Electrostatics and Gravitation

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

Electrons are move strongly bound with *B* than *A.*

Also, electrons are move strongly bound with *A* than *C.*

Thus, electrons will be more strongly bound with *B* than *C.*

2. **Ans. (B)**

Opposite charge of rod is induced on water and it gets attracted.

3. **Ans. (C)**

By Newton's 3rd law.

4. **Ans. (C)**

By super position of forces.

5. **Ans. (D)**

They will not experience any force if $|\vec{F_G}| = |\vec{F_e}|$

$$\Rightarrow \; G\frac{m^2}{(16\times10^{-2})^2} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q^2}{(16\times10^{-2})^2}$$

$$\Rightarrow \; \frac{q}{m} = \sqrt{4\pi\varepsilon_0 G}$$

6. **Ans. (A)**

Since $F = \frac{kq^2}{a^2}$

7. **Ans. (C)**

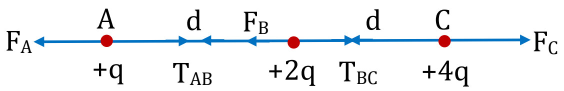
$$F' = \frac{1}{k4\pi\varepsilon_0}\frac{q_1q_2}{r_1^2} \qquad \text{...(1)}$$

$$F = \frac{q_1q_2}{4\pi_0 r^2} \qquad \text{...(2)}$$

$$\frac{q_1q_2}{k4\pi\varepsilon_0 r_1^2} = \frac{q_1q_2}{4\pi\varepsilon_0 r^2}$$

$$r_1 = \frac{r}{\sqrt{k}}$$

8. **Ans. (B)**



Force on *A*;

$$F_A = \frac{K(q)(2q)}{d^2} + \frac{K(q)(4q)}{(2d)^2} = \frac{3Kq^2}{d^2}$$

Force on B;

$$F_B = \frac{K(2q)(4q)}{d^2} - \frac{K(2q)(q)}{d^2}$$

$$\Rightarrow \quad F_B = \frac{6Kq^2}{d^2}$$

Force on Charge C;

$$F_C = \frac{K(4q)(2q)}{d^2} + \frac{K(4q)(q)}{(2d)^2}$$

$$\Rightarrow \quad F_C = \frac{9Kq^2}{d^2}$$

Net force on the system;

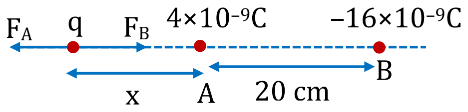
$F_{\text{net}} = F_A + F_B - F_C = 0$

hence, system is in equilibrium.

$T_{AB} = F_A$ and $T_{BC} = F_C$

$$\Rightarrow \quad \frac{T_{AB}}{T_{BC}} = \frac{F_A}{F_C} = \frac{3}{9} = \frac{1}{3}$$

9. Ans. (C)



$$\Rightarrow \quad F_A = F_B$$

$$\Rightarrow \quad \frac{Kq(4\times10^{-9})}{x^2} = \frac{K(q)(16\times10^{-9})}{(20+x)^2}$$

$$\Rightarrow \quad \frac{1}{x^2} = \frac{4}{(20+x)^2}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{2}{20+x} \quad \Rightarrow \quad 20 + x = 2x$$

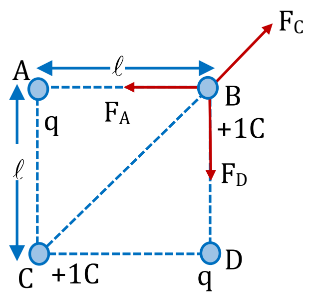
$\Rightarrow \quad x = 20\ cm$

10. Ans. (C)

If force on B is zero, them charge on A and D must be negative.

$$F_A = \frac{K(q)(1)}{\ell^2}, \quad F_D = \frac{K(q)(1)}{\ell^2}$$

$$F_C = \frac{K(1)(1)}{(\sqrt{2}\ell)^2} = \frac{K}{2\ell^2}$$



Net force in B is zero

$$\Rightarrow \quad \vec{F}_A + \vec{F}_D + \vec{F}_C = 0$$

$$\Rightarrow \quad \vec{F}_A + \vec{F}_D = -\vec{F}_C$$

$$\Rightarrow \quad \frac{\sqrt{2}Kq}{\ell^2} = \frac{K}{2\ell^2}$$

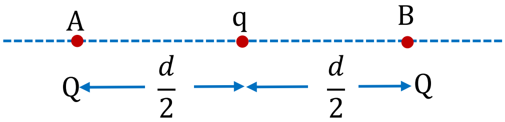
$$\Rightarrow \quad q = \frac{1}{2\sqrt{2}} = 0.35\ C$$

11. Ans. (B)

System in equilibrium $\rightarrow$ Net force on each charge must be zero.

Force on A,

$$\Rightarrow \frac{KQq}{\left(\frac{d}{2}\right)^2}+\frac{K(Q\times Q)}{d^2}=0$$



$$\Rightarrow\ 4q=-Q\Rightarrow q=-\frac{Q}{4}$$

12. Ans. (D)

$$\frac{K\times\left(5\times10^{-6}\right)^2}{r^2\times2.5}=\frac{K(5-Q)(5+Q)\times10^{-12}}{r^2}$$

13. Ans. (D)

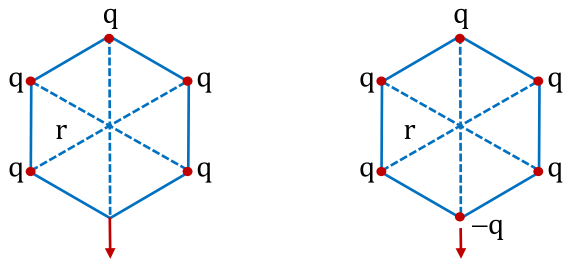
$$\Rightarrow\ \hat{F}=-\hat{r}_{AB} \qquad \Rightarrow\ -\frac{(\vec{r}_{AB})}{|\vec{r}_{AB}|}$$

$$\Rightarrow\ \hat{F}\Rightarrow-\frac{\left(\hat{i}+\hat{j}+\hat{k}\right)}{\sqrt{3}}$$

14. Ans. (B)

Use $\vec{F}_{1,2}=\frac{9\times10^9(q_1)(q_2)}{|\vec{r}_{21}|^3}\vec{r}_{21}$

15. Ans. (A)



$$E_1=\frac{Kq}{r^2} \qquad E_2=\frac{Kq}{r^2}+\frac{Kq}{r^2}=\frac{2Kq}{r^2}$$

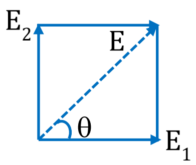
16. Ans. (B)

Due q_1, electric feed at $P(E_1)=\frac{1}{4\pi\epsilon_0}\frac{q_1}{a^2}(\hat{\imath})$

q_2 electric fed at $P(E_2)=\frac{1}{4\pi\epsilon_0}\frac{q_2}{b^2}(\hat{\jmath})$

At P,

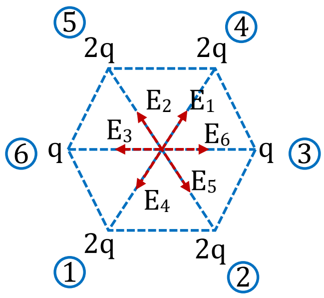
$$\therefore\ \tan\theta=\frac{E_2}{E_1}=\frac{q_2}{q_1}\frac{a^2}{b^2}=\left(\frac{1}{2}\right)\times\left(\frac{2}{1}\right)^2=2$$



17. **Ans. (D)**

$$\frac{kq_1\vec{r}_1}{r_1^3}+\frac{kq_2\vec{r}_2}{r_2^3}\Rightarrow\vec{r}_2=\frac{\vec{E}-kq_1\vec{r}_1}{kq_2}\left(|\vec{r}_1|=|\vec{r}_2|=1\right)$$

18. **Ans. (B)**



In case III,

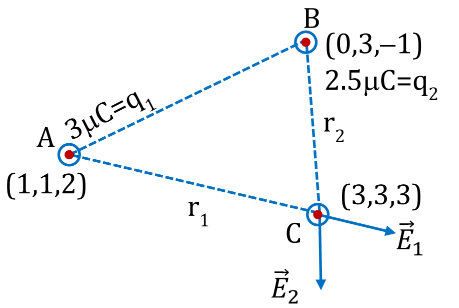
$E_1 = E_2 = E_4 = E_5$

$= \frac{2kq}{r^2}$

$E_3 = E_6 = \frac{kq}{r^2}$

$(E_0)_{mel} = 0$

19. **Ans. (D)**



$r_1 = \sqrt{(3-1)^2+(3-1)^2+(3-2)^2} = \sqrt{2^2+2^2+1^2} = 3$

$r_2 = \sqrt{(3-0)^2+(3-3)^2+[3-(-1)]^2} = \sqrt{9+0+16} = 5$

$\therefore\ \vec{E}_1 = \frac{kq_1}{r_1^3}\vec{r}_1 = \frac{9\times10^9\times3\times10^{-6}}{(3)^3}.[(3-1)\,i+(3-1)\,\hat{j}+(3-2)\,\hat{k}$

$= 10_3\,[2\,i+2\hat{j}+\hat{k}]$

$\vec{E}_2 = \frac{kq_2}{r_2^3}\vec{r}_2 = \frac{9\times10^9\times2.5\times10^{-6}}{(5)^3}.[(3-0)\hat{\imath}+(3-3)\hat{j}+\{(3-(-1)\}\hat{k}]$

$= \frac{9\times2.5}{125}\times10^3\,[3\hat{\imath}+0\hat{j}+4\hat{k}]$

$= \frac{9}{50}\times1000\,[3\hat{\imath}+4\hat{k}] = 180\,[3\hat{\imath}+4\hat{k}]$

$\therefore\ \vec{E}_{net} = \vec{E}_1+\vec{E}_2$

$= [2000\hat{\imath}+2000\hat{j}+1000\hat{k}]+[540\hat{\imath}+720\hat{k}]$

$= [2540\hat{\imath}+2000\,\hat{j}+1720\hat{k}]\ N/C.$

20. **Ans. (B)**

$\tau = 0$ *abt* bottom

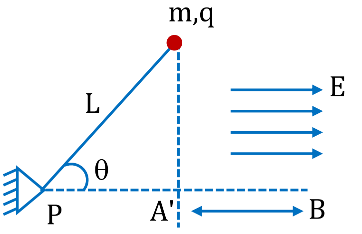
$\Rightarrow$ $qE\,(2R)\sin\theta = (mg\sin\theta)\,R$

21. **Ans. (A)**

Since Electric field is rightwards A is at highest potential

$V_A - V_B = (E)\,(A'\,B)$

$= E\ell\,(1 - \cos\theta) = E\ell\,(1 - \cos\theta)$



$U_A - U_B = q\,(V_A - V_B) = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$

$V_0 = 0$

$E\ell\,(1 - \cos\theta)\,(q) = \frac{1}{2}mv^2$

$v = \left(\frac{2qEL(1-\cos\theta)}{m}\right)^{1/2}$

22. **Ans. (A)**

$T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$; where, $g_{eff} = \frac{mg - qE}{m}$

$= g - \frac{qE}{m}$

$\therefore$ Time period increases.

23. **Ans. (C)**

The electron deviates by an angle $\theta = \tan^{-1}\frac{eE\ell}{mv_0^2}$ (from x axis) = 45°

Let final velocity is $V_x\hat{\imath} + V_y\hat{\jmath}$

Then $V_x = v_0$ and $t = \lambda/v_0$

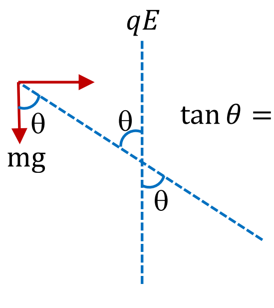
$V_y = a_y t = \frac{eE\ell}{mv_0}$; where $\left(a_y = \frac{eE}{m}\right)$

$\therefore$ $\tan\theta = \frac{V_y}{V_x} = \frac{eE\ell}{mv_0^2} = \frac{1.6\times10^{-19}\times91\times10^{-6}\times1}{9.1\times10^{-31}\times16\times10^{6}} = 1$

$\Rightarrow$ $\theta = 45°$

24. **Ans. (D)**

$\tan\theta = \frac{qE}{mg}$



$\tan\theta = \frac{qE}{mg}$

25. Ans. (B)

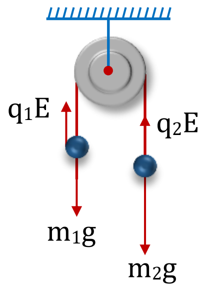
$m_2g - T - q_2E = m_2a$

$-(T + q_1E - m_1g = m_1a)$

$(m_1 + m_2)g - (q_1 + q_2)E - 2T = (m_2 - m_1)a$

$$T = \frac{(m_1 - m_2)a}{2}$$

$$a_c = \frac{2T}{m_1 + m_2} \pm 0$$



26. Ans. (D)

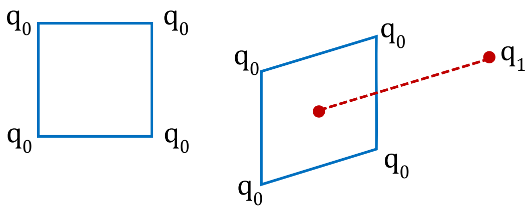
Both electric and gravitational fields accelerate the electron. The acceleration due to the electric field is opposite the direction of the field because the electron is negatively charged.

