Modern Physics

SOLUTIONS

EXERCISE - O

1. **Ans. (D)**

$$P=\frac{h}{\lambda};\ c=\nu\lambda \Rightarrow \boxed{P=\frac{h\nu}{C}}$$

2. **Ans. (B)**

$(E_r)_A > (E_r)_B$

$\therefore\quad P_A > P_B$

3. **Ans. (A)**

From Einstein's equation of PEE

$$\frac{1}{2}mv_1^2 = 2h\nu_0 - h\nu_0 \Rightarrow \frac{1}{2}mv_1^2 = h\nu_0 \quad ...(1)$$

$$\frac{1}{2}mv_2^2 = 5h\nu_0 - h\nu_0 \Rightarrow \frac{1}{2}mv_2^2 = 4h\nu_0 \quad ...(2)$$

Equation (1) ÷ (2)

$$\frac{v_1^2}{v_2^2}=\frac{1}{4}\Rightarrow\frac{v_1}{v_2}=\sqrt{\frac{1}{4}}=\frac{1}{2}$$

4. **Ans. (D)**

$$h(2\nu) = h\nu + \frac{1}{2}mv^2_{\max} \Rightarrow v_{\max}=\sqrt{\frac{2h\nu}{m}}$$

5. **Ans. (B)**

$$KE_1=\frac{hc}{\lambda}-\phi$$

$$KE_2=\frac{hc}{\lambda/2}-\phi=\frac{2hc}{\lambda}-\phi$$

$KE_2 = 3KE_1$

$$\Rightarrow \frac{2hc}{\lambda}-\phi=3\left(\frac{hc}{\lambda}-\phi\right)$$

$$\Rightarrow 2\phi=\frac{hc}{\lambda}\Rightarrow \phi=\frac{hc}{2\lambda}$$

6. **Ans. (B)**

$$eV_s=\frac{1}{2}mv^2_{\max}=h\nu-\phi_0$$

$2 = 5 - \phi_0 \Rightarrow \phi_0 = 3\ eV$

In second case,

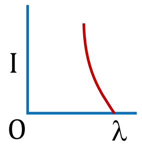
$eV_s = 6 - 3 = 3\ eV \Rightarrow V_s = 3\ V$

$\therefore\quad V_{AC} = -3\ V$

7. **Ans. (B)**

Since saturation current is different.

8. **Ans. (B)**



If λ is increased, there will be a value of λ above which photo-electron will be cease to come out, so, photo current will become zero.

9. **Ans. (D)**

Intensity is inversely proportional to square of it's distance.

$\therefore$ $d = 0.6\ m$,

Cut off voltage = $0.6\ V$

$$Is = 18\times\left(\frac{0.2}{0.6}\right)^2$$

$= 2mA$

Stopping potential would remain same.

10. **Ans. (C)**

$$E_{photon} = \frac{12400}{\lambda} = \frac{12400}{2000} = 6.2eV$$

$K.E_{\max} = E_{\text{photon}} - \phi = 1.6\ eV$

11. **Ans. (D)**

$E_{max} = 13.6\ eV$

$\therefore$ $KE_m = 9.6\ eV$

$\therefore$ $-10\ V$ will make photocurrent zero.

12. **Ans. (C)**

$$KE_{\max_1} = \frac{hC}{\lambda_1} - \phi$$

Now $\phi = \dfrac{KE_{\max_1}}{3} = 2eV$ (given)

So $\dfrac{hC}{\lambda_1}$ = 6 + 2 = 8 eV

and $\lambda_1 T_1 = \lambda_2 T_2$

So $\dfrac{hC}{\lambda_2} = 2\dfrac{hC}{\lambda_1}$ = 16 eV

$$KE_{\max_2} = \frac{hC}{\lambda_2} - \phi$$

$KE_{\max_2}$ = 16 −2 = 14 eV

13. Ans. (A)

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda' = \frac{h}{\sqrt{2m(E-U)}}$$

$$\frac{\lambda'}{\lambda} = \left(1 - \frac{U}{E}\right)^{\frac{-1}{2}}$$

$$\lambda' = \lambda\left(1 - \frac{U}{E}\right)^{\frac{-1}{2}}$$

14. Ans. (D)

By conservation of momentum

$$P_{m_1} = P_{3m_1}$$

$\therefore \quad \lambda_3 = \lambda$

15. Ans. (A)

$P = VI = 1000\ W$

16. Ans. (C)

$\lambda > \lambda_{cut}$ off

17. Ans. (A)

Characteristic X-ray have discrete λ only.

18. Ans. (D)

From Moseley's law, $\sqrt{v} = a(Z-b)$

$\frac{1}{\lambda} \propto k(Z-b)^2$ {k is a constant}

For K_α : $b = 1$

$$\frac{\frac{1}{\lambda_{K_{\alpha\, copper}}}}{\frac{1}{\lambda_{K_{\alpha\, impurity}}}}\left(\frac{Z_{Cu}-b}{Z_{imp}-b}\right)^2 \Rightarrow \frac{\lambda_{K_{\alpha\, impurity}}}{\lambda_{K_{\alpha\, icopper}}} = \left(\frac{Z_{Cu}-b}{Z_{imp}-b}\right)^2 \qquad \text{so } Z_{imp} = 26$$

19. Ans. (C)

Lithium (Li) can not emit K_β characteristic X-rays but can emit K_α characteristic X-rays

20. Ans. (B)

$E_L = -2keV$; $E_K = -20\ keV$

$E_L - E_K = (-2) - (-20) = 18\ keV$

$$\lambda = \frac{12400}{18000} = 0.688\mathring{A}$$

21. Ans. (D)

$I = I_0 e^{-kx}$

$\ln\left(\frac{I}{I_0}\right) = -kx$

In first case

$\ln(2^{-3}) = -k \times 36$

$3\ln 2 = k \times 36$

In second case, $\ln\left(\frac{1}{2}\right) = -kx$

$\ln 2 = kx$

$3 \times (kx) = k \times 36$

$x = 12$

22. Ans. (D)

Vacancy in inner shell shows there is vacancy in outer shell but vacancy in outer shell doesn't mean vacancy in inner shell.

23. Ans. (D)

$\frac{\lambda_{K\alpha}}{\lambda_{K\beta}} = \frac{(78-3)}{(78-12)}$

24. Ans. (D)

$V \propto \frac{1}{\lambda_{\min}}$

25. Ans. (A)

$P = \frac{h}{\lambda}; k.E = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2}$

Now $h\nu = K.E \Rightarrow \frac{hc}{\lambda_0} = \frac{h^2}{2m\lambda^2} \Rightarrow \lambda_0 = \frac{2m\lambda^2 C}{h}$

26. Ans. (B)

Cut off wavelength $\lambda_0 = \frac{hc}{ev_0}$

Independent of '*z*'

27. Ans. (B)

$\frac{n(m-1)}{2} = 6$

n = 4 (Final excited state)

= 3 wavelengths are shorter then λ_0

n = 2 (Initial excited state)

28. Ans. (A)

$mvr = \frac{h}{2\pi}$

$mv = \frac{h}{2\pi r}$

$\lambda = \frac{h}{m_0} = 2\pi r$

29. **Ans. (B)**

Frequency $\propto \frac{1}{n^3}$; Current = $ef \propto \frac{1}{n^3}$; $r \propto n^2$; $\vec{B} = \frac{\mu_0 i}{2r} \propto \frac{1}{n^5}$

30. **Ans. (A)**

For H-like atoms

$$v = \frac{Z}{n} \times 2.188 \times 10^6\ m/s$$

Here, $Z = 2$, $n = 3$

$$v = 1.46 \times 10^6\ m/s$$

31. **Ans. (C)**

The total energy in ground state

$$E_0 = -13.6\ eV$$

and P.E. = $2E_0$ = $-27.2\ eV$

If P.E. of ground state is zero then 27.2 eV will be added to each energy level.

So, total energy of first excited state is = $\frac{-13.6}{4} + 27.2\ eV = 23.8\ eV$

32. **Ans. (A)**

$$\frac{(n-1)(n-2)}{2} = 10$$

$\Rightarrow$ $n = 6$

33. **Ans. (C)**

$m_\mu = 207 m_e$, $M_{\text{nucleus}} = 1836\ m_e$

Reduced mass

$$\mu = \frac{mM}{M+m} = \frac{207m_e \times 1836m_e}{207m_e + 1836m_e} = 186m_e$$

$\because$ $r_1 = \frac{n^2h^2}{4\pi^2 mkze^2}$ = 0.51 Å (Given in Question)

Radius of first orbit of new atom

$$r_1 = \frac{m_e r_1}{\mu} = \frac{m_e}{186m_e} \times 0.51\text{Å} = 2.56 \times 10^{-13}\ m$$

$$E_1 = \frac{\mu}{m} E_1 = \frac{186\ m_e}{m_e}(-13.6\ eV)$$

= $-2.5\ keV$ (according to given options)

34. **Ans. (A)**

Energy of first Balmer line of hydrogen = $13.6 \times \left[\frac{1}{4} - \frac{1}{9}\right]$ ...(i)

Energy in the transition in helium = $13.6 \times (Z)^2 \left[\frac{1}{n^2} - \frac{1}{m^2}\right]$ ...(ii)

From (i) and (ii) we have

$$n = 4, m = 6$$

35. Ans. (B)

Energy of same line of Be^{+3} = 4 × energy in He^{+}.

36. Ans. (A)

One photon of 10.2 eV and an electron of energy 1.4 eV

37. Ans. (B)

$E_C - E_B = \dfrac{hc}{\lambda_1}$ ...(1)

$E_B - E_A = \dfrac{hc}{\lambda_2}$ ...(2)

$E_C - E_A = \dfrac{hc}{\lambda_3}$ ...(3)

On add equation (1) and (2)

$$E_C - E_A = hc\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)$$

$$\frac{hc}{\lambda_3}=hc\left(\frac{1}{\lambda_1}+\frac{1}{\lambda_2}\right)\Rightarrow \lambda_3=\frac{\lambda_1\lambda_2}{\lambda_1+\lambda_2}$$

38. Ans. (A)

$n = 4 \ \dfrac{n(n-1)}{2}=6\Rightarrow n=4$

$\Rightarrow$ Maximum wavelength emitted in $n = 4 \rightarrow n = 3$

39. Ans. (C)

Absorption spectra observed for electron transition from ground state to excited state.

i.e. 1, 2, 3 spectral lines.

40. Ans. (D)

$$\frac{1}{\lambda}=R_H Z^e\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]\Rightarrow \lambda\propto\frac{1}{z^2}$$

41. Ans. (D)

$$\frac{1}{\lambda}=R(1)^2\left[\frac{1}{4}-\frac{1}{9}\right]$$

$$\frac{1}{\lambda}=R\left[\frac{5}{36}\right]$$

Momentum of photon = $\dfrac{h}{\lambda}=Rh\left(\dfrac{5}{36}\right)$

$$MV=Rh\left(\frac{5}{36}\right)$$

$$V=\frac{Rh}{M}\left(\frac{5}{36}\right)$$

42. Ans. (B)

$$\Delta PE = -Ke^2\left(\frac{1}{R_2}-\frac{1}{R_1}\right),\ \frac{Mv^2}{r}=\frac{Ke^2}{r^2}$$

$$\Delta KE = Ke^2\left(\frac{1}{2R_2}-\frac{1}{2R_1}\right) \Rightarrow \frac{1}{2}mv^2=\frac{Ke^2}{2r}$$

Total decrease in energy = $\dfrac{Ke^2}{2}\left[\dfrac{1}{R_2}-\dfrac{1}{R_1}\right]$

43. Ans. (A)

$$v = v_0 \ln\left(\frac{r}{r_0}\right)$$

$$F = \left(\frac{-dv}{dr}\right) = \frac{-v_0 r_0}{r}$$

$$F \propto \frac{1}{r}$$

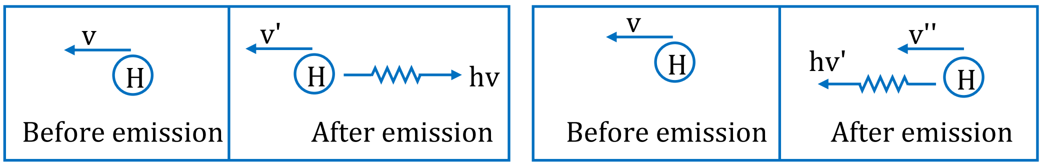
$$\frac{mv^2}{r}=\frac{k}{r}\ ;\ mvr=\frac{nh}{2\pi}$$

$$v=\sqrt{\frac{K}{m}} \Rightarrow r=\frac{nh}{2\pi m\sqrt{\frac{K}{m}}}$$

$$\boxed{r \propto n}$$

44. Ans. (B)

Case (i) Case (ii)



From momentum conservation, $v' > v''$, thus $E_2 > E_1$.

