01 Geometrical Optics

Introduction:

Optics is a branch of physics which deals with the behaviour of light. Under many circumstances, the wavelength of light is negligible compared with the dimensions of the device as in the case of ordinary mirrors and lenses. A light beam can then be treated as a ray whose propagation is governed by simple geometric rules. The part of optics that deals with such phenomenon is known as geometrical optics.

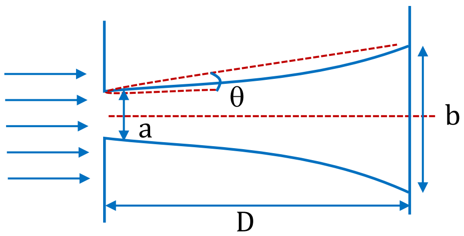
Condition for rectilinear propagation of Light:

The assumption that the light travel in a straight line is correct if

(i) the medium is isotropic, i.e., its behaviour is same in all directions and

(ii) the obstacle past which the light moves or the opening through which the light moves is not very small.

Consider a slit of width *'a'* through which monochromatic light rays pass and strike a screen, placed at a distance *D* as shown.



It is found that the light strikes in a band of width *'b'* more than *'a'*. This bending is called **diffraction**.

Light bends by $(b\text{-}a)/2$ on each side of the central line.

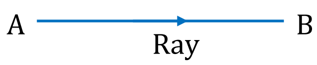
It can be shown by wave theory of light that $\sin\theta = \frac{\lambda}{a}$ ...(A), where θ is shown in figure.

This formula indicates that the **bending is considerable only when a ≃ λ**. Diffraction is more pronounced in sound because its wavelength is much more than that of light and it is of the order of the size of obstacles or apertures. Formula (A) gives $\frac{b-a}{2D} \approx \frac{\lambda}{a}$.

It is clear that the bending is negligible if $\frac{D\lambda}{a} << a$ or $a >> \sqrt{D\lambda}$. If this condition is fulfilled, light is said to move rectilinearly. In most of the situations including geometrical optics the conditions are such that we can safely assume that light moves in straight line and bends only when it gets reflected or refracted.

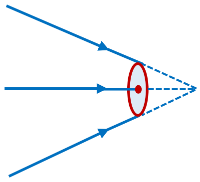
Thus, geometrical optics is an approximate treatment in which the light waves can be represented by straight lines which are called rays. A **ray** of light is the straight-line path of transfer of light energy. Arrow represents the direction of propagation of light.

Figure shows a ray which indicates light is moving from *A* to *B*.

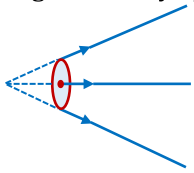


Beam : A bundle or bunch of rays is called a beam. It is of following three types :

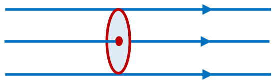
(a) **Convergent beam :** In this case diameter of beam decreases in the direction of ray.



(b) **Divergent beam :** It is a beam in which all the rays meet at a point when produced backward and the diameter of beam goes on increasing as the rays proceed forward.



(c) **Parallel beam :** It is a beam in which all the rays constituting the beam move parallel to each other and diameter of beam remains same.



Parallel beam of light

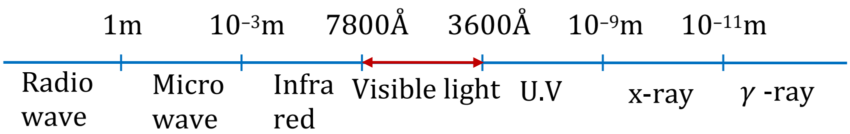
Properties of Light:

(i) Speed of light in vacuum, denoted by $c = 3 \times 10^8\ m/s$ approximately.

(ii) Light is electromagnetic wave (proposed by Maxwell). It consists of varying electric field and magnetic field.

(iii) Light carries energy and momentum.

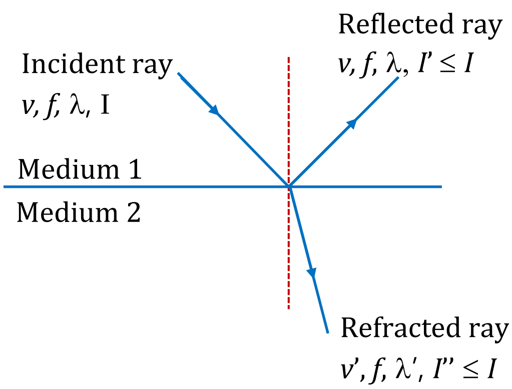
(iv) The formula $v = f\lambda$ is applicable to light.



Electromagnetic spectrum

(v) When light gets reflected in same medium, it suffers no change in frequency, speed and wavelength.

(vi) Frequency of light remains unchanged when it gets reflected or refracted.

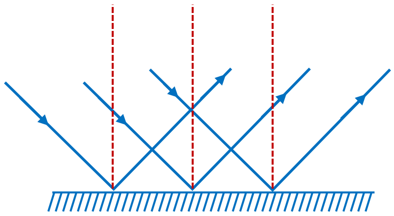


Reflection of Light:

When light rays strike the boundary of two media such as air and glass, a part of light is turned back into the same medium. This is called Reflection of Light.

(a) Regular Reflection:

When the reflection takes place from a perfect plane surface it is called **Regular Reflection**. In this case the reflected light has large intensity in one direction and negligibly small intensity in other directions.



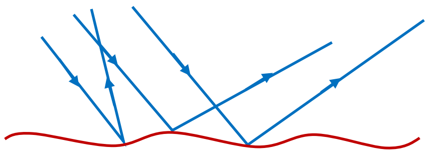
Regular Reflection

(b) Diffused Reflection

When the surface is rough, we do not get a regular behavior of light. Although at each point light ray gets reflected irrespective of the overall nature of surface, difference is observed because even in a narrow beam of light there are many rays which are reflected from different points of surface, and it is quite possible that these rays may move in different directions due to irregularity of the surface. This process enables us to see an object from any position.

Such reflection is called as **diffused reflection**.

For example, reflection from a wall, from a newspaper etc. This is why you cannot see your face in newspaper and in the wall.



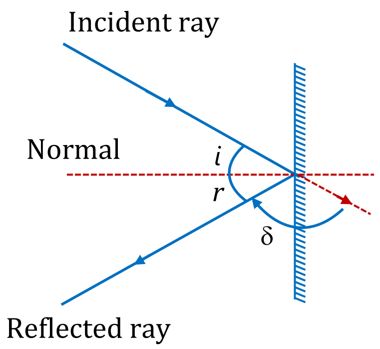
Diffused Reflection

A few terminologies regarding reflection are the following:

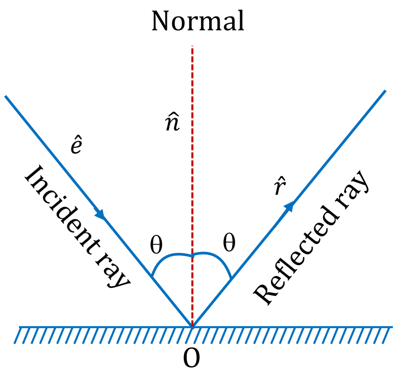
Angle of incidence: The angle that the incident ray makes with the normal to the reflecting surface is known as the angle of incidence, and it is denoted by i.

Angle of reflection: The angle that the reflected ray makes with the normal to the reflecting surface is known as angle of reflection, and it is denoted by r.

Angle of deviation: The angle of deviation is the angle between the direction of the reflected ray and the direction of incident ray in the absence of the interface, and it is denoted by δ.

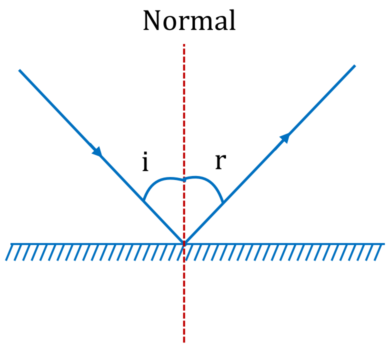


Laws of Reflection:



(a) The incident ray, the reflected ray and the normal at the point of incidence lie in the same plane. This plane is called the **plane of incidence (or plane of reflection)**. This condition can be expressed mathematically as $\hat{e}.(\hat{n}\times\hat{r})=0$

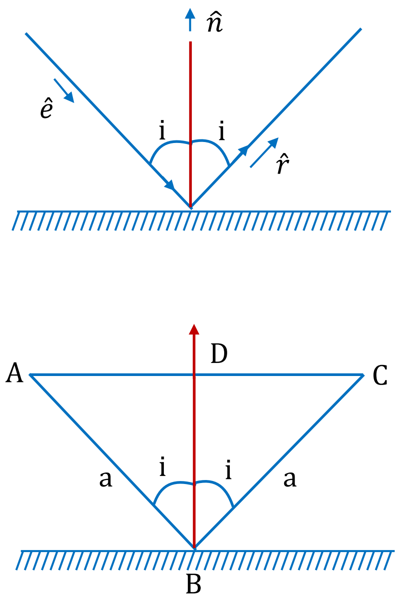
(b) The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal, i.e. $\angle i=\angle r$



In vector form both laws can be expressed as

In Vector form $\boxed{\hat{r}=\hat{e}-2(\hat{e}.\hat{n})\hat{n}}$

Vector form of Law of Reflection:



$\overrightarrow{AB}+\overrightarrow{BD}=\overrightarrow{AD}$

$\overrightarrow{CB}+\overrightarrow{BD}=\overrightarrow{CD}$

$\overrightarrow{AD}=-\overrightarrow{CD}$

$\overrightarrow{AB}+\overrightarrow{BD}=\overrightarrow{BC}+\overrightarrow{DB}$

$a\hat{e}+2(a\cos i\hat{n})=\hat{r}a$ ...(i)

$\hat{r}=\hat{e}+2\cos i\hat{n}$

$\hat{e}.\hat{n}=\cos(\pi-i)$

$\hat{e}.\hat{n}=-\cos i$ ...(ii)

From (i) & (ii)

$\hat{r}=\hat{e}-2(\hat{e}.\hat{n})\hat{n}$

Newton's corpuscular theory: according to this theory, light is made of particles which on reflection, collide elastically with the reflecting surface. Although This theory was unable to explain refraction phenomenon but it can explain the reflection of light. Using this theory, we can say that component of unit vector along normal, reverts and that tangent to surface remain same.

Illustration 1:

A light ray travelling along $3\hat{i}-4\hat{j}$ is reflected by a plane mirror reflected in *x*-*z* plane. Find unit vector along reflected ray.

Solution:

As mirror is in *x*-*z* plane so normal lies along *y*-axis

So $\quad \hat{n}=\hat{j}$

and $\quad \hat{e}=3\hat{i}-4\hat{j}$

Using $\quad \hat{r}=\hat{e}-2(\hat{e}.\hat{n})\hat{n}$

$$\hat{r}=3\hat{i}-4\hat{j}-2(-4)\hat{j}$$

$$\hat{r}=3\hat{i}+4\hat{j}$$

Alternate:

As mirror is in *x*-*z* plane so normal lies along *y*-axis so only component of unit vector along normal gets reversed. Hence in this case only y component of $\hat{e}$ gets reversed.

So $\quad \hat{r}=3\hat{i}+4\hat{j}$

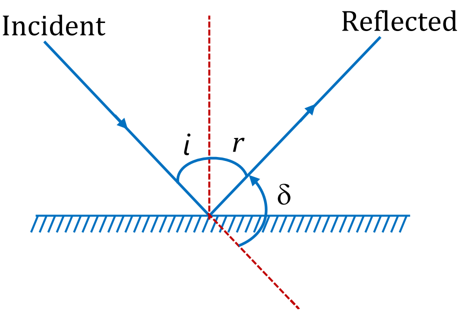
Deviation by plane mirror:

A plane mirror deviates the light through an angle

$$\delta=180°-2i$$

where i is the angle of incidence. The deviation is maximum for normal incidence.

$$\delta_{max}=180°$$



Deviation δ produced by a plane mirror

Deviation by two plane mirrors:

Total deviation produced by the combination of two plane mirrors which are inclined at an angle θ from each other.

$$\delta=\delta_1+\delta_2=180-2\alpha+180-2\beta$$

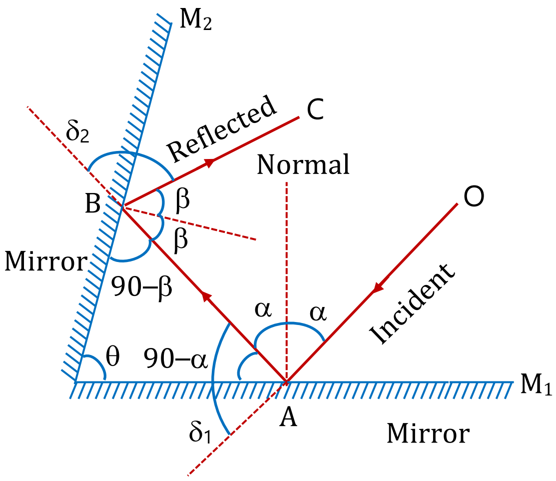
$$=360-2(\alpha+\beta) \quad \text{...(i)}$$

From ΔQAB, $\theta+90-\alpha+90-\beta=180$

$$\Rightarrow \quad \theta=\alpha+\beta \quad \text{...(ii)}$$

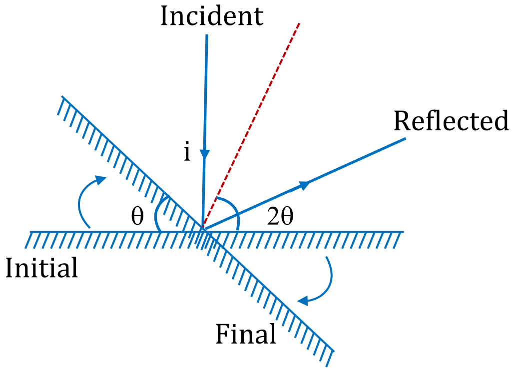
Putting the value of θ in (i) from (ii),

$$\delta=360-2\theta$$



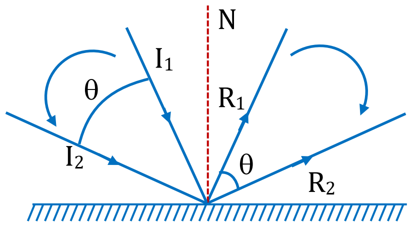
Rotation of reflected rays:

(i) If the direction of the incident ray is kept constant and the mirror is rotated through an angle θ about an axis in the plane of mirror, then the reflected ray rotates through an angle 2θ.



As the mirror turns through θ, the reflected ray turns through 2θ.

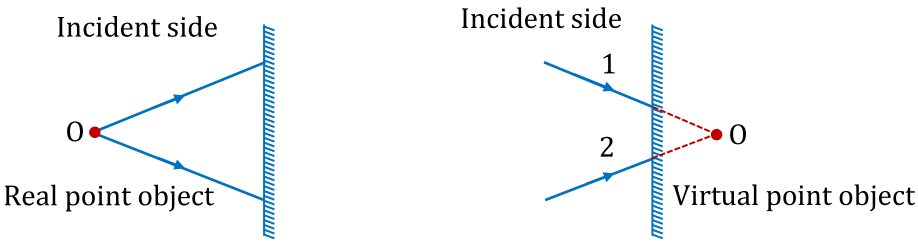
(ii) If mirror is kept fixed and incident ray is rotated then reflected ray will rotate in opposite sense by same angle.



Rotation of incident ray

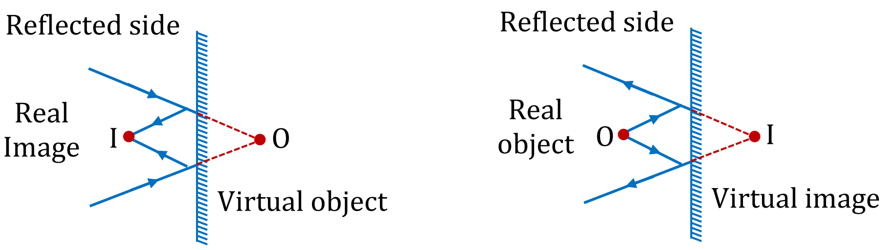
Object and Image:

(a) **Object (O): Object** is defined as point of intersection of **incident** rays.



Intersection point of diverging rays is called **real object** and that of converging rays is called **virtual object** as shown in above figure.

(b) **Image (I) : Image** is defined as point of intersection of **reflected** rays (in case of reflection) or **refracted** rays (in case of refraction).



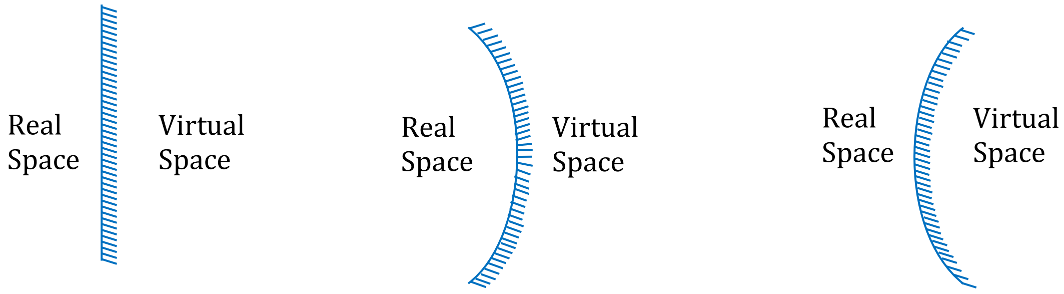
Intersection point of converging reflected/refracted rays is called **real image** and that of diverging reflected/refracted rays is called **virtual image** as shown in above diagram.

Real and Virtual Spaces:

A mirror, plane or spherical divides the space into two;

(a) Real space, a side where the reflected rays exist.

(b) Virtual space is on the other side where the reflected rays do not exist.

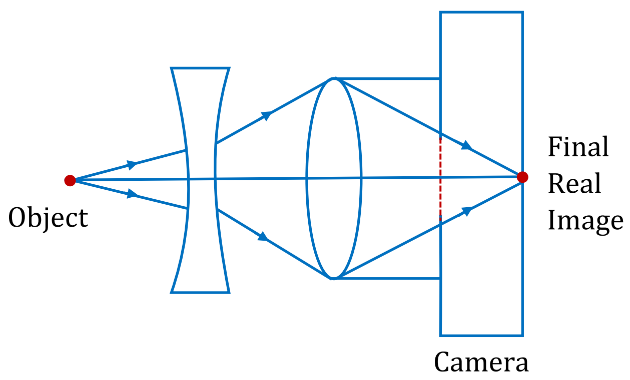


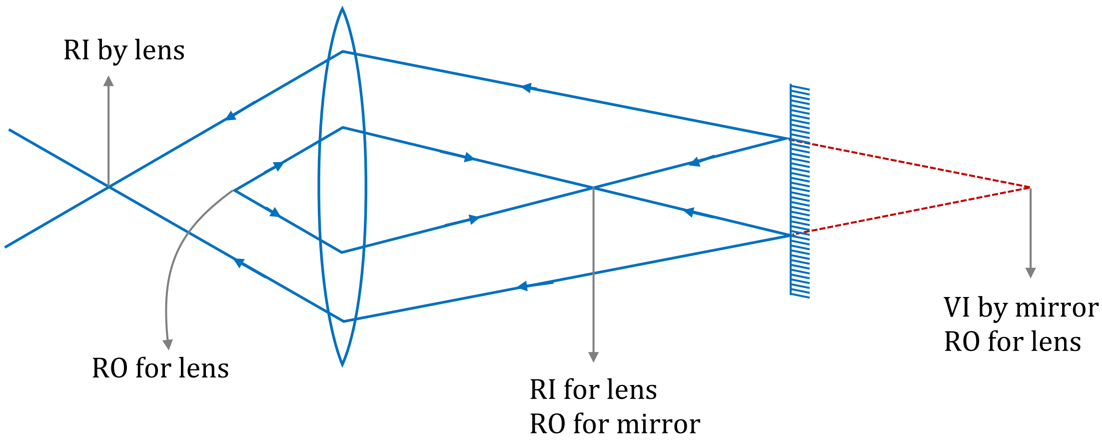
Object/Image	Criteria
Real Object	When incident rays are of diverging nature.
Virtual Object	When Incident rays are of converging nature. Incident rays converge to a point behind the optical element.
Real Image	When rays after interacting with the optical element are of converging nature.
Virtual Image	When rays after interacting with the optical element are of diverging nature.

A real image can be captured on a screen. Preferably the surface should be white in color so that we can see the image clearly. This image on the screen can be perceived by the eye.

How about virtual images? Since they cannot be captured on a screen is it possible to see them. From practical experience we know that we can see ourselves in a plane mirror which is indeed a virtual image. So, it is possible to see a virtual image. But how is it done? There is a lens in our eye that focuses the diverging set of rays on to the retina forming a real image inside our eyes.

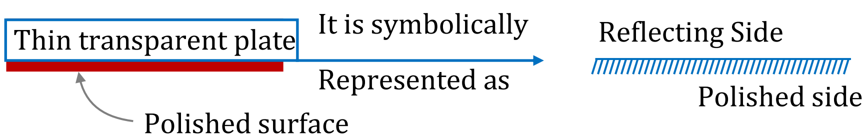
In a similar manner, a camera can also convert a virtual image into a real image which is captured on the film (figure)





Plane Mirror:

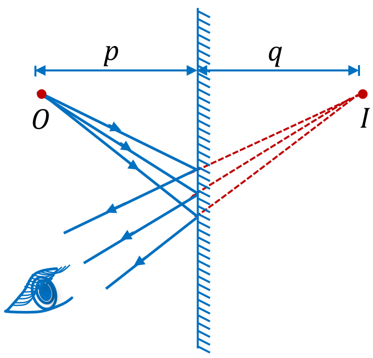
Plane mirror is formed by polishing one surface of a plane thin glass plate. It is also said to be silvered on one side.



Plane mirror

A beam of parallel rays of light, incident on a plane mirror will get reflected as a beam of parallel reflected rays.

Images Formed by Flat Mirrors: Image formation by mirrors can be understood through the behaviour of light rays as described by the wave under reflection analysis model. Let's begin with an image you see every day: your face in the bathroom mirror. This image is formed by the simplest possible mirror, the flat mirror. Consider a point source of light placed at O in Figure, a distance p in front of a flat mirror. The mirror is perpendicular to the page, so we see the inter- section of the mirror with the page. The distance p is called the **object distance,** the name anticipating that we will place objects in front of mirrors and study their images. Diverging light rays leave the source and are reflected from the mirror, obeying the law of reflection. Upon reflection, the rays continue to diverge. The dashed lines in Figure are backward extensions of the diverging rays back to a point of intersection at I. The diverging rays appear to the viewer to originate at the point I behind the mirror. Point I, which is a distance q behind the mirror, is called the **image** of the object at O. The distance q is called the **image distance.** Regard- less of the system under study, images can always be located by extending diverging rays back to a point at which they intersect.



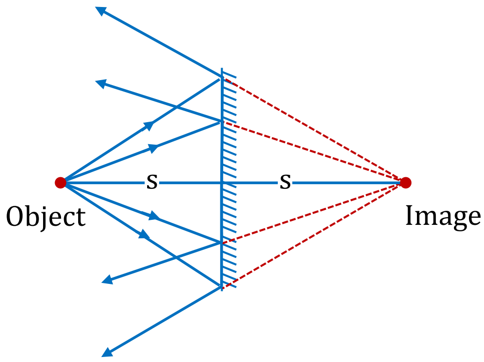
Mirror

The image point I is located behind the mirror at distance q from the mirror. The image is virtual.

An image formed by reflection from a flat mirror.

Properties of image formed by a plane mirror:

Point object



Characteristics of image due to reflection by a plane mirror :

(i) Distance of object from the mirror = Distance of image from the mirror.

(ii) All the incident rays from a point object will meet at a single point after reflection from a plane mirror which is called image.

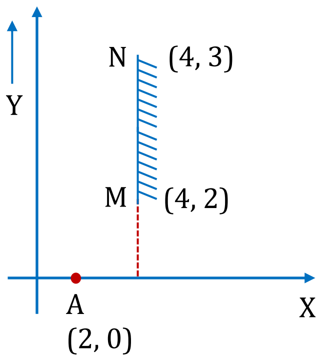
(iii) The line joining a point object and its image is normal to the reflecting surface.

(iv) For a real object the image is virtual and for a virtual object the image is real

(v) The region in which observer's eye must be present in order to view the image is called **field of view**.

Illustration 2:

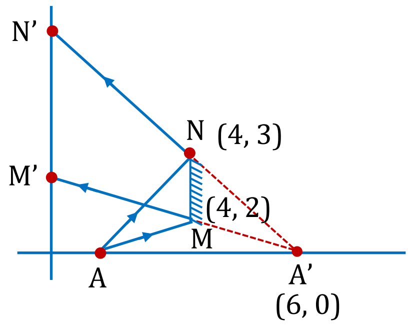
Find the region on Y axis in which reflected rays are present. Object is at A (2, 0) and MN is a plane mirror, as shown.



Solution:

The image of point A, in the mirror is at A' (6, 0).

Join $A'M$ and extend to cut Y axis at M' (Ray originating from A which strikes the mirror at M gets reflected as the ray MM' which appears to come from A'). Join $A'N$ and extend to cut Y axis at N' (Ray originating from A which strikes the mirror at N gets reflected as the ray NN' which appears to come from A').



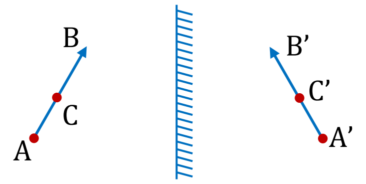
From geometry.

$M' \equiv (0, 6)$

$N' \equiv (0, 9)$. $M'N'$ is the region on Y axis in which reflected rays are present.

Extended object:

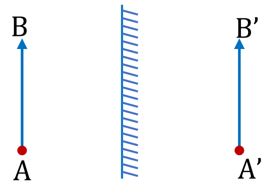
An extended object like *AB* shown in figure is a combination of infinite number of point objects from *A* to *B*. Image of every point object will be formed individually and thus infinite images will be formed. *A´* will be image of *A*, *C´* will be image of *C*, *B´* will be image of *B* etc. All point images together form extended image. Thus, extended image is formed of an extended object.



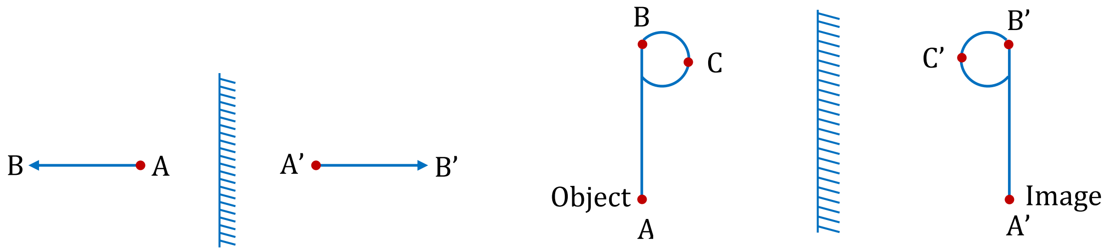
Properties of image of an extended object, formed by a plane mirror:

(i) Size of extended object = Size of extended image.

(ii) The image is erect, if the extended object is placed parallel to the mirror.



(iii) The image is inverted if the extended object lies perpendicular to the plane mirror.



(iv) If an extended horizontal object is placed infront of a mirror inclined 45° with the horizontal, the image formed will be vertical. See figure.

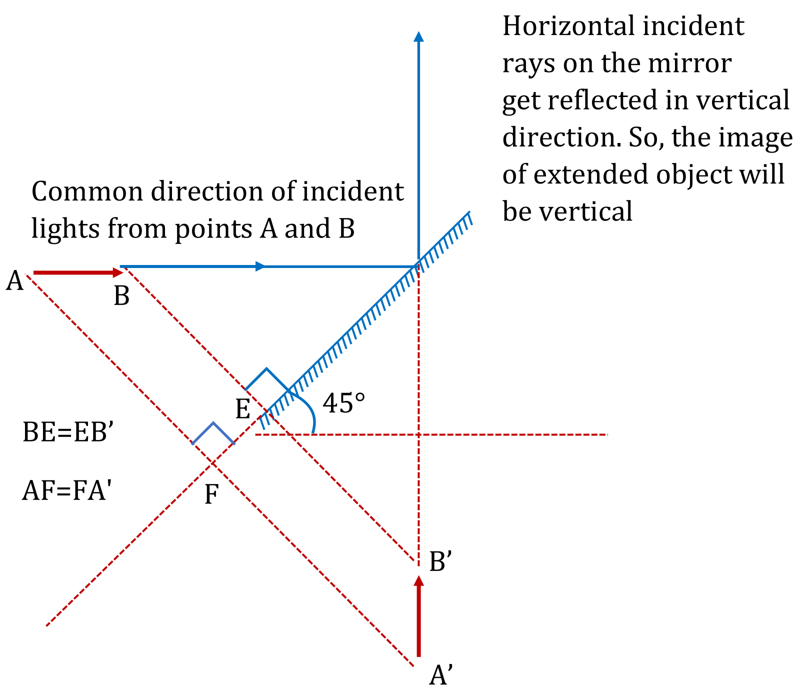
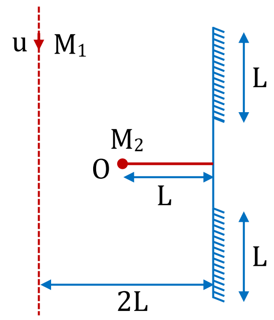


Illustration 3:

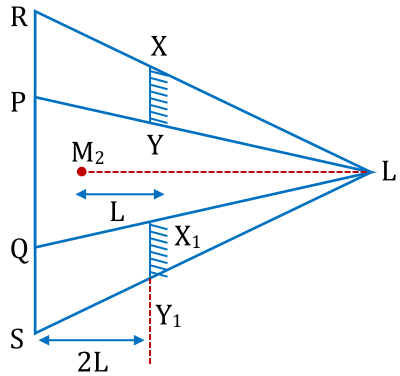
Two plane mirrors of lengths L are separated by distance L and a man M_2 is standing at distance L from the connecting line of mirrors as shown in figure. A man M_2 is walking along a straight line at distance $2L$ parallel to mirrors at speed u, then man M_2 at O will be able to see image of M_1 for total time:



(A) $\frac{4L}{u}$ (B) $\frac{3L}{u}$ (C) $\frac{6L}{u}$ (D) $\frac{9L}{u}$

Solution:

When the man M_1 is in the region RP and QS, then man M_1 sees the image of M_2, from the similar triangle.



