

CIRCLES

ASSIGNMENT.2

TANGENTS

- 1). The point of contact of tangents drawn from P (3, 2) to the circle

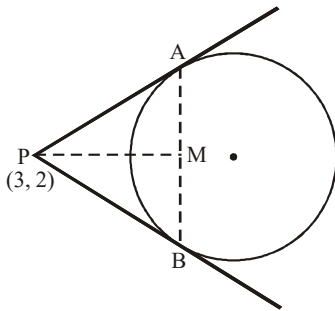
$$x^2 + y^2 + 4x + 6y + 8 = 0 \text{ is}$$

- 1) (0, 4) 2) (0, -4) 3) (-3, -1) 4) (-3, 1)

Key (2,3)

Sol

equation of AB is $x + y + 4 = 0$



$$M \text{ is } \left(-\frac{3}{2}, -\frac{5}{2} \right)$$

$$AP = \sqrt{S_1} = \sqrt{45}$$

$$PM = \frac{9}{\sqrt{2}}$$

$$\Rightarrow AM = \frac{3}{\sqrt{2}}$$

$$A \text{ \& B are given by } \left(-\frac{3}{2} \pm \frac{3}{\sqrt{2}} \cos\left(-\frac{\pi}{4}\right), -\frac{5}{2} \pm \frac{3}{\sqrt{2}} \sin\left(-\frac{\pi}{4}\right) \right)$$

$$\Rightarrow (0, -4) \text{ and } (-3, -1).$$

- 2). The line $y = x$ is a tangent at (0,0) to a circle of radius is 1, then possible centre of the circle is

$$1) \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \quad 2) (1, 2) \quad 3) \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right) \quad 4) \left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}} \right)$$

Key (3)

Sol Required circle is $x^2 + y^2 + \lambda(y - x) = 0$

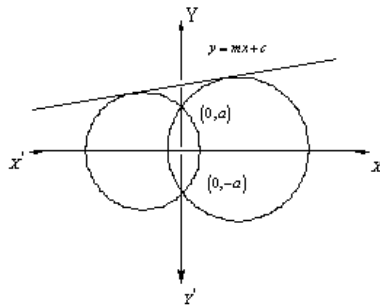
$$r^2 = \frac{\lambda^2}{4} + \frac{\lambda^2}{4} = 1 \Rightarrow \lambda = \sqrt{2} \therefore \text{centre} = \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \right)$$

3). The two circles which passes through $(0, a)$ and $(0, -a)$ and touch the line $y = mx + c$ will intersect each other at right angle, if

$$\begin{array}{lll} 1) a^2 = c^2(2m+1) & 2) a^2 = c^2(2+m^2) & 3) \\ c^2 = a^2(2+m^2) & 4) c^2 = a^2(2m+1) & \end{array}$$

Key (3)

Sol



Equation of family circles through $(0, a)$ and $(0, -a)$ is $[x^2 + (y-a)(y+a)] + \lambda x = 0, \lambda \in R$

$$\Rightarrow x^2 + y^2 + \lambda x - a^2 = 0$$

$$\text{and } \sqrt{\left(\frac{\lambda}{2}\right)^2 + a^2} = \frac{\frac{-m\lambda}{2} + c}{\sqrt{1+m^2}}$$

$$\Rightarrow (1+m^2) \left[\frac{\lambda^2}{4} + a^2 \right] = \left(\frac{m\lambda}{2} - c \right)^2$$

$$\text{Hence, } c^2 = a^2(2+m^2)$$

4). If $a > 2b > 0$ then the positive value of m for which $y = mx - b\sqrt{1+m^2}$ is a

Common tangent to $x^2 + y^2 = b^2$ and $(x-a)^2 + y^2 = b^2$ is

$$1) \frac{2b}{\sqrt{a^2 - 4b^2}} \quad 2) \frac{\sqrt{a^2 - 4b^2}}{2b} \quad 3) \frac{2b}{a - 2b} \quad 4) \frac{b}{a - 2b}$$

Key (1)

Sol

$y = mx - b\sqrt{1+m^2}$ is a tangent to the circle $x^2 + y^2 = b^2$ for all values of m . If it also

touches the circle $(x - a)^2 + y^2 = b^2$, then the length of the perpendicular from its

centre $(a, 0)$ on this line is equal to the radius b of the circle, which gives

$$\frac{ma - b\sqrt{1+m^2}}{\sqrt{1+m^2}} = \pm b$$

Taking negative value on R.H.S. We get $m = 0$, So we neglect it.

