02 Units, Dimensions and Measurement

Physical World

What is Physics?

A precise definition of this discipline is neither possible nor necessary. We can broadly describe physics as a study of the basic law's of nature and their manifestation in different natural phenomena.

Unification

To explain diverse physical phenomena in terms of a few concepts and laws. The effort to see the physical world as manifestation of some universal laws in different domains and conditions is called unification.

Example :- The attempts to unify fundamental forces in nature.

Reduction

A related effort is to derive the properties of bigger, more complex, system from properties and interaction of its constituent simpler part is called reductionism.

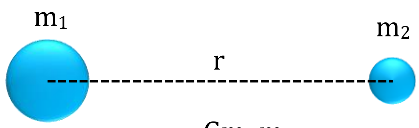
Scope of Physics

1. **Macroscopic: -**
   Macroscopic domain includes phenomena at the laboratory, terrestrial and astronomical scales. These phenomena are studied in "classical Physics" which includes mechanics, thermodynamics, optics and electrodynamics.
2. **Microscopic**
   The microscopic domain includes atomic, molecular and nuclear phenomena. These phenomena are governed by "Quantum Physics".

Fundamental Forces

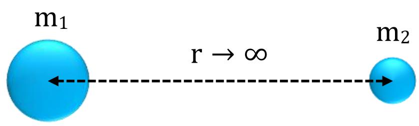
Few fundamental forces in nature are :-

1. Gravitational Force: -



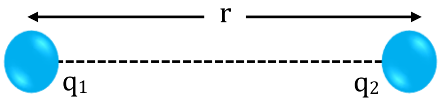
$$F_g = \frac{Gm_1m_2}{r^2}$$

- Gravitational force is weakest force in nature.
- It is the force of mutual attraction between any two objects by virtue of their masses.
- It is a universal force.
- It plays a key role in the large scale phenomena of universe such as formation and evolution of stars, galaxies and galactic clusters.
- The gravitational force is appreciable only when at least one of the two bodies has a large mass.
- They are always attractive in nature.
- It is weakest force and its range is infinite.



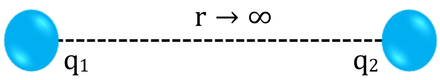
$$F_g = \frac{Gm_1m_2}{r^2} \approx 0$$

2. Electromagnetic Force :-



$$F = \frac{kq_1q_2}{r^2} = \frac{1}{4\pi \in_0}\frac{q_1q_2}{r^2}$$

- Electromagnetic force is the force between charge particles.
- When charges are at rest, the force is given by coulomb's law.
- When charges are in motion, they produce magnetic field giving rise to a force on a moving charge.
- Electric and magnetic effects are in general inseparable; hence the name electromagnetic force.
- Like the gravitational force, electromagnetic force act over large distances and does not need any intervening medium.
- It is quite strong compared to gravity.
- For example electric force between two protons is 10^{36} times the gravitational force between them, for a certain distance.
- They are attractive as well as repulsive in nature.
  its range is infinite.



$$F = \frac{kq_1q_2}{r^2} = \frac{1}{4\pi \in_0}\frac{q_1q_2}{r^2} \approx 0$$

3. Strong Nuclear Force :-

- The strong nuclear force binds protons and neutrons in a nucleus. It is evident that without some attractive force, a nucleus will be unstable due to electric repulsion between protons.
- The strong nuclear force is the strongest of all fundamental forces.
- It is charge independent.
- It is equal for protons and neutrons.
- Its range is extremely small of the order nuclear dimensions (10^{-15} m).
- It is responsible for the stability of nuclei.
- Recent developments have however indicated that protons and neutrons are composed of still more elementary constituents called quarks.
- It binds protons and neutrons in a nucleus.
- It is the strongest among all fundamental forces.

4. Weak Nuclear Force :-

- The weak nuclear force appears only in certain nuclear process β -decay of a nucleus.
- In β-decay the nucleus emits an electron and an uncharged particle called anti-neutrino.
- The weak nuclear force is not as weak as the gravitational force but much weaker than strong nuclear force.
- The range of weak nuclear force is exceedingly small of the order 10^{-16} m

Range of Fundamental Forces

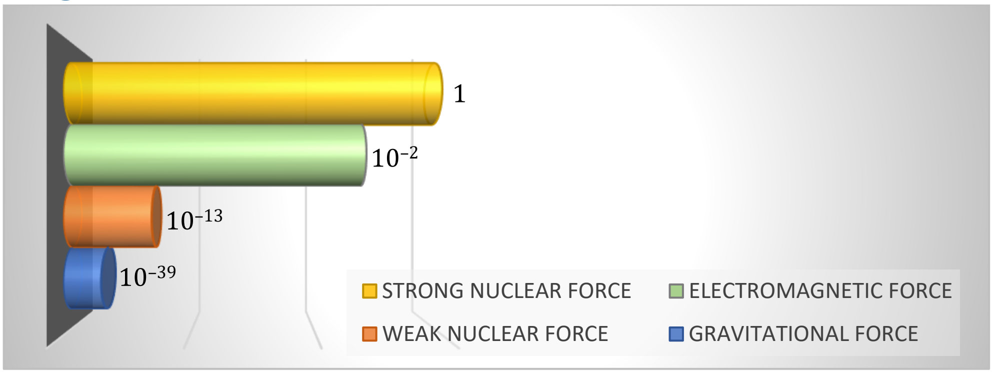
GRAVITATIONAL FORCE $\longrightarrow$ INFINITE

ELECTROMAGNETIC FORCE $\longrightarrow$ INFINITE

STRONG NUCLEAR FORCE $\longrightarrow$ Short (Nuclear Size: 10^{-15} m)

WEAK NUCLEAR FORCE $\longrightarrow$ Very Short (10^{-16} m)

Relative Strength



Physical Quantities

In Physics, all the quantities which are used to describe the laws of physics and can be measured are called Physical Quantities.

Classification On Basis Of Directional Properties

PHYSICAL QUANTITIES

SCALAR | VECTOR

- The physical quantities which have only magnitude but no direction are called scalar quantities.
  Example: mass, distance, density, volume, time, etc.

- The physical quantities which have both magnitude and direction are called vector quantities.
  Example: Displacement, Velocity, Force etc.

Classification on Basis of Dependency

PHYSICAL QUANTITIES

INDEPENDENT OF EACH OTHER → FUNDAMENTAL

DEPENDENT ON EACH OTHER → DERIVED

Fundamental or Base Quantities

The quantities which do not depend upon other quantities for their complete definition are known as fundamental or base quantities.

(1) Mass (2) Length (3) Time (4) Temperature
(5) Current (6) Luminous Intensity (7) Amount of Substance

Derived Quantities

The quantities which can be expressed in terms of the fundamental quantities are known as derived quantities. (1) Area (2) Force (3) Density

Dimensions and Dimensional Formula

Dimensions

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to express that quantity.

Dimensional Formula

The dimensional formula of any physical quantity is that expression which represents how and which of the base quantities are included in that quantity.

It is written by enclosing the symbols for base quantities with appropriate powers in square brackets.

Mass	$[M^1L^0T^0]$
Momentum	$[M^1L^1T^{-1}]$
Force	$[M^1L^1T^{-2}]$

Dimensional Equation

The equation obtained by equating a physical quantity with its dimensional formula is called a dimensional equation.

For example: [MASS] = $[M^1L^0T^0]$ is a Dimensional Equation.

Mass	$[M^1L^0T^0]$
LENGTH	$[M^0L^1T^0]$
TIME	$[M^0L^0T^1]$
TEMPERATURE	$[M^0L^0T^0K^1]$
CURRENT	$[M^0L^0T^0A^1]$
LUMINOUS INTENSITY	$[M^0L^0T^0Cd^1]$
AMOUNT OF SUBSTANCE	$[M^0L^0T^0mol^1]$

Dimensions of differential coefficients and integrals

In General $\left[\dfrac{d^ny}{dx^n}\right]=\left[\dfrac{y}{x^n}\right]$ and $[\int ydx]=[yx]$

Dimensions of a physical quantity are the powers (or exponents) to which the base quantities are raised to express that quantity.

