02 Electromagnetic Waves and Wave Optics

Ampere Law:

The general form of Ampere's law (sometimes called the **Ampere-Maxwell law**) as

$$\oint \vec{B}\cdot d\vec{\ell}=\mu_0\left(I+I_0\right)=\mu_0\left(I+\epsilon_0\frac{d\Phi_E}{dt}\right)$$

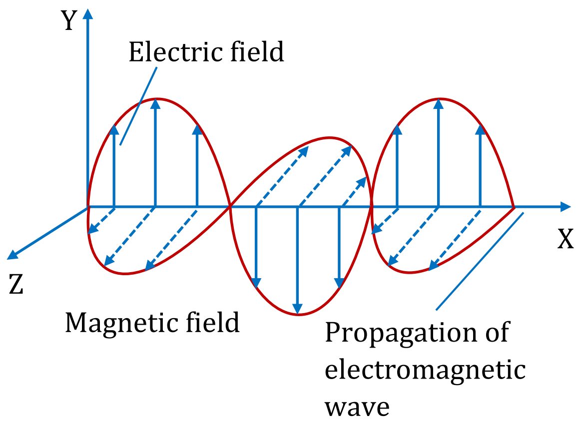
Hence, the displacement current through any surface is given by

$$I_d=\epsilon_0\frac{d\Phi_E}{dt}$$

By considering surface S_2, we can identify the displacement current as the source of the magnetic field.

Electromagnetic Waves:

In electromagnetic waves, both the field vectors ($\vec{E}$ and $\vec{B}$) vary with time and space and have the same frequency and same phase. In figure, the electric field vector ($\vec{E}$) and magnetic field vector ($\vec{B}$) are vibrating along Y and Z directions and propagation of electromagnetic wave is shown in X-direction.



According to Maxwell the electromagnetic waves are of transverse in nature and they can pass through vacuum with the speed of light (= $3\times10^8\ ms^{-1}$).

The velocity of eletromagnetic wave in a medium is given by

$$v=\frac{1}{\sqrt{\mu_0\mu_r\,\epsilon_0\epsilon_r}}$$

where, μ_0, μ_r = Absolute permeability of space and relative permeability of medium,

ϵ_0, ϵ_r = Absolute permittivity of space and relative permittivity of medium

The velocity of electromagnetic waves of different frequency in vacuum is same but in a medium is different. It is more for red light and less for violet light.

The energy is shared equally betwe en electric field vector and magnetic field vector.

$$U_{av} = \frac{1}{2}\in_0 E_0^2 = \frac{1}{2}\frac{B_0^2}{\mu_0}$$

It has been found that the velocity (c) of electromagnetic wave in free space is equal to the ratio of amplitude of electric field vector (E_0) and magnetic field vector (B_0) i.e $c = E_0/B_0$.

It was found that the accelerated charge or oscillating charge is a source of electromagnetic waves.

Light is a form of energy which generally gives the sensation of sight.

Different Theories:

Newtons corpuscular theory	Huygens wave theory	Maxwell's EM wave theory	Einstein's quantum theory	Dual theory of light
(i) Based on Rectilinear propagation of light	(i) Light travels in a hypothetical medium ether (high elasticity very low density) as waves	(i) Light travels in the form of EM waves with speed in free space $c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$	(i) Light is produced, absorbed and propagated as packets of energy called photons	(i) Light propagates both as particles as well as waves
(ii) Light propagates in the form of tiny particles called Corpuscles. Colour of light is due to different size of corpuscles	(ii) He proposed that light waves are of longitudinal nature. Later on it was found that they are transverse	(ii) EM waves consists of electric and magnetic field oscillation and they do not require material medium to travel	(ii) Energy associated with each photon $E = h\nu = \frac{hc}{\lambda}$ h = planks constant $= 6.6 \times 10^{-34} J$-sec ν = frequency λ = wavelength	(ii) Wave nature of light dominates when light interacts with light. The particle nature of light dominates when the light interacts with matter (microscopic particles)

Optical phenomena explained (✓) or not explained (×) by the different theories of light:

S.No.	Phenomena	Theory				
		Corpuscular	Wave	E.M. wave	Quantum	Dual
(i)	Rectilinear Propagation	✓	✓	✓	✓	✓
(ii)	Reflection	✓	✓	✓	✓	✓
(iii)	Refraction	✓	✓	✓	✓	✓
(iv)	Dispersion	×	✓	✓	×	✓
(v)	Interference	×	✓	✓	×	✓
(vi)	Diffraction	×	✓	✓	×	✓
(vii)	Polarisation	×	✓	✓	×	✓
(viii)	Double refraction	×	✓	✓	×	✓
(ix)	Doppler's effect	×	✓	✓	×	✓
(x)	Photoelectric effect	×	×	×	✓	✓

Wavefronts:

Consider a wave spreading out on the surface of water after a stone is thrown in. Every point on the surface oscillates. At any time, a photograph of the surface would show circular rings on which the disturbance is maximum. Clearly, all points on such a circle are oscillating in phase because they are at the same distance from the source. Such a locus of points which oscillate in phase is an example of a wavefront.

A wavefront is defined as a surface of constant phase. The speed with which the wavefront moves outwards from the source is called the wave speed. The energy of the wave travels in a direction perpendicular to the wavefront.

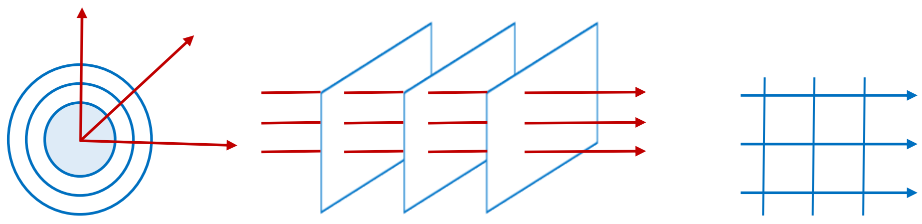
Figure (a) shows light waves from a point source forming a spherical wavefront in three dimensional space. The energy travels outwards along straight lines emerging from the source. i.e., radii of the spherical wavefront. These lines are the rays. Notice that when we measure the spacing between a pair of wavefronts along any ray, the result is a constant. This example illustrates two important general principles which we will use later:

(i) Rays are perpendicular to wavefronts.

(ii) The time taken by light to travel from one wavefront to another is the same along any ray.

If we look at a small portion of a spherical wave, far away from the source, then the wavefronts are like parallel planes. The rays are parallel lines perpendicular to the wavefronts. This is called a plane wave and is also sketched in Figure (b).

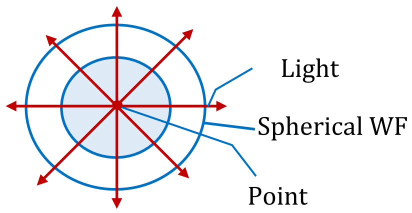
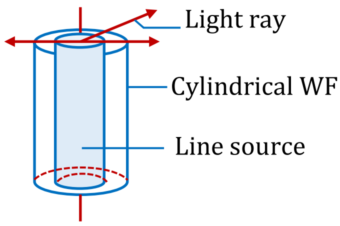
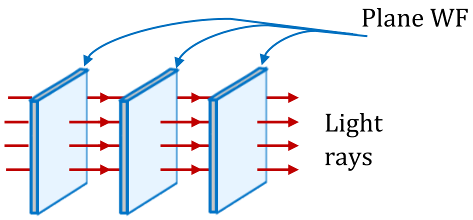
A linear source such as a slit illuminated by another source behind it will give rise to cylindrical wavefronts. Again, at larger distance from the source, these wavefronts may be regarded as planar.



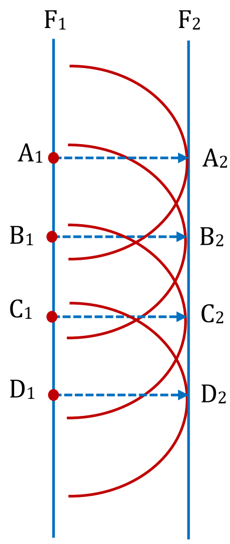
(a) (b)

Figure: Wavefronts and the corresponding rays in two cases: (a) diverging spherical wave, (b) plane wave

Table : Different types of wavefront

Type of wavefront		Intensity	Amplitude
Spherical		$I \propto \frac{1}{r^2}$	$A \propto \frac{1}{r}$
Cylindrical		$I \propto \frac{1}{r}$	$A \propto \frac{1}{\sqrt{r}}$
Plane		$I \propto r^0$	$A \propto r^0$

Huygens' Construction:



Huygens, the Dutch physicist and astronomer of the seventeenth century, gave a beautiful geometrical description of wave propagation. We can guess that he must have seen water waves many times in the canals of his native place Holland. A stick placed in water and oscillated up and down becomes a source of waves. Since the surface of water is two dimensional, the resulting wavefronts would be circles instead of spheres. At each point on such a circle, the water level moves up and down. Huygens' idea is that we can think of every such oscillating point on a wavefront as a new source of waves. According to Huygens' principle, what we observe is the result of adding up the waves from all these different sources. These are called secondary waves or wavelets.

Huygens' principle is illustrated in figure in the simple case of a plane wave.

(i) At time $t = 0$, we have a wavefront F_1, F_1 separates those parts of the medium which are undisturbed from those where the wave has already reached.

(ii) Each point on F_1 acts like a new source and sends out a spherical wave. After a time *'t'* each of these will have radius vt. These spheres are the secondary wavelets.

(iii) After a time t, the disturbance would now have reached all points within the region covered by all these secondary waves. The boundary of this region is the new wavefront F_2. Notice that F_2 is a surface tangent to all the spheres. It is called the forward envelope of these secondary wavelets.

(iv) The secondary wavelet from the point A_1 on F_1 touches F_2 at A_2. Draw the line connecting any point A_1 on F_1 to the corresponding point A_2 on F_2. According to Huygens, $A_1 A_2$ is a ray. It is perpendicular to the wavefronts F_1 and F_2 and has length vt. This implies that rays are perpendicular to wavefronts. Further, the time taken for light to travel between two wavefronts Is the same along any ray. In our example, the speed *'v'* of the wave has been taken to be the same at all points in the medium. In this case, we can say that the distance between two wavefronts is the same measured along any ray.

(v) This geometrical construction can be repeated starting with F_2 to get the next wavefront F_3 a time t later, and so on. This is known as Huygens' construction.

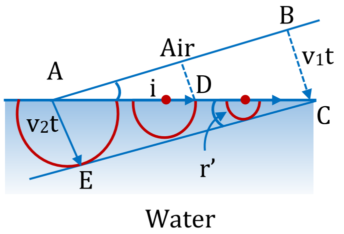
Huygens' construction can be understood physically for waves in a material medium, like the surface of water. Each oscillating particle can set its neighbours into oscillation, and therefore acts as a secondary source. But what if there is no medium, as for light travelling in vacuum? The mathematical theory, which cannot be given here, shows that the same geometrical construction works in this case as well.

Reflection and Refraction:

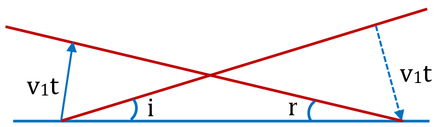
We can use a modified form of Huygens' construction to understand reflection and refraction of light. Figure (a) shows an incident wavefront which makes an angle *'i'* with the surface separating two media, for example, air and water. The phase speeds in the two media are v_1 and v_2. We can see that when the point A on the incident wavefront strikes the surface, the point B still has to travel a distance $BC = AC \sin i$, and this takes a time $t = BC/v_1 = AC(\sin i) / v_1$. After a time t, a secondary wavefront of radius v_2t with A as centre would have travelled into medium 2. The secondary wavefront with C as centre would have just started, i.e. would have zero radius. We also show a secondary wavelet originating from a point D in between A and C. Its radius is less than v_2t. The wavefront in medium 2 is thus a line passing through C and tangent to the circle centred on A. We can see that the angle r' made by this refracted wavefront with the surface is given by $AE = v_2t = AC \sin r'$. Hence, $t = AC(\sin r') / v_2$. Equating the two expressions for *'t'* gives us the law of refraction in the form $\sin i/\sin r' = v_1/v_2$. A similar picture is drawn in Fig. (b) for the reflected wave which travels back into medium 1. In this case, we denote the angle made by the reflected wavefront with the surface by r, and we find that $i = r$. Notice that for both reflection and refraction, we use secondary wavelets starting at different times.

The preceding argument gives a good physical picture of how the refracted and reflected waves are built up from secondary wavelets. We can also understand the laws of reflection and refraction using the concept that the time taken by light to travel along different rays from one wavefront to another must be the same. Fig. (c) shows the incident and reflected wavefronts when a parallel beam of light falls on a plane surface. One ray *POQ* is shown normal to both the reflected and incident wavefronts. The angle of incidence *i* and the angle of reflection r are defined as the angles made by the incident and reflected rays with the normal. As shown in Fig. (c), these are also the angles between the wavefront and the surface.

Huygens' construction for the (a) refracted wave, (b) Reflected wave.

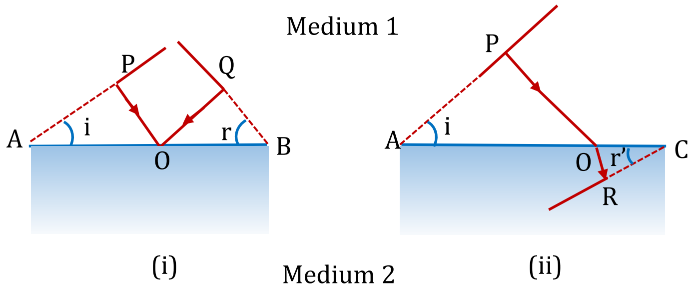


(a)



(b)

Calculation of propagation time between wavefronts in (i) reflection and (ii) refraction.



(c)

We now calculate the total time to go from one wavefront to another along the rays. From Fig. (c), we have total time for light to reach from *P* to *Q*.

$$= \frac{PO}{v_1} + \frac{OQ}{v_1} = \frac{AO\sin i}{v_1} + \frac{OB\sin r}{v_1}$$

$$= \frac{OA\sin i + (AB - OA)\sin r}{v_1} = \frac{AB\sin r + OA(\sin i - \sin r)}{v_1}$$

Different rays normal to the incident wavefront strike the surface at different points *O* and hence have different values of *OA*. Since the time should be the same for all the rays, the right side of equation must actually be Independent of *OA*. The condition, for this to happen is that the coefficient of *OA* in equation should be zero, i.e., sin *i* = sin *r*. We, thus have the law of reflection, *i* = *r*. Figure also shows refraction at a plane surface separating medium 1 (speed of light v_1) from medium 2 (speed of light v_2). The incident and refracted wavefronts are shown, making angles *i* and *r′* with the boundary. Angle *r′* is called the angle of refraction. Rays perpendicular to these are also drawn. As before, let us calculate the time taken to travel between the two wavefronts along any ray.

Time taken from *P* to *R* $= \dfrac{PO}{v_1} + \dfrac{OR}{v_2}$

$$= \frac{OA\sin i}{v_1} + \frac{(AC - OA)\sin r'}{v_2} = \frac{AC\sin r'}{v_2} + OA\left(\frac{\sin i}{v_1} - \frac{\sin r'}{v_2}\right)$$

This time should again be independent of which ray we consider. The coefficient of *OA* in Equation is, therefore, zero, that is, $\dfrac{\sin i}{\sin r'} = \dfrac{v_1}{v_2} = n_{21}$

where n_{21} is the refractive index of medium 2 with respect to medium 1.

n_{21} is the ratio of speed of light in the first medium (v_1) to that in the second medium (v_2).

This is the Snell's law of refraction Equation is, known as the Snell's law of refraction. If the first medium is vacuum, we have $\frac{\sin i}{\sin r\prime} = \frac{c}{v_2} = n_2$

where n_2 is the refractive index of medium 2 with respect to vacuum, also called the absolute refractive index of the medium. A similar equation defines absolute refractive index n_1 of the first medium. From equation we then get

$$n_{21} = \frac{v_1}{v_2} = \left(\frac{c}{n_1}\right) / \left(\frac{c}{n_2}\right) = \frac{n_2}{n_1}$$

The absolute refractive index of air is about 1.0003, quite close to 1. Hence, for all practical purposes, absolute refractive index of a medium may be taken with respect to air. For water, n_1 = 1.33, which means $v_1 = \frac{c}{1.33}$, i.e. about 0.75 times the speed of light in vacuum. The measurement of the speed of light in water by Foucault (1850) confirmed this prediction of the wave theory.