PIQ and $XIX_1 \quad \Rightarrow \quad \frac{PQ}{XX_1} = \frac{3L}{L} \quad \Rightarrow \quad PQ = 3L$

From the similar triangle,

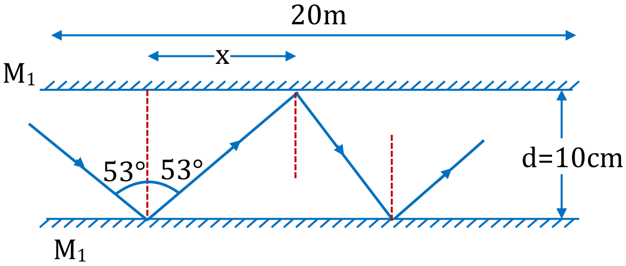
RIS and $XIY_1 \quad \Rightarrow \quad \frac{RS}{XY_1} = \frac{3L}{L} \quad \Rightarrow \quad RS = 9L$

Then $RP + QS = RS - PQ = 9L - 3L = 6L$

$\therefore \quad \text{Time} = \frac{6L}{u}$

Illustration 4:

Two plane mirrors M_1 and M_2 have a length of 20 m each and are 10cm apart. A ray of light is incident on one end of mirror M_2 at an angle 53°. Calculate the number of reflections light undergoes before reaching the other end $\left(\tan 53° = \frac{4}{3}\right)$



Solution:

Let after each reflection it covers a distance x along the mirror

$$x = d\tan 53° = 10 \times \frac{4}{3} = \frac{40}{3}$$

Total number of reflections = $\frac{20}{40/3} \times 100 + 1 = 150 + 1 = 151$

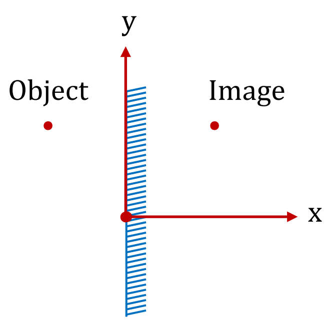
Relation between velocity of object and image:

From mirror property :

$x_{im} = -x_{om}$, $y_{im} = y_{om}$ and $z_{im} = z_{om}$

Here x_{im} means 'x' coordinate of image with respect to mirror.

Similarly, others have corresponding meaning.



Differentiating w.r.t time, we get

$v_{(im)x} = -v_{(om)x}$; $v_{(im)y} = v_{(om)y}$; $v_{(im)z} = v_{(om)z,}$

$\Rightarrow$ for x axis $v_{iG} - v_{mG} = -(v_{oG} - v_{mG})$

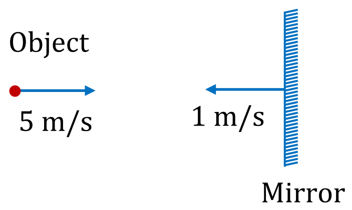
But, for y axis and z axis $v_{iG} - v_{mG} = (v_{oG} - v_{mG})$ or $v_{iG} = v_{oG}$

Here v_{iG} = velocity of image with respect to ground.

Illustration 5:

An object moves with 5 m/s towards right while the mirror moves with 1m/s towards the left as shown. Find the velocity of image.

Solution:



Take $\longrightarrow$ as + direction.

$v_i - v_m = v_m - v_0$

$v_i - (-1) = (-1) - 5$

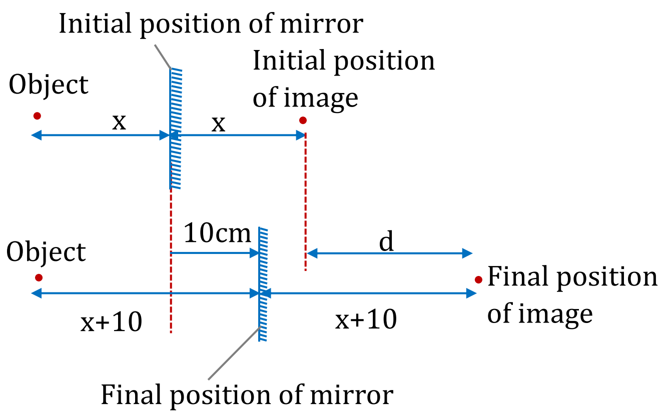
$\therefore$ $v_i = -7 m/s.$

$\Rightarrow$ 7 m/s and direction towards left.

Illustration 6:

There is a point object and a plane mirror. If the mirror is moved by 10 cm away from the object find the distance which the image will move.

Solution:



We know that $x_{im} = -x_{om}$ or $x_i - x_m = x_m - x_o$

or $\Delta x_i - \Delta x_m = \Delta x_m - \Delta x_o$.

In this question $\Delta x_o = 0$; $\Delta x_m = 10$ cm.

Therefore, $\Delta x_i = 2\Delta x_m - \Delta x_o = 20$ cm.

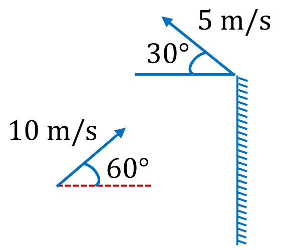
Alter :

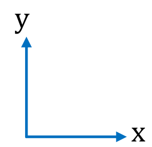
$2(x + 10) = 2x + d$

$\therefore$ $d = 20$ cm

Illustration 7:

In the situation shown in figure, find the velocity of image.





Solution:

Along x direction, applying

$$v_i - v_m = -(v_0 - v_m)$$

$$v_i - (-5\cos 30°) = -(10\cos 60° - (-5\cos 30°))$$

$\therefore \quad v_i = -5(1+\sqrt{3})\ m/s$

Along y direction $v_0 = v_i$

$\therefore \quad v_i = 10\sin 60° = 5\sqrt{3}\ m/s$

$\therefore \quad$ Velocity of the image $= -5(1+\sqrt{3})\hat{i} + 5\sqrt{3}\hat{j}\ m/s.$

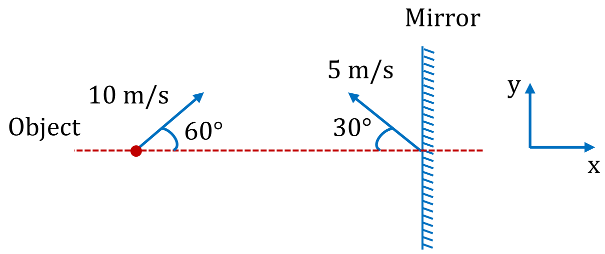


Illustration 8:

In the situation shown in the figure, find the velocity of the image.
(velocities are measured w.r.t. ground frame)

Solution:

First of all, consider the right direction (+ ve x-direction) to be the positive one.

Along x-axis in the ground frame,

The velocity of the object is $\vec{v}_{OG} = 10\cos 60° = 5ms^{-1}$

The velocity of the mirror is $\vec{v}_{MG} = -5\cos 30° = -\dfrac{5\sqrt{3}}{2}ms^{-1}$

Therefore, from equation (ii) we get,

$$\vec{v}_{MG} = \frac{\vec{v}_{OG} + \vec{v}_{IG}}{2}$$

$\Rightarrow \quad \vec{v}_{IG} = 2\vec{v}_{MG} - \vec{v}_{OG}$

$\Rightarrow \quad \vec{v}_{IG} = 2\times\left(-\dfrac{5\sqrt{3}}{2}\right) - 5$

$\Rightarrow \quad \vec{v}_{IG} = -5\sqrt{3} - 5$

$\Rightarrow \quad \vec{v}_{IG} = -5(\sqrt{3}+1)\ ms^{-1}$

Along y-axis in the ground frame,

The velocity of the object is $\vec{v}_{OG} = 10\sin 60° = 5\sqrt{3}\, ms^{-1}$

From equation (iii), we get,

$$\vec{v}_{IG} = \vec{v}_{OG}$$

$$\Rightarrow \quad \vec{v}_{IG} = 5\sqrt{3}\ ms^{-1}$$

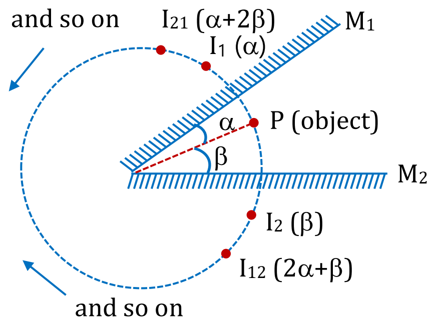
Therefore, the velocity of the image w.r.t. the ground frame is,

$$\vec{v}_I = -5\left(1+\sqrt{3}\right)\hat{i} + 5\sqrt{3}\hat{j}$$

Images formed by two plane mirrors:

If rays after getting reflected from one mirror strike second mirror, the image formed by first mirror will function as an object for second mirror, and this process will continue for every successive reflection.

Locating all the images formed by two plane mirrors:



Consider two plane mirrors M_1 and M_2 inclined at an angle $\theta = \alpha + \beta$ as shown in figure.

Point P is an object kept such that it makes angle α with mirror M_1 and angle β with mirror M_2. Image of object P formed by M_1, denoted by I_1, will be inclined by angle α on the other side of mirror M_1. This angle is written in bracket in the figure besides I_1. Similarly, image of object P formed by M_2, denoted by I_2, will be inclined by angle β on the other side of mirror M_2. This angle is written in bracket in the figure besides I_2.

Now I_2 will act as an object for M_1 which is at an angle $(\alpha+2\beta)$ from M_1. Its image will be formed at an angle $(\alpha+2\beta)$ on the opposite side of M_1. This image will be denoted as I, and so on. Think when this will process stop. Hint: The virtual image formed by a plane mirror must not be in front of the mirror or its extension.

Number of images formed by two inclined mirrors

(i) If $\frac{360°}{\theta}$ = even number; number of image = $\frac{360°}{\theta} - 1$

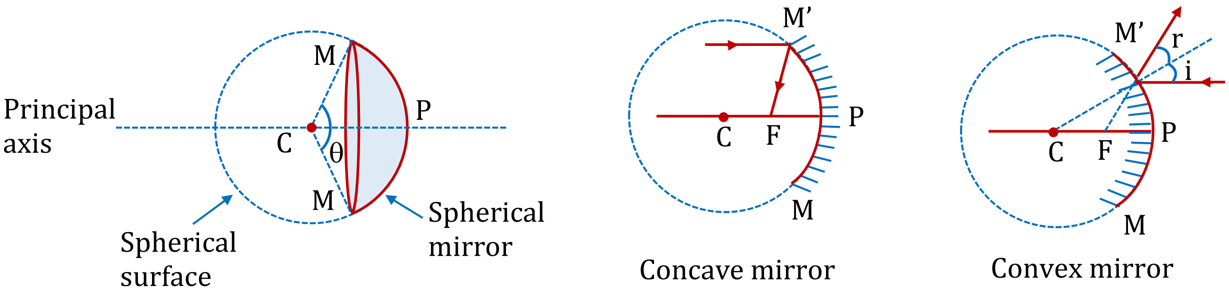
(ii) If $\frac{360°}{\theta}$ = odd number; number of image = $\frac{360°}{\theta} - 1$, if the object is placed on the angle bisector.

(iii) If $\frac{360°}{\theta}$ = odd number; number of image = $\frac{360°}{\theta}$, if the object is not placed on the angle bisector.

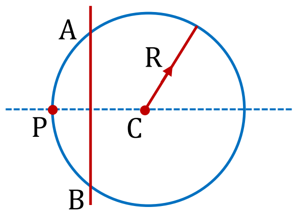
(iv) If $\frac{360°}{\theta} \neq$ integer, then count the number of images as explained above.

Spherical Mirrors

Curved mirror is part of a hollow sphere. If reflection takes place from the inner surface then the mirror is called concave and if its outer surface acts as reflector it is convex.



Important terms related with spherical mirrors:



A spherical shell with the center of curvature, pole aperture and radius of curvature identified

(a) Center of Curvature (C) :

The center of the sphere from which the spherical mirror is formed is called the center of curvature of the mirror. It is represented by *C* and is indicated in figure.

(b) Pole (P) :

The center of the mirror is called as the Pole. It is represented by the point *P* on the mirror *APB* in figure.

(c) Principal Axis :

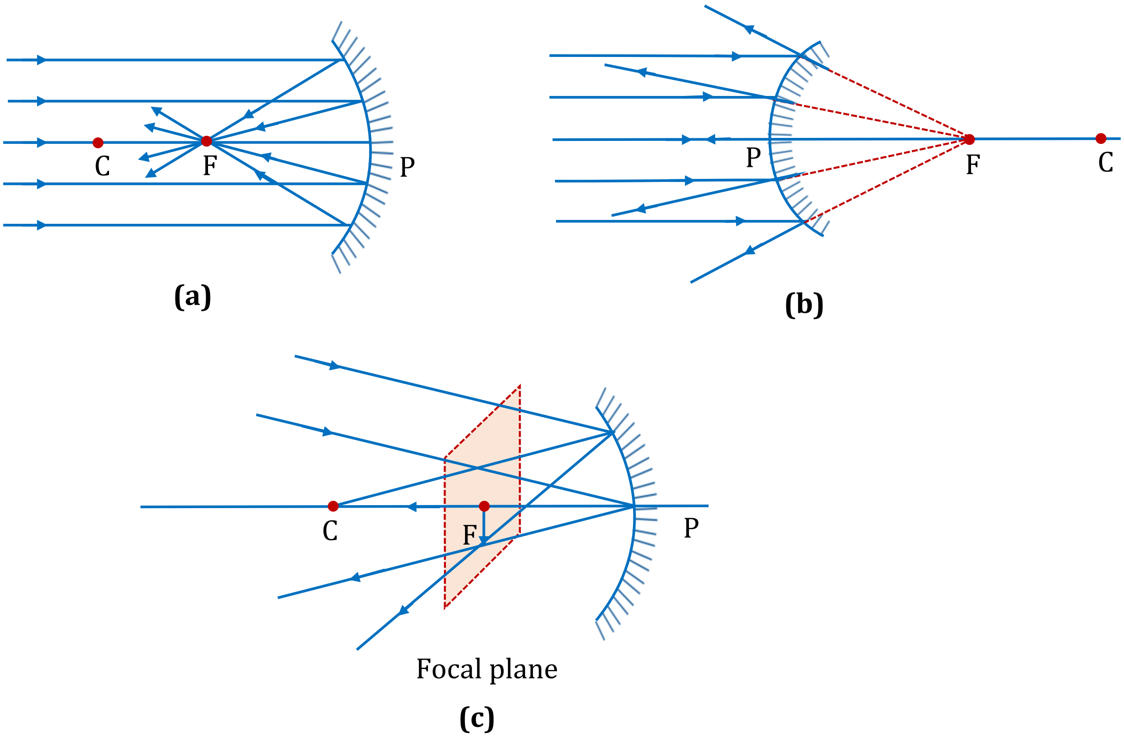
The Principal Axis is a line which is perpendicular to the plane of the mirror and passes through the pole. The Principal Axis can also be defined as the line which joins the Pole to the Center of Curvature of the mirror.

(d) Aperture (A) :

The aperture is the segment or area of the mirror which is available for reflecting light. In figure. *APB* is the aperture of the mirror.

Focal length of spherical mirrors:

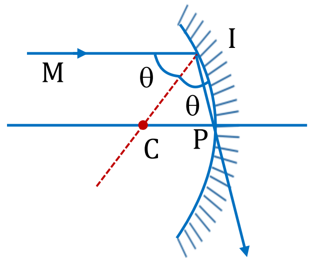
Figure shows what happens when a parallel beam of light is incident on (a) a concave mirror, and (b) a convex mirror. We assume that the rays are paraxial, i.e., they are incident at points close to the pole *P* of the mirror and make small angles with the principal axis. The reflected rays converge at a point *F* on the principal axis of a concave mirror [Fig. (a)]. For a convex mirror, the reflected rays appear to diverge from a point F on its principal axis [Fig. (b)]. The point *F* is called the principal focus of the mirror. If the parallel paraxial beam of light were incident, making some angle with the principal axis, the reflected rays would converge (or appear to diverge) from a point in a plane through *F* normal to the principal axis. This is called the focal plane of the mirror [Fig. (c)].



Focus of a concave and convex mirror.

Illustration 9:

Find the angle of incidence of ray for which it passes through the pole, given that $MI \parallel CP$.



Solution:

$\angle MIC = \angle CIP = \theta$

$MI \parallel CP$

$\angle MIC = \angle ICP = \theta$

$CI = CP$

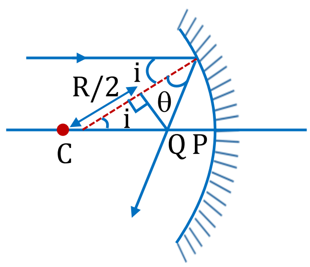
$\angle CIP = \angle CPI = \theta$

$\therefore$ In ΔCIP all angle are equal

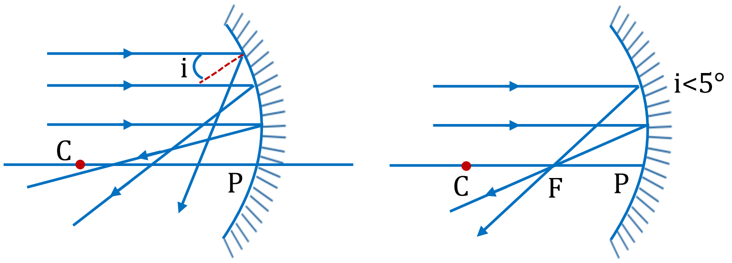
$3\theta = 180° \quad \Rightarrow \quad \theta = 60°$

Illustration 10:

Find the distance CQ if incident light ray parallel to principal axis is incident at an angle i. Also, find the distance CQ if $i \to 0$.



Solution:



$$\cos i = \frac{R}{2CQ}$$

$$\Rightarrow \quad CQ = \frac{R}{2\cos i}$$

As i increases $\cos i$ decreases.

Hence CQ increases

If i is a small angle $\cos i \approx 1$

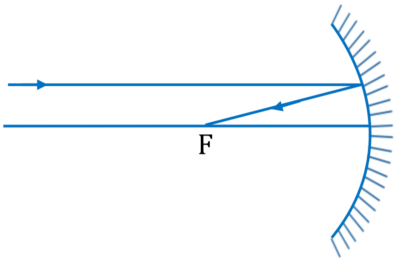
$\therefore \quad CQ = R/2$

So, paraxial rays meet at a distance equal to R / 2 from center of curvature, which is called focus.

Ray tracing:

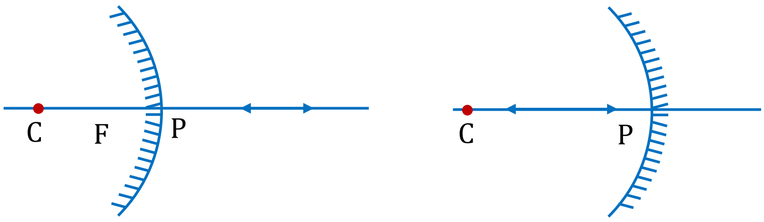
Following facts are useful in ray tracing.

(i) If the incident ray is parallel to the principle axis, the reflected ray passes through the focus.

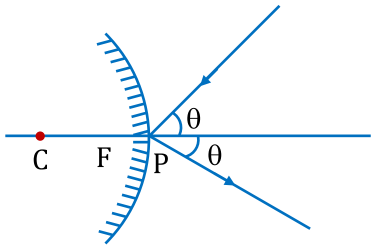


(ii) If the incident ray passes through the focus, then the reflected ray is parallel to the principle axis.

(iii) Incident ray passing through centre of curvature will be reflected back through the centre of curvature (because it is a normally incident ray).



(iv) It is easy to make the ray tracing of a ray incident at the pole as shown in below.



Sign Convention

We are using co–ordinate sign convention.

(i) Take origin at pole (in case of mirror) or at optical centre (in case of lens). Take X axis along the Principal Axis, taking **positive direction along the incident light.**

u*, *v*, *R and ***f*** indicate the x coordinate of object, image, centre of curvature and focus respectively.

(ii) y-coordinates are taken positive above Principle Axis and negative below Principle Axis' $\boldsymbol{h_1}$ and $\boldsymbol{h_2}$ denote the y coordinates of object and image respectively.

Note:

This sign convention is used for reflection from mirror, refraction through flat or curved surfaces or lens

Mirror formulae:

(i) In terms of Cartesian sign convention mirror formula may be expressed as

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

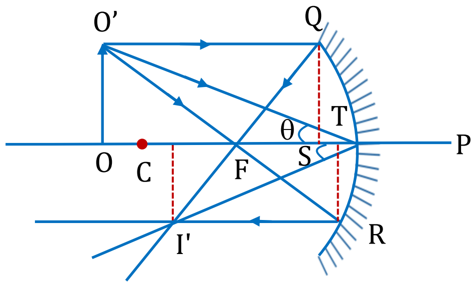
Where u, v and f represents object distance, image distance and focal length respectively.

(ii) Lateral Magnification

$$m=\frac{h_i}{h_0}=-\frac{v}{u}$$

Where h_i and h_o represents height of the image and object respectively.

Derivation:



Here, $\Delta\, QSF$ and $\Delta\, FI\,I'$ are similar triangles

Thus $\quad \dfrac{QS}{I\,I'}=\dfrac{SF}{FI}$ ...(i)

Again $\quad \Delta\, TRF$ and $\Delta\, FO'O$ are similar triangles

Thus $\quad \dfrac{OO'}{TR}=\dfrac{OF}{FT}$ ...(ii)

Since, $\quad QS = OO'$

and $\quad TR = I\,I'$

we get $\quad \dfrac{SF}{FI}=\dfrac{OF}{FT}$ or, $\quad \dfrac{PF-PS}{PI-PF}=\dfrac{OP-FP}{FP-TP}$

Due to very small aperture and paraxial rays, S and T are very close to P so that $SP = TP = 0$ (zero).

$$\frac{f}{v-f}=\frac{u-f}{f} \quad\Rightarrow\quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

For Magnification

$\Delta OPO'$ and $\Delta\, I\,PI'$ are similar

$$\frac{OO'}{OP}=\frac{II'}{IP}$$

$$\frac{h_i}{v}=-\frac{h_f}{u} \quad\Rightarrow\quad \frac{h_i}{h_f}=-\frac{v}{u} \qquad \therefore \quad m=-\frac{v}{u}$$

If m = +ve then image and object are on same side of PA.

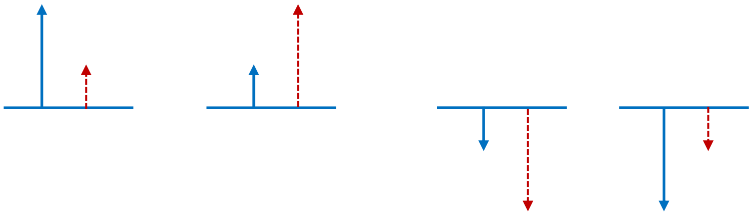


Image is called Erect.

If m=–ve then image and object are on opposite side of PA.

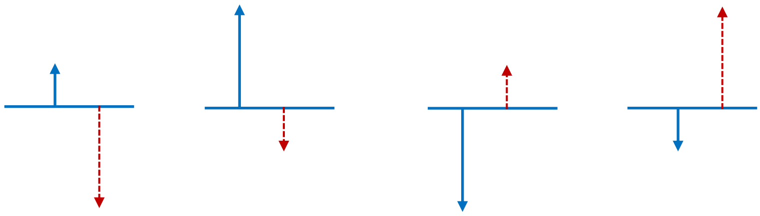
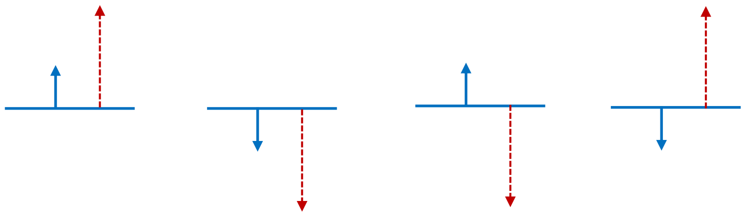
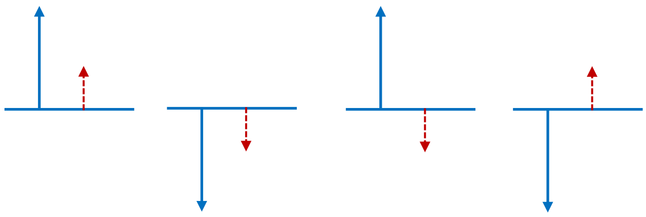


Image is called Inverted.

If $|m|>1$ image is called Enlarged.



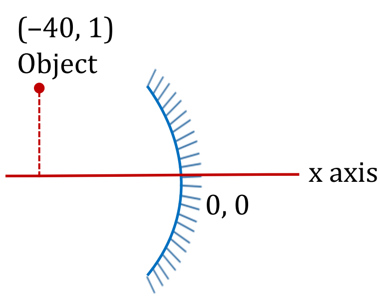
If $|m|<1$ image is called Diminished.



Magnification	Image	Magnification	Image
$\|m\|>1$	enlarged	$\|m\|<1$	diminished
$m<0$	inverted	$m>0$	erect

Illustration 11:

Figure shows a spherical concave mirror with its pole at (0, 0) and principal axis along x axis. There is a point object at (–40 cm, 1cm), find the position of image.



R.O.C = 10 cm

Solution:

According to sign convention,

$u = -40\ cm$

$h_1 = +1\ cm$

$f = -5\ cm.$

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

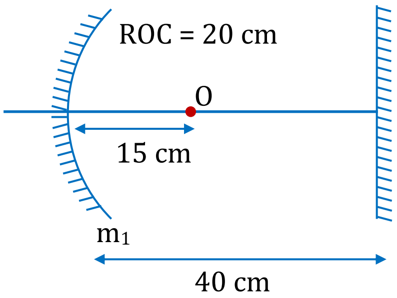
$\Rightarrow \quad \frac{1}{v}+\frac{1}{-40}=\frac{1}{-5}$; $v=\frac{-40}{7}\ cm.$; $\frac{h_2}{h_1}=\frac{-v}{u}$

$\Rightarrow \quad h_2 = \frac{-v}{u} \times h_1 = \frac{-\left(-\frac{40}{7}\right)\times 1}{-40} = -\frac{1}{7}\ cm.$

$\therefore \quad$ The position of image is $\left(\frac{-40}{7}cm, -\frac{1}{7}cm\right)$

Illustration 12:

Find the position of final image after three successive reflections taking first reflection on m_1.



Solution:

I reflection :

Focus of mirror = – 10 cm

$\Rightarrow \quad u = -15\ cm$

Applying mirror formula:

$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

$\Rightarrow \quad v = -30\ cm.$

For II reflection on plane mirror :

$u = -10\ cm$

$\therefore \quad v = 10\ cm$

For III reflection on curved mirror again :

$u = -50\ cm$; $f = -10\ cm$

Applying mirror formula:

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; $v = -12.5\ cm.$

Illustration 13:

An extended object is placed perpendicular to the principal axis of a concave mirror of radius of curvature 20 *cm* at a distance of 15 *cm* from pole. Find the lateral magnification produced.

Solution:

$u = -15\ cm$

$f = -10\ cm$

Using $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$ we get, $v = -30\ cm$

$\therefore \quad m=-\frac{v}{u}=-2$

Aliter : $m=\frac{f}{f-u}=\frac{-10}{-10(-15)}=-2$

Illustration 14:

A person looks into a spherical mirror. The size of image of his face is twice the actual size of his face. If the face is at a distance 20 *cm* then find the nature and radius of curvature of the mirror.

Solution:

Person will see his face only when the image is virtual. Virtual image of real object is erect.

Hence $m = 2$

$\therefore \quad \frac{-v}{u}=2 \quad \Rightarrow \quad v = 40\ cm$

Applying $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$; $f = -40\ cm$ or $R = -80\ cm$ (concave)

$\therefore \quad R.O.C. = 80\ cm$

Alter: $m=\frac{f}{f-u} \quad \Rightarrow \quad 2=\frac{f}{f-(-20)}$

$\Rightarrow \quad f = -40\ cm$ or $R = -80 cm$ (concave)

$\therefore \quad R.O.C. = 80\ cm$

Illustration 15:

An image of a candle on a screen is found to be double its size. When the candle is shifted by a distance 5 *cm* then the image become triple its size. Find the nature and *ROC* of the mirror.

Solution:

Since the images formed on screen it is real. Real object and real image implies concave mirror.

