Solution of Triangles

SOLUTIONS

EXERCISE - O

1. **Ans. (D)**

$\sin^3 A + \sin^3 B + \sin^3 C = 3\sin A \sin B \sin C$

property if $a^3 + b^3 + c^3 = 3abc$ then $a + b + c = 0$ or $a = b = c$

$\Rightarrow \sin A + \sin B + \sin C = 0$ OR $\sin A = \sin B = \sin C \Rightarrow \sin A + \sin B + \sin C = 0$, $A = B = C = 60°$

$\Rightarrow 4\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = 0$

A or B or $C = \pi$ (Not possible)

2. **Ans. (D)**

Using sine Rule

$$\frac{c}{\sin 105°} = \frac{b}{\sin 45°}$$

$$c = \frac{b.\sin 105°}{\sin 45°} = \frac{2\cos 15°}{\sin 45°}$$

$$c = \frac{2\left(\sqrt{3}+1\right)}{2\sqrt{2}} \times \frac{\sqrt{2}}{1} = \sqrt{3}+1$$

3. **Ans. (A)**

$a = 13, b = 14, c = 15$

area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

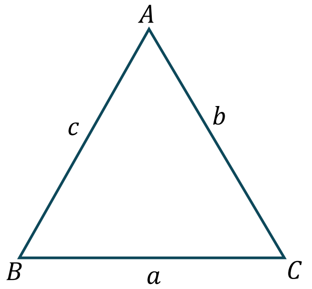
$$s = \frac{13+14+15}{2} = \frac{42}{2} = 21$$

area = $\sqrt{21(8)(7)(6)} = 84$

Now $\frac{1}{2}\ bc\sin A = 84 \Rightarrow \frac{1}{2} \times 14 \times 15\sin A = 84$

$$\sin A = \frac{84}{7\times 15} = \frac{12}{15} = \frac{4}{5}$$

$$A = \sin^{-1}\left(\frac{4}{5}\right)$$



4. **Ans. (C)**

Using cosine rule in ΔABC

$$\cos A = \frac{8^2+6^2-7^2}{2.8.6} = \frac{51}{2.8.6} \quad ...(1)$$

cosine rule in ΔADC

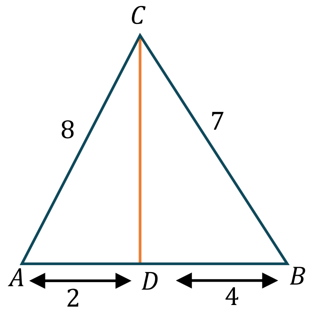
$$\cos A = \frac{8^2+2^2-CD^2}{2.8.2} \quad ...(2)$$

from (1) and (2)

$$\frac{8^2+2^2-CD^2}{2.8.2} = \frac{51}{2.8.6}$$

$68 - CD^2 = 17$

$CD^2 = 51$ or $CD = \sqrt{51}$



5. **Ans. (C)**

$2b = a + c$ and $A - C = 90°$

using sine rule $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}=\dfrac{c}{\sin C}=2R$

$2(2R\sin B) = 2R\sin A + 2R\sin C$

$\Rightarrow 2(2R\sin B) = 2R\left[2\sin\left(\dfrac{A+C}{2}\right)\cos\left(\dfrac{A-C}{2}\right)\right] \Rightarrow \sin B=\sin\left(90-\dfrac{B}{2}\right)\cos\left(\dfrac{90°}{2}\right)$

$\Rightarrow 2\sin\dfrac{B}{2}\cos\dfrac{B}{2}=\cos\dfrac{B}{2}.\dfrac{1}{\sqrt{2}}$ $\qquad\left[\cos\dfrac{B}{2}\neq 0\right]$

$\Rightarrow \sin\dfrac{B}{2}=\dfrac{1}{2\sqrt{2}}$

$\Rightarrow \sin B=2\sin\dfrac{B}{2}\cos\dfrac{B}{2}=2.\dfrac{1}{2\sqrt{2}}\times\dfrac{\sqrt{7}}{2\sqrt{2}}=\dfrac{\sqrt{7}}{4}$

6. **Ans. (C)**

$a^3 + b^3 + c^3 = c^2(a + b + c)$

$\Rightarrow a^3 + b^3 + c^3 = c^2a + c^2b + c^3$

$\Rightarrow a^3 + b^3 = c^2(a + b)$

$\Rightarrow (a + b)(a^2 - ab + b^2) = c^2(a + b)$

$\therefore a^2 - ab + b^2 = c^2$

$\therefore \dfrac{a^2+b^2-c^2}{2ab}=\dfrac{1}{2} \Rightarrow \cos C=\dfrac{1}{2}$ or $\angle C = 60°$

and it is not necessary if $\angle C = 60°$ then $a = b = c$

7. **Ans. (C)**

$\dfrac{1}{a+c}+\dfrac{1}{b+c}=\dfrac{3}{a+b+c} \Rightarrow \dfrac{a+b+c}{a+c}+\dfrac{a+b+c}{b+c}=3$

$\Rightarrow 1+\dfrac{b}{a+c}+\dfrac{a}{b+c}+1=3 \Rightarrow \dfrac{b}{a+c}+\dfrac{a}{b+c}=1$

$\Rightarrow b(b + c) + a(a + c) = (a + c)(b + c)$

$\Rightarrow b^2 + bc + a^2 + ac = ab + ac + bc + c^2$

$\Rightarrow b^2 + a^2 = ab + c^2$

or $\dfrac{a^2+b^2-c^2}{2ab}=\dfrac{1}{2}$ (By transforming)

$\cos C=\dfrac{1}{2}$ or $\angle C = 60°$

8. **Ans. (C)**

$b=(\sqrt{3}-1)a$ and $\angle C = 30°$

$\dfrac{a}{b}=\dfrac{1}{\sqrt{3}-1}$

$\dfrac{a-b}{a+b}=\dfrac{1-(\sqrt{3}-1)}{1+\sqrt{3}-1}=\dfrac{2-\sqrt{3}}{\sqrt{3}}$

Now using tangent rule

$$\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\frac{C}{2}\ =\frac{2-\sqrt{3}}{\sqrt{3}}.\cot\left(\frac{30°}{2}\right)$$

$$\tan\left(\frac{A-B}{2}\right)=\frac{2-\sqrt{3}}{\sqrt{3}}\times2+\sqrt{3}=\frac{1}{\sqrt{3}}$$

$$\frac{A-B}{2}=30° \text{ or } A-B=60°$$

9. **Ans. (C)**

$\because$ Angles A, B, C are in A.P.

$\Rightarrow 2B = A + C$ ...(1)

$\because$ In a triangle $A + B + C = 180°$ ...(2)

from (1) & (2) $\Rightarrow 3B = 180° \Rightarrow B=60°$

from sine rule

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}$$

$$\because\ \frac{b}{c}=\sqrt{\frac{3}{2}}\ \Rightarrow\ \frac{\sin B}{\sin C}=\sqrt{\frac{3}{2}}$$

$$\Rightarrow\ \frac{\sqrt{3}}{2\sin C}=\sqrt{\frac{3}{2}}$$ $(\because B = 60°)$

$$\Rightarrow \sin C=\frac{1}{\sqrt{2}}\ \Rightarrow\ \angle C=\frac{\pi}{4}$$

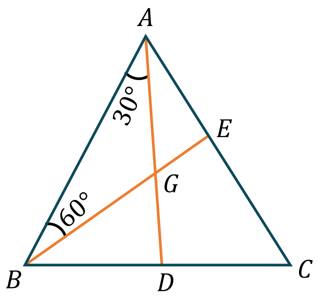
$\because A + B + C = \pi$

$$\Rightarrow\ A+\frac{\pi}{3}+\frac{\pi}{4}=\pi\ \Rightarrow\ \angle A=\frac{5\pi}{12}$$

10. **Ans. (D)**

Given $AD = 4$

$\angle DAB = \frac{\pi}{6} = 30°$



$\angle ABE = \frac{\pi}{3} = 60°$

Let medians intersect at G

Thus : $AG = \frac{2}{3}\ AD = \frac{2}{3} \times 4 = \frac{8}{3}$

$AG = \frac{8}{3}$ ...(1)

Also in ΔAGB

$\therefore \angle GAB + \angle ABG + \angle AGB = 180°$

$30° + 60° + \angle AGB = 180°$

$\angle AGB = 90°$

Thus ΔAGB = Right angled triangle

$$\frac{BG}{AG} = \cot 60°$$

$$BG = \frac{8}{3} \times \frac{1}{\sqrt{3}} = \frac{8}{3\sqrt{3}}$$

$$\text{ar}\Delta ABC = 3ar\Delta AGB = 3\left[\frac{1}{2}.BG.AG\right] = \frac{3}{2} \times \frac{8}{3\sqrt{3}} \times \frac{8}{3} = \frac{32\sqrt{3}}{9}$$

11. Ans. (B)

$a + b + c = 2s$

$a + b - c = 2s - 2c$; $b + c - a = 2s - 2a$;

$c + a - b = 2s - 2b$

Now

$$(a + b + c)(a + b - c)(b + c - a)(c + a - b) = \frac{8a^2b^2c^2}{a^2 + b^2 + c^2}$$

$$\Rightarrow 2s(2s - 2c)(2s - 2a)(2s - 2b) = \frac{8a^2b^2c^2}{a^2 + b^2 + c^2} \Rightarrow 16s(s - a)(s - b)(s - c) = \frac{8a^2b^2c^2}{a^2 + b^2 + c^2}$$

$$\Rightarrow 16\Delta^2 = \frac{8a^2b^2c^2}{a^2 + b^2 + c^2} \quad \left\{R = \frac{abc}{4\Delta} \Rightarrow a^2 + b^2 + c^2 = 8\left(\frac{abc}{4\Delta}\right)^2\right.$$

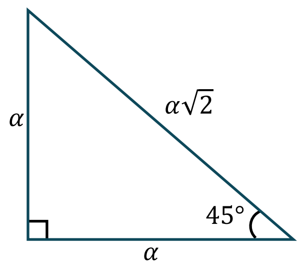
$\Rightarrow a^2 + b^2 + c^2 = 8R^2$

If $a^2 + b^2 + c^2 = 8R^2$ triangle is right angled Δ

12. Ans. (C)

$$R = \frac{\alpha}{\sqrt{2}}, \; r = \frac{\Delta}{s} = \frac{\frac{1}{2}\alpha^2}{\frac{\alpha\sqrt{2}\left(\sqrt{2}+1\right)}{2}} = \frac{\alpha}{\sqrt{2}\left(\sqrt{2}+1\right)}$$

$$\frac{r}{R} = \frac{1}{\sqrt{2}+1} = \sqrt{2} - 1 = \tan\frac{\pi}{8}$$



13. Ans. (C)

$\angle AKB = 105°$

Applying sine rule in triangle AKB

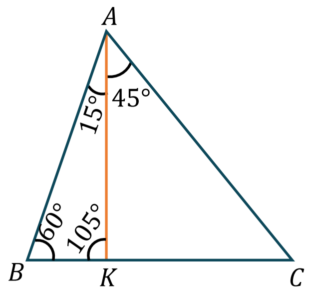
$$\frac{AB}{\sin 105°} = \frac{AK}{\sin 60°}$$

$$\Rightarrow \frac{AK}{AB} = \frac{\sin 60°}{\sin 105°}$$

$$\left(\because \sin 105° = \frac{\sqrt{3}+1}{2\sqrt{2}} \;\&\; \sin 60° = \frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \frac{AK}{AB} = \frac{\sqrt{3}\left(2\sqrt{2}\right)}{2\left(\sqrt{3}+1\right)} = \frac{\sqrt{6}}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$$

$$\Rightarrow \frac{AK}{AB} = \frac{\sqrt{6}}{2}\left(\sqrt{3}-1\right) = \frac{\sqrt{2}\left(3-\sqrt{3}\right)}{2}$$



14. Ans. (B)

$\because AB^2 + BC^2 = AC^2$

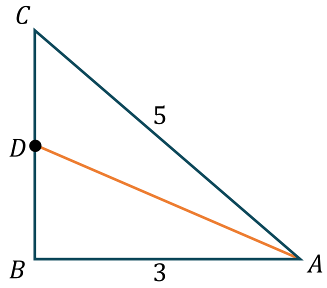
$\Rightarrow 9 + BC^2 = 25 \Rightarrow BC = 4$

Given that $\frac{BD}{DC} = \frac{AB}{AC} = \frac{3}{5}$

Let $BD = 3k$ & $DC = 5k$

$\because BD + DC = BC = 4 \Rightarrow 3k + 5k = 4 \Rightarrow k = \frac{1}{2}$

$\therefore BD = \frac{3}{2}$



In ΔABD : $AD^2 = AB^2 + BD^2 \Rightarrow AD = \sqrt{9 + \frac{9}{4}} = \frac{\sqrt{45}}{2}$

Hence $AD = \frac{3\sqrt{5}}{2}$

15. Ans. (D)

$a : b : c = 1 : \sqrt{3} : 2$

Let $a = k$, $b = \sqrt{3}k$, $c = 2k$

By cosine rule :

$\cos A = \frac{b^2 + c^2 - a^2}{2bc} = \frac{3k^2 + 4k^2 - k^2}{2(\sqrt{3}k)(2k)} \Rightarrow \cos A = \frac{6k^2}{4\sqrt{3}k^2} = \frac{\sqrt{3}}{2} \Rightarrow \angle A = 30°$

Similarly $\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{k^2 + 4k^2 - 3k^2}{2(k)(2k)}$

$\Rightarrow \cos B = \frac{1}{2} \Rightarrow \angle B = 60°$

$\because \angle A + \angle B + \angle C = 180°$

$\Rightarrow \angle C = 90°$

$\Rightarrow A : B : C = 30° : 60° : 90° = 1 : 2 : 3$

16. Ans. (A)

$\frac{\sin A}{\sin C} = \frac{\sin(A - B)}{\sin(B - C)}$ $\qquad$ $(\because \sin A = \sin(\pi - (B + C)) = \sin(B + C))$

$\Rightarrow \frac{\sin(B + C)}{\sin(A + B)} = \frac{\sin(A - B)}{\sin(B - C)}$

$\Rightarrow \sin(B + C).\sin(B - C) = \sin(A - B)\sin(A + B)$

$(\because \sin(A + B)\sin(A - B) = \sin^2 A - \sin^2 B)$

$\Rightarrow \sin^2 B - \sin^2 C = \sin^2 A - \sin^2 B \Rightarrow 2\sin^2 B = \sin^2 A + \sin^2 C$

$\because \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \Rightarrow 2\left(\frac{b}{k}\right)^2 = \left(\frac{a}{k}\right)^2 + \left(\frac{c}{k}\right)^2$

$\Rightarrow 2b^2 = a^2 + c^2$

Hence, a^2, b^2, c^2 are in A.P.