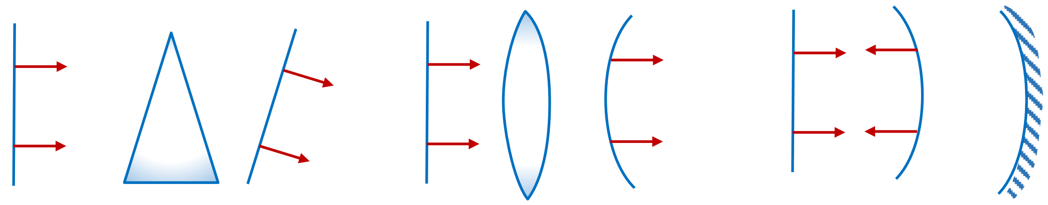
Once we have the laws of reflection and refraction, the behaviour of prisms. lenses, and mirrors can be understood. These topics are discussed in detail in the previous Chapter. Here we just describe the behaviour of the wavefronts in these three cases (Fig.)

(i) Consider a plane wave passing through a thin prism. Clearly, the portion of the incoming wavefront which travels through the greatest thickness of glass has been delayed the most. Since light travels more slowly in glass. This explains the tilt in the emerging wavefront.

(ii) Similarly, the central part of an incident plane wave traverses the thickest portion of a convex lens and is delayed the most. The emerging wavefront has a depression at the centre. It is spherical and converges to a focus.

(iii) A concave mirror produces a similar effect. The centre of the wavefront has to travel a greater distance before and after getting reflected, when compared to the edge. This again produces a converging spherical wavefront.

(iv) Concave lenses and convex mirrors can be understood from time delay arguments in a similar manner. One interesting property which is obvious from the pictures of wavefronts is that the total time taken from a point on the object to the corresponding point on the image is the same measured along any ray (Fig.). For example, when a convex lens focuses light to form a real image, it may seem that rays going through the centre are shorter. But because of the slower speed in glass, the time taken is the same as for rays travelling near the edge of the lens.



(a) (b) (c)

Coherence:

The phase relationship between two light waves can very from time to time and from point to point in space. The property of definite phase relationship is called coherence.

In a conventional light source, however, light comes from a large number of individual atoms, each atom emitting a pulse lasting for about 1 *ns*. Even if atoms were emitting under similar conditions, waves from different atoms would differ in their initial phases. Consequently light coming from two such sources have a fixed phase relationship for about 1*ns*, hence interference pattern will keep changing every billionth of a second. The eye can notice intensity changes which lasts at least one tenth of a second. Hence, we will observe uniform intensity on the screen which is the sum of the two individual intensities. Such sources are said to be incoherent. Light beam coming from two such independent sources do not have any fixed phase relationship and they do not produce any stationary interference pattern. For such sources, resultant intensity at any point is given by

$$I = I_1 + I_2$$

Methods of obtaining coherent sources:

Two coherent sources are produced from a single source of light by two methods (i) By division of wavefront and (ii) By division of amplitude

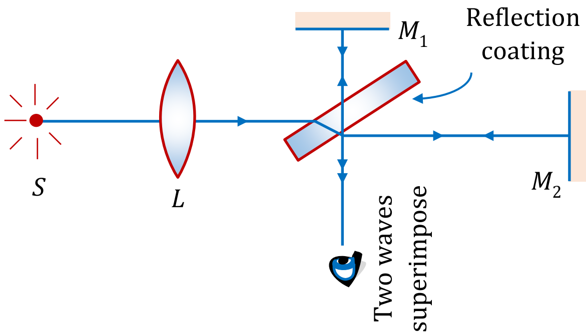
(i) **Division of wave front:** The wave front emitted by a narrow source is divided in two parts by reflection, refraction or diffraction.

The coherent sources so obtained are imaginary. They are produced in Fresnel's biprism, Llyod's mirror Youngs' double slit etc.

(ii) **Division of amplitude:** In this arrangement light wave is partly reflected (50%) and partly transmitted (50%) to produce two light rays.

The amplitude of wave emitted by an extend source of light is divided in two parts by partial reflection and partial refraction.

The coherent sources obtained are real and are obtained in Newton's rings, Michelson's interferrometer, colours in thin films.



Principle of superposition:

When two or more waves simultaneously pass through a point, the disturbance of the point is given by the sum of the disturbances each wave would produce in absence of the other wave(s). In case of wave on string disturbance means displacement, in case of sound wave it means pressure change, in case of Electromagnetic Waves. it is electric field or magnetic field. Superposition of two light travelling in almost same direction results in modification in the distribution of intensity of light in the region of superposition. This phenomenon is called interference.

Superposition of two sinusoidal waves:

Consider superposition of two sinusoidal waves (having same frequency), at a particular point.

Let, $x_1(t) = a_1 \sin \omega t$

and, $x_2(t) = a_2 \sin (\omega t + \phi)$

represent the displacement produced by each of the disturbances. Here we are assuming the displacements to be in the same direction. Now according to superposition principle, the resultant displacement will be given by,

$$x(t) = x_1(t) + x_2(t)$$
$$= a_1 \sin \omega t + a_2 \sin (\omega t + \phi)$$
$$= A \sin (\omega t + \phi_0)$$

where $A^2 = a_1^2 + a_2^2 + 2a_1 . a_2 \cos \phi$ ...(i)

and $\tan \phi_0 = \dfrac{a_2 \sin\phi}{a_1 + a_2 \cos\phi}$ ...(ii)

Illustration 1:

S_1 and S_2 are two sources of light which produce individually disturbance at point P given by $E_1 = 3 \sin \omega t$, $E_2 = 4 \cos \omega t$. Assuming $\vec{E}_1$ and $\vec{E}_2$ to be along the same line, find the resultant after their superposition.

Solution:

$$E = 3 \sin \omega t + 4 \sin(\omega t + \frac{\pi}{2})$$

$$A^2 = 3^2 + 4^2 + 2(3)(4) \cos \frac{\pi}{2} = 5^2$$

$$\tan\phi_0 = \frac{4\sin\frac{\pi}{2}}{3+4\cos\frac{\pi}{2}} = \frac{4}{3} \Rightarrow \phi_0 = 53°$$

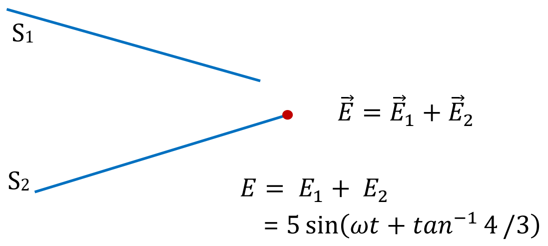
$$E = 5 \sin[\omega t + 53°]$$

Superposition of progressive waves with path difference:

Let S_1 and S_2 be two sources producing progressive waves (disturbance travelling in space given by y_1 and y_2)

At point P,

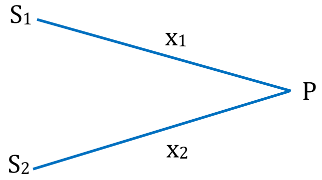
$$y_1 = a_1 \sin (\omega t - kx_1 + \theta_1)$$
$$y_2 = a_2 \sin (\omega t - kx_2 + \theta_2)$$
$$y = y_1 + y_2 = A \sin (\omega t + \Delta\phi)$$



Here, the phase difference,

$$\Delta\phi = (\omega t - kx_1 + \theta_1) - (\omega t - kx_2 + \theta_2)$$
$$= k (x_2 - x_1) + (\theta_1 - \theta_2) = k\Delta p - \Delta\theta$$

where $\Delta\theta = \theta_2 - \theta_1$



Here $\Delta p = \Delta x$ is the path difference

Clearly, phase difference due to path difference = k (path difference)

where $k = \frac{2\pi}{\lambda}$ $\Rightarrow$ $\Delta\phi = k\Delta p = \frac{2\pi}{\lambda}\Delta x$

For Constructive Interference:

$\Delta\phi = 2n\pi$, $n = 0, 1, 2 ...$ or $\Delta x = n\lambda$

$A_{max} = A_1 + A_2$

Intensity, $\sqrt{I_{max}} = \sqrt{I_1} + \sqrt{I_2}$ $\Rightarrow$ $I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$ ...(i)

For Destructive interference:

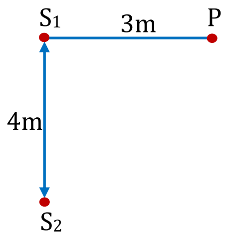
$\Delta\phi = (2n + 1)\pi$, $n = 0, 1, 2 ....$

or, $\Delta x = (2n + 1)\lambda/2$

$A_{min} = |A_1 - A_2|$

Intensity, $\sqrt{I_{\min}} = \left|\sqrt{I_1} - \sqrt{I_2}\right|$ $\Rightarrow$ $I_{min.} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$ ...(ii)

Illustration 2:



S_1 and S_2 are two coherent sources of frequency 'f' each.

[Given: $\theta_1 = \theta_2 = 0°$ and $V_{sound} = 330m/s.$]

(i) So that constructive interference at 'p'

(ii) So that destructive interference at 'p'

Solution:

For constructive interference

$k\Delta x = 2n\pi$

$\frac{2\pi}{\lambda} \times 2 = 2n\pi$

$\lambda = \frac{2}{n}$; $V = \lambda f$ $\Rightarrow$ $V = \frac{2}{n}f$

$f = \frac{330}{2} \times n$

For destructive interference

$k\Delta x = (2n + 1)\pi$

$\frac{2\pi}{\lambda}.2 = (2n + 1)\pi$

$\frac{1}{\lambda} = \frac{(2n+1)}{4}$; $f = \frac{V}{\lambda} = \frac{330 \times (2n+1)}{4}$

Illustration 3:

If two waves represented by $y_1 = 4\sin\omega t$ and $y_2 = 3\sin\left(\omega t + \frac{\pi}{3}\right)$ interfere at a point, the amplitude of the resulting wave will be about

(A) 7 (B) 6 (C) 5 (D) 3

Ans. (B)

Solution:

By using $A = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\phi} \Rightarrow A = \sqrt{(4)^2 + (3)^2 + 2\times4\times3\cos\frac{\pi}{3}} = \sqrt{37} \approx 6$.

Illustration 4:

Two interfering wave (having intensities are $9I$ and $4I$) path difference between them is 11 λ. The resultant intensity at this point will be

(A) I (B) $9\,I$ (C) $4\,I$ (D) $25\,I$

Ans. (D)

Solution:

Path difference $\Delta x = \frac{\lambda}{2\pi}\times\phi \Rightarrow \frac{2\pi}{\lambda}\times 11\lambda = 22\pi$ *i.e.* constructive interference obtained at the same point.

So, resultant intensity $I_R = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{9I} + \sqrt{4I})^2 = 25I$.

Illustration 5:

In interference if $\frac{I_{max}}{I_{min}} = \frac{144}{81}$ then what will be the ratio of amplitudes of the interfering wave

(A) $\frac{144}{81}$ (B) $\frac{7}{1}$ (C) $\frac{1}{7}$ (D) $\frac{12}{9}$

Ans. (B)

Solution:

By using $\frac{a_1}{a_2} = \left(\frac{\sqrt{\frac{I_{\max}}{I_{\min}}}+1}{\sqrt{\frac{I_{\max}}{I_{\min}}}-1}\right) = \left(\frac{\sqrt{\frac{144}{81}}+1}{\sqrt{\frac{144}{81}}-1}\right) = \left(\frac{\frac{12}{9}+1}{\frac{12}{5}-1}\right) = \frac{7}{1}$

Illustration 6:

Two interfering waves having intensities x and y meets a point with time difference $3T/2$. What will be the resultant intensity at that point ?

(A) $(\sqrt{x} + \sqrt{y})$ (B) $(\sqrt{x} + \sqrt{y} + \sqrt{xy})$ (C) $x + y + 2\sqrt{xy}$ (D) $\frac{x+y}{2xy}$

Ans. (C)

Solution:

Time difference $T.D. = \frac{T}{2\pi}\times\phi \Rightarrow \frac{3T}{2} = \frac{T}{2\pi}\times\phi \Rightarrow \phi = 3\pi$. This is the condition of constructive interference.

So resultant intensity, $I_R = (\sqrt{I_1} + \sqrt{I_2})^2 = (\sqrt{x} + \sqrt{y})^2 = x + y + 2\sqrt{xy}$.

Illustration 7:

The maximum intensity in case of interference of n identical waves, each of intensity I_0, if the interference is (i) coherent and (ii) incoherent respectively are

(A) $n^2 I_0, nI_0$ (B) $nI_0, n^2 I_0$

(C) nI_0, I_0 (D) $n^2 I_0, (n-1)I_0$

Ans. (A)

Solution:

In case of interference of two wave $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

(i) In case of coherent interference ϕ does not vary with time and so I will be maximum when $\cos\phi = \max = 1$

i.e. $(I_{\max})_{co} = I_1 + I_2 + 2\sqrt{I_1 I_2} = (\sqrt{I_1} + \sqrt{I_2})^2$

So, for n identical waves each of intensity I_0, $(I_{max})_{co} = (\sqrt{I_0} + \sqrt{I_0} + \ldots)^2 = (n\sqrt{I_0})^2 = n^2 I_0$

(ii) In case of incoherent interference at a given point, ϕ varies randomly with time, so $(\cos\phi)_{av} = 0$ and hence $(I_R)_{Inco} = I_1 + I_2$

So, in case of n identical waves $(I_R)_{Inco} = I_0 + I_0 + \ldots\ldots = nI_0$

Young's Double Slit Experiment (Y.D.S.E.)

In 1802 Thomas Young devised a method to produce a stationary interference pattern. This was based upon division of a single wavefront into two these two wavefronts acted as if they emanated from two sources having a fixed phase relationship. Hence when they were allowed to interfere, stationary interference pattern was observed.

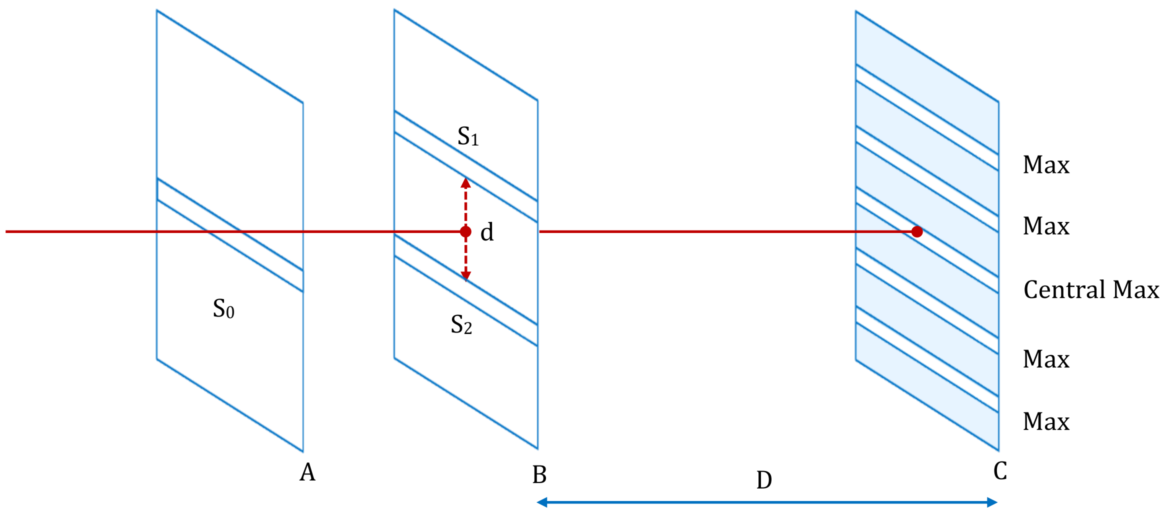


Figure: Young's arrangement to produce stationary interference pattern by division of wave front S_0 into S_1 and S_2

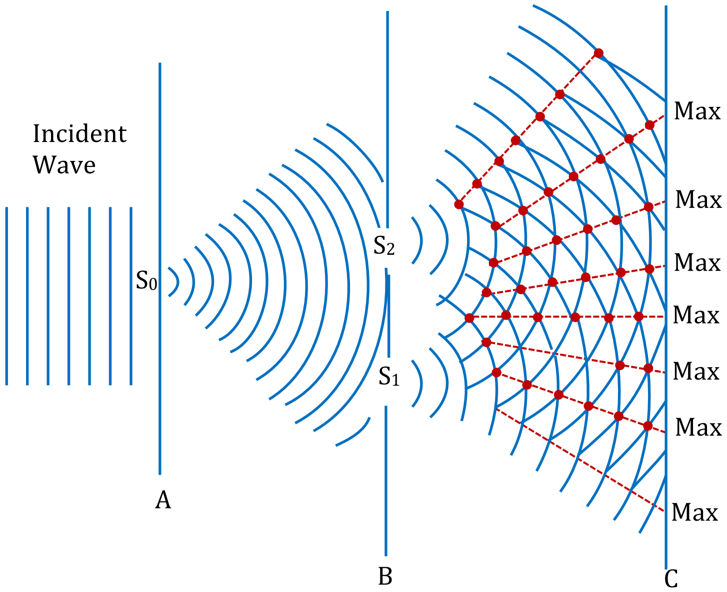


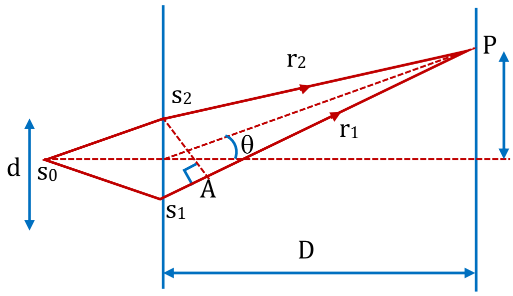
Figure: In Young's interference experiment, light diffracted from pinhole S_0 encounters pinholes S_1 and S_2 in screen *B*. Light diffracted from these two pinholes overlaps in the region between screen *B* and viewing screen C_1 producing an interference pattern on screen *C*.

