01 Ray Optics and Optical Instruments

Introduction

Optics: Optics is the branch of physics in which we study the behaviour and properties of light, including its interactions with matter.

Light is that part of electromagnetic waves which gives sensation to our eyes.

Radio waves   Microwaves   Infrared   Visible Region   Ultraviolet   X-rays   Gamma rays

f ↑ λ ↓

Properties of light

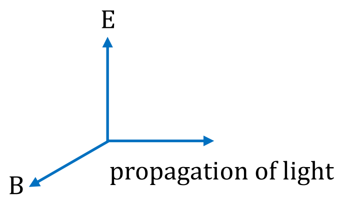
1. Speed of light in vacuum, denoted by $c = 3 \times 10^8$ m/s approximately.

2. Light carries energy and momentum.

3. Light is electromagnetic wave (proposed by Maxwell). It consists of varying electric and magnetic field.

4. Light is an electromagnetic wave and transverse in nature.

5. The formula $v = f\lambda$ is applicable to light.



Note: Eye is most sensitive for yellowish green colour and least sensitive for violet and red colours.

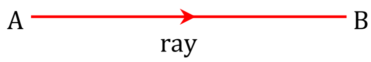
Rectilinear propagation of light

Rectilinear propagation describes the tendency of electromagnetic waves (light) to travel in a straight line path in homogenous medium.

Ray

A **ray** of light is the straight-line path of transfer of light energy. Arrow represents the direction of propagation of light.

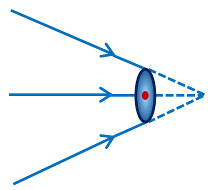
Figure shows a ray which indicates light is moving from A to B.



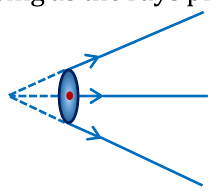
Beam

A bundle or bunch of rays is called a beam. It is of following three types :

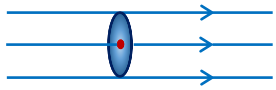
(a) **Convergent beam :** In this case diameter of beam decreases in the direction of ray.



(b) **Divergent beam :** It is a beam in which all the rays meet at a point when produced backward and the diameter of beam goes on increasing as the rays proceed forward.



(c) **Parallel beam :** It is a beam in which all the rays constituting the beam move parallel to each other and diameter of beam remains same.



Parallel beam of light

OPTICS

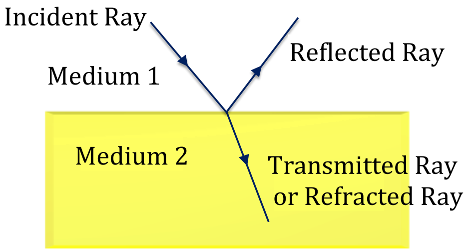
Ray Optics (Geometrical Optics) — Study of light in the form of ray

Wave Optics — Study of light in the form of wave

Under many circumstances, the wavelength of light is negligible compared with the dimensions of the device as in the case of ordinary mirrors and lenses. A light beam can then be treated as a ray whose propagation is governed by simple geometric rules. The part of optics that deals with such phenomenon is known as geometrical optics.

Reflection of Light

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called Reflection of Light.



Laws of Reflection

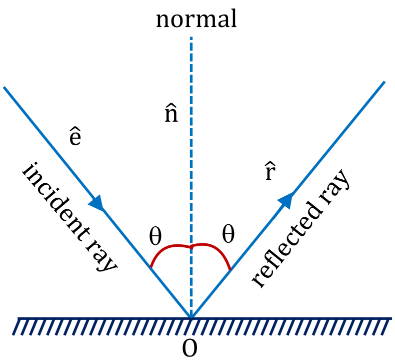
(1) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the **plane of incidence (or plane of reflection)**.

This condition can be expressed mathematically as $\hat{e}.(\hat{n}\times\hat{r})=0$

Where; $\hat{e}$ is a unit vector along incident ray.

$\hat{n}$ is a unit vector along normal.

$\hat{r}$ is a unit vector along reflected ray.



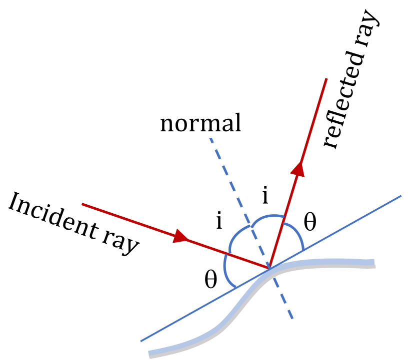
(2) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e. $\boxed{\angle i = \angle r}$

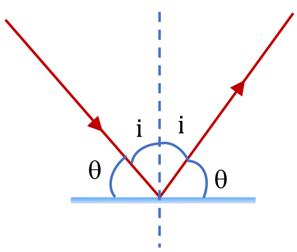
Note:

(a) The frequency, wavelength and speed of light do not change after reflection.

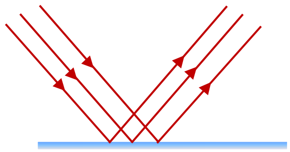
(b) If we draw a tangent at the point of incident then the angle between incident ray and tangent is equal to the angle between reflected ray and tangent.

In case of plane mirror angle between incident ray and plane mirror is same as angle between reflected ray and plane mirror.

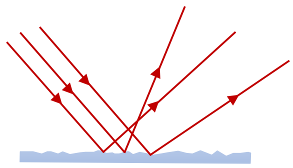




Types of Reflection



1. **Regular Reflection:** When the reflection takes place from a perfect plane surface it is called **Regular Reflection**. In this case the reflected light has large intensity in one direction and negligibly small intensity in other directions.
   Eg :- reflection from mirror.



2. **Diffused Reflection:** When the surface is rough, we do not get a regular behaviour of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface, and it is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position. Such a reflection is called as **diffused reflection**.
   Eg :- reflection from a wall, newspaper etc.

Object and Image

Object: Light rays incident on an optical element is responsible to define object.
Object is defined as the point of intersection of incident rays.

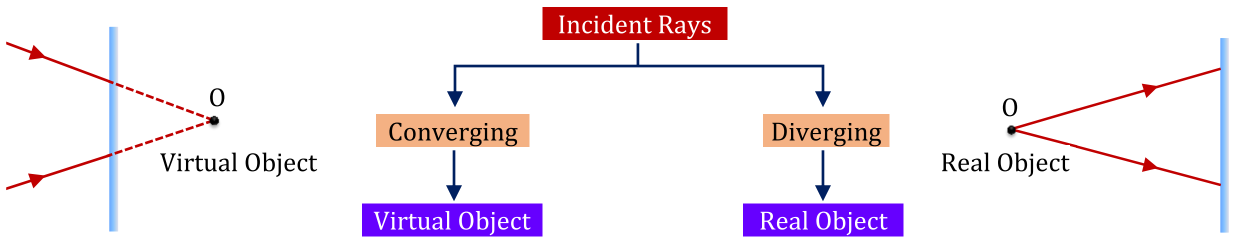
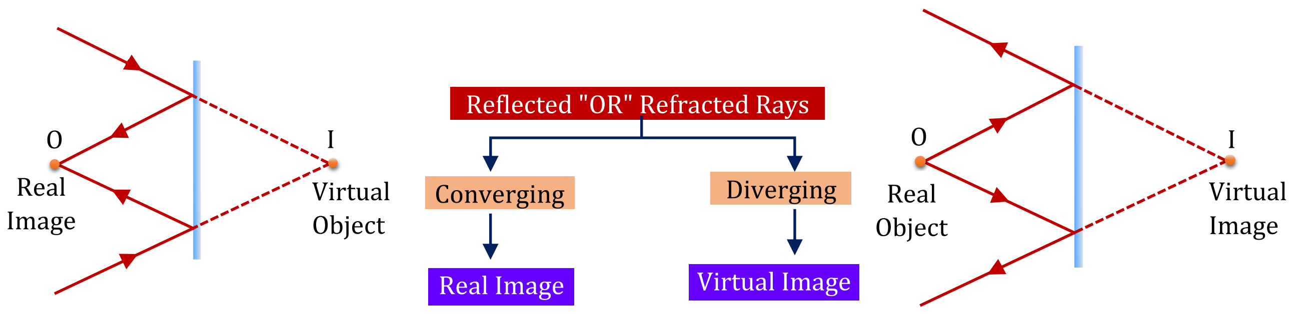


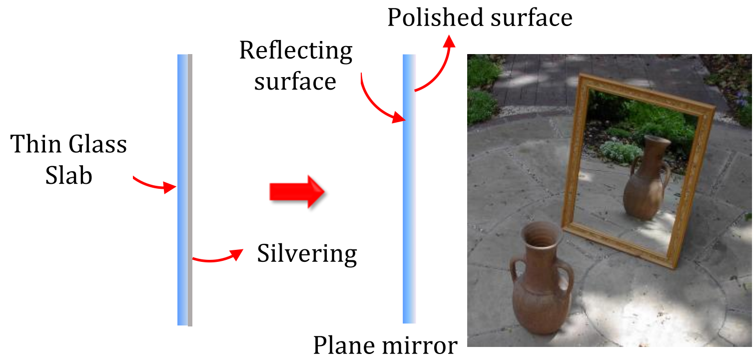
Image: Light rays formed after interaction (either reflection or refraction) with an optical element is responsible to define image.
Image is defined as the point of intersection of reflected or refracted rays.



Note: A real image can be captured on a screen. This image on the screen can be perceived by the eye.
What about virtual images? Since they cannot be captured on a screen is it possible to see them.
From practical experience we know that we can see ourselves in a plane mirror which is indeed a virtual image. So, it is possible to see a virtual image. But how is it done?
There is a lens in our eye that focuses the diverging set of rays on to the retina forming a real image inside our eyes.
In a similar manner, a camera can also convert a virtual image into a real image which is captured on the film.

Reflection from Plane Mirror

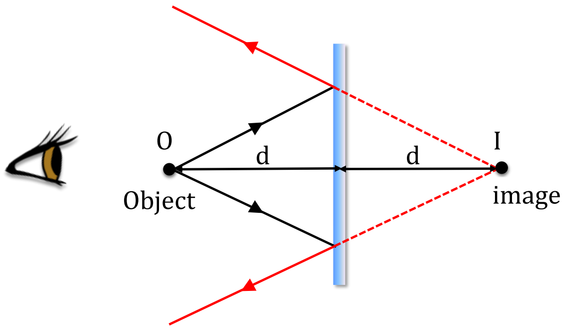
A plane mirror is a mirror with a flat (planar) reflective surface.



A plane mirror behaves like a window to a virtual world.

Image formation in plane mirror

Diverging light rays leave the source (object) and are reflected from the mirror, obeying the law of reflection. Upon reflection, the rays continue to diverge. The dashed lines in figure are backward extensions of the diverging rays back to a point of intersection at I. The diverging rays appear to originate at the point I behind the mirror to the observer. Point I, which is behind the mirror, is called the **image** of the object at O.



Perpendicular distance of object from plane mirror is equal to perpendicular distance of image from plane mirror.

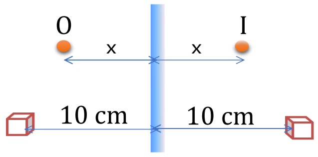
Characteristics of image due to reflection by a plane mirror:

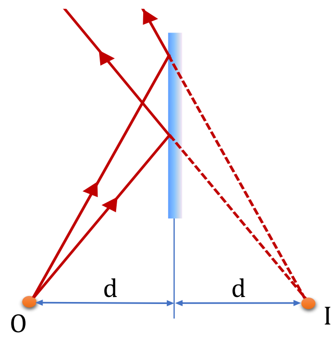
(i) Distance of object from mirror = Distance of image from the mirror.

(ii) All the incident rays from a point object will meet or appear to meet at a single point after reflection from a plane mirror which is called image.

(iii) The line joining a point object and its image is normal to the reflecting surface.

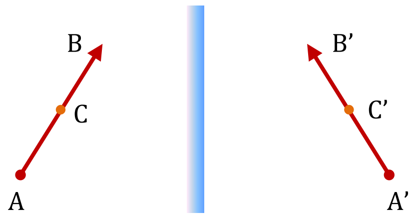
For example:





Extended object

An extended object like AB shown in figure is a combination of infinite number of point objects from A to B. Image of every point object will be formed individually and thus infinite images will be formed. A´ will be image of A, C´ will be image of C, B´ will be image of B etc. All point images together form extended image. Thus, extended image is formed of an extended object.

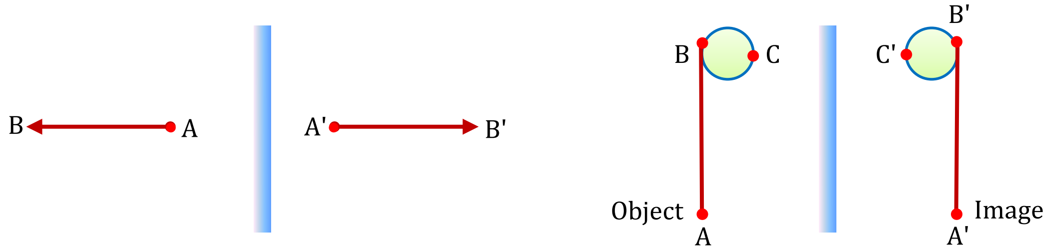


Properties of image of an extended object, formed by a plane mirror

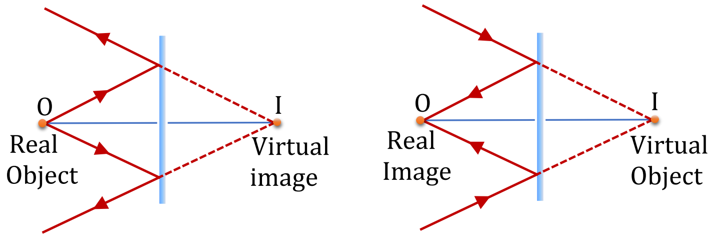
(i) Size of extended object = Size of extended image.

(ii) The image is virtual & erect, if the real extended object is placed parallel to the mirror.

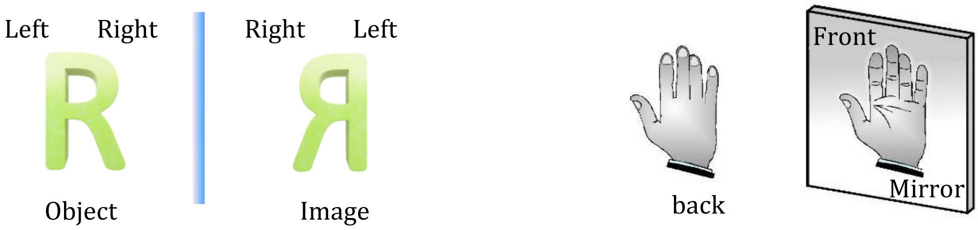
(iii) The image is inverted if the extended object lies perpendicular to the plane mirror.



(iv) For a real object the image is virtual and for a virtual object the image is real.



Note: The image formed by a plane mirror suffers lateral–inversion, i.e., left is turned into right and vice–versa with respect to object in the image formed by a plane mirror.



Some capital English alphabets do not show lateral inversion.

Example : A, H, I, M, O, T, U, V, W, X and Y

Analog clock and plane mirror

When plane mirror is placed in front of the analog clock then the time observed by the observer in image of clock is different from the actual time.

(a) If actual time is in **HH : 00** then time observed in the image of clock is, $\begin{array}{r} 12:00 \\ -\,HH:00 \\ \hline xx:00 \end{array}$	(b) If actual time is in **HH : MM** then time observed in the image of clock is, $\begin{array}{r} 11:60 \\ -\,HH:MM \\ \hline xx:yy \end{array}$	(c) If actual time is in **HH : MM : SS** then time observed in the image of clock is, $\begin{array}{r} 11:59:60 \\ -\,HH:MM:SS \\ \hline xx:yy:zz \end{array}$

Illustration 1

An unnumbered wall clock shows time 04 : 25 : 37, where 1st term represents hours, 2nd represents minutes & the last term represents seconds. What time will its image in a plane mirror show?

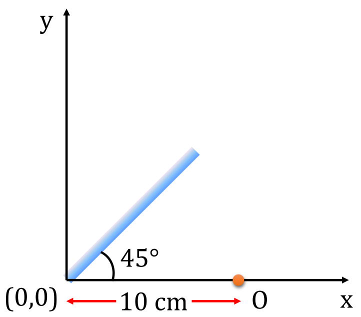
Solution:

$$\begin{array}{r} 11:59:60 \\ -\;04:25:37 \\ \hline 7:34:23 \end{array}$$

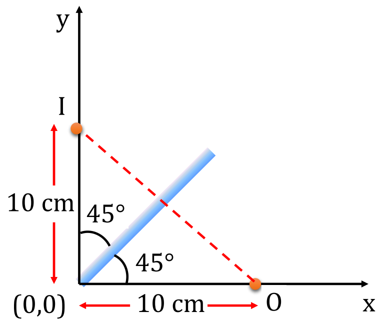
Note: The angle that an image makes from the plane mirror is equal to the angle that an object makes from the plane mirror.

Illustration 2

Find out position of image formed after reflection.



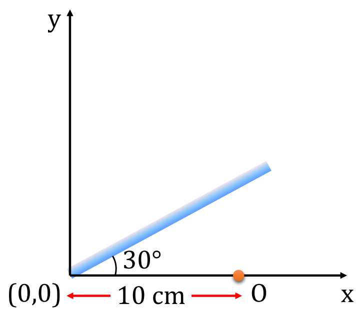
Solution:



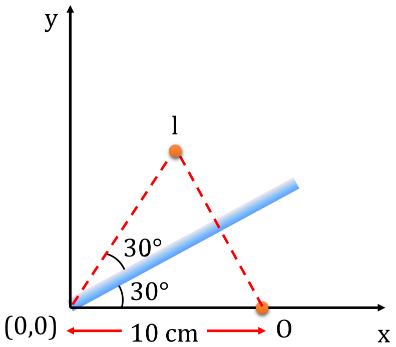
Co-ordinate of image is (0, 10)

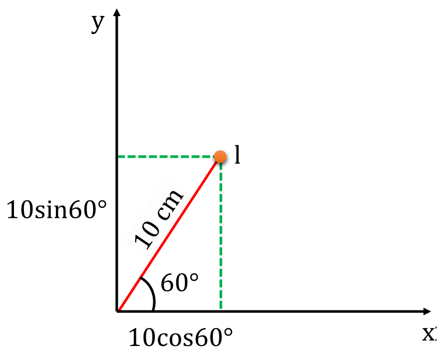
Illustration 3

Find out position of image formed after reflection.



Solution:

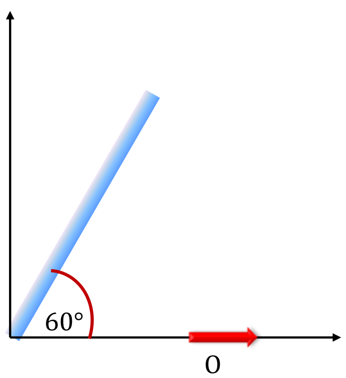




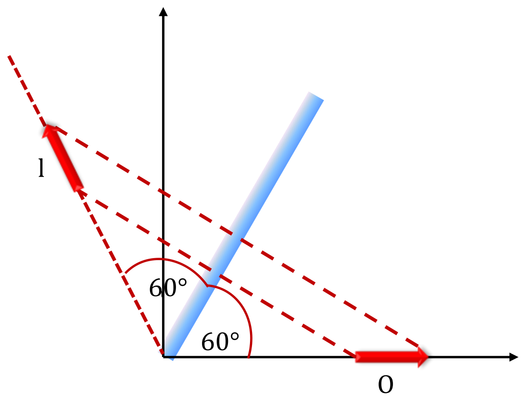
Co-ordinate of image is (10 cos 60°, 10 sin 60°) = $(5, 5\sqrt{3})$

Illustration 4

Draw image of an object as shown in figure.



Solution:



Angle of Deviation & Rotation of Plane Mirror

Angle of deviation (δ) in plane mirror: When light ray is incident on plane mirror then due to reflection its path get deviated, so the angle by which it got deviated is known as angle of deviation.

δ v/s *i* curve

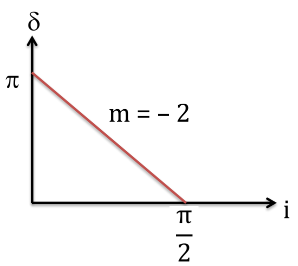
$\delta = \pi - 2i$

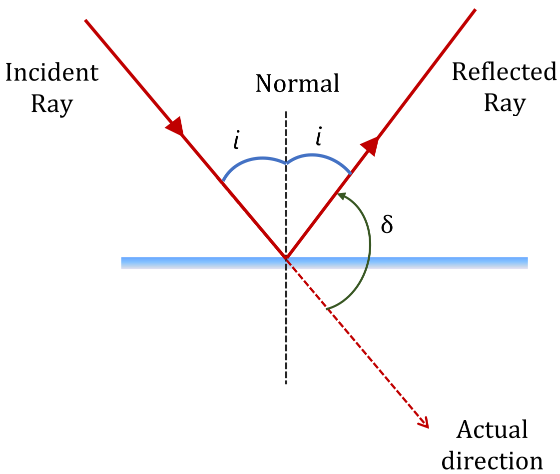
Rearrange the above relation,

$\delta = -2i + \pi$

Compare the above relation with straight line relation y = mx + c

Then graph between δ and i is a straight line with negative slope.





$\delta = \pi - 2i$

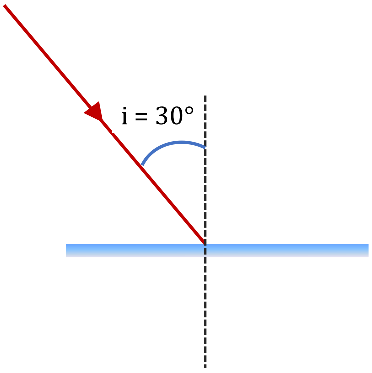
or

$\delta = 180 - 2i$

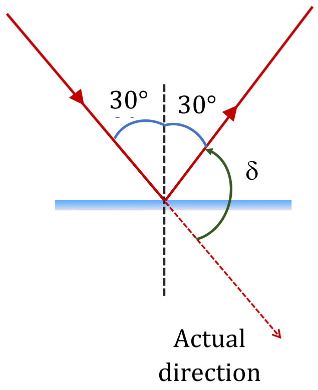
Note: When light strikes the mirror perpendicularly (normally) then after reflection it retrace its path.

Illustration 5

Find out angle of deviation?

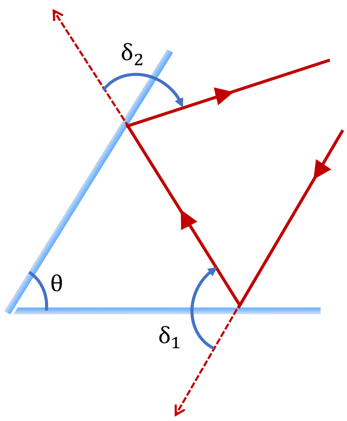


Solution:



$\delta = \pi - 2i = 180° - 2\,(30°) = 120°$ (anticlockwise)

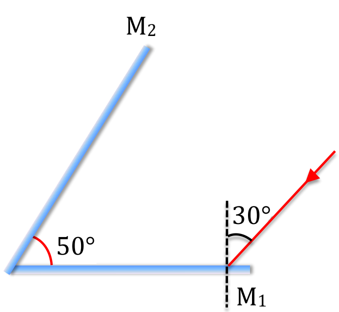
Note: When reflection are more than one then add angle of deviation of all reflection along with sign.



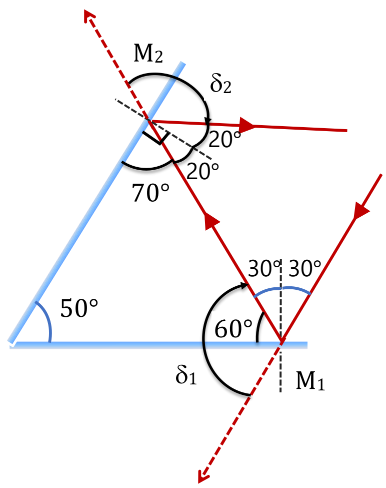
$\delta_{net} = \delta_1 + \delta_2$

Illustration 6

Find net angle of deviation after two reflection?



Solution:



For mirror M_1

$\delta_1 = 180° - 2i$

$\delta_1 = 180° - 2(30°)$

$\delta_1 = 120°$ (clockwise)

For mirror M_2

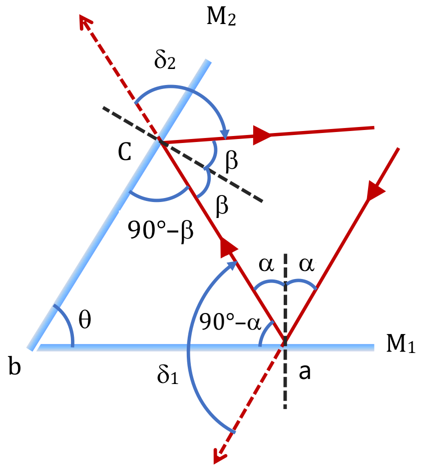
$\delta_2 = 180° - 2i$

$\delta_2 = 180° - 2(20°)$

$\delta_2 = 140°$ (clockwise)

Total deviation $\delta_{net} = \delta_1 + \delta_2 = 120° + 140° = 260°$ (clockwise)

Note:



For mirror M_1, $\delta_1 = 180° - 2\alpha$

For mirror M_2, $\delta_2 = 180° - 2\beta$

$\delta_{net} = \delta_1 + \delta_2$

$\delta_{net} = 360° - 2(\alpha + \beta)$

In Δabc,

$90° - \alpha + 90° - \beta + \theta = 180°$

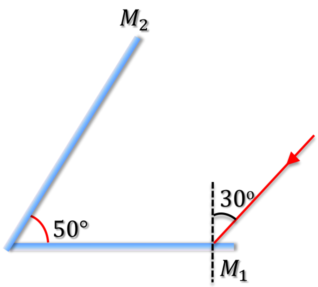
$\alpha + \beta = \theta$

Now, $\boxed{\delta_{net} = 360° - 2\theta}$

So,

- Total deviation after two reflections, $\delta_{net} = 360° - 2\theta$ (clockwise) (θ is the angle between both the mirror.)
- δ_{net} for two reflection is independent of angle of incidence.

Illustration 7



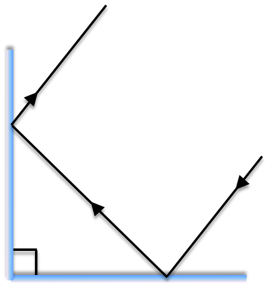
Find net angle of deviation after two reflections?

Solution:

$\delta_{net} = 360° - 2(50°) = 260°$

Illustration 8

Find net angle of deviation after two reflections?

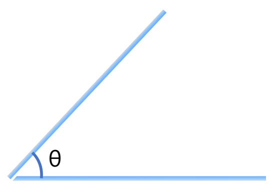


Solution:

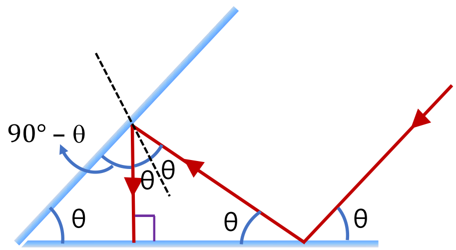
$\delta_{net} = 360° - 2(90) = 180°$

Note: When two mirror are perpendicular to each other then reflected ray is antiparallel to incident ray after two reflection.

Illustration 9



Two mirrors are inclined at an angle θ as shown in the figure. Light ray is incident parallel to one of the mirrors. The ray will start retracing its path after third reflection if -



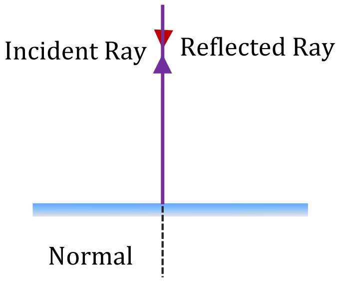
Solution:

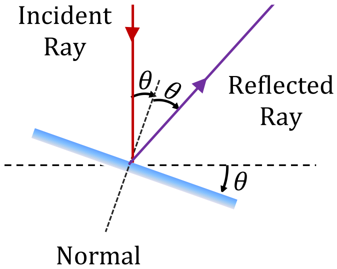
$3\theta + 90° = 180°$

$\theta = 30°$

Rotation of plane mirror

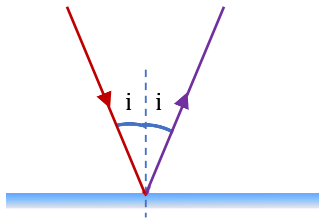
If the direction of the incident ray is kept constant and the mirror is rotated through an angle θ about an axis in the plane of mirror, then the reflected ray rotates through an angle 2θ in the same sense.





Rotation of Incident Ray

If mirror is kept fixed and incident ray is rotated, then reflected ray will rotate in opposite sense by same angle.



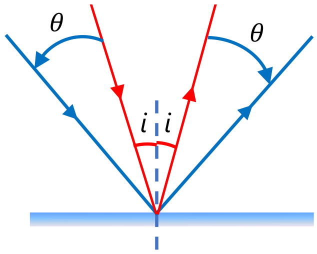
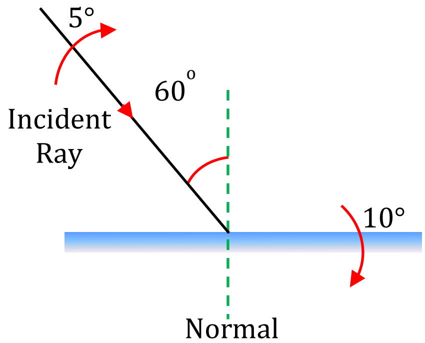


Illustration 10

Figure shows a plane mirror on which a light ray is incident. If the incident light ray is turned by 5° and the mirror by 10°, as shown, find the angle turned by the reflected ray.



Solution:

By incident ray: θ = 5° (Anticlockwise)

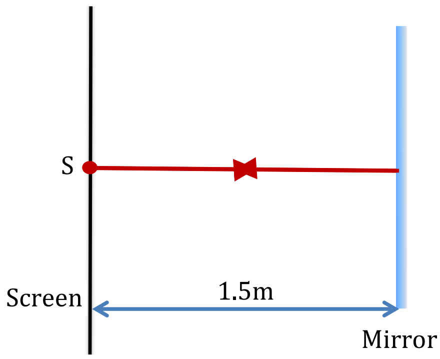
By mirror: 2θ = 2 × 10° = 20° (clockwise)

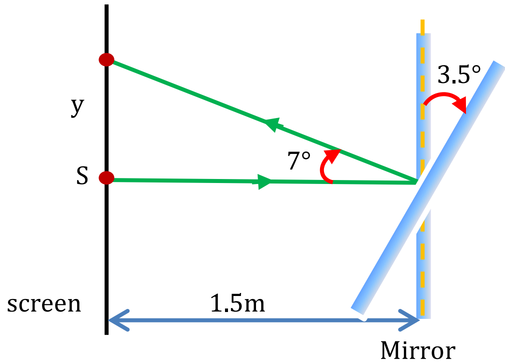
Net rotation = 20° – 5° = 15° (clockwise)

Illustration 11

A ray of light from a source S is incident normally on a plane mirror fixed at a distance 1.5 m from the source. When the mirror is rotated through a small angle 3.5°, the spot of the light is found to move through a distance y on the screen. Find out the value of y.

Solution:

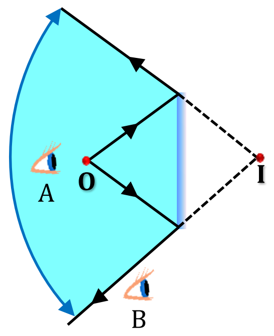




$\tan 7° = \frac{y}{1.5}$ (Angle is very small so, $\tan\theta \approx \theta$)

$7° \times \frac{\pi}{180} = \frac{y}{1.5} \Rightarrow y = 0.18$ m

Field of View



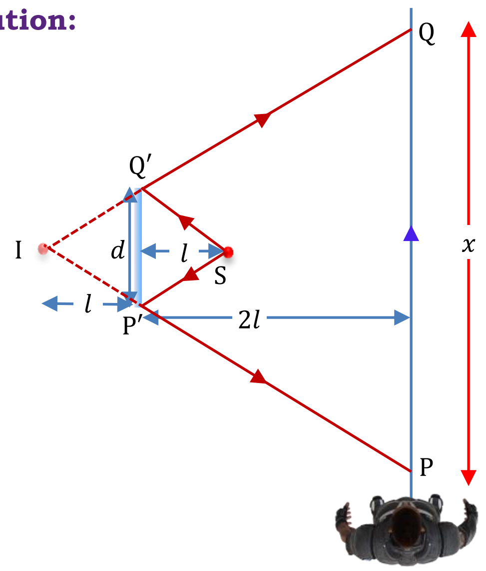
The region in which observer's eye must be present in order to view the image is called field of view.

Observer A will able to see the image because he receive reflected rays but observer B can not see the image.

Illustration 12

A point source of light S is placed at a distance l in front of the center of a mirror of width d hung vertically on a wall. A man walks in front of the mirror along a line parallel to the mirror at a distance $2l$ from it as shown in the figure. Find the greatest distance over which he can see the image of the light source from the mirror.

Solution:



Locate the position of image with the help of ray diagram in which light rays reflect from the ends of the mirror. By the concept of Similar Triangles

$$\Delta P'Q'I \sim \Delta PQI$$

$$\frac{x}{3\ell} = \frac{d}{\ell} \Rightarrow x = 3d$$

Illustration 13

A boy of height H with his eye-level at height h from the ground stands before a mirror fixed on a wall. Indicate by means of a ray diagram how the mirror should be positioned so that he can view himself fully. What should be the minimum length and position of the mirror?

Solution:

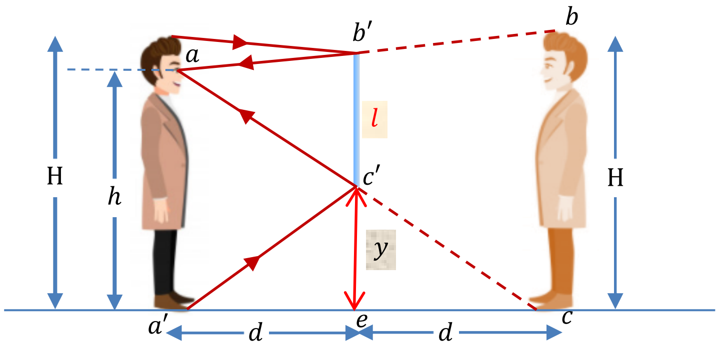
$\Delta ab'c' \sim \Delta abc$

$$\frac{l}{d} = \frac{H}{2d}$$

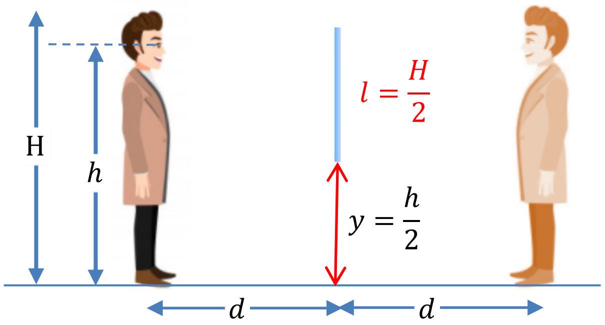
$$l = \frac{H}{2}$$

$\Delta aa'c \sim \Delta c'ec$

$$\frac{h}{2d} = \frac{y}{d} \Rightarrow y = \frac{h}{2}$$



Conclusion



- To see the complete image in a plane mirror the minimum length of plane mirror should be half the height of a person.
- Height of mirror from ground

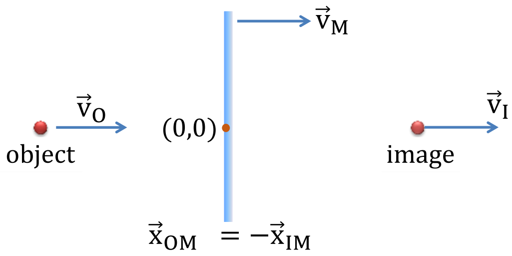
  $$y = \frac{\text{Height of eye level from ground}}{2}$$

- This result does not depend on position of eye (height of the eye from ground).
- This result is independent of the distance of person from the mirror.

