Units, Dimensions and Measurement

SOLUTIONS

Exercise-I (Conceptual Questions)

1. SI system has seven fundamental units.

2. Density is a derived quantity

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

3. Kilogram is a unit

4. (a) 10^{-6} sec (b) 366 days (c) 27.3 days

(d) 3.08×10^{16} m (not the unit of time)

5. Energy = Work = (Force) (Displacement)

= N × m = Joule

6. Theory-Kelvin

7. Momentum = mv = $M^1L^1T^{-1}$ kg × m/sec.

8. $\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{N}{m^2}$

9. $F = Ma = Kg \times \frac{m}{sec^2}$

10. $\frac{N}{Kg} = \frac{\text{Force}}{\text{Mass}} = \text{Acceleration}$

11. Velocity = $[L^1T^{-1}]$

= $(10^{-6}$ m) $(10^{-6}$ sec$)^{-1}$ = m/sec.

12. $\text{Power} = \frac{\text{Eerngy}}{\text{time}}$ = J/sec. = Watt or Kilowatt

13. Kilowatt hour = Power × time = Energy

14. 1 nanometer = 10^{-9} m = 10^{-7} cm

15. Angstrom = 10^{-10}m, nanometre = 10^{-9}m

Fermi = 10^{-15}m, attometre = 10^{-18}m

16. 1 Fermi = 10^{-15} meter

17. Velocity = $[L^1T^{-1}] \rightarrow$ derived

18. Temperature being a fundamental quantity itself cannot be expressed in terms of any of these.

19. $\text{Velocity} = \frac{\text{displacement}}{\text{time}} \frac{(L^1)}{(T^1)}$

It can be derived from other two so it is a derived.

20. d = 10^3 kg/m^3

$$= \frac{10^3 \times 10^3 \text{gram}}{10^{-6} \text{cm}^3} = 1 \text{ gram}/\text{cm}^3$$

21. 1 dyne = 10^{-5} N

1 cm = 10^{-2} m

$$\text{Surface Tension} = \frac{70 \text{ dyne}}{\text{cm}} = \frac{70 \times 10^{-5} \text{ N}}{10^{-2} \text{ m}}$$

= 7×10^{-2} N/m

22. (a) $[\text{Time}] = \left[\frac{\text{Length}}{\text{Velocity}}\right] = \text{Unchanged}$

(b) $[\text{Mass}] = \left[\frac{\text{Force}}{\text{Length} \times \text{Time}^{-2}}\right] = \text{Unchanged}$

(c) $[\text{Momentum}] = \left[\text{Mass} \times \frac{\text{Length}}{\text{Time}}\right] \propto [\text{Length}]$

so double

(d) $[\text{Energy}] = \left[\frac{\text{Mass} \times \text{Length}^2}{\text{Time}^2}\right] \propto [\text{Length}^2]$

So becomes four times of initial

23. [Power] = [Force × Velocity]

so becomes one fourth

24. Angle = Dimensionless but unit = radian

relative density = Dimensionless and is unitless so dimensionless quantity may have a unit.

25. For unitless quantity dimensions are always zero i.e. they never have non zero dimension.

26. Coefficient of friction is unitless & dimenssionless.

27. $\text{Relative density} = \frac{\text{Density of substance}}{\text{Density of water}}$

$$= \frac{kgm^{-3}}{kgm^{-3}} = \text{dimensionless}$$

28. 1 Fermi = 10^{-15} m; 1 Micron = 10^{-6} m

1 angstrom = 10^{-10} m; radian $\rightarrow$ Angle

29. Force $\rightarrow [M^1L^1T^{-2}]$
Pressure $\rightarrow [M^1L^{-1}T^{-2}]$
Gravitational constant $\rightarrow [M^{-1}L^3T^{-2}]$

30. Energy = Work = (Force) (displacement)
$= [M^1L^1T^{-2}]$

31. Impulse = Force × Time
$$= \left(\frac{\text{Linear momentum}}{\text{time}}\right)\times \text{time}$$
= linear momentum

32. Torque= (Force) × (distance)
Energy = Work = (Force) × (displacement)

33. Dimensional analysis cannot be used to derive relations involving trigonometric, exponential functions and relations involving (+) & (–) sign

34. In e^x, x must be dimensionless so in $e^{-A/B}$, A & B must have same dimensions also A+B and A–B are not possible because quantities with different dimensions can't be added or subtracted.