45. Ans. (A)

Let us discuss first in perfectly inelastic collision.

Conserving linear momentum, $\sqrt{2mK}+0=\sqrt{2\times(2m)K'}$ or $K'=\dfrac{K}{2}$

Loss in $KE = K - K' = K/2$ and this loss is maximum in perfectly inelastic collision $\dfrac{K}{2}\geq 10.2eV$

Hence, this collision is not possible. So, in inelastic collision loss will be less than $\dfrac{K}{2}$.

Hence, K should more than in the case of perfectly inelastic collision. Hence, collision is $1-D$ elastic collision and KE of neutron after the collision will be zero.

46. Ans. (A)

$P_i = P_p$

$$\frac{h}{\lambda_i}=\frac{h}{\lambda_f}+\frac{h}{\lambda_e}$$

$$\frac{1}{\lambda_i}>\frac{1}{\lambda_f}$$

$$\lambda_i<\lambda_f \quad [\lambda_i<\lambda_f]$$

47. Ans. (B)

By COLM

$p_f - p_i = 0$

$\Rightarrow\ p_{He} - p_{Th} = 0 \qquad \left[K.E.=\frac{p^2}{2m}\right]$

$\Rightarrow\ p_{He} = p_{Th}$

But, $K \propto \frac{1}{m}$ and $m_{He} < m_{Th}$

So, $K_{He} > K_{Th}$

48. Ans. (B)

$n + x = Y + hf + hf'$

$(A + 1)\ BE - 8A = 3$

$$BE=\frac{8A+3}{A+1}$$

49. Ans. (C)

In option (A) charge is not conserved.

i.e. $(5 + 2 \neq 7 + 1) \Rightarrow$ not possible

In (B) this reaction is possible only at very high temperature (or ${}^{23}_{11}Na$ is a very stable isotope and given nuclear reaction is very difficult).

In (C) and (D) $\bar{\nu}$ is emitted with β^- so option (D) is also wrong so correct answer is (C).

50. Ans. (C)

$C \longrightarrow B + A$

Energy released $= (m_C - m_A - m_B)C^2$

$= BE_A + BE_B - BE_C$

$= (300 + 1600 - 1200)\ MeV$

$= 700\ MeV$

51. Ans. (C)

If $Q = +ve$; energy is released.

$Q = (\text{B.E.})_f - (\text{B.E.})_i$

(C) $Q = 2(\text{B.E.})_Y - (\text{B.E.})_W$

$= 2\times510 - 900$

$= 120\ eV = +ve$

52. Ans. (D)

BE of ${}_2He^4$ = 4 × 7.06 = 28.24 MeV

BE of ${}_3^7Li$ = 7 × 5.60 = 39.20 MeV

$${}_3^7Li + {}_1^1H \longrightarrow {}_2He^4 + {}_2He^4 + Q$$

39.20 28.24 × 2

Q = 56.48 – 39.20 = 17.28 MeV

53. Ans. (B)

Let the number of α-particles emitted be x and the number of β-particles emitted be y.

Now, difference in mass number = $4x$ = 238 – 206 = 32

So, $x = 8$

Now, difference in charge number = $2x - y$ = 92 – 82 = 10

$\Rightarrow$ $16 - y = 10$

$\Rightarrow$ $y = 6$

So, Option (B) is correct.

54. Ans. (B)

The notation means that on being bombarded with neutrons the element breaks into Lithium and an α⁻ particle.

$${}_A^ZX + {}_0^1n \Rightarrow {}_{A-2}^{Z-3}Y + {}_2^4He$$

Now,

$${}_{A-2}^{Z-3}Y = {}_3^7Li$$

$\therefore$ $Z = 10$, $A = 5$

So, the element is Boron and X is ${}_5B^{10}$.

55. Ans. (C)

The range of β particle lie between zero to some maximum value.

During beta decay, electrons or photons are released. Because a neutron or an antineutron is emitted simultaneously, there is an emission spectrum of electrons or positions depending on the ratio or Q reaction energies boron by the large particle. The shape of this energy curve can be predicted from the Fermi theory of beta decay.

56. Ans. (A)

When ${}_{92}^{236}U$ breaks into different fragments then it imparts the kinetic energy to the fragments formed. Hence the rest mass energy of initial state (${}_{92}^{236}U$) is greater than the total rest mass energy of the fragments formed (final state).

57. Ans. (C)

$$4(2He^4) \Rightarrow {}_8O^{16}$$

Mass defect, $\Delta m = 4 \times 4.0026 - 15.9994 = 0.011 amu$

Energy released per oxygen nuclei will be :

$$E = 0.011 \times 931.48\, MeV = 10.24\, MeV$$

58. Ans. (B)

Let n be number of fissions per second.

Each fission produces 200 MeV.

$n \times 200 \times 10^6 eV$ is produced in one second by n fissions

But $1\,eV = 1.6 \times 10^{-19} J$

Hence, power produce $n \times 200 \times 10^6 \times 1.6 \times 10^{-19}$ Joule per second.

Also 1 J/s = 1 W and

1000 J/s = 1 kW.

The required power is 1 kW.

Hence, $\dfrac{n \times 200 \times 10^6 \times 1.6 \times 10^{-19}}{10^3} = 1$

$$n = \frac{10^3}{2 \times 1.6 \times 10^{-11}} = \frac{10^{14}}{3.2}$$

$$= \frac{10}{3.2} \times 10^{13} = 3.125 \times 10^{13}$$

59. Ans. (A)

As end products are stable so they don't have any decay constant.

60. Ans. (C)

Let initial amount of Cu be N.

Fraction decayed in 1st half life = $\dfrac{1}{2}N$

Fraction decayed in 2nd half lives = $\dfrac{1}{4}N$

Fraction decayed in 3 half lives = $\dfrac{1}{8}N$

Total amount decayed in 3 half lives = $\dfrac{1}{2}N + \dfrac{1}{4}N + \dfrac{1}{8}N = \dfrac{7}{8}N$

$\therefore$ 3 half lives = 15 minutes

$\therefore$ Half life of the Cu is 5 minutes.

61. Ans. (C)

According to question

$N_P = N_Q$

$\dfrac{4N_0}{2^{t/1}} = \dfrac{N_0}{2^{t/2}} \Rightarrow t = 4$ minute

Now number of active nuclei R is

N_R = (decay nuclei of P) + (decay nuclei of Q)

$$= \left(4N_0 - \frac{4N_0}{2^4}\right) + \left(N_0 - \frac{N_0}{2^2}\right)$$

$$= \left(4N_0 - \frac{N_0}{4}\right) + \left(N_0 - \frac{N_0}{4}\right) = \frac{9}{2}N_0$$

62. Ans. (B)

$M_{01} = 40g$ $M_{02} = 160g$

$T_1 = 20s$ $T_2 = 10s$

$M_1 = M_2$

$$M_{01}\left[\frac{1}{2}\right]^{\frac{t}{T_1}} = M_{02}\left[\frac{1}{2}\right]^{\frac{t}{T_2}}$$

$$\frac{40}{160} = \frac{\left[\frac{1}{2}\right]^{\frac{t}{10}}}{\left[\frac{1}{2}\right]^{\frac{t}{20}}} \Rightarrow \left[\frac{1}{2}\right]^2 = \left[\frac{1}{2}\right]^{\frac{t}{20}}$$

$t = 40s$

63. Ans. (B)

Ten years means $\frac{10}{5} = 2$ half – lives so just put $n = 2$ in $N = N_0\left(\frac{1}{2}\right)^n$

We get remaining nuclei, $N = N_0/4$

i.e., $\frac{1}{4}^{th}$ of initial nuclei.

So probability of decay will be $\frac{N_0 - N}{N_0} = 0.75$

64. Ans. (B)

$N_1 = N_0 e^{-10\lambda_0 t}$

$N_2 = N_0 e^{-\lambda_0 t} \Rightarrow \frac{N_1}{N_2} = e^{-9\lambda_0 t}$

When $\frac{N_1}{N_2} = \frac{1}{e}$

$\Rightarrow \quad t = \frac{1}{9\lambda_0}$

65. Ans. (C)

Radius $R = \frac{3.2}{2} \times 10^{-3}$ meter

So, to make the potential 1Volt we need $\frac{Q}{4\pi\varepsilon_0 R} = 1$

So, the charge needed is $Q = 4\pi\varepsilon_0 R = \frac{1.6\times10^{-3}}{10^9} = \frac{1.6\times10^{-12}}{9}$ Coulomb

So, the number of electrons (β – particle) needed to be ejected is $n = \frac{Q}{e}$

Putting $e = 1.6\times10^{-19}$ Coulomb, we get $n = \frac{10^7}{9}$

Time needed is $n = \frac{n}{rate} = \frac{10^7}{9\times6.25\times10^{10}} = .01775\times10^{-3}\,\text{sec} = 17.75\,\mu\text{sec}$ nearly 18μ sec.

66. Ans. (C)

$X \longrightarrow Y$ (stable)

$N_x \qquad\qquad N_y$

$$\frac{N_x}{N_y}=\frac{1}{7} \Rightarrow \frac{N_x}{N_x+N_y}=\frac{N}{N_0}=\frac{1}{8}$$

By using $N = N_0e^{-\lambda t}$ we have

$$\frac{N_0}{8}=N_0e^{-\lambda t}$$

$\Rightarrow \quad t = 3 \times 20$ years = 60 years

67. Ans. (A)

$$\frac{2}{3}N_0=N_0e^{-\lambda t}$$

Or $\frac{1}{3}N_0=N_0e^{-\lambda t_2}$

$$2=e^{\lambda(t_2-t_1)}$$

$$\lambda(t_2-t_1)=\ln 2$$

$$(t_2-t_1)=\frac{\ln 2}{\lambda}=20\,\text{min}$$

$$\frac{2}{3}N_0=N_0e^{-\lambda t_1}$$

$$\frac{1}{3}N_0=N_0e^{-\lambda t_2}$$

$$2=e^{\lambda(t_2-t_1)}$$

$$\lambda(t_2-t_1)=\ln 2$$

$$(t_2-t_1)=\frac{\ln 2}{\lambda}=20\,\text{min}$$

68. Ans. (B)

$A = \lambda N \;\Rightarrow\; A_1 = \lambda N_1$ and $A_2 = \lambda N_2$

$so\ (A_1 - A_2) = \lambda(N_1 - N_2) \Rightarrow (N_1 - N_2) = \dfrac{(A_1-A_2)}{\lambda}$

69. Ans. (B)

Given

Half-life, $T_{1/2}=\dfrac{0.693}{\lambda}$

$$\Rightarrow \quad \lambda=\frac{0.693}{1620\times 365\times 24}hr^{-1}$$

In time (t = 5 hours), $\lambda t=\dfrac{0.693\times 5}{1620\times 365\times 24}=2.44\times 10^{-7}$

$$N_0=\frac{mN_A}{M_0}=\frac{5\times\left(6.023\times 10^{23}\right)}{233}=1.292\times 10^{22}\text{ nuclei}$$

Number of radium nuclei decay in 5 hours is

$$(N_0 - N) = N_0(1 - e^{-\lambda t})$$

$$(N_0 - N) = 1.292 \times 10^{22}(1 - e^{-2.44\times10^{-7}})$$

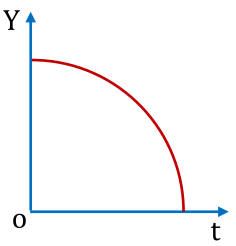
$$(N_0 - N) = 3.15 \times 10^{15}$$

Number of radium nuclei decay in 5 hours is 3.15×10^{15} nuclei.

70. **Ans. (D)**

Rate of decay = $\lambda N_0 e^{-\lambda t}$

Rate of formation of $Y = \lambda N_0 (1 - e^{-\lambda t})$



71. **Ans. (C)**

A radioactive substance is show is dissolved in a liquid and the solution is heated. The activity of the solution is equal to that of element.