Rule of Dimensions

Only **SAME physical quantities** can be added or subtracted.

$$A + B = C - D$$

Dimensionless Quantities

Dimensionless Quantities are:

- Ratio of physical quantities with same dimensions.
- All mathematical constants.
- All standard mathematical functions and their inputs (exponential, logarithmic, trigonometric & inverse trigonometric).

Physical quantity	Relationship with other physical quantities	Dimensions	Dimensional formula
Area	Length breadth	$[L^2]$	$[M^0 L^2 T^0]$
Volume	Length × breadth × height	$[L^3]$	$[M^0L^3T^0]$
Mass density	Mass/volume	$[M]/[L^3]$ or $[M L^{-3}]$	$[ML^{-3}T^0]$
Frequency	1/time period	$1/[T]$	$[M^0 L^0 T^{-1}]$
Velocity, speed	Displacement/time	$[L]/[T]$	$[M^0LT^{-1}]$
Acceleration	Velocity /time	$[LT^{-1}]/[T]$	$[M^0LT^{-2}]$

Force	Mass × acceleration	$[M][LT^{-2}]$	$[M LT^{-2}]$
Impulse	Force × time	$[M LT^{-2}][T]$	$[M LT^{-1}]$
Work, Energy	Force × distance	$[MLT^{-2}][L]$	$[M L^2 T^{-2}]$
Power	Work/time	$[ML^2 T^{-2}]/[T]$	$[ML^2 T^{-3}]$
Momentum	Mass × velocity	$[M] [LT^{-1}]$	$[MLT^{-1}]$
Pressure, stress	Force/area	$[MLT^{-2}]/[L^2]$	$[ML^{-1}T^{-2}]$
Strain	$\frac{\text{Change in dimension}}{\text{Original dimension}}$	$[L] / [L]$ or $[L^3]/[L^3]$	$[M^0L^0 T^0]$
Surface tension	Force/length	$[MLT^{-2}/[L]$	$[ML^0 T^{-2}]$
Modulus of elasticity	Stress/strain	$\frac{[ML^{-1}T^{-2}]}{[M^0L^0T^0]}$	$[ML^{-1} T^{-2}]$
Surface energy	Energy/area	$[ML^2T^{-2}]/[L^2]$	$[ML^0T^{-2}]$
Velocity gradient	Velocity/distance	$[LT^{-1}] / [L]$	$[M^0L^0T^{-1}]$
Pressure gradient	Pressure/distance	$[ML^{-1}T^{-2}]/[L]$	$[ML^{-2}T^{-2}]$
Pressure energy	Pressure × volume	$[ML^{-1}T^{-2}] [L^3]$	$[ML^2 T^{-2}]$
Coefficient of viscosity	Force/(area× velocity gradient)	$\frac{[MLT^{-2}]}{[L^2][LT^{-1}/L]}$	$[ML^{-1}T^{-1}]$
Angle, Angular displacement	Arc/radius	$[L]/[L]$	$[M^0L^0T^0]$
Trigonometric ratio	Length/length	$[L]/[L]$	$[M^0L^0T^0]$
Angular velocity	Angle/time	$[L^0]/[T]$	$[M^0L^0T^{-1}]$
Angular acceleration	Angular velocity/time	$[T^{-1}]/[T]$	$[M^0L^0T^{-2}]$
Radius of gyration	Distance	$[L]$	$[M^0LT^0]$
Moment of inertia	Mass × (radius of gyration)^2	$[M] [L^2]$	$[ML^2 T^0]$
Angular momentum	Moment of inertia × angular × velocity	$[ML^2] [T^{-1}]$	$[ML^2 T^{-1}]$
Moment of force (Couple)	Force × distance	$[MLT^{-2}] [L]$	$[ML^2 T^{-2}]$
Torque	Angular momentum/time, Or Force × distance	$[ML^2 T^{-1}]/[T]$ or $[MLT^{-2}]$ $[L]$	$[ML^2 T^{-2}]$
Angular frequency	2π × Frequency	$[T^{--1}]$	$[M^0L^0T^{-1}]$
Wavelength	Distance	$[L]$	$[M^0LT^0]$
Intensity of wave	(Energy/time)/area	$[ML^2 T^{-2}/T]/[L^2]$	$[ML^0T^{-3}]$
Radiation pressure	$\frac{\text{Intensity of wave}}{\text{Speed of light}}$	$[MT^{-3}]/[LT^{-1}]$	$[ML^{-1}T^{-2}]$
Energy density	Energy/volume	$[ML^2 T^{-2}]/ [L^3]$	$[ML^{-1} T^{-2}]$
Critical velocity	$\frac{\text{Reynold's number × coefficient of viscocity}}{\text{Mass density × radius}}$	$\frac{[M^0L^0T^0][ML^{-1} T^{-1}]}{[ML^{-3}][L]}$	$[M^0LT^{-1}]$
Escape velocity	(2 × acceleration due to gravity × earth's radius)$^{1/2}$	$[LT^{-2}]^{1/2}$× $[L]^{1/2}$	$[M^0LT^{-1}]$

Heat energy, internal energy	Work (= Force × distance)	$[MLT^{-2}]$ $[L]$	$[ML^2 T^{-2}]$
Kinetic energy	(1/2) mass × (velocity)^2	$[M]$ $[LT^{-1}]^2$	$[ML^2T^{-2}]$
Potential energy	Mass × acceleration due to gravity × height	$[M]$ $[LT^{-2}]$ $[L]$	$[ML^2 T^{-2}]$
Rotational kinetic energy	1/2 × moment of inertia (angular velocity)^2	$[M^0L^0T^0]$ $[ML^2]$ × $[T^{-1}]^2$	$[ML^2 T^{-2}]$
Efficiency	$\frac{\text{Output work or energy}}{\text{Input work or energy}}$	$\frac{[ML^2T^{-2}]}{[ML^2T^{-2}]}$	$[M^0L^0T^0]$
Angular impulse	Torque × time	$[ML^2T^{-2}]$ $[T]$	$[ML^2T^{-1}]$
Gravitational constant	$\frac{\text{Force} \times (\text{distance})^2}{\text{mass} \times \text{mass}}$	$\frac{[MLT^{-2}][L^2]}{[M][M]}$	$[M^{-1}L^3T^{-2}]$
Planck constant	Energy/frequency	$[ML^2 T^{-2}]$ /$[T^{-1}]$	$[ML^2T^{-1}]$
Heat capacity, entropy	Heat energy /temperature	$[ML^2T^{-2}]/[K]$	$[ML^2T^{-2}K^{-1}]$
Specific heat capacity	$\frac{\text{Heat Energy}}{\text{Mass} \times \text{temperature}}$	$[ML^2T^{-2}]/[M][K]$	$[M^0L^2T^{-2}K^{-1}]$
Latent heat	Heat energy/mass	$[ML^2 T^{-2}]/[M]$	$[M^0L^2T^{-2}]$
Thermal expansion coefficient or Thermal expansivity	$\frac{\text{Change in dimension}}{\text{Original dimension} \times \text{temperature}}$	$[L]/[L][K]$	$[M^0L^0K^{-1}]$
Thermal conductivity	$\frac{\text{Heat Energy} \times \text{thickness}}{\text{Area} \times \text{temperature} \times \text{time}}$	$\frac{[ML^2T^{-2}][L]}{[L^2][K][T]}$	$[MLT^{-3} K^{-1}]$
Bulk modulus or (compressibility)$^{-1}$	$\frac{\text{Volume} \times (\text{Change in pressure})}{\text{Change in volume}}$	$\frac{[L^3][ML^{-1}T^{-2}]}{[L^3]}$	$[ML^{-1}T^{-2}]$
Centripetal acceleration	(Velocity)^2/radius	$[LT^{-1}]^2/[L]$	$[M^0LT^{-2}]$
Stefan constant	$\frac{(\text{Energy / area} \times \text{time})}{(\text{Temperature})^4}$	$\frac{[ML^2T^{-2}]}{[L^2][T][K]^4}$	$[ML^0T^{-3}K^{-4}]$
Wien constant	Wavelength × temperature	$[L]$ $[K]$	$[M^0LT^0K]$
Boltzmann constant	Energy/temperature	$[ML^2T^{-2}]/[K]$	$[ML^2T^{-2}K^{-1}]$
Universal gas constant	$\frac{\text{Pressure} \times \text{volume}}{\text{mole} \times \text{temperature}}$	$\frac{[ML^{-1}T^{-2}][L^3]}{[mol][K]}$	$[ML^2T^{-2}K^{-1} mol^{-1}]$
Charge	Current × time	$[A][T]$	$[M^0L^0TA]$
Current density	Current/area	$[A]/[L^2]$	$[M^0L^{-2} T^0A]$
Voltage, electric potential, electromotive force	Work/charge	$[ML^2T^{-2}]/[AT]$	$[ML^2 T^{-3} A^{-1}]$
Resistance	$\frac{\text{Potential difference}}{\text{Current}}$	$\frac{[ML^2T^{-3}A^{-1}]}{[A]}$	$[ML^2T^{-3}A^{-2}]$

