01 Magnetic Effect of Current and Magnetism

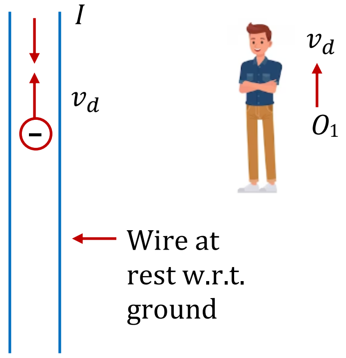
MAGNETIC EFFECT OF CURRENT

Magnetic effects of current (and moving charge)

It was observed by OERSTED that a current carrying wire produces magnetic field nearly it. It can be tested by placing a magnet in the near by space, it will show some movement. This observation shows that current or moving charge produces magnetic field.

Frame Dependence of $\vec{B}$.

(a) The motion of anything is a relative term. A charge may appear at rest by an observer (say O_1) and moving with velocity $\vec{v}_1$ with respect to observer O_2 and at velocity $\vec{v}_2$ with respect to observers O_3 then $\vec{B}$ due to that charge w.r.t. O_1 will be zero and w.r.t. to O_2 and O_3 it will be $\vec{B}_1$ and $\vec{B}_2$ (that means different)



(b) In a current carrying wire electron move in the opposite direction to that of the current and +*ve* ions (of the metal) are static w.r.t. the wire. Now if some observer (O_1) moves with velocity V_d in the direction of motion of the electrons then electrons will have zero velocity and +*ve* ions will have velocity V_d in the downward direction w.r.t. O_1. The density (n) of +*ve* ions is same as the density of free electrons and their charges are of the same magnitudes.

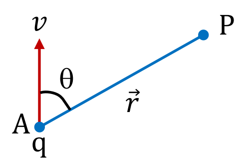
So, w.r.t. O_1 electrons will produce zero magnetic field but +*ve* ions will produce +*ve* same $\vec{B}.\vec{B}$ due to the current carrying wire does not depend on the reference frame (this is true for any velocity of the observer).

Due to a point charge:

A charge particle 'q' has velocity v as shown in the figure. It is at position 'A' at some time. $\vec{r}$ is the position vector of point 'P' w.r.t position of the charge. Then $\vec{B}$ at P due to q is

$B = \left(\frac{\mu_0}{4\pi}\right)\frac{qv\sin\theta}{r^2}$; here θ = angle between $\vec{v}$ and $\vec{r}$

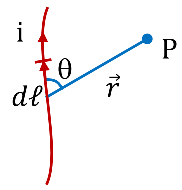
$\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{q\vec{v}\times\vec{r}}{r^3}$; $\Rightarrow \vec{B} \perp \vec{v}$ and also $\vec{B} \perp \vec{r}$.



Direction of $\vec{B}$ will be found by using the rules of vector product.

Biot-Savart's Law ($\vec{B}$ Due to a Wire)

It is an experimental law. A current steady current '*i*' flows in a wire (may be straight or curved). Due to '$d\ell$' length of the wire the magnetic field at '*P*' is



$$dB \propto id\ell$$

$$dB \propto \frac{1}{r^2}$$

$\Rightarrow \quad dB \propto \sin\theta$

$\Rightarrow \quad dB \propto \dfrac{id\ell \sin\theta}{r^2} \quad \Rightarrow \quad dB = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{id\ell \sin\theta}{r^2} \quad \Rightarrow d\vec{B} = \left(\dfrac{\mu_0}{4\pi}\right)\dfrac{i\overrightarrow{d\ell}\times\vec{r}}{r^3}$

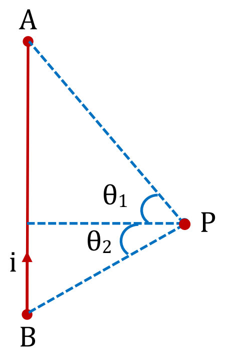
Here $\quad \vec{r}$ = Position vector of the test point w.r.t. $d\vec{\ell}$

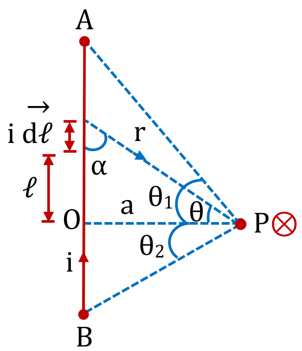
θ = Angle between $d\vec{\ell}$ and $\vec{r}$. The resultant $\vec{B} = \int d\vec{B}$

Using this fundamental formula, we can derive the expression of $\vec{B}$ due to a long wire.

Application of BIOT-SAVART LAW

Magnetic field surrounding a thin straight current carrying conductor





AB is a straight conductor carrying current *I* from *B* to *A*.

At a point *P*, whose perpendicular distance from *AB* is *OP* = *a*, the direction of field is perpendicular to the plane of paper, inwards (represented by a cross)

$\ell = a\tan\theta \Rightarrow dl = a\sec^2\theta d\theta \qquad$ ...(i)

$\alpha = 90° - \theta$ and $r = a\sec\theta$

By Biot-Savart's law

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{id\ell\sin\alpha}{r^2} \otimes$$

$\Rightarrow \quad B = dB = \int \dfrac{\mu_0}{4\pi}\dfrac{id\ell \sin\alpha}{r^2}$ (due to wire *AB*)

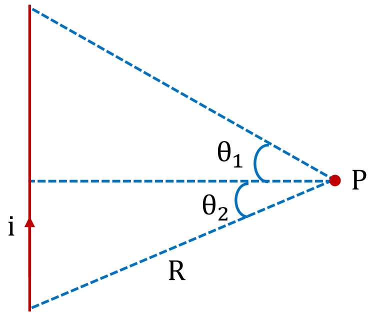
$\therefore \quad B = \dfrac{\mu_0 i}{4\pi} \int \cos\theta d\theta$

Taking limits of integration as $-\phi_2$ to ϕ_1

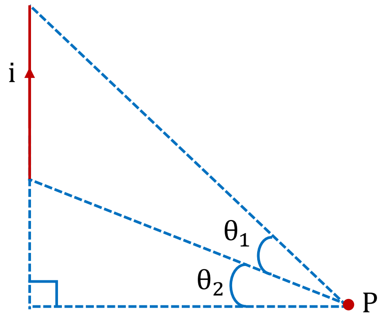
$$B = \frac{\mu_0 i}{4\pi}\int_{-\phi_2}^{\phi_1} \cos\theta d\theta = \frac{\mu_o i}{4\pi a}[\sin\theta]_{-\phi_2}^{\phi_1} = \frac{\mu_o i}{4\pi a}[\sin\phi_1 + \sin\phi_2] \text{ (inwards)}$$

How to take limit in $\boxed{B = \frac{\mu_0 i}{4\pi R}\int \cos\theta d\theta}$

R is perpendicular distance of P from wire.

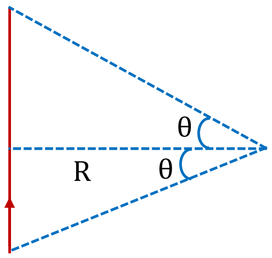
(a) 

$$B = \frac{\mu_0 i}{4\pi R}\int_{-\theta_2}^{\theta_1} \cos\theta d\theta$$

(b) 

$$B = \frac{\mu_0 i}{4\pi R}\int_{\theta_2}^{\theta_1+\theta_2} \cos\theta d\theta$$

Perpendicular bisector:

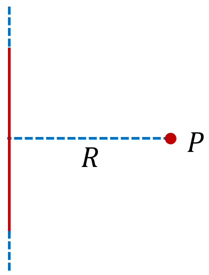


$$B = \frac{\mu_0 i}{4\pi R}\int_{-\theta}^{\theta} \cos\theta d\theta \Rightarrow B = \frac{\mu_0 i \sin\theta}{2\pi R}$$

(i) In the symmetric case where $\theta_2 = -\theta_1$, the field point P is located along the perpendicular bisector. If the length of the rod is $2L$, then $\theta_1 = L/\sqrt{L^2 + a^2}$ and the magnetic field is

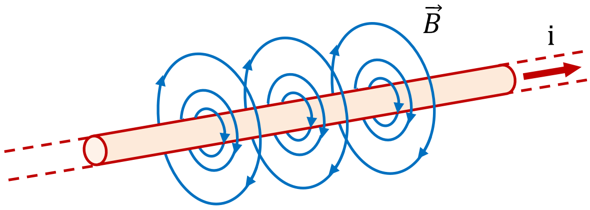
$$B = \frac{\mu_0 i}{2\pi a}\cos\theta_1 = \frac{\mu_0 i}{2\pi a}\frac{L}{\sqrt{L^2+a^2}}$$

Magnetic Field due to ∞ wire:



$$B = \frac{\mu_0 i}{4\pi R}\int_{-\pi/2}^{\pi/2} \cos\theta d\theta = \frac{\mu_o i(2)}{4\pi R} = \frac{\mu_o i}{2\pi R} \Rightarrow B = \frac{\mu_0 i}{2\pi R}$$

Magnetic field lines due to an infinite wire carrying current I :



In fact, the direction of the magnetic field due to a long straight wire can be determined by the right-hand rule (figure).

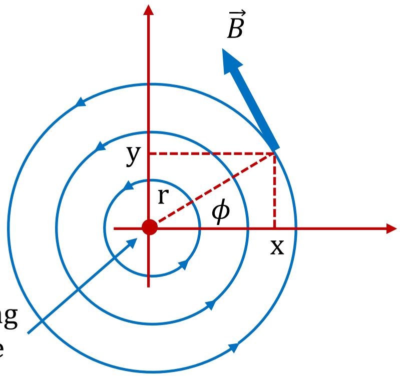
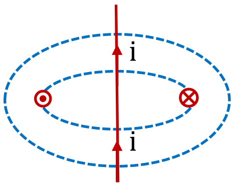


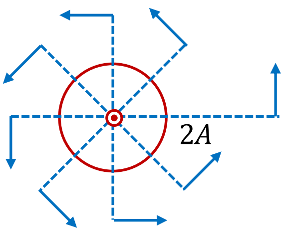
Figure: Direction of the magnetic field due to an infinite straight wire

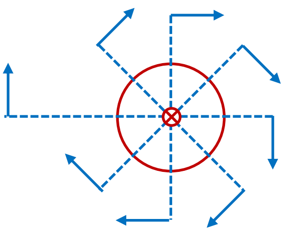
If you direct your right thumb along the direction of the current in the wire, then the fingers of your right-hand curl in the direction of the magnetic field. In cylindrical coordinates (r, ϕ, z) where the unit vectors are related by $\hat{r} \times \hat{\phi} = \hat{z}$, if the current flows in the $+z$ −direction, then, using the Biot-Savart law, the magnetic field must point in the ϕ-direction.

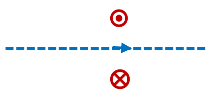
Note that in this limit, the system possesses cylindrical symmetry, and the magnetic field lines are circular.

Direction of MFI due to Straight Wire:



(eg.) 

(eg.) 

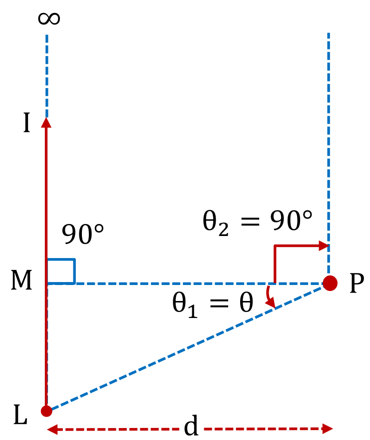
(eg.) 

In this case magnetic field lines are circular.

Hold the wire with right hand with thumb pointing along current then direction of curling of finger will give direction of MFI.

Illustration 1:

Find the magnetic field due to semi infinite length wire at point 'P'



Solution:

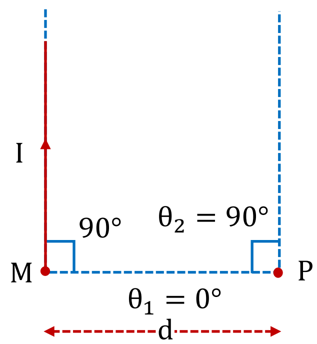
$$B_P = \frac{\mu_0 I}{4\pi d}\int_{-\theta}^{90^\circ} \cos\theta d\theta$$

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin\theta + \sin 90^\circ]$$

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin\theta + 1]$$

Illustration 2:

Find the magnetic field due to special semi-infinite length wire at point 'P'.



Solution:

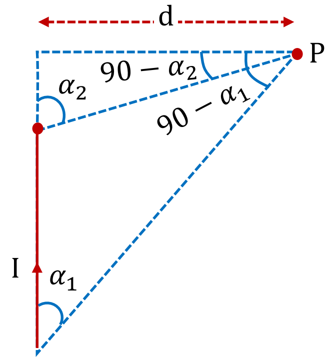
$$B_P = \frac{\mu_0 I}{4\pi d}\int_{0^\circ}^{90^\circ} \cos\theta d\theta$$

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin 0^\circ + \sin 90^\circ]$$

$$B_P = \frac{\mu_0 I}{4\pi d}$$

Illustration 3:

Find the magnetic field at point 'P' if point 'P' lies outside the line of wire.



Solution:

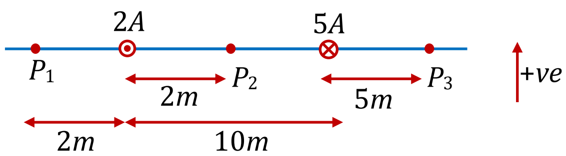
$$B_P = \frac{\mu_0 I}{4\pi d}\int_{-(90^\circ-\alpha_1)}^{-(90^\circ-\alpha_2)} \cos\theta d\theta$$

$$B_P = \frac{\mu_0 I}{4\pi d}[\sin(90^\circ-\alpha_1)-\sin(90^\circ-\alpha_2)]$$

$$= \frac{\mu_0 I}{4\pi d}[\cos\alpha_1 - \cos\alpha_2]$$

Illustration 4:

Find magnetic field at P_1, P_2 and P_3.



Solution:

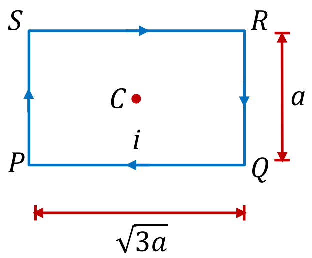
Magnetic field at $P_1 = -\frac{\mu_0(2)}{2\pi(2)} + \frac{\mu_0(5)}{2\pi(12)}$

Magnetic field at $P_2 = \frac{\mu_0(2)}{2\pi(2)} + \frac{\mu_0(5)}{2\pi(8)}$

Magnetic field at $P_3 = \frac{\mu_0(2)}{2\pi(15)} - \frac{\mu_0(5)}{2\pi(5)}$

Illustration 5:

Find resultant magnetic field at 'C' in the figure shown.



Solution:

It is clear that 'B' at 'C' due all the wires is directed ⊗. Also, B at 'C' due PQ and SR is same.
Also due to QR and PS is same,

$\therefore \quad B_{res} = 2(B_{PQ} + B_{SP})$

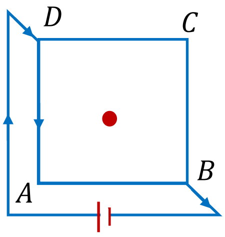
$$B_{PQ} = \frac{\mu_0 i}{4\pi\frac{a}{2}}(\sin 60° + \sin 60°)$$

$$B_{SP} = \frac{\mu_0 i}{4\pi\frac{\sqrt{3}a}{2}}(\sin 30° + \sin 30°)$$

$$\Rightarrow \quad B_{res} = 2\left(\frac{\sqrt{3}\mu_0 i}{2\pi a} + \frac{\mu_0 i}{2\pi a\sqrt{3}}\right) = \frac{4\mu_0 i}{\sqrt{3}\pi a}$$

Illustration 6:

Figure shows a square loop made from a uniform wire. Find the magnetic field at the centre of the square if a battery is connected between the points B and D as shown in the figure.

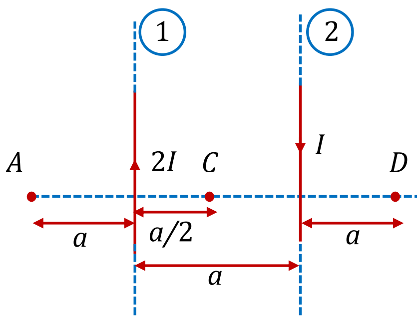


Solution:

The current will be equally divided at D. The fields at the centre due to the currents in the wires DA and DC will be equal in magnitude and opposite in direction. The resultant of these two fields will be zero. Similarly, the resultant of the fields due to the wires AB and CB will be zero. Hence, the net field at the centre will be zero.

Illustration 7:

In the figure shown there are two parallel long wires (placed in the plane of paper) are carrying currents $2I$ and I consider points A, C, D on the line perpendicular to both the wires and also in the plane of the paper. The distances are mentioned. Find



(i) $\vec{B}$ at A, C, D

(ii) position of point on line A, C, D where $\vec{B}$ is O

Solution:

(i) Let us call $\vec{B}$ due to (1) and (2) as $\vec{B}_1$ and $\vec{B}_2$ respectively. Then

At A: $\vec{B}_1$ is ⊙ and $\vec{B}_2$ is ⊗

$$B_1 = \frac{\mu_0 2I}{2\pi a} \text{ and } B_2 = \frac{\mu_0 I}{2\pi 2a}$$

$\therefore \quad B_{res} = B_1 - B_2 = \frac{3}{4}\frac{\mu_0 I}{\pi a}$ ⊙ **Ans.**

At C: $\vec{B}_1$ is ⊗ and $\vec{B}_2$ also ⊗

$\therefore \quad B_{res} = B_1 + B_2 = \frac{\mu_0 2I}{2\pi\frac{a}{2}} + \frac{\mu_0 I}{2\pi\frac{a}{2}} = \frac{6\mu_0 I}{2\pi a} = \frac{3\mu_0 I}{\pi a}$ ⊗ **Ans.**

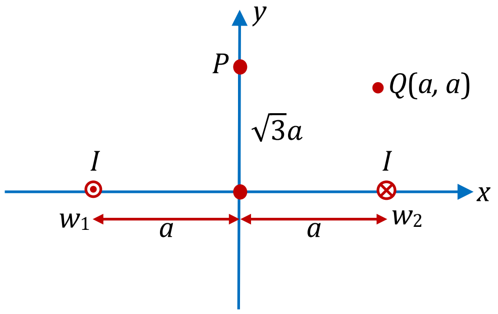
At D: $\vec{B}_1$ is ⊗ and $\vec{B}_2$ is ⊙ and both are equal in magnitude.

$\therefore \quad B_{res} = 0$ **Ans.**

(ii) It is clear from the above solution that $B = 0$ at point 'D'.

Illustration 8:

In the figure shown two long wires W_1 and W_2 each carrying current I are placed parallel to each other and parallel to z-axis. The direction of current in W_1 is outward and in W_2 it is inwards. Find the $\vec{B}$ at 'P' and 'Q'.



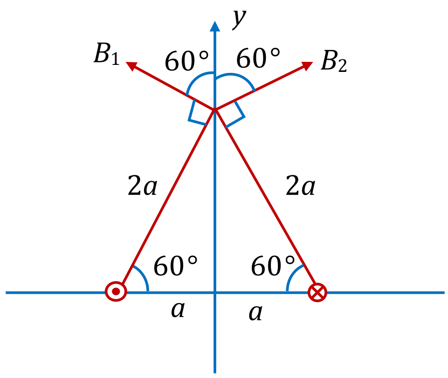
-Solution:

Let $\vec{B}$ due to W_1 be $\vec{B}_1$ and due to W_2 be $\vec{B}_2$.

By symmetry $|\vec{B}_1| = |\vec{B}_2| = B$

$$B_p = 2\,B\cos 60° = B = \frac{\mu_0 I}{2\pi 2a} = \frac{\mu_0 I}{4\pi a}$$

$$\therefore \quad \vec{B_p} = \frac{\mu_0 I}{4\pi a}\hat{j} \textbf{ Ans.}$$

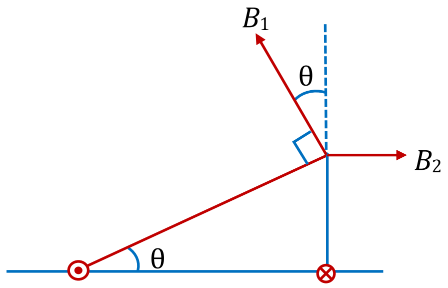


For $\quad QB_1 = \dfrac{\mu_0 I}{2\pi\sqrt{5}a} \quad \Rightarrow \quad B_2 = \dfrac{\mu_0 I}{2\pi a}$

$$\tan\theta = \frac{a}{2a} = \frac{1}{2}$$

$$\sin\theta = \frac{1}{\sqrt{5}}$$

$$\cos\theta = \frac{2}{\sqrt{5}}$$



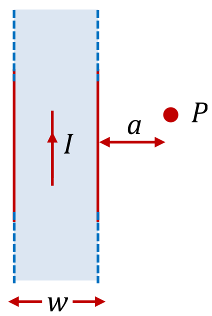
$$\Rightarrow \quad \vec{B} = (B_1\cos\theta\hat{j}) + (B_2 - B_1\sin\theta)\hat{i}$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 I}{5\pi a}\hat{j} + \left(\frac{\mu_0 I}{2\pi a} - \frac{\mu_0 I}{10\pi a}\right)\hat{i}$$

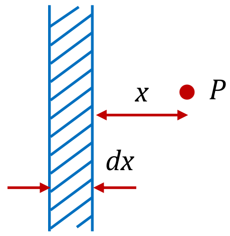
$$\Rightarrow \quad \vec{B} = \frac{2\mu_0 I}{5\pi a}\hat{i} + \frac{\mu_0 I}{5\pi a}\hat{j}$$

Illustration 9:

In the figure shown a large metal sheet of width 'w' carries a current I (uniformly distributed in its width 'w'). Find the magnetic field at point 'P' which lies in the plane of the sheet.



Solution:



To find 'B' at 'P' the sheet can be considered as collection of large number of infinitely long wires. Take a long wire distance 'x' from 'P' and of width 'dx'. Due to this the magnetic field at 'P' is 'dB'

$$dB = \frac{\mu_0\left(\frac{I}{w}dx\right)}{2\pi x} \otimes$$

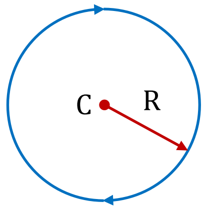
due to each such wire $\vec{B}$ will be directed inwards

$$\therefore \quad B_{res} = \int dB = \frac{\mu_0 I}{2\pi w}\int_{x=a}^{a+w}\frac{dx}{x} = \frac{\mu_0 I}{2\pi w}.\ln\frac{a+w}{a}$$

$\vec{B}$ Due to Circular Loop

(a) **At centre:** Due to each $\vec{d\ell}$ element of the loop $\vec{B}$ at 'c' is inwards (in this case).