Note: If height of eyes is not given in question, then consider the eyes at top of the head and solve the question

Velocity of Image in Plane Mirror

Case-1: When object is moving perpendicular to the plane mirror:



From mirror property: $\vec{x}_{OM} = -\vec{x}_{IM}$

Differentiate with respect to time,

$\vec{v}_{OM} = -\vec{v}_{IM}$

$\vec{v}_O - \vec{v}_M = -(\vec{v}_I - \vec{v}_M)$

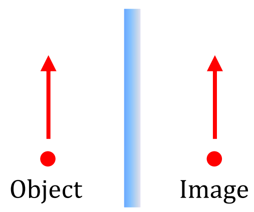
$\boxed{\vec{v}_I = 2\vec{v}_M - \vec{v}_O}$

$\vec{v}_I$ = velocity of image with respect to ground and perpendicular to the plane of mirror

$\vec{v}_o$ = velocity of object with respect to ground and perpendicular to the plane of mirror

$\vec{v}_M$ = velocity of mirror with respect to ground and perpendicular to the plane of mirror

Case-2: When object is moving parallel in front of plane mirror:



When object moves parallel to the plane mirror then the velocity of image is same as velocity of object.

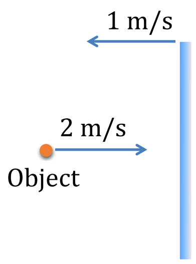
$\boxed{\vec{v}_i = \vec{v}_o}$

$\vec{v}_i$ = velocity of image with respect to ground and parallel to mirror.

$\vec{v}_o$ = velocity of object with respect to ground and parallel to mirror.

Illustration 14

Find velocity of image with respect to ground?



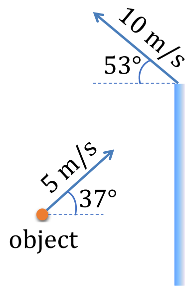
Solution:

$\vec{V}_M = -1 \text{ m/s} \Rightarrow \vec{V}_o = 2 \text{ m/s}$

$\vec{V}_I = 2\vec{V}_M - \vec{V}_o \Rightarrow \vec{V}_I = 2(-1) - 2 = -4 \text{ m/s}$

Illustration 15

Find velocity of image with respect to ground?



Solution:

For perpendicular component

$\vec{V}_I = 2\vec{V}_M - \vec{V}_o$

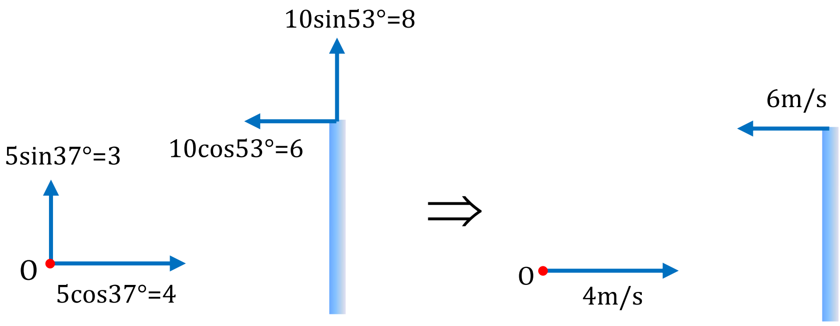
$\vec{V}_I = 2(-6) - 4 = -16$ m/s

For parallel component

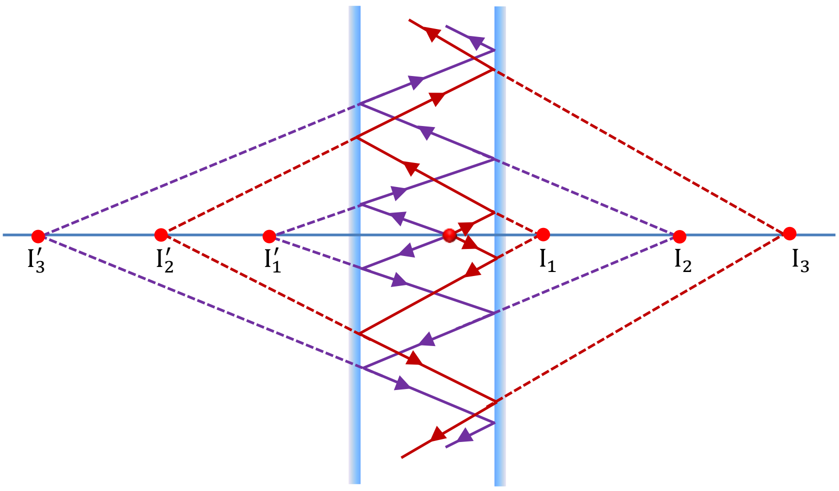
$\vec{V}_I = \vec{V}_o = 3$ m/s

So final answer

$\vec{V}_{I_F} = -16\hat{i} + 3\hat{j}$



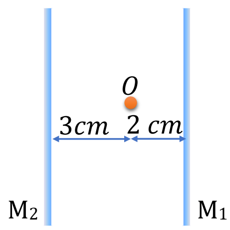
Multiple Reflections (Number of Images)



If rays after getting reflected from one mirror strike second mirror, the images formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.
For example: Image I_1 will act as an object for image I'_2 and I'_2 will act as an object for image I_3.

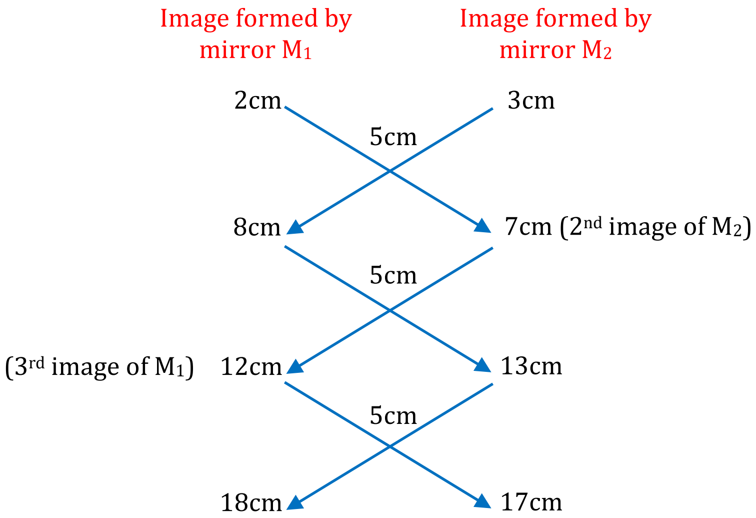
Illustration 16

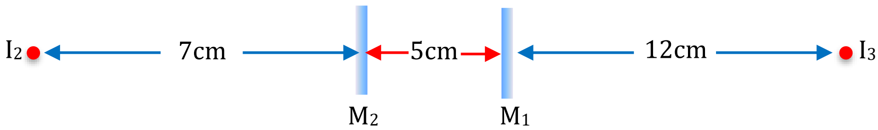
Find out distance between 3rd image of M_1 and 2nd image of M_2.



Solution:

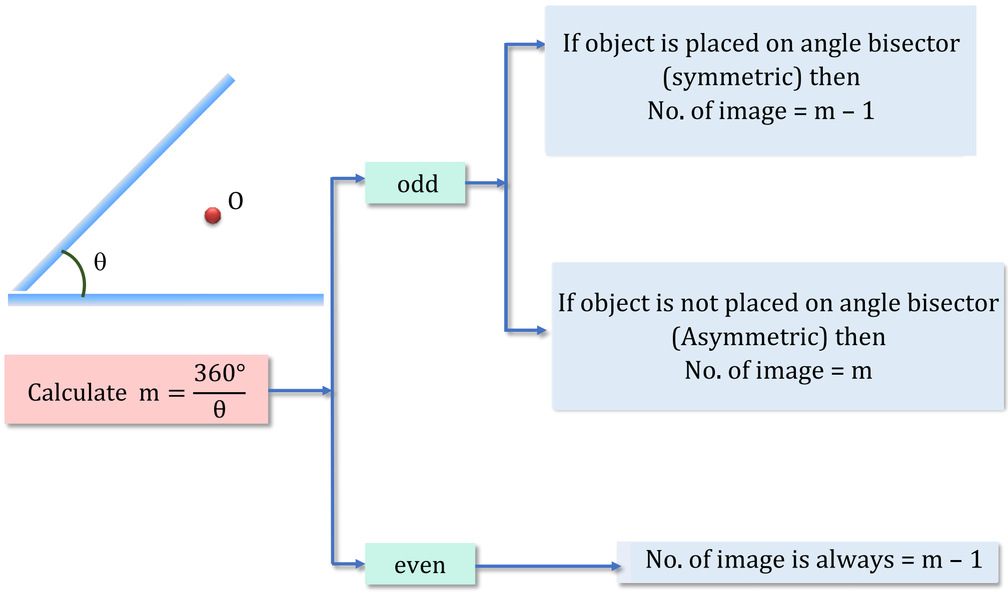
To solve the question, we use criss-cross method instead of ray-diagram.





Distance between them = 7 + 5 + 12 = 24 cm

Number of images when mirrors are inclined to each other



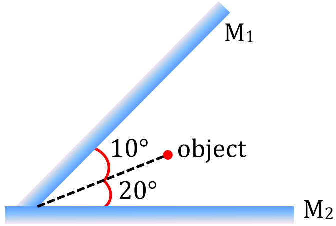
- Previous method is not useful if "m" is in fraction.

Illustration 17

Two mirrors are inclined by an angle 30°. An object is placed making 10° with the mirror M_1. Find the positions of first two images formed by each mirror. Find the total number of images using (i) direct formula and (ii) counting the images.

Solution:

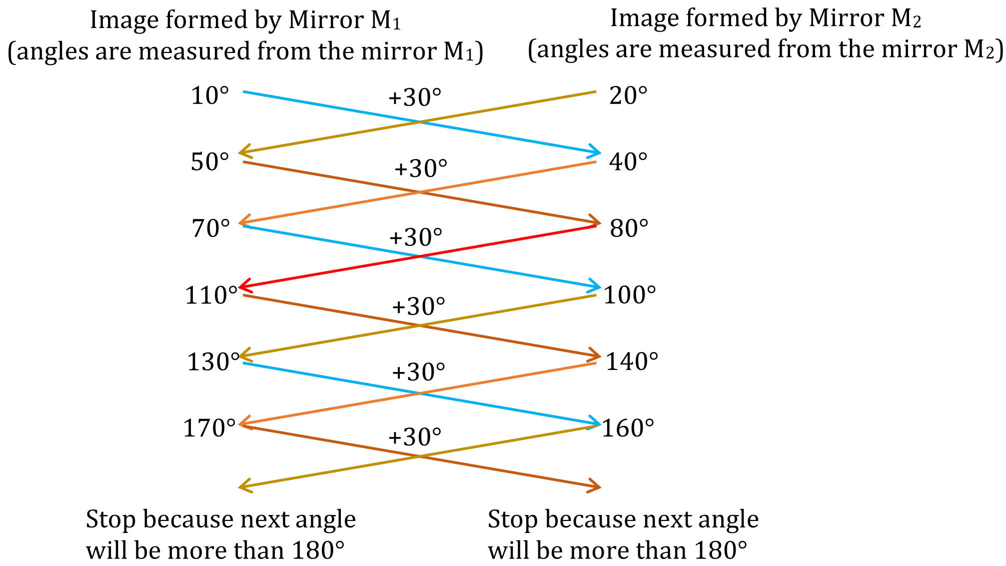
Number of images



(i) Using direct formula: $\frac{360°}{30°}$ = 12 (even number)

$\therefore$ number of images = 12 – 1 = 11

(ii) By counting. See the following table



To check whether the final images made by the two mirrors coincide or not: add the last angles and the angle between the mirrors. If it comes out to be exactly 360°, it implies that the final images formed by the two mirrors coincide.

Here last angles made by the mirrors + the angle between the mirrors = 160° + 170°+30° = 360°. Therefore, in this case the last images coincide. Therefore, the number of images = number of images formed by mirror M_1+ number of images formed by mirror M_2-1 (as the last images coincide) = 6 + 6 – 1 = 11.

Note: **1.** Criss-cross method is applicable for any angle between both the mirror.

2. If two mirrors are placed parallel and in front of each other then infinite images are formed of an object placed in between them.

3. When two plane mirrors are inclined to each other, then all the images including object always lie on a circle.

BEGINNER'S BOX-1

1. A boy is 1.8 m tall and can see his image in a plane mirror fixed on a wall. His eyes are 1.6 m from the floor level. The minimum length of the mirror to see his full image is :-

(1) 0.9 m (2) 0.85 m (3) 0.8 m (4) can't be determined

PRI180

2. An object shaped as 'L' is placed between two parallel plane mirrors as shown. In the first seven closest images, how many laterally inverted images are formed?

(1) 1 (2) 2 (3) 3 (4) 4

PRI181

3. A plane mirror is placed horizontally on level ground at a distance of 60 m from the foot of a tower. Light rays from the top of tower falling just on the edge of the mirror suffers a deviation of 90°. The height of the tower is: -

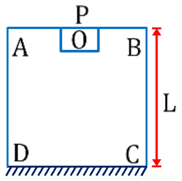
(1) 30 m (2) 60 m (3) 90 m (4) 120 m

PRI182

4. A plane mirror makes an angle of 30° with the horizontal. If a vertical ray strikes the mirror, find the angle between the mirror and the reflected ray?

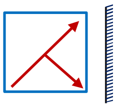
PRI183

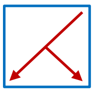
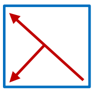
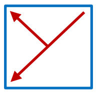
5. Figure shows the plane (top view) of a cubical room, with the wall CD as a plane mirror; each side of the room is 3 metres in length. A camera P is placed at the mid point of the wall AB. At what distance should the camera be focused to photograph the image of an object placed at A?



PRI184

6. The correct mirror–image of the figure given is :-

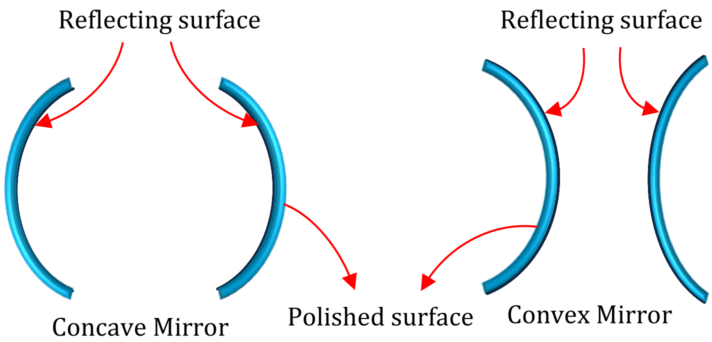


(1)  (2)  (3)  (4) 

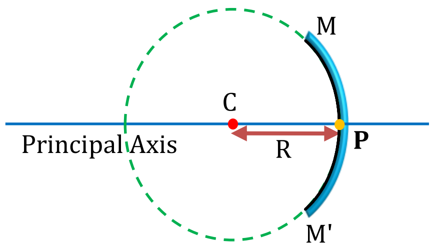
PRI185

Reflection from Spherical Surface (Spherical Mirror)

A spherical mirror is a mirror that has the shape of a piece cut out of a spherical surface. Curved mirror is part of a hollow sphere. If reflection takes place from the inner surface, then the mirror is called concave and if its outer surface acts as reflector it is convex.



Basic term related to Spherical mirror



C = Centre of curvature

R = Radius of curvature

P = Pole (geometric centre of mirror)

(i) **Pole** is any point on the reflecting surface of the mirror. For convenience we take it as the central point, P of the mirror.

(ii) **Principal–section** is any section of the mirror such as MM' passing through the pole.

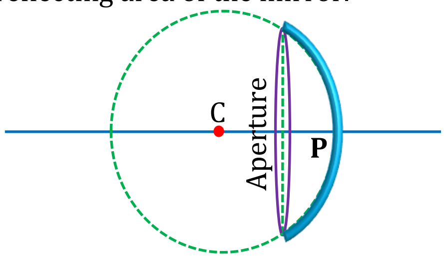
(iii) **Centre of curvature** is the centre C of the sphere of which the mirror is a part.

(iv) **Radius of curvature** is the radius R of the sphere of which the mirror is a part.

(v) **Principal–axis** is the line CP, joining the pole and centre of curvature of the mirror.

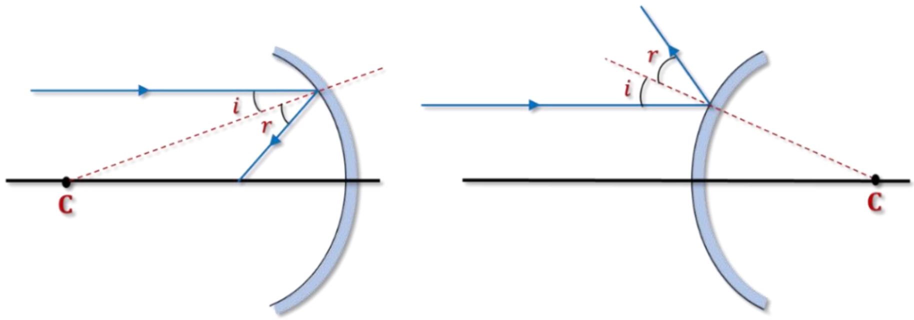
Aperture

It is the effective diameter of the light reflecting area of the mirror.



How to draw the normal in spherical mirror:

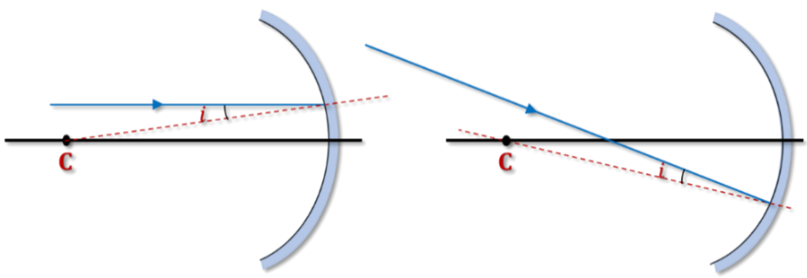
We can draw normal to a spherical mirror at a particular point by joining the particular point with the centre of curvature of the mirror.



Concave mirror (converging mirror)

Convex mirror (diverging)

Paraxial Rays: Those rays which make very small angle with normal at point of incidence is called paraxial rays.

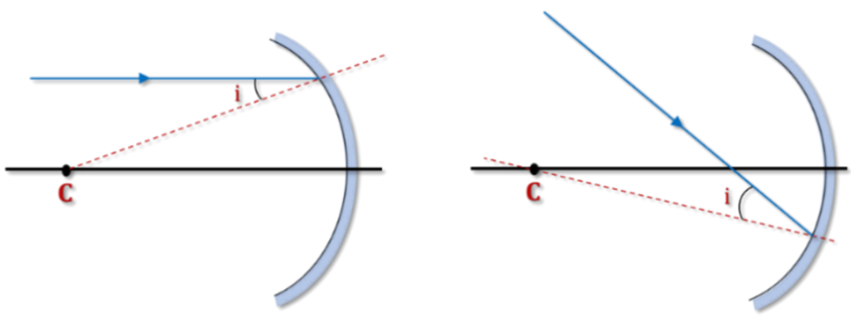


∠i is small

Paraxial Rays need not to be parallel to principle axis.

Marginal Rays: Those rays which make large angle with normal at a point of incidence is called marginal rays.

∠i is large



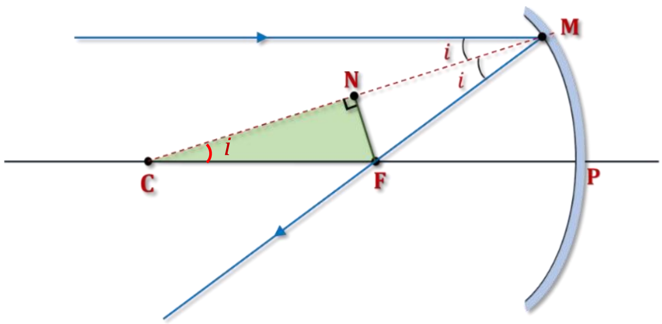
Focal length of a spherical mirror

What happens when a parallel beam of light is incident on a concave mirror, or a convex mirror. We assume that the rays are paraxial, i.e., they are incident at points closed the pole P of the mirror and make small angles with the principal axis. The reflected rays converge at a point F on the principal axis of a concave mirror. For a convex mirror, the reflected rays appear to diverge from a point F on its principal axis. The point F is called the principal focus of the mirror.

Derivation:

$\Delta CNF \sim \Delta NFM$

Then, $CN = NM = \dfrac{R}{2}$



$\cos i = \dfrac{CN}{CF} = \dfrac{\left(\frac{R}{2}\right)}{CF}$

$CF = \dfrac{R}{2\cos i}$

$FP = CP - CF = R - \dfrac{R}{2\cos i}$

If i is very small i.e. rays are paraxial, then $\cos i \approx 1$

$FP = R - \dfrac{R}{2} \Rightarrow \boxed{FP = \dfrac{R}{2} = f}$ (Here, f is called focal length of mirror).

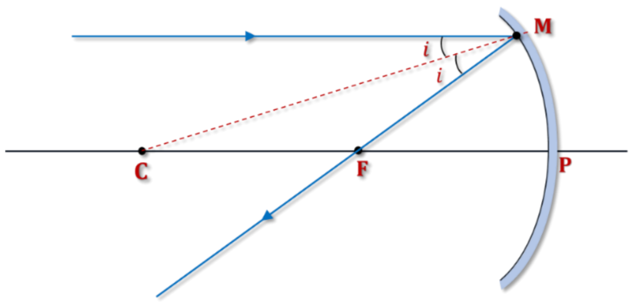
Conclusion:

For marginal rays,

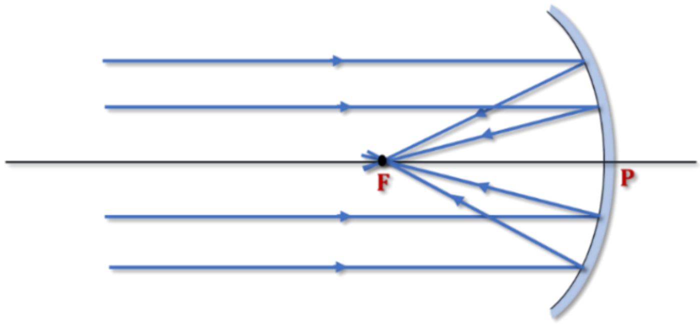
$\mathbf{FP = R - \dfrac{R}{2\cos i}}$

If rays are paraxial then,

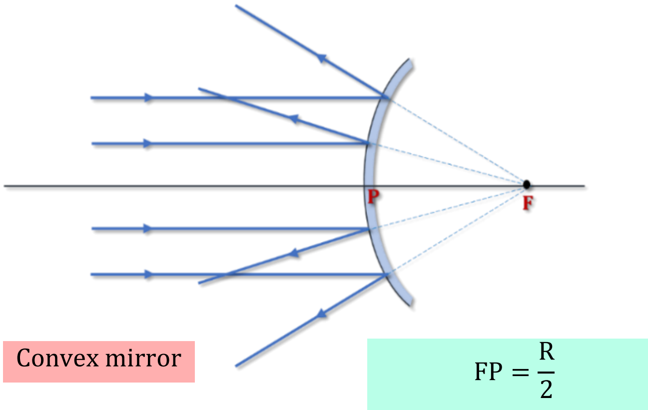
$\mathbf{FP = f = \dfrac{R}{2}}$



➢ When paraxial light rays parallel to principal axis are incident on a mirror then the point where they meet (concave mirror) or appears to meet (convex mirror) after reflection is knows as focus (F).



Concave mirror $FP = \dfrac{R}{2}$



Focal Plane

If the parallel paraxial beam of light were incident, making some angle with the principal axis, the reflected rays would converge (or appear to diverge) from a point in a plane through F normal to the principal axis. This is called the focal plane of the mirror.

If the parallel paraxial beam of light were incident, making some angle with plane passing through focus and perpendicular to the principal axis,

$\tan\alpha = \dfrac{h}{FP}$

Rays are paraxial so,

$\tan\alpha \approx \alpha$

$\alpha = \dfrac{h}{f}$

$\boxed{h = f\alpha}$

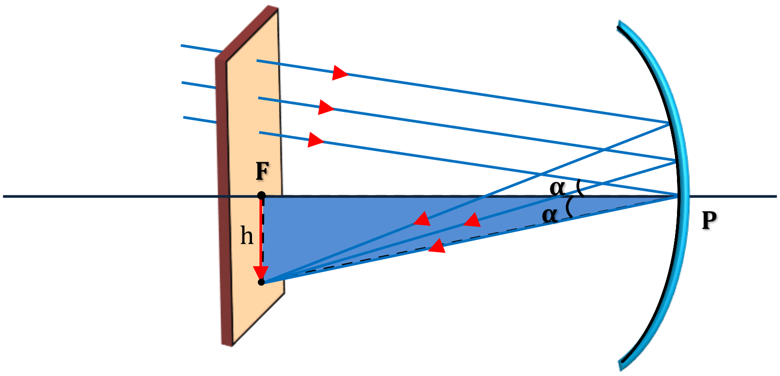
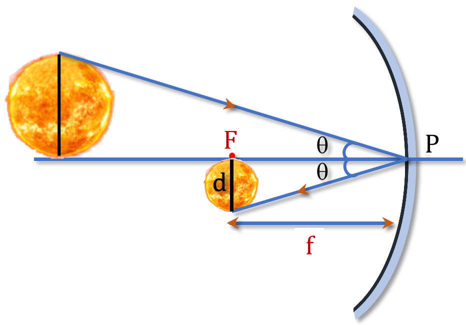


Illustration 18

The sun subtends an angle θ radians at the pole of a concave mirror of focal length f. What is the diameter of the image of the sun formed by the mirror?

Solution:



$\tan\theta = \dfrac{d}{FP}$

Rays are paraxial so,

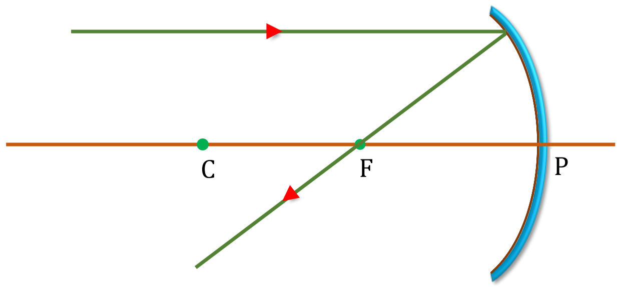
$\tan\theta \approx \theta$

$\theta = \dfrac{d}{f} \Rightarrow d = \theta f$

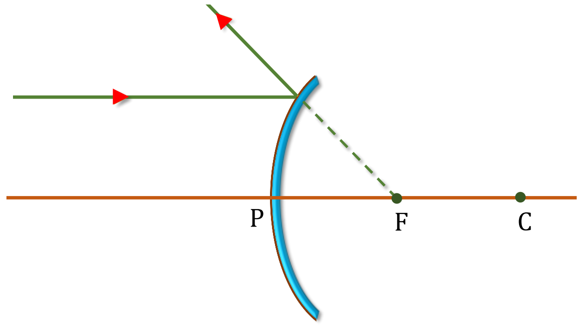
$\therefore$ Diameter of image = θf

Rules for image formation: (for paraxial rays only, based on the laws of reflection)

(1) A light ray parallel to the principal axis after reflection from the mirror passes or appears to pass through its focus (by definition of focus).

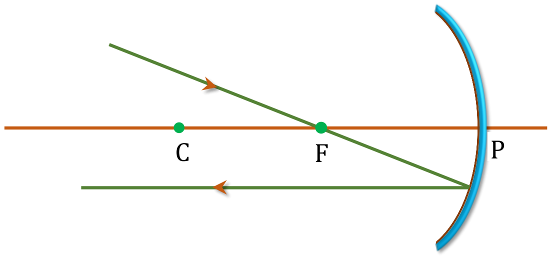


Concave mirror

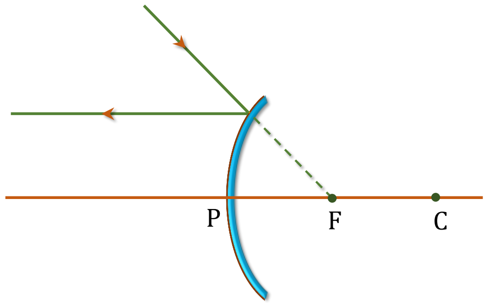


Convex mirror

(2) A light ray passing through or directed towards focus, becomes parallel to the principal axis after reflection from the mirror.

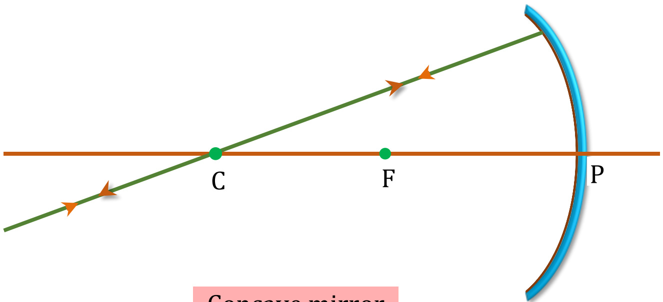


Concave mirror

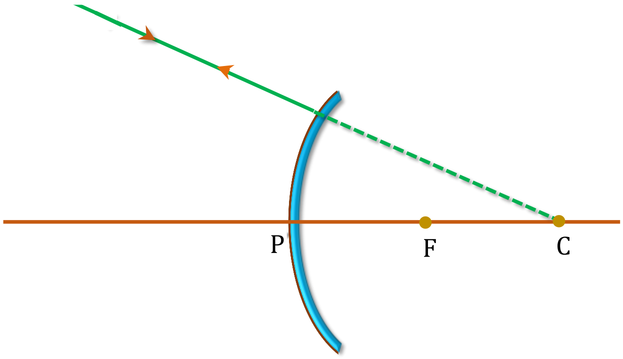


Convex mirror

(3) A ray passing through or directed towards centre of curvature, retraces its path (as for it $\angle i = 0$ and so $\angle r = 0$) after reflection from the mirror.

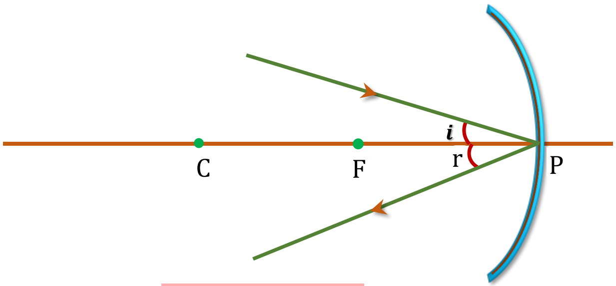


Concave mirror

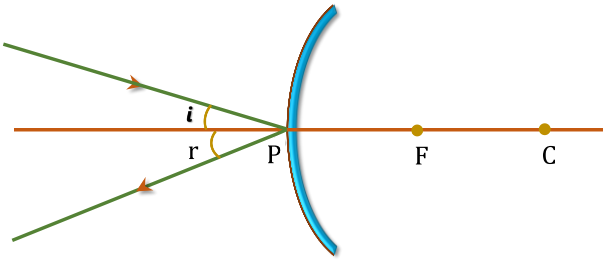


Convex mirror

(4) Incident and reflected rays at the pole of a mirror are symmetrical about the principal axis $\angle i = \angle r$.

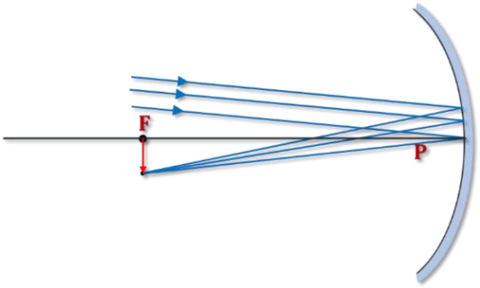
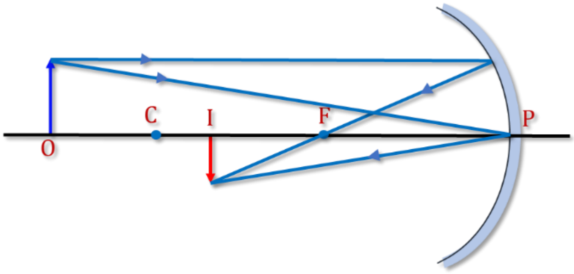
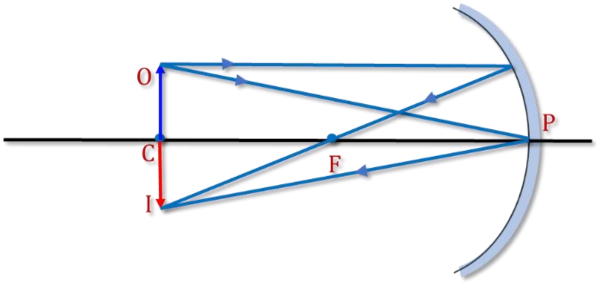
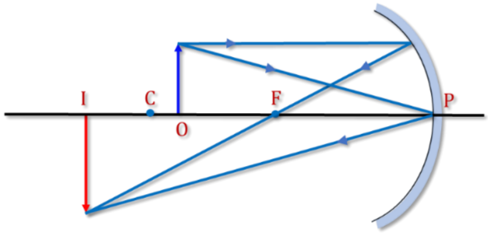
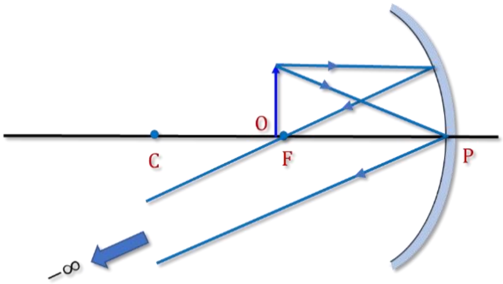


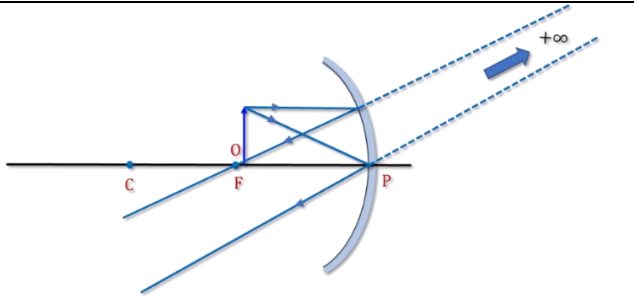
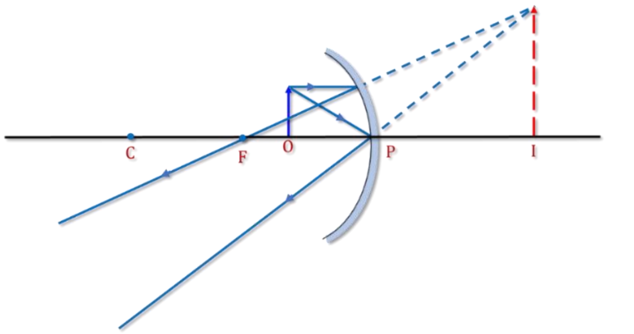
Concave mirror



Convex mirror

Image Formation by Concave Mirror

S.N.	Position of Object	Position of Image	Nature	Size	Ray Diagram
1.	Place at infinity	Formed at F	Real, Inverted	Highly diminished	
2.	Placed in between infinity and C	Formed in between C and F	Real, Inverted	Diminished	
3.	Placed at C	Formed at C	Real, Inverted	Equal in size	
4.	Placed in between F and C	Formed beyond C	Real, Inverted	Enlarged	
5.	Placed between F and C, very near to F	Formed at negative infinity	Real, Inverted	Highly enlarged	

6.	Placed between F and pole, very near to F	Formed at positive infinity	Virtual, erect	Highly enlarged	
7.	Placed between F and P	Formed behind the mirror	Virtual, erect	Enlarged	

Things to Keep in mind while solving the problems:

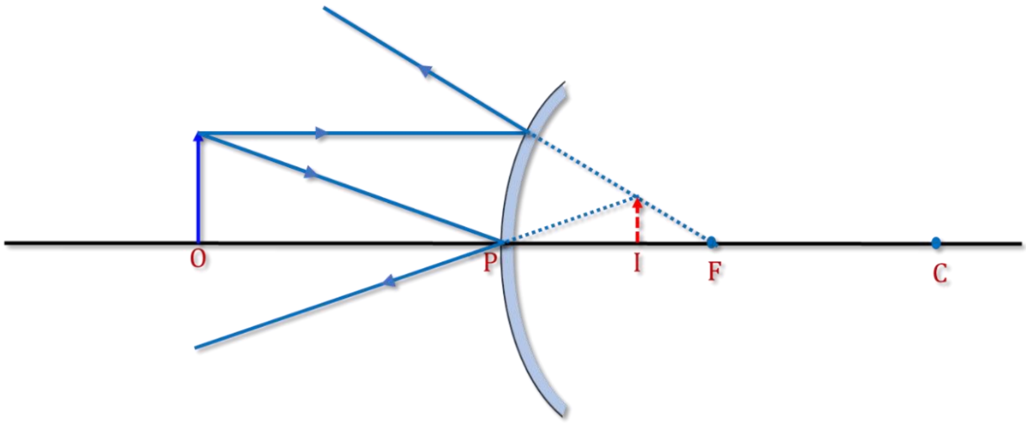
Real Object → **Concave mirror**

- Real image → Inverted → Small / Same / Large
- Virtual image → Erect → Large

Note: Concave mirror always form real image for virtual object.

