01 Current Electricity and Capacitance

CURRENT ELECTRICITY

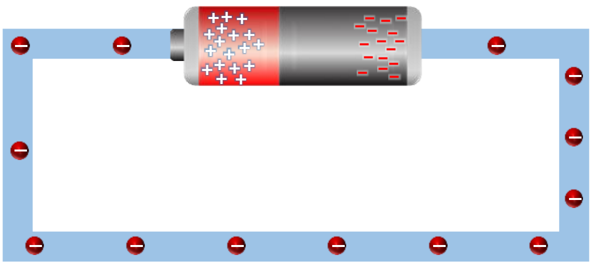
Current Microscopic Analysis

Electric Current

Time rate of flow of charge through a cross sectional area is called Current. If Δq charge flows in time interval Δt then average current is given by

$$I_{av} = \frac{\Delta q}{\Delta t}$$

Instantaneous current; $I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$



$\longleftarrow$ Direction of electron flow

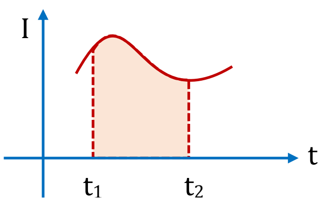
(Conventionally, direction of current is shown from positive to negative)

Direction of current is along the direction of flow of positive charge or opposite to the direction of flow of negative charge. But the current is a scalar quantity.

$\longrightarrow$ I I $\longleftarrow$

q$\oplus \longrightarrow$ velocity q $\ominus \longrightarrow$ velocity

- SI unit of current is ampere and 1 Ampere = 1 coulomb/sec
- It is a scalar quantity because it does not obey the law of vectors.
- Area under *I-t* curve provides total charge flow.



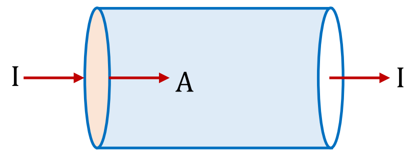
Area = $\int_{t_1}^{t_2} I dt = \Delta q$

Current, velocity and current density

Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.

- Current density $\vec{J}$ is a vector quantity. It's direction is same as that of $\vec{E}$. It's S.I. unit is ampere/m^2 and dimensions $[L^{-2}A]$.

- Current density at point P is given by $\vec{J} = \frac{dI}{dA}\vec{n}$



- If the cross–sectional area is not normal to the current, but makes an angle θ with the direction of current then

$J = \frac{dI}{dA\cos\theta} \Rightarrow dI = JdA\cos\theta = \vec{J}.d\vec{A} \Rightarrow I = \int \vec{J}.\overrightarrow{dA}$

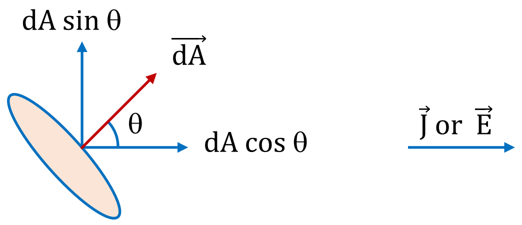


Illustration 1:
Current through a wire decreases uniformly from 4 A to zero in 10 s. Calculate charge flown through the wire during this interval of time.

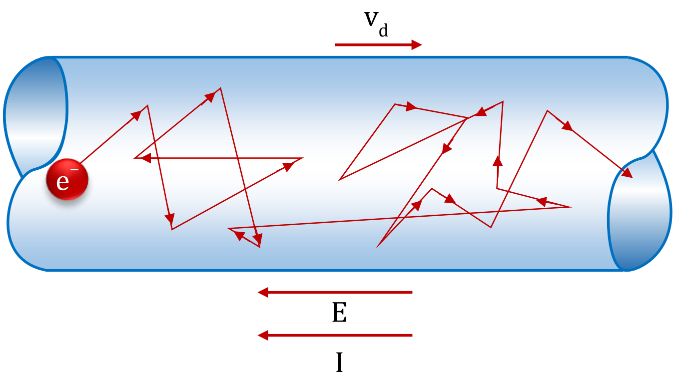
Solution:
Charge flown = average current × time

$= \left[\frac{4+0}{2}\right] \times 10 = 20C$

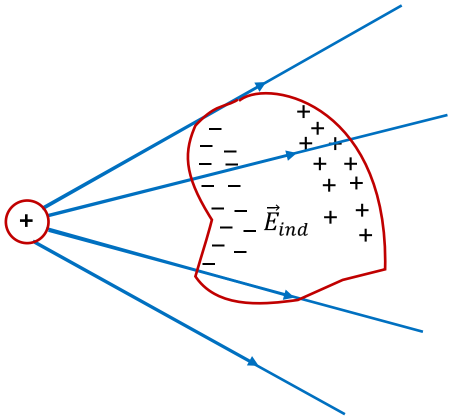
Movement of Electrons Inside Conductor
All the free electrons are in random motion due to the thermal energy and relationship is given by

$\frac{3}{2}KT = \frac{1}{2}mv^2$

At room temperature its speed is around 10^6 m/sec or 10^3 km/sec



But the average velocity is zero so current in any direction is zero.
When a conductor is placed in an electric field, then for a small duration, electron do have an average velocity but its average velocity becomes zero within short interval of time.



Mean Free Path (λ)

The distance travelled by a conduction electron during relaxation time is known as mean free path λ.

Mean free path of conduction electron = Thermal velocity × Relaxation time

$$\lambda = \frac{\lambda_1 + \lambda_2 \ldots\ldots + \lambda_N}{N}$$

Order of λ = 10Å.

Relaxation Time (τ)

It is defined as average time elapsed between two successive collisions.

It is of the order of 10^{-14} s. It is a temperature dependent characteristic of the material of the conductor. It decreases with increase in temperature.

$$\tau = \frac{\tau_1 + \tau_2 + \tau_3 + \ldots\ldots \tau_N}{N}$$

Thermal Speed

Conductor contains a large number of free electrons, which are in continuous random motion.

Due to random motion, the free electrons collide with positive metal ions with high frequency and undergo change in direction at each collision. So, the thermal velocities are randomly distributed in all possible directions.

$\vec{u}_1, \vec{u}_2, \ldots \vec{u}_N$ are the individual thermal velocities of the free electrons at any given time.

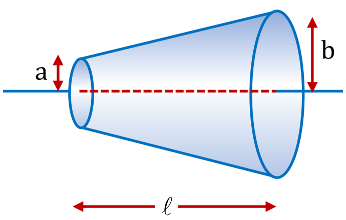
The total number of free electrons in the conductor = N

$$\text{Average velocity } \vec{u}_{avg} = \left[\frac{\vec{u}_1 + \vec{u}_2 + \ldots \vec{u}_N}{N}\right] = 0$$

The average velocity is zero but average speed is non-zero.

Illustration 2:

Figure shows a conductor of length ℓ carrying current i and having a circular cross - section. The radius of cross section varies linearly from a to b. Assuming that $(b - a) << \ell$ calculate current density at distance 'x' from left end.



Solution:

Since radius at left end is a and that of right end is b, therefore increase in radius over length ℓ is $(b - a)$.

Hence rate of increase of radius per unit length $= \left(\frac{b-a}{\ell}\right)$

Increase in radius over length $x = \left(\frac{b-a}{\ell}\right)x$

Since radius at left end is a, radius at distance x is :

$$r = a + \left(\frac{b-a}{\ell}\right)x$$

Area at this particular section $A = \pi r^2 = \pi\left[a + \left(\frac{b-a}{\ell}\right)x\right]^2$

Hence current density $= \frac{i}{A} = \frac{i}{\pi r^2} = \frac{i}{\pi\left[a+\frac{x(b-a)}{\ell}\right]^2}$

Illustration 3:

The current through a wire depends on time as $i = i_0 + \alpha \sin \pi t$, where $i_0 = 10\,A$ and $\alpha = \frac{\pi}{2}A$. Find the charge crossed through a section of the wire in 3 seconds, and average current for that interval.

Solution:

$$I = \frac{dq}{dt}$$

$$dq = Idt$$

$$q = \int_0^3 (i_0 + \alpha \sin \pi t)dt$$

$$= \left[i_0 t + \frac{\alpha}{\pi}(-\cos \pi t)\right]_0^3$$

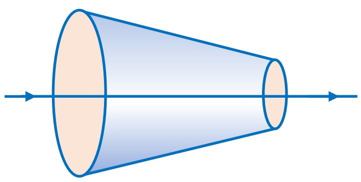
$$q = 3i_0 + \frac{2\alpha}{\pi} = 31C$$

Average current is :

$$I_{avg} = \frac{q}{\Delta t} = \frac{31C}{3s} \qquad \Rightarrow \quad I_{avg} = \frac{31}{3}A$$

Illustration 4:

For non-uniform cross-sectional area compare current density at 1 and 2 cross section



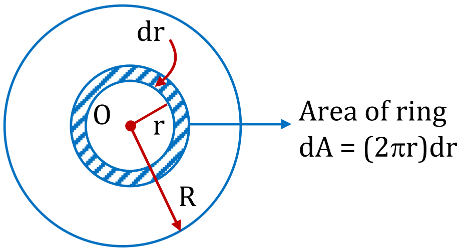
Solution:

$$J_1 = \frac{I}{A_1}\,;\, J_2 = \frac{I}{A_2}\,;\; A_2 < A_1 \Rightarrow J_1 < J_2$$

Illustration 5:

The current density across a cylindrical conductor of radius R varies in magnitude according to the equation $J = J_0\left(1-\frac{r}{R}\right)$ where r is the distance from the central axis. Thus, the current density is maximum J_0 at the axis (r = 0) and decreases linearly to zero at the surface ($r = R$). The current in terms of J_0 and conductor's cross-sectional area A is:

Solution:



$$I = \int \vec{J}\cdot\overrightarrow{dA}$$

$$= \int J dA\cos\theta$$

$$= \int_0^R J_0\left(1-\frac{r}{R}\right)(2\pi r dr)$$

$$= 2\pi J_0\int_0^R\left(r-\frac{r^2}{R}\right)dr = 2\pi J_0\left[\frac{r^2}{2}-\frac{r^3}{3R}\right]_0^R$$

$$= 2\pi J_0\left[\frac{R^2}{2}-\frac{R^3}{3R}\right]$$

$$I = \frac{\pi J_0 R^2}{3}$$

$$A = \pi R^2$$

So $\quad I = \frac{J_0 A}{3}$

Drift Velocity $(\vec{V}_d)$

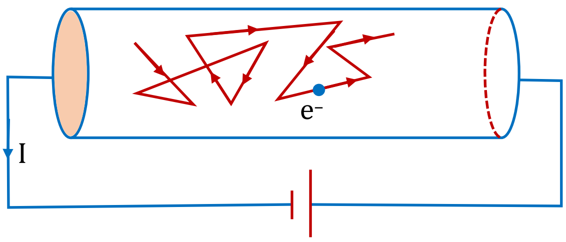
Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied electric field.

When the ends of a conductor are connected to a source of emf, an electric field E is established in the conductor, such that $E = \frac{V}{\ell}$

Where V = the potential difference across the conductor and ℓ= the length of the conductor.

The electric field $\vec{E}$ exerts an electrostatic force $-e\vec{E}$ on each electron in the conductor.

The acceleration of each electron $\vec{a} = \frac{-e\vec{E}}{m}$



Under the action of electric field:
Random motion of an electron
with superimposed drift

m = mass of electron

e = charge of electron

In addition to its thermal velocity, due to this acceleration, the electron acquires, a velocity component in a direction opposite to the direction of the electric field.

The gain in velocity due to the applied field is very small and is lost in the next collision.

At any given time, an electron has a velocity, $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$

Where, $\vec{u}_1$ = the thermal velocity

$\vec{a}\tau_1$ = the velocity acquired by the electron under the influence of the applied electric field.

τ_1 = the time that has elapsed since the last collision.

Similarly, the velocities of the other electrons are

$$\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2, \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, \ldots, \vec{v}_N = \vec{u}_N + \vec{a}\tau_N$$

The average velocity of all the free electrons in the conductor is equal to the drift velocity $\vec{v_d}$ of the free electrons.

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + \ldots \vec{v}_N}{N} = \frac{(\vec{u}_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + \ldots + (\vec{u}_N + \vec{a}\tau_N)}{N}$$

or $\vec{v}_d = \frac{(\vec{u}_1 + \vec{u}_2 + \ldots \vec{u}_N)}{N} + \vec{a}\frac{(\tau_1 + \tau_2 + \ldots + \tau_N)}{N}$ order of drift velocity is 10^{-4} m/s

$\because$ $\frac{\vec{u}_1 + \vec{u}_2 + \ldots + \vec{u}_N}{N} = 0$

$\therefore$ $\vec{v}_d = \vec{a}\frac{\tau_1 + \tau_2 + \ldots + \tau_N}{N} r$

$\Rightarrow$ $\vec{v}_d = \frac{e\vec{E}}{m}\tau$

Mobility

As we have seen, conductivity arises as a result of mobile charge carriers. In metals, these mobile charge carriers are electrons; in an ionised gas, they are electrons and positively charged ions; in an electrolyte, these can be both positive and negative ions.

An important quantity is the mobility μ defined as the magnitude of the drift velocity per unit electric field:

$$\mu = \frac{|v_d|}{E}$$

The SI unit of mobility is $m^2/V\text{-}s$ and is 10^4 times the mobility in practical units ($cm^2/V\text{-}s$). Mobility is positive.

$$v_d = \frac{e\tau}{m}E \quad \Rightarrow \mu = \frac{V_d}{E} = \frac{e\tau}{m}$$

Where τ is the relaxation time for electrons.

Note:

On increasing temperature of conductor, frequency of collision increases and hence average relaxation time decreases.

Temperature $\uparrow, \tau \downarrow$

$$V_d = \frac{eE\tau}{m}, V_d \downarrow$$

$$\sigma = \frac{ne^2\tau}{m}, \sigma \downarrow, \rho \uparrow$$ (Where ρ is resistivity of the material.)

$$\mu = \frac{e\tau}{m}, \mu \downarrow$$

Illustration 6:

Find the approximate total distance travelled by an electron in the time-interval in which its displacement is one meter along the wire.

Solution:

Time = $\dfrac{\text{displacement}}{\text{drift velocity}} = \dfrac{S}{V_d}$

∴ $V_d = 1\ mm/s = 10^{-3}\ m/s$

(Normally the value of drift velocity is 1 mm/s)

$S = 1\ m$

time = $\dfrac{1}{10^{-3}} = 10^3 s$

distance travelled = speed × time

∵ speed = $10^6\ m/s$

So, required distance

$= 10^6 \times 10^3 m = 10^9 m$

Illustration 7:

A current of 1.34 A exists in a copper wire of cross-section area 1.0 mm^2. Assuming each copper atom contributes one free electron. Calculate the drift speed of the free electrons in the wire. The density of copper is 8990kg/m^3 and atomic mass = 63.50.

Solution:

Mass of $1m^3$ volume of the copper is = 8990 kg

= 8990×10^3 g

Number of moles in $1m^3 = \dfrac{8990\times10^3}{63.5} = 1.4 \times 10^5$

Since each mole contains 6×10^{23} atoms therefore number of atoms in $1m^3$

$n = (1.4 \times 10^5) \times (6 \times 10^{23}) = 8.4 \times 10^{28}$ = electron density

$i = neAv_d$

$v_d = \dfrac{i}{neA} = \dfrac{1.34}{8.4\times10^{28}\times1.6\times10^{-19}\times10^{-6}}$ $(\because 1mm^2 = 10^{-6}m^2) = 10^{-4}m/s$

Illustration 8:

Two wires each of radius of cross-section r but of different materials are connected together end to end (in series). If the densities of charge carriers in the two wires are in the ratio 1 : 4, the drift velocity of electrons in the two wires will be in the ratio:

(A) 1 : 2 (B) 2 : 1 (C) 4 : 1 (D) 1 : 4

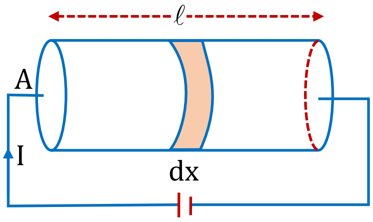
Solution:

$I = neAv_d$

⇒ $v_d \propto \dfrac{1}{n}$

⇒ $\dfrac{v_{d_1}}{v_{d_2}} = \dfrac{n_2}{n_1} = \dfrac{4n_1}{n_1} = 4 : 1$

Ohm's Law



Let the number of free electrons per unit volume in a conductor = n

Total number of electrons in dx distance = $n\ (Adx)$

Total charge, $dQ = n\ (Adx)e$

Cross sectional area = A

Current $\quad I = \frac{dQ}{dt} = nAe\frac{dx}{dt} \quad \Rightarrow \quad I = neAv_d$

Current density $\quad J = \frac{I}{A} = nev_d \quad \Rightarrow \quad J = ne\left(\frac{eE}{m}\right)\tau$

$$v_d = \left(\frac{eE}{m}\right)\tau$$

$$J = \left(\frac{ne^2\tau}{m}\right)E \quad \Rightarrow \quad J = \sigma E,\ \sigma \text{ is called conductivity } (\sigma) = \frac{ne^2\tau}{m}$$

In vector form $\quad \vec{J} = \sigma\vec{E}$

σ depends only on the material of the conductor and its temperature.

As temperature $(T)\uparrow$, $\tau\downarrow$

Now $\quad J = \frac{I}{A}$, $\sigma = \frac{ne^2\tau}{m}$ and $E = \frac{V}{\ell}$

$$I = \frac{nAe^2\tau}{m\ell}V$$

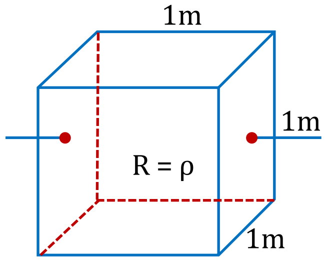
$$V = \frac{m\ell}{nAe^2\tau}I$$

$$V = IR$$

So current in conductors is proportional to potential difference applied across its ends. This is Ohm's Law.

R has S.I unit ohm (Ω).

Resistivity



It is the property of substance, defined as

$$\rho = \frac{RA}{\ell}, \text{ if } \ell = 1\text{m}, A = 1\text{m}^2 \text{ then } \rho = R$$

It's unit is Ω-m

The specific resistance of a material is equal to the resistance of the wire of that material with unit cross – section area and unit length.

Resistivity depends on:

(i) Nature of material

(ii) Temperature of material

ρ does not depend on the size and shape of the material because it is the characteristic property of the conductor material.

$$\rho_{alloy} > \rho_{semiconductor} > \rho_{conductor}$$

Conductivity

It is the measure of the ease at which an electric charge can pass through a material.

It's SI unit is Siemens per meter (S/m)

It is the inverse of resistivity $\sigma = \frac{1}{\rho}$

Limitations of Ohm's Law

Although Ohm's law has been found valid over a large class of materials, there do exist materials and devices used in electric circuits where the proportionality of V and I does not hold. The deviations broadly are one or more of the following types:

(a) V ceases to be proportional to I (Fig. 1).

(b) The relation between V and I depends on the sign of V. In other words, if I is the current for a certain V, then reversing the direction of V keeping its magnitude fixed, does not produce a current of the same magnitude as I in the opposite direction (Fig. 2). This happens, for example, in a diode

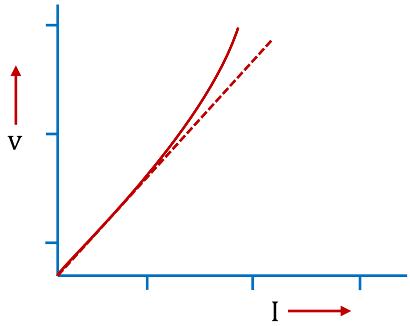


Fig-1 : The dashed line represents the linear ohm's law. The solid line is the voltage V versus current I for a good conductor.

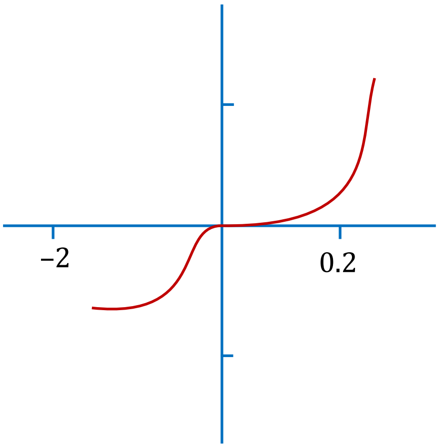


Fig -2 : Characteristics curve of a diode. Note the different scales for negative and positive values of the voltage and current.

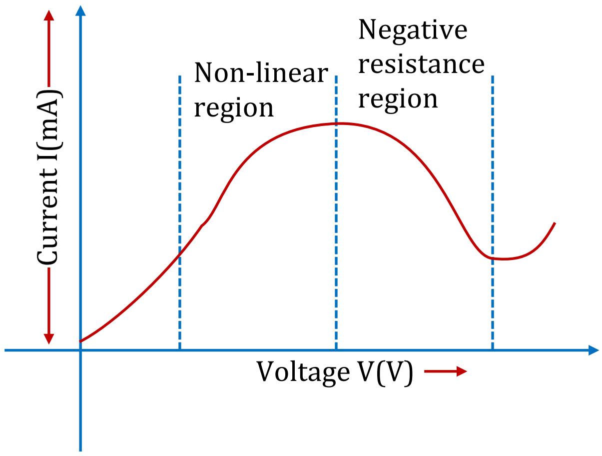


Fig.3 Variation of current versus voltage for GaAs.

(c) The relation between V and I is not unique, i.e., there is more than one value of V for the same current I (Fig. 3). A material exhibiting such behaviour is GaAs.

Materials and devices not obeying Ohm's law in the form $V = IR$, are actually widely used in electronic circuits.

Illustration 9:

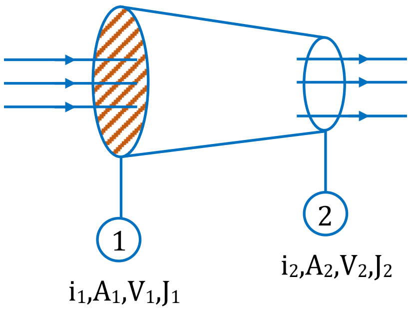
Current is flowing from a conductor of non-uniform cross section area if $A_1 > A_2$. Then find relation between

(a) i_1 and i_2

(b) J_1 and J_2

(c) V_1 and V_2 (drift velocity)

where i is current, J is current density and V is drift velocity.



Solution:

(a) i = Charge flowing through a cross-section per unit time.

$\therefore \quad i_1 = i_2$

(b) $J = \frac{i}{A}$

as $A_1 > A_2$ then $J_1 < J_2$

(c) $J = nev_d$

$v_d = \frac{J}{ne}$

as $J_1 < J_2$ then, $V_1 < V_2$

Resistance and it's Temperature Dependence

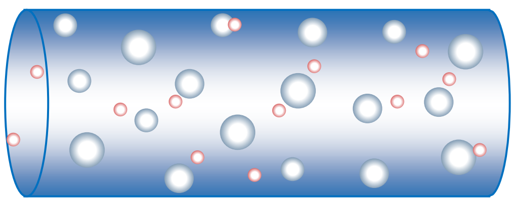
Electrical Resistance

The property of a substance by virtue of which it opposes the flow of electric current through it is termed as electrical resistance. Electrical resistance depends on the size, geometry, temperature and internal structure of the conductor.

We have, $I = \frac{nAe^2\tau}{m\ell}V$

$I = \frac{V}{R}$

$R = \frac{m\ell}{nAe^2\tau}$



Hence, $R = \frac{m}{ne^2\tau} \cdot \frac{\ell}{A}$

So, here $\boxed{R = \frac{\rho\ell}{A}}$

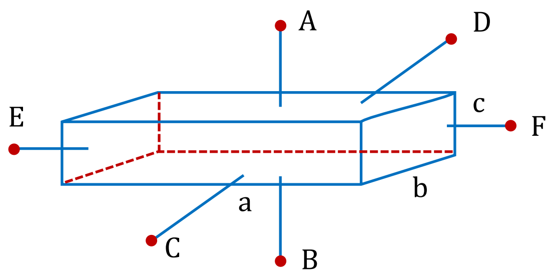
$\Rightarrow \quad V = I \times \frac{\rho\ell}{A}$

$\Rightarrow \quad \frac{V}{\ell} = \frac{I}{A}\rho \quad \Rightarrow \quad E = J\rho \quad \Rightarrow \quad J = \frac{I}{A}$ = current density

ρ is called resistively (it is also called specific resistance), and $\rho = \frac{m}{ne^2\tau} = \frac{1}{\sigma}$, σ is called conductivity. S.I. unit of resistivity is ohm-m (Ω-m). S.I. unit of conductivity is Ω^{-1}-m^{-1} also called siemens.

Illustration 10:

The dimensions of a conductor of specific resistance ρ are shown below. Find the resistance of the conductor across AB, CD and EF.



Solution:

For a condition

$R = \frac{\rho\, l}{A} = \frac{\text{Resistivity} \times \text{length}}{\text{Area of cross section}} \quad \Rightarrow R_{AB} = \frac{\rho c}{ab}, R_{CD} = \frac{\rho b}{ac}, R_{EF} = \frac{\rho a}{bc}$

Dependence of Resistance on Various Factors

From previous discussion we have:

$$R = \rho\frac{\ell}{A} = \frac{m}{ne^2\tau}\times\frac{\ell}{A}$$

Therefore, resistance depends on

(1) length of the conductor ($R \propto \ell$)

(2) area of cross - section of the conductor $R \propto \frac{1}{A}$

(3) nature of material of the conductor $R = \frac{\rho\ell}{A}$

Results:

(a) On stretching a wire (volume constant), if length of wire is taken into account then $\frac{R_1}{R_2} = \frac{\ell_1^2}{\ell_2^2}$

(b) If radius of cross section is taken into account then $\frac{R_1}{R_2} = \frac{r_2^4}{r_1^4}$, where R_1 and R_2 are initial and final resistances and ℓ_1, ℓ_2, are initial and final lengths and r_1 and r_2 initial and final radii respectively.

(c) Effect of percentage change in length of wire

$$\frac{R_2}{R_1} = \frac{\ell^2[1+\frac{x}{100}]^2}{\ell^2}$$, where ℓ - original length and x- % increment

If x is quite small (say < 5%) then % change in R is

$$\frac{R_2 - R_1}{R_1}\times 100 = \left(\frac{\left(1+\frac{x}{100}\right)^2 - 1}{1}\right)\times 100 \cong 2x\%$$

Illustration 11:

If a wire is stretched to double its length, find the new resistance if original resistance of the wire was R.

Solution:

As we know that $R = \frac{\rho\ell}{A}$ $\Rightarrow$ in case $R' = \frac{\rho\ell'}{A'}$

$\ell' = 2\ell$

$A'\ell' = A\ell$ (Volume of the wire remains constant)

$A' = \frac{A}{2}$ $\Rightarrow$ $R' = \frac{\rho\times 2\ell}{A/2} = 4\frac{\rho\ell}{A} = 4R$

Illustration 12:

The wire is stretched to increase the length by 1%. Find the percentage change in the resistance.

Solution:

As we known that,

$\therefore$ $R = \frac{\rho\ell}{A}$

$\Rightarrow \quad \frac{\Delta R}{R} = \frac{\Delta \rho}{\rho} + \frac{\Delta \ell}{\ell} - \frac{\Delta A}{A}$ and $\frac{\Delta \ell}{\ell} = -\frac{\Delta A}{A}$

$\frac{\Delta R}{R} = 0 + 1 + 1 = 2$

Hence percentage increase in the Resistance = 2%.

Note:

Above method is applicable when % change is very small.

Illustration 13:

Resistance of a wire is 20 Ω, it is stretched upto three times of its length, then its new resistance will be

(A) 6.67 Ω (B) 60 Ω (C) 120 Ω (D) 180 Ω

Solution:

Wire is stretched, therefore its volume remains unchanged

$A_1 \ell_1 = A_2 \ell_2 \Rightarrow A_1 \ell = A_2 \times 3\ell \quad \Rightarrow A_2 = \frac{A_1}{3}$

Ratio of resistance, $\frac{R_1}{R_2} = \frac{\ell_1}{\ell_2} \times \frac{A_2}{A_1} = \frac{\ell}{3\ell} \times \frac{1}{3} = \frac{1}{9} \quad \Rightarrow \frac{20}{R_2} = \frac{1}{9} \quad \Rightarrow R_2 = 180\ \Omega$

Temperature Dependence of Resistivity and Resistance:

The resistivity of a material is found to be dependent on the temperature over a limited range of temperatures (not too large). The resistivity of a metallic conductor is approximately given by:

$\rho_T = \rho_0(1 + \alpha(T - T_0))$

Where ρ_T is the resistivity at a temperature T and ρ_0 is the resistivity at a reference temperature T_0. Here, α is called the temperature co-efficient of resistivity.

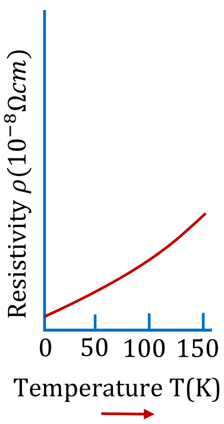


Figure - 1
Resistivity ρ_T of Copper as a function of temperature T.

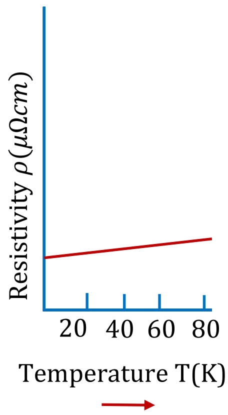


Figure - 2
Resistivity ρ_T of Nichrome as a function of absolute temperature T.

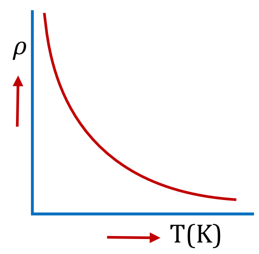


Figure - 3
Temperature dependence of resistivity for a typical semiconductor.

Unit of α is °C^{-1}.

Resistance corresponding to temperature difference (ΔT) is given as $R_t = R_0\,(1 + \alpha\Delta T)$

Where R_t = Resistance at $t°C$, R_0 = Resistance at 0°C

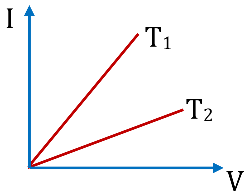
ΔT = Change in temperature, α = Temperature coefficient of resistance.

[For metals: α is positive but for semiconductors and insulators: α is negative]

Resistance of the conductor decreases linearly (figure-1) with decrease in temperature and becomes zero at a specific temperature. This temperature is called critical temperature. Below this temperature a conductor becomes a superconductor.

Illustration 14:

The current-voltage graphs for a given metallic wire at two different temperature T_1 and T_2 are shown in the figure. Which one is higher, T_1 or T_2.



Solution:

Slope of $I - V$ Curve is $\dfrac{1}{R}$

$$\left(\frac{1}{R}\right)_1 > \left(\frac{1}{R}\right)_2 \Rightarrow R_2 > R_1 \Rightarrow T_2 > T_1$$

Illustration 15:

The resistance of a thin silver wire is 1.0 Ω at 20°C. The wire is placed in liquid bath and its resistance rises to 1.2 Ω. What is the temperature of the bath? (Here α = 10^{-2} /°C)

Solution:

Here change in resistance is small so we can apply

$R = R_0(1 + \alpha\Delta\theta)$

⇒ $1.2 = 1 \times (1 + 10^{-2}\,\Delta\theta)$

⇒ $\Delta\theta = 20°C$

⇒ $\theta - 20 = 20$

⇒ $\theta = 40°\,C$ **Ans.**

Colour Coding of Carbon Resistor

Resistors in the higher range are made mostly from carbon. Carbon resistors are compact, inexpensive and thus find extensive use in electronic circuits. Carbon resistors are small in size and hence their values are given using a colour code.

The resistors have a set of co-axial coloured rings on them whose significance are listed in table. The first two bands from the end indicate the first two significant figures of the resistance in ohms. The third band indicates the decimal multiplier (as listed in Table). The last band stands for tolerance or possible variation in percentage about the indicated values. Sometimes, this last band is absent and that indicates a tolerance of 20%.

Digit	1st	2nd	Multiplier colour	Multiplier
0	Black	Black	Black	10^0
1	Brown	Brown	Brown	10^1
2	Red	Red	Red	10^2
3	Orange	Orange	Orange	10^3
4	Yellow	Yellow	Yellow	10^4
5	Green	Green	Green	10^5
6	Blue	Blue	Blue	10^6
7	Violet	Violet	Violet	10^7
8	Grey	Grey	Grey	10^8
9	White	White	White	10^9
			Gold	10^{-1}
			Silver	10^{-2}

Tolerance	
Gold	5%
Silver	10%
No Colour	20%

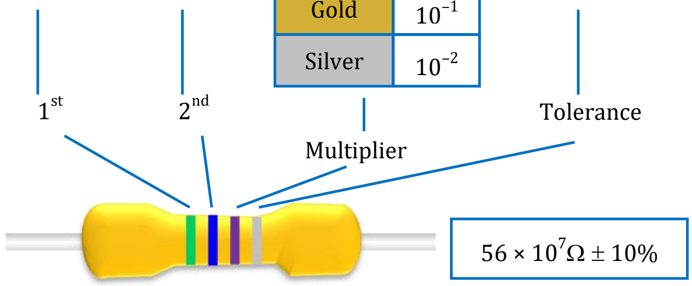


Fig. Colour Code for Carbon Resistors

To memorise the sequence of colour, learn **BB ROY** of **Great Britain** had a **Very Good Wife**.

Illustration 16:

Draw a colour code for 42 k Ω ± 10% carbon resistance.

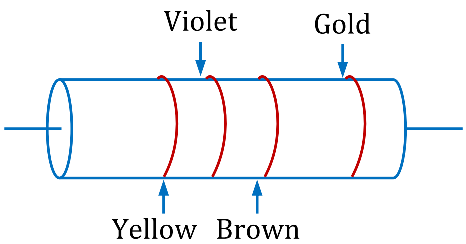
Solution:

According to colour code colour for digit 4 is yellow, for digit 2 it is red, for 3 colour is orange and 10% tolerance is represented by silver colour.

So, colour code should be yellow, red, orange and silver.

Illustration 17:

What is resistance of following resistor.



Solution:

Number for yellow is 4, Number of violet is 7.

Brown colour gives multiplier 10^1, Gold gives a tolerance of ± 5%.

So resistance of resistor is $47\times10^1\ \Omega \pm 5\%$ or $470 \pm 5\%\ \Omega$.

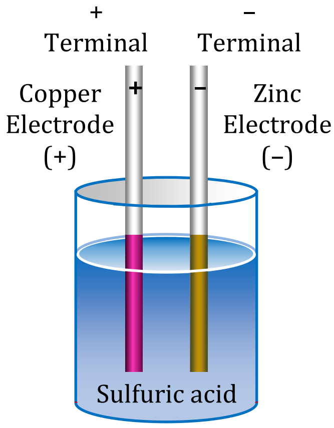
Cell, E.M.F and Internal Resistance

Battery (Cell)

A battery is a device which maintains a potential difference across its two terminals *A* and *B*. Dry cells, secondary cells, generator and thermocouple are the devices used for producing potential difference in an electric circuit. Arrangement of cell or battery is shown in figure.

Electrolyte provides continuity for current.

It is often prepared by putting two rods or plates of different metals in a chemical solution. Some internal mechanism exerts force ($\vec{F}_n$)on the ions (positive and negative) of the solution. This force drives positive ions towards positive terminal and negative ions towards negative terminal. As positive charge accumulates on anode and negative charge on cathode a potential difference and hence an electric field $\vec{E}$ is developed from anode to cathode. This electric field exerts an electrostatic force $\vec{F} = q\vec{E}$on the ions. This force is opposite to that of $\vec{F}_n$. In equilibrium (steady state) $F_n = F_e$ and no further accumulation of charge takes place.



Simple Electric Cell

When the terminals of the battery are connected by a conducting wire, an electric field is developed in the wire.

The free electrons in the wire move in the opposite direction and enter the battery at positive terminal. Some electrons are withdrawn from the negative terminal. Thus, potential difference and hence, F_e decreases in magnitude while F_n remains the same. Thus, there is a net force on the positive charge towards the positive terminal. With this the positive charge rush towards positive terminal and negative charge rush towards negative terminal. Thus, the potential difference between positive and negative terminal is maintained.

Internal resistance (r):

The potential difference across a real source in a circuit is not equal to the emf of the cell. The reason is that charge moving through the electrolyte of the cell encounters resistance. We call this the internal resistance of the source.

Internal resistance is always connected in series with the source.

The internal resistance of a cell depends on the distance between electrodes ($r \propto d$), area of electrodes ($r \propto \frac{1}{s}$) and nature, concentration ($r \propto c$) and temperature of electrolyte ($r \propto \frac{1}{\text{Temp.}}$).

Illustration 18:

What is the meaning of 10 Amp. hr?

Solution:

It means if the 10 A current is withdrawn then the battery will work for 1 hour.

$$10 \text{ Amp} \longrightarrow 1 \text{ hr}$$

$$1 \text{ Amp} \longrightarrow 10 \text{ hr}$$

$$\tfrac{1}{2} \text{ Amp} \longrightarrow 20 \text{ hr}$$

Electromotive force : (E.M.F)

Definition I : Electromotive force is the capability of the system to make the charge flow.