72. **Ans. (B)**

For radioactive decay, $\ln\left(\frac{C_0}{C}\right) = kt$

$$\Rightarrow \quad \ln\left(\frac{C_0}{6000}\right) = k(280)$$

Also,

$$\ln\left(\frac{C_0}{3000}\right) = k(280+140)$$

Subtracting the two equation,

$$\ln 2 = k(140)$$

Using this first equation gives

$$C_0 = 24000 dps$$

73. **Ans. (A)**

Given : $R_1 = 5\mu ci, R_2 = 10\mu ci, N_1 = 2N_2$

By, $\lambda N = R$

$$\frac{0.693}{T} N = R$$

$$T = \frac{0.693N}{R}$$

For sample S_1 :

$$T_1 = \frac{0.693\,N_1}{5}$$

$$T_1 = \frac{0.693 \times 2N_2}{5} \qquad ...(1)$$

For sample S_2 :

$$T_2 = \frac{0.693\, N_2}{10} \qquad ...(2)$$

By dividing equation (1) by equation (2), we get

$$\frac{T_1}{T_2} = \frac{4}{1}$$

It is clear that the ratio of half lives of the two samples is 4 :1, therefore half lives can be 20(4k) years and 5 (1k) years respectively.

74. Ans. (C)

Mean life T = 40 min = 40 × 60s = 2400 s

So, decay constant $\lambda = \frac{1}{2400} / s$

At steady state $\frac{-dN}{dt}$ (rate of decay) should be equal to rate of production

But rate of decay es equal to $N\lambda$ also.

So, $N\lambda$ = 1000/s

So, $N = \frac{1000}{\lambda} = 2400 \times 1000 = 24 \times 10^5$

75. Ans. (C)

$N = N_0 \left(1 - e^{-\lambda t}\right)$

76. Ans. (B)

According to equation, assume original percentage of x = 16% (14+2)

If only 2% of x resume 0% of original mass decayed $= \frac{2}{16} \times 100 = 12.5\%$

$100 \rightarrow 50 \rightarrow 1$ half live

$50 \rightarrow 25 \rightarrow 2$ half live

$25 \rightarrow 12.5 \rightarrow 3$ half live

For natural to get to 12.5% = 3 half live shave passed

$\Rightarrow \quad 3 \times 45 = 135\ years$

77. Ans. (B)

$$\frac{A_0}{\sqrt{3}} = A_0 e^{-e1 \times \lambda} \qquad ...(1)$$

$$A' = A_0 e^{-4\lambda} = A_0 \left(e^{-\lambda}\right) 4 = A_0 \left(\frac{1}{\sqrt{3}}\right)^4 = \frac{A_0}{9}$$

78. **Ans. (C)**

Given,

$$N_1 = N_0, N_2 = 2N_0$$

$$t_1 = \frac{1}{\lambda}\ln\frac{\lambda N_0}{A_1}$$

$$t_2 = \frac{1}{\lambda}\ln\frac{\lambda 2N_0}{A_2}$$

Hence, $t_2 - t_1 = \frac{1}{\lambda}\ln\frac{A_2}{2A_2}$

Using $T = \frac{\ln 2}{\lambda}$

$$t_2 - t_1 = \frac{T}{\ln 2}\ln\frac{A_2}{2A_1}$$

79. **Ans. (ABCD)**

Energy possessed by each photon $E = h\nu = (6.63 \times 10^{-34})\left(\frac{1}{6.63}\times 10^{14}\right) = 1 \times 10^{-20} J$

Photon emission rate = $\frac{P}{E} = \frac{2\times 10^{-3}}{1\times 10^{-20}}$ = $2 \times 10^{17}\ s^{-1}$

Fraction of area intercepted by the plate = $\frac{1\times 10^{-4}}{4\pi\times\left(\frac{1}{\sqrt{\pi}}\right)^2} = \frac{1}{4}\times 10^{-4}$

Rate of photons striking the plate = $(2\times 10^{17})\left(\frac{1}{4}\times 10^{-4}\right) = 5\times 10^{12}$

80. **Ans. (BCD)**

KE_{max} increases

i_{sat} decreases ($\frac{I}{h\nu}$ decreases)

Photoelectric current may become zero if voltage is applied in reverse direction.

81. **Ans. (ABC)**

Photoelectric effect supports quantum nature because.

Below minimum frequency photoelectric effect is not observed

KE_{max} depends only on frequency

Photoelectric effect does not depend on intensity.

82. **Ans. (AC)**

V_s depend on frequency ν and ϕ emitter's property.

83. **Ans. (ABD)**

$f_a = f_b < f_c$

$I_a < I_b$

$$\frac{I_c}{f_c} = \frac{I_b}{f_b}$$

$\therefore\ I_c > I_b$

84. **Ans. (BCD)**

Light of wavelength λ_1 and λ_2 are falling on two metal surface A and B. Far wave length λ_1 electron ejected from both the surfaces and far wavelength λ_2 electron ejected from only surface B. So, more energy is required for ejection of electron from metal A.

85. **Ans. (AC)**

$$\phi = \frac{hc}{\lambda_0}$$

$(\lambda_1)_0 = 1000\ nm$

$(\lambda_2)_0 = 500\ nm$

$(\lambda_3)_0 = 250\ nm$

$\phi_1 : \phi_2 : \phi_3 = 0.001 : 0.002 : 0.004 = 1 : 2 : 4$

$$ev = \frac{hc}{\lambda} - \phi$$

$$v = \frac{hc}{e} \times \frac{1}{\lambda} - \frac{\phi}{e}$$

$$\tan\theta = \frac{hc}{e}\ (k)$$

$$\tan\theta \propto \frac{hc}{e}$$

86. **Ans. (AB)**

$P = V\, i_s$ where V = accelerating voltage ; i_s = saturation photocurrent

$$i_s = \frac{\text{Power of source of light} \times \text{Quantum efficiency} \times \lambda(\text{in Å})}{12400} = \frac{100 \times .01 \times 124}{12400} = 0.1\ A$$

$\therefore$ Power = 100 watt

Maximum energy of incoming electron = $\frac{hc}{\lambda} - \phi + eV = \left(\frac{12400}{124} - 10 + 10{,}000\right) eV = 10{,}090\ eV$

$$\lambda_{min} = \frac{12400}{10090} = 1.23\ Å$$

Power provided by accelerating potential = 100 W

87. **Ans. (ACD)**

$KE_{max} = hf - \phi_{wf}$

Saturation current $\propto$ intensity (I) : $V_S = \frac{KE_{max}}{e}$

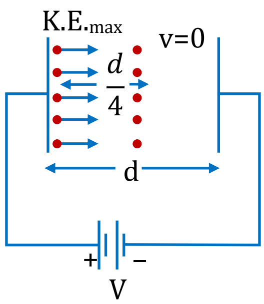
88. **Ans. (ABD)**

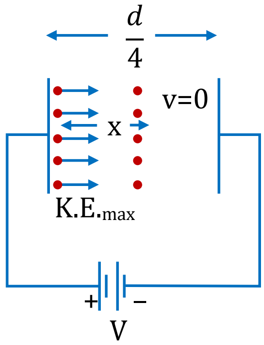
The photoelectron will stop emanating when potential grows to the KE_{max} of the photons (independent of radius)

$V = \frac{KQ}{r} \Rightarrow V$ is const. thus Q depends on radius.

As electrons emanate from the surface a net $+ve$ charge is developed this KE_{max} of photo electron decline.

89. **Ans. (BC)**





$K.E._{max} = \dfrac{eV}{4}$ $\qquad$ $K.E._{max} = e\left(\dfrac{4V}{d}.x\right)$

$\Rightarrow \; x = \dfrac{d}{16} = \dfrac{1}{4}\left(\dfrac{d}{4}\right)$

As $K.E._{max} = h\nu - \phi_e$, $K.E._{max}$ does not depend on V.

90. **Ans. (ABCD)**

λ_2 remains same.

λ_1 may increase, decrease or even be same.

91. **Ans. (BD)**

$\lambda = \dfrac{h}{P}$

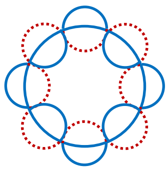
Thus λ $P\downarrow$.

92. **Ans. (ACD)**

$\dfrac{8\lambda}{2} = 2\pi R$

$4\lambda = 2\pi R \quad \Rightarrow \; n\lambda$

$n = 4$



93. **Ans. (ACD)**

Some λ is absent in A

B contains those λ absent in A along with emission in visible & R range.

94. **Ans. (ABCD)**

For ionization, electron from $n = 1$ must be moved to $n = \infty$

$\therefore$ Ionization potential = 15.6 V. λ_{min} for series terminating at $n = 2$ is $\dfrac{hc}{5.3}$

For excitation to $n = 3$, $\Delta E = 12.52\ eV$

For transition $n = 3$ to $n = 1$.

$\Delta E = 12.52\ eV$.

$\therefore \; \lambda = \dfrac{hc}{12.52}$

$\Rightarrow$ Wave no. $\left(\dfrac{1}{\lambda}\right) = \dfrac{12.52}{hc}$

95. Ans. (BC)

$$E_0z^2\left(1-\frac{1}{9}\right)-E_0z^2\left(\frac{1}{4}-\frac{1}{9}\right)=3E_0$$

$z = 2$

$\lambda_1/\lambda_2 = 3$

$KE_1 = E_0\left(1-\frac{1}{9}\right)-\phi$

$KE_2 = E_0z^2\left(1-\frac{1}{4}\right)-\phi$

$KE \propto \frac{1}{\lambda^2}=8.5eV$

96. Ans. (BD)

(a) $r \propto \frac{n^2}{\mu}$

$r_D < r_H$

(b) velocity $\propto \frac{z^2}{n^2\mu}$

$V_D < V_H$

(c) $\frac{1}{\lambda}=Rz^2\left(\frac{1}{n_1^2}-\frac{1}{n_2^2}\right)$

And $R \propto \mu$

So, $\frac{1}{\lambda}\propto\mu$

$\lambda_H > \lambda_D$

(d) Angular momentum = $\frac{nh}{2\pi}$

Same for both.

97. Ans. (ACD)

$Z=\sqrt{\frac{122.4}{13.6}} = 3$

$(E_{ex})_{min} = 9 \times 10.2$

$= 91.8\ eV$

For $KE_{initial} = 125\ eV$

$KE_{final} = 2.6\ eV$

$E_r = 122.4\ eV = E_{1\to3}$

98. Ans. (AC)

If $KE < 20.4\ eV$, elastic collision occurs.

If $KE > 20.4\ eV$, inelastic collision may occur.

Perfectly inelastic collision is possible.

99. **Ans. (ACD)**

$2.55\ eV = E4 \rightarrow 2$

Other photon has $E = 10.2\ eV$

$\therefore\ E_{net} = 12.75\ eV$

$\therefore\ KE_{min} = 25.5\ eV$

100. **Ans. (CD)**

$Be\ 10m_p + 10m_n - M_1$

$$M_1 < 10m_P + 10m_n$$

Similarly,

$$M_2 < 20m_P + 20m_n$$

$$(M_2 < 2M_1)$$

101. **Ans. (BD)**

In fusion, 2 or more lighter nuclei combine to make a comparatively heavier nucleus. In fission, a heavy nucleus breaks into 2 or more comparatively lighter nuclei.

102. **Ans. (ABC)**

K.E. of *B*-particle cannot exceed *E*.

K.E. of anti neutrino emitted lies between Zero and *E*.

N/Z ratio of the nucleus is altered.

103. **Ans. (AD)**

The rest mass of a stable nucleus is less than the sum of the rest masses of its separated nucleons.

In nuclear fusion, energy is released by fragmentation of a very heavy nucleus.

104. **Ans. (A)**

$$\lambda = \sqrt{\frac{150}{V}} = \sqrt{\frac{150}{150}}\mathring{A} = 1\mathring{A}$$

105. **Ans. (C)**

$$\lambda = \frac{h}{\sqrt{2mE}}$$

EXERCISE – S

1. **Ans.** 4

Number of ejected photo electrons $\propto \dfrac{1}{d^2}$

$$\frac{n_1}{n_2} = \frac{d_2^2}{d_1^2}$$

Let, $n_1 = n$, $d_1 = 0.5\ m$

$n_2 = ?$, $d_2 = 1\ m$

$$n_2 = \frac{n}{4}$$

2. **Ans.** 5

Since photoemission efficiency is constant $\Rightarrow$ maximum photo current will be achieved when number of photon is maximum.

For photo-emission photon must have energy of $2eV$.