35. Dimensionally correct equation may be correct or incorrect but dimensionally incorrect equation is always incorrect e.g. $p=\frac{mv}{t}$ is dimensionally as well as experimentally wrong.

36. Dimensionally correct equation may be correct or incorrect but dimensionally incorrect equation is always incorrect e.g. $p=\frac{mv}{t}$ is dimensionally as well as experimentally wrong.

37. $x = at + bt^2$
$[x] = [at] = [bt^2]$
$$b=\frac{x}{t^2}=m/hr^2$$

38. $F = at + bt^2$
dimensions of at and bt^2 are same as that of F
$at = F \Rightarrow a[T] = [M^1L^1T^{-2}]$
$a = [M^1L^1T^{-3}]$
$bt^2 = F \Rightarrow b[T]^2 = [M^1L^1T^{-2}]$
$b = [M^1L^1T^{-4}]$

39. $S = 1/3\ ft^3$
$$[S] = [ft^3] \Rightarrow [f] = \frac{[S]}{[t^3]}=[L^1T^{-3}]$$

40. $P=P_0e^{-\alpha t^2}$
αt^2 must be dimensionless
Thus dimensions of $\alpha = \frac{1}{t^2}=[T^{-2}]$

41. $F = Pt^{-1} + Q \Rightarrow [F] = [PT^{-1}]$
$$P=\frac{[M^1L^1T^{-2}]}{[T^{-1}]}=[M^1L^1T^{-1}] = \text{Momentum}$$

42. $$\eta=\frac{F}{6\pi av}=\frac{M^1L^1T^{-2}}{[L^1][L^1T^{-1}]}=M^1L^{-1}T^{-1}$$

43. θ in $\cos\theta$ and $\sin\theta$ is dimensionless
$Bx = M^0L^0L^0 \Rightarrow BL^1 = M^0L^0T^0 \Rightarrow B = M^0L^{-1}T^0$
$Dt = M^0L^0T^0 \Rightarrow DT^1 = M^0L^0T^0 \Rightarrow D = M^0L^0T^{-1}$
$$\frac{D}{B}=\frac{M^0L^0T^{-1}}{M^0L^{-1}T^0}=L^1T^{-1}$$

44. $x = at^2 + b$
$[x] = [at^2] = [b]$
$a = [L^1T^{-2}];\ b = [L^1]$
$$\sqrt{ab}=\sqrt{[L^2T^{-2}]}=[L^1T^{-1}]$$

45. $y = r\sin(\omega t - kx)$
$[\omega t - kx] = [M^0L^0T^0]$
$[\omega t= [kx] = [M^0L^0T^0]$
$$\left[\frac{\omega}{k}\right]=\left[\frac{x}{t}\right]=[L^1T^{-1}]$$

46. $y = a\sin[At - Bx + C]$
$[At - Bx + C] = [M^0L^0T^0]$
$[At] = [Bx] = [C] = [M^0L^0T^0]$
$[A] = [T^{-1}]; [B] = [L^{-1}]; [C] = [M^0L^0T^0]$

47. at + 3 = dimensionless
$\Rightarrow$ at = dimensionless $\Rightarrow aT^1 = M^0L^0T^0$
$a = M^0L^0T^{-1}$

48. $$\frac{dv}{dt} = AT - Bv$$
$$\left[\frac{dv}{dt}\right] = [AT] = [Bv]$$
$[L^1T^{-2}] = [AT]$; $[L^1T^{-2}] = [B]\ [L^1T^{-1}]$
$[A] = [L^1T^{-3}]$ $[B] = [T^{-1}]$

