Rotation Motion

SOLUTIONS

Exercise-I (Conceptual Questions)

1. Physical quantity and its units.

2. Particles present on the axis of rotation are not performing circular motion.

3. During rotation angular velocity is constant linear velocity will be more if distance from the axis of rotation of earth will be more.

4. For rotational motion we use moment of inertia.

5. Moment of inertia depends on :

(a) mass of the body

(b) mass distribution of the body

(c) position of axis of rotation

6. M.I is proportional mass distribution of the body

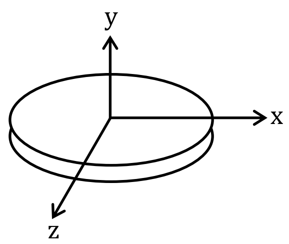
So, to increase M.I of a fly wheel its mass is concentrated at its rim.

7. $\frac{ML^2}{12}+\frac{MR^2}{4}=\frac{MR^2}{2}\Rightarrow\frac{L^2}{12}=\frac{R^2}{4}\Rightarrow L=\sqrt{3}\ R$

8. $I = I_{disc} + I_{Masses} = \frac{MR^2}{2} + 4mR^2$

9. For perpendicular axes

$I_y = I_x + I_z$



10. The theorem of perpendicular axis is applicable only for 2-D objects.

11. As disc is lying in the x-z plane, so applying perpendicular axis theorem :-

$I_x + I_z = I_y$

$30 + I_z = 40$

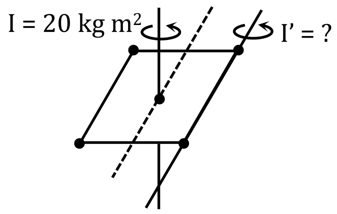
$\Rightarrow \quad I_z = 40 - 30 = 10\ kg\ m^2$

12. given $I_{solid\ sphere} = I_{hollow\ sphere}$

$\Rightarrow \quad \frac{2}{5}Mr_1^2=\frac{2}{3}Mr_2^2 \Rightarrow \frac{r_1^2}{r_2^2}=\frac{5}{3}$

$\Rightarrow \quad \frac{r_1}{r_2}=\sqrt{5}:\sqrt{3}$

13.

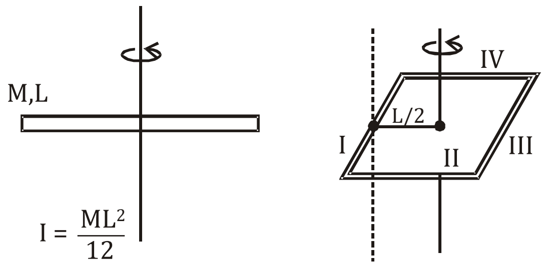


as, $\quad I=\frac{ML^2}{6}=20 \quad \Rightarrow \quad ML^2 = 120$

$I'=I_c+M\left(\frac{L}{2}\right)^2=\frac{ML^2}{12}+\frac{ML^2}{4}$

$\Rightarrow \quad I'=\frac{ML^2}{3}=\frac{120}{3} = 40\ kg\ m^2$

14.



for one rad, $I_1=\frac{ML^2}{12}+M\left(\frac{L}{2}\right)^2=\frac{4ML^2}{12}$

$I_{total} = 4I_1 = 4\times\frac{4ML^2}{12}=\frac{16ML^2}{12}$

so, $\quad I_{total} = 16I$

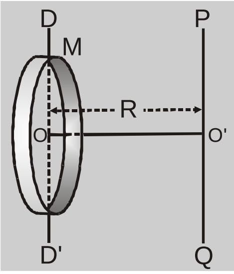
15. Using parallel axis theorem

$I = \frac{2}{5}MR^2+MX^2$

Graph is parabola

For $X = 0$, $I = \frac{2}{5}MR^2$

$\therefore$ Correct option (2)

16. 

Applying parallel axis theorem

$I_{PQ} = I_C + Md^2$

$\Rightarrow \quad I_{PQ} = \frac{MR^2}{2} + M(R)^2 = \frac{3}{2}MR^2$

17. $I = m(a)^2 + m(a)^2 + m(0)^2 + m(2a)^2$

$\Rightarrow \quad I = 6ma^2$

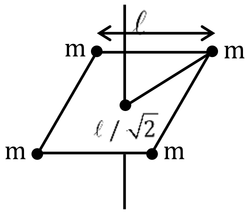
18. $I_{(1)} = \frac{2}{3}MR^2$, $I_{(2)} = \frac{2}{5}MR^2$, $I_{(3)} = \frac{MR^2}{2}$, $I_{(4)} = MR^2$

19. $I = MK^2 \Rightarrow I = 10(0.4)^2 = 1.6$ kg-m^2

20. $I_1 = \frac{2}{5}MR^2$; $I_2 = \frac{2}{3}MR^2$; $I_3 = MR^2$

21. $I_z = \frac{ML^2}{3} + \frac{ML^2}{3} + 0 \Rightarrow I_z = \frac{2}{3}ML^2$