$\therefore$ Time when photo current is maximum = 80 sec after switching on the circuit. (within the time interval of 2 min)

Thus number of photon received per second = $\dfrac{100}{2\times1.6\times10^{-19}}$

Thus number of electron received = $\dfrac{0.01}{100}\times\dfrac{100}{2\times1.6\times10^{-19}}$

Thus maximum photocurrent = $\dfrac{0.01}{100}\times\dfrac{100}{2\times1.6\times10^{-19}}$ × 1.6 × 10^{-19} = $\dfrac{1}{200}A$ = $5mA$

3. **Ans.** 1

Energy of 1 photon = $\dfrac{hc}{\lambda}=\dfrac{12400}{3540}\ eV\approx 3.5\ eV$.

Thus $K.E._{max.}$ of e^- ejected = 1 eV

Thus energy lost per electron = 1 eV = 1 × 1.6 × $10^{-19}\ J$

Total energy gained by target metal per second = 1.6 × 10^{-19} × $10^{18}\ J$

= 0.16 J

Thus 0.16 = $ms\dfrac{dT}{dt}\Rightarrow\dfrac{dT}{dt}=\dfrac{0.16}{10^{-3}\times160}{}^\circ C/\sec$

4. **Ans.** 159

Radiation αT^4.

So $T_2 = 2T_1$ and by Wein's displacement law $\lambda\propto\dfrac{1}{T}$

So $\lambda_2 = \dfrac{\lambda_1}{2}$ = 3000 Å; by Eeinstein's photoelectric equation $\dfrac{hc}{\lambda} = eV_s + \phi$

$\phi = \dfrac{hc}{\lambda} - eV_s = \dfrac{hc}{3000\mathring{A}} - (13.6\ eV)1^2\left(\dfrac{1}{2^2}-\dfrac{1}{4^2}\right) = 4.14 - 2.55$

$\phi = 1.59\ eV = 1.59 = \left(\dfrac{\alpha}{100}\right)$ or $\alpha = 159$

5. **Ans.** 255

Energy of the photon = $\dfrac{hc}{\lambda}=\dfrac{1240}{6200}\times51=10.2\ eV$

Since six spectral lines are obtained, thus transition is from ground state to n =4.

Also since the atom is not hydrogen, thus only possible atom is He^+.

[Photon of $\lambda=\dfrac{6200}{51}nm$ corresponds to transition from 4 to 2]

Thus ΔE in collision = 51 eV

For minimum kinetic energy of neutron, collision must be perfectly inelastic.

$$\frac{1}{2}\mu v_{rel}^2=51\ eV\Rightarrow\frac{1}{2}\frac{4}{5}mV^2=51\ eV\Rightarrow\frac{1}{2}mV^2=63.75\ eV$$

[m = mass of neutron, V = velocity of neutron]

6. **Ans.** $\sqrt{2}$

$\lambda = \frac{h}{\sqrt{2mk}}$

$KE = TE - PE$

$KE = \begin{cases} E_0 \ ; 0<x<1 \\ 2E_0 \ ; x>1 \end{cases}$

$\lambda \propto \frac{1}{\sqrt{KE}}$

$\frac{\lambda_1}{\lambda_2} = \sqrt{2}$

7. **Ans.** $\lambda = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

$\vec{V}_{COM} = V_1\hat{i} + V_2\hat{j}$

Velocity of first particle w.r.t. COM ; $\vec{V}_{1c} = \vec{V}_1 - \vec{V}_c = \frac{V_1\hat{i} - V_2\hat{j}}{2}$

Velocity of second particle w.r.t. COM ; $\vec{V}_{2c} = \vec{V}_2 - \vec{V}_c = \frac{V_2\hat{j} - V_1\hat{i}}{2}$

So, $\lambda_{1c} = \frac{h}{mv_{1C}} = \frac{2h}{m\sqrt{v_1^2+v_2^2}}$

$\Rightarrow \quad \lambda_{1c} = \frac{2h}{m\sqrt{\left(\frac{h}{m\lambda_1}\right)^2 + \left(\frac{h}{m\lambda_2}\right)^2}} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

and also $\lambda_{2C} = \frac{2\lambda_1\lambda_2}{\sqrt{\lambda_1^2+\lambda_2^2}}$

8. **Ans.** 6210 eV

$\lambda_{k\alpha} = 2A^\circ$

$\lambda_{k\beta} = 1A^\circ$

$E_{L\alpha} = E_{k\beta} - E_{k\alpha}$

$E_{L\alpha} = 12420\left[1 - \frac{1}{2}\right] = 6210\,ev$

9. **Ans.** 41

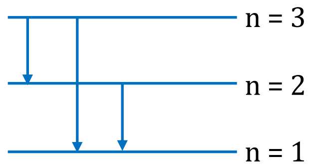
Let cut-off wavelength be λ_C and K_α line wavelength be $\lambda_{K\alpha}$.

Given $(\lambda_{K\alpha} - \lambda_C) = 4(\lambda_{K\alpha} - 2\lambda_C) \Rightarrow \frac{3}{7}\lambda_{K\alpha} = \lambda_C \Rightarrow \frac{1}{\lambda_C} = \frac{7}{3\lambda_{K\alpha}} \Rightarrow \frac{hc}{\lambda_C} = \frac{7hc}{3\lambda_{K\alpha}}$

$\Rightarrow \quad 38.08\times10^3 = \frac{7}{3}Rhc(Z-1)^2\left[\frac{3}{4}\right] \Rightarrow (Z-1) = 40$ so, $Z = 41$

10. **Ans.** 2

There are only 3 different transition, the final state is $n = 3$



$$E_n = -\frac{13.6}{n^2}(eV)$$

If n is the principal quantum number of initially excited state.

$$E_3 - E_n = -\frac{13.6}{3^2} - \left(-\frac{13.6}{n^2}\right)$$

$$= 13.6\left(\frac{1}{n^2} - \frac{1}{9}\right) \qquad ...(1)$$

$$E_3 - E_n = 1.89\ eV \text{ (given)} \qquad ...(2)$$

Atom will jump from n level to upper excited level.

From equations (1) and (2), $n = 2$.

11. **Ans.** 16

$$E_{ph} = 13.6\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 10.2\ eV$$

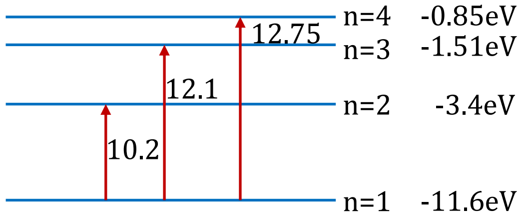
$$E_{ph} = eV_0 + h\nu_0 \Rightarrow 10.2 = 3.57 + h\nu_0$$

$$\nu_0 = \frac{6.63\times1.6\times10^{-19}}{6.63\times10^{-34}} = 1.6\times10^{15} Hz$$

12. **Ans.** 6

$$kE_{\max} = \frac{12420\,\mathring{A}\ eV}{1242\,\mathring{A}} - 2\ eV = 8\ eV$$

The electrons are emitted with kinetic energy varying from zero to 8 eV. When accelerated with 5volt potential difference their energies increases by 5eV. Hence hydrogen well get photons of energies in the range from 5 eV to 13 eV.



So max possible transactions are upto $n = 4$. Hence number of spectra lines is ${}^4C_2 = 6$

13. **Ans.** 255

Let mass of hydrogen atom be m and initial velocity of helium be u.

Therefore, from momentum conservation (for maximum loss of energy)

$$4mu = 4mv_1 + mv_1 \Rightarrow v_1 = \frac{4u}{5}$$

Loss in kinetic energy = $\frac{1}{2}4mu^2 - \frac{1}{2}5m\left(\frac{4u}{5}\right)^2 = \left(\frac{1}{2}4mu^2\right)\left(1-\frac{4}{5}\right) = \frac{1}{5}(KE_i)$

Loss in K.E. must be equal to the sum of excitation energy for helium and hydrogen atom

$$\therefore \quad 10.2\ eV + 40.8\ eV = \frac{1}{5}KE_i \Rightarrow KE_i = (51)\times 5\ eV = 255\ eV$$

14. **Ans.** 3.4 eV

In this picture, the third electron moves in the field of a total charge + 3e – 2e = + e. Thus, the energies are the same as that of hydrogen atoms. The lowest energy is :

$$E_2 = \frac{E_1}{4} = \frac{-13.6 \text{ eV}}{4} = -3.4\ eV$$

Thus, the ionization energy of the atom in this picture is 3.4 eV.

15. **Ans.** $r = \left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}$

The force at a distance r is, $F = -\frac{dU}{dr} = -2ar$

Suppose r be the radius of n^{th} orbit. The necessary centripetal force is provided by the above force. Thus,

$$\frac{mv^2}{r} = 2ar$$

Further, the quantization of angular momentum gives,

$$mvr = \frac{nh}{2\pi}$$

Solving equations (i) and (ii) for r, we get

$$r = \left(\frac{n^2h^2}{8am\pi^2}\right)^{1/4}$$

16. **Ans.** (a) $\frac{n^2h^2\varepsilon_0}{400\ \pi me^2}$ (b) 3.0 $\AA$

(a) We have, $\frac{m_pv^2}{r_n} = \frac{1}{4\pi\varepsilon_0}\frac{ze^2}{r_n^2}$ ...(i)

The quantization of angular momentum gives,

$$m_pvr_n = \frac{nh}{2\pi} \quad \text{...(ii)}$$

Solving equation (i) and (ii), we get

$$r = \frac{n^2h^2\varepsilon_0}{z\pi m_pe^2}$$

Substituting, $m_p = 100\ m$

where, m = mass of electron and $z = 4$

we get, $r_n = \frac{n^2h^2\varepsilon_0}{400\ \pi me^2}$

(b) As we know,

Energy of hydrogen atom in ground state = – 13.60 eV

and $E_n \propto \left(\frac{z^2}{n^2}\right)\ m$

For the given particle,

$$E_4 = \frac{(-13.60)\ (4)^2}{(4)^2} \times 100 = -1360\ eV$$

$$\text{and } E_2 = \frac{(-13.60)\ (4)^2}{(2)^2} \times 100 = -5440\ eV$$

$$\Delta E = E_4 - E_2 = 4080\ eV$$

$$\therefore\ \lambda\ (\text{in } \mathring{A}) = \frac{12400}{4080} = 3.0\ \mathring{A}$$

17. **Ans.** (a) $\sqrt{\frac{nh}{2\pi Be}}$ (b) $\frac{nhBe}{4\pi m}$ (c) $\frac{nheB}{4\pi m}$ (d) $\frac{nheB}{2\pi m}$

(a) Radius of circular path

$$r = \frac{mv}{Be} \qquad \text{...(i)}$$

$$mvr = \frac{nh}{2\pi} \qquad \text{...(ii)}$$

Solving these two equations, we get

$$r = \sqrt{\frac{nh}{2\pi Be}} \text{ and } v = \sqrt{\frac{nhBe}{2\pi m^2}}$$

(b) $K = \frac{1}{2}mv^2 = \frac{nhBe}{4\pi m}$

(c) $M = iA = \left(\frac{e}{T}\right)(\pi r^2) = \frac{evr}{2} = \frac{e}{2}\sqrt{\frac{nh}{2\pi Be}}\sqrt{\frac{nhBe}{2\pi m^2}} = \frac{nhe}{4\pi m}$

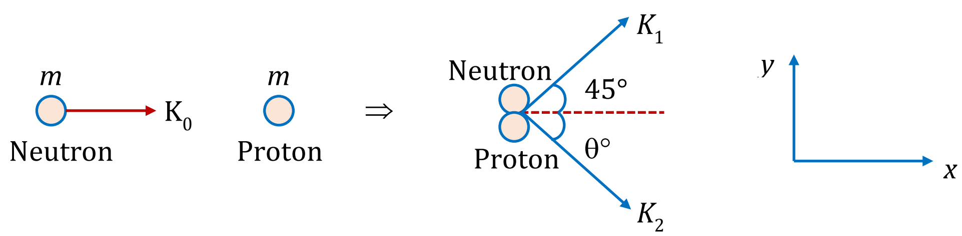
Now potential energy,

$$U = -M.B = \frac{nheB}{4\pi m}$$

(d) $E = U + K = \frac{nheB}{2\pi m}$

18. **Ans.** 24

Mass of neutron ≈ mass of proton = m



From conservation of momentum in y-direction

$$\sqrt{2mK_1}\ \sin 45° = \sqrt{2mK_2}\ \sin\theta \qquad \text{...(i)}$$

In x-direction,

$$\sqrt{2mK_0} - \sqrt{2mK_1}\cos 45° = \sqrt{2mK_2}\cos\theta \quad ...(ii)$$

Squaring and adding equation (i) and (ii), we have

$$K_2 = K_1 + K_0 - \sqrt{2K_0K_1} \quad ...(iii)$$

From conservation of energy

$$K_2 = K_0 - K_1 \quad ...(iv)$$

Solving equations (iii) and (iv), we get

$$K_1 = \frac{K_0}{2}$$

i.e., after each collision energy remains half. Therefore, after n collisions,

$$K_n = K_0\left(\frac{1}{2}\right)^n$$

$$\therefore \quad 0.23 = (4.6 \times 10^6)\left(\frac{1}{2}\right)^n \quad ; \quad 2^n = \frac{4.6\times10^6}{0.23}$$

Taking log and solving, we get

$$n \approx 24$$

19. **Ans.** 2

Radius, $R = R_0 A^{1/3}$

Surface area = $4\pi R^2$

$= 4\pi \times A^{2/3}$

$K = 2$

20. **Ans.** 100

$$K\alpha = \left(\frac{A-4}{A}\right)Q$$

$$48MeV = \left(\frac{A-4}{A}\right)\times 50\,MeV$$

$$A = 100$$

21. **Ans.** 0.2806 MeV

If the product nucleus ^{198}Hg is formed in its ground state, the kinetic energy available to the electron and the antineutrino is $Q = [m(^{198}Au) - m(^{198}Hg)]c^2$.