Analysis of Interference Pattern:

We have insured in the above arrangement that the light wave passing through S_1 is in phase with that passing through S_2. However, the wave reaching *P* from S_2 may not be in phase with the wave reaching *P* from S_1, because the latter must travel a longer path to reach *P* than the former. We have already discussed the phase-difference arising due to path difference.

If the path difference is equal to zero or is an integral multiple of wavelengths, the arriving waves are exactly in phase and undergo constructive interference.

If the path difference is an odd multiple of half a wavelength, the arriving waves are out of phase and undergo fully destructive interference. Thus, it is the path difference Δx, which determines the intensity at a point *P*.



Path difference

$$\Delta x = S_1P - S_2P = \sqrt{\left(y+\frac{d}{2}\right)^2 + D^2} - \sqrt{\left(y-\frac{d}{2}\right)^2 + D^2}$$

Approximation I : For $D >> d$, we can approximate rays 1 and 2 as being approximately parallel, at angle θ to the principle axis.

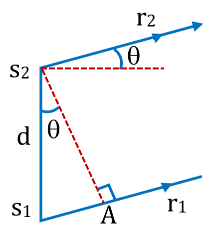
Now, $S_1P - S_2P = S_1A = S_1S_2 \sin\theta$

$\Rightarrow$ Path difference = $d \sin\theta$ ...(i)

Approximation II : Further if θ is small, i.e., $y << D$,

$$\sin\theta = \tan\theta = \frac{y}{D}$$

and hence, path difference = $\frac{dy}{D}$ ...(ii)



Figure

For maxima (constructive interference),

$$\Delta x = \frac{d.y}{D} = n\lambda$$

$\Rightarrow$ $$y = \frac{n\lambda D}{d}, \; n = 0, \pm 1, \pm 2, \pm 3 \qquad \text{...(iii)}$$

Here $n = 0$ corresponds to the central maxima

$n = \pm 1$ correspond to the 1*st* maxima

$n = \pm 2$ correspond to the 2*nd* maxima and so on.

For minima (destructive interference).

$$\Delta x = \pm\frac{\lambda}{2}, \pm\frac{3\lambda}{2}, \pm\frac{5\lambda}{2}$$

$\Rightarrow$ $$\Delta x = \begin{cases} (2n-1)\dfrac{\lambda}{2} & n = 1, 2, 3... \\ (2n+1)\dfrac{\lambda}{2} & n = -1, -2, -3... \end{cases}$$

Consequently, $$y = \begin{cases} (2n-1)\dfrac{\lambda D}{2d} & n = 1, 2, 3... \\ (2n+1)\dfrac{\lambda D}{2d} & n = -1, -2, -3... \end{cases} \qquad \text{...(iv)}$$

Here $n = \pm 1$ corresponds to first minima,

$n = \pm 2$ corresponds to second minima and so on.

Fringe width:

It is the distance between two maxima of successive order on one side of the central maxima. This is also equal to distance between two successive minima.

Fringe width $\beta = \dfrac{\lambda D}{d}$ ...(v)

Notice that it is directly proportional to wavelength and inversely proportional to the distance between the two slits.

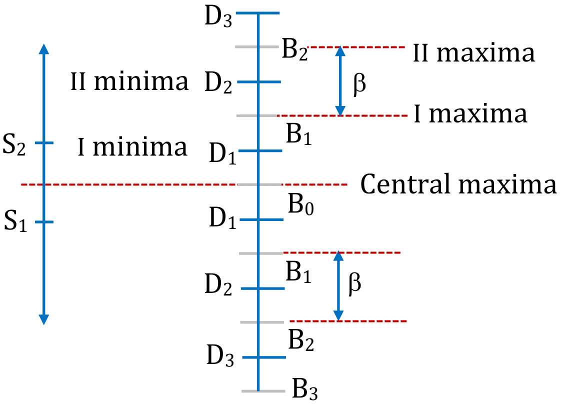


Figure: Fringe pattern in YDSE

Maximum order of Interference Fringes:

We obtained, $y = \dfrac{n\lambda D}{d}$, $n = 0, \pm 1, \pm 2$ .....

For interference maxima, n cannot take infinitely large values, as that would violate the approximation (II).

i.e., θ is small or $y << D$

$\Rightarrow$ $$\frac{y}{D} = \frac{n\lambda}{d} << 1$$

Hence the above formula (iii and iv) for interference maxima/minima are applicable when

$$n << \frac{d}{\lambda}$$

when n becomes comparable to $\dfrac{d}{\lambda}$ path difference can no longer be given by equation (i) but by (ii)

Hence for maxima

$$\Delta x = n\lambda \quad \Rightarrow \quad d \sin\theta = n\lambda \quad \Rightarrow \quad n = \frac{d\sin\theta}{\lambda}$$

Hence highest order of interference maxima,

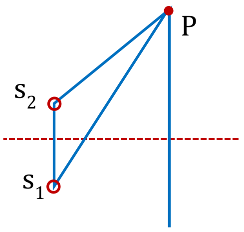
$$n_{max} = \left[\frac{d}{\lambda}\right]$$

where [ ] represents the greatest integer function.

Similarly, highest order of interference minima,

$$n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right]$$

Alter:



$$\Delta x = S_1P - S_2P$$

$$\Delta x \le d \quad \Rightarrow \quad \Delta x_{max} = d$$

(3rd side of a triangle is always greater than the difference in length of the other two sides)

Intensity:

Suppose the electric field components of the light waves arriving at point P (in the Figure) from the two slits S_1 and S_2 vary with time as

$$E_1 = E_0 \sin \omega t \text{ and } E_2 = E_0 \sin(\omega t + \phi)$$

Here $\phi = k\Delta x = \dfrac{2\pi}{\lambda}\Delta x$

and we have assumed that intensity of the two slits S_1 and S_2 are same (say I_0) hence waves have same amplitude E_0.

then the resultant electric field at point P is given by,

$$E = E_1 + E_2 = E_0 \sin \omega t + E_0 \sin(\omega t + \phi) = E_0' \sin(\omega t + \phi')$$

where $E_0'^2 = E_0^2 + E_0^2 + 2E_0 \,.\, E_0 \cos\phi = 4\,E_0^2 \cos^2 \phi/2$

Hence the resultant intensity at point P,

$$I = 4I_0 \cos^2 \frac{\phi}{2} \qquad \text{...(i)}$$

$$I_{max} = 4I_0 \text{ when } \frac{\phi}{2} = n\pi \quad ; \quad n = 0, \pm1, \pm2, \ldots$$

$$I_{min} = 0 \text{ when } \frac{\phi}{2} = \left(n - \frac{1}{2}\right)\pi \quad ; \quad n = 0, \pm1, \pm2, \ldots$$

Here $\phi = k\Delta x = \dfrac{2\pi}{\lambda}\Delta x$

If $D >> d$,

$$\phi = \frac{2\pi}{\lambda} d \sin\theta$$

If $\quad D >> d$ and $y << D$,

$$\phi = \frac{2\pi}{\lambda} d \frac{y}{D}$$

However, if the two slits were of different intensities I_1 and I_2,

say $\quad E_1 = E_{01} \sin \omega t$ and $E_2 = E_{02} \sin (\omega t + \phi)$

then resultant field at point P,

$$E = E_1 + E_2 = E_0 \sin (\omega t + \theta)$$

where $\quad E_0^2 = E_{01}^2 + E_{02}^2 + 2E_{01} E_{02} \cos \phi$

Hence resultant intensity at point P,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos \phi \qquad \text{...(ii)}$$

Illustration 8:

In a *YDSE*, $D = 1m$, $d = 1mm$ and $\lambda = 1/2\ mm$

(i) Find the distance between the first and central maxima on the screen.

(ii) Find the no of maxima and minima obtained on the screen.

Solution:

(i) $D >> d$

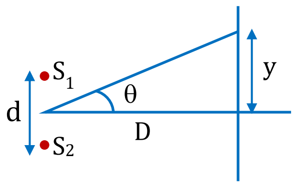
Hence $\quad \Delta p = d \sin \theta$

$$\frac{d}{\lambda} = 2$$

Clearly, $n << \frac{d}{\lambda} = 2$ is not possible for any value of n.

Hence $\quad \Delta x = \frac{dy}{D}$ cannot be used

For I[st] maxima,



$$\Delta x = d \sin \theta = \lambda$$

$$\Rightarrow \quad \sin \theta = \frac{\lambda}{d} = \frac{1}{2}$$

$$\Rightarrow \quad \theta = 30°$$

Hence, $y = D \tan \theta = \frac{1}{\sqrt{3}}$ meter

(ii) Maximum path difference $\Delta x_{max} = d = 1\ mm$

$\Rightarrow \quad$ Highest order maxima, $n_{max} = \left[\frac{d}{\lambda}\right] = 2$

and $\quad$ Highest order minima $n_{min} = \left[\frac{d}{\lambda} + \frac{1}{2}\right] = 2$

Total number of maxima $= 2n_{max} + 1^* = 5 \qquad$ *(central maxima)

Total number of minima $= 2n_{min} = 4$

Illustration 9:

Monochromatic light of wavelength 5000 $A°$ is used in *Y.D.S.E.*, with slit-width, d = 1*mm*, distance between screen and slits, D = 1*m*. If intensity at the two slits are, $I_1 = 4\,I_0$, $I_2 = I_0$, find

(i) Fringe width β

(ii) Distance of 5*th* minima from the central maxima on the screen

(iii) Intensity at $y = \frac{1}{3}$ *mm*

(iv) Distance of the 1000^{th} maxima from the central maxima on the screen.

(v) Distance of the 5000^{th} maxima from the central maxima on the screen.

Solution:

(i) $\beta = \frac{\lambda D}{d} = \frac{5000\times10^{-10}\times1}{1\times10^{-3}} = 0.5\ mm$

(ii) $y = (2n-1)\frac{\lambda D}{2d}$, $n = 5 \quad \Rightarrow \quad y = 2.25\ mm$

(iii) At $y = \frac{1}{3}$ *mm*, $y << D$

Hence $\Delta p = \frac{d.y}{D}$

$$\Delta\phi = \frac{2\pi}{\lambda}\Delta p = 2\pi\frac{dy}{\lambda D} = \frac{4\pi}{3}$$

Now resultant intensity

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\Delta\phi = 4I_0 + I_0 + 2\sqrt{4I_0^2}\cos\Delta\phi = 5I_0 + 4I_0\cos\frac{4\pi}{3} = 3I_0$$

(iv) $\frac{d}{\lambda} = \frac{10^{-3}}{0.5\times10^{-6}} = 2000$

$n = 1000$ is not $<< 2000$

Hence now $\Delta p = d\sin\theta$ must be used

Hence, $d\sin\theta = n\lambda = 1000\,\lambda \Rightarrow \sin\theta = 1000\,\frac{\lambda}{d} = \frac{1}{2} \Rightarrow \theta = 30°$

$y = D\tan\theta = \frac{1}{\sqrt{3}}$ meter

(v) Highest order maxima

$n_{max} = \left[\frac{d}{\lambda}\right] = 2000$

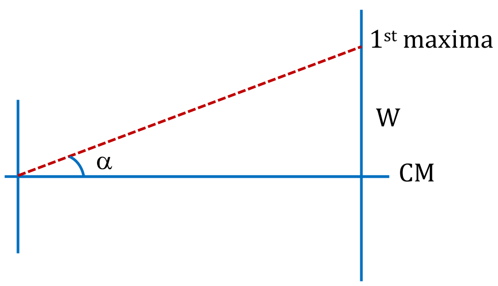
Hence, $n = 5000$ is not possible.

Angular Fringe Width (α):

$$\tan\alpha = \frac{w}{D} = \frac{\lambda D}{dD} = \boxed{\frac{\lambda}{d}}$$

If α small then

$$\boxed{\tan\alpha \approx \frac{\lambda}{d}} \qquad \{\because \alpha = \text{Angular fringe width}\}$$



Angular Fringe Width $\alpha = \frac{\beta}{D}, \alpha = \frac{\lambda}{d} \quad \left[\because \frac{\beta}{D} = \frac{\lambda}{d}\right]$

Illustration 10:

In Young's double slit experiment using sodium light (λ = 5898Å), 92 fringes are seen. If given colour (λ = 5461Å) is used, how many fringes will be seen

(A) 62 (B) 67 (C) 85 (D) 99

Ans. (D)

Solution:

By using $n_1\lambda_1 = n_2\lambda_2$

$\Rightarrow \quad 92 \times 5898 = n_2 \times 5461$

$\Rightarrow \quad n_2 = 99$

Illustration 11:

Two waves being produced by two sources S_1 and S_2. Both sources have zero phase difference and have wavelength λ. The destructive interference of both the waves will occur of point P if $(S_1P - S_2P)$ has the value.

(A) 5λ (B) $\frac{3}{4}\lambda$ (C) 2λ (D) $\frac{11}{2}\lambda$

Ans. (D)

Solution:

For destructive interference, path difference the waves meeting at P (i.e. $S_1P - S_2P$) must be odd multiple of $\lambda/2$. Hence option (d) is correct.

Illustration 12:

In Young's double-slit experiment, an interference pattern is obtained on a screen by a light of wavelength 6000 Å, coming from the coherent sources S_1 and S_2. At certain point P on the screen third dark fringe is formed. Then the path difference $S_1P - S_2P$ in microns is

(A) 0.75 (B) 1.5 (C) 3.0 (D) 4.5

Ans. (B)

Solution:

For dark fringe path difference $\Delta x = (2n-1)\frac{\lambda}{2}$; here n = 3 and λ = 6000 × 10^{-10} m

So $\Delta x = (2\times3-1)\times\frac{6\times10^{-7}}{2} = 15\times10^{-7} m = 1.5$ microns.

Illustration 13:

In a Young's double slit experiment, the slit separation is 1 mm and the screen is 1 m from the slit. For a monochromatic light of wavelength 500 nm, the distance of 3rd minima from the central maxima is

(A) 0.50 mm (B) 1.25 mm (C) 1.50 mm (D) 1.75 mm

Ans. (B)

Solution:

Distance of n^{th} minima from central maxima is given as $x = \frac{(2n-1)\lambda D}{2d}$

So here $x = \frac{(2\times3-1)\times500\times10^{-9}\times1}{2\times10^{-3}} = 1.25 mm$

Illustration 14:

The two slits at a distance of 1 mm are illuminated by the light of wavelength $6.5 \times 10^{-7} m$. The interference fringes are observed on a screen placed at a distance of 1 m. The distance between third dark fringe and fifth bright fringe will be

(A) 0.65 mm (B) 1.63 mm (C) 3.25 mm (D) 4.88 mm

Ans. (B)

Solution:

Distance between n^{th} bright and m^{th} dark fringe ($n > m$) is given as

$$x=\left(n-m+\frac{1}{2}\right)\beta=\left(n-m+\frac{1}{2}\right)\frac{\lambda D}{d} \quad \Rightarrow \quad x=\left(5-3+\frac{1}{2}\right)\times\frac{6.5\times10^{-7}\times1}{1\times10^{-3}}=1.63mm$$

Illustration 15:

Two identical sources emitted waves which produces intensity of k unit at a point on screen where path difference is λ. What will be intensity at a point on screen at which path difference is $\lambda/4$?

