03 Wave Motion

Introduction and Classification of Waves

What is wave motion?

A wave is a disturbance which transports energy and momentum both from one point to another without the transport of matter.

Few examples of waves:

The ripples on a pond (water waves), the sound we hear, visible light, radio and T.V. signals etc.

Wave Classification According to

- Necessity of medium
  - (i) Elastic or mechanical waves
  - (ii) E.M. waves or non-mechanical
- Propagation of energy
  - (i) Progressive
  - (ii) Stationary
- Dimensions
  - (i) One dimensional
  - (ii) Two dimensional
  - (iii) Three dimensional
- Vibration of particle
  - (i) Transverse
  - (ii) Longitudinal

(i) **Based on necessity of medium:** A wave may or may not require medium for its propagation

1. The waves which do not require medium for their propagation are called **non-mechanical** or electromagnetic waves.

Examples : Light, radio waves etc.

2. The wave which require medium for their propagation are called **mechanical** wave.

Examples : Waves in string etc.

Note: In propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as elastic waves.

(ii) **Based on energy propagation:** Wave can be divided into two parts on the basis of energy propagation.

1. **Progressive Wave:** The waves in which energy propagate in a medium or space with constant speed.

2. **Stationary wave:** The waves in which the particles of medium vibrates with different amplitude but energy does not propagate.

(iii) **Based on dimensions:** Wave can be one, two or three dimensional according to the number of dimensions in which they propagate energy.

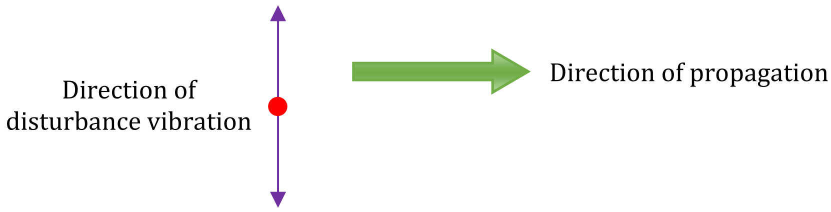
1. **One dimensional wave (1-D):** Waves moving along strings are one dimensional wave.

2. **Two dimensional wave (2-D):** Surface waves or ripple on water is two dimensional waves.

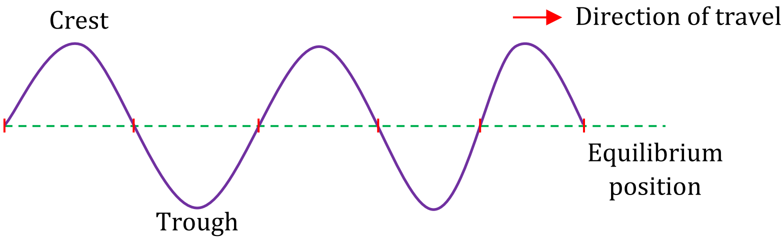
3. **Three dimensional wave (3-D):** Sound waves, light waves are three dimensional waves.

(iv) **Based on vibration of particle:** Waves are of two types on the basis of motion of particle of the medium.

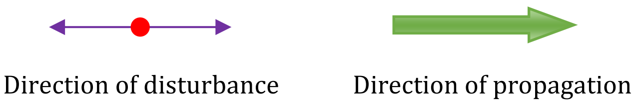
(1) Transverse wave : In transverse wave the direction of vibration of particle is perpendicular to the direction of propagation of wave.



- Mechanical transverse waves are produced in such type of medium which have shearing property, so they are known as shear wave or S-wave.
- Shearing is the property of a body by which it changes its shape on application of force.
- Mechanical transverse waves are generated only in solids and surface of liquid. This wave propagates in form of crest and trough.



(2) Longitudinal Wave : In this wave the direction of vibration of particle is along the direction of propagation of wave.



- Oscillatory motion of the medium particles produces region of compression (high pressure) and rarefaction (low pressure) which propagated in medium with time.
- The region of high particle densities are called compressions and region of low particle densities are called rarefactions.

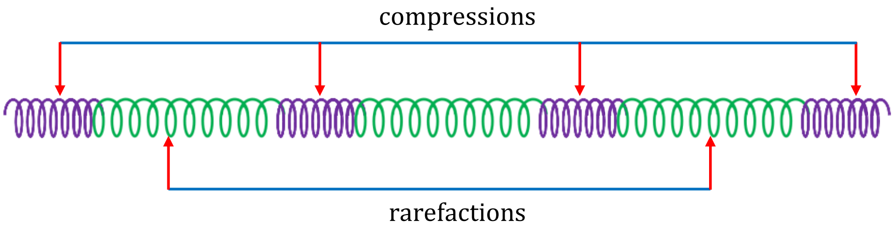


Illustration 1

Given below are some example of wave motion. State in each case whether the wave motion is transverse, Longitudinal or a combination of both?

(1) Motion of kink in a long coil spring produced by displacing one end of the spring sideways.

(2) Wave produced in a cylinder containing a liquid by moving its piston back and forth.

(3) Wave produced by a motorboat sailing in water.

(4) Ultrasonic waves in air produced by a vibrating quartz crystal.

Solution:

(1) Transverse and longitudinal (2) Longitudinal

(3) Transverse and longitudinal (4) Longitudinal

Illustration 2

Mechanical waves in gaseous medium are:

(1) Transverse
(2) Neither transverse nor longitudinal
(3) Longitudinal
(4) Either transverse or longitudinal

Solution:

A wave moving in a gas must be longitudinal and not transverse because longitudinal sound wave can travel through a gas. Hence (3) is correct.

Illustration 3

Water waves are of the nature:

(1) Transverse
(2) Longitudinal
(3) Sometime longitudinal and sometimes transverse and longitudinal both.
(4) Neither transverse nor longitudinal

Solution:

Water molecules displace along the propagation direction as well as along perpendicular direction. Hence both longitudinal and transverse. Hence (3) is correct.

Illustration 4

Which of the following statement is correct?

(1) Both light and sound waves in air are transverse.
(2) The sound waves in air are longitudinal while the light waves are transverse.
(3) Both light and sound waves in air are longitudinal
(4) Both light and sound waves can travel in vacuum.

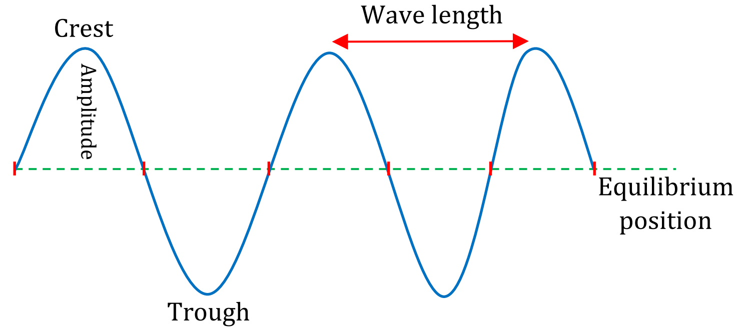
Solution:

Light waves are E.M. wave.

Sound wave is longitudinal wave. Hence (2) is correct.

Some important terms connected with wave motion.

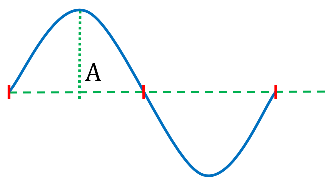
(1) **Wave length (λ) :** It is the distance between two successive crest or two successive troughs. It can also be defined as the distance travelled by the wave during the time interval in which any one particle of medium completes one cycle about its mean position.



(2) **Frequency (f/n) :** Number of cycle (number of complete wavelengths) completed by a particle in unit time.

(3) **Time Period (T) :** Time taken by wave to travel a distance equal to the wavelength.

(4) **Amplitude (A) :** Maximum displacement of vibrating particle from its equilibrium position.



(5) **Angular frequency (ω) :** It is defined as, $\omega = \frac{2\pi}{T} = 2\pi n$.

(6) **Phase:** Phase is a quantity which contains all information related to any vibrating particle in a wave.
For equation $y = A\sin(\omega t - kx)$; where $(\omega t - kx) \rightarrow$ Phase.

(7) **Angular wave number (k) :** Angular wave number or propagation constant is denoted by $k = \frac{2\pi}{\lambda}$.
The S.I. unit of 'k' is radian/meter.

(8) **Wave number ($\bar{\nu}$):** It is defined as number of waves in unit length of the wave pattern.

$$\bar{\nu} = \frac{1}{\lambda} = \frac{k}{2\pi}$$

(9) **Wave velocity (v):**
The velocity with which the disturbance travel through the medium is called wave velocity.

$$V = \frac{\lambda}{T}$$

Above formula can be written as:

$$v = \frac{\lambda}{\frac{2\pi}{\omega}} \qquad \because \left(\omega = \frac{2\pi}{T}\right)$$

$$v = \frac{\omega}{\frac{2\pi}{\lambda}} \quad \Rightarrow \quad v = \frac{\omega}{k} \qquad \because \left(k = \frac{2\pi}{\lambda}\right)$$

Graph between angular frequency (ω) and wave number ($\bar{\nu}$):

$$\bar{\nu} = \frac{1}{\text{wave length } (\lambda)}$$

As we know that,
$\omega = 2\pi n$; $v = n\lambda$

$$\omega = 2\pi\frac{v}{\lambda} \quad \Rightarrow \quad \omega \propto \frac{1}{\lambda} \quad \Rightarrow \quad \omega \propto \bar{\nu}$$

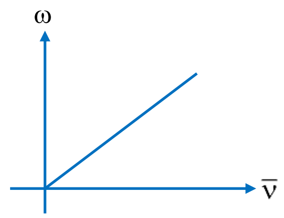


Illustration 5

On the average human heart is found to beat 75 times in a minute. Calculate its frequency of heart and its period?

Solution:

The beat frequency of heart $= \frac{75}{60} = 1.25$ Hz and Time period $T = \frac{1}{f} = \frac{1}{1.25} = 0.8$ sec

Illustration 6

If angular wave number is given 6π rad/m. Find out.
(1) Wave length (2) Angular frequency (3) frequency (4) Time period
If velocity of wave 300 m/s.

Solution:

We know that

(1) $k = \frac{2\pi}{\lambda}$ (given $k = 6\pi$) $\Rightarrow 6\pi = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{1}{3}$ m.

(2) $v = \frac{\omega}{k} \Rightarrow \omega = v \times k \Rightarrow \omega = (300)(6\pi) = 1800\,\pi$ radian/sec

(3) $\omega = 2\pi f \Rightarrow 1800\pi = 2\pi f \Rightarrow f = 900$Hz

(4) $T = \frac{1}{f} \Rightarrow T = \frac{1}{900}$ sec

Important Points:

(1) The velocity with which a wave travels is different from the velocity of the particle with which they vibrate about their mean positions.

(2) For the propagation of a mechanical wave the medium must have the properties of inertia, elasticity & minimum friction among its particles.

(3) Mechanical wave can be transverse (wave on string) and longitudinal (sound wave) and both.

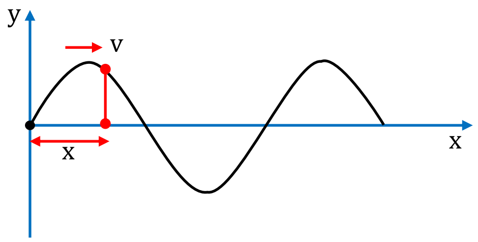
(4) Distance between two consecutive crest or two consecutive troughs is equal to wavelength of the wave.

(5) The energy or disturbance passes in the form of wave without any net displacement of medium.

Wave Equation and Characteristics of Waves

Equation of Plane Progressive Wave:

If a wave is travelling with constant speed v then it must reach the considered particle in time $\frac{x}{v}$ from origin. The particle at origin was vibrating similarly to the considered particle at time $'t-\frac{x}{v}'$. So, its equation of displacement is given as: -



$$y = A\sin\left(\omega\left(t-\frac{x}{v}\right)+\phi\right)$$

$$y = A\sin\left(\omega t-\frac{\omega x}{v}+\phi\right), \text{ where } k=\frac{\omega}{v}$$

$y = A\sin(\omega t - kx + \phi)$

$y = A\sin(\omega t - kx)$ (when $\phi = 0$)

This is the equation of a simple harmonic wave travelling along +x direction.

Note: If the wave is along –x direction, then there will be plus sign instead of minus sign inside the brackets in the above equation

$y = A\sin(\omega t + kx)$

Understand the Equation of Progressive Wave

$y = A\sin(\omega t - kx)$

- t → time at which displacement of particle is considered
- k → Angular wave number or propagation constant
- x → Represent the location of particle and also axis of propagation
- ω → Angular frequency
- A → Amplitude of particle
- y → Vibrational axis of particle

$y = A\sin(\omega t - kx)$

$\Downarrow$

If sign of both ωt and kx are different then wave is propagating on +ve x-axis.
If sign of both ωt and kx are same then wave is propagating on –ve x-axis.

$y = A\sin(\omega t - kx)$

$\Downarrow$

If vibrational and propagation axis are perpendicular then wave is Transverse.
If vibrational and propagation axis are parallel then wave is Longitudinal.

Particle Velocity (v_p) in Plane Progressive Wave

Particle velocity: - The rate of change of position (y) with respect to time (t) is known as particle velocity.

We have an equation of simple harmonic wave, $y = A\sin(\omega t - kx)$ ...(i)

The particle velocity, $\frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$ ...(ii)

$v_p = A\omega\cos(\omega t - kx)$, where v_p is the velocity of vibrating of particle.

When particle is crossing the mean position its velocity is maximum. $v_p = A\omega$, (maximum velocity)

Slope of S.H.W. (Simple Harmonic Wave)

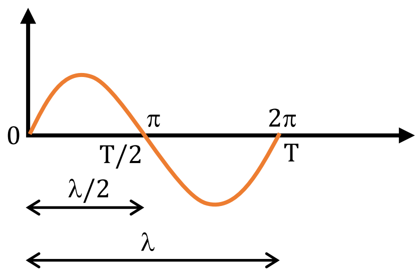
$y = A\sin(\omega t - kx)$ $\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx)$ ...(iii)

Now, from (ii) and (iii) $\frac{\frac{\partial y}{\partial t}}{\frac{\partial y}{\partial x}} = \frac{A\omega\cos(\omega t - kx)}{-Ak\cos(\omega t - kx)} \Rightarrow \frac{\partial y}{\partial t} = \left(-\frac{\omega}{k}\right)\left(\frac{\partial y}{\partial x}\right)$ $v_p = -v\left(\frac{\partial y}{\partial x}\right)$ $\because \left(v = \frac{\omega}{k}\right)$

Particle velocity = –(Wave velocity) × (Slope of the tangent at that point on the wave at that instant)

Relation between phase difference, path difference & Time difference

Phase Difference ($\Delta\phi$)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Path Difference (Δx)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ
Time difference (Δt)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T



$$\frac{\Delta\phi}{2\pi} = \frac{\Delta x}{\lambda} = \frac{\Delta t}{T}$$

Acceleration of particle in plane progressive wave

We have an equation of simple harmonic wave

$y = A\sin(\omega t - kx)$ $\Rightarrow$ $\frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$ $\Rightarrow$ $\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(\omega t - kx)$

$a_p = -A\omega^2\sin(\omega t - kx)$ $\Rightarrow$ $a_p = -\omega^2 y$ ...(i)

Differential equation of wave

$y = A\sin(\omega t - kx)$

$\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx)$

$\frac{\partial^2 y}{\partial x^2} = -Ak^2\sin(\omega t - kx)$ ...(i)

$y = A\sin(\omega t - kx)$

$\frac{\partial y}{\partial t} = A\omega\cos(\omega t - kx)$

$\frac{\partial^2 y}{\partial t^2} = -A\omega^2\sin(\omega t - kx)$ ...(ii)

Divide both the equation to obtain differential equation of wave

$\frac{\frac{\partial^2 y}{\partial x^2}}{\frac{\partial^2 y}{\partial t^2}} = \frac{-Ak^2\sin(\omega t - kx)}{-A\omega^2\sin(\omega t - kx)}$ After solving, $\frac{\partial^2 y}{\partial t^2} = \frac{\omega^2}{k^2}\frac{\partial^2 y}{\partial x^2} \Rightarrow \frac{\partial^2 y}{\partial t^2} = v^2\frac{\partial^2 y}{\partial x^2}$

$\frac{\partial^2 y}{\partial x^2} = \left(\frac{1}{v^2}\right)\left(\frac{\partial^2 y}{\partial t^2}\right)$ is known as differential equation of wave motion.

Golden Key Points

- Equation $y = A\sin(\omega t - kx)$ represents displacement of all medium particles at any time 't' in other words it can give us displacement of any specific particle at each and every time so it is called equation of progressive wave.
- For a plane progressive wave $y = A\sin(\omega t-kx)$ producing transverse vibration $\frac{\partial y}{\partial x} = -\frac{1}{v}\left(\frac{\partial y}{\partial t}\right)$

  i.e., velocity of particle at given point = – wave velocity × slope of wave at that point.
- $\frac{\partial^2 y}{\partial x^2} = -\frac{1}{v^2}\left(\frac{\partial^2 y}{\partial t^2}\right)$ is known as differential equation of wave motion.
- Relations: $\omega = 2\pi n$, $v = n\lambda$, $k = \frac{2\pi}{\lambda}$, $\omega = vk$
- Other form of wave equation: -

  $y = A\sin\left(2\pi ft - \frac{2\pi x}{\lambda}\right)$ $(\because v = f\lambda)$ $\Rightarrow y = A\sin\left(2\pi\left(\frac{vt}{\lambda} - \frac{x}{\lambda}\right)\right) \Rightarrow y = A\sin\left(\frac{2\pi}{\lambda}(vt - x)\right)$.