As $^{198}Hg^*$ has energy 1.088 MeV more than ^{198}Hg in ground state, the kinetic energy actually available is

$$Q = [m(^{198}Au) - m(^{198}Hg)]c^2 - 1.088\,MeV$$

$$= (197.968233\,u - 197.966760\,u)\left(931\frac{MeV}{u}\right) - 1.088\,MeV$$

$$= 1.3686\,MeV - 1.088\,MeV = 0.2806\,MeV.$$

This is also the maximum possible kinetic energy of the electron emitted.

22. **Ans.** 25

$$N = N_0\left(\frac{1}{2}\right)^n$$

n = Number of half life

So, $n = \frac{10}{5} = 2$

$$N = \frac{N_0}{4} \quad \text{(Amount of sample undecayed)}$$

% probability of decay will be

$$\frac{N_0 - \frac{N_0}{4}}{N_0} \times 100 = \frac{3N_0}{4N_0} \times 100 = 75\%$$

So, n = 25

23. **Ans.** 6.04 × 10^9 *yrs*

In nature, $\frac{N_{U^{238}}}{N_{U^{235}}} = \frac{140}{1} \quad (\text{at } t = t)$ ...(1)

At the time of earth formation,

$$\frac{N_{0_{U^{238}}}}{N_{0_{U^{235}}}} = \frac{1}{1} \quad (\text{at } \ t = 0\ ) \qquad ...(2)$$

Divide equation (2) by equation (1)

$$\therefore \quad \frac{N_{0_{U^{238}}}}{N_{0_{U^{235}}}} \times \frac{N_{0_{U^{235}}}}{N_{0_{U^{238}}}} = \frac{1}{140} \qquad ...(3)$$

$$\therefore \quad U^{238} : \frac{N_{0U^{238}}}{N_{U^{238}}} = e^{\lambda 238 t} \qquad ...(4)$$

For $U^{235} : \frac{N_{0U^{235}}}{N_{U^{235}}} = e^{\lambda 235 t}$ ...(5)

Divide equation (4) by equation (5)

$$\therefore \quad \frac{N_{0_{U^{238}}}}{N_{0_{U^{235}}}} \times \frac{N_{U^{235}}}{N_{U^{238}}} = e^{(\lambda^{238} - \lambda^{235})t}$$

Substitute equation (3) in the above equation

$$\frac{1}{140} = e^{(\lambda^{238} - \lambda^{235})t}$$

$$(\lambda^{238} - \lambda^{235})t = \log_e 1 - \log_e 140$$

But $\lambda = \frac{0.693}{t_{1/2}}$

Hence,

$$\left[\frac{0.693}{4.5\times10^{9}}-\frac{0.693}{7.13\times10^{8}}\right]t=-2.303\log_{10}140$$

Hence, the age of the earth is $t=6.04\times10^{9}$ yrs.

24. **Ans.** 1

$$A=\lambda N\Rightarrow10^{10}\lambda N\Rightarrow N=\frac{10^{10}}{\lambda}=\left(10^{10}\right)\tau=10^{10}\times10^{9}=10^{19}$$

$$M=Nm=\left(10^{19}\right)\left(10^{-25}\right)=10^{-6}kg=1mg$$

25. **Ans.** 0.259

Let initially there be N_0 uranium atoms.

Uranium converts to lead.

After time $t=1.5\times10^{9}\,yrs=\frac{4.5\times10^{9}}{3}\,yrs=\frac{t_{1/2}}{3}$

Converted atoms of Uranium be N_0

Left uranium atoms be N

n = number of half lives = 1/3

$$N=\frac{N_0}{2^{n}}$$

$$N=\frac{N_0}{2^{1/3}}=\frac{N_0}{1.259}$$

Lead atoms now = $18.N_0 - N$

Uranium atoms now = N

Ratio $=\frac{N_0-N_1}{N_1}=\frac{N_0}{N_1}-1=1.259-1$

= 0.259

Ratio $=259\times10^{-k}=0.259$

$=\frac{259}{0.259}=10^{k}$

$10^{3}=k^{k}$

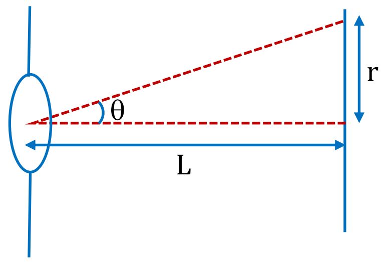
$k=3\rightarrow$ ans

26. **Ans.** 391eV

$$\frac{hC}{\lambda_{x-ray}}=E$$

Now, $I_{X-ray}=I_{electron}=\frac{h}{\sqrt{2m_eK}}$

27. **Ans.** 2.44



$$\sin\theta = \frac{1.22\lambda}{D}$$

$$\frac{r}{L} = \frac{1.22h}{mvD}$$

$$r = \frac{1.22hL}{mvD}$$

$$= \frac{1.22\times 6.4\times 10^{-34}\times 1}{1.6\times 10^{-27}\times 10^{3}\times 2\times 10^{-7}}$$

= 2.44 *mm*

EXERCISE - JEE (Main) PYQ

1. **Ans. (4)**

In magnetic field, Radius, $R = \frac{\sqrt{2m(K.E.)}}{qB}$

$$K.E. = \frac{q^2B^2R^2}{2m}$$

$K.E. = 0.80\ ev$

Energy of photon for transition from 3 $\rightarrow$ 2 in hydrogen atom

$$E = 13.6\left[\frac{1}{2^2}-\frac{1}{3^2}\right] = 1.88\ ev$$

From Einstein photoelectric equation

$E = K.E_{max} + \phi$

$\Rightarrow$ $1.88 = 0.8 + \phi$

$\Rightarrow$ $\phi = 1.08\ ev$

$\phi \approx 1.1\ ev$

2. **Ans. (1)**

In Bohr model

$$\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$\Rightarrow$ $\lambda \propto \frac{1}{z^2}$

$$\lambda_1 : \lambda_2 : \lambda_3 : \lambda_4 :: \frac{1}{1}:\frac{1}{1}:\frac{1}{4}:\frac{1}{9}$$

$\Rightarrow$ $\lambda_1 = \lambda_2 = 4\lambda_3 = 9\lambda_4$

3. **Ans. (3)**

$K = +13.6\frac{z^2}{n^2}$ as n decreases k increases

$$\left.\begin{array}{l} U = -27.2\frac{z^2}{n^2} \\ -13.6\frac{z^2}{n^2} \end{array}\right\} \begin{array}{l} \text{as } n \text{ decrease} \\ U \;\&\; T \text{ decrease} \end{array}$$

4. **Ans. (2)**

$E = (KE)_{max} + f$

$$\left[\frac{hc}{\lambda} = (KE)_{\max} + \phi\right] \quad \text{...(1)}$$

$$\frac{4}{3}\frac{hc}{\lambda} = \left(\frac{4}{3}KE_{\max} + \frac{\phi}{3}\right) + \phi$$

$(KE)_{max}$ for fastest emitted electron = $\frac{1}{2}mv'^2 + \phi$

$$\frac{1}{2}mv'^2 = \frac{4}{3}\left(\frac{1}{2}mv^2\right) + \frac{\phi}{3}$$

$$\boxed{v' > v\left(\frac{4}{3}\right)^{1/2}}$$

5. **Ans. (1)**

$t = 80\ min = 4\ T_A = 2T_B$

$\therefore$ Number of nuclei of A decayed = $N_0 - \frac{N_0}{2^4} = \frac{15N_0}{16}$

$\therefore$ Number of nuclei of B decayed = $N_0 - \frac{N_0}{2^2} = \frac{3N_0}{4}$

Required ratio = $\frac{5}{4}$

6. **Ans. (3)**

$$\frac{hc}{\lambda_{\min}} = eV$$

$$\frac{1}{\lambda_{\min}} = \frac{eV}{hc}$$

$$\lambda_{\min}V = \frac{hc}{e}$$

$$\log(\lambda_{\min}V) = \log\left(\frac{hc}{e}\right)$$

$$\ell nV + \ell n(\lambda_{min}) = \ell n\frac{hc}{e}$$

$$\ell n(\lambda_{\min}) = -\ell nV + \ell n\left(\frac{hc}{e}\right)$$

It is a straight line with $-ve$ slope.

7. Ans. (4)

At time t

$$\frac{N_B}{N_A} = 0.3 \Rightarrow N_B = 0.3\ N_A$$

Also, let initially there are total N_0 number of nuclei

$$N_A + N_B = N_0$$

$$N_A = \frac{N_0}{1.3}$$

Also, as we know

$$N_A = N_0 e^{-\lambda t}$$

$$\frac{N_0}{1.3} = N_0 e^{-\lambda t}$$

$$\frac{1}{1.3} = e^{-\lambda t} \Rightarrow \ell n(1.3) = \lambda t \text{ or } t = \frac{\ell n(1.3)}{\lambda}$$

$$t = \frac{\ell n(1.3)}{\frac{\ell n(2)}{T}} = \frac{\ell n(1.3)}{\ell n(2)} T$$

8. Ans. (3)

$$hv = E_0\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For Lyman series for series limit $n_2 = \infty$, $n_1 = 1$

$$hv_L = E_0[1] \quad ...(1)$$

For P-fund series

For series limit $n_2 = \infty$, $n_1 = 5$

$$hv_p = E_0\left[\frac{1}{25}\right] \quad ...(2)$$

By dividing equation (1) and (2)

$$\frac{v_L}{v_p} = \frac{25}{1} \Rightarrow v_p = v_L/25$$

9. Ans. (4)

$$\lambda_n = \frac{h}{mu} = \frac{h}{\sqrt{2mk_n}}$$

$$\Rightarrow \quad k_n = \frac{h^2}{2m\lambda_n^2}\ ;\ k_g = \frac{h^2}{2m\lambda_g^2}$$

$$\Rightarrow \quad k_g - k_n = \frac{h^2}{2m}\left[\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right]$$

$$E_n = -k_n$$

For emitted photon

$$\frac{hc}{\Lambda_n} = E_n - E_g = K_g - K_n$$

$$\frac{1}{\Lambda_n} = \frac{K_g - K_n}{hc}$$

$$\Lambda_n = \frac{hc}{K_g - K_n}$$

$$\Rightarrow \quad \Lambda_n = \frac{hc}{\frac{h^2}{2m}\left[\frac{1}{\lambda_g^2} - \frac{1}{\lambda_n^2}\right]}$$

$$\Lambda_n = \frac{2mc}{h\left(\frac{\lambda_n^2 - \lambda_g^2}{\lambda_g^2 \lambda_n^2}\right)}$$

$$\Lambda_n = \frac{2mc\lambda_g^2\lambda_n^2}{h\left(\lambda_n^2 - \lambda_g^2\right)}$$

