$

$$k = mh + \sqrt{a^2m^2 + b^2}$$

$$(a^2 - h^2)m^2 + 2hkm + b^2 - k^2 = 0$$

Now given that $\tan \alpha + \tan \beta = \lambda \Rightarrow m_1 + m_2 = \lambda$

$$\Rightarrow \text{locus is } \lambda(x^2 - a^2) = 2xy$$

73). A circle of radius 2 is concentric with the ellipse $\frac{x^2}{7} + \frac{y^2}{3} = 1$ then inclination of common tangent with X-axis

- 1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $\frac{\pi}{6}$

Key(4)

Sol

tangent is $y = mx + \sqrt{7m^2 + 3}$

$$\frac{x^2}{7} + \frac{y^2}{3} = 1$$

$$x^2 + y^2 = 4$$

It is also touching $x^2 + y^2 = 4$

$$\left| \frac{\sqrt{7m^2 + 3}}{\sqrt{m^2 + 1}} \right| = 2$$

$$7m^2 + 3 = 4m^2 + 4 \quad m^2 = \frac{1}{3} \Rightarrow m = \frac{1}{\sqrt{3}}$$

$$\therefore \tan \theta = \frac{1}{\sqrt{3}} \quad \theta = \frac{\pi}{6}$$

74). The four common tangents to the ellipses $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and $\frac{x^2}{4} + \frac{y^2}{9} = 1$

forms a square of area

- 1) 13 Sq. Units 2) 26 Sq. units 3) 25 Sq. units 4) none

Key(2)

Sol

Equation if tangents are

$$y = mx + \sqrt{9m^2 + 4}$$

$$\&x = m^1 y + \sqrt{am^1 + 4}$$

$$\text{Comparing } \frac{m}{-1} = \frac{1}{m^1} = \frac{\sqrt{9m^2 + 4}}{\sqrt{9m^1^2 + 4}}$$

$$mm^1 = 1 \quad m^2 = \frac{9m^2 + 4}{9m^{1^2} + 4}$$

$$\Rightarrow m = 1, -1$$

$$\text{Tangents ax } y = x \pm \sqrt{13}$$

$$\& x = -y \pm \sqrt{13}$$

$$\text{Area} = 26 \text{ sq. Units}$$

75). The line $2px + y\sqrt{1-p^2} = 1$ ($|p| < 1$) for different value of p touches.

1) An ellipse of eccentricity $\frac{2}{\sqrt{3}}$ 2) An ellipse of eccentricity $\frac{\sqrt{3}}{2}$

3) Hyperbola of eccentricity 2 4) None

Key(2)

$$y = \frac{-2p}{\sqrt{1-p^2}}x + \frac{1}{\sqrt{1-p^2}}$$

$$m = -\frac{2p}{\sqrt{1-p^2}} \Rightarrow p^2 = \frac{m^2}{4+m^2}$$

$$y = mx + \frac{1}{\sqrt{1-\frac{m^2}{4+m^2}}} \Rightarrow y = mx + \sqrt{\frac{4+m^2}{4}}$$

$$\Rightarrow y = mx + \sqrt{1 + \frac{1}{4}m^2}$$

$$\text{It touches } \frac{x^2}{1/4} + \frac{y^2}{1} = 1, e = \frac{\sqrt{3}}{2}$$

76). If a tangent of slope 2 of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is normal to the circle

$$x^2 + y^2 + 4x + 1 = 0 \text{ then the maximum value of } ab \text{ is}$$

1) 2

2) 4

3) 6

4) Can n't be found

Key(2)

Sol

A tangent of slope 2 is $y = 2x \pm \sqrt{4a^2 + b^2}$ this is normal to

$$x^2 + y^2 + 4x + 1 = 0$$

$$\text{then } 0 = -4 \pm \sqrt{4a^2 + b^2} \Rightarrow 4a^2 + b^2 = 16 \text{ using } Am \geq GM$$

$$ab \leq 4$$

77). If a tangent of slope $\frac{1}{3}$ of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) is normal to the circle $x^2 + y^2 + 2x + 2y + 1 = 0$, then

- 1) maximum value of ab is $\frac{2}{3}$ 2) $a \in \left(\sqrt{\frac{2}{5}}, 2\right)$
 3) $a \in \left(\frac{2}{3}, 2\right)$ 4) maximum value of ab is 1

Key(1,2)

Sol

$$y = mx \pm \sqrt{a^2 m^2 + b^2} \quad y = \frac{1}{3}x \pm \sqrt{\frac{a^2}{9} + b^2}$$

$$3y = x \pm \sqrt{a^2 + 9b^2} \text{ passes through } (-1, -1)$$

$$-3 = -1 \pm \sqrt{a^2 + 9b^2}$$

$$-2 = \pm \sqrt{a^2 + 9b^2}, \quad a^2 + 9b^2 = 4, \quad \frac{a^2 + 9b^2}{2} \geq \sqrt{9a^2 b^2}$$

$$2 \geq 3ab, \quad ab \leq \frac{2}{3} \quad \because a > b \quad a^2 + 9a^2 > 4 \quad a^2 > \frac{4}{10}$$

$$a > \sqrt{\frac{2}{5}} \text{ but } a \leq 2$$

78). The focal chord to $y^2 = 16x$ is tangent to $(x-6)^2 + y^2 = 2$, then slope of focal chord is

- 1) 1 2) $\frac{1}{2}$ 3) $-\frac{1}{2}$ 4) -1

Key(1,4)

Sol

$$(x-6)^2 + y^2 = 2 \rightarrow \text{tangent is } y = m(x-6) + \sqrt{2m^2 + 2}$$

It is passing through (4, 0) focus of parabola

$$0 = -2m + \sqrt{2m^2 + 2} \Rightarrow 2m^2 + 2 = 4m^2$$

$$m^2 = 1 \Rightarrow m = \pm 1$$

- 79). The number of values of c such that the straight line $y = 4x + c$ touches the curve $\frac{x^2}{4} + y^2 = 1$ is K then $K = \dots\dots\dots$

Key(2)

Sol If $y = mx + c$ is tangent to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ then

$$C^2 = a^2 m^2 + b^2$$

$$y = 4x + c, \quad \frac{x^2}{4} + \frac{y^2}{1} = 1$$

$$C = \pm\sqrt{65}$$

- 80). The equation of common tangent in first quadrant to

$\frac{x^2}{25} + y^2 = 1$ and $\frac{x^2}{25} + \frac{y^2}{4} = 1$ makes a length of intercept between the coordinate axes is $\frac{14}{\sqrt{k}}$ then $k =$

Key (3)

Sol The tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with slope m is $y = mx + \sqrt{a^2 m^2 + b^2}$

TANGENTS IN PARAMETRIC FORM

- 81) $p(\theta), D\left(\theta + \frac{\pi}{2}\right)$ are two points on the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

Then the locus of point of intersection of the two tangents at P and D to the ellipse is

$$1) \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{4} \quad 2) \frac{x^2}{a^2} + \frac{y^2}{b^2} = 4 \quad 3) \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2 \quad 4) \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$$

Key (3)

Sol

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1 \rightarrow 1\text{eq}$$

$$\frac{x}{a} \cos \left(\frac{\pi}{2} + \theta \right) + \frac{y}{b} \sin \left(\frac{\pi}{2} + \theta \right) = 1 \rightarrow 2\text{eq}$$

Eliminate θ from 1 and 2

82). Locus of the point of intersection of tangents drawn to the given

ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, at points $P(\theta_1)$ and $Q(\theta_2)$; such that

$$\theta_1 - \theta_2 = \frac{2\pi}{3}, \text{ is}$$

$$1) \frac{a^2}{x^2} + \frac{b^2}{y^2} = 4 \quad 2) \frac{a^2}{x^2} + \frac{b^2}{y^2} = 2 \quad 3) \frac{x^2}{a^2} + \frac{y^2}{b^2} = 4 \quad 4) \frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$

Key (3)

Sol

If the point of intersection be $R(h, k)$, then

$$h = \frac{a \cos \left(\frac{\theta_1 + \theta_2}{2} \right)}{\cos \left(\frac{\theta_1 - \theta_2}{2} \right)}, k = \frac{b \sin \left(\frac{\theta_1 + \theta_2}{2} \right)}{\cos \left(\frac{\theta_1 - \theta_2}{2} \right)}$$

$$\Rightarrow h = \frac{a \cos \left(\frac{\theta_1 + \theta_2}{2} \right)}{\frac{1}{2}}$$

$$k = \frac{b \sin \left(\frac{\theta_1 + \theta_2}{2} \right)}{\frac{1}{2}}$$

$$\text{Thus, } \frac{h^2}{a^2} + \frac{k^2}{b^2} = 4$$

$$\text{Hence, locus is, } \frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$$

83). The tangent drawn to the ellipse $\frac{x^2}{16} + \frac{11y^2}{256} = 1$ at the point $P(\theta)$,

touches the circle $(x-1)^2 + y^2 = 16$. Then ' θ ' is equal to

1) $\frac{\pi}{6}$

2) $\frac{\pi}{3}$

3) $\frac{\pi}{4}$

4) $\frac{\pi}{2}$

Key (2)

Sol

Equation of tangent at point ' θ ' is,

$$\frac{x}{4} \cos \theta + \frac{\sqrt{11}}{16} y \sin \theta = 1$$

It will touch the given circle, if

$$\frac{\left| \frac{\cos \theta}{4} - 1 \right|}{\sqrt{\frac{\cos^2 \theta}{16} + \frac{11 \sin^2 \theta}{256}}} = 4$$

$$\Rightarrow 4 \cos^2 \theta + 8 \cos \theta - 5 = 0$$

$$\Rightarrow \cos \theta = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

84). If $(\sqrt{3})bx + ay = 2ab$ is tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

then eccentric angle θ is

1) $\frac{\pi}{4}$

2) $\frac{\pi}{6}$

3) $\frac{\pi}{2}$

4) $\frac{\pi}{3}$

Key (2)

Sol

Equation of tangent at a point $(a \cos \theta, b \sin \theta)$ is $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$

But, it is the same as $\frac{x}{a} \frac{\sqrt{3}}{2} + \frac{y}{b} \frac{1}{2} = 1$

$$\therefore \cos \theta = \frac{\sqrt{3}}{2}, \sin \theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$$

85). The area of the parallelogram formed by the tangents at the

points whose eccentric angles are $\theta, \theta + \frac{\pi}{2}, \theta + \pi, \theta + \frac{3\pi}{2}$ on the

ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

1) ab

2) $4ab$

3) $3ab$

4) $2ab$

Key (2)

Sol Put $\theta = 0$

86). If the tangent at Point P to the ellipse $16x^2 + 11y^2 = 256$ is also the tangent to the circle $x^2 + y^2 - 2x = 15$, then the eccentric angle of point P is

1) $\pm \frac{\pi}{2}$

2) $\pm \frac{\pi}{4}$

3) $\pm \frac{\pi}{3}$

4) $\pm \frac{\pi}{6}$

Key(3)

Sol The equation of tangent at point $P\left(4\cos\theta, \frac{16}{\sqrt{11}}\sin\theta\right)$ to the ellipse

$$16x^2 + 11y^2 = 256 \text{ is}$$

$$16x(4\cos\theta) + 11y\left(\frac{16}{\sqrt{11}}\sin\theta\right) = 256$$

$$4x\cos\theta + \sqrt{11}y\sin\theta = 16$$

$$\text{This touches the circle } (x+1)^2 + y^2 = 16$$

$$\text{So, } \frac{|4\cos\theta - 16|}{\sqrt{16\cos^2\theta + 11\sin^2\theta}} = 4$$

$$\Rightarrow (\cos\theta - 4)^2 = 11 + 5\cos^2\theta$$

$$4\cos^2\theta + 8\cos\theta - 5 = 0$$

$$\therefore \cos\theta = \frac{1}{2}$$

$$\therefore \theta = \pm \frac{\pi}{3}$$

87). If the tangent to the ellipse $x^2 + 4y^2 = 16$ at the point ' α ' is a

normal to the circle $x^2 + y^2 - 8x - 4y = 0$ then $\alpha =$

$$1) \frac{\pi}{2}$$

$$2) \frac{\pi}{4}$$

$$3) \frac{\pi}{3}$$

$$4) -\frac{\pi}{4}$$

Key(1)

Sol

Normal at ' α ' is

$$\frac{x}{4} \cos \alpha + \frac{y}{2} \sin \alpha - 1 = 0$$

It passes through (4,2)

$$\Rightarrow \cos \alpha + \sin \alpha = 1$$

88). A variable tangent of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, meets the coordinate

Axes at the points A and B. Minimum area of circumcircle of triangle OAB, 'O' being the center of the ellipse, is

$$1) \frac{\pi}{4}(a-b)^2 \quad 2) \pi(a^2+b^2) \quad 3) \frac{\pi}{4}(a^2+b^2) \quad 4) \frac{\pi}{4}(a+b)^2$$

Key(4)

Sol

$$bx \cos \theta + ay \sin \theta - ab = 0$$

$$A \equiv (a \sec \theta, 0), B \equiv (0, b \operatorname{cosec} \theta)$$

$$AB^2 = a^2 \sec^2 \theta + b^2 \operatorname{cosec}^2 \theta$$

$$= a^2(1 + \tan^2 \theta) + b^2(1 + \cot^2 \theta)$$

$$= a^2 + b^2 + a^2 \tan^2 \theta + b^2 \cot^2 \theta$$

$$= (a+b)^2 + (a \tan \theta - b \cot \theta)^2$$

$$\geq (a+b)^2$$

$$\Rightarrow AB|_{\text{minimum}} = (a+b)$$

Clearly, AB is the diameter of circumcircle of triangle OAB.

Thus minimum area of the circumcircle of triangle OAB

$$= \frac{\pi}{4}(a+b)^2$$

89). The tangent at the point ' α ' on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the auxiliary circle in two points which subtends a right angle at the centre, then the eccentricity ' e ' of the ellipse is given by the equation :

- | | |
|---------------------------------|---|
| 1) $e^2(1 + \cos^2 \alpha) = 1$ | 2) $e^2(\operatorname{cosec}^2 \alpha - 1) = 1$ |
| 3) $e^2(1 + \sin^2 \alpha) = 1$ | 4) $e^2(1 + \tan^2 \alpha) = 1$ |

Key(3)

Sol

Equation of auxillary circle is $x^2 + y^2 = a^2$

Let P is $(a \cos \alpha, b \sin \alpha)$

Equation of AB is $\frac{x}{a} \cos \alpha + \frac{y}{b} \sin \alpha = 1$

$$b \cos \alpha \cdot x + a \sin \alpha \cdot y = ab \quad \dots (i)$$

To get combined equation of CA and CB, homogenize equation of circle

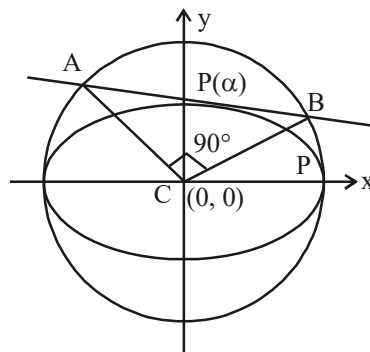
with equation (i),

$$x^2 + y^2 - a^2 \left(\frac{b \cos \alpha x + a \sin \alpha y}{ab} \right)^2 = 0$$

$$b^2 x^2 + b^2 y^2 - (b \cos \alpha \cdot x + a \sin \alpha \cdot y)^2 = 0$$

since $\angle BCA = 90^\circ$

$\therefore$ coefficient of x^2 + coefficient of $y^2 = 0$



$$\Rightarrow b^2 + b^2 - b^2 \cos^2 \alpha - a^2 \sin^2 \alpha = 0$$

$$b^2(\sin^2 \alpha + 1) = a^2 \sin^2 \alpha \quad \Rightarrow \quad \frac{b^2}{a^2} = \frac{\sin^2 \alpha}{1 + \sin^2 \alpha}$$

Now

$$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{\sin^2 \alpha}{1 + \sin^2 \alpha} = \frac{1}{1 + \sin^2 \alpha}$$

$$\therefore e^2(1 + \sin^2 \alpha) = 1.$$

90) .If 'CF' is the perpendicular from the centre C of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

on the tangent at any point P and G is the point where the normal at P meets the major axis, then CF.PG is

- 1) b^2 2) $2b^2$ 3) $\frac{b^2}{2}$ 4) $3b^2$

Key(1)

Sol $CF = \frac{ab}{\sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}} \quad PG = \frac{b}{a} \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$

91). A triangle is formed by a tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the coordinate axes. The area of the triangle cannot be less than

- 1) $\frac{a^2 + b^2}{2}$ sq units 2) $\frac{a^2 + ab + b^2}{3}$ sq units
3) $\frac{a^2 + 2ab + b^2}{2}$ sq units 4) ab sq units

Key(4)

Sol Equation of tangent at θ is $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$

Area with the axes is $\frac{ab}{\sin 2\theta} \geq ab$

92). P_1 and P_2 are the foot of altitudes drawn from the foci S_1 and S_2

respectively of the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ on one of its variable tangent

Maximum value of $(S_1P_1)(S_2P_2)$ is equal to

1) 9

2) 16

3) 25

4) 7

Key(1)

Sol

$$S_1 = (ae, 0), S_2 = (-ae, 0)$$

Let the tangent be

$$x \cos \theta + y \sin \theta = a$$

$$S_1P_1 = \frac{|ae \cos \theta - a|}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}}$$

$$\Rightarrow S_1P_1 = \frac{ab(1 - e \cos \theta)}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}}$$

$$\text{Similarly, } S_2P_2 = \frac{ab(1 + e \cos \theta)}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}}$$

$$\begin{aligned} \Rightarrow S_1P_1 \cdot S_2P_2 &= \frac{a^2 b^2 (1 - e^2 \cos^2 \theta)}{b^2 \cos^2 \theta + a^2 \sin^2 \theta} \\ &= \frac{a^2 b^2 (1 - e^2 \cos^2 \theta)}{a^2 + (b^2 - a^2) \cos^2 \theta} \\ &= \frac{a^2 b^2 (1 - e^2 \cos^2 \theta)}{a^2 - a^2 e^2 \cos^2 \theta} = b^2 \end{aligned}$$

Which is clearly a constant.