17. Ans. (A)

Given tanA : tanB : tanC = 1: 2: 3

Let $\tan A = k, \tan B = 2k$ & $\tan C = 3k$

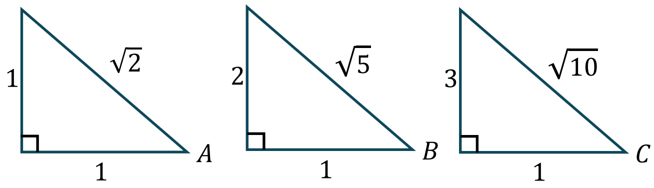
We know that in a triangle

$\tan A + \tan B + \tan C = \tan A.\tan B.\tan C$

$\Rightarrow k + 2k + 3k = (k)(2k)(3k)$

$\Rightarrow k = 1$

$\Rightarrow \tan A = 1, \tan B = 2$ & $\tan C = 3$



$\sin A = \frac{1}{\sqrt{2}}$, $\sin B = \frac{2}{\sqrt{5}}$, $\sin C = \frac{3}{\sqrt{10}}$

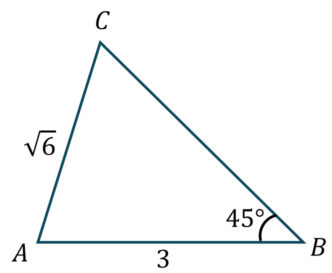
$\because a^2 : b^2 : c^2 = \sin^2 A : \sin^2 B : \sin^2 C$

(from sine rule)

$\Rightarrow a^2 : b^2 : c^2 = \frac{1}{2} : \frac{4}{5} : \frac{9}{10} = 5:8:9$

18. Ans. (C)

from sine rule



$$\frac{AC}{\sin B} = \frac{AB}{\sin C} \Rightarrow \frac{\sqrt{6}}{\sin 45°} = \frac{3}{\sin C}$$

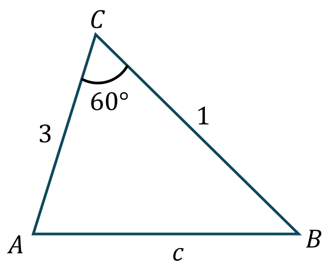
$$\Rightarrow \sqrt{12} = \frac{3}{\sin C} \Rightarrow \sin C = \frac{3}{2\sqrt{3}} = \frac{\sqrt{3}}{2}$$

$\Rightarrow \angle C = 60°$

$\because A + B + C = 180°$

$\Rightarrow A = 75°$

19. Ans. (A)



from cosine rule : $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$$\Rightarrow \cos 60° = \frac{(1)^2 + (3)^2 - c^2}{2(1)(3)}$$

$\Rightarrow 3 = 1 + 9 - c^2 \Rightarrow c^2 = 7 \Rightarrow c = \sqrt{7}$

from sine rule : $\frac{b}{\sin B} = \frac{c}{\sin C}$

$$\Rightarrow \frac{3}{\sin B} = \frac{\sqrt{7}}{\sin 60°} \Rightarrow \sin B = \frac{3\sqrt{3}}{2\sqrt{7}} \Rightarrow \sin^2 B = \frac{27}{28}$$

20. Ans. (C)

Given : $3B - C = 30°$ ...(1)

$A + 2B = 120°$

$\because A + B + C = 180°$

$A + B + B = 120°$

$180° - C + B = 120°$

$\Rightarrow C - B = 60°$ ...(2)

$(1) + (2) \Rightarrow 2B = 90°$

$\Rightarrow B = 45° \Rightarrow A = 30°$ & $C = 105°$

from sine rule : $\dfrac{a}{\sin 30°}=\dfrac{b}{\sin 45°}=\dfrac{c}{\sin 105°}=k$

$\Rightarrow a=\dfrac{k}{2},\ b=\dfrac{k}{\sqrt{2}}$ & $c=\dfrac{k\left(\sqrt{3}+1\right)}{2\sqrt{2}}$

Given : $s=3+\sqrt{3}+\sqrt{2}=\dfrac{a+b+c}{2}$

$\Rightarrow a+b+c=\dfrac{k}{2}+\dfrac{k}{\sqrt{2}}+\dfrac{k\left(\sqrt{3}+1\right)}{2\sqrt{2}}\ =6+2\sqrt{3}+2\sqrt{2}$

$k\sqrt{2}+2k+k\left(\sqrt{3}+1\right)=2\sqrt{2}\left(6+2\sqrt{3}+2\sqrt{2}\right)$

$k\sqrt{2}+3k+\sqrt{3}k=12\sqrt{2}+4\sqrt{6}+8$

$k\left(\sqrt{2}+3+\sqrt{3}\right)=4\sqrt{2}\left(3+\sqrt{3}+\sqrt{2}\right)\ \Rightarrow\ k=4\sqrt{2}$

$\because$ largest angle is C

$\Rightarrow$ largest side = $c=\dfrac{k\left(\sqrt{3}+1\right)}{2\sqrt{2}}$

$c=\dfrac{4\sqrt{2}\left(\sqrt{3}+1\right)}{2\sqrt{2}}\Rightarrow\ c=2\left(\sqrt{3}+1\right)$

21. **Ans. (A,B,D)**

$\sin C=\dfrac{4}{5}\Rightarrow\ \cos C=\dfrac{3}{5}$

Also $\tan A=\dfrac{24}{7}\ \Rightarrow \sin A=\dfrac{24}{25}$ & $\cos A=\dfrac{7}{25}$

Using sine law in ΔABC

$\dfrac{a}{\sin A}=\dfrac{c}{\sin C}\ \Rightarrow\ \dfrac{a}{\left(\frac{24}{25}\right)}=\dfrac{50}{\left(\frac{4}{5}\right)}$ (Given $c = 50$)

$\Rightarrow a = 60$

Now $\sin B = \sin(\pi - (A + C)) = \sin(A + C)$

$= \sin A.\cos C + \cos A.\sin C = \left(\dfrac{24}{25}\right)\left(\dfrac{3}{5}\right)+\left(\dfrac{7}{25}\right)\left(\dfrac{4}{5}\right)=\dfrac{100}{125}=\dfrac{4}{5}$

Now area of ΔABC = $\dfrac{1}{2}ac\sin B$

$= \dfrac{1}{2}(60)(50)\left(\dfrac{4}{5}\right) = 1200$

Using $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}\Rightarrow\dfrac{60}{\left(\frac{24}{25}\right)}=\dfrac{b}{\left(\frac{4}{5}\right)}\ \Rightarrow b = 50$

So ΔABC is an isosceles.

In isosceles Δ, circumcentre, centroid, orthocentre and incentre are collinear.

22. **Ans. (A,D)**

Given $\angle A = 120°$

$a + b = 20$ ...(1)

and $a + c = 21$ ...(2)

subtract (1) from (2)

$a + c - a - b = 1$

$c - b = 1$

$c = b + 1$

$\Rightarrow c > b \Rightarrow AB > AC \Rightarrow AB \neq AC$

$\Rightarrow \angle C \neq \angle B \Rightarrow$ so it is not isosceles Δ

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos 120° = \frac{b^2 + (b+1)^2 - (20-b)^2}{2b(b+1)}$$

$$-\frac{1}{2} = \frac{b^2 + (b+1)^2 - (20-b)^2}{2b(b+1)}$$

$-b^2 - b = 2b^2 + 1 + 2b - 400 - b^2 + 40b$

$2b^2 + 43b - 399 = 0$

$$b = \frac{-43 \pm \sqrt{5041}}{4} = \frac{-43 \pm 71}{4}$$

$$\therefore\ b = \frac{-43+71}{4} = 7$$

Area of Δ = $\frac{1}{2} bc\sin A$

= $\frac{1}{2} \times 7 \times 8 \times \sin 120°$ = $28 \times \frac{\sqrt{3}}{2} = 14\sqrt{3}$

23. **Ans. (B,C)**

(A) $(a + c - b)(a - c + b) = 4bc$

$(a + (c - b))(a - (c - b)) = 4bc$

$a^2 - (c - b)^2 = 4bc$

$a^2 - (c^2 + b^2 - 2bc) = 4bc$

$a^2 = b^2 + c^2 + 2bc$

$\Rightarrow b^2 + c^2 - a^2 = -2bc$

$$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = -1$$

$\Rightarrow \cos A = -1$

which is not possible.

(B) $b^2 \sin 2C + c^2 \sin 2B = ab$

$\Rightarrow (4R^2 \sin^2 B) 2\sin C \cos C + (4R^2 \sin^2 C) 2\sin B \cos B = ab$

$\Rightarrow 8R^2 \sin B \sin C[\sin B \cos C + \sin C \cos B]$ = $(2R\sin A)(2R\sin B)$

$\Rightarrow 2\sin C \sin(B + C) = \sin A$

$\Rightarrow 2\sin C = 1$

$\Rightarrow \ \sin C = \frac{1}{2}$

$\Rightarrow \ \angle C = \frac{\pi}{6}$ or $\frac{5\pi}{6}$ so Δ is possible.

(C) For $a = 3, b = 5, c = 7$ and $C = \frac{2\pi}{3}$

$$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{5^2+7^2-3^2}{2(5)(7)} = \frac{65}{70} < 1$$

similarly

$$\cos B = \frac{a^2+c^2-b^2}{2ac} = \frac{3^2+7^2-5^2}{2(3)(7)} = \frac{33}{42} < 1$$

So in this case triangle is possible

(D) As, $\cos\left(\frac{A-C}{2}\right) = \cos\left(\frac{A+C}{2}\right)$

$\Rightarrow \ \cos\left(\frac{A-C}{2}\right) - \cos\left(\frac{A+C}{2}\right) = 0$

$\Rightarrow \ 2\sin\frac{A}{2}.\sin\frac{C}{2} = 0$

which is not possible in ΔABC

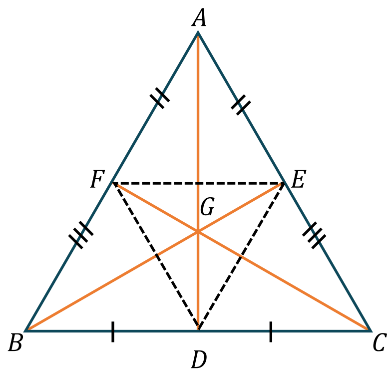
24. **Ans. (A,C,D)**

Given $AD = 6, BE = 8, CF = 10$

Area of ΔABC whose median lengths are given

$= \frac{4}{3}\sqrt{s(s-a)(s-b)(s-c)}$

$= \frac{4}{3}\sqrt{(12)(12-6)(12-8)(12-10)} = 32$



& Area of ΔDEF = $\frac{1}{4}$ (Area of ΔABC) = $\frac{1}{4}(32) = 8$

Now Area of ΔABD = $\frac{1}{2}$ (Area of ΔABC) = $\frac{1}{2}(32) = 16$

Now Area of ΔABD = $\frac{1}{2}$ × Base × height

Area $= \frac{1}{2} \times AD \times BG$ = $\frac{1}{2} \times 6 \times \frac{16}{3} = 16$

It means AD is perpendicular to BE

25. **Ans. (B,D)**

Given $A : B : C = 1 : 2 : 3$

Let $A = x, B = 2x, C = 3x$

$\Rightarrow A + B + C = \pi \Rightarrow 6x = \pi \Rightarrow \ x = \frac{\pi}{6} \ \Rightarrow A = \frac{\pi}{6}\ , \ B = \frac{\pi}{3}, \ C = \frac{\pi}{2}$

(A) $\dfrac{c}{\sin C}=2R \Rightarrow \quad R=\dfrac{1}{2}.\dfrac{c}{\sin\frac{\pi}{2}}=\dfrac{c}{2}$

(B) $a : b : c = \sin A : \sin B : \sin C$

$= \sin\dfrac{\pi}{6} : \sin\dfrac{\pi}{3} : \sin\dfrac{\pi}{2}$

$= \dfrac{1}{2} : \dfrac{\sqrt{3}}{2} : 1$

$= 1 : \sqrt{3} : 2$

(C) Let $a = k,\ b=\sqrt{3}k\ ,c = 2k$

$\Rightarrow$ perimeter = $(a + b + c) = k + \sqrt{3}k + 2k$

$= \left(3+\sqrt{3}\right)k$

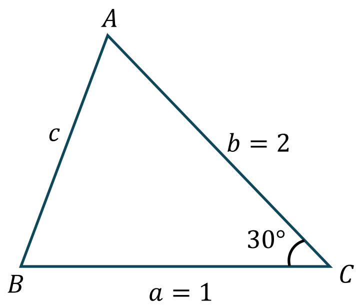
(D) Area of ΔABC = $\dfrac{1}{2}ab$

$=\dfrac{1}{2}(k).\left(\sqrt{3}k\right)=\dfrac{\sqrt{3}}{2}k^2$

$= \dfrac{\sqrt{3}}{2}\left(\dfrac{c}{2}\right)^2 \qquad \left(\because k=\dfrac{c}{2}\right)$

$= \dfrac{\sqrt{3}}{8}c^2$

26. Ans. (A,C,D)



(A) Use sine law $\dfrac{a}{\sin A}=\dfrac{b}{\sin B}$

$\Rightarrow \dfrac{1}{\sin A}=\dfrac{2}{\sin B}$

$\Rightarrow$ 2sinA = sinB

(B) Use cosine law in ΔABC

$\cos 30°=\dfrac{1^2+2^2-c^2}{2(1)(2)} \Rightarrow \dfrac{\sqrt{3}}{2}=\dfrac{5-c^2}{4}$

$\Rightarrow \quad c=\sqrt{5-2\sqrt{3}}$

(C) As $c > a \Rightarrow \angle C > \angle A$

$\therefore$ Angle B must be obtuse

$\Rightarrow$ Measure of angle A is less than 30°

(D) $\Delta=\dfrac{1}{2}ab\sin C=\dfrac{1}{2}(1)(2)\sin 30°=\dfrac{1}{2}$

$\because \ R=\dfrac{abc}{4\Delta}=\dfrac{(1)(2)(c)}{4\left(\frac{1}{2}\right)}=c$

$\Rightarrow$ Circumradius of ΔABC is equal of side AB.