Illustration 7

Which of the following functions represent travelling wave?

(1) $(x - vt)^2$ (2) $\ell n(x - vt)$ (3) $e^{-(x-vt)^2}$ (4) $\frac{1}{x+vt}$

Solution:

Although all four functions are written in the form $f(ax \pm bt)$, only (3) among the four function is finite everywhere at all times. Hence only (3) represents a travelling wave.

Illustration 8

If equation of plane progressive wave is given by $y = 10\sin\left[200\pi t - \frac{\pi x}{9}\right]$, where x is in cm and t in seconds

Find out

(a) DOP (b) Nature (c) Amplitude (d) Angular frequency
(e) Time period (f) Wave speed (g) Wave length (h) Maximum velocity

Solution:

$y = 10\sin\left[200\pi t - \frac{\pi x}{9}\right]$

(a) (+x) direction (b) Transverse (c) 10 cm (d) $\omega = 200\pi$

(e) $T = \frac{2\pi}{\omega} = \frac{2\pi}{200\pi} = \frac{1}{100} = 0.01\,s$ (f) $v = \frac{\omega}{k} = \frac{200\pi}{\frac{\pi}{9}} = 1800\frac{cm}{s}$

(g) $k = \frac{\pi}{9} \Rightarrow \frac{2\pi}{\lambda} = \frac{\pi}{9} \Rightarrow \lambda = 18$ cm (h) $V_{max} = A\omega = 10 \times 200\pi = 2\pi \times 10^3$ cm/s

Illustration 9

Find out VOP, DOP and nature of wave in following

(a) $y = A\sin(\omega t - kx)$ (b) $y = A\sin(\omega t - ky)$ (c) $y = A\sin(-\omega t - kz)$
(d) $x = A\cos(\omega t + kx)$ (e) $y = A\sin(kx - \omega t)$

Solution:

(a) $y = A\sin(\omega t - kx)$
DOP = +x
VOP = ±y
Transverse

(b) $y = A\sin(\omega t - ky)$
DOP = +y
VOP = ±y
Longitudinal

(c) $y = A\sin(-\omega t - kz)$
DOP = –z
VOP = ±y
Transverse

(d) $x = A\cos(\omega t + kx)$
DOP = –x
VOP = ±x
Longitudinal

(e) DOP = +x
Transverse
$y = A\sin(kx - \omega t)$
VOP = y ±

Illustration 10

For a sound wave path difference of 40 cm is equivalent to phase difference 1.6π rad. If speed is 330 m/s. Find n.

Solution:

$$\frac{\Delta\phi}{2\pi}=\frac{\Delta x}{\lambda} \Rightarrow \frac{1.6\pi}{2\pi}=\frac{40}{\lambda\times100} \Rightarrow \lambda=\frac{40\times2\times10}{100\times16} \Rightarrow \lambda=\frac{1}{2}\text{m}$$

$\therefore$ $v = n\lambda \Rightarrow n = 330\times2 \Rightarrow n = 660$ Hz

BEGINNER'S BOX-1

1. A wave has a speed of 300 m/sec and frequency 500 Hz. The phase difference between two adjacent points is π/3 radian. What will be the path difference between them?

PWA177

2. A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top, given that the speed of sound in air is 340 ms^{-1}? ($g = 9.8\ ms^{-2}$)

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Energy Power and Intensity of Waves

Energy : Wave transport energy when they propagate through a medium. The total kinetic energy K associated with moving particles in one wavelength is $K=\frac{1}{4}\mu\lambda(\omega A)^2$

In addition to kinetic energy, each element of the string has potential energy associated with it due to its displacement from the equilibrium position and the restoring forces from neighbouring elements. The total potential energy U associated in one wavelength is exactly the same result $U=\frac{1}{4}\mu\lambda(\omega A)^2$

Total energy is one wavelength of the wave is the sum of the potential and kinetic energies:

$$E=U+K=\frac{1}{2}\mu\lambda(\omega A)^2$$

Illustration 11

If potential energy of wave in one wavelength is 10J. Find its kinetic energy and total energy in one wave length?

Solution:

U = 10 J so, K.E. = 10 J and T.E. = 10 + 10 = 20 J

Power (P): As the wave moves along the string, this amount of energy passes by a given point on the string during a time interval of one period of the oscillation. Thus, the power, or rate of energy transfer, associated with the wave is

$$P=\frac{\Delta E}{\Delta t}=\frac{E}{T} \Rightarrow P=\frac{\frac{1}{2}\mu\lambda(A\omega)^2}{T}=\frac{\frac{1}{2}\mu A^2\omega^2\lambda}{T} \Rightarrow P=\frac{1}{2}\mu v A^2\omega^2 \quad \because \frac{\lambda}{T}=v$$

Illustration 12

If energy of wave is 20 J. Find the time when its power is 10 watts?

Solution:

$$P=\frac{E}{t} \Rightarrow t=\frac{E}{P} \Rightarrow t=\frac{20}{10} \Rightarrow t=2\text{sec.}$$

Intensity

Intensity is defined to be the power per unit area carried by a wave. Power is the rate at which energy is transferred by the wave.

$$I=\frac{\text{Power}}{\text{Area}} \Rightarrow I=\frac{\frac{1}{2}\mu v A^2\omega^2}{\text{Area}} \Rightarrow I=\frac{1}{2}\rho v A^2\omega^2$$

From the above result following conclusion can be made

(a) If medium remains same then ρ and v is constant $I \propto A^2\omega^2 \Rightarrow I \propto A^2n^2$ $(\omega=2\pi n)$

(b) If medium and frequency both are constant then $I \propto A^2$

Illustration 13

Two mechanical waves $y_1=2\sin 2\pi(50t-2x)$ and $y_2=4\sin 2\pi(ax+100t)$ propagate in medium with same speed, intensities ratio of waves.

Solution:

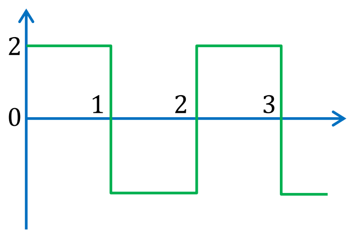
$$\frac{I_1}{I_2}=\frac{A_1^2f_1^2}{A_2^2f_2^2} \Rightarrow \frac{I_1}{I_2}=\frac{(2)^2}{(4)^2}\frac{(50)^2}{(100)^2}=\frac{4}{16}\left(\frac{1}{2}\right)^2 \Rightarrow \frac{I_1}{I_2}=\frac{1}{16}$$

Illustration 14

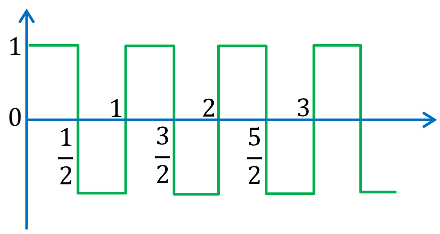
Calculate the ratio of intensity of wave train A to wave train B.

Solution:

Wave train A



Wave train B



$$\text{Intensity} \propto A^2f^2 \Rightarrow \frac{I_A}{I_B}=\frac{A_1^2f_1^2}{A_2^2f_2^2}=\left(\frac{2}{1}\right)^2\left(\frac{1}{2}\right)^2 \Rightarrow \frac{I_A}{I_B}=\frac{1}{1}$$

Golden Key Points

- Kinetic energy associated with one wavelength $K=\frac{1}{4}\mu\lambda(\omega A)^2$
- Potential energy associated with one wavelength $U=\frac{1}{4}\mu\lambda(\omega A)^2$
- Total energy associated with one wavelength $E=U+K=\frac{1}{2}\mu\lambda(A\omega)^2$
- $\text{Power}=\frac{\text{Enegy}}{\text{Time}}$
- Power associated with the wave $P=\frac{1}{2}\mu v A^2\omega^2$

- In general intensity = $\dfrac{\text{Power}}{\text{Area}}$
- If medium is same and v is constant then, intensity(I) $\propto \omega^2 A^2$
- If medium and frequency both constant then $I \propto A^2$
- Intensity due to point source $I = \dfrac{P}{4\pi r^2}$

Speed of Mechanical Wave on String

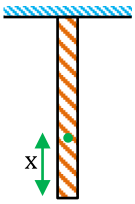
Velocity of Transverse Mechanical Waves: Velocity of a transverse wave propagating along a string having tension T and mass per unit length μ, is given by $v = \sqrt{\dfrac{T}{\mu}}$

If r is the radius of the string, ℓ is its length and ρ is its density, then

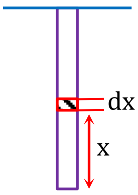
$$\mu = \frac{\rho\pi r^2 \ell}{\ell} = \rho\pi r^2 \quad \Rightarrow v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\pi r^2 \rho}}$$

Illustration 15

A uniform rope of mass M kg and length ℓ hangs from a ceiling as shown, this rope is under tension due to its own weight. What is the wave speed at 'x' distance from the bottom on the rope.



Solution:



μ is the mass per unit length of rope. At a distance x from the lower end the tension will be,

$$T = \left(\frac{M}{L}\right)xg \qquad \Rightarrow T = \mu xg$$

Wave speed $v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{\mu xg}{\mu}} = \sqrt{xg}$

Illustration 16

For two different strings ratio of velocities of transverse mechanical waves is $\dfrac{5}{4}$. If both strings have same linear mass density then find ratio of tension in strings?

Solution:

$$\frac{v_1}{v_2} = \frac{5}{4} \ ; \ \frac{\mu_1}{\mu_2} = \frac{1}{1} \ \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{\frac{T_1}{\mu_1}}{\frac{T_2}{\mu_2}}} \ \Rightarrow \frac{5}{4} = \sqrt{\frac{T_1}{T_2}} \ \Rightarrow \frac{T_1}{T_2} = \frac{25}{16}.$$

Illustration 17

A string of linear density 4 gm/cm is vibrating according to the equation, $y = A \sin\left(120\pi t - \dfrac{2\pi}{5}x\right)$, Where x & y is in cm & t in sec. Find tension (In N) in string -

Solution:

$$v = \frac{\omega}{k} = \frac{120\pi \times 5}{2\pi} = 300 \text{ cm/s}$$

$$v = \sqrt{\frac{T}{\mu}} \Rightarrow T = v^2 \mu = 9 \times 10^4 \times 4 \text{ g/cm} = 36 \times 10^4 \text{ dyne} \Rightarrow T = 3.6 \text{ N}$$

BEGINNER'S BOX-2

1. A travelling wave in stretched string is given by the equation : $y = 40 \cos(3x - 5t)$ cm where t is in sec. Determine the maximum speed of particles of medium.

PWA179

2. The displacement produced by a simple harmonic wave is $y = \frac{10}{\pi} \sin\left(200\pi t - \frac{\pi x}{17}\right)$. Then find the time period and maximum velocity of the particle. ($x \to$ m, $y \to$ cm, $t \to$ sec.)

PWA180

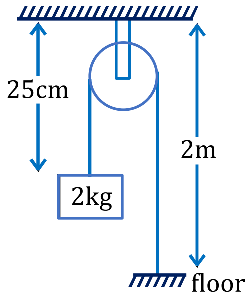
3. You have learnt that a travelling wave in one dimension is represented by a function $y = f(x, t)$ where x and t must appear in the combination $x - vt$ or $x + vt$, i.e. $y = F(x \pm vt)$. Is the converse true? Examine if the following functions for y can possibly represent a travelling wave :

(a) $(x - vt)^2$ (b) $\log[(x + vt)/x_0]$

(c) $\exp[-(x + vt)/x_0]$ (d) $1/(x + vt)$

PWA181

4. In the given figure the string has mass 4.5 g. Find the time taken by a transverse pulse produced at the floor to reach the pulley. ($g = 10 \text{ ms}^{-2}$).



PWA182

5. A wave moves with speed 300 ms^{-1} on a wire which is under a tension of 500 N. Find how much the tension must be changed to increase the speed to 360 ms^{-1}.

PWA183

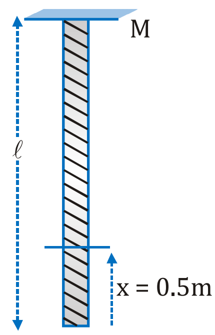
6. A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end?

PWA184

7. A uniform rope of mass 0.1 kg and length 2.45 m hangs from a ceiling as shown in diagram.

(a) find speed of transverse wave in the rope at a point 0.5 m distance from the lower end.

(b) Calculate time taken by a transverse wave to travel the full length of rope.



PWA185

Sound Waves and it's Characteristics

Classification of sound waves

Sound waves can be classified in three groups according to their range of frequencies.

(1) Infrasonic Waves: Longitudinal wave having frequencies below 20 Hz are called infrasonic waves. They cannot be heard by human beings. They are produced during earthquakes. Infrasonic waves can be heard by snakes.

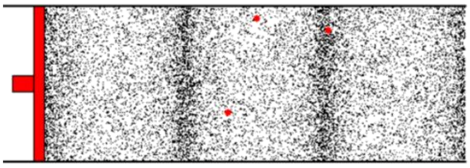
(2) Sonic Waves/Audible Waves: Longitudinal waves having frequencies lying between 20 Hz-20000 Hz are called audible waves.

(3) Ultrasonic Waves: Longitudinal waves having frequencies above 20000 Hz are called ultrasonic waves. They are produced and heard by bats. They have a large energy content.

Equations of sound wave

A longitudinal mechanical wave can be described in two ways:

Longitudinal Wave



(A) Displacement wave form : When sound wave is described in term of longitudinal displacement suffered by particles of the medium, it is called displacement wave. $s = s_{max}\cos(kx - \omega t)$

Where "s" represents the displacement of particle and "s_{max}" is the amplitude of particle or maximum displacement of particle.

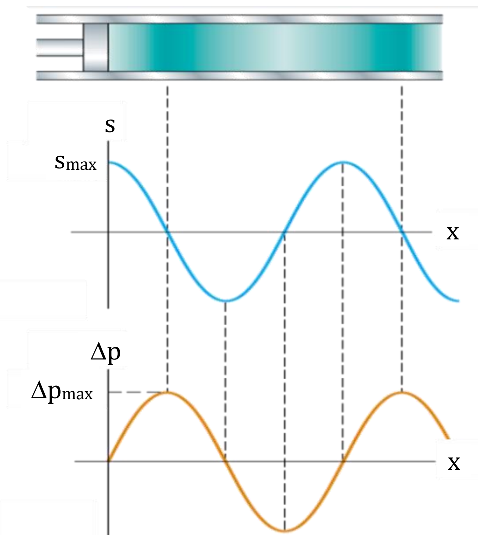
(B) Pressure Wave Form : When sound wave is described in term of excess pressure due to compression and rarefaction called pressure wave. $\Delta p = \Delta p_{max}\sin(kx - \omega t) \Rightarrow \Delta p_{max} = Bks_{max}$

(B = Bulk Modulus, k = Propagation constant, s_{max} = Amplitude of particle or maximum displacement of particle)

Maximum pressure in medium = $P_{atm} + \Delta p_{max}$

Minimum pressure in medium = $P_{atm} - \Delta p_{max}$

Pressure variation in medium = $2\Delta p_{max}$



$\Delta p_{max} = -B\dfrac{dy}{dx}$

Note: The phase difference between pressure wave form and displacement wave form is 90° and path difference is $\dfrac{\lambda}{4}$.

Displacement will be maximum when pressure is minimum and vice versa.

- When we consider the interference of sound as pressure wave then there is no change in phase when reflected from a "rigid boundary" but have a phase change of π when reflected from free end.
- A sound sensor-e.g. Ear, mike or listener, observer detects change in pressure. So, in this case we prefer pressure wave.

Illustration 18

A sound wave of wavelength π m is travelling in medium of bulk modulus $10^5\ \dfrac{N}{m^2}$ and it creates pressure variation of $0.08\dfrac{N}{m^2}$. Calculate pressure amplitude and displacement amplitude of the wave.

Solution:

Pressure variation = $2\Delta p_{max} = 0.08 \Rightarrow \Delta p_{max} = 0.04\ N/m^2$

Δp_{max}= Bks_{max} $\Rightarrow 0.04 = 10^5 \times \frac{2\pi}{\pi} \times s_{max}\left(k = \frac{2\pi}{\lambda}\right) \Rightarrow s_{max} = 2\times10^{-7}\ m$

Speed of Sound Wave

Speed of the longitudinal waves in the medium depends on the elastic and inertial properties of the medium.

$v = \sqrt{\frac{\text{Elastic property}}{\text{inertial property}}}$ $v = \sqrt{\frac{E}{\rho}}$

(a) In Solids : $v_s = \sqrt{\frac{Y}{\rho}}$; Y = Young's Modulus of Elasticity of solids and ρ = density.