Capacitance	Charge/potential difference	$\frac{[AT]}{[ML^2T^{-3}A^{-1}]}$	$[M^{-1}L^{-2}T^4A^2]$
Electrical resistivity or (electrical conductivity)$^{-1}$	$\frac{\text{Resistance} \times \text{area}}{\text{length}}$	$[ML^2T^{-3}A^{-2}][L^2]/[L]$	$[ML^3T^{-3}A^{-2}]$
Electric field	Electrical force/charge	$[MLT^{-2}]/[AT]$	$[MLT^{-3}A^{-1}]$
Electric flux	Electric field × area	$[MLT^{-3}A^{-1}][L^2]$	$[ML^3T^{-3}A^{-1}]$
Electric dipole moment	Torque/electric field	$\frac{[ML^2T^{-2}]}{[MLT^{-3}A^{-1}]}$	$[M^0LTA]$
Electric field strength or electric field intensity	$\frac{\text{Potential difference}}{\text{distance}}$	$\frac{[ML^2T^{-3}A^{-1}]}{[L]}$	$[MLT^{-3}A^{-1}]$
Magnetic field, magnetic flux density, magnetic induction	$\frac{\text{Force}}{\text{Current} \times \text{length}}$	$[MLT^{-2}]/[A][L]$	$[ML^0T^{-2}A^{-1}]$
Magnetic flux	Magnetic field × area	$[MT^{-2}A^{-1}][L^2]$	$[ML^2T^{-2}A^{-1}]$
Inductance	$\frac{\text{Magnetic flux}}{\text{Current}}$	$\frac{[ML^2T^{-2}A^{-1}]}{[A]}$	$[ML^2T^{-2}A^{-2}]$
Magnetic dipole moment	Torque/magnetic field or current × area	$[ML^2T^{-2}]/[MT^{-2}A^{-1}]$ or $[A][L^2]$	$[M^0L^2T^0A]$
Magnetic field strength, magnetic intensity or magnetic moment density	$\frac{\text{Magnetic moment}}{\text{Volume}}$	$\frac{[L^2A]}{[L^3]}$	$[M^0L^{-1}T^0A]$
Permittivity constant (of free space)	$\frac{\text{Charge} \times \text{charge}}{4\pi \times \text{electricforce} \times (\text{distance})^2}$	$\frac{[AT][AT]}{[MLT^{-2}][L]^2}$	$[M^{-1}L^{-3}T^4A^2]$
Permeability constant (of free space)	$\frac{2\pi \times \text{force} \times \text{distance}}{\text{current} \times \text{current} \times \text{length}}$	$\frac{[M^0L^0T^0][MLT^{-2}][L]}{[A][A][L]}$	$[MLT^{-2}A^{-2}]$
Refractive index	$\frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$	$[LT^{-1}]/[LT^{-1}]$	$[M^0L^0T^0]$
Wave number	2π/ wavelength	$[M^0L^0T^0]/[L]$	$[M^0L^{-1}T^0]$
Mass defect	(sum of masses of nucleons)- (mass of the nucleus)	$[M]$	$[ML^0T^0]$
Binding energy of nucleus	Mass defect × (speed of light in vacuum)^2	$[M][LT^{-1}]^2$	$[ML^2T^{-2}]$
Decay constant	0.693/half life	$[T^{-1}]$	$[M^0L^0T^{-1}]$
Resonant frequency	$(\text{Inductance} \times \text{capacitance})^{-1/2}$	$[ML^2T^{-2}A^{-2}]^{-\frac{1}{2}}[M^{-1}L^{-2}T^4A^2]^{-\frac{1}{2}}$	$[M^0L^0A^0T^{-1}]$

Quality factor or Q- factor of coil	$\frac{\text{Resonant frequency} \times \text{inducatance}}{\text{Resistance}}$	$\frac{[T^{-1}][ML^2T^{-2}A^{-2}]}{[ML^2T^{-3}A^{-2}]}$	$[M^0L^0T^0]$
Power of lens	(Focal length)$^{-1}$	$[L^{-1}]$	$[M^0L^{-1}T^0]$
Magnification	$\frac{\text{Image distance}}{\text{Object distance}}$	[L]/[L]	$[M^0L^0T^0]$
Fluid flow rate	$\frac{(\pi/8)\,(\text{pressure}) \times (\text{radius})^4}{(\text{viscosity coefficient}) \times \text{length}}$	$\frac{[ML^{-1}T^{-2}][L^4]}{[ML^{-1}T^{-1}][L]}$	$[M^0L^3T^{-1}]$
Capacitive reactance	(Angular frequency capacitance)$^{-1}$	$[T^{-1}]^{-1}\ [M^{-1}L^{-2}\ T^4A^2]^{-1}$	$[ML^2T^{-3}A^{-2}]$
Inductive reactance	(Angular frequency inductance)	$[T^{-1}]\ [ML^2\ T^{-2}A^{-2}]$	$[ML^2T^{-3}A^{-2}]$

Applications of Dimensional Formula

1. Conversion between System of Units

To convert a physical quantity from one system of units to other:

This is based on the fact that magnitude of a physical quantity remains same whatever system is used for measurement.

Magnitude = numeric value (n) × unit (u) = constant

$n_1u_1 = n_2u_2$

$$n_2 = n_1\left(\frac{u_1}{u_2}\right)$$

So, if a quantity is represented by $[M^a\ L^b\ T^c]$ then: $n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$

n_1 = numerical value in I system

n_2 = numerical value in II system

L_1 = unit of length in I system

L_2 = unit of length in II system

M_1 = unit of mass in I system

M_2 = unit of mass in II system

T_1 = unit of time in I system

T_2 = unit of time in II system

Illustration 1:

Convert 1 newton (SI unit) into dyne (CGS unit)?

Solution:

$$n_1u_1 = n_2u_2 \Rightarrow n_2 = n_1\left(\frac{u_1}{u_2}\right)$$

$$n_2 = 1\frac{[M_1L_1T_1^{-2}]}{[M_2L_2T_2^{-2}]} = 1\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]\left[\frac{T_1}{T_2}\right]^{-2} = 1\left[\frac{kg}{gm}\right]\left[\frac{m}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 1\left[\frac{1000gm}{gm}\right]\left[\frac{100cm}{cm}\right]\left[\frac{s}{s}\right]^{-2} = 10^5$$

Illustration 2:

Convert 2J to a system where unit of mass, length and time are 10gm, 20m and 10s respectively?