Applying $m=\frac{f}{f-u}$ or $-2=\frac{f}{f-(u)}$ ...(i)

After shifting $-3=\frac{f}{f-(u+5)}$ ...(ii)

[Why $u + 5$?, why not $u - 5$: In a concave mirror, the size of real image will increase, only when the real object is brought closer to the mirror. In doing so, its *x* coordinate will increase]

From (i) & (ii) we get,

$f = -30\ cm$ or $R = -60\ cm$ (concave) and $R.O.C. = 60 cm$

Positions, Size and Nature of Images

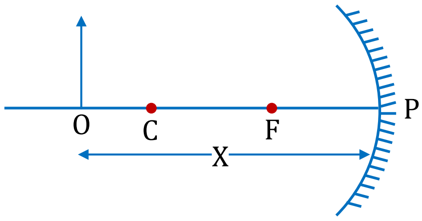
Concave Mirror:

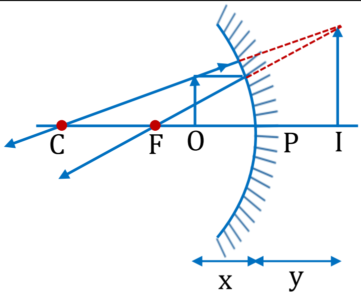
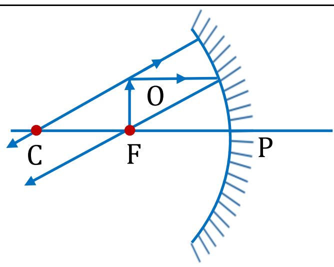
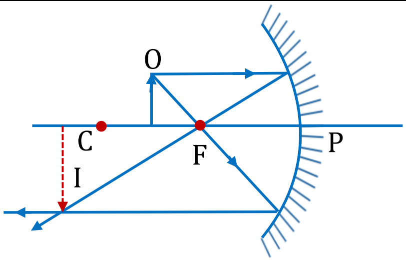
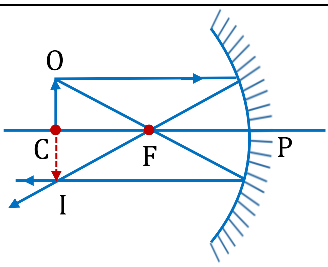
Let, $u = -x$ then

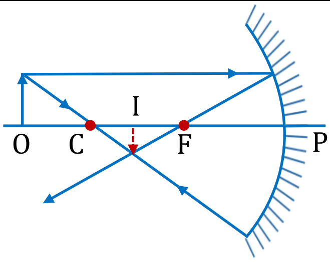
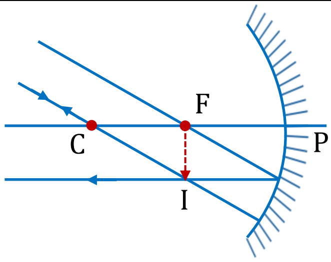
$$\frac{1}{v} + \frac{1}{-x} = \frac{1}{-f_o}$$

$$\Rightarrow \quad v = \frac{x\, f_o}{f_o - x}$$

and $$m = \frac{f_o}{f_o - x}$$



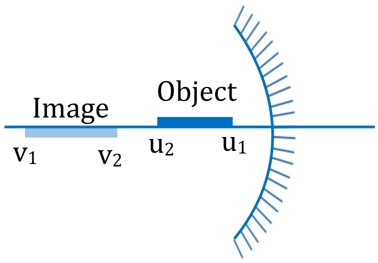
S.No	Position of Object	Position of Image	Nature	Size	Ray Diagram
1.	Between Pole and Focus	Behind Mirror	Virtual, Erect	Magnified	
2.	At Focus	In front of the Mirror at Infinity	Real, Inverted	Highly Magnified	
3.	Between Focus & centre of curvature	In front of the Mirror, beyond Centre of curvature	Real, Inverted	Magnified	
4.	At Centre of Curvature	At Centre of curvature	Real, Inverted	Equal size	

S.No	Position of Object	Position of Image	Nature	Size	Ray Diagram
5.	Between Centre of Curvature and Infinity	Between Focus & Centre of curvature	Real, Inverted	Diminished	
6.	At Infinity	At focus	Real, Inverted	Highly Diminished	

Longitudinal magnification

If one dimensional object is placed with its length along the principal axis then linear magnification is called longitudinal magnification.

Longitudinal magnification: $m_L = \frac{\text{length of image}}{\text{length of object}} = \left|\frac{v_2 - v_1}{u_2 - u_1}\right|$



For small objects only: $m_L = -\frac{dv}{du}$

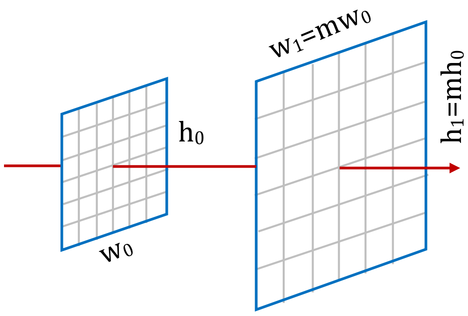
Differentiations of $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$ gives us $-\frac{dv}{v^2} - \frac{du}{u^2} = 0 \Rightarrow -\frac{dv}{du} = \left[\frac{v}{u}\right]^2$

So $\quad m_L = -\frac{dv}{du} = \left[\frac{v}{u}\right]^2 = m^2$

Superficial magnification

If two dimensional object placed with its plane perpendicular to principal axis its magnification is known as superficial magnification

Linear magnification $m = \frac{h_i}{h_0} = \frac{w_i}{w_0}$



$h_i = mh_0$

$w_i = mw_0$ and $A_{obj} = h_o \times w_o$

Area of image : $A_{image} = h_i \times w_i = mh_0 \times mw_0 = m^2 A_{obj}$

Superficial magnification $m_S = \frac{\text{Area of image}}{\text{Area of object}} = \frac{(ma)\times(mb)}{(a \times b)} = m^2$

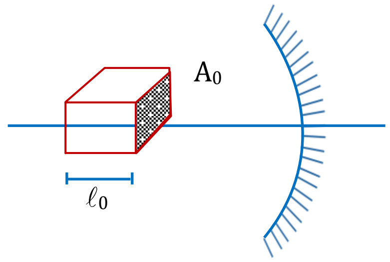
Volume Magnification:

For small cubical objects only, all dimensions will be magnified equally because all dimensions are almost at the same distance from the mirror, hence final image is also a cube.

Area of object is perpendicular to the principle axis. Hence,

Area of image $(A_i)=m^2A_0$

Length of image (for small object) $\Rightarrow \ell_i = m^2\ell_0$



Then, volume of image = $(A_i)\times(\ell_i)$

$$=(m^2A_0)(m^2\ell_0)$$

$\Rightarrow \quad m^4(A_0\ell_0)$

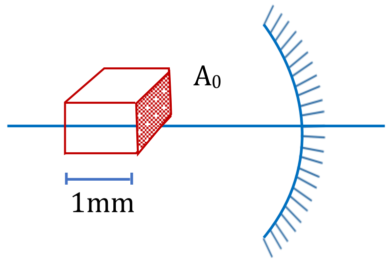
Volume of image = m^4V_0

Volume magnification $m_V = \dfrac{\text{Volume of image}}{\text{Volume of object}} = m^4$

Illustration 16:

A cube of side length 1 *mm* is placed on the axis of concave mirror at a distance of 45 *cm* from the pole as shown in the figure. One edge of the cube is parallel to the axis. The focal length of the mirror is 30 *cm*. Find the volume of the image.

Solution:



$$m = \frac{f}{f-u} = \frac{-30}{-30-(-45)} = -2$$

Volume of image = m^4 (volume of object)

$=16\times1=16\ mm^3$

Illustration 17:

A thin rod of length $\dfrac{f}{3}$ is placed along the principal axis of a concave mirror of focal length f such that its image which is real and elongated, just touches the rod. What is magnification?

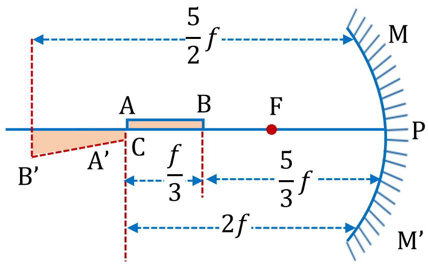
Solution:

Image is real and enlarged, the object must be between C and F.

One end A' of the image coincides with the end A of rod itself.

So $\quad v_A = u_A$

$$\frac{1}{v_A}+\frac{1}{v_A}=\frac{1}{-f}$$



i.e. $\quad v_A = u_A = -2f$

So it clear that the end A is at C.

$\because \quad$ The length of rod is $\dfrac{f}{3}$

$\therefore \quad$ Distance of the other end B from P is $u_B = 2f - \dfrac{f}{3} = \dfrac{5}{3}f$

If the distance of image of end B from P is v_B then

$$\frac{1}{v_B}+\frac{1}{\frac{-5}{3}f}=\frac{1}{-f}$$

$$v_B=\frac{-5}{2}f$$

$\therefore$ The length of the image $|v_B|-|v_A|=\frac{5}{2}f-2f=\frac{1}{2}f$

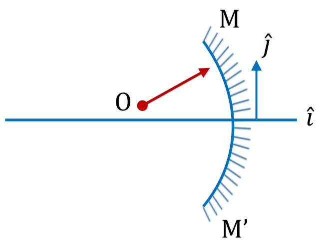
Magnification $m=\frac{|v_B|-|v_A|}{|u_B|-|u_A|}=\frac{\frac{1}{2}f}{-\frac{1}{3}f}=-\frac{3}{2}$

Negative sign implies that image is inverted with respect to object and so it is real.

Velocity of Image of Moving Object (Spherical Mirror)

(a) Velocity component along axis (Longitudinal velocity)

When an object is coming from infinite towards the focus of concave mirror



$\because \quad \frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\Rightarrow \quad -\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0 \qquad \Rightarrow \quad \vec{v}_{ix}=-\frac{v^2}{u^2}\vec{v}_{ox}=-m^2\vec{v}_{ox}$

$$\left[v_{ix}=\frac{dv}{dt}=\text{velocity of image along principal-axis; } v_{ox}=\frac{du}{dt}=\text{velocity of object along principal-axis}\right]$$

(b) Velocity component perpendicular to axis (Transverse velocity)

$$m=\frac{h_i}{h_0}=-\frac{v}{u}=\frac{f}{f-u} \qquad \Rightarrow \quad h_i=\left(\frac{f}{f-u}\right)h_0$$

$$\frac{dh_i}{dt}=\left(\frac{f}{f-u}\right)\frac{dh_0}{dt}+\frac{fh_0}{(f-u)^2}\frac{du}{dt};$$

$$\vec{v}_y=\left[m\vec{v}_{oy}+\frac{m^2h_0}{f}\vec{v}_{ox}\right]$$

$$\left[\frac{dh_i}{dt}=\text{velocity of image }\perp\text{' to principal-axis; }\frac{dh_o}{dt}=\text{ velocity of object }\perp\text{' to principal-axis}\right]$$

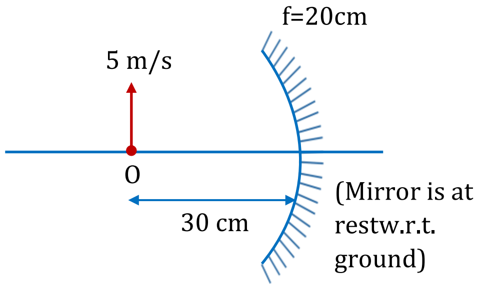
Note: Here principal axis has been taken to be along x-axis.

(c) Optical power of a mirror (in Dioptre) = $-\frac{1}{f}$

f = focal length with sign and in meters.

Illustration 18:

Find velocity of image w.r.t. ground.



Solution:

Apply mirror equation:

$$m=\frac{f}{f-u}=\frac{(-20)}{(-20)-(-30)}=-2$$

$$v=-mu=(-)(-2)(-30)=-60cm$$

For the velocity component along the principle axis:

$$(V_{IM})_{||}=-\frac{v^2}{u^2}(V_{om}) \quad \Rightarrow \quad (V_{IM})_{||}=0 \quad \left[\text{Since } (V_{om})_{||}=0\right]$$

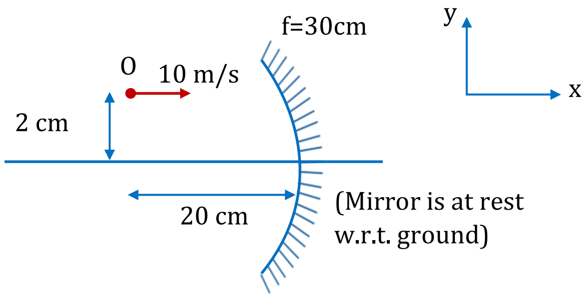
For the velocity component perpendicular to the principle axis:

$$(V_{IM})_{\perp}=-m(V_{om})_{\perp}+h_0\frac{dm}{dt} \quad \Rightarrow \quad (V_{IM})_{\perp}=(-2)(5)=-10m/s \quad \left[\text{Since } (h_0)=0\right]$$

$$\vec{V}_{IG}=\vec{V}_{IM}+\vec{V}_{MG} \quad \Rightarrow \quad -10+0=-10m/s \quad \Rightarrow \quad 10\ m/s \text{ (moving downwards)}$$

Illustration 19:

Find velocity of image w.r.t. ground.



Solution:

By apply mirror formula:

$$m=\frac{f}{f-u}=\frac{-30}{(-30)-(-20)}=3$$

$\Rightarrow$ $v=60$

$$(V_{IM})_{||}=-\frac{v^2}{u^2}(V_{OM})_{||}$$

$\Rightarrow$ $$(V_{IM})_{||}=\left(\frac{60}{-20}\right)^2(10)=-90m/s$$

$$(V_{IM})_{\perp}=-m(V_{OM})_{\perp}+h_0\frac{dm}{dt}$$

$\Rightarrow$ As $(V_{OM})_{||}=0$ but $\frac{dm}{dt}\neq 0$

$$\frac{dm}{dt}=-\left[\frac{u(V_{IM})_{||}-v(V_{OM})_{||}}{u^2}\right] \quad \Rightarrow \quad \frac{dm}{dt}=-\left[\frac{(-20)\times10^{-2}\times(-90)-(60)\times10^{-2}(10)}{(-20\times10^{2})^2}\right]$$

$\Rightarrow$ $\frac{dm}{dt}=-3\times10^2$ per sec.

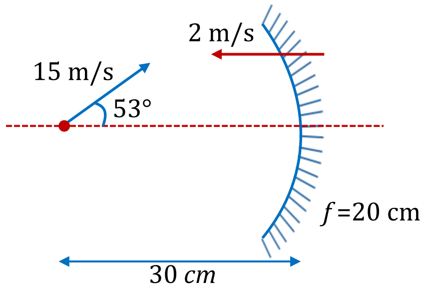
Then, $(V_{IM})_{\perp}=m(V_{OM})_{\perp}+h_0\frac{dm}{dt} \quad \Rightarrow \quad (V_{IM})_{\perp}=0+2\times10^{-2}\times(-3\times10^2)=-6m/s$

$$\vec{V}_{IM}=-90\hat{i}-6\hat{j} \quad \Rightarrow \quad \vec{V}_{IG}=\vec{V}_{IM}+\vec{V}_{MG}$$

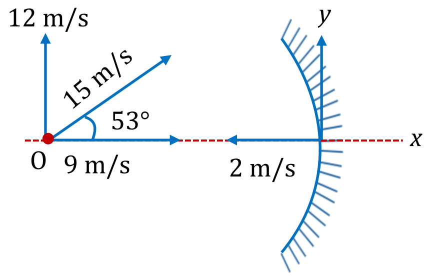
$\therefore$ $\vec{V}_{IG}=-90\hat{i}-6\hat{j}$ m/s

Illustration 20:

Find the velocity of image in situation as shown in figure.



Solution:



$\vec{V}_O =$ Velocity of object$=\left(9\hat{i}+2\hat{j}\right)$m/s

$\vec{V}_m =$ Velocity of mirror $=-2\hat{i}$ m/s

$$m=\frac{f}{f-u}=\frac{-20}{-20-(-30)}=-2$$

For velocity component parallel to optical axis

$$\left(\vec{V}_{I/M}\right)_{||}=-m^2\left(\vec{V}_{O/M}\right)_{||}$$

$$\left(\vec{V}_{I/M}\right)_{||}=(-2)^2 11\hat{i}=-44\hat{i} \text{ m/s}$$

For velocity component perpendicular to optical axis

$$\left(\vec{V}_{I/M}\right)_{\perp}=\left(\vec{V}_{O/M}\right)_{\perp}=(-2)12\hat{j}=-24\hat{j} \text{ m/s}$$

$\therefore$ $\quad \vec{V}_{I/m} =$ Velocity of image w.r.t. mirror

$$=\left(\vec{V}_{I/M}\right)_{||}=\left(\vec{V}_{I/M}\right)_{\perp}=(-2)12\hat{j}=-24\hat{j} \text{ m/s}$$

Also, $\quad \left(\vec{V}_{I/M}\right)=\vec{V}_1-\vec{V}_m$

or $\quad \left(\vec{V}_I\right)=\left(-44\hat{i}-24\hat{j}\right)-2\hat{i}$

$$=\left(-46\hat{i}-24\hat{j}\right) \text{ m/s}$$

(h) Newton's Formula: $XY = f^2$

X and Y are the distances (along the principal axis) of the object and image respectively from the principal focus. This formula can be used when the distances are mentioned or asked from the focus.

Refraction of Light:

When the light changes its medium, some changes occurs in its properties, the phenomenon is known as refraction.

- If the light is incident at an angle ($0° < i < 90°$) then it deviates from its actual path. It is due to change in speed of light as light passes from one medium to another medium.
- If the light is incident normally then it goes to the second medium without bending, but still it is called refraction.

- Refraction without reflection at any interface is never possible.
- As angle of incidence increases fraction of energy reflected increases.
- **Refractive index** of a medium is defined as the factor by which speed of light reduces as compared to the speed of light in vacuum $\mu = \frac{c}{v} = \frac{\text{Speed of light in vacuum}}{\text{Speed of light in medium}}$.

  More (less) refractive index implies less (more) speed of light in that medium, which therefore is called optical denser (rarer) medium.

Relative refractive index:

When light passes from one medium to the other, the refractive index of medium 2 relative to 1 is written as ${}_1\mu_2$ and is defined as

$$ {}_1\mu_2 = \frac{\mu_2}{\mu_1} = \frac{(c/v_2)}{(c/v_1)} = \frac{v_1}{v_2} $$

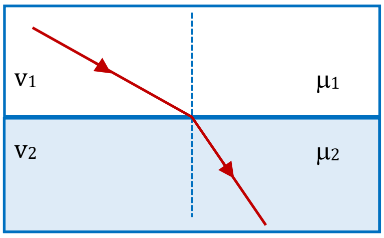


Illustration 21:

If refractive index of water and glass is $\frac{4}{3}$ and $\frac{3}{2}$ respectively and v_w= 2.25×10^8 *m/s.* Find speed of light in glass.

Solution:

$$ v_g \times \mu_{glass} = c $$

$$ v_w \times \mu_{water} = c \quad \Rightarrow \quad v_g = v_w \times \frac{\mu_{water}}{\mu_{glass}} $$

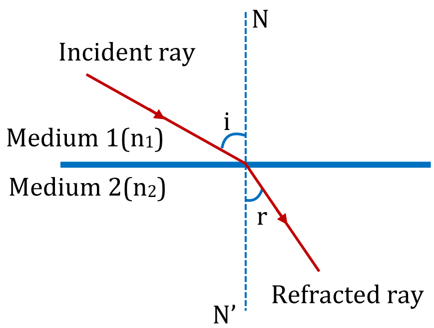
$$ v_g = \frac{4}{3\times3}(2)\times2.25\times10^8 = 8\times\frac{225}{9}\times10^6 = 200\times10^6 $$

$v_g = 2 \times 10^8$ *m/s*

Laws of Refraction:

(a) The incident ray, the normal to any refracting surface at the point of incidence and the refracted ray all lie in the same plane called the plane of incidence or plane of refraction.

(b) $\frac{\sin i}{\sin r}$ = Constant for any pair of media and for light of a given wave length. This is known as *Snell's* Law.



Also, $\dfrac{\sin i}{\sin r} = \dfrac{n_2}{n_1} = \dfrac{v_1}{v_2} = \dfrac{\lambda_1}{\lambda_2}$

For applying in problems remember

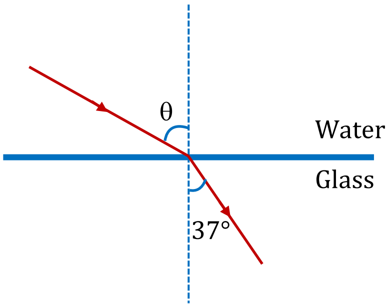
$n_1 \sin i = n_2 \sin r$

$\dfrac{n_2}{n_1} = {}_1n_2$ = Refractive Index of the second medium with respect to the first medium.

c = speed of light in air (or vacuum) = $3 \times 10^8\ m/s.$

Illustration 22:

If a beam of light is incident from water to glass and gets refracted at 37°. What is the angle of incidence if refractive index of water and glass is $\dfrac{4}{3}$ and $\dfrac{3}{2}$ respectively?



Solution:

$$\mu_w \sin\theta = \mu_g \sin 37^\circ$$

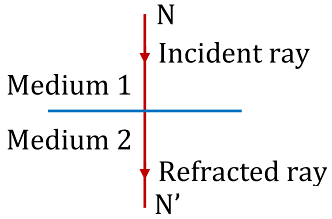
$$\frac{4}{3}\sin\theta = \frac{3}{2}\left(\frac{3}{5}\right)$$

$$\sin\theta = \frac{27}{4(10)} = \frac{6.75}{10} \quad \Rightarrow \quad \sin\theta = 0.0675$$

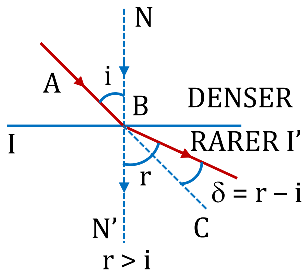
$$\boxed{\theta = \sin^{-1} = \frac{27}{40}}$$

Special cases :

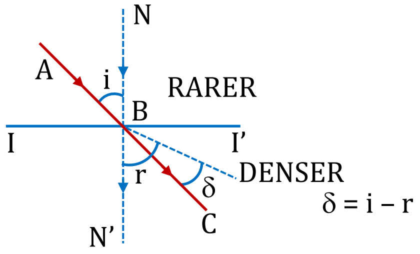
- Normal incidence : $i = 0$; from Snell's law : $r = 0$



- When light moves from denser to rarer medium it bends away from normal.



- When light moves from rarer to denser medium it bends towards the normal.



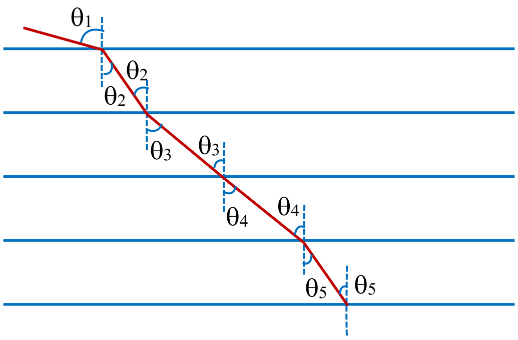
Note:

- Higher the value of *R.I.*, denser (optically) is the medium.
- Frequency of light does not change during refraction.
- Refractive index of the medium relative to vacuum = $\sqrt{\mu_r \in_r}$

  $n_{vacuum} = 1$; $n_{air} = \gtrsim 1$; n_{water} (average value) = 4/3 ; n_{glass} (average value)= 3/2

Refraction through Multiple Parallel Slabs

Consider a stack of five parallel slabs placed vertically one after the other. Since the light ray falls on the topmost slab at an angle of θ_1 with the normal, the angle of refraction at the topmost slab is θ_2 as shown in the figure.



Suppose that the refractive indices of the five slabs are n_1, n_2, n_3, n_4 and n_5, respectively, from top to bottom. Since the angle of refraction at the topmost slab is θ_2, the angle of incidence at the second slab is the same. Now, if the angle of refraction at the second slab is θ_3, the angle of incidence for the third slab is the same and this process continues as shown in the figure.

By applying Snell's law of refraction at the interface between the top two slabs, we get,

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \qquad \text{...(i)}$$

Now, by applying Snell's law of refraction at the interface between the second and third slab from the top we get,

$$n_2 \sin\theta_2 = n_3 \sin\theta_3 \qquad \text{...(ii)}$$

By equating equations (i) and (ii), we get,

$$n_1 \sin\theta_1 = n_3 \sin\theta_3$$

By going through the same procedure, we get the following result:

$$n_1 \sin\theta_1 = n_5 \sin\theta_5$$

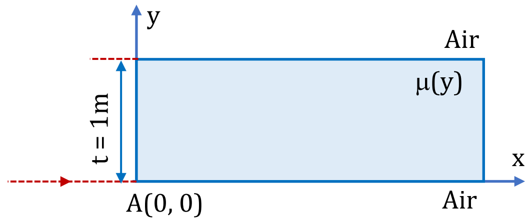
In place of five parallel slabs, if there are m numbers of parallel slabs, then we can consider the following equation;

$$n_1 \sin\theta_1 = n_m \sin\theta_m$$

Where n_m is the refractive index of the m^{th} slab, and θ_m is the angle of incidence at the m^{th} slab.

Illustration 23:

A ray of light travelling in air is incident at grazing angle (incident angle ≅ 90°) on a long rectangular slab of a transparent medium of thickness t=1.0 m (as shown in the figure). The point of incidence is origin A(0, 0). The medium has a variable index of refraction n(y) given by:



$$n(y)=\left[Ky^{\frac{3}{2}}+1\right]^{\frac{1}{2}}, \text{where } K=1m^{-\frac{3}{2}}$$

(a) Obtain a relation between the slope of the trajectory of the ray at a point B(x, y) in the medium and the incident angle at the that point.

(b) Obtain an equation for the trajectory y(x) of the ray in the medium.

(c) Determine the coordinates (x_1, y_1) of point P, where the ray intersects the upper surface of the slab-air boundary.

(d) Indicate the path of the ray subsequently.

Solution:

Given, angle of incidence of the ray, $i \cong 90°$

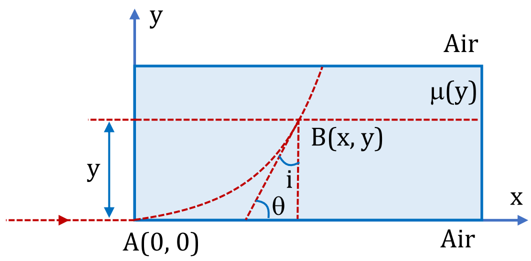
Thickness of the slab, t = 1 m

Since the medium has a variable index of refraction n(y), it is given by,

$$n(y)=\left[Ky^{\frac{3}{2}}+1\right]^{\frac{1}{2}}$$

Since the refractive index of the slab is gradually increasing along the y-axis, the slab can be considered as a composition of many slabs with infinitesimally small thickness. We know that when the light goes from a rarer to a denser medium, it bends towards the normal, in this case, the light is incident on the slab of lower refractive index (because the value of y is lower). Therefore, just above the point of incidence, the value of the refractive index is higher (since y is increasing along the positive y-axis). Since the light bends towards the positive y-axis gradually, it looks like it traces a curved path as shown in the figure.

(a) Consider any point B(x, y) and assume a slab of very small thickness at a distance y from the x-axis as shown in the figure.



Since the angle of incidence at point B is i and the tangent at point B makes angle θ with the x-axis, we get,

θ = (90°–i)

tan θ = tan (90° – i) = cot i

Now, the slope of the trajectory of the ray is nothing but the angle made by the tangent at point B with the x-axis. Thus, the slope is given as follows:

$$\frac{dy}{dx}=\tan\theta=\cot i$$

(b) Now, we have,

$$\frac{dy}{dx}=\cot i \qquad \text{...(i)}$$

By applying Snell's law of refraction through multiple slabs, we get

$$n_1 \sin\theta_1 = n_m \sin\theta_m$$

Here, $n_1 = n_{air} = 1 \quad ; \quad n_m = n(y) = n$

$\theta_1 = 90° \quad ; \quad \theta_m = i$

Thus, we get,

$$(1)\times\sin(90°) = n\sin i$$

$\Rightarrow \quad \sin i = \dfrac{1}{n}$ ...(ii)

Therefore, we get,

$$\cos i = \sqrt{1-\sin^2 i} = \frac{\sqrt{n^2-1}}{n}$$

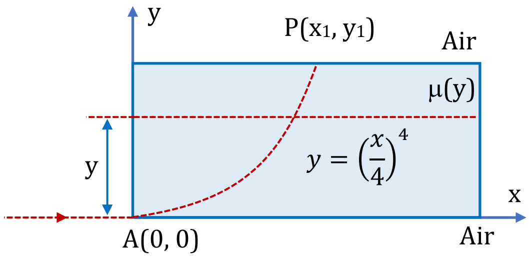
Hence, we get,

$$\cot i = \frac{\cos i}{\sin i} \Rightarrow \frac{dy}{dx} = \frac{\sqrt{n^2-1}}{n}\times\frac{n}{1} \Rightarrow \frac{dy}{dx} = \sqrt{n^2-1}$$

We have assumed that $n(y) = n$, hence we get the following:

$$\frac{dy}{dx} = \sqrt{\left(y^{\frac{3}{2}}+1\right)-1}$$

$\Rightarrow \quad \dfrac{dy}{dx} = y^{\frac{3}{4}}$



$\Rightarrow \quad \displaystyle\int \frac{dy}{y^{\frac{3}{4}}} = \int dx$

$\Rightarrow \quad \displaystyle\int y^{-\frac{3}{4}} dy = \int dx$

$\Rightarrow \quad \dfrac{y^{-\frac{3}{4}+1}}{-\frac{3}{4}+1} = x + c$

$\Rightarrow \quad 4y^{\frac{1}{4}} = x$ [Since the light ray passes through (0, 0), c=0]

$\Rightarrow \quad y = \left(\dfrac{x}{4}\right)^4$

This is the equation of the trajectory of the ray in the given slab.