(b) In Liquids : $v_\ell = \sqrt{\frac{B}{\rho}}$; B = Bulk Modulus of Elasticity of Fluid and ρ = density.

Velocity of sound wave in air according to Newton

The formula for velocity of sound in air was first obtained by Newton. He assumed that when sound propagates through air, temperature remains constant (i.e. the process is isothermal).

In isothermal process B (Bulk modulus) = P (Atmospheric pressure)

$v_g = \sqrt{\frac{P}{\rho}}$ At NTP, $P = 1.01\times10^5 \frac{N}{m^2}$; $\rho_{air} = 1.3\frac{kg}{m^3}$

$v_s = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{1.01\times10^5}{1.3}} \simeq 279\ m/sec$

However, the experimental value of sound in air is 332 m/s which is much higher than that given by Newton's formula.

Laplace's Correction

In order to remove the discrepancy between theoretical and experimental values of velocity of sound, Laplace modified Newton's formula assuming that propagation of sound in air is adiabatic process, i.e.

$B = \gamma P$

$v_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$ At NTP, $P = 1.01\times10^5 \frac{N}{m^2}$; $\rho_{air} = 1.3\frac{kg}{m^3}$; $\gamma_{air} = \frac{7}{5}$

$v_s = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{7\times1.01\times10^5}{5\times1.3}} \simeq 332\ m/sec$

Which is in good agreement with the experimental value (332 m/s). This establishes that sound propagates adiabatically through gases.

Note: The velocity of sound in air at NTP is 332 m/s which is much lesser than that of light and radio-waves (= 3×10^8 m/s). This implies that –

- If we set our watch by the sound of a distant siren it will be slow.
- If we record the time in a race by hearing sound from starting point it will be lesser than actual.
- In a cloud-lightening, though light and sound are produced simultaneously but as c > v, light precede thunder.
- $v_s = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma PV}{M}} = \sqrt{\frac{\gamma\mu RT}{M}} = \sqrt{\frac{\gamma RT}{M_w}}$

Effect of various quantities on speed of sound wave:

(a) Effect of temperature: -

The speed of sound also depends on the temperature of the medium.

$$v_s = \sqrt{\frac{\gamma RT}{M_w}} \Rightarrow v_s \propto \sqrt{T} \quad \text{(Temperature(T) is in Kelvin)}$$

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} \Rightarrow \frac{v_t}{v_0} = \sqrt{\frac{t+273}{273}} \quad \text{(Temperature(t) is in degree Celsius)}$$

$$v_t = v_0\left[1+\frac{t}{273}\right]^{\frac{1}{2}} \Rightarrow v_t = v_0\left[1+\frac{t}{273}\right]^{\frac{1}{2}} \quad \text{(applicable for any medium)}$$

For air v_o = 332 m/s and ($t <<< 273$)

$$v_t = v_0\left[1+\frac{t}{546}\right] \Rightarrow v_t = (332 + 0.61t) \text{ m/s}$$

For every 1° degree rise in temperature (up to 30°C), speed of sound increases by 0.61 m/s.

(b) Effect of Relative Humidity: - With increase in humidity, density of air decreases so from $v_s = \sqrt{\frac{\gamma P}{\rho}}$ we conclude that with rise in humidity velocity of sound increases. This is why sound travels faster in humid air (rainy season) than in dry air (summer) at same temperature.

Due to this in rainy season the sound of factories siren and whistle of train can be heard more than summer.

(c) Effect of Pressure: - $v_s = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M_W}}$

So, pressure has no effect on velocity of sound in a gas as long as temperature remain constant. This is why while going up in the atmosphere, though both pressure and density decrease, velocity of sound remains constant as long as temperature remains constant.

(d) Effect of Wind: - If air is blowing then the speed of sound changes. If the actual speed of sound is v and the speed of air is w, then the speed of sound in the direction in which air is blowing will be (v + w), and in the opposite direction it will be (v – w).

(e) Effect of Frequency: - There is no effect of frequency on the speed of sound. Sound waves of different frequencies travel with the same speed in air although their wavelength in air are different. If the speed of sound were dependent on the frequency, then we could not have enjoyed orchestra.

Illustration 19

What is the speed of sound wave in Helium gas at 57°C.

Solution:

$$v = \sqrt{\frac{\gamma RT}{M_W}} \Rightarrow v = \sqrt{\frac{5\times25\times330\times1000}{3\times3\times4}} \Rightarrow v = 1070.4 \text{ m/s} \quad \left(R = \frac{25}{3} JK^{-1}mol^{-1}\right)$$

Illustration 20

What is the speed of sound wave in oxygen gas at 1 atm pressure and density 40kg/m^3?

Solution:

$$v=\sqrt{\frac{\gamma P}{\rho}} \Rightarrow v=\sqrt{\frac{7\times10^5}{5\times40}} \Rightarrow v=59.15\,m/s$$

Illustration 21

Find out the ratio of speed of sound in oxygen & Helium gas at same temperature

Solution:

$$\frac{\text{Speed of oxygen}}{\text{Speed of Helium}}=\frac{v_{oxygen}}{v_{Helium}}=\sqrt{\frac{\frac{\gamma_{O_2}RT}{M_{W_{O_2}}}}{\frac{\gamma_{H_e}RT}{M_{W_{He}}}}} \Rightarrow \frac{v_{oxygen}}{v_{Helium}}=\sqrt{\frac{\left(\frac{7}{5\times32}\right)}{\left(\frac{5}{3\times4}\right)}} \Rightarrow \frac{v_{Oxygen}}{v_{Helium}}=\sqrt{\frac{21}{200}}$$

Illustration 22

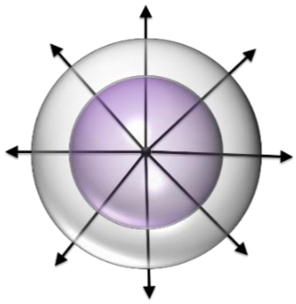
Speed of sound wave in air at 0°C is 332 m/s, then find the speed of sound at 20° C.

Solution:

$$v_t=v_0+0.61\,t \Rightarrow v_t=332+0.61(20) \Rightarrow v_t=344.2\,m/s$$

Wavefront

It is the locus of all the particle vibrating in the same phase.



Spherical Wavefront

1. Spherical wavefront originates from point source.
2. Intensity (I) $\propto \frac{1}{Area}$ $\quad \therefore I\propto\frac{1}{r^2}$ $\quad (Area=4\pi r^2)$
3. Intensity $\propto$ (Amplitude)² $\therefore \frac{1}{r^2} \propto A^2 \Rightarrow A\propto\frac{1}{r}$

Illustration 23

What is the intensity at a distance of 8 m from the point source of sound of power 25.12 Watt.

Solution:

$$Intensity=\frac{Power}{Area} \Rightarrow I=\frac{25.12}{4\pi\times8^2} \Rightarrow I=\frac{1}{32}\,Watt/m^2$$

Illustration 24

Intensity at a distance of 5 m from point source of sound is I_0. What is the intensity at a distance of 30m from the source?

Solution:

$$\frac{I_1}{I_2}=\left(\frac{r_2}{r_1}\right)^2 \Rightarrow \frac{I_0}{I_2}=\left(\frac{30}{5}\right)^2=36 \Rightarrow I_2=\frac{I_0}{36}$$

Characteristics of Sound

Sound is characterized by the following three parameters:

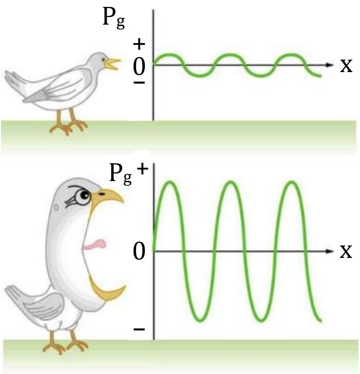
- **Loudness** • **Pitch** • **Quality/Timbre**

- **Loudness:** The quality of sound on the basis of which, sound is said to be high or low.
  It depends on, (a) Shape and size of the source (b) Intensity of sound
  According to Weber - Fechner the loudness of a sound of intensity I is given by:
  $L \propto \log_{10} I$

$$L_2 - L_1 = \Delta L = 10\log_{10}\frac{I_2}{I_1}$$

Unit of loudness - decibel (dB)

For zero decibel (0 dB) Loudness, Intensity is called threshold of hearing (I_0).



$$L_2 - L_1 = \Delta L = 10\log_{10}\frac{I_2}{I_1}$$

$$L - 0 = 10\log_{10}\frac{I}{I_0}$$

$$L = 10\log_{10}\frac{I}{I_0} \quad \left(\text{Where } I_0 = 10^{-12} W/m^2\right)$$

Note: (1) If $I = 10^{-12}$ W/m^2 then Loudness(L) = 0 dB (Threshold of hearing)

(2) If I = 1 W/m^2 then Loudness(L) = 120 dB (Non-Tolerable)

Illustration 25

Find sound intensity for 70 dB loudness?

Solution:

$$L = 10\log_{10}\left(\frac{I}{I_0}\right) \Rightarrow 70 = 10\log_{10}\left(\frac{I}{I_0}\right) \Rightarrow \frac{I}{I_0} = 10^7 \Rightarrow I = 10^7 I_0$$

Illustration 26

The power of sound from a speaker is 27 mW. By turning the knob of the volume, the power of sound is increased to 270 mW. The power increase in decibels levels is: -

Solution:

$$\Delta L = 10\ \log_{10}\left(\frac{P_2}{P_1}\right) \quad (\text{Intensity(I)} \propto \text{Power (P)})$$

$$\Delta L = 10\ \log_{10}\left(\frac{270}{27}\right) \Rightarrow \Delta L = 10\ \log_{10} 10 \Rightarrow \Delta L = 10\ [1] = 10\text{ dB}$$

Illustration 27

Loudness of sound source is 60 dB at 3m from a point source. What is the maximum distance up to which this sound can be heard if threshold of hearing is 20 db.

Solution:

$$\Delta L = 10\log_{10}\left(\frac{I_2}{I_1}\right) \qquad \left(I \propto \frac{1}{r^2} \text{ for a point source}\right)$$

$$L_2 - L_1 = 10\log_{10}\left(\frac{r_1}{r_2}\right)^2 \Rightarrow 60 - 20 = 20\log_{10}\left(\frac{r_1}{r_2}\right) \Rightarrow 40 = 20\log_{10}\left(\frac{r_1}{3}\right)$$

$$2 = \log_{10}\left(\frac{r_1}{3}\right) \Rightarrow \left(\frac{r_1}{3}\right) = 10^2 \text{ meter} \Rightarrow r_1 = 300\text{ m}$$

Pitch: It is the sensation received by the ear due to frequency and is the characteristic which distinguishes a shrill (or sharp) sound from a grave (or flat) sound. As pitch depends on frequency, higher the frequency higher will be the pitch and shriller will be the sound.

For example: The buzzing of a bee or humming of a mosquito has high pitch but low loudness while the roar of a lion has large loudness but low pitch.

Due to more harmonics usually, the pitch of female voice is higher than male.

Quality (or timbre): - It is the sensation received by the ear due to 'waveform'. Two sounds of same intensity and frequency as shown in figure will produce different sensation on the ear if their waveforms are different.

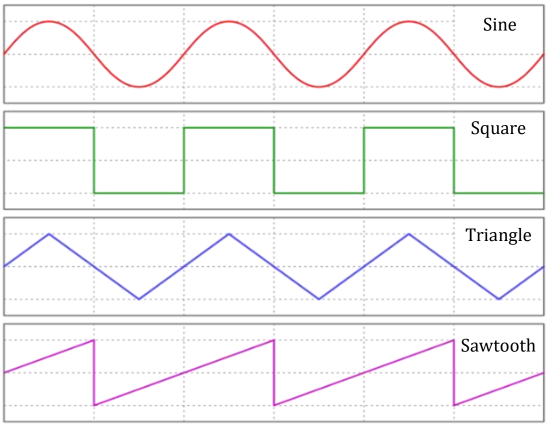


Illustration 28

(a) Speed of sound in air is 332 m/s at NTP. What will be the speed of sound in hydrogen at NTP if the density of hydrogen at NTP is (1/16) that of air.

(b) Calculate the ratio of the speed of sound in neon to that in water vapour at any temperature. [Molecular weight of neon = 2.02×10^{-2} kg/mol and for water vapours = 1.8×10^{-2} kg/mol]

Solution:

The velocity of sound in air is given by $v = \sqrt{\dfrac{E}{\rho}} = \sqrt{\dfrac{\gamma P}{\rho}} = \sqrt{\dfrac{\gamma RT}{M_w}}$

(a) In terms of density and pressure $= \dfrac{v_H}{v_{air}} = \sqrt{\dfrac{P_H}{\rho_H} \times \dfrac{\rho_{air}}{P_{air}}} = \sqrt{\dfrac{\rho_{air}}{\rho_H}}$ [as $P_{air} = P_H$]

$$\Rightarrow v_H = v_{air} \times \sqrt{\frac{\rho_{air}}{\rho_H}} = 332 \times \sqrt{\frac{16}{1}} = 1328 \text{ m/s}$$

(b) In terms of temperature and molecular weight $\dfrac{v_{Ne}}{v_w} = \sqrt{\dfrac{\gamma_{Ne}}{M_{Ne}} \times \dfrac{M_w}{\gamma_w}}$ [as $T_{Ne} = T_w$]

Now as neon is mono atomic $\left(\gamma = \dfrac{5}{3}\right)$ while water vapours poly atomic $\left(\gamma = \dfrac{4}{3}\right)$

$$\text{So, } \frac{v_{Ne}}{v_w} = \sqrt{\frac{\left(\frac{5}{3}\right) \times 1.8 \times 10^{-2}}{\left(\frac{4}{3}\right) \times 2.02 \times 10^{-2}}} = \sqrt{\frac{5}{4} \times \frac{1.8}{2.02}} = 1.055.$$

Illustration 29

Sound can be heard at great distance after rainfall. Explain?

Solution:

On a rainy day, air contains a large amount of water vapour. This decreases the density of air. Hence, sound travels faster in air and can be heard over longer distances.

Golden Key Points

- The density of a solid is much larger than that of a gas but the elasticity is larger by a greater factor. Hence longitudinal waves in a solid travel much faster than that in a gas.
- In a liquid the speed lies in between the two i.e. $v_{solid} > v_{liquid} > v_{gas}$
- Speed of longitudinal waves in a solid rod: $v = \sqrt{\frac{Y}{\rho}}$
- The property that determines the extent to which an element of a medium changes in volume when the pressure on it changes is the bulk modules (B) $B = \frac{-\Delta P}{\left(\Delta V/V\right)}$

  $\frac{\Delta V}{V}$ = Fractional change in volume produced by a change in pressure (ΔP)
- Speed of longitudinal waves in a liquid $v = \sqrt{\frac{B}{\rho}}$ Where B is the bulk modulus of elasticity.
- The velocity of mechanical waves or sound waves in gases is given by:

  $v = \sqrt{\frac{\gamma P}{\rho}}$ (where P = Pressure, ρ = Density)

  γ = ratio of specific heats of the gas at constant pressure and constant volume.
  This is known as Laplace's formula.

BEGINNER'S BOX-3

1. A 1000 m long rod of density 10.0×10^4 kg/m^3 and having young's modulus $Y = 10^{11}$ Pa, is clamped at one end. It is hammered at the other free end as shown in the figure. The longitudinal pulse goes to right end, gets reflected and again returns to the left end. How much time (in sec) the pulse will take to go back to initial point?

PWA186

2. If the bulk modulus of water is 4000 MPa, what is the speed of sound in water?

PWA187

3. The minimum intensity of audibility of sound is 10^{-12} watt/m^2. If the intensity of sound is 10^{-9} watt/m^2, then calculate the intensity level of this sound in decibels.

PWA188

4. Sound can be heard at great distances after rainfall. Explain.

PWA189

5. A sound wave is propagating in given gaseous medium then plot graph between

(a) v_{gas} & T (b) v_{gas} & P (c) v_{gas} & $\sqrt{T}$

[v_{gas} = Speed of sound, P = Pressure, T = Temperature] Also write down proper units on axis.

PWA190

6. Why are temple bells made of large size?

PWA191

7. Explain why sound travels faster in warm air than in cool air?

PWA192

8. Explosion on other planets are not heard on earth. Explain why?

PWA193

9. Calculate velocity of sound in mixture of 1 mole He and 1 mole O_2 at 127°C.

PWA194

10. Use the formula $v = \sqrt{\frac{\gamma P}{\rho}}$ to explain why the speed of sound in air

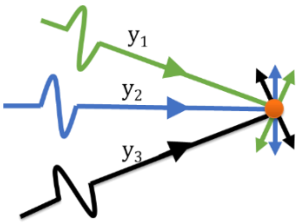
(a) is independent of pressure. (b) increases with temperature.

(c) increases with humidity.