As $\lambda_n \propto n$

$\lambda_n >> \lambda_g$

$$\Lambda_n = \frac{2mc\lambda_g^2}{h}\left[1 - \left(\frac{\lambda_g}{\lambda_n}\right)^2\right]^{-1}$$

$$\Lambda_n = \frac{2mc\lambda_g^2}{h}\left[1 + \left(\frac{\lambda_g}{\lambda_n}\right)^2 + \text{higher powers of } \frac{\lambda_g}{\lambda_n}\right]$$

$$\Lambda_n \approx A + \frac{B}{\lambda_n^2}$$

Where $A = \dfrac{2mc\lambda_g^2}{h}$

& $B = \dfrac{2mc\lambda_g^4}{h}$

10. Ans. (1)

$$N_A = N_{OA}e^{-\lambda t} = \frac{N_{OA}}{2^{t/t_{1/2}}} = \frac{N_{OA}}{2^6}$$

$\therefore$ Number of nuclei decayed

$$= N_{OA} - \frac{N_{OA}}{2^6} = \frac{63N_{OA}}{64}$$

$$N_B = N_{OB}e^{-\lambda t} = \frac{N_{OB}}{2^{t/t_{1/2}}} = \frac{N_{OB}}{2^3}$$

$\therefore$ Number of nuclei decayed

$$= N_{OB} - \frac{N_{OB}}{2^3} = \frac{7N_{OB}}{8}$$

Since, $N_{OA} = N_{OB}$

$\therefore$ Ratio of decayed numbers of nuclei

$$A \text{ and } B = \frac{63N_{OA} \times 8}{64 \times 7N_{OB}} = \frac{9}{8}$$

11. Ans. (2)

Number of uranium atoms in 2*kg*

$$=\frac{2\times6.023\times10^{26}}{235}$$

energy from one atom is 200 × 10^6 e.v. hence total energy from 2 *kg* uranium

$$=\frac{2\times6.023\times10^{26}}{235}\times200\times10^{6}\times1.6\times10^{-19}\,J$$

2 *kg* uranium is used in 30 days hence this energy is received in 30 days hence energy received per second or power is

$$\text{Power}=\frac{2\times6.023\times10^{26}\times200\times10^{6}\times1.6\times10^{-19}}{235\times30\times24\times3600}$$

Power = 63.2 × 10^6 watt or 63.2 Mega Watt

12. Ans. (2)

$${}^{7}_{3}Li+{}^{1}_{1}H\rightarrow2\left({}^{4}_{2}He\right)$$

$$\Delta m\Rightarrow[m_{Li}+m_{H}]-2[M_{He}]$$

Energy released in 1 reaction ⇒ Δmc^2

In use of 7.016 *u Li* energy is Δmc^2

In use of 1gm *Li* energy is $\frac{\Delta mc^2}{m_{Li}}$

In use of 20 gm energy is $\frac{\Delta mc^2}{m_{Li}}\times20gm$

$$\Rightarrow\quad\frac{\left[(7.016+1.0079)-2\times4.0026\right]u\times c^2}{7.016\times1.6\times10^{-24}\,gm}\times20gm$$

$$\Rightarrow\quad\left(\frac{0.0187\times1.6\times10^{-19}\times10^{9}}{7.016\times1.6\times10^{-24}\,gm}\times20gm\right)\text{Joule}$$

⇒ $0.05\times10^{+14}\,J\Rightarrow1.4\times10^{+6}\,kwh$

$[1\,J\Rightarrow2.778\times10^{-7}\,kwh]$

13. Ans. (2)

$$\lambda=\frac{h}{mv}=\frac{h}{\sqrt{2mE}},\quad mv=\sqrt{2mE}$$

$$\lambda\propto\frac{1}{\sqrt{E}}$$

$$\frac{\lambda_2}{\lambda_1}=\sqrt{\frac{E_1}{E_2}}=\frac{3}{4},\ \lambda_2=0.75\,\lambda_1$$

$$\frac{E_1}{E_2}=\left(\frac{3}{4}\right)^2$$

$$E_2=\frac{16}{9}E_1=\frac{16}{9}E\qquad(E_1=E)$$

Extra energy given = $\frac{16}{9}E-E=\frac{7}{9}E$

14. Ans. (3)

$A \to B$, $B \to C$

$$\frac{dN_B}{dt} = \lambda N_A - \lambda N_B$$

$$\frac{dN_B}{dt} = 2\lambda N_{B_0} e^{-\lambda t} - \lambda N_B$$

$$e^{+\lambda t}\left(\frac{dN_B}{dt} + \lambda N_B\right) = 2\lambda N_{B_0} e^{-\lambda t} \times e^{\lambda t}$$

$\frac{d}{dt}\left(N_B e^{\lambda t}\right) = 2\lambda N_{B_0}$, on integrating

$$N_B e^{\lambda t} = 2\lambda t N_{B_0} + N_{B_0}$$

$$N_B = N_{B_0}[1 + 2\lambda t]e^{-\lambda t}$$

$\frac{dN_B}{dt} = 0$ at $-\lambda[1 + 2\lambda t]e^{-\lambda t} + 2\lambda e^{-\lambda t} = 0$

$N_{B_{\max}}$ at $t = \frac{1}{2\lambda}$

15. Ans. (2)

$$\frac{1}{2} m V_{\max}^2 = E - \phi$$

$$V_{\max} = \sqrt{\frac{2}{m}(E - \phi)}$$

$$\left(\frac{V_1}{V_2}\right)_{\max} = \sqrt{\frac{E_1 - \phi}{E_2 - \phi}}$$

$$= \sqrt{\frac{3.8 - 0.6}{1.4 - 0.6}} = \sqrt{\frac{3.2}{0.8}} = 2$$

16. Ans. (2)

$K = +13.6\frac{z^2}{n^2}$ as n decreases k increases

$$\left.\begin{aligned} U &= -27.2\frac{z^2}{n^2} \\ &-13.6\frac{z^2}{n^2} \end{aligned}\right\} \begin{aligned} &\text{as } n \text{ decrease} \\ &U \,\&\, T \text{ decrease} \end{aligned}$$

17. Ans. (50)

$$N_t = N_0 (0.5)^{\frac{t}{t_{1/2}}}$$

$$= \frac{m}{M} \times N_A (0.5)^{\frac{t}{t_{1/2}}}$$

$$\frac{N_1}{N_2} = \frac{M_2}{M_1}(0.5)^{t\left[\frac{1}{T_A} - \frac{1}{T_B}\right]}$$

$$= 2(0.5)^{16 \times \frac{1}{8}} = \frac{2}{4} = \frac{1}{2} = \frac{x}{100}$$

18. Ans. (4)

$$\Delta E = 13.6Z^2\left[\frac{1}{2^2}-\frac{1}{4^2}\right]eV$$
$$= 13.6 \times (4)^2\left(\frac{1}{4}-\frac{1}{16}\right)eV$$
$$= 13.6[4-1]eV$$
$$= 13.6 \times 3 = 40.8 \text{ eV}$$

19. Ans. (300)

$$\frac{dN_1}{dt} = -\lambda_1 N \qquad \frac{dN_2}{dt} = -\lambda_2 N$$
$$\frac{dN}{dt} = -(\lambda_1+\lambda_2)N$$
$$\Rightarrow \quad \lambda_{eq} = \lambda_1 + \lambda_2$$
$$\Rightarrow \quad \frac{1}{t_{\bullet}} = \frac{1}{300}+\frac{1}{30} = \frac{11}{300}$$
$$\Rightarrow \quad t_{\bullet} = \frac{300}{11}$$

20. Ans. (2)

$$\lambda_D = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$$
$$\therefore \lambda \propto \frac{1}{\sqrt{m}}$$
$$\because m_\alpha > m_p > m_e$$
$$\lambda_e > \lambda_p > \lambda_\alpha$$

21. Ans. (3)

(A) Stopping potential depends on both frequency of light and work function.

(B) Saturation current $\propto$ intensity of light

(C) Maximum KE depends on frequency

(D) Photoelectric effect is explained using particle theory

22. Ans. (4)

$$\lambda = \frac{hc}{\Delta E}$$

$\Delta E_A = 2.2\ eV$

$\Delta E_B = 5.2\ eV$

$\Delta E_C = 3\ eV$

$\Delta E_D = 10\ eV$

$$\lambda_A = \frac{6.62\times10^{-34}\times3\times10^{8}}{2.2\times1.6\times10^{-19}}$$
$$= \frac{12.41\times10^{-7}}{2.2}m$$
$$= \frac{1241}{2.2}nm = 564\ nm$$
$$\lambda_B = \frac{1241}{5.2}nm = 238.65nm$$
$$\lambda_C = \frac{1241}{3}nm = 413.66nm$$
$$\lambda_D = \frac{1241}{10} = 124.1\ nm$$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (A)**

$${}^{210}_{84}Po \rightarrow {}^{206}_{82}Pb + {}^{4}_{2}He + Q$$

Total energy released = $\left(M_{Po} - M_{Pb} - M_{He}\right)C^2$

= [(209.982876) – (205.974455 + 4.002603)] × 932 MeV

= [0.005818] × 932 MeV = 5.422376 MeV

Kinetic energy of α particle = $\left(\frac{A-4}{A}\right)Q = \left(\frac{206}{210}\right)5.422376\,MeV$ = 5.319 MeV = 5319 KeV

2. **Ans. (C)**

Only in option (C); sum of the masses of product is less than sum of the masses of reactant

for reaction ${}^{2}_{1}H + {}^{4}_{2}He \rightarrow {}^{6}_{3}Li$

$M_{Li} < M_{\text{Deutron}} + M_{\text{Alpha}}$

3. **Ans. (4)**

Given $T_{½}$ = 1386 sec.; $\left|\frac{dN}{dt}\right| = 10^3$

Fraction = $\frac{N_0 - N}{N_0} = \frac{N_0 - N_0 e^{-\lambda t}}{N_0} = 1 - e^{-\lambda t}$

= $1 - e^{\frac{-\ln 2}{1386}\times 80}$ = $1 - e^{\frac{-4}{100}} \simeq 1 - \left[1 - \frac{4}{100}\right]$ = $\frac{4}{100} \Rightarrow 4\%$

4. **Ans. (C)**

Completing reaction in list II

(1) ${}^{15}_{8}O \rightarrow {}^{15}_{7}N + {}^{0}_{1}e$ (β⁺ decay Q)

(2) ${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}He$ (Alpha decay P)

(3) ${}^{185}_{83}Bi \rightarrow {}^{184}_{82}Pb + {}^{1}_{1}H$ (Proton emission S)

(4) ${}^{239}_{94}Pu \rightarrow {}^{140}_{57}La + {}^{99}_{37}Rb$ (Fission R)

5. **Ans. (B)**

$\sqrt{\frac{C}{\lambda}} = a(Z-b)$; $b = 1$

$$\sqrt{\frac{\lambda_{Cu}}{\lambda_{M_0}}} = \left(\frac{Z_{M_0} - 1}{Z_{Cu} - 1}\right)$$

$$\frac{\lambda_{Cu}}{\lambda_{M_0}} = \left(\frac{41}{28}\right)^2 = 2.14$$

6. Ans. (A-R or R,T), (B-P,S), (C-Q,T), (D-R)

As per the information given in NCERT.

7. Ans. (9)

$${}_{5}^{12}B \rightarrow {}_{6}^{12}C + {}_{-1}^{0}e + \bar{v}$$

Mass defect = (12.014 – 12) u

$\therefore$ Released energy = 13.041 $M\,eV$

Energy used for excitation of ${}_{6}^{12}C$ = 4.041 $M\,eV$

$\therefore$ Energy converted to KE of electron

= 13.041 – 4.041 = 9 $M\,eV$

8. Ans. (C)

Electrostatic energy = $BE_N - BE_O$

$= \left[[7M_H + 8M_n - M_N] - [8M_H + 7M_n - M_O]\right] \times C^2$

$= [-M_H + M_n + M_O - M_N]C^2$

$= [-1.007825 + 1.008665 + 15.003065 - 15.000109] \times 931.5$

$= +\,3.5359\ M\,eV$

$$\Delta E = \frac{3}{5} \times \frac{1.44 \times 8 \times 7}{R} - \frac{3}{5} \times \frac{1.44 \times 7 \times 6}{R} = 3.5359$$

$$R = \frac{3 \times 1.44 \times 14}{5 \times 3.5359} = 3.42\,fm$$

9. Ans. (5)

$$U = -\frac{Z^2}{n^2}E_0$$

$Z = 1$

$$U = -\frac{E_0}{n^2}$$

$$\frac{U_i}{U_f} = \frac{n_f^2}{n_i^2} = 6.25$$

Taking,

$n_i = 2$

$n_f = 5$

10. Ans. (AC)

${}_{90}^{232}Th$ is converting into ${}_{82}^{212}Pb$

Change in mass number (A) = 20

$\therefore$ Number of α particle = $\frac{20}{4} = 5$

Due to 5 α particle, z will change by 10 unit.