Image formation by convex mirror

Image is always virtual and erect, whatever be the position of the real object.

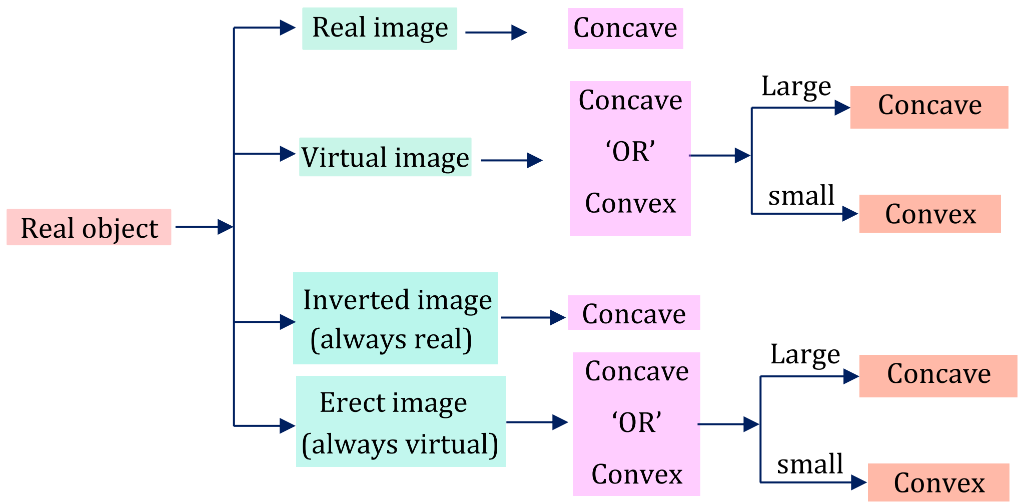


Keep in mind during solving the problems

Real Object → Convex mirror → Virtual image → Erect → Small

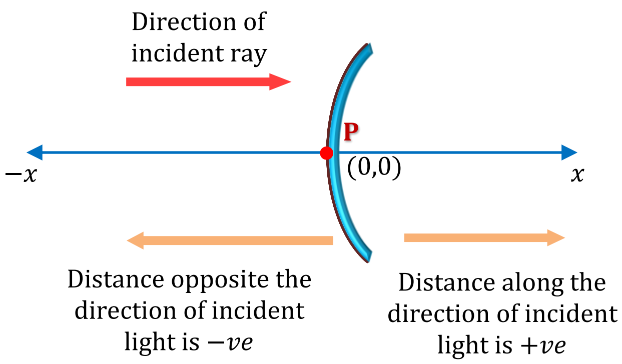
Convex mirror form both real & virtual Image of virtual object, depends on the position of virtual object.

Things to keep in mind while solving a problem:

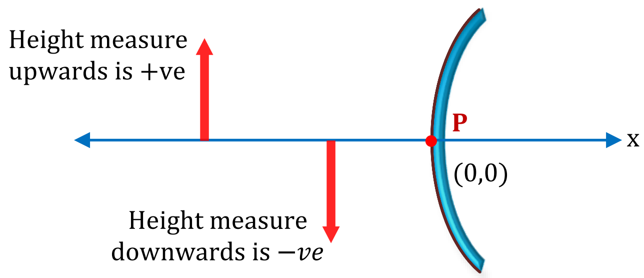


Sign Convention & Mirror Formula

Sign Convention: To derive the relevant formulae for reflection by spherical mirrors and refraction by spherical lenses, we must first adopt a sign convention for measuring distances. We shall follow the Cartesian sign convention. According to this convention, all distances are measured from the pole of the mirror or the optical centre of the lenses.



- Along the principal axis, distances are measured from the pole (Pole is taken as the origin).
- Distances in the direction of incident light are taken positive while those along opposite directions are taken negative.
- The distances above the principal axis are taken positive while below it is negative.
- Whenever and wherever possible, incident light is taken to travel from left to right.



Things to Keep in mind while solving a problem:

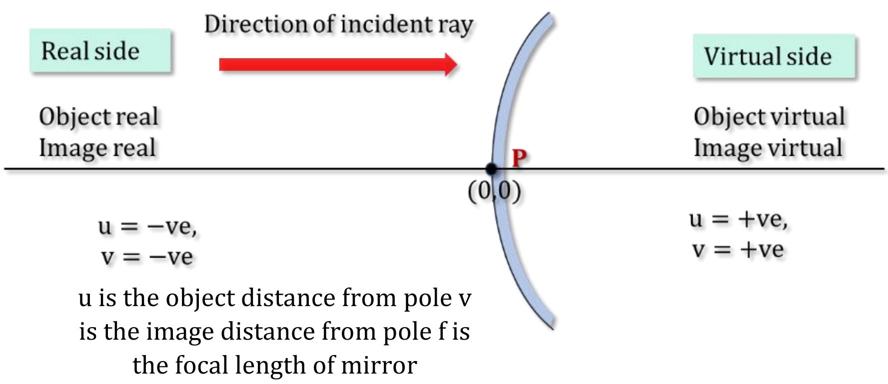
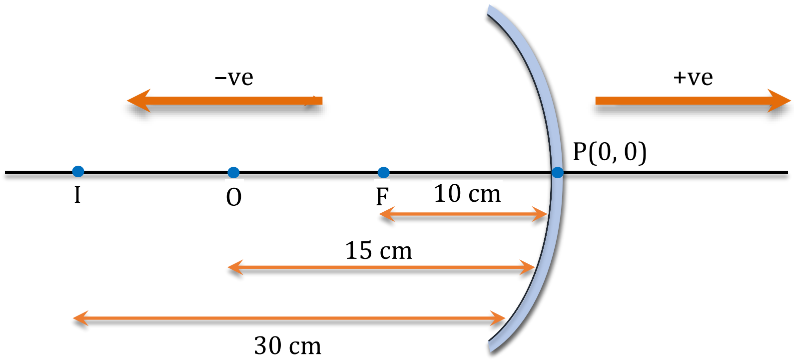


Illustration 19

A point object is placed at distance 15 cm in front of concave mirror of focal length 10 cm and its image formed at distance 30 cm in front of mirror. Comment on sign of u, v and f :

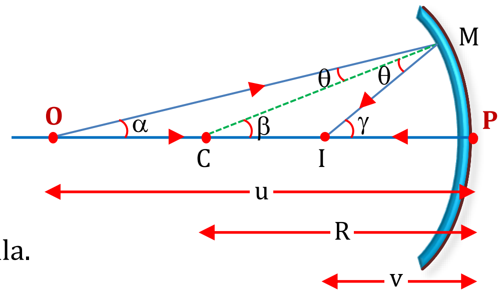
Solution:



U = negative, v = Negative, f = Negative

Mirror Formula: Relation between u, v and f in spherical mirror.

An object is placed at a distance u from the pole of a mirror for small angles and its image is formed at a distance v (from the pole).



If angle is very small : $\alpha = \frac{MP}{u}, \beta = \frac{MP}{R}, \gamma = \frac{MP}{v}$

From ΔCMO, $\beta = \alpha + \theta \Rightarrow \theta = \beta - \alpha$

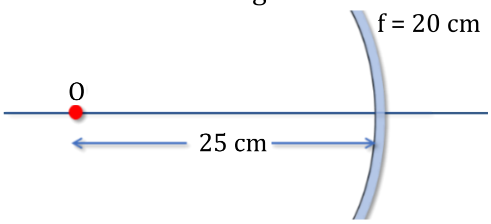
From ΔCMI, $\gamma = \beta + \theta \Rightarrow \theta = \gamma - \beta$

So we can write $\beta - \alpha = \gamma - \beta \Rightarrow 2\beta = \gamma + \alpha$

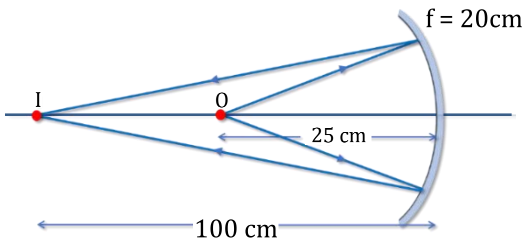
$\therefore \frac{2}{R} = \frac{1}{v} + \frac{1}{u} \Rightarrow \boxed{\frac{1}{f} = \frac{1}{u} + \frac{1}{v}}$ This equation is called mirror formula.

Illustration 20

Find out position and nature of image?



Solution:



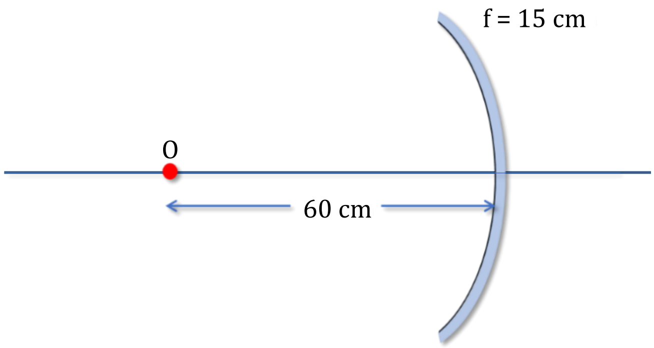
$u = -25$ cm, $f = -20$ cm, $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} + \frac{1}{-25} = \frac{1}{-20}$$

$\Rightarrow v = -100$ cm, real image.

Illustration 21

Find out position and nature of image?



Solution:

$u = -60$ cm, $f = -15$ cm,

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

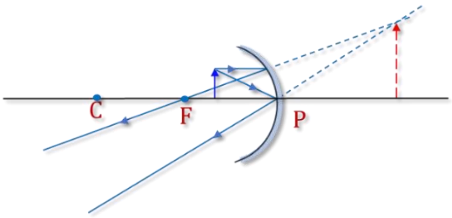
$$\frac{1}{v} + \frac{1}{-60} = \frac{1}{-15}$$

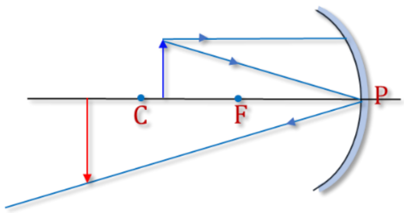
$\Rightarrow v = -20$ cm, real image.

Transverse or Lateral Magnification

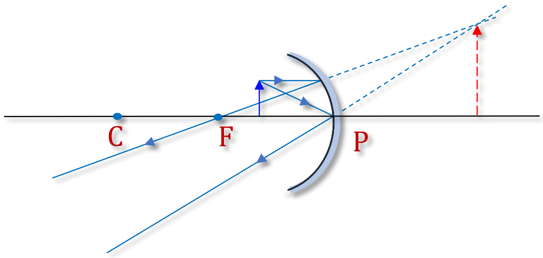
If one dimensional object is placed perpendicular to the principal axis then ratio of image height and object height is called **transverse or lateral magnification**.

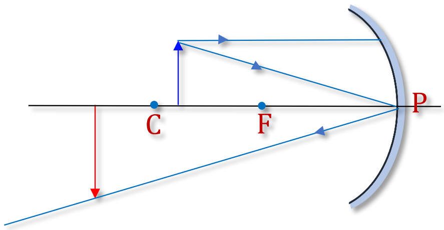
$$\boxed{m_t = \frac{h_i}{h_o}}$$



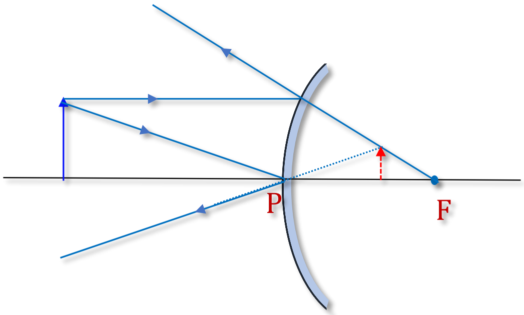


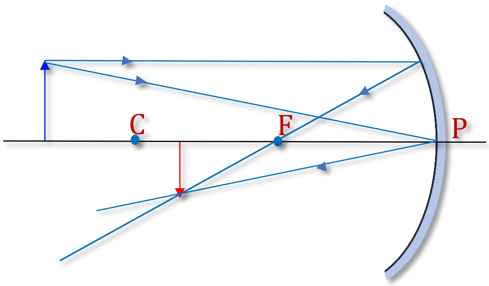
If m_t is positive means erect image is formed   If m_t is negative means inverted image is formed



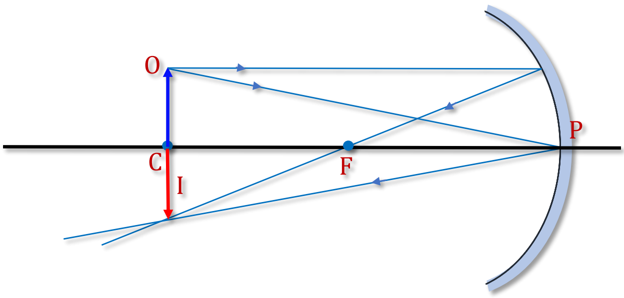


$|m_t| > 1$





$|m_t| < 1$



$|m_t| = 1$

Derivation of Transverse Magnification:

$$\frac{h_i}{h_o} = \frac{v}{u}$$

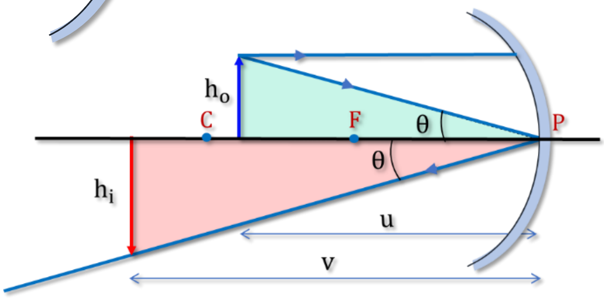
Put the value with sign,

$u \rightarrow -u$, $v \rightarrow -v$, $h_i \rightarrow -h_i$ & $h_o \rightarrow +h_o$, then

$$\frac{-h_i}{h_o} = \frac{-v}{-u}$$

$$\frac{h_i}{h_o} = -\frac{v}{u}$$

$$\boxed{m_t = \frac{h_i}{h_o} = -\frac{v}{u}}$$



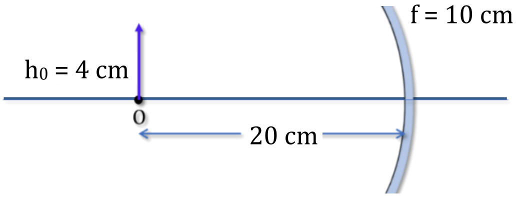
$$\left.\begin{array}{l} \tan\theta = \dfrac{h_o}{u} \\ \tan\theta = \dfrac{h_i}{v} \end{array}\right\} \frac{h_i}{h_o} = \frac{v}{u}$$

Important Results: $m_t = \frac{h_i}{h_o}$

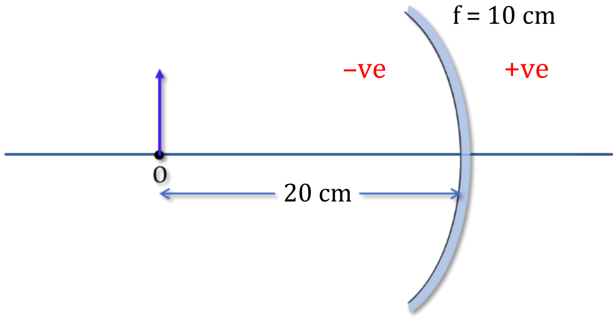
- $m_t = \frac{h_i}{h_o} = -\frac{v}{u}$
- $m_t = \frac{h_i}{h_o} = \frac{f}{f-u} = \frac{f-v}{f}$

Illustration 22

Find out position, nature & size of image of the object as shown in figure.



Solution:



u = –20 cm ; f = – 10 cm

$m = \frac{f}{f-u} = \frac{-v}{u}$

$m = \frac{-10}{-10-(-20)} = \frac{-v}{-20}$

V = –20 cm

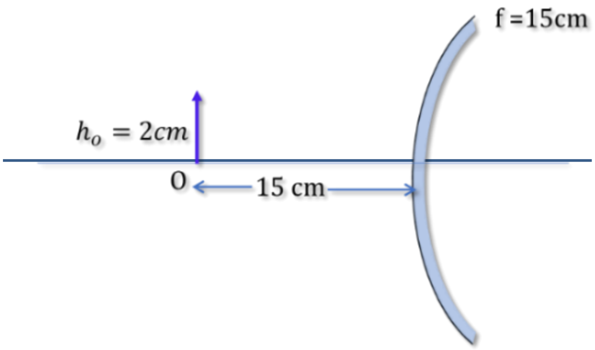
$m = \frac{h_I}{h_O} = \frac{h_I}{4}$

$h_I = -4$ cm

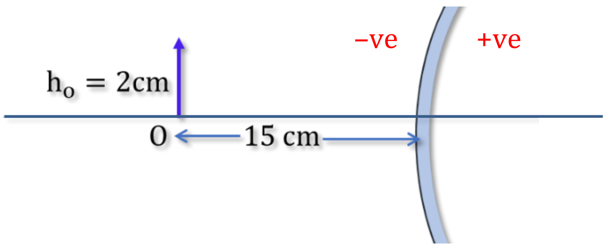
Real inverted and of same size as that of object.

Illustration 23

Find out position, nature & size of image of the object as shown in figure.



Solution:



Given; u = –15 cm ; f = 15 cm

$m = \frac{f}{f-u} \Rightarrow m = \frac{15}{15-(-15)} = \frac{1}{2}$

$m = \frac{-v}{u} \Rightarrow \frac{1}{2} = \frac{-v}{-15}$

v = 7.5 cm

$h_i = m \times h_o$

$h_i = \frac{1}{2} \times 2 = 1$ cm

Virtual erect and smaller than object.

Illustration 24

Find out position of an object placed in front of concave mirror of focal length 30cm so that 2 times magnified image is formed.

Solution:

Case-1

Given; $m = -2$

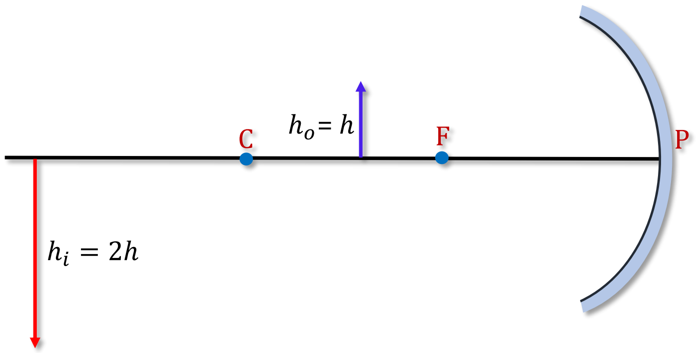
$$-2 = \frac{f}{f-u}$$

$$-2 = \frac{-30}{-30-u}$$

$$60 + 2u = -30$$

$$2u = -90$$

$$u = -45 \text{ cm}$$



Case-2

Given; $m = +2$

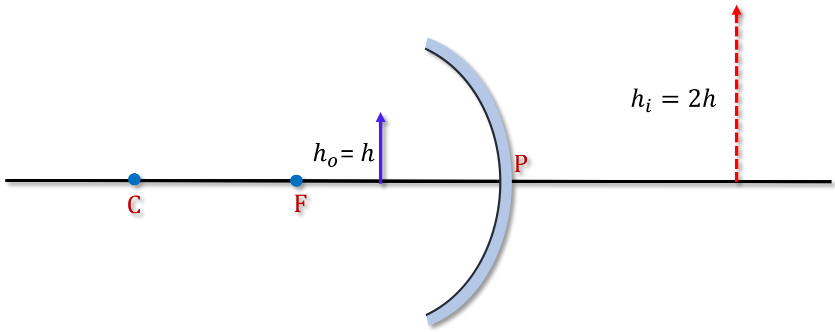
$$+2 = \frac{f}{f-u}$$

$$2 = \frac{-30}{-30-u}$$

$$-60 - 2u = -30$$

$$2u = -30$$

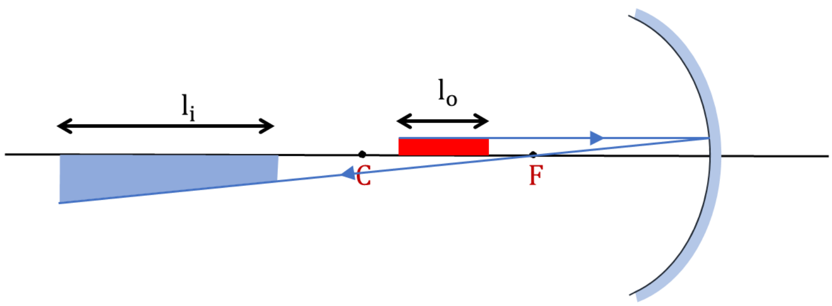
$$u = -15 \text{ cm}$$



Longitudinal Magnification

If an object is placed along the principal axis then ratio of length of image and length of object is called longitudinal or axial magnification.

$$m_L = \frac{\text{length of image}}{\text{length of object}} = \frac{\ell_i}{\ell_o}$$



$$\boxed{m_L = \frac{\text{length of image}}{\text{length of object}} = \left|\frac{v_1 - v_2}{u_2 - u_1}\right|}$$

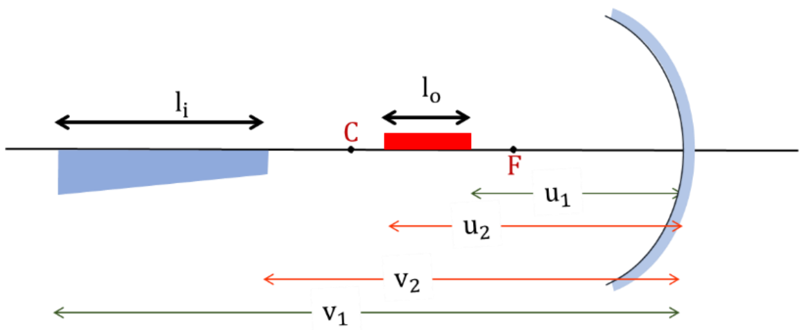
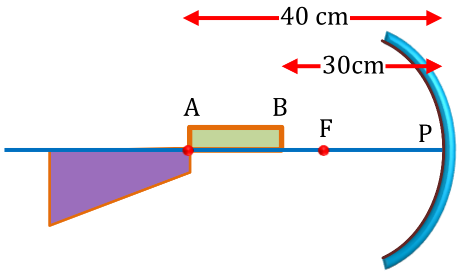


Illustration 25

A thin rod of length 10 cm is placed along the principal axis of a concave mirror of focal length 20 cm such that its image which is real and elongated, just touches one end of the rod. Find out the length of image?

Solution:



For point A; u = –40 cm, f = –20 cm

$$v_A = \frac{u \times f}{u - f} = \frac{-40 \times -20}{-40 + 20} = -40\text{cm}$$

For point B; u = –30 cm, f = –20 cm

$$v_A = \frac{uf}{u - f} = \frac{-30 \times -20}{-30 + 20} = -60 \text{ cm}$$

Length of image = 60 – 40 = 20 cm.

Longitudinal magnification for small objects ($L_o << f$):

object length = du; Image length = dv

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

Differentiating w.r.t . u

$$-\frac{1}{v^2}\frac{dv}{du} - \frac{1}{u^2} = 0 \Rightarrow \frac{dv}{du} = -\frac{v^2}{u^2}$$

If we use only magnitude then $\frac{dv}{du} = \frac{v^2}{u^2}$

$$\boxed{M_L = \frac{\ell_i}{\ell_o} = \frac{v^2}{u^2} = m_t^2}$$

Illustration 26

A point object is placed at a distance of 15 cm from a concave mirror of radius of curvature 20 cm. If the object is made to oscillate along the principal axis with amplitude 2 mm. Find out the amplitude of its image.

Solution:

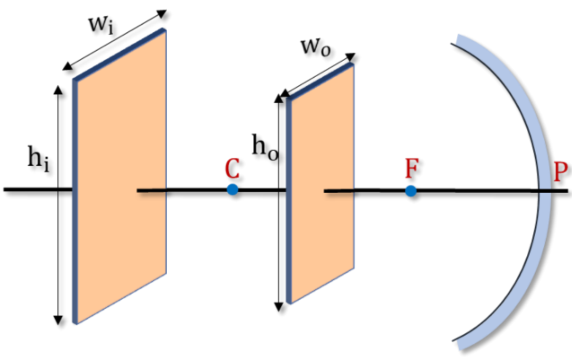
Amplitude of object |du| = 2 mm

$$m = \frac{f}{f - u} = \frac{-10}{-10 - (-15)} = -2$$

amplitude of image $|dv| = m^2 |du| = (-2)^2 \times 2 = 8$ mm

Superficial Magnification

If two-dimensional object is placed with its plane perpendicular to the principal axis then its magnification is known as superficial magnification.



$$m_s = \frac{A_i}{A_o}$$

$A_o = w_o h_o$ and $A_i = w_i h_i$

$$m_t = \frac{h_i}{h_o} = \frac{w_i}{w_o}$$

$$\frac{A_i}{A_o} = \frac{w_i h_i}{w_o h_o}$$

$$m_t = \frac{h_i}{h_o} = \frac{w_i}{w_o}$$

$$\boxed{\frac{A_i}{A_o} = m_t^2 = m_s}$$

Illustration 27

A square of side 4 mm is placed at a distance of 30 cm from a concave mirror of focal length 10 cm. The centre of the square is at the axis of the mirror and the plane is normal to the axis. Find the area enclosed by the image?

Solution:

$$m_t = \frac{f}{f-u} = \frac{-10}{-10-(-30)} = -\frac{1}{2}$$

$$\frac{A_i}{A_o} = m_t^2 \Rightarrow \frac{A_i}{(4mm)^2} = \left(-\frac{1}{2}\right)^2 \Rightarrow A_i = 4mm^2$$

Newton's Formula

x_1 : Object distance from focus.

x_2 : Image distance from focus.

The relation between x_1, x_2, f is,

$\boxed{f = \sqrt{x_1 x_2}}$

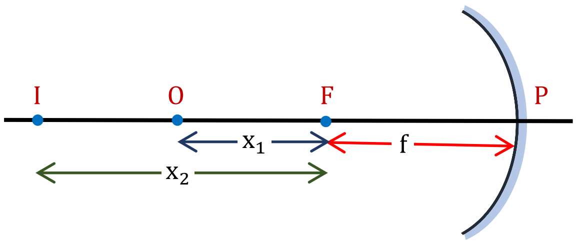


Illustration 28

If an object is placed at a distance 30 cm from focus of a concave mirror of focal length 60 cm, find distance between image and focus of mirror.

Solution:

Given, $x_1 = 30$ cm; $f = 60$ cm

So, $f^2 = x_1x_2 \Rightarrow (60)^2 = 30\ x_2 \Rightarrow x_2 = 120$ cm

Velocity of image in spherical mirror

Case (1): When object is moving along the principal axis of spherical mirror.

From equation $\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$

Differentiate with respect to time,

$$-\frac{1}{u^2}\frac{du}{dt} + \frac{-1}{v^2}\frac{dv}{dt} = 0$$

$$\frac{1}{v^2}\frac{dv}{dt} = -\frac{1}{u^2}\frac{du}{dt}$$

$$\frac{dv}{dt} = -\frac{v^2}{u^2}\frac{du}{dt}$$

$\boxed{\vec{V}_{IM} = -m_t^2\,\vec{V}_{OM}}$ All velocities are instantaneous.

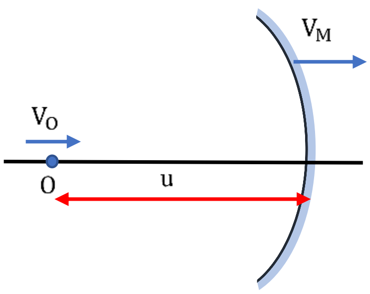
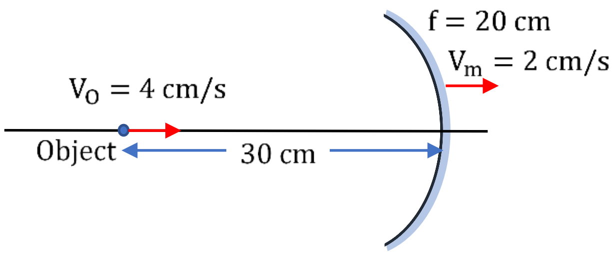


Illustration 29

Find out velocity of image.



Solution:

$$m_t = \frac{f}{f-u} = \frac{-20}{-20-(-30)} = -2$$

$$\vec{V}_{IM} = -m_t^2\,\vec{V}_{OM}$$

$$\vec{V}_I - \vec{V}_M = -m_t^2\left(\vec{V}_O - \vec{V}_M\right)$$

$$\vec{V}_I - 2 = -4(4-2)$$

$\vec{V}_I = -6$ cm/s ; So, image moves with speed 6 cm/s away from the mirror.

Case (2): When object is moving perpendicular to the principal axis of spherical mirror.

From equation $\frac{h_i}{h_o} = -\frac{v}{u}$

Differentiate with respect to time,

$$\frac{dh_i}{dt} = -\frac{v}{u}\frac{dh_o}{dt}$$

$$\boxed{\vec{V}_{IM} = m_t \vec{V}_{OM}}$$

All velocities are instantaneous.

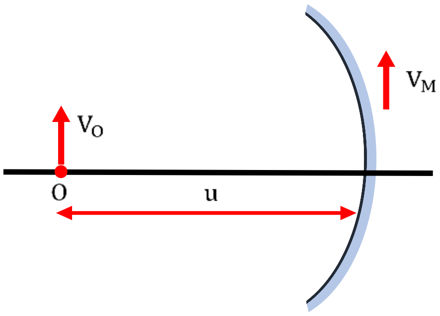
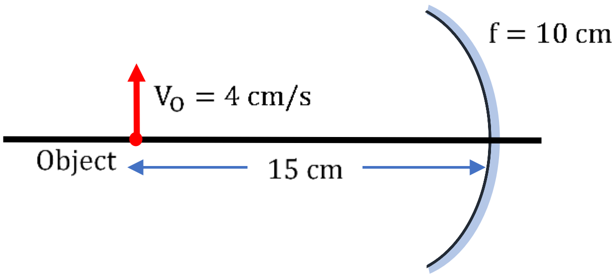


Illustration 30

Find out velocity of image.



Solution:

$$m_t = \frac{f}{f-u} = \frac{-10}{-10+15} = -2$$

$$\vec{V}_{IM} = m_t \vec{V}_{OM}$$

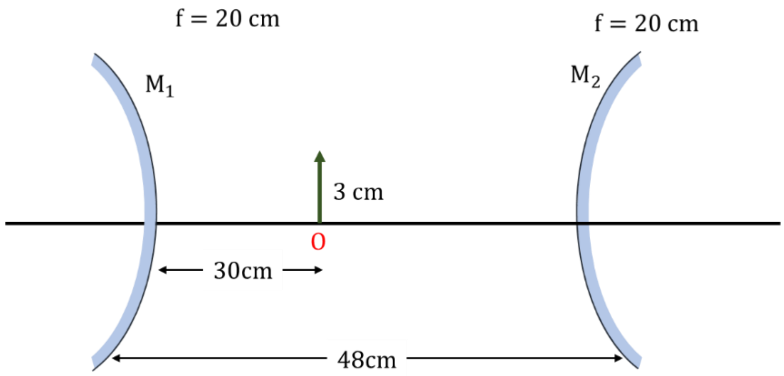
$$\vec{V}_I - \vec{V}_M = m_t\left(\vec{V}_O - \vec{V}_M\right)$$

$$\vec{V}_I - 0 = -2(4) \Rightarrow \vec{V}_I = -8 \text{ cm/s}$$

Combination of Mirrors

Illustration 31

Find out height of image formed after second reflection (consider first reflection on mirror M_1)



Solution:

For mirror M_1 (First reflection)

$u = -30$ cm, $f = 20$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{v} + \frac{1}{-30} = \frac{1}{20}$$

$v = 12$ cm

$$\frac{h_{I_1}}{h_o} = -\frac{v}{u} \Rightarrow \frac{h_{I_1}}{3} = -\frac{12}{-30}$$

$$h_{I_1} = 1.2$$

$$\frac{h_{I_2}}{h_{I_1}} = -\frac{15}{-60} \Rightarrow \frac{h_{I_2}}{1.2} = -\frac{15}{-60}$$

$$h_{I_2} = 0.3 \text{ cm}$$

for mirror M_2

$u = -60$ cm, $f = 20$ cm

$$\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} + \frac{1}{-60} = \frac{1}{20}$$

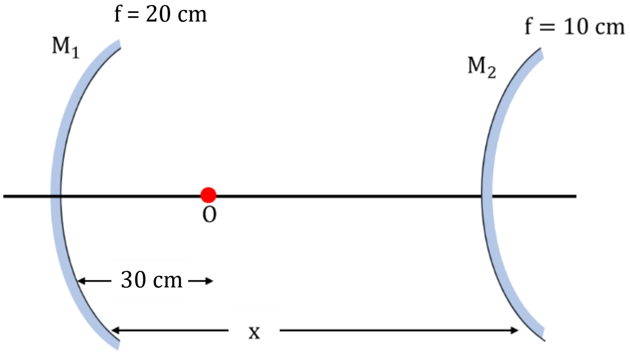
$v = 15$ cm

$$\text{Total magnification} = \frac{\text{Height of final image}}{\text{Height of object}} = \frac{0.3}{3} = \frac{1}{10}$$

Note: In case of successive reflection from mirrors, the overall lateral magnification is given by $m_1 \times m_2 \times m_3$ …….., where m_1, m_2 etc. are lateral magnifications produced by individual mirrors.

Illustration 32

Find out the value of x such that light ray retraces its path after reflection. (Consider first reflection on mirror M_1)



Solution:

For mirror M_1, u = –30 cm, f = –20 cm

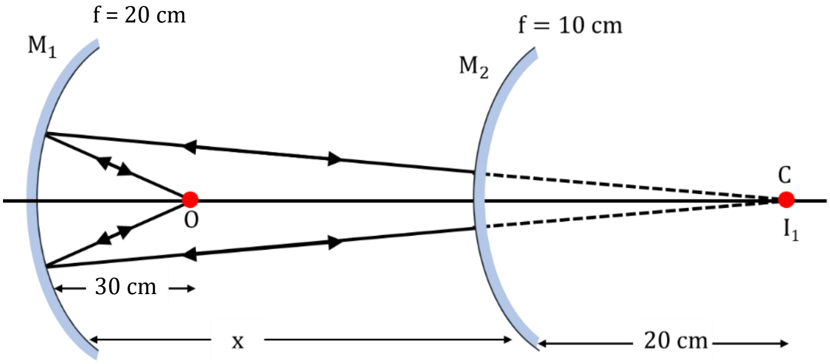
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}+\frac{1}{-30}=\frac{1}{-20}$$

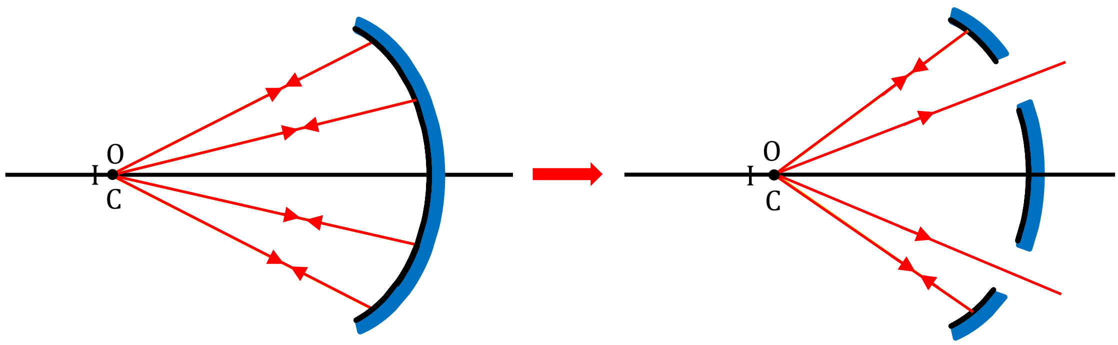
v = – 60 cm

For the light ray to retrace it's path, image of mirror M_1 should formed at centre of curvature of mirror M_2

$\therefore$ x + 20 = 60 $\Rightarrow$ x = 40 cm



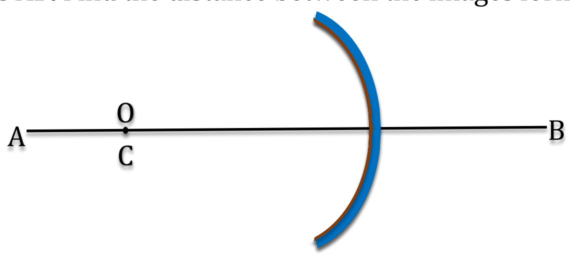
Cutting of mirror



All the parts of mirror have same hollow sphere, so its centre of curvature is same therefore number of images found is one. Intensity of image decreases.