(A) $\frac{k}{4}$ (B) $\frac{k}{2}$ (C) k (D) Zero

Ans. (B)

Solution:

By using phase difference $\phi=\frac{2\pi}{\lambda}(\Delta x)$

For path difference λ, phase difference $\phi_1 = 2\pi$ and for path difference $\lambda/4$, phase difference $\phi_2 = \pi/2$.

Also, by using $I=4I_0\cos^2\frac{\phi}{2} \Rightarrow \frac{I_1}{I_2}=\frac{\cos^2(\phi_1/2)}{\cos^2(\phi_2/2)} \Rightarrow \frac{k}{I_2}=\frac{\cos^2(2\pi/2)}{\cos^2\left(\frac{\pi/2}{2}\right)}=\frac{1}{1/2} \Rightarrow I_2=\frac{k}{2}$

Illustration 16:

Two identical radiators have a separation of $d = \lambda/4$, where λ is the wavelength of the waves emitted by either source. The initial phase difference between the sources is $\pi/4$. Then the intensity on the screen at a distance point situated at an angle $\theta = 30°$ from the radiators is

(Here I_0 is the intensity at that point due to one radiator)

(A) I_0 (B) $2I_0$ (C) $3I_0$ (D) $4I_0$

Ans. (B)

Solution:

Initial phase difference $\phi_0=\frac{\pi}{4}$; Phase difference due to path difference $\phi'=\frac{2\pi}{\lambda}(\Delta x)$

where $\Delta x = d\sin\theta$

$$\Rightarrow \quad \phi'=\frac{2\pi}{\lambda}(d\sin\theta)=\frac{2\pi}{\lambda}\times\frac{\lambda}{4}(\sin30^o)=\frac{\pi}{4}$$

Hence total phase difference $\phi=\phi_0+\phi'=\frac{\pi}{2}$. By using $I=4I_0\cos^2(\phi/2)=4I_0\cos^2\left(\frac{\pi/2}{2}\right)=2I_0$.

YDSE with white light:

The central maxima will be white because all wavelengths will constructively interference here. However slightly below (or above) the position of central maxima fringes will be coloured. For example, if P is a point on the screen such that $S_2P - S_1P = \frac{\lambda_{\text{violet}}}{2} = 190\ nm,$

completely destructive interference will occur for violet light. Hence, we will have a line devoid of violet colour that will appear reddish.

And if $S_2P - S_1P = \frac{\lambda_{\text{red}}}{2} = 350\ nm,$

completely destructive interference for red light results and the line at this position will be violet. The coloured fringes disappear at points far away from the central white fringe for these points there are so many wavelengths which interfere constructively, that we obtain a uniform white illumination. For example, if

$$S_2P - S_1\,P = 3000\ nm,$$

then constructive interference will occur for wavelengths $\lambda = \frac{3000}{n}\ nm$. In the visible region these wavelengths are 750 *nm* (red), 600 *nm* (yellow), 500 *nm* (greenish–yellow), 428.6 *nm* (violet). Clearly such a light will appear white to the unaided eye.

Thus, with white light we get a white central fringe at the point of zero path difference, followed by a few coloured fringes on its both sides, the color soon fading off to a uniform white.

In the usual interference pattern with a monochromatic source, a large number of identical interference fringes are obtained and it is usually not possible to determine the position of central maxima. Interference with white light is used to determine the position of central maxima in such cases.

Illustration 17:

A beam of light consisting of wavelengths 6000Å and 4500Å is used in a *YDSE* with $D = 1m$ and $d = 1\ mm$. Find the least distance from the central maxima, where bright fringes due to the two wavelengths coincide.

Solution:

$$\beta_1 = \frac{\lambda_1 D}{d} = \frac{6000 \times 10^{-10} \times 1}{10^{-3}} = 0.6\ mm;$$

$$\beta_2 = \frac{\lambda_2 D}{d} = 0.45\ mm$$

Let n_1th maxima of λ_1 and n_2th maxima of λ_2 coincide at a position y.

then, $y = n_1\beta_1 = n_2\beta_2 = LCM$ of β_1 and β_2

$\Rightarrow$ $y = LCM$ of 0.6 *mm* and 0.45 *mm*

$y = 1.8\ mm$

At this point 3*rd* maxima for 6000 Å and 4*th* maxima for 4500 Å coincide

Illustration 18:

White light is used in a *YDSE* with $D = 1m$ and $d = 0.9$ *mm*. Light reaching the screen at position $y = 1$ *mm* is passed through a prism and its spectrum is obtained. Find the missing lines in the visible region of this spectrum.

Solution:

$\Delta x = \dfrac{yd}{D} = 9 \times 10^{-4} \times 1 \times 10^{-3}\ m = 900\ nm$

For minima $\Delta x = (2n - 1)\lambda/2$

$$\Rightarrow \qquad \lambda = \frac{2\Delta x}{(2n-1)} = \frac{1800}{(2n-1)} = \frac{1800}{1}, \frac{1800}{3}, \frac{1800}{5}, \frac{1800}{7} \ \ldots\ldots$$

of these 600 *nm* and 360 *nm* lie in the visible range. Hence these will be missing lines in the visible spectrum.

Illustration 19:

In double slit experiment, the angular width of the fringes is 0.20° for the sodium light (λ = 5890 Å). In order to increase the angular width of the fringes by 10%, the necessary change in the wavelength is

(A) Increase of 589 Å (B) Decrease of 589 Å

(C) Increase of 6479 Å (D) Zero

Ans. (A)

Solution:

By using $\theta = \dfrac{\lambda}{d} \Rightarrow \dfrac{\theta_1}{\theta_2} = \dfrac{\lambda_1}{\lambda_2} \Rightarrow \dfrac{0.20^o}{(0.20^o + 10\% \text{of } 0.20)} = \dfrac{5890}{\lambda_2} \Rightarrow \dfrac{0.20}{0.22} = \dfrac{5890}{\lambda_2} \Rightarrow \lambda_2 = 6479$

So, increase in wavelength = 6479 – 5890 = 589 Å.

Illustration 20:

In Young's experiment, light of wavelength 4000 Å is used, and fringes are formed at 2 metre distance and has a fringe width of 0.6 *mm*. If whole of the experiment is performed in a liquid of refractive index 1.5, then width of fringe will be

(A) 0.2 *mm* (B) 0.3 *mm* (C) 0.4 *mm* (D) 1.2 *mm*

Ans. (C)

Solution:

$$\beta_{\text{medium}} = \frac{\beta_{\text{air}}}{\mu}$$

$$\Rightarrow \qquad \beta_{\text{medium}} = \frac{0.6}{1.5} = 0.4mm$$

Illustration 21:

In *YDSE*, distance between the slits is 2×10^{-3} *m*, slits are illuminated by a light of wavelength 2000 Å –9000 Å. In the field of view at a distance of 10^{-3} *m* from the central position which wavelength will be observe. Given distance between slits and screen is 2.5 *m*.

(A) 4000 Å (B) 4500 Å (C) 5000 Å (D) 5500 Å

Ans. (A)

Solution:

$$y=\frac{n\lambda D}{d}\Rightarrow \lambda=\frac{yd}{nD}=\frac{10^{-3}\times 2\times 10^{-3}}{n\times 2.5}\Rightarrow \frac{8\times 10^{-7}}{n}m=\frac{8000}{n}\mathring{A}$$

For n = 1, 2, 3....... λ = 8000 Å, 4000 Å, $\frac{8000}{3}\mathring{A}$.........

Hence only option (A) is correct.

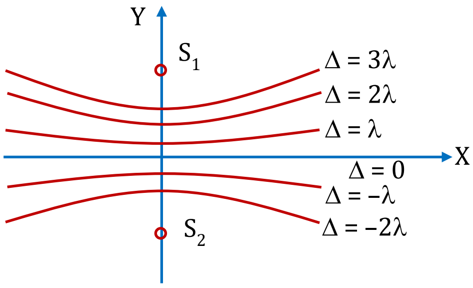
Shape of Interference fringes in YDSE:

We discuss the shape of fringes when two pinholes are used instead of the two slits in YDSE.

Fringes are locus of points which move in such a way that its path difference from the two slits remains constant.

$$S_2P - S_1P = \Delta = \text{constant}$$

Above equation represents a hyperbola with its two foci at S_1 and S_2



If $\Delta x = \pm\frac{\lambda}{2}$, the fringe represents 1st minima.

If $\Delta x = \pm\frac{3\lambda}{2}$ it represents 2nd minima.

If $\Delta x = 0$ it represents central maxima.

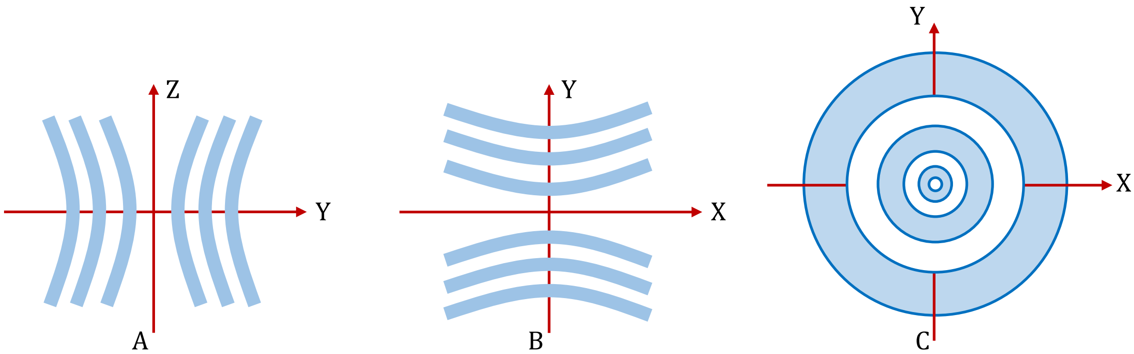
If $\Delta x = \pm\lambda$, it represents 1st maxima etc.

The interference pattern which we get on screen is the section of hyperboloid of revolution when we revolve the hyperbola about the axis S_1S_2.

A. If the screen is perpendicular to the X axis, i.e., in the YZ plane, as is generally the case, fringes are hyperbolic with a straight central section.

B. If the screen is in the XY plane, again fringes are hyperbolic.

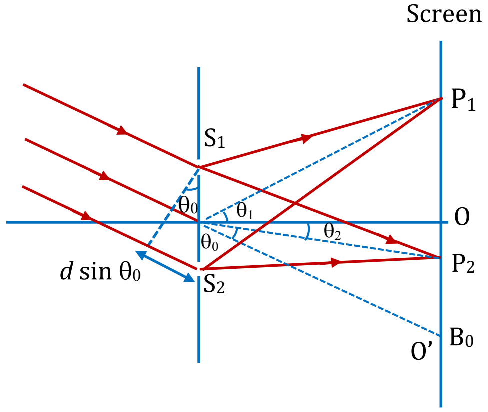
C. If screen is perpendicular to Y axis (along S_1S_2), i.e., in the XZ plane, fringes are concentric circles with center on the axis S_1S_2; the central fringe is bright if $S_1S_2 = n\lambda$ and dark if $S_1S_2 = (2n-1)\frac{\lambda}{2}$.



YDSE with Oblique incidence:

In *YDSE*, ray is incident on the slit at an inclination of θ_0 to the axis of symmetry of the experimental set-up for points above the central point on the screen, (say for P_1)

$$\Delta x = d\sin\theta_0 + (S_2P_1 - S_1P_1)$$



$\Rightarrow \quad \Delta x = d \sin \theta_0 + d \sin \theta_1$ (If $d << D$)

and for points below O on the screen, (say for P_2)

$\Delta x = |d \sin \theta_0 + S_2P_2) - S_1P_2| = |d \sin \theta_0 - (S_1P_2 - S_2P_2)|$

$\Rightarrow \quad \Delta x = |d \sin \theta_0 - d \sin \theta_2|$ (if $d << D$)

We obtain central maxima at a point where, $\Delta p = 0$.

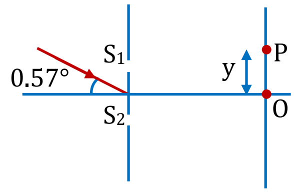
$(d \sin \theta_0 - d \sin \theta_2) = 0$ or $\theta_2 = \theta_0$

This corresponds to the point O' in the diagram.

Hence, we have finally for path difference.

$$\Delta x = \begin{cases} d(\sin\theta_0 + \sin\theta) - \text{for points above O} \\ d(\sin\theta_0 - \sin\theta) - \text{for points between O and O'} \\ d(\sin\theta - \sin\theta_0) - \text{for points below O'} \end{cases}$$

Illustration 22:



In $YDSE$ with $D = 1m$, $d = 1mm$, light of wavelength 500 nm is incident at an angle of 0.57° w.r.t. the axis of symmetry of the experimental set up. If centre of symmetry of screen is O as shown.

(i) Find the position of central maxima

(ii) Intensity at point O in terms of intensity of central maxima I_0.

(iii) Number of maxima lying between O and the central maxima.

Solution:

(i) $\theta = \theta_0 = 0.57°$

$\Rightarrow \quad y = -D \tan \theta$

$= -D\theta = -1 \text{ meter} \times \left(\frac{0.57}{57} rad\right)$

$\Rightarrow \quad y = -1cm$

(ii) For point O, $\theta = 0$

Hence, $\Delta x = d \sin \theta_0$; $d\theta_0 = 1\ mm \times (10^{-2} \text{ rad}) = 10{,}000\ nm = 20 \times (500\ nm)$

$\Rightarrow \quad \Delta x = 20\ \lambda$

Hence point O corresponds to 20th maxima

$\Rightarrow$ Intensity at $O = I_0$

(iii) 19 maxima lie between central maxima and O, excluding maxima at O and central maxima.

Illustration 23:

The slits in a Young's double slit experiment have equal widths and the source is placed symmetrically relative to the slits. The intensity at the central fringes is I_0. If one of the slits is closed, the intensity at this point will be

(A) I_0 (B) $I_0/4$ (C) $I_0/2$ (D) $4I_0$

Ans. (B)

Solution:

By using $I_R = 4I\cos^2\frac{\phi}{2}$ {where I = Intensity of each wave}

At central position $\phi = 0^o$, hence initially $I_0 = 4I$.

If one slit is closed, no interference takes place so intensity at the same location will be I only *i.e.* intensity become $\frac{I_0}{4}$.

Illustration 24:

In *YDSE* a source of wavelength 6000 Å is used. The screen is placed 1 *m* from the slits. Fringes formed on the screen, are observed by a student sitting close to the slits. The student's eye can distinguish two neighbouring fringes. If they subtend an angle more than 1 minute of arc. What will be the maximum distance between the slits so that the fringes are clearly visible

(A) 2.06 *mm* (B) 2.06 *cm* (C) 2.06×10^{-3} *mm* (D) None of these

Ans. (A)

Solution:

According to given problem angular fringe width $\theta = \frac{\lambda}{d} \geq \frac{\pi}{180 \times 60}$ [As $1' = \frac{\pi}{180 \times 60} rad$]

i.e. $d < \frac{6 \times 10^{-7} \times 180 \times 60}{\pi}$ *i.e.* $d < 2.06 \times 10^{-3} m \Rightarrow d_{max} = 2.06 mm$

Illustration 25:

The width of one of the two slits in a Young's double slit experiment is double of the other slit. Assuming that the amplitude of the light coming from a slit is proportional to the slit width. The ratio of the maximum to the minimum intensity in interference pattern will be

(A) $\frac{1}{9}$ (B) $\frac{9}{1}$ (C) $\frac{2}{1}$ (D) $\frac{1}{2}$

Ans. (B)

Solution:

$A_{max} = 2A + A = 3A$ and $A_{min} = 2A - A = A$.