Since given change is 8, therefore no. of β particle is 2.

11. Ans. (135)

$$Ra^{226} \longrightarrow Rn^{222} + \alpha$$

Q = (226.005 – 222–4) 931 MeV = 4.655 MeV

$$K_\alpha = \frac{A-4}{A}\left(Q - E_\gamma\right)$$

$$4.44\ MeV = \frac{222}{226}\left(Q - E_\gamma\right)$$

$$Q - E_\gamma = (4.44)\left(\frac{226}{222}\right)MeV$$

E_γ = 4.655 – 4.520

= 0.135 MeV = 135 keV

12. Ans. (AC)

$$\lambda_{\min} = \frac{hc}{eV}$$

$$\Rightarrow \quad \lambda_{\min} \alpha \frac{1}{V} \quad \Rightarrow (\lambda_{\min})_{new} = \frac{\lambda_2}{2}$$

$$\because \quad I = \frac{dN}{dt} \times \frac{hc}{\lambda}$$

$\because$ $\dfrac{dN}{dt}$ decreases

Hence, I decreases.

13. Ans. (D)

Out of 1000 nuclei of Q 60% may go α-decay

$\Rightarrow$ 600 nuclei may have α-decay

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{20}$$

t = 1 hour = 60 minutes

Using,

$$N = N_0 e^{-\lambda t} = 600 \times e^{-\frac{\ln 2}{20} \times 60}$$

N = 75

$\Rightarrow$ 75 Nuclei are left after one hour

So, No. of nuclei decayed = 600 – 75 = 525

14. Ans. (6)

For P & Q

$E_1 - 4 = E_P$

$E_1 - 4.5 = E_Q$

$E_P = 2\ E_Q$

$E_1 - 4 = 2\ (E_1 - 4.5)$

$E_1 = 5\ eV$

$E_P = 1\ eV$, $E_Q = E_R = 0.5\ eV$

For $E_2 - 5.5 = 0.5$

$E_2 = 6\ eV$

15. **Ans. (2.32 to 2.33)**

$^{16}_{7}N \quad + \quad ^{4}_{2}He \rightarrow \ ^{1}_{1}H \ + \ ^{19}_{8}O$

16.006 4.003 1.008 19.003

$4v_0 = 1v_1 + 19v_2 = 20v_2$ (By momentum conservation) (For maximum loss of kinetic energy)

v_0, v_1 and v_2 are the velocity of $^{4}_{2}He$, $^{1}_{1}H$, $^{19}_{8}O$ respectively

$$v_2 = \frac{v_0}{5}$$

E required = (1.008 + 19.003 – 16.006 – 4.003) × 930 = 1.86 MeV

$$\frac{1}{2}4v_0^2 - \frac{1}{2}20v^2 = 1.86 \text{ – (By energy conservation)}$$

$$\frac{1}{2}4v_0^2 - \frac{10}{25}20v_0^2 = 1.86$$

$$2v_0^2 - \frac{2}{5}v_0^2 = 1.86$$

$$\frac{8}{5}v_0^2 = 1.86$$

$$v_0^2 = \frac{1.86\times 5}{8}$$

$$KE = \frac{1}{2}4v_0^2 = 2v_0^2 = \frac{18.6\times 5}{4}\ MeV = 2.325\ MeV$$

16. **Ans. (ABD)**

Binding energy of proton & neutron due to nuclear force is same. So difference in binding energy is only due to electrostatic P.E. and it is positive

$E_0^P - E_0^n$ = electrostatic P.E.

= Z × P.E. of one proton

$$= Z\times\frac{1}{4\pi\varepsilon_0}\frac{(Z-1)e^2}{R}$$

Where $R = R_0 A^{1/3}$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Z(Z-1)e^2}{R_0A^{\frac{1}{3}}}$$

Ans. (A,B,D)

17. **Ans. (2)**

$$Th^{230}_{90} \rightarrow Po^{214}_{84} + n\alpha^4_2 + m\beta^0_{-1}$$

230 = 214 + 4n

$$n = \frac{16}{4} = 4$$

90 = 84 + n × 2 – m × 1

90 = 84 + 4 × 2 – m × 1

m = 92 – 90 = 2

Hence $\frac{n}{m} = \frac{4}{2} = 2$ **Ans.**

18. **Ans. (3)**

$n = 4 \;\; n = 3$

$-1.51Z^2 eV, \;\; -0.85\, Z^2\, eV$

$E = E_4 - E_3 = 0.66\, Z^2\, eV$

$K_{max} = E - W$

$0.66\, Z^2 \;\; 1.95 + 4 = 5.95$

$$W = 0.66Z^2 - 1.95 = \frac{hc}{\lambda} = \frac{1240}{310}$$

$\therefore \quad Z = 3$

19. **Ans. (16)**

$$\begin{array}{lll} & S_1 & S_2 \\ t=0 & A_0 & A_0 \\ t=\tau & A_1 & A_2 \end{array}$$

$$\frac{A_1}{A_2} = \frac{A_0(0.5)^{t/(t_{1/2})_1}}{A_0(0.5)^{t/(t_{1/2})_2}} = \frac{(0.5)^3}{(0.5)^7} = 2^4 = 16$$

JEE (Main) Practice Paper

SECTION–A

1. **Ans. (3)**

The electrons produced as a result of the decay of neutrons inside the nucleus.

2. **Ans. (1)**

$$KE_m = \frac{1}{2} \times 9.1 \times 10^{-31} \times 36 \times 10^{10}$$

$= 165 \times 1014\, J$

$$\therefore \quad V_0 = 7.21 \times 10^{24} - \frac{1.65 \times 10^{-19}}{6.63 \times 10^{-34}}$$

$$-7.21 \times 10^{14} - \frac{16.5}{6.63} \times 10^4$$

$= 4.73 \times 10^{14}\, Hz$

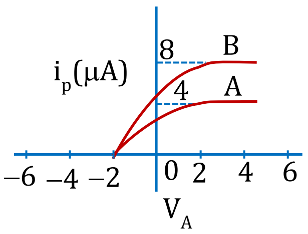
3. **Ans. (3)**

$$V_s = \frac{h(V - V_0)}{e} = \frac{6.63 \times 10^{-34} \times 9.9 \times 10^{14}}{1.6 \times 10^{-19}} \simeq 2V$$

4. **Ans. (1)**

Stopping potential, $V = (h\nu - W)/e = 2V$

In each case, the stopping potential is 2 V as energy of incident photon is the same. As the photon intensity is doubled, the saturation current is also doubled. The variation of i_p against V_a is sketched in the figure



5. Ans. (1)

$h\nu = \phi + ev_0$ and $\nu = \frac{c}{\lambda}$

$\Rightarrow \quad \lambda = \frac{hC}{\phi + ev_0} = \frac{1200}{(6.2+5)} nm = 1.13\times10^{-7} m$

6. Ans. (2)

During fusion, binding energy of daughter nucleus is always grater than the total binding energy of the parent nuclei. The difference of binding energy is released Hence,

$Q = E_2 - 2E_1$

7. Ans. (2)

$\left|\frac{dN}{dt}\right| = \lambda N_0 e^{-\lambda t} \quad \therefore \ell n\left|\frac{dN}{dt}\right| = \ell n(N_0\lambda) - \lambda t \ \therefore \ell n\left|\frac{dN}{dt}\right| = \ell n(N_0\lambda) - \lambda t \Rightarrow = slope = \frac{1}{2} yr^{-1} \Rightarrow \lambda = \frac{1}{2} yr^{-1}$

$\therefore t_{1/2} = \frac{0.693}{\lambda} = 1.386$ given time is 3 times of $t_{1/2}$ $\therefore$ value of p is 8.

8. Ans. (3)

$N = N_0 e^{-\lambda t}$

Mean life $t = \frac{1}{\lambda}$

$\Rightarrow \quad N = N_0 e^{-1} = \frac{N_0}{e}$

$\Rightarrow \quad \frac{N}{N_0} = \frac{1}{e}$

9. Ans. (2)

$0.9N_0 = N_0 e^{-\lambda t} \quad = e^{-\lambda t} = 0.9$

Let $N = N_0 = N_0 e^{-\lambda(2t)} = N_0\left(e^{-\lambda t}\right)^2 = N_0(0.9)^2 = 0.81N_0$

i.e. 19% will decay.

10. Ans. (2)

The half life of X is $t_{1/2} = \frac{0.693}{\lambda_x}$ where λ_x be the decay constant of X.

Mean life of Y is $\tau = \frac{1}{\lambda_x}$

Here, $t_{1/2} = \tau \; or \; \frac{0.693}{\lambda_x} = \frac{1}{\lambda_x}$

Or $\lambda_x = 0.693\lambda_x \Rightarrow \lambda_y > \lambda_x$

Since, $N = N_0 e^{-\lambda t}$, so Y will decay faster than X.

11. Ans. (2)

For each α emission z decreases by 2;

For each β^+ emission, z decreases by 1 and that for each β^- it increases by 1.

No of α emitted = 8

No of β^+ emitted = 2

No of β^- emitted = 4

$\therefore$ The resulting nucleus has atomic number $z = z$ (original) – 2 (8) – 2 (1) + 1 (4)

$\Rightarrow$ $z = 92 - 16 - 2 + 4 = 78$

12. Ans. (4)

From conservation of Momentum.

$$m_1 v_1 = m_2 v_2$$

$$\left(\frac{m_1}{m_2}\right) = \left(\frac{v_2}{v_1}\right) = \frac{1}{2}$$

$$\frac{r_1^3}{r_2^3} = \frac{1}{2}$$

$$\left(\frac{r_1}{r_2}\right) = \left(\frac{1}{2}\right)^{1/3}$$

13. Ans. (2)

$E_{ph} = 1.02\ MeV + KE$

$E_{ph} = 1.02\ MeV + 0.78\ MeV$

$E_{ph} = 1.8\ MeV$

14. Ans. (2)

$$\frac{1}{\lambda} = R\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right] \text{w}$$

$$\frac{1}{122} = R\left[1 - \frac{1}{4}\right] = \frac{3R}{4}$$

$$R = \frac{4}{3 \times 122 nm}$$

$$\frac{1}{\lambda} = R\left[\frac{1}{3^2} - \frac{1}{\infty}\right] = \frac{R}{9}$$

$$\lambda = \frac{9}{R}$$

$$\Rightarrow \quad \frac{9 \times 3 \times 122}{4} = 823\ nm$$

15. Ans. (3)

Energy of projectile = $\dfrac{k(z_1 e)(z_2 e)}{r_0}$

16. Ans. (1)

$P = mv \propto \frac{1}{n}$

$r \propto n^2$

$KE \propto \frac{1}{n^2}$

$L \propto n$

17. Ans. (1)

$\sum = 13.6 \times \frac{9}{4} = 30.6eV$

18. Ans. (3)

$\frac{1}{\lambda} = k(z-1)^2$

$\frac{1}{\lambda} = k(100)$

$\frac{1}{4\lambda} = k(25) = k(6-1)^2$

$\boxed{z = 6}$

19. Ans. (3)

For K_α: $E_0(Z-1)^2 \times \frac{3}{4} = \frac{hc}{\lambda_{K_\alpha}}$

For K_γ: $E_0(Z-1)^2 \times \frac{15}{16} = \frac{hc}{\lambda_{K_\gamma}}$

20. Ans. (2)

Spectral lines of Balmer series are in visible range.

Balmer series lines are obtained due to transition of an electron to $n = 2$ state from any higher state.