Thus, $(S_1P_1)(S_2P_2) = b^2$.

93). OA is the perpendicular drawn from center O of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ to the tangent at any point P on the ellipse.}$$

If the normal drawn to the ellipse at the point P meets the x-axis at B, then (OA).(PB) is equal to

- 1) a^2 2) $a\sqrt{a^2 + b^2}$ 3) b^2 4) $b\sqrt{a^2 + b^2}$

Key(3)

Sol

Let $P \equiv (a \cos \theta, b \sin \theta)$.

Equation of tangent at 'P' is

$$bx \cos \theta + ay \sin \theta - ab = 0$$

$$\Rightarrow OA = \frac{ab}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}}$$

Equation of normal at P is

$$ax \sec \theta - by \csc \theta = (a^2 - b^2)$$

$$\Rightarrow B \equiv \left(\frac{(a^2 - b^2) \cos \theta}{a}, 0 \right)$$

$$\Rightarrow PB^2 = b^2 \sin^2 \theta + \left(a \cos \theta - \frac{(a^2 - b^2) \cos \theta}{a} \right)^2$$

$$= b^2 \sin^2 \theta + \frac{b^4 \cos^4 \theta}{a^2}$$

$$= \frac{b^2}{a^2} (a^2 \sin^2 \theta + b^2 \cos^2 \theta)$$

$$\Rightarrow PB = \frac{b\sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}}{a}$$

$$\Rightarrow (OA)(PB) = b^2$$

- 94). A tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ at any point P meet the line $x = 0$ at a point Q. Let R be the image of Q in the line $y = x$, then circle whose extremities of a diameter are Q and R passes through a fixed point. The fixed point is
- 1) (3, 0) 2) (5, 0) 3) (0, 0) 4) (4, 0)

Key(3)

Sol Equation of the tangent to the ellipse at P $(5 \cos \theta, 4 \sin \theta)$ is

$$\frac{x \cos \theta}{5} + \frac{y \sin \theta}{4} = 1$$

It meets the line $x = 0$ at Q $(0, 4 \operatorname{cosec} \theta)$

Image of Q in the line $y = x$ is R $(4 \operatorname{cosec} \theta, 0)$

$\therefore$ Equation of the circle is

$$X(x - 4 \operatorname{cosec} \theta) + y(y - 4 \operatorname{cosec} \theta) = 0$$

$$\text{i.e. } x^2 + y^2 - 4(x + y) \operatorname{cosec} \theta = 0$$

$\therefore$ each member of the family passes through the intersection of $x^2 + y^2 = 0$

and $x + y = 0$

i.e. the point $(0, 0)$.

- 95). The locus of the point of intersection of tangents at the extremities of a chord

whose eccentric angles are differ by $\frac{\pi}{2}$, of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricity 'e' is another ellipse of the eccentricity.

- 1) e 2) 2e 3) 3e 4) $\frac{e}{2}$

Key(2)

Sol $p(\theta) = (a \cos \theta, b \sin \theta)$ and $Q\left(\frac{\pi}{2} + \theta\right) = (-a \sin \theta, b \cos \theta)$

Eliminate θ from equation of tangents at P and CQ.

We get the locus $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

- 96). On the ellipse $4x^2 + 9y^2 = 1$, the point at which the tangent is parallel to the line $8x = 9y$ is

1. $(2/5, 1/5)$ 2. $(-2/5, 1/5)$ 3. $(-2/5, -1/5)$ 4. None of these

Key(2)

Sol

Let the point be $((1/2)\cos\theta, (1/3)\sin\theta)$,

then the slope of the tangent is

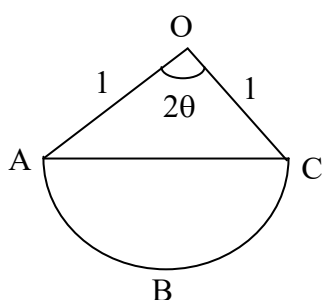
$$-\frac{1/3}{1/2}\cot\theta = \frac{8}{9} \Rightarrow \tan\theta = -\frac{3}{4} \Rightarrow \sin\theta = \pm\frac{3}{5} \text{ and } \cos\theta = \mp\frac{4}{5}$$

$$\text{And the required point can be } \left(-\frac{4}{5} \times \frac{1}{2}, \frac{3}{5} \times \frac{1}{3}\right) = \left(-\frac{2}{5}, \frac{1}{5}\right)$$

- 97). From a point P, perpendicular tangents PQ and PR are drawn to ellipse $x^2 + 4y^2 = 4$ Locus of circumcentre of triangle PQR is

- 1) $x^2 + y^2 = \frac{16}{5} (x^2 + 4y^2)^2$ 2) $x^2 + y^2 = \frac{5}{16} (x^2 + 4y^2)^2$
 3) $x^2 + 4y^2 = \frac{16}{5} (x^2 + y^2)^2$ 4) $x^2 + 4y^2 = \frac{5}{16} (x^2 + y^2)^2$

Key(2)



Sol $x^2 + 4y^2 = 4$

P lies on $x^2 + y^2 = 5$

Let $P(\sqrt{5} \cos\theta, \sqrt{5} \sin\theta)$

Comparing chord of contact with chord with middle point

$$\frac{xh}{h^2 + 4k^2} + \frac{y4k}{h^2 + 4k^2} = 1$$

$$\frac{x\sqrt{5} \cos\theta}{4} + \frac{y\sqrt{5} \sin\theta}{1} = 1$$

Eliminating θ

$$\Rightarrow x^2 + y^2 = \frac{5}{16} (x^2 + 4y^2)^2$$

98). A tangent to the ellipse $4x^2 + 9y^2 = 36$ is cut by the tangent at the extremities of the major axes at T and T^1 . The circle on TT^1 as diameter always passes through the point

- 1) $(\sqrt{5}, 1)$ 2) $(-\sqrt{5}, 0)$ 3) $(0, 0)$ 4) $(3, 2)$

Key(2)

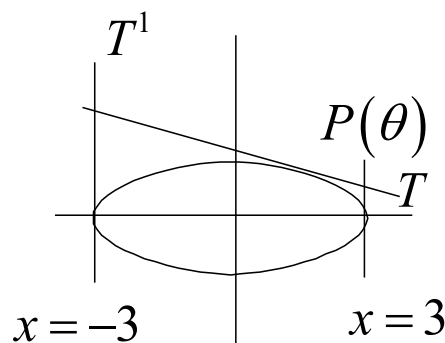
Sol

Let $P(\theta)$ be the point on ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

$$\text{Equation of Tangent, } \frac{x \cos \theta}{3} + \frac{y \sin \theta}{2} = 1 \text{ ----- (1)}$$

$$\text{at } x = 3 \Rightarrow y = \frac{2(1 - \cos \theta)}{\sin \theta} \Rightarrow T = \left(3, \frac{2(1 - \cos \theta)}{\sin \theta} \right)$$

$$\text{at } x = -3 \Rightarrow y = \frac{2(1 + \cos \theta)}{\sin \theta} \Rightarrow T^1 = \left(-3, \frac{2(1 + \cos \theta)}{\sin \theta} \right)$$



Circle with TT^1 as diameter,

$$x^2 + y^2 - (x_1 + x_2)K - (y_1 + y_2)y + (x_1x_2 + y_1y_2) = 0$$

$$\Rightarrow (x^2 + y^2 - 5) + \lambda y = 0$$

It passes through $(\pm\sqrt{5}, 0)$

99). If a variable tangent to the circle $x^2 + y^2 = 1$ intersects the ellipse

$x^2 + 2y^2 = 4$ at points P and Q, then the locus of the point of intersection of tangents to the ellipse at P and Q is a conic whose

1) eccentricity is $\frac{\sqrt{3}}{2}$

2) eccentricity is $\frac{\sqrt{5}}{2}$

3) latus -rectum is of length 2 units 4) foci are $(\pm 2\sqrt{5}, 0)$

Key(1,3)

Sol A tangent to the circle $x^2 + y^2 = 1$ is $x \cos \theta + y \sin \theta = 1$. $R(x_o, y_o)$ is the point of intersection of the tangents to the ellipse at P and Q

$$\Leftrightarrow x \cos \theta + y \sin \theta = 1 \text{ and } x_o x + 2y_o y = 4 \text{ represent the same line}$$

$$\Leftrightarrow x_o = 4 \cos \theta \text{ and } y_o = 2 \sin \theta$$

$$\Leftrightarrow \frac{x_o^2}{16} + \frac{y_o^2}{4} = 1. \text{ Hence, locus of P is the ellipse } \frac{x^2}{16} + \frac{y^2}{4} = 1$$

100). Tangent is drawn to ellipse $x^2/27 + y^2 = 1$ at $(3\sqrt{3} \cos \theta, \sin \theta)$

(where $\theta \in (0, \pi/2)$).

Then the value of θ such that sum of intercepts on coordinate axes

made by this tangent is least is $\frac{\pi}{K}$ then K =

Key(6)

Sol $\frac{x^2}{27} + \frac{y^2}{1} = 1, P(3\sqrt{3} \cos \theta, \sin \theta)$

$$\frac{3\sqrt{3} \cos \theta}{27} + \frac{\sin \theta}{1} = 1$$

$$A\left(\frac{3\sqrt{3}\cos\theta}{27}, 0\right), B = \left(0, \frac{1}{\sin\theta}\right)$$

$$f(\theta) = 3\sqrt{3}\sec\theta + \operatorname{cosec}\theta$$

$$f'(\theta) = \frac{3\sqrt{3}\sin\theta}{\cos^2\theta} - \frac{\cos\theta}{\sin^2\theta} = 0$$

$$\Rightarrow \tan^3\theta = \frac{1}{3\sqrt{3}} = \left(\frac{1}{\sqrt{3}}\right)^3 \quad \theta = \frac{\pi}{6}$$

101). A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse $x^2 + 2y^2 = 6$

at P and Q. Tangents at P and Q are included at an angle $\frac{\pi}{k}$.

The value of k is.....

Key(2)

Sol

$$A(2\cos\theta, \sin\theta) \text{ lies on } x^2 + 4y^2 = 4$$

$$\text{Tangent at A is } x\cos\theta + 2y\sin\theta = 2. \quad \dots(1)$$

$T(h, k)$ is point of intersection of tangents at P, Q ; PQ , i.e.,

$$(1) \text{ is same as } xh + 2ky = 6$$

$$\text{(the chord of contact) } \dots(2) \therefore \frac{\cos\theta}{h} = \frac{\sin\theta}{k} = \frac{1}{3}; \quad h^2 + k^2 = 9$$

$$\therefore T \text{ lies on director circle of } x^2 + 2y^2 = 6 \therefore \text{Angle of intersection} = \frac{\pi}{2}$$

102).

If the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point

$(a\cos\theta, b\sin\theta)$ meets the auxiliary circle in two points A, B such that

the chord AB subtends a right angle at the centre,

then the eccentricity

$$\text{of the ellipse is } \frac{1}{\sqrt{\alpha + \beta \sin^2\theta}} \text{ where } (\alpha + \beta)^2 =$$

Key(4)

Sol Tangent equation in $\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} = 1 \rightarrow (1)$

Auxillallary circle is $x^2 + y^2 = a^2$

Homogonising we get $x^2 + y^2 - a^2 \left(\frac{x \cos \theta}{a} + \frac{y \sin \theta}{b} \right)^2 = 0$

These lines are perpendicular $\Rightarrow a^2 \left(\frac{\cos^2 \theta}{a^2} + \frac{\sin^2 \theta}{b^2} \right) = 2$

$$\Rightarrow \cos^2 \theta + \frac{a^2}{b^2} \sin^2 \theta = 2$$

$$\frac{a^2}{b^2} \sin^2 \theta - \sin^2 \theta = 1 \Rightarrow \frac{a^2}{b^2} = \frac{1 + \sin^2 \theta}{\sin^2 \theta}$$

$$\therefore e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{1 - \left(\frac{\sin^2 \theta}{1 + \sin^2 \theta} \right)} = \sqrt{\frac{1}{1 + \sin^2 \theta}}$$

$$\therefore (\alpha + \beta)^2 = 4$$

103). If L be the length of common tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{4} = 1$

and the circle $x^2 + y^2 = 16$ intercepted by the coordinate

axis then $\frac{\sqrt{3}L}{2}$ is

Key(7)

Sol The equation of the tangent at $(5 \cos \theta, 2 \sin \theta)$ is $\frac{x}{5} \cos \theta + \frac{y}{2} \sin \theta = 1$

If it is a tangent to the circle then $\frac{1}{\sqrt{\frac{\cos^2 \theta}{25} + \frac{\sin^2 \theta}{4}}} = 4$

$$\Rightarrow \cos \theta = \frac{10}{4\sqrt{7}}, \sin \theta = \frac{\sqrt{3}}{2\sqrt{7}}$$

Let A and B be the points where the tangent meets the coordinate axis

$$\text{then } A\left(\frac{5}{\cos \theta}, 0\right), B\left(0, \frac{2}{\sin \theta}\right)$$

$$L = \sqrt{\frac{25}{\cos^2 \theta} + \frac{4}{\sin^2 \theta}} = \frac{14}{\sqrt{3}}$$

104). A tangent to the ellipse $x^2 + 4y^2 = 4$ meets the ellipse

$x^2 + \lambda y^2 = 6$ at P and Q. If tangents at P and Q of ellipse

$x^2 + \lambda y^2 = 6$ are at right angles, then $\lambda =$

Key(2)

Sol Smaller ellipse is $\frac{x^2}{4} + \frac{y^2}{1} = 1$ larger is $\frac{x^2}{6} + \frac{\lambda y^2}{6} = 1$.