Also $\frac{I_{\max}}{I_{\min}} = \left(\frac{A_{\max}}{A_{\min}}\right)^2 = \left(\frac{3A}{A}\right)^2 = \frac{9}{1}$

Geometrical path & Optical path:

Actual distance travelled by light in a medium is called geometrical path (Δx). Consider a light wave given by the equation

$$E = E_0 \sin(\omega t - kx + \phi)$$

If the light travels by Δx, its phase changes by $k\Delta x = \frac{\omega}{v} \Delta x$, where ω, the frequency of light does not depend on the medium, but v, the speed of light depends on the medium as $v = \frac{c}{\mu}$.

Consequently, change in phase $\Delta\phi = k\Delta x = \frac{\omega}{c} (\mu\Delta x)$.

It is clear that a wave travelling a distance Δx in a medium of refractive index μ suffers the same phase change as when it travels a distance $\mu\Delta x$ in vacuum. i.e., a path length of Δx in medium of refractive index μ is equivalent to a path length of $\mu\Delta x$ in vacuum.

The quantity $\mu\Delta x$ is called the optical path length of light, Δx_{opt}. And in terms of optical path length, phase difference would be given by,

$$\Delta\phi = \frac{\omega}{c}\ \Delta x_{opt} = \frac{2\pi}{\lambda_0}\Delta x_{opt} \qquad \text{...(i)}$$

where λ_0 = wavelength of light in vacuum. However, in terms of the geometrical path length Δx,

$$\Delta\phi = \frac{\omega}{c}\ (\mu\Delta x) = \frac{2\pi}{\lambda}\Delta x \qquad \text{...(ii)}$$

where λ = wavelength of light in the medium ($\lambda = \frac{\lambda_0}{\mu}$).

Displacement of fringe:

On introduction of the thin glass-slab of thickness t and refractive index μ, the optical path of the ray S_1P increases by $t(\mu - 1)$. Now the path difference between waves coming form S_1 and S_2 at any point P is

$$\Delta x = S_2P - (S_1P + t(\mu - 1)) = (S_2P - S_1P) - t(\mu - 1)$$

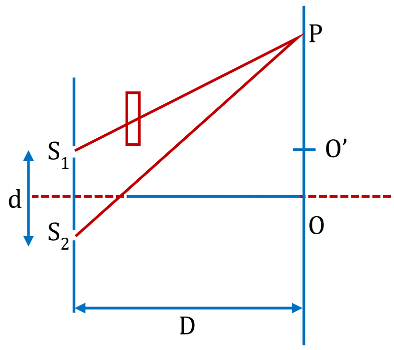
$\Rightarrow \quad \Delta x = d\sin\theta - t(\mu - 1)\quad$ if $d << D$

and $\quad \Delta x = \frac{yd}{D} - t(\mu - 1)$

If $\quad y << D$ as well

For central bright fringe, $\Delta x = 0 \;\Rightarrow\; \frac{yd}{D} = t(\mu - 1)$.

$\Rightarrow \quad y = OO' = (\mu - 1)\,t\,\frac{D}{d} = (\mu - 1)\,t.\,\frac{\beta}{\lambda}$



Figure

The whole fringe pattern gets shifted by the same distance

$$\Delta x = (\mu - 1).\frac{D}{d} = (\mu - 1)\,t\,\frac{\beta}{\lambda}$$

Note:

This shift is in the direction of the slit before which the glass slab is placed. If the glass slab is placed before the upper slit, the fringe pattern gets shifted upwards and if the glass slab is placed before the lower slit the fringe pattern gets shifted downwards.

Illustration 26:

In a *YDSE* with $d = 1mm$ and $D = 1m$, slabs of ($t = 1\mu m$, $\mu = 3$) and ($t = 0.5\ \mu m$, $\mu = 2$) are introduced in front of upper and lower slit respectively. Find the shift in the fringe pattern.

Solution:

Optical path for light coming from upper slit S_1 is

$$S_1P + 1\mu m\,(3 - 1) = S_1P + 2\ \mu m$$

Similarly, optical path for light coming from S_2 is

$S_2P + 0.5\ \mu m\ (2 - 1) = S_2P + 0.5\ \mu m$

Path difference: $\Delta x = (S_2P + 0.5\ \mu m) - (S_1P + 2\mu m) = (S_2P - S_1P) - 1.5\ \mu m. = \dfrac{yd}{D} - 1.5\ \mu m$

For central bright fringe $\Delta x = 0$

$\Rightarrow \quad y = \dfrac{1.5\mu m}{1mm} \times 1m = 1.5\ mm.$

The whole pattern is shifted by 1.5 *mm* upwards.

Illustration 27:

A thin mica sheet of thickness $2 \times 10^{-6}\ m$ and refractive index (μ = 1.5) is introduced in the path of the first wave. The wavelength of the wave used is 5000Å. The central bright maximum will shift

(A) 2 fringes upward (B) 2 fringes downward

(C) 10 fringes upward (D) None of these

Ans. (A)

Solution:

By using shift $\Delta x = \dfrac{\beta}{\lambda}(\mu - 1)t \Rightarrow \Delta x = \dfrac{\beta}{5000 \times 10^{-10}}(1.5 - 1) \times 2 \times 10^{-6} = 2\beta$

Since the sheet is placed in the path of the first wave, so shift will be 2 fringes upward.

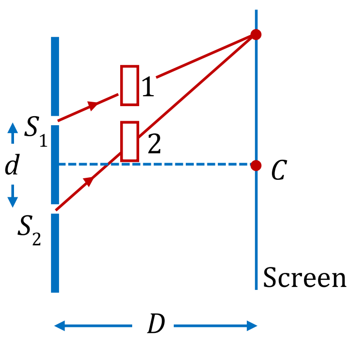
Illustration 28:

In a *YDSE* fringes are observed by using light of wavelength 4800 Å, if a glass plate (μ = 1.5) is introduced in the path of one of the wave and another plates is introduced in the path of the (μ = 1.8) other wave. The central fringe takes the position of fifth bright fringe. The thickness of plate will be

(A) 8 micron (B) 80 micron (C) 0.8 micron (D) None of these

Ans. (A)

Solution:



Shift due to the first plate $x_1 = \dfrac{\beta}{\lambda}(\mu_1 - 1)t$ (Upward)

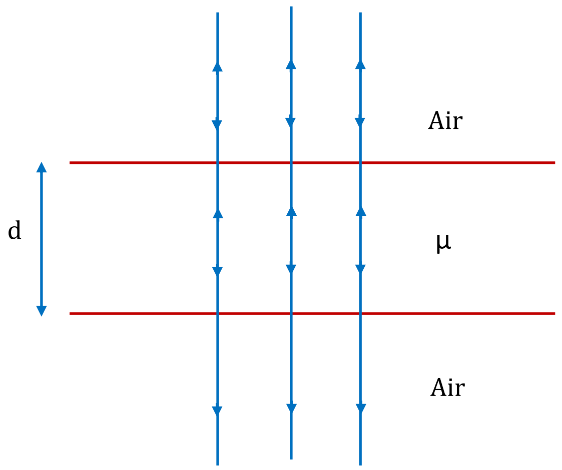
And shift due to the second $x_2 = \dfrac{\beta}{\lambda}(\mu_2 - 1)t$ (Downward)

Hence net shift = $x_2 - x_1 = \dfrac{\beta}{\lambda}(\mu_2 - \mu_1)t$

$\Rightarrow \quad 5\beta = \dfrac{\beta}{\lambda}(1.8 - 1.5)t$

$\Rightarrow \quad t = \dfrac{5\lambda}{0.3} = \dfrac{5 \times 4800 \times 10^{-10}}{0.3} = 8 \times 10^{-6}\ m = 8\ micron$

Thin-Film interference:



In *YDSE* we obtained two coherent source from a single (incoherent) source by division of wave-front. Here we do the same by division of Amplitude (into reflected and refracted wave).

When a plane wave (parallel rays) is incident normally on a thin film of uniform thickness d then waves reflected from the upper surface interfere with waves reflected from the lower surface.

Clearly the wave reflected from the lower surface travel an extra optical path of $2\mu d$, where μ is refractive index of the film.

Further if the film is placed in air the wave reflected from the upper surface (from a denser medium) suffers a sudden phase change of π, while the wave reflected from the lower surface (from a rarer medium) suffers no such phase change.

Consequently, condition for constructive and destructive interference in the reflected light is given by,

$$2\mu d = n\lambda \text{ for destructive interference}$$

and $$2\mu d = \left(n + \frac{1}{2}\right)\lambda \text{ for constructive interference} \qquad \text{...(i)}$$

where $n = 0, 1, 2$ .....,

and λ = wavelength in free space.

Interference will also occur in the transmitted light and here condition of constructive and destructive interference will be the reverse of (i)

i.e. $$2\mu d = \begin{cases} n\lambda & \text{for constructive interference} \\ \left(n+\frac{1}{2}\right)\lambda & \text{for destructive interference} \end{cases} \qquad \text{...(ii)}$$

This can easily be explained by energy conservation (when intensity is maximum in reflected light it has to be minimum in transmitted light) However the amplitude of the directly transmitted wave and the wave transmitted after one reflection differ substantially and hence the fringe contrast in transmitted light is poor. It is for this reason that thin film interference is generally viewed only in the reflected light.

In deriving equation (i) we assumed that the medium surrounding the thin film on both sides is rarer compared to the medium of thin film.

If medium on both sides are denser, then there is no sudden phase change in the wave reflected from the upper surface, but there is a sudden phase change of π in waves reflected from the lower surface. The conditions for constructive and destructive interference in reflected light would still be given by equation (i).

However, if medium on one side of the film in denser and that on the other side is rarer, then either there is no sudden phase in any reflection, or there is a sudden phase change of π in both reflection from upper and lower surface. Now the condition for constructive and destructive interference in the reflected light would be given by equation (ii) and not equation (i).

Illustration 29:

White light, with a uniform intensity across the visible wavelength range 430 – 690 *nm*, is perpendicularly incident on a water film, of index of refraction μ = 1.33 and thickness d = 320 *nm*, that is suspended in air. At what wavelength λ is the light reflected by the film brightest to an observer?

Solution:

Solving for λ and inserting the given data, we obtain

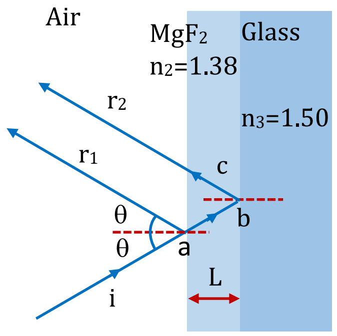
$$\lambda = \frac{2\mu d}{m+1/2} = \frac{(2)(1.33)(320\,nm)}{m+1/2} = \frac{851\,nm}{m+1/2}$$

For m = 0, this give us λ = 1700 *nm*, which is in the infrared region. For m = 1, we find λ = 567 *nm,* which is yellow-green light, near the middle of the visible spectrum. For m = 2, λ = 340 *nm*, which is in the ultraviolet region. So, the wavelength at which the light seen by the observer is brightest is λ = 567 *nm.*

Illustration 30:

A glass lens is coated on one side with a thin film of magnesium fluoride (MgF_2) to reduce reflection from the lens surface (figure). The index of refraction of MgF_2 is 1.38 and that of the glass is 1.50. What is the least coating thickness that eliminates (via interference) the reflections at the middle of the visible spectrum (λ = 550 *nm*)? Assume the light is approximately perpendicular to the lens surface.

Solution:



The situation here differs as $n_3 > n_2 > n_1$. The reflection at point a still introduces a phase difference of π but now the reflection at point b also does the same. Unwanted reflections from glass can be, suppressed (at a chosen wavelength) by coating the glass with a thin transparent film of magnesium fluoride of a properly chosen thickness which introduces a phase change of half a wavelength. For this, the path length difference $2L$ within the film must be equal to an odd number of half wavelengths:

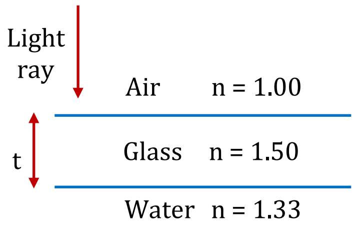
$$2L = (m + 1/2)\lambda_{n2} \quad \text{or} \quad \text{with } \lambda_{n2} = \lambda/n_2,\ \ 2n_2 L = (m + 1/2)\lambda.$$

We want the least thickness for the coating, that is, the smallest L. Thus, we choose m = 0, the smallest value of m. Solving for L and inserting the given data, we obtain

$$L = \frac{\lambda}{4n_2} = \frac{550\,nm}{(4)(1.38)} = 96.6\ nm$$

Illustration 31:

Thickness of the glass so that the reflected light gives destructive interference (λ_{air} = 600 *nm*) ?



(A) 50 *nm* (B) 100 *nm* (C) 150 *nm* (D) 200 *nm*

Ans. (D)

Solution:

For reflected light to have orangish color, rays from A, C, E must be out of phase for $\lambda = 600$

or $\quad \delta = (2n - 1)\dfrac{\lambda}{2}$

or $\quad 2\mu_g t - \dfrac{\lambda}{2} = (2n - 1)\dfrac{\lambda}{2}$

or $\quad t = \dfrac{n\lambda}{2\mu_g}$

or $\quad t_{min} = \dfrac{\lambda}{2\mu_g} = 200\ nm$

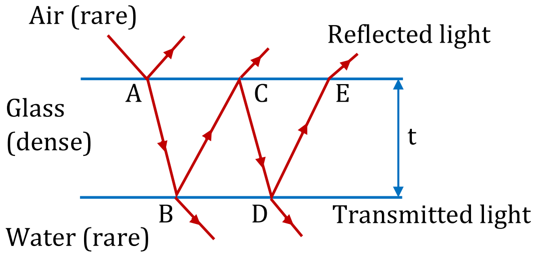


Illustration 32:

I is the intensity due to a source of light at any point P on the screen. If light reaches the point P via two different paths (a) direct (b) after reflection from a plane mirror then path difference between two paths is $3\lambda/2$, the intensity at P is

(A) I (B) Zero (C) $2I$ (D) $4I$

Ans. (D)

Solution:

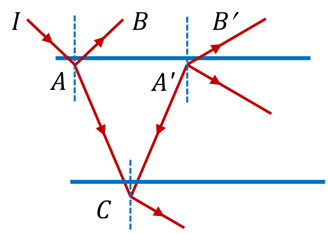
Reflection of light from plane mirror gives additional path difference of $\lambda/2$ between two waves

$\therefore \quad$ Total path difference $= \dfrac{3\lambda}{2} + \dfrac{\lambda}{2} = 2\lambda$

Which satisfies the condition of maxima. Resultant intensity $= (\sqrt{I} + \sqrt{I})^2 = 4I$.

Illustration 33:

A ray of light of intensity I is incident on a parallel glass-slab at a point A as shown in figure. It undergoes partial reflection and refraction. At each reflection 25% of incident energy is reflected. The rays AB and $A'B'$ undergo interference. The ratio I_{max}/I_{min} is



(A) 4 : 1 (B) 8 : 1

(C) 7 : 1 (D) 49 : 1

Ans. (D)

Solution:

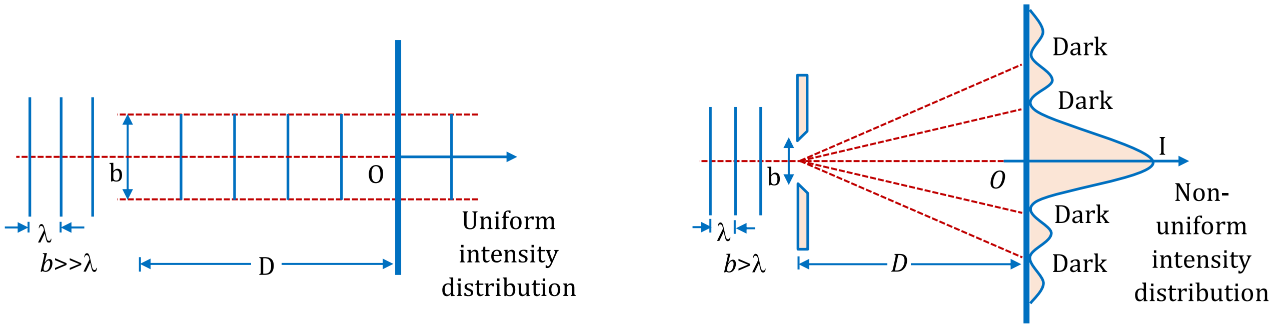
From figure $I_1 = \dfrac{I}{4}$ and $I_2 = \dfrac{9I}{64} \Rightarrow \dfrac{I_2}{I_1} = \dfrac{9}{16}$

By using $\dfrac{I_{\max}}{I_{\min}} = \left(\dfrac{\sqrt{\dfrac{I_2}{I_1}}+1}{\sqrt{\dfrac{I_2}{I_1}}-1}\right) = \left(\dfrac{\sqrt{\dfrac{9}{16}}+1}{\sqrt{\dfrac{9}{16}}-1}\right) = \dfrac{49}{1}$

Diffraction of Light:

The phenomenon of diffraction was first discovered by Girmaldi. It's experimental study was done by Newton and Young. The theoretical explanation was first given by Fresnel.