Taking the positive value on R.H.S we get $ma = b\sqrt{1+m^2}$

$$\Rightarrow m^2(a^2 - 4b^2) = 4b^2 \Rightarrow m = \frac{2b}{\sqrt{a^2 - 4b^2}}$$

5). The angle between a pair of tangents drawn from a point P to the circle

$$x^2 + y^2 + 4x - 6y + 9\sin^2 \alpha + 13\cos^2 \alpha = 0 \text{ is } 2\alpha. \text{ The equation of the locus}$$

of the point P is

$$1) x^2 + y^2 + 4x - 6y + 4 = 0$$

$$2) x^2 + y^2 + 4x - 6y - 9 = 0$$

$$3) x^2 + y^2 + 4x - 6y - 4 = 0$$

$$4) x^2 + y^2 + 4x - 6y + 9 = 0$$

Key (4)

Sol

Let PA and PB be the tangents drawn from the point $P(h, k)$ to the given circle with centre $C(-2, 3)$.

So that $\angle APB = 2\alpha$ and $\angle APC = \angle CPB = \alpha$

$$\angle PAC = \angle PBC = 90^\circ$$

From triangle PCA ,

$$\Rightarrow \sin \alpha = \frac{CA}{CP} \text{ and } CA = \sqrt{4 + 9 - (9 \sin^2 \alpha + 13 \cos^2 \alpha)}$$

$$= 2 \sin \alpha \Rightarrow CP = 2$$

$$\Rightarrow 4 = h^2 + k^2 + 4h - 6k + 13$$

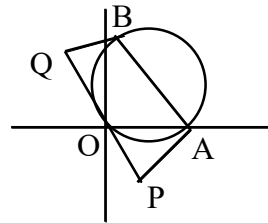
The locus of P(h, k) is therefore $x^2 + y^2 + 4x - 6y + 9 = 0$

- 6). A line meets the coordinate axes in A and B. A circle is circumscribed about the triangle OAB. If m and n are distances of tangent to circle at origin from the points A and B respectively then diameter of the circle is

- 1) $m + n$ 2) $m - n$ 3) mn 4) m/n

Key (1)

Sol



$$PA = PO, QO = QB$$

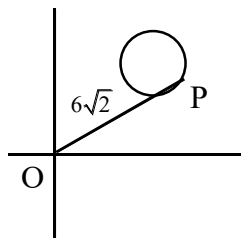
$$\text{And diameter } AB = OP + OQ = m + n$$

- 7). If the line $y = x$ touches the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ at P where $OP = 6\sqrt{2}$ then $c =$

- 1) 36 2) 72 3) 18 4) 144

Key (2)

Sol



$$\text{Length of tangent} = OP = 6\sqrt{2} = \sqrt{S_{11}} = \sqrt{C}$$

- 8). If $4l^2 - 5m^2 + 6l + 1 = 0$ and the line $lx + my + 1 = 0$ touches a fixed circle, then

- 1) The centre of the circle is at the point (4, 0)
- 2) the radius of the circle is equal to $\sqrt{5}$
- 3) the circle passes through origin
- 4) none of the above

Key (2)

Sol Let the circle be $(x-h)^2 + (y-k)^2 = r^2$

$$\text{then } r = \frac{|lh + mk + 1|}{\sqrt{l^2 + m^2}}$$

$$\text{or } l^2(h^2 - r^2) + m^2(k^2 - r^2) + 2lmhk + 2lh + 2mk + 1 = 0$$

also

$$4l^2 - 5m^2 + 6l + 1 = 0$$

Comparing the coefficients of similar terms

$$h^2 - r^2 = 4, \quad k^2 - r^2 = -5, \quad hk = 0$$

$$2h = 6, \quad 2k = 0$$

$$\therefore k = 0, h = 3 \text{ and } 9 - r^2 = 4$$

$$\therefore r = \sqrt{5}$$

Hence, centre (3, 0) $C = 72$

9). The centre of a circle of radius $4\sqrt{5}$ lies on the line $y = x$ and satisfies the inequality $3x + 6y > 10$. If the line $x + 2y = 3$ is a tangent to the circle, then the equation of the circle is

$$1) \left(x - \frac{23}{3}\right)^2 + \left(y - \frac{23}{3}\right)^2 = 80$$

$$2) \left(x + \frac{17}{3}\right)^2 + \left(y + \frac{17}{3}\right)^2 = 80$$

$$3) \left(x + \frac{23}{3}\right)^2 + \left(y - \frac{23}{3}\right)^2 = 80$$

$$4) \left(x - \frac{17}{3}\right)^2 + \left(y - \frac{17}{3}\right)^2 = 80$$

Key (1)

Sol $c = (a, a)$

radius $= 4\sqrt{5} = \text{length of the } \perp \text{ from } (a, a) \text{ to the line}$

$$\text{i.e., } \frac{|a + 2(a) - 3|}{\sqrt{4 + 1}} = \pm 4\sqrt{5} \Rightarrow a = \frac{23}{3}, \frac{-17}{3}$$

$$\therefore \text{Centre} \left(\frac{23}{3}, \frac{23}{5} \right) \text{ or } \left(\frac{-17}{3}, \frac{-17}{3} \right)$$

$$3x + 6y > 10$$

$$C = \left(\frac{23}{3}, \frac{23}{3} \right)$$

$$\therefore \left(x - \frac{23}{3} \right)^2 + \left(y - \frac{23}{3} \right)^2 = 80$$