Solution:

$$n_2 = n_1\left(\frac{u_1}{u_2}\right) \Rightarrow n_2 = 2\frac{[M_1L_1^2T_1^{-2}]}{[M_2L_2^2T_2^{-2}]}$$

$$n_2 = 2\left[\frac{M_1}{M_2}\right]\left[\frac{L_1}{L_2}\right]^2\left[\frac{T_1}{T_2}\right]^{-2} = 2\left[\frac{kg}{10gm}\right]\left[\frac{m}{20m}\right]^2\left[\frac{s}{10s}\right]^{-2} = 2\left[\frac{1000gm}{10gm}\right]\left[\frac{m}{20m}\right]^2\left[\frac{s}{10s}\right]^{-2} = 50$$

BEGINNER'S BOX-1

1. The value of Gravitational constant G in MKS system is 6.67×10^{-11} N-m^2/kg^2. What will be its value in CGS system ? **PPM152**

2. Match the type of unit (column A) with its corresponding example (column B)

(A)	(B)
(a) Base unit	(i) N
(b) Derived SI unit	(ii) hp
(c) Improper unit	(iii) kg-wt
(d) Practical unit	(iv) rad
(e) Supplementary unit	(v) kg

PPM153

2. Law of Homogeneity and Dimensions of Unknown Quantities

To check the dimensional correctness of a given physical relation:

If in a given relation, terms on both the sides have the same dimensions, then the relation is dimensionally correct.

This is known as the **Principle of Homogeneity** of Dimensions.

Illustration 3:

Check the accuracy of the relation $T = 2\pi\sqrt{\frac{L}{g}}$ for a simple pendulum using dimensional analysis?

Solution:

The dimensions of LHS = the dimension of T = $[M^0 L^0 T^1]$

The dimensions of RHS = $\left(\frac{\text{dimensions of length}}{\text{dimensions of acceleration}}\right)^{1/2}$ ($\because 2\pi$ is a dimensionless constant)

$$= \left[\frac{L}{LT^{-2}}\right]^{1/2} = [T^2]^{1/2} = [T] = [M^0 L^0 T^1]$$

Since the dimensions are same on both the sides, the relation is correct.

Illustration 4:

Check the dimensional correctness of : $p = \frac{mv}{2}$ p :- Momentum, m :- mass, v :- speed

Solution:

The dimension of LHS = $[M^1L^1T^{-1}]$

Dimension of RHS = $[M^1L^1T^{-1}]$

Note:- A Dimensionally Correct equation may be Physically Incorrect.

Illustration 5:

Find the dimensions of unknown quantities p, q in the following:

$U = p\cos\left(qt + \frac{\pi}{6}\right)$;   where, U: - Potential energy, t :- Time

Solution:

Dimensions of potential energy [U] = $[M^1L^2T^{-2}]$

Inside cos function $\rightarrow$ dimensionless

qt $\rightarrow$ dimensionless

[qt] = $[M^0L^0T^0]$

$[q] = \frac{[M^0L^0T^0]}{[T^1]}$ = $[T^{-1}]$ and [U] = [P] = $[M^1L^2T^{-2}]$

Illustration 6:

If $\alpha = \frac{F}{v^2}\sin\beta t$, find dimensions of α and β. Here v = velocity, F = force and t = time.

Solution:

Here sin βt and βt must be dimensionless

So $[\beta t]=1 \Rightarrow [\beta]=\left[\frac{1}{t}\right]=[T^{-1}];[\alpha]=\left[\frac{F}{v^2}\sin\beta t\right]=\left[\frac{F}{v^2}\right]=\left[\frac{MLT^{-2}}{L^2T^{-2}}\right]=[ML^{-1}]$

3. To Derive relationship between different physical quantities:

Using the same principle of homogeneity of dimensions new relations among physical quantities can be derived if the dependent quantities are known.

Illustration 7:

It is known that the time of revolution T of a satellite around the earth depends on the universal gravitational constant G, the mass of the earth M, and the radius of the circular orbit R. Obtain an expression for T using dimensional analysis.

Solution:

We have $[T] \propto [G]^a [M]^b [R]^c$

$\Rightarrow [M]^0 [L]^0 [T]^1 = [M]^{-a} [L]^{3a} [T]^{-2a} \times [M]^b \times [L]^c = [M]^{b-a} [L]^{c+3a} [T]^{-2a}$

Comparing the exponents

For [T] : $1 = -2a \Rightarrow a = -\frac{1}{2}$

For [M] : $0 = b - a \Rightarrow b = a = -\frac{1}{2}$

For [L] : $0 = c + 3a \Rightarrow c = -3a = \frac{3}{2}$

Putting the values we get $T \propto G^{-1/2}M^{-1/2}R^{3/2} \Rightarrow T \propto \sqrt{\frac{R^3}{GM}}$

The actual expression is $T = 2\pi\sqrt{\frac{R^3}{GM}}$

Illustration 8:

If force (F), acceleration (a) and time(T) are taken as fundamental quantities, then dimensions of length is?

Solution:

$L \propto F^P a^Q T^R$

$$\left[L^1\right]=\left[MLT^{-2}\right]^P\left[L^1T^{-2}\right]^Q\left[T^1\right]^R=\left[M^P\right]\left[L^{P+Q}\right]\left[T^{-2P-2Q+R}\right]$$

$\because$ P = 0 ...(i)

P + Q = 1 $\Rightarrow$ Q = 1 ...(ii)

–2P –2Q + R = 0 $\Rightarrow$ R = 2Q $\Rightarrow$ R = 2 ...(iii)

$L \propto F^0a^1T^2 \Rightarrow L = kF^0a^1T^2$

Limitations of Dimensional Analysis method

1. In Mechanics the formula for a physical quantity depending on more than three physical quantities cannot be derived. It can only be checked.
2. This method can be used only if the dependency is of multiplication type.
3. The formulae containing exponential, trigonometrical and logarithmic functions cannot be derived using this method.
4. Formulae containing more than one term which are added or subtracted like $s = ut + \left(\frac{1}{2}\right)at^2$ also cannot be derived.
5. The relation derived from this method gives no information about the dimensionless constants.
6. If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.
7. It gives no information whether a physical quantity is a scalar or a vector.

BEGINNER'S BOX-2

1. Match the following :

(i) Dimensional variable	(a) π
(ii) Dimensionless variable	(b) Force
(iii) Dimensional constant	(c) Angle
(iv) Dimensionless constant	(d) Gravitational constant

PPM154

2. Find the dimensions of the following quantities :

(a) Temperature (b) Kinetic energy (c) Pressure (d) Angular speed

PPM155

3. Find the dimensions of Planck's constant (h). **PPM156**
4. Centripetal force (F) on a body of mass (m) moving with uniform speed (v) in a circle of radius (r) depends upon m, v and r. Derive a formula for the centripetal force using theory of dimensions. **PPM157**

Significant Figures, Rounding off and Order of Magnitude

Significant Figures or Digits

Significant figures (SF) in a measurement are the figures or digits that are known with certainty plus one that is uncertain (i.e. Last digit).

Significant figures in a measured value of a physical quantity tell the number of digits in which we have confidence. Larger the number of significant figures obtained in a measurement, greater is its accuracy and vice versa.

Rules to find out the number of significant figures

I Rule : All the non-zero digits are significant e.g. 1984 has 4 SF.

II Rule : All the zeros between two non-zero digits are significant. e.g. 10806 has 5 SF.

III Rule : All the zeros to the left of first non-zero digit are not significant. e.g.00108 has 3 SF.

IV Rule : If the number is less than 1, zeros on the right of the decimal point but to the left of the first non-zero digit are not significant. e.g. 0.002308 has 4 SF.

V Rule : The trailing zeros (zeros to the right of the last non-zero digit) in a number with a decimal point are significant. e.g. 01.080 has 4 SF.

VI Rule : The trailing zeros in a number without a decimal point may not be significant e.g. 010100 has 3 SF.

VII Rule : When the number is expressed in exponential form, the exponential term does not affect the number of S.F. For example, in x = 12.3 = 1.23×10^1 = 0.123×10^2 = 0.0123×10^3 = 123×10^{-1} each term has 3 SF only.