$\therefore \quad \overrightarrow{B_{res}}$ at 'c' is $\otimes$



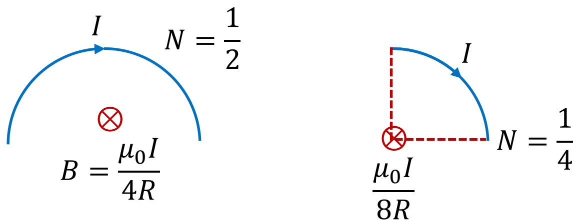
$$B = \frac{\mu_0 NI}{2R}$$

N = Number of turns in the loop

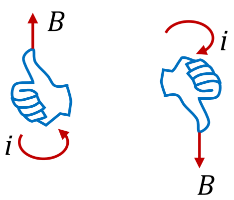
$= \frac{\ell}{2\pi R}$; ℓ = length of the loop

N can be fraction $\left(\frac{1}{4}, \frac{1}{3}, \frac{11}{3} etc.\right)$ or integer

Semi-circular and Quarter of a circle:



Direction of $\vec{B}$: The direction of the magnetic field at the centre of a circular wire can be obtained using the right-hand thumb rule. If the fingers are curled along the current, the stretched thumb will point towards the magnetic field (figure).

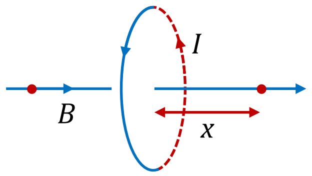


Another way to find the direction is to look into the loop along its axis. If the current is in anticlockwise direction, the magnetic field is towards the viewer. If the current is in clockwise direction, the field is away from the viewer.

(b) On the axis of the loop :

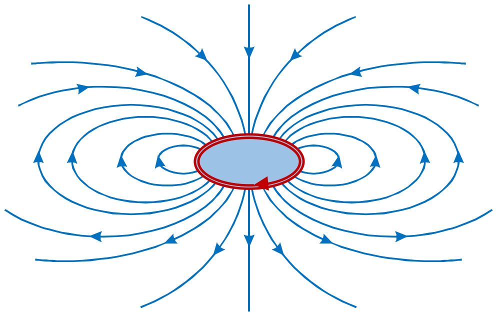
$$B = \frac{\mu_0 N I R^2}{2(R^2+x^2)^{3/2}}$$

N = No. of turns (integer)



Direction of $\vec{B}$: Direction can be obtained by right hand thumb rule. Curl your fingers in the direction of the current then the direction of the thumb points in the direction of at the points on the axis.

The magnetic field at a point not on the axis is mathematically difficult to calculate. We show qualitatively in figure the magnetic field lines due to a circular current which will give some idea of the field.



Ratio of B_{centre} and B_{axis}

The ratio of magnetic field at the centre of circular coil and on it's axis is given by

$$\frac{B_{\text{centre}}}{B_{\text{axis}}} = \left(1+\frac{x^2}{r^2}\right)^{3/2}$$

(i) If $x = \pm a, B_c = 2\sqrt{2}B_a$, $x = \pm\frac{a}{2}, B_c = \frac{5\sqrt{5}}{8}B_a$, $x = \pm\frac{a}{\sqrt{2}}, B_c = \left(\frac{3}{2}\right)^{3/2} B_a$

(ii) If $B_a = \frac{B_c}{n}$, then $x = \pm r\sqrt{(n^{2/3}-1)}$ and if $B_a = \frac{B_c}{\sqrt{n}}$, then $x = \pm r\sqrt{(n^{1/3}-1)}$

(c) Magnetic field at very large/very small distance from the centre

(i) If $x >> r$ (very large distance) $\Rightarrow B_{\text{axis}} = \frac{\mu_0}{4\pi}.\frac{2\pi N i r^2}{x^3} = \frac{\mu_0}{4\pi}.\frac{2NiA}{x^3}$

where $A = \pi r^2$ = Area of each turn of the coil.

(ii) If $x << r$ (very small distance) $\Rightarrow B_{\text{axis}} \neq B_{\text{centre}}$, but by using binomial theorem and neglecting higher power of $\frac{x^2}{r^2}$.

$$B_{\text{axis}} = B_{\text{centre}}\left(1-\frac{3}{2}\frac{x^2}{r^2}\right)$$

***B-x* curve**

The variation of magnetic field due to a circular coil as the distance x varies as shown in the figure.

B varies non-linearly with distance x as shown in figure and is maximum when $x^2 = min = 0$, i.e., the point is at the centre of the coil and it is zero at $x = \pm\infty$.

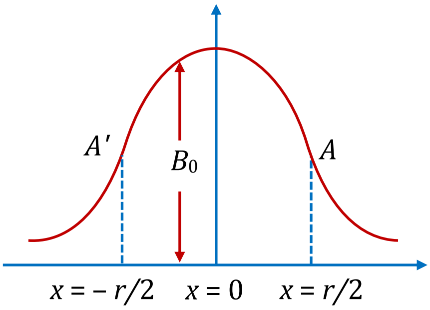
Point of inflection (A and A') : Also known as points of curvature change or points of zero curvature.

(i) At these points B varies linearly with $x \Rightarrow \frac{dB}{dx} = \text{constant} \Rightarrow \frac{d^2B}{dx^2} = 0$.

(ii) They locates at $x = \pm\frac{r}{2}$ from the centre of the coil.

(iii) Separation between point of inflection is equal to radius of coil (r).

(iv) Application of points of inflection is "Helmholtz coils" arrangement.



Note:

The magnetic field at $x = \frac{r}{2}$ is $B = \frac{4\mu_0 Ni}{5\sqrt{5}r}$

Helmholtz Coils

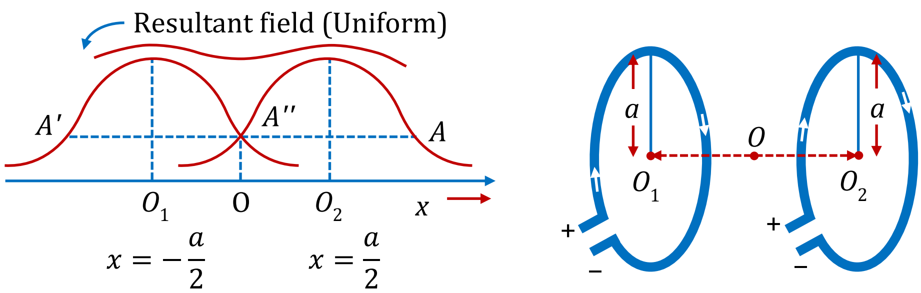
(i) This is the set-up of two coaxial coils of same radius such that distance between their centres is equal to their radius.

(ii) These coils are used to obtain uniform magnetic field of short range which is obtained between the coils.

(iii) At axial mid point O, magnetic field is given by $B = \frac{8\mu_0 Ni}{5\sqrt{5}R} = 0.716\frac{\mu_0 Ni}{R} = 1.432B$, where $B = \frac{\mu_0 Ni}{2R}$.

(iv) Current direction is same in both coils otherwise this arrangement is not called Helmholtz's coil arrangement.

(v) Number of points of inflection $\Rightarrow$ Three (A, A', A'')

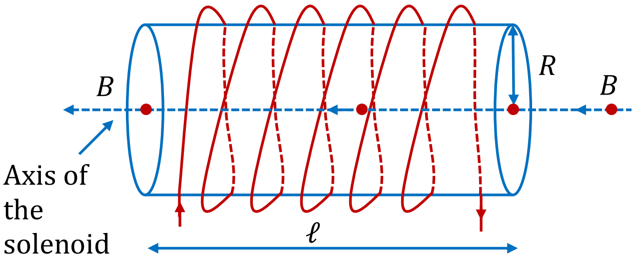


Note:

The device whose working principle based on this arrangement and in which uniform magnetic field is used called as "Helmholtz galvanometer".

Solenoid:

(i) Solenoid contains large number of circular loops wrapped around a non-conducting cylinder. (it may be a hollow cylinder or it may be a solid cylinder)



(ii) The winding of the wire is uniform direction of the magnetic field is same at all points of the axis.

(iii) Turns should be very close to each other.

$\vec{B}$ on axis

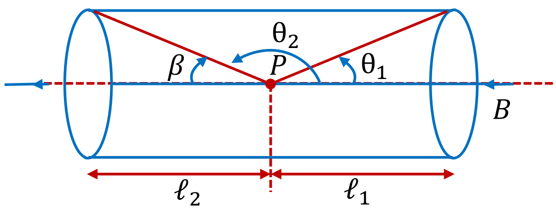
$$B = \frac{\mu_0 ni}{2}(\cos\theta_1 - \cos\theta_2)$$

where n : number of turns per unit length.

$$\cos\theta_1 = \frac{\ell_1}{\sqrt{\ell_1^2+R^2}}\ ;\ \cos\theta = \frac{\ell_2}{\sqrt{\ell_2^2+R^2}} = -\cos\theta_2$$

$$B = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2+R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2+R^2}}\right]$$

$$= \frac{\mu_0 ni}{2}(\cos\theta_1 + \cos\beta)$$

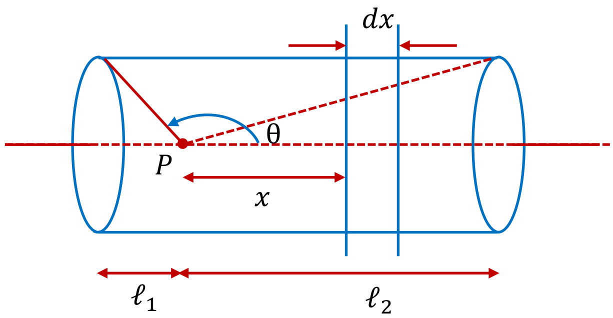


Note:

Use right hand rule for direction (same as the direction due to loop).

Derivation:

Take an element of width dx at a distance x from point P. [Point P is the point on axis at which we are going to calculate magnetic field]. Total number of turns in the element $dn = ndx$ where n : number of turns per unit length.



$$dB = \frac{\mu_0 iR^2}{2(R^2+x^2)^{3/2}}(ndx)$$

$$B = \int dB = \int_{-\ell_1}^{\ell_2}\frac{\mu_0 i\,R^2\,ndx}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 ni}{2}\left[\frac{\ell_1}{\sqrt{\ell_1^2+R^2}} + \frac{\ell_2}{\sqrt{\ell_2^2+R^2}}\right] = \frac{\mu_0 ni}{2}[cos\theta_1 - cos\theta_2]$$

(iv) $\vec{B}$ at the midpoint for 'Ideal Solenoid' :

For $\ell >> R$ or length is infinite

$\theta_1 \to 0°\ ;\ \theta_2 \to \pi$

$$B = \frac{\mu_0 ni}{2}[1-(-1)]$$

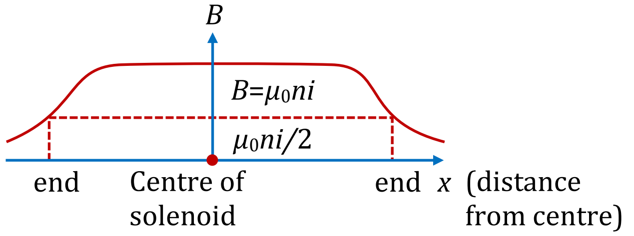
$$B = \mu_0 ni$$

If material of the solid cylinder has relative permeability 'μ_r' then $B = \mu_0 \mu_r n i$

At the ends $B = \frac{\mu_0 n i}{2}$

(v) Comparison between ideal and real solenoid:

(a) Ideal Solenoid (b) Real Solenoid



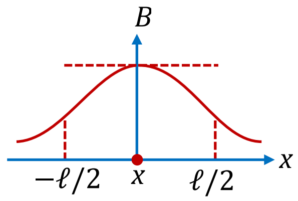


Illustration 10:

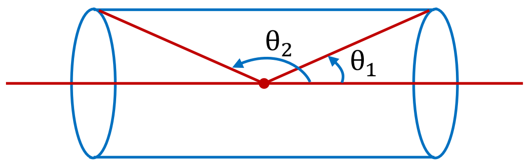
A solenoid of length 0.4 m and diameter 0.6 m consists of a single layer of 1000 turns of fine wire carrying a current of 5.0×10^{-3} ampere. Find the magnetic field on the axis at the middle and at the ends of the solenoid. (Given $\mu_0 = 4\pi \times 10^{-7} \frac{V-s}{A-m}$).

Solution:

$B = \frac{1}{2}\mu_0 n i\,[\cos\theta_1 - \cos\theta_2] \quad \Rightarrow \quad n = \frac{1000}{0.4} = 2500$ per meter

$i = 5 \times 10^{-3}\ A.$

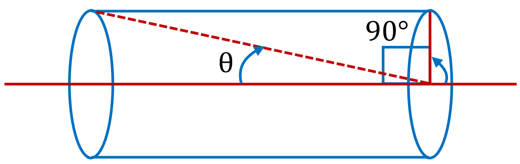
(i) $\cos\theta_1 = \frac{0.2}{\sqrt{(0.3)^2+(0.2)^2}} = \frac{0.2}{\sqrt{0.13}}$



$\cos\theta_2 = \frac{-0.2}{\sqrt{0.13}}$

$\Rightarrow \quad B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3}\ \frac{2\times 0.2}{\sqrt{0.13}} = \frac{\pi \times 10^{-5}}{\sqrt{13}} T$

(ii) At the end



$\cos\theta_1 = \frac{0.4}{\sqrt{(0.3)^2+(0.4)^2}} = 0.8$

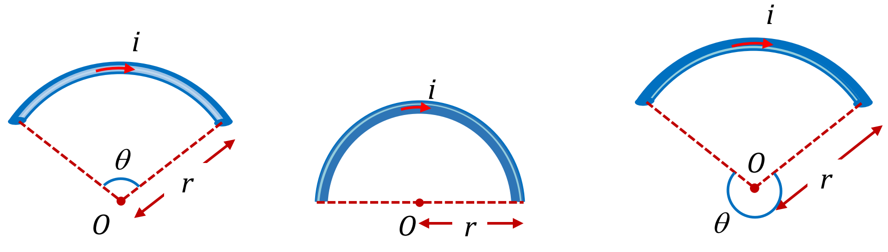
$\Rightarrow \quad \cos\theta_2 = \cos 90° = 0$

$B = \frac{1}{2} \times (4 \times \pi \times 10^{-7}) \times 2500 \times 5 \times 10^{-3} \times 0.8$

$\Rightarrow \quad B = 2\pi \times 10^{-6} Wb/m^2$

Magnetic field due to current carrying circular arc

Magnetic field at centre O



$B = \frac{\mu_0}{4\pi} \cdot \frac{\theta i}{r}$ $\quad B = \frac{\mu_0}{4\pi} \cdot \frac{\pi i}{r} = \frac{\mu_0 i}{4r}$ $\quad B = \frac{\mu_0}{4\pi} \cdot \frac{(2\pi - \theta) i}{r}$

Special results

If magnetic field at the centre of circular coil is denoted by $B_0 \left(= \frac{\mu_0}{4\pi} \cdot \frac{2\pi i}{r}\right)$.

Magnetic field at the centre of arc which is making an angle θ at the centre is $B_{arc} = \left(\frac{B_0}{2\pi}\right) . \theta$

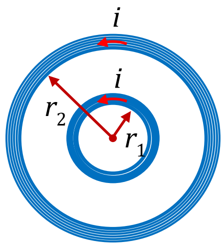
Angle at centre	Magnetic field at centre in term of B_0
360° (2π)	B_0
180° (π)	B_0 / 2
120° (2π/3)	B_0 / 3
90° (π/2)	B_0 / 4
60° (π/3)	B_0 / 6
30° (π/6)	B_0 / 12

Concentric circular loops ($N = 1$)

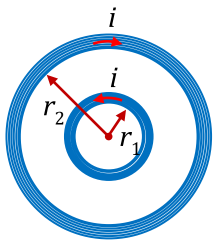
Coplanar and concentric:

It means both coils are in same plane with common centre

(a) Current in same direction (b) Current in opposite direction



$$B_1 = \frac{\mu_0}{4\pi} 2\pi i \left(\frac{1}{r_1} + \frac{1}{r_2}\right) = B_1 = \frac{\mu_0}{2} i \left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$



$$B_2 = \frac{\mu_0}{4\pi} 2\pi i \left[\frac{1}{r_1} - \frac{1}{r_2}\right] = B_2 = \frac{\mu_0}{2} i \left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

Note:

$$\frac{B_1}{B_2} = \left(\frac{r_2 + r_1}{r_2 - r_1}\right)$$

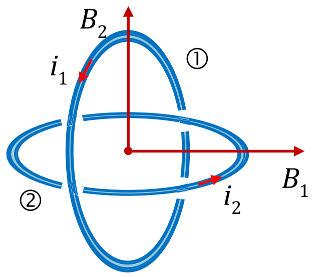
Non-coplanar and concentric:

Plane of both coils are perpendicular to each other.

Magnetic field at common centre,

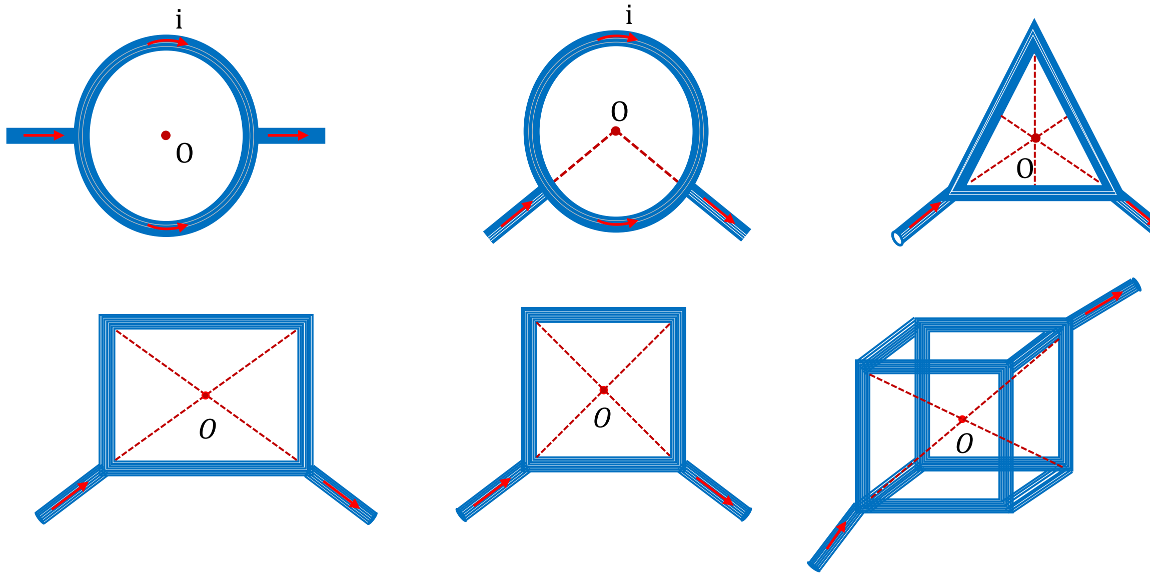
$$B = \sqrt{B_1^2 + B_2^2}$$

$$= \frac{\mu_0}{2r}\sqrt{i_1^2 + i_2^2}$$



Zero magnetic field:

If in a symmetrical geometry, current enters from one end and exists from the other, then magnetic field at the centre is zero.



In all cases at centre $B = 0$

Ampere's Law

Ampere's law gives another method to calculate the magnetic field due to a given current distribution. Consider any closed, plane curve (as shown in figure). Assign a sense to the curve by putting an arrow on the curve. Using the right-hand thumb rule, assign one side of the plane as positive and the other as negative. If you curl the fingers of the right hand along the arrow on the curve, the stretched thumb gives the positive side. The positive side can also be determined by looking into the loop along its axis. If the arrow on the loop is anticlockwise, the positive side is towards the viewer. If the arrow is clockwise, the positive side is away from the viewer. In figure, the positive side is going into the plane of the diagram.

Take a small length-element $d\vec{l}$ on the curve and let $\vec{B}$ be the resultant magnetic field at the position of $d\vec{l}$. Calculate the scalar product $\vec{B}.d\vec{l}$ and integrate by varying $d\vec{l}$ on the closed curve. This integration is called line integral or circulation of $\vec{B}$ along the curve and is represented by the symbol

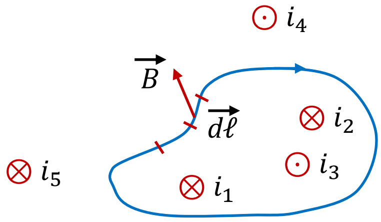
$$\oint \vec{B}.d\vec{l}.$$

Now look at the currents crossing the area bounded by the curve. A current directed towards the positive side of the plane area is taken as positive and a current directed towards the negative side is take as negative. Ampere's law then states:

The circulation $\oint \vec{B}.d\vec{l}$. of the resultant magnetic field along a closed, plane curve is equal to μ_0 times the total current crossing the area bounded by the closed curve provided the electric field inside the loop remains constant. Thus,

$$\oint \vec{B}.d\vec{l} = \mu_0 i$$

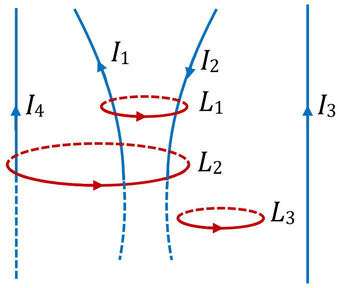
In figure, the positive side is going into the area of the diagram so that i_1 and i_2 are positive and i_3 is negative. Thus, the total current crossing the area is $i_1 + i_2 - i_3$. Any current outside the area is not included in writing the right-hand side of above equation. The magnetic field $\vec{B}$ on the left-hand side is the resultant field due to all the currents existing anywhere.



Ampere's law may be derived from the Biot-Savart law and Biot-Savart law may be derived from the Ampere's law. Thus, the two are equivalent in scientific content. However, Ampere's law is more useful under certain symmetrical conditions. In such cases, the mathematics of finding the magnetic field becomes much simple if we use the Ampere's law.

Illustration 11:

Find the values of $\oint \vec{B}.d\vec{\ell}$ for the loops L_1, L_2, L_3 in the figure shown. The sense of $d\vec{\ell}$ is mentioned in the figure.