27. **Ans. (C)**

28. **Ans. (D)**

(Sol. 27 and 28)

$Let \tan\frac{A}{2}=x;\tan\frac{B}{2}=y;\tan\frac{C}{2}=z$

$\therefore x=\frac{r}{AR},y=\frac{r}{BP},z=\frac{r}{CQ}$

Put the values of AR, BP and CQ in given relation, we get

$\frac{2x}{r}+\frac{5y}{r}+\frac{5z}{r}=\frac{6}{r}$

$\rightarrow 2x+5y+5z=6$ ...(1)

Also in the triangle, $xy+yz+zx=1$ ...(2)

It we interchange y and z then both equations (1) and (2) remain unchanged

$\Rightarrow ABC$ is isoceles with $\angle B=\angle C \Rightarrow y=z$

So form (1) and (2), we get

$x=3-5y$ ...(3)

and $2xy+y^2=1$ ...(4)

Solving we get, $x=\frac{4}{3},y=z=\frac{1}{3}$

$\therefore AR=\frac{3r}{4},BP=3r,CQ=3r$

Now perimeter $2s=2(AR)+2(BP)+2(CQ)=\frac{27r}{2}$

Given perimeter is smallest integer $\rightarrow r=2$

$\therefore s=\frac{27}{2}$

$\therefore \Delta=rs=2\times\frac{27}{2}=27sq.$ unit.

29. **Ans. (A→S;B→P;C→Q;D→R)**

(A) $r=\frac{\Delta}{s},r_1=\frac{\Delta}{s-a},r_2=\frac{\Delta}{s-b},r_3=\frac{\Delta}{s-c}$

$rr_1r_2r_3=\frac{\Delta^4}{s(s-a)(s-b)(s-c)}=\frac{\Delta^4}{\Delta^2}$

$=\Delta^2$

(A) → (S)

(B) $r_1r_2r_3=\frac{\Delta^3}{(s-a)(s-b)(s-c)}\times\frac{s}{s}$

$r_1r_2r_3=\frac{\Delta^3}{\Delta^2}\times s$

$r_1r_2r_3=\Delta s,r=\frac{\Delta}{s}\Rightarrow\Delta=rs$

$r_1r_2r_3=rs^2$

(B) → (P)

(C) $r_1 + r_2 - r_3 + r$

$$r_1 = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}$$

$$r_2 = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2}$$

$$r_3 = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2}$$

$$r = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}$$

$$r_1 + r_2 = 4R\cos\frac{C}{2}\left\{\sin\left(\frac{A+B}{2}\right)\right\}$$

$$r - r_3 = 4R\sin\frac{C}{2}\left\{-\cos\left(\frac{A+B}{2}\right)\right\}$$

$$r_1 + r_2 + r - r_3 = 4R\left\{\sin\left(\frac{A+B-C}{2}\right)\right\}$$

$$= 4R\left\{\sin\left(\frac{\pi-2C}{2}\right)\right\}$$

$$r_1 + r_2 - r_3 + r = 4R\cos C$$

(C) $\rightarrow$ (Q)

(D) $$\frac{1}{bc}+\frac{1}{ca}+\frac{1}{ab}=\frac{a+b+c}{abc}=\frac{2s}{abc}$$

$$=\frac{2\left(\frac{s}{\Delta}\right)}{\left(\frac{abc}{\Delta}\right)}=\frac{2/r}{4R}$$

$$\therefore\ \frac{1}{bc}+\frac{1}{ca}+\frac{1}{ab}=\frac{1}{2Rr}$$

(D) $\rightarrow$ (R)

30. Ans. (A$\rightarrow$R; B$\rightarrow$P; C$\rightarrow$T; D$\rightarrow$Q)

(A) $$r = \frac{\Delta}{s} = \frac{\frac{1}{2}\times 6\times 8}{\left(\frac{6+8+10}{2}\right)} = 2$$

(B) $$r_1 = \frac{\Delta}{s-a} = \frac{24}{12-6} = 4$$

(C) $$R = \frac{10}{2} = 5$$

(D) $$\Delta = \frac{1}{2}\times 6\times 8 = 24$$

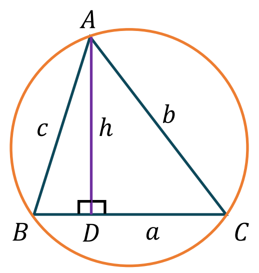
EXERCISE - S

1. **Ans. (3)**

AD = altitude = h

Now using the concept of area

$\text{ar}\Delta ABC = \frac{1}{2} \times \text{base} \times h = \frac{1}{2}\ ah$ ...(1)



Also

$\text{ar}\Delta ABC = \frac{1}{2} bc\sin A = \frac{1}{2} bc.\frac{a}{2R}$ $\left\{\frac{a}{\sin A} = 2R\right\}$

$\text{ar}\Delta ABC = \frac{abc}{4R}$ ...(2)

$\frac{1}{2} ah = \frac{abc}{4R}$

$h = \frac{bc}{2R}$ or $R = \frac{bc}{2h} = \frac{1\times 1}{2.\sqrt{\frac{2}{3}}} = \frac{\sqrt{3}}{2\sqrt{2}}$

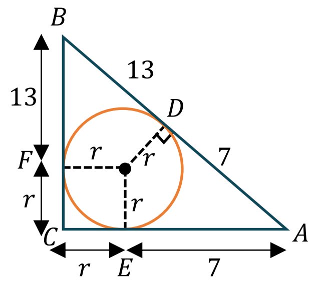
2. **Ans. (91)**

V.Imp. :

$BF = BD = 13$

$AE = AD = 7$

let $CE = CF = r$



Area of ΔABC

$= \frac{1}{2} AC \times BC = \frac{1}{2}(7+r)(13+r)$

or $\Delta ABC = \frac{1}{2}[r^2 + 20r + 91]$ ...(1)

Also $(r+7)^2 + (13+r)^2 = 20^2$

$2r^2 + 40r + 218 = 400$

$2r^2 + 40r = 182$

$r^2 + 20r = 91$ ...(2)

Put (2) in (1)

$\text{ar}\Delta ABC = \frac{1}{2}[91+91] = 91$

3. **Ans. (4)**

$\cos\alpha = \frac{a}{b+c}$, $\cos\beta = \frac{b}{c+a}$, $\cos\gamma = \frac{c}{a+b}$

To find : $\tan^2\frac{\alpha}{2} + \tan^2\frac{\gamma}{2}$

Also a, b, c are in A.P.

$\therefore 2b = a + c$

$\tan^2\frac{\alpha}{2} + \tan^2\frac{\gamma}{2}$

$$\Rightarrow \frac{1-\cos\alpha}{1+\cos\alpha}+\frac{1-\cos\gamma}{1+\cos\gamma} \Rightarrow \frac{1-\frac{a}{b+c}}{1+\frac{a}{b+c}}+\frac{1-\frac{c}{a+b}}{1+\frac{c}{a+b}}$$

$$\Rightarrow \frac{b+c-a}{a+b+c}+\frac{a+b-c}{a+b+c} \Rightarrow \frac{2b}{a+b+c}=\frac{2b}{b+2b}=\frac{2}{3}\ (a+c=2b)$$

4. **Ans. (6)**

Given $a = 2b$ and $A = 3B$

From sine rule : $\frac{a}{\sin A}=\frac{b}{\sin B}$

$$\Rightarrow \frac{2b}{\sin 3B}=\frac{b}{\sin B} \Rightarrow \frac{\sin 3B}{\sin B}=2$$

$$\Rightarrow \frac{3\sin B-4\sin^3 B}{\sin B}=2 \quad (\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta)$$

$$\Rightarrow 3-4\sin^2 B = 2 \Rightarrow \sin B=\frac{1}{2} \Rightarrow \angle B = 30^\circ$$

$\because A = 3B = 90^\circ$

$\because A + B + C = 180^\circ \Rightarrow \angle C = 60^\circ$

$$\frac{c}{b}=\frac{\sin C}{\sin B}=\frac{\sin 60^\circ}{\sin 30^\circ}=\sqrt{3}$$

5. **Ans. (3)**

Given : A, B, C are in A.P.

$\Rightarrow 2B = A + C$ ...(1)

$\because A + B + C = 180^\circ$ ...(2)

from (1) & (2) $\Rightarrow 3B = 180^\circ \Rightarrow B = 60^\circ$

$$E=\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A = \frac{a}{c}2\sin C\cos C+\frac{c}{a}(2\sin A\cos A)$$

from sine rule $\frac{c}{\sin C}=\frac{a}{\sin A} \Rightarrow E=2\left(a\frac{\sin C}{c}\cos C+c\frac{\sin A}{a}\cos A\right)$

$$=2\left(a\frac{\sin A}{a}\cos C+c\frac{\sin C}{c}\cos A\right) = 2(\sin A\cos C + \cos A\sin C)$$

$$E = 2\sin(A + C) = 2\sin B = 2\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow \boxed{E=\sqrt{3}}$$

6. **Ans. (7)**

$3a + 3b = 4c$

$$\Rightarrow a+b=\frac{4c}{3} \Rightarrow s=\frac{a+b+c}{2}=\frac{\frac{4c}{3}+c}{2} \Rightarrow \frac{7c}{6}$$

$$\cot\frac{A}{2}=\sqrt{\frac{s(s-a)}{(s-b)(s-c)}}$$

$$\cot\frac{B}{2}=\sqrt{\frac{s(s-b)}{(s-a)(s-c)}}$$

$$\cot\frac{A}{2}\cot\frac{B}{2}=\frac{s}{s-c}=\frac{7c/6}{\frac{7c}{6}-c}$$

$$\Rightarrow\frac{7c/6}{c/6}=7$$

7. **Ans. (1)**

$r_1=\Delta\,, r_1=r+r_2+r_3$

$$\Rightarrow\frac{\Delta}{s-a}=\frac{\Delta}{s}+\frac{\Delta}{s-b}+\frac{\Delta}{s-c}\Rightarrow\frac{1}{s-a}-\frac{1}{s}=\frac{1}{s-b}+\frac{1}{s-c}$$

$$\Rightarrow\frac{s-s+a}{s(s-a)}=\frac{s-b+s-c}{(s-b)(s-c)}\Rightarrow\frac{a}{s(s-a)}=\frac{2s-(b+c)}{(s-b)(s-c)}$$

$$\Rightarrow s=\frac{a+b+c}{2}\Rightarrow 2s-(b+c)=a$$

$$\Rightarrow\frac{a}{s(s-a)}=\frac{a}{(s-b)(s-c)}\Rightarrow(s-b)(s-c)=s(s-a)$$

$$\Rightarrow r_1=\Delta\Rightarrow\frac{\Delta}{s-a}=\Delta\Rightarrow s-a=1\Rightarrow s=(s-b)(s-c)$$

$$\Delta=\sqrt{s(s-a)(s-b)(s-c)}=\sqrt{s\times s}$$

$$\Rightarrow\Delta=s\Rightarrow\frac{\Delta}{s}=1$$

$\boxed{r=\frac{\Delta}{s}=1}$:-

8. **Ans. (3)**

$$\frac{12}{13}=\frac{1-\tan^2\left(\frac{A-B}{2}\right)}{1+\tan^2\left(\frac{A-B}{2}\right)}$$

$$\Rightarrow\tan^2\left(\frac{A-B}{2}\right)=\frac{1}{25}$$

$$\tan\left(\frac{A-B}{2}\right)=\frac{1}{5}$$

$$\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$\frac{1}{5}=\frac{3-2}{3+2}\cot\left(\frac{C}{2}\right)\Rightarrow\cot\frac{C}{2}=1$$

$C=90°$

Area = $\frac{1}{2}\times ab\sin C=\frac{1}{2}\times2\times3=3$

9. **Ans. (9)**

$\because\ a = 6, b = 3$ and $\cos(A - B) = 4/5$

$\because\ \tan\left(\frac{A-B}{2}\right) = \frac{a-b}{a+b}\cot\frac{C}{2}$ .... (i)

$\because\ \tan^2\left(\frac{A-B}{2}\right) = \frac{1-\cos(A-B)}{1+\cos(A-B)} = \frac{1}{9}$

$\therefore\ \tan\left(\frac{A-B}{2}\right) = \frac{1}{3}$

$\therefore$ from (i), we get $\frac{1}{3} = \frac{1}{3}\cot\frac{C}{2} \Rightarrow C = 90°$

$\therefore$ Area $= \frac{1}{2}ab = 9$ sq. unit

10. **Ans. (8)**

$\because\ \Delta = 100\ cm^2$

$r_1 = \frac{\Delta}{s-a} = 10 \Rightarrow s - a = 10$ ...(i)

$r_2 = \frac{\Delta}{s-b} = 50 \Rightarrow s - b = 2$ ...(ii)

(i) – (ii)

$\therefore\ b - a = 8$

EXERCISE - JEE (Main) PYQ

1. **Ans. (4)**

Case-1

$5 + 5r > 5r^2$

$r^2 - r - 1 < 0$

$\Rightarrow \left(r - \frac{1+\sqrt{5}}{2}\right)\left(r - \frac{1-\sqrt{5}}{2}\right) < 0 \Rightarrow r \in \left(\frac{1-\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$

Case-2

$5r^2 + 5r > 5$

$\Rightarrow r^2 + r - 1 > 0 \Rightarrow r < \frac{-1-\sqrt{5}}{2}, r > \frac{-1+\sqrt{5}}{2}$

Case-3

$5 + 5r^2 > 5r$

$\Rightarrow r^2 - r + 1 > 0$

$r \in R$

from case-1, case-2 and case-3

$r \in \left(\frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}\right)$

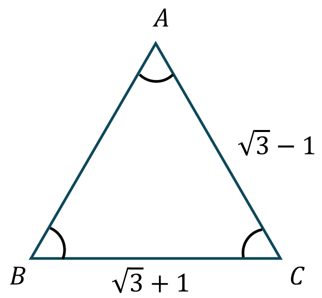
Option (4) is correct.