49. $\frac{d^2y}{dx^2}=\left[\frac{L^1}{T^2}\right]=[L_1T^{-2}]$

50. $M = E^aV^bF^c$

$[M^1] = [ML^2T^{-2}]^a\ [L^1T^{-1}]^b\ [M^1L^1T^{-2}]c$

$[M^1L^0T^0] = [M^{a+c}\ L^{2a+b+c}\ T^{-2a-b-2c}]$

$a + c = 1;\ 2a + b + c = 0;\ -2a - b - 2c = 0$

51. $F = D^aV^bA^c$

$[M^1L^1T^{-2}] = [M^1L^{-3}]^a\ [L^1T^{-1}]^b\ [L^2]^c$

$[M^1L^1T^{-2}] = [M^a\ L^{-3a+b+2c}\ T^{-b}]$

$a = 1;\ -3a + b + 2c = 1;\ -b = -2$

54. According to the rules of significant figures $0.007\ m^2$ has one significant figures.

2.64×10^{24} kg has three significant figures.

$0.0006032\ m^2$ has four significant figures.

6.3200 J has five significant figures.

55. $P = 0.0030 \Rightarrow 2$ SF

$Q = 2.40 \Rightarrow 3$ SF

$R = 3000\ m \Rightarrow 4$ SF

56. $4621 = 4.621 \times 10^3$

$\approx 4.6 \times 10^3 = 4600$

57. L+B = 4.431 cm = 4.4 cm

(Final answer must be upto as many decimal places as in number with least decimal places)

58.
$$\begin{array}{r} 5.27 \\ -0.2 \\ \hline \end{array}$$

$5.07 \simeq 5.1$ J

59. $17.8 \times 3.1143 = 55.4354$

In product & division the number of S.F. in the answer would be equal to least S.F. figure of one of the individual qunaity.

Answer will have 3 S.F.

60. $1013 = 1.013 \times 10^3 \approx 10^3$ order of magnitude = 3

61. Sum of lengths of two rods = $L_1+L_2 = (22.10 \pm 0.10)$cm

62. Thickness of cylinder can be written as

$t = t \pm \Delta t$

$t = r_2 - r_1 = 5.23 - 4.89 = 0.34$

$\Delta t = \Delta r_1 + \Delta r_2 = 0.015 + 0.005 = 0.02$

$t = (0.34 \pm 0.02)$ cm

63. $R=\frac{V}{I} \Rightarrow \frac{\Delta R}{R}=\frac{\Delta V}{V}+\frac{\Delta I}{I}$

$\Rightarrow \frac{\Delta R}{R}\times 100=\left(\frac{5}{100}\times 100+\frac{0.2}{10}\times 100\right)=7\%$

64. If % change in radius = x%

then % change in area = 2x%

and % change in volume = 3x%

65. Circumference (S) = $2\pi r$

$\frac{\Delta S}{S}\times 100=\frac{\Delta r}{r}\times 100=2\%$

66. $V=\frac{4}{3}\pi r^3=\frac{4}{3}\pi\left(\frac{d}{2}\right)^3=\frac{\pi d^3}{6}$

$\%V = 3\%d \Rightarrow 1.5 = 3\%d \Rightarrow \%d = 0.5\%$

67. Volume of cube V = $(\text{side})^3$

$\therefore$ V = a^3. Take log and differentiate.

$\frac{\Delta V}{V}\times 100=3\frac{\Delta a}{a}\times 100 = 3(\pm 2\%) = \pm 6\%$.