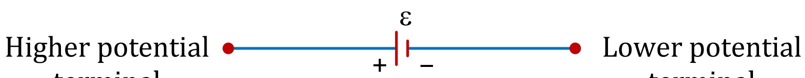
Definition II : It is the work done by the battery for the flow of 1 coloumb charge from lower potential terminal to higher potential terminal inside the battery.

Definition III : Electromotive force of a cell is equal to potential difference between its terminals when no current is passing through the circuit.

Representation for Battery:

- **Ideal cell :**

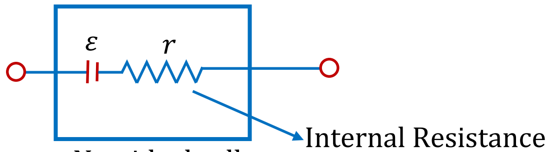
  Cell in which there is no heating effect.



Ideal cell

- **Non-ideal cell :**

  Cell in which there is heating effect inside due to opposition to the current flow internally



Non ideal cell

Terminal Potential Difference (TPD or V)

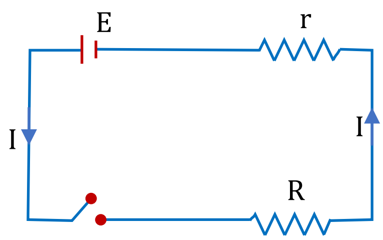
The potential difference between the two electrodes of a cell in a closed circuit i.e., when current is being drawn from the cell is called terminal potential difference.

(a) When cell is discharging:

When cell is discharging current inside the cell is from cathode to anode.

Current $I = \frac{E}{r+R}$ or $E = IR + Ir = V + Ir$ or $V = E - Ir$

When current is drawn from the cell potential difference is less than emf of cell. Greater is the current drawn from the cell smaller is the terminal voltage. When a large current is drawn from a cell its terminal voltage is reduced.

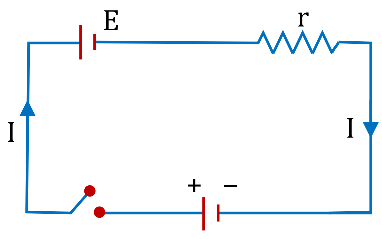


(b) When cell is charging:

When cell is charging current inside the cell is from anode to cathode.

Current $I = \frac{V-E}{r}$ or $V = E + Ir$

During charging terminal potential difference is greater than emf of cell.



(c) When cell is in open circuit:

In open circuit, $R = \infty$ $\therefore$ $I = \frac{e}{R+r} = 0$ So, $V = E$

In open circuit terminal potential difference is equal to emf and is the maximum potential difference which a cell can provide.

(d) When cell is short circuited:

In short circuit, $R = 0$ So, $I = \frac{E}{R+r} = \frac{E}{r}$ and $V = 0$

In short circuit, current from cell is maximum and terminal potential difference is zero.

Note:

- The potential at all points of a wire of zero resistance will be same.
- Earthing : If some point of circuit is earthed then its potential is assumed to be zero.

Important point

1. At the time of charging a cell. When current is supplied to the cell, the terminal voltage is greater than the e.m.f. E.

   $V = E + Ir$
2. Series combination is useful when internal resistance is less than external resistance of the cell.
3. Parallel combination is useful when internal resistance is greater than external resistance of the cell.
4. Power in R (given resistance) is maximum, if its value is equal to net resistance of remaining circuit.
5. Internal resistance of ideal cell = 0.
6. If external resistance is zero than current given by circuit is maximum.

Value of External Resistance	Current from Cell	Terminal Potential Difference (TPD)
R	$I = \frac{E}{R+r}$	$V = E - Ir$
$R = 0$	$I = \frac{E}{r}$	$V = E - \frac{E}{r}r$
Short circuit	Maximum	$V = 0$
Open circuit $R = \infty$	$I = 0$	$V = E - 0$ $V = E$ ($TPD = EMF$)

Illustration 19:

By a cell a current of 0.9 A flows through 2 ohm resistor and 0.3 A through 7 ohm resistor. The internal resistance of the cell is

(A) 0.5Ω (B) 1.0 Ω (C) 1.2 Ω (D) 2.0 Ω

Solution:

$0.9 = \frac{\varepsilon}{(2+R)}$ ...(1)

$0.3 = \frac{\varepsilon}{(7+R)}$ ...(2)

On solving equation (1) and (2)

$R = 0.5\Omega$

Illustration 20:

A cell of e.m.f. E is connected with an external resistance R, then p.d. across cell is V. The internal resistance of cell will be

(A) $\frac{(E-V)R}{E}$ (B) $\frac{(E-V)R}{V}$ (C) $\frac{(V-E)R}{V}$ (D) $\frac{(V-E)R}{E}$

Solution:

$$I = E/(R+r)$$

Potential difference across R is same as that of battery.

$$V = IR$$

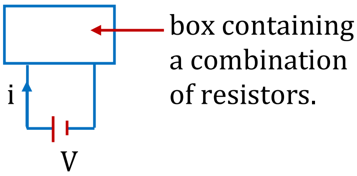
$$V = ER/(R+r) \quad \Rightarrow \quad r = \frac{ER}{V} - R$$

Series and Parallel Combination of Resistance

Combination of Resistances:

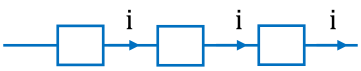
A number of resistances can be connected and all the complicated combinations can be reduced to two different types, namely series and parallel.

The equivalent resistance of a combination is defined as $R_{eq} = \frac{V}{i}$



Series Combination:

When the resistances are connected end to end then they are said to be in series. In series current through each element is same.



Resistances in series carry equal current but reverse may not be true.

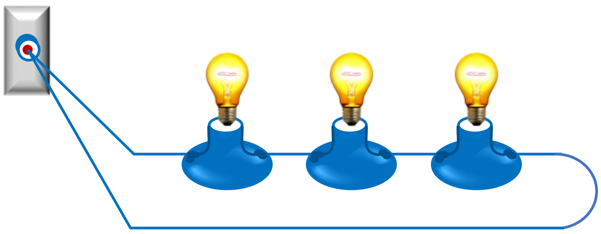
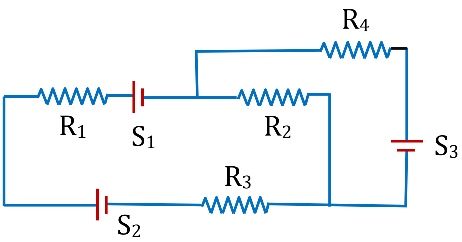


Illustration 21:

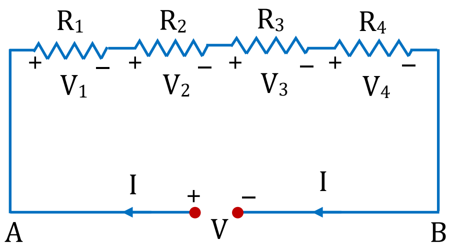
Which electrical elements are connected in series.



Solution:

Here S_1, S_2, R_1, R_3 connected in one series and R_4, S_3 connected in different series.

Equivalent Resistance of Series Combination



The potential difference across a resistor is proportional to the resistance.

$\because$ $V = IR_{eq}$ where I is same through any of the resistor.

and $V = V_1 + V_2 + V_3 + \ldots\ldots + V_n$ where

$$V_1 = \frac{R_1}{R_1+R_2+\ldots\ldots+R_n}V$$

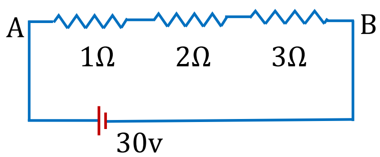
$$V_2 = \frac{R_2}{R_1+R_2+\ldots\ldots+R_n}V \text{ and so on}$$

The effective resistance appearing across the battery (or between the terminals A and B) is

$R_{eq} = R_1 + R_2 + R_3 + \ldots\ldots + R_n$ (this means R_{eq} is greater than any resistor)

Illustration 22:

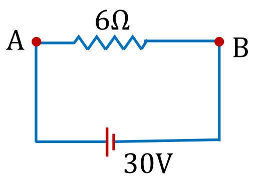
Find the current in the circuit.



Solution:

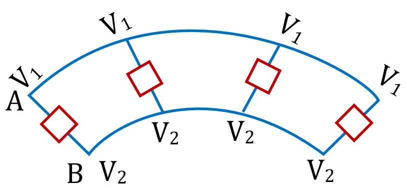
$R_{eq} = 1 + 2 + 3 = 6\Omega$ the given circuit is equivalent to

current $i = \dfrac{v}{R_{eq}} = \dfrac{30}{6} = \mathbf{5A}$ **Ans.**

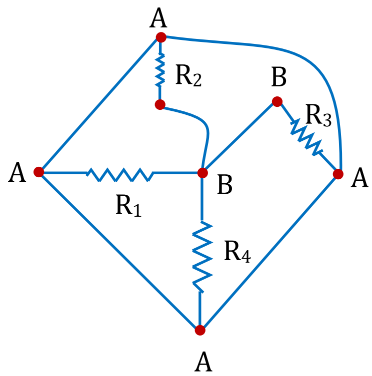


Parallel Combination:

A parallel circuit of resistors is one in which the same voltage is applied across all the components in a parallel grouping of resistors $R_1, R_2, R_3, \ldots\ldots, R_n$.



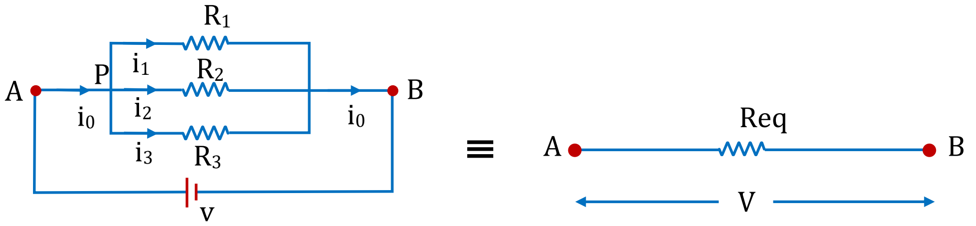
(a)



(b)

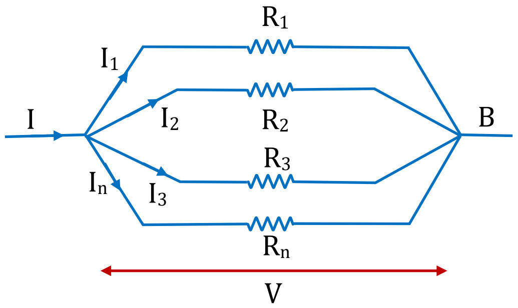
In the figure (a) and (b) all the resistors are connected between points A and B so they are in parallel.

Equivalent Resistance of Parallel Combination



At junction point P,

$i_0 = i_1 + i_2 + i_3$



Therefore, $\dfrac{V}{R_{eq}} = \dfrac{V}{R_1} + \dfrac{V}{R_2} + \dfrac{V}{R_3}$

$\Rightarrow \qquad \dfrac{1}{R_{eq}} = \dfrac{1}{R_1} + \dfrac{1}{R_2} + \dfrac{1}{R_3}$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots + \frac{1}{R_n}$$

Conclusions: (about parallel combination)

(a) Potential difference across each resistor is same.

(b) $I = I_1 + I_2 + I_3 + \ldots\ldots I_n$

(c) Effective resistance (R) then,

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

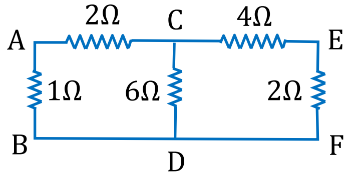
(R is less than each resistor).

(d) Current in different resistors is inversely proportional to the resistance.

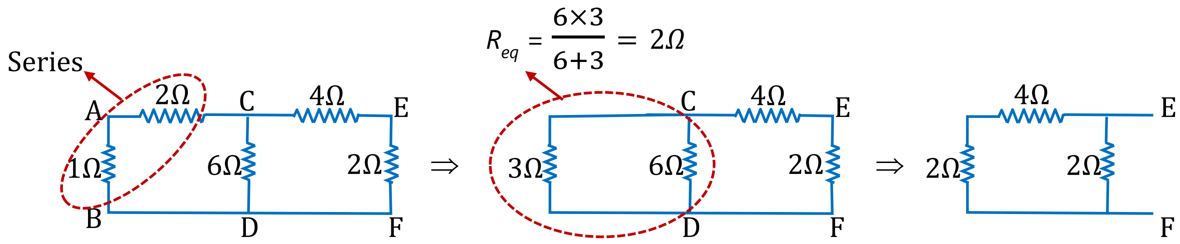
$$I_1 : I_2 : \ldots : I_n = \frac{1}{R_1} : \frac{1}{R_2} : \frac{1}{R_3} : \ldots : \frac{1}{R_n}$$

Illustration 23:

Find the equivalent resistance of the circuit given in figure between E and F.



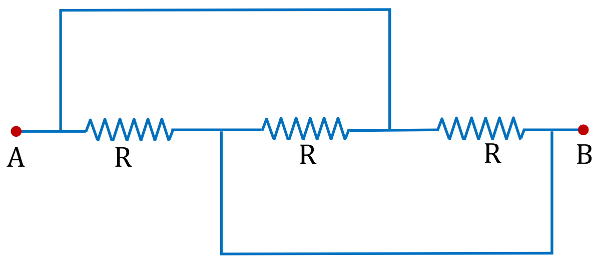
Solution:



$\Rightarrow \quad R_{EF} = \frac{6\times2}{6+2} = \frac{3}{2}\Omega$

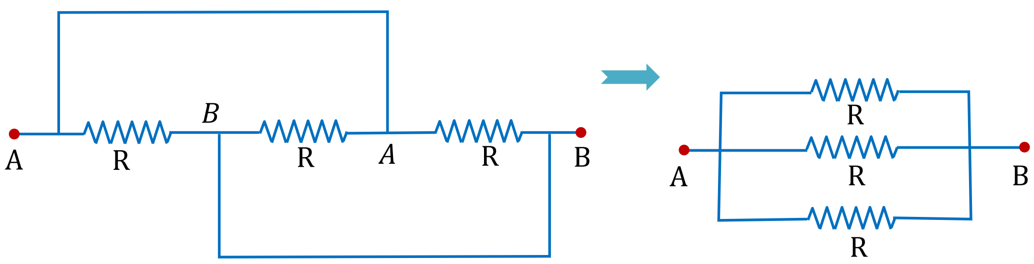
Illustration 24:

What is the equivalent resistance between A and B?



Solution:

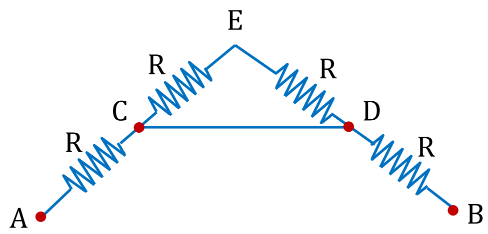
All the three resistors can be considered in parallel as shown below:



$\therefore \quad R_{eq} = \frac{R}{3}$

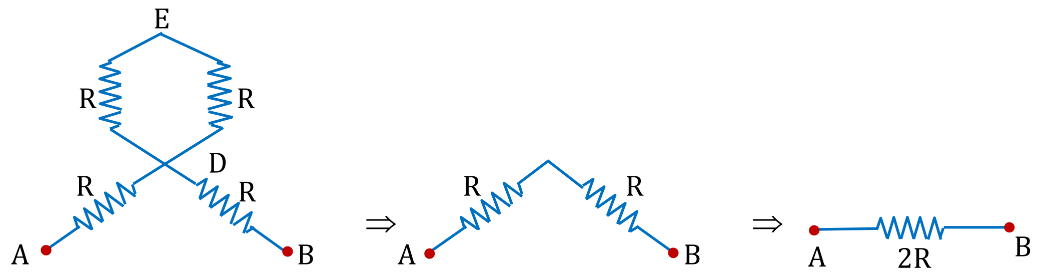
Illustration 25:

Four identical resistances each having value R are arranged as shown in figure. Find the equivalent resistance between A and B.



Solution:

Since C and D are connected with a zero resistor, they are equipotential. Then superimpose C and D to obtain the simplified circuit as shown. Since no current flows in the branches CE and ED, cut and then throw them to have



$\Rightarrow \qquad R_{AB} = R + R = 2R$

If we get a closed loop of resistors without any battery, it carries no current. Then remove the total loop to get a simpler circuit or if the current in any branch is zero, remove it.

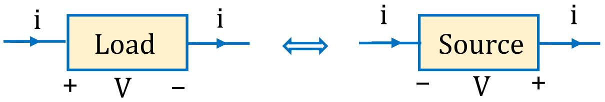
Electric Power and Heating Effects of Current

Electrical Power:

Energy liberated per second in a device is called its power. The electrical power P delivered or consumed by an electrical device is given by $P = Vi$, where V = Potential difference across the device and i = Current.

$$\text{Power } (P) = \frac{V.dq}{dt} = V\,i$$

If the current enters the higher potential point of the device then electric power is consumed by it (i.e. acts as load). If the current enters the lower potential point then the device supplies power (i.e. acts as source).



Heating Effect of Current

Cause of Heating:

The potential difference applied across the two ends of conductor sets up electric field. Under the effect of electric field, electrons accelerate and as they move, they collide against the ions and atoms in the conductor, the energy of e^- transferred to the atoms and ions appears as heat.

Joules's Law of Heating

Let voltage 'V' is applied across the conductor of resistance R and a charge dq flows through it in time dt then work done by the battery is

$$dw = dqV$$

$$= idt\,V$$

$$\int dw = \int i(iR)dt = \int i^2 Rdt$$

Since Heat energy released from the conductor is equal to work done by the battery unit,

$$H = \text{Heat released} = W = \int i^2\,Rdt$$

If $\qquad i$ = constant, $\qquad H = i^2 Rt = \dfrac{V^2}{R}t = (Vi)t$

Heat produced in conductor does not depend upon the direction of current.

SI UNIT : joule ; 1 kWh = 3.6×10^6 joule = 1 unit ; 1 BTU (British Thermal Unit) = 1055 J

Power delivered to the load resistance

$$P = \frac{dw}{dt}$$

$$P = \frac{i^2Rdt}{dt} = i^2R = \frac{V^2}{R} = Vi$$

Fusing of an Electrical Device

Every electrical device like bulb, heater, geyser, etc. comes with a design/rated voltage (V_0) and design/rated wattage or power (P_0).

If applied voltage (V) across a device becomes greater than V_0, then it will not work or we can say device will be fused.

Resistance of an electrical device can be found out using

$$P_0 = \frac{V_0^2}{R}$$

i.e $$R = \frac{V_0^2}{P_0}$$

If V is the applied voltage across the device and if P is the power consumed by it, then

$$P = \frac{V^2}{R} \text{ i.e } P = \left(\frac{V^2}{V_0^2}\right) P_0$$

Illustration 26:

If bulb rating is 100 watt and 220 V then determine:

(a) Resistance of filament

(b) Current through filament

(c) If bulb operate at 110-volt power supply then find power consumed by bulb.

Solution:

Bulb rating is 100 W and 220 V bulb means when 220 V potential difference is applied between the two ends then the power consumed is 100 W

Here, V = 220 Volt

$P = 100\ W$

$$\frac{V^2}{R} = 100 \text{ ; So, } R = 484\ \Omega$$

Since Resistance depends only on material hence it is constant for bulb

$$I = \frac{V}{R} = \frac{220}{22\times22} = \frac{5}{11} \text{ Amp.}$$

Power consumed at 110 V

$\therefore$ Power consumed $= \dfrac{110\times110}{484} = 25\ W$

Illustration 27:

Two wires of same mass, having ratio of lengths 1 : 2, density 1 : 3, and resistivity 2 : 1, are connected one by one to the same voltage supply. The rate of heat dissipation in the first wire is found to be 10W. Find the rate of heat dissipation in the second wire.

Solution:

Given, $\frac{l_1}{l_2} = \frac{1}{2}, \frac{d_1}{d_2} = \frac{1}{3}, \frac{\rho_1}{\rho_2} = \frac{2}{1}$

$$M = A_1 l_1 d_1 = A_2 l_2 d_2$$

$\therefore \quad \frac{P_2}{P_1} = \frac{V^2/R_2}{V^2/R_1}$

$$= \frac{R_1}{R_2} = \frac{\rho_1 l_1 A_2}{\rho_2 l_2 A_1} = \frac{\rho_1 l_1^2 d_1}{\rho_2 l_2^2 d_2}$$

$$= \frac{2}{1} \times \left(\frac{1}{2}\right)^2 \times \frac{1}{3} = \frac{1}{6}$$

Or $\quad P_2 = \frac{P_1}{6} = \frac{10}{6} = \frac{5}{3} W$

Fuse wire

The fuse wire for an electric circuit is chosen keeping in view the value of safe current through the circuit.

The fuse wire should have high resistance per unit length and low melting point.

However, the melting point of the material of fuse wire should be above the temperature that will be reached on the passage of the current through the circuit.

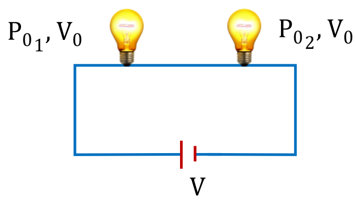
A fuse wire is made of alloys of lead (Pb) and tin (Sn).

Length of fuse wire is immaterial.

The material of the filament of a heater should have high resistivity and high melting point.

Home Appliances (Bulb, Heater, Geyser)

Equivalent power in combination of Bulbs



(i) Series combination

Let, 2 bulbs with V_0 but $P_{0_1} > P_{0_2}$ are connected in series.

Here, $P_{0_1} > P_{0_2} \Rightarrow R_1 < R_2$

Now, since series $\Rightarrow P_1 = I^2 R_1$ and $P_2 = I^2 R_2$

$\therefore \quad P_1 < P_2$

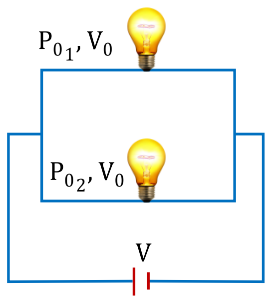
In series combination of bulbs, since Brightness $\propto$ Power consumed by bulb $\propto R \propto \frac{1}{P_{\text{rated}}}$, bulb of lesser wattage will shine more.

$$P = \text{total power of combination} = \frac{V^2}{R_1 + R_2} = \frac{V^2}{\frac{V_0^2}{P_{0_1}} + \frac{V_0^2}{P_{0_2}}} = \left(\frac{V}{V_0}\right)^2 \left(\frac{P_{0_1} P_{0_2}}{P_{0_1} + P_{0_2}}\right)$$

If $\quad V = V_0$, then $\frac{1}{P} = \frac{1}{P_{0_1}} + \frac{1}{P_{0_2}}$

(ii) Parallel combination

Similarly, let two bulbs with V_0 but $P_{0_1} > P_{0_2}$ are connected in parallel.



Since, $P_{0_1} > P_{0_2}$

$\therefore \quad R_1 < R_2$

Now $P_1 = \frac{V^2}{R_1}; P_2 = \frac{V^2}{R_2}$

$\therefore \quad P_1 > P_2$

In parallel combination of bulbs since Brightness $\propto$ Power consumed by bulb $\propto I \propto \frac{1}{R} \propto P_{\text{rated}}$, so bulb of greater wattage will shine more.

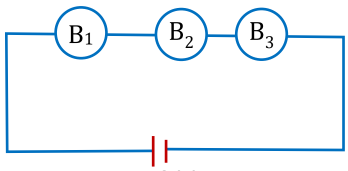
$$P_{\text{total}} = \frac{V^2}{R_1} + \frac{V^2}{R_2} = \left(\frac{V}{V_0}\right)^2 [P_1 + P_2]$$

If $\quad V = V_0$

$$P_{\text{total}} = P_1 + P_2$$

Illustration 28:

In the figure shown B_1, B_2 and B_3 are three bulbs rated as (200V, 50 W), (200V, 100W) and (200 V, 25W) respectively. Find the current through each bulb and which bulb will give more light?



Solution:

$$R_1 = \frac{(200)^2}{50}; R_2 = \frac{(200)^2}{100}; R_3 = \frac{(200)^2}{25}$$

the current flowing through each bulb is

$$= \frac{200}{R_1+R_2+R_3} = \frac{200}{(200)^2\left[\frac{2+1+4}{100}\right]} = \frac{100}{200\times7} = \frac{1}{14}\text{A}$$

Since, $R_3 > R_1 > R_2$

$\therefore$ Power consumed by bulb = i^2R

$\therefore$ If the resistance is of higher value then it will give more light.

$\therefore$ Here Bulb B_3 will give more light.

Illustration 29:

If two heater coils boil same amount of water in time t_1 and t_2 separately, find time taken by the combination of these coils if (i) there are connected in series and (ii) they are connected in parallel?

Solution:

(i) If connected in series then total power P to water will be given as

$$\frac{1}{P} = \frac{1}{P_1} + \frac{1}{P_2}$$

$$\frac{t_s}{H} = \frac{t_1}{H} + \frac{t_2}{H}$$

$$t_s = t_1 + t_2$$

(ii) Similarly, in parallel, we know total power, $P = P_1 + P_2$

So, $\frac{H}{t_P} = \frac{H}{t_1} + \frac{H}{t_2} \qquad \therefore \quad t_P = \frac{t_1t_2}{t_1+t_2}$

Illustration 30:

A heater coil is rated 100 W, 200V. It is cut into two identical parts. Both parts are connected together in parallel, to the same source of 200V. Calculate the energy liberated per second in the new combination.

Solution:

$$P = \frac{V^2}{R}$$

$$\therefore \qquad R = \frac{V^2}{P} = \frac{(200)^2}{100} = 400\Omega$$

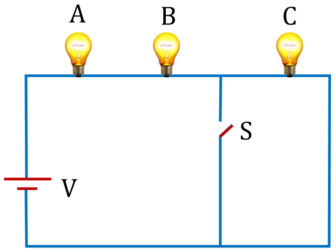
Resistance of half piece $= \dfrac{400}{2} = 200\Omega$

Resistance of pieces connected in parallel $= \dfrac{200}{2} = 100\Omega$

Energy liberated/second $= P = \dfrac{V^2}{R} = \dfrac{200\times200}{100} = 400\text{W}$

Illustration 31:

Three identical bulbs are connected in a circuit as shown. What will happen to the brightness of bulbs A and B if the switch is closed?



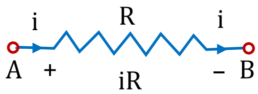
Solution:

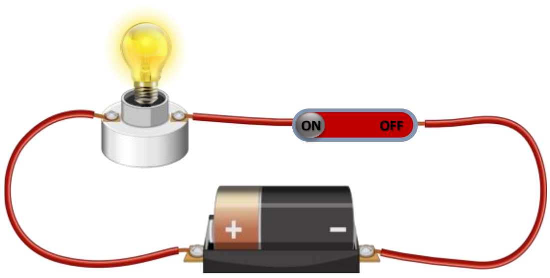
When switch S is closed, current does not pass through bulb C. Hence total resistance decreases and current passing through bulbs A and B increases. Brightness of bulbs A and B also increases.

Single Loop Circuit and Power Output, R for Maximum Power Output and Maximum Current

Electric current in resistance

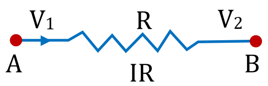
In a resistor current flows from high potential to low potential





High potential is represented by positive (+) sign and low potential is represented by negative (–) sign.

$$V_A - V_B = iR$$



If $\qquad V_1 > V_2$

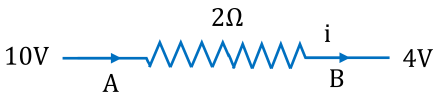
Then current will flow from A to B, and $i = \dfrac{V_1 - V_2}{R}$

If $\qquad V_1 < V_2$

Then current will go from B to A, and $i = \dfrac{V_2 - V_1}{R}$

Illustration 32:

Calculate current (i) flowing in part of the circuit shown in figure?

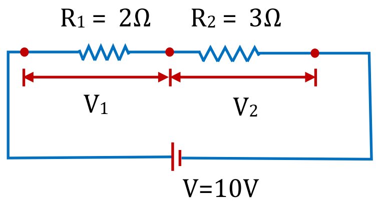


Solution:

$V_A - V_B = i \times R \quad \Rightarrow \; i = \frac{6}{2} = 3A$ **Ans.**

Illustration 33:

In the circuit given, find the potential difference across the resistance R_1.



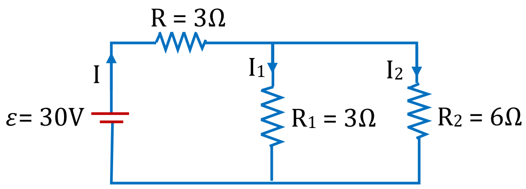
Solution:

The potential difference across R_1

$$V_1 = V\left(\frac{R_1}{R_1 + R_2}\right) = 10\left(\frac{2}{2+3}\right) = 4V$$

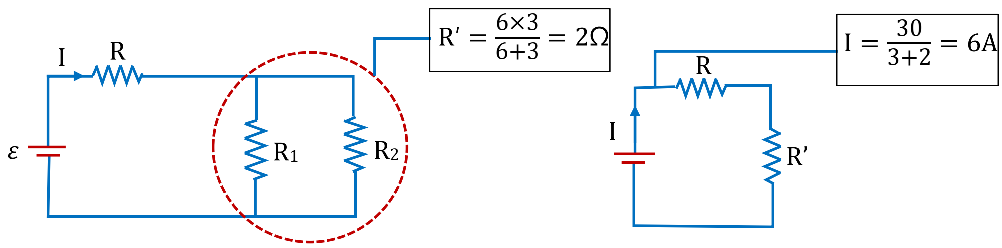
Illustration 34:

In the circuit given, find the current I, I_1 and I_2 in the circuit.



Solution:

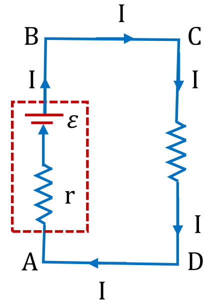
The resistance R_1 and R_2 are connected in parallel.



$\therefore \qquad I_1 = I\left[\frac{R_2}{R_1+R_2}\right] = 6\left[\frac{6}{3+6}\right] = 4A$

and $\qquad I_2 = I\left[\frac{R_1}{R_1+R_2}\right] = 6\left[\frac{3}{3+6}\right] = 2A$

Maximum Power Output



In the given electric circuit.

If we assume that potential at A is zero then potential at B is $\varepsilon - Ir$.

Now since the connecting wires are of zero resistance

$\therefore \quad V_D = V_A = 0 \qquad \Rightarrow \quad V_C = V_B = \varepsilon - Ir$

Now current through CD is also I $\quad$ ($\because$ it's in series with the cell)

$\therefore \quad I = \frac{V_C - V_D}{R} = \frac{(\varepsilon - Ir) - 0}{R} \qquad \Rightarrow \quad$ Current $I = \frac{\varepsilon}{r+R}$

Power output in R $\qquad \Rightarrow \quad P = I^2R = \frac{\varepsilon^2}{(r+R)^2} . R$

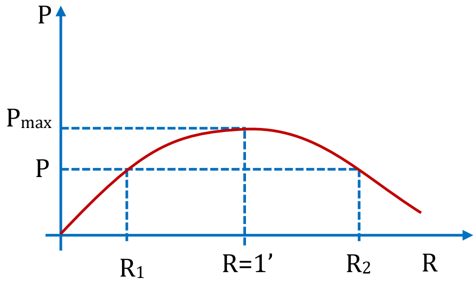
For maximum power supply $\qquad \Rightarrow \quad \frac{dP}{dR} = \frac{\varepsilon^2}{(r+R)^2} - \frac{2\varepsilon^2 R}{(r+R)^3} = \frac{\varepsilon^2}{(R+r)^3}[R + r - 2R]$

For maximum power supply

$$\frac{dP}{dR} = 0 \Rightarrow \quad r + R - 2R = 0 \quad \Rightarrow r = R$$

Here for maximum power output external resistance should be equal to internal resistance.

Now, graph between 'P' and 'R'.



Maximum power output at R

$$P_{max} = \frac{\varepsilon^2}{4R} \qquad \Rightarrow \quad I = \frac{\varepsilon}{r+R} = \frac{\varepsilon}{2R}$$

Now, from graph we can see that for a given power output there exists two values of external resistance.

$$P = \frac{\varepsilon^2 R}{(r+R)^2}$$

$$P(r^2 + 2rR + R^2) = \varepsilon^2 R$$

$$R^2 + \left(2r - \frac{\varepsilon^2}{P}\right)R + r^2 = 0$$

Above quadratic equation in R has two roots R_1 and R_2 for given values of ε, P and r such that

$\therefore \quad R_1R_2 = r^2 \qquad$ (product of roots)

$\qquad r^2 = R_1R_2$

Illustration 35:

How will you connect (series and parallel) 24 cells each of internal resistance 1Ω to get maximum power output across a load of 10 Ω?

Solution:

Suppose there are m rows and each row has n cells. The total number of cells is $mn = 24$.

Here $\quad I = \frac{ne}{R + (nr/m)}$

For maximum power, internal resistance equals the load resistance

$$\frac{nr}{m} = R \text{ or } n = 10m \text{ or } 10m^2 = 24 \text{ or } m = \sqrt{2.4} = 1.55$$

As, m should be an integer. Therefore,

If $m = 1, n = 24, I = 24e/34$

then $P_1 = \left(\frac{24e}{34}\right) \times 10 = 4.98e^2$

If $m = 2, n = 12, I = 12e/16$

then $P_2 = \left(\frac{12e}{16}\right)^2 \times 10 = 5.625e^2$

So, we have two rows (m = 2) each containing 12 cells (n = 12) in series.

Kirchhoff's Law

There are two laws given by Kirchhoff for determination of potential difference and current in different branches of any complicated network.

First law or junction rule

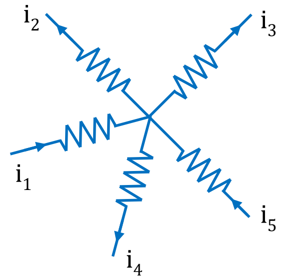
In an electric circuit, the algebraic sum of the current meeting at any junction in the circuit is zero.

or

Sum of the currents entering the Junction is equal to sum of the current leaving the Junction.

$$\sum i = 0$$

$$i_1 - i_2 - i_3 - i_4 + i_5 = 0 \Rightarrow i_1 + i_5 = i_2 + i_3 + i_4$$



This is based on law of conservation of charge.

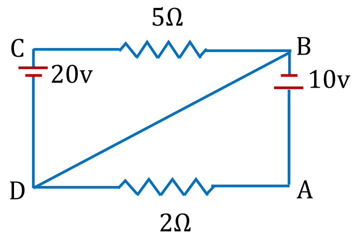
Relative Potential:

While solving an electric circuit it is convenient to choose a reference point and assigning its voltage as zero, then all other potentials are measured with respect to this point. This point is also called the common point.

Once potential has been allotted to each point, we write equations following Kirchhoff's Current Law (Outgoing Current through a point will be equal to the incoming current). These equations can be solved to give the potential at each point.

Illustration 36:

In the figure given beside find out the current in the wire BD



Solution:

Let at point D potential V = 0 and write the potential of other points.

Then current in wire $AD = \frac{10}{2} = 5\ A$ from A to D.

Current in wire $CB = \frac{20}{5} = 4A$ from C to B.

$\therefore$ Current in wire BD = 1 A from D to B.

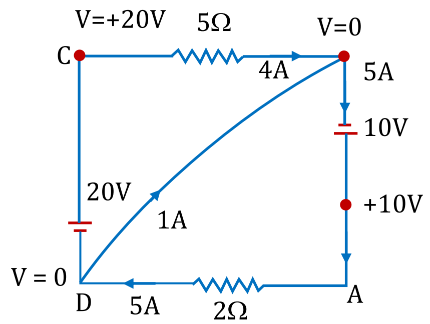
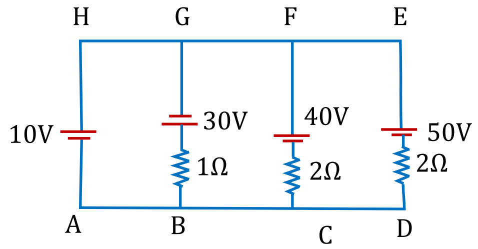


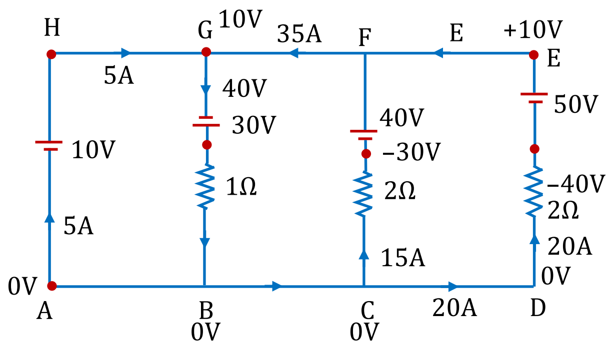
Illustration 37:

Find the current in each wire.