10). If the lengths of the tangents from the point (1, 2) to the circles $x^2 + y^2 + x + y - 4 = 0$ and $3x^2 + 3y^2 - x - y - \lambda = 0$ are in the ratio 4 : 3 then $\lambda =$

1) $\frac{23}{4}$

2) $\frac{21}{4}$

3) $\frac{17}{4}$

4) $\frac{19}{4}$

Key (2)

Sol $\frac{\sqrt{S_{11}}}{\sqrt{S_{11}^1}} = 4/3$

11). If a tangent drawn from the point (4, 0) to the circle $x^2 + y^2 = 8$ touches it at a point A in the first quadrant, then the coordinates of another point B on the circle such that AB = 4 are

1) (2, -2) or (-2, 2) 2) (1, -2) or (-2, 1) 3) (-1, 1) or (1, -1) 4) (3, -2) or (-3, 2)

Key (1)

Sol equation of tangent through (4, 0)

$$x + y - 4 = 0$$

Point of contact = (2, 2)

$$AB = 4 \Rightarrow B = (2 + 4 \cos \theta, 2 + 4 \sin \theta)$$

$$\theta = \pi, \theta = \frac{3\pi}{2}$$

$$(-2, 2) \quad (2, -2)$$

- 12). The equation of the circle which is touched by $y = x$ has its centre on the positive direction of the x-axis and cuts off a chord of length 2 unit along the line $\sqrt{3}y - x = 0$ is

1) $x^2 + y^2 - 4x + 2 = 0$

2) $x^2 + y^2 - 4x + 1 = 0$

3) $x^2 + y^2 - 8x + 8 = 0$

4) $x^2 + y^2 - 4y + 2 = 0$

Key (3)

Sol Let $c = (a, 0) (a > 0)$

$$\text{Radius } (r) = \frac{a}{\sqrt{2}}$$

$$d = \text{distance from } (a, 0) \text{ to } \sqrt{3}y - x = 0 \text{ is } \frac{a}{2}$$

$$\text{Length of the chord} = 2\sqrt{r^2 - d^2} = 2 \Rightarrow a = 2$$

$$\text{Centre } (2, 0), \text{ radius} = \sqrt{2}$$

- 13). If a circle of radius r is touching the lines $x^2 - 4xy + y^2 = 0$ in the first quadrant at points A and B, then area of triangle OAB (O being the origin) is

1) $\frac{3\sqrt{3}r^2}{4}$

2) $\frac{\sqrt{3}r^2}{4}$

3) $\frac{3r^2}{4}$

4) r^2

Key (2)

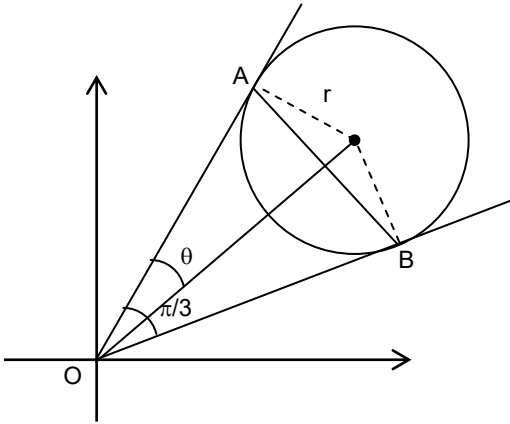
Sol

Sol: (A)

$$\text{Here } \tan 2\theta = \frac{2\sqrt{4-1}}{2} = \sqrt{3} \Rightarrow \theta = \frac{\pi}{6}$$

$$\text{Area of } \Delta OAB = \frac{1}{2}(r \cot \theta)^2 (\sin 2\theta)$$

$$= \frac{1}{2}(r\sqrt{3})^2 \frac{\sqrt{3}}{2}.$$



14). If O is the origin and OP , OQ are distinct tangents to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$, the circumcentre of the triangle OPQ is

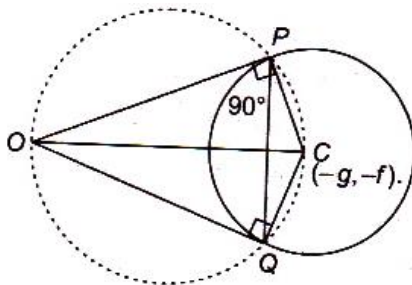
- 1) $(-g, -f)$ 2) (g, f) 3) $(-f, -g)$ 4) *none of these*

Key (4)