22. so, $I = m \times \left(\frac{\ell}{\sqrt{2}}\right)^2 \times 4$



also, $I = MK^2$

{Where M = 4m}

so, $m \times \frac{\ell^2}{2} \times 4 = 4m \times K^2$

$\therefore K = \frac{\ell}{\sqrt{2}}$

23. Using parallel axes theorem

$I = I_{CM} + Md^2$

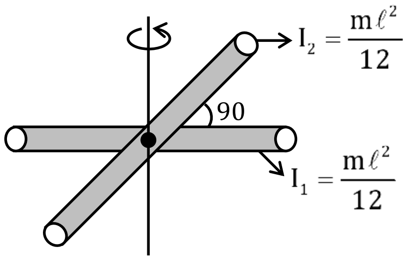
$I = \frac{ML^2}{12} + M\left(\frac{L}{2}\right)^2$

$I = \frac{ML^2}{3}$

24. M.I is proportional mass distribution of the body

So, to increase M.I of the disc its heavier mass is concentrated far from the axis of rotation.

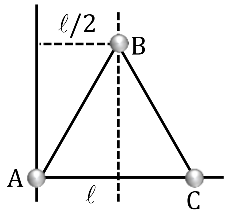
25. The moment of inertia in rotational motion is equivalent to mass of linear motion.

26. 

So $I = I_1 + I_2 = \frac{m\ell^2}{12} + \frac{m\ell^2}{12} = \frac{m\ell^2}{6}$

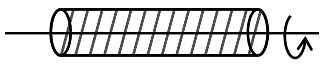
27. $K = \sqrt{\frac{I}{M}}$

$\frac{K_1}{K_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{\frac{5}{4}MR^2}{\frac{3}{2}MR^2}} = \sqrt{\frac{5}{6}}$

28. 

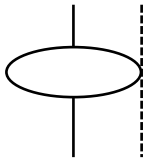
$I = I_A + I_B + I_C$

$\Rightarrow \quad I = 0 + m\left(\frac{\ell}{2}\right)^2 + m\ell^2 = \frac{5}{4}m\ell^2$

29. $I = \frac{MR^2}{2}$ 

$= \frac{20 \times (0.2)^2}{2} = 0.4$ kg-m^2

30. Using parallel axis theorem

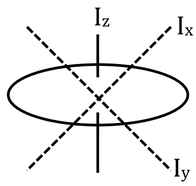


$I = MR^2 + MR^2$

$I = 2MR^2$

MR^2

31. Along diameter in the plane.



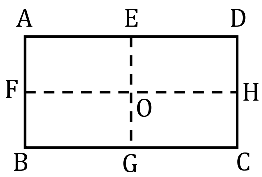
Using Axis theorem

$I_x + I_y = I_z$

$2I_x = I_z$

$I_x = \frac{MR^2}{2}$

32. The moment of inertia is minimum about FH because mass distribution is at minimum distance from FH.



33. $I_R = MR^2$, $I_D = MR^2/2$

$I_{HS} = \frac{2}{3}MR^2$ $I_{SS} = \frac{2}{5}MR^2$

34. $\tau = I\alpha$

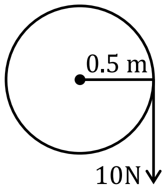
$\tau \propto I$ i.e, moment of inertia

35. $\vec{\tau} = I\vec{\alpha}$ like $\vec{F} = m\vec{a}$

$\vec{\tau} = \frac{d\vec{L}}{dt}$ like $\vec{F} = \frac{d\vec{p}}{dt}$

36. Angular acceleration will be more if M.I will be less. ($\vec{\tau} = I\vec{\alpha}$)

37. So, torque = 10 × 0.5 = 5 N-m



38. $\tau = I\alpha \Rightarrow \quad \alpha = \frac{\tau}{I}$

so, $\alpha = \frac{5}{2}$ = 2.5 rad/sec^2

39. $\omega = \omega_0 + \alpha t$

$\Rightarrow \quad \omega = 0 + 2.5 \times 2 = 5$ rad/sec

40. as, $\tau = \frac{\Delta L}{\Delta t}$

so, $\Delta L = \tau \times \Delta t$

$\Rightarrow \quad \Delta L$ = change in angular momentum = 5 × 2 = 10 N-m sec

41. $\theta = \omega_0 t + \frac{1}{2}\alpha t^2$

$\Rightarrow \quad \theta = 0 \times 2 + \frac{1}{2} \times 2.5 \times (2)^2 = 5$ radian