2. Ans. (1)

$A + B = 120°$

$$\tan\frac{A-B}{2} = \frac{a-b}{a+b}\cot\left(\frac{C}{2}\right)$$

$$= \frac{\sqrt{3}+1-\sqrt{3}+1}{2(\sqrt{3})}\cot(30°) = \frac{1}{\sqrt{3}}.\sqrt{3} = 1$$



$$\frac{A-B}{2} = 45° \quad \Rightarrow \quad \begin{aligned} A-B &= 90° \\ A+B &= 120° \end{aligned}$$

$$\overline{}$$

$$2A = 210°$$

$$A = 105°$$

$$B = 15°$$

$\therefore$ Option (1)

3. Ans. (2)

Given $a + b = x$ and $ab = y$

If $x^2 - c^2 = y \Rightarrow (a+b)^2 - c^2 = ab$

$$\Rightarrow a^2 + b^2 - c^2 = -ab \Rightarrow \frac{a^2+b^2-c^2}{2ab} = -\frac{1}{2}$$

$$\Rightarrow \cos C = -\frac{1}{2} \Rightarrow \angle C = \frac{2\pi}{3}$$

$$R = \frac{c}{2\sin C} = \frac{c}{\sqrt{3}}$$

4. Ans. (3)

$b + c = 11\lambda, c + a = 12\lambda, a + b = 13\lambda$

$\Rightarrow a = 7\lambda, b = 6\lambda, c = 5\lambda$

(using cosine formula)

$$\cos A = \frac{1}{5}, \cos B = \frac{19}{35}, \cos C = \frac{5}{7}$$

$\alpha : \beta : \gamma \Rightarrow 7 : 19 : 25$

5. Ans. (3)

$a < b < c$ are in A.P.

$\angle C = 2\angle A$ (Given)

$\Rightarrow \sin C = \sin 2A \Rightarrow \sin C = 2\sin A.\cos A$

$$\Rightarrow \frac{\sin C}{\sin A} = 2\cos A \Rightarrow \frac{c}{a} = 2\frac{b^2+c^2-a^2}{2bc}$$

put a = b – λ, c = b + λ, λ > 0

$$\Rightarrow \lambda = \frac{b}{5} \Rightarrow a = b - \frac{b}{5} = \frac{4}{5}b, c = b + \frac{b}{5} = \frac{6b}{5}$$

$\Rightarrow$ required ratio = 4 : 5 : 6

6. Ans. (3)

Let $\angle A = \theta - \lambda, \angle B = \theta, \angle C = \theta + \lambda$

So, $3\theta = 180 \Rightarrow \theta = 60°$

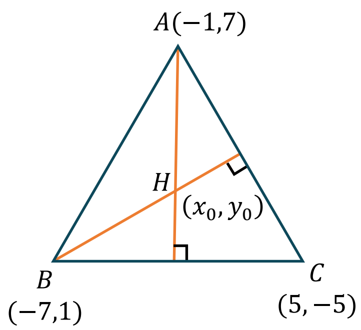
$\sin A = \frac{1}{2}$, by Sine Rule

$\Rightarrow A = 30°, a = 2, b = 2\sqrt{3}, c = 4, \angle C = 90°$

$\Delta = \frac{1}{2} \times 2\sqrt{3} \times 2 = 2\sqrt{3}$ sq. cm

7. Ans. (3)

Let orthocentre is $H(x_0, y_0)$



$m_{AH} . m_{BC} = -1$

$\Rightarrow \left(\frac{y_0 - 7}{x_0 + 1}\right)\left(\frac{1+5}{-7-5}\right) = -1$

$\Rightarrow 2x_0 - y_0 + 9 = 0$ ...(1)

and $m_{BH} . m_{AC} = -1$

$\Rightarrow \left(\frac{y_0 - 1}{x_0 + 7}\right)\left(\frac{7-(-5)}{-1-5}\right) = -1$

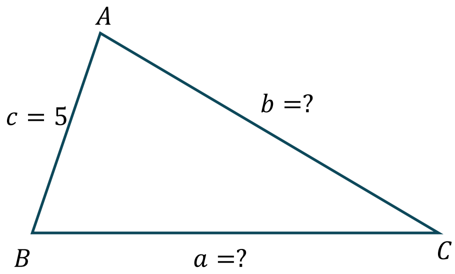
$\Rightarrow x_0 - 2y_0 + 9 = 0$ ...(2)

Solving equation (1) and (2) we get

$(x_0, y_0) = (-3, 3)$

8. Ans. (3)

As, $\cos B = \frac{3}{5} \Rightarrow B = 53°$



As, $R = 5 \Rightarrow \frac{c}{\sin c} = 2R$

$\Rightarrow \frac{5}{10} = \sin c \Rightarrow C = 30°$

Now, $\frac{b}{\sin B} = 2R \Rightarrow b = 2(5)\left(\frac{4}{5}\right) = 8$

Now, by cosine formula

$\cos B = \frac{a^2 + c^2 - b^2}{2ac} \Rightarrow \frac{3}{5} = \frac{a^2 + 25 - 64}{2(5)a} \Rightarrow a^2 - 6a - 39 = 0$

$\therefore a = \frac{6 \pm \sqrt{192}}{2} = \frac{6 \pm 8\sqrt{3}}{2}$

$\Rightarrow a = 3 + 4\sqrt{3}$;(Reject $a = 3 - 4\sqrt{3}$)

Now, $\Delta = \frac{abc}{4R} = \frac{(3+4\sqrt{3})(8)(5)}{4(5)} = 2(3+4\sqrt{3}) \Rightarrow \Delta = (6+8\sqrt{3})$

9. Ans. (15)

$$\Delta = \frac{1}{2}\cdot 5\cdot 12\sin A = 30$$

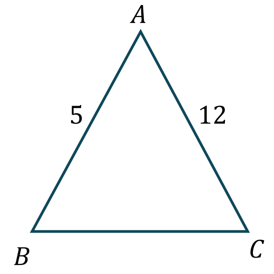
$\sin A = 1$

$A = 90° \Rightarrow BC = 13$

$BC = 2R = 13$

$$r = \frac{\Delta}{s} = \frac{30}{15} = 2$$

$2R + r = 15$



10. Ans. (3)

$a = 20 - 2x^2, b = 10 + x^2, c = 10 + x^2$

$$s = \frac{a+b+c}{2} = 20$$

$$\Delta = \sqrt{s(s-a)(s-b)(s-c)} = \sqrt{20(2x^2)(10-x^2)(10-x^2)}$$

$$= 2\sqrt{10}\sqrt{x^2(10-x^2)^2} = 2\sqrt{10}\left|10x - x^3\right|$$

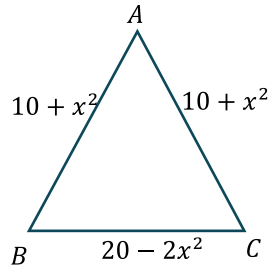
Let $y = 10x - x^3$

$$\frac{dy}{dx} = 10 - 3x^2$$

$$\frac{dy}{dx} = 0 \Rightarrow x^2 = \frac{10}{3}$$

$3x^2 = 10$

$\Rightarrow 3k^2 = 10.$



11. Ans. (3)

$$\cos A + \cos C = 2(1 - \cos B)$$

$$2\cos\frac{A+C}{2}\cos\frac{A-C}{2} = 4\sin^2 B/2$$

$$\text{as } \cos\left(\frac{A+C}{2}\right) = \sin\frac{B}{2}$$

$$\text{so } \cos\frac{A-C}{2} = 2\sin\frac{B}{2}$$

$$2\cos B/2 \cos\frac{A-C}{2} = 4\sin B/2 \cos B/2$$

$$2\sin\left(\frac{A+C}{2}\right)\cos\left(\frac{A-C}{2}\right) = 4\sin B/2\cos B/2$$

$\sin A + \sin C = 2\sin B$

$a + c = 2b \Rightarrow a = 3, c = 7, b = 5$

$$\cos A - \cos C = \frac{b^2+c^2-a^2}{2bc} - \frac{a^2+b^2-c^2}{2ab}$$

$$= \frac{25+49-9}{70} - \frac{9+25-49}{30} = \frac{65}{70} + \frac{1}{2} = \frac{20}{14} = \frac{10}{7}$$

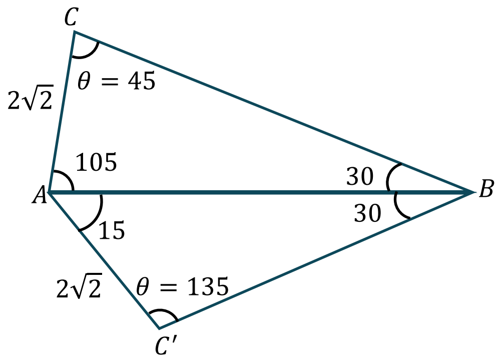
EXERCISE - JEE (Advanced) PYQ

1. **Ans. (4)**

Applying sine-law in ΔABC

$$\frac{4}{\sin\theta} = \frac{2\sqrt{2}}{\sin 30°}$$

$\sin\theta = \dfrac{1}{\sqrt{2}} \Rightarrow \theta = 45°, 135°$



Area of ΔABC – Area $\Delta AC'B$

$= \dfrac{1}{2} \times 2\sqrt{2} \times 4\,(\sin 105° - \sin 15°)$

$= 4\sqrt{2} \times 2\cos 60° \times \sin 45° = 4\sqrt{2} \times 2 \times \dfrac{1}{2} \times \dfrac{1}{\sqrt{2}} = 4.$

2. **Ans. (D)**

$A, B, C \to AP,$

$A + B + C = 180°$

$B = 60°, A + C = 120°$

$\dfrac{a}{c}.2\sin C\cos C + \dfrac{c}{a}.2\sin A\cos A$

$2a\left(\dfrac{\sin A}{a}\right)\cos C + 2c\left(\dfrac{\sin C}{c}\right)\cos A$

$2\sin(A+C) = 2\sin 120° = \sqrt{3}$

3. **Ans. (3)**

$15\sqrt{3} = \dfrac{1}{2}ab\sin C \to \sin C = \dfrac{\sqrt{3}}{2}$

$\to C = \dfrac{2\pi}{3}$ (C is obtuse)

$\cos C = -\dfrac{1}{2} = \dfrac{36+100-c^2}{2(6)(10)} \to c^2 = 196 \to c = 14$

$s = 15$

$r = \dfrac{\Delta}{s} = \dfrac{15\sqrt{3}}{15} = \sqrt{3}$

$r^2 = 3$

4. **Ans. (B)**

$\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

$\dfrac{\sqrt{3}}{2} = \dfrac{(x^2+x+1)^2+(x^2-1)^2-(2x+1)^2}{2(x^2+x+1)(x^2-1)}$

$\Rightarrow x^4(2-\sqrt{3})+x^3(2-\sqrt{3})-3x^2-x(2-\sqrt{3})+(\sqrt{3}+1)=0$

$\Rightarrow (x^2-1)[(2-\sqrt{3})x^2+(2-\sqrt{3})x-(\sqrt{3}+1)]=0$

Now $\begin{cases} x^2+x+1+x^2-1>2x+1 \\ x^2+x+1+2x+1>x^2-1 \\ 2x+1+x^2-1>x^2+x+1 \end{cases}$ ($\because$ sum of two sides is greater than third side)

$\Rightarrow x > 1 \Rightarrow x = 1+\sqrt{3}$

Alternate :

$\tan\frac{\pi}{12}=\sqrt{\frac{(s-b)(s-a)}{s(s-c)}}$

$s=x^2+\frac{3x+1}{2}$, $s-b=\frac{3(x+1)}{2}, s-a=\frac{x-1}{2}, s-c=x^2-\frac{(x+1)}{2}$

$\tan\frac{\pi}{12}=\sqrt{\frac{\frac{3}{2}(x+1)\frac{(x-1)}{2}}{\left(x^2+\frac{3x+1}{2}\right)\left(x^2-\frac{x+1}{2}\right)}}$

Simiplying

$2-\sqrt{3}=\frac{\sqrt{3}}{2x+1}$

$x=\sqrt{3}+1$

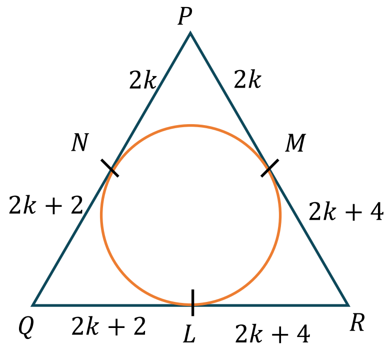
5. **Ans. (C)**

$\frac{2\sin P-2\sin P\cos P}{2\sin P+2\sin P\cos P}=\frac{(1-\cos P)}{(1+\cos P)}=\tan^2\frac{A}{2}=\frac{\Delta^2}{s^2(s-a)^2}=\frac{((s-b)(s-c))^2}{\Delta^2}$

$s = 4$

$=\left(\frac{\left(4-\frac{7}{2}\right)\left(4-\frac{5}{2}\right)}{\Delta}\right)^2=\left(\frac{3}{4\Delta}\right)^2$

6. **Ans. (B,D)**



$PQ = 4k + 2$

$QR = 4k + 6$

$PR = 4k + 4$

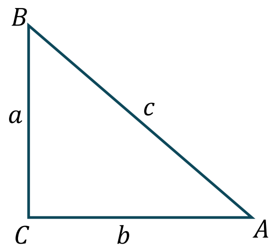
$\cos P=\frac{1}{3}=\frac{(4k+2)^2+(4k+4)^2-(4k+6)^2}{2(4k+2)(4k+4)}$