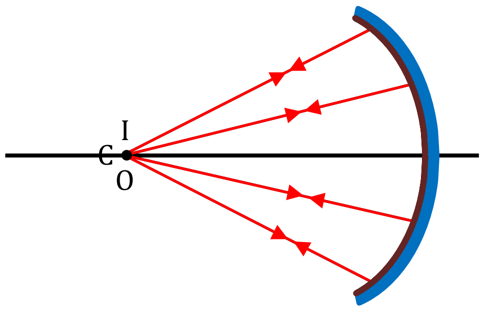
Illustration 33

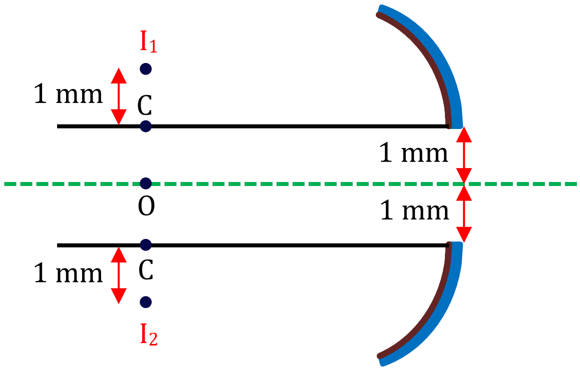
An object is placed at the Centre of curvature of concave mirror of focal length f as shown in figure. Now concave mirror is cut into two parts from the middle and the two parts are moved perpendicularly by a distance 1mm from the previous principal axis AB. Find the distance between the images formed by the two parts?



Solution:

A concave mirror is cut and each part is shifted by 1 mm. Then centre of curvature of each part shift by 1 mm and each part behaves as two independent concave mirror with its centre of curvature at the new position therefore two images are found.

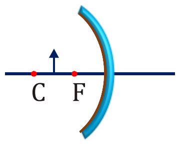
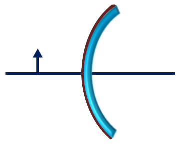
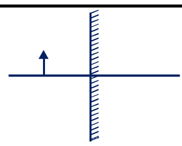
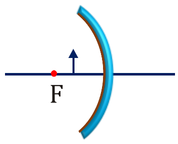




Distance between images = 4 mm

BEGINNER'S BOX-2

1. Column-I contains a list of mirrors along with the position of object. Match this with Column-II describing the nature of the image.

Column I	Column II
(A) 	(P) real, inverted, enlarged
(B) 	(Q) virtual, erect, enlarged
(C) 	(R) virtual, erect, diminished
(D) 	(S) virtual, erect

PRI186

2. A man has a shaving mirror of focal length 0.2 m. How far should the mirror be held from his face in order to give an image of two fold magnification?

PRI187

3. A convex mirror has a focal length f. A real object is placed at a distance of $\frac{f}{2}$ from the pole. Find out the position, magnification and nature of the image.

PRI188

4. A motor car is fitted with a rear view mirror of focal length 20 cm. A second motor car 2 m broad and 21.6 m high is 6 m away from first car. Find the position of second car as seen in the mirror of the first car.

PRI189

5.	A virtual image three times the size of the object is obtained with a concave mirror of radius of curvature 36 cm. Find the distance of the object from the mirror.	**PRI190**
6.	The focal length of a concave mirror is 30 cm. Where should an object be placed so that its image is three times magnified, real and inverted?	**PRI191**
7.	A small candle 2.5 cm in size is placed 27 cm in front of a concave mirror of radius of curvature 36 cm. At what distance from the mirror should a screen be placed in order to receive a sharp image ? Describe the nature and size the image. If the candle is moved closer to the mirror, how should the screen have to be moved?	**PRI192**

Refractive index (μ)

The refractive index of the material is an indicator of its optical density.

Optical density $\uparrow \Rightarrow \mu\uparrow \Rightarrow v\downarrow$

Absolute Refractive Index or simply refractive index of a medium is defined as the ratio of the speed of light in a vacuum (c) to the speed of light in that medium (v).

$$\boxed{\mu = \frac{c}{v}} \qquad \mu \geq 1 \text{ (always)}$$

Example: for vacuum $\mu = 1$; for air $\mu_a \approx 1$

for glass $\mu_g = 3/2$; for water $\mu_w = 4/3$

- μ is a scalar quantity and has no unit and dimension.

Relative refractive index

When light passes from one medium to another medium, then refractive index of second medium with respect to the first medium is known as relative refractive index.

$${}_1\mu_2 = \mu_{21} = \frac{\mu_2}{\mu_1} \qquad \mu_{21} = \text{Refractive index of medium-2 with respect to medium-1.}$$

$${}_2\mu_1 = \mu_{12} = \frac{\mu_1}{\mu_2} \qquad \mu_{12} = \text{Refractive index of medium-1 with respect to medium-2.}$$

$$\Rightarrow \boxed{\mu_{21} = \frac{1}{\mu_{12}}} \quad \Rightarrow \quad \boxed{\mu_{21} \times \mu_{12} = 1}$$

Refractive index of water with respect to glass, ${}_g\mu_w = \mu_{wg} = \frac{\mu_w}{\mu_g} = \frac{4/3}{3/2} = \frac{8}{9}$

Refractive index of glass with respect to water, ${}_w\mu_g = \mu_{gw} = \frac{\mu_g}{\mu_w} = \frac{3/2}{4/3} = \frac{9}{8}$

From the above example we can say that $\mu_{relative}$ may be greater than or less than 1.

Note: (1) Both denser and rarer are relative term

- The medium with a **higher refractive index** in which **speed of light** is comparatively **less** is known as **optically denser** medium.
  For example: Water is denser medium as compared to air.
- The medium with **lower refractive index** in which the **speed of light** is comparatively **more** is known as **optically rarer** medium.
  For example: Water is rarer medium as compared to glass.

(2) Optical density should not be confused with mass density, which is mass per unit volume. It is possible that mass density of an optically denser medium may be less than that of an optically rarer medium (optical density is the ratio of the speed of light in two media) i.e. both optical density and mass density are different term.

For example:

turpentine

Water

mass density (ρ_T) < mass density (ρ_W)

Optical Density $(O.D.)_T$ > Optical Density $(O.D.)_W$

v_T < v_W

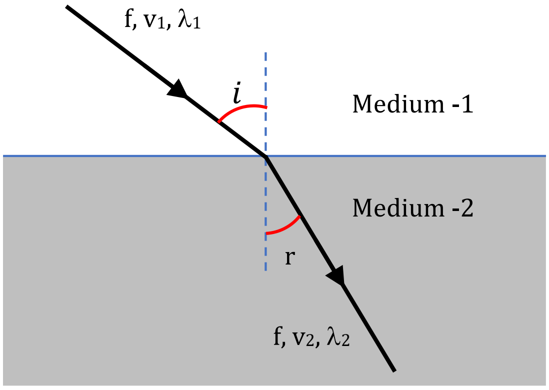
- If ε_0 and μ_0 are electric permittivity and magnetic permeability of free space respectively while ε and μ are those of a given medium, then according to electromagnetic theory,

$$\boxed{c = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} \text{ and } v_m = \frac{1}{\sqrt{\varepsilon\mu}} \Rightarrow n_m = \frac{c}{v_m} = \sqrt{\frac{\varepsilon\mu}{\varepsilon_0\mu_0}} = \sqrt{\varepsilon_r \mu_r}}$$

- As in vacuum or free space, speed of light of all wavelengths is maximum and equal to c, so for all wavelengths the refractive index of free space is minimum and is $\mu = \frac{c}{v_m} = \frac{c}{c} = 1$.

Refraction of Light

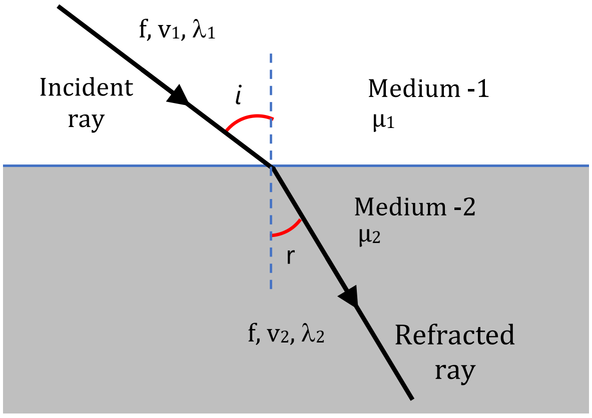
Refraction is the phenomenon in which ray of light bends when it passes obliquely from one medium (such as air) into another medium (such as glass) in which its velocity is different.



i is the angle of incidence.
r is the angle of refraction.

When light rays goes from one medium to another medium then speed (v) and wavelength (λ) changes but frequency (f) remains same.

Laws of Refraction

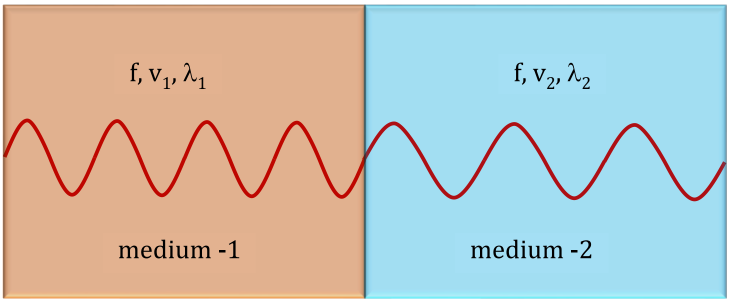


(i) Incident ray, refracted ray and normal always lie in the same plane.

(ii) The ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant, for the light of a given colour and for the given pair of media.

$$\boxed{\frac{\sin i}{\sin r} = \text{constant}} \quad (\text{Snell's Law})$$

$$\frac{\sin i}{\sin r} = \mu_{21} \Rightarrow \frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \boxed{\mu_1 \sin i = \mu_2 \sin r}$$



From, $f = \dfrac{v}{\lambda}$

in medium-1 $\quad f = \dfrac{v_1}{\lambda_1}$

in medium-2 $\quad f = \dfrac{v_2}{\lambda_2}$

Now $\quad \dfrac{v_1}{\lambda_1} = \dfrac{v_2}{\lambda_2}$ ...(i)

$\left(\text{From, } v = \dfrac{c}{\mu}\right)$

$$\frac{c/\mu_1}{\lambda_1} = \frac{c/\mu_2}{\lambda_2} \Rightarrow \boxed{\lambda_1\mu_1 = \lambda_2\mu_2}$$

$\dfrac{\mu_1}{\mu_2} = \dfrac{\lambda_2}{\lambda_1}$ ...(ii)

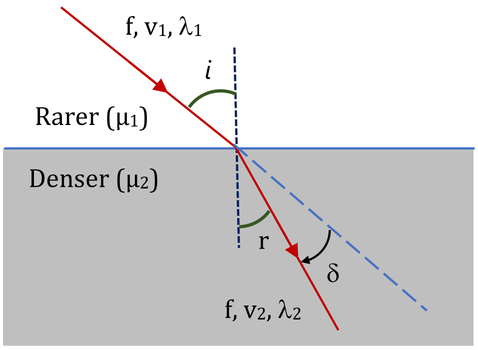
From Snell's law, $\mu_1 \sin i = \mu_2 \sin r$

$$\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} \Rightarrow \boxed{\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1} = \frac{\lambda_1}{\lambda_2} = \frac{v_1}{v_2}}$$

Bending of light ray

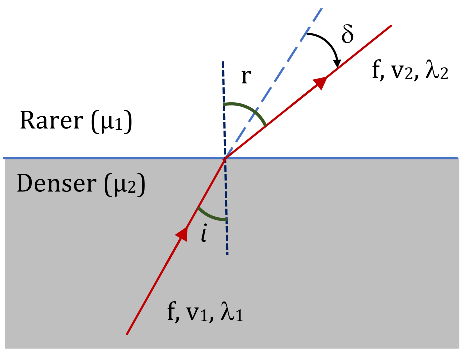
Case-1 : When light rays goes from rarer medium to denser medium ($\mu_2 > \mu_1$).

- Light rays bend towards the normal after refraction.
- $\angle i > \angle r$
- $v_1 > v_2$
- $\lambda_1 > \lambda_2$
- $\delta = i - r$



Case-2 : When light rays goes from denser medium to rarer medium ($\mu_2 > \mu_1$)

- Light rays bend away from the normal after refraction.
- $\angle i < \angle r$
- $v_1 < v_2$
- $\lambda_1 < \lambda_2$
- $\delta = r - i$

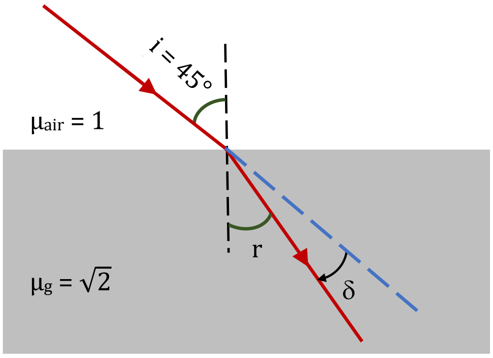


Note: If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.

Illustration 34

A light ray incident on a glass surface of refractive index $\sqrt{2}$ from air at an angle of incidence 45°.

Find out: (i) Angle of refraction (ii) Angle of deviation.



Solution:

(i) From, Snell's law,

$\mu_1 \sin i = \mu_2 \sin r$

$1 \sin 45° = \sqrt{2} \sin r \Rightarrow \frac{1}{2} = \sin r$

$r = 30°$

(ii) Angle of deviation $\delta = i - r = 45° - 30°$

$\delta = 15°$ (CW)

Illustration 35

A light ray is moving from air to denser medium of refractive index μ, it is observed that the angle of refraction is half of the angle of incidence then find out angle of incidence.

Solution:

By Snells law, $\mu_1 \sin i = \mu_2 \sin r$

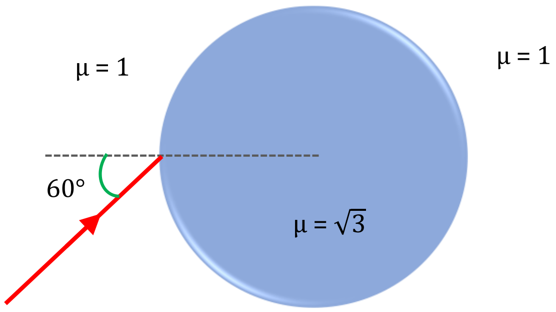
$1 \sin i = \mu \sin r \qquad \left(\text{Given, } r = \frac{i}{2}\right)$

$\sin i = \mu \sin\frac{i}{2} \qquad (\because \sin 2\theta = 2\sin\theta\cos\theta)$

$2\sin\frac{i}{2}\cos\frac{i}{2} = \mu\sin\frac{i}{2} \Rightarrow \cos\frac{i}{2} = \frac{\mu}{2} \Rightarrow \frac{i}{2} = \cos^{-1}\left(\frac{\mu}{2}\right) \Rightarrow i = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

Illustration 36

Find out total angle of deviation after two refraction through glass sphere of refractive index $\sqrt{3}$.



Solution:

1st Refraction: by snell's law

$1 \times \sin 60° = \sqrt{3} \sin r$

$r = 30°$

$\delta_1 = i - r = 60° - 30° = 30°$ (Clockwise)

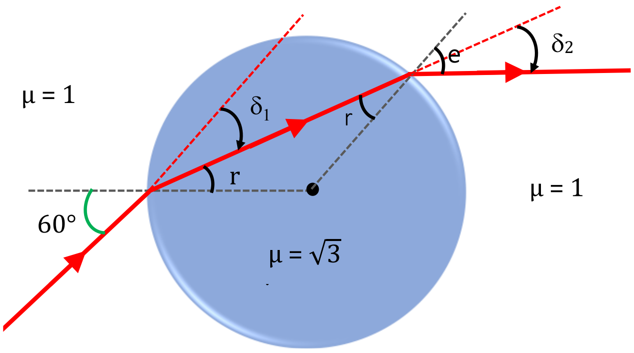
2nd Refraction: by snell's law

$\sqrt{3} \sin r = 1 \times \sin e$

$\sqrt{3} \sin 30° = \sin e \Rightarrow e = 60°$

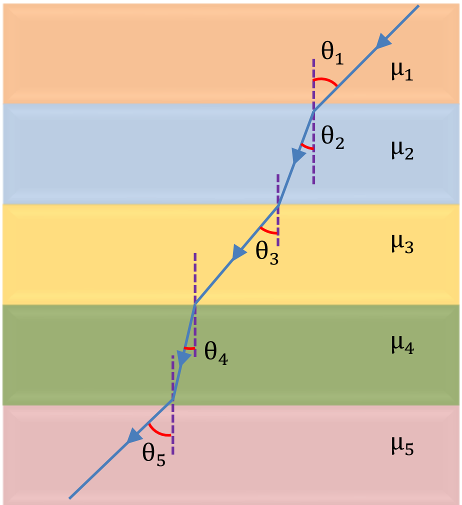
$\delta_2 = e - r = 60° - 30° = 30°$ (Clockwise)

Total deviation, $\delta = \delta_1 + \delta_2 = 30° + 30° = 60°$ (Clockwise)



Note:

(a) **Refraction through Multiple Parallel Slabs:** Consider a stack of five parallel slabs placed vertically one after the other. Since the light ray falls on the topmost slab at an angle of θ_1 with the normal, the angle of refraction at the topmost slab is θ_2 as shown in the figure.



Suppose that the refractive indices of the five slabs are μ_1, μ_2, μ_3, μ_4 and μ_5, respectively, from top to bottom. Since the angle of refraction at the topmost slab is θ_2, the angle of incidence at the second slab is the same. Now, if the angle of refraction at the second slab is θ_3, the angle of incidence for the third slab is the same and this process continues as shown in the figure.

By applying Snell's law of refraction at the interface between the top two slabs, we get,

$\mu_1 \sin\theta_1 = \mu_2 \sin\theta_2$ ...(i)

Now, by applying Snell's law of refraction at the interface between the second and third slab from the top we get,

$\mu_2 \sin\theta_2 = \mu_3 \sin\theta_3$ ...(ii)

By equating equations (i) and (ii), we get,

$\mu_1 \sin\theta_1 = \mu_3 \sin\theta_3$

By going through the same procedure, we get the following result:

$\mu_1 \sin\theta_1 = \mu_4 \sin\theta_4$

In place of five parallel slabs, if there are m numbers of parallel slabs, then we can consider the following equation; $\boxed{\mu_1 \sin\theta_1 = \mu_m \sin\theta_m}$

Where μ_m is the refractive index of the mth slab, and θ_m is the angle of incidence at the mth slab.

(b) When first and last medium are same then the incident ray and emergent ray are parallel to each other.

Refraction through single plane surface (Apparent Depth)

If a point object in denser medium is observed from rare medium and boundary is plane.

From Snell's Law

$\mu_1 \sin i = \mu_2 \sin r$

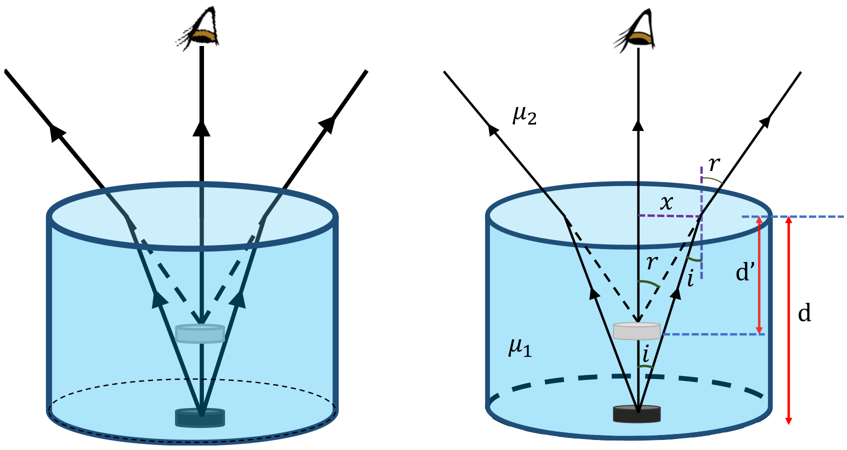
$\sin r \approx \tan r = \dfrac{x}{d'}$

$\sin i \approx \tan i = \dfrac{x}{d}$

$\mu_1 \dfrac{x}{d} = \mu_2 \dfrac{x}{d'}$

$\boxed{\dfrac{d'}{d} = \dfrac{\mu_2}{\mu_1}}$

$\boxed{\dfrac{\text{Apparent distance}}{\text{Actual distance}} = \dfrac{d'}{d} = \dfrac{\mu_2}{\mu_1}}$

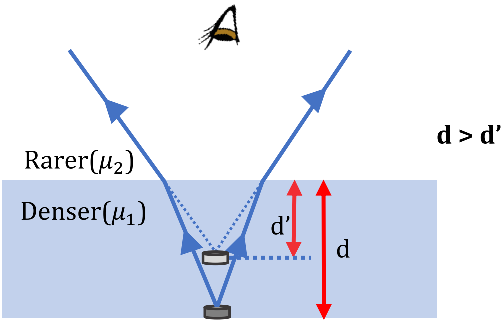


Important points

1. Rays should be paraxial. (Rays which are very near to the line joining the object and observer).
2. Both d and d' are measured from boundary.
3. μ_2 is the refractive index of that medium in which light rays enter after refraction and μ_1 is the refractive index of that medium in which light rays moves before refraction.

Different situations of observer

Case 1: If observer is in rarer medium and object is in denser medium,



Case 2: If observer is in denser medium and object is in rarer medium

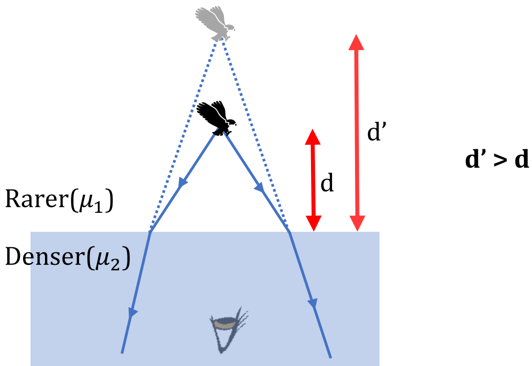
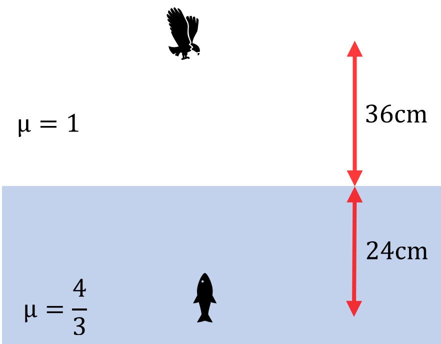
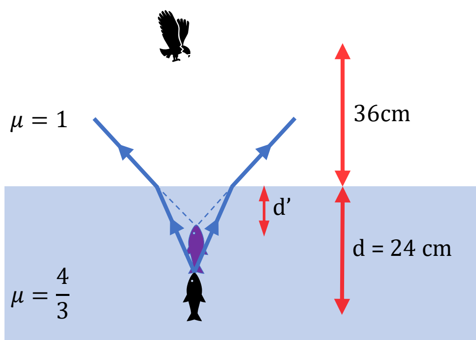


Illustration 37

Find out apparent distance between bird and fish as seen by bird.



Solution:



In this case object is fish and observer is bird.

$$\frac{d'}{d} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{d'}{24} = \frac{1}{4/3}$$

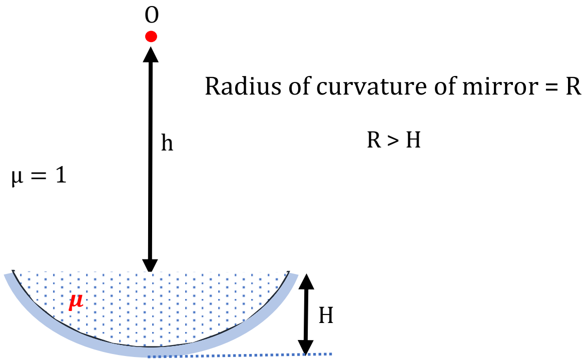
d' = 18 cm (from the surface)

Apparent distance between bird and fish as seen by bird

= d' + 36 = 18 + 36 = 54 cm

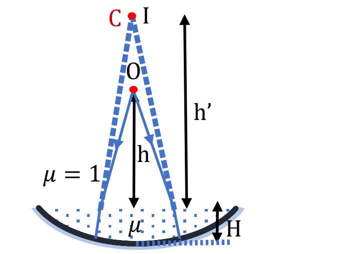
Illustration 38

Find out the value of h such that light ray retrace its path after reflection.



Solution:

1st Refraction,



$$\frac{d'}{d} = \frac{\mu_2}{\mu_1} \Rightarrow \frac{h'}{h} = \frac{\mu}{1} \Rightarrow h' = \mu h$$

Light ray retrace its path after reflection when image formed by refraction is at the centre of curvature of the mirror.

So, $H + h' = R$

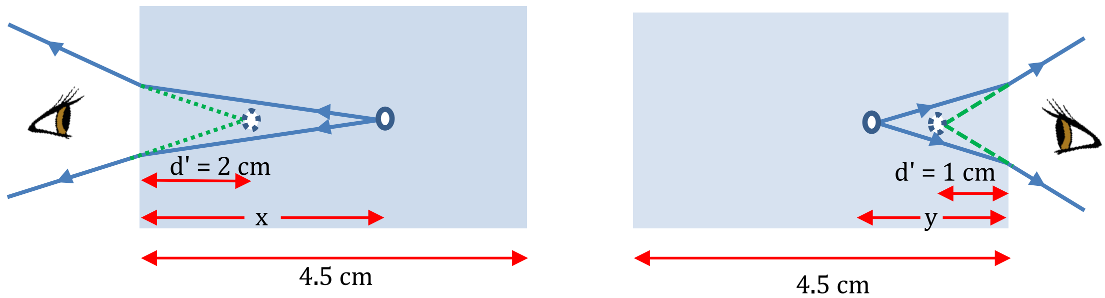
$H + \mu h = R$

$$h = \frac{R - H}{\mu}$$

Illustration 39

An air bubble inside a cubical block of glass of side 4.5 cm seems to be at 2 cm from one face and 1 cm from the other face opposite to the first when viewed normally. What is the real distance of bubble from the first face.

Solution:



$$\frac{d'}{d} = \frac{\mu_2}{\mu_1} \qquad \frac{d'}{d} = \frac{\mu_2}{\mu_1}$$

$$\frac{2}{x} = \frac{1}{\mu} \qquad \frac{1}{y} = \frac{1}{\mu}$$

$$x = 2\mu \qquad y = \mu$$

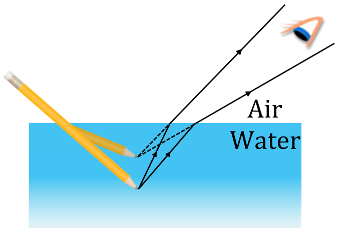
$x + y = 4.5$

$2\mu + \mu = 4.5 \Rightarrow \mu = 1.5$

Now, $x = 2\mu = 2 \times 1.5 = 3$ cm

Examples of refraction

Bending of an object: When pencil in a denser medium is seen from a rarer medium it appears to be bent.



Twinkling of stars: Due to fluctuations in the refractive index of different layers of atmosphere, the refraction becomes irregular so, that the light sometimes reaches the eye and sometimes it does not. This gives the effect of twinkling of stars.

Velocity of image in plane refraction

$$\frac{d'}{d} = \frac{\mu_2}{\mu_1} \Rightarrow d' = \frac{\mu_2}{\mu_1} d$$

Differentiate with respect to time,

$$\frac{d(d')}{dt} = \frac{\mu_2}{\mu_1}\frac{d(d)}{dt}$$

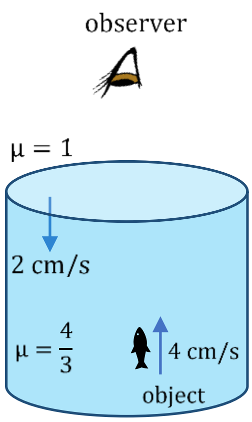
$$\boxed{\vec{V}_{IS} = \frac{\mu_2}{\mu_1}\vec{V}_{OS}}$$

$\vec{V}_{IS}$ = velocity of image with respect to surface (boundary).

$\vec{V}_{OS}$ = velocity of object with respect to surface (boundary).

Illustration 40

Find out speed of fish as seen by observer if water level decreases with speed 2 cm/s and fish is moving upward with speed 4 cm/s.



Solution:

$$\vec{V}_{IS} = \frac{\mu_2}{\mu_1}\vec{V}_{OS}$$

$$\vec{V}_I - \vec{V}_S = \frac{\mu_2}{\mu_1}\left(\vec{V}_O - \vec{V}_S\right)$$

$$\vec{V}_I - (-2) = \frac{1}{4/3}\left(4-(-2)\right)$$

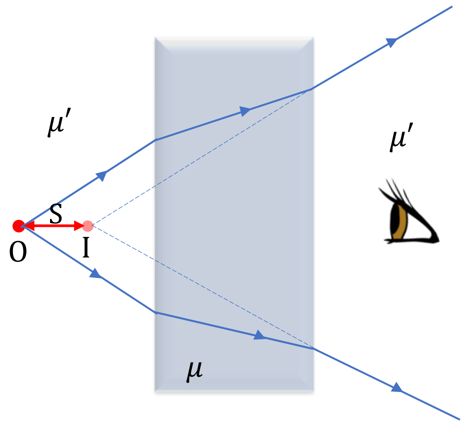
$$\vec{V}_I + 2 = \frac{3}{4}\times 6 = \frac{9}{2}$$

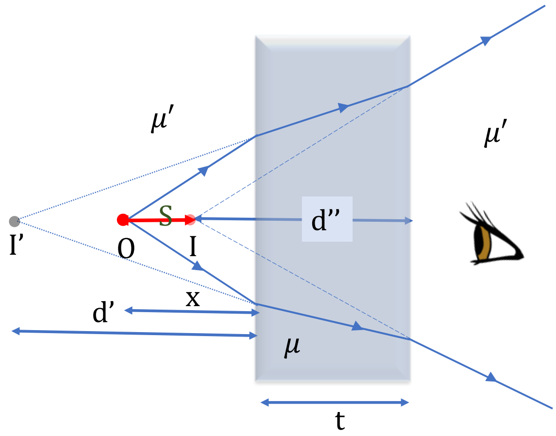
$$\vec{V}_I = \frac{9}{2} - 2 = \frac{5}{2}\,\text{cm/s}$$

$\vec{V}_I$ is the velocity of image with respect to ground but here velocity of observer is zero so, velocity of fish as seen by observer = $\frac{5}{2}$ cm/s (upward).

Refraction through glass slab (Normal Shift)

Normal Shift: When an object is placed in front of a transparent slab then it appears for an observer shift (S) as shown in figure. Distance between object and image is S or we can say object appears to shift at a distance S. This distance is called Normal shift.





Calculation of normal shift,

From $\frac{d'}{d} = \frac{\mu}{\mu'}$

1st Refraction

$\frac{d'}{x} = \frac{\mu}{\mu'} \Rightarrow d' = \frac{\mu}{\mu'}x$ (distance I' from 1st surface)

2nd Refraction: For second refraction image of first refraction will behave as an object.

$\frac{d''}{d'+t} = \frac{\mu'}{\mu} \Rightarrow d'' = \frac{\mu'}{\mu}(d'+t)$

Distance between object and its image, $S = d'+t-d'' = d'+t-\frac{\mu'}{\mu}(d'+t)$

$S = t\left(1-\frac{\mu'}{\mu}\right)$

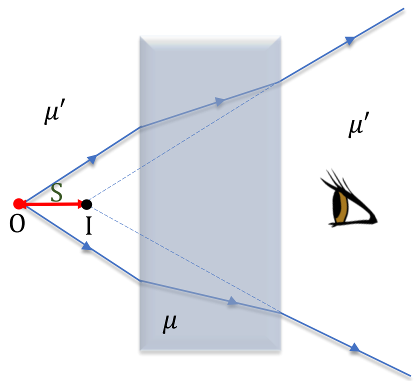
Conclusion

Shift $\boxed{S = t\left[1-\frac{\mu_{\text{surrounding}}}{\mu_{\text{slab}}}\right]}$

$S = t\left[1-\frac{\mu'}{\mu}\right]$

If surrounding medium is air then, $\mu' = 1$

$\boxed{S = t\left[1-\frac{1}{\mu}\right]}$



Important points

1. Rays should be paraxial.
2. Both object and observer are in same medium.
3. Shift is independent of distance between object and slab.
4. If shift comes to be positive then object appears to shift towards the direction of incident ray and vice versa.

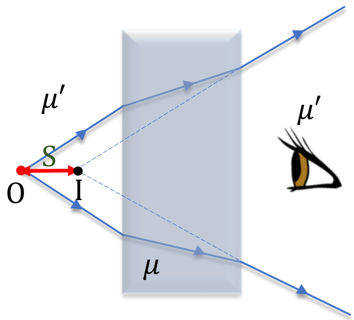


Illustration 41

Find out normal shift.

Solution:

Normal shift $S = t\left(1-\frac{1}{\mu}\right) = 15\left(1-\frac{1}{3/2}\right) = 5\text{cm}$ (Positive)

Image formed towards the slab or object appears to shift 5 cm towards the slab for an observer.

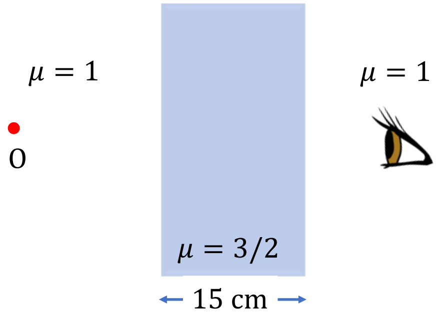


Illustration 42

A microscope is focused on a coin lying at the bottom of a beaker. The microscope is now raised up by 1 cm. To what depth should the water be poured into the beaker so that coin is again in the focus. (μ_w= 4/3)

Solution:

$S = t\left(1-\frac{1}{\mu}\right)$

$1 = t\left(1-\frac{1}{4/3}\right)$

$t = 4$ cm

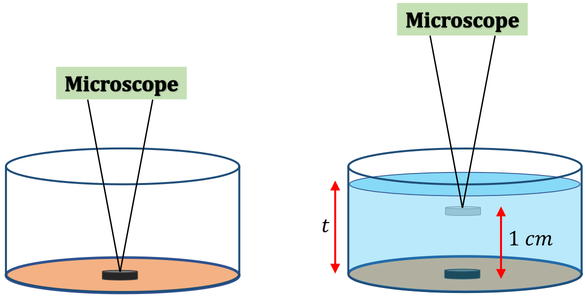
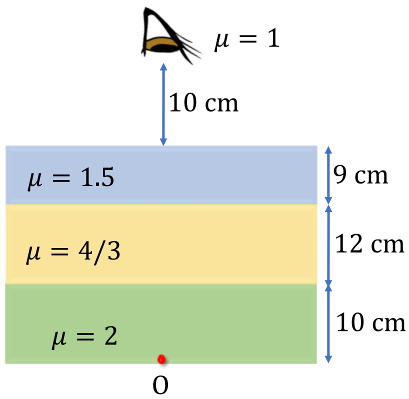


Illustration 43

Find out apparent distance between observer and object.



Solution:

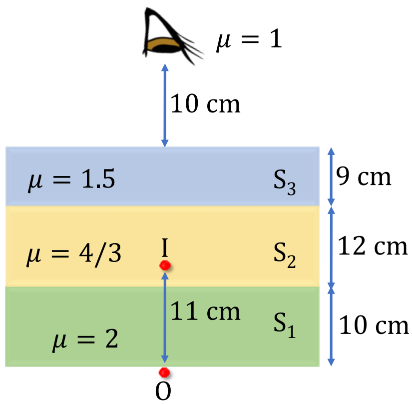
$S_1 = t\left(1-\frac{1}{\mu}\right) = 10\left(1-\frac{1}{2}\right) = 5$ cm

$S_2 = 12\left(1-\frac{1}{4/3}\right) = 3$ cm

$S_3 = 9\left(1-\frac{1}{15}\right) = 3$ cm

Total shift S = 5 + 3 + 3 = 11 cm

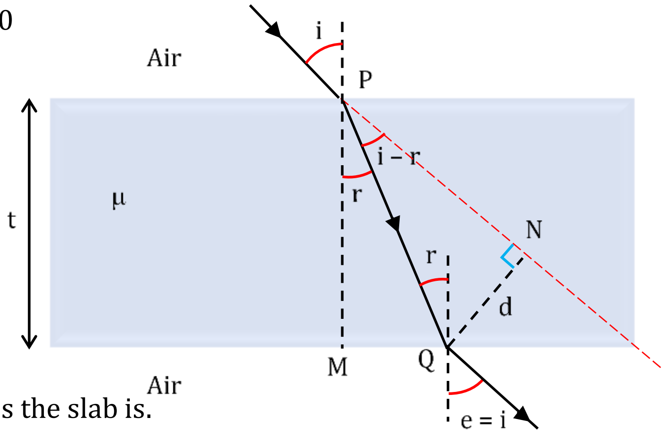
Apparent distance between object & observer = 41 – 11 = 30 cm



Refraction Through Glass Slab (Lateral Shift)

Lateral Shift: For a rectangular slab, refraction takes place at two interfaces. It is easily seen from the dig. that the emergent ray is parallel to the incident ray-there is no deviation, but it does suffer lateral displacement/shift with respect to the incident ray. The perpendicular distance between incident and emergent ray is known as lateral shift (d).