(c) The equation of the ray in the slab is given by.

$$y = \left(\frac{x}{4}\right)^4$$

As the upper surface of the slab, the y-coordinate is 1m.

By substituting this value of y in the equation, we get,

$$1 = \left(\frac{x}{4}\right)^4$$

$\Rightarrow \quad 1 = \dfrac{x}{4} \quad \Rightarrow \quad x = 4$

(d) For refraction through multiple slabs, Snell's law becomes the following:

$$n_1 \sin\theta_1 = n_m \sin\theta_m$$

Now, for incident ray, we get,

$$n_1 = n_{air} = 1$$

$$\theta_i = 90°$$

$$\theta_m = i$$

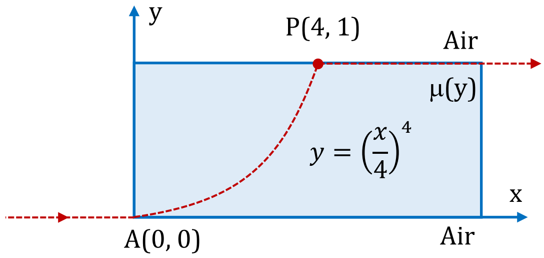
For emergent ray,

$$n_m = n_{air} = 1$$

Thus, we get the following:

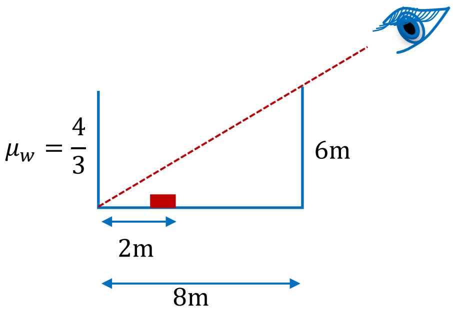
$$(1)\times\sin(90°) = (1)\sin\theta_m$$

$$\Rightarrow \quad \theta_m = 90°$$



Since the angle of incidence is approximately equal to 90°, the emergent ray is parallel to the incident ray.

Illustration 24:



Find till what minimum height water be filled so that observer can see coin?

Solution:

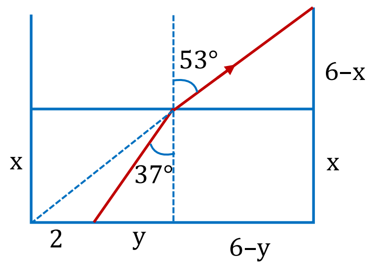
$$\mu_i \sin i = \mu_r \sin r$$

$$\frac{4}{3}\sin i = 1\left(\frac{4}{5}\right)$$

$$i = 37°$$

$$\tan 37° = \frac{y}{x}$$

$$\tan 53° = \frac{6-y}{6-x}$$



Principle of Reversibility of Light Rays:

(a) A ray travelling along the path of the reflected ray is reflected along the path of the incident ray.

(b) A refracted ray reversed to travel back along its path will get refracted along the path of the incident ray. Thus, the incident and refracted rays are mutually reversible.

Apparent Depth and shift of Submerged Object:

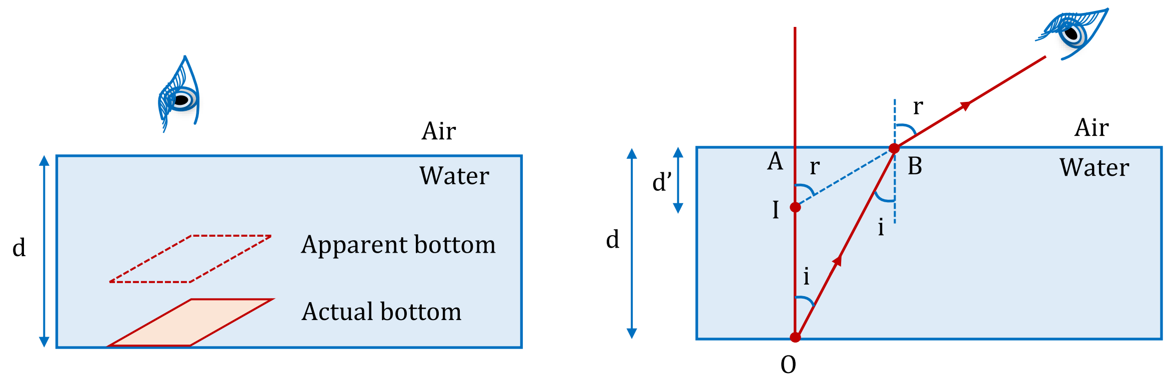
From our daily experience, we have seen that when we look at the bottom of a tank filled with water, the depth of the bottom seems to be less than the actual depth (refer to the figure).

Consider a point object (O) at the bottom of the tank. The observer sees the object's image at the point I as shown in the figure. Since the rays are assumed to be paraxial, the image is shifted only in the vertical direction, otherwise, the horizontal shift would also be there along with the vertical shift.

The main reason behind this phenomenon is that light is refracting from an optically denser medium (water) to an optically rarer medium (air). Therefore, in this case, the incident medium is the water and the refracting medium is the air.

Thus, $n_i = n_{water}$ and $n_r = n_{air}$.

Let the actual depth of the object be *d* and the apparent depth be *d'*.



From ΔOAB, we get,

$$\tan i = \frac{AB}{OA} = \frac{AB}{d}$$

From ΔIAB, we get,

$$\tan r = \frac{AB}{IA} = \frac{AB}{d'}$$

Therefore,

$$\frac{\tan i}{\tan r} = \frac{\left(\frac{AB}{d}\right)}{\left(\frac{AB}{d'}\right)} \qquad \Rightarrow \qquad \frac{\tan i}{\tan r} = \frac{d'}{d}$$

Since we have assumed all light rays to be paraxial as long as geometrical optics is concerned, we can write the following:

$$\frac{\tan i}{\tan r} \approx \frac{\sin i}{\sin r}$$

$$\Rightarrow \qquad \frac{n_r}{n_i} = \frac{d'}{d}$$

$$\Rightarrow \qquad d' = \frac{d}{\left(\frac{n_i}{n_r}\right)}$$

Defining relative refractive index as $n_{rel} = \frac{n_i}{n_r}$, we get,

$$d' = \frac{d}{n_{rel}}$$

Therefore, at near normal incidence (small angle of incidence i) apparent depth (d') is given by:

$$d' = \frac{d}{n_{relative}} \quad \text{and} \quad v' = \frac{v}{n_{relative}}$$

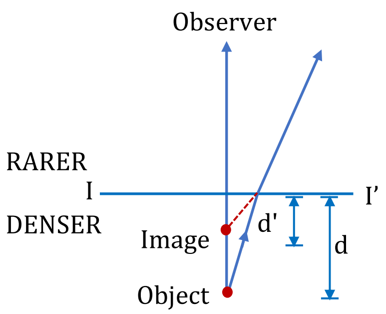
where, $n_{\text{relative}} = \dfrac{n_i \text{ (R.I. of medium of incidence)}}{n_r \text{ (R.I. of medium of refraction)}}$

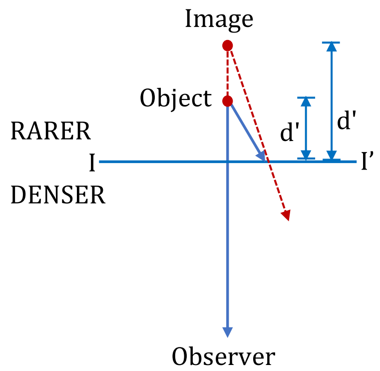
d = Distance of object from the interface = Real depth

d' = Distance of image from the interface = Apparent depth

v = Velocity of object perpendicular to interface relative to surface.

v' = Velocity of image perpendicular to interface relative to surface.





$$\text{Apparent shift} = d\left(1 - \frac{1}{n_{rel}}\right)$$

Illustration 25:

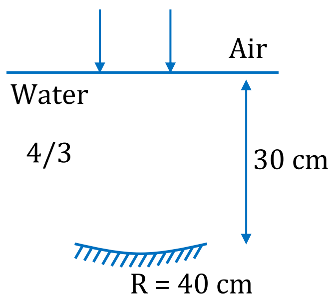
An object lies 100 *cm* inside water. It is viewed from air nearly normally. Find the apparent depth of the object. Given $n_{\text{water}} = 4/3$.

Solution:

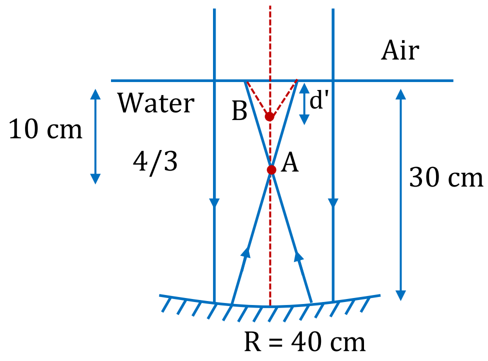
$$d' = \frac{d}{n_{relative}} = \frac{100}{\frac{4/3}{1}} = 75\ cm$$

Illustration 26:

A concave mirror is placed inside water with its shining surface upwards and principal axis vertical as shown. Rays are incident parallel to the principal axis of concave mirror. Find the position of final image. Given $n_{\text{water}} = 4/3$.



Solution:

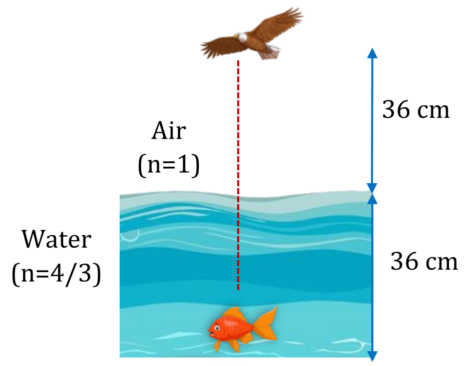


The incident rays will pass undeviated through the water surface and strike the mirror parallel to its principal axis. Therefore, for the mirror, object is at ∞. Its image A (in figure) will be formed at focus which is 20 *cm* from the mirror.

Now for the interface between water and air, d = 10 *cm.*

$$\therefore \qquad d' = \frac{d}{\left(\frac{n_w}{n_a}\right)} = \frac{10}{\left(\frac{4/3}{1}\right)} = 7.5\ cm.$$

Illustration 27:



A fish is in the pond at a depth of 36 *cm* from water surface and a bird is at a height of 36 *cm* from water surface.

(i) Find apparent height of the bird.

(ii) Find apparent depth of fish.

(iii) At what distance will the bird appear to the fish?

(iv) At what distance will the fish appear to the bird?

(v) If the velocity of bird is 12 *cm*/sec downward and the fish is 12 *cm*/sec in upward direction, then find out their relative velocities with respect to each other.

Solution:

(i) $$d'_B = \frac{36}{\frac{1}{\left(\frac{4}{3}\right)}} = \frac{36}{3/4} = 48\ cm$$

(ii) $$d'_F = \frac{36}{4/3} = 27\ cm$$

(iii) For fish : $d_B = 36 + 48 = 84\ cm$; $d_B = 36 + 48 = 84\ cm$

(iv) For bird : $d_F = 27 + 36 = 63\ cm.$; $d_F = 27 + 36 = 63\ cm.$

(v) Velocity of fish with respect to bird $= 12 + \left(\frac{12}{4/3}\right) = 21\ cm/\text{sec}.$

Velocity of bird with respect to fish $= 12 + \left(\frac{12}{3/4}\right) = 28\ cm/\text{sec}.$

Refraction through a Parallel Slab:

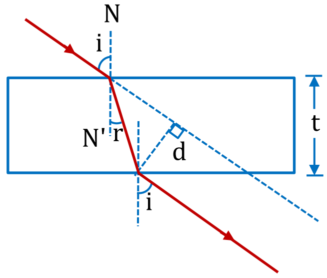
When light passes through a parallel slab, having same medium on both sides, then

(a) Emergent ray is parallel to the incident ray.

Note:

Emergent ray will not be parallel to the incident ray if the medium on both the sides of slab are different.

(b) Light is shifted laterally, given by

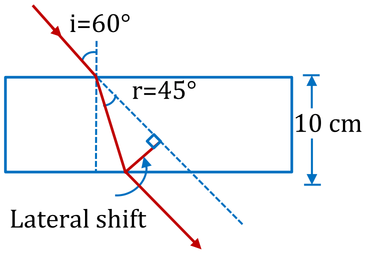


$$d = \frac{t\sin(i-r)}{\cos r} \quad ; \quad t = \text{thickness of slab}$$

Illustration 28:

Find the lateral shift of light ray while it passes through a parallel glass slab of thickness 10 *cm* placed in air. The angle of incidence in air is 60° and the angle of refraction in glass is 45°.

Solution:



$$d = \frac{t\sin(i-r)}{\cos r}$$

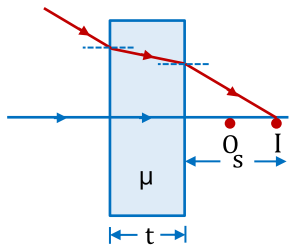
$$= \frac{10\sin(60°-45°)}{\cos 45°}$$

$$= \frac{10\sin 15°}{\cos 45°}$$

$$= 10\sqrt{2}\ \sin 15° cm$$

Apparent Shift:

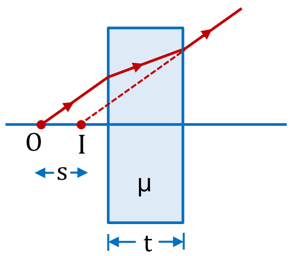
(a) for converging rays:



When a slab of thickness t and refractive index μ is placed in the path of a convergent beam, then the point of convergence is shifted by

$$S = \left(1 - \frac{1}{\mu}\right)t$$

(b) for diverging rays:



When the same slab is placed in the path of divergent beam, then the point of divergence is shifted by,

$$S = \left(1 - \frac{1}{\mu}\right)t$$

Note:

(i) The shift 'S' is always in the direction of light.

(ii) If the slab is made of air and surrounding medium is of refractive index μ, Then the apparent shift would be $S = t(\mu - 1)$

(iii) If n number of slabs with different thickness and refractive index are placed between the observer and the object, then the total apparent shift is equal to the summation of the individual shifts.

$\therefore$ $S = S_1 + S_2 + \dots\dots + S_n$

$$= t_1\left(1-\frac{1}{\mu_1}\right) + t_2\left(1-\frac{1}{\mu_2}\right) + \dots\dots + t_n\left(1-\frac{1}{\mu_n}\right) = \sum_{i=1}^{n} t_i\left(1-\frac{1}{\mu_i}\right)$$

(iv) If there are n number of slabs with different thickness and refractive index, one over the other then

$$d_{AC} = t_1 + t_2 + \dots\dots + t_n$$

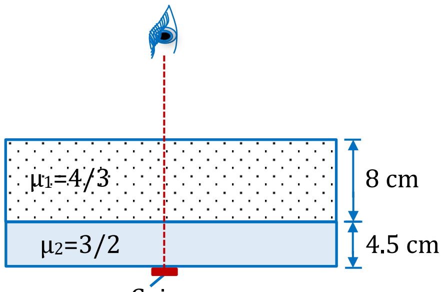
And $d_{AP} = \dfrac{t_1}{\mu_1} + \dfrac{t_2}{\mu_2} + \dots\dots + \dfrac{t_n}{\mu_n}$

So, $\mu = \dfrac{d_{AC}}{d_{AP}}$

$$= \frac{t_1 + t_2 + \dots\dots + t_n}{\left(\frac{t_1}{\mu_1}\right) + \left(\frac{t_2}{\mu_2}\right) + \dots\dots + \left(\frac{t_n}{\mu_n}\right)} = \frac{\Sigma t_i}{\Sigma\left(\frac{t_i}{\mu_i}\right)}$$

Illustration 29:

Determine the apparent shift in the position of the coin. Also, find the effective refractive index of the combination of the glass and water slab.



Solution:

Total apparent shift is

$$S = t_1\left(1-\frac{1}{\mu_1}\right) + t_2\left(1-\frac{1}{\mu_2}\right)$$

or $S = 8\left(1-\dfrac{1}{4/3}\right) + 4.5\left(1-\dfrac{1}{3/2}\right)$

or $S = 2 + 1.5 = 3.5\ cm$

The apparent depth of the coin from the top is

$$t = (8 + 4.5) - 3.5 = 9\ cm$$

and, the real depth of the coin is

$$t_1 + t_2 = 8 + 4.5 = 12.5$$

$\therefore$ The effective refractive index is

$$\mu_{eff} = \frac{t_1 + t_2}{t} = \frac{12.5}{9} = 1.39$$

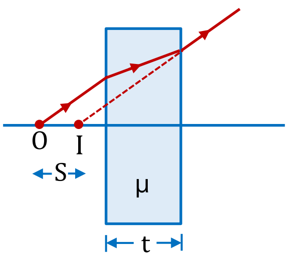
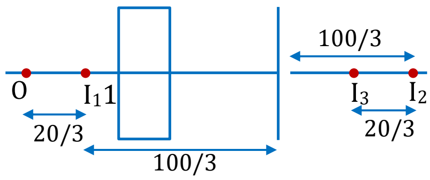


Illustration 30:

A 20 *cm* thick glass slab of refractive index 1.5 is kept in front of a plane mirror. Find the position of the image (relative to mirror) as seen by an observer through the glass slab when a point object is kept in air at a distance of 40 *cm* from the mirror.

Solution:



After event 1, I_1 will formed at $t - \frac{t}{\mu}$ from O

$$OI_1 = d\left[1-\frac{1}{\mu}\right] = 20\left[1-\frac{2}{3}\right] = \frac{20}{3}\ cm$$

I_1 will serve as object for event 2 I_2 is image of I_1.

I_2 serve as object for event 3 and I_3 is image.

$\therefore \qquad I_3 I_2 = \frac{20}{3}$

$\therefore \qquad$ Distance of I_3 from mirror $= \frac{100}{3} - \frac{20}{3} = \frac{80}{3}\ cm.$

Critical Angle and Total Internal Reflection (T. I. R.):

Critical angle is the angle made in denser medium for which the angle of refraction in rarer medium is 90°. When angle in denser medium is more than critical angle, then the light ray reflects back in denser medium following the laws of reflection and the interface behaves like a perfectly reflecting mirror.

In the figure

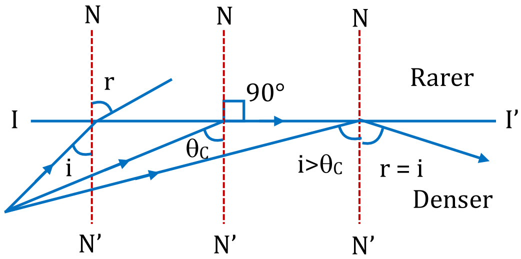
O = Object

NN' = Normal to the interface

II' = Interface

θ_C = Critical angle;

AB = reflected ray due to T. I. R.



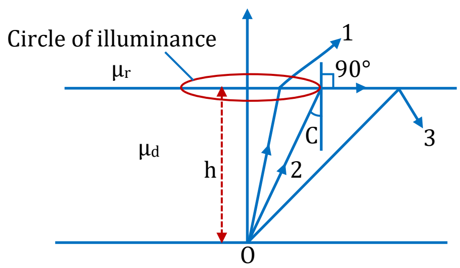
When $\quad i = \theta_C$ then $\ r = 90°$

$\therefore \qquad \theta_C = \sin^{-1} \frac{n_r}{n_d}$

Conditions of T. I. R.

(a) light is incident on the interface from denser medium.

(b) Angle of incidence should be greater than the critical angle ($i > \theta_C$). Figure shows a luminous object placed in denser medium at a distance h from an interface separating two media of refractive indices μ_r and μ_d. Subscript r and d stand for rarer and denser medium respectively.



In the figure, ray 1 strikes the surface at an angle less than critical angle θ_C and gets refracted in rarer medium. Ray 2 strikes the surface at critical angle and grazes the interface. Ray 3 strikes the surface making an angle more than critical angle and gets internally reflected. The locus of points where ray strikes at critical angle is a circle, called **circle of illuminance**. All light rays striking inside the circle of illuminance get refracted in rarer medium. If an observer is in rarer medium, he/she will see light coming out only from within the circle of illuminance. If a circular opaque plate covers the circle of illuminance, no light will get refracted in rarer medium and then the object can not be seen from the rarer medium. Radius of C.O.I can be easily found.

Illustration 31:

Find the angle of refraction in a medium ($\mu = 2$) if light is incident in vacuum, making angle equal to twice the critical angle.

Solution:

Since the incident light is in rarer medium. Total Internal Reflection can not take place.

$$\theta_C = \sin-1\frac{1}{\mu} = 30° \qquad \therefore \quad i = 2\theta_C = 60°$$

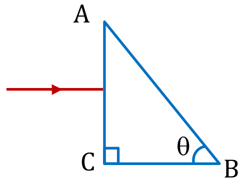
Applying Snell's Law

$$1 \sin 60° = 2 \sin r$$

$$\sin r = \frac{\sqrt{3}}{4} \qquad \Rightarrow \quad r = \sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$$

Illustration 32:

What should be the value of angle θ so that light entering normally through the surface AC of a prism (n=3/2) does not cross the second refracting surface AB ?

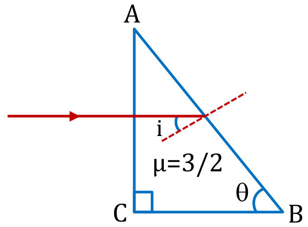


Solution:

Light ray will pass the surface AC without bending since it is incident normally. Suppose it strikes the surface AB at an angle of incidence i.

$$i = 90 - \theta$$

For the required condition : $90° - \theta > \theta_C$



or $\quad \sin(90° - \theta) > \sin\theta_C \quad$ or $\quad \cos\theta > \sin\theta_C = \dfrac{1}{3/2} = \dfrac{2}{3}$

or $\quad \theta < \cos^{-1}\dfrac{2}{3}$

Illustration 33:

What should be the value of refractive index n of a glass rod placed in air, so that the light entering through the flat surface of the rod does not cross the curved surface of the rod?

Solution:

It is required that all possible r' should be more than critical angle. This will be automatically fulfilled if minimum r' is more than critical angle ...(A)

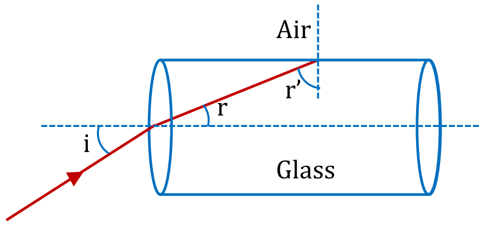
Angle r' is minimum when r is maximum i.e. θ_C.

Therefore, the minimum value of r' is $90 - \theta_C$.

From condition (A) :

$$90° - \theta_C > \theta_C \quad \text{or} \quad \theta_C < 45°$$

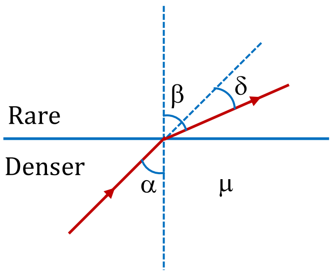
$$\sin\theta_C < \sin 45°; \quad \frac{1}{n} < \frac{1}{\sqrt{2}} \quad \text{or} \quad n > \sqrt{2}$$



Deviation (δ):

(A) A light ray travelling from a denser to a rarer medium at an angle $\alpha < \theta_C$ then deviation.

$\delta = \beta - \alpha = \sin^{-1}(\mu \sin\alpha) - \alpha$

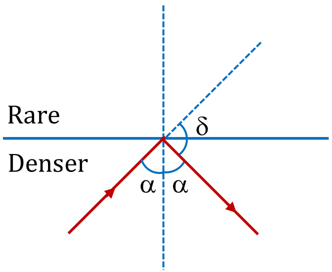


$\alpha < \theta_c$

and $\delta_{max} = \dfrac{\pi}{2} - \theta_C$

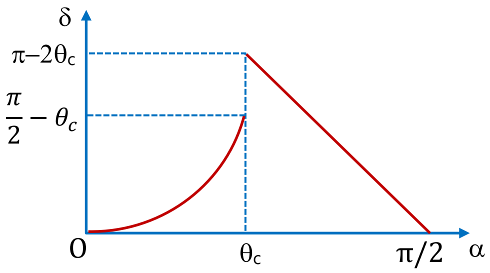
(B) If light is incident at an angle $\alpha > \theta_C$, Then the angle of deviation is

$\delta = \pi - 2\alpha$ and $\delta_{max} = \pi - 2\theta_C$



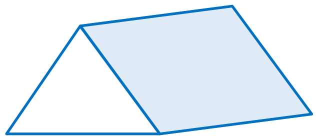
$\alpha > \theta_c$

(C) Graphically the relation between δ & α can be shown as



Characteristics of a prism:

A homogeneous solid transparent and refracting medium bounded by two plane surfaces inclined at an angle is called a prism :



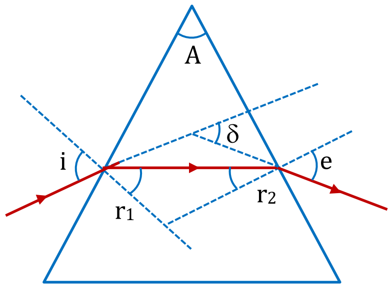
3-D view

Refraction through a prism:

PQ and PR are refracting surfaces.

$\angle QPR = A$ is called refracting angle or the angle of prism (also called Apex angle).

δ = angle of deviation



View from one side

Angle of Deviation (δ):

It is the angle between the emergent and the incident ray. In other words, it is the angle through which incident ray turns while passing through a prism.

$$\delta = (i - r_1) + (e - r_2)$$
$$= i + e - (r_1 + r_2)$$
$$= i + e - A$$

Condition of No Emergence:

The light will not emerge out of a prism for all values of angle of incidence if at face AB for $i = max = 90°$, at face AC $r_2 > \theta_C$

From Snell's law at AB,

$$1 \times \sin 90° = \mu \sin r_1$$

$\Rightarrow$ $r_1 = \sin^{-1}(1/\mu)$,

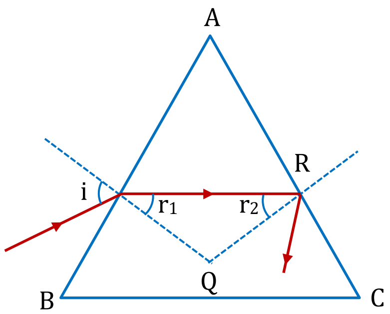
$r_1 = \theta_C$

$\therefore$ $r_1 + r_2 > 2\theta_C$ but $r_1 + r_2 = A$

$A > 2\theta_C$

or, $\sin A/2 > \sin \theta_C$

$\therefore$ $\mu > \dfrac{1}{\sin\frac{A}{2}}$ or $\mu > \text{cosec}\,(A/2)$



i.e., a ray of light will not emerge out of a prism (what ever be the angle of incidence) if

$$A > 2\theta_C$$

i.e., if $\mu > \text{cosec}\,(A/2)$

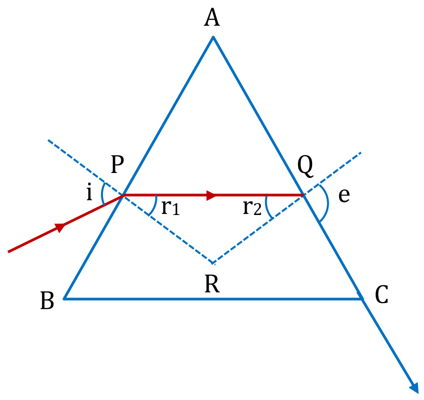
Critical Angle:

It is the angle of prism above which incident light ray on first surface will not emerge out from the second surface for all possible values of angle of incidence

$$A = 2\,\theta_C$$

Condition of Grazing Emergence:

If a ray can emerge out of a prism, the value of angle of incidence i for which angle of emergence $e = 90°$ is called condition of grazing emergence.



i.e., $r_2 = \theta_C \quad \because \quad r_1 = A - r_2 = A - \theta_C \quad (\text{As } r_1 + r_2 = A)$

Now from Snell's law at AB,

$$\sin i = \mu \sin r_1$$

$$\sin i = \mu \sin (A - \theta_C)$$

$$= \mu [\sin A \cos \theta_C - \cos A \sin \theta_C]$$

$$= \mu [\sin A \sqrt{1-\sin^2\theta_c} - \cos A \sin \theta_C]$$

$$= \mu [\sin A \sqrt{1-\frac{1}{\mu^2}} - \frac{1}{\mu}\cos A] = \left[\sqrt{\mu^2-1}\,\sin A - \cos A\right]$$

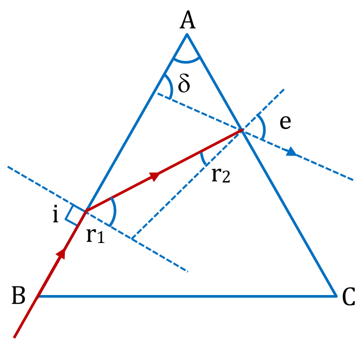
$$i = \sin^{-1}\left[\sqrt{\mu^2-1}\,\sin A - \cos A\right]$$

Note:

The light will emerge out of a given prism only if the angle of incidence is greater than the condition of grazing emergence.

Condition of maximum deviation:

Deviation will be maximum when



$$i_{max} = 90°$$

So, $\delta_{max} = i_{max} + e - A$

$$= 90° + e - A$$

However, when $i = 90°$,

For "AB"

$$1 \times \sin 90° = \mu \sin r_1$$

$\Rightarrow \quad \sin r_1 = \dfrac{1}{\mu}$

$$r_1 = \theta_C$$

Again, $r_1 + r_2 = A \quad \Rightarrow \quad r_2 = A - \theta_C$

Now for 'AC'

$\mu \sin r_2 = 1 \sin e \quad \Rightarrow \quad \sin e = \mu \sin (A - \theta_C)$

$\therefore \quad e = \sin^{-1} [\mu \sin (A - \theta_C)]$

Thus, $\delta_{max} = 90° + \sin^{-1}[\mu \sin(A - \theta_C)] - A$

Note:

This situation is reverse of grazing emergence and may also be viewed as deviation at grazing incidence.