27. Ans. (A)

If B is pushed down, upwards electrostatic repulsion on it increase and hence it return back and vice versa.

28. Ans. (C)

q_1 is in unstable *eq* if q_0 and q_1 have same signs.



As charge moves always toward the (+)ve z-axis (–ve z-axis) then net force on charge will be along the displace direction and would not come ever.

q is in stable equilibrium if q_1 and q_0 have difference signs.

In this case as charge moves away from centre then net force on the charge opposite to displace direction.

29. Ans. (A)

At right of q_1, $E = +ve$ so, charge will be positive at left of q_2, $E = -ve$ so, charge will be $+ve$ also, E is zero nearer to q, so strength of q_2 is more $(q_2 > q_1)$.

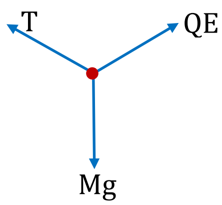
30. Ans. (C)

Since the path is parabolic thus acceleration must be constant and in the direction of change in velocity.

31. Ans. (B)

$\vec{T} + M\vec{g} + Q\vec{E} = 0$ (Initially)

$\Rightarrow \vec{T} = -\left(M\vec{g} + Q\vec{E}\right)$



When the string is cut,

$T = 0$

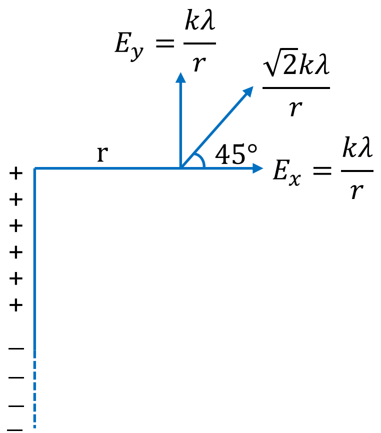
$\Rightarrow \vec{F}_{net} = M\vec{g} + Q\vec{E}$

Now, net force equal to $M\vec{g} + Q\vec{E}$, which is equal and opposite to $\vec{T}$.

Particle will move in direction of net force i.e. at q angle from, vertical.

Hence, path II indicated trajectory.

32. **Ans. (C)**



33. **Ans. (A)**

$$E_x = \frac{k\lambda}{d}(\sin\theta_1 + \sin\theta_2)$$

$$E_x = \frac{k\lambda}{2}(\sin 0^\circ + \sin 60^\circ) = \frac{\sqrt{3}k\lambda}{2d}$$

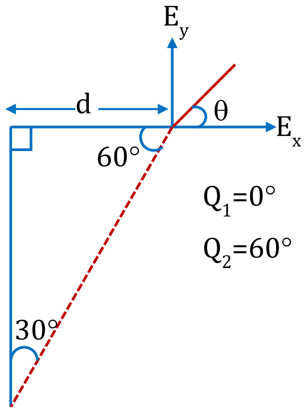
$$E_y = \frac{k\lambda}{d}(\cos\theta_1 + \cos\theta_2)$$

$$E_x = \frac{k\lambda}{d}(\cos 0^\circ + \cos 60^\circ) = \frac{k\lambda}{2d}$$

$$\tan\theta = \frac{E_y}{E_x} = \frac{\frac{k\lambda}{2d}}{\frac{\sqrt{3}k\lambda}{2d}} = \frac{1}{\sqrt{3}}$$

$$\tan\theta = \frac{1}{\sqrt{3}} = 30^\circ$$

30° with x-axis.



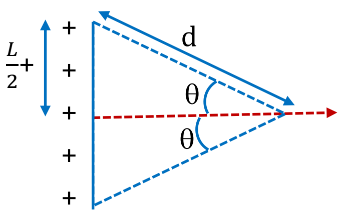
34. **Ans. (B)**

$$E = \frac{2k\lambda}{r}\sin\theta \qquad \lambda = \frac{\theta}{L}$$

$$E = \frac{2K\lambda}{r}\frac{L}{2d}$$

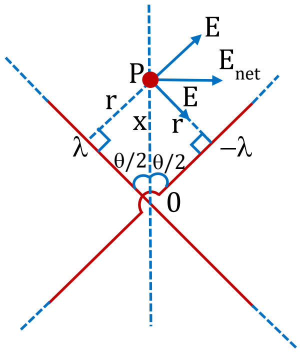
$$E = \frac{KQ}{r \times d}$$

$$= \frac{KQ}{r\sqrt{r^2 + \frac{L^2}{4}}} \qquad L\downarrow E\uparrow$$



35. Ans. (B)

As shown in figure



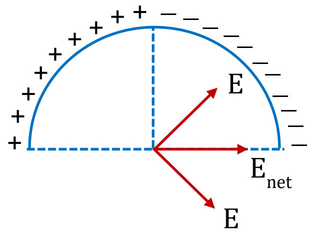
$E_{net} = 2E\cos\theta/2$

$2 = \left[\frac{2k\lambda}{r}\right]\left(\cos\frac{\theta}{2}\right) = \frac{4k\lambda}{r}.\sin\theta/2\cot\theta/2$

$= \frac{4k\lambda}{r}.\frac{r}{x}.\cot\theta/2$

$= \frac{4k\lambda}{x}\cot\theta/2$; perpendicular to OP as shown.

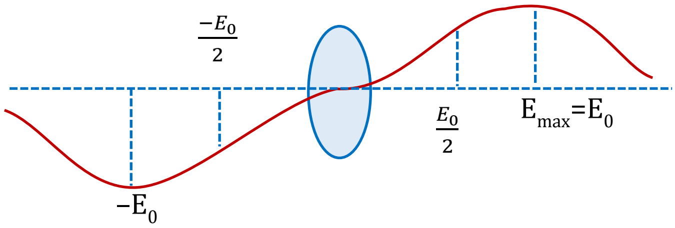
36. Ans. (C)



37. Ans. (A)

$$E = \frac{K(q)}{R^2} = \frac{K}{R^2}\left[\frac{Qd}{2\pi R}\right] \propto \frac{1}{R^3}$$

38. Ans. (D)



So, $\left(\frac{E_0}{2}\right)$ comes two times.

39. Ans. (A)

$$\frac{2K\lambda}{R}Q = ma$$

40. Ans. (B)

$$F = \left(\frac{k\lambda_1}{R}\right)\times(2\pi R)\lambda_2 = (k\lambda_1\lambda_2)2\pi = \frac{\lambda_1\lambda_2}{2\epsilon_0}$$

41. Ans. (D)

$$E_{axis} = \frac{K(\Sigma Q)x}{(a^2+x^2)^{3/2}}$$

$\because \quad \Sigma Q = 0$

$E_{axis} = 0$

42. Ans. (A)

$$E_{Arc} = \frac{2k\lambda_1}{R} \sin\frac{\theta}{2} \qquad (\theta = 180°)$$

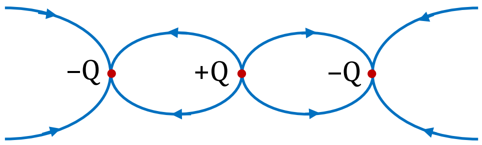
$$E_{\text{Infinite wire}} = \frac{2k\lambda_2}{d} \qquad (d = R)$$

$\lambda_1 = \lambda_2$

43. Ans. (C)

$$\frac{Q_1}{Q_2} = \frac{\text{No. of lines from } Q_1}{\text{No. of lines from } Q_2} = \frac{+10}{-5} = -2$$

44. Ans. (D)



45. Ans. (B)

Electric field lines originate from positive charge and terminate at negative charge.

46. Ans. (A)

Both cuts equal number of electric lines.

47. Ans. (D)

By Gauss's law, $\phi_{net} = 0$

Thus $\phi_{hemisphere} = -\phi_{plane}$

$= -(-E\pi R^2)$

$= E(\pi R^2)$

48. Ans. (C)

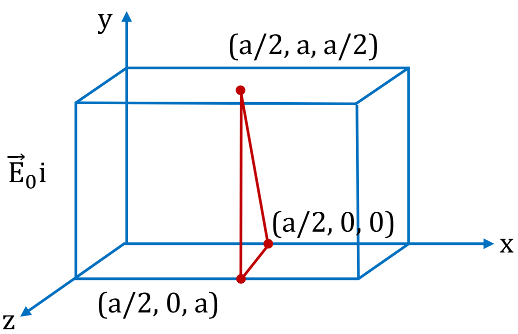
$$\phi = \vec{E}\cdot\vec{A}$$

$$= (E_0\hat{i} + E_0\hat{j})\times(-a^2\cos30°\hat{k} + a^2\sin30°\hat{i})$$

$$= \frac{E_0a^2}{2}$$

49. Ans. (B)

The flux of y-component of $\vec{E}$ is zero, through this loop



$$\therefore \quad \phi = E_0 \times \frac{1}{2} \times a \times a$$

$$\phi = \frac{E_0a^2}{2}$$

50. Ans. (B)

$$\frac{1}{2}\left[\frac{q_{en}}{\varepsilon_0}\right]=\phi$$

$$\phi=\frac{4q}{\varepsilon_0}$$

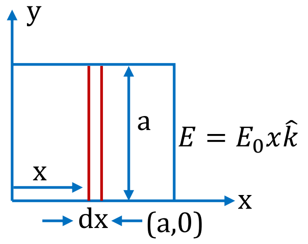
51. Ans. (C)

$\phi = \vec{E}.\vec{A}$

$= (2\hat{\imath} - 1\,0\hat{\jmath} + 5\hat{k}).(10\hat{k})$

$= 50\ N\,m^2/C$

52. Ans. (B)

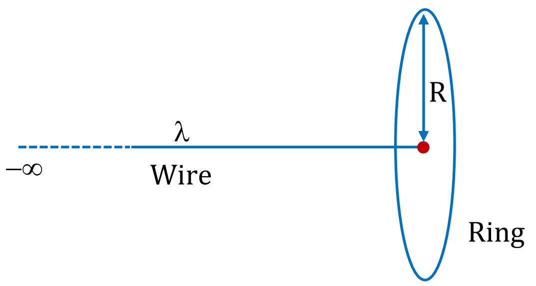


Flux through differential element $d\phi = E_0\ x\ a\ dx$.

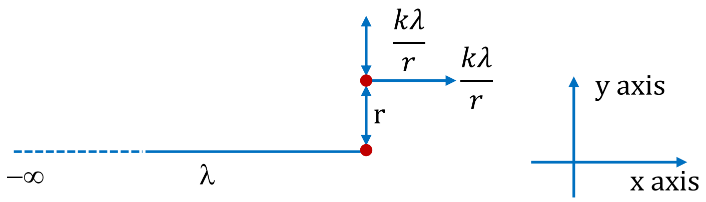
$\therefore$ Net flux

$$\Rightarrow \phi = E_0\ a \int_0^a x.dx\ = \frac{E_0 a^3}{2}$$

53. Ans. (B)



Let a wire and ring are placed as shown in figure. Due to semi-infinite wire, electric field at one of its end at distance r is as shown

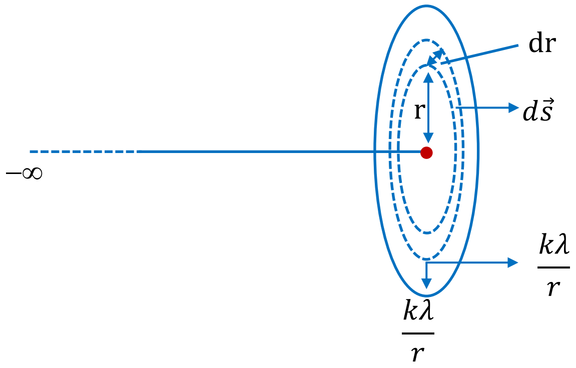


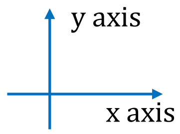
So, the Electric field = $\frac{k\lambda}{r}\ \hat{i} + \frac{k\lambda}{r}\ \hat{j}$

Now we take a ring element of radius r and thickness dr.

Let $d\vec{s}$ = Area vector of this ring element

$\therefore\ \ d\vec{s} = 2\pi r dr\ \hat{\imath}$





Now electric flux due to this element :

$$= \int \frac{k\lambda}{r}\,\hat{\imath}\,d\vec{s} + \int \frac{k\lambda}{r}\hat{\jmath}\,d\vec{s} = \int_o^R \frac{k\lambda}{r}.2\pi r\,\mathrm{dr} + 0 = 2\,\pi\,k\,\lambda\,R\, = \frac{R\lambda}{2\,\epsilon_o}$$

$$= \frac{R(16\epsilon_0)}{2\epsilon_0} = 8N/cm^2$$

54. Ans. (A)

Here $q_{enc} = q_1 + q_2 + q_3$

So q_{enc} = 10 + (–10) + (–5) = – 5 nC

$$\therefore\ \phi = \frac{q_{enc}}{\epsilon_0} = \frac{-5\times10^{-9}}{8.85\times10^{-12}} = -564.97\ Nm^2/C$$

55. Ans. (A)

Since charge enclosed by Gaussian surface is

$$\phi_{enc.} = (2\times10^{-6} - 5\times10^{-6} - 3\times10^{-6} + 6\times10^{-6}) = 0\ \text{ so }\ \phi = 0$$

56. Ans. (D)

$$\frac{\Sigma q}{\epsilon_0} = \phi_{net}$$

$$= q = \left(-4\times10^3\right)\epsilon_0$$

57. Ans. (B)

In case of net charge is zero then incoming lines = outgoing lines.