Solution:

Let potential at point A is 0 volt then potential of other points is shown in figure.



Current in $BG = \dfrac{40-0}{1} = 40\ A$ from G to B

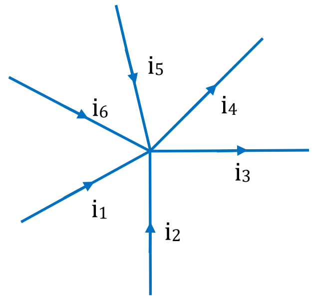
Current in $FC = \dfrac{0-(-30)}{2} = 15\ A$ from C to F

Current in $DE = \dfrac{0-(-40)}{2} = 20\ A$ from D to E

Current in wire $AH = 40 - 35 = 5\ A$ from A to H

Illustration 38:

Find relation in between current i_1, i_2, i_3, i_4, i_5 and i_6.

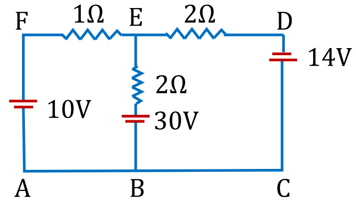


Solution:

$$i_1 + i_2 - i_3 - i_4 + i_5 + i_6 = 0$$

Illustration 39:

Find the current in each wire.



Solution:

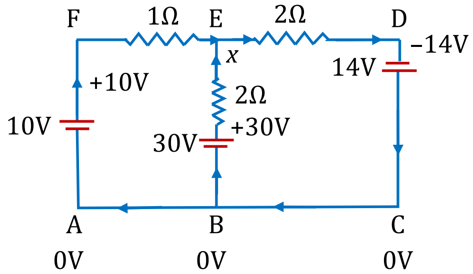
Let potential at point $B = 0$. Then potential at other points are mentioned.

$\therefore$ Potential at E is not known numerically.

Let potential at $E = x$

Now applying Kirchhoff's current law at junction E.

(This can be applied at any other junction also).



$$\frac{x-10}{1}+\frac{x-30}{2}+\frac{x+14}{2}=0$$

$4x = 36 \quad \Rightarrow x = 9$

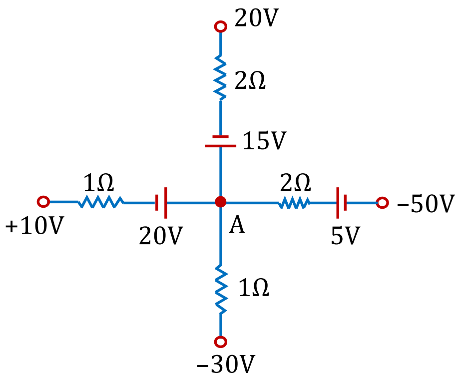
Current in $EF = \frac{10-9}{1} = 1\,A$ from F to E

Current in $BE = \frac{30-9}{2} = 10.5$ A from B to E

Current in $DE = \frac{9-(-14)}{2} = 11.5$ A from E to D

Illustration 40:

Find the potential at point A.



Solution:

Let potential at $A = x$, applying Kirchhoff current law at junction A

$$\frac{x-20-10}{1}+\frac{x-15-20}{2}+\frac{x+45}{2}+\frac{x+30}{1}=0$$

$$\Rightarrow \quad \frac{2x-60+x-35+x+45+2x+60}{2}=0$$

$\Rightarrow \quad 6x + 10 = 0$

$\Rightarrow \quad x = -5/3$

Potential at $A = \frac{-5}{3}V$

Second law or loop rule

In any closed circuit the algebraic sum of e.m.f.'s and algebraic sum of potential drops is zero.

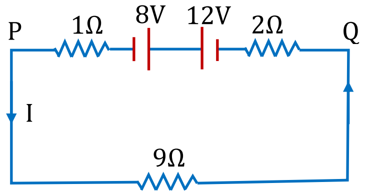
$$\sum IR+\sum E=0$$

while moving from negative to positive terminal inside the cell. e.m.f. is taken as positive while moving in the direction of current in a circuit the potential drop (i.e. IR) across resistance is taken as negative.

This law is based on law of conservation of energy.

Illustration 41:

In the given circuit calculate potential difference between the points *P* and *Q*.



Solution:

Applying Kirchhoff's voltage law (Moving clockwise starting from *P*)

$$I + 8 - 12 + 2I + 9I = 0$$

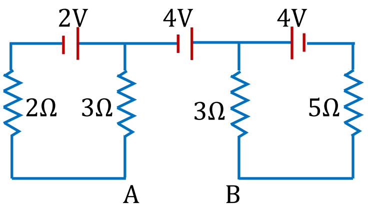
$$I = \frac{1}{3}A$$

Potential difference between the points *P* and *Q* = $9 \times \frac{1}{3} = 3$ volt

or $\quad V_P + I + 8 - 12 + 2I = V_Q \Rightarrow V_P - V_Q = 3$ *volt.*

Illustration 42:

In the given circuit calculate potential difference between *A* and *B*.



Solution:

First applying KVL on left mesh

$$2 - 3I_1 - 2I_2 = 0$$

$$I_1 = 0.4 \ amp$$

Now applying KVL on right mesh.

$$4 - 5I_2 - 3I_2 = 0$$

$$I_2 = 0.5 \ amp.$$

Potential difference between points *A* and *B*

$$V_A + 3I_1 + 4 - 3I_2 = V_B$$

$$V_A - V_B = -3 \times 0.4 - 4 + 3 \times 0.5 = -3.7 \ Volt.$$

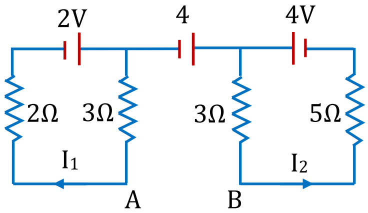
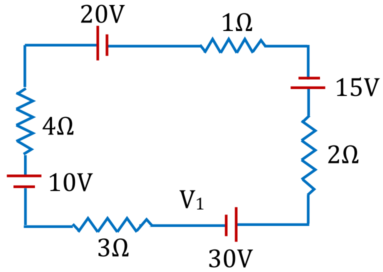


Illustration 43:

Find current in the circuit.



Solution:

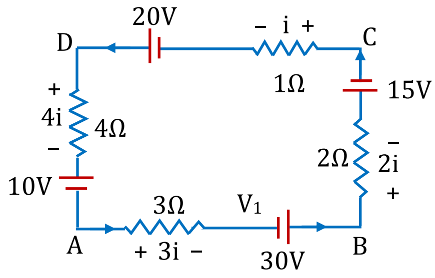
All the elements are connected in series current is all of them will be same let current = i

Applying Kirchhoff voltage law in $ADCBA$ loop

$$10 + 4i - 20 + i + 15 + 2i - 30 + 3i = 0$$

$$10\,i = 25$$

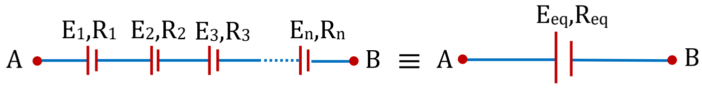
$$i = 2.5\ A$$



Combination of Cells

Grouping of Cells

Cells in Series:



Equivalent EMF

$$E_{eq} = E_1 + E_2 + \ldots.. + E_n \qquad \text{[write EMF's with polarity]}$$

Equivalent internal resistance

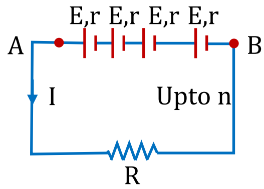
$$r_{eq} = r_1 + r_2 + r_3 + r_4 + \ldots. + r_n$$

If n cells each of emf E and internal resistance r are connected in series with an external resistance R, then total emf = nE, so current in the circuit is

$$I = \frac{nE}{R + nr}$$

If $nr << R$ then $I = \dfrac{nE}{R}$ $\longrightarrow$ Series combination is advantageous.

If $nr >> R$ then $I = \dfrac{E}{r}$ $\longrightarrow$ Series combination is not advantageous.

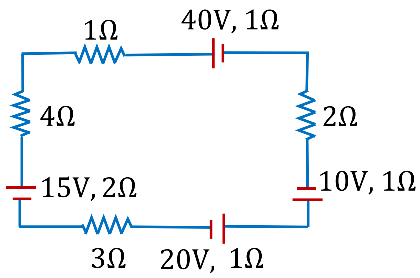


Note:

If polarity of m cells is reversed, then equivalent emf = $(n\text{-}2m)\ E$ while the equivalent resistance is still $nr + R$, so current in R will be $i = \dfrac{(n-2m)E}{nr+R}$.

Illustration 44:

Find the current in the loop.



Solution:

The given circuit can be simplified as

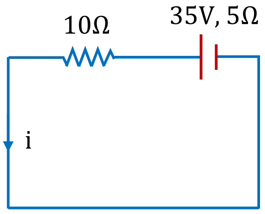
$\varepsilon_{eq} = 40 - 15 + 20 - 10 = 35V$

$r_{eq} = 1 + 2 + 1 + 1 = 5\Omega$

$R_{eq} = 1 + 4 + 3 + 2 = 10\Omega$

So $\quad i = \dfrac{35}{10+5} = \dfrac{35}{15}$

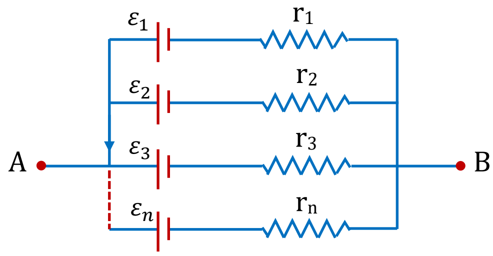
$\Rightarrow \quad i = \dfrac{7}{3} A$



Cells in Parallel:

$$\varepsilon_{eq} = \frac{\varepsilon_1/r_1 + \varepsilon_2/r_2 + \ldots + \varepsilon_n/r_n}{1/r_1 + 1/r_2 + \ldots + 1/r_n} \quad \text{[Use emf's with polarity]}$$

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} + \ldots + \frac{1}{r_n}$$



If m cells each of emf ε and internal resistance r be connected in parallel and if this combination is connected to an external resistance (R).

Then equivalent emf of the circuit = ε

Internal resistance of the circuit = $\dfrac{r}{m}$

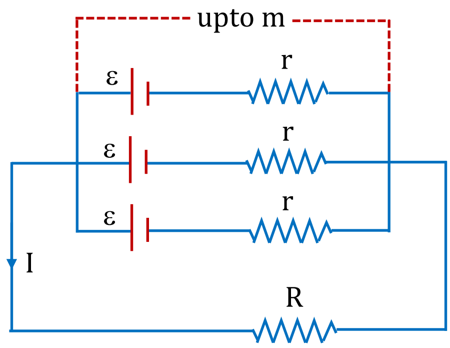
and $\quad I = \dfrac{\varepsilon}{R + \frac{r}{m}} = \dfrac{m\varepsilon}{mR + r}$

If $\quad mR << r$

$I = \dfrac{m\varepsilon}{r} \quad \rightarrow$ Parallel combination is advantageous.

If $\quad mR >> r$

$I = \dfrac{\varepsilon}{R} \quad \rightarrow$ Parallel combination is not advantageous.



Mixed combination:

mn = number of identical cells

n = number of rows

m = number of cells in each row

If internal resistance of each cell is r, then the combination of cells is equivalent to single cell of

$emf = mE$

and $\quad$ Internal resistance = $\dfrac{mr}{n}$

Current, $I = \dfrac{mE}{R + \frac{mr}{n}}$

For maximum current $nR = mr$

or $\quad R = \dfrac{mr}{n}$ = internal resistance of the equivalent battery.

$I_{max} = \dfrac{nE}{2r} = \dfrac{mE}{2R}$

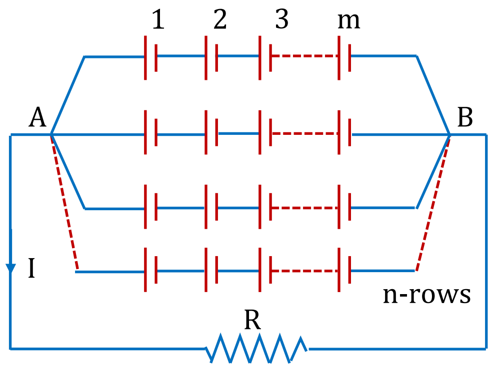
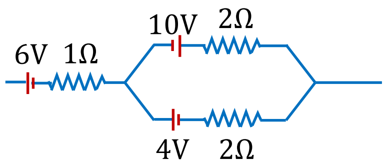


Illustration 45:

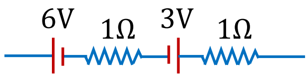
Find the emf and internal resistance of a single battery which is equivalent to a combination of three batteries as shown in figure.



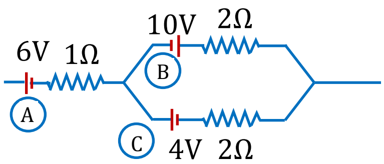
Solution:

Battery (B) and (C) are in parallel combination with opposite polarity. So, their equivalent

$$\varepsilon_{BC} = \frac{\frac{10}{2}+\frac{-4}{2}}{\frac{1}{2}+\frac{1}{2}} = \frac{5-2}{1} = 3V, r_{BC} = 1\Omega$$



Now,



$$\varepsilon_{ABC} = 6-3 = 3V,\ r_{ABC} = 2\Omega$$

Illustration 46:

n rows each containing m cells in series, are joined in parallel. Maximum current is taken from this combination in a 3Ω resistance. If the total number of cells used is 24 and internal resistance of each cell is 0.5 Ω, find the value of m and n.

Solution:

Total number of cell, $mn = 24$

For maximum current,

$$\frac{mr}{n} = R$$

$\Rightarrow$ $\quad 0.5m = 3n$

$$m = \frac{3n}{0.5} = 6n$$

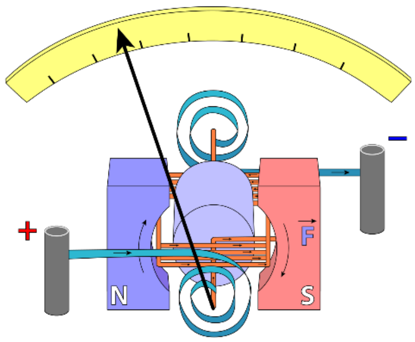
$\therefore$ $\quad 6n \times n = 24$

$\Rightarrow$ $\quad n = 2$

and $\quad m \times 2 = 24$

$\Rightarrow$ $\quad m = 12$

Galvanometer

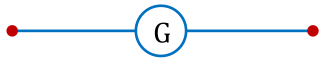


The instrument used to measure strength of current, by measuring the deflection of the coil due to torque produced by a magnetic field, is known as galvanometer.

It consists of a pivoted coil placed in the magnetic field of a permanent magnet. A spring is attached to the coil. In the equilibrium position, with no current in the coil, the pointer is at zero and spring is relaxed. When there is a current in the coil, the magnetic field exerts a torque on the coil that is proportional to current. As the coil turns, the spring exerts a restoring torque that is proportional to the angular displacement. Thus, the angular deflection of the coil and pointer is directly proportional to the coil current and the device can be calibrated to measure current.

$$\boxed{I \propto \phi}$$

Galvanometer is represented in circuit as follows:



The rate of variation of deflection depends upon the magnitude of deflection itself and so the accuracy of the instrument.

- **Current sensitivity**

  The ratio of deflection to the current i.e. deflection per unit current is called current sensitivity (CS) of the galvanometer

  $$CS = \frac{\phi}{I}$$

- **Voltage sensitivity**

  The ratio of deflection to potential difference i.e deflection per unit potential difference is called voltage sensitivity (VS) of the galvanometer

  $$VS = \frac{\phi}{V}$$

Shunt

The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer, is known as shunt.

Merits of shunt:

(i) To protect the galvanometer coil from burning.

(ii) Any galvanometer can be converted into ammeter of desired range with the help of shunt.

(iii) The range of an ammeter can be changed by using shunt resistance of different values.

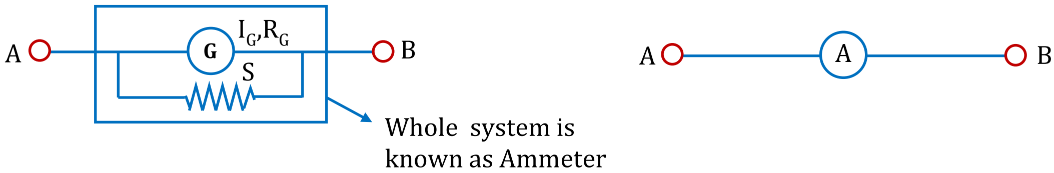
Demerits of shunt:

Shunt resistance decreases the sensitivity of galvanometer.

Conversion of galvanometer into ammeter

A galvanometer can be converted into an ammeter by connecting low resistance in parallel to its coil.

Ammeter is represented as follow -

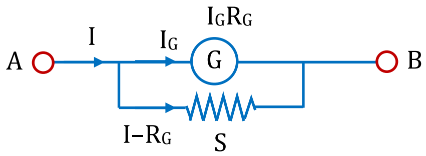


If maximum value of current to be measured by ammeter is I then

$$I_G \cdot R_G = (I - I_G)S$$

$$S = \frac{I_G . R_G}{I - I_G}$$

where I = Maximum current that can be measured using the given ammeter.



For measuring the current the ammeter is connected in series.

In calculation it is simply a resistance

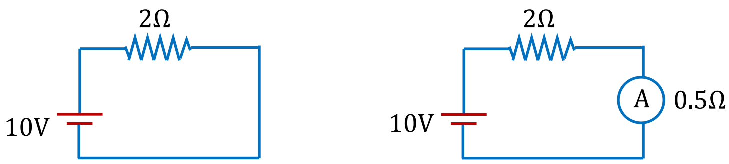
Resistance of ammeter $R_A = \frac{R_G . S}{R_G + S}$

Note:

An ideal ammeter has zero resistance.

Illustration 47:

Find the current in the circuit (a) and (b) and also determine percentage error in measuring the current through an ammeter.



(a) (b)

Solution:

Current in circuit (a) $I = \frac{10}{2} = 5A$

Current in circuit (b) $I' = \frac{10}{2.5} = 4A$

Percentage error in measurement of current = $\frac{I - I'}{I} \times 100 = 20\%$

Here we see that due to ammeter the current has reduced. A good ammeter has very low resistance as compared with other resistors, so that due to its presence in the circuit the current is not affected.

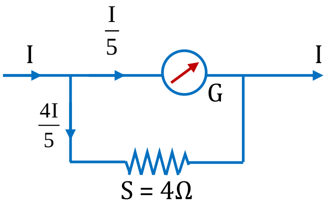
Illustration 48:

When a shunt of 4Ω is attached to a galvanometer, the deflection reduces to 1/5th. If an additional shunt of 2Ω is attached what will be the deflection?

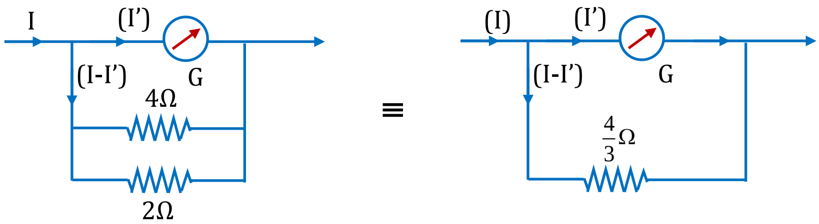
Solution:

Initial condition: When shunt of 4Ω is used,

$$\frac{I}{5} \times G = \frac{4}{5} I \times 4 \Rightarrow G = 16\Omega$$



When additional shunt of 2Ω is used,



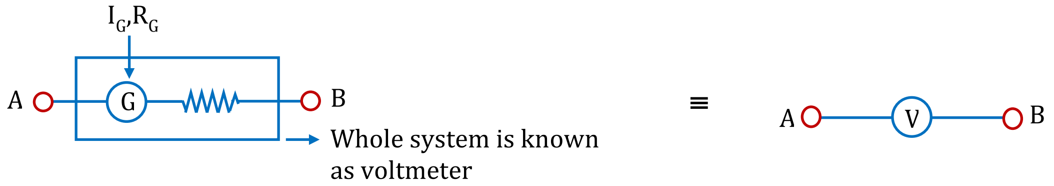
$$I' \times 16 = \frac{4}{3}(I - I') \Rightarrow I' = \frac{I}{13}$$

∴ It will reduce to $\frac{I}{13}$ of the initial deflection.

Conversion of galvanometer into voltmeter:

The galvanometer can be converted into voltmeter by connecting high resistance in series with its coil.

It is used to measure potential difference across a resistor in a circuit.



For maximum potential difference,

$$V = I_G R + I_G R_G$$

$$R = \frac{V}{I_G} - R_G$$

For calculation it is simply a resistance

Resistance of voltmeter,

$$R_V = R_G + R$$

$$I_G = \frac{V}{R_G + R}$$

Note:

A good voltmeter has high value of resistance.

$$R_V = R_G + R \simeq R$$

Ideal voltmeter $\Rightarrow$ $R \to \infty$

Illustration 49:

A 100-volt voltmeter of 20 kΩ resistance is connected in series to a very high resistance R. When it is joined in a line of 110 volt, it reads 5 volt. What is the magnitude of resistance R?

Solution:

When voltmeter is connected in 110-volt line, current through the voltmeter $I = \frac{110}{(20\times10^3+R)}$

The potential difference across the voltmeter $V = IR_v$

$\Rightarrow$ $$5 = \frac{110\times20\times10^3}{(20\times10^3+R)}$$

$\Rightarrow$ $$20 \times 10^3 + R = 440 \times 10^3$$

$\Rightarrow$ $$R = 420 \times 10^3 \Omega$$

Illustration 50:

A galvanometer having 30 divisions has current sensitivity of 20 μA/division. It has a resistance of 25Ω.

(i) How will you convert this galvanometer into an ammeter of range 1 ampere?

(ii) How will you convert this galvanometer into a voltmeter of range 1 volt?

Solution:

The current required for full scale deflection, $I_g = 20\mu A \times 30 = 600\mu A = 6 \times 10^{-4} A$

(i) To convert it into ammeter, a shunt is required in parallel with it

Shunt resistance $R_s' = \frac{I_g R_g}{(I - I_g)} = \left(\frac{6\times10^{-4}}{1-6\times10^{-4}}\right) 25 = 0.015\Omega$

(ii) To convert galvanometer in voltmeter, a high resistance is required in series with it

Series resistance $R = \frac{V}{I_g} - R_g = \frac{1}{6\times10^{-4}} - 25 = 1666.67 - 25 = 1641.67\Omega$

Illustration 51:

How can we convert a galvanometer with R_g = 20 Ω and I_g = 1.0 mA into a voltmeter with a maximum range of 10 V?

Solution:

$$V = I_g R_S + I_g R_g$$

$$10 = 1 \times 10^{-3} \times R_S + 1 \times 10^{-3} \times 20$$

$$R_S = \frac{10-0.02}{1\times10^{-3}} = \frac{9.98}{10^{-3}} = 9980\ \Omega$$

Verification of Ohm's Law Experiment

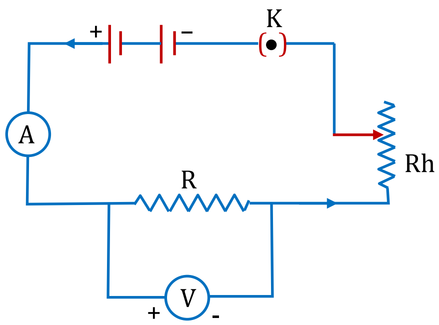
Verification of Ohm's law using voltmeter and ammeter

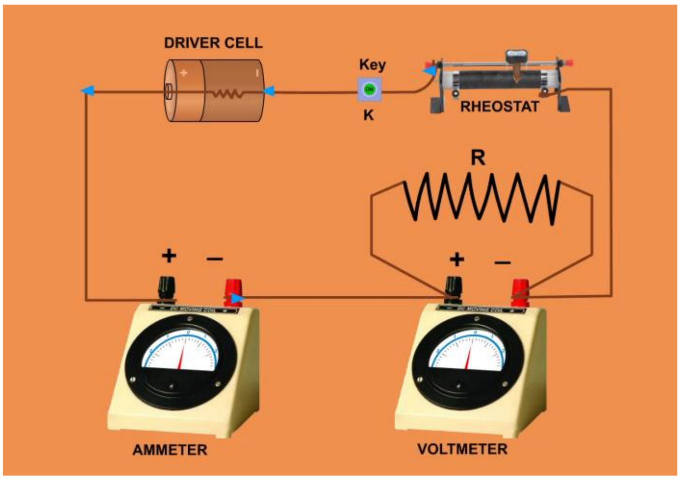
Ohm's law states that the electric current I flowing through a conductor is directly proportional to the potential difference (V) across its ends provided that the physical conditions of the conductor (such as temperature, dimensions, etc.) are kept constant. Mathematically,

$$V \propto I \text{ or } V = IR$$

Here R is a constant known as resistance of the conductor and depends on the nature and dimensions of the conductor.

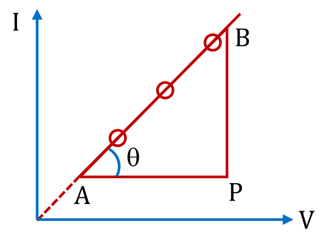
Circuit Diagram: The circuit diagram is as shown below:





Procedure: By shifting the rheostat contact, reading of ammeter and voltmeter are noted down. At least six set of observations are taken. Then a graph is plotted between potential difference (V) across R and current (I) through R. The graph comes to be a straight line as shown in figure.

Result: It is found from the graph that the ratio V/I is constant. Hence, current voltage relationship is established, i.e. $V \propto I$. It means Ohm's law is established.



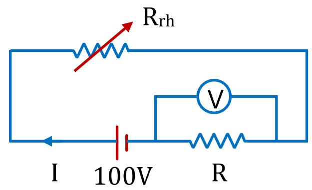
As $I = \left(\frac{1}{R}\right) V$, find the slope of $I - V$ curve and equate it to $\frac{1}{R}$.

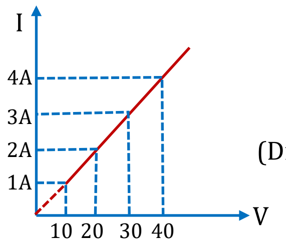
$$\text{Slope} = \frac{BP}{AP} = \frac{1}{R}$$

Get $R = \dfrac{1}{\text{Slope of } I\text{–}V \text{ graph}}$

Illustration 52:

If emf of battery is 100 V, then what was the resistance of Rheostat adjusted at 2nd reading ($I = 2A$, $V = 20V$).





(Drawn across R)

(A) 10Ω (B) 20Ω (C) 30Ω (D) 40Ω

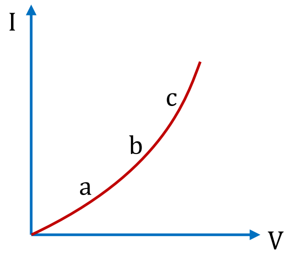
Solution:

From the curve slope $= \dfrac{I}{V} = \dfrac{1}{R} = \dfrac{1}{10}$; $R = 10\ \Omega$

For second reading, $I = \dfrac{Emf}{R+R_{rh}} \Rightarrow 2 = \dfrac{100}{10+R_{rh}} \Rightarrow R_{rh} = 40\Omega$

Illustration 53:

I v/s V curve for a non-ohmic resistance is shown. The dynamic resistance is maximum at point



(A) a (B) b (C) c (D) same for all

Solution:

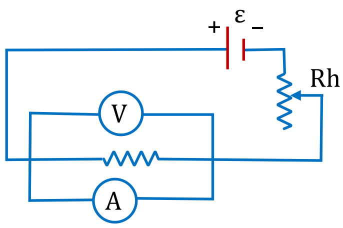
Dynamic resistance, $R = \dfrac{dv}{dI} = \dfrac{1}{dI/dv} = \dfrac{1}{\text{slope}}$

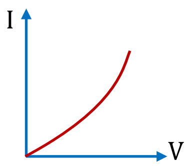
At point a, slope is min, so R is max.

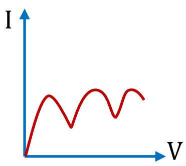
Illustration 54:

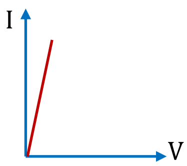
If by mistake, Ammeter is connected parallel to the resistance then I-V curve expected is

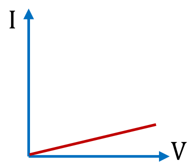
(Here I = reading of ammeter, V = reading of voltmeter)



(A)


(B)


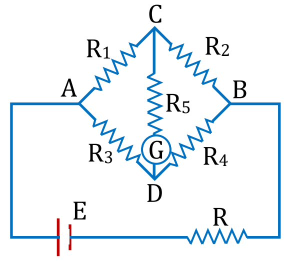
(C)


(D)


Solution:

As ammeter has very low resistance most of current will pass through the ammeter so reading of ammeter (I) will be very large. Voltmeter has very high resistance so reading of voltmeter will be very low. Therefore, most appropriate answer is option C.

Wheatstone Bridge



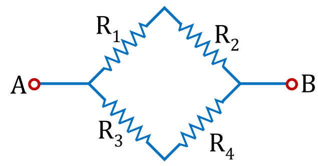
The arrangement as shown in figure of five resistors is known as Wheatstone bridge.

Here there are four terminals in which except two all are connected to each other through resistive elements.

In this circuit

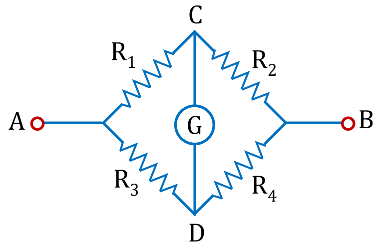
If $R_1R_4 = R_2R_3$ or $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$ then $V_C = V_D$ and current in $R_5 = 0$. This is called balance point or null point i.e. current through the galvanometer is zero.

Here in this case products of opposite arms are equal.



Potential difference between C and D at null point is zero and wheatstone arrangement is said to be balanced. Hence, here the circuit can be assumed to be following for calculating R_{eq} across AB.

Note:

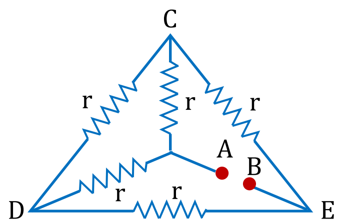


The null point is not affected by resistance R_5, E and R.

It is not affected even if the positions of Galvanometer and battery (E) are interchanged.

- If $\dfrac{R_1}{R_2} < \dfrac{R_3}{R_4}$ then $V_C > V_D$ and current will flow from C to D.
- If $\dfrac{R_1}{R_2} > \dfrac{R_3}{R_4}$ then $V_C < V_D$ and current will flow from D to C.

Illustration 55:



In the adjoining network of resistors each is of resistance r Ω. Find the equivalent resistance between point A and B.

Solution:

Given circuit is balanced Wheatstone Bridge.

$$\therefore \quad \frac{1}{R_{AB}} = \frac{1}{2r} + \frac{1}{2r} = \frac{1}{r}$$

$$\therefore \quad R_{AB} = r$$

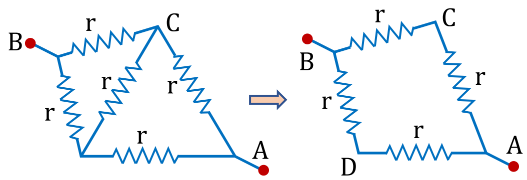


Illustration 56:

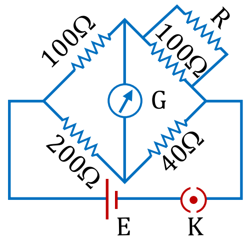
For the following diagram the galvanometer shows zero deflection then what is the value of R?

Solution:

For balanced Wheatstone bridge

$$\therefore \quad \frac{100}{\frac{100R}{(100+R)}} = \frac{200}{40} \qquad \Rightarrow \frac{100+R}{R} = 5$$

$$100 + R = 5R \qquad \Rightarrow R = \frac{100}{4} = 25\Omega$$

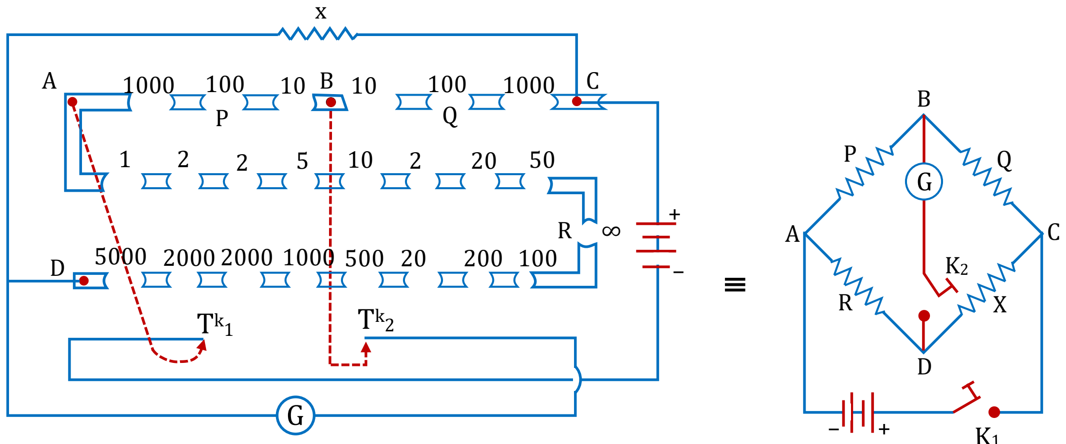


Post Office Box

A post office box is a compact form of Wheatstone bridge with the help of which we can measure the value of the unknown resistance correctly up to 2nd decimal place, i.e., up to 1/100th of an ohm correctly.

In figure (a), each of the arms AB and BC contains three resistances of 10, 100 and 1000 ohm. These arms are called the ratio arms. The third arm AD, called the resistance arm, is a complete resistance box containing resistances from 1Ω to 5,000 Ω. The unknown resistance X constitutes the fourth arm CD. Thus, the four arms AB, BC, CD and AD are infact the four arms of the Wheatstone bridge (figure (b)).

A cell is connected between A and C with key K_1 and Galvanometer is connected between B and D with key K_2.



(a) (b)

Working: The working of the post office box involves broadly the following four steps:

I. Keeping R zero, each of the resistances P and Q are made equal to 10 ohm by taking out suitable plugs from the arms AB and BC respectively. After pressing the battery key first and then the galvanometer key, the direction of deflection of the galvanometer coil is noted. Now, making R infinity, the direction of deflection is again noted. If the direction is opposite to that in the first case, then the connections are correct.

II. Keeping both P and Q equal to 10Ω, the value of R is adjusted, beginning from 1Ω, till 1Ω increase reverses the direction of deflection. The 'unknown' resistance clearly lies somewhere between the two final values of R.

$$\left[X = R\frac{Q}{P} = R\frac{10}{10} = R\right]$$

As an illustration, suppose with 4Ω resistance in the arm AD, the deflection is towards left and with 5Ω, it is towards right. The unknown resistance lies between 4Ω and 5Ω.

III. Making P 100 Ω and keeping Q 10 Ω, we again find those values of R between which direction of deflection is reversed. Clearly, the resistance in the arm AD will be 10 times the resistance X of the wire.

$$\left[X = R\frac{Q}{P} = R\frac{10}{100} = \frac{R}{10}\right]$$

In the illustration considered in step II, the resistance in the arm AD will now lie between 40 Ω, and 50 Ω. So, in this step, we have to start adjusting R from 40 Ω onwards. If 42 Ω and 43 Ω are the two values of R which give opposite deflections, then the unknown resistance lies between 4.2 Ω and 4.3 Ω.

IV. Now, P is made 1000 Ω and Q is kept at 10 Ω. The resistance in the arm AD will now be 100 times the 'unknown' resistance.

$$\left[X = R\frac{10}{1000} = \frac{R}{100}\right]$$

In the illustration under consideration, the resistance in the arm AD will lie between 420 Ω and 430 Ω. Suppose the deflection is to the right for 426 ohm, towards left for 424 ohm and zero deflection for 425 Ω then, the unknown resistance is 4.25 Ω.

<table>
<tr><th rowspan="2">No. of Obs.</th><th colspan="2">Resistance in the Ratio arms</th><th rowspan="2">Resistance in arm AD (R) (Ohm)</th><th rowspan="2">Direction of deflection left or right</th><th rowspan="2">Unknown resistance $X = \frac{Q}{P} \times R$ (Ohm)</th></tr>
<tr><th>AB (P) (Ohm)</th><th>BC(Q) (Ohm)</th></tr>
<tr><td rowspan="2">1.</td><td rowspan="2">10</td><td rowspan="2">10</td><td>4</td><td>left</td><td rowspan="2">(4–5)</td></tr>
<tr><td>5</td><td>Right</td></tr>
<tr><td rowspan="4">2.</td><td rowspan="4">100</td><td rowspan="4">10</td><td>40</td><td>Left (large)</td><td rowspan="4">(4.2 – 4.3)</td></tr>
<tr><td>50</td><td>Right (large)</td></tr>
<tr><td>42</td><td>Left</td></tr>
<tr><td>43</td><td>Right</td></tr>
<tr><td rowspan="4">3.</td><td rowspan="4">1000</td><td rowspan="4">10</td><td>420</td><td>Left</td><td rowspan="4">4.25</td></tr>
<tr><td>424</td><td>Left</td></tr>
<tr><td>425</td><td>No deflection</td></tr>
<tr><td>426</td><td>Right</td></tr>
</table>

Illustration 57:

While using a post office box the keys should be switched on in the following order:

(A) First cell key and then galvanometer key. (B) First the galvanometer key and then cell key.