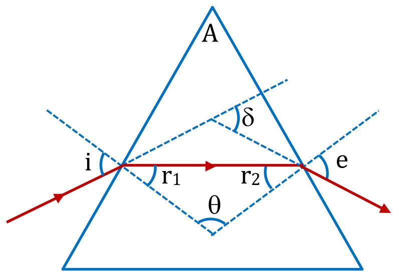
Condition for minimum Deviation:

The minimum deviation occurs when the angle of incidence is equal to the angle of emergence

i.e., $i = e$

As $\delta = i + e - A$

$\delta_{min} = 2i - A$



Using Snell 's law

$1 \times \sin i = \mu \sin r_1$

and $\mu \sin r_2 = 1 \times \sin i$

$\therefore$ $\sin r_1 = \sin r_2$

$\Rightarrow$ $r_1 = r_2 = r$

$\therefore$ $\delta_{min} = (2i - A)$

where $r = A/2$

$$\mu = \frac{\sin\ i}{\sin\ r} = \frac{\sin\left(\frac{\delta_{\min} + A}{2}\right)}{\sin(A/2)}$$

Note:

In the condition of minimum deviation the light ray passes through the prism symmetrically, i.e., the light ray in the prism becomes parallel to its base.

Proof: We know, $\delta + A = i + e$

and $r_1 + r_2 = A$

again $\sin i = \mu \sin r_1$

and $\sin e = \mu \sin r_2$

for δ_{min}, $\frac{d\delta}{di} = 0$

$\therefore$ $\frac{d\delta}{di} = \frac{di}{di} + \frac{de}{di} = 1 + \frac{de}{di} = 0$

i.e $de = -di$

also $dr_1 = -dr_2$; $\cos i\, di = \mu \cos r_1\, dr_1$

and $\cos e\, de = \mu \cos r_2\, dr_2$

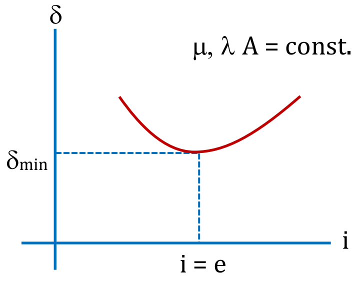
$\therefore$ $\frac{\cos i}{\cos e}\frac{di}{de} = \frac{\cos r_1}{\cos r_2}\frac{dr_1}{dr_2}$

$\Rightarrow$ $$\frac{\cos^2 i}{\cos^2 e} = \frac{\cos^2 r_1}{\cos^2 r_2} = \frac{1-\sin^2 r_1}{1-\sin^2 r_2} = \frac{1-\frac{1}{\mu^2}\sin^2 i}{1-\frac{1}{\mu^2}\sin^2 e} = \frac{1-\sin^2 i}{1-\sin^2 e} = \frac{\mu^2-\sin^2 i}{\mu^2-\sin^2 e}$$

$\Rightarrow$ $\sin^2 i = \sin^2 e$

$\therefore$ $i = e$

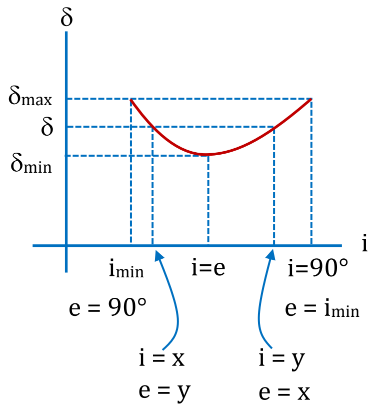
Graphical Representation of angle of deviation:



The deviation produced by a prism depends on angle of incidence i, angle of prism A, refractive index of material μ.

(i) δ is first decreasing and then increasing for $i < e$ and $i > e$ respectively.

(ii) For one δ (except δ min) there are two values of angle of incidence. If i and e are interchanged then we get the same value of δ because of reversibility principle of light



(iii) Before $i = e$, δ decreases more rapidly than it increases with i after $i = e$.

Thin Prisms:

In thin prism the distance between the refracting surfaces is negligible and the angle of prism (A) is very small.

Since $A = r_1 + r_2$, therefore, r_1 and r_2 both are small and the same is true for i_1 and i_2

According to Snell's law :

$\sin i = \mu \sin r_1$

$\Rightarrow$ $i = \mu r_1$

$\sin e = \mu \sin r_2$

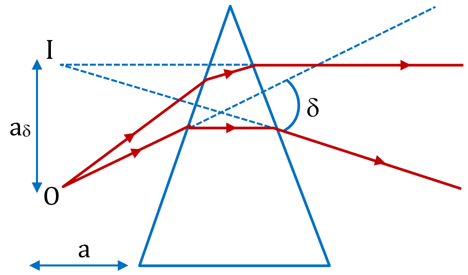
$\Rightarrow$ $e = \mu\, r_2$

$\therefore$ Deviation, $\delta = (i_1 - r_1) + (e - r_2) = (r_1 + r_2)(\mu - 1) = A(\mu - 1)$

Note:

The deviation for a small angled prism is independent of the angle of incidence.

Image formation by small angled prism



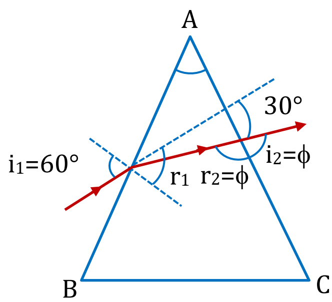
If object is real

1. Image is virtual
2. Shifting of image is OI

Illustration 34:

A ray of light is incident at an angle of 60° on one face of a prism which has an angle of 30°. The ray emerging out of the prism makes an angle of 30° with the incident ray. Show that the emergent ray is perpendicular to the face through which it emerges and calculate the refractive index of the material of the prism.

Solution:



According to given problem

$A = 30°, i_1 = 60°$

and $\delta = 30°$ and as in a prism

$\delta = (i_1 + i_2) - A$, $30° = (60 + i_2) - 30$ i.e., $i_2 = 0$

So the emergent ray is perpendicular to the face from which it emerges.

Now as $i_2 = 0$, $r_2 = 0$

But as $r_1 + r_2 = A$,

$r_1 = A = 30°$

So at first face

$1 \times \sin 60° = \mu \sin 30°$ i.e., $\mu = \sqrt{3}$

Illustration 35:

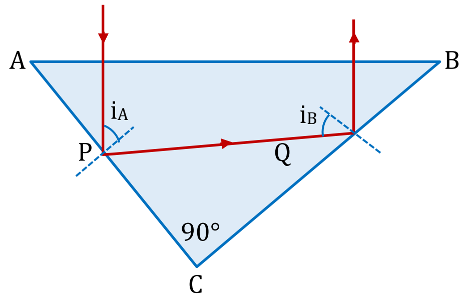
A right prism is to be made by selecting a proper material and angle A and B ($\leq A$) as shown in Fig. It is desired that a ray of light incident on the face AB emerges parallel to the incident direction after two internal reflections.

(a) What should be the minimum refractive index μ for this to be possible

(b) For $\mu = (5/3)$ is it possible to achieve this with angle $B = 30°$.

Solution:

(a) As $\angle C = 90°$, $\angle A + \angle B = 90°$ with $B \leq A$. Now TIR at P will take place if $i_A > \theta_C$ and TIR at Q will take place if $i_B > \theta_C$



So that $i_A + i_B > 2\theta_C$

But from geometry of Fig. $i_A = A$ and $i_B = B$

So, $A + B > 2\theta_C$

or $90° > 2\theta_C$ [*as* $A + B = 90°$]

or $45° > \theta_C$

i.e., $\sin 45° > \sin \theta_C$

or $\frac{1}{\sqrt{2}} > \frac{1}{\mu}$ or $\mu > \sqrt{2}$ i.e., $\mu_{min} = \sqrt{2}$

(b) For $B = 30°$ *TIR* at *Q* will take place only if $30° > \theta_C$

or $\sin 30° > \sin \theta_C$

or $\frac{1}{2} > \frac{I}{\mu}$

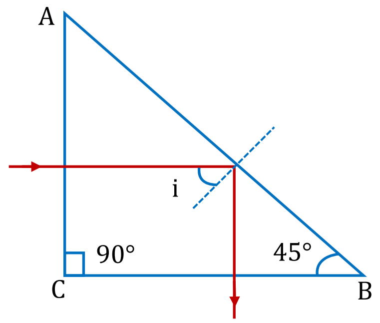
$\Rightarrow$ $\mu > 2$ $\Rightarrow$ $\mu_{min} = 2.$

But as here $\mu = (5/3) < 2$. So, *TIR* at *Q* is not possible with $\mu = (5/3)$ and $B = 30°$.

Illustration 36:

A ray of light incident normally on one of the face of a right angle isosceles prism is found to be totally reflected as shown in Fig. What is the minimum value of the refractive index of the material of the prism? When prism is immersed in water, trace the path of the emergent ray for the same incident ray, indicating the values of the angles.

Solution:



(A)

According to given problem the situation is depicted in Fig. (A).

For total internal reflection to take place at *AB*

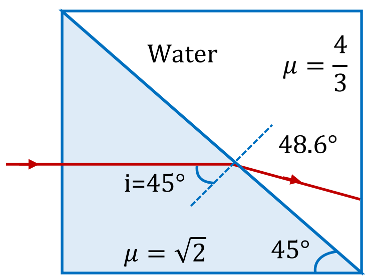
$i > \theta_C$ i.e., $\sin i > \sin \theta_C$

But as here $i = 45°$ and $\sin \theta_C = (1/\mu)$

$\frac{1}{\sqrt{2}} > \frac{1}{\mu}$ i.e., $\mu > \sqrt{2}$ so $(\mu)_{min} = \sqrt{2}$

When the prism is immersed in water

$$\mu = \frac{\mu_D}{\mu_g} = \frac{\sqrt{2}}{(4/3)} = \frac{4.2}{4}$$



(B)

So that $\theta_C = \sin^{-1}(4/4.2) = 72.25°$

and as $i = 45°$ *i.e.,* $i < \theta_C$,

TIR will not take place. In this situation from Snell's law

i.e., $\mu_1 \sin i = \mu_2 \sin r$

we have $\sqrt{2} \times \sin 45° = (4/3) \times \sin r$

i.e., $\sin r = (3/4)$ or $r = \sin^{-1}(3/4) = 48.6°$

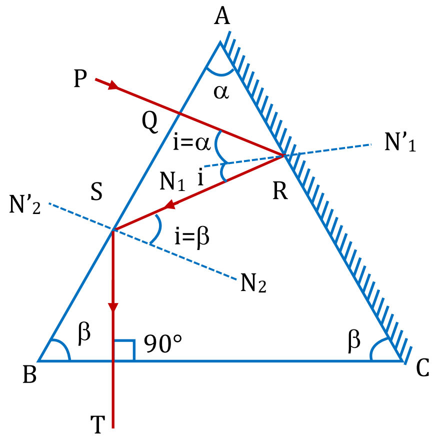
This all is shown in Fig. (B).

Illustration 37:

The cross-section of a glass prism has the form of an isosceles triangle. One of the equal faces is coated with silver. A ray is normally incident on another unsilvered face and being reflected twice emerges through the base of the prism perpendicular to it. Find the angles of the prism.

Solution:

Suppose refracting angle of prism be α and other two base angles of the isosceles prism be β. The light ray *PQ*, incident normally on the face *AB*, is refracted undeviated along *QR.* The refracted ray *QR* strikes the silvered face *AC* and gets reflected from it. The reflected ray *RS* now strikes the face *AB* from where it is again reflected along *ST* and emerges perpendicular to base *BC*.



It follows from fig. that angle of incidence on face $AC = i = \alpha$ and also angle of incidence on face $AB = \theta = \beta$ As $N_2 N_2'$ is parallel to *PR*, hence $\theta = 2i$

i.e., $\beta = 2\alpha$ ...(1)

Also $\alpha + 2\beta = 180°$ or $\alpha + 2(2\alpha) = 180°$

or $\alpha = 36°$ so $\beta = 2\alpha = 72°$

Dispersion of Light:

The angular splitting of a ray of white light into a number of components and spreading in different directions is called Dispersion of Light. [It is for whole Electro Magnetic Wave in totality]. This phenomenon is because waves of different wavelength move with same speed in vacuum but with different speeds in a medium.

Therefore, the refractive index of a medium depends slightly on wavelength also. This variation of refractive index with wavelength is given by Cauchy's formula.

Cauchy's formula $\mu(\lambda) = a + \frac{b}{\lambda^2}$ where a and b are positive constants of a medium.

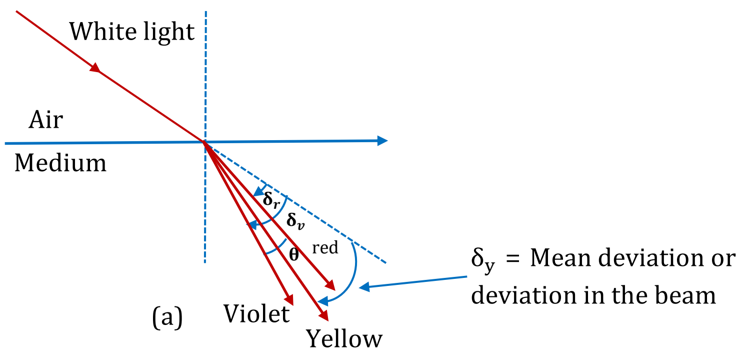
Note:

- Such phenomenon is not exhibited by sound waves.

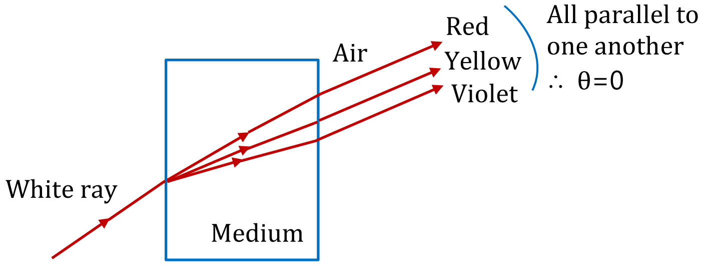
  Angle between the rays of the extreme colours in the refracted (dispersed) light is called **angle of dispersion.**

  $\theta = \delta_v - \delta_r$ (Fig. (a))

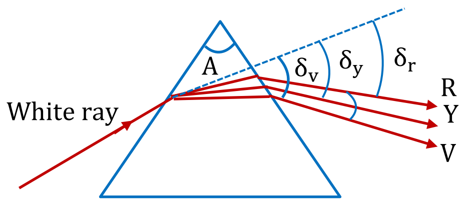
  Fig (a) and (c) represents dispersion, whereas in fig. (b) there is no dispersion.



(a)



(b)



(c)

For prism of small 'A' and with small 'i' :

$\theta = \delta_v - \delta_r = (\mu_v - \mu_r)A$

Deviation of beam (also called mean deviation)

$\delta = \delta_y = (\mu_y - 1)A$

μ_v, μ_r and μ_y are *R. I.* of material for violet, red and yellow colours respectively.

Illustration 38:

The refractive indices of flint glass for red and violet light are 1.613 and 1.632 respectively. Find the angular dispersion produced by a thin prism of flint glass having refracting angle 5°.

Solution:

Deviation of the red light is $\delta_r = (\mu_r - 1)A$ and deviation of the violet light is $\delta_v = (\mu_v - 1)A$.

The dispersion $= \delta_v - \delta_r = (\mu_v - \mu_r)A = (1.632 - 1.613) \times 5° = 0.095°$.

Note:

Numerical data reveals that if the average value of μ is small $\mu_v - \mu_r$ is also small and if the average value of μ is large $\mu_v - \mu_r$ is also large. Thus, larger the mean deviation, larger will be the angular dispersion.

Dispersive power (ω):

Dispersive power (ω) of the medium of the material of prism is given by: $\omega = \dfrac{\mu_v - \mu_r}{\mu_y - 1}$

ω is the property of a medium.

For small angled prism ($A \le 10°$) with light incident at small angle i :

$$\frac{\mu_v - \mu_r}{\mu_y - 1} = \frac{\delta_v - \delta_r}{\delta_y} = \frac{\theta}{\delta_y} = \frac{\text{Angular dispersion}}{\text{Deviation of mean ray (yellow)}}$$

[$\mu_y = \dfrac{\mu_v + \mu_r}{2}$ if μ_y is not given in the problem]

- $\mu - 1$ = Refractive index of the medium for the corresponding colour.

Illustration 39:

Refractive index of glass for red and violet colours are 1.50 and 1.60 respectively. Find :

(a) the refractive index for yellow colour, approximately

(b) dispersive power of the medium.

Solution:

(a) $\mu_y \simeq \dfrac{\mu_v + \mu_R}{2} = \dfrac{1.50 + 1.60}{2} = 1.55$

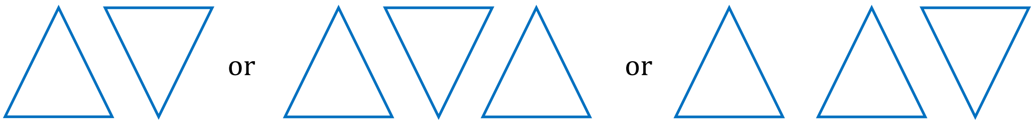
(b) $\omega = \dfrac{\mu_v - \mu_R}{\mu_y - 1} = \dfrac{1.60 - 1.50}{1.55 - 1} = 0.18.$

Dispersion without deviation (Direct Vision Combination)

The condition for direct vision combination is :

$$\left[\mu_y - 1\right] A = \left[\mu_y - 1\right] A' \Leftrightarrow \left[\frac{\mu_v + \mu_r}{2} - 1\right] A = \left[\frac{\mu'_v + \mu'_r}{2} - 1\right] A'$$

Two or more prisms can be combined in various ways to get different combination of angular dispersion and deviation.



Deviation without dispersion (Achromatic Combination)

Condition for achromatic combination is: $(\mu_v - \mu_r) A = (\mu'_v - \mu'_r) A'$

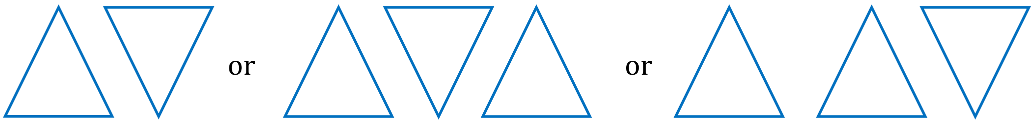
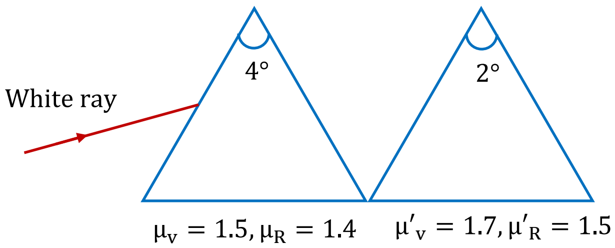


Illustration 40:

If two prisms are combined, as shown in figure, find the total angular dispersion and angle of deviation suffered by a white ray of light incident on the combination.



Solution:

Both prisms will turn the light rays towards their bases and hence in same direction. Therefore, turnings caused by both prisms are additive.

Total angular dispersion = $\theta + \theta' = (\mu_V - \mu_R) A + (\mu'_V - \mu'_R) A'$

$= (1.5 - 1.4)\, 4° + (1.7 - 1.5) 2° = 0.8°$

Total deviation = $\delta + \delta'$

$$= \left(\frac{\mu_V + \mu_R}{2} - 1\right) A + \left(\frac{\mu'_V + \mu'_R}{2} - 1\right) A' = \left(\frac{1.5 + 1.4}{2} - 1\right) 0.4° + \left(\frac{1.7 + 1.5}{2} - 1\right) 0.2°$$

$= (1.45 - 1)\, 0.4° + (1.6 - 1)\, 0.2° = 0.45 \times 0.4° + 0.6 \times 0.2° = 1.80 + 1.2 =$ **3.0°** **Ans.**

Illustration 41:

Two thin prisms are combined to form an achromatic combination. For I prism A = 4°, μ_R = 1.35, μ_Y = 1.40, μ_V = 1.42. for II prism μ'_R = 1.7, μ'_Y = 1.8 and μ'_V = 1.9 find the prism angle of II prism and the net mean deviation.

Solution:

Condition for achromatic combination. $\theta = \theta'$

$$(\mu_V - \mu_R)A = (\mu'_V - \mu'_R)A' \qquad \therefore \quad A' = \frac{(1.42-1.35)4^{\circ}}{1.9-1.7} = 1.4^{\circ}$$

$$\delta_{Net} = \delta - \delta' = (\mu_Y - 1)A - (\mu'_Y - 1)A' = (1.40 - 1)\,4^{\circ} - (1.8 - 1)\,1.4^{\circ} = 0.48^{\circ}.$$

Illustration 42:

A crown glass prism of angle 5° is to be combined with a flint prism in such a way that the mean ray passes without deviation. Find:

(a) The apex angle of the flint glass prism needed.

(b) The angular dispersion produced by the given combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.5, 1.6 and 1.7 respectively for crown glass and 1.8, 2.0 and 2.2 for flint glass.

Solution:

The deviation produced by the crown prism is $\delta = (\mu - 1)A$ and by the flint prism is $\delta' = (\mu' - 1)A'$.

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions.

Thus, the net deviation is :

$$D = \delta - \delta' = (\mu - 1)A - (\mu' - 1)A'. \qquad \text{...(i)}$$

(a) If the net deviation for the mean ray is zero,

$$(\mu - 1)A = (\mu' - 1)A' \quad \text{or,} \quad A' = \frac{(\mu-1)}{(\mu'-1)}A = \frac{1.6-1}{2.0-1}\times 5^{0} = 3^{\circ}$$

(b) The angular dispersion produced by the crown prism is : $\delta_v - \delta_r = (\mu_v - \mu_r)A$

and that by the flint prism is, $\delta'_v - \delta'_r = (\mu'_v - \mu'_r)A$

The net angular dispersion is, $(\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A = (1.7 - 1.5) \times 5 - (2.2 - 1.8) \times 3^{\circ} = -0.2^{\circ}$.

The angular dispersion has magnitude 0.2°

Illustration 43:

A crown glass prism of angle 5° is to be combined with a flint glass prism in such a way that the mean ray passes undeviated. Find

(a) The angle of the flint glass prism needed.

(b) The angular dispersion produced by the combination when white light goes through it. Refractive indices for red, yellow and violet light are 1.514, 1.517 and 1.523 respectively for crown glass and 1.613, 1.620 and 1.632 for flint glass.

Solution:

The deviation produced by the crown prism is

$$\delta = (\mu - 1)\,A$$

and by the flint prism is

$$\delta' = (\mu' - 1) A'$$

The prisms are placed with their angles inverted with respect to each other. The deviations are also in opposite directions.

Thus, the net deviation is

$$D = \delta - \delta'$$

$$= (\mu - 1) A - (\mu' - 1) A' \quad \text{...(i)}$$

(a) If the net deviation for the mean ray is zero,

$$(\mu - 1) A = (\mu' - 1) A'$$

or, $$A' = \frac{(\mu - 1)}{(\mu' - 1)} A$$

$$= \frac{1.517 - 1}{1.620 - 1} \times 5° = 4.2°.$$

(b) The angular dispersion produced by the crown prism is

$$\delta_v - \delta_r = (\mu_v - \mu_r) A$$

and that by the flint prism is

$$\delta'_v - \delta'_r = (\mu'_v - \mu'_r) A'$$

The net angular dispersion is,

$$\delta = (\mu_v - \mu_r)A - (\mu'_v - \mu'_r)A'$$

$$= (1.523 - 1.514) \times 5° - (1.632 - 1.613) \times 4.2°$$

$$= -0.0348°.$$

The angular dispersion has magnitude 0.0348°.

Illustration 44:

Calculate the dispersive power for crown glass from the given data $\mu_v = 1.5230$, $\mu_r = 1.5145$.

Solution:

$\mu_v = 1.5230$, $\mu_r = 1.5145$

Mean refractive index,

$$\mu = \frac{\mu_v + \mu_r}{2} = \frac{1.5230 + 1.5145}{2}$$

$$\mu = 1.5187$$

$$\omega = \frac{\mu_v - \mu_r}{\mu - 1} = \frac{1.5230 - 1.5145}{1.5187 - 1} = \frac{0.0085}{0.5187} = 0.0163$$

Illustration 45:

Find the angle of a prism of dispersive power 0.021 and refractive index 1.53 to form an achromatic combination with prism of angle 4.2° and dispersive power 0.045 having refractive index 1.65. Find the resultant deviation.

Solution:

Here, $\omega = 0.021$; $\mu = 1.53$; $\omega' = 0.045$; $\mu' = 1.65$; $A' = 4.2°$

For no dispersion, $\omega \delta + \omega' \delta' = 0$

or $\omega A(\mu-1)+\omega' A'(\mu-1)=0$

or $A=-\dfrac{\omega' A'(\mu'-1)}{\omega(\mu-1)}=-\dfrac{0.045\times 4.2\times(1.65-1)}{0.021\times(1.53-1)}=-11.04°$

Net deviation,

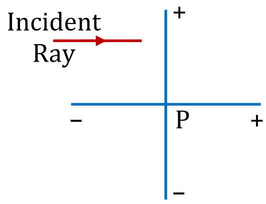
$$\delta+\delta' = A(\mu-1)+A'(\mu'-1)$$
$$= -11.04\,(1.53-1)+4.2\,(1.65-1)$$
$$= -11.04\times 0.53+4.2\times 0.65$$
$$= -5.85+2.73 = 3.12°$$

Law of refraction at spherical surface:

When light passes from a medium of refractive index μ_1 to a medium of refractive index μ_2 by a spherical surface of radius of curvature R then the relation between object distance u and image distance v is given by

$$\frac{\mu_2}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R}$$

Cartesian sign convention



(1) All distances are measured from the pole (P).

(2) Distances measured in the direction of incident rays are taken as positive.

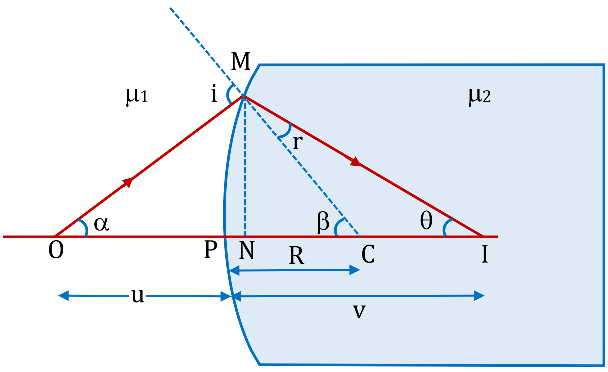
(3) Distances above the principal axis are taken as positive.

Derivation

Suppose AB is the spherical surface which separates the two medium of refractive index μ_1 and μ_2. O and I are the position of object and image in medium of $R.I.$ μ_1 and μ_2 respectively.

From, $\Delta MOC \Rightarrow i = \alpha+\beta$

$\Delta MCI \Rightarrow \beta = r+\theta$



For small aperture, i and r are small

from Snell, s law, $\mu_1 \sin i = \mu_2 \sin r$

We have $\mu_1(\alpha+\beta)=\mu_2(\beta-\theta)$

or, $\mu_2\theta+\mu_1\alpha = (\mu_2-\mu_1)\beta$

or, $\mu_2\tan\theta+\mu_1\tan\alpha=(\mu_2-\mu_1)\tan\beta$

or, $\mu_2\dfrac{MN}{NI}+\mu_1\dfrac{MN}{NP}=(\mu_2-\mu_1)\dfrac{MN}{NC}$

or, $\dfrac{\mu_2}{v}-\dfrac{\mu_1}{u}=\dfrac{\mu_2-\mu_1}{R}$

[For small aperture P & N are the same point]

Terms related to refraction at spherical surfaces

(A) Centre of curvature (C)

It is the centre of sphere of which the surface is a part.

(B) Radius of curvature (R)

It is the radius of the sphere of which the surface is a part.

(C) Pole (P)

It is the geometrical centre of the spherical refracting surface.

(D) Principal Axis

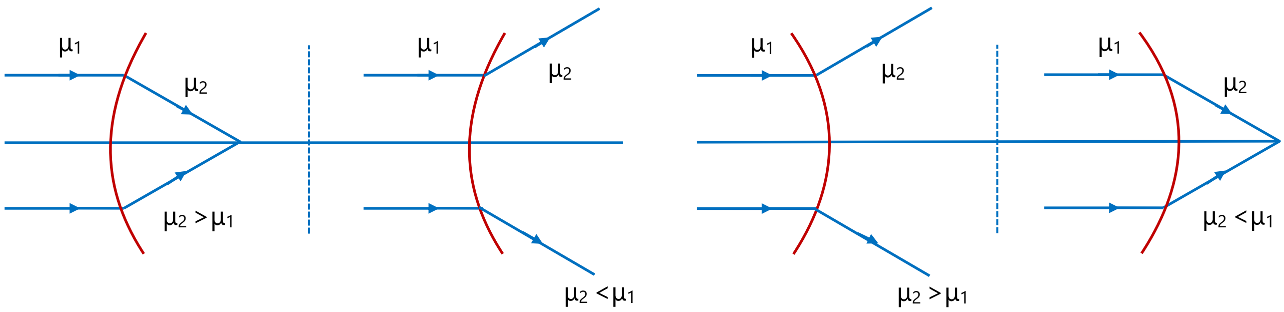
It is the straight line joining the centre of curvature to the pole.