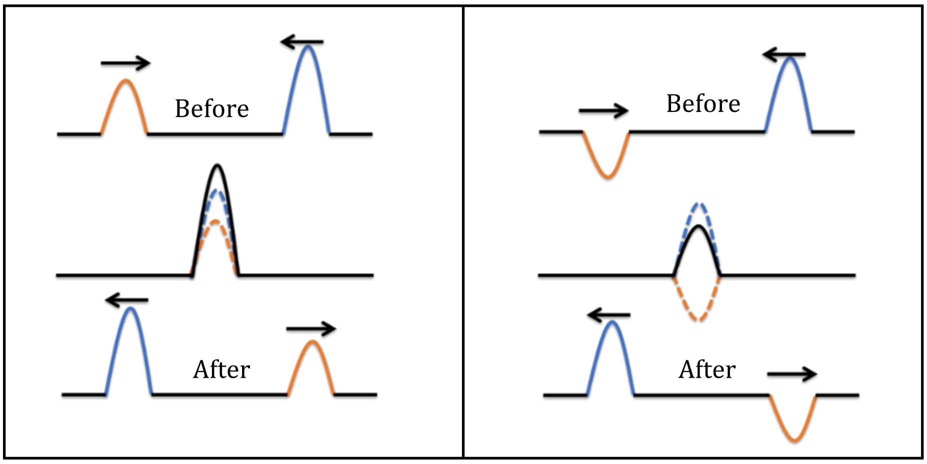
PWA195

Principle of Superposition

When two or more waves arrive at a point in the medium at the same time, then resultant displacement of medium particles is the vector addition of the individual displacements due to individual waves. This is called superposition phenomenon.



$\vec{Y}_{net} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3$



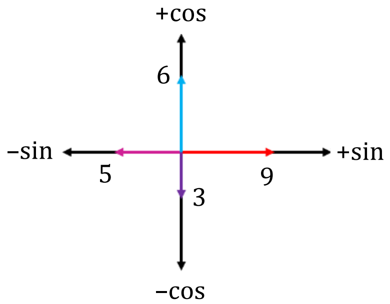
Principle of superposition

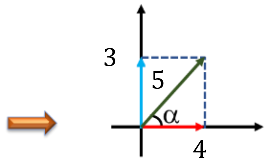
Note: Existence of one wave is not affected due to presence or absence of another wave in the medium, waves independently propagate in the medium.

Illustration 30

Find equation of resultant wave if all the below mentioned wave superimpose at a point $Y_1 = 9\sin(\omega t - kx)$; $Y_2 = 6\cos(\omega t - kx)$; $Y_3 = -5\sin(\omega t - kx)$; $Y_4 = -3\cos(\omega t - kx)$

Solution:





$\tan\alpha = \frac{3}{4} \Rightarrow \alpha = 37°$

$y_{res} = 5\sin(\omega t - kx + \phi) = 5\sin(\omega t - kx + 37°)$

Illustration 31

If equation of wave is given $y = 3\sin(\omega t - kx) + 4\cos(\omega t - kx)$m. Find the amplitude of resultant wave.

Solution:

$y = 5\times\left[\frac{3}{5}\sin(\omega t - kx) + \frac{4}{5}\cos(\omega t - kx)\right]$ $\quad$ $y = 5[\cos 53°\sin(\omega t - kx) + \sin 53°\cos(\omega t - kx)]$

$y = 5[\sin(\omega t - kx + 53°)]$ ...(i) $\quad$ $y = A[\sin(\omega t - kx + 53°)]$ ...(ii)

Comparing (i) and (ii), Amplitude = 5m.

Alternative method:

$y = 3\sin(\omega t - kx) + 4\cos(\omega t - kx)$

Amplitude of wave, $\sqrt{A^2 + B^2} = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$m

Interference of Waves

Interference for Sound Waves : When the waves emitting from the coherent sources propagating in same direction are superposed over one another, then redistribution of energy takes place in the space and this phenomenon is called as interference.

It is based on energy conservation principle. Total energy of the waves remains constant, only redistribution of energy takes place. Here intensity varies as the function of position.

Coherent source

- Sources for which phase difference remains constant with respect to time are known as coherent sources.
- Two waves are said to be coherent if their phase difference does not depend on time.

Mathematical Analysis

Let the two waves are $y_1 = A_1\sin(\omega t - kx_1)$ and $y_2 = A_2\sin(\omega t - kx_2)$

$$\Delta\phi = (\omega t - kx_1) - (\omega t - kx_2) = k(x_2 - x_1) = \frac{2\pi}{\lambda}\Delta x$$

$$A_{res} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\phi} \Rightarrow A_{res}^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\Delta\phi$$

We know that, $I \propto A^2 \Rightarrow I = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$

Constructive Interference	Destructive Interference
$\Delta x = 0, \lambda, 2\lambda, 3\lambda \ldots \ldots n\lambda$ Here n = 0, 1, 2, 3…….	$\Delta x = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2} \ldots \ldots \frac{(2n+1)\lambda}{2}$ Here n = 0, 1, 2, 3…….
$\Delta\phi = 0, 2\pi, 4\pi \ldots \ldots 2n\pi$	$\Delta\phi = \pi, 3\pi, 5\pi \ldots \ldots (2n+1)\pi$
$A_{max} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos 0°}$ $A_{max} = A_1 + A_2$	$A_{min} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2 \cos \pi}$ $A_{min} = A_1 - A_2$
$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$	$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$

$$\frac{I_1}{I_2} = \left(\frac{A_1}{A_2}\right)^2 \Rightarrow \frac{I_{max}}{I_{min}} = \left(\frac{A_{max}}{A_{min}}\right)^2 \Rightarrow \frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$$

Average intensity of interference pattern = $\frac{I_{max}+I_{min}}{2} = I_1 + I_2$

Degree of interference pattern (f) = f represents contrast effect (clarity of interference pattern)

$$f = \frac{I_{max}-I_{min}}{I_{max}+I_{min}} \times 100\%$$

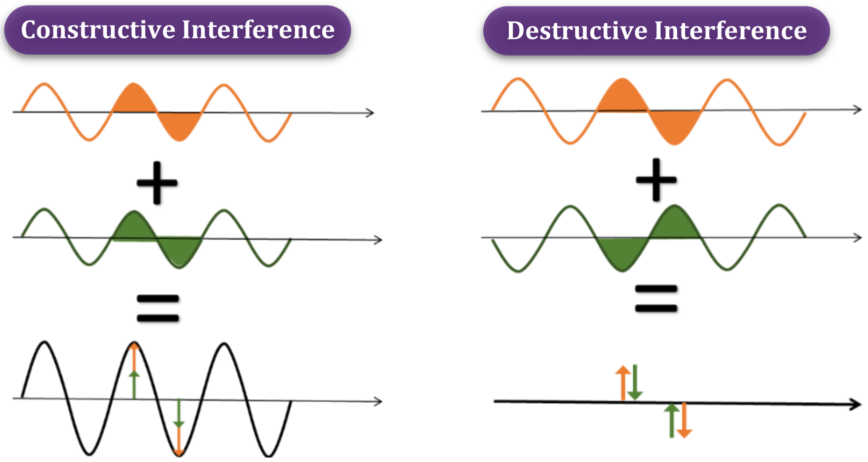
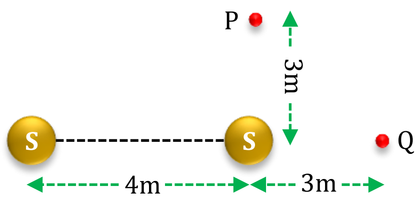


Illustration 32

If λ = 3m and the two coherent sources of same intensity are arranged as shown, then find ratio of resultant intensity at point P and Q in the given situation?



Solution:

$\Delta x = 7 - 3 = 4$ m

$$\Delta\phi = \frac{2\pi}{\lambda} \times \Delta x = \frac{2\pi}{3} \times 4 \Rightarrow \Delta\phi = \frac{8\pi}{3}$$

$$I_Q = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\Delta\phi$$

$$I_Q = I + I + 2\sqrt{I}\sqrt{I}\left(-\frac{1}{2}\right) \Rightarrow I_Q = I \Rightarrow \frac{I_p}{I_Q} = 1$$

Illustration 33

If in an interference pattern $\frac{I_{max}}{I_{min}} = \frac{64}{49}$ then find $\frac{I_1}{I_2}$ and $\frac{A_1}{A_2}$?

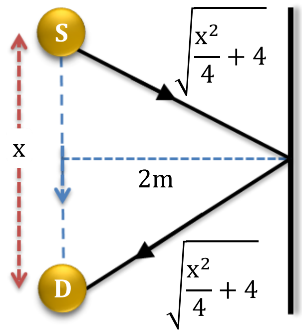
Solution:

$$\left(\frac{A_1+A_2}{A_1-A_2}\right)^2 = \frac{64}{49} \Rightarrow \frac{A_1+A_2}{A_1-A_2} = \frac{8}{7} \Rightarrow \frac{A_1}{A_2} = \frac{15}{1} \Rightarrow I \propto A^2 \Rightarrow \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} = \frac{225}{1}$$

Illustration 34

A source emitting sound of frequency 180 Hz is placed in front of a wall at a perpendicular distance of 2 m from it. A detector is also placed in front of the wall at the same distance from it. Find the minimum distance between the source and the detector for which the detector detects a maximum of sound. Speed of sound in air = 360 m/s?

Solution:



$$\Delta x = \left[2\sqrt{\frac{x^2}{4}+4} - x\right]$$

$\Delta x = 0, \lambda, 2\lambda$ ......$n\lambda$, for this question considering $\Delta x = \lambda$

$$\lambda = \frac{v}{n} = \frac{360}{180} = 2 \Rightarrow 2 = \left[2\sqrt{\frac{x^2}{4}+4} - x\right]$$

by solving we get, x = 3

Illustration 35

Amplitude of two waves are a_1 and a_2, where $a_1 = a_2 = a$ and both are superimposed at a point where path difference between them is $\frac{\lambda}{2}$. Find the amplitude of resultant wave?

Solution:

$$\Delta x = \frac{\lambda}{2} \Rightarrow \Delta\phi = \pi = 180°,\ A_{res} = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} \Rightarrow A_{res} = \sqrt{a^2 + a^2 + 2aa\cos 180°} = 0.$$

Illustration 36

In interference phenomena if the degree of interference is 60% then find the ratio of intensity and amplitudes of interfering wave form?

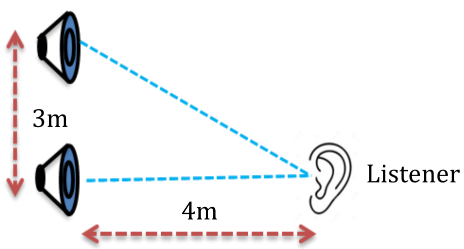
Solution:

$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{60}{100} = \frac{3}{5} \quad \Rightarrow \quad \frac{I_{max}}{I_{min}} = \frac{5+3}{5-3} = \frac{4}{1}$$

Thus $\frac{a_1+a_2}{a_1-a_2} = \frac{2}{1}$ and $\frac{a_1}{a_2} = \frac{2+1}{2-1} = \frac{3}{1}$ thus $\frac{I_1}{I_2} = \frac{9}{1}$

Illustration 37

Two loudspeakers as shown in fig. below separated by a distance 3 m, are in phase. Assume that the amplitudes of the sound from the speakers is approximately same at the position of a listener, who is at a distance 4.0 m in front of one of the speakers. For what frequencies does the listener hear minimum signal? Given that the speed of sound in air is 330 ms^{-1}.



Solution:

The distance of the listener from the second speaker = $\sqrt{(3)^2 + (4)^2}$ = $\sqrt{25}$ = 5 m

Path difference = (5 – 4) m = 1 m

For fully destructive interference 1 m = (2n + 1) $\frac{\lambda}{2}$ Hence, $\lambda=\left(\frac{2}{2n+1}\right)$m

The corresponding frequencies are given by $n=\frac{330\times(2n+1)}{2}$Hz , for n = 0, 1, 2, 3, 4, ...

n = 165 (2n + 1) Hz, for n = 0, 1, 2, 3, 4, ...

Therefore, the frequencies for which the listener would hear a minimum intensity 165 Hz. 495 Hz, 825 Hz, ...

★ Golden Key Points ★

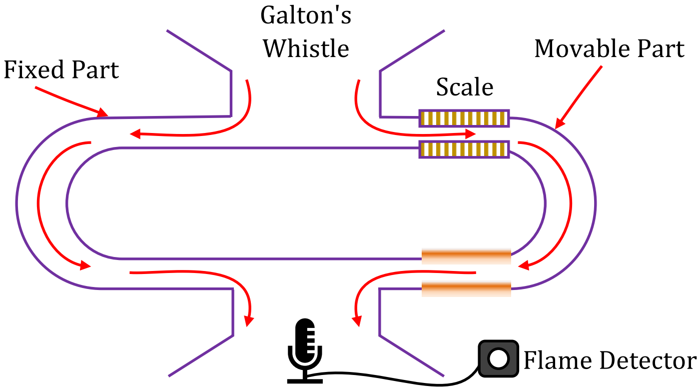
- Superposition phenomena is applicable for all types of waves.
- For the resultant wave in interference phenomenon frequency, wavelength & velocity is identical to initial or super-posing waves but, its amplitude and initial phase is changed.
- $\frac{I_{max}}{I_{min}}=\left(\frac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2$ = $\frac{(a_1+a_2)^2}{(a_1-a_2)^2}$ and $\frac{I_1}{I_2}=\left(\frac{a_1}{a_2}\right)^2$
- Average intensity of interference pattern

  I_{av} or $<I>=\frac{I_{max}+I_{min}}{2}$ or $I_{av}=I_1+I_2$

  I_{av} = sum of intensities of individual waves
- **Degree of interference pattern (f) =** f represents contrast effect (clarity of interference pattern) $f=\frac{I_{max}-I_{min}}{I_{max}+I_{min}}\times100\%$
- Condition for perfect interference

  If $a_1=a_2=a$ and $I_1=I_2=I$; $I_{max}=4I$; $I_{min}=0$
- **Interference: -** Permanent intensity pattern and it is the function of position. $[I=f(x)]$

Quincke's tube experiment



(1) It gives practical proof of interference of sound.

(2) Quincke's tube is practical method for finding the speed of sound in gaseous medium.

(3) (a) As we slide movable part of tube by ℓ unit path difference will become 2ℓ so $\Delta x = 2\ell$.

(b) If detector detects maximum to minimum or minimum to maximum then $\Delta x=\frac{\lambda}{2}$.

(c) Quincke's tube is used as a sound filter.

Illustration 38
In Quincke's tube experiment the movable part is shifted by amount of 1.5 mm, then Sound detector receives intensity from maximum to minimum. If frequency of source is 30 kHz, find speed of wave –

Solution:

$\Delta x=\frac{\lambda}{2}=2\ell \quad \because \ \ell=1.5 \ \Rightarrow \Delta x = 3 \text{ mm} \Rightarrow \frac{\lambda}{2}=3 \text{ mm} \Rightarrow \lambda = 6 \text{ mm}$

$v = f\lambda \Rightarrow v = 30 \times 10^{3} \times 6 \times 10^{-3} \Rightarrow v = 180 \text{ m/s}$

Illustration 39
In Quincke's tube experiment the movable part is shifted by amount of 1.5 mm, then Sound detector receives intensity from maximum to maximum. If frequency of source is 30 kHz, find speed of wave –

Solution:

$\Delta x = \lambda = 2\ell \ \Rightarrow \lambda = 3 \text{ mm} = 3 \times 10^{-3} \text{ m}$ So, $v = f\lambda = 30 \times 10^{3} \times 3 \times 10^{-3} = 60 \text{ m/s}$

Beats
When two sound waves of nearly equal (but not exactly equal) frequencies travel in same direction, at a given point due to their superposition, intensity alternatively increases and decreases periodically. This periodic waxing and waning of sound at a given position is called beats.

Mathematical Analysis

$y_1 = A\sin\omega_1 t,\ y_2 = A\sin\omega_2 t$

For superposition $y = y_1 + y_2 = A\sin\omega_1 t + A\sin\omega_2 t$

$$y=\left(2A\cos\left(\frac{\omega_1-\omega_2}{2}\right)t\right)\sin\left(\frac{\omega_1+\omega_2}{2}\right)t$$

$$y=\left[2A\cos 2\pi\left(\frac{f_1-f_2}{2}\right)t\right]\sin 2\pi\left(\frac{f_1+f_2}{2}\right)t$$

Beat frequency $\Delta f = |f_2 - f_1|$

Beat Time period $T_{beat} = \frac{1}{|f_2-f_1|}$

$f_{res} = \frac{f_1+f_2}{2}$

Time for I_{max} to I_{min}, $t = \frac{T_{beat}}{2} = \frac{1}{2|f_2-f_1|}$

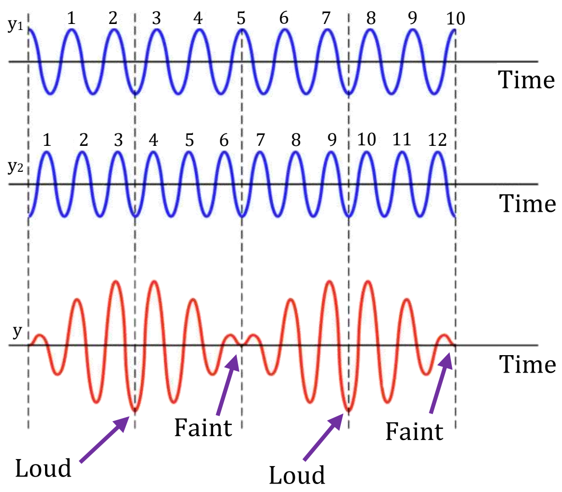


Illustration 40
Two sound waves $y_1 = 2\sin(476\pi(t - x/330))$ and $y_2 = 1.6\sin(486\pi(t - x/330))$produce beat phenomenon, then find:

(1) Beat Frequency (2) Time taken to complete 10 beats

Solution:

$y_1 = 2\sin(476\pi(t-x/330))$ $\quad$ $y_2 = 1.6\sin(486\pi(t-x/330))$

$\omega_1 = 476\pi \Rightarrow 2\pi n_1 = 476\pi$ $\quad$ $\omega_2 = 486\pi \Rightarrow 2\pi n_2 = 486\pi$

$n_1 = 238 \text{ Hz}$ $\quad$ $n_2 = 243 \text{ Hz}$

(1) $\Delta n = n_2 - n_1 = 5 \text{ beats/sec}$

(2) $T = \frac{1}{5}$ sec = Time period of 1beat $\quad$ So, time taken by 10 beats = $10T = \frac{10}{5} = 2$sec

Illustration 41
Two sound waves $y_1 = 2\sin(476\pi(t - x/300))$; $y_2 = 1.6\sin(488\pi(t - x/300)$, produce beat phenomenon, then find frequency of resultant wave?