Any tangent to smaller is $\frac{x}{2} \cos \theta + y \sin \theta = 1$ ----- (1)

Let it meet larger at P and Q. Let tangent at P, Q intersect at 'A' (h, k)

PQ is chord of contact wrt A $\therefore PQ = \frac{hx}{6} + \frac{\lambda yk}{6} = 1$ ----- (2)

Comparing (1) and (2) $h = 3 \cos \theta$ $k = \frac{6 \sin \theta}{\lambda}$

$$(h, k) \in x^2 + y^2 = \frac{6 + 6}{\lambda} \Rightarrow 9 \cos^2 \theta + \frac{6^2}{\lambda^2} \sin^2 \theta = 6 + \frac{6}{\lambda} \Rightarrow \lambda = 2$$

105). If the locus of middle points of portions of tangents intercepted

between the axes of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is $\frac{a^2}{x^2} + \frac{b^2}{y^2} = \lambda$,

then the numerical value of λ is

Key(4)

Sol Tangent at ' α ' is $\frac{x}{a} \cos \alpha + \frac{y}{b} \sin \alpha = 1$

Let A $(a \sec \alpha, 0)$ B $(0, b \csc \alpha)$

(h, k) is mid point

$$h = \frac{1}{2} a \sec \alpha \quad k = \frac{b}{2} \csc \alpha$$

$$\Rightarrow \frac{a^2}{4x^2} + \frac{b^2}{4y^2} = 1$$

TANGENTS IN POINT FORM

106). Tangents are drawn from the points on the line $x - y - 5 = 0$ to $x^2 + 4y^2 = 4$, then all the chords of contact pass through a fixed point, whose co-ordinates are

- 1) $\left(\frac{1}{5}, \frac{4}{5}\right)$ 2) $\left(\frac{4}{5}, -\frac{1}{5}\right)$ 3) $\left(\frac{2}{5}, \frac{2}{5}\right)$ 4) $(5, 0)$

Key(2)

Sol $x - y - 5 = 0$... (1)

$$\frac{x^2}{4} + \frac{y^2}{1} = 1 \quad \dots (2)$$

$\therefore$ any point P on (1) is $(h, h-5)$

$\therefore$ equation chord of contact is :

$$\Rightarrow xh + 4hy - 20y = 4$$

$$\Rightarrow h(x + 4y) - 4(5y + 1) = 0 \dots (3)$$

$\therefore$ (3) always pass through point of intersection of $5y + 1 = 0$ and $x + 4y = 0$

$$\Rightarrow \text{required fixed point } \left(\frac{4}{5}, -\frac{1}{5}\right)$$

107). The locus of point of intersection of the two tangents to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ which makes an angle 60° with one another is

$$1) 4(x^2 + y^2 - a^2 - b^2)^2 = 3(b^2x^2 + a^2y^2 - a^2b^2)$$

$$2) 3(x^2 + y^2 - a^2 - b^2)^2 = 4(b^2x^2 + a^2y^2 - a^2b^2)$$

$$3) 3(x^2 + y^2 - a^2 - b^2)^2 = 2(b^2x^2 + a^2y^2 - a^2b^2)$$

$$4) 3(x^2 + y^2 - a^2 - b^2)^2 = (b^2x^2 + a^2y^2 - a^2b^2)$$

Key(2)

Sol

$$\tan \theta = \frac{2ab\sqrt{S_{11}}}{x_1^2 + y_1^2 - a^2 - b^2}$$

108). Tangents to the ellipse $b^2x^2 + a^2y^2 = a^2b^2$ makes angles θ_1 and θ_2 with major axis such that $\cot \theta_1 + \cot \theta_2 = kA$. Then the locus of the point of intersection is

$$1) xy = 2k(y^2 + b^2) \quad 2) 2xy = k(y^2 - b^2) \quad 3) 4xy = k(y^2 - b^2) \quad 4) 8xy = k(y^2 - b^2)$$

Key(2)

Sol Apply sum of the slopes = $\frac{2x_1y_1}{x_1^2 - a^2}$

109). Tangents PA and PB are drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ from the point P(0,5). Area of triangle PAB is equal to

$$1) \frac{16}{5} \text{ sq. units} \quad 2) \frac{256}{25} \text{ sq. units} \quad 3) \frac{32}{5} \text{ sq. units} \quad 4) \frac{1024}{25} \text{ sq. units}$$

Key(2)

Sol

Equation of AB is

$$T = 0,$$

$$\text{i.e., } \frac{5y}{9} = 1$$

$$\Rightarrow y = \frac{9}{5}$$

Putting $y = \frac{9}{5}$ in the equation of ellipse, we get $\frac{x^2}{16} + \frac{81}{25 \cdot 9} = 1$

$$\Rightarrow x = \pm \frac{16}{5} \quad \text{Thus, } AB = \frac{32}{5}$$

$$\text{Distance of P from } AB = 5 - \frac{9}{5} = \frac{16}{5}$$

$$\text{Thus, area of } \Delta_{PAB} = \frac{1}{2} \cdot \frac{32}{5} \cdot \frac{16}{5} = \frac{256}{25} \text{ sq.units.}$$

110). The straight line $x - 2y + 4 = 0$ is one of the common tangents of the parabola $y^2 = 4x$ and $\frac{x^2}{4} + \frac{y^2}{b^2} = 1$. The equation of the another common tangent of these curves is

- 1) $x + 2y + 4 = 0$ 2) $x + 2y - 4 = 0$ 3) $x + 2y + 2 = 0$ 4) $x + 2y - 2 = 0$

Key(1)

So Other common tangent will clearly be the reflection of $x - 2y + 4 = 0$ in the x-axis

Thus other common tangent will be $x + 2y + 4 = 0$

111). The sum of the slopes of the tangents to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ drawn from the point (6, -2) is

- 1) 0 2) $\frac{3}{4}$ 3) $-\frac{6}{7}$ 4) $-\frac{8}{9}$

Key(4)

Sol Sum of the slopes = $\frac{2x_1y_1}{x_1^2 - a^2} = -\frac{8}{9}$

112). An ellipse passes through the point (4, - 1) and it's axes are along the axes of coordinates. If the line $x + 4y - 10 = 0$ is a tangent to it then it's equation can be

1) $\frac{x^2}{100} + \frac{y^2}{5} = 1$ 2) $\frac{x^2}{80} + \frac{y^2}{5/4} = 1$ 3) $\frac{x^2}{20} + \frac{y^2}{5} = 1$ 4) none of these

Key(2,3)

Sol Let the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$,

Using the point (4,-1) and the tangent $x+4y-10=0$

; $\frac{16}{a^2} + \frac{1}{b^2} = 1$; $a^2 + 16b^2 = 100$ solving we get options (2),(3)

113). If $x+y=2$ is a tangent to the ellipse foci are (2 ,3) & (3 ,5) is

1) length of minor axis is 6 2) length of minor axis is 3
3) length of major axis is $\frac{\sqrt{41}}{2}$ 4) eccentricity of ellipse is $\frac{\sqrt{5}}{\sqrt{41}}$

Key(1,4)

Sol $b^2 = (\text{semi minor axis})^2 = \frac{2+3-2}{\sqrt{2}} \cdot \frac{3+5-2}{\sqrt{2}} = 9$; $b=3$; $2ae = \sqrt{5} \Rightarrow e = \frac{\sqrt{5}}{\sqrt{41}}$ & $a = \frac{\sqrt{41}}{2}$

114). Tangents are drawn from points on the line $x - y + 2 = 0$ to the

ellipse $x^2 + 2y^2 = 2$, then all the chords of contact pass through

the point whose distance from $\left(2, \frac{1}{2}\right)$ is

Key(3)

Sol

Consider any point $(t_1, t+2)$, $t \in \mathbb{R}$ on the line $x - y + 2 = 0$

The chord of contact of ellipse with respect to this point is
 $x(t) + 2y(t+2) - 2 = 0$

$\Rightarrow (4y-2) + t(x+2y) = 0$, $y = \frac{1}{2}$, $x = -1$

Hence, the point is $\left(-1, \frac{1}{2}\right)$, Where distance from $\left(2, \frac{1}{2}\right)$ is 3.

115). Tangents are drawn to the ellipse $\frac{x^2}{9} + \frac{y^2}{5} = 1$ at ends of latusrectum.

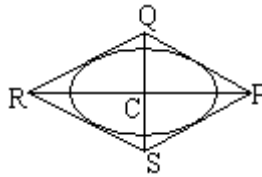
If the area of an quadrilateral be λ sq.units, then the value of $\lambda/9 =$

Key(3)

Sol

One end of latusrectum $\left(2, \frac{5}{3}\right)$

Equation of tangent at $\left(2, \frac{5}{3}\right)$ is $\frac{2x}{9} + \frac{y}{3} = 1$



Area of quadrilateral $= 4 \times (\Delta CPQ) = 4 \times \frac{1}{2} \times \frac{9}{2} \times 3 = 27 = \lambda$ (Given)

$$\Rightarrow \frac{\lambda}{9} = 3$$

116). Tangents are drawn from any point on the circle $x^2 + y^2 = 41$ to

the Ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ then the angle between the two tangents is

1) $\frac{\pi}{4}$

2) $\frac{\pi}{3}$

3) $\frac{\pi}{6}$

4) $\frac{\pi}{2}$

Key(4)

Sol Director circle

117). The locus of poles of tangents to the Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

w.r.to the circle $x^2 + y^2 = a^2$ is

- 1) $a^4x^2 + b^4y^2 = a^2$ 2) $a^2x^2 + b^4y^2 = a^6$ 3) $a^2x^2 + b^2y^2 = a^4$ 4) $a^4x^2 + b^4y^2 = a^4$

Key(3)

Sol Polar of circle touches ellipse

118). An ellipse whose length of major axis is 4, length of minor axis is 2 slides between the coordinate axis in I quadrant (two perpendicular lines) then the centre of ellipse lies on

- 1) A conic which is ellipse.
2) A circle having radius of circle is 3
3) A conic which is Parabola
4) A circle having radius of circle is $\sqrt{5}$.

Key(2)

Sol Coordinate axis is perpendicular tangent to ellipse, so director circle

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Locus is $x^2 + y^2 = a^2 + b^2$

119). If a quadrilateral formed by the four tangents to the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

is a square, then the area of the square, is

- 1) 26 2) 13 3) $\frac{13}{2}$ 4) $\frac{13}{4}$

Key(1)

Sol

$$\frac{x^2}{9} + \frac{y^2}{4} = 1$$

Director circle is $x^2 + y^2 = 13$

Diagonal of square = Diameter of circle = $2\sqrt{13}$

Let side = a $a\sqrt{2} = 2\sqrt{13}$

$a = \sqrt{26}$ $\therefore$ Area of square = 26

i.e. $2y - x - 4 = 0$ and $2y + x + 4 = 0$

- 120).** Number of points on the ellipse $\frac{x^2}{50} + \frac{y^2}{20} = 1$ from which pair of perpendicular tangents may be drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ is
- 1)0 2)2 3)1 4)4

Key(4)

Sol For ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ equation of director circle is $x^2 + y^2 = 25$.

This director circle will cut the ellipse $\frac{x^2}{50} + \frac{y^2}{20} = 1$ at 4 points

Hence number of point = 4.

- 121).** $S(3,4)$ and $S^1(9,12)$ are the foci of an ellipse and the foot of the perpendicular from S to a tangent to the ellipse is $(1,-4)$. Then the eccentricity of the ellipse is

- 1) $\frac{3}{13}$ 2) $\frac{4}{13}$ 3) $\frac{5}{13}$ 4) $\frac{1}{\sqrt{2}}$

Key(3)

Sol $2ae + SS^1 = 10$

$C = (6,8), P = (1,-4)$

$a = CP = 13$

- 122).** An ellipse has the point (1, -1) and (2, -1) as its foci and $x + y = 5$ as one of its tangent then value of $a^2 + b^2$ where a, b are the length of semimajor and semiminor axis of ellipse respectively, is
- 1) $\frac{41}{2}$ 2) 10 3) 19 4) $\frac{81}{4}$

Key(4)

Sol $2ae = SS^1 = 1$

$p_1 p_2 = b^2$, where p_1 & p_2 are the length of perpendicular from S & S^1 to the tangent

$$\frac{5}{\sqrt{2}} \cdot \frac{4}{\sqrt{2}} = b^2 \Rightarrow b^2 = 10 \Rightarrow b^2 = 10 = a^2 - e^2 a^2 \Rightarrow a^2 = \frac{41}{4}$$

- 123).** From the point A(4, 3) tangents are drawn to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

to touch the ellipse at B and C. EF is tangent to the ellipse parallel to the line BC and towards the point A. The distance of A from EF is equal to

1) $\frac{4\sqrt{18}-12}{5}$ 2) $\frac{24-4\sqrt{18}}{5}$ 3) $\frac{12-2\sqrt{18}}{5}$ 4) $\frac{2\sqrt{18}-3}{5}$

Key(2)

Sol The equation of the chord of contact is $\frac{x}{4} + \frac{y}{3} = 1$

$$\Rightarrow \text{slope of line EF is } \frac{-3}{4} \Rightarrow y = -\frac{3}{4}x + \sqrt{18}$$

$$\Rightarrow \text{distance (4, 3) from EF is } \frac{|24-4\sqrt{18}|}{5}$$

- 124).** An ellipse slides between two lines at right angles to one another.

The locus of its centre is

- 1) a parabola 2) an ellipse 3) a circle 4) straight line

Key(3)

Sol Let Ellipse be $\frac{(x - \alpha)^2}{a^2} + \frac{(y - b)^2}{b^2} = 1$

Director circle is $(x - \alpha)^2 + (y - \beta)^2 = a^2 + b^2$ -----(1)

Now (0,0) lies on (1)

$$\Rightarrow \alpha^2 + \beta^2 = a^2 + b^2$$

Locus of c (α, β) is $x^2 + y^2 = a^2 + b^2$

125) The total number of real tangents that can be drawn to the ellipse $3x^2 + 5y^2 = 32$ and to $25x^2 + 9y^2 = 450$ and passing through (3, 5) is

- 1) 4 2) 3 3) 2 4) 1

Key(2)

Sol For the 1st ellipse (3,5) lies outside and for the 2nd Ellipse it lies on the Ellipse.

$\therefore$ In total there will be 3 tangents.

126). The angle between the curves $\frac{x^2}{25} + \frac{y^2}{16} = 1$ and $\frac{x^2}{9} - \frac{y^2}{27} = \frac{1}{4}$ at their points of intersections is equal to

- 1) $\tan^{-1} \frac{3}{5}$ 2) $\frac{\pi}{2}$ 3) $\frac{\pi}{4}$ 4) 0

Key(2)

Sol If the ellipse and hyperbola have the same foci, then whenever they intersect at a angle is 90°

127). (2, 6) and (12, k) are the foci of an ellipse which touches both the coordinate Axes Then its

- 1) Point of contact with x – axis is (8, 0)
2) Point of contact with y – axis is $\left(0, \frac{40}{7}\right)$

3) Eccentricity of the ellipse is $\frac{\sqrt{13}}{5}$

4) Length of latus rectum of the ellipse is $\frac{24\sqrt{2}}{5}$

Key(1,2,3,4)

Sol Product of perpendicular distance on any tangent is equal to b^2

$$2 \times 12 = k \times 6$$

$$\frac{K = 4}{b^2 = 24}$$

128). If the curve $x^2 + 3y^2 = 9$ subtends an obtuse angle at the point $(2\alpha, \alpha)$, then a possible value of α^2 is

1) 1

2) 2

3) 3

4) 4

Key(2)

Sol The given curve is $\frac{x^2}{9} + \frac{y^2}{3} = 1$, whose director circle is $x^2 + y^2 = 12$.