Solution:

For $\quad L_1 = \oint \vec{B}.d\vec{\ell}\ \ \mu_0(I_1 - I_2)$

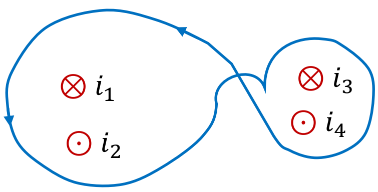
Here I_1 is taken positive because magnetic lines of force produced by I_1 is anti clockwise as seen from top. I_2 produces lines of $\vec{B}$ in clockwise sense as seen from top. The sense of $d\vec{\ell}$ is anticlockwise as seen from top.

For $\quad L_2 : \oint \vec{B}.d\vec{\ell} = \mu_0(I_1 - I_2 + I_4)$

for $\quad L_3 : \oint \vec{B}.d\vec{\ell} = 0$

Illustration 12:

Find the values of $\oint \vec{B}.d\vec{\ell}$ for the loops



Solution:

$$\oint \vec{B}.d\vec{l} = \mu_0(i_2 - i_1 + i_3 - i_4)$$

Application of Ampere's Law:

For several applications, a much-simplified version of above equation proves sufficient. We shall assume that, in such cases, it is possible to choose the loop (called an Amperian loop) such that at each point of the loop, either

(i) B is tangential to the loop and is a non-zero constant B, or

(ii) B is normal to the loop, or

(iii) B vanishes

Now, let L be the length (part) of the loop for which B is tangential. Let I_e be the current enclosed by the loop. Then, by Ampere's law

$$BL = \mu_0 I_e$$

Magnetic field due to infinite long thin current carrying straight conductor:

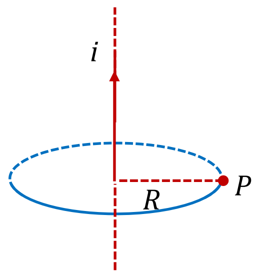
$\vec{B}$ will have circular lines. $\vec{d\ell}$ is also taken tangent to the circle.

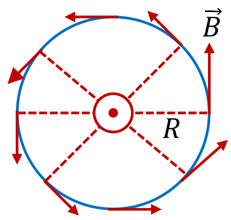
$$\oint \vec{B}.\vec{d\ell} = \oint Bd\ell$$

$\because$ $\theta = 0°$ so $B\ d \oint d\ell = B\ 2\pi R$ ($\because B$ = const.)

Now by amperes law: $B\ 2\pi R = \mu_0 i$

$\therefore$ $B = \dfrac{\mu_0 i}{2\pi R}$





Magnetic field due to Hollow current carrying infinitely long cylinder:

(I is uniformly distributed on the whole circumference)

(i) For $r \geq R$

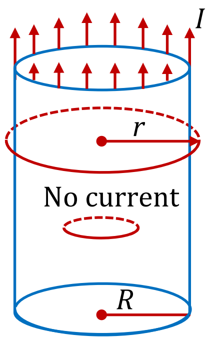
By symmetry the amperian loop is a circle

$$\oint \vec{B}.\vec{d\ell} = \oint Bd\ell$$

$\because$ $\theta = 0$

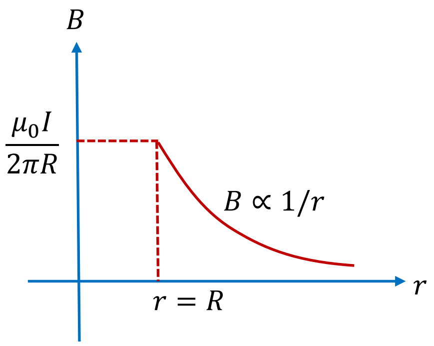
$$= B\int_0^{2\pi r} d\ell \quad \because B = \text{const.}$$

$\Rightarrow$ $B = \dfrac{\mu_0 I}{2\pi r}$



(ii) $r < R$

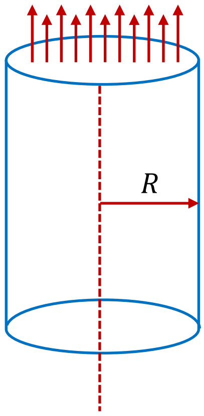
$$\oint \vec{B}.\vec{d\ell} = \oint Bd\ell$$



$$= B(2\pi\, r) = 0 \Rightarrow B_{in} = 0$$

Magnetic field due to Solid infinite current carrying cylinder:

Assume current is uniformly distributed on the whole cross section area



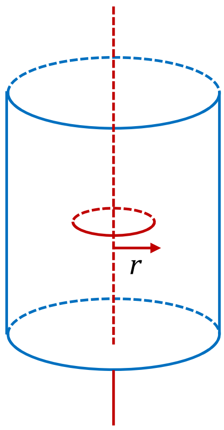
Current density $J = \frac{I}{\pi R^2}$

Case (I) : $r \leq R$

Take an amperian loop inside the cylinder. By symmetry it should be a circle whose centre is on the axis of cylinder and its axis also coincides with the cylinder axis on the loop.

$$\oint \vec{B}.d\vec{\ell} = \oint B.d\ell = B\oint d\ell = B.2\pi r = \mu_0 \frac{I}{\pi R^2}\pi r^2$$

$$B = \frac{\mu_0 I r}{2\pi R^2} = \frac{\mu_0 J r}{2} \Rightarrow \vec{B} = \frac{\mu_0 \vec{J}\times\vec{r}}{2}$$



Case (II) : $r \geq R$

$$\oint \vec{B}.d\vec{\ell} = \oint B.d\ell = B\oint d\ell = B.2\pi r = \mu_0.I$$

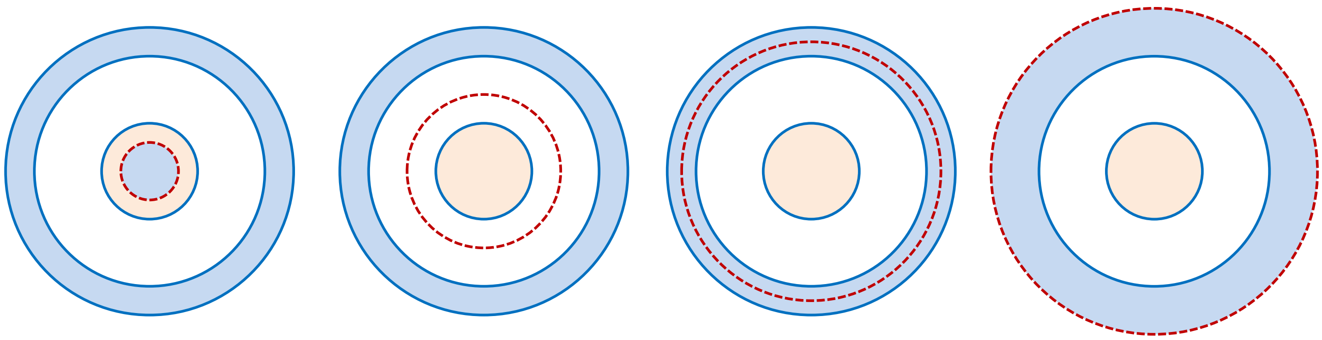
$$\Rightarrow \quad B = \frac{\mu_0 I}{2\pi r}$$

Also, $\vec{B} = \frac{\mu_0 I}{2\pi r}(\hat{J} \times \hat{r}) = \frac{\mu_0 J \pi R^2}{2\pi r}$

Illustration 13:

Consider a coaxial cable which consists of an inner wire of radius a surrounded by an outer shell of inner and outer radii b and c respectively. The inner wire carries an electric current i_0 and the outer shell carries an equal current in same direction. Find the magnetic field at a distance x from the axis where (a) $x < a$, (b) $a < x < b$ (c) $b < x < c$ and (d) $x > c$. Assume that the current density is uniform in the inner wire and also uniform in the outer shell.

Solution:



(a) (b) (c) (d)

A cross-section of the cable is shown in figure. Draw a circle of radius x with the centre at the axis of the cable. The parts a, b, c and d of the figure correspond to the four parts of the problem. By symmetry, the magnetic field at each point of a circle will have the same magnitude and will be tangential to it. The circulation of B along this circle is, therefore,

$$\oint \vec{B}.d\vec{\ell} = B2\pi x$$

in each of the four parts of the figure.

(a) The current enclosed within the circle is

$$\frac{i_0}{\pi a^2}.\pi x^2 = \frac{i_0}{a^2}x^2$$

Ampere's law

$$\oint \vec{B}.d\vec{\ell} = \mu_0 i$$

$$B.2\pi x = \frac{\mu_0 i_0 x^2}{a^2} \quad \text{or,} \quad B = \frac{\mu_0 i_0 x}{2\pi a^2}$$

The direction will be along the tangent to the circle.

(b) The current enclosed within the circle in part b is i_0 so that

$$B\,2\pi x = \mu_0\, i_0 \text{ or, } B = \frac{\mu_0 i_0}{2\pi x}$$

(c) The area of cross-section of the outer shell is $\pi c^2 - \pi b^2$. The area of cross-section of the outer shell within the circle in part c of the figure is $\pi x^2 - \pi b^2$

Thus, the current through this part is $\frac{i_0(x^2-b^2)}{(c^2-b^2)}$. This is in the same direction to the current i_0 in the inner wire. Thus, the net current enclosed by the circle is

$$i_{net} = i_0 + \frac{i_0(x^2-b^2)}{c^2-b^2} = \frac{i_0(c^2+x^2-2b^2)}{c^2-b^2}$$

From Ampere's law,

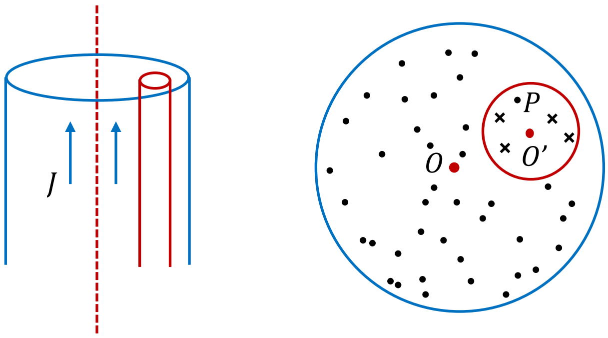
$$B\,2\pi x = \frac{i_0(c^2+x^2-2b^2)}{c^2-b^2} \text{ or } B = \frac{\mu_0 i_0(c^2+x^2-2b^2)}{2\pi x(c^2-b^2)}$$

(d) The net current enclosed by the circle in part d of the figure is $2i_0$ and hence

$$B\,2\pi x = \mu_0 2i_0 \text{or } B = \frac{\mu_0 i_0}{\pi x}$$

Illustration 14:

$\vec{B}$ inside cylindrical cavity inside solid cylinder, at P?

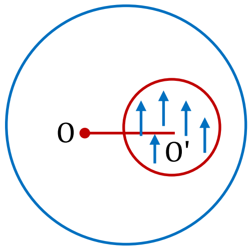


Solution:

$$B = \frac{\mu_0}{2}\vec{j} \times O\vec{P} + \frac{\mu_0}{2}(-\vec{j} \times O'\vec{P})$$

$$B = \frac{\mu_0}{2}\vec{j} \times (O\vec{P} + P\overrightarrow{O'})$$

$$B = \frac{\mu_0}{2}\vec{j} \times (\overrightarrow{OO'})$$



Non-Uniform Current Density:

Consider long, cylindrical conductor of radius R carrying a current I with a non-uniform current density $J = \alpha r$.

Where α is a constant.

The problem can be solved by using the Ampere's law:

$$\oint \vec{B}.d\vec{s} = \mu_0 I_{enc}$$

Where the enclosed current I_{enc} is given by

$$I_{enc} = \int \vec{J}.d\vec{A} = \int (\alpha r')(2\pi r' dr')$$

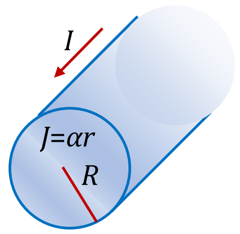


Figure : Non-uniform current density

(a) For $r < R$, the enclosed current is

$$I_{enc} = \int_0^r 2\pi\alpha r' 2 dr' = \frac{2\pi\alpha r^3}{3}$$

Applying Ampere's law, the magnetic field at P_1 is given by

$$B_1(2\pi r) = \frac{2\mu_0 \pi \alpha r^3}{3} \quad \text{or} \quad B_1 = \frac{\alpha\mu_0}{3} r^2$$

This direction of the magnetic field $\vec{B}_1$ is tangential to the Amperian loop which encloses the correct.

(b) For $r > R$, the enclosed current is :

$$I_{enc} = \int_0^R 2\pi\alpha r'^2 dr' = \frac{2\pi\alpha R^3}{3}$$

Which yields $B_2(2\pi r) = \dfrac{2\mu_0 \pi \alpha R^3}{3}$

Thus, the magnetic field at a point P_2 outside the conductor is

$$B_2 = \frac{\alpha\mu_0 R^3}{3r}$$

A plot of B as a function of r is shown in figure.

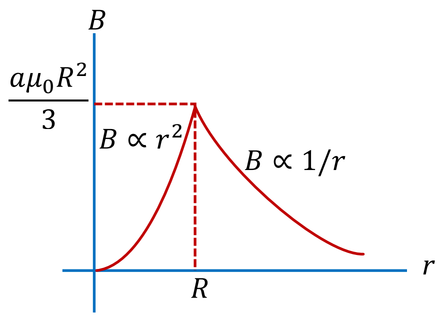
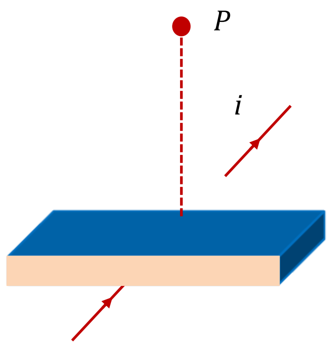
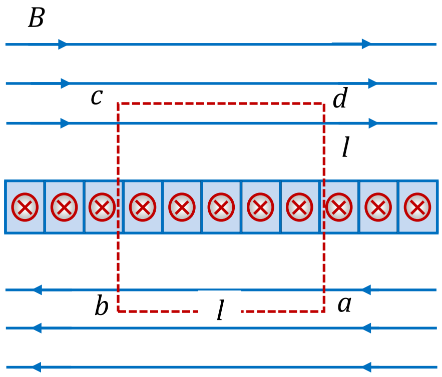


Figure : The magnetic field as a function of distance away from the conductor

Magnetic Field Due to an Infinite Sheet Carrying Current

The figure shows an infinite sheet of current with linear current density j (A/m). Due to symmetry the field line pattern above and below the sheet is uniform. Consider a square loop of side l as shown in the figure.





$$\int_a^b \vec{B}\,.\,\vec{dl} + \int_b^c \vec{B}\,.\,\vec{dl} + \int_c^d \vec{B}\,.\,\vec{dl} + \int_d^a \vec{B}\,.\,\vec{dl} = \mu_0 i \quad \text{(By Ampere's law)}$$

Since $B \perp dl$ along the path $b \to c$ and $d \to a$, therefore, $\int_b^c \vec{B}\,.\,\vec{dl} = 0;\ \int_d^a \vec{B}\,.\,\vec{dl} = 0$

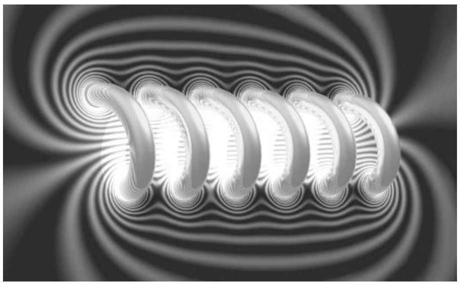
Also, $B \,||\, dl$ along the path $a \to b$ and $c \to d$, thus $\int_a^b \vec{B}\,.\,\vec{dl} + \int_d^a \vec{B}\,.\,\vec{dl} = 2Bl$

The current enclosed by the loop is $i = jl$.

Therefore, according to Ampere's law $2Bl = \mu_0(jl)$ or $B = \frac{\mu_0 j}{2}$

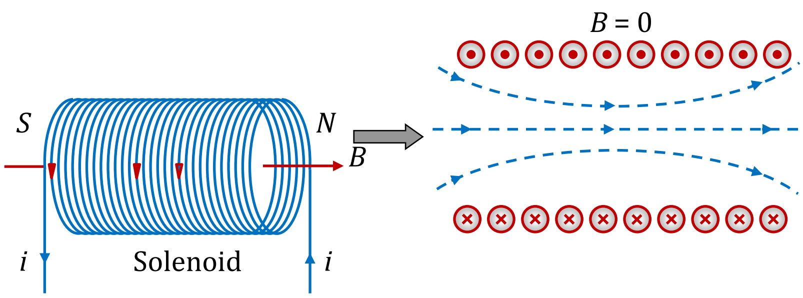
Solenoid:

It is a coil which has length and used to produce uniform magnetic field of long range. It consists a conducting wire which is tightly wound over a cylindrical frame in the form of helix. All the adjacent turns are electrically insulated to each other. The magnetic field at a point on the axis of a solenoid can be obtained by superposition of field due to large number of identical circular turns having their centres on the axis of solenoid.



Magnetic field due to a long solenoid:

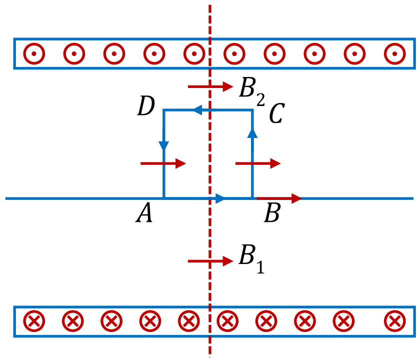
A solenoid is a tightly wound helical coil of wire. If length of solenoid is large, as compared to its radius, then in the central region of the solenoid, a reasonably uniform magnetic field is present. Figure shows a part of long solenoid with number of turns/length n. We can find the field by using Ampere circuital law. Ideal solenoid: For an ideal solenoid its length must be very larger is comparison to its radius. For an ideal solenoid $r \ << \ L$. 'n' turns per unit length.



A magnetic field is produced around and within the solenoid. The magnetic field within the solenoid is uniform and parallel to the axis of solenoid.

From previous formula:

$$B_{\text{inside}} = \frac{\mu_0 ni}{2}[\sin 90° + \sin 90°] = \boxed{(\mu_0 ni)}$$



Also

$$\int_A^B \vec{B}.d\vec{\ell} + \int_B^C \vec{B}.d\vec{\ell} + \int_C^D \vec{B}.d\vec{\ell} + \int_D^A \vec{B}.d\vec{\ell} = \mu_0(0)$$

$$B_1\ell + 0 + B_2\ell\cos 180° = 0°$$

$$B_1 - B_2 = 0$$

$$\boxed{B_1 = B_2}$$

Consider a rectangular loop $ABCD$. For this loop $\oint_{ABCD}\vec{B}.d\vec{\ell} = \mu_0 i_{enc}$

Now,

$$\oint_{ABCD}\vec{B}.d\vec{\ell} = \oint_{AB}\vec{B}.d\vec{\ell} + \oint_{BC}\vec{B}.d\vec{\ell} + \oint_{CD}\vec{B}.d\vec{\ell} + \oint_{DA}\vec{B}.d\vec{\ell} = B \times a$$

This is because

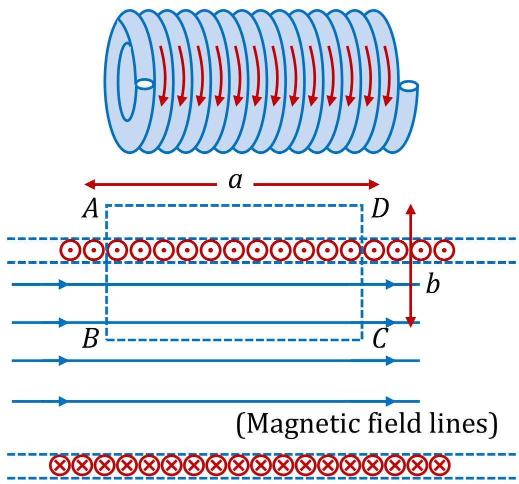
$$\oint_{AB}\vec{B}.d\vec{\ell} = \oint_{CD}\vec{B}.d\vec{\ell} = 0, \vec{B} \perp d\vec{\ell}$$

And $\oint_{DA}\vec{B}.d\vec{\ell} = 0$

($\because \vec{B}$ outside the solenoid is negligible)

Now, $i_{enc} = (n \times a) \times i \Rightarrow B \times a = \mu_0(n \times a \times i)$

$$\Rightarrow B = \mu_0 ni$$

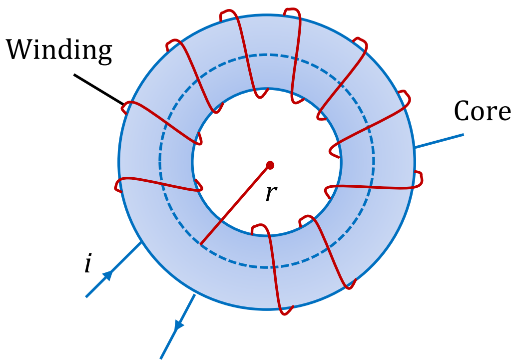


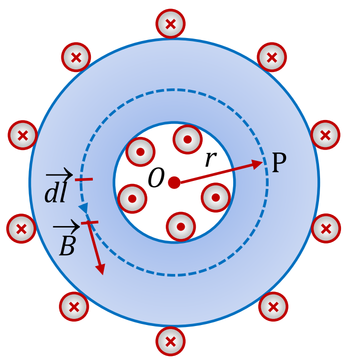
For ideal solenoid:

1. MFI inside is uniform and axial
2. MFI outside solenoid is zero

Toroid:

A toroid can be considered as a ring-shaped closed solenoid. Hence it is like an endless cylindrical solenoid.





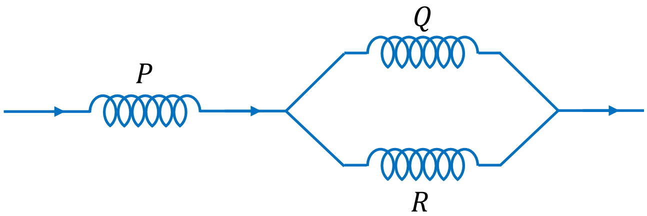
Consider a toroid having n turns per unit length. Magnetic field at a point P in the figure is given as

$$B = \frac{\mu_0 Ni}{2\pi r} = \mu_o ni$$

where $n = \frac{N}{2\pi r}$

Illustration 15:

Three identical long solenoids P, Q and R are connected to each other as shown in figure. If the magnetic field at the centre of P is $4T$, what would be the field at the centre of Q ? Assume that the field due to any solenoid is confined within the volume of that solenoid only.



Solution:

As the solenoids are identical, the currents in Q and R will be the same and will be half the current in P. The magnetic field within a solenoid is given by $B = \mu_0 ni$. Hence the field in Q will be equal to the field in R and will be half the field in P i.e., will be $2T$.