58. Ans. (B)

$$\oint \vec{E}.ds = \frac{q}{\epsilon_0}$$

59. Ans. (B)

Given that charge per cm length of the wire is Q.

Since 100 cm length of the wire is enclosed so $Q_{enc} = 100Q$

$\Rightarrow$ Electric flux emerging through cylindrical surface $\phi = \frac{100Q}{\varepsilon_0}$.

60. Ans. (C)

Flux coming out of the cube $\phi_1 = \frac{\lambda.a\sqrt{3}}{\varepsilon_0}$ ...(i)

and from sphere $\phi_2 = \frac{\lambda\cdot 2a}{\varepsilon_0}$ ...(ii)

$$\therefore\ \frac{\phi_1}{\phi_2} = \frac{\sqrt{3}}{2}$$

61. **Ans. (D)**

$$\phi_A + \phi_C + \phi_B = \frac{q}{\epsilon_0}$$

$$\because \quad \phi_A = \phi_C$$

$$\therefore \quad \phi_A + \phi_A + \phi = \frac{q}{\epsilon_0}$$

$$\therefore \quad \phi_A = \frac{1}{2}\left(\frac{q}{\epsilon_0} - \phi\right)$$

62. **Ans. (D)**

By symmetry we can cover the charge with five extra square of same size so that it makes a cube with charge at its centre so.

$$\phi_{\text{closed cube}} = \frac{q}{\epsilon_0}$$

$$\phi_{\text{square}} = \frac{q}{6\epsilon_0}$$

63. **Ans. (D)**

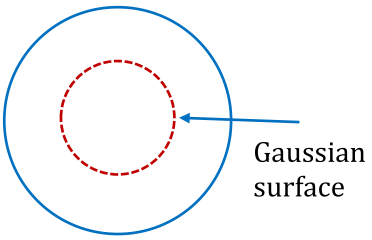
Using Gauss's law, $\phi = \frac{q}{\varepsilon_0} = \frac{\sigma\pi\left(R^2 - x^2\right)}{\varepsilon_0}$

64. **Ans. (C)**

Calculate flux through curve surface and circular cross section both.

65. **Ans. (A)**

For spherical charge distribution we can apply Gauss theorem



$$\int E \cdot ds = \frac{q_{in}}{\epsilon_0}$$

$$E4\pi r^2 = \frac{q_{in}}{\epsilon_0}\left[r = \frac{R}{2}\right]$$

$$q_{in} = \int_0^{R/2} \rho dv$$

$$= \frac{Ar}{R} 4\pi r^2 dr$$

$$q_{in} = \frac{A4\pi}{R}\left[\frac{r^4}{4}\right]_0^{R/2} = \frac{A\pi R^3}{16}$$

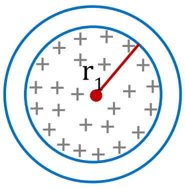
$$\frac{E4\pi R^2}{4} = \frac{A\pi R^3}{16\ \epsilon_0}$$

$$\frac{AR}{16\ \epsilon_0} = \frac{2R}{\epsilon_0} \Rightarrow A = 2$$

66. **Ans. (C)**

$$P(r) = \frac{Q}{\pi R^4}\ r$$

From Gauss law,



$$\oint E.ds = \frac{q_{en}}{\varepsilon_0} = \frac{\int \rho V dV}{\varepsilon_0}$$

$$= \int_O^{r_1} \frac{Qr4\pi r^2 dr}{\pi R^4} / \varepsilon_0$$

$$E.4\pi r_1^2 = \frac{Q}{\pi R^4}\ 4\pi\ \frac{r_1^4}{4}$$

$$E = \frac{Qr_1^2}{4\pi\varepsilon_0 R^4}$$

67. Ans. (B)

For calculating the electric field outside the ball, we should calculate first the total charge present in the ball, i.e.

$$Q = 4\pi\rho_0 \int_0^R \left[1 - \frac{r}{R}\right] r^2 dr$$

$$= 4\pi\rho_0 \left[\frac{R^3}{3} - \frac{R^4}{4R}\right] = 4\,\pi\rho_0 \left(\frac{R^3}{12}\right)$$

Again, to find the electric field outside the ball, total charge Q will be considered to be concentrated at the centre. Hence, electric field at a distance r from the centre of ball.

$$E = \frac{1}{4\pi\varepsilon}\frac{Q}{r^2} = \frac{1}{4\pi\varepsilon} 4\pi\rho_0 \left(\frac{R^3}{12}\right) \times \frac{1}{r^2} = \frac{\rho_0 R^3}{12\varepsilon r^2}$$

68. Ans. (C)

$$\int E.dr = \frac{q_{in}}{\epsilon_0}$$

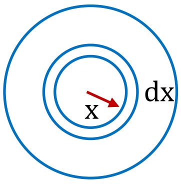
$$\Rightarrow\; E\,(4pr^2) = \frac{Q + \int_R^r \frac{\alpha}{r} 4\pi r^2 dr}{\epsilon_0} = \frac{Q + 2\pi\alpha(r^2 - R^2)}{\epsilon_0}$$

$$\Rightarrow\; E = \frac{\alpha}{2\epsilon_0} + \frac{Q - 2\pi\alpha R^2}{4\pi\epsilon_0 r^2}$$

$$\Rightarrow\; Q = 2\pi\alpha R^2$$

69. Ans. (B)

We can consider all the charge inside the sphere to be concentrated at the centre of sphere, consider an elementary shell of radius x and thickness dx.



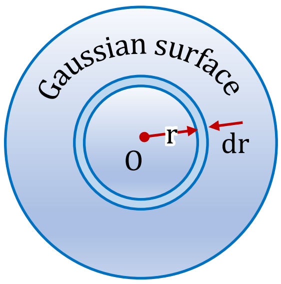
$$E = \frac{k\int dq}{r^2} = \frac{k\int_0^r 4\pi x^2 dx(\propto x)}{r^2} = \frac{\propto r^2}{4\varepsilon_0}$$

70. Ans. (A)

By using Gauss law

$$\oint \vec{E}\cdot d\vec{S} = \frac{q}{\epsilon_0}$$

$$\Rightarrow\; (E)(4\pi r^2) = \frac{\int \rho(4\pi r^2 dr)}{\epsilon_0}$$



(**Note :** Check dimensionally that $\rho \propto r^6$)

$$(kr^7)(4\pi r^2) = \frac{\int \rho(4\pi r^2 dr)}{\epsilon_0}$$

$$\Rightarrow\; k\varepsilon_0 r^9 = \int \rho r^2 dr$$

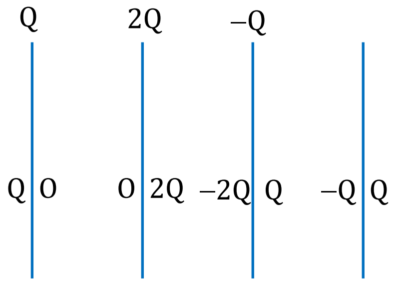
Differentiating above equation w.r.t. r we get $k\varepsilon_0\,(9r^8) = \rho r^2 \;\Rightarrow\; \rho = 9k\varepsilon_0 r^6$

71. Ans. (C)

$$I_{net} = \left[\frac{\sigma}{2\varepsilon_0} + \frac{2\sigma}{2\varepsilon_0} + \frac{\sigma}{2\varepsilon_0}\right]\left(-\hat{k}\right)$$

$$E_{net} = \frac{2\sigma}{2\varepsilon_0}\left(-\hat{k}\right)$$

72. Ans. (D)



73. Ans. (C)

Let the charge distribution on the plates is as shown in the figure.

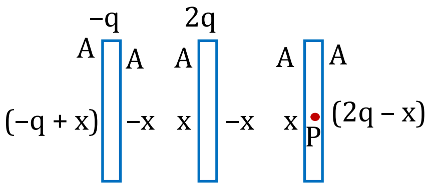
Equating electric field at point P inside the right most plate:

$$\frac{-q}{A\epsilon_0} + 0 + \frac{x}{A\epsilon_0} = \left(\frac{2q-x}{A\epsilon_0}\right)$$

$\therefore \quad -q + x = 2q - x \qquad \text{or} \quad x = \frac{3q}{2}$

$\therefore$ Charge on outer surface of leftmost plate is

$$-q + \frac{3q}{2} = \frac{q}{2}$$



74. Ans. (A)

Where, $x = (2\varepsilon_0 E + 3\sigma)/2$

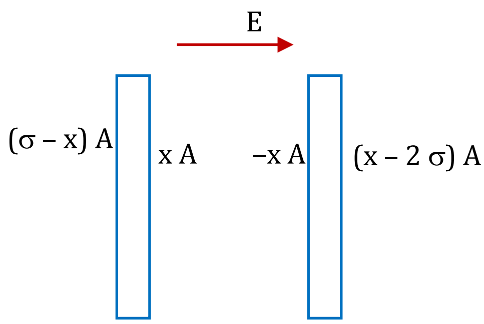
Let charge distribution on each surface at plates are shown in figure (after applying the electric field)

Now, at point P:

$$\frac{(\sigma - x)}{2\epsilon_0} - \frac{(x - 2\sigma)}{2\epsilon_0} = E$$

$\Rightarrow \quad 2x + 3\sigma = E.2\varepsilon_0$

$$\therefore \quad x = \frac{3\sigma + 2\varepsilon_0 E}{2}$$



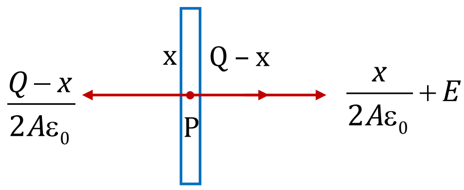
75. Ans. (C)

Let there is x charge on left side of plate and $Q - x$ charge on right side of plate

$\because \quad E_P = 0$

$$\therefore \quad \frac{x}{2A\varepsilon_0} + E = \frac{Q-x}{2A\varepsilon_0} \text{ or } \frac{x}{A\varepsilon_0} = \frac{Q}{2A\varepsilon_0} - E$$

$$\therefore \quad x = \frac{Q}{2} - EA\varepsilon_0 \text{ and } Q - x = \frac{Q}{2} + EA\varepsilon_0$$



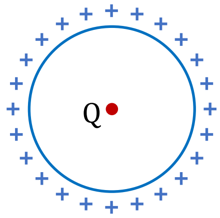
So, charge on one side is $\frac{Q}{2} - EA\varepsilon_0$ and other side $\frac{Q}{2} + EA\varepsilon_0$

76. Ans. (B)

If y is fixed then there is an external force so L.M. is not conserved but E is conserved as there is no work is done by that force.

77. Ans. (B)

$$U = \frac{KqQ}{a} + \frac{KqQ}{\sqrt{2}a} + \frac{Kqq}{a} = 0$$



$$Q + \frac{Q}{\sqrt{2}} = -q$$

$$Q\left(\frac{-\sqrt{2}+1}{\sqrt{2}}\right) = -q$$

$$Q = -\frac{\sqrt{2}q}{\sqrt{2}+1}$$

$$Q = -\frac{2q}{2+\sqrt{2}}$$

78. Ans. (A)

$$U_2 = U_1 + qV \Rightarrow V = \frac{U_2 - U_1}{q}$$

79. Ans. (A)

$$W = U_f - U_i = \frac{Kq^2}{a}$$

80. Ans. (C)

As Mechanical Energy is conserved. So, it will move along the Electric field so, K.E. ↑ and P.E. ↓.

81. Ans. (B)

$W = q\Delta V$

$= q(V_\infty - V_C)$

$$= q\left(0 - 3\frac{Kq}{(a/\sqrt{3})}\right) = \frac{-3\sqrt{3}Kq^2}{a}$$

82. Ans. (B)

Apply linear momentum conservation and total mechanical energy conservation.