For the required condition $(2\alpha, \alpha)$ should lie inside the circle and outside

the ellipse

$$\text{i.e., } (2\alpha)^2 + 3\alpha^2 - 9 > 0 \text{ and } (2\alpha)^2 + \alpha^2 - 12 < 0 \quad \text{i.e., } \frac{9}{7} < \alpha^2 < \frac{12}{5}.$$

129) Equation of the locus at point of intersection of perpendicular tangents to

$$\frac{(x+y-2)^2}{9} + \frac{(x-y)^2}{16} = 1 \text{ is}$$

1) $(x-1)^2 + (y-1)^2 = \frac{25}{2}$

2) $(x-1)^2 + (y-1)^2 = 50$

3) $(x-1)^2 + (y-1)^2 = 25$

4) $(x+y-2)^2 + (x-y)^2 = 25$

Key

(1,4)

Sol

$$\frac{\left(\frac{x+y-2}{\sqrt{2}}\right)^2}{\frac{9}{2}} + \frac{\left(\frac{x-y}{\sqrt{2}}\right)^2}{8} = 1 \text{ (which is in standard form)}$$

$$\Rightarrow (x-1)^2 + (y-1)^2 = \frac{9}{2} + 8 \text{ or } \frac{(x+y-2)^2 + (x-y)^2}{2} = \frac{9}{2} + 8 \text{ is required locus.}$$

- 130). If the tangent and normal at a point P on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ intersect the major axis at the points T and N respectively in such a way that the ratio of the areas of $\triangle PTN$ and $\triangle PSS^1$ is $\frac{91}{60}$ and if the area of $\triangle PSS^1$ is $2K$ (S and S^1 are foci of ellipse) then the value of $[K]$ ($[]$ is GIF)

Key(5)

Sol

$$\frac{\frac{1}{2}(NT)y_1}{\frac{1}{2}(6)y_1} = \frac{91}{60} \Rightarrow NT = \frac{91}{10}$$

$$\Rightarrow \frac{25}{x_1} - \frac{9x_1}{25} = \frac{91}{10} \Rightarrow x_1 = \frac{5}{2} \quad \Rightarrow y_1 = 2\sqrt{3}$$

$$\therefore \triangle PSS^1 = \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3}$$

$$\Rightarrow 2k = 6\sqrt{3} \Rightarrow k = \sqrt{27} \quad [K] = 5$$

- 131). If F_1, F_2 are the feet of the perpendiculars from foci S_1, S_2 of the ellipse

$$16x^2 + 25y^2 = 400 \text{ on the tangent at any point P on the ellipse then}$$

minimum value of $S_1F_1 + S_2F_2$ is

Key(8)

Sol

The minimum perpendiculars from two foci upon any tangent is b^2

$$S_1F_1 \cdot S_2F_2 = 16$$

$$AM \geq GM \Rightarrow \frac{S_1F_1 + S_2F_2}{2} \geq \sqrt{S_1F_1 \times S_2F_2} \Rightarrow S_1F_1 + S_2F_2 \geq 8$$

132). An ellipse slides between two perpendicular straight lines

$x = 0$ and $y = 0$ then, locus of its foci is

- | | |
|---------------|------------------|
| 1) a parabola | 2) an ellipse |
| 3) a circle | 4) none of these |

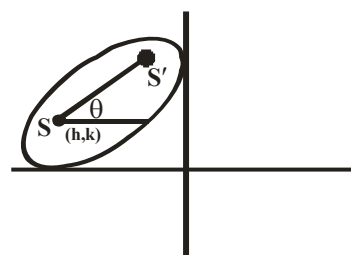
Key(4)

Sol

$$(h + 2ae \cos \theta)h = b^2 \dots\dots (1)$$

$$(k + 2ae \sin \theta)k = b^2 \dots\dots (2)$$

$$2ae \cos \theta = \frac{b^2 - h^2}{h}$$



$$2ae \sin \theta = \frac{b^2 - k^2}{k}$$

$$4a^2 e^2 (\cos^2 \theta + \sin^2 \theta) = \left(\frac{b^2 - h^2}{h} \right)^2 + \left(\frac{b^2 - k^2}{k} \right)^2$$

133). Tangent at any point 'P' of ellipse $9x^2 + 16y^2 - 144 = 0$ is drawn.

Eccentric angle of 'P' is $\theta = \frac{1}{2} \sin^{-1} \left(\frac{1}{7} \right)$. If 'N' is the foot of perpendicular

From center 'O' of the ellipse to this tangent then $\angle PON$

- | | | | |
|--|--|---------------------|--------------------|
| 1) $\tan^{-1} \left(\frac{1}{12} \right)$ | 2) $\tan^{-1} \left(\frac{1}{24} \right)$ | 3) $\frac{\pi}{12}$ | 4) $\frac{\pi}{3}$ |
|--|--|---------------------|--------------------|

Key(2)

Sol

$$\tan \phi = \sin 2\theta \left(\frac{a^2 - b^2}{2ab} \right)$$

$$|\tan \phi| = \frac{16-9}{2 \times 4 \times 3} \times \frac{1}{7} \Rightarrow \phi = \tan^{-1}\left(\frac{1}{24}\right)$$

134) Let S be the circle with centre at (1,0) and touches the ellipse

$$x^2 + 4y^2 = 16 \text{ then}$$

1) The radius of S is $\sqrt{\frac{10}{3}}$

2) The radius of S is $\sqrt{\frac{11}{3}}$

3) S makes an intercept of length $4\sqrt{\frac{2}{3}}$ on the y-axis

4) S makes an intercept of length $2\sqrt{\frac{7}{3}}$ on the y-axis

Key(2,3)

Sol The circle and the ellipse have a normal in common

$$\text{let } \frac{4x}{\cos \theta} - \frac{2y}{\sin \theta} = 12 \text{ be its}$$

equation. It passes through the centre (1,0) of the circle. $\therefore \cos \theta = \frac{1}{3}$

135). The tangents are drawn from a point 'P' to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

are perpendicular to each other, the possible coordinates of 'P' are

1) (3, 4)

2) (4, 3)

3) (-4, 3)

4) (-3, -4)

Key(1,2,3,4)

Sol Required points lies on director circle $x^2 + y^2 = 25$

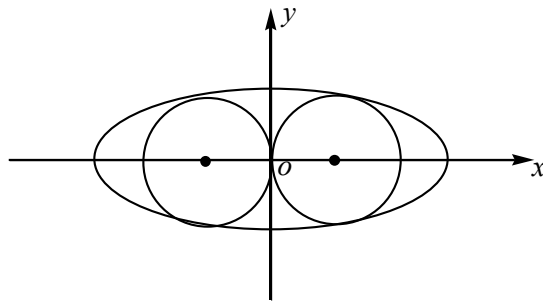
$$\Leftrightarrow \frac{x_0^2}{16} + \frac{y_0^2}{4} = 1. \text{ Hence, locus of P is the ellipse } \frac{x^2}{16} + \frac{y^2}{4} = 1$$

136.If two circles touching externally at origin also touch the ellipse $\frac{x^2}{10} + \frac{y^2}{8} = 1$

as shown and if the square of the length of their direct common tangent is $\frac{a}{b}$ where

$G, C, D(a, b) = 1, a, b \in N$ then the value of $a - b^2$ equals

Key(7)



Sol

$$\frac{x^2}{10} + \frac{y^2}{8} = 1, (x-r)^2 + y^2 = r^2$$

$$\text{Solving (2)} \Rightarrow x^2 - 2rx + r^2 + y^2 = r^2 \Rightarrow x^2 - 2rx + 8\left(1 - \frac{x^2}{10}\right) = 0$$

$$\Rightarrow \frac{1}{5}x^2 - 2rx + 8 = 0 \text{ for which } \Delta = 0 \Rightarrow 4r^2 = 4\left(\frac{1}{5}\right)(8) \Rightarrow 4r^2 = \frac{32}{5}$$

$$\therefore \text{Square of length of direct common tangent} = (2r)^2 = \frac{32}{5}$$

$$\therefore \left. \begin{matrix} a = 32 \\ b = 5 \end{matrix} \right\} \Rightarrow a - b^2 = 7$$

137). A tangent to the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ at any point P meet the

line $x = 0$ at a point Q. Let R be the image of Q in the line $y = x$, then circle whose extremities of a diameter are Q and R passes through a fixed point.

The fixed point is

1)(3, 0)

2)(5, 0)

3)(0, 0)

4)(4, 0)

Key (3)

Sol Equation of the tangent to the ellipse at P ($5 \cos \theta$, $4 \sin \theta$) is

$$\frac{x \cos \theta}{5} + \frac{y \sin \theta}{4} = 1$$

It meets the line $x = 0$ at Q (0 , $4 \operatorname{cosec} \theta$)

Image of Q in the line $y = x$ is R ($4 \operatorname{cosec} \theta$, 0)

$\therefore$ Equation of the circle is

$$X(x - 4 \operatorname{cosec} \theta) + y(y - 4 \operatorname{cosec} \theta) = 0$$

i.e.
$$x^2 + y^2 - 4(x + y) \operatorname{cosec} \theta = 0$$

$\therefore$ each member of the family
passes through the intersection of $x^2 + y^2 = 0$ and $x + y = 0$

i.e. the point $(0, 0)$.

138). Q is a point on the auxiliary circle corresponding to the point P of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. If T is the foot of the perpendicular dropped

from the focus S on to the tangent to the auxiliary circle at Q then

the ΔSPT is :

1) isosceles

2) equilateral

3) right angled

4) right isosceles

Key (1)

Sol: Tangent at Q is $x \cos \theta + y \sin \theta = a$

$$ST = |a e \cos \theta - a| = a(1 - e \cos \theta)$$

$$\text{Also } SP = e PM = e \left(\frac{a}{e} - a \cos \theta \right) = a(1 - e \cos \theta)$$

$$ST = SP \Rightarrow \text{isosceles.}$$

139). If the curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$) and $x^2 - y^2 = c^2$ ($a > c$)

cut at right angles then

$$1) a^2 + b^2 = 2c^2$$

$$2) b^2 - a^2 = 2c^2$$

$$3) a^2 - b^2 = 2c^2$$

$$4) a^2 b^2 = 2c^2$$

Key (3)

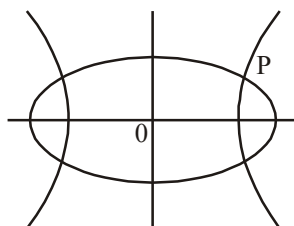
Sol: Let P on the ellipse is $(a \cos \theta, b \sin \theta)$

Slope of tangent at P on the ellipse $m_1 = -\frac{b \cos \theta}{a \sin \theta}$

Slope of tangent at P on the hyperbola $x^2 - y^2 = c^2$, is

$$m_2 = \frac{a \cos \theta}{b \sin \theta}$$

Since these curves are intersecting at right angle



$$\therefore m_1 m_2 = -1$$

$$-\frac{b}{a} \times \frac{\cos \theta}{\sin \theta} \times \frac{a}{b} \times \frac{\cos \theta}{\sin \theta} = -1 \Rightarrow \tan^2 \theta = 1$$

$P(a \cos \theta, b \sin \theta)$ also lies on hyperbola

$$\therefore a^2 \cos^2 \theta - b^2 \sin^2 \theta = c^2$$

$$a^2 - b^2 \tan^2 \theta = c^2 + c^2 \tan^2 \theta$$

$$\Rightarrow a^2 - b^2 = c^2 + c^2 \quad [\because \tan^2 \theta = 1]$$

$$a^2 - b^2 = 2c^2$$

140). If a variable tangent to the circle $x^2 + y^2 = 1$ intersects the ellipse $x^2 + 2y^2 = 4$ at points P and Q, then the locus of the point of intersection of tangents to the ellipse at P and Q is a conic whose

- 1) eccentricity is $\frac{\sqrt{3}}{2}$
- 2) eccentricity is $\frac{\sqrt{5}}{2}$
- 3) latus -rectum is of length 2 units
- 4) foci are $(\pm 2\sqrt{5}, 0)$

Key (1,3)

Sol

A tangent to the circle $x^2 + y^2 = 1$ is $x \cos \theta + y \sin \theta = 1$. $R(x_0, y_0)$ is the point of intersection of the tangents to the ellipse at P and Q
 $\Leftrightarrow x \cos \theta + y \sin \theta = 1$ and $x_0 x + 2y_0 y = 4$ represent the same line
 $\Leftrightarrow x_0 = 4 \cos \theta$ and $y_0 = 2 \sin \theta$
 $\Leftrightarrow \frac{x_0^2}{16} + \frac{y_0^2}{4} = 1$. Hence, locus of P is the ellipse $\frac{x^2}{16} + \frac{y^2}{4} = 1$

NORMALS

141). The line $y = mx + c$, will be a normal to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, if

$$1) \frac{a^2}{m^2} + b^2 = \frac{(b^2 - a^2)^2}{c^2}$$

$$2) \frac{c^2}{m^2} + b^2 = \frac{(b^2 - a^2)^2}{a^2}$$

$$3) a^2 + \frac{b^2}{m^2} = \frac{(b^2 - a^2)^2}{c^2}$$

$$4) c^2 + \frac{b^2}{m^2} = \frac{(b^2 - a^2)^2}{a^2}$$

Key (1)

Sol Let the foot of normal is $(a \cos \theta, b \sin \theta)$, then $\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$

and the given lines will be identical.

$$\text{Thus, } \frac{m \cos \theta}{a} = \frac{\sin \theta}{b} = \frac{c}{(b^2 - a^2)}$$

$$\Rightarrow \cos \theta = \frac{ac}{m(b^2 - a^2)}, \sin \theta = \frac{bc}{(b^2 - a^2)}$$

$$\Rightarrow \frac{a^2}{m^2} + b^2 = \frac{(b^2 - a^2)^2}{c^2}$$

142). The number of points outside the ellipse on its major axis from which a normal (other than x-axis) can be drawn to ellipse is

- 1) 0 2) 1 3) 2 4) 4

Key (0)

Sol

Equation of normal $ax \sec \theta - by \operatorname{cosec} \theta = a^2 - b^2$

If normal intersects x-axis at $(\alpha, 0)$

$$\Rightarrow \alpha = \frac{a^2 - b^2}{a \sec \theta} = \frac{ae}{\sec \theta} \Rightarrow |\alpha| < |a|$$

143). The normal at an end of a latus rectum of the ellipse $x^2/a^2 + y^2/b^2 = 1$ passes through an end of the minor axis if

- 1) $e^4 + e^2 = 1$ 2) $e^3 + e^2 = 1$ 3) $e^2 + e = 1$ 4) $e^3 + e = 1$

Key (1)

Sol. Given ellipse equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Let $P\left(ae, \frac{b^2}{a}\right)$ be one end of latus rectum.

Slope of normal at $P\left(ae, \frac{b^2}{a}\right) = \frac{1}{e}$

Equation of normal is

$$y = \frac{b^2}{a} = \frac{1}{e}(x - ae)$$

It passes through $B'(0, b)$ then

$$b - \frac{b^2}{a} = -a$$

$$a^2 - b^2 = -ab$$

$$a^4 e^4 = a^2 b^2$$

$$e^4 + e^2 = 1$$

144). How many tangents to the circle $x^2 + y^2 = 3$ are there which are normal to the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$

1) 3

2) 2

3) 1

4) 0

Key (4)

Sol. Equation of normal at $p(3\cos\theta, 2\sin\theta)$ is $3x \sec\theta - 2y \operatorname{cosec}\theta = 5$

$$\frac{5}{\sqrt{9\sec^2\theta + 4\operatorname{cosec}^2\theta}} = \sqrt{3}$$

But Min. of $9\sec^2\theta + 4\operatorname{cosec}^2\theta = 25$

$\therefore$ no such θ^{-1} exists.