(1) The phenomenon of bending of light around the corners of an obstacle/aperture of the size of the wave length of light is called diffraction.



(A) Size of the slit is very large compared to wavelength

(B) Size of the slit is comparable to wavelength

(2) The phenomenon resulting from the superposition of secondary wavelets originating from different parts of the same wave front is define as diffraction of light.

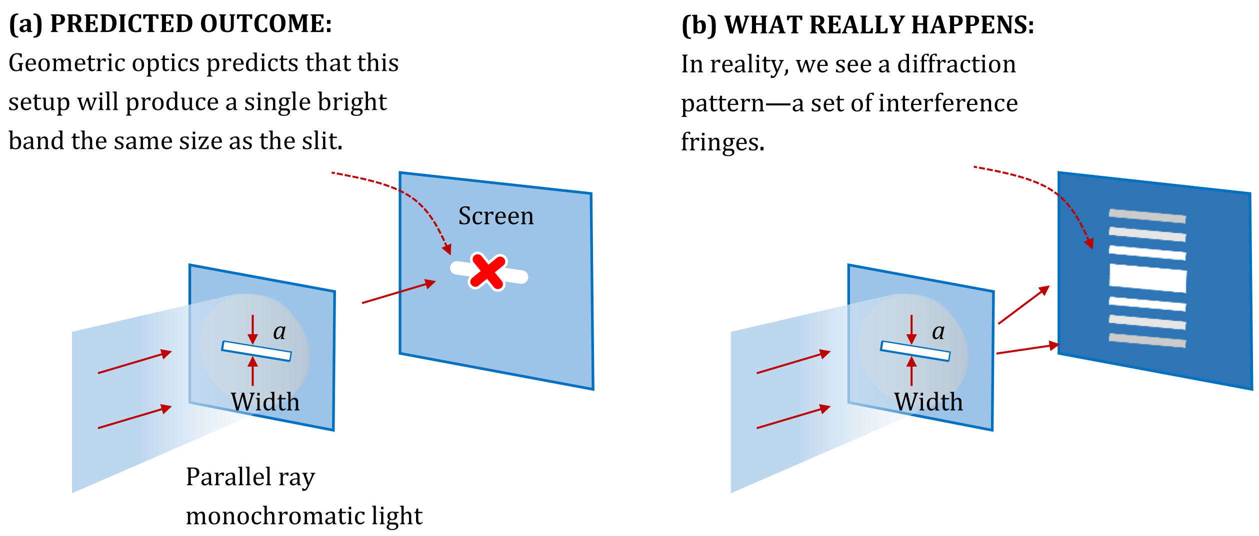
(3) Diffraction is the characteristic of all types of waves.

(4) Greater the wave length of wave higher will be it's degree of diffraction.

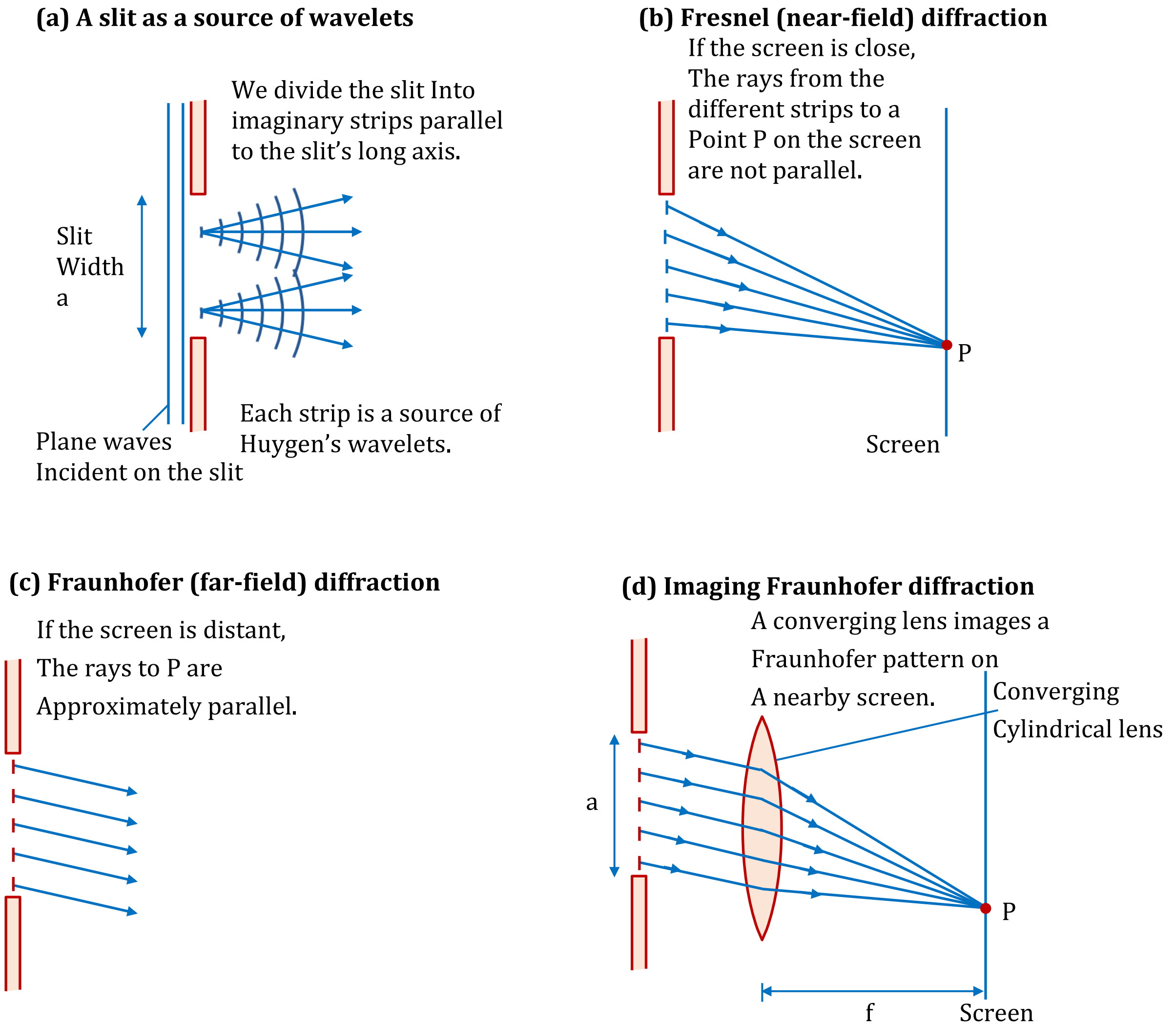
Diffraction from a Single Slit

In this section we'll discuss the diffraction pattern formed by plane-wave (parallel ray) monochromatic light when it emerges from a long, narrow slit, as shown in Fig. We call the narrow dimension the width, even though in this figure it is a vertical dimension. According to geometric optics, the transmitted beam should have the same cross section as the slit, as in Fig. a. What is actually observed is the pattern shown in Fig. b. The beam spreads out vertically after passing through the slit. The diffraction pattern consists of a central bright band, which may be much broader than the width of the slit, bordered by alternating dark and bright bands with rapidly decreasing intensity. About 85% of the power in the transmitted beam is in the central bright band, whose width is inversely proportional to the width of the slit. In general, the smaller the width of the slit, the broader the entire diffraction pattern. (The horizontal spreading of the beam in Fig. b is negligible because the horizontal dimension of the slit is relatively large.) You can observe a similar diffraction pattern by looking at a point source, such as a distant street light, through a narrow slit formed between your two thumbs held in front of your eye; the retina of your eye corresponds to the screen.

In figure below (a) The "shadow" of a horizontal slit as incorrectly predicted by geometric optics. (b) A horizontal slit actually produces a diffraction pattern. The slit width has been greatly exaggerated.



Diffraction by a single rectangular slit. The long sides of the slit are perpendicular to the figure.



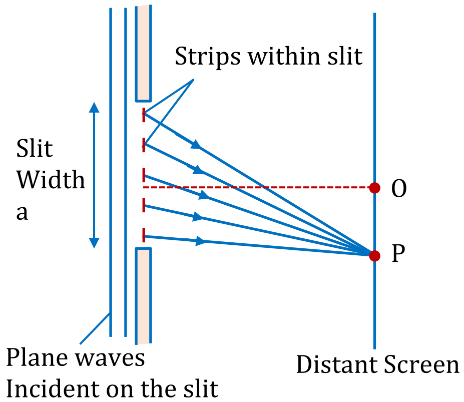
Intensity in the Single-Slit Pattern:

We can derive an expression for the intensity distribution for the single-slit diffraction pattern by the phasor-addition method. We imagine a plane wave front at the slit subdivided into many strips. We superpose the contributions of the Huygens wavelets from all the strips at a point P on a distant screen at an angle from the normal to the slit plane (Fig. a). To do this, we use a phasor to represent the sinusoidally varying field from E field from each individual strip. The magnitude of the vector sum of the phasors at each point P is the amplitude E_P of the total $\vec{E}$ field at the point. The intensity at P is proportional to E_P^2. At the point O shown in fig. a corresponding to the center of the pattern where $\theta = 0$, there are negligible path differences for $x >> a$. The phasors are all essentially in phase (that is, have the same direction). In fig. b we draw the phasors at time $t = 0$ and denote the resultant amplitude at O by E_0. In this illustration we have divided the slit into 15 strips.

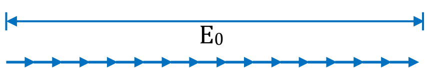
Now consider wavelets arriving from different strips at point P in fig. a at an angle θ from O. Because of the differences in path length, there are now phase differences between wavelets coming from adjacent strips; the corresponding phasor diagram is shown in fig. c. The vector sum of the phasors is now part of the perimeter of a many sided polygon, and E_P, the amplitude of the resultant electric field at P, is the chord. The angle β is the total phase difference between wave from the top strip of fig. a and the wave from the bottom strip; that is β is the phase of the wave received at P from the top strip with respect to the wave received at P from the bottom strip.

Using phasor diagrams to find the amplitude of the $\vec{E}$ field in single-slit diffraction. Each phasor represents the $\vec{E}$ fields from a single strip within the

(a)

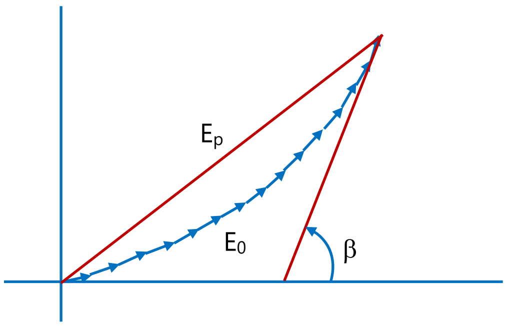


(b) At the center of the diffraction pattern (point O), the phasors from all strips within the slit are in phase.

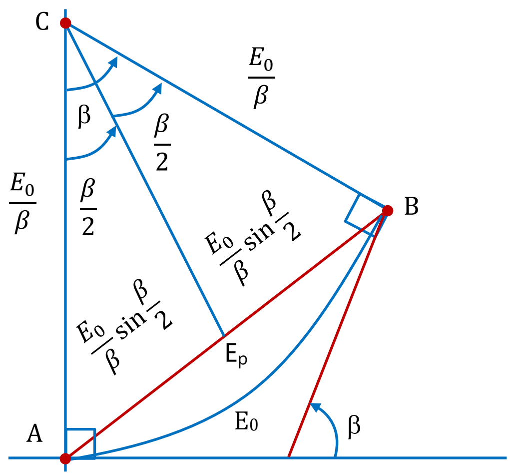


(c) Phasor diagram at a point slightly off the center of the pattern

β = total phase difference between the first and last phasors.



(d) As in (c), but in the limit that the slit is Subdivided into infinitely many strips



We may imagine dividing the slit into narrower and narrower strips. In the limit that there is an infinite number of infinitesimally narrow strips, the curved trails of phasors becomes an arc of a circle (fig. d). with are length equal to the length E_0 in fig. b. The center C of this arc is found by constructing perpendiculars at A and B. From the relationship among arc length, radius, and angle, the radius of the arc is E_0/β. The amplitude E_P of the resultant electric field at P is equal to the chord AB, which is 2 (E_0/β) sin $(\beta/2)$.

(Note that β must be in radians)

We then have

$$E_P = E_0 \frac{\sin(\beta/2)}{\beta/2} \quad \text{(amplitude in single slit diffraction)} \quad ...(i)$$

The intensity at each point on the screen is proportional to the square of the amplitude given by Eq. (i), if I_0 is the intensity in the straight-ahead direction where θ = 0 and β = 0, then the intensity I at any point is

$$I = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \quad \text{(intensity in signal slit diffraction)} \quad ...(ii)$$

We can express the phase difference β in terms of geometric quantities, as we did for the two-slit pattern. The phase difference is $2\pi/\lambda$ times the path difference. The path difference between the ray from the top of the slit and the ray from the middle of the slit is $(a/2)$ sinθ. The path difference between the rays from the top of the slit and the bottom of the slit is twice this, so

$$\beta = \frac{2\pi}{\lambda} a \sin\theta \quad ...(iii)$$

And eq. (ii) becomes

$$I = I_0 \left\{\frac{\sin\left[\pi a(\sin\theta)/\lambda\right]}{\pi a(\sin\theta)/\lambda}\right\}^2 \quad \text{(intensity in single slit diffraction)} \quad ...(iv)$$

This equation expresses the intensity directly in terms of the angle θ. In many calculations it is easier first to calculate the phase angle β, using equation (iii) and then to use equation (ii).

Equation (iv) is plotted in figure below. Note that the central intensity peak is much larger than any of the others. This means that most of the power in the wave remains within an angle θ from the perpendicular to the slit, where $\sin\theta = \lambda/a$ (the first diffraction minimum).

Note:

The peak intensities in figure below decrease rapidly as we go away from the center of the pattern. (Compare figure which shows a single slit diffraction pattern for light.)

The dark fringes in the pattern are the places where I = 0. These occur at point for which the numerator of equation is zero so that β is a multiple of 2π. From equation this corresponds to

$$\frac{a\sin\theta}{\lambda} = m \quad (m = \pm1, \pm2, ...)$$

$$\sin\theta = \frac{m\lambda}{a} \quad (m = \pm1, \pm2, ...) \quad ...(v)$$

Note again that β = 0 (corresponding to θ = 0) is not a minimum. Equation is indeterminate at β = 0, but we can evaluate the limit as β → 0 using L'Hopital's rule. We find that at β = 0, $I = I_0$, as we should expect.

Intensity Maxima in the Single-Slit Pattern

We can also use equation (ii) from above to calculate the positions of the peaks, or intensity maxima, and intensities at these peaks. This is not quite as simple as it may appear. We might expect the peaks to occur where the sine function reaches the value ±1 –– namely, where $\beta = \pm\pi, \pm 3\pi, -5\pi$ or in general,

$$\beta \approx \pm(2m+1) \quad (m = 0, 1, 2, ...) \qquad ...(vi)$$

This is approximately correct, but because of the factor $(\beta/2)^2$ in the denominator of eq. (ii), the maxima don't occur precisely at these points. When we take the derivative of eq. (ii) with respect to β and set it equal to zero to try to find the maxima and minima, we get a transcendental equation that has to be solved numerically. In fact there is no maximum near $\beta = \pm\pi$. The first maxima on either side of the central maximum, near $\beta = \pm 3\pi$, actually occur at $\pm 2.860\pi$. The second side maxima, near $\beta = \pm 5\pi$, are actually at $\pm 4.918\pi$ and so on. The error in equation (vi) vanishes in the limit of large m––that is, for intensity maxima far from the center of the pattern.