(E) Focus (F)

When a narrow beam of rays of light, parallel to the principal axis and close to it (known as paraxial rays) is incident on a refracting spherical boundary, the refracted beam is found to converge or appear to diverge [depending upon the nature of the boundary (concave/convex) and the refractive index of two media] from a point on the principal axis. This point is called focus.

Note:

(1) It is not always necessary that for convex boundary the parallel rays always converge. Similarly, for concave boundary the incident parallel ray may converge or diverge depending upon the refractive index of two media,



(2) Laws of refraction are valid for spherical surface also.

(3) Pole, centre of curvature, Radius of curvature, Principal axis etc. are defined as spherical mirror except for the focus.

Lateral Magnification:

It is defined as

$$m = \frac{h_i}{h_o} = \frac{\text{height of image}}{\text{height of object}}$$

$$= -\frac{v/\mu_2}{-u/\mu_1} = \frac{\mu_1}{\mu_2} \cdot \frac{v}{u}$$

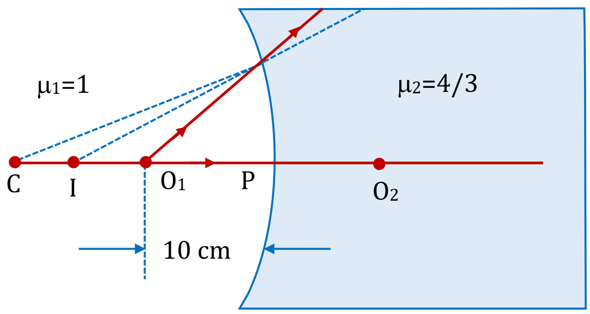
Illustration 46:

There are two objects O_1 and O_2 at same distance of 10 *cm* from a spherical refracting boundary of radius 30 *cm* as shown in Fig. If the indices of refraction of the media on two sides of the boundary are 1 and (4/3) respectively, (a) where will object O_1 will appear if seen from O_2 and (b) where will object O_2 appear if seen from O_1 ?

Solution:

In case of refraction from curved boundary

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$$



(A)

(a) As in this situation:

$\mu_1 = 1$; $\mu_2 = (4/3)$;

$R = -30cm$

and $u_1 = -10cm$,

So, $$\frac{(4/3)}{v_1} - \frac{1}{(-10)} = \frac{(4/3)-1}{(-30)}$$

i.e., $v_1 = -12\ cm$

i.e., object O_1 (which is 10 *cm* from *P*) will appear at 12 *cm* from *P* towards *C*, when seen from O_2.

(b) In this situation,

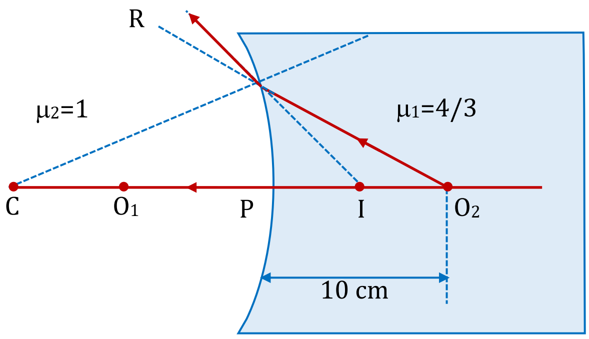
$\mu_1 = (4/3)$; $\mu_2 = 1$; $R = 30cm$

and $u_2 = -10cm$

So $$\frac{1}{v_2} - \frac{(4/3)}{(-10)} = \frac{1-(4/3)}{30}$$

i.e., $$v_2 = -\frac{90}{13} = -6.92\ cm$$

i.e., object O_2 (which is 10 *cm* from *p*) will appear at a distance of 6.92 *cm*. from *P* towards O_2 when seen from O_1

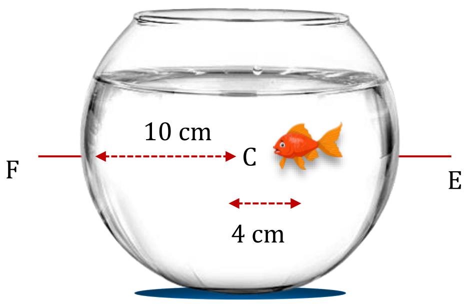


(B)

Illustration 47:

In a thin spherical fish bowl of radius 10 *cm* filled with water of refractive index (4/3), there is a small fish at a distance 4 *cm* from the centre *C* as shown in figure.

Where will the fish appear to be, if seen from (a) *E* and (b) *F* (neglect the thickness of glass)?



Solution:

In case of refraction from curved surface

$$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{(\mu_2 - \mu_1)}{R}$$

(a) As here

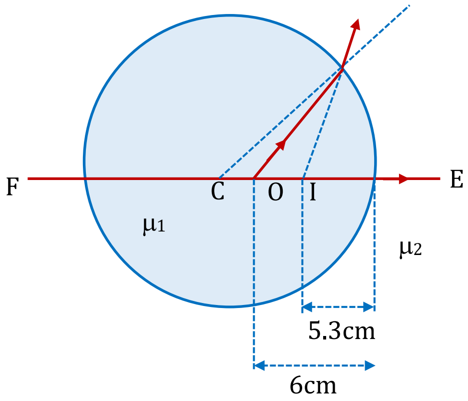
$\mu_1 = (4/3)$; $\mu_2 = 1$; $R = -10cm$

and $u = -(10 - 4) = -6cm$

So $\frac{1}{v} - \frac{(4/3)}{(-6)} = \frac{1-(4/3)}{(-10)}$

i.e. $v = -\frac{90}{17} = -5.3\ cm$

i.e. fish will appear at a distance 5.3 *cm* from *E* towards *F* (lesser than actual distance i.e., 6 *cm*)



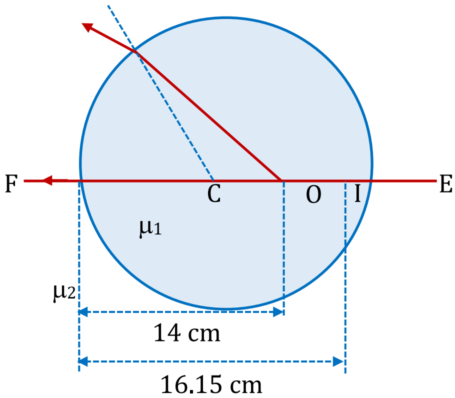
(b) As here $\mu_1 = (4/3)$; $\mu_2 = 1$; $R = -10\ cm$

and $u = -(10 + 4) = -14cm$

So $\frac{1}{v} - \frac{(4/3)}{(-14)} = \frac{1-(4/3)}{(-10)}$

i.e. $v = \frac{-210}{13} = -16.15\ cm$

i.e. fish will appear at a distance 16.15 *cm* from *F* towards *E*, (more than actual distance, i.e., 14 *cm*)



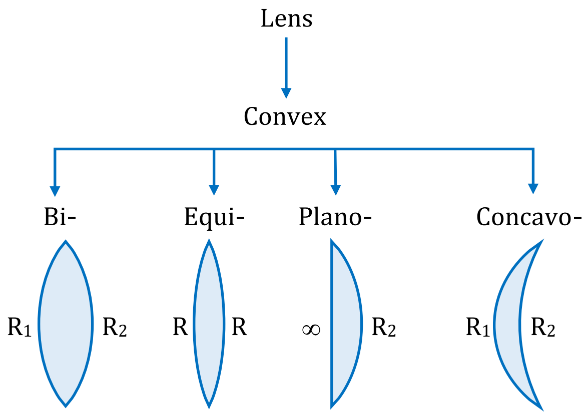
Lens Theory:

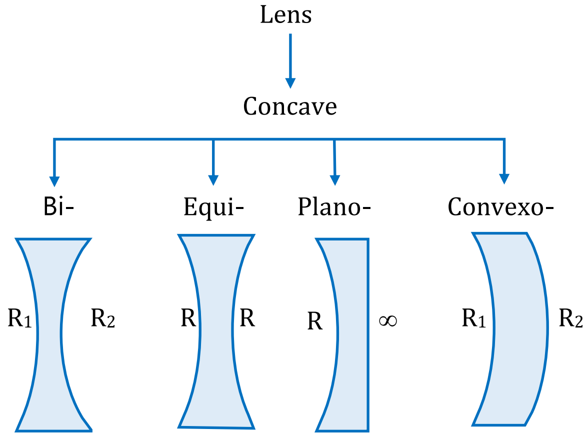
Definition:

A lens is a piece of transparent material with two refracting surfaces such that at least one is curved and refractive index of its material is different from that of the surroundings.

Types of lenses :

Depending upon the shape of the refracting surfaces following types of lenses can be formed:

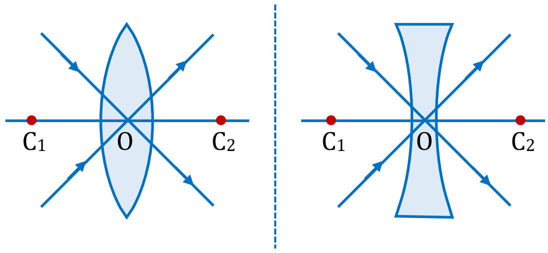




Terms related to thin spherical lenses:

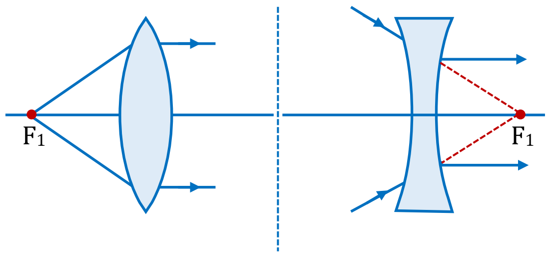
(A) Optical centre (O)

It is a point for a given lens through which any ray passes undeviated.



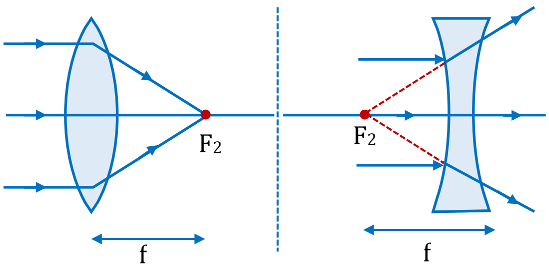
(B) Principal Axis (C_1C_2)

It is a line passing through optical centre and perpendicular to the lens. The centre of curvature of curved surfaces always lies on the principal axis.



(C) Principal Focus:

A lens has two surfaces causing two focal points



(i) **First focal point**

It is an object point on the principal axis whose image is formed at infinity.

(ii) **Second focal point**

It is an image point on the principal axis whose object lies at infinity.

(D) Focal length (*f*)

It is defined as the distance between optical centre of a lens and the point where the parallel beam of light converges or appear to converge.

(E) Aperture:

In reference to a lens, aperture means the effective diameter of its light transmitting area. So, the brightness, i.e., intensity of image formed by a lens which depends on the light passing through the lens will depend on the square of aperture, i.e., $I \propto (\text{aperture})^2$

Lens-Maker's formula:

It relates the focal length of the lens to the relative refractive index μ of the lens material and the radii of curvature of the two surfaces

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Where, $\mu=\dfrac{\mu_2}{\mu_1}=\dfrac{\text{Refractive index of lens}}{\text{Refractive index of surrounding}}$

R_1 is the radius of curvature of first surface and R_2 is the radius of curvature of the second surface from where light emerges out in the first medium.

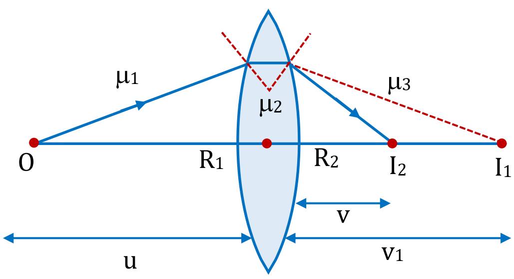
Derivation

In case of image formation by a lens the incident ray is refracted twice, at first and second surface respectively. The image formed by first surface, I_1 acts as an object for the second and makes final image I_2. And for thin lens thickness of lens $t \cong 0$. Thus for first surface,

$$\frac{\mu_2}{v_1}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1} \qquad ...(i)$$

While for second surface,

$$\frac{\mu_3}{v}-\frac{\mu_2}{v_1}=\frac{\mu_3-\mu_2}{R_2} \qquad ...(ii)$$



Adding equation (1) and (2)

$$\frac{\mu_2}{v_1}+\frac{\mu_3}{v}-\frac{\mu_1}{u}-\frac{\mu_2}{v_1}=\frac{\mu_2-\mu_1}{R_1}+\frac{\mu_3-\mu_2}{R_2}$$

or, $$\frac{\mu_3}{v}-\frac{\mu_1}{u}=\frac{\mu_2-\mu_1}{R_1}+\frac{\mu_3-\mu_2}{R_2}$$

For special case $\mu_3 = \mu_1$ we have

$$\frac{\mu_1}{v}-\frac{\mu_1}{u}=(\mu_2-\mu_1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

or, $$\frac{1}{v}-\frac{1}{u}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Now if object is at infinity, image will be formed at focus, i.e., for $u = -\infty$, $v = f$ so that above equation becomes

$$\frac{1}{f}=(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Note:

(1) The **Lensmaker's formula** is applicable for thin lenses only. The value of R_1 and R_2 are to be put in accordance with the cartesian sign convention.

(2) Position of object and image are interchangable. These positions are called **conjugate position.**

Lens formula:

The object and image distances of a lens are related to each other as:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f} \quad ; \text{ where, } \frac{1}{f}=\left(\frac{\mu_2}{\mu_1}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Lateral magnification:

$$m=\frac{h_i}{h_o}=\frac{v}{u}$$

Note:

(1) If converging ray's fall, the focus is on the other side of the direction of incidence and for diverging ray's focus is on the same region of the direction of incidence.

(2) m has negative and positive value for real virtual pair.

(3) Use cartesian sign convention with optical centre of lens as origin.

Ray diagram:

Graphically we can locate the position of image for a given object by drawing any two of the following three rays.

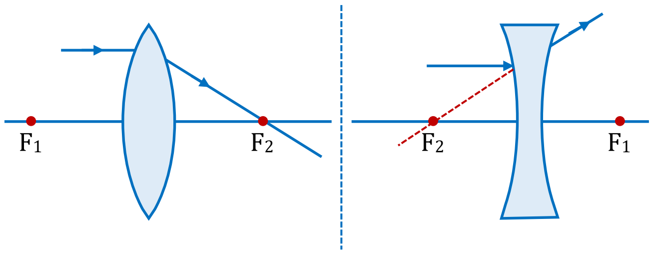


Fig. (1)

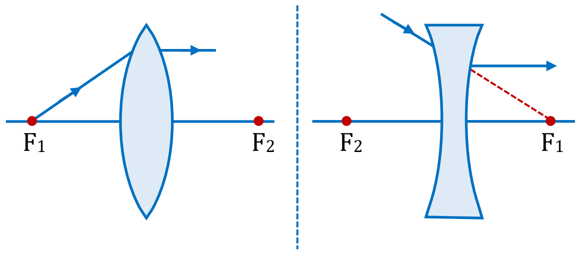


Fig. (2)

(A) A ray, initially parallel to the principal axis will pass (or appear to pass) through focus. (see fig.1)

(B) A ray which initially passes (or appear to pass) through focus will emerge from the lens parallel to the principal axis. (see fig.2)

(C) A ray passing through the optical centre of the lens goes undeviated as it passes through the lens region.

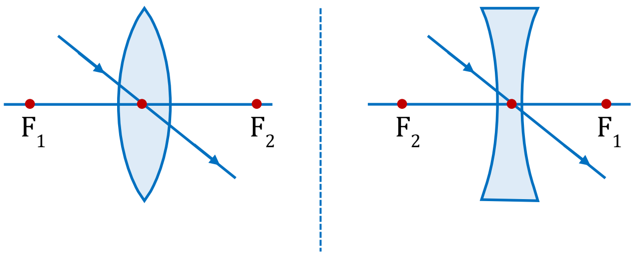


Fig. (3)

Position, size and nature of the images formed by a lens:

(A) Convex lens:

Suppose a real object is placed at a distance x from a convex lens of focal length f_0. The lens formulae may be modified as

$$\frac{1}{v}-\frac{1}{-x}=\frac{1}{f_0}$$

$$\Rightarrow \qquad v=\frac{xf_0}{x-f_0}$$

And, lateral magnification,

$$m=\frac{v}{u}=\frac{v}{-x}=-\frac{f_0}{x-f_0}$$

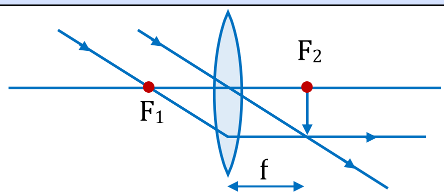
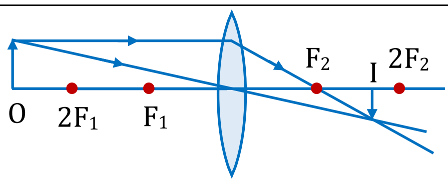
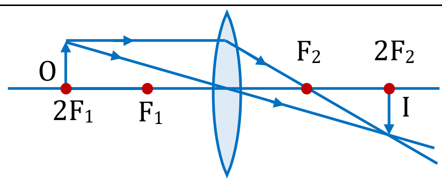
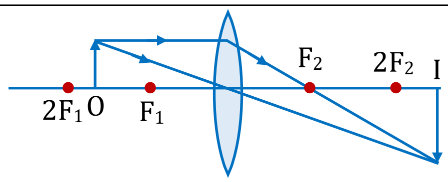
Note:

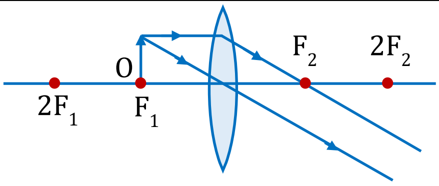
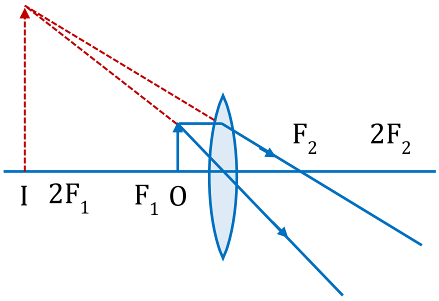
(1) A convex lens will form a real image for a real object when the object is placed beyond focus ($x > f_0$).

(2) When the object comes within focus i.e., $x < f_0$, then a virtual image is formed for the real object.

(3) The real image formed is always inverted while the virtual image is always erect.

(a) For Convergent or Convex Lens

S.No.	Position of Object	Position of image	Ray - Diagram	Nature (size)
1.	At infinity	At focus		Real, inverted, [Highly Diminished] ($m << -1$)
2.	Between ∞ and 2F	Between F & 2F		Real, inverted, [Diminished ($m < -1$)]
3.	At 2F	At 2F		Real, inverted, [Equal (m = –1)]
4.	Between 2F and F	Between 2F & ∞		Real, inverted, [Enlarged (m > –1)]

S.No.	Position of Object	Position of image	Ray - Diagram	Nature (size)
5.	At F	At ∞		Real, inverted, [Highly Enlarged (m >> – 1)]
6.	Between F and P	Between ∞ and O on same side		Virtual, erect [Enlarged (m > +1)]

(B) Concave lens

Suppose a real object is placed at a distance x in front of a concave lens of focal length f_o. Then the lens formulae can be modified as

$$\frac{1}{v}-\frac{1}{-x}=\frac{1}{-f_0} \quad \Rightarrow \quad v=\frac{-xf_0}{x+f_0}$$

And, transverse magnification,

$$m=\frac{v}{u}=\frac{v}{-x}=\frac{f_0}{x+f_0}$$

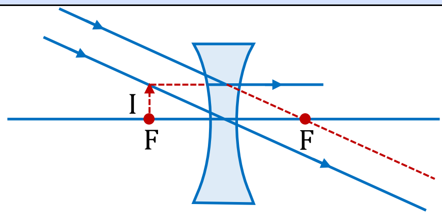
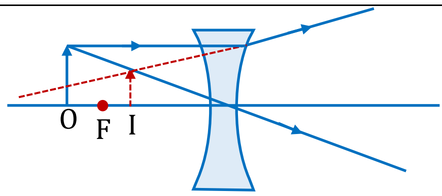
Note:

(1) A concave lens always form a virtual image for a real object.

(2) The image formed by a concave lens is always erect and diminished in size as compared to real object.

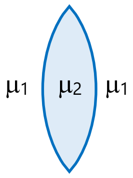
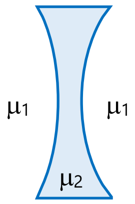
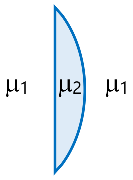
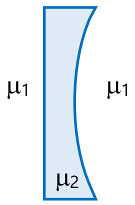
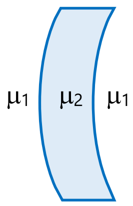
(3) A concave lens can form a real image if the object is virtual.

(a) For Divergent or Concave Lens

S. No.	Position of Object	Position of Image	Ray - Diagram	Nature of Image	Size
1.	At infinity	At *F*		Virtual, erect	Highly diminished ($m << +1$)
2.	In front of lens	Between *F* & optical centre		Virtual, erect	Diminished ($m < +1$)

Nature of various lenses depending upon their surroundings:

If μ_1 is the R.I of surrounding and μ_2 is that of lens then

Shape of lens	Nature of lens	
	For $\mu_1 < \mu_2$	For $\mu_1 > \mu_2$
	Converging	Diverging
	Diverging	Converging
	Converging	Diverging
	Diverging	Converging
	Depends on the radius of curvature of the first and second surface	

Power of a lens:

When focal length is written in metre then $P = \frac{\mu}{f} D$ is known as the power of the lens. Where D is (diopter) unit of power and μ is the refractive index of the medium in which the lens is placed.

Representation of lens:

Converging lens is represent as 

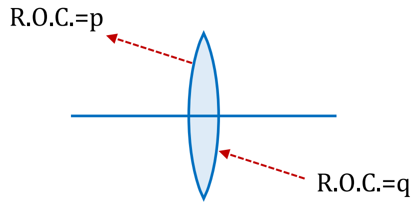
while diverging lens in represented as 

Illustration 48:

Show that the factor $\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ (and therefore focal length) does not depend on which surface of the lens light strike first.

Solution:

Consider a convex lens of radii of curvature p and q as shown.



Case 1: Suppose light is incident from left side and strikes the surface with radius of curvature p, first. Then

$R_1 = +p$; $R_2 = -q$ and $\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1}{p}-\frac{1}{-q}\right)$

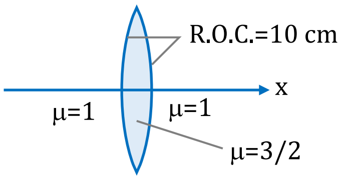
Case 2: Suppose light is incident from right side and strikes the surface with radius of curvature q, first. Then

$R_1 = +q$; $R_2 = -p$ and $\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{1}{q}-\frac{1}{-p}\right)$

Though we have shown the result for biconvex lens, it is true for every lens.

Illustration 49:

Find the focal length of the lens shown in the figure.



Converging lens

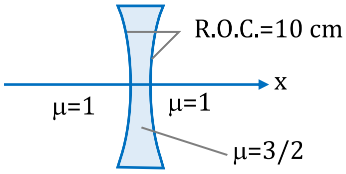
Solution:

$$\because \quad \frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

$$\Rightarrow \quad \frac{1}{f} = (3/2 - 1)\left(\frac{1}{10}-\frac{1}{(-10)}\right) \quad \Rightarrow \quad \frac{1}{f} = \frac{1}{2}\times\frac{2}{10} \quad \Rightarrow \quad f = +\,10\ cm.$$

Illustration 50:

Find the focal length of the lens shown in figure

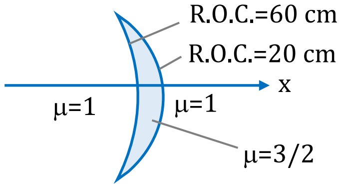


Solution:

$$\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right) = \left(\frac{3}{2}-1\right)\left(\frac{1}{-10}-\frac{1}{10}\right) ; f = -\,10\ cm$$

Illustration 51:

Find the focal length of the lens shown in figure



(a) If the light is incident from left side,

(b) If the light is incident from right side.

Solution:

(a) $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{-60} - \frac{1}{-20}\right)$; $f = 60\ cm$

(b) $\frac{1}{f} = (n_{rel} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \left(\frac{3}{2} - 1\right)\left(\frac{1}{20} - \frac{1}{60}\right)$; $f = 60\ cm$

Illustration 52:

Point object is placed on the principal axis of a thin lens with parallel curved boundaries i.e., having same radii of curvature. Discuss about the position of the image formed.

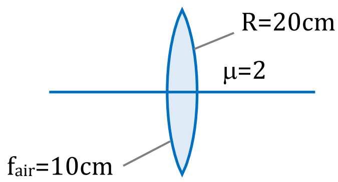
Solution:

$$\frac{1}{f} = (n_{\text{rel}} - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = 0 \qquad [\because R_1 = R_2]$$

$\frac{1}{v} - \frac{1}{u} = 0$ or $v = u$ i.e. rays pass without appreciable bending.

Illustration 53:

Focal length of a thin lens in air, is 10 cm. Now medium on one side of the lens is replaced by a medium of refractive index μ=2. The radius of curvature of surface of lens, in contact with the medium, is 20 cm. Find the new focal length.



Solution:

Let radius of I surface be R_1 and refractive index of lens be μ. Let parallel rays be incident on the lens. Applying refraction formula at first surface

$$\frac{\mu}{v_1} - \frac{1}{\infty} = \frac{\mu - 1}{R_1} \qquad \text{...(i)}$$

At II surface $\frac{2}{v} - \frac{\mu}{v_1} = \frac{2-\mu}{-20}$ ...(ii)

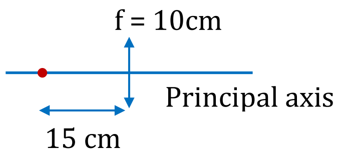
Adding (i) and (ii)

$$\frac{\mu}{v_1} - \frac{1}{\infty} + \frac{2}{v} - \frac{\mu}{v_1} = \frac{\mu-1}{R_1} + \frac{2-\mu}{-20} = (\mu - 1)\left(\frac{1}{R_1} - \frac{1}{-20}\right) - \frac{1}{20} = \frac{1}{10} - \frac{1}{20}$$

$v = 40\ cm \Rightarrow f = 40\ cm$

Illustration 54:

Figure shows a point object and a converging lens. Find the final image formed.



Solution:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

$$\frac{1}{v}-\frac{1}{-15}=\frac{1}{10}$$

$$\frac{1}{v}=\frac{1}{10}-\frac{1}{15}=\frac{1}{30}$$

$v = +\,30\ cm$

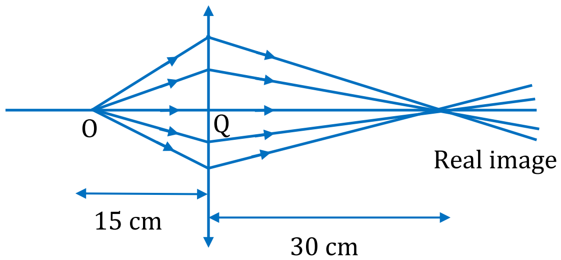
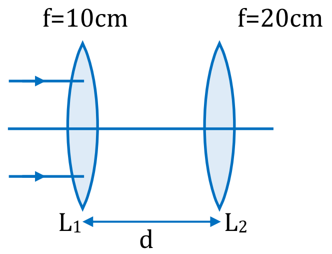


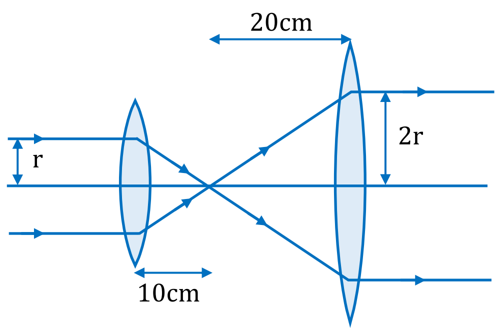
Illustration 55:

Figure shows two converging lenses. Incident rays are parallel to principal axis. What should be the value of d so that final rays are also parallel.



Solution:

Final rays should be parallel. For this the II focus of L_1 must coincide with I focus of L_2.

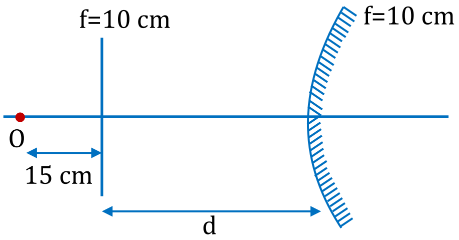


$d = 10 + 20 = 30\ cm$

Here the diameter of ray beam becomes wider.

Illustration 56:

Find the position of final image formed.



Solution:

For lens, $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$

$\frac{1}{v}-\frac{1}{-15}=\frac{1}{10}$ $\Rightarrow$ $v = +30\ cm$

Hence it is object for mirror $u = -15\ cm$

$\frac{1}{v}+\frac{1}{-15}=\frac{1}{-10}$ $\Rightarrow$ $v = -30\ cm$

Now for second time it again passes through lens $u = -15\ cm$

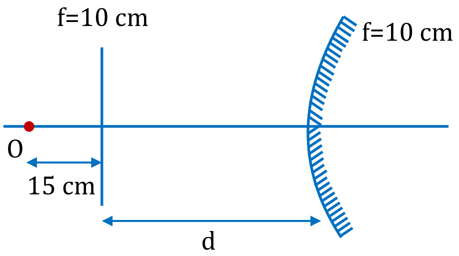
$v = ?$; $f = 10\ cm$

$\frac{1}{v}-\frac{1}{-15}=\frac{1}{10}$ $\Rightarrow$ $v = +30$

Hence final image will form at a distance 30 cm from the lens towards left.