145). Number of normals that can be drawn from the point $(0, 0)$ to $3x^2 + 2y^2 = 30$ are

1) 2

2) 4

3) 1

4) 3

Key (2)

Sol. It is centre

146). If 'CF' is the perpendicular from the centre C of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ on the tangent at any point P and G is the point where the normal at P meets the major axis, then CF.PG is

1) b^2

2) $2b^2$

3) $\frac{b^2}{2}$

4) $3b^2$

Key (1)

Sol. $CF = \frac{ab}{\sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}}$ $PG = \frac{b}{a} \sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}$

147). The normal to the curve $x^2 + 3y^2 - 4 = 0$ at the point $P(\pi/6)$ intersects the curve again at the point $Q(\theta)$, θ being the eccentric angle of the point Q , then $\theta =$

1) 0

2) $\pi/2$

3) π

4) $3\pi/2$

Key (4)

Sol. The given curve is $\frac{x^2}{4} + \frac{y^2}{4/3} = 1$. Point P is $\left(2\cos\frac{\pi}{6}, \frac{2}{\sqrt{3}}\sin\frac{\pi}{6}\right) \equiv \left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$

On differentiating the given equation, w.r.t. x , we get $x + 3y \frac{dy}{dx} = 0 \Rightarrow$

$$\left[\frac{dy}{dx}\right]_P = \left[-\frac{x}{3y}\right]_P = -1$$

The equation of normal is $y - \frac{1}{\sqrt{3}} = 1(x - \sqrt{3}) \Rightarrow x - y = \sqrt{3} - \frac{1}{\sqrt{3}}$. Normal

passes through $Q(\theta)$. Hence $2\cos\theta - \frac{2}{\sqrt{3}}\sin\theta = \sqrt{3} - \frac{1}{\sqrt{3}}$

$$\Rightarrow 2\sqrt{3}\cos\theta - 2\sin\theta = 2 \Rightarrow \sqrt{3}\cos\theta - \sin\theta = 1 \Rightarrow \theta = \frac{3\pi}{2}.$$

148)

A normal to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets the axes in L and M . The perpendiculars to the axes through L and M intersect at P . Then the equation to the locus of P is

1) $a^2x^2 - b^2y^2 = (a^2 + b^2)^2$

2) $a^2x^2 + b^2y^2 = (a^2 + b^2)^2$

3) $b^2x^2 - a^2y^2 = (a^2 - b^2)^2$

4) $a^2x^2 + b^2y^2 = (a^2 - b^2)^2$

Key (4)

Sol $P = (x_1, y_1), \frac{x}{x_1} + \frac{y}{y_1} = 1$ Apply normal condition

149. The normal to the curve $x^2 + 3y^2 - 4 = 0$ at the point P($\pi/6$) intersects the curve again at the point Q(θ), θ being the eccentric angle of the point Q, then $\theta =$

- 1) 0 2) $\pi/2$ 3) π 4) $3\pi/2$

Key (4)

Sol The given curve is $\frac{x^2}{4} + \frac{y^2}{4/3} = 1$. Point P is $\left(2 \cos \frac{\pi}{6}, \frac{2}{\sqrt{3}} \sin \frac{\pi}{6}\right) \equiv \left(\sqrt{3}, \frac{1}{\sqrt{3}}\right)$

On differentiating the given equation, w.r.t. x, we get $x + 3y$

$$\frac{dy}{dx} = 0 \Rightarrow \left[\frac{dy}{dx} \right]_P = \left[-\frac{x}{3y} \right]_P = -1$$

The equation of normal is $y - \frac{1}{\sqrt{3}} = 1(x - \sqrt{3}) \Rightarrow x - y = \sqrt{3} - \frac{1}{\sqrt{3}}$. Normal passes

through Q(θ). Hence $2 \cos \theta - \frac{2}{\sqrt{3}} \sin \theta = \sqrt{3} - \frac{1}{\sqrt{3}}$

$$\Rightarrow 2\sqrt{3} \cos \theta - 2 \sin \theta = 2 \Rightarrow \sqrt{3} \cos \theta - \sin \theta = 1 \Rightarrow \theta = \frac{3\pi}{2}.$$

150). If the normal at any given point P on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ meets its

auxiliary circle at Q and R such that $\angle QOR = 90^\circ$, where O is the centre of ellipse, then

- 1) $a^4 + 2b^4 \geq 6a^2b^2 + 3a^3b$ 2) $a^4 + 2b^4 \geq 5a^2b^2 + 2a^3b$
 3) $a^4 + 2b^4 \geq 5a^2b^2 + 4a^3b$ 4) none of these

Key (3)

Sol

Normal at P ($a \cos \theta, b \sin \theta$) is

$$ax \sec \theta - by \operatorname{cosec} \theta = a^2 - b^2.$$

Homogenizing,

$$x^2 \left(1 - \frac{a^4 \sec^2 \theta}{(a^2 - b^2)^2} \right) + y^2 \left(1 - \frac{a^2 b^2 \operatorname{cosec}^2 \theta}{(a^2 - b^2)^2} \right) + \frac{2a^3 b}{(a^2 - b^2)^2} xy \sec \theta \operatorname{cosec} \theta = 0$$

$$\angle QOR = 90^\circ$$

$$1 - \frac{a^4 \sec^2 \theta}{(a^2 - b^2)^2} + 1 - \frac{a^2 b^2 \operatorname{cosec}^2 \theta}{(a^2 - b^2)^2} = 0$$

$$a^4 - 5a^2 b^2 + 2b^4 = a^4 \tan^2 \theta + a^2 b^2 \cot^2 \theta.$$

Apply A.M. > G.M.

151). A ray emanating from the point $(5, 0)$ is incident on the hyperbola

$$9x^2 - 16y^2 = 144$$

at a point P with the abscissa 8, then the equation of reflected ray after the first reflection is (P lies in first quadrant)

$$1) \sqrt{3}x - y + 7 = 0$$

$$2) 3\sqrt{3}x - 13y + 15\sqrt{3} = 0$$

$$3) 3\sqrt{3}x + 13y - 15\sqrt{3} = 0$$

$$4) \sqrt{3}x + y - 14 = 0$$

Key (2)

Sol Coordinate of $P(8, 3\sqrt{3})$ and $e = \frac{5}{4}$ so foci $= (\pm 5, 0)$

Equation of the reflected ray after first reflection passes through $P(8, 3\sqrt{3})$ and

$$\text{focus } (-5, 0) \quad y - 0 = \frac{3\sqrt{3} - 0}{8 + 5}(x + 5)$$

$$\Rightarrow 3\sqrt{3}x - 13y + 15\sqrt{3} = 0$$

152). Q is a point on the auxiliary circle corresponding to the point P of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1. \text{ If T is the foot of the perpendicular dropped from the}$$

focus S on to the tangent to the auxiliary circle at Q then the ΔSPT is :

- | | |
|-----------------|--------------------|
| 1) isosceles | 2) equilateral |
| 3) right angled | 4) right isosceles |

Key (1)

Sol: Tangent at Q is $x \cos \theta + y \sin \theta = a$

$$ST = |a e \cos \theta - a| = a(1 - e \cos \theta)$$

$$\text{Also } SP = e PM = e \left(\frac{a}{e} - a \cos \theta \right) = a(1 - e \cos \theta)$$

$$ST = SP \Rightarrow \text{isosceles.}$$

153. If the normal to an ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ at any point P cuts the major axis at Q, locus of middle point of PQ is a conic whose length of latus rectum is

- | | | | |
|--------------------|--------------------|--------------------|--------------------|
| 1) $\frac{20}{17}$ | 2) $\frac{40}{19}$ | 3) $\frac{40}{17}$ | 4) $\frac{20}{19}$ |
|--------------------|--------------------|--------------------|--------------------|

Key (3)

Sol: $P(5 \cos \theta, 4 \sin \theta)$

Equation of normal is

$$5x \sec \theta - 4y \operatorname{cosec} \theta - 9 = 0$$

$$Q\left(\frac{9}{5 \sec \theta}, 0\right) \quad Q\left(\frac{9}{5} \cos \theta, 0\right)$$

Let midpoint of PQ is (h,k)

$$\therefore 2h = \frac{34}{5} \cos \theta \quad \Rightarrow \quad \frac{5h}{17} = \cos \theta$$

$$2k = 4 \sin \theta \quad \Rightarrow \quad \frac{k}{2} = \sin \theta$$

$$\therefore \frac{25h^2}{289} + \frac{k^2}{4} = 1 \quad \Rightarrow \quad \frac{x^2}{(289/25)} + \frac{y^2}{4} = 1$$

$$a = 17/5, b = 2$$

$$\text{Length of latus rectum} = \frac{2b^2}{a} = \frac{2 \times 4 \times 5}{17} = \frac{40}{17}$$

154. The normal drawn to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the extremity of the latus rectum passes through the extremity of the minor axis. Eccentricity of this ellipse is equal to

- 1) $\sqrt{\frac{\sqrt{5}-1}{2}}$ 2) $\frac{\sqrt{5}-1}{2}$ 3) $\sqrt{\frac{\sqrt{3}-1}{2}}$ 4) $\frac{\sqrt{3}-1}{2}$

Key (1)

Sol Equation of normal at $\left(ae, \frac{b^2}{a} \right)$ is

$$\frac{x - ae}{\frac{ae}{a^2}} = \frac{y - \frac{b^2}{a}}{\frac{b^2}{ab^2}}$$

$$\frac{a(x - ae)}{e} = (ay - b^2)$$

It should pass through (0, -b)

$$\Rightarrow \frac{a(0 - ae)}{e} = -ab - b^2$$

$$\Rightarrow a^2 = ab + b^2$$

$$\Rightarrow \frac{b^2}{a^2} + \frac{b}{a} - 1 = 0$$

155. OA is the perpendicular drawn from center O of the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ to the tangent at any point P on the ellipse. If the normal}$$

drawn to the ellipse at the point P meets the x-axis at B, then (OA).(PB) is equal to

- 1) a^2 2) $a\sqrt{a^2 + b^2}$ 3) b^2 4) $b\sqrt{a^2 + b^2}$

Key (3)

Sol: Let $P \equiv (a \cos \theta, b \sin \theta)$.

Equation of tangent at 'P' is

$$bx \cos \theta + ay \sin \theta - ab = 0$$

$$\Rightarrow OA = \frac{ab}{\sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta}}$$

Equation of normal at P is

$$ax \sec \theta - by \csc \theta = (a^2 - b^2)$$

$$\Rightarrow B \equiv \left(\frac{(a^2 - b^2) \cos \theta}{a}, 0 \right)$$

$$\Rightarrow PB^2 = b^2 \sin^2 \theta + \left(a \cos \theta - \frac{(a^2 - b^2) \cos \theta}{a} \right)^2$$

$$= b^2 \sin^2 \theta + \frac{b^4 \cos^4 \theta}{a^2}$$

$$= \frac{b^2}{a^2} (a^2 \sin^2 \theta + b^2 \cos^2 \theta)$$

$$\Rightarrow PB = \frac{b\sqrt{a^2 \sin^2 \theta + b^2 \cos^2 \theta}}{a}$$

$$\Rightarrow (OA)(PB) = b^2$$

156). Any ordinate MP of a ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$ meets the auxiliary circle in Q,

then locus of point of intersection of normals at P and Q to the respective

curves, is

$$\begin{aligned} 1) \quad & x^2 + y^2 = 8 \\ 3) \quad & x^2 + y^2 = 64 \end{aligned}$$

$$\begin{aligned} 2) \quad & x^2 + y^2 = 34 \\ 4) \quad & x^2 + y^2 = 15 \end{aligned}$$

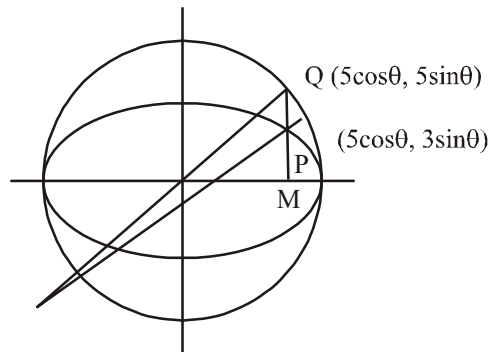
Key (3)

Sol:

Equation of normal to the ellipse at 'P' is

$$5x \sec \theta - 3y \operatorname{cosec} \theta = 16 \quad \dots(1)$$

Equation of normal to the circle $x^2 + y^2 = 25$ at point Q is –



$$y = x \tan \theta$$

Eliminating θ from (1) & (2). We get $x^2 + y^2 = 64$.

156.

If W is one of the angles between the normals to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{at the}$$

points whose eccentric angles are θ and $\frac{\pi}{2} + \theta$, then $\left| \frac{2 \cot W}{\sin 2\theta} \right|$ is

$$1) \frac{e^2}{\sqrt{1-e^2}}$$

$$2) \frac{e^2}{\sqrt{1+e^2}}$$

$$3) \frac{e}{\sqrt{1-e^2}}$$

$$4) \frac{e}{\sqrt{1+e^2}}$$

Key (1)

Sol.

$$\tan \omega = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

158).

If normal at any point P to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ meet the axes at M and

N so that $\frac{PM}{PN} = \frac{2}{3}$ then value of eccentricity e.

- 1) $\frac{1}{\sqrt{2}}$ 2) $\sqrt{\frac{2}{3}}$ 3) $\frac{1}{\sqrt{3}}$ 4) $2/3$

Key (3)

Sol.

$$\frac{PM}{PN} = \frac{2}{3} \Rightarrow \frac{b^2}{a^2} = \frac{2}{3}$$

159). If the tangent to the ellipse $x^2 + 4y^2 = 16$ at the point ' α ' is a normal to the circle

$$x^2 + y^2 - 8x - 4y = 0 \text{ then } \alpha =$$

- 1) $\frac{\pi}{2}$ 2) $\frac{\pi}{4}$ 3) $\frac{\pi}{3}$ 4) $-\frac{\pi}{4}$

Key (1)

Sol. Normal at ' α ' is

$$\frac{x}{4} \cos \alpha + \frac{y}{2} \sin \alpha - 1 = 0$$

It passes through (4,2)

$$\Rightarrow \cos \alpha + \sin \alpha = 1$$

160). The normal at a point P on the ellipse $x^2 + 4y^2 = 16$ meets the x – axis at Q. If M is midpoint of the line segment PQ. Then the locus of M intersects the latus rectum of the given ellipse at points

$$1) \left(\pm \frac{3\sqrt{5}}{2}, \pm \frac{2}{4} \right)$$

$$2) \left(\pm \frac{3\sqrt{5}}{2}, \pm \frac{\sqrt{19}}{4} \right)$$

$$3) \left(\pm 2\sqrt{3}, \pm \frac{1}{7} \right)$$

$$4) \left(\pm 2\sqrt{3}, \pm \frac{4\sqrt{3}}{7} \right)$$

Key (3)

Sol. The ellipse is $\frac{x^2}{16} + \frac{y^2}{4} = 1$

Any point P on it is $4\cos\phi, 2\sin\phi$

Normal at P is $4x \sec\phi - 2y \operatorname{cosec}\phi = 12$

Q $\equiv 3\cos\phi, 0$

Let M $\equiv h, k$

$$h = \frac{3\cos\phi + 4\cos\phi}{2} = \frac{7}{2}\cos\phi$$

$$\Rightarrow \cos\phi = \frac{2}{7}h$$

$$k = \sin\phi$$

$$\cos^2\phi + \sin^2\phi = 1$$

$$\Rightarrow \frac{4}{49}h^2 + k^2 = 1 \Rightarrow \text{locus is } \frac{4}{49}x^2 + y^2 = 1 \dots\dots(1)$$

Also, eccentricity of given circle, $e = \sqrt{1 - \frac{4}{16}} = \frac{\sqrt{3}}{2}$

$\therefore$ latus rectum are $x = \pm 4 \times \frac{\sqrt{3}}{2} = \pm 2\sqrt{3}$

Solving with (1), we get

$$\frac{48}{49} + y^2 = 1 \Rightarrow y = \pm \frac{1}{7}$$

So, the point of intersection are $\pm 2\sqrt{3}, \pm 1/7$

161). From the focus $(-5, 0)$ of the ellipse $\frac{x^2}{45} + \frac{y^2}{20} = 1$ a ray of light is sent which

makes angle $\cos^{-1}\left(\frac{-1}{\sqrt{5}}\right)$ with the positive direction of X-axis upon reacting the ellipse the ray is reflected from it. Slope of the reflected ray is

1) $-3/2$ 2) $-7/3$ 3) $-5/4$ 4) $-2/11$

Key (4)

Sol. Let $\theta = \cos^{-1}\left(\frac{-1}{\sqrt{5}}\right) \Rightarrow \cos \theta = \frac{-1}{\sqrt{5}} \Rightarrow \tan \theta = -2$