Magnetic force on moving charge

When a charge q moves with velocity $\vec{v}$, in a magnetic field $\vec{B}$, then the magnetic force experienced by moving charge is given by following formula:

$\vec{F} = q(\vec{v} \times \vec{B})$ Put q with sign.

$\vec{v}$: Instantaneous velocity

$\vec{B}$: Magnetic field at that point.

Note:

- $\vec{F} \perp \vec{v}$ and also $\vec{F} \perp \vec{B}$
- $\because$ $\vec{F} \perp \vec{v}$ $\therefore$ power due to magnetic force on a charged particle is zero.

  (Use the formula of power $P = \vec{F} \cdot \vec{v}$ for its proof).
- Since work done by magnetic force is zero in every part of the motion. The magnetic force cannot increase or decrease the speed (or kinetic energy) of a charged particle. Its can only change the direction of velocity.
- On a stationary charged particle, magnetic force is zero.
- If $\vec{V}||\vec{B}$, then also magnetic force on charged particle is zero. It moves along a straight line if only magnetic field is acting.

Illustration 16:

A charged particle of mass $5\,\mu g$ and charge $q = +2\mu C$ has velocity $\vec{v} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$. Find out the magnetic force on the charged particle and its acceleration at this instant due to magnetic field $\vec{B} = 3\hat{\jmath} - 2\hat{k}$. $\vec{v}$ and $\vec{B}$ are in m/s and Wb/m^2 respectively.

Solution:

$\vec{F} = q\vec{v} \times \vec{B} = 2 \times 10^{-6}\,(2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}) \times (3\hat{\jmath} - 2\hat{k})$

$= 2 \times 10^{-6}\,[-6\hat{\imath} + 4\hat{\jmath} + 6\hat{k}]\,N$

By Newton's Law $\vec{a} = \dfrac{\vec{F}}{m} = \dfrac{2\times10^{-6}}{5\times10^{-6}}(-6\hat{\imath} + 4\hat{\jmath} + 6\hat{k})$

$= 0.8\,(-3\hat{\imath} + 2\hat{\jmath} + 3\hat{k})\,m/s^2$

Motion of charged particles under the effect of magnetic force

- Particle released if $v = 0$ then $f_m = 0$

  $\therefore$ Particle will remain at rest

- When $\vec{V} || \vec{B}$ then $\theta = 0$ or $\theta = 180°$

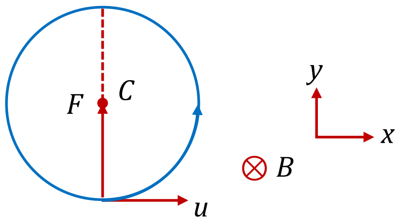
  $\therefore$ $F_m = 0$ $\therefore$ $\vec{a} = 0$ $\therefore$ $\vec{V}$ = const.

  $\therefore$ Particle will move in a straight line with constant velocity

- Initial velocity $\vec{u} \perp \vec{B}$ and $\vec{B}$ = uniform

  In this case $\because$ B is in z direction so the magnetic force in $z-$direction will be zero $\left(\because \overrightarrow{F_m} \perp \vec{B}\right)$.

  Now there is no initial velocity in z-direction.

  $\therefore$ Particle will always move in xy plane.

  $\therefore$ Velocity vector is always $\perp \vec{B}$

  $\therefore$ $F_m = quB$ = constant

  Now $quB = \frac{mu^2}{R}$ $\Rightarrow R = \frac{mu}{qB}$ = constant.

  The particle moves in a curved path whose radius of curvature is same everywhere, such curve in a plane is only a circle.

  $\therefore$ Path of the particle is circular.

  $$R = \frac{mu}{qB} = \frac{p}{qB} = \frac{\sqrt{2mk}}{qB}$$

  Here, $p =$ Linear momentum; $k =$ Kinetic energy

  Now, $v = \omega R \Rightarrow \omega = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$

  Time period, $T = 2\pi m/qB$

  Frequency, $f = qB/2\pi m$

Note:

ω, f, T are independent of velocity.

Illustration 17:

A proton (p), $\alpha-$particle and deuteron (D) are moving in circular paths with same kinetic energies in the same magnetic field. Find the ratio of their radii and time periods.

(Neglect interaction between particles).

Solution:

$$R = \frac{\sqrt{2mK}}{qB}$$

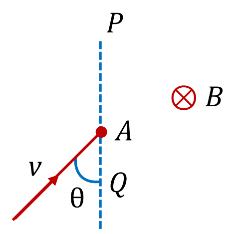
$\therefore$ $R_p : R_\alpha : R_D = \frac{\sqrt{2mK}}{qB} : \frac{\sqrt{2.4mK}}{2qB} : \frac{\sqrt{2.2mK}}{qB} = 1 : 1 : \sqrt{2}$

$T = 2\pi m/qB$

$\therefore$ $T_p : T_\alpha : T_D = \frac{2\pi m}{qB} : \frac{2\pi 4m}{2qB} : \frac{2\pi 2m}{qB} = 1 : 2 : 2$ Ans.

Illustration 18:

A positive charge particle of charge q, mass m enters into a uniform magnetic field with velocity v as shown in the figure. There is no magnetic field to the left of PQ. Find



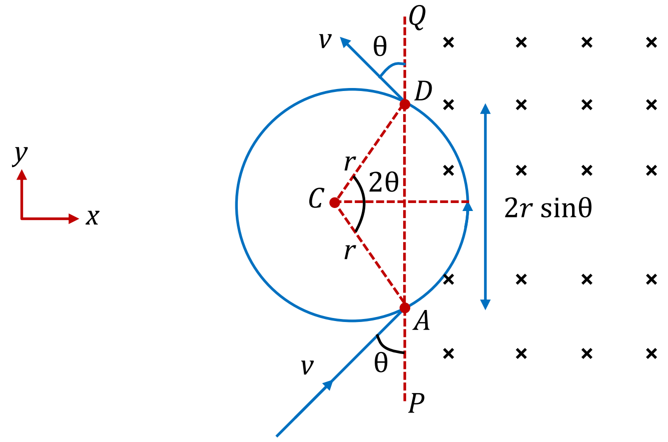
(i) time spent

(ii) distance travelled in the magnetic field

(iii) impulse of magnetic force.

Solution:

The particle will move in the field as shown



Angle subtended by the arc at the centre = 2θ

(i) Time spent by the charge in magnetic field $\omega t = 2\theta \Rightarrow \frac{qB}{m}t = 2\theta \Rightarrow t = \frac{2m\theta}{qB}$

(ii) Distance travelled by the charge in magnetic field = $r(2\theta) = \frac{mv}{qB}.2\theta$

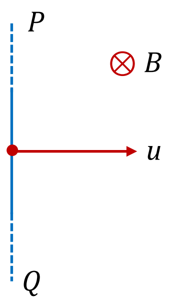
(iii) Impulse = change in momentum of the charge

$= (-mv\ \sin\theta\hat{i} + mv\ \cos\ \theta\hat{j}) - (mv\ \sin\theta\hat{i} + mv\ \cos\theta\hat{j})$

$= -2\ mv\ \sin\theta\hat{i}$

Illustration 19:

In the figure shown the magnetic field on the left of 'PQ' is zero and on the right of 'PQ' it is uniform. Find the time spent in the magnetic field.



Solution:

The path will be semi-circular time spent = $T/2 = \pi m/qB$

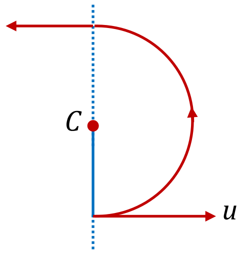
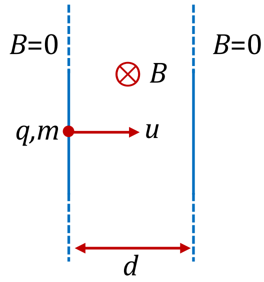


Illustration 20:

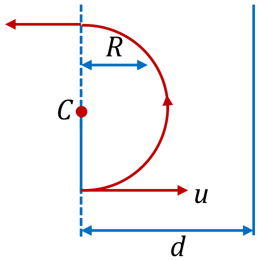
A uniform magnetic field of strength 'B' exists in a region of width 'd'. A particle of charge 'q' and mass 'm' is shot perpendicularly (as shown in the figure) into the magnetic field. Find the time spend by the particle in the magnetic field if



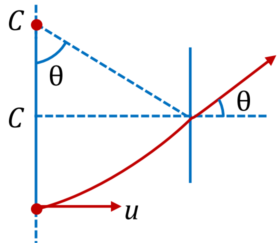
(i) $d > \frac{mu}{qB}$ (ii) $d < \frac{mu}{qB}$

Solution:

(i) $d > \frac{mu}{qB}$ means $d > R$ $\therefore\ t = \frac{T}{2} = \frac{\pi m}{qB}$



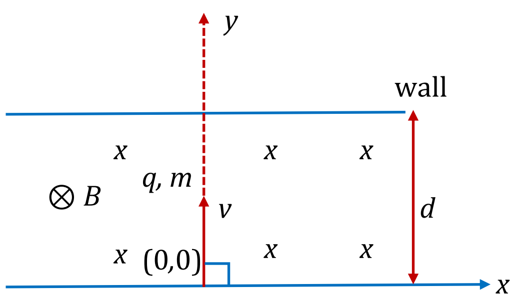
(ii) $\sin\theta = \frac{d}{R}$



$\theta = \sin^{-1}\left(\frac{d}{R}\right)$; $\omega t = \theta \Rightarrow t = \frac{m}{qB}\sin^{-1}\left(\frac{d}{R}\right)$

Illustration 21:

What should be the speed of charged particle so that it can't collide with the upper wall? Also find the coordinate of the point where the particle strikes the lower plate in the limiting case of velocity.



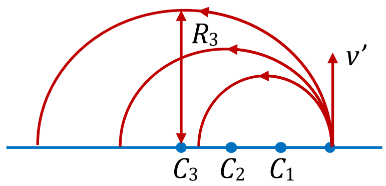
Solution:

(i) The path of the particle will be circular larger the velocity, larger will be the radius.

For particle not to strike $R < d$

$\therefore \quad \dfrac{mv}{qB} < d$

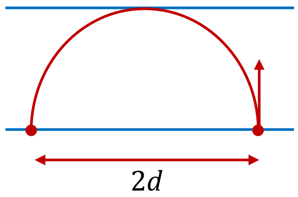
$\Rightarrow \quad v < \dfrac{qBd}{m}$



(ii) For limiting case

$v = \dfrac{qBd}{m}$; $R = d$

$\therefore \quad$ Coordinate = $(-2d, 0, 0)$

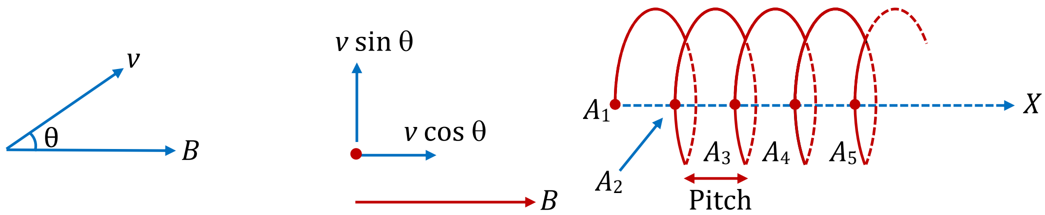


Helical path:

If the velocity of the charge is not perpendicular to the magnetic field, we can break the velocity in two components – $v_{\parallel}$, parallel to the field and $v_{\perp}$, perpendicular to the field. The components $v||$ remains unchanged as the force $q\vec{v} \times \vec{B}$ is perpendicular to it. In the plane perpendicular to the field, the particle traces a circle of radius $r = \dfrac{mv_{\perp}}{qB}$ as given by equation. The resultant path is helix.

Complete analysis:

Let a particle have initial velocity in the plane of the paper and a constant and uniform magnetic field also in the plane of the paper.



The particle starts from point A_1.

It completes its one revolution at A_2 and 2nd revolution at A_3 and so on. X-axis is the tangent to the helix points.

$A_1, A_2, A_3,$ .... all are on the x-axis.

Distance $\quad A_1A_2 = A_3A_4 = ... = v \cos\theta . T$ = pitch

where $\quad T$ = Time period

Let the initial position of the particle be $(0, 0, 0)$ and $v \sin\theta$ in $+y$ direction. Then in x :

$$F_x = 0, a_x = 0, v_x = \text{constant} = v \cos\theta, x = (v \cos\theta)t$$

In *y*-*z* plane :

From figure it is clear that $y = R \sin\beta, v_y = v \sin\theta \cos\beta$

$z = -(R - R\cos\beta)$

$v_z = v \sin\theta \sin\beta$

Acceleration towards centre = $\dfrac{(v \sin\theta)^2}{R} = \omega^2 R$

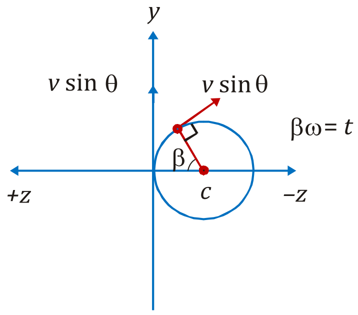
$\therefore \quad a_y = -\omega^2 R \sin\beta, a_z = -\omega^2 R \cos\beta$

At any time: the position vector of the particle (or its displacement w.r.t. initial position)

$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$, where x, y, z already found

Velocity, $\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$, where v_x, v_y, v_z already found

$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$, where a_x, a_y, a_z already found



Radius $q(v\sin\theta)B = \dfrac{m(v\sin\theta)^2}{R}$

$\Rightarrow \quad R = \dfrac{mv\sin\theta}{qB}$

$\Rightarrow \quad \omega = \dfrac{v\sin\theta}{R} = \dfrac{qB}{m} = \dfrac{2\pi}{T} = 2\pi f.$

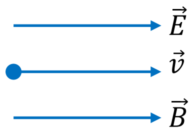
Lorentz Force on Moving Charge (F_L):

When a charge is moving in a region, where both electric field $\vec{E}$ and magnetic field $\vec{B}$ exist, then electric and magnetic forces are acting on it. The resultant of these forces called electromagnetic or Lorentz force on charge. (zero gravity).

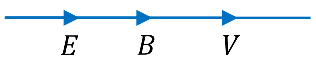
$$\vec{F}_L = \vec{F}_e + \vec{F}_m \Rightarrow \vec{F}_L = q\vec{E} + q(\vec{v} \times \vec{B})$$

Illustration 22:

A particle is moving with velocity $\vec{v}$ in a region where $\vec{E}$ and $\vec{B}$ are parallel to $\vec{v}$. Discuss the path of particle.



Solution:



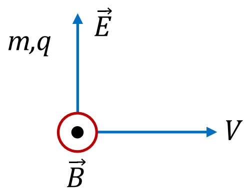
$\vec{E} \parallel \vec{B} \parallel \vec{v}$

In above situation particle passes undeviated but its velocity will change due to electric field. Magnetic force on it = 0.

Path will be straight line with increasing speed.

Illustration 23:

A particle is moving with velocity $\vec{v}$ in a region where $\vec{E}$, $\vec{B}$ and $\vec{v}$ are mutually perpendicular. Find v such that particle moves constant velocity.



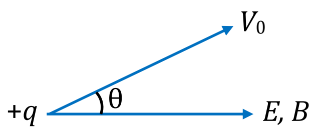
Solution:

$qE = qvB$

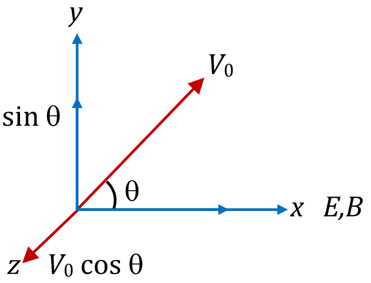
$v = \dfrac{E}{B}$

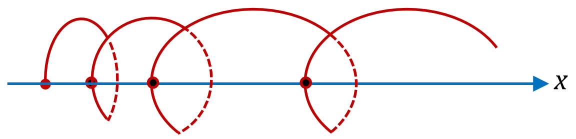
Illustration 24:

A particle is moving with velocity $\vec{v}$ enter a region where $\vec{E}$ and $\vec{B}$ are parallel to each other and $\vec{v}$ at an angle θ with it. $\vec{E}$ and $\vec{B}$ are constant and uniform and $\theta \neq 0, 180°$.



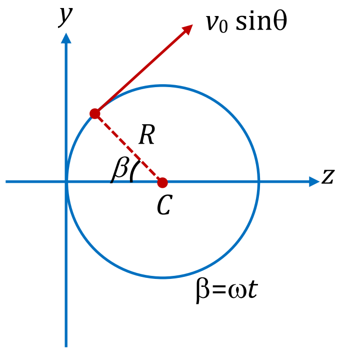
Solution:





In x: $F_x = qE, a_x = \frac{qE}{m}, v_x = v_0\cos\theta + a_x t, x = v_0 t + \frac{1}{2}a_x t^2$

In yz plane :



$$qv_0\sin\theta\, B = m(v_0\sin\theta)^2/R \Rightarrow \quad R = \frac{mv_0\sin\theta}{qB}$$

$$\omega = \frac{v_0\sin\theta}{R} = \frac{qB}{m} = \frac{2\pi}{T} = 2\pi f$$

$$\vec{r} = \left\{(v_0\cos\theta)t + \frac{1}{2}\frac{qE}{m}t^2\right\}\hat{\imath} + R\sin\omega t\hat{\jmath} + (R - R\cos\omega t)(-k)$$

$$\vec{v} = \left(v_0\cos\theta + \frac{qE}{m}t\right)\hat{\imath} + (v_0\sin\theta)\cos\omega t\hat{\jmath} + v_0\sin\theta\sin\omega t\,(-\hat{k})$$

$$\vec{a} = \frac{qE}{m}\hat{\imath} + \omega^2 R[-\sin\beta\hat{\jmath} - \cos\beta\hat{k}]$$

Illustration 25:

A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{\imath}$ and magnetic field $\vec{B} = B_0\hat{\imath}$ with velocity $\vec{v} = v_0\hat{\jmath}$. The speed of the particle will become $2v_0$ after a time:

(A) $t = \frac{2mv_0}{qE}$ (B) $t = \frac{2Bq}{mv_0}$ (C) $t = \frac{\sqrt{3}Bq}{mv_0}$ (D) $t = \frac{\sqrt{3}mv_0}{qE}$

Ans. (D)

Solution:

Charged particle will move in a Helical path.

$$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$$

$$\vec{a} = \frac{qE\hat{\imath}}{m} + \frac{qv_0\times B}{m}$$

$$v_x = \frac{qE}{b}.t$$

$$v = \sqrt{v_x^2 + v_0^2}$$

$$2v_0 = \sqrt{\left(\frac{qE}{m}t\right)^2 + v_0^2}$$

$$\sqrt{3}v_0 = \frac{qE}{m}t$$

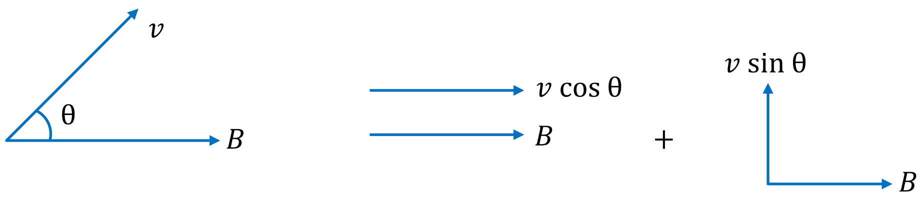
$$t = \frac{\sqrt{3}mv_0}{qE}$$

Illustration 26:

A particle of m, q accelerated through potential difference V_0 enters a uniform magnetic field B at angle θ with it. Find radius and pitch of helix.

Solution:

$$V_0 \times q = \frac{1}{2}mv^2 \quad \Rightarrow \quad v = \sqrt{\frac{2V_0}{m}q}$$



$$T = \frac{2\pi m}{qB} \qquad \Rightarrow \quad R = \frac{mv_{\perp}}{qB} = \frac{m\sqrt{2V_0q}}{qB\sqrt{m}}\sin\theta = \sqrt{\frac{2m}{q}V_0}\frac{1}{B}\sin\theta$$

$$P = v_{\parallel}\cos\theta \times T = \sqrt{\frac{2V_0q}{m}}(\cos\theta) \times \frac{2\pi m}{qB} = 2\sqrt{\frac{2mV_0}{q}}\frac{\pi}{B}\cos\theta$$

Velocity Selector:

In the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$, the total force on a charged particle is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

This is known as the Lorentz force. By combining the two fields, particles which, move with a certain velocity can be selected. This was the principle used by J.J. Thomson to measure the charge-to-mass ratio of the electrons. In figure the schematic diagram of Thomson's apparatus is depicted.

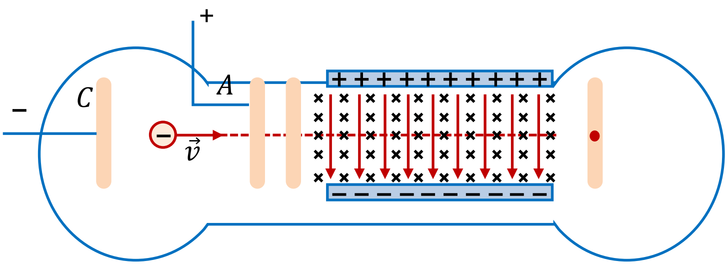


Figure: Thomson's apparatus

The electrons with charge $q = -e$ and mass m are emitted from the cathode C and then accelerated toward slit A. Let the potential difference between A and C be $V_A - V_c = \Delta V$. The change in potential energy is equal to the external work done in accelerating the electrons: $\Delta U = W_{ext} = q\Delta V = -e\Delta V$. By energy conservation, the kinetic energy gained is $\Delta K = -\Delta U = \frac{mv^2}{2}$. Thus, the speed of the electrons is given by

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

If the electrons further pass through a region where there exists a downward uniform electric field, the electrons, being negatively charged, will be deflected upward. However, if in addition to the electric field, a magnetic field directed into the page is also applied, then the electrons will move in a straight path.

From equation we see that when the condition for the cancellation of the two forces is given by $eE = evB$. Which implies

$$v = \frac{E}{B}$$

In other words, only those particles with speed $v = \frac{E}{B}$ will be able to move in a straight line. Combining the two equations, we obtain

$$\frac{e}{m} = \frac{E^2}{2(\Delta V)B^2}$$

By measuring $E, \Delta V$ and B, the change-to-mass ratio can be readily determined. The most precise measurement to date is $\frac{e}{m} = 1.758820174(71) \times 10^{11}\ C/kg$.

Mass Spectrometer:

Various methods can be used to measure the mass of an atom. One possibility is through the use of a mass spectrometer. The basic feature of a Bainbridge mass spectrometer is illustrated in Figure. A particle carrying a charge $+q$ is first sent through a velocity selector.