68. $A=\ell^2 \Rightarrow \frac{\Delta A}{A}\times 100=2\frac{\Delta \ell}{\ell}\times 100=2(2\%)=4\%$

69. $f(x) = x^2$

Relative error = $2\left(\frac{\Delta x}{x}\right)$

70. KE = $\frac{1}{2}mv^2$

$\frac{\Delta KE}{KE}=\frac{\Delta m}{m}+2\frac{\Delta v}{v}$

$\frac{\Delta KE}{KE}\times 100 = 2 + 2 \times 3 = 8\%$

71. $H = I^2Rt$

$\Rightarrow \frac{\Delta H}{H}\times 100=\frac{2\Delta I}{I}\times 100+\frac{\Delta R}{R}\times 100+\frac{\Delta t}{t}\times 100$

$=2(2\%)+1\%+1\% = 6\%$

72. $I = 2/5\ MR^2$

$\frac{\Delta I}{I}=\frac{\Delta M}{M}+2\frac{\Delta R}{R}$

$\frac{\Delta I}{I}\times 100=1\%+2\times 2\%=5\%$

73. Resistance of a wire $R=\frac{\rho l}{\pi r^2}$

$\frac{\Delta R}{R}\times 100=\frac{\Delta \rho}{\rho}\times 100+\frac{\Delta \ell}{\ell}\times 100+2\frac{\Delta r}{r}\times 100$

$= 1\% + 2\% + 2(3\%) = 9\%$

74. $P=\frac{F}{A}=\frac{F}{L^2}$ For max. error

$$\frac{\Delta P}{P}\times 100=\frac{\Delta F}{F}\times 100+\frac{2\Delta L}{L}\times 100$$

$= 4\% + 2(2\%) = 8\%$

75. $T=2\pi\sqrt{\frac{\ell}{g}}\Rightarrow g=\frac{4\pi^2\ell}{T^2}$

$$\frac{\Delta g}{g}=\frac{\Delta\ell}{\ell}+\frac{2\Delta T}{T}=y+2x$$

76. $T=2\pi\sqrt{\frac{\ell}{g}}\Rightarrow g=\frac{4\pi^2\ell}{T^2}$

$$\frac{\Delta g}{g}\times 100=\frac{\Delta\ell}{\ell}\times 100+\frac{2\Delta T}{T}\times 100$$

$=1\%+2(3\%)=7\%$

77. $X = ab^2/c^3 \Rightarrow \frac{\Delta X}{X}=\frac{\Delta a}{a}\pm 2\frac{\Delta b}{b}\pm 3\frac{\Delta c}{c}$

$$\frac{\Delta X}{X}\times 100=\pm1\%\pm2\times3\%\pm3\times2\%=\pm13\%$$

78. Given : $X=\frac{2k^3 l^2}{m\sqrt{n}}$

$$\therefore\ \frac{\Delta X}{X}=3\frac{\Delta k}{k}+2\frac{\Delta l}{l}+\frac{\Delta m}{m}+\frac{1}{2}\frac{\Delta n}{n}$$

Percentage error in X

$$\frac{\Delta X}{X}\times 100=\left(3\frac{\Delta k}{k}+2\frac{\Delta l}{l}+\frac{\Delta m}{m}+\frac{1}{2}\frac{\Delta n}{n}\right)\times 100$$

$= 3 \times 1\% + 2 \times 1\% + 3\% + \frac{1}{2} \times 4\%$

$= 3\% + 2\% + 3\% + 2\% = 10\%$

Hence, the value of X is uncertain by 12 %.

79. $\frac{\Delta X}{X}\times 100=a\frac{\Delta M}{M}\%+b\frac{\Delta L}{L}\%+c\frac{\Delta T}{T}\%$

$= a\alpha\% + b\beta\% + c\gamma\%$

$$\frac{\Delta X}{X}\times 100=(a\alpha+b\beta+c\gamma)\%$$

80. $X = [M^{-1}L^3T^{-2}]$

$$\left(\frac{\Delta x}{x}\right)100\%=\left[\frac{1\Delta M}{M}+\frac{3\Delta L}{L}+\frac{2\Delta T}{T}\right]100\%$$

$= 2\% + 3(3\%) + 2(4\%) = 19\%$

81. $PV^{3/2}=K\Rightarrow P=\frac{K}{V^{3/2}}$

$\%P = 1\%K - \frac{3}{2}\%V = 1(0) - \frac{3}{2}(-0.5) = 0.75\%$

82. Zero error is due to erroneous manufacture of instrument so it is a instrumental error $\Rightarrow$ systematic error.