42. as, torque = $I\alpha$

$\Rightarrow \quad \tau = mK^2\alpha$ {where K = radius of gyration}

43. Power, $P = \tau \times \omega$

so, $\tau = \frac{P}{\omega} = \frac{1200}{600}$ = 2 N-m

44. $\vec{\tau} = \frac{d\vec{L}}{dt}$

45. Frequency, n = 20 Hz

$\omega_i = 2\pi n = 40\pi$ rad/sec

$\omega_f = 0$, t = 10 sec

so, $\omega_f = \omega_i + \alpha t$

$0 = 40\pi + \alpha \times 10$

so, $\alpha = -4\pi$ rad/sec^2

Now, $\tau = I\alpha = 5 \times 10^{-3} \times 4\pi = 2\pi \times 10^{-2}$ N-m

46. f = 60 rpm = $\frac{60}{60}$ rps $= 1$ rps

I = 2 kg-m^2

$\omega = \omega_0 + \alpha t$

$0 = 2\pi + \alpha(60)$

$\alpha = -\frac{2\pi}{60} = -\frac{\pi}{30}$ $\therefore$ (Retardation)

$\tau = I\alpha$

$\tau = 2 \times \frac{\pi}{30} = \frac{\pi}{15}$

47. Torque = $\frac{\text{Change in angular momentum}}{\text{time}}$

$\tau = \frac{L_2 - L_1}{t} = \frac{4A_0 - A_0}{4}$

$\Rightarrow \quad \tau = \frac{3A_0}{4}$

48. $\vec{\tau} = I\vec{\alpha}$

$\vec{\tau}$ = constant

$\vec{\alpha}$ = constant, if I = constant

49. $\vec{\tau} = I\vec{\alpha} = \frac{d\vec{L}}{dt}$

$\vec{\tau} = 0 \Rightarrow \vec{L}$ = constant angular momentum

50. given, $\omega_0 = 20$ rad/sec

$\omega = 0$

I = 50 kg-m^2

t = 10 sec

$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - 20}{10} = -2$ rad/sec^2

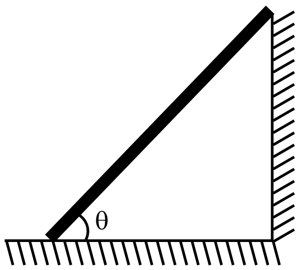
and $\tau = I\alpha = 50 \times 2 = 100$ kg-m^2/s^2 = 100 N-m

51. Torque $\vec{\tau} = \vec{r} \times \vec{F}$

$\therefore \vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 0.5 & 1 \\ 2 & 0 & -3 \end{vmatrix}$

$\vec{\tau} = \left(-1.5\hat{i} - 4\hat{j} - \hat{k}\right)$ N-m

52.



Increasing angle causes equilibrium.

53. J = Iω where 'I' is constant. So graph will be straight line.

54. Angular momentum is an axial vector.

55. $I = mR^2 = 10(0.2)^2 = 0.4$ kg-m^2

$\omega = \dfrac{1200 \times 2\pi}{60}$ rad/sec.

$\omega = 40\pi$ rad/s

Angular Momentum $L = I\omega = 16\pi$ J-s

≈ 50.25 J$-$s

56. given, $K_1 = K_2$

$\Rightarrow \dfrac{L_1^2}{2I_1} = \dfrac{L_2^2}{2I_2} \quad \Rightarrow \dfrac{L_1^2}{2 \times 9} = \dfrac{L_2^2}{2 \times 1}$

$\Rightarrow \dfrac{L_1^2}{L_2^2} = \dfrac{9}{1}$ or $\dfrac{L_1}{L_2} = \dfrac{3}{1}$

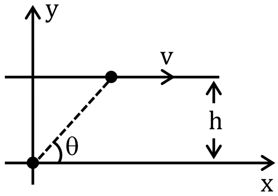
57. $f = 0.5$

$\omega = 2\pi f = \pi$

$L = I\omega$

$= 0.6\pi$ kg $\times \dfrac{m^2}{S}$

58. Constant velocity



Angular momentum = $m\vec{v}_1\vec{r}$

= mvh = constant

59. Angular momentum is an axial vector.

60. Relation between, kinetic energy (K) and angular momentum (L) is :-

$L = \sqrt{2IK}$ {where, I = moment of inertia}

so, $\dfrac{L_1}{L_2} = \sqrt{\dfrac{I_1}{I_2}} = \sqrt{\dfrac{I}{2I}} = 1 : \sqrt{2}$

61. Rotational kinetic energy of a body,

$K_{rot} = \dfrac{1}{2} I\omega^2$

or, $I\omega^2 = 2K_{rot}$ or $\omega = \sqrt{\dfrac{2K_{rot}}{I}}$ ....(i)

Angular momentum of a body $L = I\omega$

or, $L = I\sqrt{\dfrac{2K_{rot}}{I}} = \sqrt{2IK_{rot}}$ [From (i)]