Rules for arithmetical operations with significant figures

I Rule : In addition, or subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places. e.g. 12.587 – 12.5 = 0.087 = 0.1 ($\because$ second term contain lesser i.e. one decimal place)

II Rule : In multiplication or division, the number of SF in the product or quotient is same as the smallest number of SF in any of the factors. e.g. $2.4 \times 3.65 = 8.8$

BEGINNER'S BOX-3

1. Write the following in scientific notation :

(a) 3256 g (b) 0.0010 g (c) 50000 g (d) 0.3204

PPM158

2. Give the number of significant figures in the following :

(a) 0.165 (b) 4.0026 (c) 0.0256 (d) 201

(e) 0.050 (f) 2.653×10^4 (g) 6.02×10^{23} (h) 0.0006032

PPM159

3. From the point of view of significant figures which of the following statements are correct?

(i) 10.2 cm + 8 cm = 18.2 cm (ii) 2.53 m – 1.2 m = 1.33 m

(iii) 4.2 m × 1.4 m = 5.88 m^2 (iv) 3.6 m / 1.75 sec = 2.1 m/s

(1) (i) & (iv) only (2) (ii) & (iii) only (3) (iv) only (4) (ii) & (iv) only

PPM160

Rounding off

To represent the result of any computation containing more than one uncertain digit, it is rounded off to appropriate number of significant figures.

Rules for rounding off the numbers:

I Rule : If the digit to be rounded off is more than 5, then the preceding digit is increased by one. e.g. 6.87 ≈ 6.9

II Rule : If the digit to be rounded off is less than 5, then the preceding digit is left unchanged. e.g. 3.94 ≈ 3.9

III Rule : If the digit to be rounded off is 5 then the preceding digit is increased by one if it is odd and is left unchanged if it is even. e.g. 14.35 ≈ 14.4 and 14.45 ≈ 14.4

Ex. The following values can be rounded off to four significant figures as follows:

(a) 36.879 ≈ 36.88 (∵ 9 > 5 ∴ 7 is increased by one i.e. I Rule)

(b) 1.0084 ≈ 1.008 (∵ 4 < 5 ∴ 8 is left unchanged i.e. II Rule)

(c) 11.115 ≈ 11.12 (∵ last 1 is odd it is increased by one i.e.III Rule)

(d) 11.125 ≈ 11.12 (∵ 2 is even it is left unchanged i.e. III Rule)

Illustration 9:

The length, breadth and thickness of a metal sheet are 4.234 m, 1.005 m and 2.01 cm respectively. Give the area and volume of the sheet to correct number of significant figures.

Solution:

length (ℓ) = 4.234 m   breadth (b) = 1.005 m   thickness (t) = 2.01 cm = 2.01×10^{-2} m

Therefore, area of the sheet = $2(\ell \times b + b \times t + t \times \ell)$

$= 2\,(4.234 \times 1.005 + 1.005 \times 0.0201 + 0.0201 \times 4.234)\ m^2$

$= 2\,(4.3604739)\ m^2 = 8.720978\ m^2$

Since area can contain a maximum of 3 SF therefore, rounding off, we get: Area = 8.72 m^2

Likewise, volume = $\ell \times b \times t = 4.234 \times 1.005 \times 0.0201\ m^3 = 0.0855289\ m^3$

Since volume can contain 3 SF, therefore after rounding off, we get: Volume = 0.0855 m^3

BEGINNER'S BOX-4

1. Round off the following numbers as indicated:

(a) 25.653 to 3 digits   (b) 4.996×10^5 to 3 digits   (c) 0.6995 to 1digit

(d) 3.350 to 2 digits   (e) 0.03927 kg to 3 digits   (f) 4.085×10^8 s to 3 digits

PPM161

2. Calculate area enclosed by a circle of diameter 1.06 m to correct number of significant figures

PPM162

3. Subtract 2.5×10^4 from 3.9×10^5 and give the answer to correct number of significant figures.

PPM163

4. The mass of a box measured by a grocer's balance is 2.3 kg. Two gold pieces of masses 20.15 g and 20.17 g are added to the box. What is (a) total mass of the box (b) the difference in masses of gold pieces to correct significant figures. **PPM164**

Order of Magnitude

Order of magnitude of a quantity is the power of 10 required to represent that quantity. This power is determined after rounding off the value of the quantity properly. For rounding off, the last digit is simply ignored if it is less than 5 and, is increased by one if it is 5 or more than 5.

- When a number is divided by 10^x (where x is the order of the number) the result will always lie between 0.5 and 5 i.e. $0.5 \le N/10^x < 5$

Ex. Order of magnitude of the following values can be determined as follows:

(a) $49 = 4.9 \times 10^1$ ∴ Order of magnitude = 1

(b) $51 = 0.51 \times 10^2$ ∴ Order of magnitude = 2

(c) $0.049 = 4.9 \times 10^{-2}$ ∴ Order of magnitude = –2

(d) $0.050 = 0.5 \times 10^{-1}$ ∴ Order of magnitude = –1

(e) $0.051 = 0.51 \times 10^{-1}$ ∴ Order of magnitude = –1

BEGINNER'S BOX-5

1. Give the order of the following :

(a) 1
(b) 1000
(c) 499
(d) 500
(e) 501
(f) 1 AU (1.496×10^{11} m)
(g) 1 Å (10^{-10} m)
(h) Speed of light (3.00×10^{8} m/s)
(i) Gravitational constant (6.67×10^{-11} N – m^2/kg^2)
(j) Avogadro constant (6.02×10^{23} mol^{-1})
(k) Planck's constant (6.63×10^{-34} J-s)
(l) Charge on electron (1.60×10^{-19} C)
(m) Radius of H– atom (5.29×10^{-11} m)
(n) Atmospheric pressure (1.01×10^{5} Pa)
(o) Mass of earth (5.98×10^{24} kg)
(p) Mean radius of earth (6.37×10^{6} m)

PPM165

Types and Representation of Errors

Errors in Measurement

The result of every measurement by any measuring instrument contains some uncertainty. **This uncertainty is called error**.

The difference between the true value and the measured value of a quantity is known as the error in the measurement.

Error = True value – Measured value

Types of Errors

1. **Systematic Errors:-** Systematic errors are the errors whose causes are known. They can be either positive or negative. Due to the known causes these errors can be minimised. Systematic errors can further be classified into three categories:

(i) Instrumental errors (ii) Environmental errors (iii) Observational errors

(i) **Instrumental Errors :** Error due to imperfect design or calibration of the measuring instrument.

(ii) **Environmental Errors :** These errors are due to the changes in external environmental conditions such as temperature, pressure, humidity, dust, vibrations, magnetic and electrostatic fields etc.

(iii) **Observational Errors :** These errors arise due to improper setting of the apparatus or carelessness in taking observations.

2. **Random Errors:** These errors are due to unknown causes. Therefore, they occur irregularly and are variable in magnitude and sign. The causes of these errors are not known precisely, they cannot be eliminated completely.

For example: -

- when the same person repeats the same observation in the same conditions, he may get different readings different times.
- If the random error in the arithmetic mean of 100 observations is 'x' then the random error in the arithmetic mean of 500 observations will be $\frac{x}{5}$.

3. **Gross Errors:-** Gross errors arise due to human carelessness and mistakes in taking reading or calculating and recording the measurement results.

For example: -

- Reading instrument without proper initial settings.
- Taking the observations wrongly without taking necessary precautions.
- Committing mistakes in recording the observations.
- Putting improper values of observations in calculations.
- These errors can be minimised by increasing the sincerity and alertness of the observer.

Representation of Errors

1. Absolute Error (Δa)

The difference between the true value and the individual measured value of the quantity is called the absolute error of the measurement.

Absolute Error = True Value – Measured Value

$$\Delta a = a_T - a$$

If true value of a quantity is not given then mean of all the measured values is taken as true value.

$$a_m = \frac{1}{n}\sum_{i=0}^{n} a_i$$

2. Mean Absolute Error $(\Delta a)_m$

The arithmetic mean of all the absolute errors(magnitudes) is defined as the final or mean absolute error.