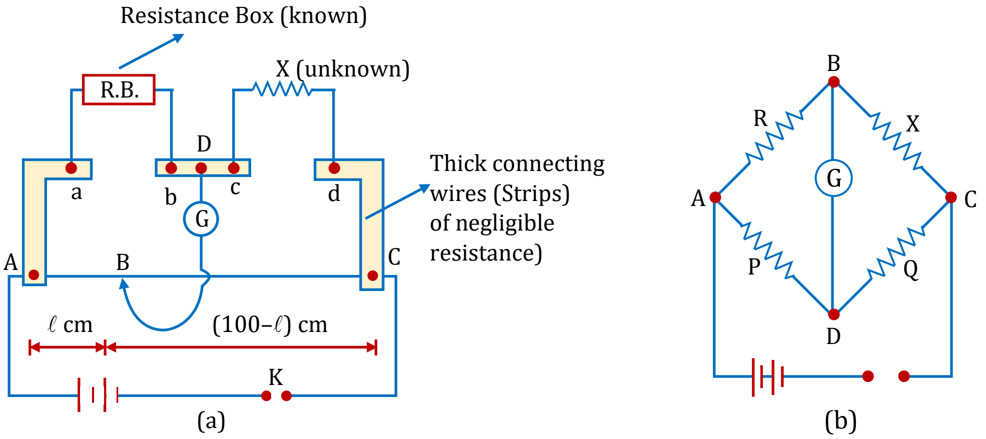
(C) Both the keys simultaneously. (D) Any key first and then the other key.

Solution:

First cell key and then galvanometer key.

Meter Bridge

- It is based on principle of Wheatstone bridge.
- It is used to find the value of unknown resistance of wire.



If $AB = \ell$ cm, then $BC = (100 - \ell)$ cm.

Resistance P of the wire between A and B will be proportional to ℓ i.e., $P \propto \ell$

[$\because$ Specific resistance ρ and cross-sectional area A are same for whole of the wire]

Similarly, if Q is resistance of the wire between B and C, then

$$Q \propto 100 - \ell$$

So, $$\frac{P}{Q} = \frac{\ell}{100-\ell}$$

Applying the condition for balanced Wheatstone bridge, we get

$$RQ = PX$$

$$\therefore \quad X = R\frac{Q}{P} \quad \text{or} \quad X = \frac{100-\ell}{\ell}R$$

Since R and ℓ are known, therefore, the value of X can be calculated.

Note:

For better accuracy, R is so adjusted that ℓ lies close to 50 cm.

End Corrections

In meter Bridge circuit, some extra length comes (is found under metallic strips) at end point A and C. So, some additional length (α and β) should be included at ends for accurate result. Hence in place of ℓ we use $\ell + \alpha$ and in place of $100 - \ell$, we use $100 - \ell + \beta$ (where α and β are called end correction). To estimate α and β, we use known resistance R_1 and R_2 at the place of R and X in meter Bridge.

Suppose we get null point at ℓ_1 distance then

$$\frac{R_1}{R_2} = \frac{\ell_1+\alpha}{100-\ell_1+\beta} \quad \text{...(i)}$$

Now we interchange the position of R_1 and R_2, and get null point at ℓ_2 distance then

$$\frac{R_2}{R_1} = \frac{\ell_2+\alpha}{100-\ell_2+\beta} \quad \text{...(ii)}$$

Solving equation (i) and (ii) get,

$$\alpha = \frac{R_2\ell_1 - R_1\ell_2}{R_1 - R_2} \text{ and } \beta = \frac{R_1\ell_1 - R_2\ell_2}{R_1 - R_2} - 100$$

These end corrections (α and β) are used to modify the observations.

Illustration 58:

If we used 100Ω and 200Ω resistance in place of R and X, we get null deflection at ℓ_1 = 33.0cm. If we interchange the Resistance, the null deflection was found to be at ℓ_2 = 67.0 cm. The end correction α and β should be :

(A) α = 1cm, β = 1cm
(B) α = 2cm, β = 1cm
(C) α = 1cm, β = 2cm
(D) None of these

Solution:

$$\alpha = \frac{R_2\ell_1 - R_1\ell_2}{R_1 - R_2} = \frac{(200)(33)-(100)(67)}{100-200} = 1\text{ cm}$$

$$\beta = \frac{R_1\ell_1 - R_2\ell_2}{R_1 - R_2} - 100 = \frac{(33)(100)-(200)(67)}{100-200} - 100 = 1cm$$

Illustration 59:

If the unknown Resistance calculated without using the end correction, is R_1 and with using the end corrections is R_2 then (assume same end correction)

(A) $R_1 > R_2$ when balanced point is in first half
(B) $R_1 < R_2$ when balanced point is in first half
(C) $R_1 > R_2$ when balanced point is in second half
(D) $R_1 > R_2$ always

Solution:

$$R_1 = X\left(\frac{\ell}{100-\ell}\right) \quad ; \quad R_2 = X\left(\frac{\ell+\alpha}{100-\ell+\beta}\right)$$

If balance point is in first half say ℓ = 40

$$R_1 = X\left(\frac{40}{60}\right) \qquad ; \quad R_2 = X\left(\frac{41}{61}\right) \text{so}, R_2 > R_1$$

If balance point is in second half say ℓ = 70

$$R_1 = X\left(\frac{70}{30}\right) \qquad ; \quad R_2 = X\left(\frac{71}{31}\right) \text{so}, R_2 < R_1$$

Potentiometer

Necessity of potentiometer

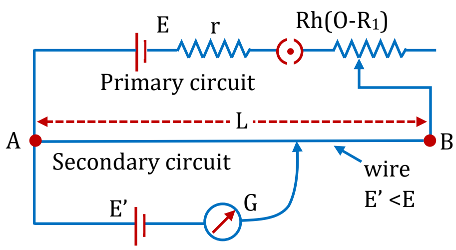
Practically voltmeter has finite resistance. (ideally it should be ∞) in other words it draws some current from the circuit. To overcome this problem potentiometer is used because at the instant of measurement, it does not draws current from the circuit.

A potentiometer is a linear conductor of uniform cross-section with a steady current set up in it. This maintains a uniform potential gradient along the length of the wire. Any potential difference which is less than the potential difference maintained across the potentiometer wire can be measured using this.

The wire should have high resistivity and low expansion coefficient.

For example: Manganin or, Constantine wire etc.

Circuit of Potentiometer



- **Primary circuit** contains constant source of voltage rheostat or Resistance Box.
- **Secondary circuit** contains galvanometer and unknown cell.
- **Potential gradient (x) (V/m)**

  The fall of potential per unit length of potentiometer wire is called potential gradient.

$$x = \frac{V}{L} = \frac{\text{current} \times \text{resitance of potentiometer wire}}{\text{length of potentiometer wire}} = I\left(\frac{R_p}{L}\right)$$

Illustration 60:

A Potentiometer wire of 10 m length and having 10 ohm resistance, emf 2 volts and a rheostat. If the potential gradient is 1 micro volt/mm, the value of resistance in rheostat in ohms will be:

(A) 1.99 (B) 19.9 (C) 199 (D) 1990

Solution:

$\ell = 10$ m, $R = 10\Omega$,

$E = 2$ volts, $\frac{dV}{d\ell} = 1\mu V/mm$

$$\frac{dV}{d\ell} = \frac{1\times10^{-6}}{1\times10^{-3}} V/m = 1 \times 10^{-3} V/m$$

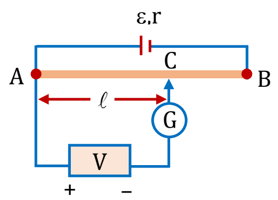
Across wire potential drop, $\frac{dV}{d\ell} \times \ell = 1 \times 10^{-3} \times 10 = 0.01$ volts

$$I = \frac{0.01}{10} = 0.001 = \frac{E}{R+R'} \quad (R' = \text{resistance of rheostat})$$

$$R' = \frac{E}{0.001} - R = \frac{2}{0.001} - 10 = 2000 - 10 = 1990\Omega$$

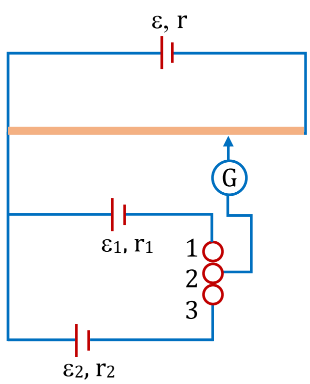
Application of Potentiometer

(A) To measure unknown emf of cell



The unknown voltage V is connected across the potentiometer wire as shown in figure. The positive terminal of the unknown voltage is kept on the same side as of the source of the top most battery. When reading of galvanometer is zero then we say that the meter is balanced. In that condition $V = x\ell$.

(B) To find emf of unknown cell and compare emf of two cells



In case I, In figure, (2) is joint to (1) then balance length = ℓ_1

$$\varepsilon_1 = x\ell_1 \quad \text{...(1)}$$

In case II, In figure, (3) is joint to (2)

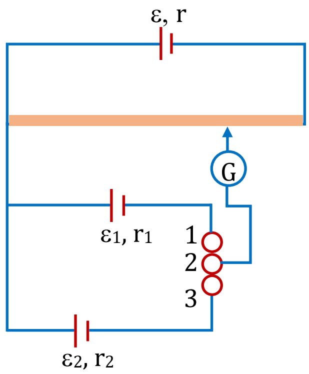
Then balance length = ℓ_2

$$\varepsilon_2 = x\ell_2 \quad \text{...(2)}$$

$$\frac{\varepsilon_1}{\varepsilon_2} = \frac{\ell_1}{\ell_2}$$

If any one of ε_1 or ε_2 is known the other can be found. If x is known then both ε_1 and ε_2 can be found.

Illustration 61:



In an experiment to determine the emf of an unknown cell, its emf is compared with a standard cell of known emf $\varepsilon_1 = 1.12\ V$. The balance point is obtained at 56 cm with standard cell and 80 cm with the unknown cell. Determine the emf of the unknown cell.

Solution:

Here, $\varepsilon_1 = 1.12\ V$; $\ell_1 = 56$ cm ; $\ell_2 = 80$ cm

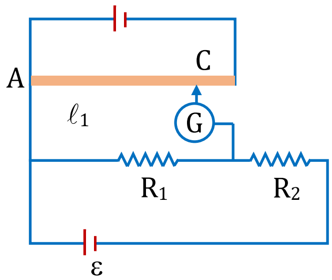
Using equation

$\varepsilon_1 = x\ell_1$ ...(1)

$\varepsilon_2 = x\ell_2$ ...(2)

We get, $\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{\ell_1}{\ell_2} \Rightarrow \varepsilon_2 = \varepsilon_1\left(\dfrac{\ell_2}{\ell_1}\right)$ or $\varepsilon_2 = 1.12\left(\dfrac{80}{56}\right) = 1.6V$ **Ans.**

(C) To find current if resistance is known



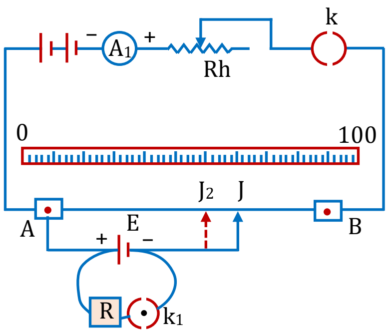
$V_A - V_C = x\ell_1$

$IR_1 = x\ell_1$; $I = \dfrac{x\ell_1}{R_1}$

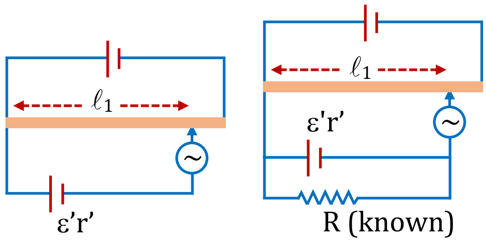
Similarly, we can find the value of R_2 also.

Potentiometer is ideal voltmeter because it does not draw any current from circuit, at the balance point.

(D) To find the internal resistance of cell



1st arrangement



2nd arrangement

By first arrangement, $\varepsilon' = x\ell_1$ ...(1)

By second arrangement, $IR = x\ell_2$

$$I = \frac{x\ell_2}{R}$$

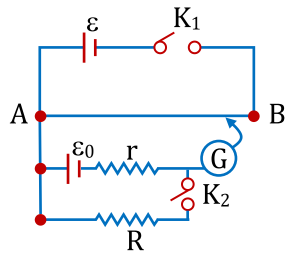
Also $I = \dfrac{\varepsilon'}{r'+R}$

$$\Rightarrow \frac{\varepsilon'}{r'+R} = \frac{x\ell_2}{R}$$

$$\Rightarrow \frac{x\ell_1}{r'+R} = \frac{x\ell_2}{R}$$

$$\Rightarrow r' = \left[\frac{\ell_1 - \ell_2}{\ell_2}\right]R$$

Illustration 62:



The internal resistance of a cell is determined by using a potentiometer. ln an experiment, an external resistance of 60Ω is used across the given cell. When the key is closed, the balance length on the potentiometer decreases from 72 cm to 60 cm. calculate the internal resistance of the cell.

Solution:

According to equation,

$$\varepsilon_0 = x\ell_0 \qquad \text{...(i)}$$

$$V = IR = x\ell \qquad \text{...(ii)}$$

$$I = \frac{\varepsilon_0}{R+r} \qquad \text{...(iii)}$$

From equation (i), (ii) and (iii) we get

$$r = R\left(\frac{\ell_0}{\ell} - 1\right)$$

Here ℓ_0 = 72cm; ℓ = 60 cm; R = 60 Ω

$\therefore \qquad r = (60)\left(\frac{72}{60} - 1\right)$or $r = 12\Omega.$

Note:

(a) If same polarity of 1 cell and 2 cell are not connected with each other then balancing condition (null deflection) cannot be obtained.

(b) If potential drop across wire is less than potential drop of secondary circuit then balancing condition cannot be obtained.

Difference between potentiometer and voltmeter	
Potentiometer	**Voltmeter**
• It measures the unknown emf very accurately • While measuring potential difference the resistance of potentiometer becomes infinite. • It is based on zero deflection method. • It has a high sensitivity. • It is used for various applications like measurement of internal resistance of cell, calibration of ammeter and voltmeter, measurement of thermo emf, comparison of emfs etc.	• It measures the unknown emf approximately. • While measuring emf it draws some current from the source of emf. • Its sensitivity is low. • While measuring unknown potential difference the resistance of voltmeter is high but finite. • It is based on deflection method. • It is only used to measure unknown potential difference.

Sensitivity:

It is the ability of potentiometer to read very small value of potential difference easily.

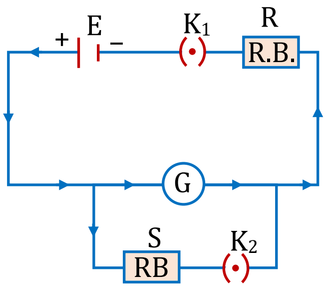
So, sensitivity is inversely proportional to potential gradient of potentiometer wire.

$$\text{Sensitivity} \propto \frac{1}{P.G.} \propto \frac{\ell}{V} \propto \frac{\text{Length of wire}}{\text{potential drop across wire}}$$

Half Deflection Method

Aim:

To find the resistance of a galvanometer by half deflection method and find its figure of merit.



Theory:

The connections for finding the resistance of a galvanometer by the half deflection method are shown in Fig. When the key K_1 is closed, keeping the key K_2 open, the current I_g through the galvanometer is given by

$I_g = \frac{E}{R+G}$, where E = *E.M.F.* of the cell.

R = Resistance from the resistance box R.B.

G = Galvanometer resistance.

If θ is the deflection produced, then

$$\frac{E}{R+G} = k\theta \qquad ...(1)$$

If now the key K_2 is closed and the value of the shunt resistance S is adjusted so that the deflection is reduced to half of the first value, then current flowing through the galvanometer I'_g is given by

$$I'_g = \frac{E}{R + \frac{GS}{(G+S)}}\left(\frac{S}{G+S}\right) = \frac{k\theta}{2}$$

or $$I'_g = \frac{ES}{R(G+S)+GS} = \frac{k\theta}{2} \qquad ...(2)$$

Comparing (1) and (2), we get

$$(R+G)2S = R(G+S) + GS$$

or $$(R-S)G = RS \quad \text{or} \quad G = \frac{RS}{R-S}$$

If the value of R is very large as compared to S, then $\frac{R}{R-S}$ is nearly equal to unity. Hence

$$\boxed{G \approx S}$$

Figure of Merit:

Figure of merit of a galvanometer is that much current sent through the galvanometer in order to produce a deflection of one division on the scale.

If k is the figure of merit of the galvanometer and 'θ' be the number of divisions on the scale, then current (I_g) through the galvanometer is given by

$$\boxed{I_g = k\theta}$$

Precautions:

1. The value of 'R' should be large
2. To decrease the deflection, the shunt resistance should be decreased and vice-versa.
3. In this method it is assumed that the deflection is proportional to the current. This is possible only in a Weston type moving coil galvanometer.
4. The connections must be tight and the ends of connecting wires should be cleaned.

Illustration 63:

The series combination of variable resistance R, one 100 Ω resistor and moving coil galvanometer is connected to a mobile phone charger having negligible internal resistance. The zero of the galvanometer lies at the centre and the pointer can move 30 division full scale on either side depending on the direction of current. The reading of the galvanometer is 10 divisions and the voltages across the galvanometer and 100 Ω resistor are respectively 12 mV and 16 mV.

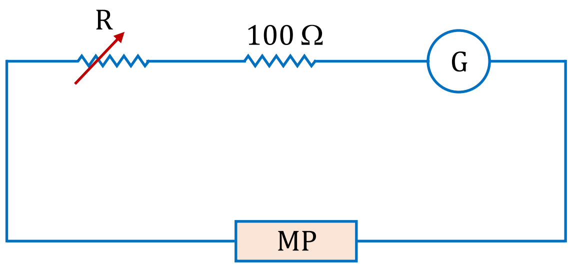
1. The figure of merit of the galvanometer in microampere per division is:

(A) 16 (B) 20 (C) 32 (D) 10

2. The resistance of the galvanometer (in Ω) is:

(A) 50Ω (B) 75 Ω (C) 100 Ω (D) 80 Ω

Solution:



$10I_0G = 12 \times 10^{-3}$

$10I_0(100) = 16 \times 10^{-3} \quad \Rightarrow \quad 10^3 I_0 = 16 \times 10^{-3} \quad \Rightarrow \quad I_0 = 16 \times 10^{-6} A$

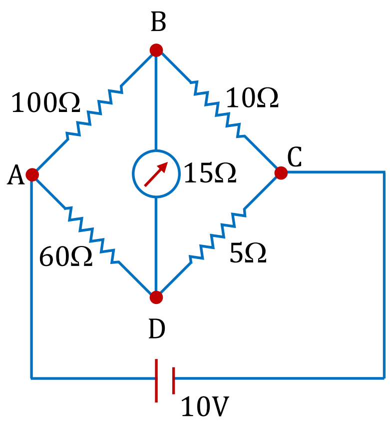
$\dfrac{G}{100} = \dfrac{3}{4} \quad \Rightarrow \quad G = 75\Omega$

Resistance Calculation of Complex Circuits

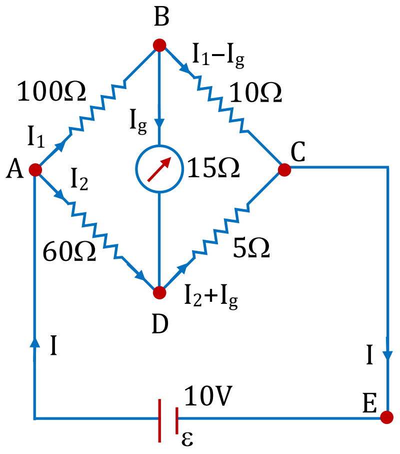
Unbalanced Wheatstone Bridge

Illustration 64:

The four arms of a Wheatstone bridge (Fig.) have the following resistances: AB = 100Ω, BC = 10Ω, CD = 5Ω, and DA = 60Ω. A galvanometer of 15Ω resistance is connected across BD. Calculate the current through the galvanometer when a potential difference of 10V is maintained across AC.



Solution:



Consider the mesh BADB, we have

$$100I_1 + 15I_g - 60I_2 = 0$$

Or $$20I_1 + 3I_g - 12I_2 = 0 \quad ...(1)$$

Considering the mesh BCDB, we have

$$10(I_1 - I_g) - 15I_g - 5(I_2 + I_g) = 0$$

$$10I_1 - 30I_g - 5I_2 = 0$$

$$2I_1 - 6I_g - I_2 = 0 \quad ...(2)$$

Considering the mesh ADCEA,

$$60I_2 + 5(I_2 + I_g) = 10$$

$$13I_2 + I_g = 2 \quad ...(3)$$

Multiplying equation (2) by 10

$$20I_I - 60I_g - 10I_2 = 0 \quad ...(4)$$

From equations (4) and (1) we have

$$63I_g - 2I_2 = 0$$

$$I_2 = 31.5I_g \quad ...(5)$$

Substituting the value of I_2 into equation (3), we get

$$13(31.5I_g) + I_g = 2$$

$$410.5I_g = 2$$

$$I_g = 4.87mA$$

Symmetrical Circuits:

Some circuits can be modified to have simpler solution by using symmetry rules. One should note that if these are solved by traditional method of KVL and KCL then it would take much more time.

Mirror symmetry:

(1) It is applied along perpendicular bisector of line AB (A and B are the points about which $R_{eq.}$ is to be calculated)

(2) We can remove resistances and junctions which are on the line of symmetry.

(3) Point on line of symmetry are equipotential points.

Folding symmetry:

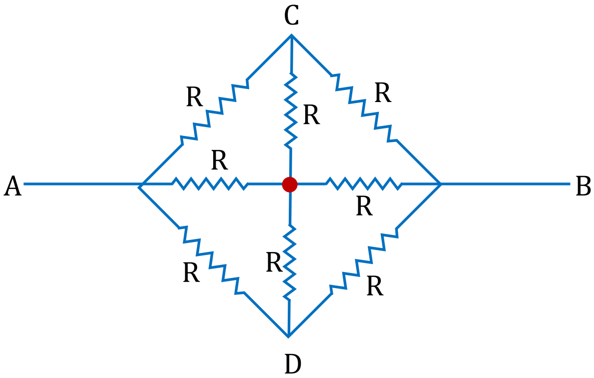
(1) It is applied along line AB.

(2) We can fold the circuit about line of symmetry

(3) When we fold the circuit the parallel combination of resistance can be solved easily.

Illustration 65:

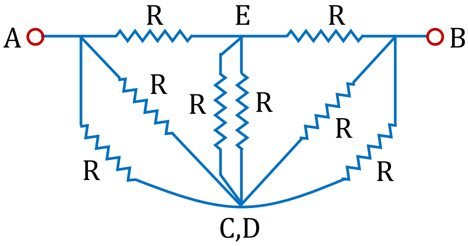
Find the equivalent Resistance between A and B



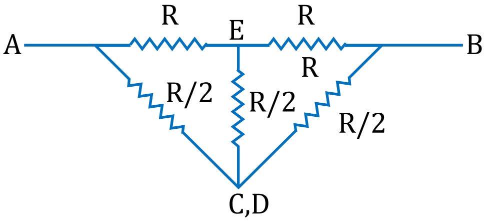
Solution:

Let's use folding symmetry about line AB.

Circuit can be simplified as per folding symmetry.



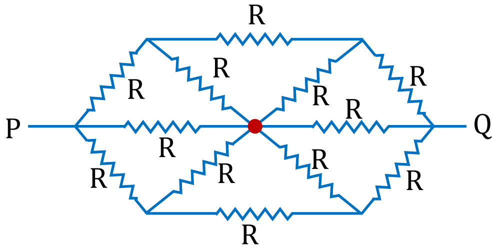
Now, it is Balanced Wheatstone bridge



$$R_{eq} = \frac{2R \times R}{2R+R} = \frac{2R}{3}$$

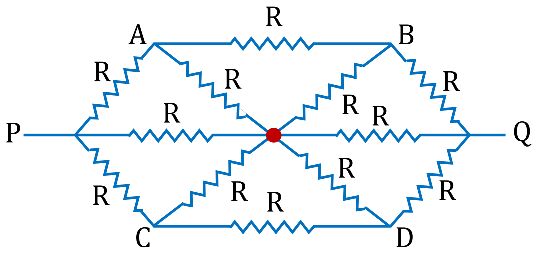
Illustration 66:

Find the equivalent Resistance between P and Q

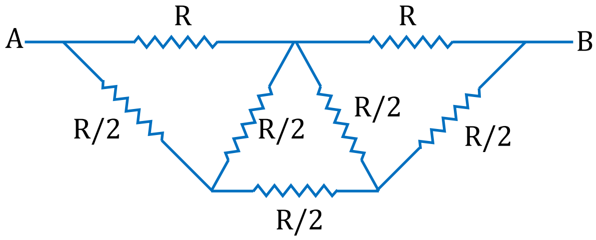


Solution:

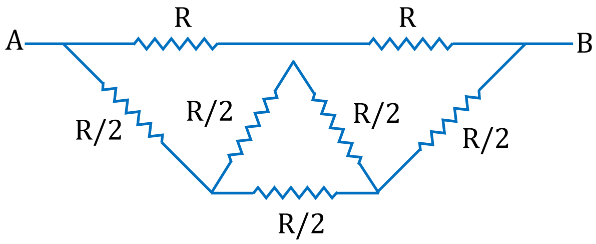
Here both folding and mirror symmetries exists. Let's start solving using folding symmetry.



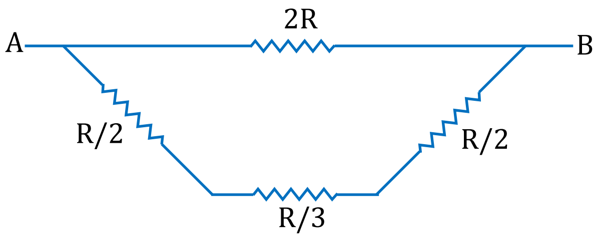
Here $V_A = V_C$ and $V_B = V_D$

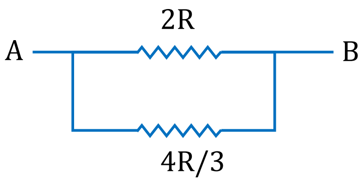


Here the circuit can be further simplified using mirror symmetry and junction can be removed.



This circuit can be simplified as





$$R_{eq} = \frac{2R \times \frac{4R}{3}}{\frac{10R}{3}} = \frac{4R}{5}$$

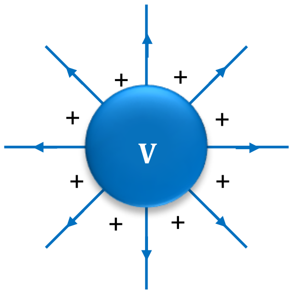
CAPACITANCE

Introduction

A capacitor can store energy in the form of potential energy in an electric field. In this chapter we'll discuss the capacity of conductors to hold charge and energy.

Capacitance of an isolated conductor

When a conductor is charged its potential increases. For an isolated conductor (conductor should be of finite dimension, so that potential of infinity can be assumed to be zero) potential of the conductor is proportional to charge given to it.



q = charge on conductor

V = potential of conductor

$q \propto V \Rightarrow q = CV$

Here C is proportionality constant called capacitance of the conductor.

Illustration 67:

When 30μC charge is given to an isolated conductor of capacitance 5μF. Find out the Potential of the conductor.

Solution:

$$Q = 30\mu C$$

$$C = 5\mu F$$

$$V = \frac{Q}{C} = \frac{30}{5} = 6V$$

Illustration 68:

Two insulated conductors are charged by transferring electrons from one conductor to another. The potential difference of 100 V was produced on transferring 6.25×10^{15} electrons from one conductor to another. The capacity of the system will be.

Solution:

$$Q = CV \Rightarrow C = \frac{Q}{V} = \frac{ne}{V}$$

Given $V = 100$ volt ; $n = 6.25 \times 10^{15}$

$\therefore \quad C = \dfrac{6.25\times10^{15}\times1.6\times10^{-19}}{100} = 10\mu F$

Definition of capacitance:

Capacitance of conductor is defined as charge required to increase the potential of conductor by one unit.

Unit and dependency of capacitance of an isolated conductor:

(i) It is a scalar quantity.

(ii) Unit of capacitance is farad in SI units and its dimensional formula is $M^{-1}L^{-2}I^{2}T^{4}$.

(iii) 1 Farad: 1 Farad is the capacitance of a conductor for which 1 coulomb charge increases potential by 1 volt.

$$1 \text{ Farad} = \frac{1 \text{ Coulomb}}{1 \text{ Volt}}$$

$$1\ \mu F = 10^{-6}\ F, 1 nF = 10^{-9}\ F \text{ or } 1\ pF = 10^{-12} F$$

(iv) Capacitance of an isolated conductor depends on following factors:

(a) Shape and size of the conductor: On increasing the size, capacitance increases.

(b) On surrounding medium: With increase in dielectric constant *K*, capacitance increases.

(c) Presence of other conductors: When a neutral conductor is placed near a charged conductor, capacitance of conductors increases.

(v) Capacitance of a conductor does not depend on

(a) Charge on the conductor

(b) Potential of the conductor

Capacitor

Definition

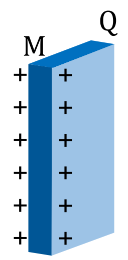
A capacitor or condenser consists of two conductors separated by an insulator or dielectric. Having opposite charges on which sufficient quantity of charge may be accommodated.

Principle

It is based on the fact that if an identical earthed conductor is placed in the vicinity of a charged conductor then the capacitance of the charged conductor increases appreciable.

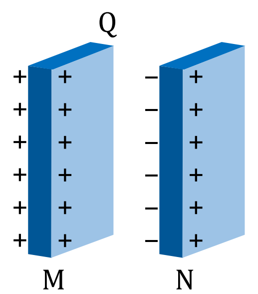
Consider a conducting plate *M* which is given a charge *Q*

Such that its potential rises to *V* then,



$$C = \frac{Q}{V}$$

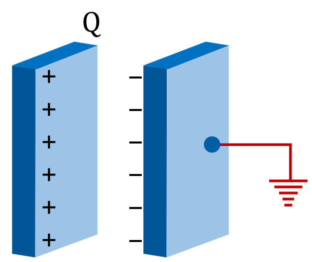
Let us place another identical conducting plate *N* parallel to it such that charge is induced on plate *N* (as shown in figure). If V_- is the potential at *M* due to induced negative charge on *N* and V_+ is the potential at *M* due to induced positive charge on *N*, then



$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$

Since $V' < V$ (as the induced negative charge lies closer to the plate *M* in comparison to induced positive charge). $\Rightarrow C' > C$ Further, if *N* is earthed from the outer side (see figure) then $V'' = V_+ - V_-$

(∵ The entire positive charge flows to the earth)



$$C'' = \frac{Q}{V''} = \frac{Q}{V - V_-} \Rightarrow C'' >> C'$$

Circuit Representation:

The capacitor is represented as following:

Type

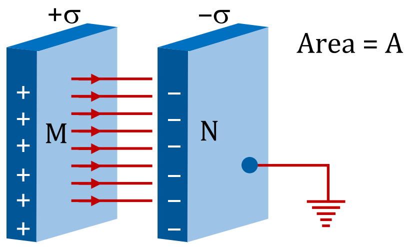
Based on shape and arrangement of capacitor plates there are various types of capacitors.

(a) Parallel plate capacitor (b) Spherical capacitor (c) Cylindrical capacitor

Parallel Plate Capacitor:

Capacitance

It consists of two metallic plates M and N each of area A at separation d. Plate M is positively charged and plate N is earthed. If ε_r is the dielectric constant of the material medium and E is the field at a point P that exists between the two plates, then



Step- I : Finding electric field,

$$E = E_+ + E_- = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r} \quad [\varepsilon = \varepsilon_0\varepsilon_r]$$

Step- II : Finding potential difference,

$$V = Ed = \frac{\sigma}{\varepsilon_0\varepsilon_r} d = \frac{qd}{A\varepsilon_0\varepsilon_r} \left(\because E = \frac{V}{d} \text{ and } \sigma = \frac{q}{A}\right)$$

Step- III : Finding capacitance,

$$C = \frac{q}{V} = \frac{\varepsilon_r\varepsilon_0 A}{d}$$

Dependency:

Capacitance of a capacitor depends on

(a) Area of plates. (b) Distance between the plates. (c) Dielectric medium between the plates.

Illustration 69:

The capacitance of a parallel plate capacitor is 10 μF when distance between its plates is 8 cm. If distance between the plates is reduced to 4 cm, its capacitance will be-

Solution:

$$C = \frac{\varepsilon_0 A}{d}; C' = \frac{\varepsilon_0 A}{d'}$$

$$d' = \frac{d}{2}$$

$$C' = 2C = 20\mu F$$

Illustration 70:

The capacitance of a parallel plate capacitor is $12\mu F$. If the distance between its plates is reduced to half and the area of plates is doubled, then the capacitance of the capacitor will become

Solution:

$C = 12\ \mu F$

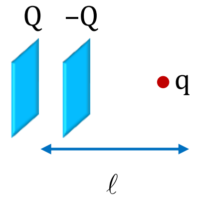
$d' = \dfrac{d}{2}$

$A' = 2A$

$\dfrac{C'}{C} = \dfrac{A'}{A}\dfrac{d}{d'} = 2 \times 2 = 4$

$\Rightarrow \quad C' = 4C = 4 \times 12\mu F = 48\mu F$

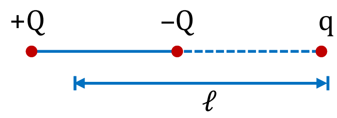
Illustration 71:



The plates of small size of a parallel plate capacitor are charged as shown. The magnitude of force on the charged particle of 'q' at a distance 'l' from the capacitor is : (Assume that the distance between the plates is $d << l$)

Solution:

The two plates acts as a dipole



$\therefore$ The magnitude of force on charge q ;

$$|\vec{F}| = |\vec{E}|q = \left(\frac{2kQd}{\ell^3}\right).q = \frac{Qqd}{2\pi\varepsilon_0\ell^3}$$

Types of Capacitors

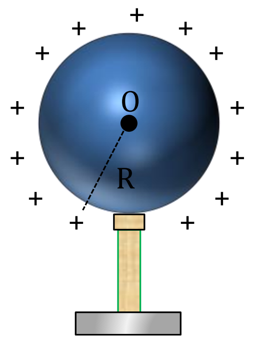
Based on shape and arrangement of capacitor plates there can be various other types of capacitors :

(i) Spherical capacitor

(ii) Cylindrical capacitor

Spherical Capacitance

Capacitance of an Isolated Spherical Conductor



Let there is charge Q on sphere of radius R.

$\because$ Potential $V = \dfrac{KQ}{R}$

Hence by formula:

$Q = CV \rightarrow Q = \dfrac{CKQ}{R} \rightarrow C = 4\pi\epsilon_0 R$

Capacitance of an isolated spherical conductor $\rightarrow C = 4\pi\epsilon_0 R$

(a) If the medium around the conductor is vacuum or air.

$C_{\text{vacuum}} = 4\pi\epsilon_0 R$ [R = Radius of spherical conductor. (may be solid or hollow.)]

If the medium around the conductor is a dielectric of constant K from surface of sphere to infinity then

$C_{\text{medium}} = 4\pi\epsilon_0 KR$

(b) $\dfrac{C_{\text{medium}}}{C_{\text{air/vaccum}}} = K$ = dielectric constant

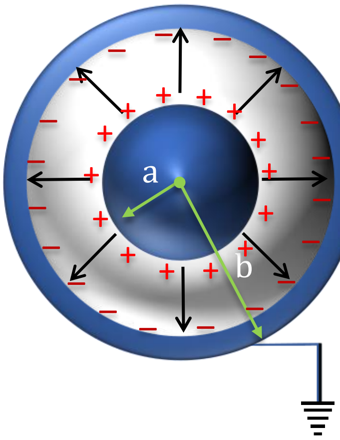
Illustration 72:

Find out the capacitance of the earth? (Radius of the earth = 6400 km)

Solution:

$$C = 4\pi\epsilon_0 R = \frac{6400\times10^3}{9\times10^9} = 711\ \mu F$$

Capacitance of spherical capacitor



(a) Outer sphere is earthed

When a charge Q is given to inner sphere it is uniformly distributed on its surface, A charge $-Q$ is induced on inner surface of outer sphere. The charge $+Q$ induced on outer surface of outer sphere flows to earth as it is grounded.

$E = 0$ for $r < a$

and $E = 0$ for $r > b$

This arrangement is known as spherical capacitor.