The applied electric and magnetic fields satisfy the relation $E = vB$ so that the trajectory of the particle is a straight line. Upon entering a region where a second magnetic fields $\vec{B}_0$ pointing into the page has been applied, the particle will move in a circular path with radius r and eventually strike the photographic plate. Using equation, we have

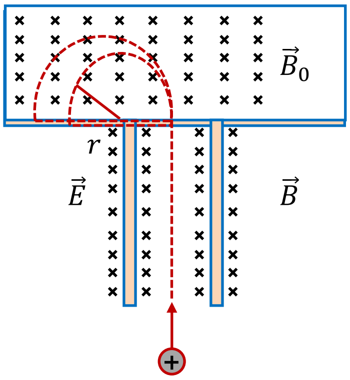


Figure: A Bainbridge mass spectrometer

$$r = \frac{mv}{qB_0}$$

Since $v = E/B$, the mass of the particle can be written as $m = \frac{qB_0 r}{v} = \frac{qB_0 Br}{E}$

Illustration 27:

Particle A with charge q and mass m_A and particle B with charge $2q$ and mass m_B are accelerated from rest by a potential difference ΔV and subsequently deflected by a uniform magnetic field into semi-circular paths. The radii of the trajectories by particle A and B are R and $2R$, respectively. The direction of the magnetic field is perpendicular to the velocity of the particle. What is their mass ratio?

Solution:

The kinetic energy gained by the charges is equal to

$$\frac{1}{2}mv^2 = q\Delta V$$

Which yields, $v = \sqrt{\frac{2q\Delta V}{m}}$

The charges move in semicircles, since the magnetic force points radially inward and provides the source of the centripetal force:

$$\frac{mv^2}{r} = qvB$$

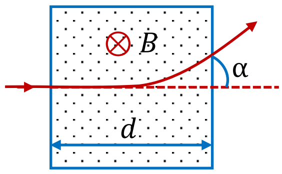
The radius of the circle can be obtained as:

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$

Which gives $\frac{m_A}{m_B} = \frac{1}{8}$

Illustration 28:

A proton accelerated by a potential difference $V = 500\,kV$ flies through a uniform transverse magnetic field with induction $B = 0.51\,T$. The field occupies a region of space $d = 10\,cm$ in thickness. Find the angle α through which the proton deviates from the initial direction of its motion.



Solution:

From the figure,

$$\sin\alpha = \frac{d}{R} = \frac{dqB}{mv}$$

As radius of the arc $R = \frac{mv}{qB}$, where v is the velocity of the particle, when it enters into the field.

From initial condition of the problem,

$$qV = \frac{1}{2}mv^2 \text{ or } v = \sqrt{\frac{2qV}{m}}$$

Hence, $\sin\alpha = \frac{dqB}{m\sqrt{\frac{2qV}{m}}} = dB\sqrt{\frac{q}{2mV}}$

And on putting the values. $\alpha = \sin^{-1}\left(dB\sqrt{\frac{q}{2mV}}\right) = 30°$

Illustration 29:

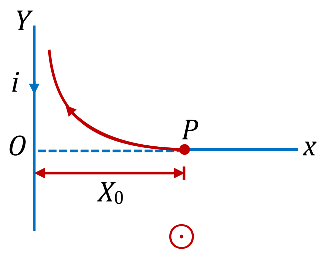
A long, straight wire carries a current i. A particle having a positive charge q and mass m kept at a distance x_0 from the wire is projected towards it with a speed v as shown in figure. Find the minimum separation between the wire and the particle

Solution:

Let the particle be initially at P (figure). Take the wire as the Y −axis and the foot of perpendicular from P to the wire as the origin. Take the line OP as the X-axis. We have, $OP = x_0$. The magnetic field B at any point to the right of the wire is along the positive Z −axis. The magnetic force on the particle is, therefore, in the $X - Y$ plane. As there is no initial velocity along the Z-axis, the motion will be in the $X - Y$ plane. Also, its speed remains unchanged. As the magnetic field is not uniform, the particle does not go along a circle. The force at time t is $\vec{F} = q\vec{v} \times \vec{B}$

$$= q(v_x\hat{\imath} + v_y\hat{\jmath}) \times \left(\frac{\mu_0 i}{2\pi x}\hat{k}\right)$$

$$= -\hat{\jmath}qv_x\frac{\mu_0 i}{2\pi x} + \hat{\imath}qv_y\frac{\mu_0 i}{2\pi x}.$$



Thus $a_x = \frac{F_x}{m} = \frac{\mu_0 qi}{2\pi m} = \lambda\frac{v_y}{x}$ ...(i)

where $\lambda = \frac{\mu_0 qi}{2\pi m}$

Also, $a_x = \frac{dv_x}{dt} = \frac{dv_x}{dx}\frac{dx}{dt} = \frac{v_x dv_x}{dx}$ ...(ii)

As, $v_x^2 + v_y^2 = v^2$,

Giving $v_x\, dv_x = -\, v_y\, dv_y$. ...(iii)

From (i), (ii) and (iii), $\dfrac{v_y dv_y}{dx} = \dfrac{-\lambda v_y}{x}$ or $\dfrac{dx}{x} = \dfrac{-dv_y}{\lambda}$

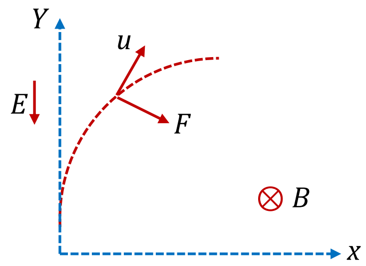
Initially $x = x_0$ and $v_y = 0$. At minimum separation from the wire, $v_x = 0$, so that $v_y = v$.

Thus $\int_{x_0}^{x} \dfrac{dx}{x} = \int_0^v \dfrac{dv_y}{-\lambda}$ or, $\ln \dfrac{x}{x_0} = \dfrac{v}{\lambda}$ or, $x = x_0\, e^{v/\lambda} = x_0 e^{\frac{2\pi m v}{\mu_0 q i}}$

Illustration 30:

An electron is released from the origin at a place where a uniform electric field E and a uniform magnetic field B exist along the negative Y-axis and the negative Z-axis respectively. Find the displacement of the electron along the Y-axis when its velocity becomes perpendicular to the electric field for the first time.

Solution:



Let us take axes as shown in figure. According to the right-handed system, the Z-axis is upward in the figure and hence the magnetic field is shown downwards. At any time, the velocity of the electron may be written as

$$\vec{u} = u_x\hat{\imath} + u_y\hat{\jmath}$$

The electric and magnetic fields may be written as

$$\vec{E} = -E\hat{\jmath} \text{ and } \vec{B} = -B\vec{k}$$

Respectively, the force on the electron is $\vec{F} = -e(\vec{E} + \vec{u}\times\vec{B}) = eE\hat{\jmath} + eB(u_y\hat{\imath} - u_x\hat{\jmath})$

Thus, $F_x = eu_y B$ and $F_y = e(E - u_x B)$

The components of the acceleration are

$$a_x = \frac{du_x}{dt} = \frac{eB}{m}u_y \qquad \text{...(i)}$$

and $$a_y = \frac{du_y}{dt} = \frac{e}{m}(E - u_x B) \qquad \text{...(ii)}$$

We have,

$$\frac{d^2u_y}{dt^2} = -\frac{eB}{m}\frac{du_x}{dt} = -\frac{eB}{m}.\frac{eB}{m}u_y = -\omega^2 u_y$$

where $\omega = \dfrac{eB}{m}$ ...(iii)

This equation is similar to that for a simple harmonic motion. Thus,

$$u_y = A\sin(\omega t + \delta) \qquad \text{...(iv)}$$

and hence, $\dfrac{du_y}{dt} = A\omega\cos(\omega t + \delta)$ ...(v)

At $t = 0, u_y = 0$ and $\dfrac{du_y}{dt} = \dfrac{eE}{m}$

Putting in (iv) and (v),

$$\delta = 0 \text{ and } A = \frac{eE}{m\omega}\frac{E}{B}$$

Thus, $u_y = \dfrac{E}{B}\sin\omega t$

The path of the electron will be perpendicular to the Y-axis when $u_y = 0$. This will be the case for the first time at t where

$$\sin\omega t = 0 \quad \text{or} \quad \omega t = \pi \quad \text{or} \quad t = \frac{\pi}{\omega} = \frac{\pi m}{eB}$$

Also, $u_y = \frac{dy}{dt} = \frac{E}{B}\sin\omega t$

or $\int_0^y dy = \frac{E}{B}\sin\omega\, t\, dt$

or $y = \frac{E}{B\omega}(1 - \cos\omega t)$.

At $t = \frac{\pi}{\omega}$, $y = \frac{E}{B\omega}(1 - \cos\pi) = \frac{2E}{B\omega}$

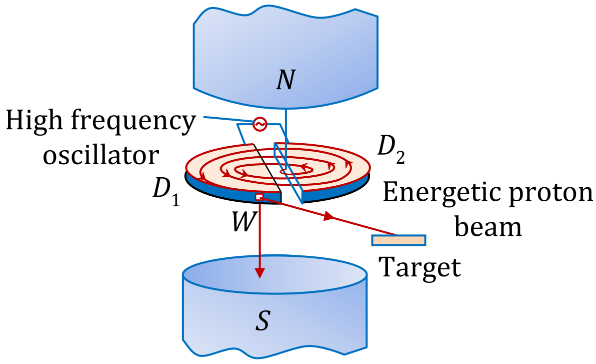
Thus, the displacement along the $Y -$ axis is $\frac{2E}{B\omega} = \frac{2Em}{BeB} = \frac{2Em}{eB^2}$. **Ans.**

Cyclotron

Cyclotron is a device used to accelerate positively charged particles like, α-particles, deuterons *etc*. to acquire enough energy to carry out nuclear disintegration etc.

It is based on the fact that the electric field accelerates a charged particle and the magnetic field keeps it revolving in circular orbits of constant frequency.

It consists of two hollow D –shaped metallic chambers D_1 and D_2 called dees. The two dees are placed horizontally with a small gap separating them. The dees are connected to the source of high frequency electric field. The dees are enclosed in a metal box containing a gas at a low pressure of the order of 10^{-3} mm mercury. The whole apparatus is placed between the two poles of a strong electromagnet NS as shown in fig. The magnetic field acts perpendicular to the plane of the dees.



(1) **Cyclotron frequency:** Time taken by ion to describe a semi-circular path is given by

$$t = \frac{\pi r}{v} = \frac{\pi m}{qB}$$

If T = time period of oscillating electric field then $T = 2t = \frac{2\pi m}{qB}$ the cyclotron frequency $\nu = \frac{1}{T} = \frac{Bq}{2\pi m}$

(2) **Maximum energy of particle :** Maximum energy gained by the charged particle,

$$E_{max} = \left(\frac{q^2B^2}{2m}\right)r_0^2$$

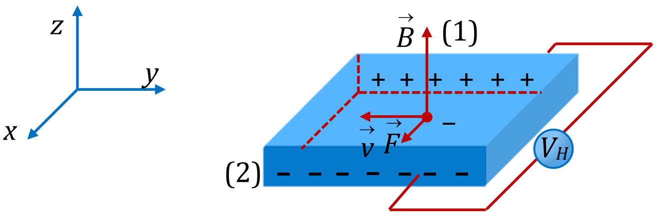
where r_0 = maximum radius of the circular path followed by the positive ion.

Hall Effect

The Phenomenon of producing a transverse emf in a current carrying conductor on applying a magnetic field perpendicular to the direction of the current is called Hall effect.

Hall effect helps us to know the nature and number of charge carriers in a conductor.

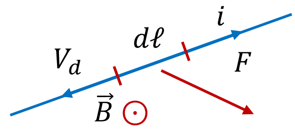
Consider a conductor having electrons as current carriers. The electrons move with drift velocity $\vec{v}$ opposite to the direction of flow of current.



Force acting on electron $F_m = -e(\vec{v} \times \vec{B})$. This force acts along x-axis and hence electrons will move towards face (2) and it becomes negatively charged.

Magnetic force on a current carrying wire (Ampere force)

Suppose a conducting wire, carrying a current i, is placed in a magnetic field $\vec{B}$. Consider a small element $d\ell$ of the wire. The free electrons drift with a speed v_d opposite to the direction of the current. The relation between the current i and the drift speed v_d is



$$i = jA = nev_dA \qquad \text{...(i)}$$

Here A is the area of cross-section of the wire and n is the number of free electrons per unit volume. Each electron experiences an average magnetic force

$$\vec{F} = -e\vec{v}_d \times \vec{B}$$

The number of free electrons in the small element considered in $nAd\ell$. Thus, the magnetic force on the wire of length $d\ell$ is

$$d\vec{F} = (nAd\ell)(-e\vec{v}_d \times \vec{B})$$

If we denote the length $d\ell$ along the direction of the current by $d\vec{\ell}$, the above equation becomes

$$d\vec{F} = nAev_d d\vec{\ell} \times \vec{B}$$

Using (i), $d\vec{F} = id\vec{\ell} \times \vec{B}$

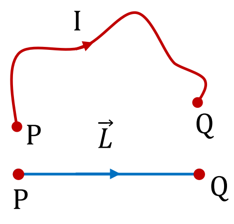
The quantity $id\vec{\ell}$ is called a current element

$$\vec{F}_{\text{res}} = \int d\vec{F} = \int id\vec{\ell} \times \vec{B} = i \int d\vec{\ell} \times \vec{B} \; (\because \; i \text{ is same at all points of the wire.})$$

If $\vec{B}$ is uniform then, $\vec{F}_{res} = i(\int d\vec{\ell}) \times \vec{B}$

$$\vec{F}_{\text{res}} = i\vec{L} \times \vec{B}$$

Here $\vec{L} = \int d\vec{\ell}$ = vector length of the wire = vector connecting the end points of the wire.

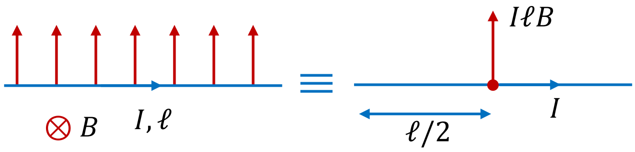
⇒ 

Note:

If a current loop of any shape is placed in a uniform $\vec{B}$ then $\vec{F}_{\text{res}})_{\text{magnetic}}$ on it = 0 ($\because$ $\vec{L}$ = 0).

Point of application of magnetic force :

On a straight current carrying wire the magnetic force in a uniform magnetic field can be assumed to be acting at its mid point.



This can be used for calculation of torque.

Illustration 31:

A wire is bent in the form of an equilateral triangle PQR of side 20 cm and carries a current of 2.5 A. It is placed in a magnetic field B of magnitude 2.0 T directed perpendicularly to the plane of the loop. Find the forces on the three sides of the triangle.

Solution:

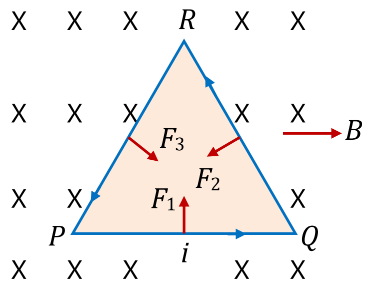
Suppose the field and the current have directions as shown in figure. The force on PQ is

$$\vec{F}_1 = i\vec{\ell} \times \vec{B}$$

or $\quad F_1 = 2.5\,A \times 20\,cm \times 2.0\,T = 1.0\,N$

The rule of vector product shows that the force F_1 is perpendicular to PQ and is directed towards the inside of the triangle.

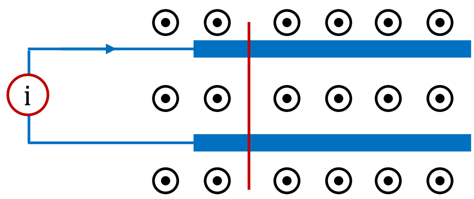
The forces $\vec{F}_2$ and $\vec{F}_3$ on QR and RP can also be obtained similarly. Both the forces are 1.0 N directed perpendicularly to the respective sides and towards the inside of the triangle.



The three forces $\vec{F}_1, \vec{F}_2$ and $\vec{F}_3$ will have zero resultant, so that there is no net magnetic force on the triangle. This result can be generalised. Any closed current loop, placed in a homogeneous magnetic field, does not experience a net magnetic force.

Illustration 32:

Figure shows two long metal rails placed horizontally and parallel to each other at a separation y. A uniform magnetic field B exists in the vertically upward direction. A wire of mass m can slide on the rails. The rails are connected to a constant current source which drives a current i in the circuit. The friction coefficient between the rails and the wire is μ.



(a) What should be the minimum value of μ which can prevent the wire from sliding on the rails?

(b) Describe the motion of the wire if the value of μ is half the value found in the previous part.

Solution:

(a) The force on the wire due to the magnetic field is

$$\vec{F} = i\vec{\ell} \times \vec{B} \quad \text{or} \quad F = iyB$$

It acts towards right tin the given figure. If the wire does not slide on the rails, the force of friction by the rails should be equal to F. If μ_0 be the minimum coefficient of friction which can prevent sliding, this force is also equal to $\mu_0\, mg$. Thus,

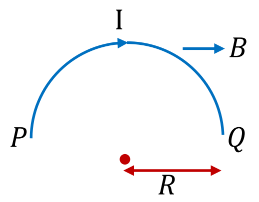
$$\mu_0\, mg = iyB \quad \text{or} \quad \mu_0 = \frac{iyB}{mg}$$

(b) If the friction coefficient is $\mu = \frac{\mu_0}{2} = \frac{iyB}{2mg}$, the wire will slide towards right. The frictional force by the rails is $f = \mu mg = \frac{iyB}{2}$ towards left.

The resultant force is $iyB - \frac{iyB}{2} = \frac{iyB}{2}$ towards right. The acceleration will be $a = \frac{iyB}{2m}$. The wire will slide towards right with this acceleration.

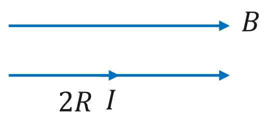
Illustration 33:

In the figure shown a semi-circular wire is placed in a uniform $\vec{B}$ directed toward right. Find the resultant magnetic force and torque on it.



Solution:

The wire is equivalent to

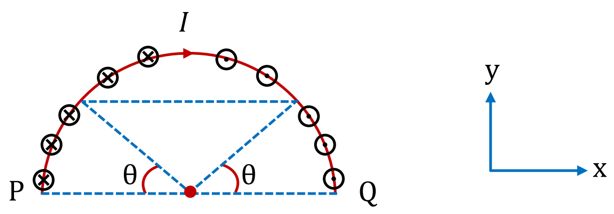


$\because \quad \theta = 0 \qquad \therefore \quad F_{res} = 0$

Forces on individual parts are marked in the figure by ⊗ and ⊙.

By symmetry their will be pair of forces forming couples.

$$\tau = \int_0^{\pi/2} i(Rd\theta)B\sin(90-\theta).2R\cos\theta$$

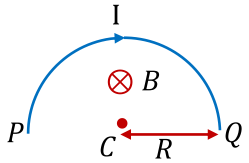


$$\tau = \frac{i\pi R^2}{2}B$$

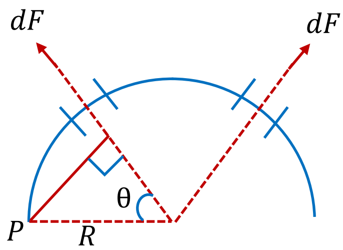
$$\Rightarrow \quad \vec{\tau} = \frac{i\pi R^2}{2}B(-\hat{j}) \text{ Ans.}$$

Illustration 34:

In the figure shown find the resultant magnetic force and torque about 'C', and 'P'.



Solution:



$\vec{F}_{nett} = I \,.\, 2R \,.\, B$

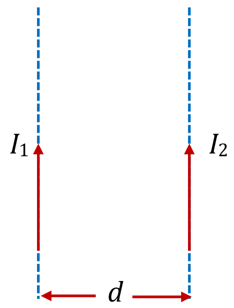
$\because$ Wire is equivalent to

Force on each element is radially outward: $\tau_c = 0$

Torque about $P = \int_0^{\pi}[I(Rd\theta)B \sin 90^0]\ R \sin\theta = 2IBR^2$

Illustration 35:

Prove that magnetic force per unit length on each of the infinitely long wire due to each other is $\mu_0 I_1 I_2 / 2\pi d$. Here it is attractive also.



Solution:

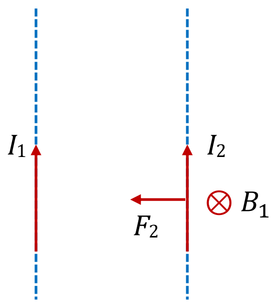
On (2), B due to (1) is $= \dfrac{\mu_0 I_1}{2\pi d} \otimes$

$\therefore$ F on (2) on $1m$ length

$= I_2 . \dfrac{\mu_0 I_1}{2\pi d} . 1$ towards left it is attractive

$= \dfrac{\mu_0 I_1 I_2}{2\pi d}$ (hence proved)

Similarly, on the other wire also.



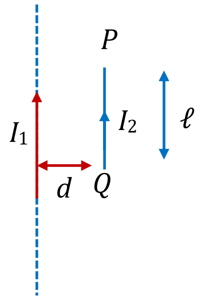
Note:

- Definition of ampere (fundamental unit of current) using the above formula.

  If $I_1 = I_2 = 1A, d = 1m$ then $F = 2 \times 10^{-7}\ N$

  $\therefore$ "When two very long wires carrying equal currents and separated by $1m$ distance exert on each other a magnetic force of $2 \times 10^{-7}\ N$ on $1m$ length then the current is 1 ampere."

- The above formula can also be applied if to one wire is infinitely long and the other is of finite length. In this case the force per unit length on each wire will not be same.



Force per unit length on PQ $= \frac{\mu_0 I_1 I_2}{2\pi d}$(attractive)

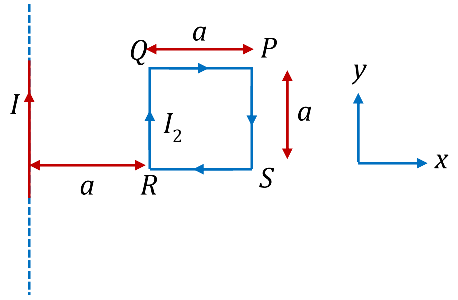
- If the currents are in the opposite direction then the magnetic force on the wires will be repulsive.

Illustration 36:

Find the magnetic force on the loop '$PQRS$' due to the loop wire.