Sol

OC is the diameter of the circle

$$\begin{aligned} \therefore \text{Circumcentre} &\equiv \left(\frac{0-g}{2}, \frac{0-f}{2} \right) \\ &\equiv \left(-\frac{g}{2}, -\frac{f}{2} \right) \end{aligned}$$



15). If from any point on the circle $x^2 + y^2 = a^2$, tangents are drawn to the circle $x^2 + y^2 = b^2$ ($a > b$) then the angle between tangents is

- 1) $\sin^{-1}\left(\frac{b}{a}\right)$ 2) $2\sin^{-1}\left(\frac{a}{b}\right)$ 3) $2\sin^{-1}\left(\frac{b}{a}\right)$ 4) $\sin^{-1}\left(\frac{a}{b}\right)$

Key (3)

Sol

$$\sin \theta = \frac{b}{a} \Rightarrow \theta = \sin^{-1} \frac{b}{a}$$

Angle between the $2\theta = 2\sin^{-1} \frac{b}{a}$

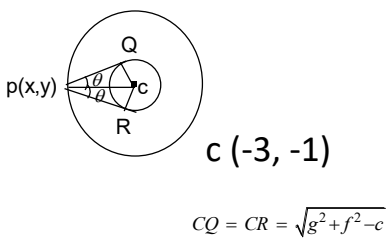
16). If from any point on the circle $x^2 + y^2 + 2gx + 2fy + c = 0$ tangents are drawn to the circle $x^2 + y^2 + 2gx + 2fy + c\sin^2 \alpha + (g^2 + f^2)\cos^2 \alpha = 0$ then the angle between the tangents is

- 1) 2α 2) α 3) $\alpha/2$ 4) $\alpha/4$

Key (1)

Sol

Take any Pt on the first circle (x_1, y_1) and write the pair of tangents

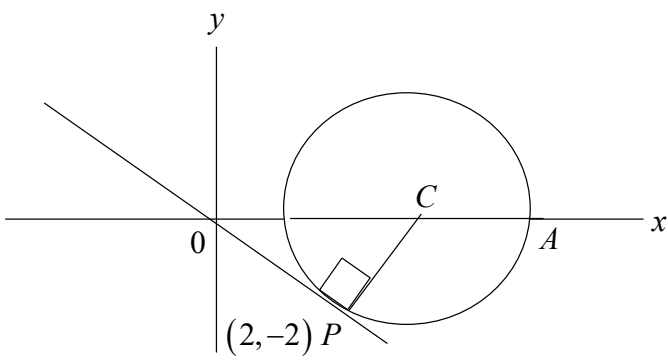


17). If (a, b) is a point on the circle whose centre is on the x-axis and which touches the line $x + y = 0$ at $(2, -2)$ then the greatest value of 'a' is

- 1) $4 + 2\sqrt{2}$ 2) $2 + 2\sqrt{2}$ 3) $4 + \sqrt{2}$ 4) $2 + \sqrt{2}$

Key (1)

Sol



$\angle COP = 45^\circ$
 $OP = 2\sqrt{2} = CP$

$$OC = 4 \Rightarrow a = OA = OC + CA$$

18). $x^2 + y^2 + 6x + 8y = 0$ and $x^2 + y^2 - 4x - 6y - 12 = 0$ are the equation of the two circles. Equation of one of their common tangent is

- 1) $7x - 5y - 1 - 5\sqrt{74} = 0$ 2) $7x - 5y - 1 + 5\sqrt{74} = 0$
 3) $7x - 5y + 1 - 5\sqrt{74} = 0$ 4) $5x - 7y + 1 - 5\sqrt{74} = 0$

Key (3)

Sol

Both the circles have radius = 5 and they intersect each other, therefore their common tangent is parallel to the line joining their centres.

Equation of the line joining their centre is $7x - 5y + 1 = 0$.

$\therefore$ Equation of the common tangent is $7x - 5y = c$

$$\therefore \left| \frac{c+1}{\sqrt{74}} \right| = 5 \Rightarrow c = \pm 5\sqrt{74} - 1$$

$\therefore$ equation is $7x - 5y + 1 \pm 5\sqrt{74} = 0$.

19). With respect to a variable point on the line $x + y = 2a$, chord of contact of the circle $x^2 + y^2 = a^2$ is drawn. If it passes through a fixed point F, then.

- 1) F is $\left(\frac{a}{2}, \frac{a}{2} \right)$
 2) Chord of circle with F is mid point is parallel to $x + y = 2a$
 3) Chord of circle with F is mid point is perpendicular to $x - y = 2a$
 4) Length of chord whose mid point F is $\frac{a}{\sqrt{2}}$.