Potential of inner sphere, $V_1 = \dfrac{Q}{4\pi\varepsilon_0 a} + \dfrac{-Q}{4\pi\varepsilon_0 b} \Rightarrow \dfrac{Q}{4\pi\varepsilon_0}\left(\dfrac{b-a}{ab}\right)$

As outer surface is earthed so potential

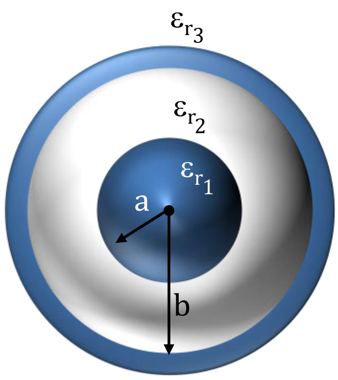
$V_2 = 0$

$$V_1 - V_2 = \left[\frac{KQ}{a} - \frac{KQ}{b}\right] - \left[\frac{KQ}{b} - \frac{KQ}{b}\right] = \frac{KQ}{a} - \frac{KQ}{b}$$

$$C = \frac{Q}{V_1 - V_2} = \frac{Q}{\frac{KQ}{a} - \frac{KQ}{b}} = \frac{ab}{K(b-a)} = \frac{4\pi\epsilon_0 ab}{b-a}$$

$$C = \frac{4\pi\epsilon_0 ab}{b-a}$$

If $b >> a$ then $C = 4\pi\epsilon_0 a$ (Like isolated spherical capacitor)



If dielectric mediums are filled as shown then: $C = \dfrac{4\pi\epsilon_0\epsilon_{r_2} ab}{b-a}$

Illustration 73:

The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km.

Solution:

The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

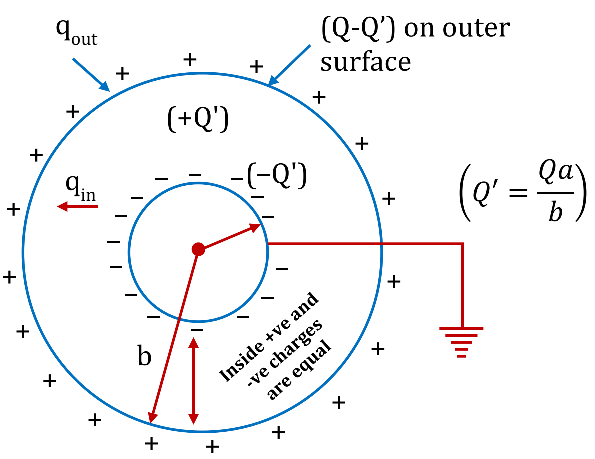
b = radius of the top of stratosphere layer = 6400 km + 50 km = 6450 km = 6.45 × 10^6 m

a = radius of earth = 6400 km = 6.4 × 10^6 m

$\therefore \quad C = \frac{1}{9\times10^9} \times \frac{6.45\times10^6\times6.4\times10^6}{6.45\times10^6 - 6.4\times10^6} = 0.092\ F$

(b) Inner sphere is earthed

Here the system is equivalent to a spherical capacitor of inner and outer radii a and b respectively and a spherical conductor of radius b in parallel. This is because charge Q given to outer sphere distributes in such a way that for the outer sphere charge on the inner side is $\frac{a}{b}Q$ and



Charge on the outer side is

$Q - \frac{a}{b}Q = \frac{(b-a)}{b}Q$

So total capacitance of the system

$C = 4\pi\varepsilon_0\frac{ab}{b-a} + 4\pi\varepsilon_0 b$

$$\boxed{C = \frac{4\pi\varepsilon_0 b^2}{b-a}}$$

Cylindrical Capacitance

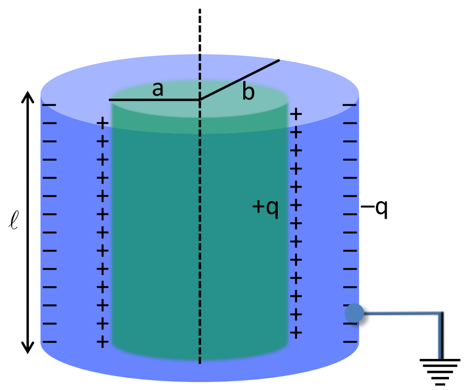
There are two co-axial conducting cylindrical surfaces where $\ell >> a$ and $\ell >> b$, where a and b are radius of cylinders.

When a charge Q is given to inner cylinder it is uniformly distributed on its surface. A charge $-Q$ is induced on inner surface of outer cylinder. The charge $+Q$ induced on outer surface of outer cylinder flows to earth as it is grounded.

Electrical field between cylinders, $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{Q/\ell}{2\pi\varepsilon_0 r}$

Potential difference between plates

$$V = \int_a^b \frac{Q}{2\pi\varepsilon_0 r\ell}dr = \frac{Q}{2\pi\varepsilon_0\ell}\ell n\left(\frac{b}{a}\right)$$



Capacitance per unit length

$$C = \frac{\lambda}{V} = \frac{\lambda}{2K\lambda\ell n\frac{b}{a}} = \frac{4\pi\epsilon_0}{2\ell n\frac{b}{a}} = \frac{2\pi\epsilon_0}{\ell n\frac{b}{a}}$$

Capacitance per unit length = $\frac{2\pi\epsilon_0}{\ell n\frac{b}{a}}\ F/m$

Illustration 74:

A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder.

Solution:

ℓ = 15 cm = 15 × 10^{-2} m; a = 1.4cm = 1.4 × 10^{-2} m

b =1.5 cm = 1.5 × $10^{-2}$$m$; q = 3.5 μC = 3.5 × $10^{-6}$$C$

$$\text{Capacitance } C = \frac{2\pi\varepsilon_0\ell}{2.303\,log_{10}\left(\frac{b}{a}\right)} = \frac{2\pi\times8.854\times10^{-12}\times15\times10^{-2}}{2.303\,log_{10}\frac{1.5\times10^{-2}}{1.4\times10^{-2}}}$$

$= 1.21 \times 10^{-10}\, F$

Since the outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them. Potential of inner cylinder, is

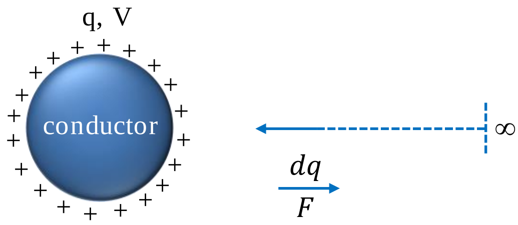
$$V = \frac{q}{C} = \frac{3.5\times10^{-6}}{1.2\times10^{-10}} = 2.89 \times 10^4 V$$

Work done by external agent to charge a conductor

Work has to be done to charge a conductor because charge present on it will repel the next incoming charged element.

Let at any instant charge of conductor is q and potential is V.

Work done by external agent to bring small charged element dq from infinity to surface of conductor.



$dw = dq\,[V_f - V_i]$, $V_i = V_\infty = 0$, $V_f = V$

$dw = dq\,[V-0]$

$dw = V\,dq$

$\int dw = \int V.dq$

∵ $V = q/C$

Work done by external agent to charge the conductor from q_1 to q_2 is

$$W = \int_{q_1}^{q_2} \frac{q}{C}.dq \Rightarrow W = \frac{1}{C}\left(\frac{q^2}{2}\right)_{q_1}^{q_2} \Rightarrow W = \frac{q_2^2-q_1^2}{2C}$$

$q_1 = CV_1$, $q_2 = CV_2$

Work done by external agent to increase the potential of conductor from V_1 to V_2

$$W = \frac{C}{2}(V_2^2 - V_1^2)$$

For 0 to V

$$w = \frac{C}{2}(V^2 - 0^2) = \frac{1}{2}CV^2$$

So, $w = \frac{1}{2}\,CV^2 = \frac{1}{2}\,QV = \frac{1}{2}\,\frac{Q^2}{C}$

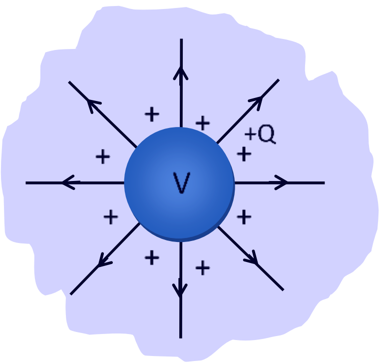
Potential energy of a charged conductor

Since electrostatic field is conservative field and in conservative field work done by external agent is stored in form of potential energy.

* Potential energy of conductor will be stored in form of electric field.
* Potential energy of a conductor which is charged by V potential is given by

$$P.E. = W_{ext}, \qquad U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

Potential Energy of Conducting Sphere:



$$U = \frac{1}{2}CV^2 = \frac{1}{2}QV = \frac{Q^2}{2C}$$

$\because \quad C = 4\pi\varepsilon_0 R$

$$U = \frac{1}{2}\frac{Q^2}{4\pi\varepsilon_0 R}$$

$$U = \frac{KQ^2}{2R}$$

Self-potential energy of spherical conductor.

This potential energy is stored in the form of electric field.

Illustration 75:

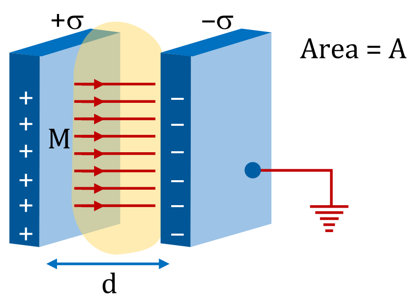
When $30\mu C$ charge is given to an isolated conductor of capacitance $5\mu F$. Find out the energy stored in form of electric field in the conductor.

Solution:

$Q_1 = 30\mu C, \; C_1 = 5\mu F$

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}\frac{(30\times10^{-6})^2}{(5\times10^{-6})} = 90\ \mu J$$

Energy of parallel plate capacitor:



Energy stored in capacitor $= \frac{Q^2}{2C}$ putting the value of capacitance for parallel plate capacitor

$$U = \frac{1}{2}\frac{Q^2}{C} = \frac{(A\sigma)^2}{2} \times \frac{d}{\varepsilon_0 A}$$

$$U = \frac{1}{2}(Ad)\frac{\sigma^2}{\epsilon_0}$$

Energy density

$$\frac{U}{V} = \frac{\sigma^2}{2\epsilon_0} = \frac{1}{2}\epsilon_0 E^2 \text{ (here } V \text{ is volume i.e. A.d and } E = \frac{\sigma}{\epsilon_0})$$

Once it is established that a region containing electric field E has energy $\frac{1}{2}\epsilon_0 E^2$ per unit volume, the result can be used for any electric field whether it is due to a capacitor or otherwise.

Illustration 76:
The work done against electric forces in increasing the potential difference of a condenser from 20 V to 40 V is W. The work done in increasing its potential difference from 40 V to 50 V will be

Solution:

$$W = U_f - U_i = \frac{1}{2}\ CV_f^2 - \frac{1}{2}\ CV_i^2\ = \frac{1}{2}\ C\,(40^2 - 20^2)\ W = 600\ C$$

$$W_1 = \frac{1}{2}\ C\,(50^2\ -\ 40^2)\ =\ \frac{900}{2}\ C$$

$$W_1 = \frac{900}{2}\cdot\frac{W}{600}\ =\ \frac{3}{4}\,W$$

Illustration 77:
The plate separation in a parallel plate condenser is d and plate area is A. If it is charged to V volt and battery is disconnected then the work done in increasing the plate separation to $2d$ will be:

Solution:

As battery is disconnected, charge remains constant in the work process.

Work done = final potential energy – initial potential energy

$$= \frac{Q^2}{2C'} - \frac{Q^2}{2C^0} = \frac{Q^2}{2}\left\{\frac{1}{C'} - \frac{1}{C^0}\right\}$$

Where, $Q = CV = \dfrac{A\varepsilon_0 V}{d}$; $C = \dfrac{A\varepsilon_0}{d}$; $C' = \dfrac{A\varepsilon_0}{2d}$

Now, work done = $\dfrac{\varepsilon_0 V^2 A}{2d}$

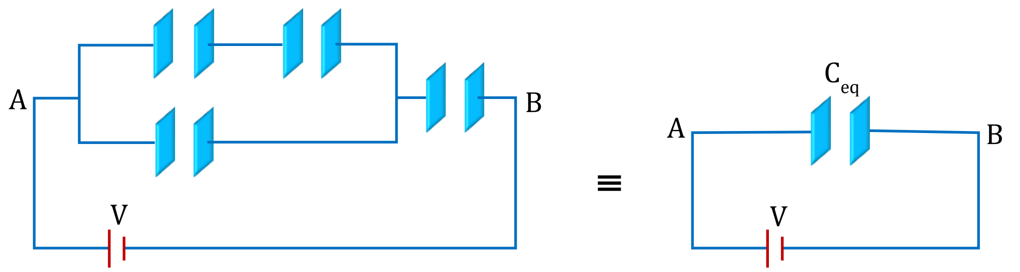
Illustration 78:
A capacitor of capacitance $\frac{1}{3}\mu F$ is connected to a battery of 300 volt and charged. Then the energy stored in capacitor is:

Solution:

$$U = \frac{1}{2}CV^2 = 1.5\times 10^{-2}J$$

Meaning of Equivalent Capacitor:
(a) Replacing the given capacitor network between two terminals by a single capacitor so that this single capacitor produces the same electrical effect when connected between the same two terminals instead of the original capacitors network.

(b) Capacitance of this single capacitor which can be used to replace the original capacitor network is termed as equivalent capacitance of the given network.



$$C_{eq} = \frac{Q}{V}$$

(c) Here same electrical effect refers to the charge flown through the battery or charge transferred between the terminals plates of capacitors connected to battery. It could also be understood in form of energy stored being same in two cases i.e. when original capacitor network is connected and the one when original network is replaced by equivalent capacitance.

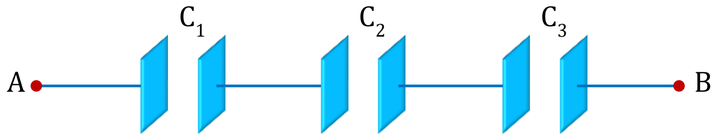
There are two standard combinations:

(1) Series Combination (2) Parallel Combination

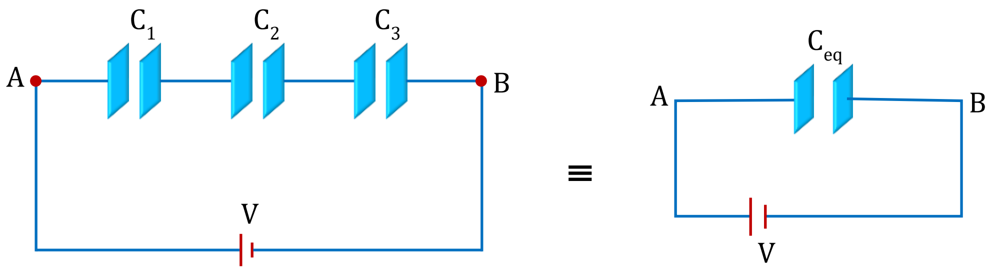
Series Combination

When initially uncharged capacitors are connected as shown so that charges do not have any alternative path(s) to flow then the combination is called series combination.

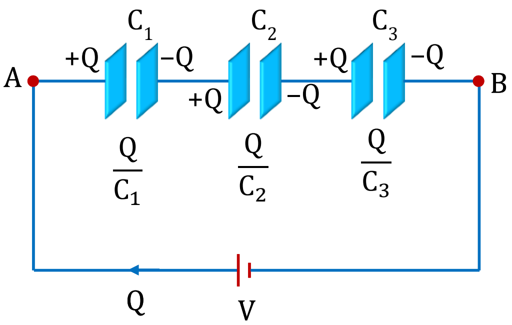
Derivation:



To find equivalent capacitance of this combination lets connect a battery across its terminals.



$$C_{eq} = \frac{Q}{V}$$



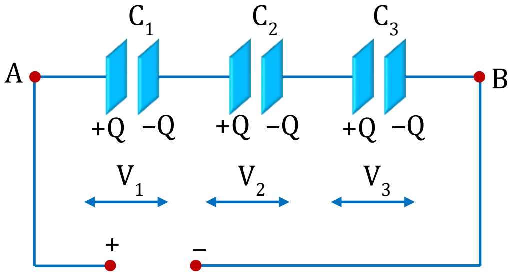
Let's assume that initially, the capacitors were uncharged and after connecting to battery, Q charge flows through the battery as shown in above figure.

Applying Kirchhoff's voltage law

$$\frac{-Q}{C_1} + \frac{-Q}{C_2} + \frac{-Q}{C_3} + V = 0. \; ; \; \frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

$$\Rightarrow \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \text{ or in general } \frac{1}{C_{eq}} = \sum_{n=1}^{n} \frac{1}{C_n}$$

Salient features of series combination



(a) All capacitors will have same charge but can have different potential difference across them.

(b) We can say that potential difference across capacitor is inversely proportional to its capacitance in series combination $\propto \frac{1}{C}$

$$V_1 = \frac{Q}{C_1}, V_2 = \frac{Q}{C_2}, V_3 = \frac{Q}{C_3} \ldots$$

(c) $V_1 : V_2 : V_3 = \frac{1}{C_1} : \frac{1}{C_2} : \frac{1}{C_3}$

In series combination the smallest capacitor gets maximum potential.

(d) $V_1 = \dfrac{\frac{1}{C_1}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots\ldots} V_2 = \dfrac{\frac{1}{C_2}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots\ldots} V$

$$V_3 = \frac{\frac{1}{C_3}}{\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\ldots.} V$$

Where $V = V_1 + V_2 + V_3$

(e) In series : $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots$

In series combination equivalent capacitance is always less the smallest capacitor of combination.

(f) Energy stored in the combination $U_{\text{combination}} = \frac{Q^2}{2C_1} + \frac{Q^2}{2C_2} + \frac{Q^2}{2C_3}$

$$U_{\text{combination}} = \frac{Q^2}{2C_{eq}}$$

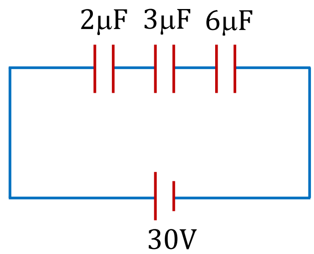
Energy supplied by the battery in charging the combination $U_{\text{battery}} = Q \times V = Q \cdot \frac{Q}{C_{eq}} = \frac{Q^2}{C_{eq}}$

$$\frac{U_{\text{combination}}}{U_{\text{battery}}} = \frac{1}{2}$$

Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (if capacitors are initially uncharged)

Illustration 79:

Three initially uncharged capacitors are connected in series as shown in circuit with a battery of emf 30 V. Find out following:



(i) charge flow through the battery

(ii) potential energy in 3μF capacitor

(iii) U_{total} in capacitors

(iv) heat produced in the circuit

Solution:

$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$$

$C_{eq} = 1\mu F$

(i) $Q = C_{eq}\, V = 30\ \mu C$

(ii) Charge on $3\mu F$ capacitor = $30\ \mu C$

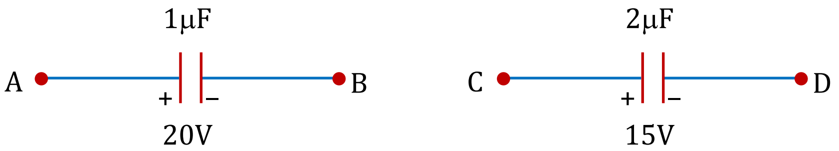
Energy = $\frac{Q^2}{2C} = \frac{30\times30}{2\times3} = 150\mu J$

(iii) $U_{\text{total}} = \frac{30\times30}{2}\ \mu J = 450\ \mu J$

(iv) Heat produced = $(30\ \mu C)\ (30) - 450\ \mu J = 450\ \mu J$.

Illustration 80:

Two capacitors of capacitance $1\ \mu F$ and $2\mu F$ are charged to potential difference $20V$ and $15V$ as shown in figure. If now terminal B and C are connected together terminal A with positive of battery and D with negative terminal of battery $30\ V$ then find out final charges on both the capacitor.



Solution:

Now applying Kirchoff voltage law $\frac{-(20+q)}{1} - \frac{30+q}{2} + 30 = 0$

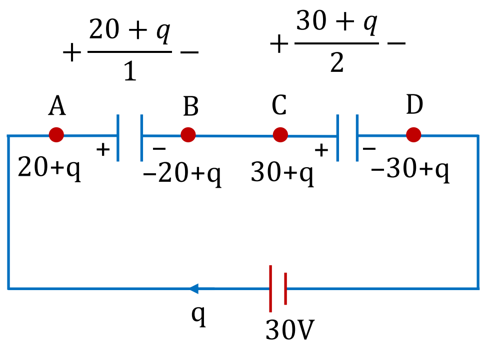
$-40 - 2q - 30 - q = -60$

$3q = -10$

Charge flow = $-10/3\ \mu C$.

Charge on capacitor of capacitance $1\mu F = 20 + q = \frac{50}{3}\mu C$

Charge on capacitor of capacitance $2\mu F = 30 + q = \frac{80}{3}\mu C$



Parallel Combination:

When one plate of each capacitors (more than one) is connected together and the other plate of each capacitor is connected together, such combination is called parallel combination.

Derivation of Equivalent Capacitance

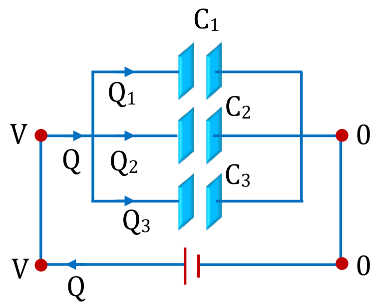
$Q = Q_1 + Q_2 + Q_3$

$Q = C_1V + C_2V + C_3V = V(C_1 + C_2 + C_3)$

$\frac{Q}{V} = C_1 + C_2 + C_3$

$C_{eq} = C_1 + C_2 + C_3$

In general $C_{eq} = \sum_{n=1}^{n} C_n$



Salient Features of Parallel Combination

(a) All capacitors have same potential difference but can have different charges.

(b) We can say that:

$Q_1 = C_1V$

Q_1 = Charge on capacitor C_1

C_1 = Capacitance of capacitor C_1

V = Potential across capacitor C_1

- $Q_1 : Q_2 : Q_3 = C_1 : C_2 : C_3$

  The charge on the capacitor is proportional to its capacitance

  $Q \propto C$

  $Q_1 = \dfrac{C_1}{C_1+C_2+C_3}Q \Rightarrow Q2 = \dfrac{C_2}{C_1+C_2+C_3}Q$

  $Q_3 = \dfrac{C_3}{C_1+C_2+C_3}Q$

  Where $Q = Q_1 + Q_2 + Q_3$ ......

  Maximum charge will flow through the capacitor of largest value.

- $C_{eq} = C_1 + C_2 + C_3$

  Equivalent capacitance is always greater than the largest capacitor of combination.

- Energy stored in the combination :

  $U_{\text{combination}} = \frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2 + \cdots = \frac{1}{2}(C_1 + C_2 + C_3 \ldots\ldots)V^2 = \frac{1}{2}C_{eq}V^2$

  $U_{\text{battery}} = QV = C_{eq}V^2$

  $\dfrac{U_{\text{combination}}}{U_{\text{battery}}} = \dfrac{1}{2}$

  Half of the energy supplied by the battery is stored in form of electrostatic energy and half of the energy is converted into heat through resistance. (If all capacitors are initially uncharged)

Illustration 81:

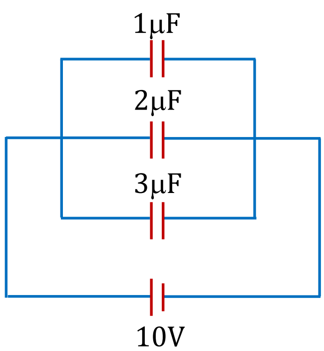
Three initially uncharged capacitors are connected to a battery of 10 V is parallel combination find out following :

(i) charge flown from the battery

(ii) total energy stored in the capacitors

(iii) heat produced in the circuit

(iv) potential energy in the 3μF capacitor.



Solution:

(i) $Q = C_{eq}V = (3 + 2 + 1)10 = 60\ \mu C$

(ii) $U_{\text{total}} = \frac{1}{2} \times 6 \times 10 \times 10 = 300\ \mu J$

(iii) heat produced = $60 \times 10 - 300 = 300\ \mu J$

(iv) $U_{3\mu F} = \frac{1}{2} \times 3 \times 10 \times 10 = 150\ \mu J$

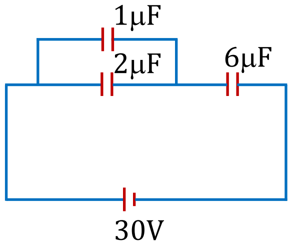
Mixed Combination:

The combination which contains mixing of series parallel combinations or other complex combinations fall in mixed category. There are two types of mixed combinations

(i) Simple (ii) Complex

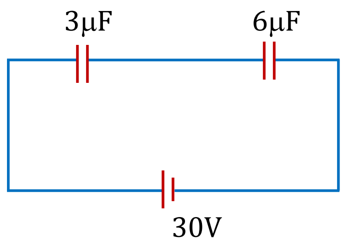
Illustration 82:

In the given circuit find out charge on $6\mu F$ and $1\mu F$ capacitor.



Solution:

It can be simplified as $C_{eq} = \frac{18}{9} = 2\mu F$



Charge flow through the cell = $30 \times 2\mu C$

$Q = 60\mu C$

Now charge on $3\mu F$ = Charge on $6\mu F$= $60\mu C$

Potential difference across $3\mu F = 60/3 = 20V$

$\therefore$ Charge on $1\mu F = 20\ \mu C$

Illustration 83:

An infinite number of capacitors of capacitance C, $4C$, $16C$... ∞ are connected in series then what will be their resultant capacitance?

Solution:

Let the equivalent capacitance of the combination $= C_{eq}$

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{4C} + \frac{1}{16C} + \ldots \infty = \left[1 + \frac{1}{4} + \frac{1}{16} + \ldots \infty\right]\frac{1}{C} \text{ (this is G.P. series)}$$

$\Rightarrow$ $S_\infty = \frac{a}{1-r}$ first term $a = 1$, common ratio $r = \frac{1}{4}$

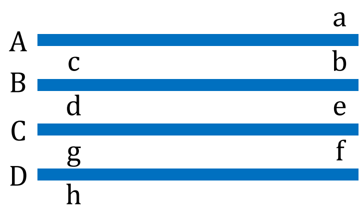
$\Rightarrow$ $\frac{1}{C_{eq}} = \frac{1}{1-\frac{1}{4}} \times \frac{1}{C} \Rightarrow C_{eq} = \frac{3}{4}C$

Combination of Parallel Plates:

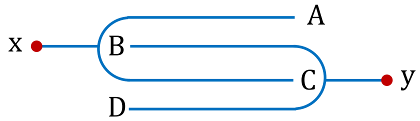
Combinations of parallel plates can be grouped and equivalent capacitance can be found easily by redrawing the circuit by connecting plates to common terminals. This has been illustrated by following example.

Four parallel plates each of area A and plate separation d. These four plates are forming three parallel plate capacitors, one between plates A and B, one between plates B and C and another between plates C and D.

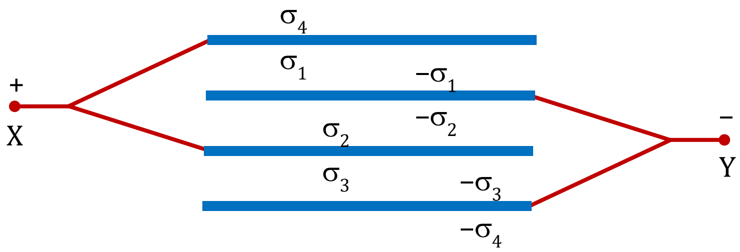
Capacitance of each of these capacitors is given as $C = \frac{\epsilon_0 A}{d}$.



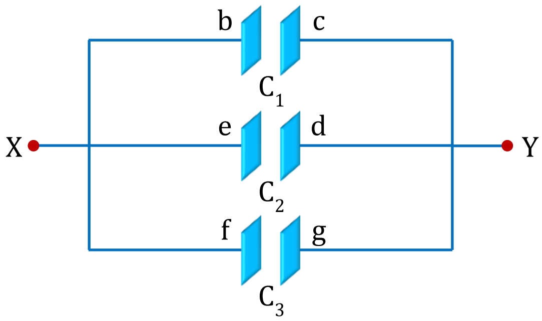
If plates A and C are connected as terminal X and plates B and D are connected as terminal Y. Then calculation of equivalent capacitance across terminal X and Y can be done by following method.



Let us assume that we have connected a battery between terminal X and Y. This battery supplies equal and opposite charges to terminal X and Y. For the detailed analysis let us say σ_1 and σ_2 are charge density on faces b and e. Similarly, σ_3 and σ_4 are charge density on faces f and a.



Now equal opposite charges will be there on opposite faces as shown in figure (as per Gauss Law). Also electric field at any point inside conductor must be zero therefore σ_4 must be zero.

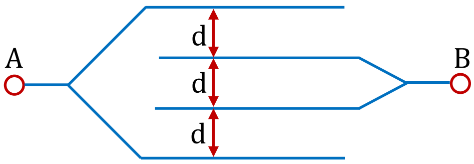


$$V_{AB} = V_{BC} = V_{CD}$$

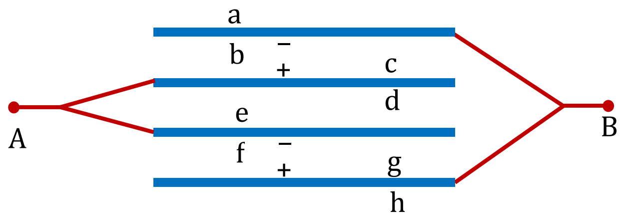
So, all the capacitors are in parallel combination.

$$C_{eq} = C_1 + C_2 + C_3 = \frac{3A\epsilon_0}{d}$$

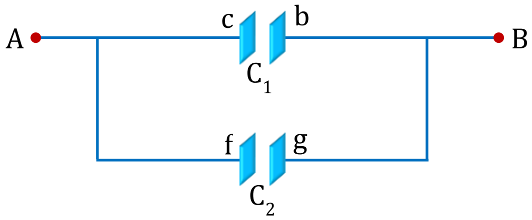
Let us try to find out equivalent capacitance between A and B. (take each plate Area = A) in following case by using the same above concept.



Assuming a battery between terminals A and B the positive and negative charges on the plates will be as shown in the figure fellow.



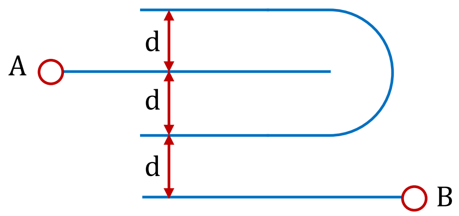
Here charges on plates d and e is zero because their potential must be same, so no electric field should be there in between these faces, so charge on faces d and e must be zero.



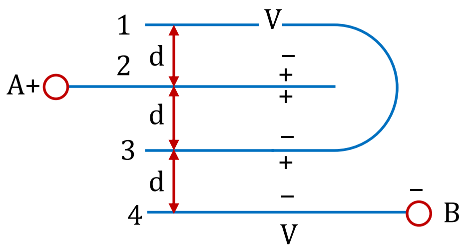
These are only two capacitors, $C_{eq} = C_1 + C_2 = \dfrac{2A\epsilon_0}{d}$

Illustration 84:

Find out equivalent capacitance between A and B. (take each plate Area = A)

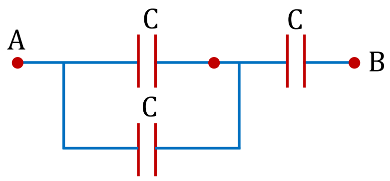


Solution:



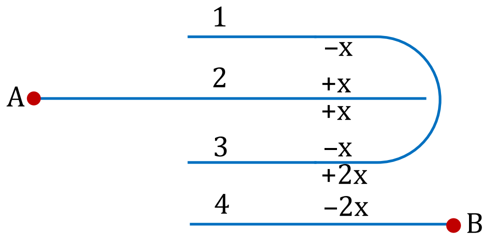
The modified circuit is

$$C_{eq} = \frac{2C}{3} = \frac{2A\epsilon_0}{3d}$$



Other method:

Let charge density as shown



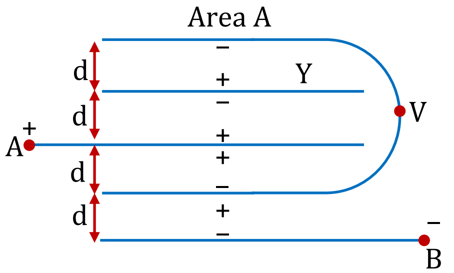
$$C_{eq} = \frac{Q}{V} = \frac{2xA}{V}$$

$$V = V_2 - V_4 = (V_2 - V_3) + (V_3 - V_4) = \frac{xd}{\epsilon_0} + \frac{2xd}{\epsilon_0} = \frac{3xd}{\epsilon_0}$$

$$\therefore \quad C_{eq} = \frac{2Ax\,\epsilon_0}{3xd} = \frac{2A\,\epsilon_0}{3d} = \frac{2C}{3}.$$

Illustration 85:

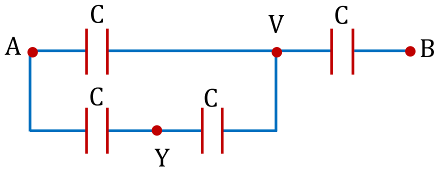
Find out equivalent capacitance between A and B.



Solution:

Let $\quad C = \dfrac{A\epsilon_0}{d}$

Equivalent circuit :

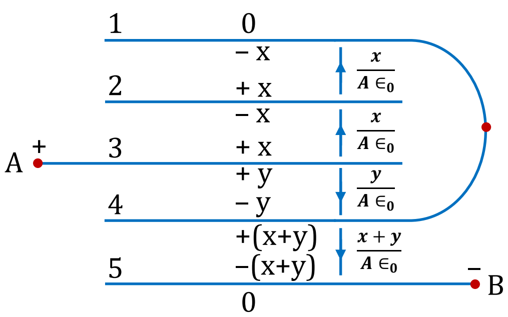


$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{2}{3C} = \frac{5}{3C}$$

$$C_{eq} = \frac{3C}{5} = \frac{3A\,\epsilon_0}{5d}$$

Alternative Method:

Let charge distributions on plates as shown:



$$C = \frac{Q}{V} = \frac{x+y}{V_{AB}}$$

Potential of 1 and 4 is same

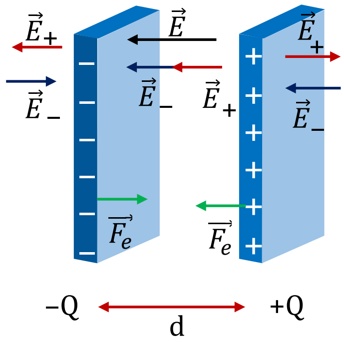
$$\frac{y}{A\epsilon_0} = \frac{2x}{A\epsilon_0} \qquad y = 2x$$

$$V = \left(\frac{2y+x}{A\varepsilon_0}\right)d$$

$$\Rightarrow \quad C = \frac{(x+2x)A\epsilon_0}{(5x)d} = \frac{3A\epsilon_0}{5d}$$

The two plates of capacitor attract each other because they are oppositely charged.

Electric field due to positive plate,



$$E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}$$

Force on negative charge $-Q$ is

$$F = -QE = -\frac{Q^2}{2\varepsilon_0 A}$$

Magnitude of force

$$F = \frac{Q^2}{2\varepsilon_0 A} = \frac{1}{2}\varepsilon_0 A E^2$$

Force per unit area or energy density or electrostatic pressure

$$= \frac{F}{A} = u = p = \frac{1}{2}\epsilon_0 E^2$$

Note:

Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.

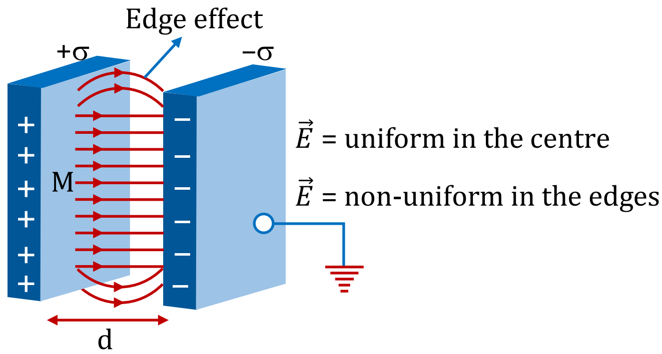


Illustration 86:

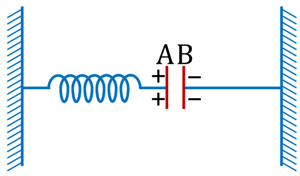


Plate A of a parallel air-filled capacitor is connected to a non-conducting spring having force constant k and plate B is fixed. If a charge $+q$ is placed on plate A and charge $-q$ on plate B then find out extension in the spring in equilibrium. Assume area of plate is 'A'.