84. 40 VSD = 38 MSD

1 MSD = $\frac{40}{38}$ VSD

L.C = 1 MSD – 1 VSD

85. 10 MSD = 1 cm

1 MSD = 0.1 cm

20 VSD = 16 MSD

1 VSD = $\frac{16}{20}$ MSD

L.C. = 1 MSD – 1 VSD

86. 10 MSD = 1 cm

1 MSD = 0.1 cm

10 VSD = 9 MSD $\Rightarrow$ 1 VSD = 0.9 MSD

1 VSD = 0.09 cm

L.C. = 1 MSD – 1 VSD = 0.1 – 0.09

= 0.01 cm

T.R. = MSR + VSR = 2.8 cm + 7(0.01 cm)

= 2.87 cm

87. The correct value of diameter is upto 3 digits after the decimal as the least count is 0.001 cm

88. $LC=\frac{\text{Pitch}}{\text{Total circular divisions}}=\frac{5\text{mm}}{100}$

= 0.05 mm

89. In 5 rotation distance moved = 5 mm

In 1 rotation distance moved = 1 mm

= Pitch

$LC=\frac{\text{Pitch}}{\text{Total circular divisions}}=\frac{1\text{mm}}{100}$

= 0.01 mm = 0.001 cm

90. Total reading (T.R.) = MSD + CSR

= (3 mm) + 25 × (L.C.)

L.C. = 0.01 mm

91. $L.C. = \dfrac{\text{Pitch}}{\text{no. of division on circular scale}}$

$= \dfrac{1}{100} = 0.01\ \text{mm}$

T.R. = MSR + CSR

= 2 × 1 mm + 25 × L.C.

= 2 × 1 mm + 25 × 0.01 mm

True value (T.V.) = T.R. – error

= 2.25 – 5 × 0.01

= 2.20 mm

Exercise-II (Previous Year Questions)

1. $[\text{mass}] = \left[\dfrac{\text{Force}}{\text{Acceleration}}\right] = \left[\dfrac{\text{Force}}{\text{Velocity / time}}\right]$

$= [F\ V^{-1}\ T]$

2. Applying dimensional analysis

$S \propto E^a V^b T^c$

$[M^1L^0T^{-2}] = k\ [M^1L^2T^{-2}]^a\ [L^1T^{-1}]^b\ [T^1]^c$

$[M^1L^0T^{-2}] = k\ [M^aL^{2a}T^{-2a}]\ [L^bT^{-b}]\ [T^c]$

$[M^1L^0T^{-2}] = k[M]^a[L]^{2a+b}[T]^{-2a-b+c}$

Comparision

$a = 1 \qquad \begin{matrix} 2a+b=0 \\ b=-2 \end{matrix} \qquad \begin{matrix} -2=-2a-b+c \\ -2=-2(1)+2+c \end{matrix}$

$c = -2$

So the dimensional formula for surface tension will be $[E^1V^{-2}T^{-2}]$

Alternate solution :

$\text{Surface Tension} = \dfrac{\text{Surface energy}}{\text{Area}}$

$[\text{Surface tension}] = \dfrac{[E]}{[V \cdot T]^2} = [E\ V^{-2}\ T^{-2}]$

3. $v_c \propto [\eta^x\ \rho^y\ r^z]$

$[L^1T^{-1}] \propto [M^1\ L^{-1}\ T^{-1}]^x\ [M^1L^{-3}]^y\ [L^1]^z$

$[L^1T^{-1}] \propto [M^{x+y}]\ [L^{-x-3y+z}]\ [T^{-x}]$

taking comparision on both size

x + y = 0, –x – 3y + z = 1, –x = –1

$\Rightarrow$ x = 1, y = –1, z = –1

4. $\ell \propto h^x G^y c^z$

$M^0L^1T^0 = (ML^2T^{-1})^x\ (M^{-1}L^3T^{-2})^y\ (LT^{-1})^z$

$= M^{x-y}\ L^{2x+3y+z}\ T^{-x-2y-z}$

Equating :