83. Ans. (D)

$$V = \frac{Kq}{x_0}\left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} \ldots\right]$$

$$\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots$$

$x = 1$

$$V = \frac{Kq}{x_0}\log_e(2)$$

84. Ans. (A)

Due to the principle of superposition, potential at O

$$V = \frac{1}{4\pi\varepsilon_0} \times \frac{28\times10^{-6}}{0.4} = 9 \times 10^9 \times \frac{28\times10^{-6}}{0.4} = 63 \times 10^4\ volt$$

85. **Ans. (B)**

$$V_{\text{centre}} = \frac{k(8q)}{\frac{a\sqrt{3}}{2}} = \frac{16kq}{a\sqrt{3}}$$

$$= \frac{4q}{\pi\varepsilon_0 a\sqrt{3}}$$

86. **Ans. (C)**

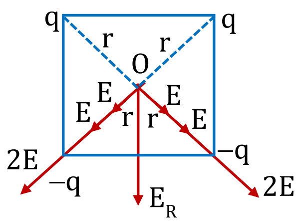
$\sum\frac{Q}{r} = 0$ at $x = 0 = y$

$$\sum\frac{Q}{r} = \frac{4}{r} + \frac{6}{r} - \frac{12}{r} + \frac{2}{r} = 0$$

87. **Ans. (D)**

$$V_0 = \frac{kq}{r} + \frac{kq}{r} - \frac{kq}{r} - \frac{kq}{r} = 0$$

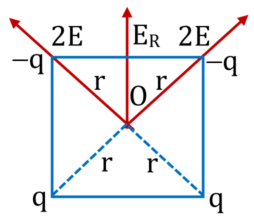
$$E_R = 2(2E)\cos 45\,(-\hat{j}) = 2\sqrt{2}\,\frac{kq}{r^2}\,(-\hat{j})$$



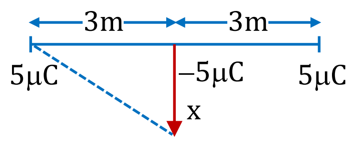
After interchanging,

$$V_0 = \frac{kq}{r} + \frac{kq}{r} - \frac{kq}{r} - \frac{kq}{r} = 0$$

$$E_R = 2\,(2E)\,cos\,45\,(+\hat{j}) = 2\sqrt{2}\,\frac{kq}{r^2}\,(+\hat{j})$$



88. **Ans. (D)**



$$\frac{-k\times5\times5\times10^{-12}}{|3|}\times2+0.6=-\frac{k\times5\times5\times10^{-12}\times2}{\sqrt{(3^2+x^2)}}$$

$$\Rightarrow\ -\frac{9\times10^{-9}\times25\times10^{-12}\times2}{3}+0.6=\frac{-9\times10^{9}\times25\times10^{-12}\times10^{-3}\times2}{\sqrt{3^2+x^2}}$$

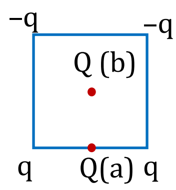
$$\Rightarrow\ -0.15+0.06=-\frac{0.45}{\sqrt{3^2+x^2}}$$

$$-0.9=-\frac{0.45}{\sqrt{3^2+x^2}}$$

$x = 4$

89. Ans. (A)

$$U_A + K_A = U_B + K_B$$



$$\frac{2KQq}{a} - \frac{2KQq}{\sqrt{a^2+4a^2}} + 0 = 0 + \frac{1}{2}mv^2$$

$$\frac{2KQq}{a} - \frac{2KQq}{\sqrt{5}a} = \frac{1}{2}mv^2$$

$$\frac{1}{4\pi\varepsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right) = K$$

90. Ans. (B)

$$V = \frac{1000}{x} + \frac{1500}{x^2} + \frac{500}{x^3}$$

$$E = -\frac{\partial V}{\partial x}\hat{i} - \frac{\partial V}{\partial y}\hat{j} - \frac{\partial V}{\partial z}\hat{k}$$

$$\frac{\partial V}{\partial x} = -\frac{1000}{x^2} - \frac{3000}{x^3} - \frac{1500}{x^4}$$

$$\frac{\partial V}{\partial y} = 0 \;\&\; \frac{\partial V}{\partial z} = 0$$

$$\therefore \quad E(x,0,0) = \left(\frac{1000}{x^2} + \frac{3000}{x^3} + \frac{1500}{x^4}\right)\hat{i}\ V/m$$

$$E(1,0,0) = (1000 + 3000 + 1500)\hat{\imath}V/m$$

$$= 5500\hat{\imath}V/m$$

91. Ans. (B)

$$E = -\frac{dv}{dr}\hat{r} = \frac{343}{7^3}(3\hat{\imath} + 2\hat{\jmath} + 6\hat{k})$$

92. Ans. (B)

Charge on sphere $q = \int_0^R \rho_0(4\pi r^2 dr) = \pi\rho_0 R^4$

By definition of potential = $V_{2R} - V_\infty = \frac{kQ}{2R}$

$\Rightarrow \; V_\infty = -\frac{kQ}{2R} = -\frac{Q}{8\pi\epsilon_0 R}$

93. Ans. (C)

$$V = -\int \vec{E}\cdot d\vec{r}$$

$$V = -\int(\sqrt{3}\hat{\imath} - \hat{\jmath})\cdot(dx\hat{\imath} + dy\hat{\jmath})$$

$$V = -[\sqrt{3}x - y]$$

$$V = -\sqrt{3}x + y$$

So, equipotential line having slope 60°.

94. Ans. (B)

$\because u = qV$

At Point C and D, V is Positive so electron will have negative potential energy.

At Point A, V is zero so potential energy = 0.

At Point B, V us negative so potential energy is Maximum.

95. **Ans. (A)**

At a great distance from a collection of charges behaves as a point charge.

96. **Ans. (B)**

$\because V_1 - V_2 = V_2 - V_3$ (given)

As we go along positive x-axis, electric field increases, E.F.L. becomes denser so distance so distance between equal potential surfaces.

Decreases is $x_1 > x_2$.

97. **Ans. (A)**

$\vec{E} = -ky\hat{\imath} + ky\hat{\jmath}$

$E^2 = k^2(y^2 + x^2)$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ -ky & kx & 0 \end{vmatrix} = 2K\hat{k}$$

So, the field is nonconservative.

98. **Ans. (B)**

$E_x = -\frac{dv}{dx}$, $E_x = -\left(\frac{2}{2\times 10^{-2}}\right) = -100\ N/C$, $E_y = -\left(\frac{-2}{1\times 10^{-2}}\right) = 200\ N/C$

$\Rightarrow\ E = \left(-1\hat{i} + 2\hat{j}\right) N/C$

99. **Ans. (C)**

$$V_B - V_A = -\int_{x=5}^{10} \vec{E}.\vec{dx}$$

$$= -\int_5^{10} \frac{20}{x^2}\, dx = \left[\frac{20}{x}\right]_5^{10} = -2V$$

100. **Ans. (A)**

Potential due to both hemisphere shells are $\frac{K2Q}{d}$

Potential due to one hemisphere shell is $\frac{KQ}{d}$

101. **Ans. (C)**

Volume of bubble = Volume of droplets

$4\pi a^2 \times t = \frac{4}{3}\pi r^3$ (r = radius of droplet)

$r = (3a^2t)^{1/3}$

The potential of bubble $V = \frac{KQ}{a}$

Q = Charge on bubble

Now the potential of droplet = $V' = \frac{KQ}{r}$

$$V' = \frac{KQ}{(3a^2t)^{1/3}} = \frac{Va}{(3a^2t)^{1/3}}$$

$$= V' \times \left(\frac{a}{3t}\right)^{1/3}$$

102. Ans. (A)

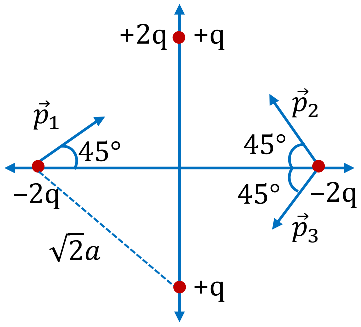
$\vec{p}_1 = 2qa(\hat{\imath}) + 2qa(\hat{\jmath})$

$\vec{p}_2 = qa(-\hat{\imath}) + qa(\hat{\jmath})$

$\vec{p}_3 = qa(-\hat{\imath}) + qa(-\hat{\jmath})$

$\vec{p}_{Net} = \vec{p}_1 + \vec{p}_2 + \vec{p}_3$

$\vec{p}_{Net} = 2qa(\hat{\jmath})$



103. Ans. (B)

$(0, a, a)$; $(0, a, -a)$; $(0, 0, -a)$

$$\vec{P} = q\left(a\hat{\jmath} + a\hat{k} - a\hat{\jmath} + a\hat{k}\right) + q\left(a\hat{\jmath} + a\hat{k} + a\hat{k}\right)$$

$$= q\left(2a\hat{k}\right) + q\left(a\hat{\jmath} + 2a\hat{k}\right)$$

$$= q\left(a\hat{\jmath} + 4a\hat{k}\right)$$

$|P| = \sqrt{17}qa$ and $\tan^{-1}\left(\frac{1}{4}\right)$ with z-axis in y-z plane.

104. Ans. (C)

$\vec{p}_1 = (qa)\hat{\jmath}$

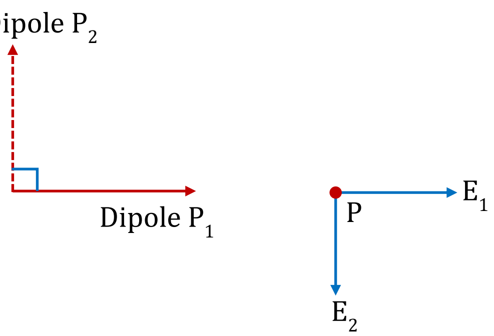
$\vec{p}_2 = (qa)\hat{\jmath}$

$\therefore \quad \vec{p} = (qa)\hat{\imath} + (qa)\hat{\jmath}$

$|\vec{p}| = \sqrt{2}qa$

Along line which joins points $(0, 0, 0)$ and $(a, a, 0)$.

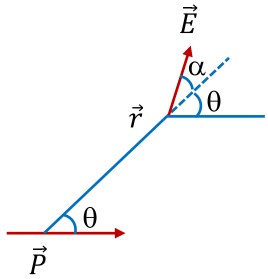
105. Ans. (A)



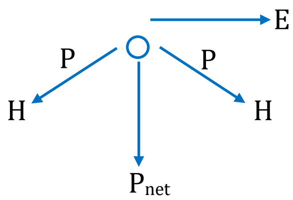
106. Ans. (C)

In shown diagram, $\vec{E}$ = Net electric field vector due to dipole. (by derivation) and $\tan\alpha = \frac{1}{2}\tan\theta$

$\therefore$ Angle made by $\vec{E}$ with x-axis is $(\theta + \alpha)$.



107. Ans. (A)



$P_{net} = \vec{P} + \vec{P}$

$\tau = \vec{P} \times \vec{E}$

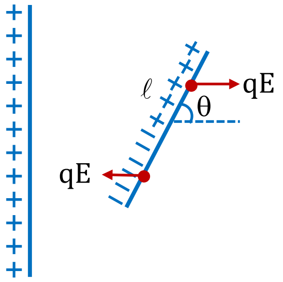
108. Ans. (C)

At $-q$ field is more so the net force towards left

$E- > E+$

$F_{net} = qE^- - qE^+$

109. Ans. (C)



$$E = \frac{\sigma}{2\epsilon_0}, q = \lambda\ell$$

$$\tau = 2qE\frac{\ell}{2}\sin\theta = 2\lambda\ell\frac{\sigma}{2\epsilon_0}\times\frac{\ell}{2}\sin\theta$$

$$\tau = \frac{\sigma\lambda\ell^2}{2\epsilon_0}\sin\theta$$

110. Ans. (A)

$$\vec{F} = \frac{2K\lambda}{r}(-q) + \frac{2K\lambda}{r+d}(+q)$$

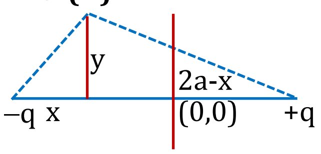
$$= \frac{2K\lambda}{r(r+d)}(-q\times d)$$

$$= \frac{\lambda P}{2\pi\varepsilon_0 r^2}(\text{towards wire})$$

111. Ans. (A)

By $V = \dfrac{2kp\cos\theta}{r^2}$

112. Ans. (D)



$$V_P = \frac{-kq}{\sqrt{y^2+x^2}} + \frac{kq}{\sqrt{y^2+(2a-x)^2}}$$

113. Ans. (D)

Properties of equipotential surface.

114. Ans. (C)

Potential inside a conductor is constant electric field inside a conductor is zero.

115. Ans. (A)

$$\frac{kq_a}{b}+\frac{kq_b}{b}=5$$

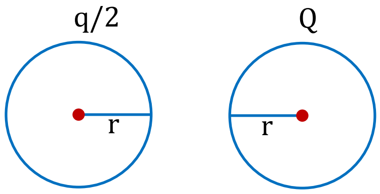
$$\frac{kq_a}{a}+\frac{kq_b}{b}=10$$

116. Ans. (C)

Let the Q charge on sphere,

$$\frac{Kq/2}{d}+\frac{KQ}{r}=0$$

$$Q=-\frac{qr}{2d}$$

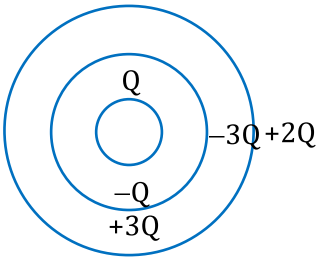


117. Ans. (A)

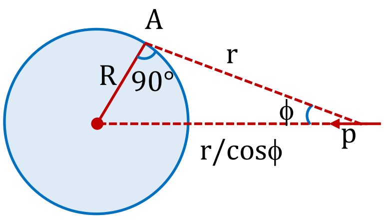
$\phi_1=\frac{kq_1}{r_1}$, $\phi_2=\frac{kq_1}{r_2}$ (All the charges reaches to surface of shell to potential difference)

$q_1=\frac{\phi_1 r_1}{k}$, $\phi_2=\frac{k}{r_2}\times\frac{\phi_1 r_1}{k}$ (Principle of generator)

118. Ans. (D)



119. Ans. (B)



$$V_p=\frac{kp\cos^2\phi}{r^2}$$

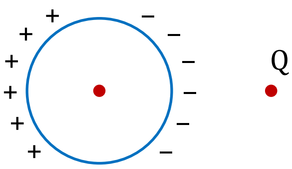
120. Ans. (D)

$$V_P=\frac{kq_0}{2a}-\frac{kq_0}{a}+\frac{kq_0}{(a/2)}=\frac{3kq_0}{2a}$$

121. Ans. (C)

Net induced charge on sphere is zero.

So potential at the center will be zero.