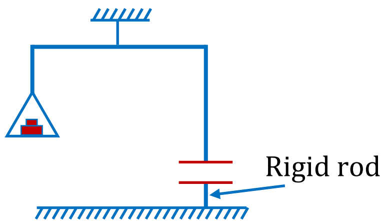
Solution:

$$F_{\text{attraction}} = \frac{q^2}{2\epsilon_0 A}; \; F_{\text{Repulsion}} = kx$$

$$|F_{\text{attraction}}| = |F_{\text{Repulsion}}|$$

$$x = \frac{q^2}{2k\epsilon_0 A}$$

Illustration 87:



The lower plate of a parallel plate capacitor is supported on a rigid rod. The upper plate is suspended from one end of a balance. The two plates are joined together by a thin wire and subsequently disconnected. The balance is then counterpoised. Now a voltage V = 5000 volt is applied between the plates. The distance between the plates is d = 5 mm and the area of each plate is A = 100 cm^2. Then find out the additional mass placed to maintain balance.

[All the elements other than plates are massless and non-conducting]

Solution:

The electric force between the plates will be balanced by the additional weight

Hence $mg = \dfrac{Q^2}{2A\epsilon_0} = \dfrac{C^2V^2}{2A\epsilon_0}$

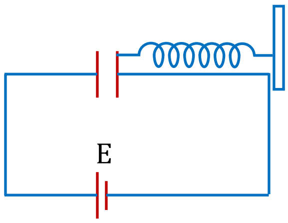
$$mg = \frac{\epsilon_0 AV^2}{2d^2}$$

$$m = \frac{\epsilon_0\ AV^2}{2d^2 g} = \frac{\epsilon_0 \times 100 \times 10^{-4}(5000)^2}{2(5\times 10^{-3})^2 \times 10}$$

$m = 4.425\ g$ **Ans.**

Illustration 88:

One plate of a capacitor is fixed and the other is connected to a spring as shown in Fig. Area of both the plates is A. In steady state (equilibrium), separation between the plates is 0.8 d (spring was unstretched and the distance between the plates was d when the capacitor was uncharged). The force constant of the spring is approximately.



Solution:

$$\frac{Q^2}{2A\varepsilon_0} = K(0.2d)$$

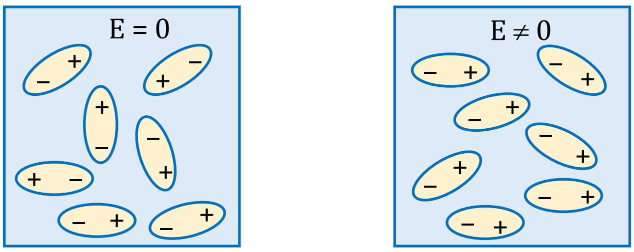
$$\frac{\left(\frac{\varepsilon_0 A}{0.8d}E\right)^2}{2A\varepsilon_0} = 0.2Kd$$

$$K = \frac{3.9\varepsilon_0 AE^2}{d^3}$$

Effect of Dielectric

- The insulators in which at microscopic level displacement of charges takes place in presence of electric field are known as **dielectrics**.
- Dielectrics are non-conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called **dielectric strength** they lose their insulating property and begin to conduct.
- **Dielectric strength** is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre. Dimensions $M^1\ L^1\ T^{-3}\ A^{-1}$.

Polar dielectrics



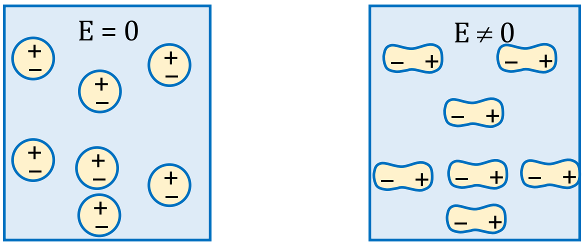
- In absence of external field, the centres of positive and negative charge do not coincide-due to asymmetric shape of molecules.

- Each molecule has permanent dipole moment.
- The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.

  In presence of external field dipoles tends to align in direction of field.

  Example: Water, Alcohol CO_2, $HC\ell$, NH_3

Non polar dielectrics



- In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In presence of external field they acquire induced dipole moment.

  Example: Nitrogen, Oxygen, Benzene, Methane

Polarisation:

The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

- **Polarisation Vector $\vec{P}$**

  This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

  $\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

  where n is number of atoms per unit volume of dielectric and P is dipole moment of an atom or molecule.

  $\vec{P} = n\vec{p} = \dfrac{q_i d}{Ad} = \left(\dfrac{q_i}{A}\right) = \sigma_i$ = induced surface charge density.

Capacitors with Dielectric:

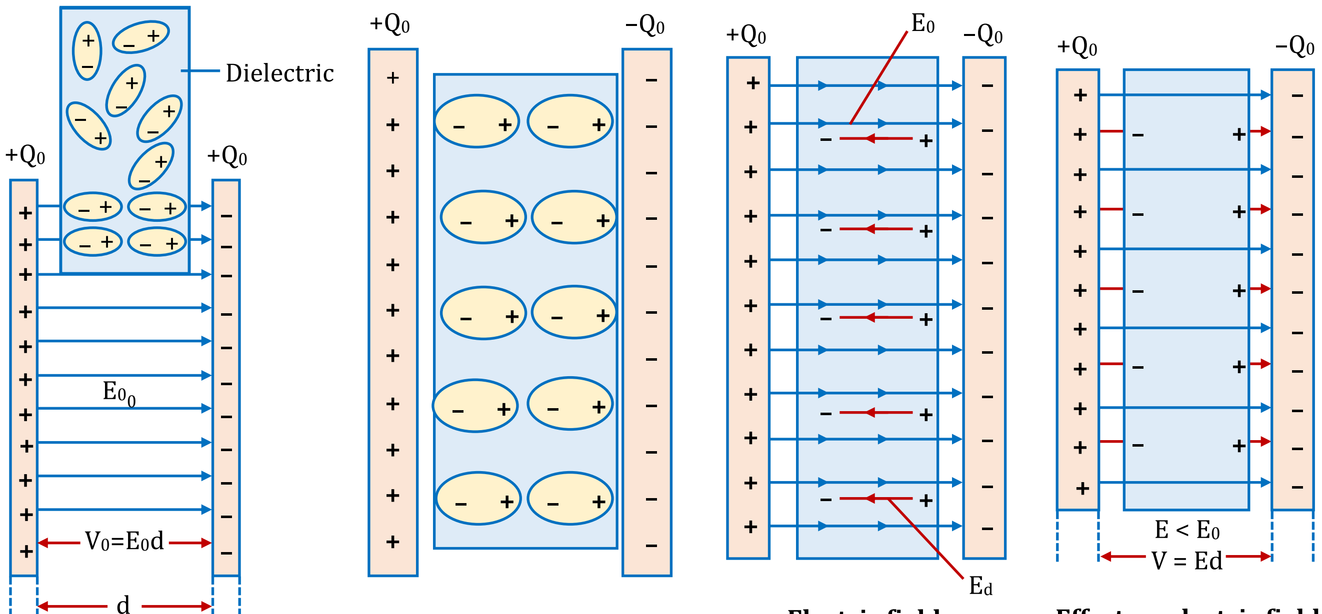
(i) In absence of dielectric $E = \dfrac{\sigma}{\epsilon_0}$

(ii) When a dielectric fills the space between the plates then molecules having dipole moment align themselves in the direction of electric field.

σ_b = induced (bound) charge density (called bound charge because it is not due to free electrons).

The induced charge also produce electric field.

Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric and E, V, C are electric field, potential difference and capacitance respectively in presence of dielectric.



Charged capacitor | **Charged capacitor With dielectric inserted** | **Electric field Inside dielectric** | **Effect on electric field and voltage**

Electric field in absence of dielectric, $E_0 = \frac{V_0}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric, $E = E_0 - E_i = \frac{\sigma - \sigma_b}{\varepsilon_0} = \frac{Q - Q_b}{\varepsilon_0} = \frac{V}{d}$

Capacitance in absence of dielectric, $C_0 = \frac{Q}{V_0}$

Capacitance in presence of dielectric, $C = \frac{Q - Q_b}{V}$

The dielectric constant or relative permittivity K or $\varepsilon_r = \frac{E_0}{E} = \frac{V_0}{V} = \frac{C}{C_0} = \frac{Q}{Q - Q_b} = \frac{\sigma}{\sigma - \sigma_b} = \frac{\varepsilon}{\varepsilon_0}$

From $K = \frac{Q}{Q - Q_b}, Q_b = Q\left(1 - \frac{1}{K}\right)$ and $K = \frac{\sigma}{\sigma - \sigma_b} \Rightarrow \sigma_b = \sigma\left(1 - \frac{1}{K}\right)$

(iii) Capacitance in the presence of dielectric

$$C = \frac{\sigma A}{V} = \frac{\sigma A}{\frac{\sigma}{K\epsilon_0}.d} = \frac{AK\epsilon_0}{d}$$

Here capacitance is increased by a factor K.

$$C = \frac{AK\epsilon_0}{d}$$

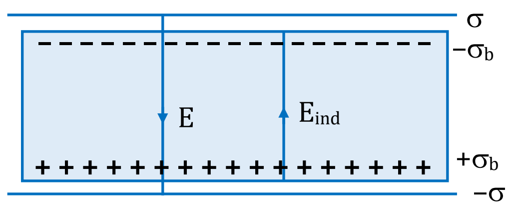


Illustration 89:

A parallel-plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9×10^7 V/m is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.

Solution:

If the charge on the capacitor = Q

the surface charge density $\sigma = \frac{Q}{A}$ and the electric field $= \frac{Q}{KA\varepsilon_0}$.

This electric field should not exceed the dielectric strength 1.9×10^7 V/m.

$\therefore$ If the maximum charge which can be given is Q

then $\dfrac{Q}{KA\varepsilon_0} = 1.9 \times 10^7\ V/m,$ $A = 100\ cm^2 = 10^{-2}\ m^2$

$\Rightarrow$ $Q = (5.0) \times (10^{-2}) \times (8.85 \times 10^{-12}) \times (1.9 \times 10^7) = 8.4 \times 10^{-6}\ C.$

Illustration 90:

Hard rubber has a dielectric constant of 2.8 and a dielectric strength (maximum electric field) of 18×10^6 volt/meter. If it is used as the dielectric material filling the full space in a parallel plate capacitor. What minimum area may the plates of the capacitor have in order that the capacitance be 7.0×10^{-2} μF and that the capacitor be able to withstand a potential difference of 4000 volts. ($\epsilon_0 = 8.85 \times 10^{-12}$ S.I unit)

Solution:

Let distance between the plates = d

$18 \times 10^6 \times d = 4000$

$d = \dfrac{4000}{18\times10^6}$

Now, $C = \dfrac{\epsilon_r A \epsilon_0}{d}$

$7.0 \times 10^{-2} \times 10^{-6} = \dfrac{8.85\times10^{-12}\times A\times2.8}{\frac{4000}{18\times10^6}}$

Solving, we get $A = 0.62\ m^2$.

Illustration 91:

A capacitor has two circular plates whose radius are $8cm$ and distance between them is $1mm$. When mica (dielectric constant = 6) is placed between the plates, calculate the capacitance of this capacitor and the energy stored when it is given potential of 150 volt.

Solution:

Area of plate $\pi r^2 = \pi \times (8 \times 10^{-2})^2 = 0.0201\ m^2$ and $d = 1mm = 1 \times 10^{-3}\ m$

Capacity of capacitor,

$$C = \frac{\varepsilon_0 \varepsilon_r A}{d} = \frac{8.85\times10^{-12}\times6\times0.0201}{1\times10^{-3}} = 1.068 \times 10^{-9}\ F$$

Potential difference,

$V = 150$ volt

Energy stored,

$$U = \frac{1}{2} CV^2 = \frac{1}{2} \times (1.068 \times 10^{-9}) \times (150)^2 = 1.2 \times 10^{-5}\ J$$

Illustration 92:

A capacitor with air as the dielectric is charged to a potential of 100 volts. If the space between the plates is now filled with a dielectric of dielectric constant 10, the potential difference (in volt) between the plates will be:

Solution:

$V = 100\ V,\quad K = 10$

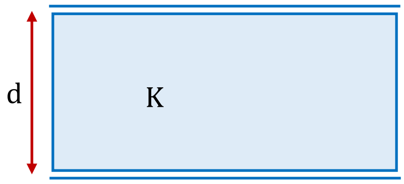
$Q = CV$ (constant), $C' = KC$

$V' = \dfrac{V}{K} = 10V$

Capacitor with Dielectric in Different Configuration

Calculation of Capacitance

When **single** dielectric is completely filled inside capacitor



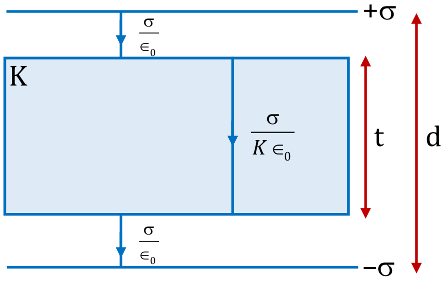
$$C = \frac{\sigma A}{V} = \frac{\sigma A}{\frac{\sigma}{K \epsilon_0}.d} = \frac{AK \epsilon_0}{d}$$

Here capacitance is increased by a factor K.

$$C = \frac{AK \epsilon_0}{d}$$

Now lets try to understand how to calculate capacitance in other configurations.

Single or multiple dielectric is partially filled



When the dielectric is filled partially between plates, the thickness of dielectric slab is t $(t < d)$. If the medium does not fill between the plates completely then electric field will be as shown in figure.

The total electric field produced by bound induced charge on the dielectric outside the slab is zero because they cancel each other and hence electric field will be only due to plate charges in these vacant spaces. Where as electric field in the dielectric space will be vector sum of electric field due to plate charges and induced charges as shown in figure.

Then $\quad V = E_0 (d-t) + Et$

If V is total potential,

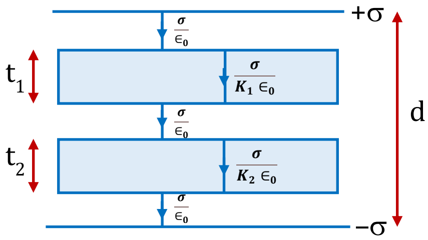
$$\Rightarrow \quad V = E_0 \left[d - t + \left(\frac{E}{E_0}\right)t\right] \frac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$$

$$\Rightarrow \quad V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right]$$

$$\Rightarrow \quad C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)}$$

Example:

If v is the potential difference between the plate,

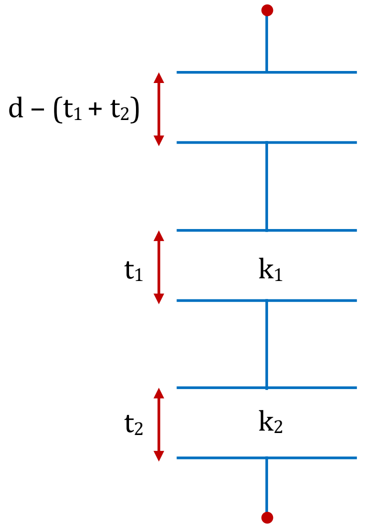


$$V = \frac{\sigma}{\varepsilon_0}(d - (t_1 + t_2)) + \frac{\sigma}{\varepsilon_0 K_1} t_1 + \frac{\sigma}{\varepsilon_0 K_2} t_2$$

$$\therefore \quad C = \frac{Q}{V} = \frac{\sigma A}{V} = \frac{\sigma A}{\frac{\sigma}{\varepsilon_0}(d - (t_1 + t_2)) + \frac{\sigma}{\varepsilon_0 K_1} t_1 + \frac{\sigma t_2}{\varepsilon_0 K_2}}$$

$$= \frac{A\varepsilon_0\ K_1 K_2}{K_2 K_1 (d - (t_1 + t_2)) + K_2 t_1 + K_1 t_2}$$

Above question can also be solved by using the concept of series and parallel combination. We can redraw the given configuration as follows.

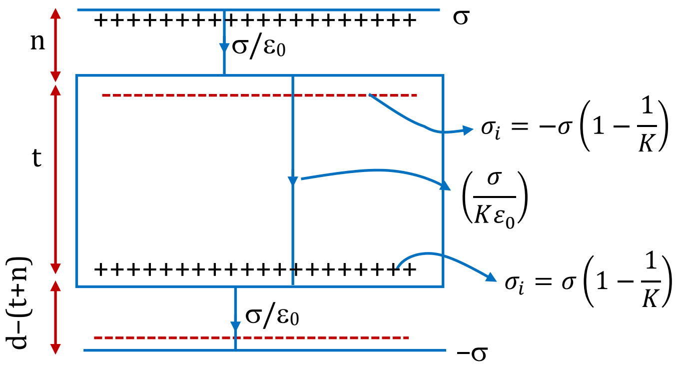


$$C_1 = \frac{A\varepsilon_0}{d-(t_1+t_2)}$$

$$C_2 = \frac{k_1 A\varepsilon_0}{t_1}$$

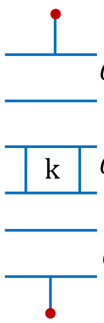
$$C_3 = \frac{k_2 A\varepsilon_0}{t_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$



Above question can also be solved by using concept of series and parallel combination.

We can redraw the given configuration as follow.



$$C_1 = \frac{A\varepsilon_0}{n}$$

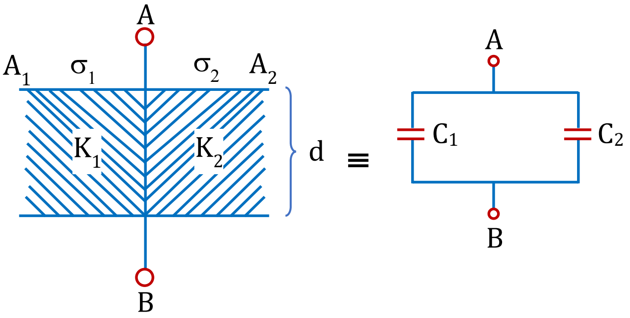
$$C_2 = \frac{kA\varepsilon_0}{t}$$

$$C_3 = \frac{A\varepsilon_0}{d-(t+n)}$$

$$\frac{1}{C_{eq.}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

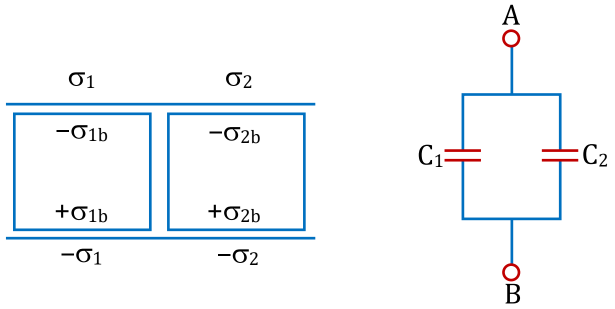
Multiple dielectrics completely fill the capacitor

Lets try to calculate capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of area A_1 and A_2 and each of thickness d are inserted between the plates of parallel plate capacitor of plate area A as shown in figure. $(A_1 + A_2 = A)$



$$C_1 = \frac{A_1K_1\epsilon_0}{d}, C_2 = \frac{A_2K_2\epsilon_0}{d}$$

As dielectrics are between same metal plates therefore potential difference and hence electric field will be same either calculated using electric field in any of dielectric region. Therefore, if dielectric constant of these two dielectric is different then charges on plate will be different in two regions.



$$E_1 = \frac{V}{d} = \frac{\sigma_1}{K_1\epsilon_0}, \quad E_2 = \frac{V}{d} = \frac{\sigma_2}{K_2\epsilon_0}$$

$$\sigma_1 = \frac{K_1\epsilon_0 V}{d}, \sigma_2 = \frac{K_2\epsilon_0 V}{d}$$

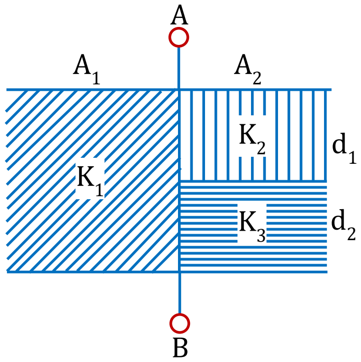
$$C = \frac{Q_1 + Q_2}{V} = \frac{\sigma_1A_1 + \sigma_2A_2}{V} = \frac{K_1\,\epsilon_0\,A_1}{d} + \frac{K_2\,\epsilon_0\,A_2}{d}$$

The combination is equivalent to : $C = C_1 + C_2$. Hence above question could have been solved by using series and parallel combination.

Illustration 93:

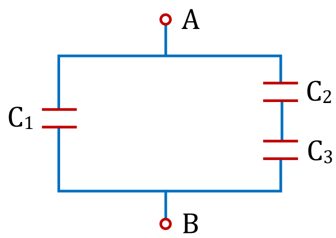
Find out capacitance between A and B if three dielectric slabs of dielectric constant K_1 of area A_1 and thickness d, K_2 of area A_2 and thickness d_1 and K_3 of area A_2 and thickness d_2 are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.

(Given distance between the two plates $d = d_1+d_2$).



Solution:

It is equivalent to $C = C_1 + \frac{C_2C_3}{C_2+C_3}$



$$C = \frac{A_1K_1\,\epsilon_0}{d_1 + d_2} + \frac{\frac{A_2K_2\,\epsilon_0}{d_1}\cdot\frac{A_2K_3\,\epsilon_0}{d_2}}{\frac{A_2K_2\,\epsilon_0}{d_1} + \frac{A_2K_3\,\epsilon_0}{d_2}}$$

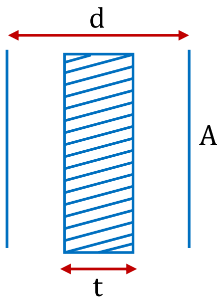
$$= \frac{A_1K_1\,\epsilon_0}{d_1 + d_2} + \frac{A_2^2K_2K_3\,\epsilon_0^2}{A_2K_2\,\epsilon_0\,d_2 + A_2K_3\,\epsilon_0\,d_1}$$

$$= \frac{A_1K_1\,\epsilon_0}{d_1 + d_2} + \frac{A_2K_2K_3\,\epsilon_0}{K_2d_2 + K_3d_1}$$

Illustration 94:

A parallel plate isolated condenser consists of two metal plates of area A and separation 'd'. A slab of thickness 't' and dielectric constant K is inserted between the plates with its faces parallel to the plates and having the same surface area as that of the plates. Find the capacitance of the system. If K = 2, for what value of t/d will the capacitance of the system be 3/2 times that of the condenser with air filling the full space? Calculate the ratio of the energy in the two cases and account for the energy change (assuming q charge on the plate to be constant).

Solution:



$$C' = \frac{A\epsilon_0}{\frac{t}{k}+d-t}$$

(i) Without dielectric, $C = \frac{A\epsilon_0}{d}$

With dielectric, $C' = \frac{A\epsilon_0}{\frac{t}{2}+d-t} = \frac{3\,C}{2} = \frac{3}{2}\frac{A\epsilon_0}{d} \Rightarrow \frac{d}{t} = \frac{3}{2}$

(ii) Energy = $\frac{Q^2}{2\,C}$

Energy in 1st case, $E_1 = \frac{q^2}{2\,C}$

Energy is 2nd case

$E_2 = \frac{q^2}{2\,C'}$

$\frac{E_1}{E_2} = \frac{C'}{C} = \frac{3}{2}$

(iii) $\Delta E = E_2 - E_1 = \frac{q^2}{2\,C'} - \frac{q^2}{2\,C} = \frac{q^2}{2}\left[\frac{2}{3C} - \frac{1}{C}\right] = -\frac{q^2}{6C} = -\frac{q^2 d}{6A\epsilon_0}$

Illustration 95:

The distance between the plates of a parallel-plate capacitor is 0.05 m. A field of 3×10^4 V/m is established between the plates of capacitor by connecting with battery. Now capacitor is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted between the plates of capacitor. Calculate new potential difference between the plates of capacitor. What would be the potential difference if a plate of same thickness and dielectric constant K = 2 is introduced in place of metal plate ?

Solution:

(i) In case of a capacitor as $E = (V/d)$, the potential difference between the plates before the introduction of metal plate

$$V = E \times d = 3 \times 10^4 \times 0.05 = 1.5\ kV$$

(ii) Now as after charging battery is removed, capacitor is isolated so q = constant. If C' and V' are the capacity and potential after the introduction of plate, $q = CV = C'V'$ i.e., $V' = \frac{C}{C'}V$

And as $C = \frac{\varepsilon_0 A}{d}$ and $C' = \frac{\varepsilon_0 A}{(d-t)+(t/K)} = V' = \frac{(d-t)+(t/K)}{d} \times V$

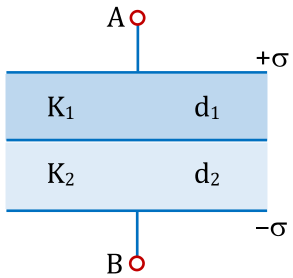
So, in case of metal plate as $K = \infty$, $V_M = \left[\frac{d-t}{d}\right] \times V = \left[\frac{0.05-0.01}{0.05}\right] \times 1.5 = 1.2$ kV

And if instead of metal plate, dielectric with K = 2 is introduced

$$V_D = \left[\frac{(0.05-0.01)+\left(\frac{0.01}{2}\right)}{0.05}\right] \times 1.5 = 1.35\ kV$$

Illustration 96:

Find out capacitance between A and B if two dielectric slabs of dielectric constant K_1 and K_2 of thickness d_1 and d_2 and each of area A are inserted between the plates of parallel plate capacitor of plate area A as shown in figure.



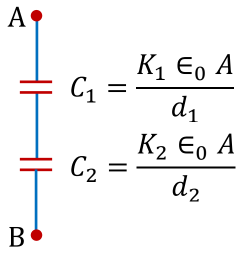
Solution:

$$C = \frac{\sigma A}{V} \qquad \left(C = \frac{Q}{V}\ ;\ q = \sigma A\right)$$

$$V = E_1\,d_1 + E_2\,d_2 = \frac{\sigma d_1}{K_1\,\epsilon_0} + \frac{\sigma d_2}{K_2\,\epsilon_0} = \frac{\sigma}{\epsilon_0}\left(\frac{d_1}{k_1} + \frac{d_2}{k_2}\right)$$

$$\therefore \qquad C = \frac{A\epsilon_0}{\frac{d_1}{K_1}+\frac{d_2}{K_2}}$$

$$\Rightarrow \qquad \frac{1}{C} = \frac{d_1}{AK_1\epsilon_0} + \frac{d_2}{AK_2\epsilon_0}$$



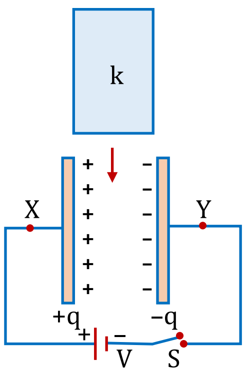
This formula suggests that the system between A and B can be considered as series combination of two capacitors.

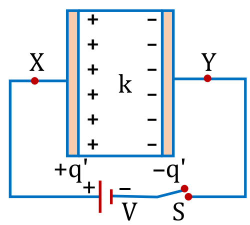
Comparison of E (Electric Field), σ (Surface Charges Density) and Other Parameters:

Comparison of E (electric field), σ (surface charge density), Q (charge), C (capacitance) before and after inserting a dielectric slab between the plates of a parallel plate capacitor

Case 1: When constant potential difference across capacitor plates is maintained.

- Parallel plate capacitor of capacitance C connected to a voltage source V due to which the capacitor is charged to the steady state charge $q = CV$. As battery remain connected during insertion of slab the potential difference across capacitor does not change in this case.





(a) Initial Capacitance, $C = \frac{\epsilon_0 A}{d}$

(b) Initial, $Q = CV$

(c) Initial electric field, $E = \frac{\sigma}{\epsilon_0} = \frac{CV}{A\epsilon_0}$

(a) Final capacitance, $C' = \frac{A\epsilon_0 K}{d}$

(b) $Q' = C'V$

(c) $E' = \frac{\sigma'}{K\epsilon_0} = \frac{CV}{A\epsilon_0}$

(d) Initially potential difference between the plates

$Ed = V$

$E = \dfrac{V}{d}$

$\dfrac{V}{d} = \dfrac{\sigma}{\epsilon_0}\ \dfrac{V}{d} = \dfrac{\sigma'}{K\epsilon_0}$

Equating both

$\dfrac{\sigma}{\epsilon_0} = \dfrac{\sigma'}{K\epsilon_0}$

$\sigma' = K\sigma$

(d) Here potential difference between the plates

$E'd = V$

$E' = \dfrac{V}{d}$

Or we can say that after inserting dielectric final charge becomes KCV which is K times the initial charge. Hence $CV(K-1)$ charge has flown through battery does a work of $CV^2(K\text{-}1)$ in the insertion process.

(e) Energy Stored in capacitor = $CV^2/2$

(e) Energy Stored in Capacitor = $KCV^2/2$

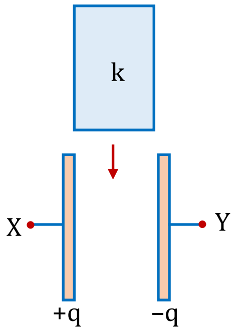
Energy equation for the capacitor circuit in dielectric insertion process will be

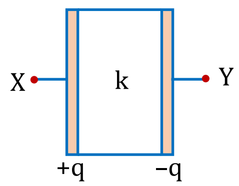
$W_{\text{batt}} + W_{\text{ext}} = \Delta U + \text{Heat.}$

If dielectric is inserted slowly and circuit has negligible resistance then heat loss will be negligible and some negative work will be done by external force.

Case 2: When plates of capacitor have constant charge:

- Parallel plate capacitor of capacitance C which is charged to a charge q and disconnected from the voltage source. When the dielectric slab is inserted in this capacitor which fills the space between plates then after insertion of dielectric the physical quantities associated with capacitor changes as listed in table below. As capacitor is charged and not connected in any circuit, its charge will remain constant and does not change while insertion of dielectric slab.





Parameter before insertion of dielectric.

(a) Initial Capacitance $C = \dfrac{\epsilon_0 A}{d}$

(b) Initial charge = Q

(c) Initial electric field = E

(d) Initial voltage = V

(e) Initial energy stored = U

Parameter after insertion of dielectric

(a) Final Capacitance = kC

(b) Final charge = Q

(c) Final electric field = E/k

(d) Final voltage = V/k

(e) Final energy stored = U/k

Illustration 97:

The parallel plates of a capacitor have an area 0.2 m^2 and are 10^{-2} m apart. The original potential difference between them is 3000 V, and it decreases to 1000 V when a sheet of dielectric is inserted between the plates filling the full space. Compute: ($\in_0 = 9 \times 10^{-12}$ S. I. units)

(i) Original capacitance C_0.
(ii) The charge Q on each plate.
(iii) Capacitance C after insertion of the dielectric.
(iv) Dielectric constant K.
(v) Permittivity $\in$ of the dielectric.
(vi) The original field E_0 between the plates.
(vii) The electric field E after insertion of the dielectric.

Solution:

(i) $C_0 = \dfrac{\in_0 A}{d} = \dfrac{0.2\in_0}{10^{-2}} = 20\in_0 = 20 \times 9 \times 10^{-12} = 180\ pF$

(ii) $Q = C_0V = 180 \times 10^{-12} \times 3000 = 5.4 \times 10^{-7}\ C$

(iii) $C_1 = \dfrac{Q}{V_1} = \dfrac{5.4\times10^{-7}}{1000} = 540\ pF$

(iv) $K = \dfrac{C_1}{C_0} = \dfrac{540}{180} = 3$

(v) $\in = \in_r\in_0 = K\in_0 = 3 \times 9 \times 10^{-12} = 27 \times 10^{-12}\ C^2N^{-1}m^{-2}$

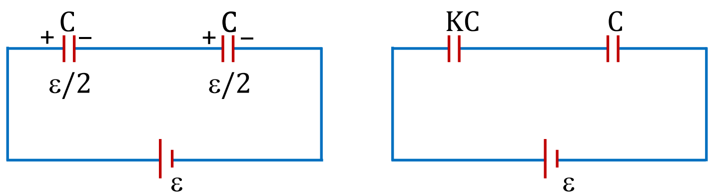
(vi) $E_0 = \dfrac{V}{d} = \dfrac{3000}{10^{-2}} = 3 \times 10^5\ V/m$

(vii) $E = \dfrac{V_1}{d} = \dfrac{1000}{10^{-2}} = 1 \times 10^5\ V/m$

Illustration 98:

Two parallel plate air capacitors each of capacitance C were connected in series to a battery with e.m.f. ε. Then one of the capacitors was filled up with uniform dielectric of relative permittivity k. How many times did the electric field strength in that capacitor decrease? What amount of charge flows through the battery?

Solution:



$V_1 = \dfrac{Q_1}{kC} = \dfrac{\varepsilon}{k+1}, E_1 = \dfrac{V_1}{d} = \dfrac{1}{k+1}\cdot\dfrac{\varepsilon}{d}$ and $E = \dfrac{Q}{dC} = \dfrac{\varepsilon}{2d}$

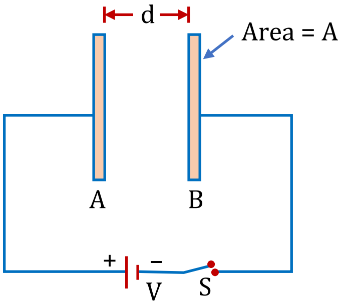
$E_1 = \dfrac{2}{(k+1)}E \Rightarrow \dfrac{1}{2}(1+k)$ times decrease

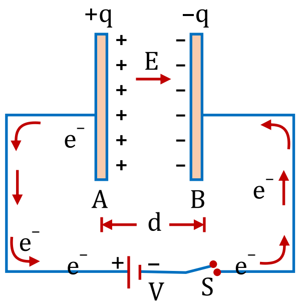
$\Delta q = Q_1 - Q = \dfrac{kC\varepsilon}{k+1} - \dfrac{C\varepsilon}{2} = \dfrac{C\varepsilon\,(2k-k-1)}{2(k+1)}$

$\Rightarrow \quad \Delta q = \dfrac{1}{2}\dfrac{C\varepsilon(k-1)}{(k+1)}$

Principle

Parallel plate capacitor with plates A and B and a battery of emf V. If we close the switch positive terminal of battery which is connected to the plate A will start pulling electrons from the plate A and will transfer these electrons to the plate B of the capacitor. On plate A positive charge will appear due to loss of electrons and as these electrons are transferred to plate B by battery a negative charge will appear on plate B.





These charges will reside only on the inner surface of plates A and B and outer surface will not carry any charge because overall charge of the system of this parallel plate capacitor is zero.

Electric Field and Steady State

Charges gets accumulated on the plates and hence gradually electric field is increasing between the plates. As electric field is uniform between the plates and plate separation is d hence potential difference between the plates $(V_A - V_B) = Ed$.

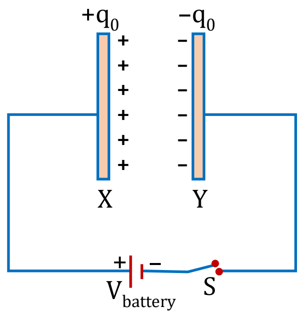
This potential difference is also increasing and as soon as this becomes equal to potential difference across the battery no further charge will flow and current will become zero in the circuit. In this situation we can say capacitor is charged having charge of magnitude $q = CV$ and as no further accumulation takes place we can say that capacitor is in steady state.

Conclusion

Whenever capacitor is connected across battery of constant potential difference then in steady state charges on capacitor will be $q = CV$ where V is the potential difference applied (may be battery of constant emf) and C is the capacitance of capacitor.

Charging of Capacitor when Capacitor has initially some charge

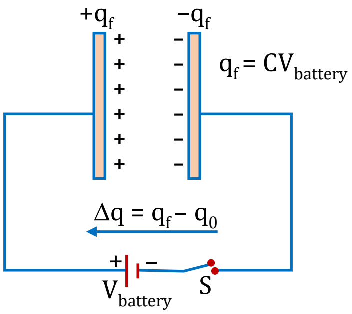
If capacitor having initial charge on its plates as q_0. In this state of initial charge on capacitor if switch is closed then battery will modify the charges on capacitor plates which will depend upon the initial charge on it.



Due to initial charge on capacitor q_0 the initial potential difference of the capacitor is $V_0 = q_0/C$.

Case 1: If $V_0 < V_{\text{battery}}$

Then on closing the switch battery will supply charges to capacitor and final charge on capacitor will increase until the potential difference across capacitor becomes equal to that of battery.

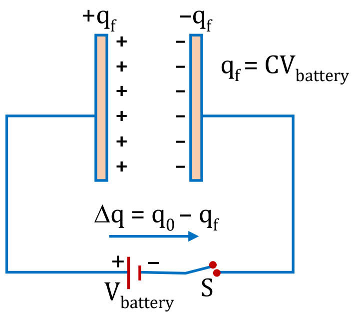


If $\quad \dfrac{q_0}{C} < V_{\text{battery}} \quad \Rightarrow q_f > q_0$

Thus, the final charge on capacitor plates will be given as $q_f = CV_f$.

Case 2: If $V_0 > V_{\text{battery}}$

Then on closing the switch charge will flow into the battery from the capacitor and final charge on capacitor will decrease until the potential difference across capacitor decreases and becomes equal to that of the battery.