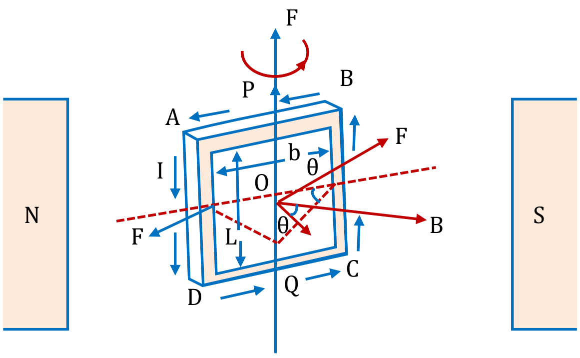
Solution:

$$F_{res} = \frac{\mu_0 I_1 I_2}{2\pi a} \text{a}(-\hat{\imath}) + \frac{\mu_0 I_1 I_2}{2\pi(2a)} a(\hat{\imath}) = \frac{\mu_0 I_1 I_2}{4\pi}(-\hat{\imath})$$



Torque on a current loop

When a current-carrying coil is placed in a uniform magnetic field the net force on it is always zero. However, as its different parts experience forces in different directions so the loop may experience a torque (or couple) depending on the orientation of the loop and the axis of rotation. For this, consider a rectangular coil in a uniform field B which is free to rotate about a vertical axis PQ and normal to the plane of the coil making an angle θ with the field direction as shown in figure (A).



Axis of rotation
(A)

The arms AB and CD will experience forces $B(NI)b$ vertically up and down respectively. These two forces together will give zero net force and zero torque (as are collinear with axis of rotation), so will have no effect on the motion of the coil.

Now the forces on the arms AC and BD will be $BINL$ in the direction out of the page and into the page respectively, resulting in zero net force, but an anticlockwise couple of value

$$\tau = F \times Arm = BINL \times (b \sin\theta)$$

i.e. $\tau = BIA \sin\theta$ with $A = NLb$ ...(i)

Now treating the current–carrying coil as a dipole of moment $\vec{M} = I\vec{A}$ Eqn. (i) can be written in vector form as

$$\vec{\tau} = \vec{M} \times \vec{B} \text{ with } \vec{M} = I\vec{A} = NIA\vec{n} \quad \text{...(ii)}$$

This is the required result and from this it is clear that :

(1) Torque will be minimum (= 0) when sin θ = min = 0, i.e., θ = 0°, i.e. 180° i.e., the plane of the coil is perpendicular to magnetic field i.e. normal to the coil is collinear with the field [fig. (A) and (C)]

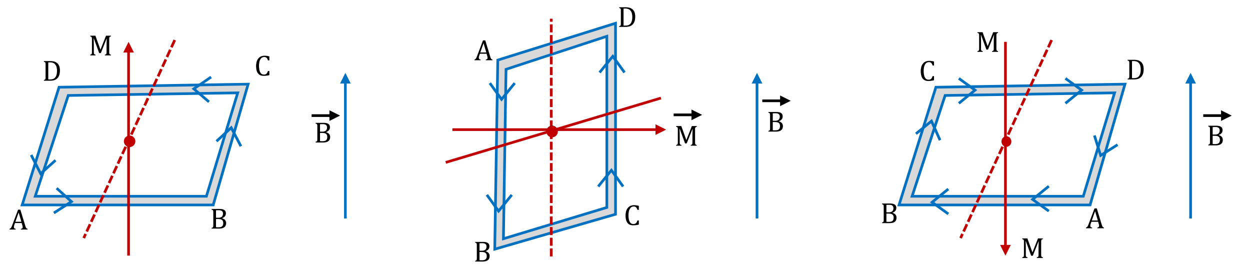
(2) Torque will be maximum (= BINA) when sin θ = max = 1, i.e., θ = 90° i.e. the plane of the coil is parallel to the field i.e. normal to the coil is perpendicular to the field. [fig.(B)].

(3) By analogy with dielectric or magnetic dipole in a field, in case of current–carrying in a field.

$$U = -\vec{M} \bullet \vec{B} \quad \text{with} \quad F = \frac{-dU}{dr}$$

and $W = MB(1-\cos\theta)$

The values of U and W for different orientations of the coil in the field are shown in fig.



θ = 0° ($\vec{M}$ is parallel to $\vec{B}$)
τ=0=min
W=0=min
U=–MB=min
Stable equilibrium
(A)

θ = 90° ($\vec{M}$ is ⊥ to $\vec{B}$)
τ=MB=max
W=MB
U=0
No equilibrium
(B)

θ = 180° ($\vec{M}$ is antiparallel to $\vec{B}$)
τ=0
W=2MB=max
U=MB=max
Unstable equilibrium
(C)

(4) Instruments such as electric motor, moving coil galvanometer and tangent galvanometers etc. are based on the fact that a current–carrying coil in a uniform magnetic field experiences a torque (or couple).

Illustration 37:

A circular coil of radius R and a current I, which can rotate about a fixed axis passing through its diameter is initially placed such that its plane lies along magnetic field B. Kinetic energy of loop when it rotates through an angle 90° is : (Assume that I remains constant)

(A) $\pi R^2 BI$ (B) $\frac{\pi R^2 BI}{2}$ (C) $2\pi R^2 BI$ (D) $\frac{3}{2}\pi R^2 I$

Ans. (A)

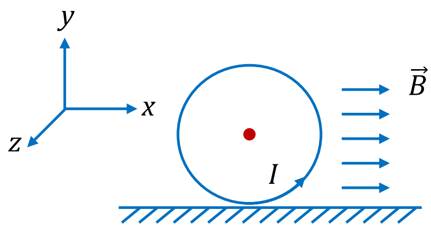
Solution:

Loss in potential energy = gain kinetic energy

$$(-MB\cos 90°) - (-MB\cos 0°) = KE$$

Illustration 38:

In the shown figure a conducting ring of mass $m = 2kg$ and radius $R = 0.5\ m$. lies on a smooth horizontal plane with its plane vertical. The ring carries a current of $I = \frac{1}{\pi}A$. A horizontal uniform magnetic field of $B = 12T$ is switched on at $t = 0$. The initial angular acceleration α in $rad./\sec^2$ of the ring will be $4x$ if x is :



Solution:

$$\tau_{y-\text{axis}} = I_{y-\text{axis}}\alpha$$

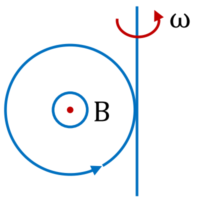
$$(I.\pi r^2)B = 1/2mr^2\alpha$$

$$\alpha = 12\ \text{rad}/\sec^2 \ \therefore\ \ x = 3$$

Illustration 39:

A current carrying ring, carrying a constant current $\frac{2}{\pi}$Amp., radius $1m$, mass $\frac{2}{3}\ kg$ and having 10 windings is free to rotate about its tangential vertical axis. A uniform magnetic field of 1 tesla is applied perpendicular to its plane. How much minimum angular velocity (in rad/sec.) should be given to the ring in the direction shown, so that it can rotate 270° in that direction. Write your answer in nearest single digit in rad/sec.

Solution:

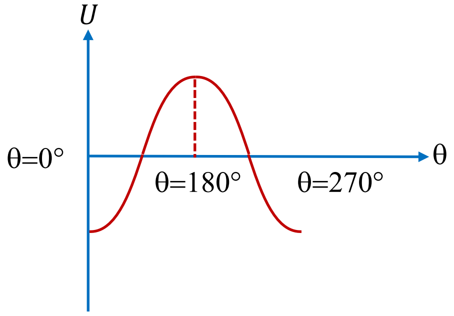


To reach θ = 270°, it has to cross the potential energy barrier at θ = 180° and to cross θ = 180° angular velocity at θ = 180° should be 0^+

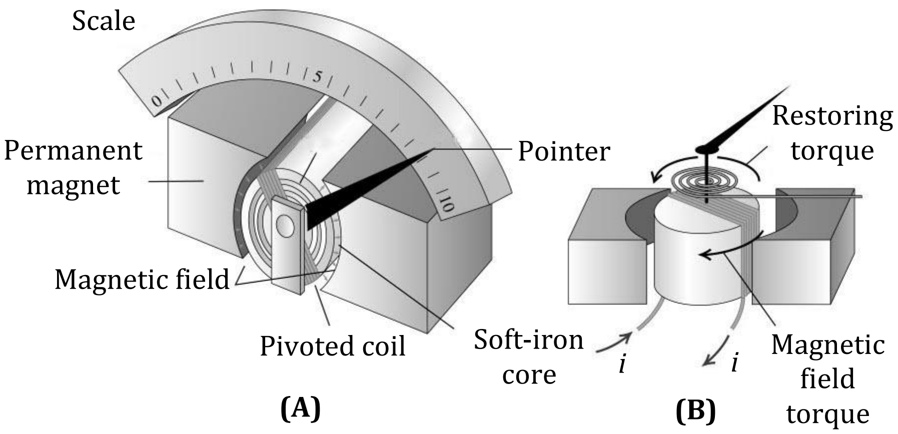
$$k_i + U_i = k_f + U_y$$

$$\frac{1}{2}\left(\frac{3}{2}MR^2\right)\omega^2 + (-Mi\,AB\cos 0°) = 0 + (-NiAB\cos 180°)$$

$$\omega = \sqrt{80} \approx\ 9\ \text{rad/sec.}$$



Moving Coil Galvanometer:



In a moving coil galvanometer the coil is suspended between the pole pieces of a strong horse-shoe magnet. The pole pieces are made cylindrical and a soft iron cylindrical core is placed within the coil without touching it. This makes the field radial. In such a field the plane of the coil always remains parallel to the field. Therefore $\theta = 90^o$ and the deflecting torque always has the maximum value.

$$\tau_{\text{def}} = NBiA \qquad \text{...(i)}$$

Coil deflects, a restoring torque is set up in the suspension fibre. If α is the angle of twist, the restoring torque is

$$\tau_{\text{rest}} = C\alpha \qquad \text{...(ii)}$$

where C is the torsional constant of the fibre.

When the coil is in equilibrium $NBiA = C\alpha \Rightarrow i = K\alpha$,

where $K = \frac{C}{NBA}$ is the galvanometer constant. This linear relationship between i and α makes the moving coil galvanometer useful for current measurement and detection.

Current sensitivity (S_i) : The current sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit current flowing through it.

$$S_i = \frac{\alpha}{i} = \frac{NBA}{C}$$

Voltage sensitivity (S_V) : Voltage sensitivity of a galvanometer is defined as the deflection produced in the galvanometer per unit voltage applied to it.

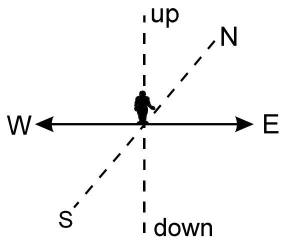
$$S_V = \frac{\alpha}{V} = \frac{\alpha}{iR} = \frac{S_i}{R} = \frac{NBA}{RC}$$

MAGNETISM

Magnet:

Two bodies even after being neutral (showing no electric interaction) may attract / repel strongly if they have a special property. This property is known as magnetism. The force with which they attract or repel is called magnetic force. Those bodies are called magnets. Later on we will see that it is due to circulating currents inside the atoms. Magnets are found in different shape but for many experimental purposes, a bar magnet is frequently used. When a bar magnet is suspended at its middle, as shown, and it is free to rotate in the horizontal plane it always comes to equilibrium in a fixed direction.

One end of the magnet (say A) is directed approximately towards north and the other end (say B) approximately towards south. This observation is made everywhere on the earth. Due to this reason the end A, which points towards north direction is called NORTH POLE and the other end which points towards south direction is called SOUTH POLE. They can be marked as 'N' and 'S' on the magnet. This property can be used to determine the north or south direction anywhere on the earth and indirectly east

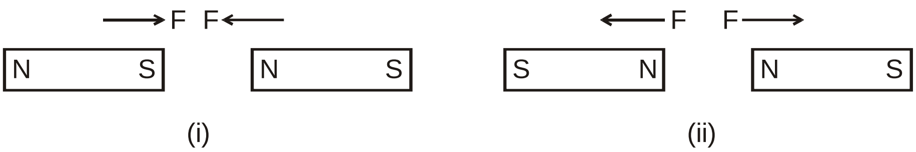


and west also if they are not known by other method (like rising of sun and setting of the sun). This method is used by navigators of ships and aeroplanes. The directions are as shown in the figure. All directions E, W, N, S are in the horizontal plane.

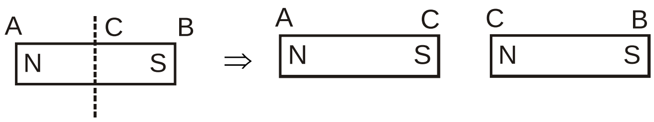
The magnet rotates due to the earths magnetic field about which we will discuss later in this chapter.

Pole strength magnetic dipole and magnetic dipole moment :

A magnet always has two poles 'N' and 'S' and like poles of two magnets repel each other and the unlike poles of two magnets attract each other they form action reaction pair.

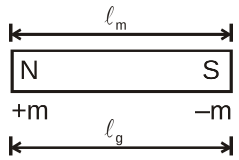


The poles of the same magnet do not come to meet each other due to attraction. They are maintained we cannot get two isolated poles by cutting the magnet from the middle. The other end becomes pole of opposite nature. So, 'N' and 'S' always exist together.



They are known as +ve and –ve poles. North pole is treated as positive pole (or positive magnetic charge) and the south pole is treated as –ve pole (or –ve magnetic charge). They are quantitatively represented by their "POLE STRENGTH" +m and –m respectively (just like we have charges +q and –q in electrostatics). Pole strength is a scalar quantity and represents the strength of the pole hence, of the magnet also).

A magnet can be treated as a dipole since it always has two opposite poles (just like in electric dipole we have two opposite charges –q and +q). It is called MAGNETIC DIPOLE and it has a MAGNETIC DIPOLE MOMENT. It is represented by $\vec{M}$. It is a vector quantity. It's direction is from –m to +m that means from 'S' to 'N')



$M = m.\ell_m$ here ℓ_m = magnetic length of the magnet. ℓ_m is slightly less than ℓ_g (it is geometrical length of the magnet = end to end distance). The 'N' and 'S' are not located exactly at the ends of the magnet. For calculation purposes we can assume $\ell_m = \ell_g$ [Actually $\ell_m/\ell_g \simeq 0.84$].

The units of m and M will be mentioned afterwards where you can remember and understand.

Magnetic field and strength of magnetic field:

The physical space around a magnetic pole has special influence due to which other pole experience a force. That special influence is called MAGNETIC FIELD and that force is called 'MAGNETIC FORCE'. This field is qualitatively represented by 'STRENGTH OF MAGNETIC FIELD' or "MAGNETIC INDUCTION" or "MAGNETIC FLUX DENSITY". It is represented by $\vec{B}$. It is a vector quantity.

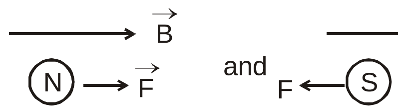
***Definition* of $\vec{B}$:** The magnetic force experienced by a north pole of unit pole strength at a point due to some other poles (called source) is called the strength of magnetic field at that point due to the source.

Mathematically, $$\vec{B} = \frac{\vec{F}}{m}$$

Here $\vec{F}$ = magnetic force on pole of pole strength m. m may be +ve or –ve and of any value.

S.I. unit of $\vec{B}$ is **Tesla** or **Weber/m²** (abbreviated as T and Wb/m²).

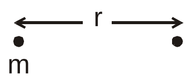
We can also write $\vec{F} = m\vec{B}$. According to this direction of on +ve pole (North pole) will be in the direction of field and on –ve pole (south pole) it will be opposite to the direction of $\vec{B}$.



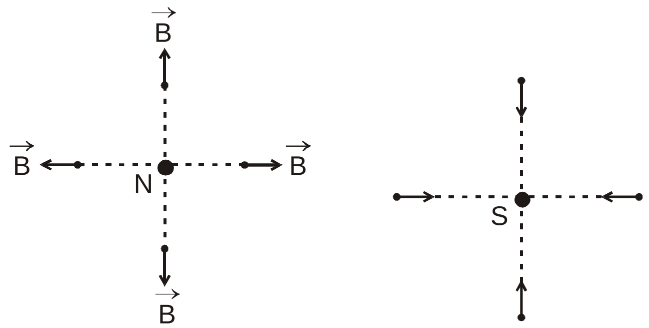
The field generated by sources does not depend on the test pole (for its any value and any sign).

(a) **$\vec{B}$ due to various source**

(i) **Due to a single pole :** (Similar to the case of a point charge in electrostatics)

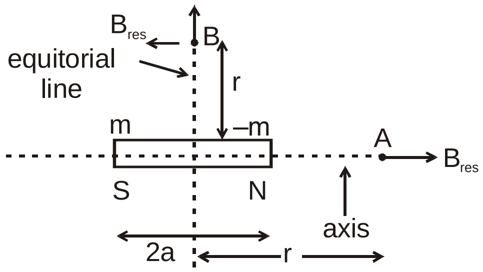


$$B = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^2}$$. This is magnitude



Direction of B due to north pole and due to south poles are as shown in vector form $\vec{B} = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{r^3}\vec{r}$ here m is with sign and $\vec{r}$ = position vector of the test point with respect to the pole.

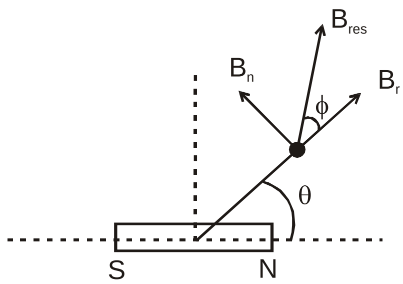
(ii) **Due to a bar magnet :** (Same as the case of electric dipole in electrostatics) Independent case never found. Always 'N' and 'S' exist together as magnet.



at A (on the axis) = $2\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r

at B (on the equatorial) = $-\left(\frac{\mu_0}{4\pi}\right)\frac{\vec{M}}{r^3}$ for a << r

At General point :



$$B_r = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\cos\theta}{r^3}$$

$$B_n = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M\sin\theta}{r^3}$$

$$B_{res} = \frac{\mu_0 M}{4\pi r^3}\sqrt{1+3\cos^2\theta}$$

$$\tan\phi = \frac{B_n}{B_r} = \frac{\tan\theta}{2}$$

Magnetic lines of force of a bar magnet :

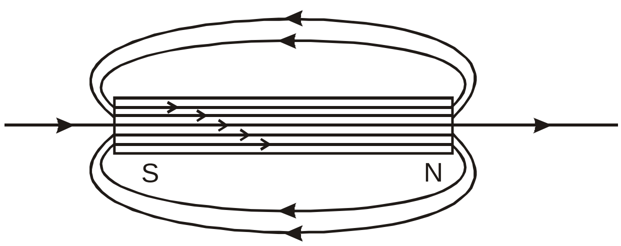
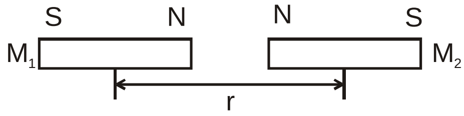


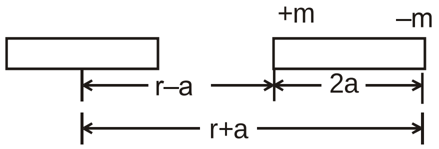
Illustration 40:

Find the magnetic force on a short magnet of magnetic dipole moment M_2 due to another short magnet of magnetic dipole moment M_1.



Solution:

To find the magnetic force we will use the formula of 'B' due to a magnet. We will also assume m and –m as pole strengths of 'N' and 'S' of M_2. Also length of M_2 as 2a. B_1 and B_2 are the strengths of the magnetic field due to M_1 at +m and –m respectively. They experience magnetic forces F_1 and F_2 as shown.



$$F_1 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r-a)^3}m \quad \text{and} \quad F_2 = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1}{(r+a)^3}m$$

$$\therefore \qquad F_{res} = F_1 - F_2 = 2\left(\frac{\mu_0}{4\pi}\right)M_1 m\left[\left(\frac{1}{(r-a)^3}\right)-\left(\frac{1}{(r+a)^3}\right)\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[\left(1-\frac{a}{r}\right)^{-3}-\left(1+\frac{a}{r}\right)^{-3}\right]$$

By using acceleration, Binomial expansion, and neglecting terms of high power we get

$$F_{res} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\left[1+\frac{3a}{r}-1+\frac{3a}{r}\right]$$

$$= 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 m}{r^3}\frac{6a}{r} = 2\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 3M_2}{r^4} = 6\left(\frac{\mu_0}{4\pi}\right)\frac{M_1 M_2}{r^4}$$

Direction of F_{res} is towards right.

Alternative Method :

$$B = \frac{\mu_0}{4\pi}\cdot\frac{2M_1}{r^3} \qquad \Rightarrow \qquad \frac{dB}{dr} = -\frac{\mu_0}{4\pi}\times\frac{6M_1}{r^4}$$

$$F = -M_2 \times \frac{dB}{dr} \qquad \Rightarrow \qquad F = \left(\frac{\mu_0}{4\pi}\right)\frac{6M_1 M_2}{r^4}$$

Illustration 41:

A magnet is 10 cm long and its pole strength is 120 CGS units (1 CGS unit of pole strength = 0.1 A-m). Find the magnitude of the magnetic field B at a point on its axis at a distance 20 cm from it.

Solution:

The pole strength is m = 120 CGS units = 12A-m.

Magnetic length is 2ℓ = 10 cm or ℓ = 0.05 m.

Distance from the magnet is d = 20 cm = 0.2 m. The field B at a point in end-on position is

$$B = \frac{\mu_0}{4\pi}\frac{2Md}{(d^2-\ell^2)^2} = \frac{\mu_0}{4\pi}\frac{4m\ell d}{(d^2-\ell^2)^2}$$

$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{4\times(12A-m)\times(0.05m)\times(0.2m)}{[(0.2m)^2-(0.05m)^2]^2}$$

$= 3.4 \times 10^{-5}$ T.

Illustration 42:

Find the magnetic field due to a dipole of magnetic moment 1.2 A-m² at a point 1 m away from it in a direction making an angle of 60° with the dipole-axis.

Solution:

Solution : The magnitude of the field is $B = \frac{\mu_0}{4\pi}\frac{M}{r^3}\sqrt{1+3\cos^2\theta}$

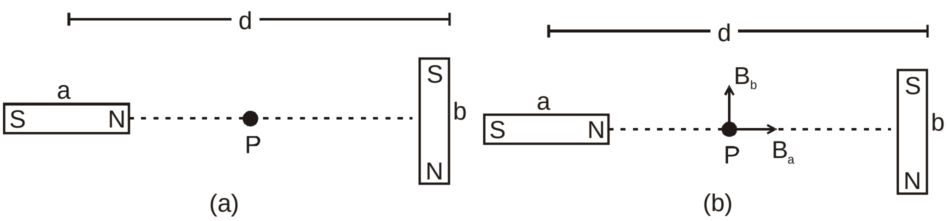
$$= \left(10^{-7}\frac{T-m}{A}\right)\frac{1.2A-m^2}{1m^3}\sqrt{1+3\cos^2 60^{\circ}} = 1.6 \times 10^{-7}\text{ T.}$$

The direction of the field makes an angle α with the radial line where

$$\tan\alpha = \frac{\tan\theta}{2} = \frac{\sqrt{3}}{2}$$

Illustration 43:

Figure shows two identical magnetic dipoles a and b of magnetic moments M each, placed at a separation d, with their axes perpendicular to each other. Find the magnetic field at the point P midway between the dipoles.