Key (1,2,3)

Sol

Point on the line is $(t, 2a - t)$

Now equation of chord of Contact

$$\text{is } xt + y(2a - t) = a^2 \Rightarrow (x - y)t + 2ay - a^2 = 0$$

$\therefore$ This passes through $\left(\frac{a}{2}, \frac{a}{2} \right)$

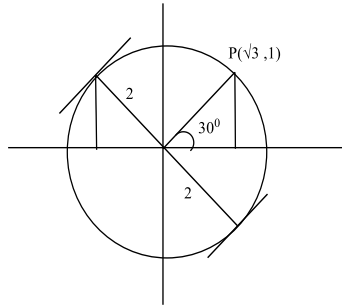
Now equation of chord whose mid point is F is

$$\frac{xa}{2} + \frac{ya}{2} = \frac{a^2}{2} \Rightarrow x + y = a.$$

20). A point $P = (\sqrt{3}, 1)$ moves on the circle $x^2 + y^2 = 4$ and after covering a quarter of the circle leaves it tangentially. The equation of a line along which the point moves after leaving the circle is _____

- 1) $y = \sqrt{3}x + 4$ 2) $\sqrt{3}y = x + 4$ 3) $y = \sqrt{3}x - 4$ 4) $\sqrt{3}y = x - 4$

Key (2,4)



Sol

$$A = (-2 \cos 60^\circ, 2 \sin 60^\circ) \text{ and } A = (2 \cos 60^\circ, -2 \sin 60^\circ)$$

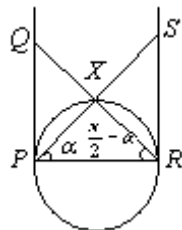
Equation of tangents at A and B

21) Let PQ and RS be tangents at the extremities of the diameter PR of a circle of radius r . If PS and RQ intersect at a point X on the circumference of the circle, then $2r$ equals

- 1) $\sqrt{PQ \cdot RS}$ 2) $\frac{PQ + RS}{2}$ 3) $\frac{2PQ \cdot RS}{PQ + RS}$ 4) $\sqrt{\frac{PQ^2 + RS^2}{2}}$

Key (1)

From figure



$$\tan \alpha = \frac{RS}{PR} = \frac{RS}{2r} \quad \dots\dots(i)$$

$$\tan\left(\frac{\pi}{2} - \alpha\right) = \frac{PQ}{PR} = \frac{PQ}{2r} \quad \dots\dots(ii)$$

Multiplying Eqs. (i) and (ii),

$$\tan \alpha \cot \alpha = \frac{RS \cdot PQ}{(2r)^2}$$

$$\therefore 2r = \sqrt{(RS)(PQ)}$$

22). If Q, S are two points on the circle $x^2 + y^2 = 4$ such that the tangents QP, SR are parallel. If PS, QR intersect at T, on the circle then $\left(\frac{QT}{PQ}\right)^2 + \left(\frac{ST}{RS}\right)^2 + PQ \cdot RS =$

1) 5

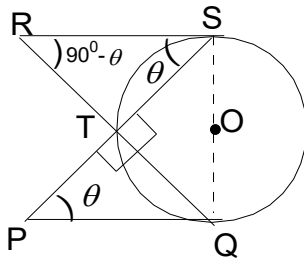
2) 10

3) 16

4) 17

Key (4)

Sol



$$\angle QTS = \angle PTQ = \frac{\pi}{2} \text{ as QS is diameter}$$

$$\frac{QT}{PQ} = \sin \theta, \frac{ST}{RS} = \cos \theta, \frac{4}{PQ} = \tan \theta, \frac{4}{RS} = \cot \theta$$

$$\therefore \left(\frac{QT}{PQ}\right)^2 + \left(\frac{ST}{RS}\right)^2 + PQ \cdot RS = \sin^2 \theta + \cos^2 \theta + (4 \cot \theta)(4 \tan \theta)$$

23). Tangents from point P are drawn one to each of the circles $x^2 + y^2 - 4x - 8y + 11 = 0$ and $x^2 + y^2 - 4x - 8y + 15 = 0$ if the tangents are perpendicular then the locus of P is

1) $x^2 + y^2 - 4x - 8y + 6 = 0$

2) $x + y + 10 = 0$

3) $y^2 = 2(x - 8)$

4) $x^2 + y^2 - 4x - 8y - 6 = 0$

Key (1)

Sol

$$\text{Let the tangent be } (x-2)\cos\theta + y-4\sin\theta = 3$$

$$(x-2)\sin\theta - y-4\cos\theta = \sqrt{5}$$

Squaring and adding we get required locus.