Illustration 57:

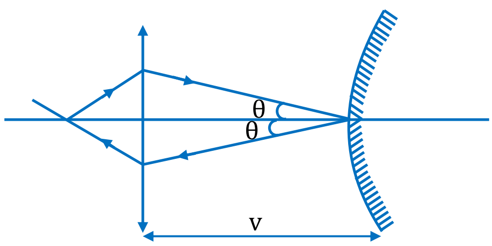
What should be the value of d so that image is formed on the object itself.



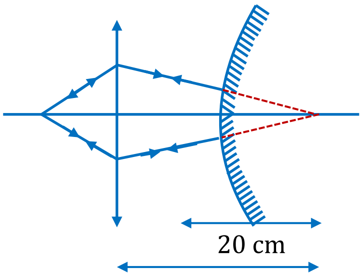
Solution:

For lens : $\frac{1}{v}-\frac{1}{-15}=\frac{1}{10}$ $v = +30\ cm$

Case I : If $d = 30$, the object for mirror will be at pole and its image will be formed there itself.



Case II : If the rays strike the mirror normally, they will retrace and the image will be formed on the object itself.



$\therefore \quad d = 30 - 20 = 10\ cm$

Illustration 58:

An extended real object of size 2 *cm* is placed perpendicular to the principal axis of a converging lens of focal length 20 *cm*. The distance between the object and the lens is 30cm.

(i) Find the lateral magnification produced by the lens.

(ii) Find the height of the image.

(iii) Find the change in lateral magnification, if the object is brought closer to the lens by 1 mm along the principal axis.

Solution:

(i) Using $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ and $m=\frac{v}{u}$, we get $m=\frac{f}{f+u}$ ...(A)

$$\therefore \quad m=\frac{+20}{+20+(-30)}=\frac{+20}{-10}=-2$$

–ve sign implies that the image is inverted.

(ii) $\frac{h_2}{h_1}=m \quad \therefore \ h_2 = mh_1 = (-2)\ (2) = -4\ cm$

(iii) Differentiating (A) we get

$$dm=\frac{-f}{(f+u)^2}\,du = \frac{-(20)}{(-10)^2}\,(0.1) = \frac{-2}{100} = -.02$$

Note that the method of differential is valid only when changes are small.

Alternate method : u (after displacing the object) = $-(30 + 0.1) = -29.9\ cm$

Applying the formula $m=\frac{f}{f+u}$

$$m=\frac{20}{20+(-29.9)} = -2.02$$

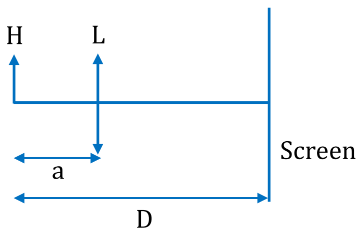
$\therefore$ Change in 'm' = – 0.02.

Since in this method differential is not used, this method can be used for any changes, small or large.

Displacement Method to find Focal length of Converging Lens:

Fix an object of small height H and a screen at a distance D from object (as shown in figure). Move a converging lens from the object towards the screen. Let a sharp image forms on the screen when the distance between the object and the lens is 'a'. From lens formula we have

$$\frac{1}{D-a}-\frac{1}{(-a)}=\frac{1}{f} \qquad \text{or} \qquad a^2 - Da + fD = 0$$



This is quadratic equation and hence two values of 'a' are possible. Thus, a_1 and a_2 are the roots of the equation. From the properties of roots of a quadratic equation,

$\therefore \quad a_1 + a_2 = D \qquad \Rightarrow \quad a_1 a_2 = fD$

Also $\quad (a_1 - a_2) = \sqrt{(a_1+a_2)^2-4a_1a_2} = \sqrt{D^2-4fD} = d$ (suppose)

'd' physically means the separation between the two position of lens.

The focal length of lens in terms of D and d.

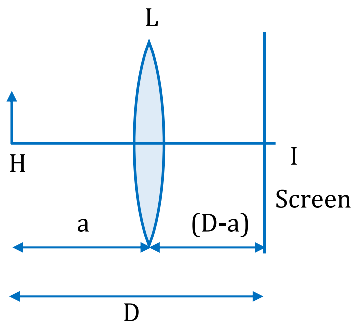
So, $\quad a_1 - a_2 = \sqrt{(a_1+a_2)^2-4a_1a_2}$

$$\sqrt{D^2-4fd} = d \Rightarrow \boxed{f=\frac{D^2-d^2}{4D}}$$

Condition, $d = 0$, i.e. the two position coincide

$$f=\frac{D^2}{4D}$$

$\therefore \quad D = 4f$



Roots of the equation $a^2 - Da + fD = 0$, become imaginary if

$$b^2 - 4ac < 0 = D^2 - 4fD < 0$$
$$= D(D-4f) < 0$$
$$= D - 4f < 0$$

For real value of a in equation $a^2 - Da + fD = 0$

$$b^2 - 4ac \geq 0 = D^2 - 4fD \geq 0$$

so, $\quad D \geq 4f \quad \Rightarrow \quad D_{min} = 4f$

Lateral magnification in displacement method:

If m_1 and m_2 be two magnifications in two positions in the displacement method

$$m_1=\frac{v_1}{u_1}=\frac{(D-a_1)}{-a_1} \qquad \text{and} \qquad m_2=\frac{v_2}{u_2}=\frac{a_1}{-(D-a_1)}$$

So $\quad m_1m_2=\frac{(D-a_1)}{-a_1}\times\frac{a_1}{-(D-a_1)}=1$

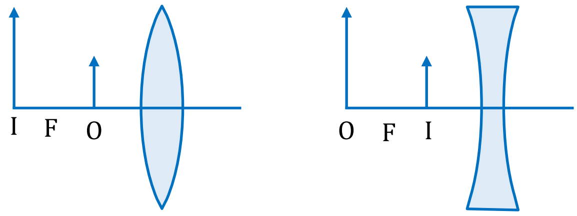
If image length are h_1 and h_2 in the two cases,

then $\quad m_1=-\frac{h_1}{H} \quad \Rightarrow m_2=-\frac{h_2}{H} \quad \Rightarrow \quad m_1 m_2 = 1$

$\therefore \quad \frac{h_1h_2}{H^2}=1 \quad \Rightarrow h_1h_2 = H^2 \quad \Rightarrow \quad H = \sqrt{h_1 \; h_2}$

Some important points

(A) For real extended objects if the image formed by a single lens is erect (i.e. m is positive) it is always virtual. In this situation if the image is enlarged the lens is converging (i.e. convex) with object between focus and optical centre and if diminished the lens is diverging (i.e. concave) with image between focus and optical centre.



(A) (B)

(B) As every part of a lens forms complete image, if a portion (say lower half) is obstructed (say covered with black paper) full image will be formed but brightness i.e. intensity will be reduced (to half). Also, if a lens is painted with black strips and a donkey is seen through it, the donkey will not appear as a zebra but will remain a donkey with reduced intensity.

(C) If L is the distance between a real object and its real image formed by a lens, then as

$$L = (|u| + |v|) = \left(\sqrt{u} - \sqrt{v}\right)^2 + 2\sqrt{uv}$$

So L will be minimum when

$$\left(\sqrt{u} - \sqrt{v}\right)^2 = \min = 0 \qquad \text{i.e.} \qquad u = v$$

On substituting $u = -u$ and $v = +u$ in lens formula, we get

$$\frac{1}{u} - \frac{1}{-u} = \frac{1}{f} \qquad \text{i.e.} \qquad u = 2f$$

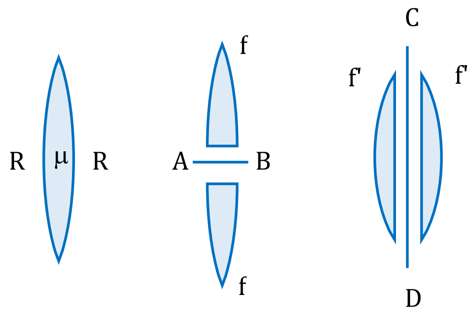
So that $(L)_{min} = 2\sqrt{2f \times 2f} = 4f$ [For L_{min}, $u = v$]

i.e. the minimum distance between a real object and its real image formed by a single lens is $4f$.

(D) If an object is moved at constant speed towards a convex lens from infinity to focus, the image will move slower in the beginning and faster later on, away from the lens. This is because in the time the object moves from infinity to $2F$, the image will move from F to $2F$ and when the object moves from $2F$ to F, the image will move from $2F$ to infinity. At $2F$ the speed of object and image will be equal. It can be shown that in case of lens, speed of image.

$$V_i = V_0\left[\frac{f}{u+f}\right]^2$$

(E) If an equi-convex lens of focal length f is cut into two equal parts by a horizontal plane AB, then as none of μ, R_1 and R_2 will change, the focal length of each part be equal to that of initial lens, i.e.



$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{-R}\right] = \frac{2(\mu - 1)}{R}$$

However in this situation as light transmitting area of each part becomes half of initial, so intensity will be reduced to half and aperture to $(1/\sqrt{2})$ times of its initial value [as I $\propto$ (Aperture)^2].

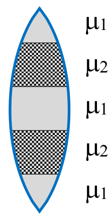
However, if the same lens is cut into two equal parts by a vertical plane CD, the focal length of each part will become.

$$\frac{1}{f'} = (\mu - 1)\left[\frac{1}{R} - \frac{1}{\infty}\right] = \frac{(\mu - 1)}{R} = \frac{1}{2f}$$

i.e., $\quad f' = 2f$

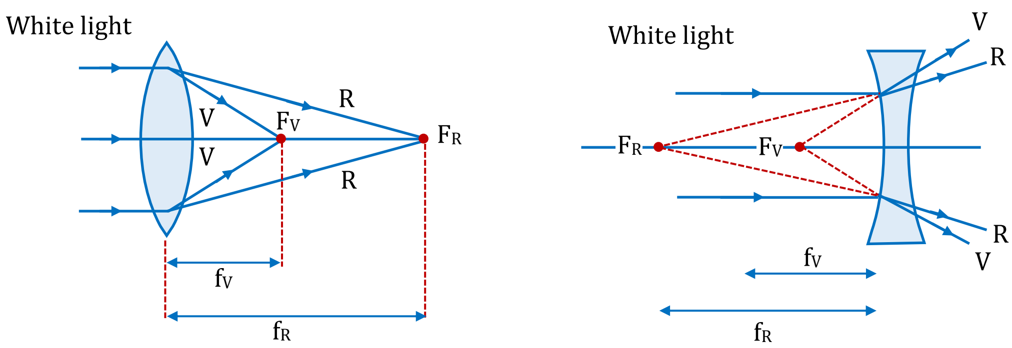
i.e., focal length of each part will be double of initial value. In this situation as the light transmitting area of each part of lens remains equal to initial, intensity and aperture will not change.

(F) If a lens is made of a number of layers of different refractive indices as shown in figure for a given wavelength of light it will have as many focal lengths or will form as many images as there are μ'*s* as



$$\frac{1}{f} \propto (\mu - 1)$$

(G) As focal length of a lens depends on μ, i.e. $(1/f) \propto (\mu - 1)$, the focal length of a given lens is different for different wavelengths and maximum for red and minimum for violet whatever be the nature of lens.



(H) If a lens of glass (μ = 3/2) is shifted from air (μ = 1) to water (μ = 4/3) then as :

$$\frac{1}{f_A} = \left[\frac{3/2}{1} - 1\right]K \text{ and } \frac{1}{f_W} = \left[\frac{(3/2)}{(4/3)} - 1\right]K \qquad \text{with } K = \left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

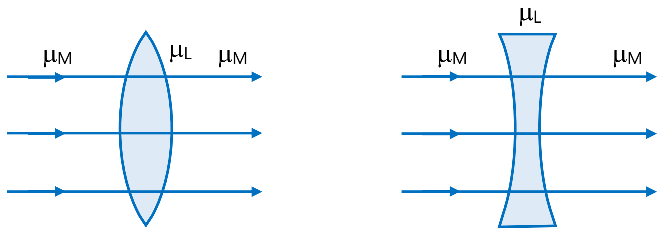
$$\frac{f_W}{f_A} = \left[\frac{8}{K}\right] \times \left[\frac{K}{2}\right] \text{ i.e. } f_W = 4f_A$$

i.e. focal length of a lens in water becomes 4 times of its value in air.

(I) If a lens is shifted from one medium to the other, depending on the refractive index of the lens and medium, the following three situations are possible.

(i) **$\mu_M < \mu_L$ but μ_M increases :** In this situation $\mu = (\mu_L/\mu_M)$ will remain greater than unity but will decrease and as $(1/f) \propto (\mu - 1)$, $(1/f)$ will decrease i.e., f will increase (without change in nature of lens)

(ii) **$\mu_M = \mu_L$:** In this situation $\mu = (\mu_L/\mu_M) = 1$, so that $(1/f) \propto (\mu - 1) = 0$, *i.e., $f = \infty$,* i.e., lens will neither converge nor diverge but will behave as a plane glass plate.



(iii) **$\mu_M > \mu_L$:** In this situation $\mu = (\mu_L / \mu_M) < 1$, so in **Lens-maker's formula** sign of f and hence nature of lens will change, i.e. a converging lens will behave as divergent and vice -versa.

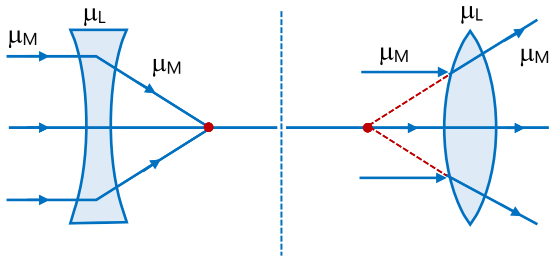


Illustration 59:

What is the refractive index of material of a plano-convex lens, if the radius of curvature of the convex surface is 10 *cm* and focal length of the lens is 30 *cm* ?

Solution:

According to Lens-maker's formula:

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

with $\mu = \dfrac{\mu_L}{\mu_M}$

Here f = 30 *cm* and R_2 =10 *cm* and $R_2 = \infty$

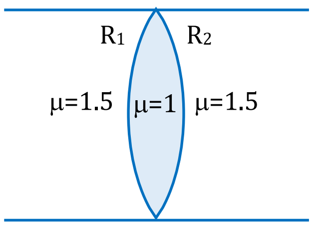
So $$\frac{1}{30} = (\mu - 1)\left[\frac{1}{10} - \frac{1}{\infty}\right]$$

$$3\mu - 3 = 1$$

or, $$\mu = (4/3)$$

Illustration 60:

As shown in Fig. a spherical air lens of radii $R_1 = R_2 = 10\ cm$ is cut in a glass ($\mu = 1.5$) cylinder. Determine the focal length and nature of air lens. If a liquid of refractive index 2 is filled in the lens, what will happen to its focal length and nature?



Solution:

According to lens-maker's formula.

$$\frac{1}{f} = (\mu - 1)\left[\frac{1}{R_1} - \frac{1}{R_2}\right]$$

with $\quad \mu = \frac{\mu_M}{\mu_L}$

Initially, $\mu = \frac{\mu_A}{\mu_G} = \frac{1}{(3/2)} = \frac{2}{3}$;

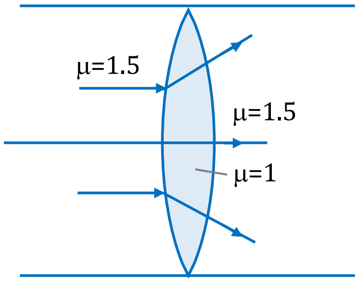
$R_1 = +\ 10\ cm$

and $\quad R_2 = -\ 10\ cm$

So $\quad \frac{1}{f} = \left(\frac{2}{3} - 1\right)\left[\frac{1}{+10} - \frac{1}{10}\right] = -\frac{2}{30}$

i.e., $\quad f = -\ 15\ cm$

i.e., the air lens in glass behaves as divergent lens of focal length 15 *cm.*



(A)

When the liquid of $\mu = 2$ is filled in the air cavity,

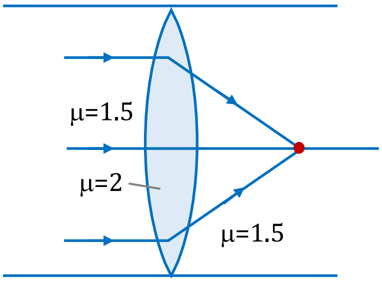
$$\mu = \frac{\mu_L}{\mu_M} = \frac{2}{1.5} = \frac{4}{3}$$

So that now

$$\frac{1}{f'} = \left(\frac{4}{3} - 1\right)\left[\frac{1}{10} - \frac{1}{10}\right] = \frac{2}{30}$$

i.e., $\quad f' = 15\ cm$

i.e., the lens behaves as convergent lens of focal length 15 *cm*.



(B)

Combination of lenses:

When lenses are in contact with each other:

When several lenses are kept co-axially, the image formation is considered one after another in steps. The image formed by the lens facing the object serves as an object for the next lens, the image formed by the second acts as an object the third and so on.

(A) Net magnification, $m = m_1 \times m_2 \times m_3 \times ......$

(B) If thin lenses are kept close together with their principal axis coincide then,

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + ...,$$

$f = (+)$ ve gives equivalent converging lens

$= (-)$ ve gives equivalent diverging lens

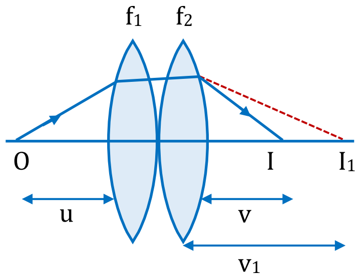
Note:

(i) Value of f_1, f_2, f_3 ...., is to be put w.r.t. the surrounding of whole system of lenses.

(ii) Value of f_1, f_2, f_3 ...., is to be put with sign.

Derivation:

Image I_1 of the object 'O' is formed due to first lens of focal length f_1



$$\frac{1}{v_1} - \frac{1}{u} = \frac{1}{f_1}$$

Now, this I_1 acts as an object for the second lens of focal length f_2

$$\frac{1}{v} - \frac{1}{v_1} = \frac{1}{f_2}$$

Adding, $\frac{1}{v} - \frac{1}{u} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f}$

Thus the combination behaves, like a single lens of equivalent focal length 'f' where.

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$$

or, $P = P_1 + P_2$

Note:

(i) If $f_1 = -f_2 = f$ i.e., lenses of equal focal length and of opposite nature, then

$f = \infty$ and $P = 0$

i.e., the system will behave as a plane glass plate

(ii) If two thin lenses are of same nature, then

$\frac{1}{f} > \frac{1}{f_1}$ and $\frac{1}{f} > \frac{1}{f_2}$

i.e., $f < f_1$ and $f < f_2$

Thus the resultant focal length will be lesser than the smallest individual.

(iii) If two thin lenses of opposite nature with different focal lengths are put in contact, the resultant focal length will be of same nature as that of the lens of shorter focal length but its magnitude will be more than that of shorter focal length.

(iv) If a lens of focal length f is divided into two equal parts as shown in Fig.(A) and each part has a focal length f' then as

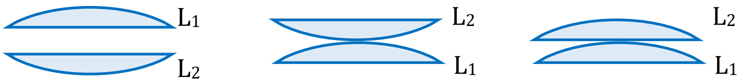
$\frac{1}{f} = \frac{1}{f'} + \frac{1}{f'}$ i.e., $f' = 2f$

i.e., each part will have focal length $2f$.

Now if these parts are put in contact as in Fig. (B) or (C) the resultant focal length of the combination will be

$$\frac{1}{F}=\frac{1}{2f}+\frac{1}{2f}$$

i.e., $F=f$ (= initial value)



(A) **(B)** **(C)**

(v) If a lens of focal length f is cut in two equal parts as shown in figure (A) each part will have focal length f. Now if these parths are put in contact as shown in figure (B) the resultant focal length will be.

(A) **(B)**

$$\frac{1}{F}=\frac{1}{f}+\frac{1}{f}$$

i.e. $F=(f/2)$

Determination of focal length of a concave mirror by u–v method:

The relation between object distance u and the image v from the pole of the mirror is given by,

$$\boxed{\frac{1}{v}+\frac{1}{u}=\frac{1}{f}}$$

Where f is the focal length of the mirror. The focal length of the concave mirror is obtained either from $\frac{1}{v}$ versus $\frac{1}{u}$ graph or from u–v graph.

$\frac{1}{v}$ versus $\frac{1}{u}$ graph:

When the image is real (of course only upon then it can be obtained on screen), the object lies between focus (f) and infinity. In such a situation u, v and f all are negative. Hence the mirror formula

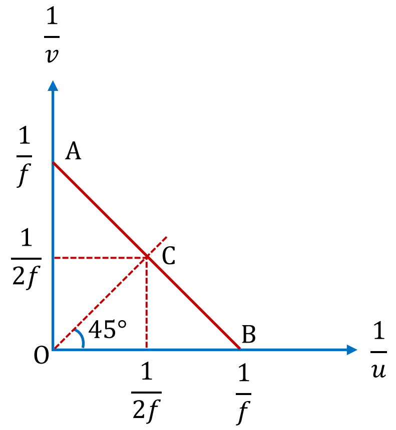
$$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$$

Becomes, $-\frac{1}{v}-\frac{1}{u}=-\frac{1}{f}$

or again, $\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

or, $\frac{1}{v}=-\frac{1}{u}+\frac{1}{f}$

Comparing with $y = mx + c$, the desired graph will be a straight line with slope -1 and intercept equal to $\frac{1}{f}$.

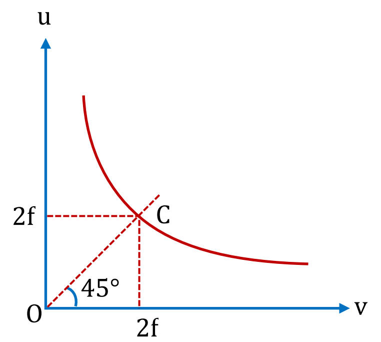


The corresponding $\frac{1}{v}$ versus $\frac{1}{u}$ graph is as shown in figure. The intercepts on the horizontal and vertical axes are equal. It is equal to $\frac{1}{f}$. A straight line OC at an angle 45° with the horizontal axis intersects line AB at C. The coordinates of point C are $\left(\frac{1}{2f}, \frac{1}{2f}\right)$.

The focal length of the mirror can be calculated by measuring the coordinates of either of the points A, B or C.

u–v graph:

The u–v graph comes out to be a hyperbola as shown in figure.
A line drawn at angle 45° from the origin intersects the hyperbola at point C. The coordinates of point C are $(2f, 2f)$.
The focal length of mirror can be calculated by measuring the coordinates of point C.



Determination of focal length of convex lens using u–v method:

The relation between u, v and f for a convex lens is,

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$\frac{1}{u}$ and $\frac{1}{v}$ graph:

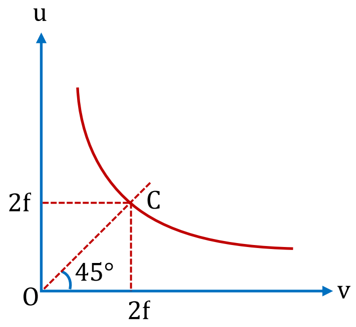
Using the proper sign convention, u is negative, u negative, v and f are positive. So, we have,

$$\frac{1}{v} - \frac{1}{-u} = \frac{1}{f} \qquad \text{or} \qquad \frac{1}{v} = -\frac{1}{u} + \frac{1}{f}$$

Comparing with $y = mx + c$, graph between $\frac{1}{v}$ and $\frac{1}{u}$ is negative with slope -1 and intercept $\frac{1}{f}$. The corresponding graph is as shown in figure.
Proceeding in the similar manner as discussed in case of a concave mirror the focal length of the lens can be calculated by measuring the coordinates of either of the points A, B and C.

u–v Graph:

The u versus v graph is a hyperbola as shown in figure. By measuring the coordinates of point C whose coordinates are $(2f, 2f)$ we can calculate the focal length of the lens.



Velocity of Image of moving object (In lens):

There are two components of velocity of image:

(a) Velocity component along the principal axis that means perpendicular to the lens.

(b) Velocity component perpendicular to the principal axis that means parallel to the lens.

(a) Velocity component along the principal (Optical) axis:

$$\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$$

Differentiating mirror formula on the both side w.r.t time we, get,

$$-\frac{1}{v^2}\frac{dv}{dt}+\frac{1}{u^2}\frac{du}{dt}=0 \text{ (Since focal length of the lens remains constant)}$$

$$\frac{dv}{dt}=\frac{v^2}{u^2}\frac{du}{dt} \quad \Rightarrow \quad (V_{IL})_{||}=+\frac{v^2}{u^2}(V_{OL})_{||}$$

$(V_{IL})_{||}$ = Velocity of image w.r.t. lens along the principal axis.

$(V_{OL})_{||}$ = Velocity of object w.r.t. lens along the principal axis.

Direction of velocity of object and image w.r.t. lens is same to each other.

When object is moving from -∞ to 2*F*, the image is coming from *F* to 2 *F*.

(b) Velocity component perpendicular to the principal axis:

$$m=\frac{v}{u}=\frac{h_i}{h_o}$$

$$h_i=\frac{v}{u}h_o$$

Differentiating both side w.r.t time we, get,

$$\frac{dh_i}{dt}=\frac{v}{u}\frac{dh_o}{dt}$$

$$(V_{IL})_{\perp}=+\frac{v}{u}(V_{OL})_{\perp}$$

$(V_{IL})_{\perp}$ = Velocity of image w.r.t. lens perpendicular the principal axis.

$(V_{OL})_{\perp}$ = Velocity of object w.r.t. lens perpendicular the principal axis.

Silvering at One Surface of Lens:

When one surface of a thin lens is silvered, then the focal length F of the effective lens-mirror combination is expressed as,

$$\frac{1}{F}=\sum\frac{1}{f_i}$$

where f_i is the focal length of the lens or mirror to be repeated as many times as the refraction or reflection respectively is repeated.

Some Cases:

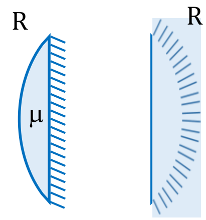
(i) Focal length of plano–convex lens when silvered at its plane surface

When an object is placed in front of such a lens. The ray first of all are refracted from the convex surface, then reflected from the polished plane surface and again refracted out from the convex surface. If f_ℓ and f_m be the focal lengths of lens and mirror.

$$\Rightarrow \quad \frac{1}{F}=\frac{1}{f_\ell}+\frac{1}{f_m}+\frac{1}{f_\ell}, \qquad \text{Here, } \frac{1}{f_\ell}=(\mu-1)\left[\frac{1}{R}\right] \text{and } \frac{1}{f_m}=\frac{2}{R_m}=\frac{2}{\infty}=0$$

$$\Rightarrow \quad \frac{1}{F}=\frac{2}{f_\ell}\Rightarrow F=\frac{f_\ell}{2}=\frac{R}{2(\mu-1)}$$

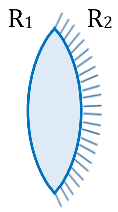
(ii) Focal length of plano–convex lens when silvered at convex surface



$$\Rightarrow \quad \frac{1}{F}=\frac{1}{f_\ell}+\frac{1}{f_m}+\frac{1}{f_\ell}, \qquad \text{Here, } \frac{1}{f_\ell}=(\mu-1)\left[\frac{1}{R}\right] \text{and } \frac{1}{f_m}=\frac{2}{R}$$

$$\Rightarrow \quad \frac{1}{F}=\frac{2}{f_\ell}+\frac{2}{R}\Rightarrow\frac{1}{F}=2\left[\frac{(\mu-1)}{R}\right]+\frac{2}{R}\Rightarrow F=\frac{R}{2\mu}$$

(iii) Focal length of convex lens where convex surface of radius R_2 is silvered



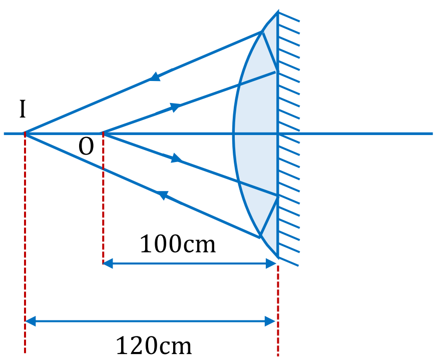
$$\Rightarrow \quad \frac{1}{F}=\frac{1}{f_l}+\frac{1}{f_m}+\frac{1}{f_\ell}, \qquad \text{Here, } \frac{1}{f_\ell}=(\mu-1)\left[\frac{1}{R_1}-\frac{1}{R_2}\right] \text{and } \frac{1}{f_m}=\frac{2}{R_2}$$

$$\Rightarrow \quad \frac{1}{F}=\frac{2}{f_\ell}+\frac{2}{R_2}\Rightarrow\frac{1}{F}=2\left[(\mu-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right]+\frac{2}{R_2}$$

Illustration 61:

An object is 1 metre in front of the curved surface of a plano–convex lens whose flat surface is silvered. A real image is formed 120 *cm* in front of the lens. What is the focal length of the lens?