122. Ans. (A)

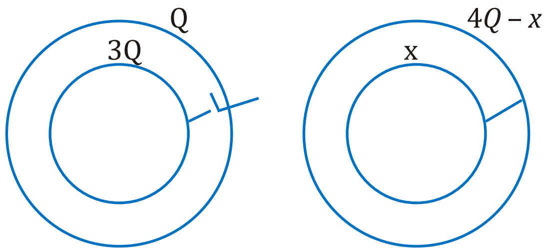
$v_{in} = v_{int}$

$\frac{Kx}{R} + \frac{K(4Q-x)}{2R} = \frac{K(x+4Q-x)}{2R}$

$x + \frac{4Q-x}{2} = \frac{4Q}{2}$

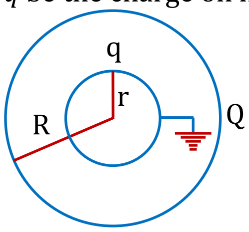
$2Q - x = 2Q$

$x = 0$



123. Ans. (D)

Let q be the charge on inner sphere then potential at the surface of inner surface



$V_{\text{inner}} = \frac{KQ}{R} + \frac{Kq}{r} = 0$ ($\because$ Earthed) $q = -\frac{r}{R}Q$

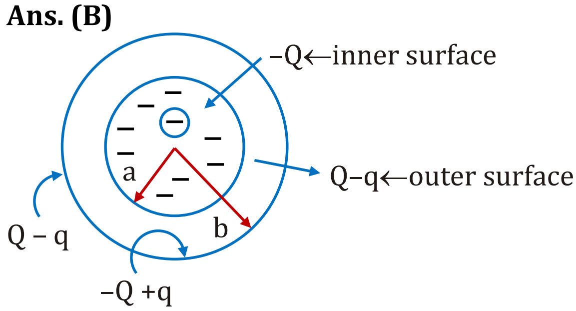
124. Ans. (A)

$V = \frac{kQ}{d}$

125. Ans. (A)

Property of conductor.

126. Ans. (B)



127. Ans. (C)

$\frac{U}{\text{volume}} = \frac{1}{2}\varepsilon_0 E^2$ and $\vec{E}.\vec{E} = E^2$

$U = \frac{\varepsilon_0}{2}\int_R^{2R} E^2 4\pi r^2 dr$

128. Ans. (A)

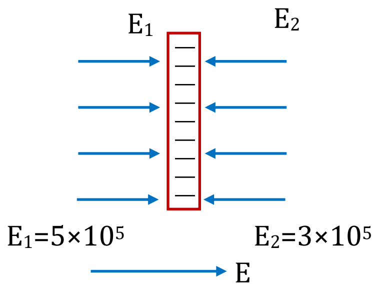
$E_1 = \frac{\sigma}{\varepsilon_0} + E$

$E_2 = \frac{\sigma}{\varepsilon_0} - E$

$E_1 + E_2 = \frac{2\sigma}{\varepsilon_0} = \frac{2q}{A\varepsilon_0}$

$F = q\ (2 \times E)$

$0.08 = q(2 \times 10^5)$



129. Ans. (D)

$F \propto \frac{1}{r^2}$. If r becomes double then F reduces to $\frac{F}{4}$.

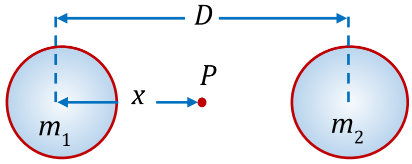
130. Ans. (D)

$F = m\omega^2 r = 6 \times 10^{24} \times (2 \times 10^{-7})^2 \times 1.5 \times 10^8 \times 10^3 = 36 \times 10^{21}\ N$

131. Ans. (D)

Force will be zero at the point of zero intensity

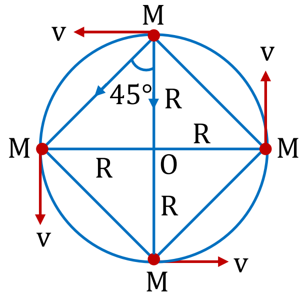
$$x = \frac{\sqrt{m_1}}{\sqrt{m_1}+\sqrt{m_2}}D = \frac{\sqrt{81M}}{\sqrt{81M}+\sqrt{M}}D = \frac{9}{10}D$$



132. Ans. (D)

Gravitational force on each due to other three particles provides the necessary centripetal force.

$$\therefore\ 2\frac{GM^2}{(\sqrt{2}R)^2}\cos 45° + \frac{GM^2}{(2R)^2} = \frac{Mv^2}{R}$$



Simplifying it, we get,

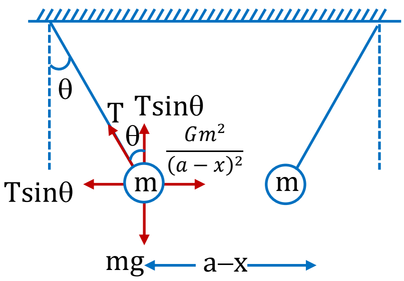
$$v = \sqrt{\frac{GM}{R}\left(\frac{2\sqrt{2}+1}{4}\right)}$$

133. Ans. (B)

$$T\sin\theta = \frac{Gm^2}{(a-x)^2}$$

$$T\cos\theta = mg$$

Dividing we get $\tan\theta = \frac{mG}{(a-x)^2 g}$



134. Ans. (A)

Gravitational potential at mid point

$$V = \frac{-GM_1}{d/2} + \frac{-GM_2}{d/2}$$

Now, $PE = m \times V = \frac{-2Gm}{d}(M_1 + M_2)$ [m = mass of particle]

So, for projecting particle from mid point to infinity

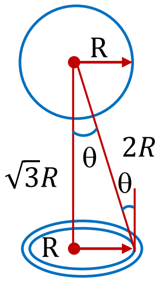
$$TE_i = TE_f$$

$$\frac{1}{2}mv^2 + PE = 0 + 0$$

$$\Rightarrow \frac{1}{2}mv^2 = \frac{2Gm}{d}(M_1 + M_2)$$

$$\Rightarrow v = 2\sqrt{\frac{G(M_1+M_2)}{d}}$$

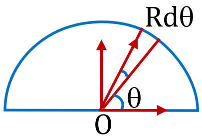
135. Ans. (D)



$$F = \frac{GMm}{(2R)^2}\cos\theta = \frac{GMm}{(2R)^2}\times\frac{\sqrt{3}}{2} = \frac{\sqrt{3}GMm}{8R^2}$$

136. Ans. (C)

y component of field,



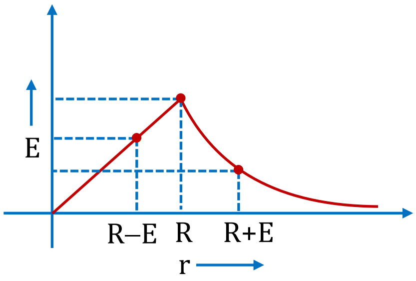
$$= \frac{Gdm\sin\theta}{R^2} = \frac{G\lambda}{R}\int_0^{\pi}\sin\theta d\theta = \frac{2G\lambda}{R}$$

137. Ans. (C)

Apply principle of superposition, $0 + \dfrac{G\left(\dfrac{M}{8}\right)}{\dfrac{R^2}{4}} = \dfrac{GM}{2R^2}$

138. Ans. (D)

Graph of gravitational field versus distance from axis for a solid cylinder.



139. Ans. (B)

$$g = \frac{4\pi}{3}G\rho R$$

140. Ans. (C)

Velocity of body in inter planetary space $v' = \sqrt{v^2 - v_{es}^2}$

where $v_{es} =$ Escape velocity and

v = Velocity of projection

$\therefore \quad v' = \sqrt{(2v_{es})^2 - v_{es}^2} = \sqrt{3v_{es}^2} \Rightarrow v' = \sqrt{3}\,v_{es}$

141. Ans. (B)

$$v_e = \sqrt{\frac{2GM}{R}} = R\sqrt{\frac{8}{3}\pi G\rho}$$

If mean density is constant then $v_e \propto R$

$$\frac{v_e}{v_p} = \frac{R_e}{R_p} = \frac{1}{2} \quad \Rightarrow \quad v_e = \frac{v_p}{2}$$

142. Ans. (A)

h = height above the earth surface.

From energy conservation between surface and maximum height.

$$-\frac{GM_e m_p}{R_e} + \frac{1}{2}m_p\left(KV_e\right)^2 = \frac{-Gm_e m_p}{\left(R_e + h\right)}$$

and $V_e = \sqrt{\frac{2Gm_e}{R_e}}$

143. Ans. (C)

$$T = \frac{2\pi}{\sqrt{GM}}r^{3/2}$$

$$\frac{T}{2} = \frac{2\pi}{\sqrt{GM}}r_1^{3/2}$$

$$2 = \left(\frac{r}{r_1}\right)^{3/2}$$

$$4^{4/3} = \frac{r}{r_1}$$

144. Ans. (B)

$$5MV = -MV + 4MV_1$$

$$V_1 = 1.5V$$

$$V_1 > V_{\text{escape}}$$

145. Ans. (D)

$$TE = \frac{PE}{2} = -KE = -\frac{GM}{2r}$$

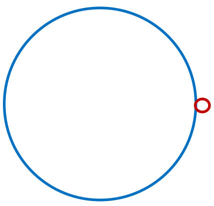
146. Ans. (D)

$$E_1 = \frac{-GMm}{R}$$

$$E_2 = \frac{-GMm}{4R}$$

$$W = E_2 - E_1 = \frac{GMm}{R} - \frac{GMm}{4R} = \frac{3}{4}\frac{GMm}{R}$$

$$= \frac{3}{4}mgR$$



147. Ans. (D)

$$U_1 = \frac{-GMm}{4R}$$

$$U_2 = \frac{-GMm}{6R}$$

$$W = U_2 - U_1 = \frac{GMm}{4Re} - \frac{GMm}{6Re} = \frac{GMm}{12Re}$$

148. Ans. (A)

$U = \frac{-GMm}{r}$ $\quad r \uparrow \quad U \uparrow$

$\omega = \frac{\sqrt{GM}}{r^{3/2}}$ $\quad r \uparrow \quad \omega \downarrow$

$U = \frac{v^2}{r}$ $\quad r \uparrow \quad a_c \downarrow$

149. Ans. (D)

Orbital radius of satellites,

$r_1 = R + R = 2R$

$r_2 = R + 7R = 8R$

$U_1 = \frac{-GMm}{r_1}$ and $U_2 = \frac{-GMm}{r_2}$

$K_1 = \frac{GMm}{2r_1}$ and $K_2 = \frac{GMm}{2r_2}$

$E_1 = \frac{GMm}{2r_1}$ and $E_2 = \frac{GMm}{2r_2}$

$\therefore \quad \frac{U_1}{U_2} = \frac{K_1}{K_2} = \frac{E_1}{E_2} = 4$

150. Ans. (A)

A real velocity $\frac{dA}{dt} = \frac{L}{2m}$

L = angular momentum

m = mass of planet

151. Ans. (C)

$\frac{dA}{dt} = \frac{L}{2m}$

$\frac{dA}{dt} = \frac{Vr}{2}$

$\frac{2}{r}\frac{dA}{dt} = V$

$\frac{2}{2\times10^{12}} \times 4 \times 10^{16} = v$

$4 \times 10^4 = v$

$40\ km/sec = v$

152. Ans. (A)

Areal velocity = $\frac{dA}{dt} = \frac{v_0 r}{2}$

and $\quad V_0 = \sqrt{\frac{GM}{r}}$

V_0 = Orbital speed

r = Radius of orbit

153. Ans. (C)

According to Kepler's Third Law,

$T^2 \propto r^2$

$\Rightarrow\ 5^2 \propto r^3$ ...(1)

And $(T')^2 \propto (4r)^3$ ...(2)

Dividing equation (1) and (2), we get

$$\frac{25}{(T')^2} = \frac{r^3}{64r^3}$$

or $T' = \sqrt{1600}$

or $T' = 40\ h$

154. Ans. (A)

Let M_G and m_s represent masses of the galaxy and the Sun.

According to Kepler's law,

$$t^2 = \frac{4\pi^2}{Gm_s} r^3$$

and $T^2 = \dfrac{4\pi^2}{GM_G} R^3$

$$\Rightarrow\ \frac{M_G}{m_s} = \left(\frac{R}{r}\right)^3 \left(\frac{t}{T}\right)^2 n$$

155. Ans. (C)

(i) $T_{st} = 2\pi\sqrt{\dfrac{(R+h)^3}{GM}} = 2\pi\sqrt{\dfrac{R}{g}}$ [As $h << R$ and $GM = gR^2$]

(ii) $T_{ma} = 2\pi\sqrt{\dfrac{R}{g}}$

(iii) $T_{sp} = 2\pi\sqrt{\dfrac{1}{g\left(\dfrac{1}{l}+\dfrac{1}{R}\right)}} = 2\pi\sqrt{\dfrac{R}{2g}}$ [As $l = R$]

(iv) $T_{is} = 2\pi\sqrt{\dfrac{R}{g}}$ $[As\ l = \infty]$

156. Ans. (ABC)

Incoming flux is negative and outgoing flux is positive.