Deviation (δ) between incident and emergent ray = 0



Calculation of Lateral Shift

In ΔPQM, $\cos r = \frac{PM}{PQ} \Rightarrow PQ = \frac{t}{\cos r}$

In ΔPQN, $\sin(i-r) = \frac{QN}{PQ}$

$QN = PQ\sin(i-r)$

$$\boxed{d = \frac{t}{\cos r}\sin(i-r)}$$

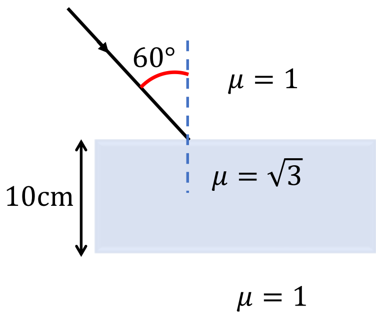
Calculation of time: Time taken by light ray to cross the slab is.

$T = \frac{\text{Distance}}{\text{Speed}} = \frac{PQ}{v}$ $\qquad \left(PQ = \frac{t}{\cos r}, v = \frac{c}{\mu}\right)$

Now, $\boxed{T = \frac{\mu t}{c\cos r}}$

Illustration 44

Find out: (i) Lateral Shift (ii) Time taken by light ray to cross the slab.



Solution:

By Snell's law; 1 sin 60° = $\sqrt{3}$ sinr

r = 30°

Lateral shift d = $\frac{t}{\cos r}\sin(i-r) = \frac{10}{\cos 30°}\sin(60° - 30°) = \frac{10}{\sqrt{3}}$ cm

Time to cross the slab $= \frac{\mu t}{c \times \cos r} = \frac{\sqrt{3} \times 0.10}{3 \times 10^8 \times \cos 30°}$

$= \frac{\sqrt{3} \times 0.10}{3 \times 10^8 \times \sqrt{3}/2} = \frac{2}{3} \times 10^{-9}$ sec

BEGINNER'S BOX-3

1. Width of a slab is 6 cm whose $\mu = \frac{3}{2}$. If its rear surface is silvered and object is placed at a distance 28 cm from the front face. Calculate the final position of the image from the silvered surface.

PRI193

2. A light ray is moving from denser (refractive index=μ) to air. If the angle of incidence is half the angle of refraction, find out the angle of refraction.

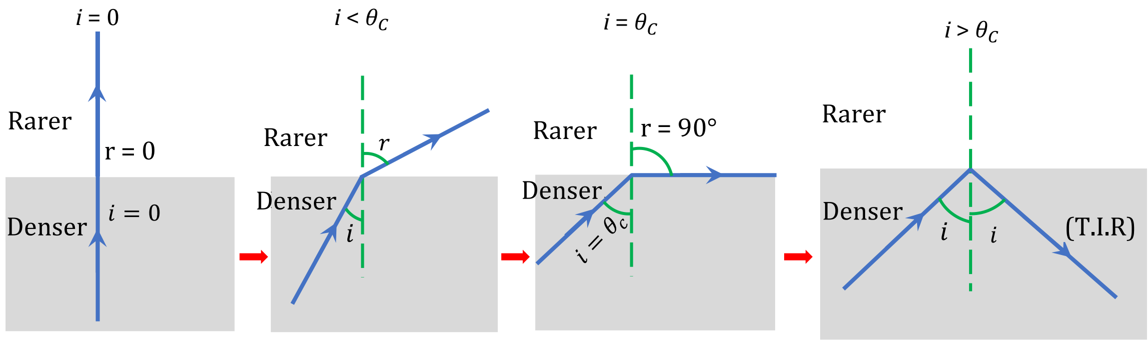
PRI194

3. Light of wavelength 8000 Å enters from air into water ($\mu_{water} = 4/3$). What is the change in the frequency of light in water?

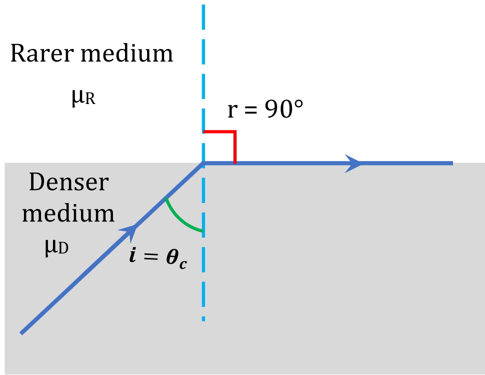
PRI195

Total Internal Reflection (T.I.R.)

When light ray travels from denser to rarer medium, it bends away from the normal. If the angle of incidence is increased, the angle of refraction also increases. At a particular value of angle, the refracted ray subtends 90° angle with the normal, this angle of incidence is known as critical angle (θ_C). If angle of incidence increases further, the ray comes back to the same medium. This phenomenon is known as total internal reflection.



Calculation of Critical Angle



Using Snell's law

$\mu_D \sin\theta_C = \mu_R \sin 90°$

$$\boxed{\sin\theta_C = \frac{\mu_R}{\mu_D}}$$

$$\boxed{\sin\theta_C = \frac{\mu_R}{\mu_D} = \frac{v_D}{v_R} = \frac{\lambda_D}{\lambda_R}}$$

Conclusion

For T.I.R. light must travel from Denser medium to Rarer medium (for example, glass to air, water to air, glass to water etc.)

$i < \theta_c$: Refraction

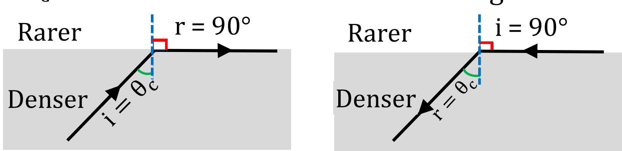
$i = \theta_c$: Grazing Emergence (refraction)

$i > \theta_c$: TIR

$$\boxed{\sin\theta_C = \frac{\mu_R}{\mu_D}}$$

If rarer medium is air, $\boxed{\sin\theta_C = \frac{1}{\mu}}$

Note: **(1)** From the ray diagram as shown in figure we conclude that in case of refraction maximum angle from normal is θ_C in denser medium and maximum angle from normal is 90° in rarer medium.



(2) In case of total internal reflection, as all (i.e. 100%) incident light is reflected back into the same medium there is no loss of intensity while in case of reflection from mirror or refraction from lenses there is some loss of intensity as the entire light cannot be reflected or refracted. Due to this reason, images formed by TIR are much brighter those than formed by mirrors or lenses.

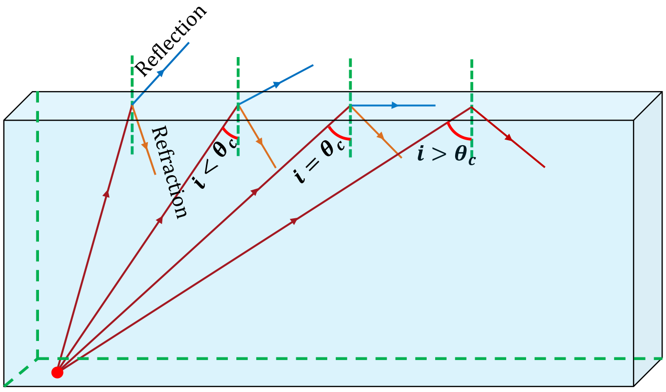


Illustration 45

If the critical angle for a certain medium and vacuum is 30°. Find out the velocity of light in the medium?

Solution:

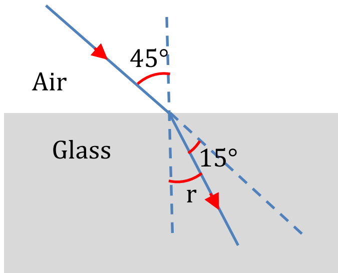
$$\sin\theta_c = \frac{1}{\mu} \Rightarrow \sin 30° = \frac{1}{\mu} \Rightarrow \mu = 2$$

$$v = \frac{c}{\mu} = \frac{3\times10^8}{2} = 1.5\times10^8 \text{ m/s}$$

Illustration 46

Calculate the critical angle for glass-air interface if a ray of light incident on a glass surface is deviated through 15° when angle of incidence is 45°?

Solution:



$\delta = i - r \Rightarrow 15° = 45° - r$

$r = 30°$

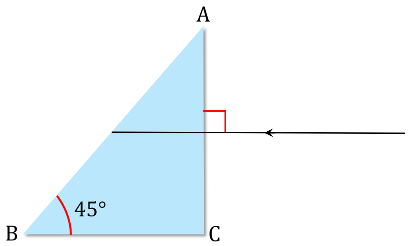
By snell's law,

$1\sin45° = \mu\sin r$

$$\frac{1}{\sqrt{2}} = \mu\sin30° \Rightarrow \mu = \sqrt{2}$$

$$\sin\theta_c = \frac{1}{\mu} = \frac{1}{\sqrt{2}} \Rightarrow \theta_c = 45°$$

Illustration 47



What should be the range of refractive index of prism so that light ray suffers total internal reflection at surface AB?

Solution:

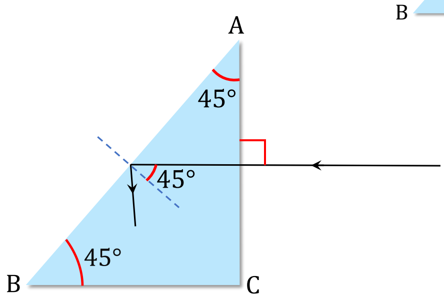
Light ray performs T.I.R. at surface AB

Then $45° > \theta_c$

$\sin45° > \sin\theta_c$

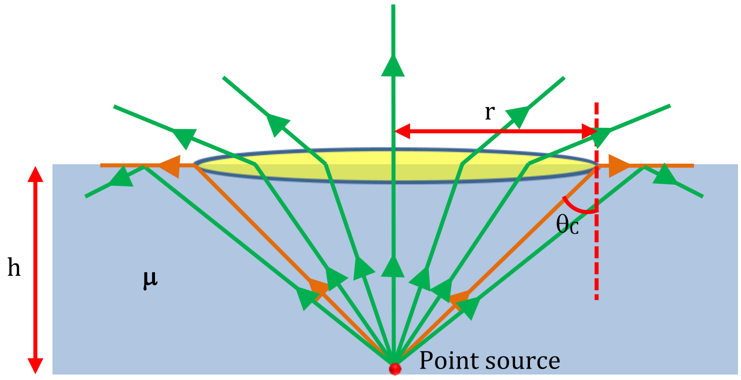
$$\frac{1}{\sqrt{2}} > \frac{1}{\mu}$$

$$\mu > \sqrt{2}$$

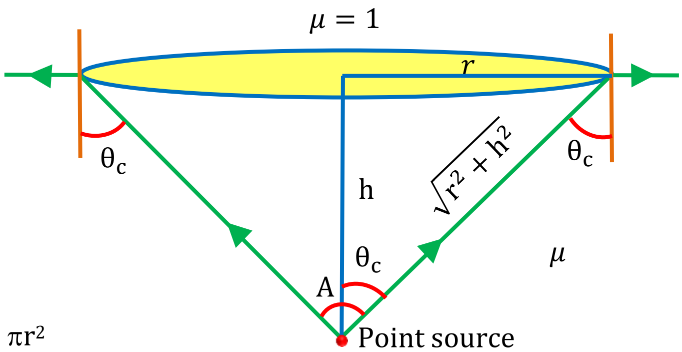


Circle of illuminance

Ray incident from denser to rarer medium undergoes total internal reflection when their angle of incidence become more than the critical angle, this situation creates a circular region from which light escapes and is called circle of illuminance.



Calculation: A point source is situated at the bottom of a tank filled with a liquid of refractive index μ upto h height. It is found that light comes out of liquid surface through a circular portion above the source.



$\sin\theta_C = \frac{r}{\sqrt{r^2+h^2}}$ and

$\sin\theta_C = \frac{1}{\mu} \Rightarrow \frac{1}{\mu} = \frac{r}{\sqrt{r^2+h^2}} \Rightarrow \frac{1}{\mu^2} = \frac{r^2}{r^2+h^2}$

$\Rightarrow \mu^2 r^2 = r^2 + h^2 \Rightarrow (\mu^2-1)r^2 = h^2$

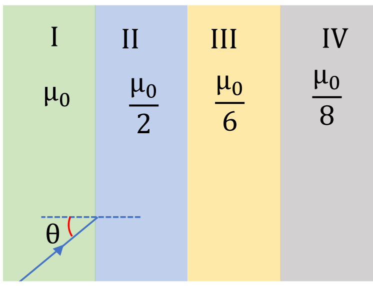
Radius of circular portion, $r = \frac{h}{\sqrt{\mu^2-1}}$ and area = πr^2

Vertex angle A = $2\theta_C$

For pure water, A = 2 × 49° = 98°

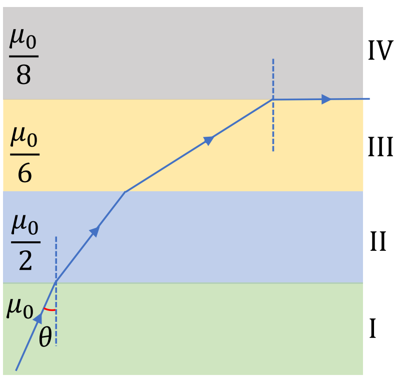
Illustration 48

Find out the angle of incidence θ for which light ray just misses entering in region IV?



Solution:

Apply Snell's Law between I & IV medium,



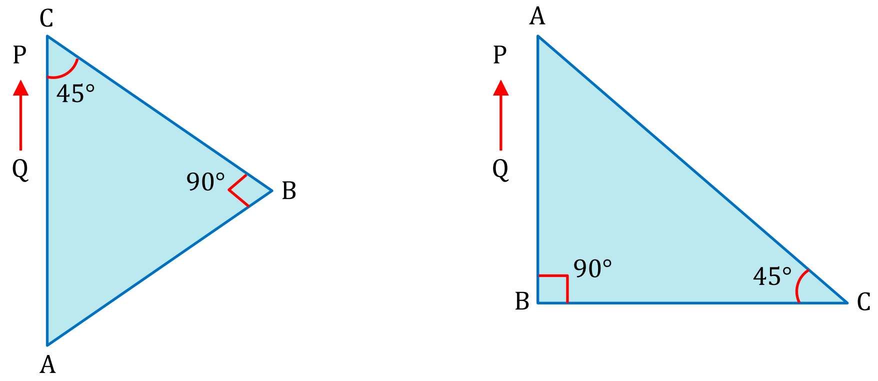
$\mu_0 \sin\theta = \frac{\mu_0}{8}\sin 90°$

$\sin\theta = \frac{1}{8}$

$\theta = \sin^{-1}\left(\frac{1}{8}\right)$

Illustration 49

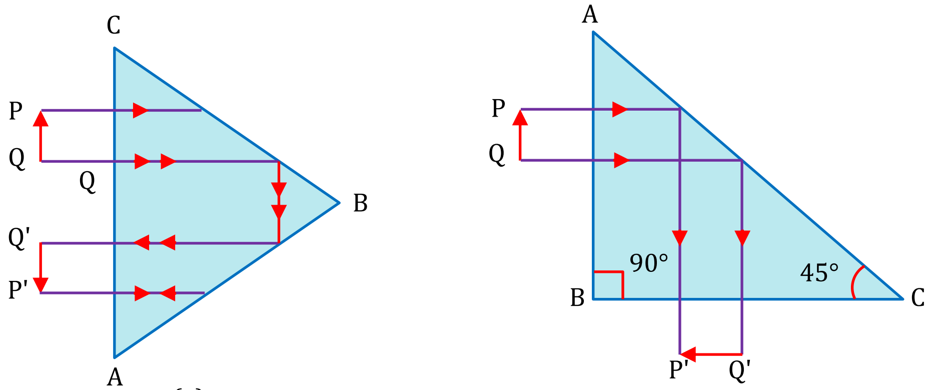
An object is placed in front of a right angled prism ABC in two positions (a) and (b) as shown. The prism is made of crown glass with critical angle 41°. Trace the path of rays starting form P and Q as shown in the figures (a) and (b) entering normal to the prism.



(a) (b)

Solution:

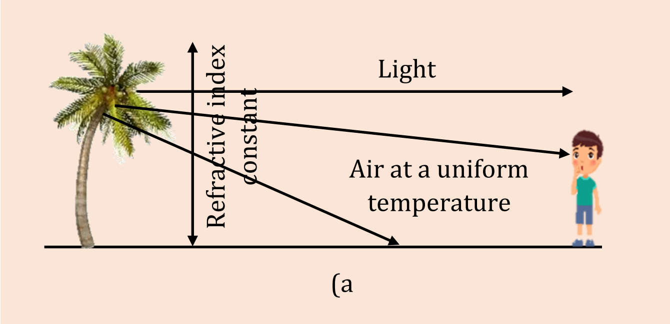
For the refraction on the inclined surfaces of the prism i = 45° (>C=41°). So, the rays undergo total internal reflection.

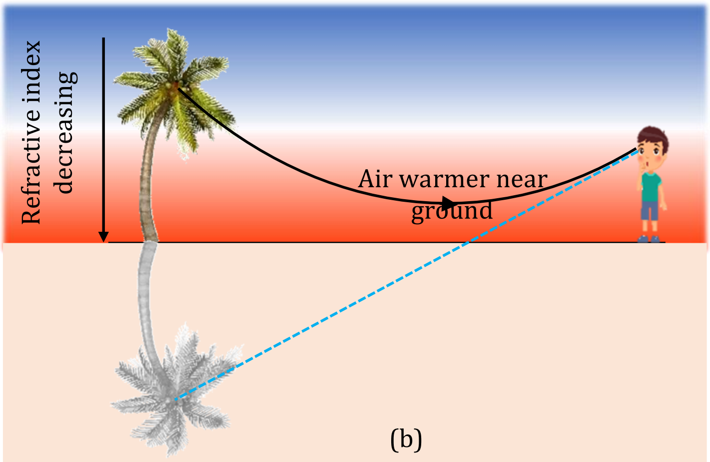


(a) (b)

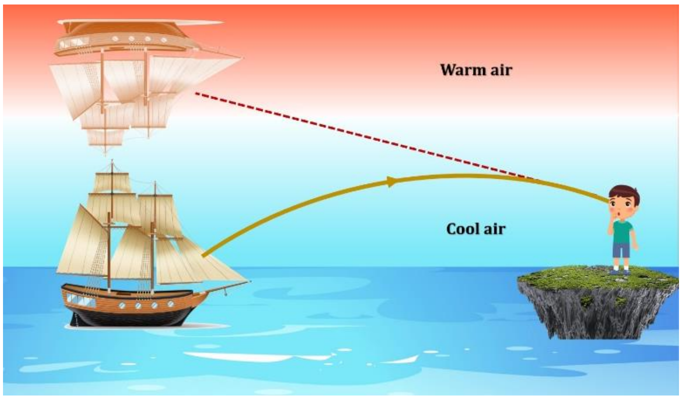
Application of TIR

1. Mirage: In hot summer days, the air near the ground becomes hotter than the air at higher levels. The refractive index of air increases with its density. Hotter air is less denser, and has smaller refractive index than the cooler air. If the air currents are small, that is, the air is still, the optical density at different layers of air increases with height. As a result, light from a tall object such as a tree, passes through a medium whose refractive index decreases towards the ground. Thus, a ray of light from such an object successively bends away from the normal and undergoes total internal reflection, if the angle of incidence for the air near the ground exceeds the critical angle. This is shown in Figure (b). To a distant observer, the light appears to be coming from somewhere below the ground. The observer naturally assumes that light is being reflected from the ground, say, by a pool of water near the tall object. Such inverted images of distant tall objects cause an optical illusion to the observer. This phenomenon is called mirage. This type of mirage is especially common in hot deserts. Some of you might have noticed that while moving in a bus or a car during a hot summer day, a distant patch of road, especially on a highway, appears to be wet. But, you do not find any evidence of wetness when you reach that spot. This is also due to mirage.





2. Looming: Similar to 'mirage' in deserts, 'optical looming' takes place in polar regions due to TIR. Here μ of different air layers decrease with height and so an image of an object is formed in the sky which appears to be suspended in air.



3. Sparkling of Diamond: The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so $\theta_c = 24.4°$. Diamond is cut in such a manner that, once the light enters into it, when it tends come out then $i > \theta_c$. So TIR will take place repeatedly inside it. The light which beams out from a few places entering into the eyes of the observer makes it sparkle.

4. Optical Fibre:

- An optical fibre is a cylindrical dielectric waveguide (nonconducting waveguide) that transmits light along its axis through the process of total internal reflection.
- Optical fibre is used as a medium for telecommunication and computer networking

Mathematical description in Optical Fibre

Snell's law at air/core interface, $1 \times \sin i = \mu_1 \sin r$

$\sin r = \frac{\sin i}{\mu_1} \Rightarrow \sin\theta_c = \frac{\mu_2}{\mu_1}$

For T.I.R. in optical fibre, $90 - r > \theta_c$

Take **sin** on both side, $\sin(90 - r) > \sin\theta_c$

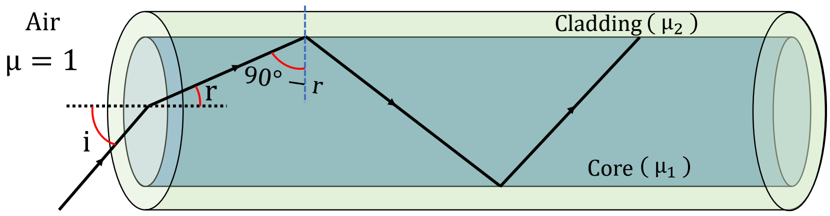
$\cos r > \sin\theta_c$

$\sqrt{1-\sin^2 r} > \sin\theta_c$

$\sqrt{1-\frac{\sin^2 i}{\mu_1^2}} > \frac{\mu_2}{\mu_1}$

$\mu_1^2 - \sin^2 i > \mu_2^2$

$\boxed{\sin i < \sqrt{\mu_1^2 - \mu_2^2}}$

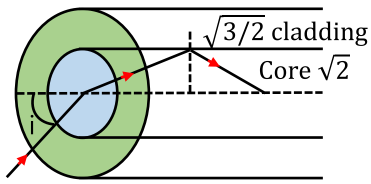


for maximum value,

$\boxed{\sin i = \sqrt{\mu_1^2 - \mu_2^2}}$ Here, i is called maximum acceptance angle.

Illustration 50

Figure shows a cross section of a 'light pipe' made of a glass fibre of refractive index $\sqrt{2}$. The outer covering of the pipe is made of a material of refractive index $\sqrt{\frac{3}{2}}$. What is maximum angle of the incident ray with the axis of the pipe for which total internal reflections inside the pipe take place?



Solution:

$\sin i < \sqrt{\mu_1^2 - \mu_2^2}$

$\sin i < \sqrt{\left(\sqrt{2}\right)^2 - \left(\sqrt{\frac{3}{2}}\right)^2}$

$\sin i < \frac{1}{\sqrt{2}} \Rightarrow i < 45°$

$i_{max} = 45°$

BEGINNER'S BOX-4

1. If light travels a distance x in time t_1 sec in air and 10x distance in time t_2 in a certain medium, then find the critical angle of the medium.

PRI196

2. Calculate the critical angle for glass-air interface if a ray of light incident on a glass surface is deviated through 15° when angle of incidence is 45°.

PRI197

3. A ray of light travels from denser medium having refractive index $\sqrt{2}$ to air, What should be the angle of incidence for the ray to emerge out ?

PRI198

Refraction Through Single Curved Surfaces

Spherical refraction

By using snell's law, $\mu_1 \sin i = \mu_2 \sin r$

We use paraxial ray so angle i and r are very small

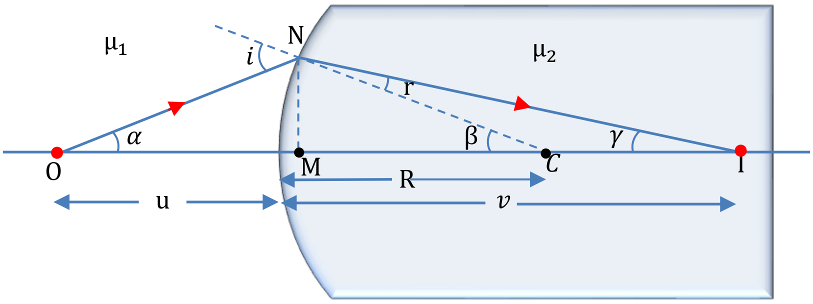
$\sin i \approx i$ & $\sin r \approx r$

$\Rightarrow \mu_1 i = \mu_2 r$ ...(i)

In ΔNOC $\alpha + \beta = I$ ...(ii)

In ΔNIC $\gamma + r = \beta$

$r = \beta - \gamma$ ...(iii)



Put the value of i and r in equation (i) from equation (ii) and (iii)

$\mu_1(\alpha + \beta) = \mu_2(\beta - \gamma)$ $\tan\alpha = \frac{MN}{OM} \approx \alpha$

$\mu_1\alpha + \mu_2\gamma = (\mu_2 - \mu_1)\beta$ $\tan\beta = \frac{MN}{MC} \approx \beta$

$\mu_1\frac{MN}{OM} + \mu_2\frac{MN}{MI} = (\mu_2 - \mu_1)\frac{MN}{MC}$ $\tan\gamma = \frac{MN}{MI} \approx \gamma$

$\frac{\mu_2}{MI} + \frac{\mu_1}{OM} = \frac{\mu_2 - \mu_1}{MC}$

Here OM, MI, MC represents magnitude of distances.

Applying the Cartesian sign

OM = –u, MI = +v, MC = +R

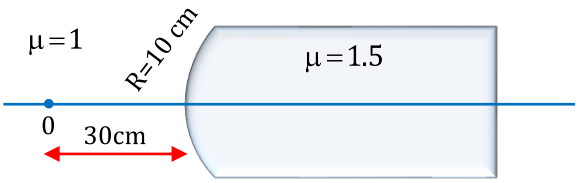
$$\boxed{\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}}$$

Important points regarding formula

1. Rays should be paraxial.
2. μ_2 is the medium in which light rays enter after refraction and μ_1 is the medium in which light rays moves before refraction.
3. v , u & R (what ever is given) should be put along with sign.

Illustration 51

Find out position and nature of image.



Solution:

Given, $\mu_2 = 1.5$; $\mu_1 = 1$; u = –30cm ; R = +10cm

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1.5}{v} - \frac{1}{-30} = \frac{1.5 - 1}{10}$$

$$\frac{3}{2v} = \frac{1}{20} - \frac{1}{30}$$

v = +90 cm (Real Image)

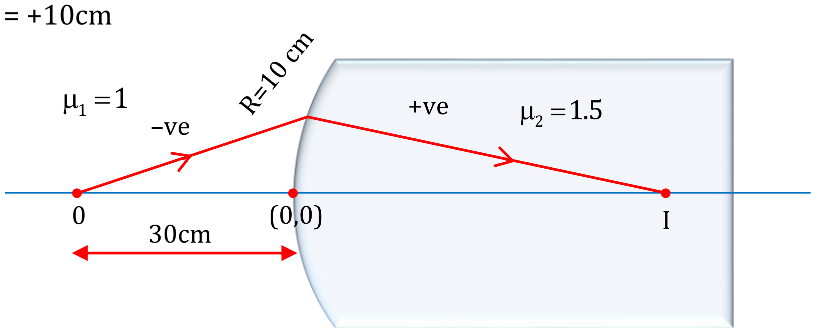
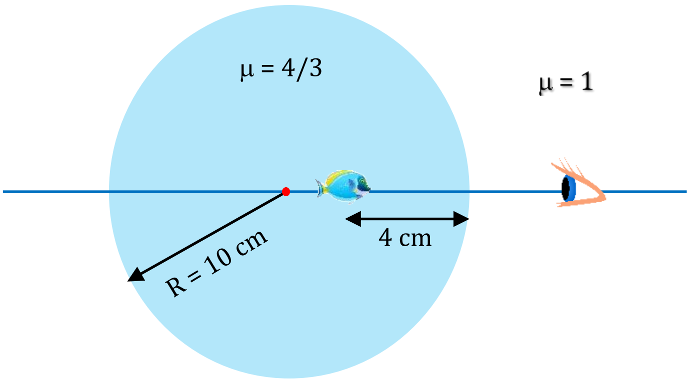


Illustration 52

Find out apparent distance of fish as seen by observer. (Radius of sphere is 10 cm)?

Solution:



Given, $\mu_2 = 1$, $\mu_1 = \frac{4}{3}$, $u = -4$ cm, $R = -10$ cm

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$

$$\frac{1}{v} - \frac{4}{3(-4)} = \frac{1 - \frac{4}{3}}{-10}$$

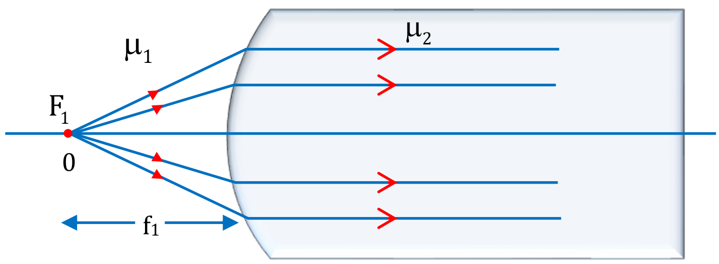
$$\frac{1}{v} + \frac{1}{3} = \frac{1}{30} \Rightarrow \frac{1}{v} = \frac{1}{30} - \frac{1}{3}$$

$$v = \frac{-30}{9} = -3.34 \text{ cm} \ \text{(virtual image).}$$

Focal length in spherical surface

A single spherical surface has two principal focal points which are as follows:

First Focus: The first principal focus is the point on the axis where an object should be placed so that the image is formed at infinity.



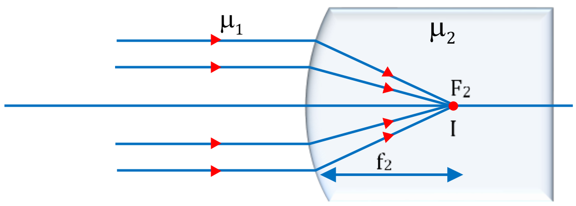
$u = f_1$, $v = \infty$, then from $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

We get $-\frac{\mu_1}{f_1} = \frac{\mu_2 - \mu_1}{R} \Rightarrow \boxed{f_1 = \frac{-\mu_1 R}{(\mu_2 - \mu_1)}}$

Second Focus: The second principal focus is the point where parallel rays get focused.

$u = -\infty$, $v = f_2$, then

$$\frac{\mu_2}{f_2} = \frac{\mu_2 - \mu_1}{R}; \boxed{f_2 = \frac{\mu_2 R}{(\mu_2 - \mu_1)}}$$



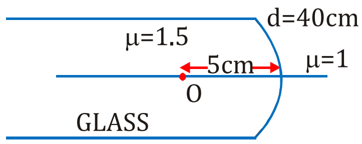
Ratio of Focal lengths: $f_1 = -\frac{\mu_1 R}{\mu_2 - \mu_1}$

$$f_2 = \frac{\mu_2 R}{\mu_2 - \mu_1}$$

$$\boxed{\frac{f_1}{f_2} = -\frac{\mu_1}{\mu_2}} \text{ or } \boxed{\frac{f_1}{\mu_1} + \frac{f_2}{\mu_2} = 0}$$

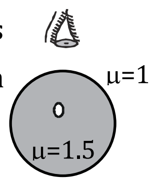
BEGINNER'S BOX-5

1. An object O in glass ($\mu = 1.5$) is situated at a distance of 5 cm from a spherical surface of diameter 40 cm as shown in the figure. Find the distance of the image from the surface.



PRI199

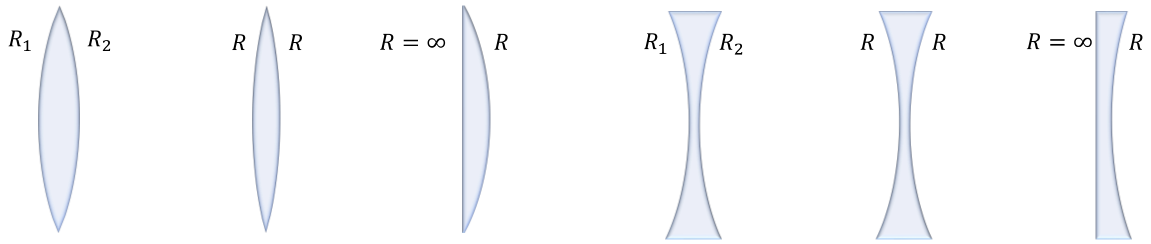
2. There is a small air bubble inside a glass sphere ($\mu = 1.5$) of radius 10 cm. The bubble is 4 cm below the surface and is viewed normally from the outside as shown in figure. Find the apparent depth of the bubble.



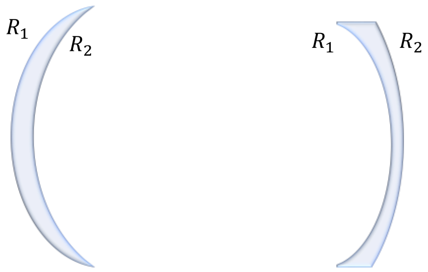
PRI200

LENS

- A lens is a portion of a transparent material with two refracting surfaces such that at least one is curved with refractive index of its material being different from that of the surroundings.
- A thin spherical lens with refractive index greater than that of surroundings behaves as a convergent or convex lens, i.e., converges parallel rays if its central (i.e. paraxial) portion is thicker than marginal one.
- However, if the central portion of a lens is thinner than marginal one, it diverges parallel rays passing through it and behaves as divergent or concave lens. This is how we classify convergent and divergent lenses.



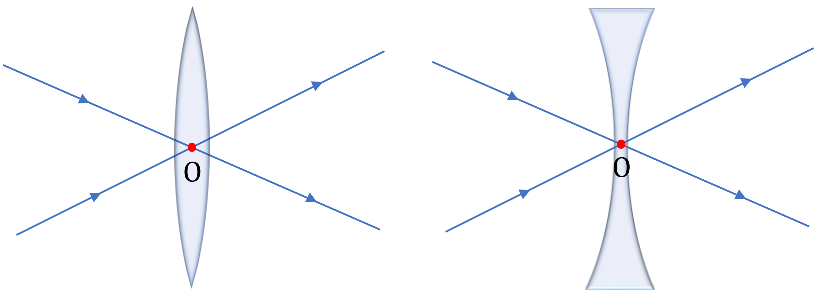
Bi convex Lens | equiconvex Lens | Plano convex Lens | Bi-concave Lens | Equiconcave Lens | Plano concave Lens



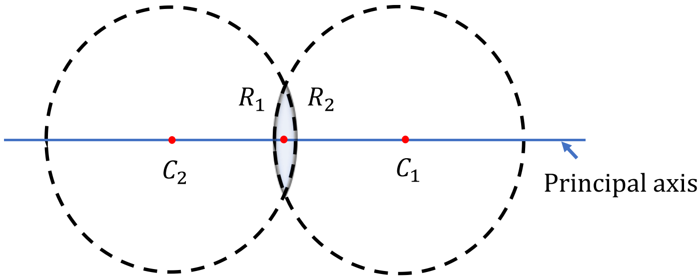
Concavo- convex Lens | Convexo- concave Lens

Basic terminologies of thin Lens

Optical Centre: It is a point O for a given thin Lens, through which any ray passes un-deviated.

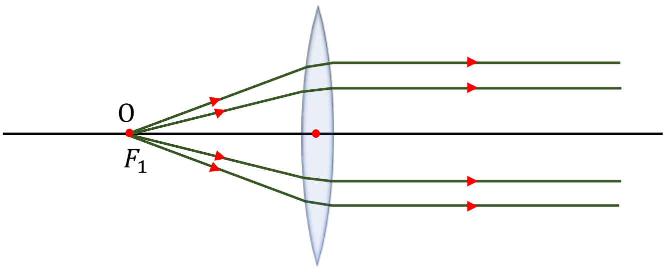


Principal Axis: C_1 C_2 is a line passing through optical Centre and perpendicular to the Lens.