62. M = 10 kg

K = 0.1 m

$\omega = 10$ rad/sec

angular momentum (L) = $I\omega$

$= MK^2\omega = 10 \times (0.1)^2 \times 10$

$\Rightarrow L = 1$ kg m^2/s

63. Here, Mass, M = 1.0 kg

Diameter, D = 2.0 m

$\therefore$ Radius, $R = \dfrac{D}{2} = 1.0$ m

The moment of inertia of the body,

$I = MR^2 = (1.0 \text{ kg})(1.0\text{m})^2 = 1.0$ kg m^2

The angular velocity of the body,

$\omega = 2\pi\upsilon = 2 \times 3.14 \times \dfrac{10}{31.4}$ rad / s

= 2 rad/s

The angular momentum of the body,

$L = I\omega = (1.0 \text{ kg m}^2)(2\text{rad/s})$

= 2 kg m^2/s

64. Centripetal force, $F_C = \dfrac{mv^2}{r}$ ...(i)

and angular momentum, L = mvr

so, $v = \dfrac{L}{mr}$

putting value of v in (i),

$F_C = \dfrac{m}{r} \times \left(\dfrac{L}{mr}\right)^2$

$\Rightarrow F_C = \dfrac{m}{r} \times \dfrac{L^2}{m^2r^2} = \dfrac{L^2}{mr^3}$

65. Since torque of string on stone is zero, the angular momentum will be conserved

66. $\omega_1 = \dfrac{2\pi}{10} = \dfrac{\pi}{5}$ rad/s.

By conservation of Angular momentum

$I_1\omega_1 = I_2\omega_2$

$\Rightarrow 100\left(\dfrac{\pi}{5}\right) = [100 + 50(2)^2]\omega$

$\Rightarrow \omega = \dfrac{\pi}{15}$ or $\dfrac{2\pi}{30}$

67. On melting water will be more at equator which increase "r", $\therefore$ moment of inertia increases.

68. On loosing atmosphere, moment of inertia of earth decreases so, ω increases and time period T decreases.

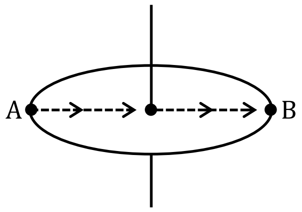
$\left\{\because T = \dfrac{2\pi}{\omega}\right\}$

69. $m\uparrow \; I\uparrow$

70. $I\omega$ = constant

$I\uparrow \; \omega\downarrow$

71.



Initially as ant move towards axis I decreases so ω increases.

After crossing axis I increases and ω decreases.

so, first ω increases then ω decreases.

72. $\vec{\tau} = \dfrac{d\vec{L}}{dt}$

$\vec{\tau} = 0$

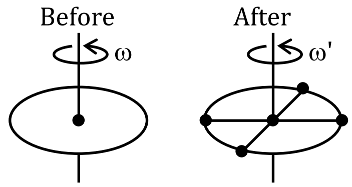
$\vec{L}$ = constant

73. As person is coming towards axis, it's distance from axis decreases. So M.I also decreases.

From conservation of angular momentum :-

as I decreases so, ω increases

74.



Applying conservation of angular momentum

$I_1\omega_1 = I_2\omega'$

$Mr^2 \times \omega = (Mr^2 + 4mr^2)\omega'$

so, $\omega' = \dfrac{Mr^2\omega}{r^2(M+4m)} = \dfrac{M\omega}{M+4m}$

75. Applying conservation of angular momentum

$I_1\omega = (I_1 + I_2)\omega'$

so, $\omega' = \dfrac{I_1\omega}{(I_1+I_2)}$

76. $\vec{\tau} = \dfrac{d\vec{L}}{dt}$

$\vec{L}$ = Angular momentum

77. As the angular momentum is conserved

$\therefore I_1\omega_1 = I_2\omega_2$

or, $mr^2 2\omega = m(r/2)^2\omega_2$ $(\omega_1 = 2\omega)$

$\omega_2 = 8\omega$.

78. Angular momentum is conserved,

i.e. L = Iω = constant

$\therefore I_1\omega_1 = I_2\omega_2$

$\dfrac{2}{5}MR^2.\dfrac{2\pi}{T_1} = \dfrac{2}{5}M\left(\dfrac{R}{2}\right)^2.\dfrac{2\pi}{T_2}$

$\Rightarrow \dfrac{R^2}{T} = \dfrac{R^2}{4}\dfrac{1}{T_2} \Rightarrow T_2 = \dfrac{T_1}{4} = \dfrac{24\text{hr.}}{4}$

T_2 = 6hr.

79. Direction of linear momentum is variable.

80. Frequency of rotation = n Hz

so, $\omega = 2\pi n$

and kinetic energy, $K = \dfrac{1}{2}I\omega^2$

so, $K = \dfrac{1}{2}\times\dfrac{mL^2}{3}\times(4\pi^2\times n^2)$

$\Rightarrow$ $K = \dfrac{2}{3}mL^2\pi^2n^2$

81. If torque is absent, Angular momentum will be conserved.

E = $\dfrac{L^2}{2I} \Rightarrow E \propto \dfrac{1}{I}$

$I \propto m$ & $I \propto K^2$

So $\dfrac{E_1}{E_2} = \dfrac{m_2}{m_1}\times\left(\dfrac{K_2}{K_1}\right)^2 = 2 \Rightarrow E_2$ $\dfrac{1}{2}$ = E_1 = 0.5 E