157. Ans. (ABC)

Total flux through cube = $\dfrac{q}{4\epsilon_0}$

$\phi_{ABCD} = \phi_{GFEH} = \dfrac{q}{32\epsilon_0}$ & $\phi_{BEFD} = \phi_{CGFD}$ & $\phi_{ABEH} = \phi_{AHGC} = 0$

$\phi_{ABCD} = \phi_{GFEH} + \phi_{BEFD} + \phi_{CGFD} = \dfrac{q}{4\epsilon_0}$

158. Ans. (BC)

(A) $V_{x=0} \neq 0$

(B) $V_{x=\frac{R}{2}} = \dfrac{\int_R^{2R} K 4\pi r^2 dr}{r} = \dfrac{3}{2}\dfrac{\ell R^2}{\varepsilon_0}$

(C) $V_{x=2R} = \dfrac{K\ell \times \frac{4}{3}\pi 7R^3}{2R} = \dfrac{7\ell R^2}{6\varepsilon_0}$

159. Ans. (AB)

$\vec{E} = x^2\hat{i} + y^2\hat{j}$

$\therefore \quad dv = -\vec{E}.d\vec{r}$

$\therefore \quad v = -\left[\int x^2 dx + \int y^2 dy\right] + C$

$\quad = -\left(\dfrac{x^3 + y^3}{3}\right) + C$

From given condition,

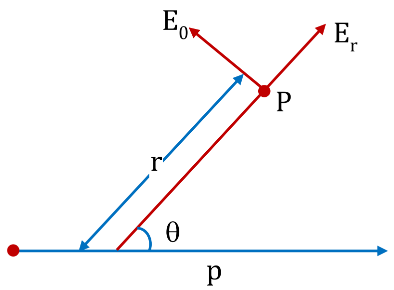
V at $x = 3, y = 3$ is equal to O

$\therefore \quad C = +18$

$\therefore \quad V = -\left(\dfrac{x^3 + y^3}{3}\right) + 18$

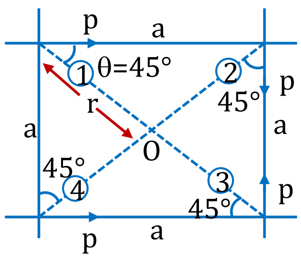
Also, $V = -\left[\int_a^b x^2 dx + \int_b^a y^2 dy\right] = 0$

160. Ans. (BD)

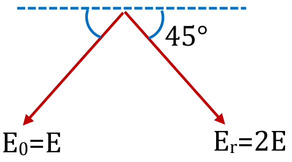


$E_r = \dfrac{2kp\cos\theta}{r^3}$ $E_\Theta = \dfrac{2kp\sin\theta}{r^3}$ $V_P = \dfrac{kp\cos\theta}{r^2}$

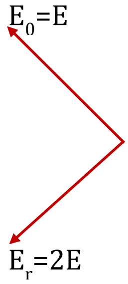
$\left[\dfrac{2kp}{r^3} = E\right]$ θ = 45°



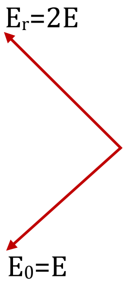
$r = \dfrac{a}{\sqrt{2}}$; $E_r = \dfrac{2kp\cos 45°}{\left(\frac{a}{\sqrt{2}}\right)^3} = \dfrac{4kp}{a^3} = 2E$; $E_\Theta = \dfrac{kp\sin 45°}{\left(\frac{a}{\sqrt{2}}\right)^3} = \dfrac{2kp}{a^3} = E$



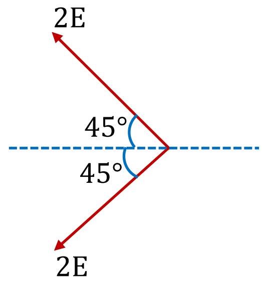
1st



2nd



3rd

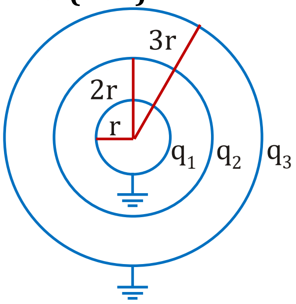


4th

$2\sqrt{2}E \ = \dfrac{2\sqrt{2}.2kp}{a^3} = \dfrac{\sqrt{2}p}{\pi\in_o a^3}$

$V = 4 \times \dfrac{k\,p\cos45^o}{\left(\frac{a}{\sqrt{2}}\right)^2}$ or $V = \dfrac{\sqrt{2}p}{\pi\in_o a^2}$

161. Ans. (ABC)



$\dfrac{Kq_1}{r} + \dfrac{kq_2}{2r} + \dfrac{kq_3}{3r} = 0$ ...(i)

$\dfrac{k(q_1+q_2+q_3)}{3r} = 0$ ...(ii)

Solving (i) and (ii)

$q_1 + q_3 = -q_2, \quad q_1 = \dfrac{-q_2}{4}, \quad \dfrac{q_3}{q_1} = 3$

162. Ans. (AD)

$V_c = V_{\text{surface}}$

$V_c = \dfrac{kq}{R+d}$

$V_s = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{R+d}$

$V_q + V_{ind} = V_c$

$\dfrac{Kq}{d} + V_{ind} = \dfrac{kq}{R+d}$

$V_{ind} = \dfrac{kq}{R+d}\dfrac{-kq}{d}$

$V_{ind} = \dfrac{-qR}{4\pi\varepsilon_0(R+d)d}$

163. Ans. (ABC)

(A) $\vec{E}_{at}\ C = 0$

(B) $V = K\dfrac{Q}{3a} - K\dfrac{3Q}{a}$

(C) $V = 0 = \dfrac{KQ}{3a} + \dfrac{KQ'}{a}$

$Q = -\dfrac{Q}{3}$

164. Ans. (BC)

$V_1 = 0 = K\dfrac{Q}{2a} + K\dfrac{q}{a} \quad \Rightarrow \quad q = -\dfrac{Q}{2}$

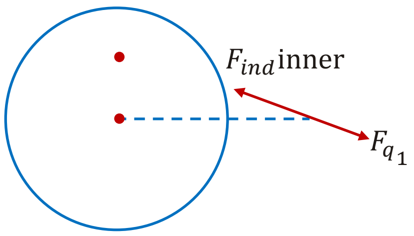
$r < a \quad \Rightarrow \quad E = 0 \qquad V = K\left(-\dfrac{Q}{2a}\right) + \dfrac{KQ}{2a} = 0$

$a < R < 2a \quad \Rightarrow \quad E = -\dfrac{KQ/2}{(r)^2} \qquad V = -\dfrac{KQ}{2r} + \dfrac{KQ}{2a}$

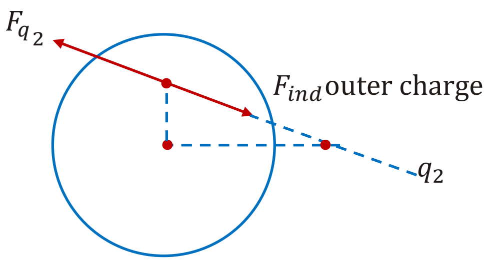
$r > 2a \quad \Rightarrow \quad E = -\dfrac{KQ/2}{r^2} + \dfrac{KQ}{r^2} = \dfrac{KQ}{2r^2} \qquad V = -\dfrac{KQ/2}{r} + \dfrac{KQ}{r} = \dfrac{KQ}{2r}$

165. Ans. (BD)

(D)



(B)

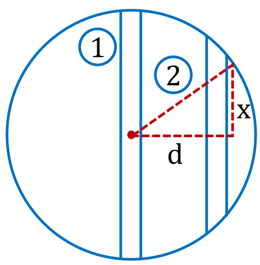


(A) Magnitude of force on q_1 due to induced charge at inner surface of conducting shell is non zero.

166. Ans. (AD)

For tunnel (1), $\omega = \sqrt{G\rho \times \dfrac{4}{3}\pi}$

For tunnel (2), $\omega = \sqrt{\dfrac{G.\rho\dfrac{4}{3}\pi\left(\sqrt{d^2+x^2}\right)^3 .x}{\left(\sqrt{d^2+x^2}\right)^3 x}}$



Since ω is same thus, $T_1 = T_2$

Also, $v_{max} = A\omega$, thus $v_1 > v_2$

167. Ans. (ABC)

$T \propto r^{3/2} \qquad r\downarrow \quad T\uparrow$

$L = mvr$ constant $\qquad r\downarrow \quad v\uparrow$

$E = \dfrac{-GMm}{2r} \qquad r\uparrow \quad E\uparrow$

168. Ans. (ACD)

There is no external torque acting in any case.

169. Ans. (B)

$$\int dq = \int_0^a \rho_0 r^3 . 2\pi r dr . \ell$$

$$q = \frac{2\pi\rho_0 \ell a^5}{5}$$

170. Ans. (A)

$$E.2\pi r\ell = \frac{2\pi\rho_0 \ell r^5}{5\varepsilon_0}$$

$$E = \frac{\rho_0 r^4}{5\varepsilon_0}$$

171. Ans. (C)

$\phi = \frac{Q}{\epsilon_0} \Rightarrow Q = 2 \times 10^5 \times 8.85 \times 10^{-12}\, C = 1.77\ \mu C$

172. Ans. (B)

$\frac{(1.77\times10^{-6} + Q_A)}{\epsilon_0} = -4 \times 10^5 \Rightarrow Q_A = -5.31 \times 10^{-6} C$

173. Ans. (D)

For all values of r, flux ϕ is non-zero i.e., no Gaussian sphere of radius r is possible in which net enclosed charge is zero.

174. Ans. (C)

$$\frac{2G\lambda}{r} \times m = \frac{mv^2}{r}$$

were λ = mass per unit long = $\rho \times A = \rho \times \pi R^2$

$$\Rightarrow \frac{2G \times \rho \times \pi R^2}{r} = \frac{v^2}{r}$$

$$\Rightarrow v = R\sqrt{2\pi G\rho}$$

175. Ans. (B)

$$\Rightarrow \int_R^{3R} \frac{2G\lambda_m}{r} dr = \frac{1}{2} mv^2 \Rightarrow v = 2R\sqrt{\pi\rho g \ell n 3}$$

176. Ans. (D)

$$T = \frac{2\pi d}{v} \Rightarrow d = RT\sqrt{\frac{G\rho}{2\pi}}$$

177. Ans. (B)

$A_{ACB} > A_{ADB}$

$t_1 > t_2$

178. Ans. (C)

$T < 0$

$K + U < 0$

$|K| < |U|$ at all points

179. Ans. (A)

Check using direction in each quadrant.

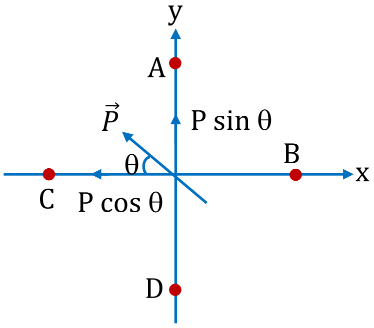
180. Ans. (A)

$$v_A = \frac{kp\sin\theta}{r^2}$$

$$v_B = \frac{-kp\cos\theta}{r^2}$$

$$v_C = \frac{kp\cos\theta}{r^2}$$

$$v_D = \frac{-kp\sin\theta}{r^2}$$



181. Ans. (A-Q,R), (B-Q,T), (C-S,T), (D-P)

(A) New Q_1, potential is positive $\Rightarrow Q_1$ is positive

New Q_2, potential is negative $\Rightarrow Q_2$ is negative

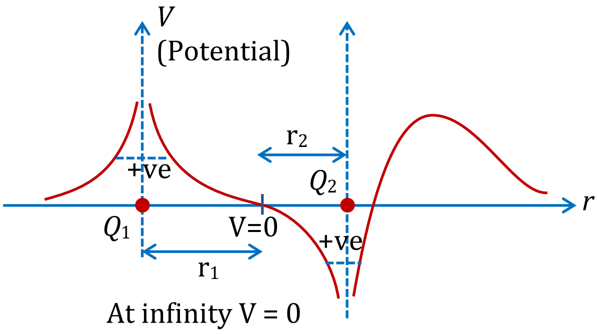
For $v = 0$ and $r_1 > r_2$

$$\frac{KQ_1}{r_1} - \frac{KQ_2}{r_2} = 0$$

$$\frac{|Q_1|}{r_1} = \frac{|Q_2|}{r_2}$$

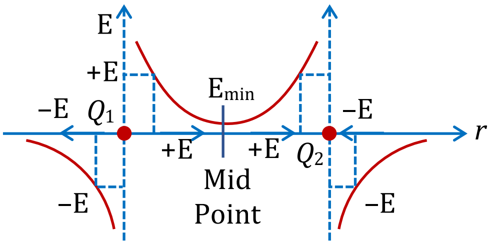
$$\Rightarrow \frac{|Q_1|}{|Q_2|} = \frac{r_1}{r_2} > 1$$

$$\Rightarrow \boxed{|Q_1| > |Q_2|}$$



At infinity V = 0

(B)



E_{mis} is at midpoint $\Rightarrow |Q_1| = |Q_2|$

Near Q_1 : $\xleftarrow{-E} \overset{Q_1}{\bullet} \xrightarrow{+E}$, So, Q_1 is positive

Near Q_2 : $\xrightarrow{+E} \overset{Q_2}{\bullet} \xleftarrow{-E}$, So, Q_2 is negative

(C) Near Q_1 : $\xrightarrow{+E} \overset{Q_1}{\bullet} \xleftarrow{-E}$, So, Q_1 is positive

Near Q_2 : $\xrightarrow{+E} \overset{Q_2}{\bullet} \xleftarrow{-E}$, So, Q_2 is negative