$\left.\begin{matrix} x-y=0 \\ 2x+3y+z=1 \\ -x-2y-z=0 \end{matrix}\right\} \Rightarrow x=\dfrac{1}{2}; y=\dfrac{1}{2}; z=-\dfrac{3}{2}$

$\Rightarrow \ell \propto \dfrac{\sqrt{hG}}{c^{3/2}}$

5. $[L] = [c]^a\ [G]^b \left[\dfrac{e^2}{4\pi\varepsilon_0}\right]^c$

$[L] = [LT^{-1}]^a\ [M^{-1}L^3T^{-2}]^b \left[ML^3T^{-2}\right]^c$

$[L] = L^{a+3b+3c}\ M^{-b+c}\ T^{-a-2b-2c}$

a + 3b + 3c =1

–b + c = 0

a + 2b + 2c = 0

On solving,

$a = -2,\ b = \dfrac{1}{2},\ c = \dfrac{1}{2}$

$\therefore\ L = \dfrac{1}{c^2}\left[G.\ \dfrac{e^2}{4\pi\varepsilon_0}\right]^{\frac{1}{2}}$

6. Reading of screw gauge

= MSR + VSR × LC + zero error

= 0.5 cm + 25 × 0.001 cm + 0.004 cm

= 0.529 cm

7. $x = \dfrac{A^2B^{1/2}}{C^{1/3}D^3}$

$\dfrac{\Delta x}{x} = \dfrac{2\Delta A}{A} + \dfrac{1}{2}\dfrac{\Delta B}{B} + \dfrac{1}{3}\dfrac{\Delta C}{C} + 3\dfrac{\Delta D}{D}$

$\Rightarrow \dfrac{\Delta x}{x} \times 100 = 2(1\%) + \dfrac{1}{2}(2\%) + \dfrac{1}{3}(3\%) + 3(4\%) = 16\%$

8. $\dfrac{dQ}{dt} = -(K)A\dfrac{dT}{dx}$

$\dfrac{J}{s} = (K)m^2\dfrac{\text{kelvin}}{m}$

(K) = watt $m^{-1}\ K^{-1}$

9. n(VSD) = (n – 1)MSD $\Rightarrow$ 1 VSD $= \dfrac{(n-1)}{n}$ MSD

Least count =1MSD –1VSD$= \left[1 - \dfrac{(n-1)}{n}\right]$MSD

Least count = $\dfrac{1}{n}$ MSD $= \dfrac{1}{n}\left(\dfrac{1}{n}\right)$ cm = $\dfrac{1}{n^2}$ cm

10. In subtraction the number of decimal places in the result should be equal to the number of decimal places of that term in the operation which contain lesser number of decimal places.

$$\begin{array}{r} 9.99 \\ -0.0099 \\ \hline 9.98 \end{array}$$

11. $[\text{stress}] = \left[\frac{\text{Force}}{\text{Area}}\right] = \left[\frac{M^1L^1T^{-2}}{L^2}\right] = M^1L^{-1}T^{-2}$

12. $\text{L.C.} = \frac{\text{Pitch}}{\text{Number of division on circular scale}}$

$\Rightarrow 0.01\text{mm} = \frac{\text{Pitch}}{50} \Rightarrow \text{Pitch} = 0.5\text{ mm}$

13. $1' = \left(\frac{1}{60}\right)^{o} = \frac{1}{60} \times \frac{\pi}{180}$ radian