82. Kinetic energy of particle

$K = \dfrac{1}{2}mv^2$ ...(i)

and angular momentum, L = mvr

so, $v = \dfrac{L}{mr}$

putting v in (i), $K = \dfrac{1}{2}m\left(\dfrac{L}{mr}\right)^2 = \dfrac{1}{2}m\times\dfrac{L^2}{m^2r^2}$

so, $K = \dfrac{L^2}{2mr^2}$

83. as, $K = \dfrac{1}{2}I\omega^2 = \dfrac{1}{2}(I\times\omega)\times\omega$

$\Rightarrow$ $K = \dfrac{1}{2}L\omega$ or $L = \dfrac{2K}{\omega}$

so, $\dfrac{L_2}{L_1} = \dfrac{K_2}{K_1}\times\dfrac{\omega_1}{\omega_2} = \dfrac{1}{2}\times\dfrac{1}{2}$

$\Rightarrow$ $\dfrac{L_2}{L_1} = \dfrac{1}{4}$ or $L_2 = \dfrac{L}{4}$

84. Revolutions per minute (N) $= \frac{3000}{\pi}$ rpm

so, frequency, $n = \frac{N}{60} = \frac{3000}{60 \times \pi} = \frac{50}{\pi}$ Hz

and $\omega = 2\pi n = 2\pi \times \frac{50}{\pi}$ = 100 rad/sec

I = 400 kg m^2

so rotational kinetic energy (K) is

$$K = \frac{1}{2} I\omega^2 = \frac{1}{2} \times 400 \times (100)^2 = 2 \times 10^6 \text{ J}$$

85. Translational KE = $\frac{1}{2}mv^2$ = E

Total KE = $\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

$= \frac{1}{2}mv^2$ (1 + 1) = 2E

86. $\frac{K_{Slide}}{K_{Roll}} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)}$

$= \frac{1}{1+1}$ = 1 : 2

87. For pure rolling

$K_{Rotation} = \frac{1}{2}mv^2 \times \frac{K^2}{R^2}$

and $K_{Total} = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

so $\frac{K_{Rotation}}{K_{Total}} = \frac{\frac{K^2}{R^2}}{\left(1 + \frac{K^2}{R^2}\right)}$

for solid sphere, $\frac{K^2}{R^2} = \frac{2}{5}$

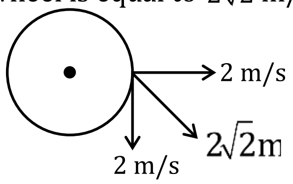
so, $\frac{K_{Rotation}}{K_{Total}} = \frac{\frac{2}{5}}{1 + \frac{2}{5}} = \frac{2}{7}$

88. $\frac{K_{Rotation}}{K_{Total}} = \frac{\frac{K^2}{R^2}}{1 + \frac{K^2}{R^2}}$

For disc, $\frac{K^2}{R^2} = \frac{1}{2}$,

so, fraction $= \frac{\frac{1}{2}}{1 + \frac{1}{2}} = 1:3$

89. Different parts of the rolloing wheel move with the same linear and angular speed. The magnitude of the linear velocity of the points at the extermities of the horizontal diameter of the wheel is equal to $2\sqrt{2}$ m/s



90. Given, $\frac{K_{Rotation}}{K_{Total}} \times 100 = 50\%$

$\Rightarrow \frac{\frac{K^2}{R^2}}{1 + \frac{K^2}{R^2}} \times 100 = 50 \quad \Rightarrow \frac{\frac{K^2}{R^2}}{1 + \frac{K^2}{R^2}} = \frac{1}{2}$

so, $\frac{K^2}{R^2} = 1$, so it is a ring.

91. Total energy = $\frac{1}{2}mv^2\left[1 + \frac{K^2}{R^2}\right]$

Rotational energy $\frac{1}{2}I\omega^2$ = or $\frac{1}{2}mv^2\left[\frac{K^2}{R^2}\right]$

92. Rotational kinetic energy = $\frac{1}{2}I\omega^2$

$K_1 = \frac{1}{2}I_1\omega_1^2 ; K_2 = \frac{1}{2}I_2\omega_2^2$

$\therefore \frac{K_2}{K_1} = \left(\frac{I_2}{I_1}\right)\left(\frac{\omega_2}{\omega_1}\right)^2 = \left(\frac{I_1}{2I_1}\right)\left(\frac{2\omega_1}{\omega_1}\right)^2 = \frac{2}{1}$

or $K_2 = 2K_1$

Rotational KE will be doubled.