If $\quad \dfrac{q_0}{C} > V_{\text{battery}} \quad \Rightarrow q_f < q_0$

The final charge on capacitor in this state will also be given by $q_f = CV_f$

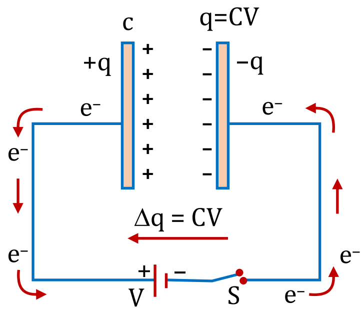
On connecting a capacitor across a battery charges flows from one plate of capacitor to another plate due to battery and as wires are conducting with very low or negligible resistance, charges flow very fast and quickly.

Capacitor plates attain the steady state charges and finally current in circuit becomes zero.

Heat Loss in Charging

Principle:

In the process free electrons in the conducting wires gain kinetic energy while transferring and finally all charges come to rest thus the kinetic energy attained by these free electrons is finally dissipated in form of heat radiations to the surrounding.



(a) Work by battery (W):

When the switch was open the initial charge on capacitor was zero and there was no energy stored in capacitor. When switch is closed battery supplies a charge on capacitor plates and in the process amount of charge flowing through battery is $q = CV$ thus the work done by battery is given as $W = qV = (CV)V = CV^2$.

(b) Energy Stored in Capacitor (ΔU):

In steady state charge on capacitor plates becomes $q = CV$. Thus, in steady state the potential difference across capacitor plates becomes V and energy stored in it is given by equation $U = \dfrac{1}{2}CV^2$.

(c) Heat Produced (H):

In the process of charging the capacitor, work done by battery is CV^2 and out of this half is stored in capacitor as electric field remaining energy which the battery supplied as work in charging is dissipated as heat which is given as

$H = W - \Delta U$

$H = CV^2 - CV^2/2 = CV^2/2$

Illustration 99:

A capacitor having a capacitance of 200 μF is charged to a potential difference of 20V. The charging battery is disconnected and the capacitor is connected to another battery of emf 10V with the positive plate of the capacitor joined with the positive terminal of the battery.

(a) Find the charges on the capacitor before and after the reconnection in steady state.

(b) Find the net charge flown through the 10 V battery

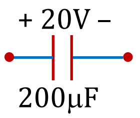
(c) Is work done by the battery or is it done on the battery? Find its magnitude.

(d) Find the decrease in electrostatic field energy.

(e) Find the heat developed during the flow of charge after reconnection.

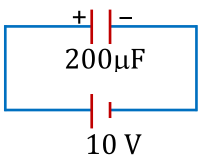
Solution:

(a) Charge on capacitor before connection



$Q_1 = CV_1 = 4000\ \mu C$

Charge on capacitor after connection



$Q_2 = CV_2 = 200 \times 10 = 2000\ \mu C$

(b) Charge flown through the 10V battery = 4000 – 2000 = 2000 μC

(c) Work is done on the battery.

(d) The decrease in electrostatic field energy.

$$= U_i - U_f = \frac{1}{2}CV_1^2 - \frac{1}{2}CV_2^2 = \frac{1}{2}\times 200 \times (20)^2 - \frac{1}{2}\times 200\ (10)^2$$

$= 30\ mJ$

(e) $W = \Delta U + \Delta H$

$-20\ mJ = -30\ mJ + \Delta H$

$[\Delta H = 10\ mJ]$

Illustration 100:

Consider the situation shown in the figure. The switch S is open for a long time and then closed and again steady state reached then

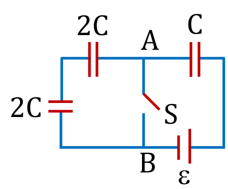
(a) Find the charge flown through the battery after the switch S is closed.

(b) Find the charge flown through the switch S from B to A.

(c) Find the work done by the battery after the switch S is closed.

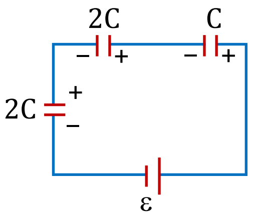
(d) Find the change in energy stored in the system of capacitors.

(e) Find the heat developed in the system after the switch S is closed.



Solution:

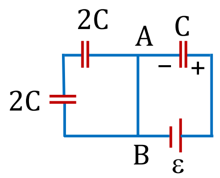
Situation when S is open



$C_{eq.} = C/2$

Charge supplied $Q = C_{eq.}\varepsilon = \dfrac{C\varepsilon}{2}$

Situation when S is closed



$C_{eq.} = C$

Charge on capacitor $C = C\varepsilon$

After S is closed, voltage across $2C$ capacitor become zero, so charge on it also become zero.

(a) So, charge flown through the battery when the switch S is closed $= C\varepsilon - \dfrac{C\varepsilon}{2} = \dfrac{C\varepsilon}{2}$

(b) The charge flown through the switch S is $\dfrac{C\varepsilon}{2} + \dfrac{C\varepsilon}{2} = C\varepsilon$ (A to B) $= -C\varepsilon$ (B to A)

(c) Work done by the battery $= Q_{\text{supplied}}\, V = \dfrac{C\varepsilon}{2}\,\varepsilon = \dfrac{C\varepsilon^2}{2}$

(d) Change in energy stored in the capacitors = energy stored in 2nd case – energy stored in 1st case

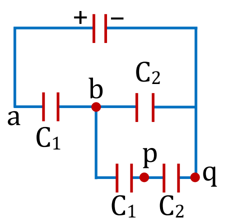
$$= \frac{1}{2}C_{eq2}V^2 - \frac{1}{2}C_{eq1}V^2 = \frac{1}{2}\cdot\left(C - \frac{C}{2}\right)\varepsilon^2 = \frac{C\varepsilon^2}{4}$$

(e) Heat developed in the system = Work done by battery – change in energy in the capacitors

$$\frac{C\varepsilon^2}{2} - \frac{C\varepsilon^2}{4} = \frac{C\varepsilon^2}{4}$$

Illustration 101:

In the given network if potential difference between p and q is $2V$ and $C_2 = 3C_1$. Then find the potential difference between a and b.



Solution:

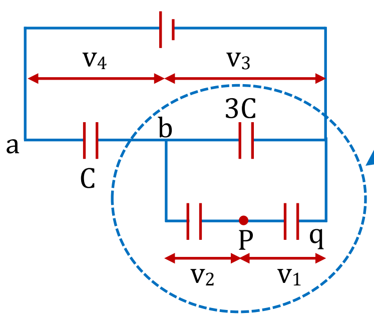
$V_1 : V_2 = \dfrac{1}{3C_1} : \dfrac{1}{C_1}$

$V_1 : V_2 = 1 : 3$

$V_1 = \dfrac{1}{4}V_3 \Rightarrow V_3 = 8V$

$V_3 : V_4 = \dfrac{4}{15C_1} : \dfrac{1}{C_1} \Rightarrow \dfrac{8}{V_4} = \dfrac{4}{15}$

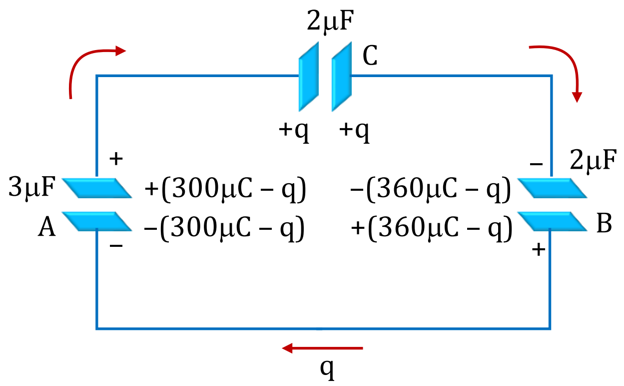
$V_4 = 30V$



$C_{eq} = \dfrac{15C_1}{4}$

KVL or Charge Flow Method in Capacitor circuit with R = 0

In this method we distribute charge *s* at various capacitors of circuit and then we write potential equation for different loops of circuit.



Analysis in circuit 1

This method is explained by using an illustration in which there are two capacitors of capacitances $4\mu F$ and $8\mu F$ initially charged at $20V$ and $5V$ respectively and connected in a single loop with two batteries of $10V$ and $20V$ along with a switch S.

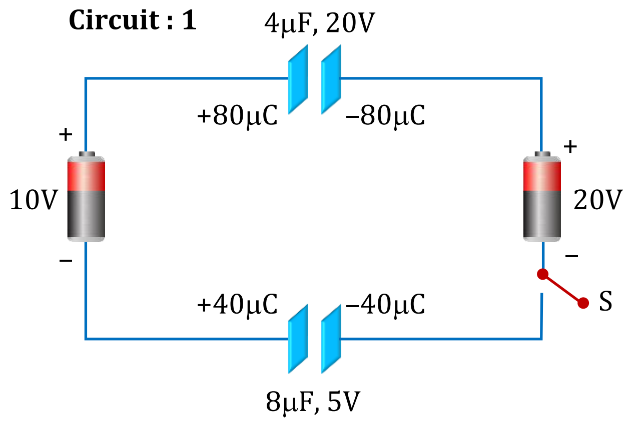
Here, we will try to find the steady state charges of the two capacitors when switch is closed.

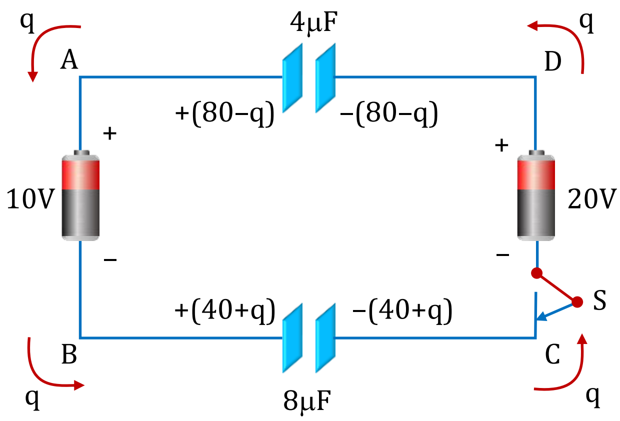
On closing the switch redistribution of charges will take place and a charge q flows through the circuit in anticlockwise direction until new steady state is achieved and the final charges on capacitor plates will be as shown in figure.

We can write equation of potential for the circuit loop starting from any one point in the loop move either clockwise or anticlockwise and come back to the same point as described.

$$V_A - 10 - \frac{40+q}{8} + 20 + \frac{80-q}{4} = V_A$$

Solving we get $q = \dfrac{200}{3}\ \mu C$

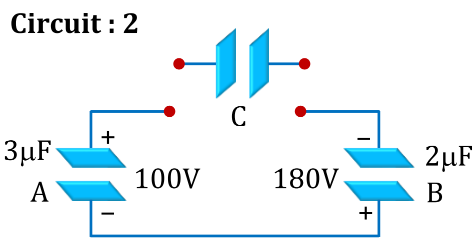




Analysis in circuit 2

Applying *KVL* in another example.

Two capacitors A and B with capacitances $3\mu F$ and $2\mu F$ are charged to potential difference of $100V$ and $180V$ respectively. The plates of the capacitors are connected as shown with one wire from each capacitor free.



The upper plate of A is positive and that of B is negative. An uncharged $2\mu F$ capacitor C with lead wires falls on the free ends to complete the circuit. We will try to calculate the final charge on the three capacitors and the amount of electrostatic energy stored in the system before and after the capacitor C falls and the circuit is closed.

Let q be the charge flowing in clockwise sense during redistribution of charge as shown. We can write the potential equation for this loop as

$$-\frac{q}{2}+\frac{360-q}{2}+\frac{300-q}{3}=0$$

$q = 210\ \mu C$

Final charges on the capacitors are

$q_A = 300 - 210 = 90\ \mu C$

$q_B = 360 - 210 = 150\ \mu C$

$q_C = 210 = 210\ \mu C$

Initial electrostatic energy stored is given as

$$U_i=\frac{1}{2}\times(3\times10^{-6})(100)^2+\frac{1}{2}\times(2\times10^{-6})(180)^2$$

$$U_i=4.74\times10^{-2}J=47.4\times10^{-3}J=47.4mJ$$

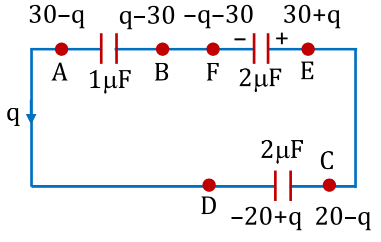
Final electrostatics energy stored is given as

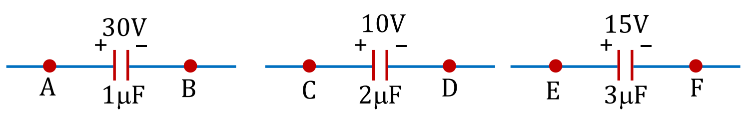
$$U_f=\frac{(90\times10^{-6})^2}{2\times(3\times10^{-6})}+\frac{(150\times10^{-6})^2}{2\times(2\times10^{-6})}+\frac{(210\times10^{-6})^2}{2\times(2\times10^{-6})}$$

$$U_f=1.8\times10^{-2}J=18mJ$$

Illustration 102:

Three capacitors as shown of capacitance $1\mu F$, $2\mu F$ and $2\mu F$ are charged upto potential difference 30 V, 10 V and 15 V respectively. If terminal A is connected with D, C is connected with E and F is connected with B. Then find out charge flow in the circuit and find the final charges on capacitors.





Solution:

Let charge flow is q.

Now applying Kirchhoff's voltage law

$$-\frac{(q-20)}{2}-\frac{(30+q)}{2}+\frac{30-q}{1}=0$$

$-2q = -25$

$q = 12.5\ \mu C$

Final charges on plates.

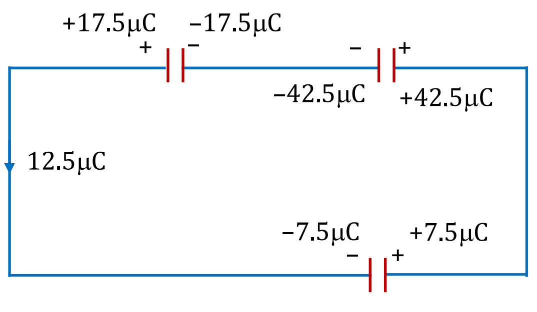
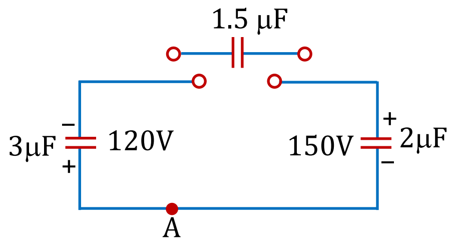
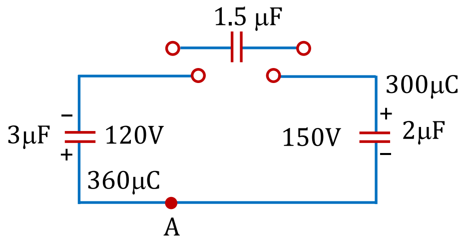


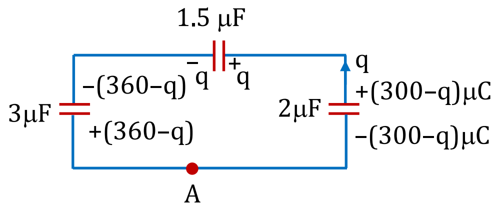
Illustration 103:



Two capacitors of 2 μF and 3 μF are charged to 150 volt and 120 volts respectively. The plates of a capacitor are connected as shown in the fig. A discharged capacitor of capacity 1.5 μF falls to the free ends of the wire and connected through the free ends of the wire, then find the charge on each capacitor at steady state.

Solution:





$$V_A + \frac{(300-q)}{2} - \frac{q}{1.5} + \frac{360-q}{3} = V_A \qquad ...(i)$$

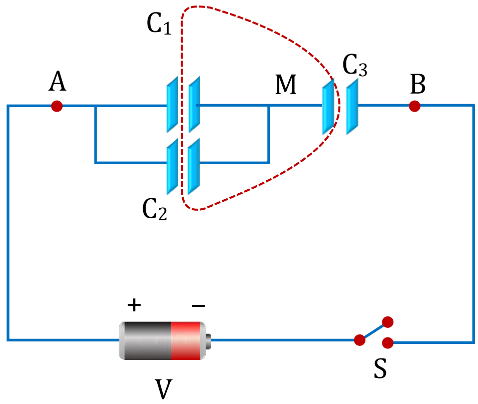
By solving this equation, we get $\Rightarrow q = 180\mu C$

Charge on 1.5μF capacitor is = 180μC

Charge on 2μF capacitor is = 300 – 180 = 120μC

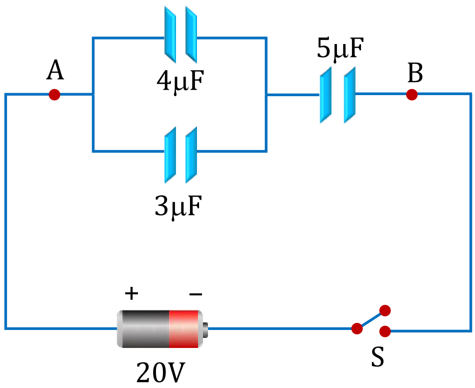
Charge on 3μF capacitor is = 360 – 180 = 180μC

Nodal Analysis



There are some cases in which capacitors are connected in such combinations which are neither series nor parallel called mixed combinations. Such cases can be easily handled by a specific method of solving capacitive circuits called 'Nodal Analysis'.

'Node' in capacitive circuit is an isolated part of circuit at which two or more plates of capacitors are connected. Shows a capacitive circuit in which C_1 and C_2 are connected in parallel and this group is connected in series with C_3 and connected across a battery of voltage V. In this circuit also, a node is shown at junction M by closed dotted line.

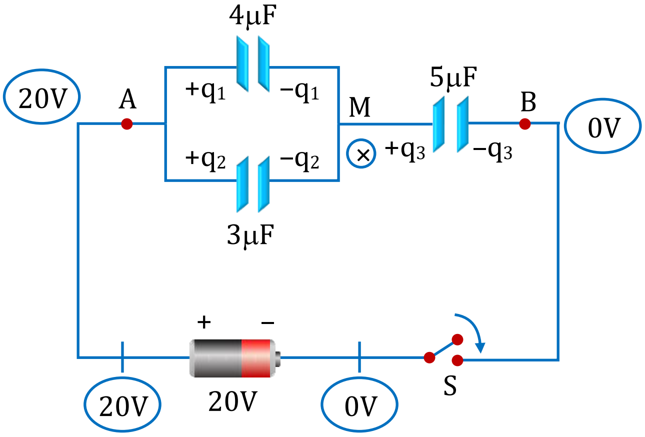


We will discuss the understanding and application of nodal analysis using an illustration shown. In this circuit we will calculate the equivalent capacitance of the circuit across terminals AB and steady state charge on each capacitor after closing the switch. There are three steps to be followed for solving this circuit using nodal analysis.

Step-I : Identifying the Nodes of Circuit

Step-II : Distribution of Potential in Circuit

Distribute potential at every point in the circuit. Starting with the battery terminals and first considering the potential at negative terminal of the battery as zero. At the node junction M as it is neither connected to left or right side of battery directly, we can consider its potential to be x. Starting with zero potential reference from negative terminal of battery and distributing potentials at all other points of circuit. For nodes where potential is unknown, we give variable potentials x, y, z etc. if more than one node are there.



Step-III : Writing Nodal Equations

On a node charges can only be induced so charges on these plates must be conserved. If capacitors are initially uncharged, net charge on a node must remain zero and if these are initially charged then sum of charges must remain same.

$$q_3 - q_1 - q_2 = 0$$

$$q_1 = 4 \times (20 - x)\mu C \qquad \text{...(1)}$$

$$q_2 = 3 \times (20 - x)\mu C \qquad \text{...(2)}$$

and $\quad q_3 = 5 \times x\mu C \qquad \text{...(3)}$

Substituting the values of charges in equation (1), we get

$$5x - 4(20 - x) - 3(20 - x) = 0 \qquad \text{...(4)}$$

$\Rightarrow \quad 12x = 140$

$\Rightarrow \quad x = \dfrac{140}{12} = \dfrac{35}{3}\,V \qquad \text{...(5)}$

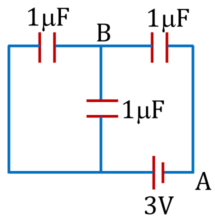
$$\Delta q_0 = |q_1 + q_2| = |q_3| = \frac{175}{3}\,\mu C$$

Thus, equivalent capacitance in this case can be given as

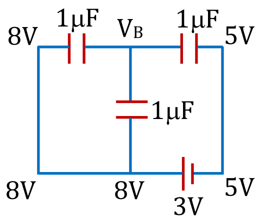
$$C_{eq} = \frac{\Delta q_0}{V_{\text{battery}}} = \frac{\left(\frac{175}{3}\right)}{20} = \frac{35}{12}\,\mu F \qquad \text{...(6)}$$

Illustration 104:

If potential of A is $5V$, then potential of B in volt is



Solution:



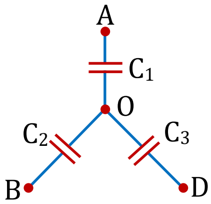
$$1(8 - V_B) + 1(8 - V_B) + 1(5 - V_B) = 0$$

$$21 = 3V_B$$

$$V_B = 7V$$

Illustration 105:

Three uncharged capacitors of capacitance $C_1 = 1\mu F$, $C_2 = 2\mu F$ and $C_3 = 3\mu F$ are connected as shown in the figure. The potential of point A, B and D are 10 volt, 25 volt and 20 volt respectively. Determine the potential at point O.



Solution:

$$(V_A - V_o)C_1 + (V_B - V_o)C_2 + (V_D - V_o)C_3 = 0$$

$$V_o = \frac{V_A C_1 + V_B C_2 + V_D C_3}{C_1 + C_2 + C_3}$$

$$= \frac{10\times1+25\times2+20\times3}{1+2+3} = 20\ V$$

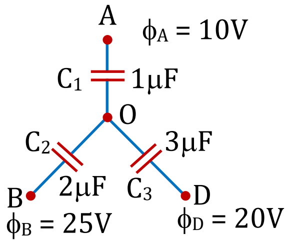
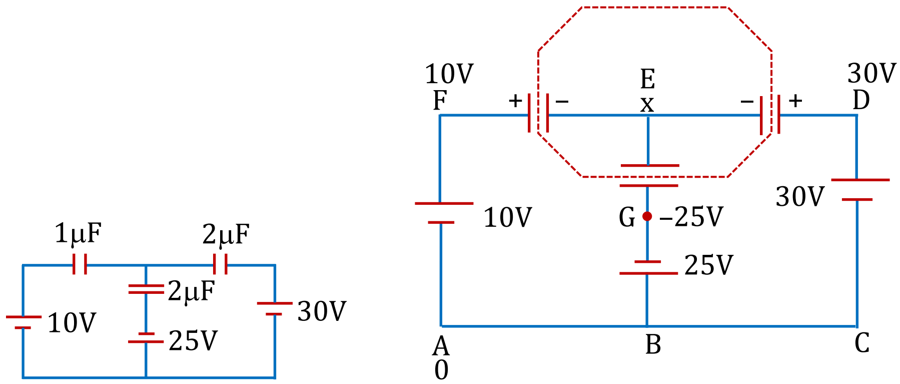


Illustration 106:

In the given circuit find out the charge on each capacitor. (Initially they are uncharged)



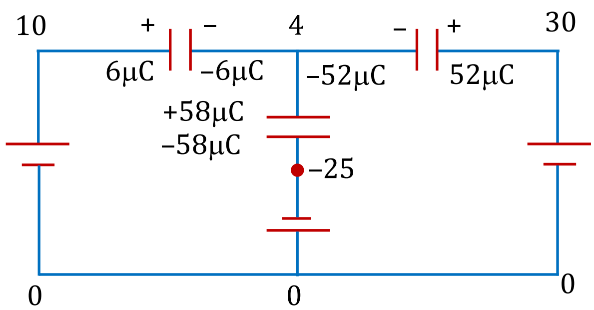
Solution:

Let potential at A is 0, so at D it is 30 V, at F it is 10 V and at point G potential is –25V and let potential at E is x. Now apply Kirchhoff's I[st] law at point E. (Total charge of all the plates connected to 'E' must be same as before i.e. 0)

$\therefore$ $(x - 10) + (x - 30)2 + (x + 25)2 = 0$

$5x = 20$

$x = 4\ V$



Final charges:

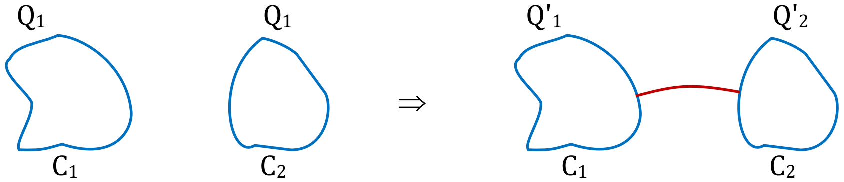
$Q_{2\mu F} = (30 - 4)2 = 52\ \mu C$

$Q_{1\mu F} = (10 - 4) = 6\mu C$

$Q_{2\mu F} = (4 - (-25))2 = 58\ \mu C$

Sharing of Charges on Joining two Conductors (By a conducting wire)

Initially **Finally**



Principle:

Whenever there is potential difference, there will be movement of charge. If only electric force act on charge then they have tendency to move from **high potential energy** to **low potential energy**. Therefore, positive charge moves from **high potential** to **low potential** and negative charge moves from **low potential** to **high potential**. This movement of charge will continue till the potential difference between the conductors vanishes.

Formulae related with redistribution of charges:

Before connecting the conductors		
Parameter	**I[st] conductor**	**II[nd] conductor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the conductors		
Parameter	**I[st] conductor**	**II[nd] conductor**
Capacitance	C_1	C_2
Charge	Q_1'	Q_2'
Potential	V	V

Calculation:

$$V = \frac{Q_1'}{C_1} = \frac{Q_2'}{C_2} \Rightarrow \frac{Q_1'}{Q_2'} = \frac{C_1}{C_2}$$

But $Q_1 + Q_2 = Q'_1 + Q'_2$

$\therefore$ $$V = \frac{Q_1 + Q_2}{C_1 + C_2} = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$\therefore$ $$Q_1' = \frac{C_1}{C_1 + C_2}(Q_1 + Q_2)$$

and $$Q_2' = \frac{C_2}{C_1+C_2}(Q_1 + Q_2)$$

Heat loss during redistribution:

$$\Delta H = \frac{1}{2}\frac{C_1C_2}{C_1+C_2}(V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.

Note:

Always put Q_1, Q_2, V_1 and V_2 with sign.

Illustration 107:

Two conductors having capacities $2\mu F$ and $5\mu F$ and potentials 2 volt and 10 volt respectively. Calculate the ratio of their charges after connecting by a wire.

Solution:

$$\frac{Q'_1}{Q'_2}=\frac{C_1V_{com}}{C_2V_{com}}=\frac{2\times10^{-6}}{5\times10^{-6}}=\frac{2}{5}$$

Illustration 108:

Two charged metal spheres of radii R and $2R$ are temporarily placed in contact and then separated. At the surface of each, which sphere has the greater value for the following:

(a) Charge (b) Charge density (c) Potential (d) Electric field

Solution:

(a) In sharing, charge divides in proportion to capacity so,

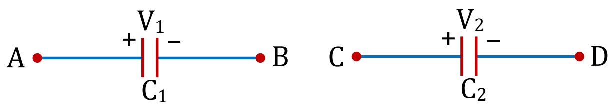
$$\frac{q_1}{q_2}=\frac{C_1}{C_2}=\frac{R_1}{R_2}=\frac{R}{2R}=\frac{1}{2}$$

(b) Charge density, $\sigma=\dfrac{q}{4\pi R^2}$ So, $\dfrac{\sigma_1}{\sigma_2}=\dfrac{q_1}{q_2}\times\left[\dfrac{R_2}{R_1}\right]^2=\dfrac{1}{2}\times\left[\dfrac{2R}{R}\right]^2=2$

(c) In case of spherical conductor, $V_S=\dfrac{q}{4\pi\varepsilon_0R}$ So, $\dfrac{V_1}{V_2}=\dfrac{q_1}{q_2}\times\dfrac{R_2}{R_1}=\dfrac{1}{2}\times\dfrac{2R}{R}=1$

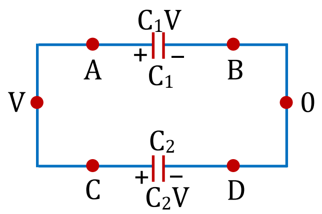
(d) In case of spherical conductor, $E_S=\dfrac{q}{4\pi\varepsilon_0R^2}$ So, $\dfrac{E_1}{E_2}=\dfrac{q_1}{q_2}\times\left[\dfrac{R_2}{R_1}\right]^2=\dfrac{1}{2}\times\left[\dfrac{2R}{R}\right]^2=2$

Sharing of Charge between two Charged Capacitor:

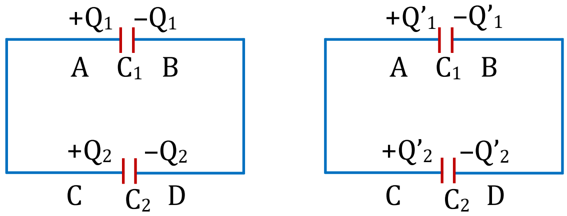


Initially let's say capacitors C_1 is charged by applying potential difference V_1 and similarly capacitor C_2 by applying potential difference V_2.

When two capacitors are C_1 and C_2 are connected as shown in figure. Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.



By charge conservation of plates A and C before and after connection



Initially Finally

Before connecting the conductors		
Parameter	**I[st] conductor**	**II[nd] conductor**
Capacitance	C_1	C_2
Charge	Q_1	Q_2
Potential	V_1	V_2

After connecting the conductors		
Parameter	**I[st] conductor**	**II[nd] conductor**
Capacitance	C_1	C_2
Charge	Q_1'	Q_2'
Potential	V	V

$Q_1 + Q_2 = C_1V + C_2V$ or $C_1V + C_2V = C_1V_1 + C_2V_2$

Which gives, $V = \frac{C_1V_1+C_2V_2}{C_1+C_2}$

This could be understood as $V = \frac{Q_1+Q_2}{C_1+C_2} = \frac{C_1V_1+C_2V_2}{C_1+C_2} = \frac{\text{Total charge}}{\text{Total capacitance}}$

(a) Q_1'(Final charge on capacitor C_1) $= C_1V = \frac{C_1}{C_1+C_2}(Q_1 + Q_2)$

$\Rightarrow$ Q_2'(Final charge on capacitor C_2) $= C_2V = \frac{C_2}{C_1+C_2}(Q_1 + Q_2)$

(b) Heat loss during redistribution:

$$\Rightarrow \Delta H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

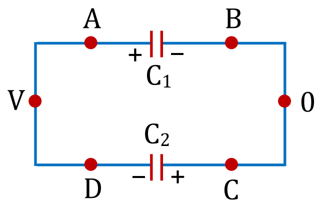
$$= \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}\frac{(C_1V_1 + C_2V_2)^2}{(C_1 + C_2)}$$

$$= \frac{1}{2}\left[\frac{C_1^2V_1^2 + C_1C_2V_1^2 + C_2C_1V_2^2 + C_2^2V_2^2 - C_1^2V_1^2 - C_2^2V_2^2 - 2C_1C_2V_1V_2}{C_1 + C_2}\right]$$

$$= \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

$$\Delta H = U_i - U_f = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 - V_2)^2$$

The loss of energy is in the form of Joule heating in the wire.



When oppositely charge terminals are connected then

$\therefore$ $C_1V + C_2V = C_1V_1 - C_2V_2$

$V = \frac{C_1V_1 - C_2V_2}{C_1 + C_2}$ and $H = \frac{1}{2}\frac{C_1C_2}{C_1 + C_2}(V_1 + V_2)^2$

These two following formula can be remembered for both the cases using proper sign convention.

$$V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}$$

$$H = \frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2 - \frac{1}{2}(C_1 + C_2)V^2$$

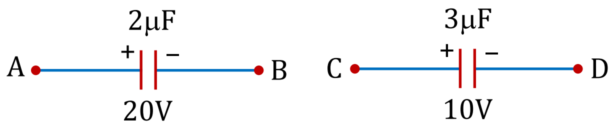
Note:

(i) When plates of similar charges are connected with each other (+ with + and – with –) then put all values (Q_1, Q_2, V_1, V_2) with positive sign.

(ii) When plates of opposite polarity are connected with each other (+ with –) then take charge and potential of one of the plate to be negative.

Illustration 109:

Find out the following if A is connected with C and B is connected with D.

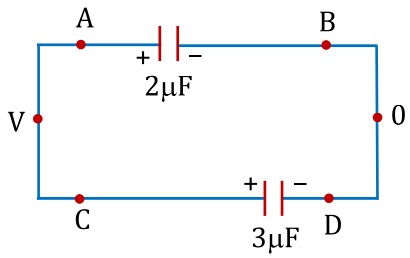


(i) How much charge flows in the circuit.

(ii) How much heat is produced in the circuit.

Solution:

(i) Let potential of B and D is zero and common potential on capacitors is V, then at A and C it will be V.



By charge conservation,

$3V + 2V = 40 + 30$

$5V = 70$

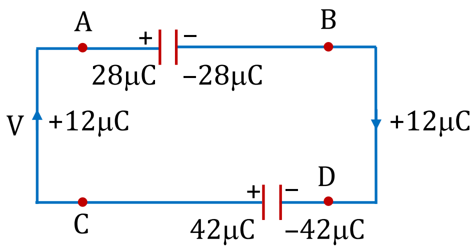
$V = 14$ volt

Charge flow

$= 40 - 28$

$= 12\ \mu C$

Now final charges on each plate is shown in the figure.



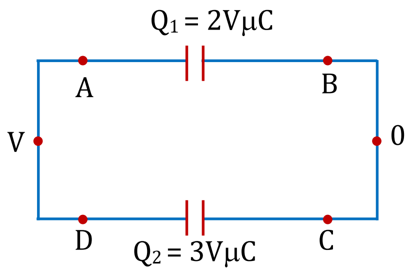
(ii) Heat produced $= \frac{1}{2} \times 2 \times (20)^2 + \frac{1}{2} \times 3 \times (10)^2 - \frac{1}{2} \times 5 \times (14)^2$

$= 400 + 150 - 490$

$= 550 - 490 = 60\ \mu J$

Illustration 110:

Repeat above question if A is connected with D and B is connected with C.



Solution:

Let potential of B and C is zero and common potential on capacitors is V, then at A and D it will be V

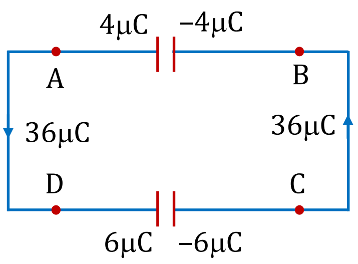
$$(2V + 3V)\ \mu C = 40 - 30 = 10\ (\mu C)$$

$\Rightarrow$ $V = 2\ volt$

$Q_1 = 2V = 4\mu C$

$Q_2 = 3V = 6\mu C$

Now charge on each plate is shown in the figure



$$\text{Heat produced} = 400 + 150 - \frac{1}{2} \times 5 \times 4$$

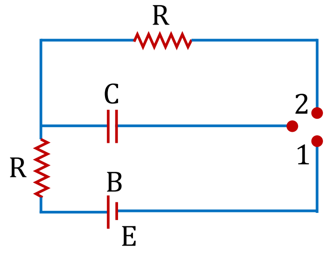
$$= 550 - 10$$

$$= 540\ \mu J$$

Charging of A Capacitor

Charging of a condenser:

(i) In the following circuit. If key 1 is closed then the condenser gets charged. Finite time is taken in the charging process. The quantity of charge at any instant of time t is given by $q = q_0[1 - e^{-(t/RC)}]$



Where q_0 = maximum final value of charge at $t = \infty$.

According to these equations the quantity of charge on the condenser increases exponentially with increase of time.

(ii) If $t = RC = \tau$ then

$$q = q_0\ [1 - e^{-(RC/RC)}] = q_0 \left[1 - \frac{1}{e}\right]$$

or $q = q_0\ (1 - 0.37) = 0.63\ q_0 = 63\%$ of q_0

(iii) Time $t = RC$ is known as time constant.

i.e. the time constant is that time during which the charge rises on the condenser plates to 63% of its maximum value.

(iv) The potential difference across the condenser plates at any instant of time is given by

$$V = V_0[1 - e^{-(t/RC)}]\ volt$$

(v) The potential curve is also similar to that of charge. During charging process an electric current flows in the circuit for a small interval of time which is known as the transient current. The value of this current at any instant of time is given by

$$I = I_0[e^{-(t/RC)}]\ \text{ampere}$$

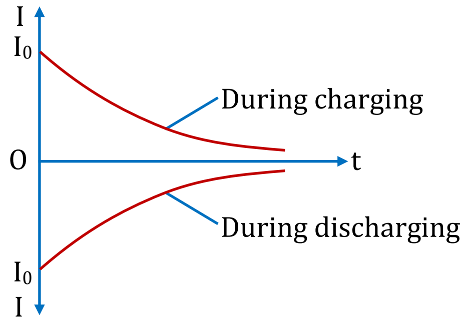
According to this equation the current falls in the circuit exponentially (Fig.).