$\Rightarrow k^2 - 3k - 4 = 0$

$\Rightarrow k = 4, k = -1$ (reject)

side lengths 18,22,20

7. **Ans. (B)**



$a + b = x$

$ab = y$

$(a + b)^2 - c^2 = ab$

$\Rightarrow a^2 + b^2 - c^2 = -ab$

$\cos C = -\frac{1}{2}\Rightarrow C=\frac{2\pi}{3}$

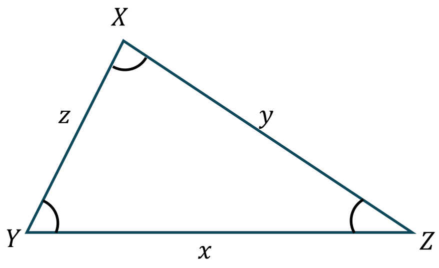
Now

$$\frac{r}{R}=\frac{\frac{\Delta}{S}}{\frac{abc}{4\Delta}}=\frac{4\Delta^2}{\frac{(a+b+c)}{2}abc}=\frac{8\left(\frac{1}{2}ab\frac{\sqrt{3}}{2}\right)^2}{(x+c)(yc)}$$

$$=\frac{\frac{1}{2}y^2.3}{(x+c)cy}=\frac{3y}{2c(x+c)}$$

8. **Ans. (A,C,D)**

Let $\frac{s-x}{4}=\frac{s-y}{3}=\frac{s-z}{2}=k$



$s-x=4k \Rightarrow y+z-x=8k$

$s-y=3k \Rightarrow x+z-y=6k$

$s-z=2k \Rightarrow x+y-z=4k$

$\Rightarrow x=5k, y=6k, z=7k \Rightarrow 3s-(x+y+z)=9k$

$s=9k$

Let r be inradius

$\Rightarrow \pi r^2=\frac{8\pi}{3}$

$\pi\left(\frac{\Delta}{s}\right)^2=\frac{8\pi}{3}$

$\Delta=\sqrt{\frac{8}{3}}.s$

$\sqrt{s(s-x)(s-y)(s-z)}=\sqrt{\frac{8}{3}}.s$

$\sqrt{9k.4k.3k.2k}=\sqrt{\frac{8}{3}}9k$

$\sqrt{24.9k^2}=\sqrt{\frac{8}{3}}.9k$

$k=1$

$\Rightarrow x=5, y=6, z=7$

$\Delta=\sqrt{\frac{8}{3}}.9k=\sqrt{\frac{8}{3}}.9=6\sqrt{6}$

R = circumradius = $\frac{xyz}{4\Delta}=\frac{5.6.7}{4.6\sqrt{6}}=\frac{35}{4\sqrt{6}}$

$$\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}=\frac{r}{4R}=\frac{\frac{\Delta}{s}}{\frac{xyz}{\Delta}}=\frac{\Delta^2}{s.xyz}=\frac{36.6}{9.5.6.7}=\frac{4}{35}$$

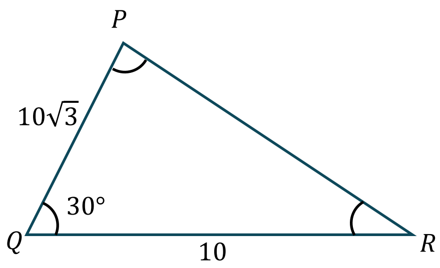
$$\cos Z=\frac{x^2+y^2-z^2}{2xy}=\frac{25+36-49}{2.5.6}=\frac{1}{5}$$

$$\sin^2\left(\frac{X+Y}{2}\right)=\cos^2\frac{Z}{2}=\frac{1+\cos Z}{2}=\frac{1+\frac{1}{5}}{2}=\frac{3}{5}$$

9. Ans. (B,C,D)

$$\cos 30° = \frac{(10\sqrt{3})^2 + (10)^2 - (PR)^2}{2\times 10\sqrt{3}\times 10}$$

$\Rightarrow PR = 10$



$\because QR = PR$

$\Rightarrow \angle PQR = \angle QPR$

$\angle QPR = 30°$

(B) area of ΔPQR

$$= \frac{1}{2}\times 10\sqrt{3}\times 10\times \sin 30° = \frac{1}{2}\times 10\times 10\sqrt{3}\times \frac{1}{2} = 25\sqrt{3}$$

$\angle QRP = 180° - (30° + 30°) = 120°$

(C) $$r = \frac{\Delta}{S} = \frac{25\sqrt{3}}{\left(\frac{10+10+10\sqrt{3}}{2}\right)} = \frac{25\sqrt{3}}{10+5\sqrt{3}}$$

$$= 5\sqrt{3}.(2-\sqrt{3}) = 10\sqrt{3} - 15$$

(D) $$R = \frac{a}{2\sin A} = \frac{10}{2\sin 30°} = 10$$

$\therefore$ Area $= \pi R^2 = 100\pi$

10. Ans. (B,C,D)

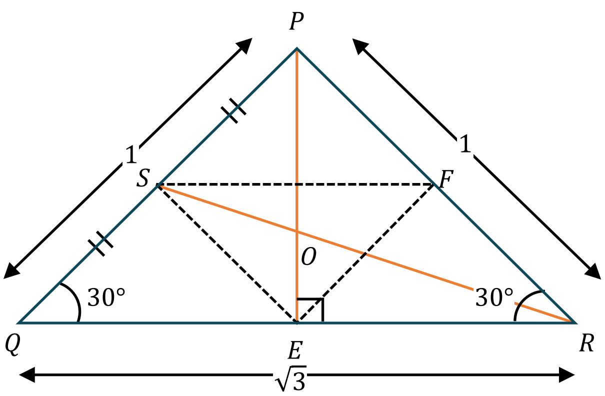
$$\frac{\sin P}{\sqrt{3}} = \frac{\sin Q}{1} = \frac{1}{2R} = \frac{1}{2}$$

$$\Rightarrow P = \frac{\pi}{3} \text{ or } \frac{2\pi}{3} \text{ and } Q = \frac{\pi}{6} \text{ or } \frac{5\pi}{6}$$

Since $p > q \Rightarrow P > Q$

So, if $P = \frac{\pi}{3}$ and $Q = \frac{\pi}{6}$

$\Rightarrow R = \frac{\pi}{2}$ (not possible)



Hence, $P = \frac{2\pi}{3}$ and $Q = R = \frac{\pi}{6}$

$$r = \frac{\Delta}{s} = \frac{\frac{1}{2}(1)(\sqrt{3})\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}+2}{2}\right)} = \frac{\sqrt{3}}{2}(2-\sqrt{3})$$

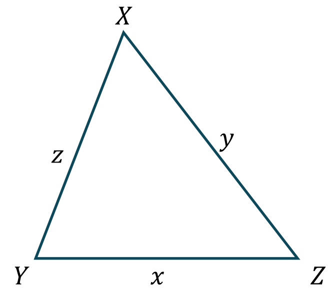
Now, area of $\Delta SEF = \frac{1}{4}$ area of ΔPQR

$\Rightarrow$ area of ΔSOE = $\frac{1}{3}$ area of $\Delta SEF = \frac{1}{12}$ area of $\Delta PQR = \frac{1}{12}.\frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{48}$

$$RS = \frac{1}{2}\sqrt{2(3)+2(1)-1} = \frac{\sqrt{7}}{2}$$

$$OE = \frac{1}{3}PE = \frac{1}{3}.\frac{1}{2}\sqrt{2(1)^2+2(1)^2-3} = \frac{1}{6}$$

11. Ans. (B,C)



$$\tan\frac{X}{2}+\tan\frac{Z}{2}=\frac{2y}{x+y+z}$$

$$\frac{\Delta}{s(s-x)}+\frac{\Delta}{s(s-z)}=\frac{2y}{2s}$$

$$\frac{\Delta}{s}\left(\frac{2s-(x+z)}{(s-x)(s-z)}\right)=\frac{y}{s}$$

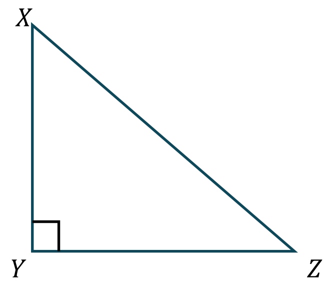
$$\Rightarrow \frac{\Delta y}{s(s-x)(s-z)}=\frac{y}{s} \Rightarrow \Delta^2 = (s-x)^2(s-z)^2$$

$\Rightarrow s(s-y) = (s-x)(s-z) \Rightarrow (x+y+z)(x+z-y)$

$= (y+z-x)(x+y-z) \Rightarrow (x+z)^2 - y^2 = y^2 - (z-x)^2$

$$\Rightarrow (x+z)^2 + (x-z)^2 = 2y^2 \Rightarrow x^2+z^2 = y^2 \Rightarrow \angle Y = \frac{\pi}{2}$$

$\Rightarrow \angle Y = \angle X + \angle Z$



$$\tan\frac{X}{2}=\frac{\Delta}{S(S-x)}$$

$$\tan\frac{X}{2}=\frac{\frac{1}{2}xz}{\frac{(y+z)^2-x^2}{4}}$$

$$\tan\frac{X}{2}=\frac{2xz}{y^2+z^2+2yz-x^2}$$

$$\tan\frac{X}{2}=\frac{2xz}{2z^2+2yz} \quad (\text{using } y^2 = x^2+z^2)$$

$$\tan\frac{X}{2}=\frac{x}{y+z}$$

12. Ans. (2)

With standard notations given

$c=\sqrt{23}, a=3, b=4$

Now,

$$\frac{\cot A+\cot C}{\cot B} = \frac{\frac{\cos A}{\sin A}+\frac{\cos C}{\sin C}}{\frac{\cos B}{\sin B}} = \frac{\frac{b^2+c^2-a^2}{2bc\ \ \sin A}+\frac{a^2+b^2-c^2}{2ab\ \ \sin C}}{\frac{c^2+a^2-b^2}{2ac\ \ \sin B}}$$

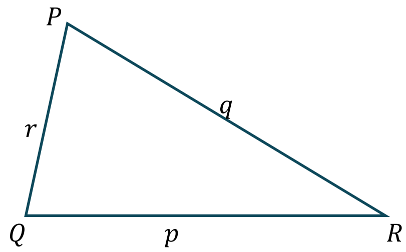
$$= \frac{\frac{b^2+c^2-a^2}{4\Delta}+\frac{a^2+b^2-c^2}{4\Delta}}{\frac{c^2+a^2-b^2}{4\Delta}} = \frac{2b^2}{a^2+c^2-b^2}$$

$$= \frac{32}{9+23-16} = 2$$

13. **Ans. (A,B)**

(1) $\cos P=\frac{q^2+r^2-p^2}{2qr}$,$q^2+r^2\geq 2qr$ $(by AM \geq GM)$

$\Rightarrow \cos P \geq 1-\frac{p^2}{2qr}$



(2) By triangle inequality $q+p>r$

by projection formula

$r\cos P+p\cos R+q\cos R+r\cos Q>p\cos Q+q\cos P$

$\Rightarrow \cos R>\frac{(q-r)}{(p+q)}\cos P+\frac{(p-r)}{(p+q)}\cos Q$

(3) By $AM \geq GM$.

$q+r\geq 2\sqrt{qr}$

$\frac{q+r}{p}\geq\frac{2\sqrt{qr}}{p}$

$\frac{\sin P}{p}=\frac{\sin Q}{q}=\frac{\sin R}{r}$ (by Sine rule)

$\frac{q+r}{p}\geq\frac{2\sqrt{\sin Q\sin R}}{\sin P}$

(4) If $\cos Q>\frac{p}{r}\Rightarrow r\cos Q>p$ ...(i)

$\cos R>\frac{p}{q}\Rightarrow q\cos R>p$ ...(ii)

Add (i) and (ii)

$r\cos Q+q\cos R>2P\Rightarrow p>2p$

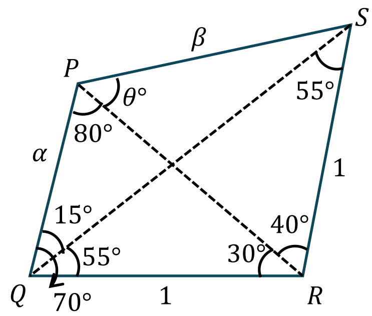
$\Rightarrow p<0$, false

14. **Ans. (A,B)**

Figure as shown, can be drawn on the basis of given data.

Applying sine rule in ΔPQR

$\frac{\alpha}{\sin 30°}=\frac{1}{\sin 80°}\quad\Rightarrow\alpha=\frac{1}{2\sin 80°}$



applying Sine rule in ΔPRS

$\frac{\beta}{\sin 40°}=\frac{1}{\sin\theta°}\ \Rightarrow \beta\sin\theta°=\sin 40°$

From (i) & (ii).