14. Mean of observation

$= \frac{1.25+1.24+1.27+1.21+1.28}{5} = 1.25\text{ s}$

Mean absolute error

$= \frac{0+0.01+0.02+0.04+0.03}{5} = 0.02\text{ s}$

% error = $\frac{0.02}{1.25} \times 100$ = 1.6%

15. $E \propto F^a A^b T^c$

$[M^1L^2T^{-2}] = [M^1L^1T^{-2}]^a [LT^{-2}]^b [T]^c$

Solving,

a = 1 ...(i)

a + b = 2 $\Rightarrow$ b = 1 ...(ii)

–2a – 2b + c = –2 $\Rightarrow$ c = 2 ...(iii)

$\Rightarrow$ a = 1 b = 1 c = 2

$E = [F]\,[A]\,[T^2]$

16. Least count = $\frac{1\text{mm}}{100} = 0.01\text{mm}$

Diameter = main scale reading + circular scale reading

Diameter = 0 + 52 × 0.01 mm

= 0.52 mm = 0.052 cm

17. E = energy = $[ML^2T^{-2}]$

G = Gravitational constant = $[M^{-1}L^3T^{-2}]$

So $\frac{E}{G} = \frac{[E]}{[G]} = \frac{ML^2T^{-2}}{M^{-1}L^3T^{-2}} = [M^2L^{-1}T^0]$

18. Plane angle and solid angle are dimensionless but have units.

19. $[MLT^{-2}A^{-2}]$ = Magnetic permeability

20. $Y = \frac{\text{stress}}{\text{strain}}$ (Strain is dimensionless)

$\therefore$ [Y] = [stress] = [Pressure]

21. $\frac{F_e}{F_G} = \frac{\frac{Kq_1q_2}{r^2}}{\frac{Gm_1m_2}{r^2}}$

$2.4 \times 10^{39} = \frac{K}{G} \times \frac{(1.6\times10^{-19})^2}{(9.11\times10^{-31}\times1.67\times10^{-27})}$

$\frac{K}{G} = \frac{2.4\times10^{39}\times15.2137\times10^{-58}}{2.56\times10^{-38}}$

$= 14.26 \times 10^{19}$

$= 1.426 \times 10^{20}$

$\approx 10^{20}$

22. Error arise due to unpredictable fluctuation in temperature and voltage supply are $\rightarrow$ random errors.

23. $\rho = \frac{M}{V}$

$\rho = \frac{M}{\pi r^2 \ell}$

$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + \frac{2\Delta r}{r} + \frac{\Delta\ell}{\ell}$

$\frac{\Delta\rho}{\rho}\% = \left[\frac{0.002}{0.4} + \frac{2(0.001)}{(0.3)} + \frac{0.02}{5}\right] \times 100\%$

$= \frac{1}{2}\% + \frac{2}{3}\% + \frac{2}{5}\%$

= 1.6%

24. Mean diameter = $\frac{d_1+d_2+d_3+d_4+d_5}{5}$

$= \frac{3.33+3.32+3.34+3.33+3.32}{5}$

$= 3.328 \approx 3.33$

25. Angular momentum = $[ML^2T^{-1}]$

Coeff of thermal conductivity = $[MLT^{-3}K^{-1}]$

Torque = $[ML^2T^{-2}]$

Gravitational constant = $[M^{-1}L^3T^{-2}]$

So, gravitational constant has power of (–1) of M.

26. Vernier Constant = MSD – VSD

$$= MSD - \frac{N}{N+1}MSD$$

$$= \frac{1}{N+1}(MSD)$$

$$= \frac{1}{N+1}(0.01)cm$$

$$= \frac{1}{100(N+1)}$$

27. Solid angle (Ω) = $\frac{A}{r^2}$

It is dimensionless quantity

So from options

Option (1) Strain & Angle both are dimensionless

28. Dimensional Formula of

$[\alpha t^2] = [F]$

$$[\alpha] = \left[\frac{MLT^{-2}}{T^2}\right] = [MLT^{-4}]$$

and $[\beta t] = [F]$

$$[\beta] = \frac{MLT^{-2}}{T} = [MLT^{-3}]$$

Option (2) is satisfied as $\frac{\alpha t}{\beta} = \frac{[MLT^{-4}][T]}{[MLT^{-3}]}$