93. $\frac{K_{Rot.}}{K_{Total}} = \frac{\frac{K^2}{R^2}}{1 + \frac{K^2}{R^2}} = \frac{\frac{2}{5}}{1 + \frac{2}{5}}$

= 2 : 7

94. as, $KE = \dfrac{L^2}{2I}$

if L = constant,

then $KE \propto \dfrac{1}{I}$

as $I_A > I_B$

so $(KE)_A < (KE)_B$

95. Kinetic energy of ring,

$$KE = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$$

$$= \frac{1}{2}0.4\times\left(\frac{10}{100}\right)^2\times(1+1)\left[\because \frac{K^2}{R^2}=1\right]$$

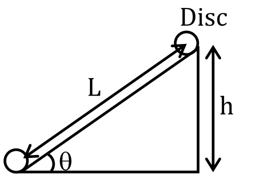
$\Rightarrow$ $KE = 4 \times 10^{-3}$ J

96. L = Angular momentum

$\Rightarrow L = I\omega$

Rot. KE = $\dfrac{1}{2}I\omega^2 = \dfrac{I^2\omega^2}{2I} = \dfrac{L^2}{2I}$

97.



Applying conservation of energy,

$$mgh = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$$

also, $h = L\sin\theta$ and $\dfrac{K^2}{R^2} = \dfrac{1}{2}$

so, $mgL\sin\theta = \dfrac{1}{2}mv^2\left(1+\dfrac{1}{2}\right)$

on solving, $v = \sqrt{\dfrac{4gL\sin\theta}{3}}$

98. Acceleration of a purely rolling object on an inclined plane is :-

$$a = \frac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}$$

for spherical shell, $\dfrac{K^2}{R^2} = \dfrac{2}{3}$

for solid cylinder, $\dfrac{K^2}{R^2} = \dfrac{1}{2}$

so, $$\frac{a_{shell}}{a_{cylinder}} = \frac{\dfrac{g\sin\theta}{\left(1+\frac{2}{3}\right)}}{\dfrac{g\sin\theta}{\left(1+\frac{1}{2}\right)}} = \frac{9}{10}$$

99. t_1 = time for sliding = $\sqrt{\dfrac{2L}{a_s}}$

t_2 = time for rolling = $\sqrt{\dfrac{2L}{a_R}}$

$\Rightarrow t_1 : t_2 = \sqrt{a_R} : \sqrt{a_s}$

$a_R = \dfrac{mg\sin\theta}{m+\dfrac{I}{R^2}} = \dfrac{g\sin\theta}{2} = \dfrac{1}{2}a_s$

$\Rightarrow$ $t_1 : t_2 = 1 : \sqrt{2}$

100. $t_{rolling} > t_{sliding}$

101. As we know, $t = \dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left(1+\dfrac{K^2}{R^2}\right)}$

For sphere, $\dfrac{K^2}{R^2} = \dfrac{2}{5}$ & For disc, $\dfrac{K^2}{R^2} = \dfrac{1}{2}$

so, $$\frac{t_{sphere}}{t_{disc}} = \frac{\dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left(1+\dfrac{2}{5}\right)}}{\dfrac{1}{\sin\theta}\sqrt{\dfrac{2h}{g}\left(1+\dfrac{1}{2}\right)}}$$

102. $v \propto \sqrt{a}$

$\dfrac{v}{v_R} = \sqrt{\dfrac{a}{a_R}}$ & $a_R = \dfrac{g\sin\theta}{1+\dfrac{I}{mR^2}} = \dfrac{g\sin\theta}{2}$

$a = g\sin\theta \Rightarrow v_R = \dfrac{v}{\sqrt{2}}$

103. Percentage of total energy which is rotational is

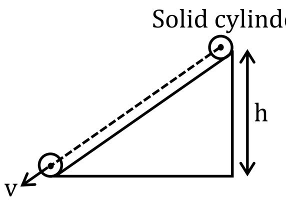
$$= \frac{\frac{K^2}{R^2}}{\left(1+\frac{K^2}{R^2}\right)}\times 100$$

$$= \frac{\frac{2}{5}}{1+\frac{2}{5}}\times 100 \left\{\because \text{for sphere}\frac{K^2}{R^2} = \frac{2}{5}\right\}$$

$$= \frac{2}{7}\times 100 \approx 28\%$$

104. For rolling we have both translational and rotational motion together.

105.



Applying conservation of energy :-

mgh = K_{Total}

so $\quad mgh = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right)$

$$mgh = \frac{1}{2}mv^2\left(1+\frac{1}{2}\right)$$

on solving, $\quad v = \sqrt{\frac{4}{3}gh}$

106. The magnitude changes and the direction of angular momentum does not change when the cylinder rolls down a slope.

107. For rolling motion on an inclined plain acceleration is given by $a_R = \dfrac{g\sin\theta}{\left(1+\frac{K^2}{R^2}\right)}$

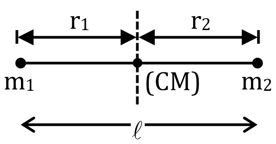
Exercise-II (Previous Year Questions)

13. $E_{sphere} = \frac{1}{2}I_s\omega^2 = \frac{1}{2}\times\frac{2}{5}MR^2\times\omega^2$

$E_{cylinder} = \frac{1}{2}I_c(2\omega)^2 = \frac{1}{2}\times\frac{MR^2}{2}\times4\omega^2$

$$\frac{E_{sphere}}{E_{cylinder}} = \frac{1}{5}$$

14.