To find the intensities at the side maxima, we substitute these values of β back into eq. (ii). Using the approximate expression in equation (vi), we get

$$I_m \approx \frac{I_0}{\left(m+\frac{1}{2}\right)^2 \pi^2} \qquad ...(vii)$$

Where I_m is the intensity of the mth side maximum and I_0 is the intensity of the central maximum. Equation (vii) gives the series of intensities

$0.0450 I_0$ $\quad$ $0.0162\ I_0$ $\quad$ $0.0083\ I_0$

and so on. As we have pointed out, this equation is only approximately correct. The actual intensities of the side maxima turn out to be

$0.0472 I_0$ $\quad$ $0.0165\ I_0$ $\quad$ $0.0083\ I_0$

Note that the intensities of the side maxima decrease very rapidly, as fig. below also shows. Even the first side maxima have less than 5% of the intensity of the central maximum.

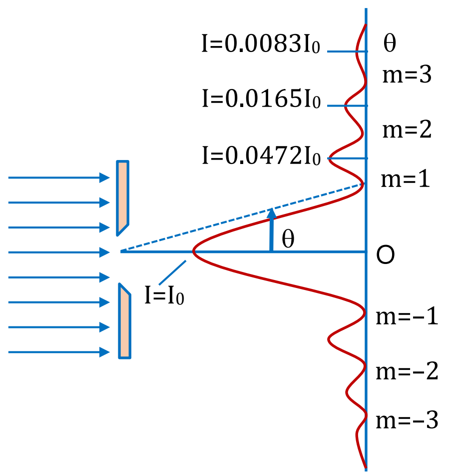


Figure: Intensity versus angle in single slit diffraction. The values of m label intensity minima given by equation (v). Most of the wave power goes into the central intensity peak (between he m = 1 and m = –1 intensity minima). Only the diffracted waves within the central intensity peak are visible the waves at larger angles are too faint to see.

Width of the Single-Slit Pattern

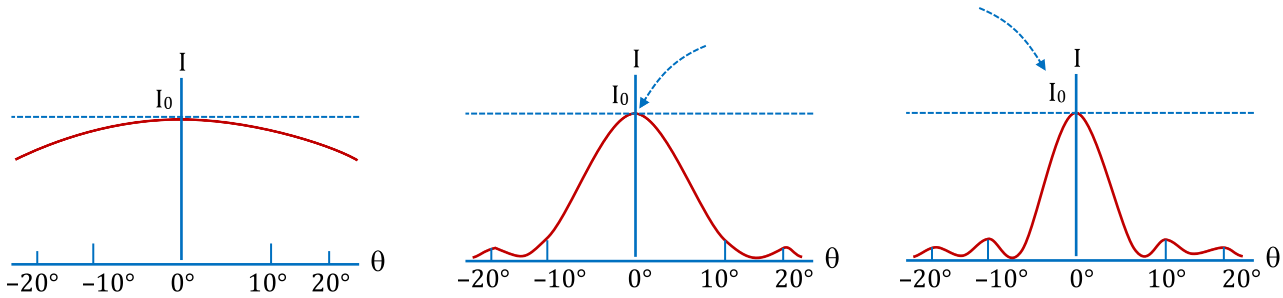
For small angles the angular spread of the diffraction pattern is inversely proportional to the slit width a or, more precisely, to the ratio of a to the wavelength λ. Figure shows graphs of intensity I as a function of the angle θ for three values of the ratio a/λ.

The single-slit diffraction pattern depends on the ratio of the slit width a to the wavelength λ.

(a) $a = \lambda$ $\qquad$ (b) $a = 5\lambda$ $\qquad$ (c) $a = 8\lambda$ΣΣΣΣΣ

If the slit width is equal to or narrower than the wavelength, only one broad maximum forms.

The wider the slit (or the shorter the wavelength), the narrower and sharper is the central peak.



With light waves, the wavelength λ is often much smaller than the slit width a, and the values of θ in equations (v) and (vi) are so small that the approximation $\sin\theta = \theta$ is very good. With this approximation the position θ_1 of the first minimum beside the central maximum, corresponding to $\beta/2 = \pi$, is from equation (vi)

$$\theta_1 = \frac{\lambda}{a} \qquad \text{...(viii)}$$

This characterizes the width (angular spread) of the central maximum, and we see that it is inversely proportional to the slit width a. When the small-angle approximation is value, the central maximum is exactly twice as wide as each side maximum. When a is of the order of a centimetre or more, θ_1 is so small that we can consider practically all the light to be concentrated at the geometrical focus. But when a is less than λ, the central maximum spreads over 180°, and the fringe pattern is not seen at all.

It's important to keep in mind that diffraction occurs for all kinds of waves, not just light. Sound waves undergo diffraction when they pass through a slit or aperture such as an ordinary doorway. The sound waves used in speech have wavelengths of about a meter or greater, and a typical doorway is less that 1 *m* wide; in this situation, a is less than λ, and the central intensity maximum extends over 180°. This is why the sounds coming through an open doorway can easily be heard by an eavesdropper hiding out of sight around the corner. In the same way, sound waves can bend around the head of an instructor who faces the blackboard while lecturing. By *c* contrast, there is essentially no diffraction of visible light through a doorway because the width a is very much greater than the wavelength λ (of order 5×10^{-7} *m*). You can hear around corners because typical sound waves have relatively long wavelengths you cannot see around corners because the wavelength of visible light is very short.

Illustration 34:

(a) The intensity at the center of a single-slit diffraction pattern is I_0. What is the intensity at a point in the pattern where there is a 66-radian phase difference between wavelets from the two edges of the slit?

(b) If this point is 7.0° away from the central maximum, how many wavelengths wide is the slit?

Solution:

(a) We have $\beta/2 = 33$ rad, so from equation (ii)

$$I = I_0\left[\frac{\sin(33rad)}{33rad}\right]^2 = \left(9.2\times10^{-4}\right)I_0$$

(b) From equation (iii)

$$\frac{a}{\lambda} = \frac{\beta}{2\pi\sin\theta} = \frac{66rad}{(2\pi rad)\sin 7.0°} = 86$$

For example, for 5550-*nm* light the slit width is

$$a = (86)\,(550nm) = 4.7\times10^{-5}m = 0.047\ mm, \text{ or roughly } \frac{1}{20}mm$$

Illustration 35:

The intensity at the center of the pattern is I_0 for a light of wave length $6.33 \times 10^{-7}m$. What is the intensity at a point on the screen 3.0 *mm* from the center of the pattern. Screen is placed at a distance of 6*m*.

Solution:

We have $y = 3mm$ and $x = 6\ m$, so $\tan\theta = y/x = (3 \times 10^{-3}m)/(6m) = 5\times10^{-4}$. This is so small that the values of $\tan\theta$, $\sin\theta$, and θ (in radians) are all nearly the same. Then, using equation (iv)

$$\frac{\pi a \sin\theta}{\lambda} = \frac{\pi\left(2.4\times10^{-4}m\right)\left(5\times10^{-4}\right)}{6.33\times10^{-7}m} = 0.60$$

Illustration 36:

Light of wavelength 6000Å is incident normally on a slit of width $24 \times 10^{-5}\ cm$. Find out the angular position of second minimum from central maximum?

Solution:

Given $\lambda = 6 \times 10^{-7}m$, $a = 24 \times 10^{-5} \times 10^{-2}m$

$$a \sin\theta = 2\lambda$$

$$\sin\theta = \frac{2\lambda}{a} = \frac{2\times6\times10^{-7}}{24\times10^{-7}} = \frac{1}{2}$$

$$\theta = 30°$$

Illustration 37:

Light of wavelength 6328Å is incident normally on a slit of width 0.2*mm*. Calculate the angular width of central maximum on a screen distance 9 *m*?

Solution:

given $\lambda = 6.328 \times 10^{-7}$m, $a = 0.2 \times 10^{-3}$m

$$w_\theta = \frac{2\lambda}{a} = \frac{2\times6.328\times10^{-7}}{2\times10^{-4}}\text{radian} = \frac{6.328\times10^{-3}\times180}{3.14} = 0.36°$$

Illustration 38:

Light of wavelength 5000Å is incident normally on a slit of width 0.1mm. Find out the width of central bright line on a screen distance 2 m from the slit?

Solution:

$$w_x = \frac{2D\lambda}{a} = \frac{2\times2\times5\times10^{-7}}{10^{-4}} = 20mm$$

Illustration 39:

The Fraunhofer diffraction pattern of a single slit is formed at the focal plane of a lens of focal length 1m. The width of the slit is 0.3 mm. If the third minimum is formed at a distance of 5mm from the central maximum then calculate the wavelength of light.

Solution:

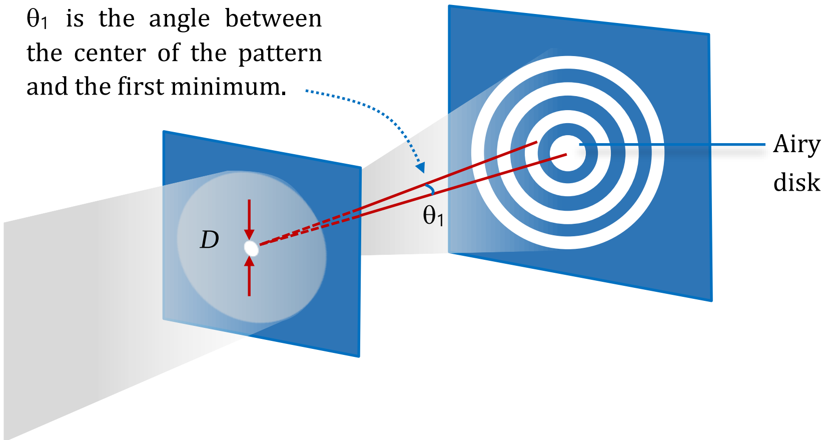
$$x_n = \frac{nf\lambda}{a}$$

$$\Rightarrow \quad \lambda = \frac{ax_n}{fn} = \frac{3\times10^{-4}\times5\times10^{-3}}{3\times1} = 5000\mathring{A} \qquad [\because n = 3]$$

Circular apertures and resolving power

We have studied in detail the diffraction patterns formed by long, thin slits or arrays of slits. But an aperture of any shape forms a diffraction pattern. The diffraction pattern formed by a circular aperture is of special interest because of its role in limiting how well an optical instrument can resolve fine details. In principle, we could compute the intensity at any point *P* in the diffraction pattern by dividing the area of the aperture into small elements, finding the resulting wave amplitude and intensity at *P*. In practice, the integration cannot be carried out in terms of elementary functions. We will simply describe the pattern and quote a few relevant numbers.

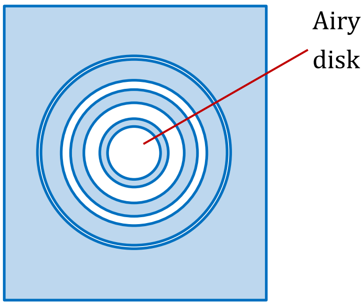
The diffraction pattern formed by a circular aperture consists of a central bright spot surrounded by a series of bright and dark rings, as Fig. below shows.



Diffraction pattern formed by a circular aperture of diameter *D*. The pattern consists of a central bright spot and alternating dark and bright rings. The angular radius θ_1 of the first dark ring is shown.

We can describe the pattern in terms of the angle θ, representing the angular radius of each ring. If the aperture diameter is D and the wavelength is λ, the angular radius θ_1 of the first dark ring is given by

$$\sin\theta_1 = 1.22\frac{\lambda}{D} \quad \text{(diffraction by a circular aperture)}$$



Diffraction pattern formed by a circular aperture.

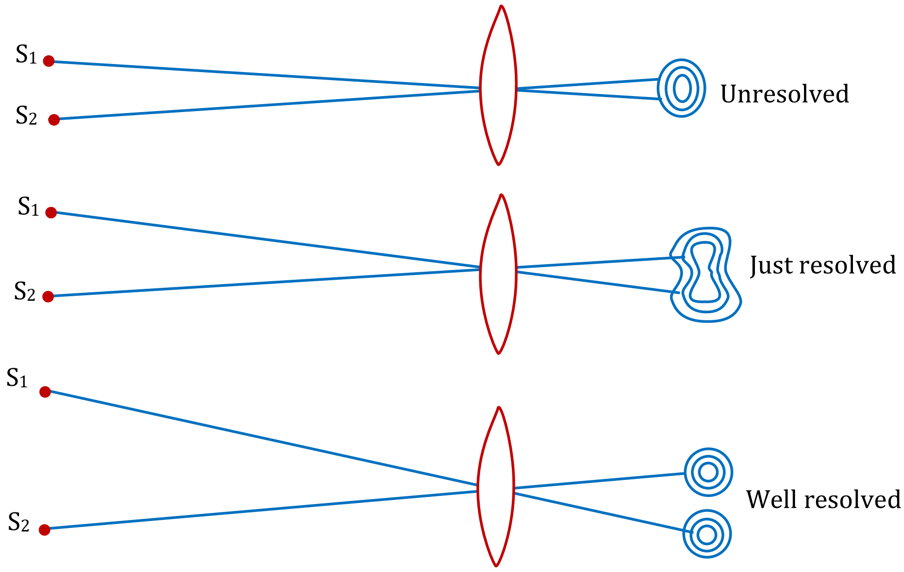
The discussion shows that a converging lens can never form a point image of a distant point source. In the best conditions, it produces a bright disc surrounded by dark and bright rings. If we assume that most of the light is concentrated within the central bright disc, we can say that the lens produces a disc image for a distant point source. This is not only true for a distant point source but also for any point source. The radius of the image disc is

$$R = 1.22\frac{\lambda}{D}d$$

Where *d* is the distance between image plane and lens.

Limit of resolution

The fact that a lens forms a disc image of a point source, puts a limit on resolving two neighbouring points imaged by a lens.



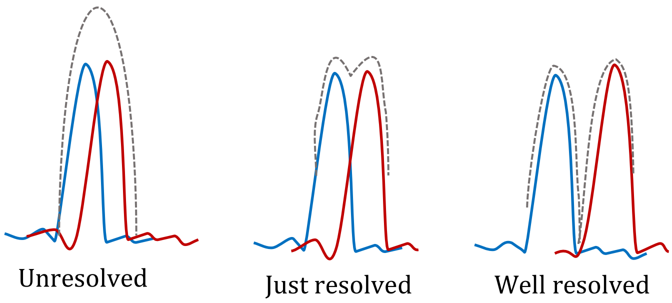
Suppose S_1 and S_2 are two-point sources placed before a converging lens (as shown in figure). If the separation between the centres of the image-discs is small in comparison to the radii of the discs, the discs will largely overlap on one another and it will appear like a single disc. The two points are then not resolved. If S_1 and S_2 are moved apart, the centres of their image-discs also move apart. For a sufficient separation, one can distinguish the presence of two discs in the pattern. In this case, we say that the points are just resolved.

The angular radius θ of the diffraction disc is given by $\sin\theta = \frac{1.22\lambda}{b}$, where b is the radius of the lens. Thus, increasing the radius of the lens improves the resolution. This is the reason why objective lenses of powerful microscopes and telescopes are kept large in size.

The human eye is itself a converging lens which forms the image of the points (we see) on the retina. The above discussion then shown that two points very close to each other cannot be seen as two distinct points by the human eye.

Rayleigh Criterion

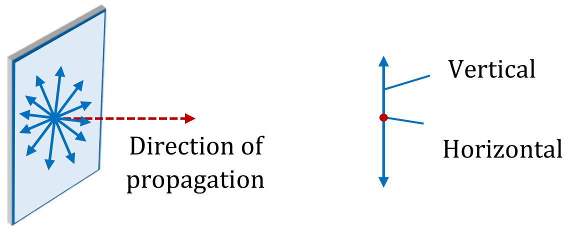
Whether two-disc images of nearby points are resolved or not may depend on the person viewing the images. Ryleigh suggested a quantitative criterion for resolution. Two images are called just resolved in this criterion if the centre of one bright disc falls one the periphery of the second. This means, the radius of each bright disc should be equal to the separation between them. In this case, the resultant intensity has a minimum between the centres of the images. Figure shows the variation of intensity when the two images are just resolved.



Polarisation of Light:

Light propagates as transverse EM waves. The magnitude of electric field is much larger as compared to magnitude of magnetic field. We generally prefer to describe light as electric field oscillations.