Mean Absolute Error = $(\Delta a)_m = \frac{1}{n}\sum_{i=0}^{n} |\Delta a_i|$

3. Relative Or Fractional Error

It is defined as the ratio of the mean absolute error to the true value or the mean value of the quantity measured.

Relative Error = $\frac{\text{Mean Absolute Error}}{\text{True Value}} \Rightarrow$ Relative Error = $\frac{(\Delta a)_m}{a_m}$

4. Percentage Error

When the relative error is expressed in percentage, it is known as percentage error.

Percentage Error = Relative Error × 100%

Percentage Error = $\frac{\text{Mean Absolute Error}}{\text{True Value}} \times 100\%$

Illustration 10:

Following observations were taken with a Vernier callipers while measuring the length of a cylinder.

3.29 cm, 3.28 cm, 3.29 cm, 3.31 cm, 3.28 cm, 3.27 cm, 3.29 cm, 3.30 cm

Then find

(a) Most accurate length of the cylinder.

(b) Absolute error in each observation.

(c) Mean absolute error

(d) Relative error

(e) Percentage error

Express the result in terms of absolute error and percentage error.

Solution:

(a) Most accurate length of the cylinder will be the mean length $(\bar{\ell})$

$$\bar{\ell} = \frac{3.29+3.28+3.29+3.31+3.28+3.27+3.29+3.30}{8} = 3.28875 \text{ cm or } \bar{\ell} = 3.29 \text{ cm}$$

(b) Absolute error in the first reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the second reading = 3.29 – 3.28 = 0.01 cm
Absolute error in the third reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the fourth reading = 3.29 – 3.31 = –0.02 cm
Absolute error in the fifth reading = 3.29 – 3.28 = 0.01 cm
Absolute error in the sixth reading = 3.29 – 3.27 = 0.02 cm
Absolute error in the seventh reading = 3.29 – 3.29 = 0.00 cm
Absolute error in the last reading = 3.29 – 3.30 = –0.01 cm

(c) Mean absolute error = $\overline{\Delta\ell} = \frac{0.00+0.01+0.00+0.02+0.01+0.02+0.00+0.01}{8}$ = 0.01 cm

(d) Relative error in length = $\frac{\overline{\Delta\ell}}{\bar{\ell}} = \frac{0.01}{3.29}$ = 0.0030395 = 0.003

(e) Percentage error = $\frac{\overline{\Delta\ell}}{\bar{\ell}}$ × 100 = 0.003 × 100 = 0.3%

So, length $\ell = 3.29 \text{ cm} \pm 0.01 \text{cm}$ (in terms of absolute error)

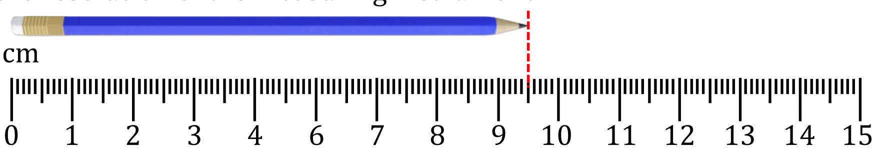
or $\ell = 3.29 \text{ cm} \pm 0.30\%$ (in terms of percentage error)

Least Count

It is the smallest value of a physical quantity that can be measured accurately by an instrument.

Least Count Error

This is related to the resolution of the measuring instrument



If the instrument has unknown least count the absolute error is taken to be equal to the least count unless otherwise stated.

Illustration 11:

An instrument having LC is 0.01cm measures the length of a rod. Which can be the reading
(1) 46.300 cm (2) 8.2 cm (3) 10.1 cm (4) 4.03 cm

Solution: (4) 4.03 cm

Propagation of Errors

Rule 1: - Addition or Subtraction of Quantities:

The maximum absolute error in the sum or difference of the two quantities is equal to the sum of the absolute errors in the individual quantities.

If X = A + B or X = A – B and if $\pm\Delta A$ and $\pm\Delta B$ represent the absolute errors in A and B respectively, then the maximum absolute error in X is $\Delta X = \Delta A + \Delta B$ and Maximum percentage error = $\frac{\Delta X}{X} \times 100\,\%$ The result will be written as $X \pm \Delta X$ (in terms of absolute error) or $X \pm \left(\frac{\Delta X}{X} \times 100\right)\%$ (in terms of percentage error)

Rule 2: - Multiplication or Division of Quantities:

The maximum fractional or relative error in the product or quotient of quantities is equal to the sum of the fractional or relative errors in the individual quantities.

If $X = AB$ or $X = \frac{A}{B}$ then $\frac{\Delta X}{X} = \pm\left(\frac{\Delta A}{A} + \frac{\Delta B}{B}\right)$

Rule 3: - The maximum fractional error in a quantity raised to a power (n) is n times the fractional error in the quantity itself, i.e.

If $X = A^n$ then $\frac{\Delta X}{X} = n\left(\frac{\Delta A}{A}\right)$

If $X = A^p B^q C^r$ then $\frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$

If $X = \frac{A^p B^q}{C^r}$ then $\frac{\Delta X}{X} = \left[p\left(\frac{\Delta A}{A}\right) + q\left(\frac{\Delta B}{B}\right) + r\left(\frac{\Delta C}{C}\right)\right]$

Illustration 12:

The initial and final temperatures of water as recorded by an observer are (40.6 ± 0.2)°C and (78.3 ± 0.3)°C. Calculate the rise in temperature with proper error limits.

Solution:

Given $\theta_1 = (40.6 \pm 0.2)$°C and $\theta_2 = (78.3 \pm 0.3)$°C

Rise in temp. $\theta = \theta_2 - \theta_1 = 78.3 - 40.6 = 37.7$°C.

$\Delta\theta = \pm(\Delta\theta_1 + \Delta\theta_2) = \pm(0.2 + 0.3) = \pm 0.5\ C$ $\quad\therefore$ rise in temperature = (37.7 ± 0.5)°C

Illustration 13:

The side of a cube is (2.00 ± 0.01) cm. The volume and surface area of cube are respectively

Solution:

Volume, $V = a^3 = 8\ cm^3$, Also $\frac{\Delta V}{V} = 3\frac{\Delta a}{a} \Rightarrow \Delta V = 3V\left(\frac{\Delta a}{a}\right) = (3)(8)\left(\frac{0.01}{2.00}\right) = 0.12 cm^3$

Therefore $V = (8.00 \pm 0.12)\ cm^3$

Surface Area $A = 6a^2 = 6\ (2.00)^2 = 24.0\ cm^2$

Also $\frac{\Delta A}{A} = 2\frac{\Delta a}{a} \Rightarrow \Delta A = 2A\left(\frac{\Delta a}{a}\right) = 2(24.0)\left(\frac{0.01}{2.00}\right) = 0.24$

Therefore $A = (24.0 \pm 0.24)\ cm^2$

Illustration 14:

A thin copper wire of length L increases in length by 2% when heated from T_1 to T_2. If a copper cube having side 10 L is heated from T_1 to T_2 what will be the percentage change in

(i) Area of one face of the cube and. (ii) Volume of the cube.