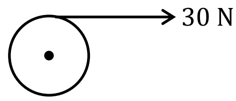
$r_1 = \frac{m_2\ell}{m_1+m_2}$, $r_2 = \frac{m_1\ell}{m_1+m_2}$

$$I_{cm} = m_1r_1^2 + m_2r_2^2 = \frac{m_1m_2}{m_1+m_2}\ell^2$$

OR

$$I_{cm} = \mu\ell^2 = \frac{m_1m_2}{m_1+m_2}\ell^2$$

15. $\tau = I\alpha$



$RF = mR^2\alpha$

$$\alpha = \frac{F}{mR} = \frac{30}{3\times\frac{40}{100}} = 25\ rad/s^2$$

16. COAM : $I\omega_1 + I\omega_2 = 2I\omega \Rightarrow \omega = \frac{\omega_1+\omega_2}{2}$

$(K.E.)_i = \frac{1}{2}I\omega_1^2 + \frac{1}{2}I\omega_2^2$

$(K.E.)_f = \frac{1}{2}\times2I\omega^2 = I\left(\frac{\omega_1+\omega_2}{2}\right)^2$

Loss in K.E. = $(K.E.)_i - (K.E)_f = \frac{I}{4}(\omega_1-\omega_2)^2$

17. Centre of mass may lie on centre of gravity net torque of gravitational pull is zero about centre of mass.

Mechanical advantage = $\frac{Load}{Effort} > 1$

$\Rightarrow$ Load > Effort

18. W = loss in KE = $\frac{1}{2}I\omega^2 \propto I$

$I_A = \frac{2}{5}MR^2 = 0.4MR^2$

$I_B = \frac{1}{2}MR^2 = 0.5MR^2$

$I_C = MR^2$

$\therefore \quad W_C > W_B > W_A$

19. Here $\vec{F} = 4\hat{i}+5\hat{j}-6\hat{k}$ and

$\vec{r} = (2-2)\hat{i} + (0-(-2))\hat{j} + (-3-(-2))\hat{k} = 2\hat{j} - 1\hat{k}$

$$\vec{r}\times\vec{F} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 2 & -1\\ 4 & 5 & -6\end{vmatrix} = -7\hat{i} - 4\hat{j} - 8\hat{k}$$

20. $K_t = \frac{1}{2}mv^2$

$K_t + K_r = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = \frac{1}{2}mv^2\left(1+\frac{2}{5}\right)$

$$\therefore \frac{K_t}{K_t+K_r} = \frac{1/2\ mv^2}{1/2\ mv^2\left(1+\frac{2}{5}\right)} = \frac{5}{7}$$

21. $\tau_{ext} = 0 \Rightarrow \frac{d\vec{L}}{dt} = 0 \Rightarrow \vec{L}$ = constant

22. $W_{all} = \Delta KE$

$\Rightarrow \quad W = 0 - \frac{1}{2}mv_{cm}^2\left(1+\frac{K^2}{R^2}\right)$

$\Rightarrow \quad W = -3J$

23. $\theta = 2\pi \times 2\pi$ radian

$\omega_0 = 3$ rpm $\Rightarrow \frac{2\pi}{60}(3)\frac{rad}{sec}$

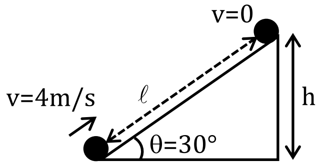
$\omega^2 = \omega_0^2 - 2\alpha\theta$

$0 = \left(\frac{3\times 2\pi}{60}\right)^2 - 2\alpha(4\pi^2)$

$\therefore \quad \alpha = \frac{1}{800}$ rad/s²

$\tau = \frac{mR^2}{2}\alpha = \frac{2}{2}\times\left(\frac{4}{100}\right)^2\times\frac{1}{800} = 2\times 10^{-6}$ Nm

24.



$\frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) = mgh$

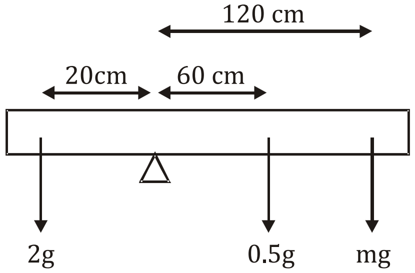
$\Rightarrow 8\left(1+\frac{1}{2}\right) = 10h \Rightarrow h = 1.2$ m

$\frac{h}{\ell} = \sin 30° \Rightarrow \ell = 2.4$ m

25. $\vec{F} = 3\hat{j}N$, $\vec{r} = 2\hat{k}$

$\vec{\tau} = \vec{r}\times\vec{F} = 2\hat{k}\times 3\hat{j} = 6(\hat{k}\times\hat{j}) = 6(-\hat{i}) = -6\hat{i}$ Nm

26.