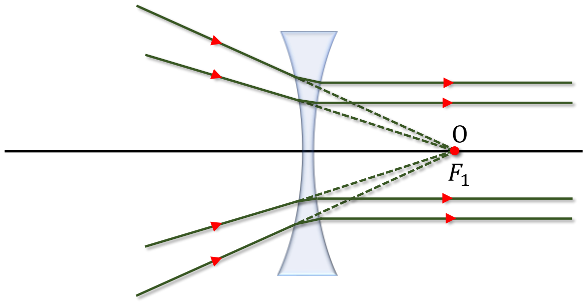


Principal Focus: A Lens has two focal points.

(i) First Focus: First focal point is an object point on the principal axis corresponding to which the image is formed at infinity.

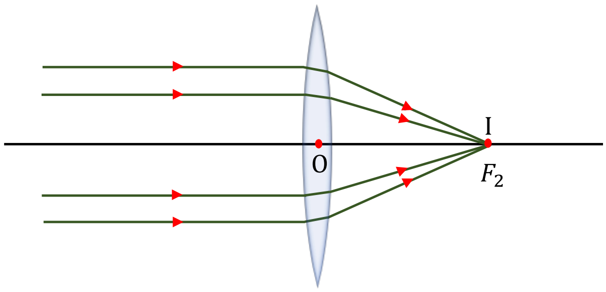


Convex Lens

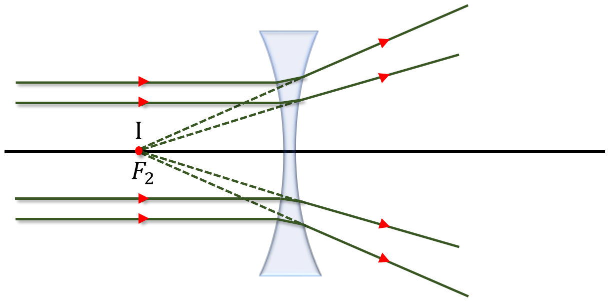


Concave Lens

(ii) Second Focus: Second focal point is an image point on the principal axis corresponding to which object lies at infinity



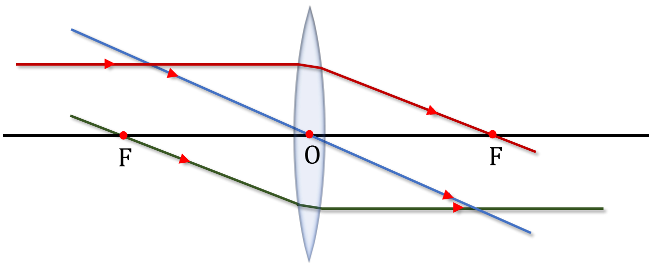
Convex Lens



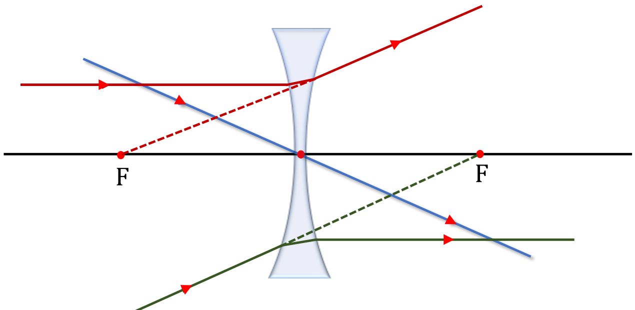
Concave Lens

Rules for image formation

- A ray passing through optical centre proceeds without deviation through the lens.
- A ray passing through first focus or directed towards it, becomes parallel to the principal axis after refraction from the lens.
- A ray passing parallel to the principal axis passes or appears to pass through F after refraction through the lens.



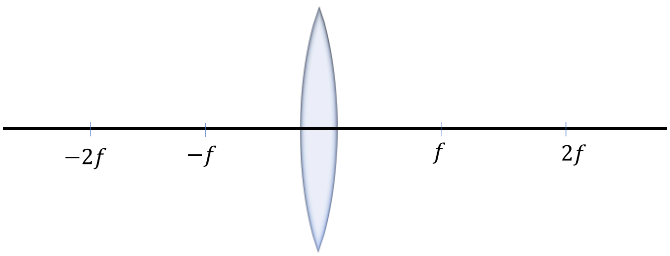
Convex Lens

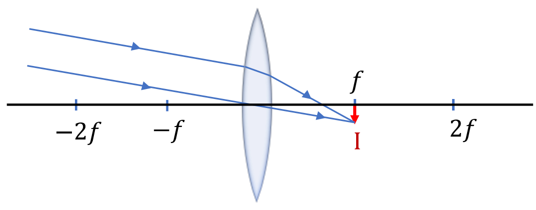
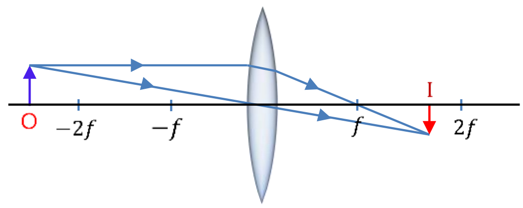
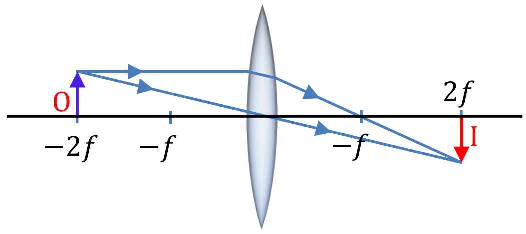
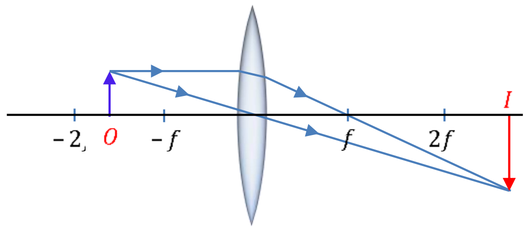
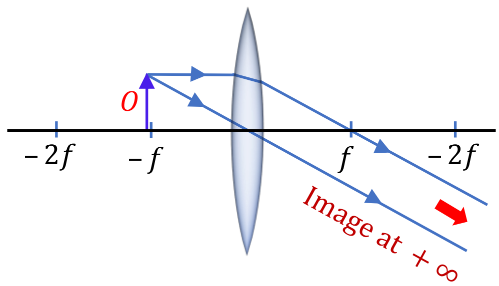
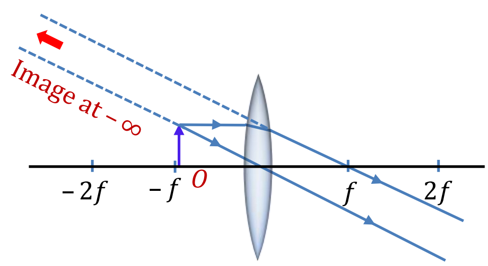


Concave Lens

Image Formation in Convex and Concave Lens

Image formation in Convex Lens:



S.No.	Position of Object	Position of Image	Nature	Size	Ray Diagram
1.	Place at infinity	Formed at f	Real, Inverted	Highly diminished,	
2.	Placed at − ∞ and 2f	Formed in between 2f and f	Real, Inverted	Diminished	
3.	Placed at − 2f	Formed at 2f	Real, Inverted	Equal in size	
4.	Placed in between −2f and −f	In between $2f$ and ∞	Real, Inverted	Enlarged	
5.	Placed in between −2f and −f (very nearer to −f)	Formed at +∞	Real, Inverted	Enlarged	
6.	Object is placed in between −f and optical centre (very nearer to −f)	Formed at −∞	Virtual, erect	Highly enlarged	

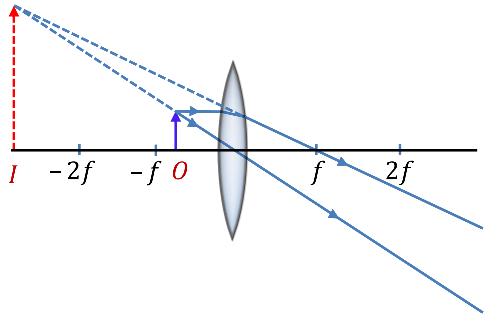
7.	Object is placed in between –f and optical centre	In between $-\infty$ and optical centre	Virtual, erect	Enlarged	

Image formation in Convex Lens

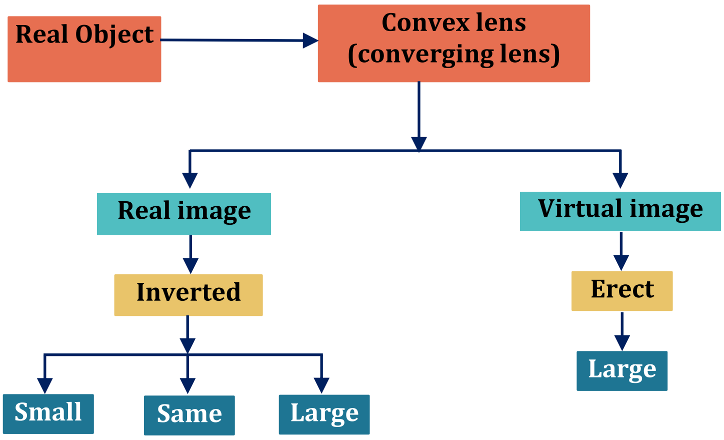


Image formation in Concave Lens:

Object: Object is placed in between $-\infty$ and optical centre

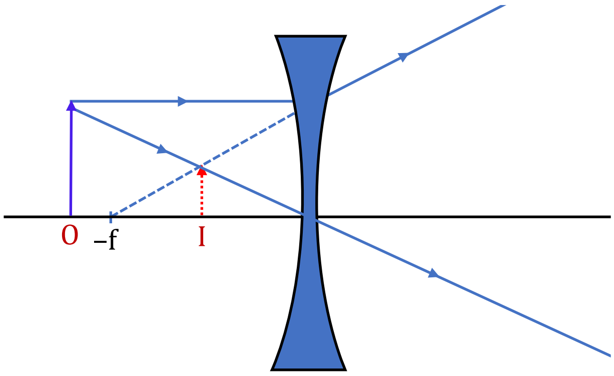


Image: in between –f and optical centre, virtual, erect, diminished

Diverging lens always forms virtual image for real object.

Lens maker's formula & Lens Equations

Lens maker's formula: Spherical Refraction from 1st surface,

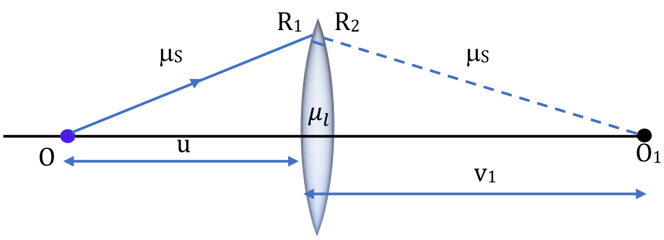
$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$\mu_2 = \mu_l$, $\mu_1 = \mu_S$

$u = u$, $v = v_1$

$R = R_1$

$$\frac{\mu_l}{v_1}-\frac{\mu_S}{u}=\frac{\mu_l-\mu_S}{R_1} \qquad \text{...(i)}$$



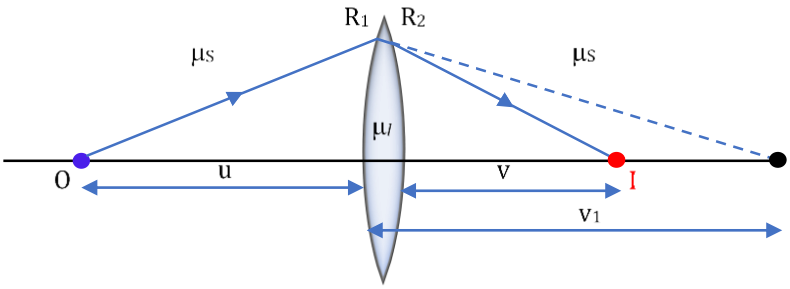
Spherical Refraction from 2nd surface,

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

$\mu_2 = \mu_S$, $\mu_1 = \mu_l$

$u = v_1$, $v = v$

$R = R_2$



$$\frac{\mu_S}{v}-\frac{\mu_l}{v_1}=\frac{\mu_S-\mu_l}{R_2} \qquad \text{...(ii)}$$

Add equation (i) & (ii) then,

$$\frac{\mu_S}{v}-\frac{\mu_S}{u}=\frac{\mu_l-\mu_S}{R_1}+\frac{\mu_S-\mu_l}{R_2}$$

$$\frac{\mu_S}{v}-\frac{\mu_S}{u}=(\mu_l-\mu_S)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_l}{\mu_S}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

If object placed at infinity i.e. $u = -\infty$ then $v = f$

$$\frac{1}{f}-\frac{1}{-\infty}=\left(\frac{\mu_l}{\mu_S}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

$$\boxed{\frac{1}{f}=\left(\frac{\mu_l}{\mu_S}-1\right)\left[\frac{1}{R_1}-\frac{1}{R_2}\right]}$$ Len's maker formula

$\boxed{\frac{1}{v}-\frac{1}{u}=\frac{1}{f}}$ Lens formula. $\Rightarrow$ $\boxed{v=\frac{uf}{u+f}}$

Important points regarding Lens formula

1. Rays should be paraxial.

2. Lenses should be thin.

3. Medium on both side of the Lens should be same.

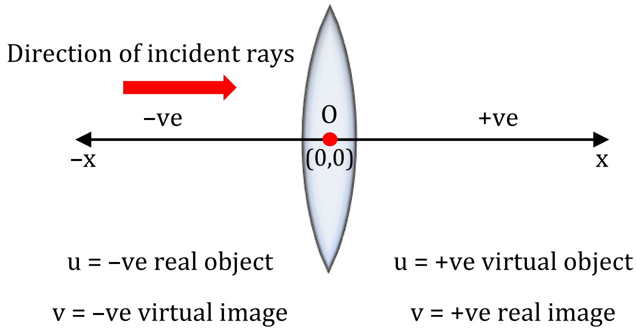
4. R_1 is the radius of curvature of that surface on which light rays incident initially and R_2 is the radius of curvature of that surface on which light rays incident after refraction from the first surface.

5. v, u, R_1, R_2, f (whatever is given in question) should be put along with sign.

Sign Convention

To derive the relevant formulae for refraction by spherical lenses, we must first adopt a sign convention for measuring distances. We shall follow the Cartesian sign convention. According to this convention, all distances are measured from the optical centre of the lenses.

- Along the principal axis, distances are measured from the optical centre of the lenses (Optical centre is taken as the origin).
- Distances in the direction of incident light are taken positive while those along opposite direction negative.
- The distances above the principal axis are taken positive while below it negative.
- Whenever and wherever possible incident light is taken to travel from left to right.

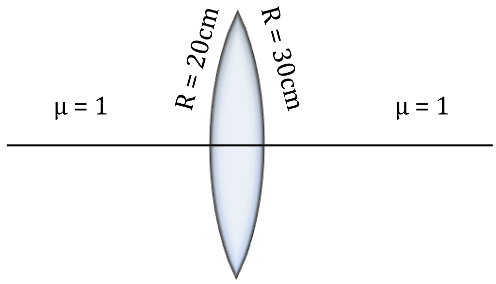


Note: (1) For converging Lens (convex Lens in air), focal length is positive.

(2) For diverging Lens (concave Lens in air), focal length is negative.

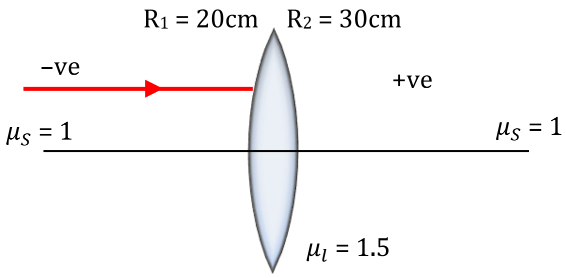
Illustration 53

Find out focal length of the following Lens. ($\mu_\ell = 1.5$)



Solution:

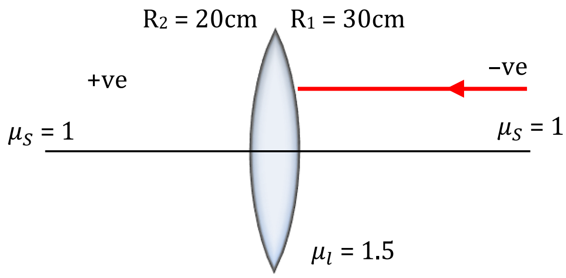
If Light rays incident from left side



$$\frac{1}{f} = \left[\frac{\mu_\ell}{\mu_S} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \left[\frac{1.5}{1} - 1\right]\left[\frac{1}{20} - \frac{1}{-30}\right]$$

f = 24 cm (converging lens)

If Light rays incident from right side



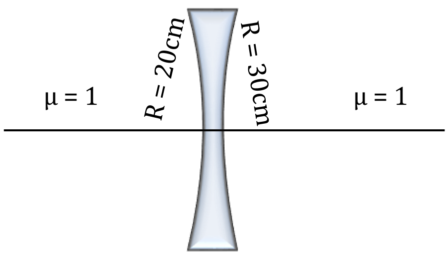
$$\frac{1}{f} = \left[\frac{\mu_\ell}{\mu_S} - 1\right]\left[\frac{1}{R_1} - \frac{1}{R_2}\right] = \left[\frac{1.5}{1} - 1\right]\left[\frac{1}{30} - \frac{1}{-20}\right]$$

f = 24 cm (converging lens)

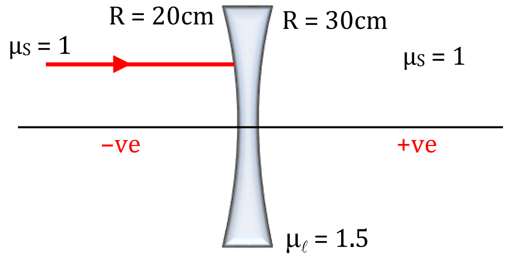
Note: From the above illustration we conclude that focal length of a lens is independent of direction of incident ray.

Illustration 54

Find out focal length of the following Lens. ($\mu_\ell = 1.5$)



Solution:



Given, $\mu_\ell = 1.5$, $\mu_S = 1$, $R_1 = -20$cm, $R_2 = 30$cm

$$\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\left[\frac{1.5}{1}-1\right]\left[\frac{1}{-20}-\frac{1}{30}\right]$$

$f = -24$cm (Diverging lens)

Illustration 55

If focal length of convex Lens ($\mu_\ell = 3/2$) in air is 20 cm. Find out focal length of same lens in water. ($\mu_w = 4/3$)

Solution:

$$\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

In Air: $\dfrac{1}{20}=\left[\dfrac{3}{2\times1}-1\right]\left[\dfrac{1}{R_1}-\dfrac{1}{R_2}\right]$ ...(i)

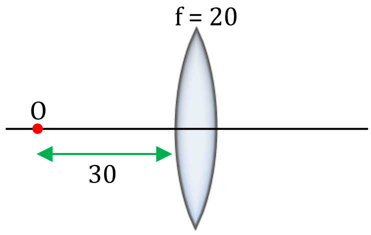
In water: $\dfrac{1}{f_w}=\left[\dfrac{3}{2\times4/3}-1\right]\left[\dfrac{1}{R_1}-\dfrac{1}{R_2}\right]$ ...(ii)

by dividing equation (i) by (ii)

$\dfrac{f_w}{20}=\dfrac{1/2}{1/8} \Rightarrow f_w = 80$ cm

Illustration 56

Find out position of image?



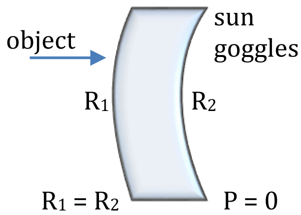
Solution:

Given, $u = -30$ cm, $f = +20$ cm, v =?

Using lens formula,

$\dfrac{1}{v}-\dfrac{1}{u}=\dfrac{1}{f} \Rightarrow \dfrac{1}{v}-\dfrac{1}{-30}=\dfrac{1}{20} \Rightarrow v = +60$ cm

Note: 1. Sun-glasses or goggles: radii of curvature of two surfaces are equal with centres of curvatures on the same side of the lens.



$R_1 = R_2 = +R$ so, $\dfrac{1}{f}=(\mu-1)\left[\dfrac{1}{R}-\dfrac{1}{R}\right]$

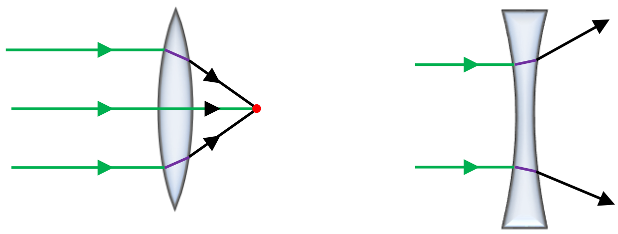
$\Rightarrow \dfrac{1}{f}=0 \Rightarrow f=\infty \Rightarrow$ and $P = 0$ $\left(\therefore P=\dfrac{1}{f}\right)$

Sun-glasses or goggles have no power.

2. If refractive index of medium < refractive index of lens

If $\mu_S < \mu_L$ then $\dfrac{\mu_L}{\mu_S}>1$ or $\left(\dfrac{\mu_L}{\mu_S}-1\right)>0$

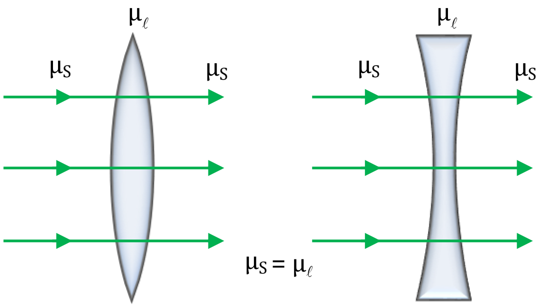
Convex lens behaves as a convergent lens. While concave lens behaves as a divergent lens.



3. Refractive index of medium = Refractive index of lens ($\mu_S = \mu_L$)

$$\frac{1}{f}=\left(\frac{\mu_L}{\mu_S}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right);\ \frac{1}{f}=0 \Rightarrow f=\infty \ \&\ P=0$$

Lens will behave as a plane transparent plate

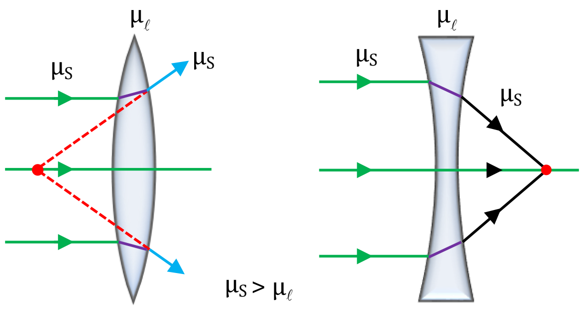


(A) (B)

4. Refractive index of surrounding medium > Refractive index of lens

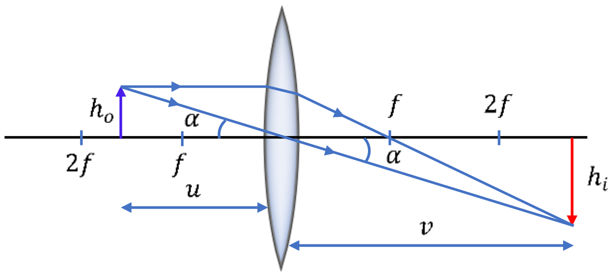
$$\mu_S > \mu_\ell \Rightarrow \frac{\mu_\ell}{\mu_S}<1 \text{ and } \left(\frac{\mu_\ell}{\mu_S}-1\right)<0$$

Convex lens will behave as a divergent lens and concave lens will behave as a convergent lens. An air bubble in water behaves as a concave lens.



Transverse or Lateral Magnification (m_t)

$$m_t=\frac{\text{height of image}}{\text{height of object}}=\frac{h_i}{h_o}$$



$$\tan\alpha=\frac{h_o}{u}=\frac{h_i}{v}$$

$$\frac{h_i}{h_o}=\frac{v}{u}$$

$$\boxed{m_t=\frac{h_i}{h_o}=\frac{v}{u}}$$

Different forms of Transverse magnification:

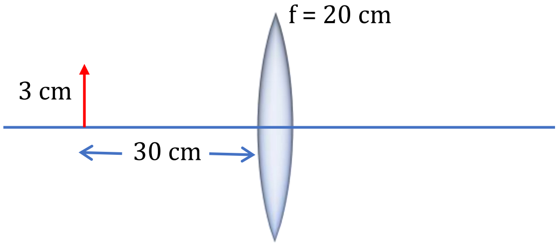
$$m_t=\frac{h_i}{h_o}$$

$$m_t=\frac{h_i}{h_o}=\frac{v}{u}$$

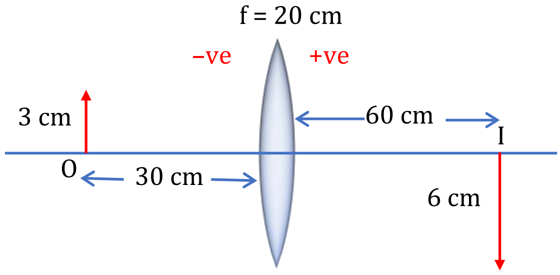
$$m_t=\frac{h_i}{h_o}=\frac{f}{f+u}=\frac{f-v}{f}$$

Illustration 57

Find out position, nature and size of image.



Solution:



Given, $u = -30$ cm, $h_o = 3$ cm, $f = 20$ cm

$$V = \frac{u \times f}{u+f} = \frac{-30\times 20}{-30+20} \Rightarrow V = 60 \text{ cm (Real)}$$

$$m_t = \frac{h_i}{h_o} = \frac{V}{u} \Rightarrow \frac{h_i}{3} = \frac{60}{-30}$$

$h_i = -6$ cm (Inverted, Large)

Illustration 58

A needle placed 45 cm away from a lens forms an image on a screen placed 90 cm away on the other side of the lens. Identify the type of lens and determine its focal length, what is the size of the image, if the size of the needle is 5 cm?

Solution:

Here, $u = -45$ cm, $v = 90$ cm, $f = ?$, $h_2 = ?$, $h_1 = 5$ cm,

$$\because \frac{1}{v} - \frac{1}{u} = \frac{1}{f} \quad \therefore \frac{1}{90} + \frac{1}{45} = \frac{1}{f} \Rightarrow \frac{1+2}{90} = \frac{1}{f} \Rightarrow f = 30 \text{ cm}$$

As f is positive, the lens is converging,

$$\because \frac{h_2}{h_1} = \frac{v}{u} \quad \therefore \frac{h_2}{5} = \frac{90}{-45} = -2 \Rightarrow h_2 = -10 \text{ cm.}$$

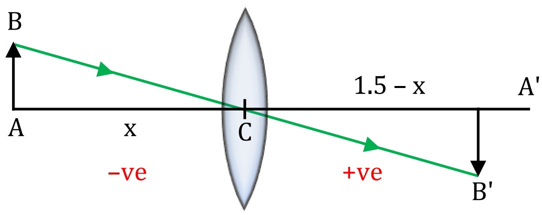
Minus sign indicates that image is real and inverted.

Illustration 59

An object is placed at a distant of 1.50 m from a screen and a convex lens placed in between produces an image magnified 4 times on the screen. What is the focal length and the position of the lens?

Solution:

$$m = \frac{h_2}{h_1} = -4$$



Let the lens be placed at a distance x from the object.

Then, $u = -x$, and $v = (1.5 - x)$

using, $m = \frac{v}{u}$, we get $-4 = \frac{1.5 - x}{-x}$

$\Rightarrow x = 0.3$ m

The lens is placed at a distance of 0.3 m from the object (or 1.20 m from the screen)

For focal length, we may use

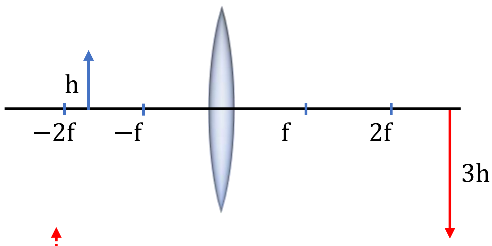
$$m = \frac{f}{f+u} \Rightarrow -4 = \frac{f}{f+(-0.3)} \Rightarrow f = \frac{1.2}{5} = 0.24 \text{ m}$$

Illustration 60

An object is placed in front of a convex Lens of focal length 10 cm. Find out position of the object such that we get three times magnified image.

Solution:

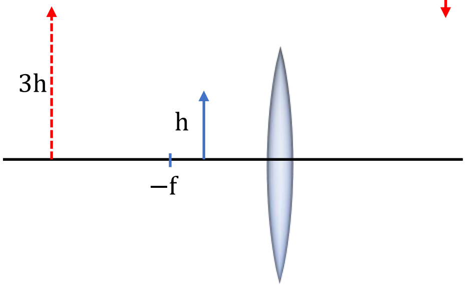
Case-1: We get magnified real, inverted image when object lies in between f and 2f



$$m_t = \frac{f}{f+u} = -3$$

$$\frac{10}{10+u} = -3 \Rightarrow u = -\frac{40}{3}\text{ cm}$$

Case-2: We get magnified virtual, erect image when object lies in between f and optical centre.

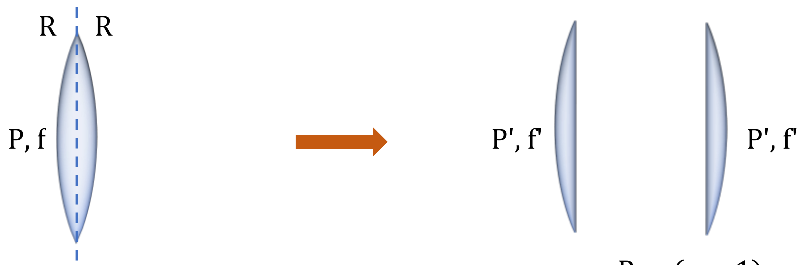


$$m_t = \frac{f}{f+u} = 3$$

$$\frac{10}{10+u} = 3 \Rightarrow u = -\frac{20}{3}\text{ cm}$$

Cutting of lens

Case (1): When we cut the lens perpendicular to the principal axis : If the equiconvex lens is cut into equal parts by a vertical plane, the focal length of each part will be double the initial value but intensity of image will remain unchanged.



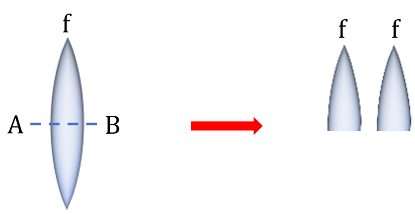
$$P = \frac{1}{f} = \frac{2(\mu-1)}{R}$$

$$f = \frac{R}{2(\mu-1)}$$

$$P' = \frac{P}{2} = \frac{(\mu-1)}{R}$$

$$f' = 2f = \frac{R}{(\mu-1)}$$

Case (2): When we cut the lens parallel to the principal axis: If an equiconvex lens of focal length f is cut into two identical parts by a horizontal plane AB then the focal length of each part will be equal to that of the initial lens; because μ, R_1 and R_2 will remain unchanged. Only intensity of image will be reduced.

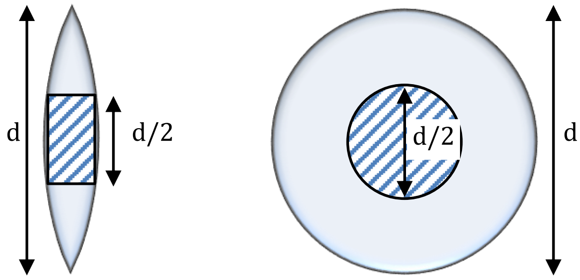


$\boxed{\therefore \text{ intensity (I)} \propto (\text{aperture})^2}$

Illustration 61

A Lens having focal length f and aperture of diameter d forms an image of intensity I. Aperture of diameter d/2 in central region of Lens is covered by a black paper. Focal length of Lens and intensity of image now will be respectively: -

Solution:



Initial area $(A_i) = \pi\left(\frac{d}{2}\right)^2$

Effective area $(A_f) = \pi\left(\frac{d}{2}\right)^2 - \pi\left(\frac{d}{4}\right)^2$

$A_f = \frac{3\pi d^2}{16} = \frac{3}{4}A_i$

Effective area is $\frac{3}{4}$ times of initial area so image intensity is also $\frac{3I}{4}$.

After covered the lens by black paper no change occurs in radius of curvature of lens, refractive index of lens and surrounding. so focal length of lens remains unchanged.

Illustration 62

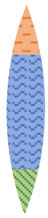
Two similar planoconvex lenses are combined together in three different ways as shown in the figure. The ratio of the focal length in three cases will be.

Solution:

In each case two planoconvex lens are placed close to each other and $\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$

So, the ratio of focal length in three cases will be $\Rightarrow 1 : 1 : 1$

Illustration 63



A convex Lens is made up of three different materials as shown in the figure. For a point object placed on its axis, the number of images formed are:

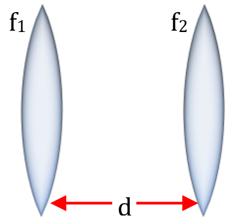
Solution:

R.I $\propto$ no. of images

So, number of images = 3

Note: Two thin coaxial lenses are placed at a small separation d

(provided incident rays are parallel to the principal axis).



$$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{d}{f_1 f_2}}$$

$$\Rightarrow \quad \boxed{P = P_1 + P_2 - d\,P_1 P_2}$$

Use proper sign convention when solving numerical.

Illustration 64

A convex lens of focal length 25 cm and a concave lens of focal length 20 cm are mounted coaxially separated by a distance d cm. If the power of the combination is zero, d is equal to :

Solution:

$P_1 = \frac{1}{f_1} = \frac{100}{25} = 4D$ and $P_2 = \frac{1}{f_2} = \frac{100}{-20} = -5D$

$P = P_1 + P_2 - d\ P_1 P_2 = 0$

$4 - 5 + d\,(4)\,(5) = 0$

$d = 0.05$ m = 5 cm

BEGINNER'S BOX-6

1. An object is placed at the distance of 30 cm in front of a convex lens of focal length 10 cm. Find the position of the image, its nature and magnification.

PRI201

2. A planoconvex lens has a focal length of 30 cm and an index of refraction 1.5. Find the radius of the convex surface.

PRI202

3. A biconvex lens (μ = 1.5) of focal length 0.2 m acts as a divergent lens of power 1D when immersed in a liquid. Find the refractive index of the liquid.

PRI203

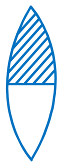
4. Two thin converging lenses of focal lengths 20 cm and 40 cm are placed in contact. Find the effective power of the combination.

PRI204

5. An object placed 20 cm in front of a convex lens has its image 40 cm behind the lens. Find the power of the lens.

PRI205

6. A lens shown in figure is made of two different materials. A point object is placed on its axis. How many images will be formed.



PRI206

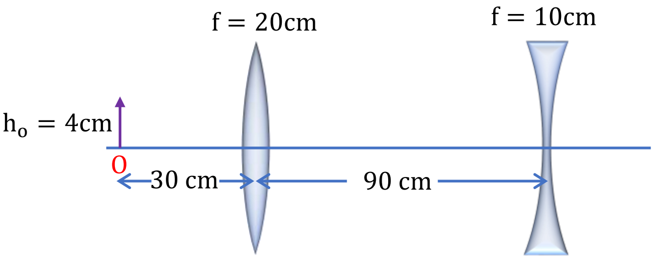
7. A convex lens of focal length f produces a real image of size is $\frac{1}{n}$ times the size of the object. Find the position of the object.

PRI207

Combination of Lens

Illustration 65

Find out height of final image formed.



Solution:

1^{st} Lens (Convex lens)

u= –30 cm

f = 20 cm

h_o = 4 cm

$$V = \frac{u \times f}{u+f} = \frac{-30 \times 20}{-30+20} = 60\,\text{cm} \quad (I_1 \text{ image})$$

$$h_i = \frac{V}{u} h_o = \frac{60}{-30} \times 4 = -8\,\text{cm}$$

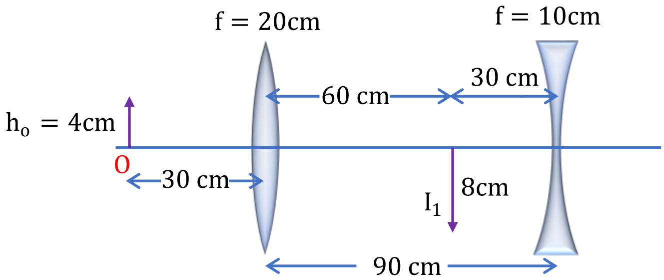


Image of convex lens will behave as an object for concave lens (2^{nd} lens).