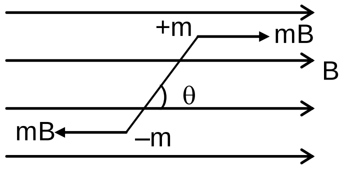
Solution:

The point P is in end-on position for the dipole (a) and in broadside-on position for the dipole (b). The magnetic field at P due to a is $B_a = \frac{\mu_0}{4\pi}\frac{2M}{(d/2)^3}$ along the axis of a, and that due to b is $B_b = \frac{\mu_0}{4\pi}\frac{M}{(d/2)^3}$ parallel to the axis of b as shown in figure. The resultant field at P is, therefore. $B = \sqrt{B_a^2 + B_b^2}$

$$= \frac{\mu_0 M}{4\pi(d/2)^3}\sqrt{1^2+2^2} = \frac{2\sqrt{5}\mu_0 M}{\pi d^3}$$

The direction of this field makes an angle α with B_a such that $\tan\alpha = B_b/B_a = 1/2$.

Magnet in an external uniform magnetic field :



(same as case of electric dipole)

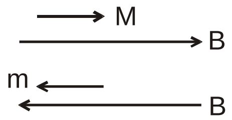
$F_{res} = 0$ (for any angle)

$\tau = MB \sin\theta$

* here θ is angle between $\vec{B}$ and $\vec{M}$

Note :

- $\vec{\tau}$ acts such that it tries to make $\vec{M} \times \vec{B}$.
- $\vec{\tau}$ is same about every point of the dipole it's potential energy is



$U = -MB\cos\theta = -\vec{M}\cdot\vec{B}$

$\theta = 0°$ is stable equilibrium

$\theta = \pi$ is unstable equilibrium

for small 'θ' the dipole performs SHM about $\theta = 0°$ position

$\tau = -MB\sin\theta$;

$I\alpha = -MB\sin\theta$

for small θ, $\sin\theta \simeq \theta$

$$\alpha = -\left(\frac{MB}{I}\right)\theta$$

Angular frequency of SHM

$$\omega = \sqrt{\frac{MB}{I}} = \frac{2\pi}{T} \qquad \Rightarrow \qquad T = 2\pi\sqrt{\frac{I}{MB}}$$

here $I = I_{cm}$ if the dipole is free to rotate $= I_{hinge}$ if the dipole is hinged

Illustration 44:

A bar magnet having a magnetic moment of 1.0×10^{-4} J/T is free to rotate in a horizontal plane. A horizontal magnetic field $B = 4 \times 10^{-5}$ T exists in the space. Find the work done in rotating the magnet slowly from a direction parallel to the field to a direction 60° from the field.

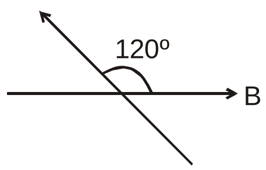
Solution:

The work done by the external agent = change in potential energy

$= (-MB\cos\theta_2) - (-MB\cos\theta_1) = -MB(\cos 60° - \cos 0°)$

$= \frac{1}{2}MB = \frac{1}{2} \times (1.0 \times 10^{4}\ \text{J/T})\ (4 \times 10^{-5}\ \text{T}) = 0.2\ \text{J}$

Illustration 45:



A magnet of magnetic dipole moment M is released in a uniform magnetic field of induction B from the position shown in the figure. Find :

(i) Its kinetic energy at $\theta = 90°$

(ii) its maximum kinetic energy during the motion.

(iii) will it perform SHM? oscillation ? Periodic motion? What is its amplitude?

Solution:

(i) Apply energy conservation at $\theta = 120°$ and $\theta = 90°$

$- MB \cos 120° + 0 = - MB \cos 90° + (K.E.) \Rightarrow KE = \frac{MB}{2}$ **Ans.**

(ii) K.E. will be maximum where P.E. is minimum. P.E. is minimum at $\theta = 0°$. Now apply energy conservation between $\theta = 120°$ and $\theta = 0°$.

$- mB \cos 120° + 0 = -mB \cos 0° + (KE)_{max}$

$\frac{3}{2}(KE)_{max} = MB$ **Ans.**

The K.E. is max at $\theta = 0°$ can also be proved by torque method. From $\theta = 120°$ to $\theta = 0°$ the torque always acts on the dipole in the same direction (here it is clockwise) so its K.E. keeps on increases till $\theta = 0°$. Beyond that τ reverses its direction and then K.E. starts decreasing

$\therefore$ $\theta = 0°$ is the orientation of M to here the maximum K.E.

(iii) Since ‘θ’ is not small.

$\therefore$ the motion is not S.H.M. but it is oscillatory and periodic amplitude is 120°.

Illustration 46:

A bar magnet of mass 100 g, length 7.0 cm, width 1.0 cm and height 0.50 cm takes $\pi/2$ seconds to complete an oscillation in an oscillation magnetometer placed in a horizontal magnetic field of 25μT.

(a) Find the magnetic moment of the magnet.

(b) If the magnet is put in the magnetometer with its 0.50 cm edge horizontal, what would be the time period?

Solution:

(a) The moment of inertia of the magnet about the axis of rotation is $I = \frac{m'}{12}(L^2 + b^2)$

$$\frac{100\times10^{-3}}{12} = [(7 \times 10^{-2})^2 + (1 \times 10^{-2})^2] \text{ kg-m}^2. = \frac{25}{6} \times 10^{-5} \text{ kg-m}^2.$$

We have, $T = 2\pi\sqrt{\frac{I}{MB}}$ or $M = \frac{4\pi^2 I}{BT^2} = \frac{4\pi^2 \times 25\times10^{-5} kg/m^2}{6\times(25\times10^{-6}T)\times\frac{\pi^2}{4}s^2} = 27 \text{ A-m}^2.$

(b) In this case the moment of inertia becomes $I' = \frac{m'}{12}(L^2 + b^2)$ where b’ = 0.5 cm.

The time period would be $T' = \sqrt{\frac{I'}{MB}}$ .... (ii)

Dividing by equation (i),

$$\frac{T'}{T} = \sqrt{\frac{I'}{I}} = \frac{\sqrt{\frac{m'}{12}(L^2+b'^2)}}{\sqrt{\frac{m'}{12}(L^2+b^2)}} = \frac{\sqrt{(7cm)^2+(0.5cm)^2}}{\sqrt{(7cm)^2+(1.0cm)^2}} = 0.992$$

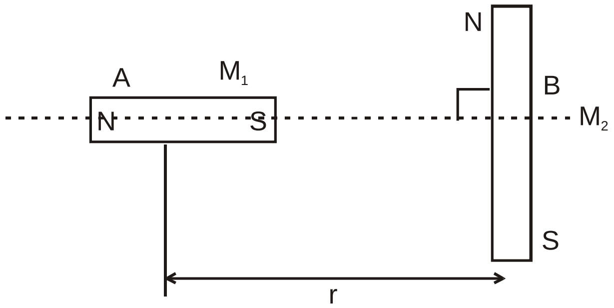
or $T' = \frac{0.992\times\pi}{2} s = 0.496\pi$ s.

Magnet in an External Non-uniform Magnetic Field :

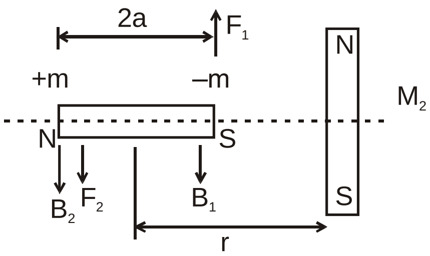
No special formulae are applied is such problems. Instead see the force on individual poles and calculate the resistant force torque on the dipole.

Illustration 47:

Find the torque on M_1 due to M_2 .



Solution:



Due to M_2, magnetic fields at 'S' and 'N' of M_1 are B_1 and B_2 respectively. The forces on –m and +m are F_1 and F_2 as shown in the figure. The torque (about the centre of the dipole m_1) will be

$$= F_1 a + F_2 a = (F_1 + F_2)a$$

$$= \left[\left(\frac{\mu_0}{4\pi}\right)\frac{M_2}{(r-a)}m + \frac{\mu_0}{4\pi}\frac{M_2}{(r+a)}m\right]a$$

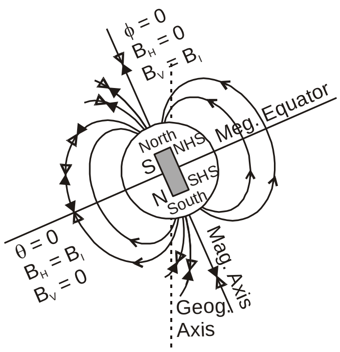
$$\cong \frac{\mu_0}{4\pi} M_2 m\left(\frac{1}{r^3}+\frac{1}{r^3}\right)a \qquad \because \ a << r$$

$$= \frac{\mu_0 M_2 m}{4\pi}\frac{2}{r^3}a = \frac{\mu_0 M_1 M_2}{4\pi r^3} \qquad \textbf{Ans.}$$

Terrestrial Magnetism (Earth's Magnetism) :

Introduction :

The idea that earth is magnetised was first suggested towards the end of the sixteen'th century by Dr. William Gilbert. The origin of earth's magnetism is still a mater of conjecture among scientists but it is agreed upon that the earth behaves as a magnetic dipole inclined at a small angle (11.5°) to the earth's axis of rotation with its south pole pointing north. The lines of force of earth's magnetic field are shown in figure which are parallel to the earth's surface near the equator and perpendicular to it near the poles. While discussing magnetism of the earth one should keep in mind that:



(a) The **magnetic meridian** at a place is not a line but a vertical plane passing through the axis of a freely suspended magnet, i.e., it is a plane which contains the place and the magnetic axis.

(b) The **geographical meridian** at a place is a vertical plane which passes through the line joining the geographical north and south, i.e., it is a plane which contains the place and earth's axis of rotation, i.e., geographical axis.

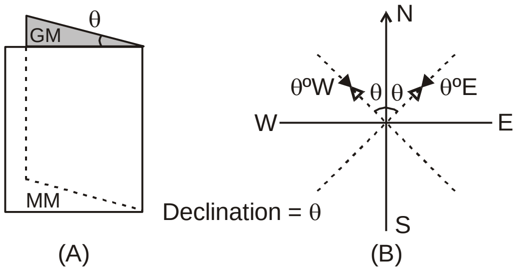
(c) The **magnetic Equator** is a great circle (a circle with the centre at earth's centre) on earth's surface which is perpendicular to the magnetic axis. The magnetic equator passing through Trivandrum in South India divides the earth into two hemispheres. The hemisphere containing south polarity of earth's magnetism is called the northern hemisphere (NHS) while the other, the southern hemisphere (SHS).

(d) The magnetic field of earth is not constant and changes irregularly from place to place on the surface of the earth and even at a given place it varies with time too.

Elements of the Earth's Magnetism :

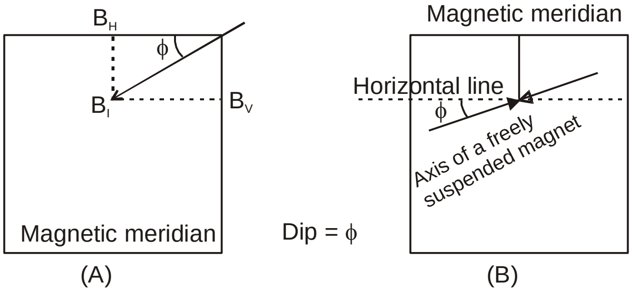
The magnetism of earth is completely specified by the following three parameters called elements of earth's magnetism :

(a) Variation or Declination θ : At a given place the angle between the geographical meridian and the magnetic meridian is called declination, i.e., at a given place it is the angle between the geographical north-south direction and the direction indicated by a magnetic compass needle, Declination at a place is expressed at $\theta°$ E or $\theta°$ W depending upon whether the north pole of the compass needle lies to the east (right) or to the west (left) of the geographical north-south direction. The declination at London is 10°W means that at London the north pole of a compass needle points 10°W, i.e., left of the geographical north.



(b) Inclination or Angle of Dip ϕ : It is the angle which the direction of resultant intensity of earth's magnetic field subtends with horizontal line in magnetic meridian at the given place. Actually it is the angle which the axis of a freely suspended magnet (up or down) subtends with the horizontal in magnetic meridian at a given place.

Here, it is worthy to note that as the northern hemisphere contains south polarity of earth's magnetism, in it the north pole of a freely suspended magnet (or pivoted compass needle) will dip downwards, i.e., towards the earth while the opposite will take place in the southern hemisphere.



Angle of dip at a place is measured by the instrument called Dip-Circle in which a magnetic needle is free to rotate in a vertical plane which can be set in any vertical direction. Angle of dip at Delhi is 42°.

(c) **Horizontal Component of Earth's Magnetic Field B_H :** At a given place it is defined as the component of earth's magnetic field along the horizontal in the magnetic meridian. It is represented by B_H and is measured with the help of a **vibration** or **deflection magnetometer**. At Delhi the horizontal component of the earth's magnetic field is 35 μT, i.e., 0.35 G.

If at a place magnetic field of earth is B_I and angle of dip ϕ, then in accordance with figure (a).

$$B_H = B_I \cos\phi \quad \text{and} \quad B_v = B_I \sin\phi \qquad .....(1)$$

so that, $\tan\phi = \dfrac{B_v}{B_H}$ and $B_I = \sqrt{B_H^2 + B_v^2}$ .....(2)

MAGNETIC MATERIALS AND THEIR PROPERTIES

Magnetic classification of substances

(A) Classification of substances according to their magnetic behaviour:

All substance show magnetic properties. An iron nail brought near a pole of a bar magnet is strongly attracted by it and sticks to it, Similar is the behaviour of steel, cobalt and nickel. Such substance are called 'ferromagnetic 'substance. Some substances are only weakly attracted by a magnet, while some are repelled by it. They are called 'paramagnetic' and 'diamagnetic 'substance respectively. All substance, solids, liquids and gases, fall into one or other of these classes.

(i) **Diamagnetic substance :** Some substance, when placed in a magnetic field, are feebly magnetised opposite to the direction of the magnetising field. These substances when brought close to a pole of a powerful magnet, are somewhat repelled away from the magnet. They are called 'diamagnetic' substances and their magnetism is called the 'diamagnetism'.

Examples of diamagnetic substances are bisuth, zinc, copper, silver, gold, lead, water, mercury, sodium chloride, nitrogen, hydrogen, etc.

(ii) **Paramagnetic substances :** Some substance when placed in a magnetic field, are feebly magnetised in the direction of the magnetising field. These substance, when brought close to a pole of a powerful magnet, are attracted towards the magnet. These are called 'paramagnetic' substance and their magnetism is called 'paramagnetism'.

(iii) **Ferromagnetic substances :** Some substance, when placed in a magnetic field, are strongly magnetised in the direction of the magnetising field. They are attracted fast towards a magnet when brought close to either of the poles of the magnet. These are called 'ferromagnetic' substances and their magnetism is called 'ferromagnetism'.

(B) Some important terms used in magnetism :

Magnetisation and Magnetic Intensity

The earth abounds with a bewildering variety of elements and compounds.In addition, we have been synthesising new alloys, compounds and evenelements. One would like to classify the magnetic properties of these substances. In the present section, we define and explain certain terms which will help us to carry out this exercise.

We have seen that a circulating electron in an atom has a magnetic moment. In a bulk material, these moments add up vectorially and theycan give a net magnetic moment which is non-zero.

Magnetisation

Magnetisation M of a sample to be equal to its net magnetic moment per unit volume:

$$M = \frac{m_{net}}{V}$$

M is a vector with dimensions L^{-1} A and is measured in a units of A m^{-1}.Consider a long solenoid of n turns per unit length and carrying a current I. The magnetic field in the interior of the solenoid was shown to be given by $B_0 = \mu_0 nI$

If the interior of the solenoid is filled with a material with non-zero magnetisation, the field inside the solenoid will be greater than B_0. The net B field in the interior of the solenoid may be expressed as

$$B = B_0 + B_m$$

where B_m is the field contributed by the material core. It turns out thatthis additional field B_m is proportional to the magnetisation M of the material and is expressed as $B_m = \mu_0 M$

where μ_0 is the same constant (permeability of vacuum) that appears in Biot-Savart's law.

Magnetic Intensity

It is convenient to introduce another vector field H, called the magnetic intensity, which is defined by

$$\Rightarrow \quad H = \frac{B}{\mu} \quad \Rightarrow \quad H = \frac{B_0 + B_m}{\mu}$$

where H has the same dimensions as M and is measured in units of A m^{-1}. Thus, the total magnetic field B is written as : $B = B_0 + B_m$

We repeat our defining procedure. We have partitioned the contributionto the total magnetic field inside the sample into two parts: one, due toexternal factors such as the current in the solenoid. This is represented by H. The other is due to the specific nature of the magnetic material,namely M.

$$\Rightarrow \quad H = \frac{\mu_0 H + \mu_0 M}{\mu} \Rightarrow \quad H = \frac{H}{\mu_r} + \frac{M}{\mu_r} \quad \Rightarrow \quad (\mu_r - 1)H = M \quad \Rightarrow \quad M = \chi H$$

The latter quantity can be influenced by external factors. This influence is mathematically expressed as

$$M = \chi H$$

where χ , a dimensionless quantity, is appropriately called the **magnetic susceptibility**. It is a measure of how a magnetic material responds to an external field. Table lists χ for some elements. It is small and positive for materials, which are called paramagnetic. It is small and negative for materials, which are termed diamagnetic.

$$\mu_r - 1 = \chi, \quad \mu_r = 1 + \chi$$

is a dimensionless quantity called the relative magnetic permeability of the substance. It is the analog of the dielectric constant in electrostatics. The magnetic permeability of the substance is μ and it has the same dimensions and units as μ_0 ; $\mu = \mu_0\mu_r = \mu_0 (1 + \chi)$.

$$\mu_r = (1 + \chi).$$

The three quantities χ, μ_r and μ are interrelated and only one of them is independent. Given one, the other two may be easily determined.

Table :1 Magnetic Susceptibility of Some Element At 300K

Diamagnetic substance	χ	Paramagnetic substance	χ
Bismuth	-1.66×10^{-5}	Aluminium	2.3×10^{-5}
Copper	-9.8×10^{-6}	Calcium	1.9×10^{-5}
Diamond	-2.2×10^{-5}	Chromium	2.7×10^{-4}
Gold	-3.6×10^{-5}	Lithium	2.1×10^{-5}
Lead	-1.7×10^{-5}	Magnesium	1.2×10^{-5}
Mercury	-2.9×10^{-5}	Niobium	2.6×10^{-5}
Nitrogen (STP)	-5.0×10^{-9}	Oxygen (STP)	2.1×10^{-6}
Silver	-2.6×10^{-5}	Platinum	2.9×10^{-4}
Silicon	-4.2×10^{-6}	Tungsten	6.8×10^{-5}

Illustration 48:

A solenoid has a core of a material with relative permeability 400. The windings of the solenoid are insulated from the core and carry a current of 2A. If the number of turns is 1000 per metre, calculate

(a) H (b) M (c) B (d) the magnetising current I_m.

Solution:

(a) The field H is dependent of the material of the core, and is

$H = nI = 1000 \times 2.0 = 2 \times 10^3$ A/m.

(b) The magnetic field B is given by

$B = \mu_r \mu_0 H$

$= 400 \times 4\pi \times 10^{-7}$ (N/A^2) $\times 2 \times 10^3$ (A/m) $= 1.0$ T

(c) Magnetisation is given by

$M = (B - \mu_0 H)/\mu_0$

$= (\mu_r \mu_0 H - \mu_0 H)/\mu_0 = (\mu_r - 1)H = 399 \times H \approx 8 \times 10^5$ A/m

(d) The magnetising current I_M is the additional current that needs to be passed through the windings of the solenoid in the absence of the core which would give a B value as in the presence of the core. Thus

$B = \mu_r n_0 (I + I_M)$. Using I = 2A, B = 1 T, we get $I_M = 794$ A.

Magnetic Properties of Materials

The discussion in the previous section helps us to classify materials as diamagnetic, paramagnetic or ferromagnetic. In terms of the susceptibility χ , a material is diamagnetic if χ is negative, para- if χ is positive and small, and ferro- if χ is large and positive.

A glance at Table 5.3 gives one a better feeling for these materials. Here ε is a small positive number introduced to quantify paramagnetic materials. Next, we describe these materials in some detail.

Table : 2

Diamagnetic	Paramagnetic	Ferromagnetic
$-1 \le \chi < 0$	$0 < \chi < \varepsilon$	$\chi >> 1$
$0 \le \mu_r < 1$	$1 < \mu_r < 1 + \varepsilon$	$\mu_r >> 1$
$\mu < \mu_0$	$\mu > \mu_0$	$\mu >> \mu_0$

Diamagnetism:

Diamagnetic substances are those which have tendency to move from stronger to the weaker part of the external magnetic field. In other words,unlike the way a magnet attracts metals like iron, it would repel adiamagnetic substance.

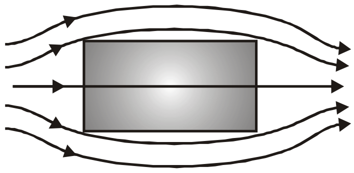
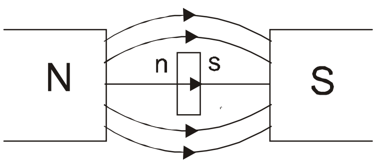


Figure shows a bar of diamagnetic material placed in an externalmagnetic field. The field lines are repelled or expelled and the field insidethe material is reduced. In most cases, as is evident from Table 1, this reduction is slight, being one part in 10^5. When placed in a non-uniform magnetic field, the bar will tend to move from high to low field.The simplest explanation for diamagnetism is as follows. Electrons in an atom orbiting around nucleus possess orbital angular momentum.These orbiting electrons are equivalent to current-carrying loop and thus possess orbital magnetic moment. Diamagnetic substances are the ones in which resultant magnetic moment in an atom is zero. When magnetic field is applied, those electrons having orbital magnetic moment in thesame direction slow down and those in the opposite direction speed up.This happens due to induced current in accordance with Lenz's law whichyou will study in Electro magnetic Induction. Thus, the substance develops a net magneticmoment in direction opposite to that of the applied field and hencerepulsion.Some diamagnetic materials are bismuth, copper, lead, silicon,nitrogen (at STP), water and sodium chloride. Diamagnetism is present in all the substances. However, the effect is so weak in most cases that it gets shifted by other effects like paramagnetism, ferromagnetism, etc.The most exotic diamagnetic materials are superconductors. These are metals, cooled to very low temperatures which exhibits both perfect conductivity and perfect diamagnetism. Here the field lines are completely expelled! $\chi = -1$ and $\mu_r = 0$. A superconductor repels a magnet and (byNewton's third law) is repelled by the magnet. The phenomenon of perfect diamagnetism in superconductors is called the Meissner effect, after the name of its discoverer. Superconducting magnets can be gainfully exploited in variety of situations, for example, for running magnetically levitated superfast trains.