By balancing torque

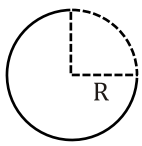
2g × 20 = 0.5 g × 60 + mg × 120

$m = \frac{0.5}{6}kg = \frac{1}{12}kg$

27. $M_{remain} = \frac{3}{4}M$

$I = M_{remain}R^2$

$= \frac{3}{4}MR^2$

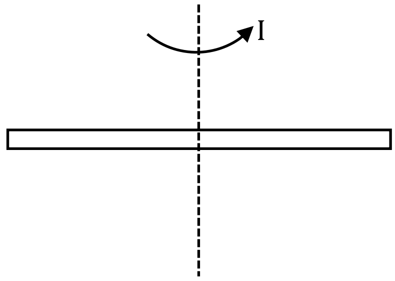


30. Radius of gyration : $K = \sqrt{\frac{I}{m}}$

$\frac{k_{solid\ sphere}}{k_{hollow\ sphere}} = \sqrt{\frac{2mR^2/5m}{2mR^2/3m}} = \sqrt{3}:\sqrt{5}$

31. I = 2400 gcm²

m = 400 g



$I = \frac{ML^2}{12}$

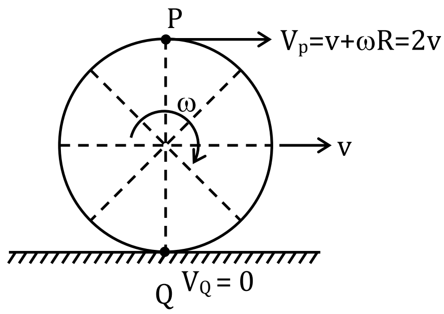
$2400 = \frac{400\times L^2}{12}$

$\Rightarrow L^2 = 72$

$\Rightarrow L = \sqrt{72} \approx 8.5$ cm

32. In case of Pure Rolling :-

$v = \omega R$



$\therefore V_p = 2V$

$V_Q = 0$

$\therefore$ Point 'P' moves faster than point Q.

33. I = MK²

$\frac{7}{5}MR^2 = MK^2$

$\Rightarrow \frac{7}{5}\frac{25X^2}{7} = (5)^2$

$x = \sqrt{5}$

Exercise-III (Analytical Questions)

3. **Axis-1:**

$(\text{M.I.})_{\text{Rod Hz.}} = \dfrac{m\ell^2}{3}$; $(\text{M.I.})_{\text{Rod Vert.}} = 0$

Axis-2:

$(\text{M.I.})_{\text{Rod Hz.}} = 0$; $(\text{M.I.})_{\text{Rod Vert.}} = \dfrac{m\ell^2}{3}$

Axis-3:

$(\text{M.I.})_{\text{Rod Hz.}} = \dfrac{m\ell^2}{3}$; $(\text{M.I.})_{\text{Rod Vert.}} = \dfrac{m\ell^2}{3}$

Axis-4:

$(\text{M.I.})_{\text{Rod Hz.}} = \dfrac{m\ell^2}{3}$; $(\text{M.I.})_{\text{Rod Vert.}} = mL^2$

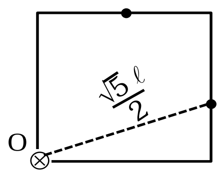
4. about (A) $10(2)\hat{k}+6(2)\hat{k}=32\hat{k}$

about (B) $10 \times 2\hat{k}-10\times2\hat{k}+6\times2\hat{k}-12\times2\hat{k}=-12\hat{k}$

about (C) $-(10)\ (2)\ \hat{k} = -20\hat{k}$

about (D) $-12(2)\hat{k}-10(2)\hat{k}-10(2)\hat{k}=-64\hat{k}$

6. (P)



M.O.I. about z-axis passing through 'O'

$$= 2\left[\frac{m\ell^2}{3}\right]+2\left[\frac{m\ell^2}{12}+\frac{5}{4}m\ell^2\right] = \frac{10}{2}m\ell^2$$

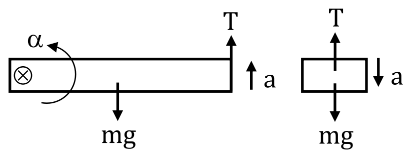
(Q) about are side $I=2\left(\dfrac{m\ell^2}{3}\right) + m\ell^2 = \dfrac{5m\ell^2}{3}$

(R) about x-axis $I=2\left(\dfrac{m\ell^2}{3}\right)+2(m)\left(\dfrac{\ell}{2}\right)^2 = \dfrac{2}{3}m\ell^2$

(S) about z-axis $I=4\left[\dfrac{m\ell^2}{12}+m\left(\dfrac{\ell}{2}\right)^2\right]$

$$=\frac{m\ell^2}{3}+m\ell^2=\frac{4m\ell^2}{3}$$

8.



$a = \ell\alpha$ ... (1)

$mg - T = ma$ ... (2)

$T\ell - mg\dfrac{\ell}{2} = I\alpha$ ... (3)

on solving (1), (2), (3)

a = 3g/8

$a = \dfrac{3g}{3\ell}$

$T = \dfrac{5mg}{8}$