- 24). A ray of light incident at the point $(-2, -1)$ gets reflected from the tangents at $(0, -1)$ to the circle $x^2 + y^2 = 1$. The reflected ray touches the circle, the equation of the line along which the incident ray moved is

1) $4x - 3y + 11 = 0$

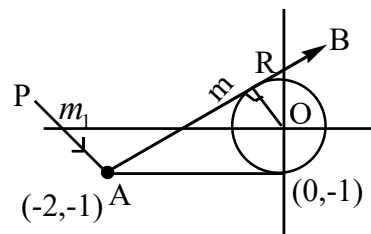
2) $4x + 3y + 11 = 0$

3) $3x + 4y + 11 = 0$

4) $4x - 3y + 5 = 0$

Key (2)

Sol



OR = radius, Let equation AB : $y + 1 = m(x + 2)$

We get $m = \frac{4}{3}$

Print $m_1 = -m = -\frac{4}{3}$

$\therefore$ Incident ray is $y + 1 = -\frac{4}{3}(x + 2) \Rightarrow 4x + 3y + 11 = 0$

- 25). Let A be the centre of the circle $x^2 + y^2 - 2x - 4y - 20 = 0$. Suppose the tangents at the points B(1, 7) and D(4, -2) on the circle meet at C. If area of the quadrilateral ABCD is Δ then Find $\frac{\Delta}{15}$.

Key (5)

Sol Equation of CD is $y + 2 = \frac{3}{2}(x - 4) \Rightarrow 4y - 3x + 20 = 0$ equation of BC is $y = 7$.

$\Rightarrow C \equiv (16, 7), BC = \sqrt{S_1} = 15$

Area of quadrilateral ABCD,

$$\Delta = 2 \left(\frac{1}{2} \times 5 \times 15 \right) = 75 \Rightarrow \frac{\Delta}{15} = 5$$

$$\Rightarrow (PA) \sin \theta = 4 = x \Rightarrow 2x = 8$$

28). $x^2 + y^2 + 6x + 8y = 0$ and $x^2 + y^2 - 4x - 6y - 12 = 0$ are the equation of the two circles. Equation of one of their common tangent is

$$1) 7x - 5y - 1 - 5\sqrt{74} = 0$$

$$2) 7x - 5y - 1 + 5\sqrt{74} = 0$$

$$3) 7x - 5y + 1 - 5\sqrt{74} = 0$$

$$4) 5x - 7y + 1 - 5\sqrt{74} = 0$$

Key (3)

Sol

Both the circles have radius = 5 and they intersect each other, therefore their common tangent is parallel to the line joining their centres.

Equation of the line joining their centre is $7x - 5y + 1 = 0$.

$\therefore$ Equation of the common tangent is $7x - 5y = c$

$$\therefore \left| \frac{c+1}{\sqrt{74}} \right| = 5 \Rightarrow c = \pm 5\sqrt{74} - 1$$

$$\therefore \text{equation is } 7x - 5y + 1 \pm 5\sqrt{74} = 0.$$

29). If the circle $x^2 + y^2 + ax + c = 0$ lies inside the circle $x^2 + y^2 + bx + c = 0$ then

$$1) ab < 0 \text{ and } c > 0$$

$$2) ab < 0 \text{ and } c < 0$$

$$3) ab > 0 \text{ and } c > 0$$

$$4) ab > 0 \text{ and } c < 0$$

Key (3)

Sol Use $c_1 c_2 < |r_1 - r_2|$

30). If the two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points then ($r > 0$)

$$1) r < 2$$

$$2) r = 2$$

$$3) r > 8$$

$$4) 2 < r < 8$$

Key (4)

Sol $|r_1 - r_2| < c_1 c_2 < r_1 + R_1$

31). If $\left(0, \frac{5}{2}\right)$ is a centre of similitude for the circles $x^2 + y^2 + 6x - 2y + 1 = 0$ and $x^2 + y^2 - 2x - 6y + 9 = 0$

then the length of the common tangent of the circles through it is

$$1) 6$$

$$2) 3$$

$$3) 2$$

$$4) 1$$

Key (3)**sol**

$(0, 5/2)$ is the internal centre of similitude, then apply the formula $\sqrt{d^2 - (r_1 + r_2)^2}$

32). Two circles with radii a and b touch each other externally such that θ is the angle between the direct common tangents ($a > b \geq 2$), then

$$\begin{array}{ll} 1) \theta = 2 \cos^{-1} \left(\frac{a-b}{a+b} \right) & 2) \theta = 2 \tan^{-1} \left(\frac{a+b}{a-b} \right) \\ \theta = 2 \sin^{-1} \left(\frac{a+b}{a-b} \right) & 4) \theta = 2 \sin^{-1} \left(\frac{a-b}{a+b} \right) \end{array} \quad 3)$$

Key (4)**Sol** Draw the diagram

33) If two circles $(x-1)^2 + (y-3)^2 = r^2$ and $x^2 + y^2 - 8x + 2y + 8 = 0$ intersect in two distinct points, then

$$1) 2 < r < 8 \quad 2) r < 2 \quad 3) r > 2 \quad 4) r = 2$$

Key (1)**Sol** $|r_1 - r_2| < C_1 C_2 < r_1 + r_2$

$$\begin{aligned} |r-3| < 5 < r+3 & \quad 5 < r+3 \Rightarrow 2 > r \\ |r-3| < 5 \\ -5 < r-3 < 5 \\ -2 < r < 8 \end{aligned}$$