Foci are $(\pm 5, 0)$

Equation of line through $(-5, 0)$ with slope -2 is $y = -29x + 5 = -2x - 10$

This line meets the ellipse above X-axis at $(-6, 2)$

$$\therefore \text{Slope} = \frac{2-0}{-6-5} = -\frac{2}{11}.$$

162). If the length of the major axis intercepted between the tangent &

normal at a point $(a \cos \theta, b \sin \theta)$ on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is equal to the length of semi-major axis, then, eccentricity of the ellipse is,

1) $\frac{\cos \theta}{\sqrt{1 - \cos \theta}}$ 2) $\frac{\sqrt{1 - \cos \theta}}{\cos \theta}$

$$3) \frac{\sqrt{1-\cos\theta}}{\sin\theta}$$

$$4) \frac{\sin\theta}{\sqrt{1-\sin\theta}}$$

Key (2)

Sol.
$$\frac{a}{\cos\theta} - \frac{(a^2-b^2)}{a} \cos\theta = a \Rightarrow e^2 \cos^2\theta = 1 - \cos\theta \Rightarrow e = \frac{\sqrt{1-\cos\theta}}{\cos\theta}$$

163). If the normals at the four points (x_i, y_i) , $i=1,2,3,4$, on the hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \text{ are concurrent, then the value of } \left(\sum x_i\right)\left(\sum \frac{1}{x_i}\right) \text{ is}$$

Key (4)

Sol

$$\frac{a^2x}{x_i} + \frac{b^2y}{y_i} = a^2 + b^2 - 1$$

Eliminating y_i from

1 and the hyperbola we have a biquadratic equation in x_i has roots

$$x_1, x_2, x_3, x_4.$$

$$\sum x_i = \frac{2a^2h}{\lambda}, \sum \frac{1}{x_i} = \frac{2\lambda}{a^2h} \text{ where } (h,k) \text{ is the point of concurrence.}$$

164). If $\phi_1, \phi_2, \phi_3, \phi_4$ are the eccentric angles of four concyclic points on the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \text{ then } \cos(\phi_1 + \phi_2 + \phi_3 + \phi_4) \text{ is equal to } \underline{\hspace{2cm}}$$

Key (1)

Sol

$$\phi_1 + \phi_2 + \phi_3 + \phi_4 = 2n\pi$$

$$\cos(\phi_1 + \phi_2 + \phi_3 + \phi_4) = 1$$

165). If the tangent and normal at a point P on the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ intersect the major axis at the points T and N respectively in such a way that the ratio of the areas of ΔPTN and ΔPSS^1 is $\frac{91}{60}$ and if the area of ΔPSS^1 is $2K$ (S and S^1 are foci of ellipse) then the value of $[K]$ ($[]$ is GIF)

Key (5)

Sol

$$\frac{\frac{1}{2}(NT)y_1}{\frac{1}{2}(6)y_1} = \frac{91}{60} \Rightarrow NT = \frac{91}{10}$$

$$\Rightarrow \frac{25}{x_1} - \frac{9x_1}{25} = \frac{91}{10} \Rightarrow x_1 = \frac{5}{2} \Rightarrow y_1 = 2\sqrt{3}$$

$$\therefore \Delta PSS^1 = \frac{1}{2} \times 6 \times 2\sqrt{3} = 6\sqrt{3} \Rightarrow 2k = 6\sqrt{3} \Rightarrow k = \sqrt{27} \quad [K] = 5$$

Passage:1

In an ellipse $(x-1)^2 + (y-2)^2 = \left(\frac{5x+4y-20}{7}\right)^2$

1). The eccentricity is

- (A) $\frac{5}{7}$ (B) $\frac{4}{7}$ (C) $\frac{7}{41}$ (D) $\frac{\sqrt{41}}{7}$

Key. D

2). Equation of major axis is

- (A) $4x-5y+8=0$ (B) $3x+4y-6=0$ (C) $4x-5y+6=0$ (D) $5x+4y-10=0$

Key. C

3). Length of latus rectum is

- (A) 1 (B) 2 (C) 3 (D) 4

Key. B

Sol:

$$e^2 = \frac{41}{49} \Rightarrow e = \frac{\sqrt{41}}{7}$$

Major axis is perpendicular to directrix and passing through focus (1, 2),
i.e., $4(x-1) - 5(y-2) = 0 \Rightarrow 4x - 5y + 6 = 0$.

The distance from focus to directrix = $\frac{b^2}{ae}$

$$\frac{b^2}{ae} = \left| \frac{5+8-20}{\sqrt{41}} \right| = \frac{7}{\sqrt{41}} \Rightarrow \frac{b^2}{a} = \frac{7}{\sqrt{41}} \times e = 1$$

$$\text{L.L.R.} = \frac{2b^2}{a} = 2.1 = 2$$

Passage:2

In an ellipse $25(3x-4y+7)^2 + 16(4x+3y-6)^2 = 10000$.

- 4). The length of major axis is
(A) 50 (B) 8 (C) 16 (D) 10

Key. D

- 5). The length of minor axis is
(A) 5 (B) 8 (C) 40 (D) 50

Key. B

- 6). Equation of minor axis of ellipse
(A) $x+7y-13=0$ (B) $7x-y+1=0$ (C) $3x-4y+7=0$ (D)
 $4x+3y-6=0$

Key. D

Sol: Length of major axis = $2b = 2 \times 5 = 10$

Length of minimum axis = $2a = 2 \times 4 = 8$.

Equation of minor axis = $4x + 3y - 6 = 0$.

Passage:3

Consider the conic defined by the equation :

$$\left| \sqrt{(x-1)^2 + (y-2)^2} - \sqrt{(x-5)^2 + (y-5)^2} \right| = 3$$

- 7). The equation of an axis of the conic is
-

a) $6x + 8y = 45$

b) $3x - 4y - 5 = 0$

c) $8x + 6y = 45$

d) $3x + 4y + 5 = 0$

Key. C

Sol: Given equation represents a hyperbola having foci $S(1,2)$ and $S'(5,5)$ & $2a = 3$

transverse axis : line SS' : $3x - 4y + 5 = 0$

Conjugate axis : perpendicular bisector of SS' : $8x + 6y = 45$

8). The distance between the directrices of the conic is

a) $9/5$

b) $3/5$

c) $5/3$

d) $5/9$

Key. A

Sol: Distance between directrices $= \frac{2a}{e} = \frac{3}{5/3} = \frac{9}{5}$

9). The eccentricity of the conic conjugate to the given one, is

a) $5/3$

b) $5/4$

c) $5/2$

d) 5

Key. B

Sol: let e' be the ecc. of conjugate hyperbola then $\frac{1}{e^2} + \frac{1}{e'^2} = 1 \Rightarrow e'^2 = \frac{25}{16}$

Passage:4

An ellipse E has its centre C (1,3), focus at S (6, 3) and passes through the point P (4, 7). Then

10). The product of the perpendicular distances of foci from tangent at P to the ellipse, is

a) 20

b) 45

c) 40

d) 60

Key. A

- a) $\left(\frac{5}{3}, 5\right)$ b) $\left(\frac{4}{3}, 3\right)$
c) $\left(\frac{8}{3}, 3\right)$ d) $\left(\frac{10}{3}, 5\right)$

12). If the normal at a variable point on the ellipse (E) meets its axes in Q and R, then the locus of the midpoint of QR is a conic with eccentricity =

- a) $3/\sqrt{10}$ b) $\sqrt{5}/3$
c) $3/\sqrt{5}$ d) $\sqrt{10}/3$

$$\text{Product} = b^2$$

Consider the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ and the parabola $y^2 = 2x$. They intersect at P and Q in the first and fourth quadrants respectively. Tangents to the ellipse at P and Q intersect the x-axis at R and tangents to the parabola at P and Q intersect the x-axis at S.

- A) 1:3 B) 1:2 C) 2:3 D)

Sol: (C)

- 14). The equation of circle touching the parabola at upper end of its latus rectum and passing through its vertex, is

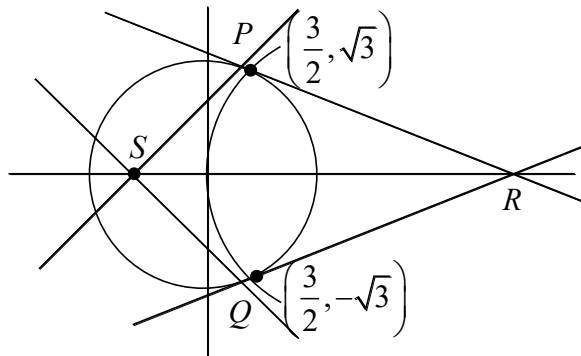
A) $2x^2 + 2y^2 - x - 2y = 0$

B) $2x^2 + 2y^2 + 4x - \frac{9}{2}y = 0$

C) $2x^2 + 2y^2 + x - 3y = 0$

D) $2x^2 + 2y^2 - 7x + y = 0$

Sol: (D)



Equation of circle, touching the parabola at upper end of its latus rectum is $\left(x - \frac{1}{2}\right)^2 + (y - 1)^2 + \lambda$ (equation of tangent to the parabola at $\left(\frac{1}{2}, 1\right) = 0$ and passing through the vertex $\Rightarrow$ " λ " value

Passage:6

$C_1 : x^2 + y^2 = r^2$ and $C_2 : \frac{x^2}{16} + \frac{y^2}{9} = 1$ intersect at four distinct points A, B, C and D. Their common tangents form a parallelogram $A^1B^1C^1D^1$.

15). If ABCD is a square then r is equal to

A) $\frac{12}{5}\sqrt{2}$

B) $\frac{12}{5}$

C) $\frac{12}{5\sqrt{5}}$

D) $\frac{12}{\sqrt{5}}$

Sol: (A)

$$x^2 + y^2 = r^2, \quad \frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$\frac{r^2 - y^2}{16} + \frac{y^2}{9} = 1$$

$$9r^2 - 9y^2 + 16y^2 = 144$$

$$y^2 = \frac{144 - 9r^2}{7}$$

$$x^2 = r^2 - y^2 \Rightarrow \frac{16r^2 - 144}{7} = x^2$$

If ABCD is square $x^2 = y^2$

$$\Rightarrow \frac{16r^2 - 144}{7} = \frac{144 - 9r^2}{7}$$

$$25r^2 = 288$$

$$r = \frac{12}{5}\sqrt{2}$$

16) If $A^1B^1C^1D^1$ is a square then ' r ' is equal to

A) $\sqrt{20}$

B) $\sqrt{12}$

C) $\sqrt{15}$

D) None of

these

Sol: (D)

$$y = mx \pm \sqrt{16m^2 + 9} \text{ is equation of tangent to } \frac{x^2}{16} + \frac{y^2}{9} = 1$$

$$y = mx + r\sqrt{1+m^2} \text{ is equation of tangent to } x^2 + y^2 = r^2$$

$$r^2(1+m^2) = 25$$

$$2r^2 = 25$$

$$r^2 = \frac{25}{2}$$

$$r = \frac{5}{\sqrt{2}}$$

Passage:7

Let $S = O$ be the equation of reflection of $\frac{(x-4)^2}{16} + \frac{(y-3)^2}{9} = 1$ about the line $x - y - 2 = 0$. P is any point of S , A and B are foci of S , then

17) Equation of minor axis of S is

- (A) $y - 2 = 0$ (B) $x - 5 = 0$ (C) $x - 4 = 0$ (D) $y - 3 = 0$

Sol: (B)

Image of $(4,3)$ on $x - y - 2 = 0$ is $(5,2)$

$$s = \frac{(x-5)^2}{16} + \frac{(y-2)^2}{9} = 1$$

Equation of minor axis is $x - 5 = 0$

18) Locus of point of intersection of perpendicular tangents of S is

- (A) $x^2 + y^2 + 10x - y - 4 = 0$ (B) $x^2 + y^2 = 25$
 (C) $x^2 + y^2 - 10x - 4y + 4 = 0$ (D) $x^2 + y^2 - 10x - 4y = 0$

Sol: (C)

$$(x-5)^2 + (y-2)^2 = 16 + 9$$

Passage:8

From $A (0,0)$ two tangents AP and AQ are drawn to $(x+2)^2 + (y+2)^2 = 4$. The line MN (M and N are mid points of AP and AQ) intersect the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ at points R and S . Circle with RS as diameter intersects ellipse at U and V .

19) Equation of circle passing through points R, S, U, V is equal to

- A) $13x^2 + 13y^2 + 8x + 8y + 3 = 0$ B) $13x^2 + 13y^2 + 10x + 8y - 13 = 0$

C) $13x^2 + 13y^2 + 18x + 8y - 59 = 0$

D) none of these

Sol: (C)

20). Equation of line UV is

A) $y - x = \frac{13}{5}$

B) $y - x = \frac{11}{5}$

C) $y - 2x = 5$

D) $y - 2x = \frac{11}{5}$

Sol: (A)

Equation of the line MN is $x + y = -1$. Now the slope of line UV should be 1 let equation of UV is $y - x = k$ so the required equation of the circle will

be $\frac{x^2}{9} + \frac{y^2}{4} - 1 + \lambda(x + y + 1)(y - x - k) = 0$

$$\lambda = -\frac{5}{72}, k = \frac{13}{5}$$

Passage:9

Many properties of an ellipse in the standard form $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ can be used easily in the general form also. Consider an ellipse with the equation.

$$(15x - 3)^2 + (15y + 5)^2 = (12x + 5y - 2)^2$$

21). Eccentricity of the ellipse will be

(A) $\frac{1}{13}$

(B) $\frac{1}{19}$

(C) $\frac{13}{15}$

(D) $\sqrt{\frac{13}{19}}$

Sol: (C)

Equation of ellipse can be rewritten as

$$\left(x - \frac{1}{5}\right)^2 + \left(y + \frac{1}{3}\right)^2 = \left(\frac{13}{15}\right)^2 \left(\frac{12x + 5y - 2}{13}\right)^2$$

So focus is $\left(\frac{1}{5}, -\frac{1}{3}\right)$, directrix $12x + 5y - 2 = 0$

and eccentricity is $\frac{13}{15}$.

22). Locus of foot of perpendicular from foci to tangents to ellipse will be a circle of radius

(A) $\frac{19}{56}$

(B) $\frac{13}{225}$

(C) $\frac{19}{169}$

(D) $\frac{38}{195}$

Sol: (A)

Distance between focus and directrix will be $\left| \frac{\frac{12}{5} - \frac{9}{3} - 2}{13} \right| = \frac{19}{195} = a \left(e - \frac{1}{e} \right)$

$$\Rightarrow a = \frac{19}{56}$$

Required locus is auxiliary circle of the ellipse.

23). Equation of major axis of ellipse is

(A) $3x + 4y - 7 = 0$

(B) $12x + 5y = 0$

(C) $5x - 12y = 0$

(D) $5x - 12y - 5 = 0$

Sol: (D)

Major axis is perpendicular to directrix and passes through focus, so it is $5x - 12y - 5 = 0$.