Solution:

(i) Area $A = 10\ L \times 10\ L = 100\ L^2 \Rightarrow A \propto L^2$

% change in area = $\frac{\Delta A}{A} \times 100 = 2 \times \frac{\Delta L}{L} \times 100 = 2 \times 2\% = 4\%$

(ii) Volume $V = 10\ L \times 10\ L \times 10\ L = 1000\ L^3 \Rightarrow V \propto L^3$

% change in volume = $\frac{\Delta V}{V} \times 100 = 3\ \frac{\Delta L}{L} = 3 \times 2\% = 6\%$

BEGINNER'S BOX-6

1. Two rods have lengths measured as (1.8 ± 0.2)m and (2.3 ± 0.1)m. Calculate their combined length with error limits.
PPM166

2. The original length of wire is (153.7 ± 0.6) cm. It is stretched to (155.3 ± 0.2) cm. Calculate the elongation in the wire with error limits.
PPM167

3. Measure of two quantities along with the precision of respective measuring instrument is :-
$A = 2.5$ m/s ± 0.5 m/s $B = 0.10$s ± 0.01 s
The value of AB will be :-
(1) (0.25 ± 0.08)m (2) (0.25 ± 0.5) m (3) (0.25 ± 0.05) m (4) (0.25 ± 0.135) m
PPM168

4. The radius of a sphere is measured to be (2.10 ± 0.05) cm. Calculate its surface area with absolute error limits. **PPM169**

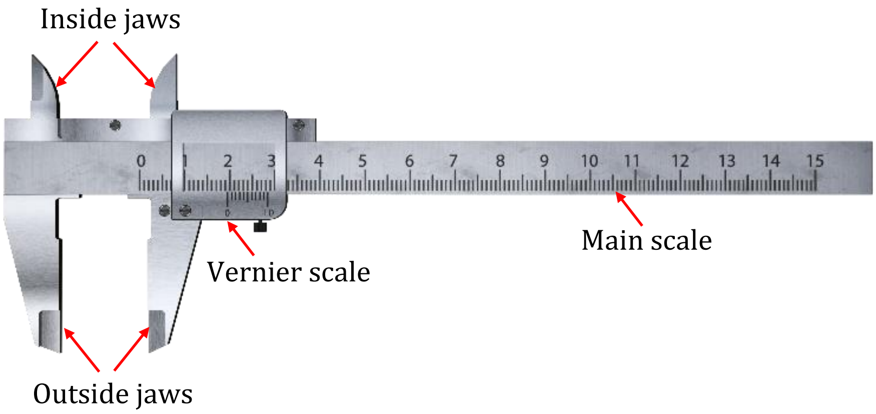
5. A physical quantity x is calculated from the relation $x = a^3b^2/\sqrt{cd}$. Calculate percentage error in x, if a, b, c and d are measured respectively with an error of 1%, 3%, 4% and 2%. **PPM170**

6. An object covers (16.0 ± 0.4) m distance in (4.0 ± 0.2)s. Find out its speed. **PPM171**

Vernier Callipers

A measuring device that is used for the measurement of linear dimensions. It is also used for the measurement of diameters of round objects with the help of the measuring jaws.

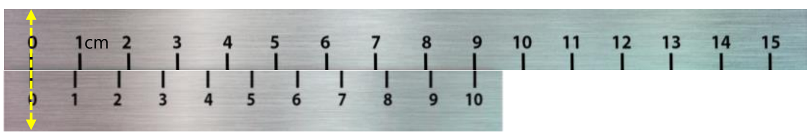
French mathematician Pierre Vernier invented the vernier scale in 1631. The main use of the vernier calliper over the main scale is to get an accurate and precise measurement.



Least Count of Vernier Callipers

Suppose the size of one main scale division (M.S.D.) is M units and that of one Vernier scale division (V. S. D.) is V units. Also let the length of 'a' main scale divisions is equal to the length of 'b' Vernier scale divisions.

$$aM = bV \Rightarrow V = \frac{a}{b}M$$



$$\therefore M - V = M - \frac{a}{b}M = \left(\frac{b-a}{b}\right)M$$

[M $\rightarrow$ MSD, V $\rightarrow$ VSD]

$$\boxed{L.C. = M - V = \left(\frac{b-a}{b}\right)M}$$

Least count (x) = Difference between MSD & VSD

10 VSD = 9 MSD

1 VSD = 0.9 MSD

x = 1 MSD – 1 VSD = 1 MSD – 0.9 MSD = 0.1 MSD

Generally, 1 MSD = 1 mm

$\therefore$ x = 0.1 mm

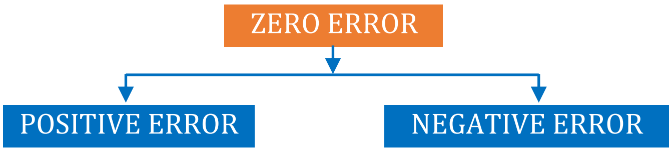
Measurement with Vernier Calliper

READING = (Main Scale Reading) + (Vernier Scale Reading)

READING = MSR + (VS Coincided division with MS × LC)

Zero Error

If the zero marking of main scale and Vernier scale **do not coincide**, necessary correction has to be made for this error which is known as zero error of the instrument.



1. Positive Zero Error

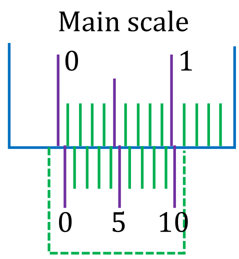
If the zero of the Vernier scale is to the **right of the zero** of the main scale the zero error is said to be positive.

The zero error is always **subtracted** from the reading to get the corrected value.

Positive Zero Error Correction

READING = (Main Scale Reading) + (Vernier Scale Reading) – ZE

Positive zero error = +VS Coincided division with any MSD × LC



Vernier Scale
With positive zero error

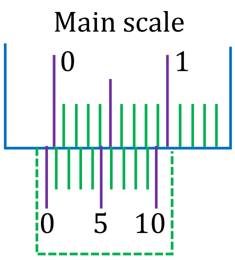
2. Negative Zero Error

If the zero of the Vernier scale is to the **left** of the zero of the main scale the zero error is said to be negative.

Note: The zero error is always **subtracted** from the reading to get the corrected value.

Negative Zero Error Correction

Negative zero error = –(Total VSD – VS Coincided division with any MSD) × LC



Vernier Scale
With negative zero

Illustration 15:

If the jaws of the vernier callipers are in contact with each other, then determine the zero error of the vernier calliper if the VSD is 3. (take its least count as 0.1mm)

Solution:

Given: The jaws of the vernier calliper are in contact with each other.

The vernier scale reading, VSR = 3

Therefore, Zero error = +VSR × LC = + 3 × 0.1 = +0.3 mm

Illustration 16:

1 mm marks are present on the main scale of the vernier scale. The total no. of divisions on the vernier scale are 20 which matches the 16 main scale divisions. Calculate the least count of this vernier scale.

Solution:

Given: One main scale division, MSD = 1 mm

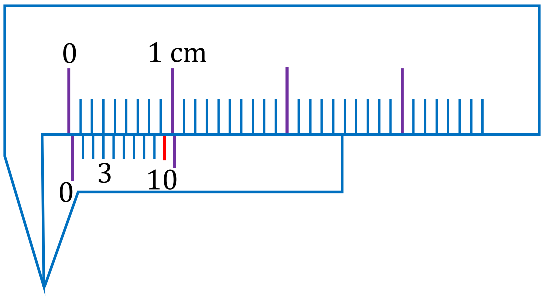
20 vernier scale divisions, VSD = 16 main scale divisions, MSD

Therefore, 1 VSD = 16/20 MSD = 0.8 mm

The least count, LC = 1 MSD – 1 VSD = 1 – 0.8 = 0.2 mm

Illustration 17:

Find the zero error in figure given



Solution:

Since, the zero of vernier scale is ahead of zero of main scale. So, the error is positive zero error.

Least count of the vernier calliper L.C. = 0.1 mm

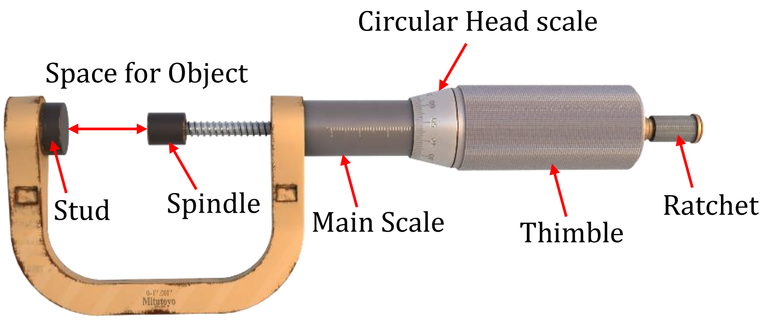
Since the 3rd division matches with the main scale division.