34) The circles $(x-a)^2 + (y-b)^2 = c^2$ and $(x-b)^2 + (y-a)^2 = c^2$ touch each other, then

$$1) a = b \pm 2c \quad 2) a = b \pm \sqrt{2}c \quad 3) a = b \pm c \quad 4) a = 2b \pm c$$

Key (2)

Sol $\sqrt{(a-b)^2 + (a-b)^2} = |c \pm c|$

35) There are two circles whose equations are $x^2 + y^2 = 9$ and $x^2 + y^2 - 8x - 6y + n^2 = 0, n \in \mathbb{Z}$. If two circles have exactly two common tangents then the number of possible values of n is

- 1) 2 2) 8 3) 9 4) 10

Key (3)

Sol centre (4, 3) radius $\sqrt{4^2 + 3^2 - n^2} \Rightarrow -5 < n < 5$

$$r_1 + r_2 > d \Rightarrow \sqrt{25 - n^2} > 2$$

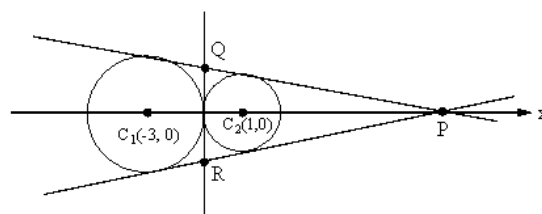
$$-\sqrt{21} < n < \sqrt{21}$$

36) If PQR is the triangle formed by the common tangents to the circles $x^2 + y^2 + 6x = 0$ and $x^2 + y^2 - 2x = 0$ then

- 1) Centroid of $\triangle PQR$ is (1, 0)
 2) In-centre of $\triangle PQR$ is (1, 0)
 3) Circum-radius of $\triangle PQR$ is 2 units
 4) In radius of $\triangle PQR$ is 1 unit

Key (1,2,3,4)

sol



$$C_1 = (-3, 0) \quad r_1 = 3$$

$$C_2 = (1, 0) \quad r_2 = 1$$

$\triangle PQR$ is equilateral Δ^{le}

$$\therefore S = I = (1, 0)$$

In radius of $\Delta PQR = r_1 = 1$

Circum radius of $\Delta PQR = 2$

37) Consider two circles $S_1: x^2 + y^2 + 8x = 0$ and $S_2: x^2 + y^2 - 2x = 0$. Let PQR be formed by the common tangents to circles S_1 and S_2 . Then which of the following **hold(s) good?**

- 1) Incentre of ΔPQR is $(1, 0)$.
- 2) The equation of radical axis of circles S_1 and S_2 is $y = 0$.
- 3) Product of slope of direct common tangents is -1 .
- 4) If transverse common tangent intersects direct common tangents at points A and B, then AB equals 4

Key (1,4)

sol

We have $S_1: x^2 + y^2 + 8x = 0$

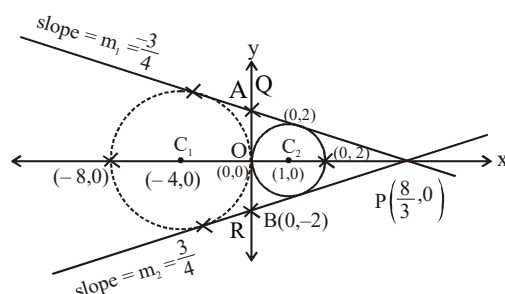
with centre $(-4, 0)$ and radius $= 4$

Also $S_2: x^2 + y^2 - 2x = 0$

with centre $(1, 0)$ and radius $= 1$

Hence circles S_1 and S_2 touch each other externally

Also $P\left(\frac{8}{3}, 0\right)$



(As P divides line joining their centres in ratio of their radii externally)

Let equation of their direct common tangent be $y = m\left(x - \frac{8}{3}\right)$.

By applying condition of tangency, we get $\frac{\left|m - \frac{8m}{3}\right|}{\sqrt{1+m^2}} = 1$

$$m^2 = \frac{9}{16} \left[\begin{array}{l} m_1 = \frac{3}{4} \\ m_2 = -\frac{3}{4} \end{array} \right]$$

38) Let $\Delta^{le}PQR$ be formed by common tangents of circles $x^2 + y^2 + 6x = 0$ and $x^2 + y^2 - 2x = 0$ then which of the following is true

1) centroid of $\Delta^{le}PQR$ is (1,0)