Passage:10

A sequence of ellipse $E_1, E_2, \dots, E_n$ are constructed as follows: Ellipse E_n is drawn so as to touch ellipse E_{n-1} at the extremities of the major axis of E_{n-1} and to have its foci at the extremities of the minor axis of E_{n-1}

24). If E_n is independent of n then eccentricity of the ellipse E_{n-2}

- A) $\frac{3-\sqrt{5}}{2}$ B) $\frac{\sqrt{5}-1}{2}$ C) $\frac{2-\sqrt{3}}{2}$ D) $\frac{\sqrt{3}-1}{2}$

Key. B

Sol: $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1 (a_n > b_n), b_n^2 = a_n^2(1-e_n^2) \dots\dots(1)$

For E_{n-1} , $a_{n-1}^2 = b_{n-1}^2(1-e_{n-1}^2)$, $b_n = b_{n-1}e_{n-1}$, $a_{n-1} = a_n$

$$b_{n-1}^2 e_{n-1}^2 = a_n^2 (1-e_n^2) = a_{n-1}^2 (1-e_n^2)$$

$$b_{n-1}^2 e_{n-1}^2 = b_{n-1}^2 (1-e_{n-1}^2)(1-e_n^2)$$

Let all the eccentricities are 'e'

$$e^2 = (1-e^2)(1-e^2) \Rightarrow e^4 - 3e^2 + 1 = 0 \Rightarrow e = \frac{\sqrt{5}-1}{2}$$

25) If eccentricity of ellipse E_n is independent of 'n' then the locus of the mid point of chords of slope '-1' of E_n (If the axis of E_n is along Y-axis)

- A) $(\sqrt{5}-1)x = 2y$ B) $(\sqrt{5}+1)x = 2y$
C) $(3-\sqrt{5})x = 2y$ D) $(3+\sqrt{5})x = 2y$

Key. B

Sol: $T = S_1 \Rightarrow \frac{xx_1}{a_n^2} + \frac{yy_1}{b_n^2} = \frac{x_1^2}{a_n^2} + \frac{y_1^2}{b_n^2}$

$$\Rightarrow \frac{-b_n^2 x_1}{a_n^2 y_1} = -1 \Rightarrow b_n^2 x_1 = a_n^2 y_1 \Rightarrow x_1(1-e^2) = y_1$$

$$x_1 = y_1 \left(1 - \frac{3 - \sqrt{5}}{2} \right) \Rightarrow 2x_1 = (\sqrt{5} - 1)y_1$$

$$2y_1 = (\sqrt{5} + 1)x_1$$

Passage:11

If $\alpha, \beta, \gamma, \delta$ are eccentric angles of 4 – points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ the normals at which are concurrent then

26) $\alpha + \beta + \lambda + \delta =$

A. $2n\pi, n \in \mathbb{Z}$ B. $(2n+1)\frac{\pi}{2}, n \in \mathbb{Z}$

C. $(2n+1)\pi, n \in \mathbb{Z}$ D. $(2n+1)\frac{\pi}{4}, n \in \mathbb{Z}$

Key. C

27) $\cos(\alpha + \beta) + \cos(\alpha + \lambda) + \cos(\alpha + \delta) + \cos(\beta + \gamma) + \cos(\beta + \delta) + \cos(\lambda + \delta) =$

A. 6 B. 3 C. 0 D. 1

Key. C

28) $\sin(\alpha + \beta) + \sin(\beta + \lambda) + \sin(\lambda + \delta) =$

A. 0 B. 1 C. -1 D.

2

Key. A

Sol:

Let $Z = \text{cis } \theta$ $\frac{1}{Z} = \cos \theta - i \sin \theta$

$$2 \cos \theta = Z + \frac{1}{Z}, \quad \cos \theta = \frac{Z^2 + 1}{2Z}$$

$$\sin \theta = \frac{Z^2 - 1}{2iZ}$$

Equation of normal is $\frac{ax}{\cos \theta} - \frac{by}{\sin \theta} = a^2 - b^2$

It is drawn from (x_1, y_1) .

$$\frac{ax_1}{\cos \theta} - \frac{by_1}{\sin \theta} = a^2 - b^2$$

$$\frac{\frac{ax_1}{\left(\frac{Z^2+1}{2t}\right)} - \frac{by_1}{\frac{Z^2-1}{2iZ}}}{2t} = a^2 - b^2$$

$$(a^2 - b^2)Z^4 - 2(ax_1 - iby_1)Z^3 + 2(ax_1 + iby_1)Z - (a^2 - b^2) = 0 \rightarrow (1)$$

Roots are Z_1, Z_2, Z_3, Z_4

$$Z_1 Z_2 Z_3 Z_4 = -1 \quad \text{cis}\alpha.\text{cis}\beta.\text{cis}\gamma.\text{cis}\delta = -1$$

$$\text{cis}(\alpha + \beta + \gamma + \delta) = -1$$

$$\cos(\alpha + \beta + \gamma + \delta) = -1, \quad \sin(\alpha + \beta + \gamma + \delta) = 0$$

$$\alpha + \beta + \gamma + \delta = (2n+1)\pi$$

$$\sum Z_1 Z_2 = 0$$

$$\sum \text{cis}\alpha.\text{cis}\beta = 0$$

$$\sum \text{cis}(\alpha + \beta) = 0$$

$$\cos(\alpha + \beta) + \cos(\alpha + \gamma) + \cos(\alpha + \delta) + \cos(\beta + \gamma) + \cos(\beta + \delta) + \cos(\gamma + \delta) = 0$$

$$\text{Ily, } \sin(\alpha + \beta) + \sin(\alpha + \gamma) + \sin(\alpha + \delta) + \sin(\beta + \gamma) + \sin(\beta + \delta) + \sin(\gamma + \delta) = 0$$

$$\sin(\alpha + \beta) = \sin(\gamma + \delta)$$

$$\sin(\beta + \gamma) = \sin(\alpha + \delta)$$

$$\sin(\gamma + \alpha) = \sin(\beta + \delta)$$

$$2(\sin(\alpha + \beta) + \sin(\beta + \gamma) + \sin(\gamma + \delta)) = 0$$

$$\sin(\alpha + \beta) + \sin(\beta + \gamma) + \sin(\gamma + \delta) = 0$$

Passage:12

Let P, Q, R be three points on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and let P', Q', R' be their corresponding points on it's auxiliary circle, then

29) The maximum area of the triangle PQR is

a) $\frac{3\sqrt{3}}{4}ab$

b) $\frac{3\sqrt{3}}{2}ab$

c) $\frac{\sqrt{3}}{4}ab$

d) πab

Key. A

Sol: Let $P = (a \cos \alpha, b \sin \alpha)$ $P^1 = (a \cos \alpha, a \sin \alpha)$

$$P = (a \cos \beta, b \sin \beta) \quad Q^1 = (a \cos \beta, a \sin \beta)$$

$$R = (a \cos \gamma, b \sin \gamma) \quad R^1 = (a \cos \gamma, a \sin \gamma).$$

Area of ΔPQR is $2ab \sin \frac{\alpha-\beta}{2} \sin \frac{\beta-\gamma}{2} \sin \frac{\gamma-\alpha}{2}$ its max value is $2ab$

$$\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{3\sqrt{3}ab}{4}$$

$$30) \quad \frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta P'Q'R'} =$$

a) $\frac{a}{b}$

b) $\frac{b}{a}$

c) $\frac{1}{2}$

d) depends on points taken

Key. B

$$\text{Sol: } \frac{\text{Area of } \Delta PQR}{\text{Area of } \Delta P^1Q^1R^1} = \frac{\frac{1}{2}ab \begin{vmatrix} \cos \alpha - \cos \gamma & \sin \alpha - \sin \gamma \\ \cos \alpha - \cos \beta & \sin \alpha - \sin \beta \end{vmatrix}}{\frac{1}{2}a^2 \begin{vmatrix} \cos \alpha - \cos \gamma & \sin \alpha - \sin \gamma \\ \cos \alpha - \cos \beta & \sin \alpha - \sin \beta \end{vmatrix}} = \frac{b}{a}$$

31). When the area of triangle PQR is maximum, the centroid of triangle $P'Q'R'$ lies at

a) one focus b) one vertex c) centre d) on one directrix

Key. C

Sol: Area of ΔPQR is max when $\alpha - \beta = \beta - \gamma = \gamma - \alpha = 120^\circ$ is $\Delta P^1Q^1R^1$ is equilateral hence its centroid is (0,0) centre of the ellipse

Passage:13

If 'P' is any point on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. S_1 and S_2 are foci of the ellipse

- 32) Locus of incentre of triangle PS_1S_2 will be
 a) a straight line b) a circle c) a parabola d) an ellipse

Key. D

- 33). If $e = \frac{1}{2}$ and $\angle PS_1S_2 = \alpha, \angle PS_2S_1 = \beta, \angle S_1PS_2 = \gamma$, then $\cot \frac{\alpha}{2}, \cot \frac{\gamma}{2}$ and $\cot \frac{\beta}{2}$ are in
 a) A.P b) G.P
 c) H.P d) None

Key. A

- 34). Maximum area of the triangle PS_1S_2 is equal to
 a) b^2e sq.units b) a^2e sq.units
 c) ab sq.units d) $abesq.$ units

Key. D

Sol: $\frac{PS_2}{S_2G} = \frac{PS_1}{GS_1} = \frac{PS_2 + PS_1}{S_2G + GS_1} = \frac{2a}{2ae} = \frac{1}{e}$

So $PI : IG = 1 : e$

$$\tan \frac{\alpha}{2} \tan \frac{\beta}{2} = \frac{1-e}{1+e} = \frac{1}{3}$$

Base S_1S_2 fixed and $PS_2 + PS_1$ is fixed, Hence area will be maximum if $PS_1 = PS_2$

Passage:14

The points P,Q,R are taken on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with eccentricities $\theta, \theta + \alpha, \theta + 2\alpha$ then

- 36). Area of the triangle PQR is independent of
 A) θ B) α C) θ & α both D) none

Key. A

- 37). If the area of triangle PQR is maximum, then
 A) $\alpha = \frac{\pi}{3}$ B) $\alpha = \frac{\pi}{2}$ C) $\alpha = \frac{2\pi}{3}$ D) none

Key. C

38). If A_1 be the area of triangle PQR and A_2 be the area of the triangle formed by corresponding points on the auxiliary circle then $\frac{A_1}{A_2}$ is ____.

A) 1

B) $\frac{a}{b}$

C) $\frac{b}{a}$

D) none

Key. C

Sol:

$$A_1 = \Delta = \frac{1}{2} \begin{vmatrix} a \cos \theta & b \sin \theta & 1 \\ a \cos(\theta + \alpha) & b \sin(\theta + \alpha) & 1 \\ a \cos(\theta + 2\alpha) & b \sin(\theta + 2\alpha) & 1 \end{vmatrix} = ab(1 - \cos \alpha) \sin \alpha$$

$$\Delta \text{ is max } \Rightarrow \alpha = \frac{2\pi}{3};$$

$$A_1 = \frac{1}{2} \begin{vmatrix} a \cos \alpha & b \sin \alpha & 1 \\ a \cos \beta & b \sin \beta & 1 \\ a \cos \gamma & b \sin \gamma & 1 \end{vmatrix} = A_2 = \frac{1}{2} \begin{vmatrix} a \cos \alpha & b \sin \alpha & 1 \\ a \cos \beta & b \sin \beta & 1 \\ a \cos \gamma & b \sin \gamma & 1 \end{vmatrix} \therefore \frac{A_1}{A_2} = \frac{b}{a}$$

Passage:15

P is any point of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. S and S' are foci and e is the eccentricity of ellipse. $\angle PSS' = \alpha$ and $\angle PS'S = \beta$

39). $\tan \frac{\alpha}{2} \tan \frac{\beta}{2}$ is equal to

(A) $\frac{2e}{1-e}$

(B) $\frac{1+e}{1-e}$

(C) $\frac{1-e}{1+e}$

(D) $\frac{2e}{1+e}$

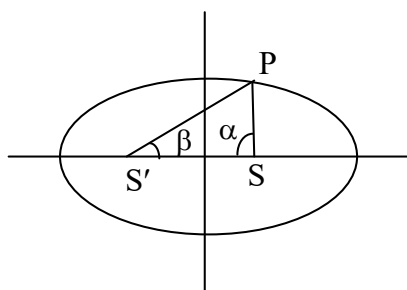
Key. C

Sol: $\frac{PS}{\sin \beta} = \frac{PS'}{\sin \alpha} = \frac{2ae}{\sin(\pi - (\alpha + \beta))}$

$$\text{or, } \frac{2a}{\sin \alpha + \sin \beta} = \frac{2ae}{\sin(\alpha + \beta)}$$

$$\text{or, } \frac{1}{e} = \frac{2 \sin \frac{\alpha + \beta}{2} \cdot \cos \frac{\alpha - \beta}{2}}{2 \sin \frac{\alpha + \beta}{2} \cos \frac{\alpha + \beta}{2}}$$

$$\therefore \frac{1-e}{1+e} = \tan \frac{\alpha}{2} \tan \frac{\beta}{2}$$



40) Locus of incentre of triangle PSS' is

(A) an ellipse

(B) hyperbola

(C) parabola

(D) circle

Key. A

Sol: $y - 0 = \tan \frac{\beta}{2} (x + ae) \dots (i)$

$$y - 0 = -\tan \frac{\alpha}{2} (x - ae) \dots (ii)$$

$$\text{or, } y^2 = -\left(\frac{1-e}{1+e}\right) [x^2 - a^2e^2]$$

$$\text{or, } \left(\frac{1-e}{1+e}\right) x^2 + y^2 = \left(\frac{1-e}{1+e}\right) a^2e^2$$

$$\text{or, } \frac{x^2}{a^2e^2} + \frac{y^2}{\left(\frac{1-e}{1+e}\right) a^2e^2} = 1$$

which is clearly an ellipse.

41). Eccentricity of conic, which is locus of incentre of triangle PSS'

(A) $\sqrt{\frac{e}{1+e}}$

(B) $\sqrt{\frac{2e}{1+e}}$

(C) $\sqrt{\frac{2e}{1-e}}$

(D) $\sqrt{\frac{e}{1-e}}$

Key. B

Sol:
$$e' = \sqrt{1 - \frac{1-e}{1+e}}$$
$$= \sqrt{\frac{2e}{1+e}}$$

Passage:16

Consider the conic defined by $x^2 + y^2 = (3x + 4y + 10)^2$.

42). If (α, β) is the centre of the conic then $4\alpha + 3\beta =$

- A) -8 B) -10 C) -6 D) -9

Key. B

43) If (p, q) is a vertex of the conic then $2p - q =$

- A) -1 B) 1 C) -3 D) 2

Key. A

44). The number of points through which a pair of real perpendicular tangents can be drawn to the conic is

- A) infinite B) 1 C) 0 D) 4

Key. C

Sol:

The given equation can be expressed as $\sqrt{x^2 + y^2} = 5 \frac{|3x + 4y + 10|}{5}$

Hence it is Hyperbola with eccentricity 5.

Focus is $(0, 0)$

Directrix is $3x + 4y + 10 = 0$

And hence the axis is $4x - 3y = 0$

Passage:17

he ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with centre "O" where $a > b > 0$. Tangent at any point

P on the ellipse meets the co-ordinate axis at X and Y and N is the root of

the perpendicular from the origin on the tangent at P. Minimum length of XY is 24 and maximum length of PN is 8.

45). The eccentricity of the ellipse is

- a) $\frac{2}{5}$ b) $\frac{3}{5}$ c) $\frac{\sqrt{3}}{2}$ d) $\frac{3}{4}$

46). maximum area of an isosceles triangle inscribed in the ellipse of one of its vertex coincides with one end of the major axis of the ellipse is

- a) $196\sqrt{3}$ b) $96\sqrt{3}$ c) 96 d) $3\sqrt{96}$

47). Maximum area of the triangle OPN is

- a) 96 b) $96\sqrt{3}$ c) $196\sqrt{3}$ d) 48

Sol: (C) $a + b = 24$, $a - b = 8$

(B) $\frac{3\sqrt{3}}{4}ab$

(D) $\frac{a^2 - b^2}{4}$

Passage:18

A, B, C, D are consecutive vertices of a rectangle whose area is 2006. An ellipse with area 2006π passes through A and C and has foci at B and D.

48). The perimeter of the rectangle is

- A) $8\sqrt{2006}$ B) $8\sqrt{1003}$ C) $6\sqrt{1003}$ D) $6\sqrt{2006}$

49). The eccentricity of the ellipse is

- A) $\sqrt{\frac{2006}{4009}}$ B) $\sqrt{\frac{3009}{4012}}$ C) $\frac{3}{11}$ D) $\sqrt{\frac{2006}{2009}}$

50). The radius of director circle of the ellipse is

- A) $\sqrt{5015}$ B) $\sqrt{4014}$ C) $\sqrt{3003}$ D) $\sqrt{2009}$

Key: B-B-A

Hint:

Let $2a, 2b$ respectively be the lengths of major axis and minor axis of the ellipse. Let the dimensions of the rectangle be x, y then by hypothesis $ab = 2006 = xy$ and $x^2 + y^2 = 4(a^2 - b^2)$.