Now for 2^{nd} lens (concave lens)

u = –30 cm, f = –10 cm, h_o = –8 cm

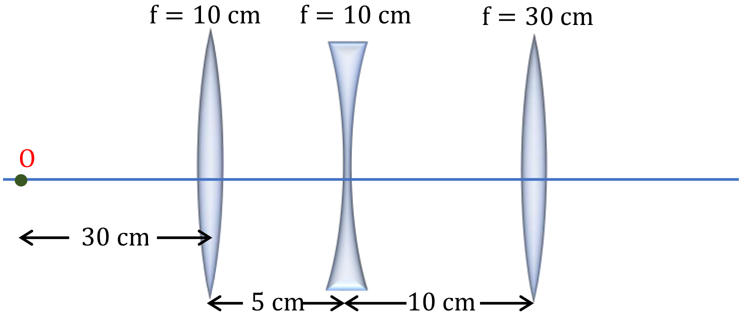
$$V = \frac{u \times f}{u+f} = \frac{-30 \times -10}{-30-10} = -\frac{300}{40} = -7.5\,\text{cm}$$

$$h_i = \frac{V}{u} h_o = \frac{-7.5}{-30} \times (-8) = -2\,\text{cm}$$

So final image formed at a distance of 7.5 cm from the concave lens and height of final image is 2 cm.

Illustration 66

Find the position of the image formed by the Lens combination given in the figure:



Solution:

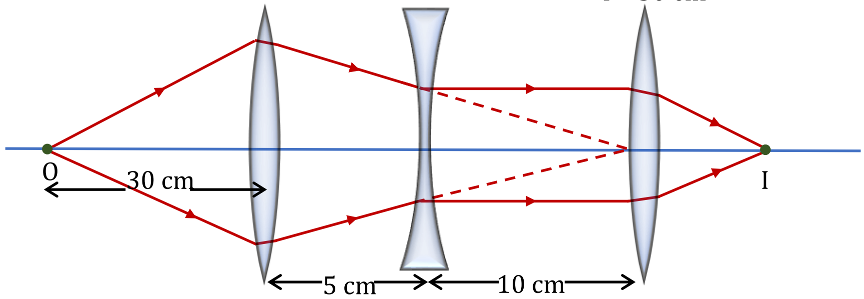


Image formed by the first lens, $\frac{1}{v_1}-\frac{1}{u_1}=\frac{1}{f_1} \Rightarrow \frac{1}{v_1}-\frac{1}{-30}=\frac{1}{10} \Rightarrow v_1 = 15$ cm

The image formed by the first lens serves as the object for the second. This is at a distance of (15 – 5) cm = 10 cm to the right of the second lens. Though the image is real, it serves as a virtual object for the second lens, which means that the rays appear to come from it for the second lens. $\frac{1}{V_2}-\frac{1}{10}=\frac{1}{-10} \Rightarrow V_2 = \infty$

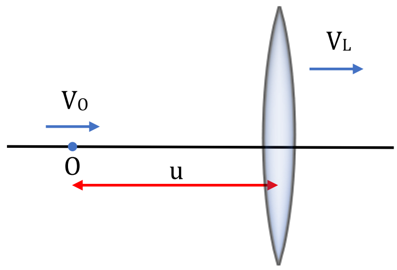
The virtual image is formed at an infinite distance to the left of the second lens. This acts as an object for the third lens. $\frac{1}{V_3}-\frac{1}{u_3}=\frac{1}{f_3} \Rightarrow \frac{1}{v_3}=\frac{1}{\infty}+\frac{1}{30} \Rightarrow V_3 = 30$ cm

The final image is formed 30 cm to the right of the third lens.

Velocity of Image in Lens

Case (1) : When the object is moving along to principal axis of the Lens.

From equation, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$



Differentiate with respect to time,

$$-\frac{1}{v^2}\frac{dv}{dt}-\frac{-1}{u^2}\frac{du}{dt}=0$$

$$-\frac{1}{v^2}\frac{dv}{dt}=\frac{-1}{u^2}\frac{du}{dt}$$

$$\frac{dv}{dt}=+\frac{v^2}{u^2}\frac{du}{dt}$$

$\boxed{\vec{V}_{IL}=m_t^2\vec{V}_{OL}}$ $\vec{V}_{IL}$ = velocity of image with respect to lens.

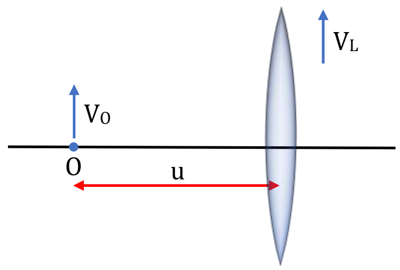
$\vec{V}_{OL}$ = velocity of object with respect to lens.

All velocities are instantaneous.

Case (2) : When the object is moving perpendicular to the principal axis of the Lens.

From equation $m_t = \frac{h_i}{h_o} = \frac{v}{u}$

$$h_i = \frac{v}{u} h_o$$



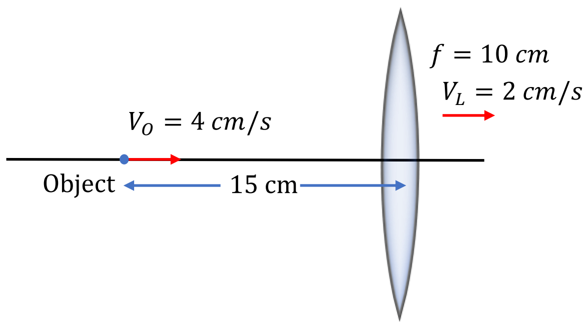
Differentiate with respect to time,

$$\frac{dh_i}{dt} = \frac{v}{u}\frac{dh_o}{dt}$$

$\boxed{\vec{V}_{IL} = m_t \vec{V}_{OL}}$ All velocities are instantaneous.

Illustration 67

Find out velocity of image.



Solution:

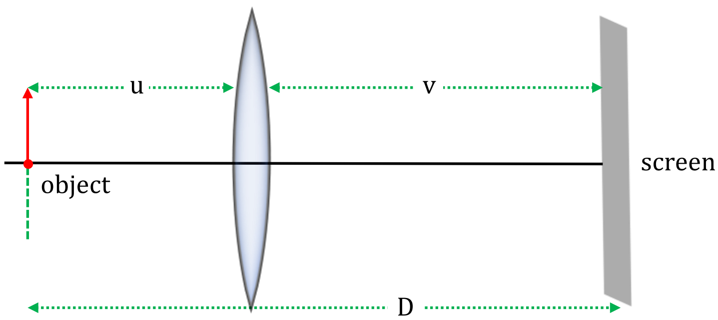
$$m_t = \frac{f}{f+u} = \frac{10}{10-15} = -2$$

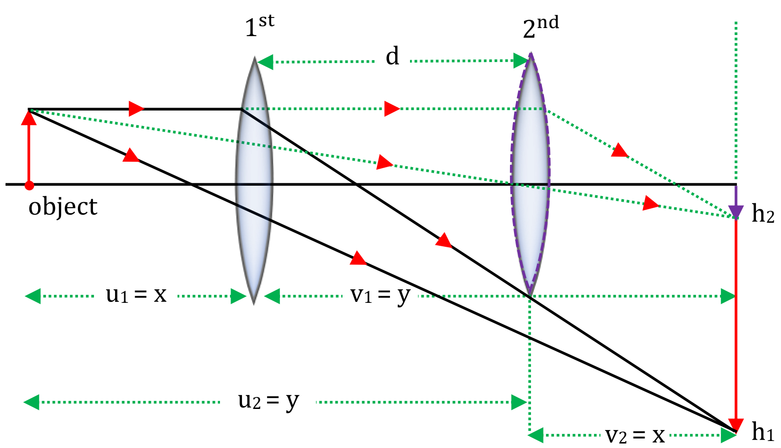
$$\vec{V}_{IL} = m_t^2 \vec{V}_{OL} \Rightarrow \vec{V}_I - \vec{V}_L = m_t^2\left(\vec{V}_O - \vec{V}_L\right) \Rightarrow \vec{V}_I - 2 = 4(4-2)$$

$$\vec{V}_I = 10\ cm/s$$

Displacement Method

It is used for determination of focal length of convex lens in laboratory. A thin convex lens of focal length f is placed between an object and a screen fixed at a distance D apart. If D > 4f there are two positions of lens corresponding to which a sharp image of the object is formed on the screen.





Calculation of focal length	Calculation of height of object
$x + y = D$ ...(i) $y - x = d$ ...(ii) From equation (i) & (ii) $y=\frac{D+d}{2}$ and $x=\frac{D-d}{2}$ When Lens is in 1st position $u = -x, v = y$ From Lens formula, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \Rightarrow \frac{1}{y}-\frac{1}{-x}=\frac{1}{f}$ $\frac{2}{D+d}+\frac{2}{D-d}=\frac{1}{f} \Rightarrow \boxed{f=\frac{D^2-d^2}{4D}}$	When Lens in 1st position $m_1=\frac{y}{x}=\frac{h_1}{h_o}$ When Lens is in 2nd position $m_2=\frac{x}{y}=\frac{h_2}{h_o}$ Multiply both equation, $m_1 \times m_2 = 1 = \frac{h_1 h_2}{h_o^2}$ $\boxed{h_o=\sqrt{h_1 h_2}}$

Note:

(1) If $D > 4f$ Two position of lens could be found to form the real image of the source on the screen.

(2) If $D = 4f$ One position of lens could be found to form the real image of the source on the screen.

(3) If $D < 4f$ No position of lens could be found to form the real image of the source on the screen.

So, for image form on the screen, $D \geq 4f \Rightarrow \boxed{f \leq \frac{D}{4}}$

Conclusion

(1) $f=\frac{D^2-d^2}{4D}$ (2) $m_1 \times m_2 = 1$ (3) $h_o=\sqrt{h_1 \times h_2}$ (4) $|m_1 - m_2| = \frac{d}{f}$

(5) $D \geq 4f$ where, d is the displacement of Lens.

(6) $f_{max}=\frac{D}{4}$ D is the distance between object and screen.

Illustration 68

A screen is placed a distance 40 cm away from an illuminated object. A converging lens is placed between the source and the screen and it is attempted to form the image of the source on the screen. If no position could be found then find out the focal length of the lens.

Solution:

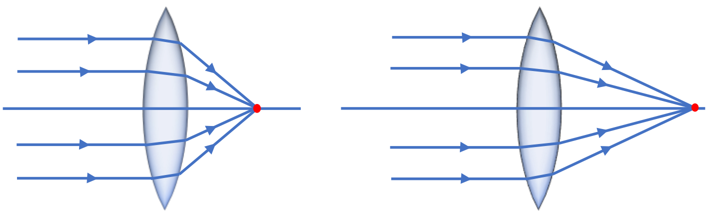
Given, D = 40 cm

If no position of lens could be found to form the image i.e. $D < 4f \Rightarrow 40 < 4f \Rightarrow f > 10$ cm

Optical Power (Lens and Mirror)

Optical power is the degree to which a lens, mirror, or other optical system converges or diverges light. Optical power also referred to as dioptric power, refractive power, focusing power, or convergence power.

Unit of optical power is diopter (m^{-1}).



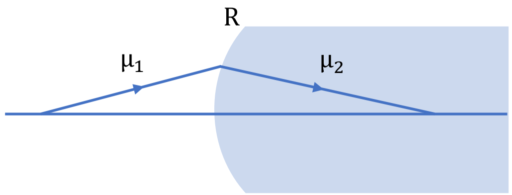
(1) (2)

Optical power of 1st lens is greater than the optical power of 2nd lens.

- **Power of any converging optical system is positive.**
  Example : Convex lens in air, concave mirror.
- **Power of any diverging optical system is negative.**
  Example : Concave lens in air, convex mirror.

Optical power of different optical system

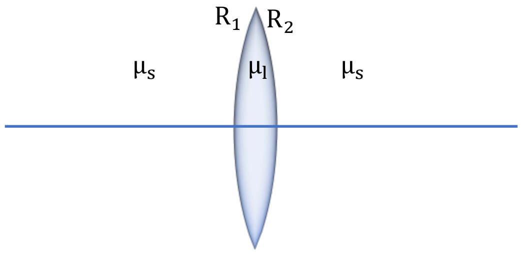
(1) Spherical surface:



$$\boxed{P = \frac{\mu_2 - \mu_1}{R}}$$

- Put the value of R with sign.
- μ_2 is the medium in which light rays enter after refraction and μ_1 is the medium in which light rays moves before refraction.

(2) Thin lens:



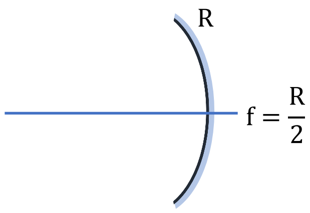
$$\boxed{P = \frac{\mu_s}{f}}$$

$$\boxed{\frac{1}{f} = \left(\frac{\mu_\ell}{\mu_s} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

$$P = \mu_S\left(\frac{\mu_\ell}{\mu_S} - 1\right)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

$$\boxed{P = (\mu_\ell - \mu_S)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]}$$

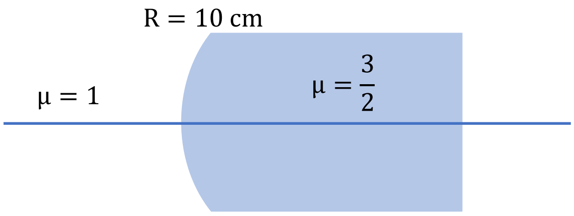
(3) Spherical Mirror:



$$\boxed{P = -\frac{1}{f}}$$

Illustration 69

Find out optical power of the spherical surface.

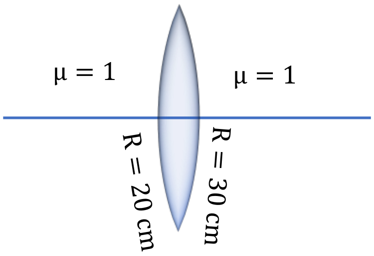


Solution:

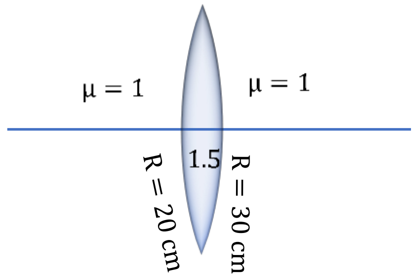
$$P = \frac{\mu_2 - \mu_1}{R} = \left[\frac{3/2 - 1}{10}\right] \times 100 = 5D \text{ (converging system).}$$

Illustration 70

Find out optical power of the lens (μ_l = 1.5).



Solution:

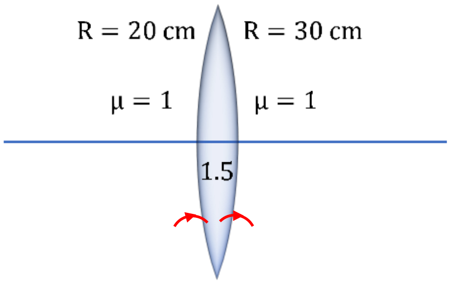


First method: $\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$

$$\frac{1}{f}=\left[\frac{1.5}{1}-1\right]\left[\frac{1}{20}-\left(-\frac{1}{30}\right)\right]$$

f = 24 cm

$$P=\frac{1}{f}=\frac{100}{24}=\frac{25}{6}D$$



Second method:

Through two spherical refraction

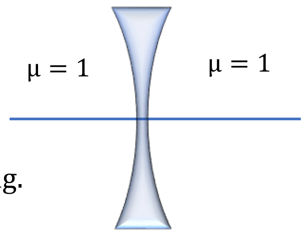
$$P=\left[\frac{1.5-1}{20}+\frac{1-1.5}{-30}\right]\times 100$$

$$P=\frac{25}{6}D$$

Illustration 71

Find out optical power of equi-concave lens of refractive index μ and radius of curvature R.

Solution:

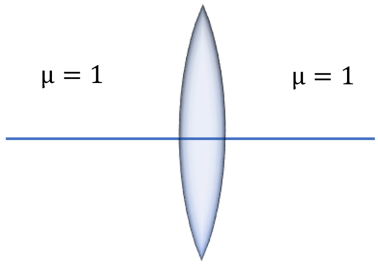


$$\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\left[\frac{\mu}{1}-1\right]\left[\frac{1}{-R}-\frac{1}{R}\right]$$

$f=-\frac{R}{2(\mu-1)}\Rightarrow \boxed{P=-\frac{2(\mu-1)}{R}}$ (Learn directly), Power is –ve, so lens is diverging.

Illustration 72

Find out optical power of equiconvex lens of refractive index μ and radius of curvature R.



Solution:

$$\frac{1}{f}=\left[\frac{\mu_\ell}{\mu_S}-1\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]=\left[\frac{\mu}{1}-1\right]\left[\frac{1}{R}-\frac{1}{-R}\right]$$

$$f=\frac{R}{2(\mu-1)}$$

$\boxed{P=\frac{2(\mu-1)}{R}}$ (Learn directly), Power is +ve, so lens is converging.

Illustration 73

Power of a convex lens (μ = 3/2) in air is 10D, now we immersed the lens in water (μ = 4/3) then find out the power of lens in water?

Solution:

$$P=[\mu_\ell-\mu_s]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$$

In medium-1: $P_1=\left[\mu_\ell-\mu_{s_1}\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$ ...(i)

In medium-2: $P_2=\left[\mu_\ell-\mu_{s_2}\right]\left[\frac{1}{R_1}-\frac{1}{R_2}\right]$ ...(ii)

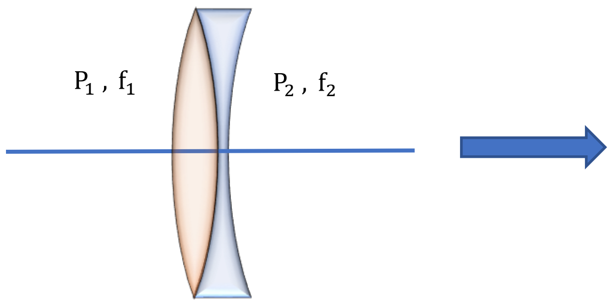
By dividing equation (i) by (ii), $\boxed{\dfrac{P_1}{P_2}=\left[\dfrac{\mu_\ell-\mu_{s_1}}{\mu_\ell-\mu_{s_2}}\right]}$

Now medium-1 is air and medium-2 is water, $\boxed{\dfrac{P_a}{P_w}=\left[\dfrac{\mu_\ell-\mu_a}{\mu_\ell-\mu_w}\right]}$

$\dfrac{10}{P_w}=\dfrac{3/2-1}{3/2-4/3}\Rightarrow P_w=\dfrac{10}{3}$ D ; Power of convex lens in water is less than that of air.

Combination of lenses

Two or more than two lens are placed in contact with each other.

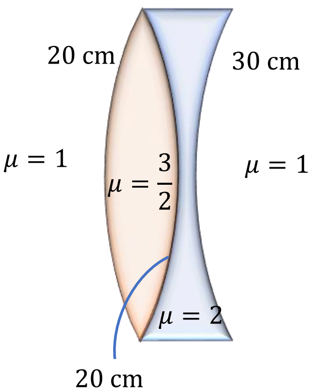


Lens (P_{eq} , f_{eq}) $\boxed{P_{eq}=P_1+P_2}\Rightarrow\boxed{\dfrac{1}{f_{eq}}=\dfrac{1}{f_1}+\dfrac{1}{f_2}}$

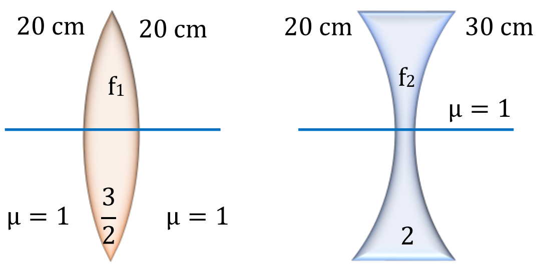
If P_{eq} is +ve then **converging lens** is formed.
If P_{eq} is –ve then **diverging lens** is formed.
If P_{eq} is zero then **plane surface** is formed.
Use sign convention while solving numerical.

Illustration 74

Find out P_{eq} & f_{eq}?



Solution:



$$\frac{1}{f_1}=\left[\frac{3}{2}-1\right]\left[\frac{1}{20}+\frac{1}{20}\right]=\frac{1}{20}$$

$$\frac{1}{f_2}=[2-1]\left[-\frac{1}{20}-\frac{1}{30}\right]$$

$$\frac{1}{f_2}=-\frac{1}{12}\Rightarrow\frac{1}{f_{eq}}=\frac{1}{f_1}+\frac{1}{f_2}$$

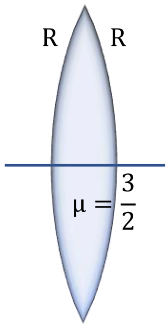
$$\frac{1}{f_{eq}}=\frac{1}{20}-\frac{1}{12}\Rightarrow f_{eq}=-30\text{cm}$$

$$P_{eq}=\frac{\mu_s}{f_{eq}}=-\frac{100}{30}=-\frac{10}{3}\text{D}$$

Illustration 75

Two identical glass ($\mu = 3/2$) equiconvex lens of focal length f each are kept in contact. The space between the two lenses is filled with water ($\mu = 4/3$). Find out the focal length of the combination?

Solution:



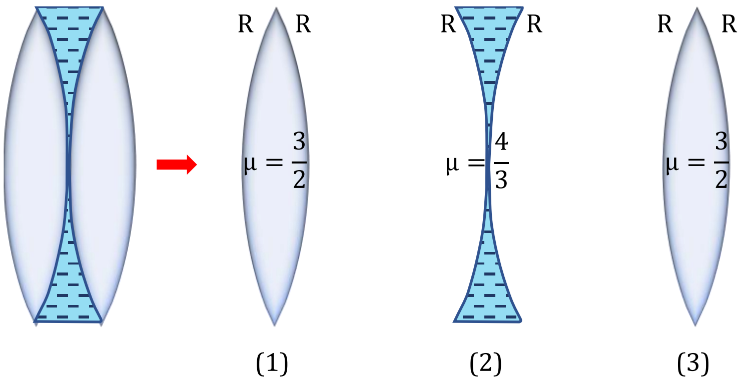
$$f = \frac{R}{2(\mu - 1)}$$

$$f = \frac{R}{2\left(\frac{3}{2} - 1\right)} = R$$

$$f_1 = f$$

$$\frac{1}{f_2} = \left[\frac{4}{3} - 1\right]\left[-\frac{2}{R}\right] = -\frac{3R}{2} \Rightarrow f_2 = -\frac{3f}{2}$$

and $f_3 = f$



So equivalent focal length,

$$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$$

$$\frac{1}{f_{eq}} = \frac{1}{f} - \frac{2}{3f} + \frac{1}{f} = \frac{4}{3f}$$

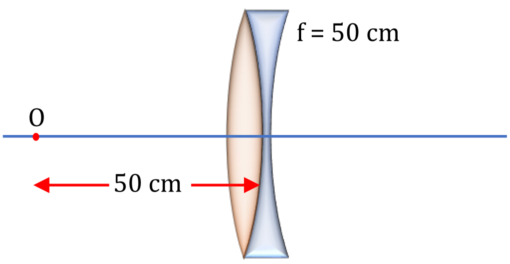
$$f_{eq} = \frac{3f}{4}$$

BEGINNER'S BOX-7

1. In the displacement method the distance between the object and the screen is 70 cm and the focal length of the lens is 16 cm, find the separation between the magnified and diminished image position of the lens.

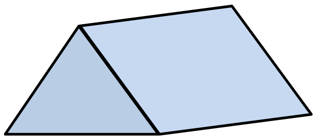
PRI208

2. Lens of power 3D and –5D are combined to form a compound lens. An object is placed at a distance of 50 cm from the lens. Calculate the position of its image



PRI209

Refraction Through Prism



An optical prism is a homogeneous solid transparent medium (such as glass) with flat surfaces that refract light. In prism light ray crosses two plane inclined surfaces. These surfaces are called the 'refracting surfaces' and the angle between them is called the 'refracting angle' or the 'angle of prism'.

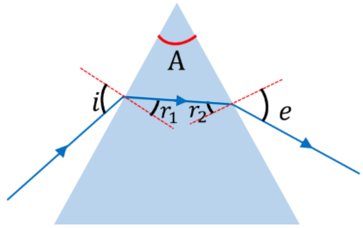
i : angle of incidence.

e : angle of emergence.

r_1 & r_2 are angle of refraction.

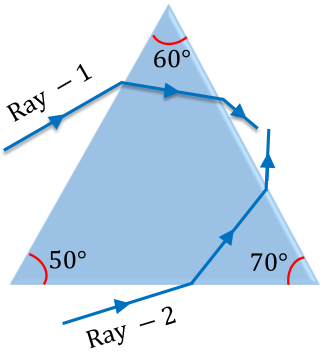
A : apex angle of prism OR prism angle

OR Refracting angle of prism.



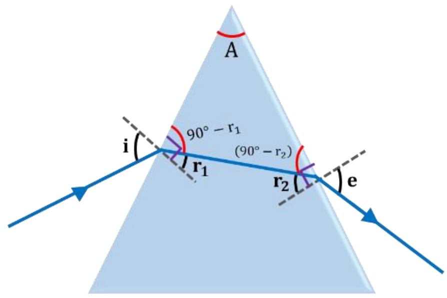
Front view

Calculation of Prism Angle



For Ray-1
A = 60°
For Ray-2
A = 70°

Important results regarding Prism

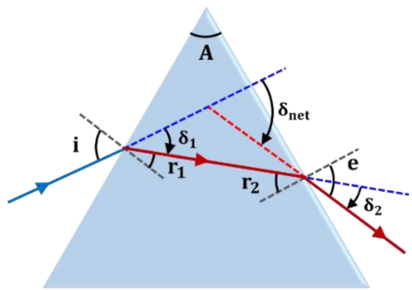


$A + (90° - r_1) + (90° - r_2) = 180°$

$\boxed{A = r_1 + r_2}$

When required, r_1 & r_2 to be found using Snell's Law.

Deviation produced by Prism



$\delta_1 = i - r_1$ (clockwise)

$\delta_2 = e - r_2$ (clockwise)

In both refraction, light ray deviated in same sense, hence,

$\delta_{net} = \delta_1 + \delta_2$ (clockwise)

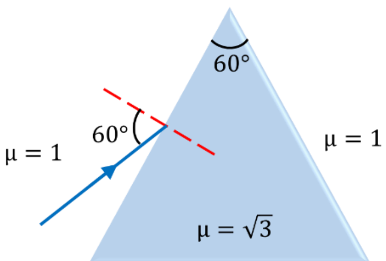
$\delta_{net} = (i - r_1) + (e - r_2)$

$\delta_{net} = (i + e) - (r_2 + r_2)$

$\boxed{\delta_{net} = (i + e) - A}$

Illustration 76

Find out deviation produce by prism in a light ray?



Solution:

Snell's law at first surface,

$1\sin 60° = \sqrt{3}\sin r_1$

$\frac{\sqrt{3}}{2} = \sqrt{3}\sin r_1 \Rightarrow r_1 = 30°$

$A = r_1 + r_2$

$60° = 30° + r_2 \Rightarrow r_2 = 30°$

Snell's law at second surface

$\sqrt{3}\, \sin r_2 = 1 \sin e$.

$\sqrt{3} \sin 30° = \sin e \Rightarrow e = 60°$

$\delta_{net} = i + e - A$

$\delta_{net} = 60° + 60° - 60° = 60°$

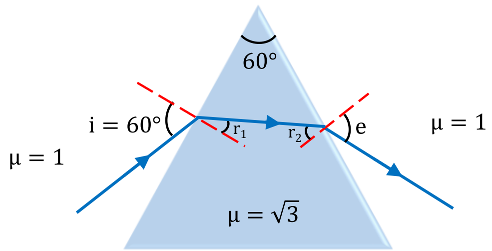
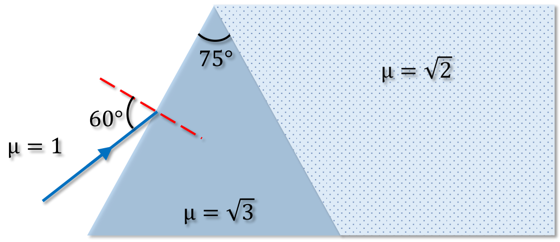


Illustration 77

Find out deviation produce by prism in a light ray?



Solution:

Snell's law at first surface,

$1 \sin 60° = \sqrt{3} \sin r_1$

$\frac{\sqrt{3}}{2} = \sqrt{3} \sin r_1 \Rightarrow r_1 = 30°$

$A = r_1 + r_2$

$75° = 30° + r_2 \Rightarrow r_2 = 45°$

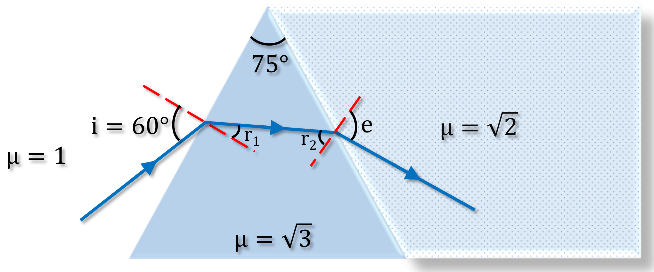
Snell's law at second surface,

$\sqrt{3} \sin r_2 = \sqrt{2} \sin e$.

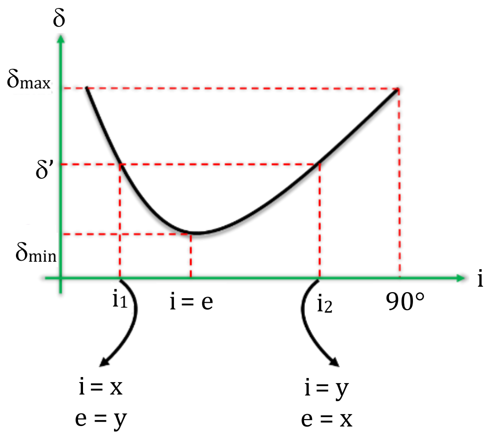
$\sqrt{3} \sin 45° = \sqrt{2} \sin e \Rightarrow e = 60°$

$\delta_{net} = i + e - A$

$\delta_{net} = 60° + 60° - 75° = 45°$



δ v/s I curve for prism



(1) The deviation produced by a prism depends on angle of incidence i, angle of prism A, refractive index of material μ.

(2) δ is first decreasing and then increasing for i < e and i > e respectively.

(3) For one δ (except δ_{min}) there are two values of angle of incidence. If i and e are interchanged then we get the same value of δ because of reversibility principle of light.

(4) There is one and only one angle of incidence for which the angle of deviation is minimum (when i = e).

(5) Before i = e, δ decreases more rapidly than it increases with i after i = e. Right hand side part of the graph is more tilted then the left hand side.

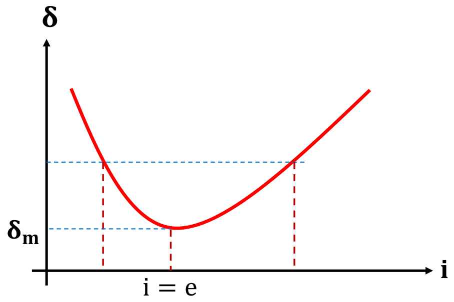
Minimum Deviation in Prism

From graph condition of minimum deviation is

$$\boxed{i = e}$$

Then,

$$\boxed{r_1 = r_2 = \frac{A}{2}}$$



Relation between Minimum deviation and Refractive index of prism

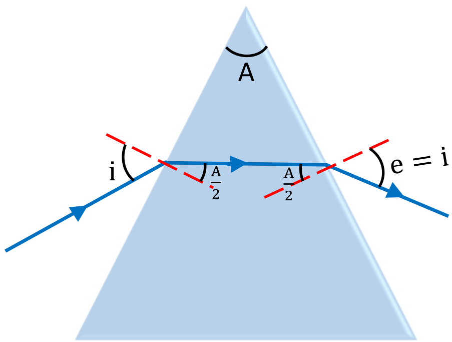
From $\delta = i + e - A$

$\boxed{\delta_m = 2i - A} \Rightarrow i = \dfrac{\delta_m + A}{2}$

Now Snell's law at first surface, $\mu_s \sin i = \mu_p \sin\left(\dfrac{A}{2}\right)$

$$\mu_s \sin\left(\frac{\delta_m + A}{2}\right) = \mu_p \sin\left(\frac{A}{2}\right)$$

$$\boxed{\frac{\mu_p}{\mu_s} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}}$$



Note: For an isosceles / equilateral prism, light goes parallel to the base inside the prism.

Illustration 78

If refractive index of prism is $\sqrt{2}$ and refracting angle is 60° then find out minimum deviation produce by the prism.

Solution:

$$\frac{\mu_p}{\mu_s} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)} \Rightarrow \frac{\sqrt{2}}{1} = \frac{\sin\left(\frac{\delta_m + 60°}{2}\right)}{\sin\left(\frac{60°}{2}\right)}$$

$$\sqrt{2} = \frac{\sin\left(\frac{\delta_m + 60°}{2}\right)}{\sin 30°} \Rightarrow \frac{1}{\sqrt{2}} = \sin\left(\frac{\delta_m + 60°}{2}\right) \Rightarrow \frac{\delta_m + 60°}{2} = 45° \Rightarrow \delta_m = 30°$$

Illustration 79

An equilateral prism provides least deviation 46° in air. Find out the refractive index of unknown liquid in which same prism gives least deviation 30°

Solution:

Using formula, $\dfrac{\mu_p}{\mu_s} = \dfrac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

For air, $\dfrac{\mu_p}{1} = \dfrac{\sin\left(\dfrac{46°+60°}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)}$ ...(i)

For liquid, $\dfrac{\mu_p}{\mu_\ell} = \dfrac{\sin\left(\dfrac{30°+60°}{2}\right)}{\sin\left(\dfrac{60°}{2}\right)}$ ...(ii)

By equation (i) and (ii)

$\mu_\ell = \dfrac{\sin 53°}{\sin 45°} = \dfrac{4\times\sqrt{2}}{5\times 1} = \dfrac{4\sqrt{2}}{5}$

Illustration 80

A ray of light passing through a prism having refractive index $\sqrt{2}$ suffers minimum deviation. It is found that angle of incidence is double the angle of refraction within the prism. Find out angle of prism.

Solution:

For minimum deviation, $r_1 = r_2 = \dfrac{A}{2}$ and $i = 2r_1 \Rightarrow i = A$

Snell's law at first surface, $1 \times \sin i = \mu_p \sin r_1$

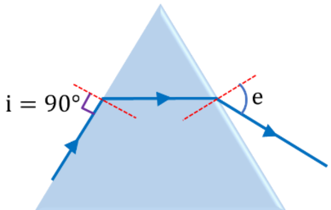
$\sin A = \sqrt{2}\sin\left(\dfrac{A}{2}\right)$

$2\sin\left(\dfrac{A}{2}\right)\cos\left(\dfrac{A}{2}\right) = \sqrt{2}\sin\left(\dfrac{A}{2}\right)$

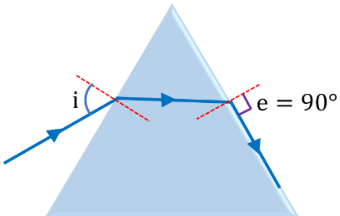
$\cos\left(\dfrac{A}{2}\right) = \dfrac{1}{\sqrt{2}} \Rightarrow \dfrac{A}{2} = 45° \Rightarrow A = 90°$

Maximum Deviation

For δ_{max} either $i = 90°$ or $e = 90°$

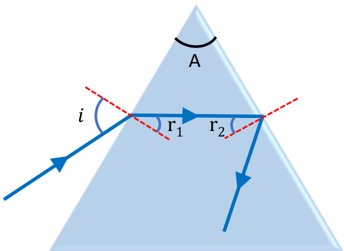


'OR'



Then use $\delta_{net} = (i + e) - A$

Condition of No emergence



$(r_2)_{min} > \theta_c$

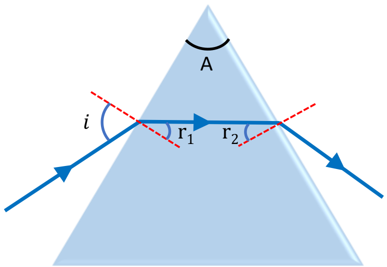
$A - r_{1\,max} > \theta_c$

When $i_{max} = 90°$ then $r_{1\,max} = \theta_c$

So, $A - \theta_c > \theta_c$

$\boxed{A > 2\theta_C}$

Condition of Always emergence



$(r_2)_{max} = \theta_c$

$A - r_{1\,min} = \theta_c$

When $i_{min} = 0°$ then $r_{1\,min} = 0°$

$A_{max} = \theta_c$

So, $\boxed{A \le \theta_C}$

Conclusion

1. if $A \le \theta_c$; then light ray always crosses the prism for any value of i.
2. if $A > 2\theta_c$; then light ray never crosses the prism for any value of i.
3. if $\theta_c \le A \le 2\theta_c$; then light ray may crosses the prism or not depends on angle of incidence.