2) Radius of circle inscribed in $\Delta^{le}PQR$ is 1

3) Area of $\Delta^{le}PQR$ is $3\sqrt{3}$

4) The y-intercept of common tangent having negative slope is 3

Key (1,2,3)

sol

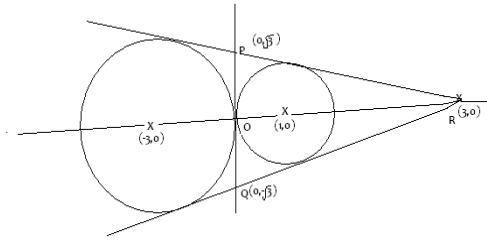
Clearly $R=(3,0)$

Equation of direct common tangents

$$y - 0 = m(x - 3)$$

$$mx - y - 3m = 0$$

$$m = \pm \frac{1}{\sqrt{3}}$$



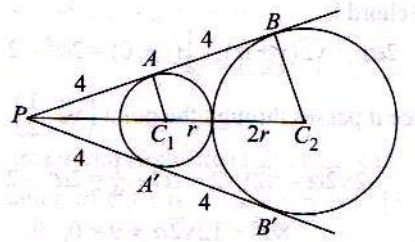
Clearly $\Delta^{le}PQR$ is an equilateral Δ^{le}

a,b,c are correct

- 39) Two circles are externally touches lines PAB and PA^1B^1 are common tangents with A and A^1 on the smaller circle and B and B^1 on the larger circle. If $PA=AB=4$, then the square of radius of circle is.

Key (2)

sol



$$\frac{AC_1}{PA} = \frac{BC_2}{PB} \Rightarrow BC_2 = 2 AC_1$$

$$PC_1 = \sqrt{16 + r^2}$$

$$\text{And } PC_2 = \sqrt{64 + 4r^2} = 2\sqrt{16 + r^2}$$

$$PC_2 - PC_1 = 3r$$

$$\Rightarrow 2\sqrt{16 + r^2} - \sqrt{16 + r^2} = 3r$$

$$\Rightarrow \sqrt{16 + r^2} = 3r$$

$$\Rightarrow 16 + r^2 = 9r^2$$

$$\Rightarrow r^2 = 2$$

- 40)** The circles $x^2 + y^2 - 6x + 6y + 17 = 0$ and $x^2 + y^2 - 6x - 2y + 1 = 0$, a common exterior tangent is drawn thus forming a curvilinear triangle. The radius of the circle inscribed in this triangle is

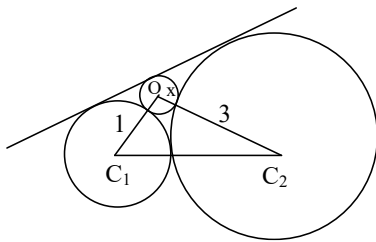
- 1) $\frac{3}{2}(2+\sqrt{3})$ 2) $\frac{3}{2}(2-\sqrt{3})$ 3) $\frac{1}{2}(2+\sqrt{3})$ 4) $\frac{1}{2}(2-\sqrt{3})$

Key (2)

sol

The given circles are touching each other externally.

$$x = \frac{3}{(1+\sqrt{3})^2} = \frac{3}{2}(2-\sqrt{3})$$



- 41.** If the common chord of the circles $x^2 + (y - \lambda)^2 = 16$ and $x^2 + y^2 = 16$ subtend a right angle at the origin, then $\lambda =$

- 1) $4\sqrt{3}$ 2) $-4\sqrt{3}$ 3) $4\sqrt{2}$ 4) $-4\sqrt{2}$

Key (3,4)

sol Common chord is $S_1 - S_2 = 0$

$$\Rightarrow \lambda(\lambda - 2y) = 0 \Rightarrow \lambda = 0, y = \frac{\lambda}{2}$$

$\lambda = 0$ is rejected because circles will be same. $\therefore y = \frac{\lambda}{2}$

Making homogeneous with $x^2 + y^2 = 16$ and $y = \frac{\lambda}{2}$, which is

$$x^2 + y^2 - 16 \left(\frac{2y}{\lambda} \right)^2 = 0 \quad \dots(i)$$

lines are perpendicular

$$\therefore 1 + 1 - \frac{64}{\lambda^2} = 0$$

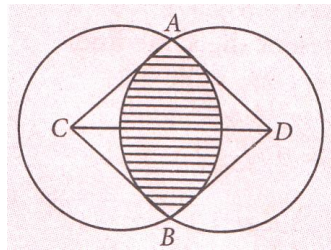
$$\Rightarrow \lambda = \pm 4\sqrt{2}.$$

- 42.** Two circles each of unit radius intersect orthogonally. The common area of the circles is $\frac{\pi}{\lambda} - 1$. Then the value of $\lambda =$

Key (2)

sol

ABCD is squares



$\therefore$ Required area = Area of sector ABC + area of sector BDA – Area of square ABCD

$$= 2\left(\frac{\pi}{4}\right) - 1 = \frac{\pi}{2} - 1$$