SECTION–B

1. Ans. (4)

$N = \frac{I\lambda}{hc} = \frac{1.388 \times 10^2 \times 550 \times 10^{-9}}{2 \times 10^{-25}}$

$= 6.94 \times 55 \times 10^{19}$

$= 4 \times 10^{21}\ ph/sec\ m^2$

2. Ans. (7)

$E_{\text{photon}} = \left(-\frac{13.6}{9}\right)eV - (-13.6\ eV) \approx 12.1\ eV$

$eV_0 = 12.1\ eV - 5.1\ eV \Rightarrow V_0 = 7V$

3. Ans. (2)

$eV_0 = h\nu - h\nu_0$

$\Rightarrow \quad V_0 = \dfrac{h(\nu - \nu_0)}{e}$

$\Rightarrow \quad V_0 = \dfrac{6.6\times10^{-34}[8.2-3.3]\times10^{14}}{1.6\times10^{-19}} \approx 2V$

4. Ans. (6)

$E = \dfrac{12400}{975} = 12.71\ eV$

$\Rightarrow$ Electron can be excited upto $n \simeq 4$

Total No. of line = $\dfrac{4\times3}{2} = 6$

5. Ans. (4)

$eV_S = E - \phi \Rightarrow V_S = \dfrac{hc}{\lambda e} - \dfrac{hc}{\lambda_0 e}$

Here, $3V_0 = \dfrac{hc}{\lambda e} - \dfrac{hc}{\lambda_0 e}$ ...(1)

and $V_0 = \dfrac{hc}{2\lambda e} - \dfrac{hc}{\lambda_0 e}$ ...(2)

Equation (1) – 3 × equation (2)

$\Rightarrow \quad 0 = -\dfrac{hc}{2\lambda e} + \dfrac{2hc}{\lambda_0 e} \Rightarrow \lambda_0 = 4\lambda$

6. Ans. (6)

$\because \quad R \times R_0\ (A)^{1/3}$

$\dfrac{R_{Ae}}{R_{Te}} = \dfrac{R_0(A_{A\ell})^{1/3}}{R_0(A_{TE})} = \dfrac{3}{5}$

$R_{Te} = \dfrac{5}{3}\times3.6 = 6\ fermi$

7. Ans. (18)

$4{}_1^1H \to {}_2^4He + 26MeV$ (Amount of energy released)

Total number of atoms $= \dfrac{(1.7\times10^{30})}{1\times10^{-3}}\times N_A = (1.7\times10^{33})N_A$

4 Hydrogen atom $\to$ 26 MeV

$1.7\times10^{33}N_A$ hydrogen atom $\to \dfrac{26}{4}\times1.7\times10^{33}N_A$ (Total amount of energy released) = E

Power $= \dfrac{\text{Energy}}{\text{Time}}$

$3.9\times10^{21} = \dfrac{E}{t}, \quad t = \dfrac{8}{3}\times10^{18}$ sec

8. Ans. (1)

The nuclear reaction is,

$${}_1^2H + {}_1^2H \rightarrow {}_2^4He + Q$$

Total binding energy of helium nucleus

$= 4 \times 7 = 28\ Mev$

Total binding energy of each deuteron

$= 2 \times 1.1 = 2.2\ MeV$

Hence, energy released when two deuteron fuse to form helium,

$= 28 - 2 \times 2.2 = 28 - 4.4 = 23.6\ MeV = 236 \times 10^{-1}\ MeV$

9. Ans. (100)

Given that,

Kinetic energy $K.E = 48 MeV$

Q value $Q = 50\ MeV$

We know that,

The kinetic energy is

$$E = \frac{A-4}{A} \times Q \qquad ...(1)$$

Now, put the value in equation (I)

$48A = (A - 4)50$

$48A - 50A = -200$

$2A = 200$

$A = 100$

Hence, the mass number of the mother nucleus is 100.

10. Ans. (3)

Let initially number of nuclei is = N_0

$N_0 \rightarrow \frac{N_0}{2}$ (After first half life)

$\frac{N_0}{2} \rightarrow \frac{N_0}{4}$ (After second half life)

Total decay = $N_0 - \frac{N_0}{4} = \frac{3N_0}{4}$

Probability = $\frac{\frac{3N_0}{4}}{N_0} = \frac{3}{4}$

JEE (Advanced) Practice Paper

1. Ans. (B)

$$p^x q^y c^z = \left(ML^{-1}T^{-2}\right)^x \left(\frac{ML^2T^{-2}}{L^2T}\right)^y \left(LT^{-1}\right)^z$$

$$= M^{x+y}\ L^{-x+z}\ T^{-2x-3y-z}$$

As the quantity is dimensionless, therefore

$x + y = 0$; $\quad \therefore \quad x = -y$

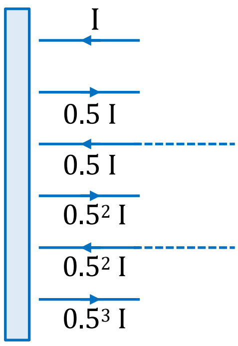
$-x + z = 0$; $\quad \therefore \quad x = z$

$-2x - 3y - z = 0$, which satisfy this equation.

Hence, $x = -y = z$

2. Ans. (D)

The force experienced by A is due to infinite number of incidence and reflection. The situation is shown in the diagram.



$$F = \frac{I}{c} + \frac{0.5\,I}{c}\times 2 + \frac{0.5^2\,I}{c}\times 2 + \frac{0.5^3\,I}{c}\times 2 + ...$$

$$= \frac{I}{c} + \frac{2\,I}{c}\left[\frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} + ...\right]$$

$$= \frac{I}{c} + \frac{2\,I}{c}\left[\frac{1/2}{1-1/2}\right] = \frac{3I}{c}$$

3. Ans. (C)

$f = mg$

$= 20 \times 10^{-3} \times 10$

$= 10^{-1} \times 2$

$= 2 \times 10^{-1}\ N$

$F_{photon} = mg$

$$\frac{P}{C} = 2\times 10^{-1}$$

$P = 2 \times 10^{-1} \times 3 \times 10^{8}$

$P = 6 \times 10^{7}\ W$

4. Ans. (A)

Be for ${}_1^2H$

Per nucleon = 1.1 MeV

Energy released = 23.6 MeV

$${}_1^2H + {}_1^2H \rightarrow {}_2^4H$$

Total Binding energy of He – 2. Total BE of Deuteron = Energy released

Total $B.E.$ of Helium = Energy + 2 × (Total Be of ${}_1^2H$)

$$= 23.6\ MeV + 2\times(1.1\times 2) = 28\ MeV$$

5. Ans. (A)

Let $m = m_0e^{-\lambda t}$ (mass after 't's)

$$-\frac{dm}{dt} = m_0\lambda e^{-\lambda t}$$

Then thrust force on the body

$$-u_r\frac{dm}{dt} = F_t$$

$$u\left(m_0\lambda e^{-\lambda t}\right) = m.\frac{dv}{dt}$$

$$um_0\lambda e^{-\lambda t} = m_0e^{-\lambda t}.\frac{dv}{dt}$$

On integrating

Both sides $v = u\lambda t$

6. Ans. (A)

Binding energy of $3B$ elements will be $3E_b$

So, by energy conservation, binding energy of three B elements is equal to the sum of A element's binding energy and the released energy.

So, equation is

$E_a + e = 3E_b$

So, correct choice is option A.

7. Ans. (D)

Conservation of angular momentum

$$\frac{M}{4}v_1 = \frac{3M}{4}v_2$$

$$\Delta mC^2 = \frac{1}{2}\frac{M}{4}v_1^2 + \frac{1}{2}\times\frac{3M}{4}v_2^2$$

$$v_1 = c\sqrt{\frac{6\Delta m}{M}}$$

8. Ans. (A)

From momentum conservation $\dfrac{h}{\lambda_{\text{photon}}} = \sqrt{2m_{\text{nucleus}}\times K}$

$\Rightarrow \quad K = 1.1 \times 10^3\ eV = 1.1\ KeV.$

9. Ans. (AB)

$$N_x = N_0e^{-(\lambda_1+\lambda_2)t}$$

$$\frac{dN_y}{dt} = \lambda_1N_0e^{-(\lambda_1+\lambda_2)t}$$

$$\frac{dN_z}{dt} = \lambda_2N_0e^{-(\lambda_1+\lambda_2)t}$$

$$N_y = \frac{\lambda_1N_0}{(\lambda_1+\lambda_2)}\left[1-e^{-(\lambda_1+\lambda_2)t}\right]$$

$$N_z = \frac{\lambda_2 N_0}{(\lambda_1+\lambda_2)}\left[1-e^{-(\lambda_1+\lambda_2)t}\right]$$

So, $\frac{\lambda_1 N_0}{\lambda_1+\lambda_2} = b;\quad \frac{\lambda_2 N_0}{\lambda_1+\lambda_2} = c$

$$\frac{\lambda_1}{\lambda_2} = \frac{b}{c};\quad \lambda_1 + \lambda_2 = \tan\theta$$

$$\lambda_1 = \frac{b\tan\theta}{b+c}$$

$$\lambda_2 = \frac{c\tan\theta}{b+c}$$

10. Ans. (ACD)

$$\lambda = \frac{1240}{50\times 10^3}\, nm$$

$$f = \frac{c}{\lambda} = \frac{3\times 10^8 \times 50\times 10^3}{1240\times 10^{-9}} = \frac{150}{124}\times 10^{49} = 1.2\times 10^{19}\, Hz$$

minimum wavelength

$$\lambda_{\min} = \frac{1240}{50\times 10^{+3}} = 24.8\times 10^{-3}\, nm$$

Rate of emission of electron

$$\frac{dN}{dt} = \frac{20\times 10^{-3}}{1.6\times 10^{-19}} = 1.25\times 10^{17}\,/\sec$$

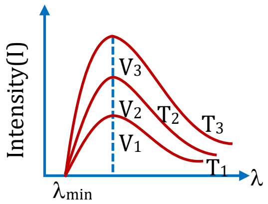
11. Ans. (CD)

Since $\lambda_{\min}$ for all the three accelerating potential is same.

So, $V_1 = V_2 = V_3$

As we know temperature of filament increases intensity increases.

So, $T_1 < T_2 < T_3$



12. Ans. (AB)

Average rate of rise of temperature $\frac{\Delta\theta}{\Delta t} = \frac{P}{ms} = \frac{(0.99)(50kV)(20mA)}{(1kg)(495\text{ J kg}^{-1}{}^\circ C^{-1})} = 2^\circ C/\sec$

The minimum wavelength of the X-rays emitted $\lambda_{\min} = \frac{hc}{eV} = \frac{12400}{50\times 10^3} = 0.248\text{Å}$

13. Ans. (ACD)

(i) 99% of applied electric power is converted into heat

$$ms\frac{\Delta T}{\Delta t} = 0.99\times VI$$

$$\Rightarrow \frac{\Delta T}{\Delta t} = \frac{0.99 \times 50 \times 103\times 10\times 10-3}{1\times 495}$$

$$\frac{\Delta T}{\Delta t} = 1\ ^\circ C/\text{sec}$$

(ii) $\lambda_{\min} = \frac{1240}{V}\, nm$

$$= \frac{1240}{50\times 10^3} \approx 0.25\times 10^{-10}\, m$$

14. Ans. (ACD)

The peak represents the characteristic x-ray, a property of atomic number.

Increasing accelerating potential will cause the electron to strike the target with more energy $\Rightarrow \lambda_m$ decreases.

15. Ans. (8)

$$\frac{mv^2}{r} = \frac{Kq_1q_2}{r^2}; \frac{v^2}{r} \propto \left(\frac{Z}{n}\right)^2 \times \frac{Z}{n^2} = \frac{Z^3}{n^4}$$

In He, $Z = 2$, $\left.\frac{v^2}{r}\right|_{He/H} = 8$

16. Ans. (2)

$$E \propto \frac{Z^2}{n^2}$$

In series limit of bracket axis, transition is for $n = \infty$ to $n = 4$

In series limit of Balmer series, transition for $n = \infty$ to $n = 2$

$$h\nu_{\text{bracket}} = 13.6Z^2\left[\frac{1}{16}\right]$$

$$h\nu_{\text{Balmer}} = 13.6\left[\frac{1}{4}\right]$$

$$\frac{13.6Z^2}{16} = \frac{13.6}{4} = Z = 2$$

17. Ans. (7)

$$\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

18. Ans. (200 Ω.s)

The activity of the sample at time t is given by

$A = A_0\, e^{-\lambda t}$

where λ is the decay constant and A_0 is the activity at time $t = 0$ when the capacitor plates are connected. The charge on the capacitor at time t is given by

$Q = Q_0\, e^{-t/CR}$

where Q_0 is the charge at $t = 0$ and $C = 100\ \mu F$ is the capacitance. Thus, $\dfrac{Q}{A} = \dfrac{Q_0}{A_0}\dfrac{e^{-t/CR}}{e^{-\lambda t}}$

It is independent of t if $\lambda = \dfrac{1}{CR}$ or $R = \dfrac{1}{\lambda C} = \dfrac{t_{av}}{C} = \dfrac{20\times10^{-3}\,s}{100\times10^{-6}\,F} = 200\ \Omega$.