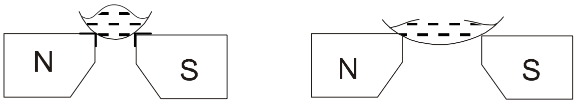
Properties of diamagnetic substance :

Diamagnetic substance show following properties.

(i) When a rod of diamagnetic material is suspended freely between two magnetic poles, then its axis becomes perpendicular to the magnetic field.

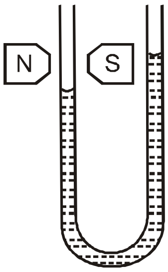


(ii) In a non-uniform magnetic field a diamagnetic substance tends to move from the stronger to the weaker part of the field.



(a) **(b)**

(iii) If a diamagnetic solution is poured into a U-tube and one arm of this U-tube is placed between the poles of a strong magnet, the level of the solution in that arm is depressed.

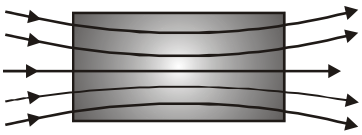


(iv) A diamagnetic gas when allowed to ascend in between the poles of a magnet spreads across the field.

(v) The susceptibility of a diamagnetic substance is independent of temperature.

Paramagnetism

Paramagnetic substances are those which get weakly magnetised when placed in an external magnetic field. They have tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get weakly attracted to a magnet.



The individual atoms (or ions or molecules) of a paramagnetic material possess a permanent magnetic dipole moment of their own. On account of the ceaseless random thermal motion of the atoms, no net magnetisation is seen. In the presence of an external field B_0, which is strong enough, and at low temperatures, the individual atomic dipole moment can be made to align and point in the same direction as B_0. Figure shows a bar of paramagnetic material placed in an external field. The field lines gets concentrated inside the material, and the field inside is enhanced. In most cases, as is evident from Table 1, this enhancement is slight, being one part in 10^5. When placed in a non-uniform magnetic field, the bar will tend to move from weak field to strong.Some paramagnetic materials are aluminium, sodium, calcium,oxygen (at STP) and copper chloride. Experimentally, one finds that the magnetisation of a paramagnetic material is inversely proportional to the absolute temperature T,

$$M = C\frac{B_0}{T}$$

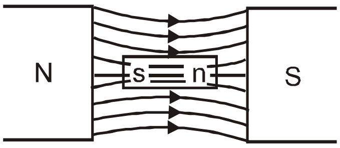
or equivalently, using Eqs $M = \chi H$ and $B_0 = \mu_0 H$

$$\chi = \frac{\mu_0}{T}$$

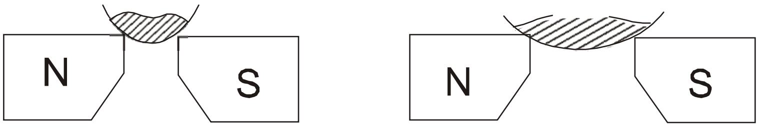
This is known as Curie's law, after its discoverer Pieree Curie (1859-1906). The constant C is called Curie's constant. Thus, for a paramagneticmaterial both χ and μ_r depend not only on the material, but also (in a simple fashion) on the sample temperature. As the field is increased or the temperature is lowered, the magnetisation increases until it reaches the saturation value M_S, at which point all the dipoles are perfectly aligned with the field. Beyond this, Curie's law is no longer valid.

Properties of Paramagnetic Substance

(i) When a rod of paramagnetic material is suspended freely between two magnetic poles, then its axis becomes parallel to the magnetic field The poles produced at the ends of the rod are opposite to the nearer magnetic poles.

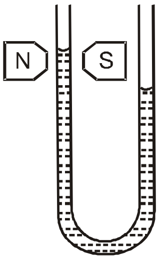


(ii) In a non-uniform magnetic field, the paramagnetic substances tend to move from weaker to stronger part of the magnetic field.



(a) (b)

(iii) If a paramagnetic solution is poured in a U-tube and one arm of the U-tube is placed between two strong poles, the level of the solution in that arm rises.



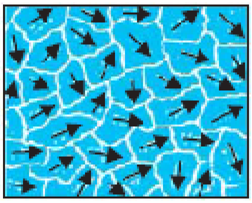
(iv) A paramagnetic gas when allowed to ascend between the pole-pieces of a magnet spreads along the field.

(v) The susceptibility of a paramagnetic substance varies inversely as the kelvin temperature of the substance, that is,

$\chi_m \propto \frac{1}{T}$. This known as curie's law.

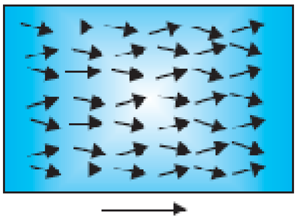
Ferromagnetism

Ferromagnetic substances are those which gets strongly magnetised when placed in an external magnetic field. They have strong tendency to move from a region of weak magnetic field to strong magnetic field, i.e., they get strongly attracted to a magnet.The individual atoms (or ions or molecules) in a ferromagnetic material possess a dipole moment as in a paramagnetic material. However, they interact with one another in such a way that they spontaneously align themselves in a common direction over a macroscopic volume called domain. The explanation of this cooperative effect requires quantum mechanics and is beyond the scope of this textbook. Each domain has a net magnetisation. Typical domain size is 1mm and the domain contains about 10^{11} atoms. In the first instant, the magnetisation varies randomly from domain to domain and there is no bulk magnetisation. This is shown in Fig.



Randomlyoriented domains,

When we apply an external magnetic field B_0, the domains orient themselves in the direction of B_0 and simultaneously the domain oriented in the direction of B_0 grow in size. This existence of domains and their motion in B_0 are not speculations. One may observe this under a microscope after sprinkling a liquid suspension of powdered ferromagnetic substance of samples. This motion of suspension can be observed. Figure (b) shows the situation when the domains have aligned and amalgamated to form a single 'giant' domain.



Aligned domains.

Thus, in a ferromagnetic material the field lines are highly concentrated. In non-uniform magnetic field, the sample tends to move towards the region of high field. We may wonder as to what happens when the external field is removed. In some ferromagnetic materials the magnetisation persists. Such materials are called hard magnetic materials or hard ferromagnets. Alnico, an alloy of iron, aluminium, nickel, cobalt and copper, is one such material. The naturally occurring lodestone is another. Such materials form permanent magnets to be used among other things as a compass needle. On the other hand, there is a class of ferromagnetic materials in which the magnetisation disappears on removal of the external field. Soft iron is one such material. Appropriately enough,such materials are called soft ferromagnetic materials. There are a number of elements, which are ferromagnetic: iron, cobalt, nickel, gadolinium, etc. The relative magnetic permeability is >1000!

The ferromagnetic property depends on temperature. At high enoughtemperature, a ferromagnet becomes a paramagnet. The domain structuredisintegrates with temperature. This disappearance of magnetisation with temperature is gradual. It is a phase transition reminding us of the melting of a solid crystal. The temperature of transition from ferromagnetic to paramagnetism is called the Curie temperature T_C. Table lists the Curie temperature of certain ferromagnets. The susceptibilityabove the Curie temperature, i.e., in the paramagnetic phase isdescribed by, $\chi = \frac{C}{T - T_c}$ $(T > T_C)$

Ferromagnetic substances : These substances which are strongly attracted by a magnet, show all the properties of a paramagnetic substance to a much higher degree. For example, they are strongly magnetised in relatively weak magnetising field in the same direction as the field. They have relative permeabilities of the order of hundreds and thousands. Similarly, the susceptibilities of ferromagnetic have large positive values.

Curie temperature : Ferromagnetism decreases with rise in temperature. If we heat a ferromagnetic substance, then at a definite temperature the ferromagnetic property of the substance "suddenly" disappears and the substance becomes paramagnetic. The temperature above which a ferromagnetic substance becomes paramagnetic is called the' Curie temperature' of the substance. The curie temperature of iron is 770°C and that of nickel is 358°C.

Illustration 49:

A domain in ferromagnetic iron is in the form of a cubeof side length 1μm. Estimate the number of iron atoms in the domainand the maximum possible dipole moment and magnetisation of the domain. The molecular mass of iron is 55 g/mole and its density is 7.9 g/cm^3. Assume that each iron atom has a dipole momentof 9.27×10^{-24} A m^2.

Solution:

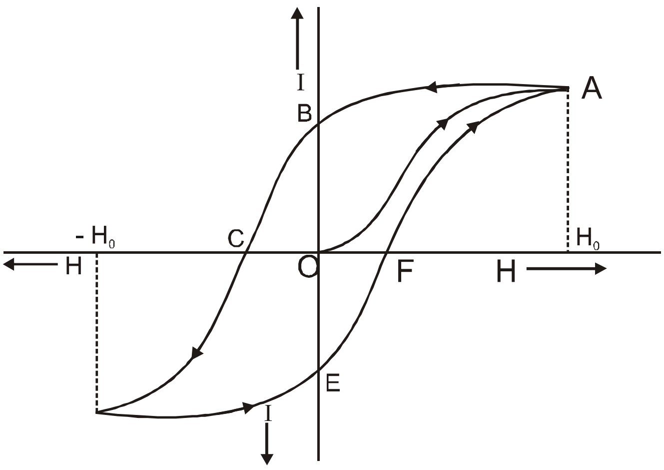
The volume of the cubic domain is$V = (10^{-6}\text{ m})^3 = 10^{-18}\text{ m}^3 = 10^{-12}\text{ cm}^3$. Its mass is volume × density $= 7.9\text{ g cm}^{-3} \times 10^{-12}\text{ cm}^3 = 7.9 \times 10^{-12}$ gIt is given that Avagadro number (6.023×10^{23}) of iron atoms have a mass of 55 g. Hence, the number of atoms in the domain is

$$N = \frac{7.9\times10^{-12}\times6.023\times10^{23}}{55} = 8.65 \times 10^{10}\text{ atoms}$$

The maximum possible dipole moment m_{max} is achieved for the(unrealistic) case when all the atomic moments are perfectly aligned.Thus,$m_{max} = (8.65 \times 10^{10}) \times (9.27 \times 10^{-24}) = 8.0 \times 10^{-13}$ A m^2

The consequent magnetisation is $M_{max} = m_{max}$/Domain volume $= 8.0 \times 10^{-13}\text{ Am}^2/10^{-18}\text{ m}^3 = 8.0 \times 10^5\text{ Am}^{-1}$

Hysterisis



Consider that a specimen of ferromagnetic material is placed in a magnetising field, whose strength and direction can be changed. Suppose that the specimen is unmagnetised initially. When the magnetising field (H) is increased, the intensity of magnetisation (I) of the material of the specimen also increases. It is found that when the magnetising field is made zero, the intensity of magnetisation does not become zero but still has some finite value. It becomes zero only, when magnetising field is increased inreversed direction. In other words, intensity of magnetisation does not become zero on making magnetising field zero but does so a little late and this effect is called hysteresis.

The lag of intensity of magnetisation behind the magnetising field during the process of magnetisation and demagnetisation of a ferromagnetic material is called hysteresis.

Fig shows the magnetisation curve of a ferromagnetic material, when it is taken over a complete cycle of magnetisation (I) is also zero. As magnetising field is increased, intensity of a mgnetisation also increases along.OA. Corresponding to point A, the intensity of magnetisation becomes maximum. The increase in value of the magnetising field beyond H_0 does not produce any increase in the intensity of magnetisation. In other words ,corresponding to point A, The specimen of the ferromagnetic material acquires a state of magnetic saturation. If magnetising field is now decreased slowly, intensity of magnetisation decreases but not a along the path AO.It decrease along the path AB. Corresponding to point B, magnetising field becomes zero but some magnetisation equal to OB is still left in the specimen. Here, OB gives the measure of retentivity of the material of the specimen.

The value of the intensity of magnetisation of a material, when the magnetising field is reduced to zero, is called retentivity of the material. It is also known residual magnetism or remanetism or remanence.

To reduce intensity of magnetisation to zero, the magnetising field has to be increased in reverse direction. As it is done so, the intensity of magnetisation decreases along BC, till it become zero corresponding to point C. Thus, to make intensity of magnetisation zero, magnetising field equal OC has to be applied in reverse direction. Here, OC gives the measure of coercivity of the material of the specimen .

The value of reverse magnetising field required so as to reduce residual magnetism to zero, is called coercivity of the material.

When the magnetising field is further increased in reverse direction, intensity of magnetisation increase along CD with the increase in magnetising field. Corresponding to point D (when the magnetising field becomes-H_0) it again acquires a saturation value, which is symmetrical to that corresponding to point A. If the magnetising field is decreased from–H_0 to zero, the intensity of magnetisation follows the path DE. Finally, when magnetising field is increased field in original direction, the point A is reached via EFA. If the magnetising field is repeatedly changed between H_0 and – H_0 , the curve ABCDEFA is retraced. The curve ABCDEFA is called the hysteresis loop. It is found that the area of the hysteresis loop (I – H curve)is proportional to the net energy absorbed per unit volume by the specimen, as it taken over a complete cycle of magnetisation and demagneti-sation. The energy so absorbed by the speciment appears as the heat energy.

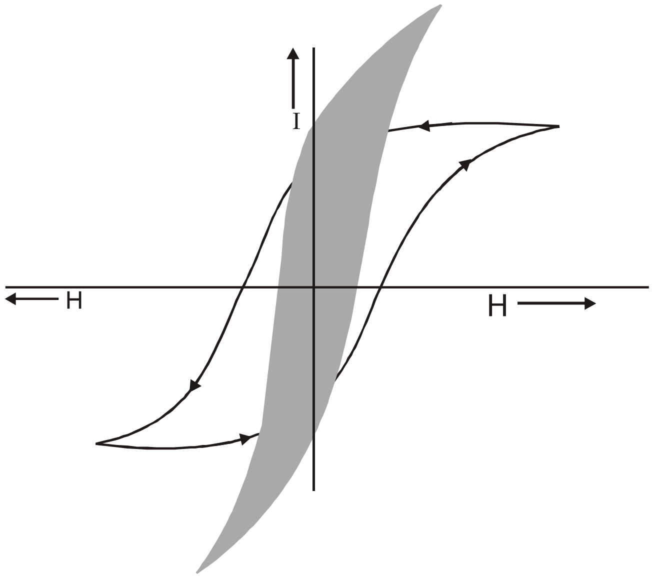
Note :- If hysteresis loop is drawn by plotting a graph between magnetic induction (B) and intensity of magnetisation, then area of the hysteresis loop is numerically equal to the work done per unit volume (or energy absorbed per unit volume) in taking the magnetic specimen over a complete cycle of magnetisation.

Comparison of hysteresis loops for soft iron and steel :-

The shape of the hysteresis loop is a characteristics of a ferromagnetic substance. It gives the idea about many important magnetic properties of the substance.

Figure Shown hysteresis loops for soft iron and steel. Whereas the hysteresis loop for soft iron is narrow, the hysteresis loop for steel is quite wide. The following conclusion can be drawn from the study of the hysteresis loops of soft iron and steel :

1. The area of hysteresis loop for soft iron is much smaller then that for steel. Therefore, loss of energy per unit volume in case of soft iron will be very small as compared to that in case of steel, when they are taken over a complete cycle of magnetisation and demagnetisation.



2. Soft iron acquires maximum intensity of magnetisation for comparatively much lesser value of magnetising field than in case of steel. In other words, soft iron is much strongly magnetised (or more susceptible to magnetisation) than steel.

3. The retentivity of soft iron is greater then that of steel. On removing magnetising field, quite a large amount of magnetisation is retained by soft iron.

4. The coercivity of steel is much larger then that of soft iron. Therefore, the residula magnetism in steel can not be destroyed that easily as in case of soft iron.

Section of magnetic materials :

The choice of a magnetic material for making permanent magnet, electromagnet, core of transformer or diaphragm of telephone ear-piece can be decided from the hysteresis curve of the material.

(i) Permanent magnets :

The material for a permanent magnet should have high retentivity so that the magnet is strong, and high coercivity so that the magnetisation is not wiped out by stray external fields, mechanical ill treatement and temperature changes. The hysteresis loss is immaterial because the material in this case is never put to cyclic changes of magnetisation. From these considerations permanent magnets are made of steel. The fact that the retentivity of soft iron is a little greater than that of steel is outweighed by its much smaller coercivity, which makes it very easy to demagnetise.

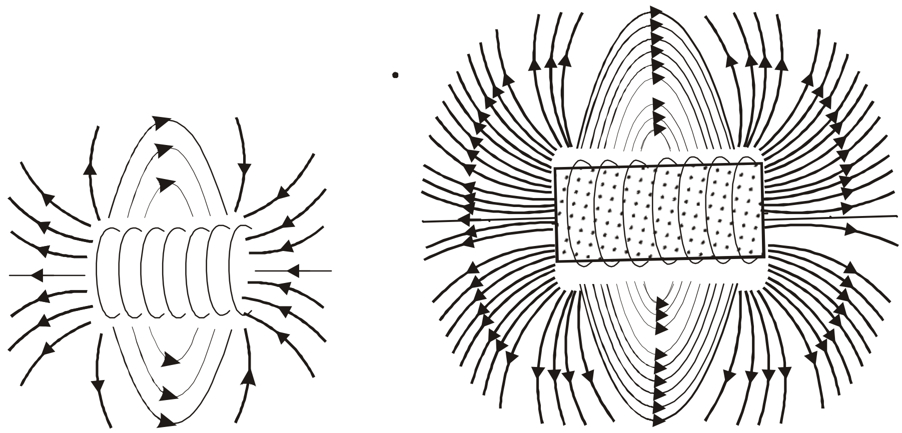
(ii) **Electromagnets :** The material for the cores of electromagnets should have high permeability (or high susceptibility), specially at low magnetising fields, and a low retentivity. Soft iron is suitable material for electromagnets).

(iii) **Transformer cores and telephone diaphragms :** In these cases the material goes through complete cycles of magnetisation continuously. The material must therefore have a low hysteresis loss to have less dissipation of energy and hence a small heating of the material (otherwise the insulation of windings may break), a high permeability (to obtain a large flux density at low field) and a high specific resistance (to reduce eddy current loses).

Soft -iron is used for making transformer cores and telephone diaphragms : More effective alloys have now been developed for transformer cores. They are permalloys, mumetals etc.

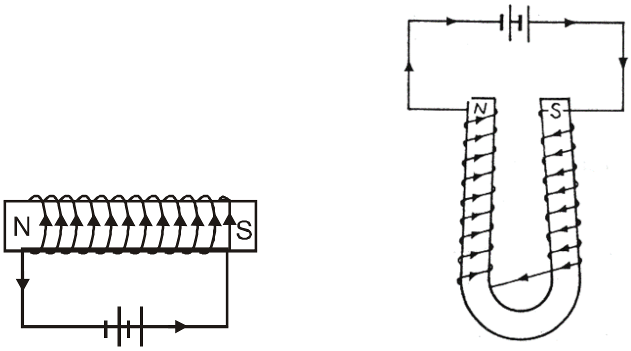
Electromagnet :

If we place a soft-iron rod in the solenoid, the magnetism of solenoid increases hundreds of time. Then the solenoid is called an 'electromagnet'. It is a temporary magnet.



(a) (b)

An electromagnet is made by winding closely a number of turns of insulated copper wire over a soft-iron straight rod or a horse-shoe rod. On passing current through this solenoid, a magnetic field is produced in the space within the solenoid.



(a) (b)

Illustration 50:

A magnetising field of 1600 Am^{-1} produces a magnetic flux of 2.4×10^{-5} wb in an iron bar of cross-sectional area $0.2 cm^2$. Calculate permeability and susceptibility of the bar.

Solution:

$$B = \frac{\Phi}{A} = \frac{2.4 \times 10^{-5}\ Wb}{0.2 \times 10^{-4}\ m^2} = 1.2\ Wb/m^2 = 1.2\ N\ A^{-1}\ m^{-1}.$$

The magnetising field (or magnetic intensity) H is 1600 Am^{-1}. Therefore, the magnetic permeability is given by -

$$\mu = \frac{B}{H} = \frac{1.2 NA^{-1} m^{-1}}{1600\ Am^{-1}} = 7.5 \times 10^{-4}\ N/A^2.$$

Now, from the relation $\mu = \mu_0 (1 + \chi_m)$, the susceptibility is given by

$$\chi_m = \frac{\mu}{\mu_0} - 1.$$

We known that $\mu_0 = 4\pi \times 10^{-7}\ N/A^2$. $\quad \therefore \quad \chi_m = \frac{7.5 \times 10^{-4}}{4 \times 3.14 \times 10^{-7}} - 1 = 596.$

Illustration 51:

The core of toroid of 3000 turns has inner and outer radii of 11 cm and 12 cm respectively. A current of 0.6 A produces a magnetic field of 2.5 T in the core. Compute relative permeability of the core.($\mu_0 = 4\pi \times 10^{-7}\ T\ m\ A^{-1}$).

Solution:

The magnetic field in the empty space enclosed by the windings of a toroid carrying a current i_0 is $\mu_0\ n\ i_0$ where n is the number of turns per unit length of the toroid and μ_0 is permeability of free space.

If the space is filled by a core of some material of permeability μ, then the field is given by

$$B = \mu\ n\ i_0$$

But $\mu = \mu_0 u_r$, where μ_r is the relative permeability of the core material. Thus,

$$B = \mu_0\ u_r\ ni_0 \quad \text{or} \quad \mu_r = \frac{B}{\mu_0 n\ i_0}$$

Here $\quad B = 2.5$ T, $i_0 = 0.7$ A and $n = \frac{300}{2\pi r}\ m^{-1}$, where r is the mean radius of the toroid

$(r = \frac{11+12}{2} = 11.5$ cm 11.5×10^{-2} m). Thus,

$$\mu_r = \frac{2.5}{(4\pi \times 10^{-7}) \times (3000 / 2\pi \times 11.5 \times 10^{-2}) \times 0.7} = \frac{2.5 \times 11.5 \times 10^{-2}}{2 \times 10^{-7} \times 3000 \times 0.7}$$

$$\mu_r = 684.5$$