(vi) If $t = RC = \tau$ = Time constant

$$I = I_0 e^{(-RC/RC)} = \frac{I_0}{e} = 0.37\, I_0$$

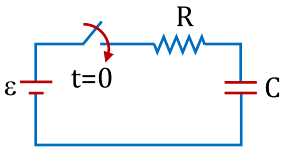
= 37% of I_0

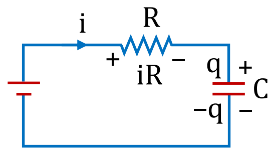
i.e. time constant is that time during which current in the circuit falls to 37% of its maximum value.

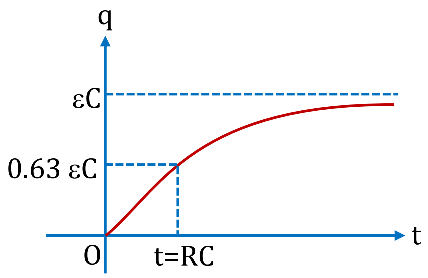


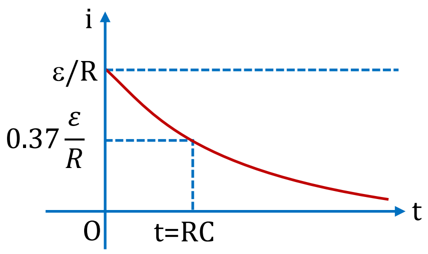
Derivation of formulae for charging of capacitor

It is given that initially capacitor is uncharged. Let at any time charge on capacitor is q. Applying Kirchoff voltage law









$$\varepsilon - iR - \frac{q}{C} = 0 \Rightarrow iR = \frac{\varepsilon C - q}{C}$$

$$i = \frac{\varepsilon C - q}{CR} \Rightarrow \frac{dq}{dt} = \frac{\varepsilon C - q}{CR}$$

$$\frac{CR}{\varepsilon C - q}\,.\,dq = dt$$

$$\int_0^q \frac{dq}{\varepsilon C - q} = \int_0^t \frac{dt}{RC} \Rightarrow -\ln(\varepsilon C - q) + \ln \varepsilon C = \frac{t}{RC}$$

$$\ln \frac{\varepsilon C}{\varepsilon C - q} = \frac{t}{RC}\ ;\ \varepsilon C - q = \varepsilon C\,.\,e^{-t/RC}$$

$$q = \varepsilon C(1 - e^{-t/RC})$$

RC = time constant of the RC series circuit.

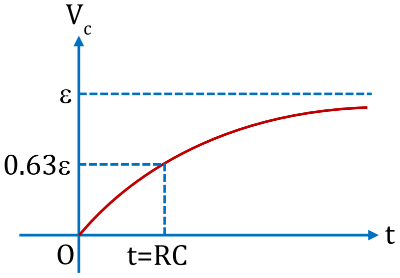
After one time constant

$$q = \varepsilon C\left(1 - \frac{1}{e}\right) = \varepsilon C\,(1 - 0.37) = 0.63\,\varepsilon C$$

Current at any time t

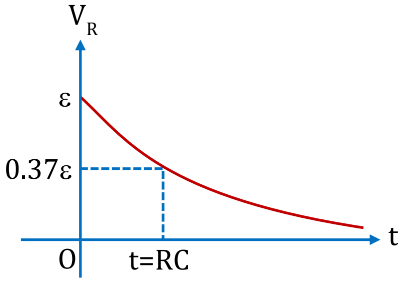
$$i = \frac{dq}{dt} = \varepsilon C\left(-e^{-t/RC}\left(-\frac{1}{RC}\right)\right) = \frac{\varepsilon}{R} e^{-t/RC}$$

Voltage across capacitor after one-time constant $V = 0.63\ \varepsilon$



$Q = CV$; $V_C = \varepsilon(1 - e^{-t/RC})$

Voltage across the resistor



$V_R = iR = \varepsilon e^{-t/RC}$

By energy conservation,

Heat dissipated = work done by battery – $\Delta U_{\text{capacitor}}$

$$= C\varepsilon(\varepsilon) - \left(\frac{1}{2}C\varepsilon^2 - 0\right) = \frac{1}{2}C\varepsilon^2$$

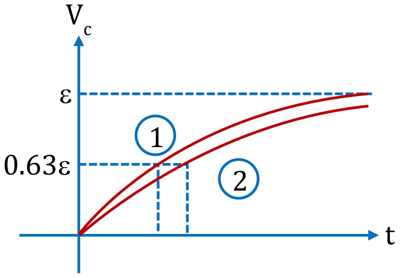
Alternatively:

$$Heat = H = \int_0^\infty i^2 R dt$$

$$= \int_0^\infty \frac{\varepsilon^2}{R^2} e^{-\frac{2t}{RC}} R\, dt = \frac{\varepsilon^2}{R}\int_0^\infty e^{-2t/RC}\, dt = \frac{\varepsilon^2}{R}\left[\frac{e^{-\frac{2t}{RC}}}{-2/RC}\right]_0^\infty$$

$$= -\frac{\varepsilon^2 RC}{2R}\left[e^{-\frac{2t}{RC}}\right]_0^\infty = \frac{\varepsilon^2 C}{2}$$

Note:



In the figure time constant of (2) is more than (1).

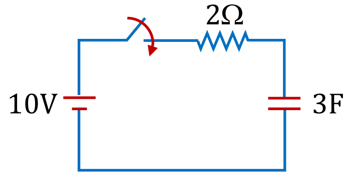
Charging of RC circuit

Illustration 111:

Find out current in the circuit and charge on capacitor which is initially uncharged in the following situations.

(a) Just after the switch is closed.

(b) After a long time when switch was closed.

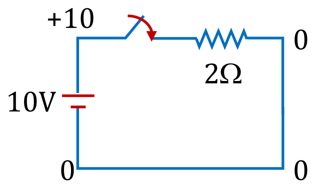


Solution:

(a) For just after closing the switch:

potential difference across capacitor = 0

$\therefore \quad Q_C = 0 \qquad \therefore \quad i = 5A$



(b) After a long time

at steady state current, $i = 0$

and potential difference across capacitor = 10 V

$\therefore \quad Q_C = 3 \times 10 = 30\ C$

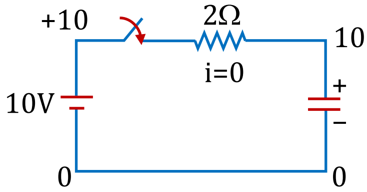
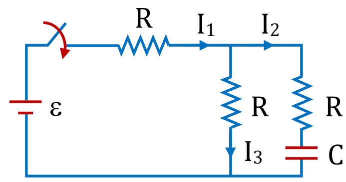


Illustration 112:

Find out current I_1, I_2, I_3, charge on capacitor and $\frac{dQ}{dt}$ of capacitor in the circuit which is initially uncharged in the following situations.

(a) Just after the switch is closed

(b) After a long time when switch is closed.



Solution:

(a) Initially the capacitor is uncharged so its behaviour is like a conductor

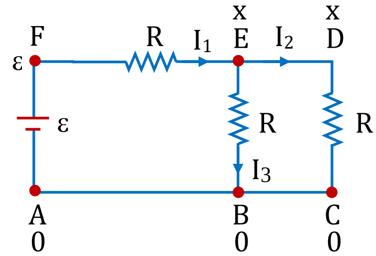
Let potential at A is zero so at B and C also zero and at F it is ε. Let potential at E is x so at D also x.

Apply Kirchhoff's Ist law at point E :

$$\frac{x-\varepsilon}{R} + \frac{x-0}{R} + \frac{x-0}{R} = 0 \Rightarrow \frac{3x}{R} = \frac{\varepsilon}{R}$$

$$x = \frac{\varepsilon}{3};\ Q_c = 0$$

$$\therefore \quad I_1 = \frac{-\varepsilon/3+\varepsilon}{R} = \frac{2\varepsilon}{3R},\ I_2 = \frac{dQ}{dt} = \frac{\varepsilon}{3R} \text{ and } I_3 = \frac{\varepsilon}{3R}$$



Alternatively:

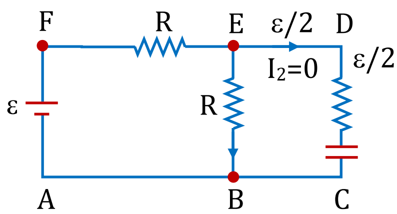
$$i_1 = \frac{\varepsilon}{R_{eq}} = \frac{\varepsilon}{R+\frac{R}{2}} = \frac{2\varepsilon}{3R},\ i_2 = i_3 = \frac{i_1}{2} = \frac{\varepsilon}{3R} \text{ and } \frac{dQ}{dt} = i_2 = \frac{\varepsilon}{3R}$$

(b) At $t = \infty$ (finally)

Capacitor completely charged so their will be no current through it.

$$I_2 = 0, \quad I_1 = I_3 = \frac{\varepsilon}{2R}$$

$$V_E - V_B = V_D - V_C = (\varepsilon/2R)R = \varepsilon/2 \Rightarrow Q_C = \frac{\varepsilon C}{2},\ \frac{dQ}{dt} = I_2 = 0$$



Time	I_1	I_2	I_3	Q	dQ/dt
$t = 0$	$\frac{2\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	$\frac{\varepsilon}{3R}$	0	$\frac{\varepsilon}{3R}$
Finally $t = \infty$	$\frac{\varepsilon}{2R}$	0	$\frac{\varepsilon}{2R}$	$\frac{\varepsilon C}{2}$	0

Illustration 113:

A capacitor is connected to a 36 V battery through a resistance of 20Ω. It is found that the potential difference across the capacitor rises to 12.0 V in 2μs. Find the capacitance of the capacitor.

Solution:

The charge on the capacitor during charging is given by $Q = Q_0(1 - e^{-t/RC})$.

Hence, the potential difference across the capacitor is $V = Q/C = Q_0/C\,(1 - e^{-t/RC})$.

Here, at $t = 2$ μs, the potential difference is $12V$ whereas the steady potential difference is

$Q_0/C = 36V$. So, $12V = 36V(1 - e^{-t/RC})$

or $\quad 1 - e^{-t/RC} = \frac{1}{3} \Rightarrow e^{-t/RC} = \frac{2}{3}$

or $\quad \frac{t}{RC} = \ell n\left(\frac{3}{2}\right) = 0.405 \Rightarrow RC = \frac{t}{0.405} = \frac{2\ \mu s}{0.405} \Rightarrow RC = 4.936\ \mu s$

or, $\quad C = \frac{4.936\mu s}{20\Omega} = 0.25\ \mu F.$

Illustration 114:

A capacitor of capacity $1\mu F$ is connected in a closed series circuit with a resistance of 10^7 ohms, an open key and a cell of 2 V with negligible internal resistance:

(i) When the key is switched on at time $t = 0$, find;

(a) The time constant for the circuit.

(b) The charge on the capacitor at steady state.

(c) Time taken to deposit charge equal to half of charge that will deposit at steady state.

(ii) If after completely charging the capacitor, the cell is shorted by zero resistance at time $t = 0$, find the charge on the capacitor at $t = 50$ s.

(Given: $e^{-5} = 6.73 \times 10^{-3}$, $ln2 = 0.693$)

Solution:

(i) (a) Time constant $\tau = RC = 10^7 \times 1 \times 10^{-6}$

$= 10$ $sec.$

(b) $Q_0 = CV = 1 \times 10^{-6} \times 2 = 2\ \mu C$

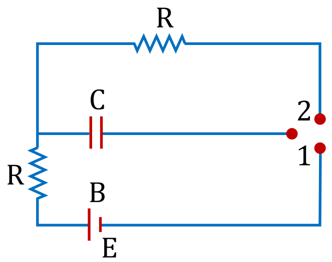
(c) $Q = \frac{Q_0}{2} = Q_0(1 - e^{-t/Rc})$

$\frac{t}{RC} = \ell n2 \quad \Rightarrow \quad t = 10\ \ell n2 = 6.93\ sec.$

(ii) $q = q_0 e^{-t/RC} = 2 \times 10^{-6}\, e^{-50/10} = 2 \times 10^{-6}\, e^{-5} = 1.348 \times 10^{-8}\ C.$

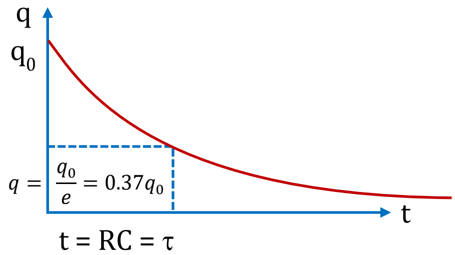
Discharging of a condenser

(i) In the circuit shown below, if key 1 is opened and key 2 is closed then the condenser gets discharged.



(Initial charge on capacitor = q_0)

(ii) The quantity of charge on the condenser at any instant of time t is given by $q = q_0\, e^{-(t/RC)}$



i.e. the charge falls exponentially.

Here q_0 = initial charge of capacitor

(iii) If $t = RC = \tau$ = time constant, then $q = \dfrac{q_0}{e} = 0.37q_0 = 37\%$ of q_0

i.e., the time constant is that time during which the charge on condenser plates in discharge process, falls to 37%.

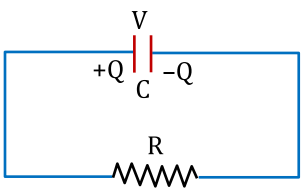
(iv) The dimensions of RC are those of time i.e. $[M^0L^0T^1]$ and the dimensions of $\dfrac{1}{RC}$ are those of frequency i.e. $[M^0L^0T^{-1}]$.

(v) The potential difference across the condenser plates at any instant of time t is given by $V = V_0e^{-(t/RC)}$ Volt.

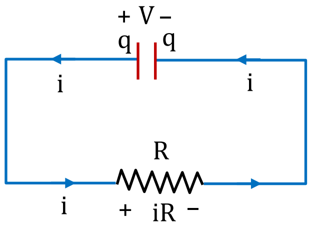
(vi) The transient current at any instant of time is given by $I = -I_0e^{-(t/RC)}$ ampere.

i.e. the current in the circuit decreases exponentially but its direction is opposite to that of charging current. (–ve only means that direction of current is opposite to that at charging current)

Derivation of equation of discharging circuit



Initially



At any time t

Applying K.V.L.

$$+\frac{q}{C} - iR = 0$$

$$i = \frac{q}{CR}, -\frac{dq}{dt} = \frac{q}{CR}$$

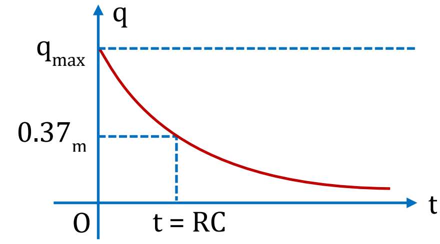
$$\int_Q^q \frac{-dq}{q} = \int_0^t \frac{dt}{CR}$$

$$-\ln\frac{q}{Q} = +\frac{t}{RC}$$

$$q = Q.\,e^{-t/RC}$$

$$i = -\frac{dq}{dt} = \frac{Q}{RC}\,e^{-t/RC} = i_0\,e^{-t/RC}$$

$$V = \frac{q}{C} = \frac{Qe^{-\frac{t}{RC}}}{C} = V_0 e^{-t/RC}$$



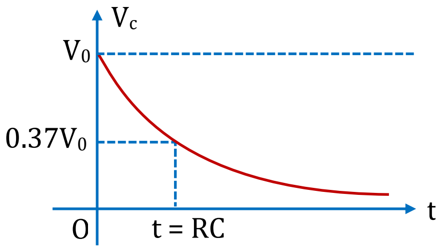


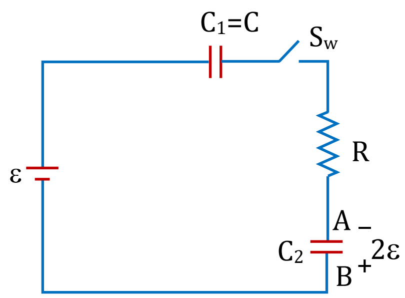
Illustration 115:

At t = 0, switch is closed, if initially C_1 is uncharged and C_2 is charged to a potential difference 2ε then find out following (Given $C_1 = C_2 = C$).

(a) Charge on C_1 and C_2 as a function of time.

(b) Find out current in the circuit as a function of time.

(c) Also plot the graphs for the relations derived in part (a).



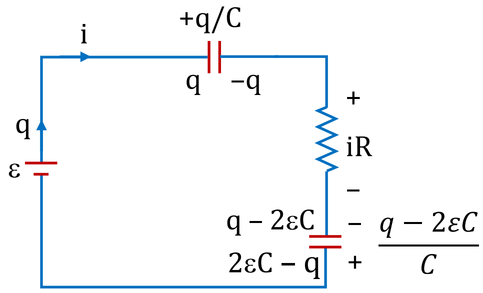
Solution:

Let q charge flow in time 't' from the battery as shown.

The charge on various plates of the capacitor is as shown in the figure.

Now applying KVL

$$\varepsilon - \frac{q}{C} - iR - \frac{q-2\varepsilon C}{C} = 0$$



$$\varepsilon - \frac{q}{C} - \frac{q}{C} + 2\varepsilon - iR = 0$$

$$3\varepsilon = \frac{2q}{C} + iR \Rightarrow 3\varepsilon - iR = \frac{2q}{C}$$

$$3\varepsilon C - iRC = 2q \quad \Rightarrow \quad \frac{dq}{dt}RC = 3\varepsilon C - 2q$$

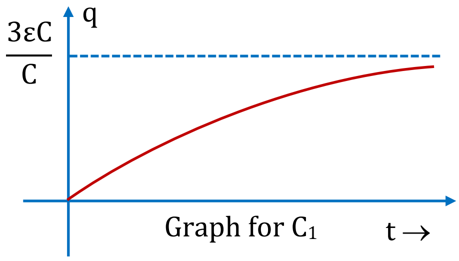
$$\int_0^q \frac{dq}{3\varepsilon C - 2q} = \int_0^t \frac{dt}{RC} \quad \Rightarrow \quad -\frac{1}{2}\ell n\left(\frac{3C\varepsilon - 2q}{3C\varepsilon}\right) = \frac{t}{RC}$$

$ln\left(\frac{3\varepsilon C-2q}{3\varepsilon C}\right) = -\frac{2t}{RC}$ $\Rightarrow$ $3\varepsilon C - 2q = 3\varepsilon Ce^{-2t/RC}$

$3\varepsilon C\,(1 - e^{-2t/RC}) = 2q$ $\Rightarrow$ $q = \frac{3}{2}\varepsilon C\,(1 - e^{-2t/RC})$

(charge on C, as function of time) **Ans.**

$i = \frac{dq}{dt} = \frac{3\varepsilon}{R}e^{-2t/RC}$ **Ans.**



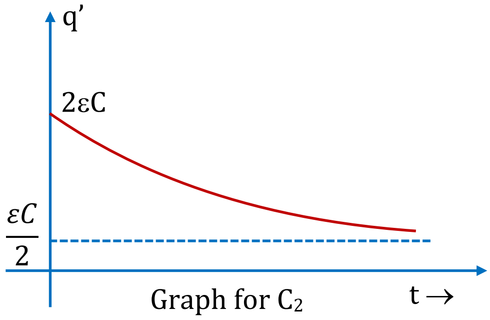
Graph for C_1

Charge on C_2 as function of time :

$q' = 2\varepsilon C - q$

$= 2\varepsilon C - \frac{3}{2}\varepsilon C + \frac{3}{2}\varepsilon Ce^{-2t/RC} = \frac{\varepsilon C}{2} + \frac{3}{2}\varepsilon Ce^{-2t/RC}$

$= \frac{\varepsilon C}{2}\left[1 + 3e^{-2t/RC}\right]$



Graph for C_2

Illustration 116:

A capacitor of capacitance 200 μF is connected across a battery of emf 10.0 V through a resistance of 40 kΩ for 16.0s. The battery is then replaced by a thick wire. What will be the charge on the capacitor 16.0 s after the battery is disconnected? (Given: $e^{-2} = 0.135$)

Solution:

For charging

$q_1 = CV(1 - e^{-t/RC}) = 20 \times 10^{-4}\left(1 - e^{-16/200\times10^{-6}\times40\times10^{3}}\right)$

$= 20 \times 10^{-4}\,(1 - e^{-2})$

For discharging

$q_2 = q_1e^{-t/RC}$

$= 20 \times 10^{-4}\,(1 - e^{-2})e^{-16/200\times10^{-6}\times40\times10^{3}}$

$= 20 \times 10^{-4}\,(1 - e^{-2})\,e^{-2} = 20 \times 10^{-4}\,\frac{(e^{2}-1)}{e^{4}}$

$= 233.55\ \mu C$ **Ans.**

Illustration 117:

A 5.0 μF capacitor having a charge of 20 μC is discharged through a wire of resistance 5.0 Ω. Find the heat dissipated in the wire between 25 to 50 μs after the connections are made. (Given : $e^{-2} = 0.135$)

Solution:

$$i_0 = \frac{20}{5} \times \frac{1}{5} = \frac{4}{5}\text{amp.}$$

$$i = i_0 e^{-t/RC}$$

$$H = \int i^2 R dt = \int_{25\mu s}^{50\mu s} \frac{16}{5} e^{-2t/25\times10^{-6}} dt$$

$$= \frac{16}{5}\left(-\frac{25}{2} \times 10^{-6}\right)\left[e^{-2t/25\times10^{-6}}\right]_{25\mu s}^{50\mu s}$$

$$= 40 \times 10^{-6}\left[\frac{e^2-1}{e^4}\right]$$

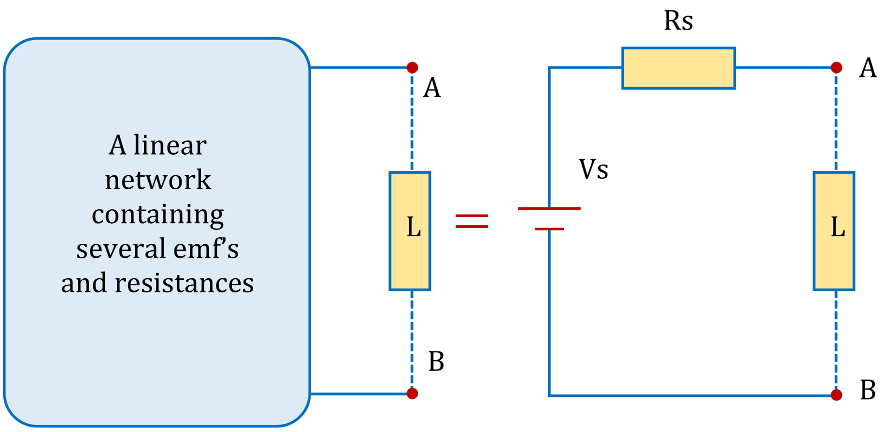
$$= 4.7\ \mu J$$

Calculation of time constant of complex circuits

Thevenins Theorem:

To calculate time constant and write charge as function of time directly in those circuits where only one capacitor is present.

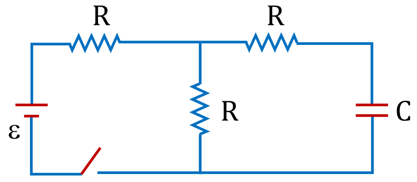
It states that any linear circuit containing any amount of emfs source and resistance can be replaced by a single voltage source called as Thevenins voltage (V_{Th}) and single series resistance connected to load which is termed as Thevenins resistance (R_{Th}).



In RC circuits we can take help of this method to simplify the circuit and calculate the time constant and charge on capacitor as function of time without actually analysing the circuit paths. In this method we have to simply open the circuit terminal where a capacitor is connected and replace the voltage sources by their internal resistances. Thus, the equivalent resistance between the terminals from where capacitor has been removed is terminals or effective resistance. The time constant hence can be written as $\tau = R_{eff.} C$.

In such cases if capacitor is initially uncharged we can directly use the result of charging RC circuit and write charge on capacitor at time t as $q = Q_{st}\left(1 - e^{-t/\tau}\right)$ where Q_{st} = steady state charge on capacitor.

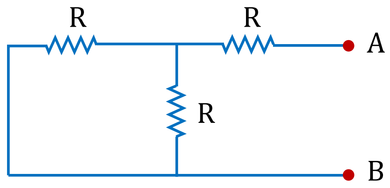
Let's try to understand the application of this method by illustrating following example



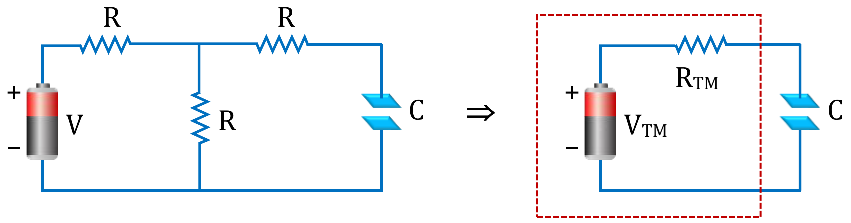
In this circuit we will try to find time constant and charge as a function of time taking help of Thevenins theorem. Calculation of $R_{\text{effective}}$.

Therefore, Effective resistance will be equivalent of the combination between terminals A and B i.e. $3R/2$.

The given circuit will have time constant as $\tau = 3RC/2$.



The given circuit can be redrawn as



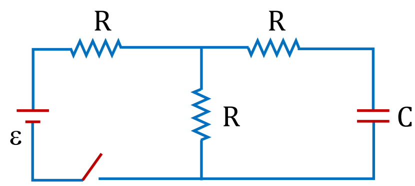
and we can write $q = Q_{st}\left(1 - e^{-t/\tau}\right)$; replacing time constant and steady state charge we will get the result of charge as function of time as $\Rightarrow q = \dfrac{\varepsilon C}{2}\left(1 - e^{-2t/3RC}\right)$.

Charging of complex RC circuits

There can be many complex RC circuit which may contain more than one resistor and capacitors. Charge as a function of time can be calculated by using KVL and KCL. This has been illustrated using KVL in simple following case.

Initially the capacitor is uncharged and here we will try to find the charge on capacitor as a function of time.

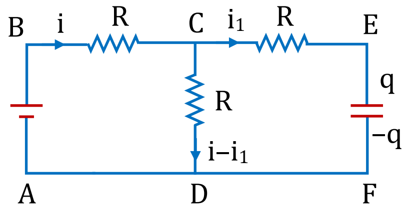
Let's assume that current through battery is i and it gets distributed in i_1 and $i - i_1$ as shown in figure. Also charge on capacitor at instant t is assumed to be q.



Applying KVL in loop ABCDA

$$\varepsilon - iR - (i - i_1)R = 0$$

$$\varepsilon - 2iR + i_1R = 0 \qquad \text{...(i)}$$



Applying KVL in loop ABCEFDA

$$\varepsilon - iR - i_1R - \frac{q}{C} = 0$$

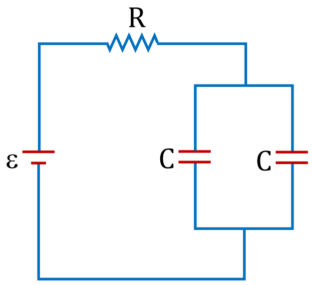
By equation (i)

$$\frac{2\varepsilon - \varepsilon - i_1R - 2i_1R}{2} = \frac{q}{C} \Rightarrow \varepsilon C - 3i_1RC = 2q$$

$$\varepsilon C - 2q = 3\frac{dq}{dt}.RC \quad \Rightarrow \int_0^q \frac{dq}{\varepsilon C - 2q} = \int_0^t \frac{dt}{3RC}$$

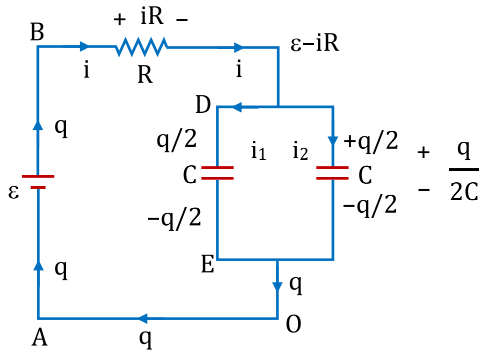
$$-\frac{1}{2}\ ln\ \frac{\varepsilon C - 2q}{\varepsilon C} = \frac{t}{3RC} \Rightarrow q = \frac{\varepsilon C}{2}\left(1 - e^{-2t/3RC}\right)$$

In this illustration we have more than one capacitor without using the formula of equivalent we will try to find out charge on capacitor and current in all the branches as a function of time.



Here we have assumed current through battery at any instant t as i which distributes as i_1 and i_2 in respectively capacitor branches as shown in diagram. Also, let's assume that charge at time t on capacitors C_1 and C_2 are q_1 and q_2 respectively.

Applying KVL in ABDEA



$$\varepsilon - iR = \frac{q}{2C}$$

$$i = \frac{\varepsilon}{R} - \frac{q}{2CR} = \frac{2C\varepsilon - q}{2CR}$$

$$\frac{dq}{2\varepsilon C - q} = \frac{dt}{2CR}$$

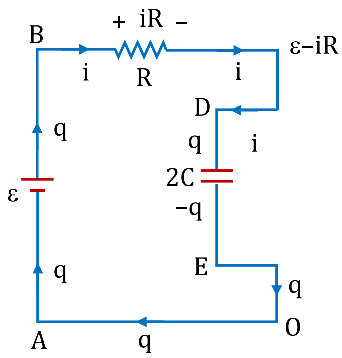
$$\int_0^q \frac{dq}{(2\varepsilon C - q)} = \frac{t}{2CR}$$

$$\frac{2\varepsilon C - q}{2\varepsilon C} = e^{-t/2RC}$$

$$q = 2\varepsilon C\left(1 - e^{-t/2RC}\right)$$

$$q_1 = \frac{q}{2} = \varepsilon C\left(1 - e^{-t/2RC}\right) \quad \Rightarrow i_1 = \frac{\varepsilon}{2R}e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\left(1 - e^{-t/2RC}\right) \quad \Rightarrow i_2 = \frac{\varepsilon}{2R}e^{-t/2RC}$$



Alternate solution:

By equivalent

Time constant of circuit = $2C \times R = 2RC$

Maximum charge on capacitor = $2C \times \varepsilon = 2C\varepsilon$

Hence equations of charge and current are as given below

$$q = 2\varepsilon C\left(1 - e^{-t/2RC}\right)$$

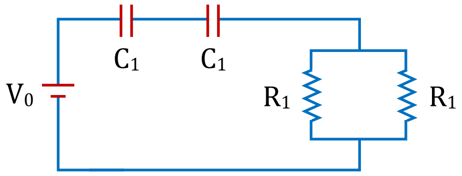
$$q_1 = \frac{q}{2} = \varepsilon C\left(1 - e^{-t/2RC}\right) \qquad \Rightarrow \; i_1 = \frac{\varepsilon}{2R} e^{-t/2RC}$$

$$q_2 = \frac{q}{2} = \varepsilon C\left(1 - e^{-t/2RC}\right) \qquad \Rightarrow \; i_2 = \frac{\varepsilon}{2R} e^{-t/2RC}$$

Illustration 118:

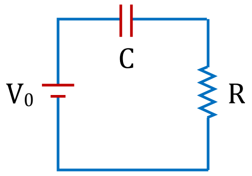
Find the time constant given circuit if

$R_1 = 4\Omega$, $R_2 = 12\Omega$, $C_1 = 3\mu F$ and $C_2 = 6\mu F$.



Solution:

Given circuit can be reduced to



$$C = \frac{C_1C_2}{C_1+C_2} = \frac{3\times6}{3+6} = 2\mu F, \quad R = \frac{R_1R_2}{R_1+R_2} = \frac{4\times12}{4+12} = 3\Omega$$

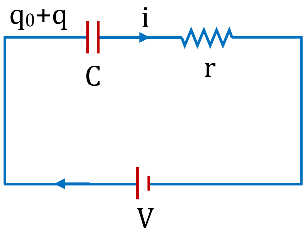
Time constant = $RC = (3)(2) = 6\ \mu s$.

Illustration 119:

A capacitor of capacitance C is charged by charge q_0. At $t = 0$, it is connected to a battery of emf V and internal resistance r. Find the charge on the capacitor at time t (positive plate of capacitor connected with positive plate of battery).

Solution:

At time t using KVL



$$-\left(\frac{q_0+q}{C}\right) - ir + V = 0$$

$$\Rightarrow \qquad -\left(\frac{q_0+q}{C}\right) - \frac{dq}{dt}r + V = 0$$

$$\Rightarrow \qquad \int_0^q \frac{dq}{(CV-(q_0+q))} = \int_0^t \frac{dt}{rC}$$

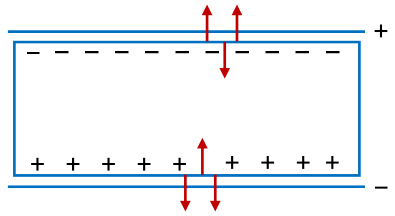
By using integration,

$$q = CV\left(1 - e^{-t/RC}\right) + q_0\, e^{-t/RC} - q_0$$

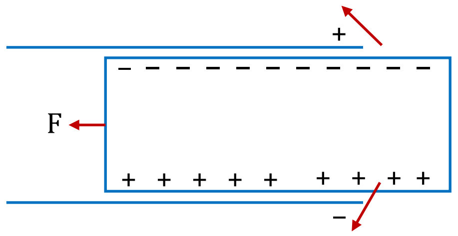
$\Rightarrow$ So, charge on capacitor = $q_0 + q = CV\left(1 - e^{-t/RC}\right) + q_0 e^{-t/RC}$

Force on a dielectric due to charged capacitor

If dielectric is completely inside the capacitor then force is equal to zero.



If dielectric is not completely inside the capacitor.



Case I : Voltage source remains connected

V = constant.

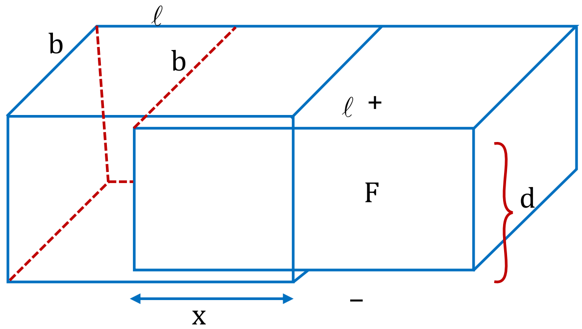
$U = \frac{1}{2}CV^2$

$|\vec{F}| = \left(\frac{dU}{dx}\right) = \frac{V^2}{2}\frac{dC}{dx}$

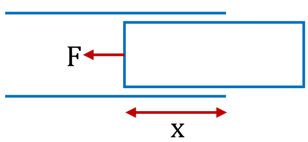
where $C = \frac{xb\epsilon_0 K}{d} + \frac{\epsilon_0(\ell - x)b}{d} \Rightarrow C = \frac{\epsilon_0 b}{d}[Kx + \ell - x]$

$\frac{dC}{dx} = \frac{\epsilon_0 b}{d}(K - 1)$

$\therefore \quad F = \frac{\epsilon_0 b(K-1)V^2}{2d}$ = constant (does not depend on x)



Case II : When charge on capacitor is constant



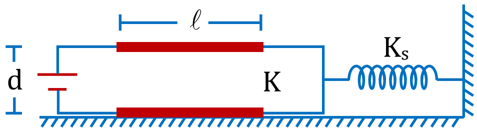
$$C = \frac{xb\in_0 K}{d} + \frac{\in_0(\ell - x)b}{d}, U = \frac{Q^2}{2C}$$

$$F = \left(\frac{dU}{dx}\right) = \frac{Q^2}{2C^2} \cdot \frac{dC}{dx} \text{ [where, } \frac{dC}{dx} = \frac{\in_0 b}{d}(K-1)]$$

$$F = \frac{Q^2}{2C^2} \times \frac{\in_0 b}{d}(K-1) = \frac{\varepsilon_0 b(k-1)Q^2}{2dC^2}$$ (here force 'F' depends on x)

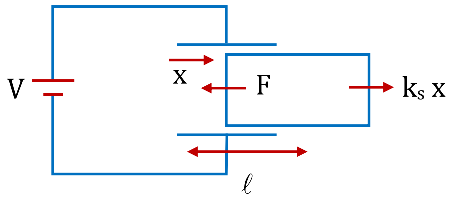
Illustration 120:

Consider the situation shown in figure. The width of each plate is b. The capacitor plates are rigidly clamped in the laboratory and connected to a battery of emf V. All surface are frictionless. Calculate the extension in the spring in equilibrium (spring is nonconducting).



Solution:

Potential energy $= \frac{1}{2}(C_1 + C_2)V^2$



$$= \frac{1}{2}\left\{\frac{\in_0 xb}{d} + \frac{\in_0 K(\ell - x)b}{d}\right\} V^2$$

$$F = -\frac{\partial U}{\partial x} = -\frac{V^2}{2}\frac{dC}{dx}$$

$$F = \frac{-1}{2}\frac{dC}{dx}V^2$$

where, $\frac{dC}{dx} = \frac{\in_0 b}{d}\{1 - K\}$

$$F = \frac{1}{2}\frac{V^2 \in_0 b(K-1)}{d}$$

At equilibrium, $F = K_s x$

$$x = \frac{F}{k_s} = \frac{\in_0 bV^2(K-1)}{2dK_S}$$