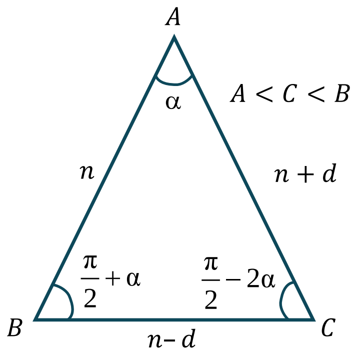
$4\alpha\beta\sin\theta°=4\cdot\frac{1}{2\sin 80°}\sin 40°=\frac{1}{\cos 40°}$

$30°<40°<45°\Rightarrow\cos 45°<\cos 40°<\cos 30°$

$\Rightarrow\frac{1}{\sqrt{2}}<\cos 40°<\frac{\sqrt{3}}{2}\ \Rightarrow\frac{2}{\sqrt{3}}<\frac{1}{\cos 40°}<\sqrt{2}$

$\Rightarrow\frac{2}{\sqrt{3}}<4\alpha\beta\sin\theta°<\sqrt{2}$

15. Ans. (1007.99 to 1008.01)



$n - d = 2\sin\alpha$ ...(1)

$n + d = 2\sin\left(\frac{\pi}{2}+\alpha\right)$

$\Rightarrow n + d = 2\cos\alpha$ ....(2)

$n = 2\sin\left(\frac{\pi}{2}-2\alpha\right)$

$\Rightarrow n = 2\cos 2\alpha$ ...(3)

$\Rightarrow 2\cos 2\alpha = \sin\alpha + \cos\alpha \Rightarrow 2(\cos\alpha - \sin\alpha) = 1$

$\Rightarrow \sin 2\alpha = \frac{3}{4}$

Then, $a = \frac{1}{2}.n.(n+d).\sin\alpha = \frac{1}{2}.2\cos 2\alpha.2\cos\alpha.\sin\alpha$

$= \sin 2\alpha.\cos 2\alpha = \frac{3}{4}\times\frac{\sqrt{7}}{4} = \frac{3\sqrt{7}}{16}$

$(64a)^2 = \left(64\times\frac{3\sqrt{7}}{16}\right)^2 = 16\times 9\times 7 = 1008$

16. Ans. (0.24 to 0.26)

From above equation in Ques. 15

$r = \frac{\Delta}{s} = \frac{1}{2}\frac{n(n+d)\sin\alpha}{\left(\frac{3n}{2}\right)} = \frac{(n+d).\sin\alpha}{3}$

$= \frac{2\cos\alpha.\sin\alpha}{3}$ (from (2))

$r = \frac{\sin 2\alpha}{3} = \frac{1}{4}$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (3)

$\because A : B : C = 3 : 5 : 4 \quad \therefore A = 45°, B = 75°, C = 60°$

$\because$ from Sine - rule

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = k \Rightarrow \frac{a}{\frac{1}{\sqrt{2}}} = \frac{b}{\frac{\sqrt{3}+1}{2\sqrt{2}}} = \frac{c}{\frac{\sqrt{3}}{2}} = k$ ($\because$ $\sin 75° = \sin(45° + 30°)$)

$\therefore a = \frac{k}{\sqrt{2}}, b = \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)k$ and $c = \frac{k\sqrt{3}}{2}$

$\therefore a + b + c\sqrt{2} = \frac{k}{\sqrt{2}} + \left(\frac{\sqrt{3}+1}{2\sqrt{2}}\right)k + \left(\frac{k\sqrt{3}}{2}\right)\sqrt{2} = \frac{k}{2\sqrt{2}}\left[2+(\sqrt{3}+1)+2\sqrt{3}\right] = \frac{3k(\sqrt{3}+1)}{2\sqrt{2}} = 3b$

2. Ans. (3)

$\because \frac{bc\sin^2 A}{\cos A+\cos B\cos C} = \frac{k^2\sin B\sin C\sin^2 A}{-\cos(B+C)+\cos B\cos C} = \frac{k^2\sin B\sin C\sin^2 A}{\sin B\sin C} = k^2\sin^2 A = a^2.$

3. **Ans. (1)**

$\because \ \sin C = \dfrac{1-\cos A\cos B}{\sin A\sin B} \le 1 \Rightarrow \cos(A-B) \ge 1 \Rightarrow \cos(A-B) = 1$

$\Rightarrow A - B = 0 \Rightarrow A = B \quad \therefore \sin C = \dfrac{1-\cos^2 A}{\sin^2 A} = 1 \Rightarrow C = 90°$

4. **Ans. (1)**

$bc = 2b^2\cos A + 2c^2\cos A - 4bc\cos^2 A$

$\Rightarrow bc = 2\cos A\,(b^2 + c^2 - 2bc\cos A) \Rightarrow bc = (2\cos A)a^2$

$\Rightarrow \dfrac{bc}{2a^2} = \cos A = \dfrac{b^2+c^2-a^2}{2bc} \Rightarrow b^2c^2 = a^2(b^2+c^2-a^2) \Rightarrow (a^2-b^2)(a^2-c^2) = 0$

Thus, triangle is isosceles

5. **Ans. (3)**

$\because \ \cot\dfrac{B}{2}\cot\dfrac{C}{2} = \sqrt{\dfrac{s(s-b)}{(s-a)(s-c)}}\sqrt{\dfrac{s(s-c)}{(s-a)(s-b)}} = \dfrac{s}{s-a} = \dfrac{2s}{2s-2a}$

$= \dfrac{a+b+c}{b+c-a} = \dfrac{4a}{2a} = 2 \qquad (\because \ b + c = 3a)$

6. **Ans. (2)**

$\because \ \dfrac{b^2-c^2}{2aR} = \dfrac{4R^2\left(\sin^2 B-\sin^2 C\right)}{2.2R\sin A.R} = \dfrac{\sin(B+C).\sin(B-C)}{\sin A} = \sin(B-C)$

7. **Ans. (2)**

$a = 3k\ ;\ b = 7k\ ;\ c = 8k$

$\because \ s = 9k.$

$\because \ \Delta = \sqrt{9k\cdot 6k\cdot 2k\cdot k} = k^2\,6\sqrt{3} \qquad \therefore \ R = \dfrac{abc}{4\Delta} = \dfrac{(3\ k)\ (7\ k)\ (8\ k)}{4\times k^2\times\ 6\sqrt{3}} = \dfrac{7\ k}{\sqrt{3}}$

$\because \ r = \dfrac{\Delta}{s} = \dfrac{k^2\ \ 6\sqrt{3}}{9\ \ k} = \dfrac{2k}{\sqrt{3}} \qquad \therefore \ R : r = 7 : 2$

8. **Ans. (2)**

$\because \ \ell = \dfrac{1}{2}\sqrt{2b^2+2c^2-a^2}$

$\therefore \ 4\ell^2 = 2b^2 + 2c^2 - a^2$

$= a^2 + 2(b^2 + c^2 - a^2) = a^2 + 2(2bc\cos A)$

$4\ell^2 = a^2 + 4bc\cos A$

9. **Ans. (2)**

$\because$ required distance $= \dfrac{2\Delta}{a}$

$\because \ \Delta = \sqrt{s(s-a)(s-b)(s-c)}$

$a = 13;\ b = 12;\ c = 5 \quad \Rightarrow \ s = 15$

$\therefore \ \Delta = \sqrt{15\times2\times3\times10} = 5\times3\times2 = 30$

$\therefore$ required distance $= \dfrac{2\times30}{13} = \dfrac{60}{13}$

10. Ans. (4)

$\because a^2 = b^2 + c^2$

$\because \tan C = \dfrac{c}{b}$

$\because \tan C = \dfrac{2\tan\frac{C}{2}}{1-\tan^2\frac{C}{2}} = \dfrac{c}{b}$

$\dfrac{2t}{1-t^2} = \dfrac{c}{b}$ where $t = \tan\dfrac{C}{2}$

$t^2(c) + (2b)t - c = 0$

$\therefore t = \dfrac{-2b \pm \sqrt{4b^2+4\times c^2}}{2c}$

$t = \dfrac{-b \pm a}{c}$

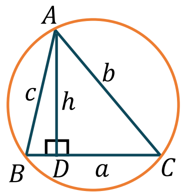
$\Rightarrow t = \dfrac{a-b}{c} = \tan\dfrac{C}{2}$

11. Ans. (4)

AD = altitude = h

Now using the concept of area

$ar\Delta ABC = \dfrac{1}{2} \times \text{base} \times h = \dfrac{1}{2}ah$ ...(1)



Also

$ar\Delta ABC = \dfrac{1}{2}bc\sin A = \dfrac{1}{2}bc.\dfrac{a}{2R}$ $\left\{\dfrac{a}{\sin A} = 2R\right\}$

$ar\Delta ABC = \dfrac{abc}{4R}$ ...(2)

$\dfrac{1}{2}ah = \dfrac{abc}{4R}$

$h = \dfrac{bc}{2R}$ or $R = \dfrac{bc}{2h} = \dfrac{1\times 1}{2.\sqrt{\frac{2}{3}}} = \dfrac{\sqrt{3}}{2\sqrt{2}}$

12. Ans. (1)

V.Imp. :

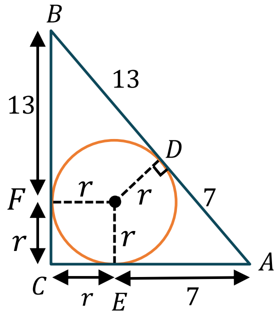
$BF = BD = 13$

$AE = AD = 7$

let $CE = CF = r$

Area of ΔABC

$= \dfrac{1}{2}AC \times BC = \dfrac{1}{2}(7+r)(13+r)$



ar $\Delta ABC = \dfrac{1}{2}[r^2 + 20r + 91]$ ...(1)

Also $(r + 7)^2 + (13 + r)^2 = 20^2$

$2r^2 + 40r + 218 = 400$

$2r^2 + 40r = 182$

$r^2 + 20r = 91$ ...(2)

Put (2) in (1)

$\text{ar}\Delta ABC = \frac{1}{2}[91 + 91] = 91$

13. Ans. (4)

$\cos\alpha = \frac{a}{b+c}$, $\cos\beta = \frac{b}{c+a}$, $\cos\gamma = \frac{c}{a+b}$

To find : $\tan^2\frac{\alpha}{2} + \tan^2\frac{\gamma}{2}$

Also a, b, c are in A.P.

$\therefore\ 2b = a + c$

$\tan^2\frac{\alpha}{2} + \tan^2\frac{\gamma}{2}$

$\Rightarrow \frac{1-\cos\alpha}{1+\cos\alpha} + \frac{1-\cos\gamma}{1+\cos\gamma} \Rightarrow \frac{1-\frac{a}{b+c}}{1+\frac{a}{b+c}} + \frac{1-\frac{c}{a+b}}{1+\frac{c}{a+b}}$

$\Rightarrow \frac{b+c-a}{a+b+c} + \frac{a+b-c}{a+b+c} \Rightarrow \frac{2b}{a+b+c} = \frac{2b}{b+2b} = \frac{2}{3}$ $(a + c = 2b)$

14. Ans. (4)

Given $a = 2b$ and $A = 3B$

From sine rule : $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\Rightarrow \frac{2b}{\sin 3B} = \frac{b}{\sin B} \Rightarrow \frac{\sin 3B}{\sin B} = 2$

$\Rightarrow \frac{3\sin B - 4\sin^3 B}{\sin B} = 2$ $(\because \sin 3\theta = 3\sin\theta - 4\sin^3\theta)$

$\Rightarrow 3 - 4\sin^2 B = 2 \Rightarrow \sin B = \frac{1}{2} \Rightarrow \angle B = 30°$

$\because A = 3B = 90°$

$\because A + B + C = 180° \Rightarrow \angle C = 60°$

$\frac{c}{b} = \frac{\sin C}{\sin B} = \frac{\sin 60°}{\sin 30°} = \sqrt{3}$

15. Ans. (3)

In ΔAHC $p^2 + (16 + x)^2 = (26)^2$ ...(i)

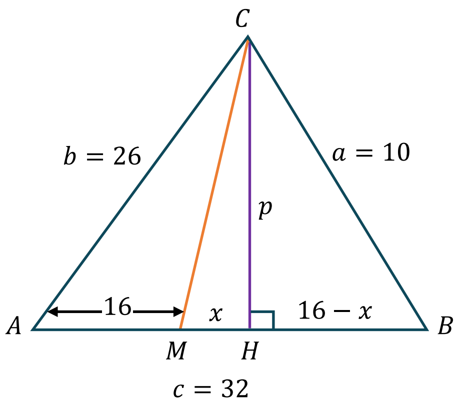
In ΔBCH $p^2 + (16 - x)^2 = (10^2$ ...(ii)

from (i) & (ii)

$\Rightarrow (16+x)^2 - (16-x)^2 = (26)^2 - (10)^2$

$\Rightarrow (32)(2x) = (36)(16)$

$\Rightarrow x = 9.$



16. Ans. (1)

$$\frac{\tan C}{\sin B} = \frac{\sin C}{\cos C \sin B} \quad \text{...(1)}$$

From sine rule :

$$\frac{\sin C}{\sin B} = \frac{c}{b} = \frac{10}{9} \quad \text{...(2)}$$

From cosine rule :

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos C = \frac{64+81-100}{2(8)(9)} = \frac{145-100}{2(8)(9)}$$

$$\Rightarrow \cos C = \frac{45}{2(8)(9)} = \frac{5}{16} \quad \text{...(3)}$$

Putting these values in equation (1)

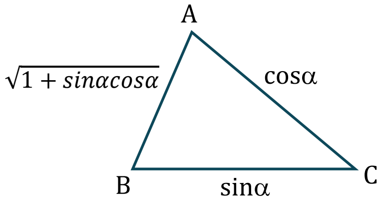
$$\frac{\sin C}{\cos C \sin B} = \frac{10}{9}\cdot\frac{16}{5} = \frac{32}{9}$$

17. Ans. (3)

$\because$ Largest angle lies opposite to largest side.