Solution:

$$f_{res} = \frac{n_1 + n_2}{2} \Rightarrow f_{res} = \frac{244 + 238}{2} \Rightarrow f_{res} = 241 \text{ Hz}$$

Illustration 42
Two sound waves $y_1 = 2\sin(476\pi(t - x/330))$; $y_2 = 1.6\sin(486\pi(t - x/330))$, produce beat phenomenon, then find Time interval during which intensity becomes maximum to minimum?

Solution:

$$\text{Time} = \frac{T_b}{2} = \frac{\left(\frac{1}{5}\right)}{2} = \frac{1}{10}\text{sec}$$

Illustration 43
Two sound $y_1 = 4\sin(476\pi(t - x/330))$; $y_2 = 3\sin(486\pi(t - x/330))$, produce beat phenomenon, then find Ratio of maximum to minimum intensity?

Solution:

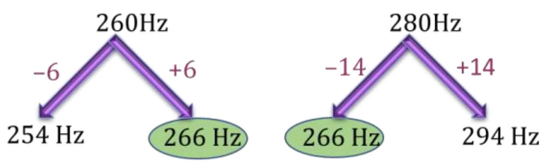
$$\frac{I_{max}}{I_{min}} = \left(\frac{A_{max}}{A_{min}}\right)^2 = \left(\frac{4+3}{4-3}\right)^2 = \frac{49}{1}$$

Illustration 44
An unknown tuning fork produce 6 beats with a tuning fork of frequency to 260 Hz and 14 beats with a tuning fork of frequency 280 Hz. Find out frequency of unknown tuning fork.

Solution:

Frequency of known tuning fork

Frequency of unknown tuning fork = 266 Hz



Important terms
1. Loading of tuning fork ⇒ Frequency of tuning fork decreases.
2. Filing of tuning fork ⇒ Frequency of tuning fork increases.
3. The ratio between the frequencies of two notes is called the musical interval. Following are the names of some musical intervals.

   Unison, $\frac{n_2}{n_1} = 1$; Octave , $\frac{n_2}{n_1} = 2$

Illustration 45
Tuning fork having f = 400 Hz produces 8 beats/sec with another tuning fork. If known tuning fork is loaded with wax, then number of beats decreases. Find frequency of unknown.

Solution:

Frequency of known tuning fork is 400 Hz. $408 \text{ Hz} \xleftarrow{+8} 400 \text{ Hz} \xrightarrow{-8} 392 \text{ Hz}$

After loading with wax frequency decrease and it makes less number of beats/sec with 392 Hz.

∴ **392 Hz** is the frequency of the unknown tuning fork.

Illustration 46
Tuning fork having f = 165 Hz produces 5 beats/sec with another tuning fork. If unknown tuning fork is filed, then beats increases. Find frequency of unknown.

Solution:

Frequency of known tuning fork is 165 Hz

170 Hz $\xleftarrow{+5}$ 165 Hz $\xrightarrow{-5}$ 160 Hz

After filling frequency increases and it makes more number of beats/sec with 170 Hz.

After filling frequency increases and it makes initially less number of beats/sec with 160 Hz then after it also produce more number of beats/sec with 160 Hz.

170 Hz and 160 Hz are the frequencies of unknown which make more beats after filling.

Illustration 47

81 tuning forks are arranged in the increasing order of their frequency and each tuning fork produce 4 beats per sec. With its nearby tuning fork. If the frequency of last tuning fork is octave of frequency of first tuning fork then find the frequency of last tuning fork.

Solution:

According to question

$n + 80b = 2n$ (octave = 2 times)

$n = 80 \times 4 = 320$ Hz (b = 4 beats)

$n_{last} = n + 80b = 320 + 320$

$n_{last} = 640$ Hz

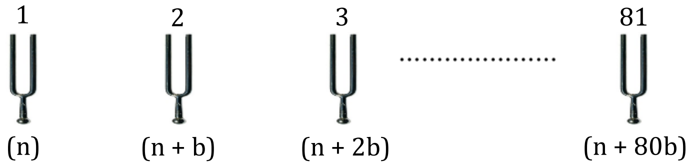


Illustration 48

A tuning fork having frequency 300 Hz produce, four beats per sec with X. If we file arm of unknown and again vibrate it, the number of beats decreases. Determine X?

Solution:

Let tuning fork A having frequency = 300 Hz

It produces 4 beats/sec with unknown tuning fork B then frequency of B is

So, answer is 296 Hz

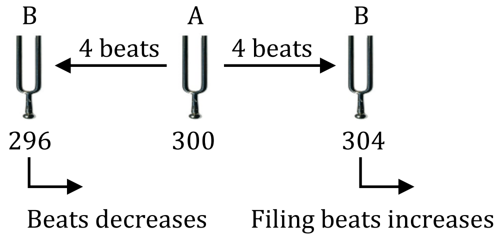


Illustration 49

Three sound sources of frequency 151 Hz, 152 Hz, 153 Hz are vibrated together and beats are produced. Find beat frequency.

Solution:

152 & 153 $\Rightarrow$ 1 beat/sec $\Rightarrow$ 1 sec

151 & 152 $\Rightarrow$ 1 beat/sec $\Rightarrow$ 1 sec Common

151 & 153 $\Rightarrow$ 2 beat/sec $\Rightarrow$ $\frac{1}{2}$, $\frac{2}{2}$

1 beat 1 beat

2 beat/sec

Illustration 50

Two plane harmonic sound waves are expressed by the following equations

$y_1(x, t) = A \sin(0.5\pi x - 100\pi t)$ $y_2(x, t) = A \sin(0.48\pi x - 96\pi t)$

(All parameters are in MKS system)

(a) How many times does an observer hear maximum intensity in 1 second?

(b) What is the speed of the sound?

(c) What is the amplitude of $y_1 + y_2$ at $x = 0$ and $t = 0.25$ s?

Solution:

(a) Beat frequency = $f_1 - f_2 = \frac{1}{2\pi}[100\pi - 96\pi] = 2$

Therefore, observer heard maximum intensity twice in one second.

(b) Speed of the sound wave = $\frac{\omega_1}{k_1} = \frac{100\pi}{0.5\pi} = 200$ m/sec

(c) At x = 0 and t = 0.25 s

$y_1 = 0$ and $y_2 = 0$ so $y_1 + y_2 = 0$

Golden Key Points

- Filing the prongs of tuning fork raises the frequency and loading decreases the frequency.
- **Vibration of tuning fork:** when tuning fork is sounded by striking its one end on rubber pad, then the ends of prongs vibrate in and out while the stem vibrates up and down or vibration of the prongs are transverse and that of the stem is longitudinal. Generally tuning fork produces fundamental tone.

BEGINNER'S BOX-4

1. Two waves having same frequency and same amplitude are superimposed at a point. Find the phase difference between two waves if resultant amplitude is :

(i) 2a (ii) $\sqrt{2}$a (iii) a (iv) Zero

PWA196

2. During interference phenomenon of two wave it is observed that maximum amplitude to minimum amplitude ratio is 9 : 7. Find the intensity ratio of waves.

PWA197

3. During interference phenomenon of two wave it is observed that amplitude ratio of waves is 5 : 4. Find the maximum to minimum intensity ratio of resulting wave.

PWA198

4. Two sound waves emitted by one sound source is approaching at a point through two path. When path difference is 36 cm or 60 cm, then points are silence point if speed of sound in air 330 m/sec then find the frequency of sound source.

PWA199

5. In a Quincke's tube there are two positions where sound becomes minimum, the sliding distance between them is 16.6 cm. Find the freq. of sound source. (Sound velocity in air = 332 m/sec.)

PWA200

6. A tuning fork having frequency 300 Hz produce, four beats per sec with x. If we file arm of unknown and again vibrate, number of beats decreases. Determine x.

PWA201

7. 81 TF are arranged in the increasing order of their frequency and each TF produce 4 beats per sec. with its near by TF. If the frequency of last TF is octave of frequency of first TF then find the frequency of last TF.

PWA202

8. Two sound waves $y_1 = 3 \sin 400\pi t$; $y_2 = 4 \sin 402\pi t$

(a) Find frequency of resultant wave (b) Beat frequency

(c) Intensity ratio of waves

PWA203

9. A Standard tuning fork contains frequency n_0, another TF n_A have 20% more compared to n_0, n_B 30% more compared to n_0, n_A & n_B have beat relation of 6 beat/sec. Then determine n_0, n_A, n_B

PWA204

10. Two tuning fork having frequency 320 Hz & 324 Hz produce beat phenomena. Determine –
(a) Beat time period
(b) Minimum time interval in which maximum intensity become minimum.
(c) Number of beat per three seconds.

PWA205

11. Two audio speakers are kept some distance apart and are driven by the same amplifier system. A person is sitting at a place 6.0 m from one of the speakers and 6.4 m from the other. If the sound signal is continuously varied from 500 Hz to 5000 Hz, what are the frequencies for which there is a destructive interference at the place of the listener? Speed of sound in air = 320 m/s.

PWA206

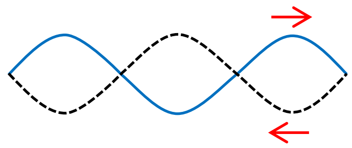
12. 51 tuning fork are arranged in a series in such a way that each fork produce five beats/sec with neighbouring tuning fork. If frequency of last is six time of first then determine frequency of first, 11^{th}, 17^{th}, 27^{th}, 33^{th} and last tuning fork.

PWA207

Stationary Waves, Standing Waves in strings & Reflection of Waves

Stationary/Standing Waves

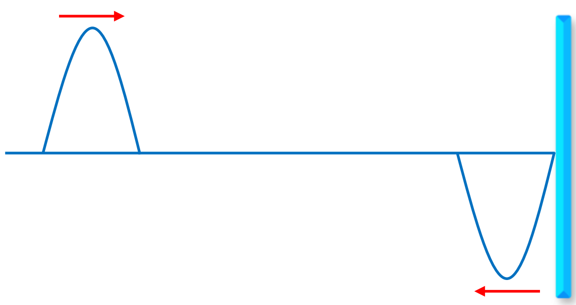
- When two identical waves (transverse or longitudinal) propagating in opposite direction superimpose in bounded medium then the resultant wave is called stationary wave or standing wave.



- It is of two types -
  (i) Transverse stationary waves (ii) Longitudinal stationary waves
- According to the nature of Reflected surface, Reflection are of two types,
  (i) Reflection from rigid end (ii) Reflection from free end

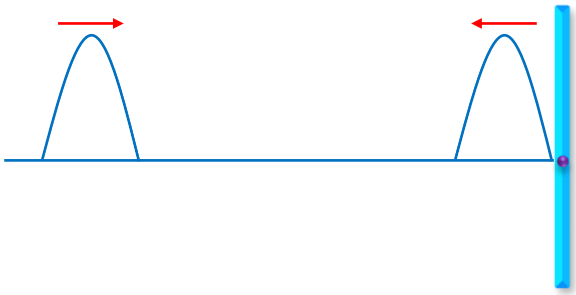
(I) Reflection from Rigid end:

- Whenever a travelling wave reaches a boundary, part or all of the wave will be reflected.
- Consider a pulse travelling on a string fixed at one end. When the pulse reaches the fixed wall, it will be reflected. Ideally it does not transmit any part of the disturbance to the wall. This must be noted that Reflected pulse is inverted.



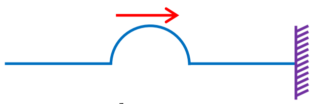
(II) Reflection from free end:

- Now consider another case when the pulse arrives at the end of a string that is free to move vertically. Again, the pulse will be reflected, but this time its displacement is not inverted. As the pulse reaches the post, it exerts a force on the free end, causing the ring to accelerate upward.



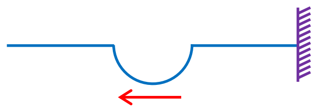
Mathematical Analysis

(a) Rigid End: - In such type of reflection incident and reflected waves have phase difference of π and direction of propagation are opposite.



Incident wave

$$y_i = a\sin(\omega t - kx)$$



Reflected wave

(The direction of propagation reverses. Also, a phase difference of π.)

$$y_r = a\sin(\omega t + kx + \pi) = -a\sin(\omega t + kx)$$

$$y_{res} = y_i + y_r$$

$$y_{res} = a[\sin(\omega t - kx) - \sin(\omega t + kx)]$$

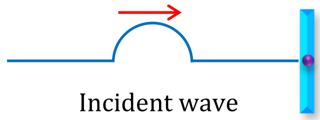
$$\left\{\sin A - \sin B = 2\sin\left(\frac{A-B}{2}\right)\cos\left(\frac{A+B}{2}\right)\right\}$$

$$y_{res} = \underbrace{-2a\sin kx}_{\text{amplitude}}\cos\omega t$$

Let A = 2a sin kx

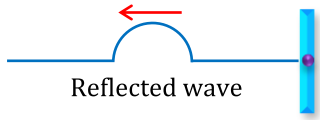
so, at x = 0, A = 0

(b) Free End: - In such type of reflection incident and reflected waves are in phase and direction of propagation are opposite.



Incident wave

$$y_i = a\sin(\omega t - kx)$$



Reflected wave

(The direction of propagation reverses. But no phase difference.)

$$y_r = a\sin(\omega t + kx)$$

$$y_{res} = y_i + y_r$$

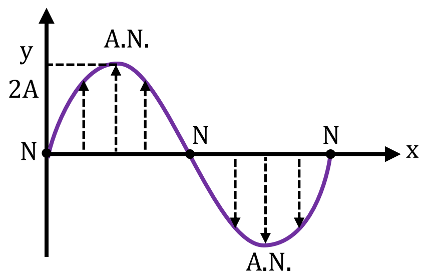
$$y_{res} = a\{\sin(\omega t - kx) + \sin(\omega t + kx)\}$$

$$\left\{\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right\}$$

$$y_{res} = \underbrace{2a\cos kx}_{\text{amplitude}}\sin\omega t$$

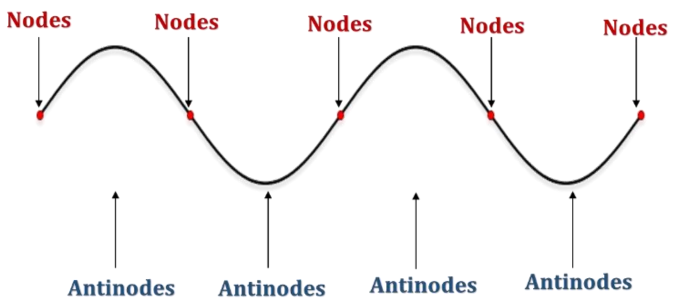
Let A = 2a cos kx

so, at x = 0, A = 2a



Important Points:

- Distance between two successive nodes or successive antinodes is $\lambda/2$.
- Distance between successive node to antinode or antinode to node is $\lambda/4$.
- All the particles between two successive nodes vibrate in same phase while either side particles of a node are in opposite phase.



- In a stationary wave pattern, all particles execute S.H.M. of same time period but of different amplitude.
- All particles reach at their extreme position simultaneously but it is different for different particles.
- Either side particles of an antinode are in same phase.
- In one complete vibration string appear straight twice.
- Since loop pattern does not transfer in stationary waves, so speed of stationary wave is zero.

Special properties of stationary waves

- **Zero wave velocity: -** No transfer of energy between two points, particle velocity is non-zero but wave velocity is zero.
- Position of antinodes and nodes in this pattern remains fix.
- All medium particles doing simple harmonic vibrations have identical time period but different vibration amplitude and because of this their maximum velocity at mean position is different.
- All medium particles pass through their mean position simultaneously but with different maximum velocity.
- In this pattern at antinode position, displacement and velocity is maximum but wave strain is minimum.
- Strain = slope of stationary wave pattern $\left(\frac{dy}{dx}\right)$

  At node position displacement and velocity is minimum but wave strain is maximum.

Illustration 51

A standing wave is created on a string of length 120 cm and it is vibrating in 6th harmonic. Maximum possible amplitude of any particle is 10 cm and maximum possible velocity will be 10 cm/s. Choose the correct statement.