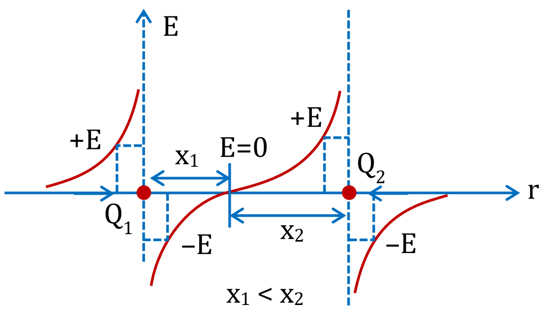
For, $E = 0$, $\boldsymbol{x_1 < x_2}$

$E_1 - E_2 = 0$

$$\frac{KQ_1}{x_1^2} - \frac{KQ_2}{x_2^2} = 0$$

$$\frac{|Q_1|}{|Q_2|} = \frac{x_1^2}{x_2^2} < 1$$

$$\boxed{|Q_1| < |Q_2|}$$



(D) Near Q_1 : $\xrightarrow{+E} \overset{Q_1}{\bullet} \xleftarrow{-E}$, So, Q_1 is positive

Near Q_2 : $\xleftarrow{-E} \overset{Q_2}{\bullet} \xrightarrow{+E}$, So, Q_2 is positive

$E = 0$ is near Q_1

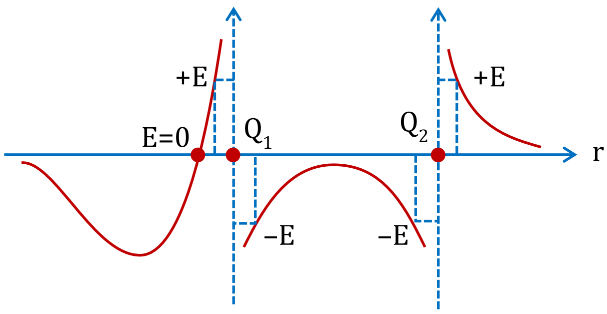
$E_1 - E_2 = 0$

$|E_1| = |E_2|$

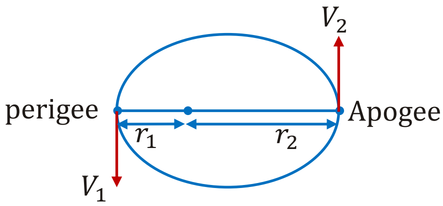
$\frac{KQ_1}{r_1^2} = \frac{KQ_2}{r_2^2}$

$\frac{|Q_1|}{|Q_2|} = \left(\frac{r_1}{r_2}\right)^2 < 1$

$\boxed{|Q_1| < |Q_2|}$



182. Ans. (A-R), (B-Q), (C-Q), (D-P)



$r_2 > r_1$ $\qquad v_2 < v_1$

$U = \frac{-GMm}{r}$ $\qquad L = \text{constant}$

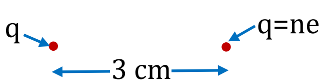
$U_2 > U_1$

EXERCISE – S

1. Ans. 625

$F = 10^{-19} N$

$\Rightarrow\ 10^{-19} = \frac{K \times q^2}{(3\times10^{-2})^2}$



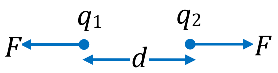
$\Rightarrow\ 10^{-19} = \frac{9\times10^9 \times q^2}{9\times10^{-4}}$

$\Rightarrow\ q^2 = 10^{-32}$ $\qquad \Rightarrow\ q = 10^{-16} C$

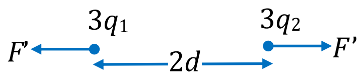
$\Rightarrow\ q = ne$ $\qquad \Rightarrow\ n = \frac{q}{e} = \frac{10^{-16}}{1.6\times10^{-19}}$

$\Rightarrow\ n = \frac{10^3}{1.6}$ = 625 electron to each body.

2. **Ans.** $\frac{9F}{4}$



$\Rightarrow \; F = \frac{kq_1q_2}{d^2}$



$\Rightarrow \; F' = \frac{K(3q_1)(3q_2)}{(2d)^2} = \frac{9}{4}\frac{Kq_1q_2}{d^2}$

$\Rightarrow \; F' = \frac{9F}{4}$

3. **Ans.** 3

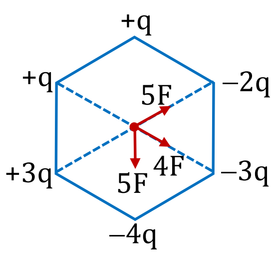
$F_e = mg$

$9 \times 10^9 \frac{q^2}{3^2} = 0.9 \times 10^{-3} \times 10$

$\Rightarrow \; q = 3 \times 10^{-6}\, C$

$= 3\, \mu C$

4. **Ans.** 9



$F_{net} = 9F$

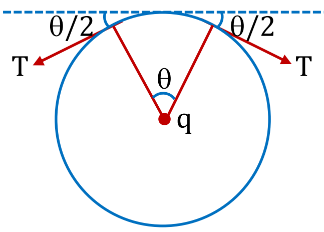
5. **Ans.** 3

$F = 3a$

$F - \frac{9\times10^9\times2\times10^{-9}}{9} = 1a = \frac{F}{3}$

$\frac{2F}{3} = 2 \Rightarrow F = 3$

6. **Ans.** 8



$2T\sin\frac{\theta}{2} = dF$

$2T\frac{\theta}{2} = \frac{1}{4\pi\in}\frac{(q)\times dQ}{r^2}$

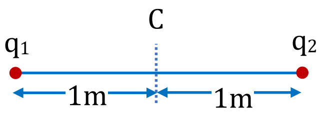
$T = \frac{Qq}{8\pi^2\in_0 r^2}$

7. **Ans.** 400

A:

$q_1 = q_2 = 50\ nc$

$\Rightarrow\ \vec{E}_C = \frac{Kq_1}{1^2} - \frac{Kq_2}{1^2} = 0$



Initially electric field at centre is zero.

B:

$\Rightarrow\ E_C = \frac{K\left(q_1^{'}\right)}{1^2} - \frac{K(50)}{(3)^2} = 0$

$\Rightarrow\ q'_1 = \frac{50}{9} nc$

Removed charged

$\Rightarrow\ \frac{\alpha}{9} = q_1 - q'_1$

$\Rightarrow\ \frac{\alpha}{9} = 50nc - \frac{50}{9}nc$

$\Rightarrow\ \frac{\alpha}{9} = \frac{8\times 50nc}{9} = \frac{400}{9}nc$

$\Rightarrow\ \alpha = 400$

8. **Ans.** 3

$$E_{\perp} = \frac{9\times 10^9 \times 2\sqrt{2}\times 10^{-9}}{12}\left[\frac{3}{5}+\frac{4}{5}\right]$$

$$E_{||} = \frac{9\times 10^9 \times 2\sqrt{2}\times 10^{-9}}{12}\left[\frac{4}{5}-\frac{3}{5}\right]$$

$$E = \sqrt{E_{\perp}^2 + E_{||}^2}$$

$$E = \frac{\sqrt{900}}{10} = 3N/C$$

9. **Ans.** 2

Due to symmetry field due to one pair will be forming the resultant field strength and equal to

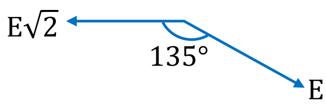
$$E = 2 \times \frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2} = 2k_eQ/r^2$$

10. **Ans.** 1

Find resultant of field due to DE, CB and FG rest on them cancel each other.

Resultant of BC and $FG = E\sqrt{2}$ and at 135° with E of DE.

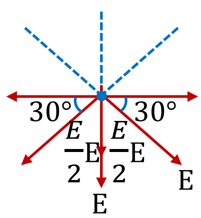
$E_{\text{net}} = E$



11. **Ans.** 2

The magnitude of electric field intensity due to each part of the hemispherical surface at the centre 'O' is same. Suppose, It is E.

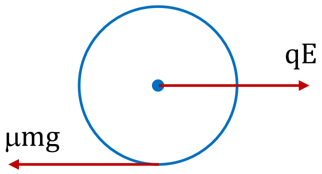
$E + \frac{E}{2} + \frac{E}{2} = E_0\,;\ 2E = E_0;\ \boxed{E = \frac{E_0}{2}}$



12. **Ans.** 12

$a_{CM} = \alpha r$

$\Rightarrow \dfrac{qE-\mu mg}{m} = \dfrac{\mu mgr}{mr^2/2}r$



$\Rightarrow qE - \mu mg = 2\mu mg \Rightarrow \dfrac{qE}{m} = 3\mu g$

$\Rightarrow E = \dfrac{3\mu gm}{q} = \dfrac{3\times 0.2\times 10\times 2}{1} = 12$

13. **Ans.** 4

$E_0 = \dfrac{2\lambda}{2\pi \in_0 d}$

$\lambda = \pi \in_0 de_0$

So $ep = E_0 \dfrac{\ell d}{4r^2}$

14. **Ans.** 6

$u = \sqrt{4g\ell} = 6$

15. **Ans.** 3

$E = \dfrac{\sigma}{4\varepsilon_0}$ due to spherical shell.

$E = \int_R^{2R} \dfrac{\rho 2\pi r^2 dr}{4\in_0}$, where $\rho = \dfrac{\rho_0 r}{R}$

16. **Ans.** 576

$\phi = (E \sin 53° \times \pi R^2)$

17. **Ans.** 1

Electric field inside spherical cavity,

$E = \dfrac{\rho(OO')}{3\varepsilon_0}$

Where OO' = distance between centre of sphere and cavity,

$E = \dfrac{2}{3}\dfrac{N}{C} \Rightarrow y = 3, x = 2$

18. **Ans.** 8

$Q = \int_0^{\infty} \rho 4\pi r^2 dr$

$Q = \int_0^{\infty} \rho_0 \dfrac{e^{-r/2a}}{r^2} 4\pi r^2 dr$

$Q = \rho_0 4\pi \int_0^{\infty} e^{-r/2a} dr$

$Q = \rho_0 8\pi a$

$\dfrac{Q}{\rho_0 \pi a} = 8$

19. **Ans.** 3

$$\phi=\frac{Q_{in}}{\epsilon_0}=\int\frac{\rho dV}{\epsilon_0}=\frac{\text{Area under }\rho\text{ v/s x graph}}{\epsilon_0}=\left(\frac{3}{4}\right)$$

20. **Ans.** 2

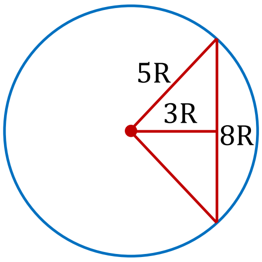
Flux through bottom circular end

$$\frac{q}{4\pi\varepsilon_0}\left(2\pi(1-\cos 37^0)\right)=\frac{q}{10\varepsilon_0}$$

Flux through curved surface is

$$\frac{q}{2\varepsilon_0}-\frac{q}{10\varepsilon_0}=\frac{2q}{5\varepsilon_0}$$

21. **Ans.** 6



$$\phi=\frac{q_{in}}{\varepsilon_0}=\frac{\rho\left(\frac{1}{2}8R.3R\right)R}{\varepsilon_0}=\frac{12\rho R^3}{\varepsilon_0}$$

22. **Ans.** (a) 200 Vm^{-1} making an angle 120° with the x-axis

(b) Radially outward, decreasing with distance as $E=\frac{6}{r^2}V/m$

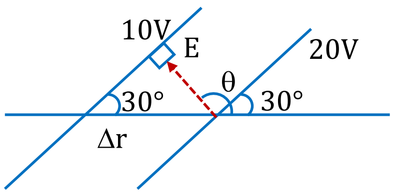
(a) Using $dv = -\vec{E}.\vec{dr}$

$\Rightarrow\ \Delta V = -E.\Delta r\cos\theta$

$\Rightarrow\ E = \frac{-\Delta V}{\Delta r\cos\theta}$

$\Rightarrow\ E = \frac{-(20-10)}{10\times10^{-2}\cos120°}$

$= \frac{-10}{10\times10^{-2}(-\sin30°)} = \frac{-10^2}{-1/2} = 200V/m$



Direction of E is perpendicular to the equipotential surface.

i.e., at 120° with x- axis

(b) As Electric field intensity is $\perp r$ to Potential surface

So, $E = \frac{kq}{r^2} = \frac{6}{r^2}V/m \quad \Rightarrow \quad \frac{kq}{0.1}=60\ V \ \Rightarrow \ kq = 6$

23. **Ans.** 2

ΔK of $q=\frac{1}{2}mv^2-0\left(\text{final zero at }\frac{a}{2}\right)$.