Passage:19

An ellipse whose distance between foci S & S' is 4 units is inscribed in the $\triangle ABC$, touching the sides AB, AC and BC at P, Q and R . If centre of ellipse is at origin 'O' and major axis along x-axis and $SP + S'P = 6$ then

51 If $\angle BAC = 90^\circ$ then locus of vertex A is

- A. $x^2 + y^2 = 12$ B. $x^2 + y^2 = 16$ C. $x^2 + y^2 = 14$ D. $x^2 + y^2 = 25$

Key. C

Sol: $2a = 6, 2ae = 4$

$$\therefore e = \frac{2}{3}$$

Ellipse equation is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$\angle BAC = 90^\circ \Rightarrow A$ lies on director circle $x^2 + y^2 = 14$

$\angle POQ = 90^\circ \Rightarrow$ Let $A(h, k)$

Equation of PQ is $S_1 = 0$

$$\frac{hx}{9} + \frac{ky}{5} = 1$$

Homogenising

$$\frac{x^2}{9} + \frac{y^2}{5} = \left(\frac{hx}{9} + \frac{ky}{5} \right)^2$$

coefficient of x^2 + coefficient of $y^2 = 1$

$$25h^2 + 81k^2 = 630$$

$$AB = BC, \angle B = 90^\circ \Rightarrow \angle A = 45^\circ$$

$$\tan 45^\circ = \frac{2ab\sqrt{S_{11}}}{x_1^2 + y_1^2 - a^2 - b^2}$$

$$(x_1^2 + y_1^2 - a^2 - b^2)^2 = (2ab\sqrt{b^2x_1^2 + ay_1^2 - a^2b^2})^2$$

$$(x^2 + y^2 - 14)^2 = 20x^2 + 36y^2 - 180$$

52. If $\angle POQ = 90^\circ$ then locus of vertex A is

A.

$$25x^2 + 81y^2 = 330$$

B.

$$81x^2 + 25y^2 = 630$$

C.

$$25x^2 + 81y^2 = 630$$

D.

$$25x^2 + 81y^2 = 230$$

Key. C

Sol: $2a = 6, 2ae = 4$

$$\therefore e = \frac{2}{3}$$

Ellipse equation is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$\angle BAC = 90^\circ \Rightarrow$ A lies on director circle $x^2 + y^2 = 14$

$\angle POQ = 90^\circ \Rightarrow$ Let A(h, k)

Equation of PQ is $S_1 = 0$

$$\frac{hx}{9} + \frac{ky}{5} = 1$$

Homogenising

$$\frac{x^2}{9} + \frac{y^2}{5} = \left(\frac{hx}{9} + \frac{ky}{5} \right)^2$$

coefficient of x^2 + coefficient of $y^2 = 1$

$$25h^2 + 81k^2 = 630$$

$$AB = BC, \angle B = 90^\circ \Rightarrow \angle A = 45^\circ$$

$$\tan 45^\circ = \frac{2ab\sqrt{S_{11}}}{x_1^2 + y_1^2 - a^2 - b^2}$$

$$(x_1^2 + y_1^2 - a^2 - b^2)^2 = (2ab\sqrt{b^2x_1^2 + ay_1^2 - a^2b^2})^2$$

$$(x^2 + y^2 - 14)^2 = 20x^2 + 36y^2 - 180$$

53. If $AB = BC$ and $\angle B = 90^\circ$ then locus of vertex A is

A. $(x^2 + y^2 - 14)^2 = 20x^2 + 36y^2 - 180$

B. $(x^2 + y^2 + 14)^2 = 20x^2 + 36y^2 - 180$

C. $(x^2 + y^2 - 14) = 20x^2 - 36y^2 - 180$

D. $(x^2 + y^2 - 14)^2 = 20x^2 - 36y^2 - 180$

Key. A

Sol: $2a = 6, 2ae = 4$

$$\therefore e = \frac{2}{3}$$

Ellipse equation is $\frac{x^2}{9} + \frac{y^2}{5} = 1$

$\angle BAC = 90^\circ \Rightarrow$ A lies on director circle $x^2 + y^2 = 14$

$\angle POQ = 90^\circ \Rightarrow$ Let A(h, k)

Equation of PQ is $S_1 = 0$

$$\frac{hx}{9} + \frac{ky}{5} = 1$$

Homogenising

$$\frac{x^2}{9} + \frac{y^2}{5} = \left(\frac{hx}{9} + \frac{ky}{5} \right)^2$$

coefficient of x^2 + coefficient of $y^2 = 1$

$$25h^2 + 81k^2 = 630$$

$$AB = BC, \angle B = 90^\circ \Rightarrow \angle A = 45^\circ$$

$$\tan 45^\circ = \frac{2ab\sqrt{S_{11}}}{x_1^2 + y_1^2 - a^2 - b^2}$$

$$(x_1^2 + y_1^2 - a^2 - b^2)^2 = (2ab\sqrt{b^2x_1^2 + ay_1^2 - a^2b^2})^2$$

$$(x^2 + y^2 - 14)^2 = 20x^2 + 36y^2 - 180$$

Passage:20

Let the equation $ax^2 + 2hxy + by^2 = 1$ represent an ellipse, then $h^2 - ab < 0$. If the equation of ellipse can be changed to $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$, by putting $x = X \cos \theta - Y \sin \theta$ and $y = X \sin \theta + Y \cos \theta$ then,

54). $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ must be equal to

(A) $\frac{1}{a^2} + \frac{1}{b^2}$

(B) $\frac{1}{a} + \frac{1}{b}$

(C) $a + b$

(D) $a - b$

Key. 3

55). $\frac{1}{\alpha^2 \beta^2}$ equals

(A) $h^2 - ab$

(B) $\sqrt{h^2 - ab}$

(C) $h^2 + ab$

(D) $ab - h^2$

Key. 4

56). If e is eccentricity, then $e^2 =$

(A) $\frac{\sqrt{(a-b)^2 + 4h^2}}{a+b}$

(B) $\frac{2\sqrt{(a-b)^2 + 4h^2}}{a+b + \sqrt{(a-b)^2 + 4h^2}}$

$$(C) \frac{\sqrt{(a-b)^2 + 4h^2}}{a+b+\sqrt{(a-b)^2 + 4h^2}}$$

$$(D) \frac{2\sqrt{(a-b)^2 + 4h^2}}{a+b}$$

Key. 2

Sol: Equation can be changed to $\frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$

$$x = X \cos \theta - Y \sin \theta \quad y = X \sin \theta + Y \cos \theta$$

$$\text{We get } a(X \cos \theta - Y \sin \theta)^2 + 2h(X \cos \theta - Y \sin \theta)(X \sin \theta + Y \cos \theta) + b(X \sin \theta + Y \cos \theta)^2$$

$$\text{Coeff. } X^2 = a \cos^2 \theta + 2h \sin \theta \cos \theta + b \sin^2 \theta$$

$$= a \left[\frac{1 + \cos 2\theta}{2} \right] + 2h \sin \theta \cos \theta + b \left[\frac{1 - \cos \theta}{2} \right]$$

$$= \frac{1}{2} [(a+b) + (a-b) \cos 2\theta + 2h \sin 2\theta]$$

$$\text{Similarly we get coeff. of } y^2 = \frac{1}{2} [(a+b) - (a-b) \cos 2\theta - 2h \sin 2\theta]$$

$$xy = -2a \sin \theta \cos \theta + 2h [\cos 2\theta] + 2b \sin \theta \cos \theta$$

$$= (b-a) \sin 2\theta + 2h \cos \theta$$

$$\Rightarrow \quad \text{or} \quad -2h \cos 2\theta = (b-a) \sin 2\theta$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{1}{2} [(a+b) + (a-b) \cos 2\theta + 2h \sin 2\theta + (a+b) - (a-b) \cos 2\theta - 2h \sin 2\theta]$$

$$= a+b$$

$$\text{Also } \frac{1}{\alpha^2 \beta^2} = \frac{1}{4} [(a+b)^2 - \{(a-b) \cos 2\theta + 2h \sin 2\theta\}^2]$$

$$= \frac{1}{4} \left[(a+b)^2 - \left\{ \frac{2h \cos^2 2\theta + 2h \sin^2 2\theta}{\sin 2\theta} \right\}^2 \right]$$

$$= \frac{1}{4} [(a+b)^2 - 4h^2 \operatorname{cosec}^2 2\theta]$$

$$\begin{aligned}\text{Also } \tan 2\theta &= \frac{2h}{a-b} \text{ we get } \operatorname{cosec}^2 2\theta = \frac{(a-b)^2 + 4h^2}{4h^2} \\ &= \frac{1}{4} [(a+b)^2 - (a-b)^2 - 4h^2] \\ &= ab - h^2\end{aligned}$$

$$\text{Also eccentricity } \frac{x^2}{\alpha^2} + \frac{y^2}{\beta^2} = 1$$

$$e^2 = \frac{\alpha^2 - \beta^2}{\alpha^2}$$

$$\alpha^2 + \beta^2 = \frac{a+b}{ab-h^2} \quad \alpha^2 \beta^2 = \frac{1}{ab-h^2}$$

$$e^2 = \frac{2\sqrt{(a-b)^2 + 4h^2}}{a+b+ab-h^2}$$

Passage:21

If $P(\theta_1)$ and $D(\theta_2)$ be the end point of two semi-conjugate diameters of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ whose centre is C, then answer the following questions

57). $\theta_1 - \theta_2 =$

- a) 45° b) 90° c) 135° d) None

Key. B

58). $CP^2 + CD^2 =$

- a) $\frac{a^2 + b^2}{4}$ b) $(a^2 + b^2)$ c) $\frac{b^4 + a^4}{b^2 + a^2}$ d) $\frac{a^4 + b^4}{2(b^2 + a^2)}$

Key. C

59). Locus of midpoint of PD is

a) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$ b) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ c) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{4}$ d) None

Key. A

Sol: $m_1 m_2 = \frac{b^2}{a^2}$

$$\cos(\theta_1 - \theta_2) = 0 \Rightarrow \theta_1 - \theta_2 = \frac{\pi}{2}$$

Let p be θ & D be $\frac{\pi}{2} + \theta$

$$Cp^2 + CD^2 = a^2 + b^2$$

$$2h = a(\cos \theta - \sin \theta)$$

$$2k - b(\sin \theta + \cos \theta)$$

$$\frac{4a^2}{a^2} + \frac{4k^2}{b^2} = 2 > \frac{1}{2}$$

Passage:22

Consider an ellipse $\frac{x^2}{4} + y^2 = \alpha$, (α is parameter > 0) and a parabola $y^2 = 8x$. If a common tangent to the ellipse and the parabola meets the coordinate axes at A and B respectively, then

60). Locus of mid point of AB is

a) $y^2 = -2x$ b) $y^2 = -x$ c) $y^2 = -\frac{x}{2}$ d)

$$\frac{x^2}{4} + \frac{y^2}{2} = 1$$

Key. B

61). If the eccentric angle of a point on the ellipse where the common tangent meets it is $\left(\frac{2\pi}{3}\right)$, then α is equal to _____

- a) 4 b) 5 c) 26 d) 36

Key. D

62). If two of the three normals drawn from the point $(h, 0)$ on the ellipse to the parabola $y^2 = 8x$ are perpendicular, then

- a) $h = 2$ b) $h = 3$ c) $h = 4$ d) $h = 6$

Key. D

Sol: Equation of tangent to $y^2 = 8x$ is $yt - x - 2t^2 = 0 \rightarrow (1)$

Equation of tangent to ellipse is

$$\frac{x \cos \theta}{2\sqrt{\alpha}} + \frac{y \sin \theta}{\sqrt{\alpha}} \rightarrow (2)$$

Comparing $\frac{\sqrt{\alpha}}{\cos \theta} = -t^2; \frac{\sqrt{\alpha}}{\sin \theta} = 2t \rightarrow (3)$

If the tangent meets the coordinate axes at A and B then

A is $\left(\frac{2\sqrt{\alpha}}{\cos \theta}, 0\right), B\left(0, \frac{\sqrt{\alpha}}{\sin \theta}\right)$

Let mid point of AB is (h, k)

$$h = \frac{\sqrt{\alpha}}{\cos \theta}; K = \frac{\sqrt{\alpha}}{2 \sin \theta}$$

$$h = -t^2; K = t \Rightarrow K^2 - h \text{ or } y^2 = -x$$

From (3)

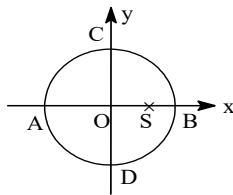
$$\frac{\alpha}{\sin^2 \theta} = \frac{-4\sqrt{\alpha}}{\cos \theta} \Rightarrow \sqrt{\alpha} = \frac{-4 \sin^2 \theta}{\cos \theta} = 6$$

Any normal is $y = mx - 4m - 2m^2$

i.e., $2m^3 + (4-h)m = 0$ i.e., $h = 6$

Passage:23

Consider an ellipse as shown in the adjacent figure, such that $OS = 6$ units and the in-radius of the triangle OCS is 1 unit, Now answer,



63). The equation of the director circle of the ellipse, is

a) $x^2 + y^2 = \frac{97}{2}$

b) $x^2 + y^2 = \sqrt{97}$

c) $x^2 + y^2 = 97$

d) $x^2 + y^2 = \sqrt{\frac{97}{2}}$

Key. C

64). The semi-perimeter of the triangle OCS is

a) 10

b) 5

c) 7.5

d) 12.5

unit

Key. C

65). The area of the ellipse is

a) $\frac{65\pi}{4}$

b) $\frac{64\pi}{5}$

c) 64π

d) 65π

Key. A

Sol:

Given, $OS = ae = 6$

Let $OB = a$, $OC = b$

$$\text{In radius of } \Delta OCS, r = \frac{OS + OC - CS}{2}$$

$$\Rightarrow 1 = \frac{6 + b - a}{2} \Rightarrow a - b = 4 \quad (1)$$

As we know

$$b^2 = a^2 - c^2$$

$$\Rightarrow (a - 4)^2 = a^2 - 36$$

$$\Rightarrow a = 13/2 \Rightarrow b = 5/2$$

Hence, director circle of the given ellipse $\therefore x^2 + y^2 = a^2 + b^2$

$$\Rightarrow \text{the required director circle is : } x^2 + y^2 = 2(a^2 + b^2)$$

$$\Rightarrow x^2 + y^2 = 97$$

$$\text{also, semi-perimeter of } \Delta OCS = \frac{OC + CS + OS}{2} = \frac{\frac{5}{2} + \frac{13}{2} + 6}{2} = \frac{15}{2}$$

$$\text{and, area of the ellipse} = \pi ab = \pi \left(\frac{65}{4} \right)$$

Passage:24

A rod AB of 20 units length with its ends 'A' on x-axis and B on the y-axis is sliding between the axes. P is a marked point on AB such that AP = 8 Answer the questions 18, 19, 20

66). The locus of P is :

(a) $\frac{x^2}{144} + \frac{y^2}{64} = 1$

(b) $\frac{x^2}{64} + \frac{y^2}{144} = 1$

(c) $\frac{x^2}{25} + \frac{y^2}{96} = 1$

(d) $\frac{x^2}{169} + \frac{y^2}{144} = 1$

Key. A

67). The eccentricity of the conic is :

(a) $\frac{4}{9}$

(b) $\frac{9}{4}$

(c) $\frac{2}{\sqrt{3}}$

(d) $\frac{\sqrt{5}}{3}$

Key. D

68). The locus of the point of intersection of perpendicular tangents is :

(a) $x^2 + y^2 = 200$

(b) $x^2 + y^2 = 208$

(c) $x^2 + y^2 = 313$

(d)

$x^2 + y^2 = 41$

Key. B

Sol: $P = (x_1, y_1) \Rightarrow x_1 = 12 \cos \theta,$

$$y_1 = 8 \sin \theta$$

$$\therefore \frac{x_1^2}{144} + \frac{y_1^2}{64} = 1$$

$$\text{Locus is } \frac{x^2}{144} + \frac{y^2}{64} = 1$$