(1) Angular wave number of two waves will be $\frac{\pi}{20}$.

(2) Time period of any particle's SHM will be 4π sec.

(3) Amplitude of interfering waves are 10 cm each.

Solution:

$3\lambda = 120\text{cm} \Rightarrow \lambda = 40\text{cm}$

Now, $A_{max} = 10\text{cm}$ and $v_{max} = 10\text{cm/sec} \Rightarrow A_{max}\omega = v_{max} \Rightarrow 10 = \omega(10) \Rightarrow \omega = 1$ rad/sec

$T = 2\pi \text{ sec} \Rightarrow K = \frac{2\pi}{\lambda} = \frac{2\pi}{40} = \frac{\pi}{20}$ per metre

Illustration 52

An incident wave equation $y = A\sin(kx - \omega t)$, if at $x = 0$ node is formed, then find the equation of reflected wave?

Solution:

At $x = 0$, node is formed therefore it is a rigid end.

For incident wave: $(y_i) = A\sin[-(\omega t - kx)] \Rightarrow y_i = -A\sin[\omega t - kx]$

For reflected wave: $(y_r) = -A\sin[\omega t + kx + \pi] \Rightarrow y_r = -A[-\sin(\omega t + kx)] = A\sin(\omega t + kx)$

Illustration 53

For a stationary wave equation $y = 20\cos\left(\frac{\pi}{24}x\right)\cos(16\pi t)$ where x and y in cm and t in second. Find?

(a) Type of reflector

(b) Equation of incident and reflected wave

Solution:

Compare the given equation with standard equation $y = 2A \cos kx \ \cos\omega t$

$\therefore 2A\cos kx = 20\cos\left(\frac{\pi}{24}x\right)$ and $\omega = 16\pi$

(a) At x = 0 amplitude is maximum therefore it represents Free end

(b) By comparing $A = 10$ cm, $\omega = 16\pi$, $k = \frac{\pi}{24}$

$y_i = A\cos(\omega t - kx)$ $\qquad$ $y_r = A\cos(\omega t + kx)$

$y_i = 10\cos\left(16\pi t - \frac{\pi}{24}x\right)$ $\qquad$ $y_r = 10\cos\left(16\pi t + \frac{\pi}{24}x\right)$

Illustration 54

For a stationary wave equation $y = 20\cos\left(\frac{\pi}{28}x\right)\cos(14\pi t)$, where x and y in cm and t in second. Find Amplitude of stationary wave at x = 7cm?

Solution:

Amplitude of stationary wave $(y_{max}) = 20\cos\left(\frac{\pi}{28}x\right) \Rightarrow y_{max} = 20\cos\left(\frac{\pi}{28}\times 7\right) \Rightarrow y_{max} = \frac{20}{\sqrt{2}} = 10\sqrt{2}$ cm

Illustration 55

For a stationary wave equation $y = 20\cos\left(\frac{\pi}{28}x\right)\cos(14\pi t)$, where x and y in cm and t in second. Find

(a) Amplitude at node and antinode positions $\qquad$ (b) Distance between two successive nodes

Solution:

(a) Node $\rightarrow$ Minimum Amplitude = 0 $\qquad$ Antinode $\rightarrow$ Maximum Amplitude = 20 cm

(b) $k = \frac{\pi}{28} \Rightarrow \frac{2\pi}{\lambda} = \frac{\pi}{28} \Rightarrow \lambda = 56$ cm $\qquad$ Distance between node and node $= \frac{\lambda}{2} = \frac{56}{2} = 28$ cm

Illustration 56

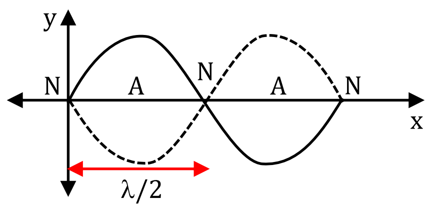
For a stationary wave equation $y = 20\cos\left(\frac{\pi}{28}x\right)\cos(14\pi t)$, where x and y in cm and t in second. Find:

(a) Length of 1 loop $\qquad$ (b) Speed of progressive wave $\qquad$ (c) speed of stationary wave

Solution:

(a) From the given equation $k = \frac{\pi}{28} = \frac{2\pi}{\lambda} \Rightarrow \lambda = 56$ cm

Length of one loop $= \frac{\lambda}{2} = \frac{56}{2} = 28$ cm



(b) $v = \frac{\omega}{k} = \frac{14\pi}{\left(\frac{\pi}{28}\right)} = 28\times 14 = 392$ cm / s.

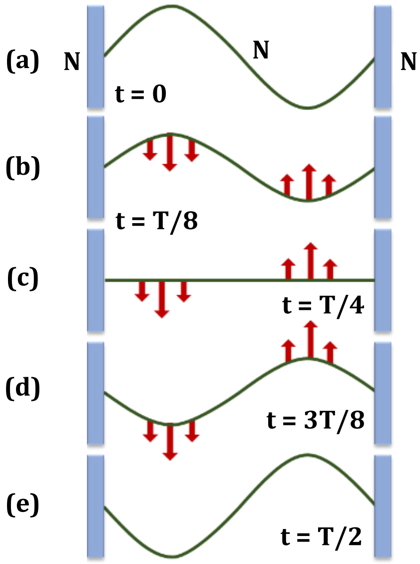
(c) Speed of stationary Wave = 0

Transverse Stationary Waves

Important terms

- **Fundamental frequency** – Minimum frequency produce by any source.
- **Overtone frequency** – Frequency higher than fundamental frequency.
- **Harmonic frequency** – Integer multiple of fundamental frequency.

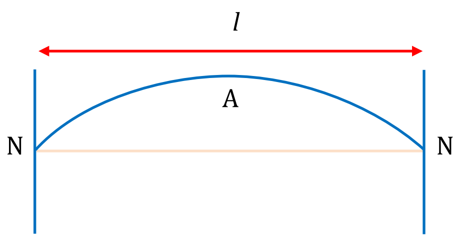
Transverse stationary wave (Standing Waves in a String Fixed at Both Ends):



- Consider a string of length L fixed at both ends and standing waves are set up in the string by a continuous superposition of waves incident on and reflected from the rigid ends.
- The ends of the string are fixed (rigid end) must have zero displacement therefore nodes are formed at the ends.
- The boundary condition results in the string having a number of natural patterns of oscillation, called normal modes.
- Figure shows one of the normal modes of oscillation of a string fixed at both ends for half time period. Except for the nodes, which are always stationary, all elements of the string oscillate vertically with the same frequency but with different amplitudes of simple harmonic motion.
- The red arrows show the velocities of various elements of the string at various times.
- All elements of the string have zero velocity at the extreme positions and elements have varying velocities at other positions.

First Normal Mode/First Harmonic:

The first normal mode that is consistent with the boundary conditions has nodes at its ends and one antinode in the middle.



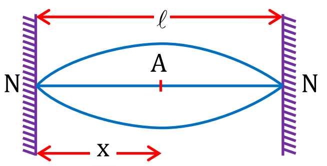
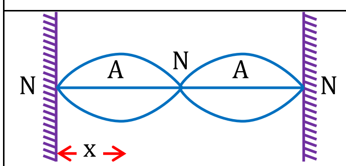
$$\frac{\lambda}{2}=\ell \Rightarrow \lambda=2\ell \text{ So, } v=f\lambda \Rightarrow v=f(2\ell) \Rightarrow f=\frac{v}{2\ell} \quad \left(v=\sqrt{\frac{T}{\mu}}\right)$$

$$\Rightarrow f_1=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}=\frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2\rho}}$$ This is the longest-wavelength mode.

This mode is also known as **first harmonic** and its frequency is known as **fundamental frequency.**

Pluck distance = $\frac{\ell}{2P}$ (P = Number of loops), Support distance = $\frac{\ell}{P}$ (P = Number of loops)

Other Modes of vibration:

Mode of vibration	Plucking distance is x(frequency)	Overtone	Harmonic	Node	Antinode
	$\ell=\frac{\lambda_1}{2}$ $\Rightarrow \lambda_1=2\ell,\ f_1=\frac{v}{2\ell}$	0	1	2	1
	$\ell=\frac{2\lambda_2}{2}$ $\Rightarrow \lambda_2=\frac{2\ell}{2},\ f_2=\frac{2v}{2\ell}$	1	2	3	2

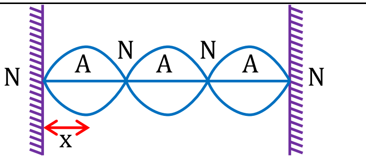
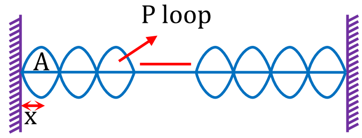
	$\ell = 3\left(\frac{\lambda_3}{2}\right)$ $\Rightarrow \lambda_3 = \frac{2\ell}{3},\ f_3 = \frac{3v}{2\ell}$	2	3	4	3
	$\ell = P\left(\frac{\lambda_p}{2}\right)$ $\Rightarrow \lambda_p = \frac{2\ell}{P},\ f_p = \frac{PV}{2\ell}$	P-1	P	P+1	P

Illustration 57

A standing wave having 5 nodes and 4 antinodes is formed between 2 Å distance then find the wavelength?

Solution:

$2\lambda = 2\text{Å}$

$\lambda = 1\text{Å}$

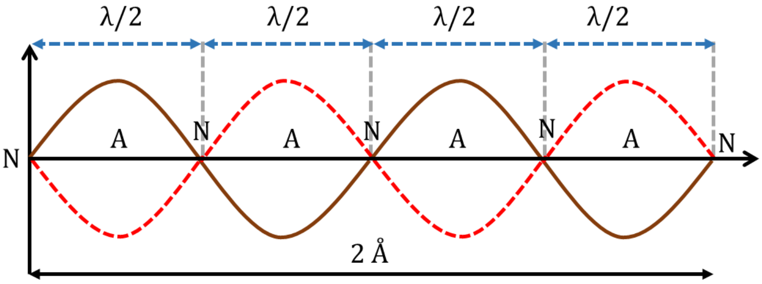


Illustration 58

A stretched string is 2 m long. Its mass per unit length is 2 g/m. It is stretched with a force of 720 N. If it is plucked at a distance of 10 cm from one end then find the frequency of note emitted by it?

Solution:

$$T = 720\text{ N},\ \ell = 2\text{m},\ \mu = 2\times10^{-3}\text{ kg/m}$$

$$\text{Pluck distance } = \frac{\ell}{2P} \Rightarrow \frac{\ell}{2P} = \frac{1}{10}, P = 10\text{ loops}$$

$$f = \frac{Pv}{2\ell} = \frac{10}{2\ell}\sqrt{\frac{T}{\mu}} \Rightarrow f = \frac{10}{2\times2}\sqrt{\frac{720\times10^3}{2}} = \frac{5}{2}\times6\times10^2 = 1500\text{ Hz}$$

Illustration 59

A string with a mass density of 9×10^{-3} kg/m is under tension of 360 N and is fixed at both ends. One of its resonance frequencies is 470 Hz. The next higher resonance frequency is 560 Hz. Find the mass of the string.

Solution:

$$\underbrace{f_1 = f \qquad f_2}_{f} \underbrace{= 2f \qquad f_3}_{f} = 3f$$

$$f = 560 - 470 = 90\text{ Hz} \Rightarrow f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = 90 \Rightarrow 90 = \frac{1}{2\ell}\sqrt{\frac{360}{9\times10^{-3}}} \Rightarrow \ell = \frac{10}{9}\text{m}$$

$$m = \mu \times \ell \Rightarrow m = 9\times10^{-3}\times\frac{10}{9} \Rightarrow m = 10^{-2}\text{ kg}$$

Illustration 60

Two identical wires under the same tension have a fundamental frequency of 350 Hz. What fractional increase in the tension of one wire will give 7 beats per second?

Solution:

$$f=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}\Rightarrow f\propto\sqrt{T}\propto T^{\frac{1}{2}}\Rightarrow\frac{\Delta f}{f}=\frac{1}{2}\left(\frac{\Delta T}{T}\right)\text{ Fractional change in tension}$$

$$\frac{7}{350}=\frac{1}{2}\left(\frac{\Delta T}{T}\right)\Rightarrow\frac{\Delta T}{T}=\frac{1}{25}\text{ and percentage change}=\frac{1}{25}\times100=4\%$$

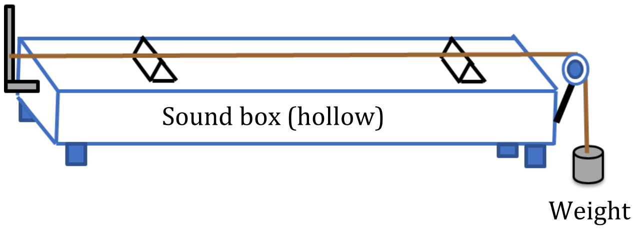
Illustration 61

A stretched wire of length 128 cm is divide into three segments whose fundamental frequencies are in the ratio 5 : 6 : 15, the length of the segments must be in the ratio

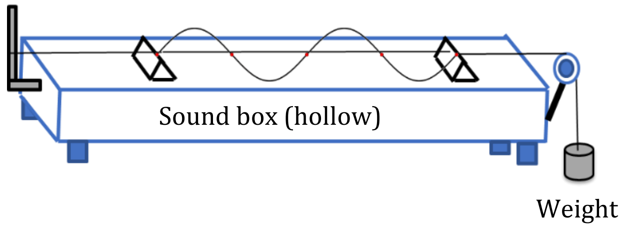
Solution:

$$f_1:f_2:f_3=5:6:15\Rightarrow f=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}\Rightarrow f\propto\frac{1}{\ell}\Rightarrow\ell_1:\ell_2:\ell_3=\frac{1}{f_1}:\frac{1}{f_2}:\frac{1}{f_3}=\frac{1}{5}:\frac{1}{6}:\frac{1}{15}\Rightarrow\ell_1:\ell_2:\ell_3=6:5:2$$

Sonometer



- Relation between frequency, length of string, Tension in string and linear mass density of string can be find out experimentally with this instrument known as sonometer.



- The portion of the wire between the movable bridges forms the "string" fixed at both ends. By sliding these bridges, the length of the experimental wire can be changed. The tension of the experimental wire can be changed by changing the weights on the hanger. One can remove the experimental wire itself and put another wire in its place thereby changing the mass per unit length.

$$f=\frac{Pv}{2\ell}=\frac{P}{2\ell}\sqrt{\frac{T}{\mu}}\quad\text{Law of length}\Rightarrow f\propto\frac{1}{\ell}\text{; Law of tension}\Rightarrow f\propto\sqrt{T}\text{; Law of Mass}\Rightarrow f\propto\sqrt{\frac{1}{\mu}}$$

Illustration 62

What should be the percentage change in tension of sonometer wire, so that it can vibrate in two octave higher frequency?

Solution:

One Octave higher means frequency get double so two octave would mean four times.

$$\text{By law of tension}\Rightarrow f\propto\sqrt{T}\Rightarrow\frac{f_1}{f_2}=\sqrt{\frac{T_1}{T_2}}\Rightarrow\frac{f}{4f}=\sqrt{\frac{T}{T_2}}\Rightarrow T_2=16T$$

$$\text{Percentage change}=\frac{T_f-T_i}{T_i}\times100=\frac{16T-T}{T}\times100=1500\ \%$$

Illustration 63

If the tension and diameter of a sonometer wire of fundamental frequency (f) is tripled and density is 1/3 times of initial density then find its new fundamental frequency.

Solution:

$$f=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}}=\frac{1}{2\ell}\sqrt{\frac{T}{\pi r^2\times\rho}} \quad ...(i) \qquad f'=\frac{1}{2\ell}\sqrt{\frac{T'}{\mu}}=\frac{1}{2\ell}\sqrt{\frac{T'}{\pi(r')^2\times\rho'}} \quad ...(ii)$$

$$T'=3T \Rightarrow r'=3r \Rightarrow \rho'=\frac{1}{3}\rho$$

Eq (ii) ÷ Eq (i), $\frac{f'}{f}=\sqrt{\frac{3T}{T}\times\frac{r^2}{(3r)^2}\times\frac{\rho}{\left(\frac{\rho}{3}\right)}} \Rightarrow \frac{f'}{f}=1$

Illustration 64

Bridges of a sonometer wire are arranged in such a way that its length segments are in ratio 3 : 2 : 1. Calculate the ratio of fundamental frequency of vibrations for these segments.

Solution:

$$f\propto\frac{1}{\ell} \Rightarrow f_1:f_2:f_3=\frac{1}{l_1}:\frac{1}{l_2}:\frac{1}{l_3} \Rightarrow f_1:f_2:f_3=\frac{1}{3}:\frac{1}{2}:\frac{1}{1} \Rightarrow f_1:f_2:f_3=2:3:6$$

Illustration 65

The fundamental frequency of a sonometer wire increases by 9 Hz, if its tension is increased by 69%, keeping the length constant. Find the change in fundamental frequency of the sonometer wire, when the length of wire is increased by 50%, keeping the original tension in the wire.