Illustration 81

Find out the maximum value of refractive index of a prism which permits the transmission of light through it when the refracting angle of the prism is 90°.

Solution:

We can transmit the light through the prism when.

$$A \le 2\theta_c \Rightarrow \frac{A}{2} \le \theta_C$$

$$\sin\left(\frac{A}{2}\right) \le \sin\theta_C \Rightarrow \sin\left(\frac{90°}{2}\right) \le \frac{1}{\mu}$$

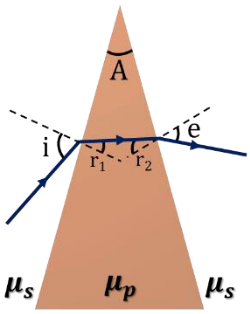
$$\mu \le \sqrt{2} \Rightarrow \mu_{max} = \sqrt{2}$$

Thin Prism

For a thin prism, prism angle (A) is very small.

In general A < 10°

In prism,

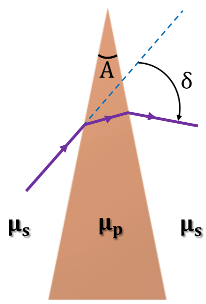


$$\frac{\mu_p}{\mu_s} = \frac{\sin\left(\frac{\delta_m + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

If prism is thin then A is very small & then we assume all angles are very small

So, all apply $\sin\theta \approx \theta$ then,

$$\frac{\mu_p}{\mu_s} = \frac{\left(\frac{\delta_m + A}{2}\right)}{\left(\frac{A}{2}\right)} \Rightarrow \frac{\mu_p}{\mu_s} \times \frac{A}{2} = \frac{\delta_m + A}{2}$$



$$\delta_m = \left(\frac{\mu_p}{\mu_s} - 1\right)A$$

General deviation or minimum deviation of thin prism is,

$$\boxed{\delta = \left[\frac{\mu_p}{\mu_s} - 1\right]A}$$

The deviation for a small angled prism is independent of the angle of incidence.

Illustration 82

A thin prism of 5° angle gives a deviation of 3·2°. Find the refractive index of the material.

Solution:

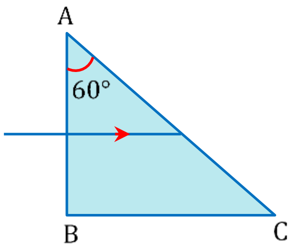
Angle of deviation $\delta = A(\mu - 1) \Rightarrow 3{\cdot}2° = 5° (\mu - 1) \Rightarrow \mu = 1{\cdot}64$

BEGINNER'S BOX-8

1. Angle of incidence is 45° in the condition of minimum deviation for a prism of refracting angle 60°. Find the angle of deviation.

PRI210

2. A light ray is incident normally on the surface AB of a prism of refracting angle 60°. If the light ray does not emerge from AC, then find the refractive index of the prism.



PRI211

3. Calculate the refractive index of the material of an equilateral prism for which the angle of minimum deviation is $\frac{\pi}{3}$ radian.

PRI212

4. A ray of light passing through a prism having $\mu = \sqrt{2}$ suffers minimum deviation. It is found that angle of incidence is double the angle of refraction within the prism. Find angle of the prism.

PRI213

5. The angle of minimum deviation measured with a prism is 30° and the angle of prism is 60°. Find the refractive index of the material of the prism.

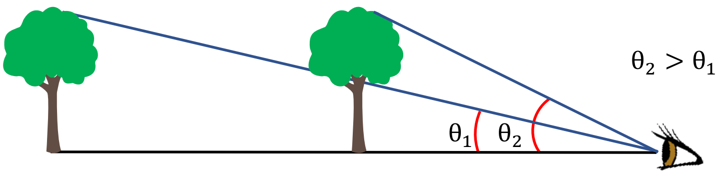
PRI214

6. A ray incident at 15° on a refracting surface of a prism of angle 30° suffers a deviation of 55°. Find the angle of emergence.

PRI215

Optical Instruments (Simple microscope, Compound microscope, Telescope)

Visual angle: Visual angle is the angle, a viewed object or image subtends at the eye. It is also called the object's angular size.



Nearer the object larger the visual angle then object appears big in size.

Note: Near point (N.P.) of normal eye D = 25cm

Far point (F.P.) of normal eye = ∞

Angular Magnification 'OR' Magnifying power

$$\boxed{\text{M.P.} = \frac{\text{visual angle of image in the presence of instrument}(\beta)}{\text{maximum visual angle of an object in the absence of instrument}(\alpha)}}$$

Simple Microscope

(1) A single converging lens (convex lens) of small focal length.

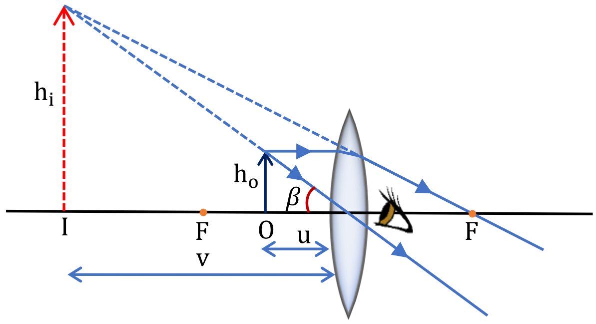
(2) When the object is placed between the focus and the optical centre a virtual, magnified and erect image is formed.

(3) Also known as magnifying glass.

$$\tan\beta = \frac{h_o}{u} = \frac{h_i}{v}$$

If angle is small then,

$$\beta = \frac{h_o}{u} = \frac{h_i}{v}$$

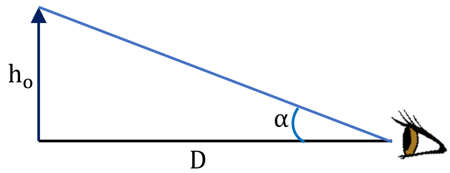


Calculation of magnifying power

$$\tan\alpha \approx \alpha = \frac{h_o}{D}$$

$$M.P. = \frac{\beta}{\alpha} = \frac{h_o / u}{h_o / D}$$

$$\boxed{M.P. = \frac{D}{|u|}}$$



Case (1): When image formed at a distance of least distance of distinct vision from the lens.

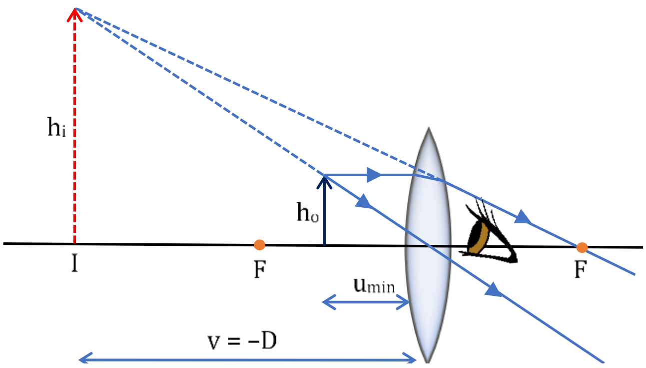
$$\frac{1}{-D} - \frac{1}{-u} = \frac{1}{f} \Rightarrow \frac{1}{u} = \frac{1}{D} + \frac{1}{f}$$

[v = –D and u = –ve]

Multiplying both the sides by D

$$\frac{D}{u} = 1 + \frac{D}{f} \Rightarrow MP = \frac{D}{u} = 1 + \frac{D}{f}$$

$$\boxed{(M.P.)_{max} = \frac{D}{u_{min}} = 1 + \frac{D}{f}}$$



Case (2): When image formed at infinite distance from the lens.

From lens equation

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{-\infty} - \frac{1}{-u} = \frac{1}{f} \Rightarrow u = f.$$

$$\text{So } MP = \frac{D}{u} = \frac{D}{f}$$

$$\boxed{(M.P.)_{min} = \frac{D}{u_{max}} = \frac{D}{f}}$$

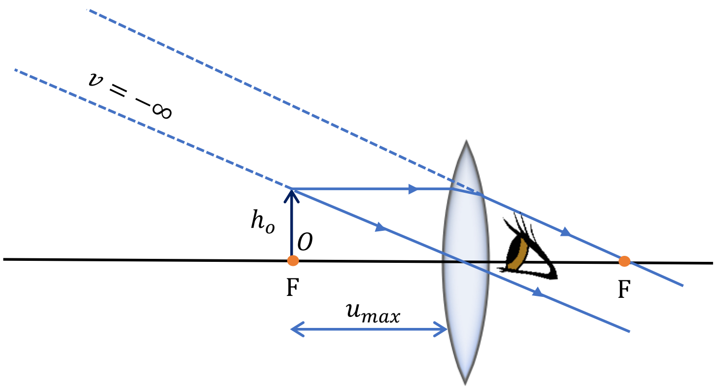


Illustration 83

Focal length of a convex lens is 2.5 cm. Find its maximum and minimum magnifying power when used as a simple microscope.

Solution:

$$(M.P.)_{max} = 1 + \frac{D}{f} = 1 + \frac{25}{2.5} = 11$$

$$\Rightarrow (M.P.)_{min} = \frac{D}{f} = \frac{25}{2.5} = 10$$

Illustration 84

A man with normal vision, reads a book with small print using a magnifying glass of focal length 5 cm. What is the closest and farthest distance at which he should hold the lens from the print so that he can read easily?

Solution:

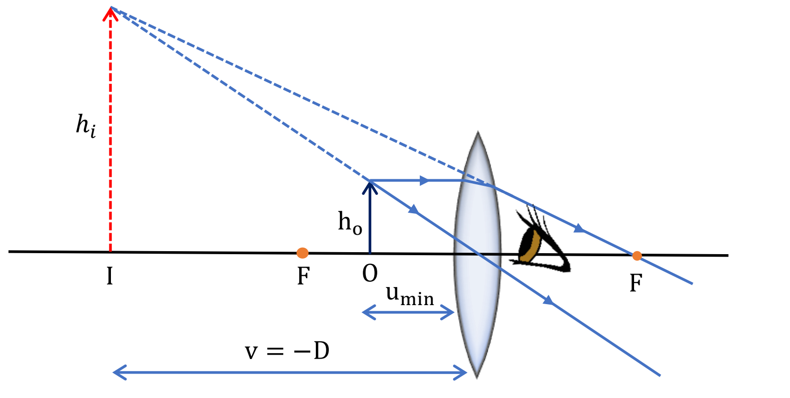
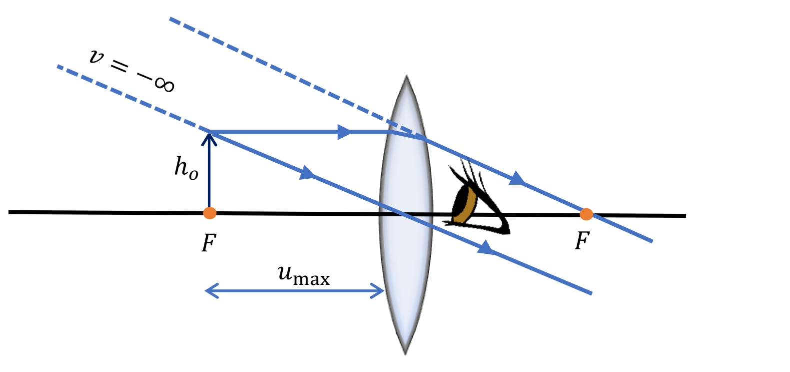
$\frac{D}{u_{min}} = 1 + \frac{D}{f}$ $\frac{25}{u_{min}} = 1 + \frac{25}{5}$ $u_{min} = \frac{25}{6}$ cm	
$\frac{D}{u_{max}} = \frac{D}{f}$ $u_{max} = f$ $u_{max} = 5$ cm	

Illustration 85

A 10 D lens is used as a magnifier. Where should the object be placed to obtain maximum angular magnification for a normal eye (near point = 25 cm)?

Solution:

1st method

$$P = \frac{1}{f} = 10$$

f = 0.1 m = 10 cm

$$(M.P)_{max} = 1 + \frac{D}{f} = 1 + \frac{25}{10}$$

$$= 3.5$$

$$M.P = \frac{D}{|u|} \Rightarrow 3.5 = \frac{25}{u} \Rightarrow u = \frac{50}{7} \text{cm}$$

2nd method

We get maximum M.P where using image formed at least distance of district vision

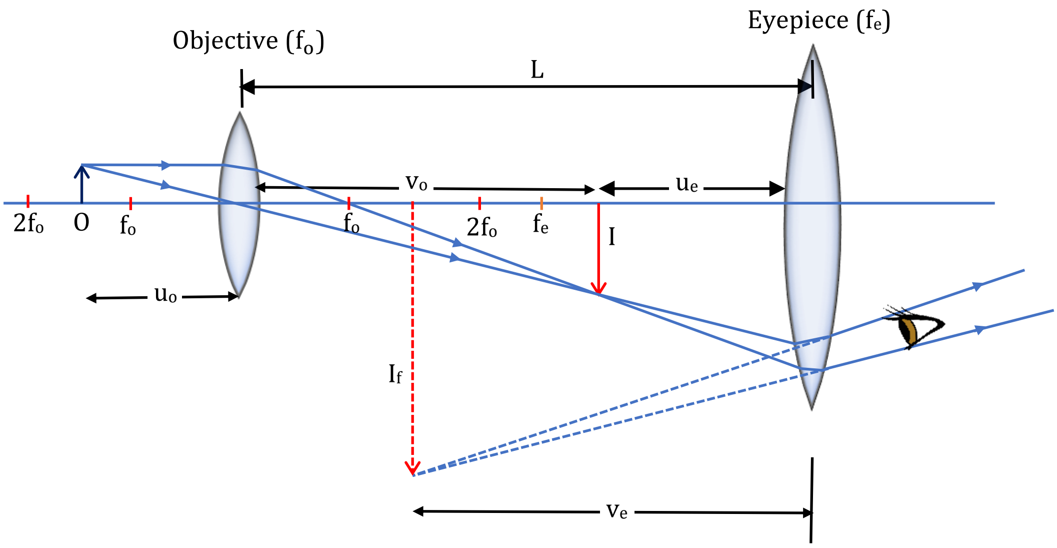
$v = -25$ cm

$$-\frac{1}{25}-\frac{1}{u}=\frac{1}{10} \Rightarrow \quad u=\frac{-50}{7}\text{cm}$$

Compound microscope

A compound microscope is a laboratory instrument with high magnification power, which is consists of more than one lenses. Compound Microscopes are used for the study of structural details of a cell, tissue, or organ in sections. A compound microscope can magnify the image of a tiny object up to 1000.

Ray diagram: Compound microscope is used to get more magnified image as compared to a simple microscope. Object is placed in front of the objective lens and the image is seen through the eye piece. The aperture of objective lens is less as compared to eye piece because object is very near so collection of more light is not required. Generally, object is placed between F – 2F due to this a real, inverted and magnified image is formed between 2F – ∞. It is known as intermediate image (I). The intermediate image acts as an object for the eye piece. Now the distance between both the lens are adjusted in such a way that intermediate image falls between the optical centre of eye piece and its focus. In this condition, the final image (I_f) is virtual, inverted and magnified.

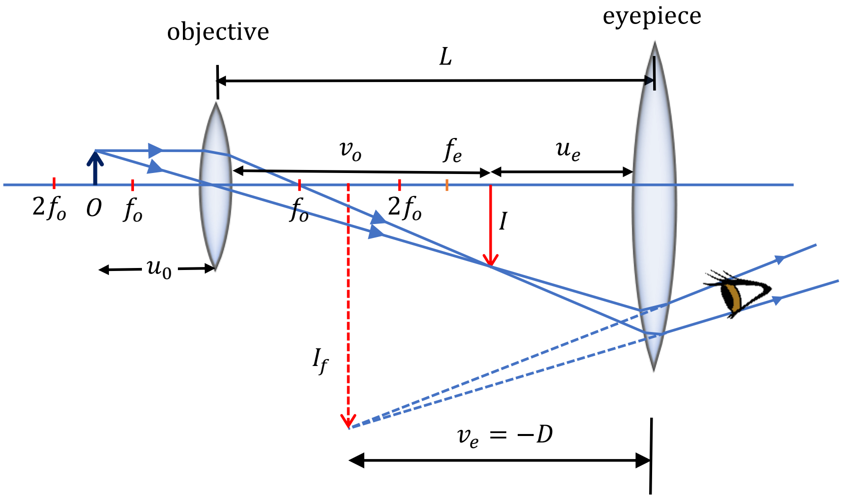


Calculation of magnifying power: Total magnifying power = Linear magnification of objective lens × angular magnification MP of eye lens

$$\boxed{\mathbf{M.P.}=\mathbf{m_o \times m_e}}$$

$$\boxed{\mathbf{M.P.}=-\left|\frac{v_o}{u_o}\frac{D}{u_e}\right|}$$

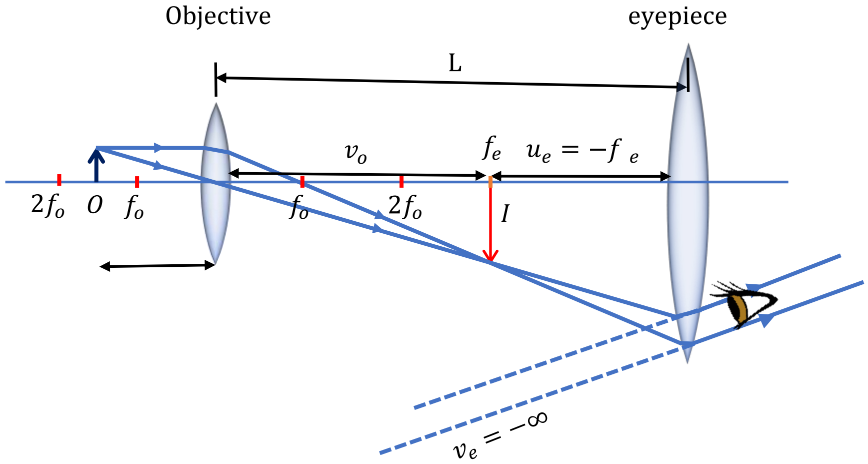
Case (1): When final image formed at least distance of distinct vision.



$$MP = \frac{v_0}{u_0}\left[1+\frac{D}{f_e}\right] = \frac{f_0}{(f_0+u_0)}\left[1+\frac{D}{f_e}\right] = \frac{f_0-v_0}{f_0}\left[1+\frac{D}{f_e}\right]$$

Distance between both the lenses, $L = v_0 + |u_e|$

Case (2) : When final image formed at infinity.



$$MP = \frac{v_0}{u_0}\left[\frac{D}{f_e}\right] = \frac{f_0}{(f_0+u_0)}\left[\frac{D}{f_e}\right] = \frac{(f_0-v_0)}{f_0}\left[\frac{D}{f_e}\right]$$

Distance between both the lenses, $L = v_0 + f_e$

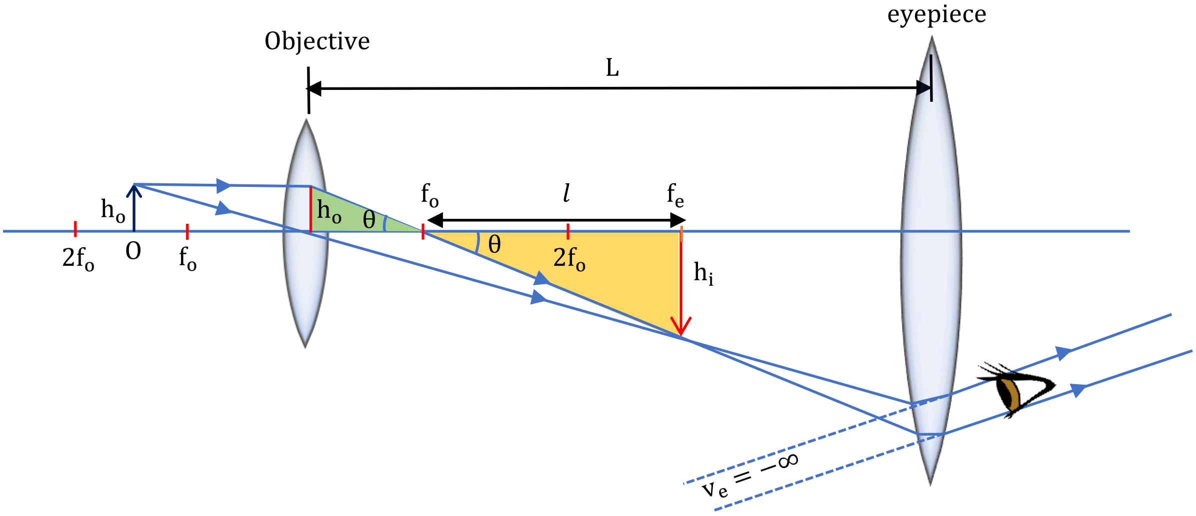
Sign convention for solving numericals, u_0 = –ve, v_0 = +ve, f_0 = +ve,

u_e = –ve, v_e = –ve, f_e = +ve, m_0 = –ve, m_e = +ve, MP = –ve

Tube Length

The distance L, i.e., the distance between the second focal point of the objective and the first focal point of the eyepiece (focal length f_e) is called the tube length of the compound microscope.

Magnifying power of compound microscope when final image formed at infinity.



$$\left.\begin{aligned}\tan\theta &= \frac{h_o}{f_o} \\ \tan\theta &= \frac{h_i}{l}\end{aligned}\right\} \Rightarrow \frac{h_o}{f_o} = \frac{h_i}{l} \Rightarrow \frac{h_i}{h_o} = \frac{l}{f_o}$$

So, M.P. of compound microscope when final image formed at ∞.

M.P. = $m_o \times m_e$

$$\boxed{\text{M.P.} = -\frac{l}{f_o} \times \frac{D}{f_e}}$$

Illustration 86

A compound microscope has a magnifying power 30X. The focal length of its eye-piece is 5 cm. Assuming the final image formed at the least distance of distinct vision (25cm). Calculate the magnification produced by objective.

Solution:

Given, M.P. = –30, f_e = 5 cm

M.P. of eyepiece when final image formed at 25 cm away,

$$m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{5} = 6$$

Now, M.P. = $m_o \times m_e$

$-30 = m_o \times 6 \Rightarrow m_o = -5$

Illustration 87

An angular magnification of 30X is desired using an objective of focal length 1.25 cm and an eye piece of focal length 5 cm. What distance will you set between Objective and Eye piece of the compound microscope for final image at infinity?

Solution:

Given, M.P. = –30, f_o = 1.25 cm, f_e = 5 cm

If final image at ∞ then $m_e = \frac{D}{f_e} = \frac{25}{5} = 5$

M.P. = $m_o \times m_e$

$-30 = m_o \times 5 \Rightarrow m_o = -6$

$m_o = \frac{f_o - V_o}{f_o} = -6 \Rightarrow \frac{1.25 - V_o}{1.25} = -6 \Rightarrow V_o = 8.75$ cm

Distance between objective and eyepiece is, L = $V_o + f_e$ = 8.75 + 5 = 13.75 cm

Illustration 88

In a compound microscope object is placed at a distance of 6 cm from objective of focal length 5 cm and focal length of eyepiece is 10 cm. Find out magnifying power of compound microscope when final image is formed at least distance of distinct vision?

Solution:

Given, u_o = –6 cm, f_o = 5 cm, f_e = 10 cm

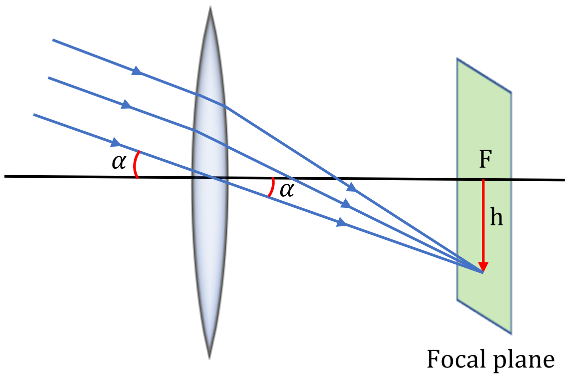
When final image formed at least distance of distinct vision then,

$m_e = 1 + \frac{D}{f_e} = 1 + \frac{25}{10} = 3.5$ & $m_o = \frac{f_o}{f_o + u_o} = \frac{5}{5-6} = -5$

M.P. = $m_o \times m_e$ = –5 × 3.5 = –17.5

Telescope

Focal Plane: When light rays are parallel and paraxial but not parallel to principal axis can incident on the lens then the plane where they meet or appears to meet after refraction is known as focal plane.



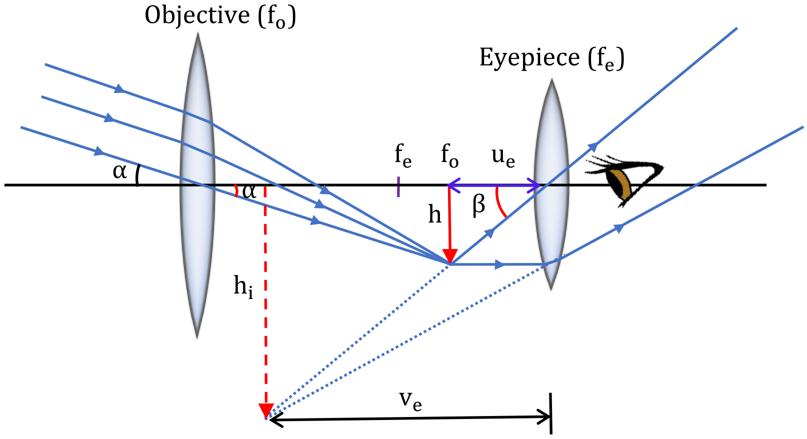
$\tan\alpha = \frac{h}{f}$

If rays are paraxial then, $\tan\alpha \approx \alpha$

$\alpha = \frac{h}{f} \Rightarrow \boxed{h = f\alpha}$

Astronomical telescope

The telescope is used to provide angular magnification of distant objects (Fig.). It also has an objective and an eyepiece. But here, the objective has a large focal length and a much larger aperture than the eyepiece. Light from a distant object enters the objective and a real image is formed in the tube at its second focal point. The eyepiece magnifies this image producing a final inverted image.



$h = f_o\alpha$

$\boxed{\alpha = \frac{h}{f_o}}$

$\beta = -\frac{h}{u_e}$

Calculation of magnifying power

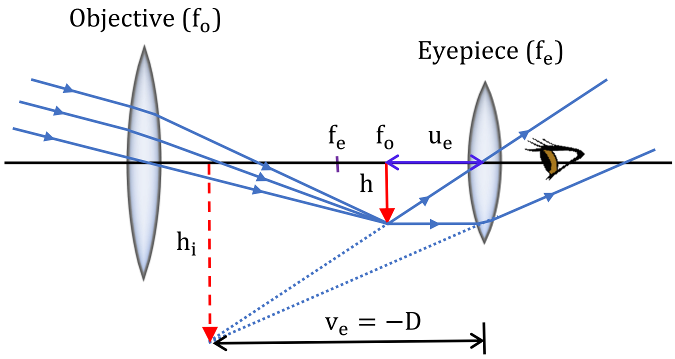
$$MP = \frac{\text{visual angle with instrument } (\beta)}{\text{visual angle for unaided eye } (\alpha)} \Rightarrow MP = \frac{\frac{h}{-u_e}}{\frac{h}{f_0}} = -\frac{f_0}{u_e} \qquad \boxed{MP = -\frac{f_o}{u_e}}$$

Case (1): When final image formed at least distance of distinct vision, $v_e = -D$, $u_e = -ve$

$$\frac{1}{-D} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow \frac{1}{u_e} = \frac{1}{f_e} + \frac{1}{D} = \frac{1}{f_e}\left[1 + \frac{f_e}{D}\right]$$

So $\boxed{MP = -\frac{f_o}{u_e} = -\frac{f_o}{f_e}\left[1 + \frac{f_e}{D}\right]}$

Length of the tube is, $L = f_0 + |u_e|$



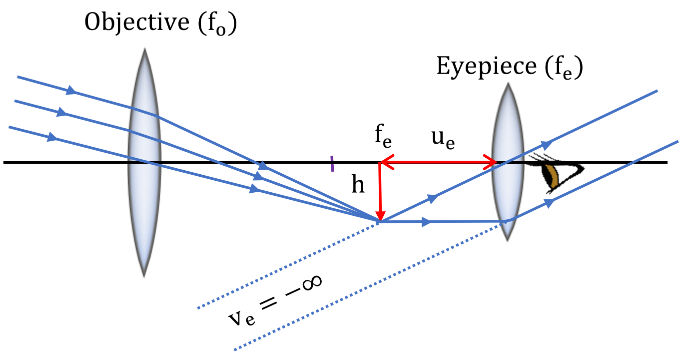
Case (2): When final image formed at infinity

$v_e = -\infty$, $u_e = -ve$

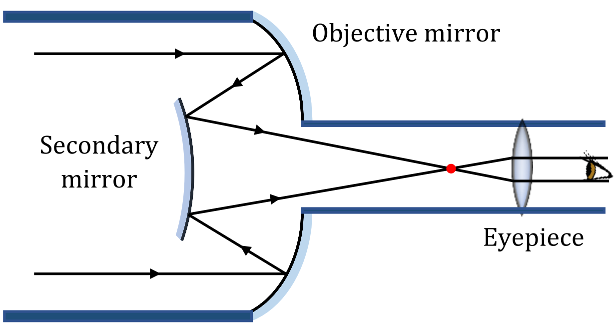
$$\frac{1}{-\infty} - \frac{1}{-u_e} = \frac{1}{f_e} \Rightarrow u_e = f_e$$

So $\boxed{MP = -\frac{f_o}{f_e}}$

Length of the tube, $L = f_0 + f_e$



Cassegrain telescope (Reflecting Telescope)



Big lenses tend to be very heavy and therefore, difficult to make and support by their edges. Further, it is rather difficult and expensive to make such large sized lenses which form images that are free from any kind of chromatic aberration and distortions. For these reasons, modern telescopes use a concave mirror rather than a lens for the objective. Telescopes with mirror objectives are called reflecting telescopes. There is no chromatic aberration in a mirror. Mechanical support is much less of a problem since a mirror weighs much less than a lens of equivalent optical quality, and can be supported over its entire back surface, not just over its rim. One obvious problem with a reflecting telescope is that the objective mirror focusses light inside the telescope tube. Another solution to the problem is to deflect the light being focussed by another mirror.

One such arrangement using a convex secondary mirror to focus the incident light, which now passes through a hole in the objective primary mirror, as shown in Figure. This is known as a Cassegrain telescope, after its inventor. It has the advantages of a large focal length in a short telescope.

Illustration 89

A small telescope has an objective of focal length 80 cm and an eyepiece of focal length 2 cm. If final image is formed at infinity.

(1) What is the magnifying power of the telescope?

(2) What is the separation between the objective and the eyepiece?

Solution:

$$\text{M.P} = \frac{f_o}{f_e} = \frac{80}{2} = 40$$

$L = f_o + f_e = 80 + 2 = 82$ cm

Illustration 90

Magnifying power of an astronomical telescope is 5. If focal length of eye piece is 4 cm. Find focal length of objective lens & also find the distance between both the lenses when final image is formed at infinity.

Solution:

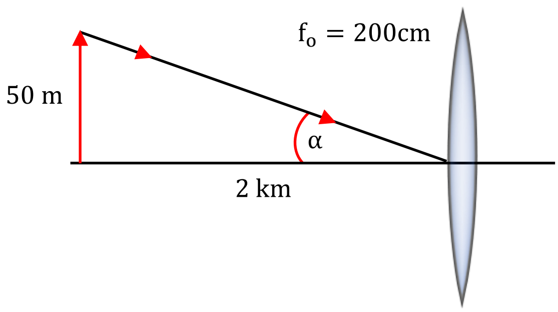
Given, M.P. = 5, $f_e = 4$ cm

$$\text{M.P.} = \frac{f_o}{f_e} \Rightarrow 5 = \frac{f_o}{4} \Rightarrow f_o = 20 \text{ cm}$$

$L = f_o + f_e = 20 + 4 = 24$ cm

Illustration 91

A telescope has an objective lens of focal length 200 cm. It is used to see a 50 m tall building at 2 km. What is height of the image formed by objective lens?



Solution:

$$\tan\alpha = \frac{50}{2000} \quad (\alpha \text{ is small angle})$$

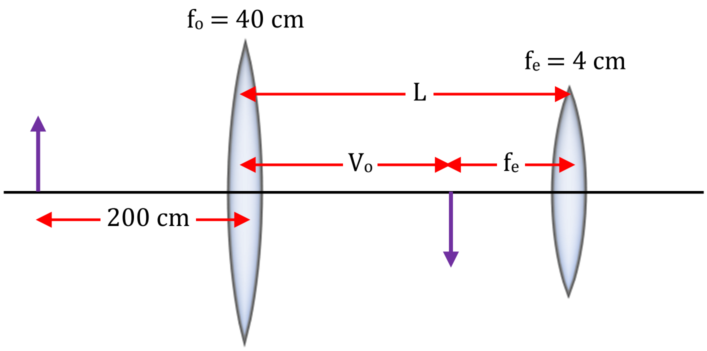
$$\alpha = \frac{1}{40}$$

$$h = f_o\alpha = 200 \times \frac{1}{40} = 5 \text{ cm}$$

Illustration 92

An astronomical telescope has objective and eyepiece of focal length 40 cm and 4 cm respectively. To view an object 200 cm away from the objective, the lens must be separated by a distance.

Solution:



Assume final image formed at ∞,

$$V_0 = \frac{u_o f_o}{u_o + f_o}$$

$$V_0 = \frac{-200 \times 40}{-200 + 40} = 50 \text{ cm}$$

$L = V_o + f_e = 50 + 4 = 54$ cm

BEGINNER'S BOX-9

1. Magnification of a compound microscope is 30. Focal length of eye piece is 5 cm and the image is formed at least distance of distinct vision. Find the magnification of objective.

PRI216

2. The powers of the lenses of a telescope are 0.5 and 20 dioptres. If the final image is formed at the minimum distance of distinct vision (25 cm) then what will be length of the tube?

PRI217

3. The focal lengths of objective and eye piece of a Galilean telescope are respectively 30 cm and 5 cm. Calculate its magnifying power and length when used to view distant objects.

PRI218

4. A telescope consisting of an objective of focal length 60 cm and an eyepiece of focal length 5 cm is focussed to a distant object in such a way that parallel rays emerge from the eye piece. If the object subtends an angle of 2° at the objective, then find the angular width of the image.

PRI219

5. The focal lengths of the objective and the eye piece of an astronomical telescope are 60 cm and 5 cm respectively. Calculate the magnifying power and the length of the telescope when the final image is formed at (i) infinity, (ii) least distance of distinct vision (25 cm).

PRI220

BEGINNER'S BOX ANSWER KEY

BEGINNER'S BOX-1

1.	0.9 m	**2.**	4	**3.**	60 m	**4.**	60°	**5.**	6.18m
6.	3								

BEGINNER'S BOX-2

1. (A)P (B)R,S (C)S (D)Q,S **2.** u = –0.1 m

3. $\frac{f}{3}$, m = $\frac{2}{3}$ so image will be virtual, erect and smaller than the object

4. 19.35 cm behind the mirror. **5.** u = –12 cm **6.** – 40 cm

7. – 54 cm, h_I = –5 cm, m = –2 real, inverted, magnified

BEGINNER'S BOX-3

1. 30 cm **2.** $2\cos^{-1}\left(\frac{\mu}{2}\right)$

3. Frequency is a characteristic of source, so it will not change with change in medium.

BEGINNER'S BOX-4

1. $\sin^{-1}\left(\frac{10t_1}{t_2}\right)$ **2.** $C=\sin^{-1}\frac{1}{\sqrt{2}}=45^0$ **3.** i < 45°

BEGINNER'S BOX-5

1. 3.63 cm **2.** –3.07 cm

BEGINNER'S BOX-6

1. 15 cm, real, inverted, small in size **2.** + 15 cm **3.** 1.66

4. 7.5 D **5.** 7.5 D **6.** 2 **7.** – f (1+n)

BEGINNER'S BOX-7

1. 20.5 cm **2.** v = –25cm

BEGINNER'S BOX-8

1. 30° **2.** $\mu>\frac{2}{\sqrt{3}}$ **3.** $\sqrt{3}$ **4.** 90°

5. $\sqrt{2}$ **6.** 70°

BEGINNER'S BOX-9

1. 5 **2.** 204.17 cm **3.** 6, 25 cm **4.** 24°

5. (i) –12, 65 cm (ii) –14.4, 64.17 cm