Solution:



Here $u = 100\ cm$

$v = 120\ cm$

$$\frac{1}{F}=\frac{1}{v}+\frac{1}{u}=\frac{1}{120}+\frac{1}{100}=\frac{11}{600}$$

Now $$\frac{1}{F}=\frac{1}{F_L}+\frac{1}{F_M}+\frac{1}{F_L}$$

Where F_L – focal length of the lens

F_M – focal length of the mirror

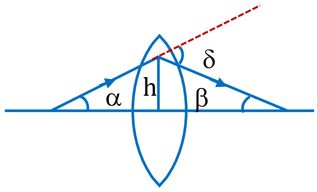
$$=\frac{2}{F_L}+\frac{1}{F_M}\quad(\because F_M=\infty)$$

$$\frac{1}{F}=\frac{2}{F_L}$$

$\therefore$ $$\frac{11}{600}=\frac{2}{F_L}$$

$$F_L=\frac{1200}{11}=109.1\,cm$$

Angle of deviation of a ray when it passes a lens



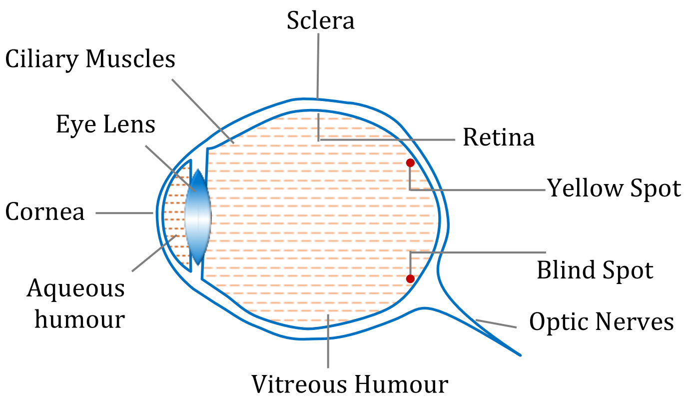
O is the object and I is the image δ is the angle of deviation.

$$\delta=\alpha+\beta\approx\tan\alpha+\tan\beta$$

$$=\frac{h}{-u}+\frac{h}{v}=h\left[\frac{1}{v}-\frac{1}{u}\right]\quad\Rightarrow\quad\delta=\frac{h}{f}$$

Optical Instruments:

Human Eye:



(1) **Eye lens :** Over all behaves as a convex lens of $\mu=1.437$

(2) **Retina :** Real and inverted image of an object, obtained at retina, brain sense it erect.

(3) **Yellow spot :** It is the most sensitive part, the image formed at yellow spot is brightest.

(4) **Blind spot :** Optic nerves goes to brain through blind spot. It is not sensitive for light.

(5) **Ciliary muscles :** Eye lens is fixed between these muscles. It's both radius of curvature can be changed by applying pressure on it through ciliary muscles.

(6) **Power of accomodation :** The ability of eye to see near objects as well as far objects is called power of accomodation.

Note: When we look distant objects, the eye is relaxed and it's focal length is largest.

(7) **Range of vision :** For healthy eye it is 25 *cm* (near point) to ∞ (far point).

A normal eye can see the objects clearly, only if they are at a distance greater than 25 *cm.* This distance is called Least distance of distinct vision and is represented by *D*.

(8) **Persistence of vision :** Is 1/10 sec. *i.e.* if time interval between two consecutive light pulses is lesser than 0.1 sec., eye cannot distinguish them separately.

(9) **Binocular vision :** The seeing with two eyes is called binocular vision.

(10) **Resolving limit :** The minimum angular displacement between two objects, so that they are just resolved is called resolving limit. For eye it is $1'=\left(\frac{1}{60}\right)^{o}$.

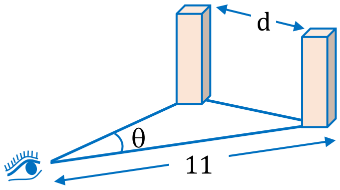
Illustration 62:

A person wishes to distinguish between two pillars located at a distance of 11 Km. What should be the minimum distance between the pillars.

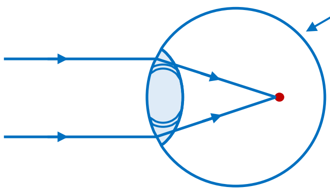
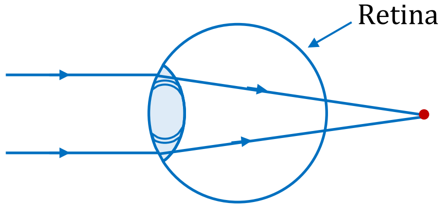
Solution :

As the limit of resolution of eye is $\left(\frac{1}{60}\right)^o$

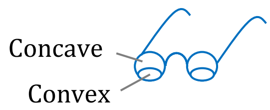
So $\theta > \left(\frac{1}{60}\right)^o \quad \Rightarrow \frac{d}{11\times10^3} > \left(\frac{1}{60}\right)\times\frac{\pi}{180} \quad \Rightarrow d > 3.2m$



Defects in eye:

Myopia (short sightedness)		Hypermetropia (long sightedness)	
(i)	Distant objects are not seen clearly but nearer objects are clearly visible.	(i)	Distant objects are seen clearly but nearer object are not clearly visible.
(ii)	Image formed before the retina. 	(ii)	Image formed behind the retina. 
(iii)	Far point comes closer.	(iii)	Near point moves away
(iv)	Reasons : (a) Focal length or radii of curvature of lens reduced or power of lens increases. (b) Distance between eye lens and retina increases.	(iv)	Reasons : (a) Focal length or radii of curvature of lens increases or power of lens decreases. (b) Distance between eye lens and retina decreases.
(v)	Removal : By using a concave lens of suitable focal length.	(v)	Removal : By using a convex lens.
(vi)	Focal length : (a) A person can see upto distance $\rightarrow x$ wants to see $\rightarrow \infty$, so focal length of used lens $f = -x$ = – (defected far point) (b) A person can see upto distance $\rightarrow x$ wants to see distance $\rightarrow y$ $(y > x)$ so $f = \frac{xy}{x-y}$	(vi)	Focal length : (a) A person cannot see before distance $\rightarrow d$ wants to see the object place at distance $\rightarrow D$ so $f = \frac{dD}{d-D}$

Presbyopia : In this defect both near and far objects are not clearly visible. It is an old age disease and it is due to the loosing power of accommodation. It can be removed by using bifocal lens.



Astigmatism : In this defect eye cannot see horizontal and vertical lines clearly, simultaneously. It is due to imperfect spherical nature of eye lens. This defect can be removed by using cylindrical lens (Torric lenses).

Microscope:

It is an optical instrument used to see very small objects. It's magnifying power is given by

$$m = \frac{\text{Visual angle with instrument}(\beta)}{\text{Visual angle when object is placed at least distance of distinct vision }(\alpha)}$$

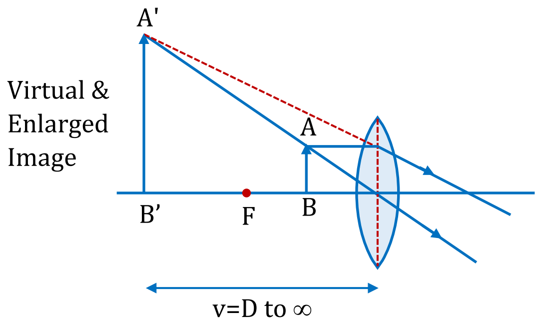
(1) Simple microscope

(i) It is a single convex lens of lesser focal length.

(ii) Also called magnifying glass or reading lens.

(iii) Magnification's, when final image is formed at D and ∞ (i.e. m_D and m_∞)

$$m_D = \left(1 + \frac{D}{f}\right)_{\max} \quad \text{and} \quad m_\infty = \left(\frac{D}{f}\right)_{\min}$$

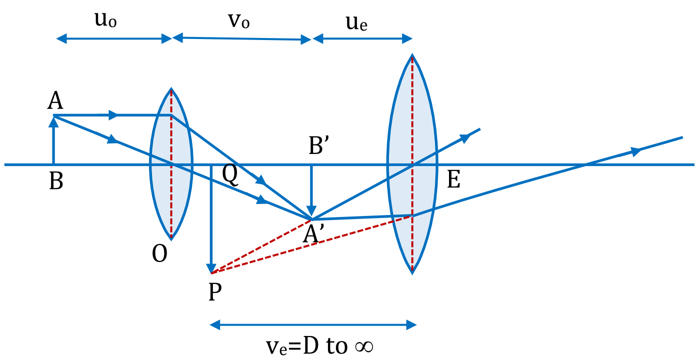


Note:

$m_{\max.} - m_{\min.} = 1$

If lens is kept at a distance a from the eye then $m_D = 1 + \dfrac{D-a}{f}$ and $m_\infty = \dfrac{D-a}{f}$

(2) Compound microscope



(i) Consist of two converging lenses called objective and eye lens.

(ii) $f_{\text{eye lens}} > f_{\text{objective}}$ and $(\text{diameter})_{\text{eye lens}} > (\text{diameter})_{\text{objective}}$

(iii) Final image is magnified, virtual and inverted.

(iv) u_0 = Distance of object from objective (O),

v_0 = Distance of image ($A'B'$) formed by objective from objective,

u_e = Distance of $A'B'$ from eye lens,

v_e = Distance of final image from eye lens,

f_0 = Focal length of objective,

f_e = Focal length of eye lens.

Magnification :

$m = m_{\text{objective}} \times m_{\text{eye lens}}$

$$m_D = -\frac{v_0}{u_0}\left(1+\frac{D}{f_e}\right) = -\frac{f_0}{(u_0-f_0)}\left(1+\frac{D}{f_e}\right) = -\frac{(v_0-f_0)}{f_0}\left(1+\frac{D}{f_e}\right)$$

$$m_\infty = -\frac{v_0}{u_0}.\frac{D}{f_e} = \frac{-f_0}{(u_0-f_0)}\left(\frac{D}{f_e}\right) = -\frac{(v_0-f_0)}{f_0}.\frac{D}{f_e}$$

Length of the tube (*i.e.* distance between two lenses)

When final image is formed at D ; $L_D = v_0 + u_e = \dfrac{u_0 f_0}{u_0 - f_0} + \dfrac{f_e D}{f_e + D}$

When final images is formed at ∞ ; $L_\infty = v_0 + f_e = \dfrac{u_0 f_0}{u_0 - f_0} + f_e$

(Do not use sign convention while solving the problems)

Note:

$$m_\infty = \frac{(L_\infty - f_0 - f_e)D}{f_0 f_e}$$

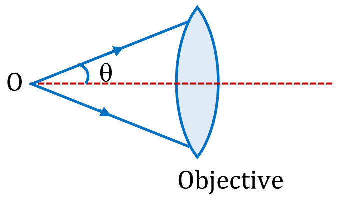
(i) For maximum magnification both f_0 and f_e must be less.

(ii) $m = m_{\text{objective}} \times m_{\text{eye lens}}$

(iii) If objective and eye lens are interchanged, practically there is no change in magnification.

(3) Resolving limit and resolving power : In reference to a microscope, the minimum distance between two lines at which they are just distinct is called Resolving limit (RL) and it's reciprocal is called Resolving power (RP)

$$R.L. = \frac{\lambda}{2\mu\sin\theta} \text{ and } R.P. = \frac{2\mu\sin\theta}{\lambda} \Rightarrow R.P. \propto \frac{1}{\lambda}$$



λ = Wavelength of light used to illuminate the object,

μ = Refractive index of the medium between object and objective,

θ = Half angle of the cone of light from the point object, $\mu\sin\theta$ = Numerical aperture.

Note:

Electron microscope : electron beam $(\lambda \approx 1\mathring{A})$ is used in it so it's *R.P.* is approx 5000 times more than that of ordinary microscope $(\lambda \approx 5000\mathring{A})$

Telescope

By telescope distant objects are seen.

(1) Astronomical telescope

(i) Used to see heavenly bodies.

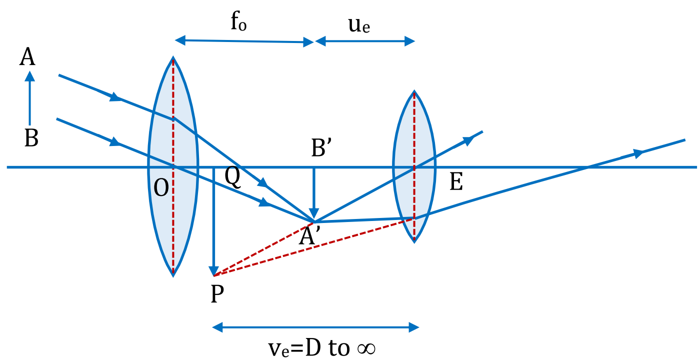
(ii) $f_{\text{objective}} > f_{\text{eyelens}}$ and $d_{\text{objective}} > d_{\text{eye lens}}$.

(iii) Intermediate image is real, inverted and small.

(iv) Final image is virtual, inverted and small.

(v) Magnification : $m_D = -\frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$ and $m_\infty = -\frac{f_o}{f_e}$

(vi) Length : $L_D = f_0 + u_e = f_0 + \frac{f_e D}{f_e + D}$ and $L_\infty = f_0 + f_e$



(2) Terrestrial telescope

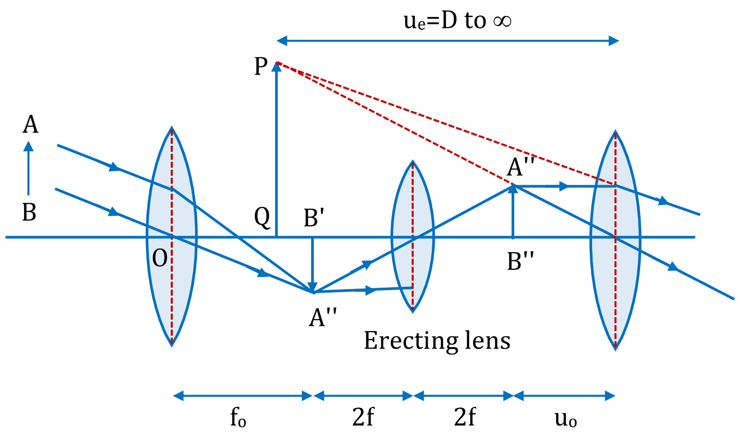
(i) Used to see far off object on the earth.

(ii) It consists of three converging lens : objective, eye lens and erecting lens.

(iii) It's final image is virtual erect and smaller.

(iv) Magnification : $m_D = \frac{f_0}{f_e}\left(1+\frac{f_e}{D}\right)$ and $m_\infty = \frac{f_0}{f_e}$

(v) Length : $L_D = f_0 + 4f + u_e = f_0 + 4f + \frac{f_e D}{f_e + D}$ and $L_\infty = f_0 + 4f + f_e$



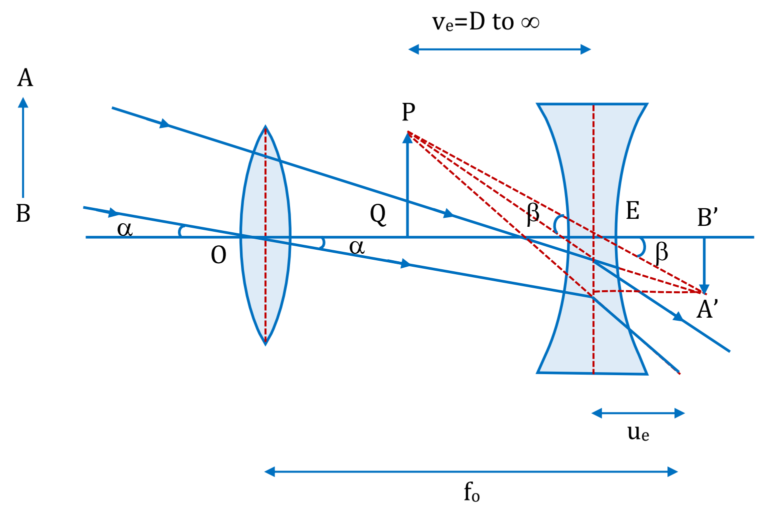
(3) Galilean telescope

(i) It is also a terrestrial telescope but of much smaller field of view.

(ii) Objective is a converging lens while eye lens is diverging lens.

(iii) Magnification : $m_D = \frac{f_0}{f_e}\left(1 - \frac{f_e}{D}\right)$ and $m_\infty = \frac{f_0}{f_e}$

(iv) Length : $L_D = f_0 - u_e$ and $L_\infty = f_0 - f_e$



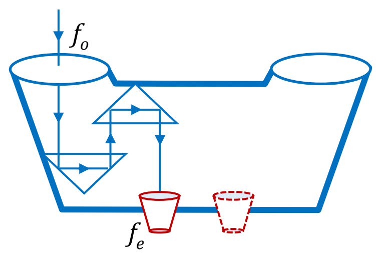
Note:

Minimum separation (d) between objects, so they can just resolved by a telescope is $d = \frac{r}{R.P.}$

where r = distance of objects from telescope.

(4) Binocular

If two telescopes are mounted parallel to each other so that an object can be seen by both the eyes simultaneously, the arrangement is called 'binocular'. In a binocular, the length of each tube is reduced by using a set of totally reflecting prisms which provided intense, erect image free from lateral inversion. Through a binocular we get two images of the same object from different angles at same time. Their superposition gives the perception of depth also along with length and breadth, *i.e.*, binocular vision gives proper three-dimensional (3D) image.



Few important points

(i) As magnifying power is negative, the image seen in astronomical telescope is truly inverted, i.e., left is turned right with upside down simultaneously. However, as most of the astronomical objects are symmetrical this inversion does not affect the observations.

(ii) Objective and eye lens of a telescope are interchanged, it will not behave as a microscope but object appears very small.

(iii) In a telescope, if field and eye lenses are interchanged magnification will change from (f_o / f_e) to (f_e / f_o), i.e., it will change from m to (1/m), i.e., will become (1/m^2) times of its initial value.

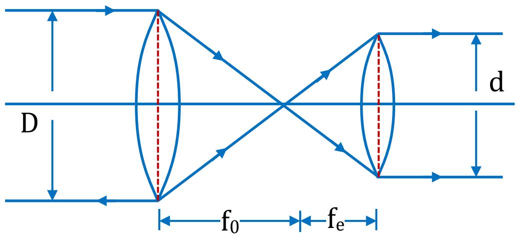
(iv) As magnification for normal setting as (f_o / f_e), so to have large magnification, f_o must be as large as practically possible and f_e small. This is why in a telescope, objective is of large focal length while eye piece of small.

(v) In a telescope, aperture of the field lens is made as large as practically possible to increase its resolving power as resolving power of a telescope $\propto (D/\lambda)^*$. Large aperture of objective also helps in improving the brightness of image by gathering more light from distant object. However, it increases aberrations particularly spherical.

(vi) For a telescope with increase in length of the tube, magnification decreases.

(vii) In case of a telescope if object and final image are at infinity then:

$$m = \frac{f_o}{f_e} = \frac{D}{d}$$



(viii) If we are given four convex lenses having focal lengths $f_1 > f_2 > f_3 > f_4$. For making a good telescope and microscope. We choose the following lenses respectively.

Telescope $f_1(o), f_4(e)$

Microscope $f_4(o), f_3(e)$

(viii) If a parrot is sitting on the objective of a large telescope and we look towards (or take a photograph) of distant astronomical object (say moon) through it, the parrot will not be seen but the intensity of the image will be slightly reduced as the parrot will act as obstruction to light and will reduce the aperture of the objective.

Illustration 63:

A man can see the objects upto a distance of one metre from his eyes. For correcting his eye sight so that he can see an object at infinity, he requires a lens whose power is

or

A man can see upto 100 *cm* of the distant object. The power of the lens required to see far objects will be

(A) +0.5 *D* (B) +1.0 *D* (C) +2.0 *D* (D) –1.0 *D*

Ans. (D)

Solution:

f = –(defected far point) = – 100 *cm*. So, power of the lens $P = \frac{100}{f} = \frac{100}{-100} = -1D$

Illustration 64:

A man can see clearly up to 3 metres. Prescribe a lens for his spectacles so that he can see clearly up to 12 metres

(A) – 3/4 *D* (B) 3 *D* (C) – 1/4 *D* (D) – 4 *D*

Ans. (C)

Solution:

By using $f = \frac{xy}{x-y} \Rightarrow f = \frac{3\times 12}{3-12} = -4m$. Hence power $P = \frac{1}{f} = -\frac{1}{4}D$

Illustration 65:

The diameter of the eye-ball of a normal eye is about 2.5 *cm*. The power of the eye lens varies from

(A) 2 *D* to 10 *D* (B) 40 *D* to 32 *D* (C) 9 *D* to 8 *D* (D) 44 *D* to 40 *D*

Ans. (D)

Solution:

An eye sees distant objects with full relaxation so $\frac{1}{2.5\times10^{-2}}-\frac{1}{-\infty}=\frac{1}{f}$ or $P=\frac{1}{f}=\frac{1}{25\times10^{-2}}=40D$

An eye sees an object at 25 *cm* with strain so $\frac{1}{2.5\times10^{-2}}-\frac{1}{-25\times10^{-2}}=\frac{1}{f}$ or $P=\frac{1}{f}=40+4=44D$

Illustration 66:

In a compound microscope, the focal lengths of two lenses are 1.5 *cm* and 6.25 *cm* an object is placed at 2 *cm* form objective and the final image is formed at 25 *cm* from eye lens. The distance between the two lenses is

(A) 6.00 *cm* (B) 7.75 *cm* (C) 9.25 *cm* (D) 11.00 *cm*

Ans. (D)

Solution:

It is given that f_o = 1.5 *cm*, f_e = 6.25 *cm*, u_o = 2 *cm*

When final image is formed at least distance of distinct vision, length of the tube

$$L_D=\frac{u_o f_o}{u_o-f_o}+\frac{f_e D}{f_e+D} \quad \Rightarrow L_D=\frac{2\times1.5}{(2-1.5)}+\frac{6.25\times25}{(6.25+25)}=11cm.$$

Illustration 67:

The focal lengths of the objective and the eye-piece of a compound microscope are 2.0 *cm* and 3.0 *cm* respectively. The distance between the objective and the eye-piece is 15.0 *cm*. The final image formed by the eye-piece is at infinity. The two lenses are thin. The distances in *cm* of the object and the image produced by the objective measured from the objective lens are respectively

(A) 2.4 and 12.0 (B) 2.4 and 15.0 (C) 2.3 and 12.0 (D) 2.3 and 3.0

Ans. (A)

Solution:

Given that $f_o=2cm$, $f_e=3cm$, $L_\infty=15cm$

By using $L_\infty=v_o+f_e \Rightarrow 15=v_o+3 \Rightarrow v_o=12cm$. Also, $\frac{v_o}{u_o}=\frac{v_o-f_o}{f_o} \Rightarrow \frac{12}{u_o}=\frac{12-2}{2} \Rightarrow u_o=2.4cm$.

Illustration 68:

The focal lengths of the objective and eye-lens of a microscope are 1 *cm* and 5 *cm* respectively. If the magnifying power for the relaxed eye is 45, then the length of the tube is

(A) 30 *cm* (B) 25 *cm* (C) 15 *cm* (D) 12 *cm*

Ans. (C)

Solution:

Given that $f_o=1cm$, $f_e=5cm$, $m_\infty=45$

By using $m_\infty=\frac{(L_\infty-f_o-f_e)}{f_o f_e} \Rightarrow 45=\frac{(L_\infty-1-5)\times25}{1\times5} \Rightarrow L_\infty=15cm$

Illustration 69:

If the focal lengths of objective and eye lens of a microscope are 1.2 *cm* and 3 *cm* respectively and the object is put 1.25 *cm* away from the objective lens and the final image is formed at infinity, then magnifying power of the microscope is

(A) –150 (B) –200 (C) –250 (D) –400

Ans. (B)

Solution:

Given that $f_o = 1.2cm$, $f_e = 3cm$, $u_o = 1.25cm$

By using $m_\infty = -\frac{f_o}{(u_o - f_o)} \cdot \frac{D}{f_e} \Rightarrow m_\infty = -\frac{1.2}{(1.25-1.2)} \times \frac{25}{3} = -200$

Illustration 70:

The magnifying power of an astronomical telescope is 8 and the distance between the two lenses is 54*cm*. The focal length of eye lens and objective lens will be respectively

(A) 6 *cm* and 48 *cm* (B) 48 *cm* and 6 *cm* (C) 8 *cm* and 64 *cm* (D) 64 *cm* and 8 *cm*

Ans. (A)

Solution:

Given that $m_\infty = 8$ and $L_\infty = 54$

By using $|m_\infty| = \frac{f_o}{f_e}$ and $L_\infty = f_o + f_e$ we get $f_o = 6cm$ and $f_e = 48cm$.

Illustration 71:

If an object subtend angle of 2° at eye when seen through telescope having objective and eyepiece of focal length $f_o = 60cm$ and $f_e = 5cm$ respectively than angle subtend by image at eye piece will be

(A) 16° (B) 50° (C) 24° (D) 10°

Ans. (C)

Solution:

By using $\frac{\beta}{\alpha} = \frac{f_o}{f_e} \Rightarrow \frac{\beta}{20} = \frac{60}{5} \Rightarrow \beta = 24°$

Illustration 72:

The focal lengths of the lenses of an astronomical telescope are 50 *cm* and 5 *cm*. The length of the telescope when the image is formed at the least distance of distinct vision is

(A) 45 *cm* (B) 55 *cm* (C) $\frac{275}{6}$ *cm* (D) $\frac{325}{6}$ *cm*

Ans. (D)

Solution:

By using $L_D = f_o + u_e = f_o + \frac{f_e D}{f_e + D} = 50 + \frac{5 \times 25}{(5+25)} = \frac{325}{6} cm$

Illustration 73:

The diameter of moon is $3.5 \times 10^3\ km$ and its distance from the earth is $3.8 \times 10^5\ km$. If it is seen through a telescope whose focal length for objective and eye lens are 4 *m* and 10 *cm* respectively, then the angle subtended by the moon on the eye will be approximately

(A) 15° (B) 20° (C) 30° (D) 35°

Ans. (B)

Solution:

The angle subtended by the moon on the objective of telescope $\alpha = \dfrac{3.5 \times 10^3}{3.8 \times 10^5} = \dfrac{3.5}{3.8} \times 10^{-2}\ rad$

Also $\quad m = \dfrac{f_o}{f_e} = \dfrac{\beta}{\alpha} \Rightarrow \dfrac{400}{10} = \dfrac{\beta}{\alpha}$

$\Rightarrow \quad \beta = 40\alpha$

$\Rightarrow \quad \beta = 40 \times \dfrac{3.5 \times 10^3}{3.8 \times 10^5} \times \dfrac{180}{\pi} = 20°$

Illustration 74:

A telescope has an objective lens of 10 *cm* diameter and is situated at a distance one *kilometre* from two objects. The minimum distance between these two objects, which can be resolved by the telescope, when the mean wavelength of light is 5000 *Å*, is of the order of

(A) 0.5 *m* (B) 5 *m* (C) 5 *mm* (D) 5*cm*

Ans. (B)

Solution:

Suppose minimum distance between objects is *x* and their distance from telescope is *r*

So, resolving limit $d\theta = \dfrac{1.22\lambda}{a} = \dfrac{x}{r}$

$\Rightarrow \quad x = \dfrac{1.22\lambda \times r}{a} = \dfrac{1.22 \times (5000 \times 10^{-10}) \times (1 \times 10^3)}{(0-1)} = 6.1 \times 10^{-3} m = 6.1 mm$

Hence, it's order is ≈ 5 *mm*.

Illustration 75:

A compound microscope has a magnifying power 30. The focal length of its eye-piece is 5*cm*. Assuming the final image to be at the least distance of distinct vision. The magnification produced by the objective will be

(A) +5 (B) – 5 (C) +6 (D) – 6

Ans. (B)

Solution:

Magnification produced by compound microscope $m = m_o \times m_e$

where m_o = ? and $m_e = \left(1 + \dfrac{D}{f_e}\right) = 1 + \dfrac{25}{5} = 6 \Rightarrow 30 = -m_o \times 6 \Rightarrow m_o = -5$.

Illustration 76:

A man is looking at a small object placed at his least distance of distinct vision. Without changing his position and that of the object he puts a simple microscope of magnifying power 10 X and just sees the clear image again. The angular magnification obtained is

(A) 2.5 (B) 10.0 (C) 5.0 (D) 1.0

Ans. (D)

Solution:

Angular magnification $= \frac{\beta}{\alpha} = \frac{\tan\beta}{\tan\alpha} = \frac{I/D}{O/D} = \frac{I}{O}$

Since image and object are at the same position, $\frac{I}{O} = \frac{v}{u} = 1 \Rightarrow$ Angular magnification = 1

Illustration 77:

A compound microscope is used to enlarge an object kept at a distance 0.03m from it's objective which consists of several convex lenses in contact and has focal length 0.02m. If a lens of focal length 0.1m is removed from the objective, then by what distance the eye-piece of the microscope must be moved to refocus the image

(A) 2.5 cm (B) 6 cm (C) 15 cm (D) 9 cm

Ans. (D)

Solution:

If initially the objective (focal length F_o) forms the image at distance v_o then

$$v_o = \frac{u_o f_o}{u_o - f_o} = \frac{3\times 2}{3-2} = 6\,cm$$

Now as in case of lenses in contact

$$\frac{1}{F_o} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots = \frac{1}{f_1} + \frac{1}{F_o'} \quad \left\{\text{where } \frac{1}{F_o'} = \frac{1}{f_2} + \frac{1}{f_3} + \ldots\ldots\right\}$$

So, if one of the lens is removed, the focal length of the remaining lens system

$$\frac{1}{F_o'} = \frac{1}{F_0} - \frac{1}{f_1} = \frac{1}{2} - \frac{1}{10}$$

$\Rightarrow \quad F_o' = 2.5\,cm$

This lens will form the image of same object at a distance v_o' such that

$$v_o' = \frac{u_o F_o'}{u_o - F_o'} = \frac{3\times 2.5}{(3-2.5)} = 15\,cm$$

So, to refocus the image, eye-piece must be moved by the same distance through which the image formed by the objective has shifted *i.e.* 15 – 6 = 9 cm.