Zero error = n × L.C. = 3 × 0.1 = 0.3 mm

Screw Gauge and Spherometer

Screw Gauge

For precise measurement of spherical or a cylindrical object, a screw gauge is the best instrument. An extremely fine-tuned screw gauge can be a bit difficult for untrained hands to use. This article's objective is to make anyone and everyone aware of the basics of handling a screw gauge.



Pitch of Screw Gauge

The pitch of the instrument is the distance between two consecutive threads of the screw which is equal to the distance moved by the screw due to one complete rotation of the cap.

Least Count of Screw Gauge

The minimum (or least) measurement (or count) of length is equal to one division on the main scale which is equal to pitch divided by the total cap divisions.

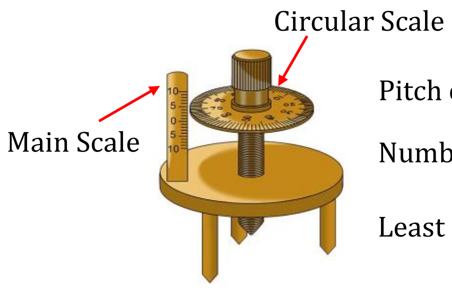
- Pitch of screw gauge = **P**
- Number of division on circular scale = **N**
- Least count of screw gauge = $\dfrac{\text{Pitch (P)}}{\text{Total no. of divisions on the circular scale (N)}}$

Length as measured by Screw Gauge

The formula for measuring the length is, $L = (n \times \text{pitch}) + (f \times \text{least count})$

Where, n = main scale reading; f = circular scale reading

Spherometer



Pitch of screw gauge = P

Number of division on circular scale = N

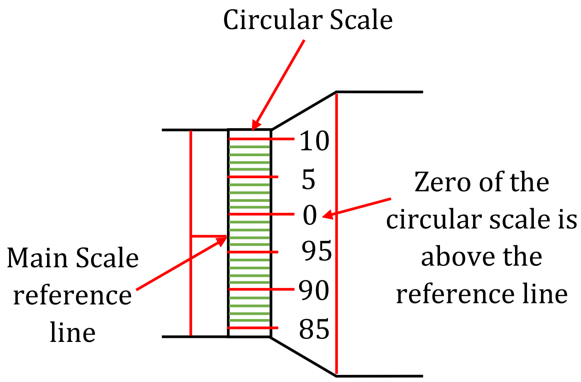
Least count of Spherometer = $\dfrac{P}{N}$

Zero Error in Screw Gauge

If there is no object between the studs (i.e. studs are in contact), the screw gauge should give zero reading. But due to extra material on studs, even if there is no object, it gives some excess reading. This excess reading is called Zero error.

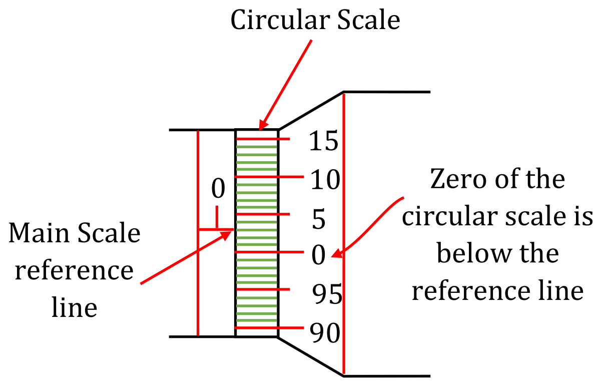
Negative Zero Error

(3 division error) i.e., –0.003 cm



Positive Zero Error

(3 division error) i.e., +0.003 cm



Calculation of zero error for screw gauge: -

Positive zero error = +CS Coincided division with MS base line × LC

Negative zero error = –(Total CSD – CS Coincided division with MS base line) × LC

Correct reading = (Reading) – (zero error)

Illustration 18:

A spherometer has 50 equal divisions marked along the periphery of its disc, and one full rotation of the disc advances on the main scale by 0.01 cm. Find the least count of the system.

Solution:

Given, Pitch = 0.01 cm

$$\therefore \text{ Least count} = \frac{\text{Pitch}}{\text{Total no. of divisions on the the circular scale}} = \frac{0.01}{50}\text{cm} = 2\times10^{-4}\ \text{cm}$$

Illustration 19:

The pitch of screw gauge is 1 mm and there are 100 divisions on the circular scale. In measuring the diameter of a sphere there are 6 divisions on the linear scale and forty division on circular scale coincide with the reference line. Find the diameter of the sphere.

Solution:

Least count of the screw gauge is L.C = pitch / number of circular divisions = 1/100 = 0.01mm

Linear scale reading (LSR) = 6 × 1mm = 6mm

Circular scale reading (CSR) = 40 × 0.01mm

Diameter of the sphere = LSR + (CSR × L.C) = 6 + (40 × 0.01) = 6.4mm

BEGINNER'S BOX-7

1. One centimetre on the main scale of vernier callipers is divided into ten equal parts. If 20 divisions of vernier scale coincide with 19 small divisions of the main scale then what will be the least count of the callipers.

PPM172

2. If the number of divisions on the circular scale is 100 and number of full rotations given to screw is 8 and distance moved by the screw is 4 mm, then what will be least count of the screw gauge?

PPM173

3. A spherometer has 250 equal divisions marked along the periphery of its disc, and one full rotation of the disc advances by 0.0625 cm. on the main scale. What is the least count of the spherometer?

PPM174

Accuracy and Precision

Accuracy: How close the measured value is to the true value.

OR

The ability of an instrument to measure the accurate value is known as accuracy. In other words, it is the closeness of the measured value to a standard or true value. Accuracy is obtained by taking small readings. The small reading reduces the error of the calculation.

Precision: How consistent our results are, regardless of proximity to true value.

OR

The closeness of two or more measurements to each other is known as the precision of a substance. If you weigh a given substance five times and get 4.3 kg each time, then your measurement is very precise but not necessarily accurate. Precision is independent of accuracy.

BEGINNER'S BOX ANSWER KEY

BEGINNER'S BOX-1

1. $(6.67 \times 10^{-8}\ cm^3/g\ s^2)$

2. (a)$\rightarrow$ (v); (b) $\rightarrow$ (i); (c) $\rightarrow$ (iii); (d) $\rightarrow$ (ii); (e) $\rightarrow$ (iv).

BEGINNER'S BOX-2

1. (i) (b), (ii) (c), (iii) (d), (iv) (a)

2. (a) $[M^0\ L^0\ T^0 K^1]$ (b) $[M\ L^2\ T^{-2}]$

(c) $[M\ L^{-1}\ T^{-2}]$ (d) $[M^0\ L^0\ T^{-1}]$

3. $[M\ L^2\ T^{-1}]$

4. $F = K\dfrac{mv^2}{r}$

BEGINNER'S BOX-3

1. (a) 3.256×10^3 g (b) 1.0×10^{-3} g

(c) 5.0000×10^4 g (d) 3.204×10^{-1}

2. (a) 3 (b) 5 (c) 3 (d) 3

(e) 2 (f) 4 (g) 3 (h) 4

3. (3)

BEGINNER'S BOX-4

1. (a) 25.7 (b) 5.00×10^5 (c) 0.7

(d) 3.4 (e) 0.0393 kg (f) 4.08×10^8 s

2. $0.882\ m^2$ (3 SF)

3. 3.6×10^5

4. (a) Total mass = 2.3 kg (b) Difference in masses = 0.02g

BEGINNER'S BOX-5

1. (a) 0 (b) 3 (c) 2 (d) 3

(e) 3 (f) 11 (g) –10 (h) 8

(i) –10 (j) 24 (k) –33 (l) –19

(m) –10 (n) 5 (o) 25 (p) 7

BEGINNER'S BOX-6

1. (4.1 ± 0.3) m **2.** (1.6 ± 0.8) cm

3. (1) **4.** $(55.39 \pm 2.64)\ cm^2$

5. $\pm 12\%$ **6.** (4.0 ± 0.3) m/s

BEGINNER'S BOX-7

1. 0.005 cm **2.** 0.005 mm

3. 2.5×10^{-4} cm