(1) **Unpolarised light:** In ordinary light (light from sun, bulb *etc.*) the electric field vectors are distributed in all directions in a light is called unpolarised light. The oscillation of propagation of light wave. This resolved into horizontal and vertical component.



(2) **Polarised light:** The phenomenon of limiting the vibrating of electric field vector in one direction in a plane perpendicular to the direction of propagation of light wave is called polarization of light.

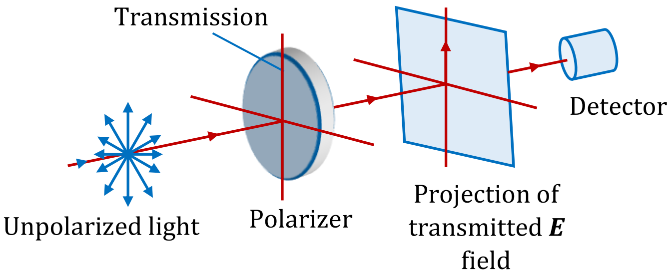
(i) The plane in which oscillation occurs in the polarised light is called plane of oscillation.

(ii) The plane perpendicular to the plane of oscillation is called plane of polarisation.

(iii) Light can be polarised by transmitting through certain crystals such as tourmaline or polaroid.

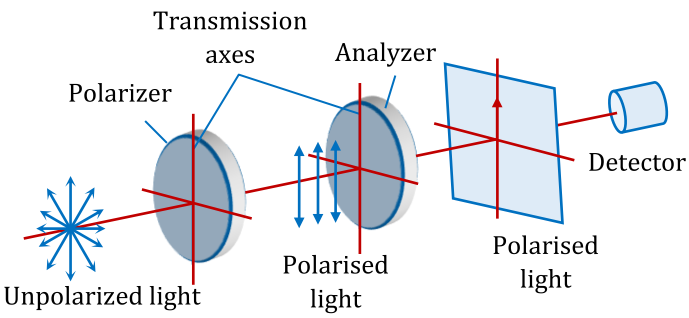
(3) **Polaroids:** It is a device used to produce the plane polarised light. It is based on the principle of selective absorption and is more effective than the tourmaline crystal.

It is a thin film of ultramicroscopic crystals of quinine iodosulphate with their optic axis parallel to each other.

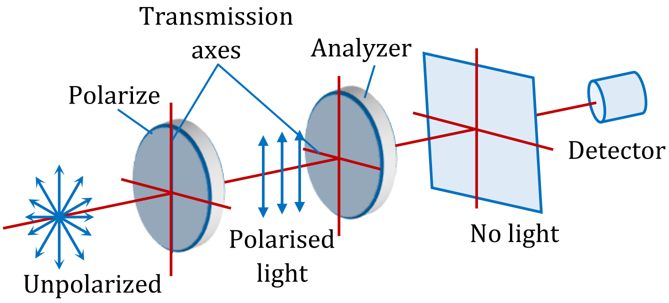


(i) Polaroids allow the light oscillations parallel to the transmission axis pass through them.

(ii) The crystal or polaroid on which unpolarised light is incident is called polariser. Crystal or polaroid on which polarised light is incident is called analyser.

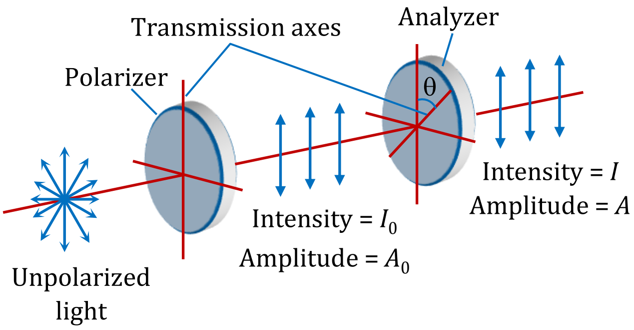


(A) Transmission axes of the polariser and analyser are parallel to each other, so whole of the polarised light passes through analyser



(B) Transmission axis of the analyser is perpendicular to the polariser, hence no light passes through the analyser

(4) **Malus law:** This law states that the intensity of the polarised light transmitted through the analyser varies as the square of the cosine of the angle between the plane of transmission of the analyser and the plane of the polariser.



(i) $I = I_0 \cos^2\theta$ and $A^2 = A_0^2 \cos^2\theta \Rightarrow A = A_0 \cos\theta$

If $\theta = 0^o, I = I_0, A = A_0$.

If $\theta = 90^o, I = 0, A = 0$.

(ii) If I_i = Intensity of unpolarised light.

So $I_0 = \frac{I_i}{2}$ *i.e.* if an unpolarised light is converted into plane polarised light (say by passing it through a Polaroid or a Nicol-prism), its intensity becomes half and $I = \frac{I_i}{2}\cos^2\theta$.

Methods of Producing Polarised Light:

(1) **Polarisation by reflection:** Brewster discovered that when a beam of unpolarised light is reflected from a transparent medium (refractive index = μ), the reflected light is completely plane polarised at a certain angle of incidence (called the angle of polarisation θ_p).

From fig. it is clear that $\theta_P + \theta_r = 90^o$

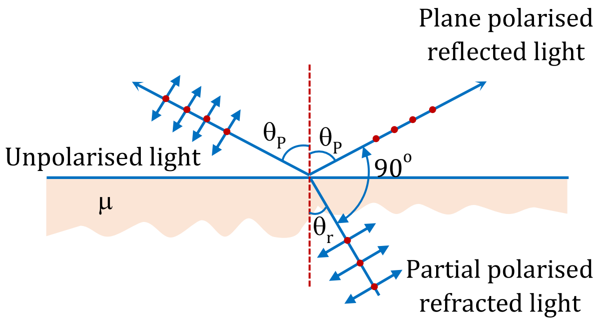
Also $\mu = \tan\theta_p$ Brewster's law

(i) For $i < \theta_P$ or $i > \theta_P$

Both reflected and refracted rays becomes partially polarised

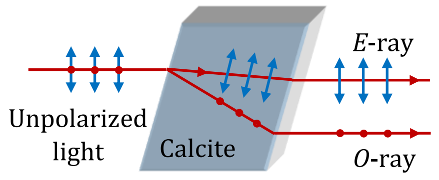
(ii) For glass $\theta_P \approx 57^o$,

for water $\theta_P \approx 53^o$



(2) By Dichroism: Some crystals such as tourmaline and sheets of iodosulphate of quinine have the property of strongly absorbing the light with vibrations perpendicular to a specific direction (called transmission axis) transmitting the light with vibrations parallel to it. This selective absorption of light is called dichroism.

(3) By double refraction: In certain crystals, like calcite, quartz and tourmaline *etc*, incident unpolarized light splits up into two light beams of equal intensities with perpendicular polarization.



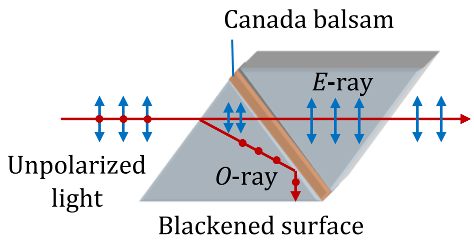
(i) One of the ray is ordinary ray (O-ray) it obeys the Snell's law. Another ray's extra ordinary ray (E-ray) it doesn't obeys the Snell's law.

(ii) Along a particular direction (fixed in the crystal, the two velocities (velocity of O-ray v_o and velocity of E-ray v_e) are equal. This direction is known as the optic axis of the crystal (crystal's known as uniaxial crystal). Optic axis is a direction and not any line in crystal.

(iii) In the direction, perpendicular to the optic axis for negative crystal (calcite) $v_e > v_o$ and $\mu_e < \mu_o$. For positive crystal $v_e < v_o, \mu_e > \mu_o$.

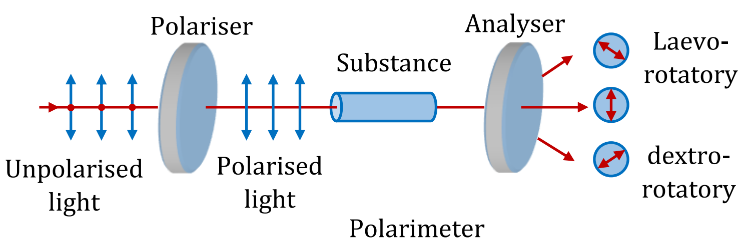
(4) Nicol prism: Nicol prism is made up of calcite crystal and in it E-ray is isolated from O-ray through total internal reflection of O-ray at Canada balsam layer and then absorbing it at the blackened surface as shown in fig.

The refractive index for the O-ray is more that for the E-ray. The refractive index of Canada balsam lies between the refractive indices of calcite for the O-ray and E-ray.



(5) By Scattering: It is found that scattered light in directions perpendicular to the direction of incident light is completely plane polarised while transmitted light is unpolarised. Light in all other directions is partially polarised.

(6) Optical activity and specific rotation: When plane polarised light passes through certain substances, the plane of polarisation of the light is rotated about the direction of propagation of light through a certain angle. This phenomenon is called optical activity or optical rotation and the substances optically active.



Polarimeter

If the optically active substance rotates the plane of polarisation clockwise (looking against the direction of light), it is said to be *dextro-rotatory* or *right-handed.* However, if the substance rotates the plane of polarisation anti-clockwise, it is called *laevo-rotatory* or *left-handed.*

The optical activity of a substance is related to the asymmetry of the molecule or crystal as a whole, *e.g.*, a solution of cane-sugar is dextro-rotatory due to asymmetrical molecular structure while crystals of quartz are dextro or laevo-rotatory due to structural asymmetry which vanishes when quartz is fused.

Optical activity of a substance is measured with help of polarimeter in terms of 'specific rotation' which is defined as the rotation produced by a solution of length 10 *cm* (1 *dm*) and of unit concentration (*i.e.* 1 g/cc) for a given wavelength of light at a given temperature. *i.e.* $[\alpha]_{t^oC}^{\lambda} = \frac{\theta}{L \times C}$

where θ is the rotation in length L at concentration C.

(7) Applications and uses of polarisation

(i) By determining the polarising angle and using Brewster's law, *i.e.* $\mu = \tan\theta_P$, refractive index of dark transparent substance can be determined.

(ii) It is used to reduce glare.

(iii) In calculators and watches, numbers and letters are formed by liquid crystals through polarisation of light called liquid crystal display **(LCD)**.

(iv) In CD player polarised laser beam acts as needle for producing sound from compact disc which is an encoded digital format.

(v) It has also been used in recording and reproducing three-dimensional pictures.

(vi) Polarisation of scattered sunlight is used for navigation in solar-compass in polar regions.

(vii) Polarised light is used in optical stress analysis known as 'photo elasticity'.

(viii) Polarisation is also used to study asymmetries in molecules and crystals through the phenomenon of 'optical activity'.

(ix) A polarised light is used to study surface of nucleic acids (DNA, RNA).

Doppler's Effect of Light:

The phenomenon of apparent change in frequency (or wavelength) of the light due to relative motion between the source of light and the observer is called Doppler's effect.

Source of light moves towards the stationary observer: When a light source is moving towards an observer with a relative velocity v then the apparent frequency (f') is greater than the actual frequency (f) of light. Thus, apparent wavelength (λ') is lesser the actual wavelength (λ).

$$f' = f\sqrt{\frac{(1+v/c)}{(1-v/c)}} \text{ and } \lambda' = \lambda\sqrt{\frac{(1-v/c)}{(1+v/c)}}$$

where f = Actual frequency, f' = Apparent frequency,

v = Speed of source *w.r.t* stationary observer, c = speed of light

For $v << c$:

(i) Apparent frequency $f' = f\left(1+\frac{v}{c}\right)$

(ii) Apparent wavelength $\lambda' = \lambda\left(1-\frac{v}{c}\right)$

(iii) Doppler's shift: Apparent wavelength < actual wavelength,

So, spectrum of the radiation from the source of light shifts towards the violet end of spectrum. This is called violet shift

Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$

(iv) The fraction decrease in wavelength $= \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$

Source of light moves away from the stationary observer: In this case $f' < f$ and $\lambda' > \lambda$

$$f' = f\sqrt{\frac{(1-v/c)}{(1+v/c)}} \text{ and } \lambda' = \lambda\sqrt{\frac{(1+v/c)}{(1-v/c)}}$$

For $v << c$:

(i) Apparent frequency $f' = f\left(1-\frac{v}{c}\right)$ and

(ii) Apparent wavelength $\lambda' = \lambda\left(1+\frac{v}{c}\right)$

(iii) Doppler's shift: Apparent wavelength > actual wavelength,

So, spectrum of the radiation from the source of light shifts towards the red end of spectrum. This is called red shift.

Doppler's shift $\Delta\lambda = \lambda.\frac{v}{c}$

(iv) The fractional increase in wavelength $= \frac{\Delta\lambda}{\lambda} = \frac{v}{c}$.

Doppler broadening: For a gas in a discharge tube, atoms are moving randomly in all directions. When spectrum of light emitted from these atoms is analysed, then due to Doppler effect (because some atoms are moving towards detector, some atoms are moving away from detector), the frequency of a spectral line is not observed as having one value, but is spread over a range.

$$\pm\Delta f = \pm\frac{v}{c}f, \quad \pm\Delta\lambda = \pm\frac{v}{c}\lambda$$

This broadens the spectral line by an amount (2 $\Delta\lambda$). It is called Doppler broadening. The Doppler broadening is proportional to v, which in turn is proportional to $\sqrt{T}$, where T is the temperature in Kelvin.

Radar: Radar is a system for locating distant object by means of reflected radio waves, usually of microwave frequencies. Radar is used for navigation and guidance of aircraft, ships *etc*.

Radar employs the Doppler effect to distinguish between stationary and moving targets. The change in frequency between transmitted and received waves is measured. If v is the velocity of the approaching target, then the change in frequency is $\Delta f = \frac{2v}{c}f$. (The factor of 2 arises due to refection of waves). For a receding target $\Delta f = -\frac{2v}{c}f$. (The minus sign indicates decrease in frequency).

Applications of Doppler effect:

(i) Determination of speed of moving bodies (aeroplane, submarine etc) in RADAR and SONAR.

(ii) Determination of the velocities of stars and galaxies by spectral shift.

(iii) Determination of rotational motion of sun.

(iv) Explanation of width of spectral lines.

(v) Tracking of satellites.

(vi) In medical sciences in echo cardiogram, sonography *etc*.

Illustration 40:

A star is moving towards the earth with a speed of $4.5 \times 10^6 m/s$. If the true wavelength of a certain line in the spectrum received from the star is 5890 Å, its apparent wavelength will be about $[c = 3 \times 10^8 m/s]$

(A) 5890 Å (B) 5978 Å (C) 5802 Å (D) 5896 Å

Ans. (C)

Solution:

By using $\lambda' = \lambda\left(1 - \frac{v}{c}\right)$

$\Rightarrow \quad \lambda' = 5890\left(1 - \frac{4.5\times10^6}{3\times10^8}\right) = 5802\,\text{Å}$

Illustration 41:

Light coming from a star is observed to have a wavelength of 3737 Å, while its real wavelength is 3700 Å. The speed of the star relative to the earth is [Speed of light $= 3 \times 10^8 m/s$]

(A) $3 \times 10^5 m/s$ (B) $3 \times 10^6 m/s$ (C) $3.7 \times 10^7 m/s$ (D) $3.7 \times 10^6 m/s$

Ans. (B)

Solution:

By using $\Delta\lambda = \lambda\frac{v}{c}$

$\Rightarrow \quad (3737\text{-}3700) = 3700 \times \frac{v}{3\times10^8}$

$\Rightarrow \quad v = 3 \times 10^6 m/s$

Illustration 42:

Light from the constellation Virgo is observed to increase in wavelength by 0.4%. With respect to Earth the constellation is

(A) Moving away with velocity $1.2 \times 10^6 m/s$ (B) Coming closer with velocity $1.2 \times 10^6 m/s$

(C) Moving away with velocity $4 \times 10^6 m/s$ (D) Coming closer with velocity $4 \times 10^6 m/s$

Ans. (A)

Solution:

By using $\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$; where $\frac{\Delta\lambda}{\lambda} = \frac{0.4}{100}$ and $c = 3 \times 10^8 m/s$

$\Rightarrow \quad \frac{0.4}{100} = \frac{v}{3\times10^8}$

$\Rightarrow \quad v = 1.2 \times 10^6 m/s$

Since wavelength is increasing *i.e.* it is moving away.