Solution:

$$T'=T+\frac{69}{100}T=1.69T \Rightarrow f\propto\sqrt{T} \Rightarrow \frac{f+9}{f}=\sqrt{\frac{1.69\,T}{T}}$$

$f+9=1.3f \Rightarrow f=30$ Hz Now, $f=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \Rightarrow \frac{f_1}{f_2}=\frac{\ell_2}{\ell_1}=\frac{\ell+0.5\ell}{\ell} \Rightarrow f_2=\frac{30}{1.5}=20$ Hz

Change in frequency = 30 – 20 = 10 Hz

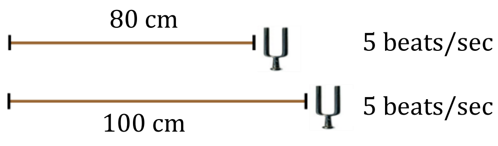
Illustration 66

Length of a sonometer wire is either 80 cm or 100 cm, in both the cases a tuning fork produces 5 beats. Then find the frequency of tuning fork.

Solution:

$$f=\frac{1}{2\ell}\sqrt{\frac{T}{\mu}} \Rightarrow f\propto\frac{1}{\ell}$$

$$\frac{f_1}{f_2}=\frac{\ell_2}{\ell_1} \Rightarrow \frac{f+5}{f-5}=\frac{100}{80}$$ (f is the frequency of tuning fork)



$f=45$ Hz

BEGINNER'S BOX-5

1. The equation of progressive wave is $y = 0.09 \sin 8\pi\, [t - (x/20)]$ after reflection of this wave from rigid end the amplitude of reflected wave becomes 2/3 of initial then (i) Find the equation of reflected wave (ii) Find the displacement of that particle which is situated at x =0, in reflected wave.

PWA208

2. The transverse displacement of a string is given by

$$y\,(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right) \cos\,(120\pi t)$$

where x and y are in m and t in s. The length of the string is 1.5m and its mass is 3.0×10^{-2} kg. Answer the following :

(a) Does the function represent a travelling wave or a stationary wave?

(b) What are the wavelength, frequency, and speed of each wave?

(c) Determine the tension in the string.

PWA209

3. For a plane progressive wave : $y = 0.02 \sin 2\pi\,(330t - x)$

(i) If this wave is reflected from rigid end and amplitude becomes 60% of initial then equation of wave will be?

(ii) If this wave is reflected from free end and amplitude becomes 75% of initial then equation of wave will be?

PWA210

4. A progressive wave the equation is $y = 0.08 \sin 2\pi\,(200t - x)$ it is reflected from rigid end and amplitude becomes half then equation of reflected wave and if it is reflected from free end then equation of reflected wave is.

PWA211

5. If a stretched string which is fixed at both ends has m nodes, then calculate its length.

PWA212

6. A sonometer wire emits a fundamental note of frequency 150 Hz. Calculate the frequency of the note emitted when the tension is changed in the ratio of 9 : 16 and length in the ratio of 1 : 2.

PWA213

7. A tuning fork is found to give 20 beats in 12 seconds when sounded in conjunction with a stretched string vibrating under a tension of 14.4 or 10 kgf. Calculate the frequency of the fork.

PWA214

8. If we increase the tension of stretched wire by 5 kg. wt. then fundamental frequency increase with ratio of 2 : 3 find the initial tension.

PWA215

9. Fill in the blanks for Sonometer.

(i) In sonometer if tension of wire and length of wire becomes double and mass per unit length of wire remain constant then fundamental frequency of wire will becomes..........

(ii) In sonometer if tension of wire and length becomes double and mass of wire remain constant then fundamental frequency of wire will becomes..........

(iii) In sonometer if tension of wire and length becomes double and mass per unit length remain constant then wave velocity will becomes..........

(iv) In a stretched wire calculate speed of wave if tension in wire is 40 N and mass for unit length is 4×10^{-3} kg/m. ....

(v) In a stretched wire calculate speed of wave in wire if tension in wire is 80 N and mass is 4×10^{-3} kg. and length of wire is 2 m. ....

(vi) In sonometer, if tensions and length both becomes four times, taking mass per unit length constant then, for vibration of same frequency Tuning fork, Number of loop will becomes...........

PWA216

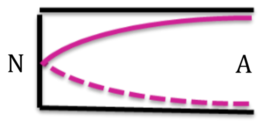
Longitudinal Stationary Waves in Organ Pipe

Longitudinal Stationary Waves

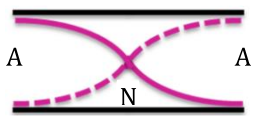
When two identical longitudinal progressive waves superpose while moving in opposite direction, then longitudinal stationary waves are formed. **Examples –** waves produced in organ pipes.

Organ pipes are of two types: -

(i) **Closed Organ Pipe (C.O.P.): -** The tube which is closed at one end and open at the another end is called closed organ pipe.



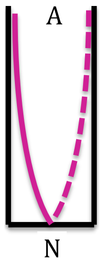
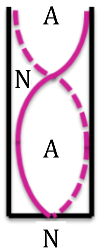
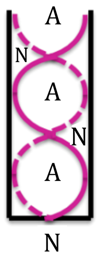
(ii) **Open Organ Pipe (O.O.P.): -** The tube which is open at both ends is called an open organ pipe.



(i) Closed Organ Pipe (COP):

- The tube which is closed at one end and open at the other end is called closed organ pipe.
- At the closed end there is a node since particles does not have freedom to vibrate whereas at open end there is an antinode because particles have greatest freedom to vibrate.

Modes of vibration of air column in closed organ pipe:

Fundamental Frequency Or First Harmonic	First overtone Or Third Harmonic	Second overtone Or Fifth Harmonic
 $\ell = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4\ell$ $f_1 = \frac{v}{\lambda_1} = \frac{v}{4\ell} \left(v = \sqrt{\frac{\beta}{\rho}} \right)$ Fundamental frequency Or First Harmonic	 $\ell = \frac{3\lambda_2}{4} \Rightarrow \lambda_2 = \frac{4\ell}{3}$ $f_2 = \frac{v}{\lambda_2} = \frac{3v}{4\ell} = 3f_1 \left(v = \sqrt{\frac{\beta}{\rho}} \right)$ First overtone Or Third Harmonic	 $\ell = \frac{5\lambda_3}{4} \Rightarrow \lambda_3 = \frac{4\ell}{5}$ $f_3 = \frac{v}{\lambda_3} = \frac{5v}{4\ell} = 5f_1 \left(v = \sqrt{\frac{\beta}{\rho}} \right)$ Second overtone Or Fifth Harmonic

Results for COP:

When closed organ pipe vibrate in m^{th} overtone then,

- Number of harmonics = (2 × overtone) + 1
- Number of antinodes = m + 1, Number of nodes = m + 1
- When closed organ pipe vibrate in m^{th} overtone, then frequency $f=(2\times\text{overtone}+1)\frac{v}{4\ell}$
- Frequency of harmonics is given by $f_1: f_2: f_3$ .... = 1: 3: 5....

Illustration 67

For a closed organ pipe if 2^{nd} overtone has frequency of 200 Hz. Find the fundamental frequency of this pipe?

Solution:

We know 2^{nd} overtone means 5^{th} harmonic

$f_5=5f_0$ (where f_0 = fundamental frequency) $\Rightarrow 200=5(f_0) \Rightarrow f_0=40$ Hz

Illustration 68

In a closed organ pipe if fundamental frequency is 100 Hz. Find the length of pipe if velocity of wave is 300 m/s?

Solution:

Fundamental frequency $f_0=\frac{v}{4\ell} \Rightarrow 100=\frac{300}{4\ell} \Rightarrow 400\ell=300 \Rightarrow \ell=\frac{300}{400} \Rightarrow \ell=\frac{3}{4}$ m.

Illustration 69

An air column in a pipe, which is closed at one end, will be in resonance with a vibrating body of frequency 83 Hz, if the length of the air column is- (v = 332 m/s)?

Solution:

$f_{cop}=\frac{v}{4\ell} \Rightarrow 83=\frac{332}{4\ell} \Rightarrow \ell=1$ m

Illustration 70

Three successive resonating frequency of an organ pipe are 255 Hz, 425 Hz, 595 Hz. If speed of sound is 340 m/s, then find nature of pipe and its length?

Solution:

255	:	425	:	595	
51	:	85	:	119	
3	:	5	:	7	(Odd Harmonics, ∴ nature of pipe is COP)

3^{rd} harmonic = 255, $3\left(\frac{v}{4\ell}\right)=255 \Rightarrow \ell=\frac{3\times340}{4\times255}=1$ m

Golden Key Points

- In closed organ pipe fundamental frequency is equal to

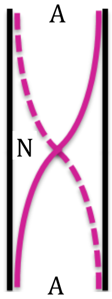
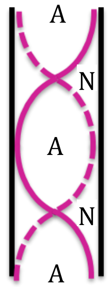
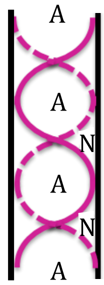
  $f_0=\frac{v}{4\ell}$
- Ratio of harmonic frequency is given by $f_1:f_2:f_3$ .... = 1 : 3 : 5 ....
- When closed organ pipe vibrate in m^{th} overtone, then frequency

  $f=(2\times\text{overtone}+1)\frac{v}{4\ell}$

(ii) Open Organ Pipe

The tube which is open at both ends is called an open organ pipe. The pipe is open at both ends by which an antinode is formed at open end. Hence on blowing air at the open-end antinodes are formed at each end and nodes in the middle.

Modes of vibration of air column in open organ pipe:

Fundamental Frequency Or First Harmonic	**First overtone Or Second Harmonic**	**Second overtone Or Third Harmonic**
		
$\ell = \frac{\lambda_1}{2}$ $\lambda_1 = 2\ell$ $f_1 = \frac{v}{\lambda_1} = \frac{v}{2\ell} \quad \left(v = \sqrt{\frac{\beta}{\rho}}\right)$ Fundamental Frequency Or First Harmonic	$\ell = \lambda_2$ $\lambda_2 = \ell$ $f_2 = \frac{v}{\lambda_2} = \frac{v}{\ell} = 2f_1 \quad \left(v = \sqrt{\frac{\beta}{\rho}}\right)$ Second Harmonic Or First overtone	$\ell = \frac{3\lambda_3}{2}$ $\lambda_3 = \frac{2\ell}{3}$ $f_3 = \frac{v}{\lambda_3} = \frac{3v}{2\ell} = 3f_1 \quad \left(v = \sqrt{\frac{\beta}{\rho}}\right)$ Third Harmonic Or Second overtone

Results for OOP:

When open organ pipe vibrates in mth overtone then

Number of harmonics = (overtone +1), Number nodes = m +1, Number of antinodes = m + 2

When open organ pipe vibrate in mth overtone, then frequency $f = \frac{(m+1)v}{2\ell}$

Ratio of frequency of harmonics is given by $f_1 : f_2 : f_3 ..... = 1 : 2 : 3 ....$

Illustration 71

In an open organ pipe 5th overtone is 90Hz. Find its 2nd overtone?

Solution:

As we know that, 5th overtone means 6th harmonic $f_6 = 6f_0$ (f_0 = fundamental frequency)

$f_0 = 15$ Hz and 2nd overtone means 3rd harmonic $f_3 = 3f_0 = 3(15) \Rightarrow f_3 = 45$ Hz

Illustration 72

In an open organ pipe, if 2nd harmonic is 200 Hz then find the length of pipe, if velocity of wave is 320 m/s?

Solution:

2nd harmonic $\Rightarrow f_2 = 2f_0 \Rightarrow 200 = 2f_0 \Rightarrow f_0 = 100\text{Hz}$ ($\because$ fundamental frequency of open organ pipe)

$\frac{v}{2\ell} = 100 \text{ Hz} \Rightarrow \frac{320}{2\ell} = 100 \Rightarrow \ell = \frac{320}{200} = 1.6 \text{ m}$

Illustration 73
If the velocity of sound in air is 350 m/s. Find the fundamental frequency of an open organ pipe of length 100 cm?

Solution:

$$f_{oop} = \frac{v}{2\ell} \Rightarrow f_{oop} = \frac{350}{2\times 1} \Rightarrow f_{oop} = 175 \text{ Hz}$$

Illustration 74
If the length of an open organ pipe is 1m and velocity of sound is 330 m/s, then find the frequency of first overtone?

Solution:

Fundamental frequency $(f) = \frac{v}{2\ell}$

First Overtone frequency $(f') = (1+1)f = 2f \Rightarrow f' = 2f = \frac{v}{\ell} \because f = \frac{v}{2\ell} \Rightarrow f' = \frac{330}{1} \Rightarrow f' = \frac{v}{\ell} \Rightarrow f' = 330 \text{ Hz}$

Illustration 75
Two open organ pipes of length 35 cm and 35.5 cm produces 5 beats/sec. Find the velocity of sound?

Solution:

$$\Delta f = f_1 - f_2 \Rightarrow 5 = \frac{v}{2\ell_1} - \frac{v}{2\ell_2} \Rightarrow 10 = v\left[\frac{1}{35} - \frac{1}{35.5}\right]\times 100 \Rightarrow 10 = v\left(\frac{0.5}{35\times 35.5}\right)\times 100 \Rightarrow v = 248.5 \text{ m/s}$$

Mix questions on Open organ pipe and Closed organ pipe
If length of OOP and COP are same and speed of sound is also same in both the pipe then $2f_{COP} = f_{OOP}$.

If OOP is partially dipped inside the water then it works like COP having considerable length equal to the length of air column.

End correction: Due to finite motion of air molecules in organ pipes reflection takes place not exactly at open end but little what above it so in an organ pipe antinode is not formed exactly at free–end but above it at a distance e = 0.6r (called end correction or Rayleigh's correction) with r being the radius of pipe.

So, for closed organ pipe $L \to L + 0.6r$ while for open $L \to L + 2 \times 0.6r$ (as both ends are open)

So that ; $f_C = \frac{v}{4(L+0.6r)}$ while $f_0 = \frac{v}{2(L+1.2r)}$

This is why for a given v and L narrower the pipe higher will the frequency or pitch and shriller will be the sound. For an organ pipe (closed or open) if v = constant. $f \propto (1/L)$

Illustration 76
For a given C.O.P. (closed organ pipe) if 8th overtone has frequency 3400 Hz, then find the fundamental frequency of same length O.O.P.?

Solution:

m = 8 $\Rightarrow$ Harmonic = 2m+1 =2(8) + 1 = 17

$17f = 3400 \Rightarrow f = 200\text{Hz} \Rightarrow f_{OOP} = 2f_{COP}$ [ℓ = same] $\Rightarrow f_{OOP} = 2 \times 200 \Rightarrow f_{OOP} = 400$ Hz

Illustration 77
An organ pipe closed at one end vibrating in its second overtone and another pipe which is open at both ends vibrating in its fourth overtone are in resonance with a given tuning fork. Find the ratio of their lengths?

Solution:

2nd overtone of COP = 4th overtone of OOP $\Rightarrow 5f_{COP} = 5f_{OOP} \Rightarrow 5\frac{v}{4\ell_{cop}} = 5\frac{v}{2\ell_{oop}} \Rightarrow \frac{\ell_{cop}}{\ell_{oop}} = \frac{1}{2}$

Illustration 78

An open organ pipe is suddenly closed with the result that the third overtone of the closed pipe is found to be higher in frequency by 100 Hz than the second overtone of the original pipe. Find the fundamental frequency of open organ pipe?

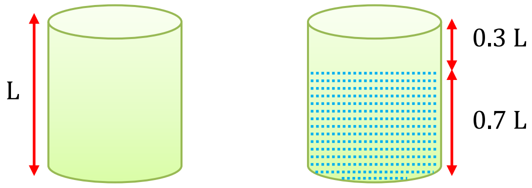
Solution:

$f_{oop} = n$, also $f_{cop} = \frac{f_{oop}}{2} = \frac{n}{2} \Rightarrow f_{cop_{3^{rd}\,OT}} = f_{oop_{2^{nd}\,OT}} + 100 \Rightarrow 7\left(\frac{n}{2}\right) = 3n + 100 \Rightarrow 0.5n = 100 \Rightarrow n = 200$ Hz

Illustration 79

If an OOP of fundamental frequency 1200 Hz, dipped 70% in water then calculate produced frequency.