Let $a = \sin\alpha$, $b = \cos\alpha$ & $c = \sqrt{1+\sin\alpha\cos\alpha}$

Largest side = $c \Rightarrow \angle C$ is largest angle



$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

$$\cos C = \frac{\sin^2\alpha+\cos^2\alpha-(1+\sin\alpha\cos\alpha)}{2(\sin\alpha)(\cos\alpha)}$$

$$\Rightarrow \cos C = -\frac{1}{2} \Rightarrow C = \frac{2\pi}{3}$$

18. Ans. (2)

From Napier's analogy :

$$\tan\left(\frac{B-C}{2}\right) = \frac{b-c}{b+c}\cot\frac{A}{2} \quad \text{...(1)}$$

Given : $b : c$

$\sqrt{3}+1 \ : 2$

Let $b = \left(\sqrt{3}+1\right)k$ and $c = 2k$

$\angle A = 60°$

putting these values in (1)

$$\tan\left(\frac{B-C}{2}\right) = \frac{\left(\sqrt{3}+1\right)k-2k}{\left(\sqrt{3}+1\right)k+2k}\cot\left(\frac{60°}{2}\right)$$

$$\tan\left(\frac{B-C}{2}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+3}\left(\sqrt{3}\right) = \frac{\sqrt{3}-1}{\sqrt{3}+1}\times\frac{\left(\sqrt{3}-1\right)}{\left(\sqrt{3}-1\right)}$$

$$\Rightarrow \tan\left(\frac{B-C}{2}\right) = 2-\sqrt{3} \qquad (\because \tan 15° = 2-\sqrt{3})$$

$$\Rightarrow \frac{B-C}{2} = 15° \Rightarrow B - C = 30°$$

19. Ans. (4)

Given $\Delta = a^2 - (b-c)^2$

$\Rightarrow \Delta = (a-b+c)(a+b-c)$

$\sqrt{s(s-a)(s-b)(s-c)} = (2s-2b)(2s-2c)$

$(\because a-b+c = 2s-2b)$

$\sqrt{s(s-a)(s-b)(s-c)} = 4(s-b)(s-c) \Rightarrow 1 = 4\sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}$

$\left(\because \tan\dfrac{A}{2} = \sqrt{\dfrac{(s-b)(s-c)}{s(s-a)}}\right)$

$\Rightarrow 1 = 4\tan\dfrac{A}{2} \Rightarrow \tan\dfrac{A}{2} = \dfrac{1}{4} \Rightarrow \sin A = \dfrac{2\tan A/2}{1+\tan^2 A/2} = \dfrac{2(1/4)}{1+1/16} = \dfrac{8}{17}$

20. Ans. (3)

Given $\cot\dfrac{A}{2} = \dfrac{b+c}{a}$

From sine rule :

$b = k\sin B$, $c = k\sin C$ & $a = k\sin A$

$\cot\dfrac{A}{2} = \dfrac{k(\sin B + \sin C)}{k\sin A}$

$\dfrac{\cos A/2}{\sin A/2} = \dfrac{2\sin\left(\dfrac{B+C}{2}\right).\cos\left(\dfrac{B-C}{2}\right)}{2\sin\dfrac{A}{2}.\cos\dfrac{A}{2}}$

$(\because B + C = \pi - A)$

$2\cos^2\dfrac{A}{2} = 2\sin\left(\dfrac{\pi - A}{2}\right)\cos\left(\dfrac{B-C}{2}\right)$

$\cos^2\dfrac{A}{2} = \cos\dfrac{A}{2}\cos\left(\dfrac{B-C}{2}\right) \Rightarrow \cos\dfrac{A}{2} = \cos\left(\dfrac{B-C}{2}\right)$

$\Rightarrow A = B - C \Rightarrow A + C = B$

$\because A + B + C = 180°$

$\Rightarrow 2B = 180° \Rightarrow B = 90°$

SECTION–B

1. Ans. (1)

$\because a : b : c = 4 : 5 : 6$

$\therefore a = 4k, b = 5k, c = 6k$

$\because \cos B = \dfrac{c^2 + a^2 - b^2}{2ac} = \dfrac{9}{16}$

$\because \cos A = \dfrac{b^2 + c^2 - a^2}{2bc} = \dfrac{25+36-16}{2\times5\times6} = \dfrac{3}{4}$

$\because \cos 3A = 4\cos^3 A - 3\cos A$

$= 4 \times \dfrac{27}{64} - 3 \times \dfrac{3}{4} = \dfrac{27}{16} - \dfrac{9}{4} = \dfrac{27-36}{16} = \dfrac{-9}{16}$

$\cos 3A = -\cos B = \cos(\pi - B)$

$\therefore 3A + B = \pi$

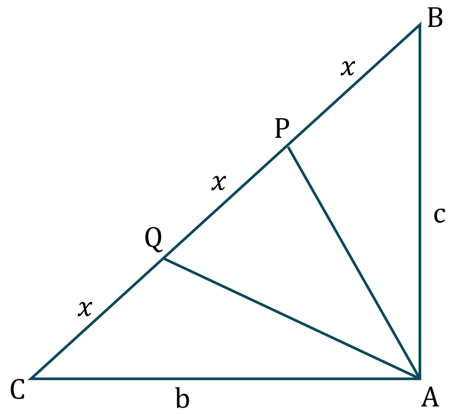
2. **Ans. (3)**

Let $BP = PQ = QC = x$

In ΔABP, using cosine rule

$9 = c^2 + x^2 - 2cx\cos B$

But $\cos B = \dfrac{c}{3x}$ ...(1)



$\Rightarrow 9 = x^2 + \dfrac{c^2}{3}$

similarly using cosine rule in ΔACQ, we get

$16 = x^2 + \dfrac{b^2}{3}$ ...(2)

Adding (1) and (2), we get $25 = 2x^2 + \left(\dfrac{b^2+c^2}{3}\right)$

$\therefore\ 25 = 2x^2 + \dfrac{(3x)^2}{3}$

$\therefore\ 25 = 2x^2 + 3x^2$

$\therefore\ x^2 = 5$

$\therefore\ BC = 3x = 3\sqrt{5}$

3. **Ans. (7)**

$\because\ s - a = 3$ ...(1)

and $s - c = 2$ ...(2)

by (1) – (2), we get

$c - a = 1$

(1) + (2), we get $2s - a - c = 5 \Rightarrow b = 5$

$\because\ \Delta ABC$ is right angled at B

$\therefore\ a^2 + c^2 = 25$ ...(3)

$\therefore\ (c-a)^2 + 2ac = 25$

$ac = 12$ ...(4)

$\therefore\ a(1 + a) = 12 \Rightarrow a^2 + a - 12 = 0$

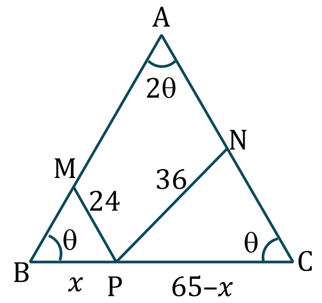
$\Rightarrow (a + 4)\ (a - 3) = 0$

$\Rightarrow a = 3$ and $c = 4$.

4. **Ans. (5)**

$A = \dfrac{1}{2} b^2 \sin 2\theta = b^2 \sin\theta\cos\theta$

Now, $\dfrac{x}{24} = \dfrac{65-x}{36}$ $(\because\ \Delta PMB \cong \Delta PNC)$



or $60x = (24)\ (65)$ or $x = 26$

$\therefore\ \sin\theta = \dfrac{12}{13}$ and $\cos\theta = \dfrac{5}{13}$

Again, $\dfrac{b}{\sin\theta} = \dfrac{65}{\sin 2\theta}$

or $b = \dfrac{65}{2\cos\theta} = \dfrac{(65)(13)}{(2)(5)} = \dfrac{13^2}{2}$

From eq. (i) we get

$A = \dfrac{13^4}{4} \times \dfrac{12}{13} \times \dfrac{5}{13} = (169)\,(15) = 2535$

5. **Ans. (1)**

We known $\Delta = sr = (70/2) \times 6 = 210$

$\Rightarrow (1/2)\,ab = 210 \Rightarrow ab = 420$

Now $(a+b)^2 = (70-c)^2$

$\Rightarrow a^2 + b^2 + 2ab = 4900 - 140c + c^2$

$\Rightarrow c = \dfrac{4900 - 2\times 420}{140} = 29$ $\quad [\because a^2 + b^2 = c^2]$

$\Rightarrow (a-b)^2 = a^2 + b^2 - 2ab = c^2 - 2ab = 841 - 840 = 1$

6. **Ans. (4)**

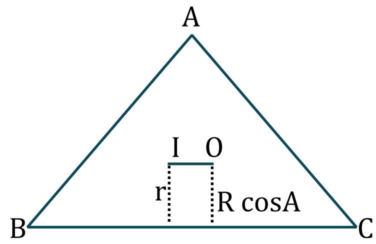
$\because\ r_1 + r_2 = \dfrac{\Delta c}{(s-a)(s-b)}$

$\therefore\ \Pi\,(r_1 + r_2) = \dfrac{\Delta^3 abc}{(s-a)^2(s-b)^2(s-c)^2} = \dfrac{\Delta^3 (abc) s^2}{\Delta^4}$

$= \dfrac{(abc)s^2}{\Delta} = \dfrac{4R\Delta s^2}{\Delta} = 4Rs^2$

$\therefore\ \dfrac{\Pi\ (r_1 + r_2)}{Rs^2} = 4$

7. **Ans. (1)**



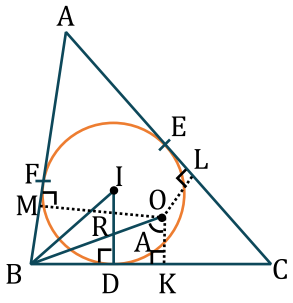
$\because\ R\cos A = r$

$R\cos A = 4R\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}$

$\cos A = \cos A + \cos B + \cos C - 1$

$\cos B + \cos C = 1$

8. **Ans. (2)**



$\because BD = s - b,\ BK = R\sin A$

$\Rightarrow DK = |BK - BD|$

$= |R\sin A - (s-b)| = \left|\dfrac{a}{2} - (s-b)\right| = \left|\dfrac{b-c}{2}\right| = \dfrac{1}{2}$

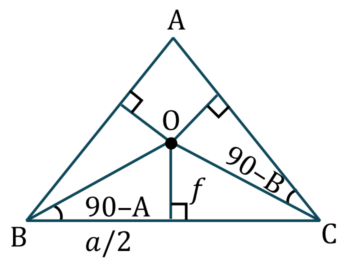
Similarly $EL = \left|\dfrac{a-c}{2}\right| = 1$ & $FM = \left|\dfrac{a-b}{2}\right| = \dfrac{1}{2}$

$\Rightarrow DK + EL + FM = 2$

9. **Ans. (9)**

Length of tangent $= s = \dfrac{4+7+7}{2} = 9$

10. Ans. (1)



$\tan A = \frac{a}{2f}$

$\tan B = \frac{b}{2g}$

$\tan C = \frac{c}{2h}$

Now $\sum \tan A = \prod \tan A$

$\sum \frac{a}{2f} = \frac{abc}{8fgh}$

$\sum \frac{a}{f} = \frac{abc}{4fgh} \Rightarrow K = 1$

JEE (Advanced) Practice Paper

1. Ans. (C)

$\because (a + b + c)(b + c - a) = kbc$

$\Rightarrow (b + c)^2 - a^2 = kbc \Rightarrow b^2 + c^2 - a^2 = (k - 2)bc$

$\Rightarrow \frac{b^2 + c^2 - a^2}{2bc} = \frac{k-2}{2} = \cos A$

$\because$ In a ΔABC $\quad -1 < \cos A < 1$

$\therefore -1 < \frac{k-2}{2} < 1$

$0 < k < 4$

2. Ans. (B)

$\Delta = (a + b - c)(a - b + c)$

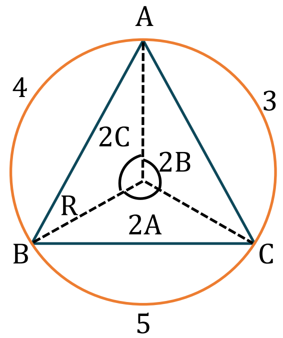
$\Delta = 4(s - c)(s - b) \Rightarrow \frac{\Delta}{(s-b)(s-c)} = 4$

$\therefore \tan\frac{A}{2} = \frac{1}{4}$

$\because \tan A = \frac{2\tan\frac{A}{2}}{1-\tan^2\frac{A}{2}}$

$\Rightarrow \tan A = \frac{8}{15}$

3. Ans. (A)



angle = $\frac{arc}{radius}$ ...(1)

$\because 4 + 5 + 3 = 2\pi R \Rightarrow R = 6/\pi$

$\because 2A = \frac{5}{R} = \frac{5\pi}{6}, 2B = \frac{3}{R} = \frac{\pi}{2}$ and $2C = \frac{4}{R} = \frac{2\pi}{3}$

Area of $\Delta ABC = \frac{1}{2}R^2\left[\sin\frac{2\pi}{3} + \sin\frac{5\pi}{6} + \sin\frac{\pi}{2}\right]$

$= \frac{R^2}{2}\left[\frac{\sqrt{3}}{2} + \frac{1}{2} + 1\right] = \frac{R^2}{2}\left[\frac{\sqrt{3}+3}{2}\right] = \frac{\sqrt{3}(\sqrt{3}+1)}{4} \times \frac{36}{\pi^2} = \frac{9\sqrt{3}(\sqrt{3}+1)}{\pi^2}$

4. Ans. (A)

$$\frac{(s-b)-(s-a)}{(s-c)-(s-b)} = \frac{s-a}{s-c} \Rightarrow \frac{\frac{\Delta}{r_2}-\frac{\Delta}{r_1}}{\frac{\Delta}{r_3}-\frac{\Delta}{r_2}} = \frac{\frac{\Delta}{r_1}}{\frac{\Delta}{r_3}} \Rightarrow \frac{(r_1-r_2)\ r_2r_3}{r_1r_2(r_2-r_3)} = \frac{r_3}{r_1}$$

$\Rightarrow 2r_2 = r_1 + r_3$

$\Rightarrow r_1, r_2, r_3$ are in A.P. $\Rightarrow r_1, r_2, r_3$

5. Ans. (B,D)

$\cos A \cos 2B + \sin A \sin 2B \sin C = 1$

$\sin A \sin 2B \sin C = 1 - \cos A \cos 2B$

$$\sin C = \frac{1-\cos A \cos 2B}{\sin A \sin 2B} \le 1$$

$\therefore 1 - \cos A \cos 2B \le \sin A \sin 2B$

$\Rightarrow \cos A \cos 2B + \sin A \sin 2B \ge 1 \Rightarrow \cos(A - 2B) \ge 1$

$\Rightarrow \cos(A - 2B) = 1 \Rightarrow A = 2B \Rightarrow \sin C = \dfrac{1-\cos^2 A}{\sin^2 A} = 1$

$\because A + B + C = 180°$

$2B + B + 90° = 180° \Rightarrow B = 30°$ & $A = 60°$

$\therefore B, A, C$ are in A.P.