$= [M^0L^0T^0]$

29. $R = \frac{V}{I} = \frac{200}{20} = 10\ \Omega$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I}$$

$$\frac{\Delta R}{10} = \frac{4}{200} + \frac{0.2}{20}$$

$$\Delta R = \frac{6}{200}(10) = 0.3$$

$\therefore (10 \pm 0.3)\Omega$

30. $LC = \frac{\text{pitch}}{\text{No. of divisions on circular scale}}$

$= \frac{1}{100}mm = 0.01\,mm$

Reading = MSR + LC × CSR

= 1 + 0.01 × 63

= 1.63 mm

= 0.163 cm

31. $V = \frac{AX^2}{\sqrt{x}+B}$

$[B] = [\sqrt{x}] \Rightarrow [B] = L^{1/2}$

$$[V] = \left[\frac{AX^2}{\sqrt{x}+B}\right] \Rightarrow ML^2T^{-2} = \frac{[A]L^2}{\sqrt{L}}$$

$ML^2T^{-2} = [A]\ L^{3/2} \Rightarrow [A] = ML^{1/2}T^{-2}$

$$\frac{A^2}{B} = \frac{M^2 \perp T^{-4}}{L^{1/2}} = M^2L^{1/2}T^{-4}$$

Exercise-III (Analytical Questions)

2. Reason is false because error in measurement can have any value.

eg. :- A student can read 23 as 25 from the device from which he is measuring a physical quantity.

3. Assertion is true because lower least count means lower error.

Since accuracy increases when least count decreases.

Reason is correct explanation because higher accuracy indicates error will be smaller.

4. Relative error is unitless and dimensionless. Absolute error has dimension.

5. Least count = $\frac{\text{Pitch}}{\text{No. of circular scale divisions}}$

$\therefore$ Statement I is false.

Statement II is true because smaller pitch means, smaller least count. $\therefore$ Greater accuracy.

6. [Stress] = [Modulus of Elasticity] = [Pressure] = [Energy density] = $[ML^{-1}T^{-2}]$

[Strain] = [Angle] = $[M^0L^0T^0]$

[Torque] = [Energy] = $[ML^2T^{-2}]$

7. Electrical resistance [Use $P = I^2R$]

$$R = \frac{[P]}{[I^2]} = \frac{[ML^2T^{-3}]}{[A^2]} = [ML^2T^{-3}A^{-2}]$$

Capacitance $\left[\text{use}\quad U = \frac{1}{2}\frac{Q^2}{C}\right]$

$$[C] = \frac{[Q^2]}{[U]} = \frac{[AT]^2}{[ML^2T^{-2}]} = [M^{-1}L^{-2}T^4A^2]$$

Magnetic field [use F = ILB]

$$[B]=\frac{[F]}{[I][L]}=\frac{[MLT^{-2}]}{[A][L]}=[MT^{-2}A^{-1}]$$

Inductance [use $U = \frac{1}{2}LI^2$]

$$[L]=\frac{[U]}{[I]^2}=\frac{[ML^2T^{-2}]}{[A^2]}=[ML^2T^{-2}A^{-2}]$$

8. Boltzmann constant (use $E = \frac{3}{2}KT$]

$[K] = ML^2T^{-2}K^{-1}$

Coefficient of viscosity $\left(\text{use } \eta=\frac{F}{A}\left(\frac{\Delta x}{\Delta v}\right)\right)$

$[\eta] = [ML^{-1}T^{-1}]$

Planck's constant (use $E = h\nu$)

$[h] = [ML^2T^{-1}]$

Thermal conductivity $\left(\text{use } K=\frac{i_t L}{A\Delta\theta}\right)$

$[K] = [MLT^{-3}K^{-1}]$

9. [Energy density] = [Young's modulus] = $[ML^{-1}T^{-2}]$

10. [h] = [Angular impulse] = [Angular momentum]

= $[ML^2T^{-1}]$