Solution:



$f_{oop} = \frac{v}{2\ell} = 1200$ Hz $\qquad \frac{v}{\ell} = 2400 \qquad$ ... (i)

After it is dipped in water then it behaves as COP

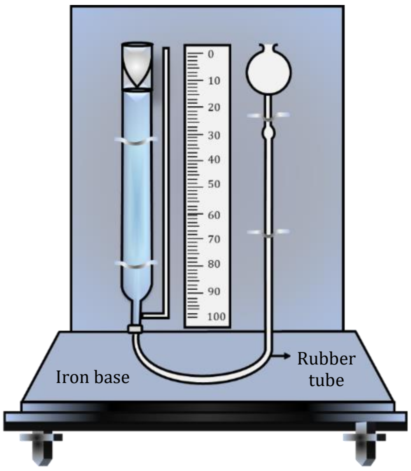
New frequency $(f_{new}) = \frac{v}{4(0.3\ell)} \Rightarrow f_{new} = \frac{v}{1.2\ell}$

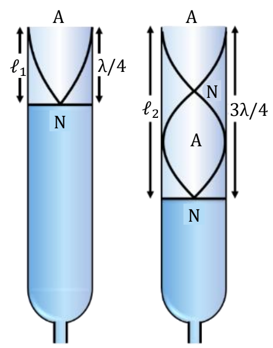
From equation (i), $f_{new} = \frac{2400}{1.2} = 2000$ Hz

Golden Key Points

- In open organ pipe fundamental frequency $f_0 = \frac{v}{2\ell}$
- Ratio of frequency of harmonics is given by $f_1 : f_2 : f_3$ ..... = 1 : 2 : 3 ....
- When open organ pipe vibrate in m^{th} overtone, then frequency $f = (\text{overtone} + 1)\frac{v}{2\ell}$

Resonance Tube Experiment:





- In this experiment the velocity of sound in air is to be found by using tuning forks of known frequency. The wavelength of the sound will be determined by making use of the resonance of an air column.
- The apparatus for the experiment consists of a long cylindrical tube attached to a water reservoir. The length of the water column may be changed by raising or lowering the water level while the tuning fork is held over the open end of the tube.
- Resonance is indicated by the sudden increase in the intensity of the sound when the column is adjusted to the proper length. The resonance is a standing wave phenomenon in the air column and occurs when the column length is $\frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$..., where λ is the wavelength of sound.
- The water surface constitutes a node of the standing wave since the air is not free to move longitudinally. The open end provides the conditions for an antinode.

Mathematical analysis of Resonance tube:

(1) For first resonance $\ell_1 = \frac{\lambda}{4}$

(2) For second resonance $\ell_2 = \frac{3\lambda}{4}$

(3) Subtract both the above resonance length to obtain wavelength of wave.

$$\ell_2 - \ell_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} \Rightarrow \ell_2 - \ell_1 = \frac{\lambda}{2} \Rightarrow \lambda = 2(\ell_2 - \ell_1)$$

(4) If the frequency of the fork be f and the temperature of the air-column be t°C, then the speed of sound at t°C is given by $\Rightarrow v_t = f\lambda = 2f(\ell_2 - \ell_1)$

The speed of sound waves at 0°C in air is $v_0 = (v_t - 0.61t)\, m/sec$

Illustration 80

In resonance tube experiment if V = 300 m/s, f = 375 Hz, L = 125 cm

(i) Maximum number of resonances?

(ii) Maximum & minimum water level kept at resonance condition?

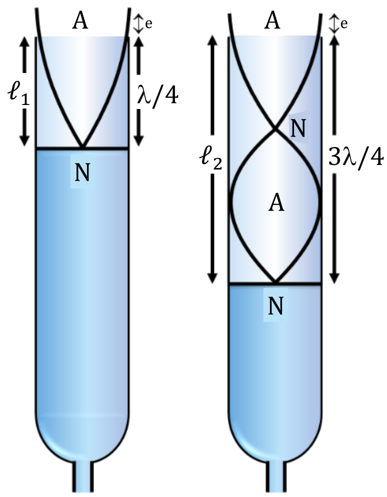
Solution:

$$\ell_1 = \frac{\lambda}{4} = \frac{300}{375 \times 4} = 20 \text{ cm}$$

S.No.	Length of air column	Length of water column	
(1)	$\frac{\lambda}{4} = 20$ cm	105 cm	Maximum water level
(2)	$\frac{3\lambda}{4} = 60$ cm	65 cm	
(3)	$\frac{5\lambda}{4} = 100$ cm	25 cm	Minimum water level
(4)	$\frac{7\lambda}{4} = 140$ cm	-	Not possible

Calculation of end Correction:

In the resonance tube, the antinode is not formed exactly at the open end but slightly outside at a distance e. Hence the length of the air column in the first and second states of resonance are $(\ell_1 + e)$ and $(\ell_2 + e)$ then



(1) For first resonance $\ell_1 + e = \frac{\lambda}{4}$

(2) For second resonance $\ell_2 + e = \frac{3\lambda}{4}$

(3) Subtract both the above resonance length to obtain wavelength of wave.

$$\ell_2 - \ell_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} \Rightarrow \ell_2 - \ell_1 = \frac{\lambda}{2} \Rightarrow \lambda = 2(\ell_2 - \ell_1)$$

(4) Put the value of λ from 3rd equation to 1st equation to find value of e

$$\ell_1 + e = \frac{2(\ell_2 - \ell_1)}{4}$$

After solving $e = \frac{\ell_2 - 3\ell_1}{2}$ [e = 0.6 r, where r is radius of tube]

Illustration 81

The length of air column for fundamental mode in resonance tube is 25 cm and that for the second resonance further is 78 cm. Find the end correction and inner diameter of tube.

{Data : $\ell_1 = 25\,\text{cm},\ \ell_2 = 78\,\text{cm}$ }

Solution:

$e = \dfrac{\ell_2 - 3\ell_1}{2} \Rightarrow e = \dfrac{78-3(25)}{2} = \dfrac{78-75}{2} \Rightarrow e = \dfrac{3}{2} = 1.5\ \text{cm} \Rightarrow e = 0.3\ d \Rightarrow d = 5\,\text{cm}$

Illustration 82

In resonance tube closed at one end by a smooth moving piston and the other end open, exhibits the first three resonance lengths of air column ℓ_1, ℓ_2 and ℓ_3 for the same tuning fork, then they are related by

Solution:

The first three resonance length of air column is ℓ_1, ℓ_2 and ℓ_3

In case of resonance condition the length of tube ℓ is satisfied $\ell_n = \frac{1}{4}(2n+1)\lambda$

$\ell_1 = \dfrac{\lambda}{4}$; $\ell_3 - \ell_2 = \dfrac{5}{4}\lambda - \dfrac{3}{4}\lambda \Rightarrow \dfrac{\lambda}{2}$

$\ell_2 = \dfrac{3\lambda}{4}$; $\ell_2 - \ell_1 = \dfrac{3}{4}\lambda - \dfrac{1}{4}\lambda \Rightarrow \dfrac{\lambda}{2}$

$\ell_3 = \dfrac{5\lambda}{4}$ So, $\ell_3 - \ell_2 = \ell_2 - \ell_1$.

Illustration 83

In resonance tube experiment the first resonance is obtained for 14 cm of air column and second for 48 cm. The end correction for this apparatus is equal to

Solution:

$\ell_1 + e = \dfrac{\lambda}{4}$ and $\ell_2 + e = \dfrac{3\lambda}{4} \Rightarrow 3\ell_1 + 3e = \dfrac{3\lambda}{4} = \ell_2 + e$

$2e = \ell_2 - 3\ell_1 \Rightarrow e = \dfrac{1}{2}(\ell_2 - 3\ell_1) \Rightarrow e = \dfrac{1}{2}(48 - 3(14)) \Rightarrow e = \dfrac{6}{2} \Rightarrow e = 3\ \text{cm}.$

Comparison of progressive and stationary waves

	Progressive waves	Stationary waves
1.	These waves travels in a medium with definite velocity.	These waves do not travel and remain confined between two boundaries in the medium.
2.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.
3.	The phase of vibration varies continuously from particle to particle.	The phase of all the particles in between two nodes is always same. But particles on adjacent side of a node differ in phase by 180°
4.	No particle of medium is permanently at rest.	Particles at nodes are permanently at rest.
5.	All particles of the medium vibrate and amplitude of vibration is same.	The amplitude of vibration changes from particle to particle. The amplitude is zero at nodes and maximum at antinodes.
6.	All the particles do not attain the maximum displacement position simultaneously.	All the particles attain the maximum displacement position simultaneously.

Golden Key Points

Wave property	Reflection	Transmission (Refraction)
v	does not change	changes
n, T, ω	do not change	do not change
λ, k	do not change	change
A, I	change	change
Phase difference ($\Delta\phi$)	$\Delta\phi$ = 0, from a rarer medium	does not change
	$\Delta\phi = \pi$, from a denser medium	

- **Echo (Based on reflection) :**

  Multiple reflection of sound is called an echo. If the distance of reflector from the source is d then, $2d = vt$

  Hence, v = speed of sound and t = the time of echo. $\therefore \quad d = \frac{vt}{2}$

  Since, the effect of ordinary sound remains on our ear for $\frac{1}{10}$ second, therefore, if the sound returns to the starting point before $\frac{1}{10}$ second, then it will not be distinguished from the original sound and no echo will be heard. Therefore, the minimum distance of the reflector is,

  $$d_{min} = \frac{v \times t}{2} = \left(\frac{330}{2}\right)\left(\frac{1}{10}\right) = 16.5m$$

- **Musical Interval**

  The ratio between the frequencies of two notes is called the musical interval. Following are the names of some musical intervals.

  (i) Unison $\frac{n_2}{n_1} = 1$ (ii) Octave $\frac{n_2}{n_1} = 2$

BEGINNER'S BOX-6

1. An organ pipe produces a sound of frequency 400 Hz. If it is blown a little harder then it produces a sound of frequency 800 Hz. It is an open pipe or a closed pipe?

PWA217

2. A 20 cm long pipe is closed at one end. Which harmonic mode of the pipe is resonantly excited by 425Hz source? Will the same source be in resonance with the pipe if both ends are open? (Speed of sound in air = 340 m/s)

PWA218

3. If frequency of third overtone of closed organ pipe is equal to frequency of sixth overtone of an open organ pipe, then determine their length ratio $\frac{\ell_c}{\ell_o}$.

PWA219

4. Third overtone of a closed organ pipe is in unison with fourth harmonic of an open organ pipe. find the ratio of lengths of the pipes.

PWA220

5. Two successive resonant frequencies in an open organ pipe are 1944 and 2592 Hz. If the speed of sound in air 324 ms^{-1}, then find the length of tube.

PWA221

6. If OOP have 10 antinodes and fundamental freq. 300 Hz. then freq at 10^{th} Antinode.

PWA222

7. The sum of frequencies of COP's first overtone and OOP's second overtone is 180 find fundamental frequencies of COP and OOP. (length is same)

PWA223

8. A tuning fork produce four beats per second with two OOP's having length 30 & 31 cm find the freq. of T.F.

PWA224

9. Two tuning fork n_1, n_2 produce two beats per sec. n_1 resonant with 15 cm COP and n_2 resonant with 30.5 cm OOP. Find the freq. of n_1 & n_2.

PWA225

10. A tube 1 m long is closed at one end. A stretched wire is placed near the open end. The wire is 0.3m long and has a mass of 0.01kg. It is held fixed at both ends and vibrates in its fundamental mode. It sets the air column in the tube into vibration at its fundamental frequency by resonance. Find –

(a) the frequency of oscillation of the air column.

(b) the tension in the wire, If speed of sound in air is 330 m/s.

PWA226

11. In resonance tube experiment if V = 300 m/s, n = 500 Hz., L = 125 cm

(i) Find out maximum order of resonance that can be established?

(ii) Maximum number of resonance?

(iii) Maximum & minimum water level kept at resonance condition?

PWA227

12. Fill in the blanks for COP and OOP.

(i) In COP if the freq of 7^{th} overtone is 600 Hz then fundamental frequency of COP is..........

(ii) In COP if the frequency of 3^{rd} overtone is 1400 Hz then fundamental frequency of same length OOP is..........

(iii) In OOP if the frequency of 7^{th} overtone is 800 Hz then fundamental frequency of OOP is..........

(iv) In COP if the frequency of 7^{th} overtone is 600 Hz then frequency of 3^{rd} overtone is..........

(v) In OOP if the frequency of 7^{th} overtone is 1600 Hz then frequency of 3^{rd} overtone is..........

(vi) In OOP if the frequency of 7^{th} overtone is 1600 Hz then fundamental frequency of same length COP is..........

(vii) In OOP if the frequency of 7^{th} overtone is 800 Hz then frequency of OOP corresponding to first overtone is..........

(viii) Length of OOP is 44 cm speed of sound 340 m/s fundamental frequency is 340 Hz then value of end correction is..........

(ix) Length of OOP is 38 cm speed of sound 340 m/s fundamental frequency is 340 Hz then value of radius of pipe is..........

PWA228

13. Fill in the blanks for Resonance tube.

(i) In resonance tube first resonating length is 25 cm then its second resonating length will be (if $e = 0$) ....

(ii) In resonance tube first resonating length is 25 cm then its second resonating length will be (if $e \neq 0$) ....

(iii) Length of resonance tube is 140 cm. How many resonance are possible for wave have wave length 40 cm. ....

(iv) Length of resonance tube is 150 cm. How many resonance are possible for wave having frequency 400 Hz and speed of sound 320 m/s. ....

(v) Length of resonance tube is 150 cm. Maximum level of liquid, in resonance condition for wave having wavelength 80 cm. is.........

(vi) Length of resonance tube is 150 cm Minimum level of liquid, in resonance condition for wave having wavelength 80 cm. is.........

(vii) In resonance tube first resonating length is 17 cm, second resonating length is 55 cm then wavelength of wave is..........

(viii) In resonance tube first resonating length is 17 cm, second resonating length is 55 cm then radius of tube is..........

PWA229

BEGINNER'S BOX ANSWER KEY

BEGINNER'S BOX-1

1. 10 cm. **2.** 8.707 s

BEGINNER'S BOX-2

1. 200 cm/s **2.** $\frac{1}{100}$ s, 20 m/s **3.** No, None **4.** 0.02 s **5.** 220 N

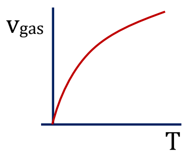
6. 0.5 s **7.** (a) 2.21 m/sec; (b) 1 sec.

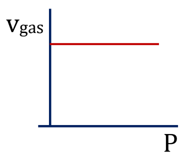
BEGINNER'S BOX-3

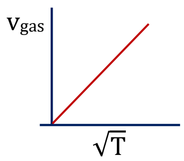
1. 2 s **2.** 2000 ms^{-1} **3.** 30dB

4. After rain fall, the humidity of air increases. This lowers the density of air hence there is increase in velocity of sound $\left(\because v \propto \frac{1}{\sqrt{\rho}}\right)$

5. We know $v_{gas} = \sqrt{\frac{\gamma RT}{M_W}}$

(a) 

(b) 

(c) 

6. Due to this loudness of sound increases.

7. Velocity of sound wave in air $v_{air} = \sqrt{\frac{\gamma RT}{M_w}} \Rightarrow v_{air} \propto \sqrt{T}$

so sound travels faster in warm air than in cool air.

8. Sound wave require material medium for their propagation.

9. $\sqrt{27 \cdot 7} \times 100$ m/s

10. (a) Independent of pressure. (b) Increases as $\sqrt{T}$ (c) Increase

BEGINNER'S BOX-4

1. 0°, 90°, 120°, 180° **2.** 64 : 1 **3.** 81 : 1 **4.** 1375 Hz **5.** 1000 Hz

6. 296 Hz **7.** 640 Hz **8.** (a) 200.5, (b) 1, (c) nearly 9/16

9. 60, 72, 78 **10.** (a) $\frac{1}{4}$ s, (b) $\frac{1}{8}$ s, (c) 12 beats

11. 1200 Hz, 2000 Hz, 2800 Hz, 3600 Hz, 4400 Hz

12. $n_1 = 50$ Hz $n_{11} = 100$ Hz

$n_{17} = 130$ Hz $n_{27} = 180$ Hz

$n_{33} = 210$ Hz $n_{51} = 300$ Hz

BEGINNER'S BOX-5

1. (i) $y = -0.06 \sin 8\pi [t + (x/20)]$, (ii) $y = -0.06 \sin 8\pi t$
2. (a) The function represents a stationary wave. (b) 3m, 60 Hz, 180 ms^{-1} (c) 648 N
3. (i) $y = -0.012 \sin 2\pi (330 t + x)$ (ii) $y = 0.015 \sin 2\pi (330 t + x)$
4. $\pm 0.04 \sin 2\pi (200 t + x)$; + = free, – = rigid
5. $(m-1)\frac{\lambda}{2}$ 6. 100 Hz 7. $\frac{55}{3}$ Hz. 8. 4 kg wt.
9. (i) $\frac{1}{\sqrt{2}}$ times (ii) No change (iii) $\sqrt{2}$ times (iv) 100 m/s (v) 200 m/s

   (vi) 2 times

BEGINNER'S BOX-6

1. Open organ pipe
2. For closed organ pipe $f_0 = \frac{v}{4\ell} = \frac{340}{4\times20\times10^{-2}} = 425$ Hz $\Rightarrow$ first harmonic

   If pipe is open, its fundamental frequency $= \frac{v}{2\ell} = 850$ Hz so it will not resonant with the given source
3. $\frac{1}{2}$ 4. $\frac{7}{8}$ 5. 0.25m 6. 2700 7. 20, 40 8. 244
9. 122,120 10. (a) 82.5 Hz; (b) 81.675 N
11. (i) forth order; (ii) 4; (iii) 110 cm, 20 cm
12. (i) 40 Hz (ii) 400 Hz (iii) 100 Hz (iv) 280 Hz (v) 800 Hz

    (vi) 100 Hz (vii) 200 Hz (viii) 3 cm (ix) 10 cm
13. (i) 75 cm (ii) Slightly more than 75 cm (iii) 7 (iv) 4

    (v) 130 cm (vi) 10 cm (vii) 76 cm (viii) 3.33 cm