Now $\cos A + \cos B + \cos C = 1 + \dfrac{r}{R}$

$\cos 60° + \cos 30° + \cos 90° = 1 + \dfrac{r}{R}$

$\dfrac{1}{2} + \dfrac{\sqrt{3}}{2} + 0 = 1 + \dfrac{r}{R}$

$$\frac{r}{R} = \frac{\sqrt{3}-1}{2} = \left(\frac{\sqrt{3}-1}{2\sqrt{2}}\right)\sqrt{2} = \sqrt{2}\sin\left(\frac{\pi}{12}\right)$$

6. Ans. (A,C)

Given $\Delta = 10\sqrt{3}$, $s = 10$, $\angle B = 60°$ & $a > b > c$

(A) $r = \dfrac{\Delta}{s} = \dfrac{10\sqrt{3}}{10} = \sqrt{3}$

(B) $r = (s-b)\tan\dfrac{B}{2}$

$\Rightarrow \sqrt{3} = (10 - b)\tan 30° \Rightarrow b = 7$

$\therefore a + c = 13$ ...(1)

$\Delta = 10\sqrt{3} = \dfrac{1}{2}ac\sin B = \dfrac{1}{2}ac\sin 60°$

$\Rightarrow ac = 40$ ...(2)

from equation (1) & (2), we get

$a^2 - 13a + 40 = 0 \Rightarrow a = 5, a = 8$

$\Rightarrow a = 8, c = 5$

(As $a > b > c$)

$\Rightarrow$ Length of longest side of triangle = 8

(C) As $\dfrac{b}{\sin B} = 2R$

$\Rightarrow \dfrac{7}{\sin 60°} = 2R \Rightarrow R = \dfrac{7}{\sqrt{3}}$

(D) $r_1 = \dfrac{\Delta}{s-a} = \dfrac{10\sqrt{3}}{10-8} = 5\sqrt{3}$

$r_2 = \dfrac{\Delta}{s-b} = \dfrac{10\sqrt{3}}{10-7} = \dfrac{10}{\sqrt{3}}$

$r_3 = \dfrac{\Delta}{s-c} = \dfrac{10\sqrt{3}}{10-5} = 2\sqrt{3}$

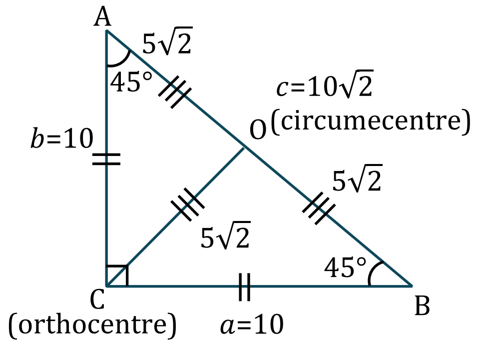
so the radius of largest escribed circle = $5\sqrt{3}$

7. Ans. (A,B,C)

$\because \dfrac{b}{\sin B} = 2R$

$\therefore \sin B = \dfrac{b}{2R} = \dfrac{10}{2\left(5\sqrt{2}\right)}$

$\sin B = \dfrac{1}{\sqrt{2}}$



$\Rightarrow B = 45°$

(Here $B \neq 135°$ as $c > b \Rightarrow \angle C > \angle B$)

Also $\dfrac{c}{\sin C} = 2R$

$\Rightarrow \sin C = \dfrac{c}{2R}$

$\sin C = \dfrac{10\sqrt{2}}{2\left(5\sqrt{2}\right)}$

$\sin C = 1 \Rightarrow C = 90°$

(A) Area of triangle ABC = $\dfrac{1}{2} \times 10 \times 10 = 50$

(B) Distance between orthocentre and circum centre = $5\sqrt{2}$

(C) Circum radius (R) = $5\sqrt{2}$ (Given)

Inradius (r) = $\dfrac{\Delta}{s}$

$$= \dfrac{50}{\dfrac{10\sqrt{2}+10+10}{2}} = \dfrac{100}{20+10\sqrt{2}} = \dfrac{10}{2+\sqrt{2}} = \dfrac{10\left(2-\sqrt{2}\right)}{2}$$

$\therefore$ Sum of $R + r$ = $5\sqrt{2} + 5\left(2-\sqrt{2}\right) = 10$

(D) Length of internal angle bisector of $\angle ACB$ is

$$=\frac{2ab\cos\frac{C}{2}}{a+b}=\frac{2\times10\times10\times\cos45^\circ}{10+10}=\frac{10}{\sqrt{2}}=5\sqrt{2}$$

8. **Ans. (A,B)**

Given $a = 4$, $b = 8$, $\angle C = 60°$

$$\cos C=\frac{1}{2}=\frac{16+64-c^2}{2.(4).(8)}\Rightarrow c^2 = 48 \Rightarrow\ c=4\sqrt{3}$$

Use sine Rule $\frac{a}{\sin A}=\frac{c}{\sin C}\ \Rightarrow\ \frac{4}{\sin A}=\frac{4\sqrt{3}}{\sqrt{3}/2}\ \Rightarrow A = 30°$

As $\angle C = 60°$, $\angle A = 30° \Rightarrow \angle B = 90°$

(A) Area of ΔABC = $\frac{1}{2}ac\sin B$

$$=\frac{1}{2}(4)\left(4\sqrt{3}\right)\sin 90^\circ=8\sqrt{3}$$

(B) $\Sigma\sin^2 A = \sin^2 A + \sin^2 B + \sin^2 C$

$= \sin^2(30°) + \sin^2(90°) + \sin^2(60°)\ =\frac{1}{4}+1+\frac{3}{4}=2$

(C) Now $r=\frac{\Delta}{s}=\frac{2\Delta}{a+b+c}=\frac{2\left(8\sqrt{3}\right)}{4+4\sqrt{3}+8}=\frac{4\sqrt{3}}{3+\sqrt{3}}$

(D) Length of internal angle bisector of angle C

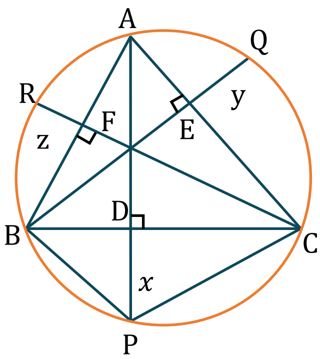
$$=\frac{2ab\cos\left(\frac{C}{2}\right)}{a+b}=\frac{2(4)(8)\cos30^\circ}{4+8}=\frac{8}{\sqrt{3}}$$

9. **Ans. (A)**

Chord AC subtends $\angle ABC$ and $\angle APC$ on circumference of circle

$\Rightarrow \angle APC = \angle ABC = B$

In ΔPDC :



$\frac{DC}{x}=\tan\angle APC$

$\Rightarrow DC = x\tan B$ ...(i)

similarly, we can show

$BD = x\tan c$ ...(ii)

(i) + (ii) : $DC + BD = BC = a = x\tan B + x\tan C$

$\Rightarrow\frac{a}{2x}=\frac{\tan B+\tan C}{2}$ ...(iii)

similarly, we can get

$\frac{b}{2y}=\frac{\tan C+\tan A}{2}$ ...(iv)

and $\frac{c}{2z}=\frac{\tan A+\tan B}{2}$ ...(v)

(iii) + (iv) + (v)

$$\frac{a}{2x}+\frac{b}{2y}+\frac{c}{2z}=\tan A+\tan B+\tan C$$

10. **Ans. (B)**

Clearly

11. **Ans. (B)**

Side of pedal triangle = $I_2I_3 \cos\theta = BC$

$I_2I_3 = \dfrac{a}{\cos\left(\dfrac{\pi}{2} - \dfrac{A}{2}\right)}$

$I_2I_3 = 4R\cos\left(\dfrac{A}{2}\right)$

12. **Ans. (B)**

$II_1 = 4R \sin\dfrac{A}{2}$

$I_2I_3 = 4R \cos\dfrac{A}{2}$

$\therefore\ II_1^2 + I_2I_3^2 = 16R^2$

13. **Ans. (A → q; B → p; C → s; D → r)**

(A) $\because\ 2B = A + C \Rightarrow B = \dfrac{\pi}{3}$ and $A + C = \dfrac{2\pi}{3}$

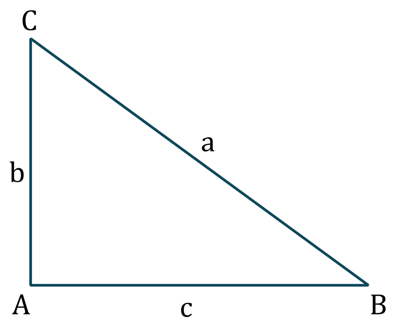
$\because\ b^2 = ac \Rightarrow \sin^2 B = \sin A.\sin C$

$\Rightarrow \sin A \sin C = \dfrac{3}{4}$

$\Rightarrow \cos(A - C) - \cos(A + C) = \dfrac{3}{2}$ $\qquad \because\ A + C = \dfrac{2\pi}{3}$

$\Rightarrow \cos(A - C) = 1 \Rightarrow A = C = \dfrac{\pi}{3} = B$

$\Rightarrow a = b = c\ \therefore\ \dfrac{a^2(a+b+c)}{3abc} = 1$



(B) $\because\ a^2 = b^2 + c^2$ and $2R = a$ $\qquad \therefore\ \dfrac{a^2+b^2+c^2}{R^2} = \dfrac{2a^2}{R^2} = 8$

(C) $\because\ \Delta = \dfrac{1}{2}bc \sin A \Rightarrow \Delta = \dfrac{1}{2}.9. \sin A = \dfrac{9}{2} \times \dfrac{a}{2R}$

$\because\ a = 2$

$\therefore\ 2R\Delta = 9$

(D) $\because\ a = 5, b = 3$ and $c = 7$

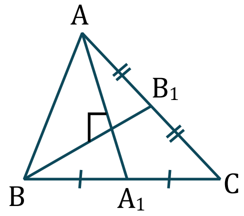
and because we know that

$b \cos C + c \cos B = a$

$\therefore\ 3 \cos C + 7 \cos B = 5$

14. Ans. (A $\to$ s; B $\to$ p; C $\to$ r; D $\to$ q)

(A) AA_1 and BB_1 are perpendicular



$\therefore\ a^2 + b^2 = 5c^2$

$\therefore\ c^2 = \dfrac{a^2+b^2}{5} = 5 \Rightarrow c = \sqrt{5}$

$\cos C = \dfrac{a^2+b^2-c^2}{2ab} = \dfrac{16+9-5}{2\times4\times3} = \dfrac{5}{6}$

$\sin C = \dfrac{\sqrt{11}}{6}$

$\therefore\ \Delta = \dfrac{1}{2}\ ab \sin C = \sqrt{11}$

$\therefore\ \Delta^2 = 11$

(B) $\because$ G.M. $\geq$ H.M.

$(r_1r_2r_3)^{1/3} \geq \dfrac{3}{\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}}$

$\Rightarrow (r_1r_2r_3)^{1/3} \geq 3r$

$\Rightarrow \dfrac{r_1r_2r_3}{r^3} \geq 27$

(C) $\tan^2 \dfrac{C}{2} = \dfrac{(s-a)(s-b)}{s(s-c)}$ $\quad \because\ a = 5, b = 4 \quad 2s = 9 + c$

$= \dfrac{(9+c-10)(9+c-8)}{(9+c)(9-c)} = \dfrac{c^2-1}{81-c^2}$

$\Rightarrow \dfrac{7}{9} = \dfrac{c^2-1}{81-c^2} \Rightarrow c^2 = 36 \Rightarrow c = 6$

(D) $\because\ 2a^2 + 4b^2 + c^2 = 4ab + 2ac.$

$\Rightarrow (a - 2b)^2 + (a - c)^2 = 0$

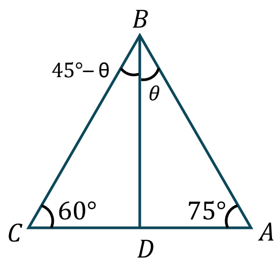
$\Rightarrow a = 2b = c$

$\because\ \cos B = \dfrac{a^2+c^2-b^2}{2ac} = \dfrac{7}{8}$

$\therefore\ 8 \cos B = 7$

15. Ans. (5)

Area of $\Delta BAD = \sqrt{3}\ \times$ Area of ΔBCD



$\Rightarrow \dfrac{1}{2} BD \times BA \sin\theta = \sqrt{3} \times \dfrac{1}{2} BC \times BD \sin(45° - \theta)$

$\dfrac{BA}{BC} = \sqrt{3}\ \dfrac{\sin(45°-\theta)}{\sin\theta}$ ...(1)

From Sine-Rule

$\dfrac{BC}{\sin 75°} = \dfrac{AB}{\sin 60°}$

$\therefore \dfrac{BA}{BC} = \dfrac{\sin 60^\circ}{\sin 75^\circ} = \dfrac{\sqrt{3}\sqrt{2}}{\sqrt{3}+1}$

$\therefore$ From equation (1)

$\dfrac{\sqrt{3}.\sqrt{2}}{(\sqrt{3}+1)} = \sqrt{3}\left[\dfrac{1}{\sqrt{2}}\cot\theta - \dfrac{1}{\sqrt{2}}\right]$

$\Rightarrow \dfrac{2}{(\sqrt{3}+1)} = \cot\theta - 1 \Rightarrow \dfrac{2\left(\sqrt{3}-1\right)}{2} = \cot\theta - 1$

$\Rightarrow \cot\theta = \sqrt{3} \Rightarrow \theta = 30^\circ \Rightarrow \angle ABD = 30^\circ$

16. Ans. (6)

(i and ii) Let a, b and $c(a < b < c)$ be the sides of given triangle.

Also, $2r = a + b - c$

When $r = 4$ then, $(a, b) = (9, 40), (10, 24), (12,16)$

$\therefore$ Greatest perimeter = 9 + 40 + 41 = 90 units

17. Ans. (5)

(i and ii) Let a, b and c $(a < b < c)$ be the sides of given triangle.

Also, $2r = a + b - c$

when $r = 5$ then $(a, b) = (11, 60)\ (12,35)\ (15,20)$

$\therefore$ Greatest area $= \dfrac{11\times 60}{2}$ = 330 sq. unit

18. Ans. (4)

L.H.S. $= \dfrac{1}{2}[a^2(b + c - a) + b^2(c + a - b) + c^2(a + b - c)]$

$= \dfrac{1}{2}[a(b^2 + c^2 - a^2) + b(c^2 + a^2 - b^2) + c(a^2 + b^2 - c^2)]$

$= \dfrac{1}{2}(2abc\cos A + 2abc\cos B + 2abc\cos C)$

$= abc\left(1 + 4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}\right)$

$= 4R\Delta\left(1 + 4\sin\dfrac{A}{2}\sin\dfrac{B}{2}\sin\dfrac{C}{2}\right)$