For q_1 work done by $\vec{E}$ is $+ve$, so kinetic energy increases by $\frac{1}{2}mv^2$

For KE $\frac{1}{2}mv^2+\frac{1}{2}mv^2=mv^2=\frac{1}{2}mv'^2\Rightarrow v'=\sqrt{2}v=\frac{2v}{\sqrt{2}}=N=2$

24. **Ans.** $-\frac{kq^2}{a}\left(3-\sqrt{2}\right)$

$W_{ext} = U_f - U_i$

25. **Ans.** 20

$q(V_{inner}) + 0 = q(V_{outer}) + \frac{1}{2}mv^2$

$\Rightarrow \; (-20)(-V) = \frac{1}{2}\times(1)\times V^2$

26. **Ans.** $2\tan^{-1}\left(\frac{\sigma q}{2\varepsilon_0 mg}\right)$

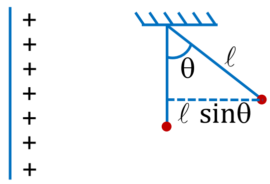
$W_g + W_{elect} = K_f - K_i$

$-mg\ell(1-\cos\theta)+\frac{\sigma}{2\varepsilon_0}q\ell\sin\theta = 0$

$-mg\ell(1-\cos\theta)=\frac{\sigma q}{2\varepsilon_0}\ell\sin\theta$

$mg2\sin^2\frac{\theta}{2}=\frac{\sigma q}{2\varepsilon_0}2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

$\frac{\theta}{2}=\tan^{-1}\left(\frac{\sigma q}{2\varepsilon_0 mg}\right)$



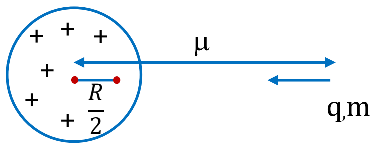
27. **Ans.** $\left[\frac{2KQq}{mR}\left(\frac{r-R}{r}+\frac{3}{8}\right)\right]^{1/2}$

$U_A + K_A = U_B + K_B$

$\frac{KQq}{r}+\frac{1}{2}mu_{min}^2=\frac{KQq}{2R^3}\left(3R^2-\frac{R^2}{4}\right)+0$

$\frac{1}{2}mu_{min}^2=\frac{11KQq}{8R}-\frac{KQq}{r}$

$u_{min} = \left[\frac{2KQq}{mR}\left(\frac{r-R}{r}+\frac{3}{8}\right)\right]^{1/2}$



28. **Ans.** 2

$\frac{W_1}{W_2}=\frac{qV_1}{qV_2}=\frac{q\left(\frac{KQ}{R}\right)}{q\left(\frac{3}{2}\frac{KQ}{R}\right)} \Rightarrow \frac{W_1}{W_2}=\frac{2}{3}$

29. **Ans.** 2

$\Delta KE + \Delta PE = 0$

$2.\left[\frac{1}{2}mv^2\right]+\left[0-\left(\frac{Kq^2}{4R}+\frac{2KqQ}{2R}\right)\right]=0$

$v = \sqrt{\frac{1}{m}\left(\frac{Kq^2}{4R}+\frac{KqQ}{R}\right)}$

30. Ans. 2

$V_A \propto \rho L^2$

Divide cube into smaller cube of length $\frac{L}{2}$

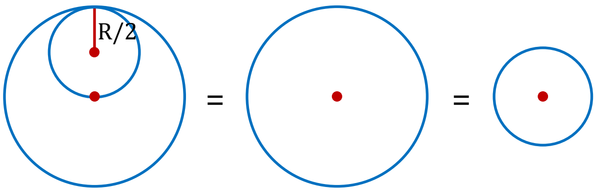
Therefore

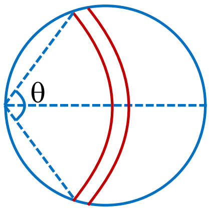
$$V_C \propto 8\cdot(\rho)\left(\frac{L}{2}\right)^2 \propto 2\rho L^2$$

$V_C \propto 2\rho L^2$

$\frac{V_A}{V_C} = \frac{1}{2}$

31. Ans. 1





$$\frac{\sigma}{2\varepsilon_0}(R-0)-\frac{\sigma R}{2\pi\varepsilon_0}$$

$$=\frac{\sigma R}{2\in_0}\left(1-\frac{1}{\pi}\right)=v_0$$

$$dv=\frac{k\sigma 2x\theta dx}{x}$$

$$x = 2R\cos\theta$$

$$dx = -2R\sin\theta\, d\theta$$

$$v=\frac{\sigma R}{2\pi\in_0}$$

32. Ans. 2

p = charge × (distance between mass centres of both the portions)

= $4\mu C \times 0.5\ m = 2\ \mu C - m$

33. Ans. 6

$\tau = 2qE\,3a\sin\theta + k2a\sin\theta\,.\,2a\cos\theta + 2k\,a\sin\theta\,a\cos\theta$

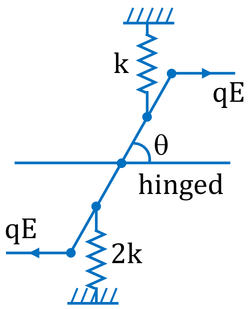
For small angular displacement

$\tau = 6aqE\,\theta + 6ka^2\theta$

$-18ma^2\frac{d^2\theta}{dt^2} = 6a(ak + qE)\theta$

$$\Rightarrow \frac{d^2\theta}{dt^2}+\frac{(ak+qE)}{3ma}\theta = 0$$

$$f=\frac{1}{2\pi}\sqrt{\frac{ak+qE}{3ma}}=\frac{1}{2\pi}\sqrt{\frac{9ka}{3ma}}=\frac{1}{2\pi}\sqrt{\frac{3k}{m}}=\frac{1}{2\pi}\sqrt{\frac{9k\pi^2}{k}}=1.5\sec^{-1}$$



34. **Ans.** 4

When connected all charge move to outer shell,

Heat = $U_i - U_f = \dfrac{KQ^2}{4a}$

35. **Ans.** 5

$$F_d = \frac{(m_1 + m_2)g}{2}$$

$$T + m_2 g = \frac{m_1 g}{2} + \frac{m_2 g}{2}$$

$$T = (m_1 - m_2)\frac{g}{2}$$

$$\therefore\ T = (2-1)\times\frac{10}{2} = 5N$$

36. **Ans.** $\left(\vec{g} = +\dfrac{\pi G\rho_0 R^3}{6}\left[\dfrac{1}{(x-(R/2))^2} - \dfrac{8}{x^2}\right], \vec{g} = -\dfrac{2\pi G\rho_0 R}{3}\hat{i}\right)$

$g(x) = g_{\text{solid sphere}} + g_{\text{hollow sphere}}$

$$= \frac{GM_{solid\ sphere}}{x^2} + \frac{GM_{hollow\ sphere}}{\left(x - \frac{R}{2}\right)^2}$$

$$= \frac{-G\rho_0\left(\frac{4}{3}\pi R^3\right)}{x^2} + \frac{G\rho_0\left[\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right]}{\left(x-\frac{R}{2}\right)^2}\hat{i}$$

$$g(x) = G\left(\frac{4}{3}\pi\rho_0 R^3\right)\left[\frac{-1}{x^2} + \frac{1}{8\left(x - \frac{R}{2}\right)^2}\right]\hat{i}$$

37. **Ans.** 2

Let mass of P, Q and S are m_1, m_2 and m_3 respectively

$$\frac{Gm_1}{R^2} = \frac{G(m_1 + m_2 + m_3)}{9R^2}$$

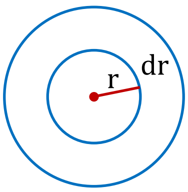
$8m_1 = m_2 + m_3$

38. **Ans.** 4

$dm = kr4\pi r^2 dr$

$m = k\pi r^4$

$E = \dfrac{Gm}{r^2}$



$$-dP \times 4\pi r^2 = G\frac{m}{r^2}dm$$

$$-dP = \frac{Gk\pi r^4 \times k4\pi r^3 dr}{4\pi r^4}$$

$-dP = Gk^2\pi r^3 dr$

$$-\int_{P=0}^{P} dP = Gk^2\pi\int_{r=R}^{r=r} r^3 dr$$

$$-P = Gk^2\pi\frac{r^4 - R^4}{4}$$

$$P = G\pi k^2\left(\frac{R^4 - r^4}{4}\right)$$

39. **Ans.** 7

$dm = \rho_0 r 4\pi r^2 dr$

Inside $g_1 = \frac{Gm}{x^2} = \frac{G\int_0^x \rho_0 \times 4\pi r^3 dr}{x^2}$

$$g_1 = \frac{G\rho_0\pi x^4}{x^2}$$

$g_1 = G\rho_0\pi x^2$

Outside-

$$g_2 = \frac{Gm}{y^2} = \frac{G\int_0^R \rho_0 \times 4\pi r^3 dr}{y^2} = \frac{G\rho_0\pi R^4}{y^2}$$

Given $g_1 = g_2 = \frac{1}{2}g_{surface}$

$g_{\text{surface}} = G\rho_0\pi R^2$

$$G\rho_0\pi x^2 = \frac{G\rho_0\pi R^4}{y^2} = \frac{1}{2}G\rho_0\pi R^2$$

$$\Rightarrow \quad x = \frac{R}{\sqrt{2}}, y = \sqrt{2}R$$

$$x + y = \frac{R}{\sqrt{2}} + \sqrt{2}R = \frac{3R}{\sqrt{2}}$$

$$\Rightarrow \quad x + y = R\sqrt{\frac{9}{2}}$$

$\alpha = 9$

$\beta = 2$

40. **Ans.** (1.6 hours if it is rotating from west to east, 24/17 hours if it is rotating from east to west)

$$\boxed{\omega = \frac{2\pi}{24} = \frac{\pi}{12}}$$

There can be two w' one from west to east and one from east to west.

$$w' = \frac{2\pi}{1.5} = \frac{4\pi}{3} \text{ rad/hr.}$$

For west to east

$w_{rel} = w - w'$

$= \frac{5\pi}{4}$ rad/hr.

$T = \frac{8}{5}$ hrs.

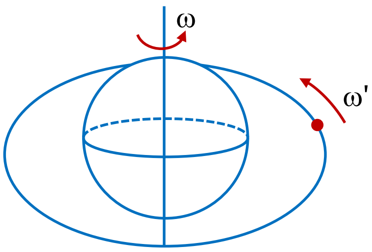
$= 1.6$ hrs.

For east to west

$w_{rel} = w + w'$

$= \frac{4\pi}{3} + \frac{\pi}{12} = \frac{17\pi}{12}$

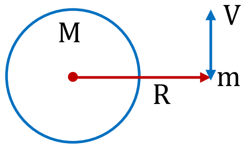
$T = \frac{24}{17}$ hrs



41. **Ans.** $1 \times 10^5 J$

$$\frac{GMm}{R^2} = \frac{mv^2}{R}$$

$\frac{1}{2}mv^2 = \frac{GMm}{2R}$ at any distance R



So for $PE = -\frac{GMm}{R}$ corresponding $KE = -\frac{PE}{2}$

$\Rightarrow$ For $U = -2 \times 10^5 J$ and $KE = -\frac{U}{2} = 10^5 J$

42. **Ans.** (a) $T = \frac{2\pi d^{3/2}}{\sqrt{3Gm}}$, (b) 2, (c) 2

(a) The COM of the system divides the distance between the stars in the inverse ratio of thin masses. If d_1 and d_2 are the distances of M and m from the COM. (COM = Centre of mass)

$$d_1 = \frac{d}{M+m} \times m \text{ and } d_2 = \frac{d}{M+m} \times M$$

The stars will rotate in circles of radii d_1 and d_2 about their COM. The same force of attraction provides the necessary centripetal force for their circular motion.

$$\therefore \quad \frac{GmM}{d^2} = M\omega_1^2 d_1 = m\omega_2^2 d_2$$

$$\text{or} \quad \omega_1^2 = \frac{Gm}{d^2 d_1} = \frac{Gm}{d^2} \times \frac{M+m}{d \times m} = \frac{G(M+m)}{d^3}$$

$$\text{and} \omega_1 = \omega_2 = \sqrt{\frac{G(M+m)}{d^3}}$$

(b) $\frac{L_m}{L_M} = \frac{md_2^2\omega}{Md_1^2\omega} = \frac{M}{m}\left(\frac{md_2}{Md_1}\right)^2$

As $md_2 = Md_1$

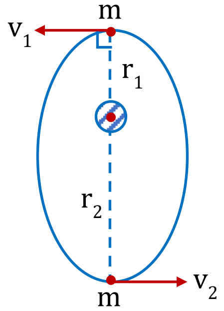
$$\Rightarrow \quad \frac{L_m}{L_M} = \frac{M}{m} = 2$$

(c) $\frac{K_M}{K_m} = \frac{\frac{1}{2}Mv_1^2}{\frac{1}{2}Mv_2^2} = \frac{M}{m} \times \frac{\omega_1^2 d_1^2}{\omega_2^2 d_2^2} = \frac{M}{m}\frac{d_1^2}{d_2^2}$

$$\Rightarrow \quad \frac{K_M}{K_m} = \frac{M}{m} \times \frac{m^2}{M^2} = \frac{m}{M}$$

43. **Ans.** 2

By conservation of angular momentum



$mv_1r_1 = mv_2r_2$

$$\frac{v_1}{v_2} = \frac{r_2}{r_1} = \frac{144 \times 10^6}{72 \times 10^6} = 2$$

44. **Ans.** 9

Circular orbits

$$T = 2\pi\sqrt{\frac{R^2}{GM_S}}$$

Binary stars

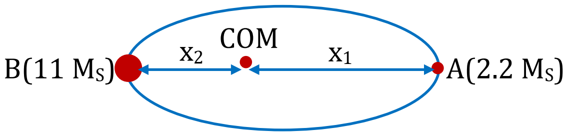
$$nT = 2\pi\sqrt{\frac{(9R)^3}{G(3M_S + 6M_S)}}$$

$$n \times 2\pi\sqrt{\frac{R^3}{GM_S}} = 9 \times 2\pi\sqrt{\frac{R^3}{GM_S}}$$

$n = 9$

45. **Ans.** 6

$x_1 + x_2 = d$ ...(i)



For COM

$M_1x_1 = M_2x_2$

$(2.2\ M_S)\ x_1 = (11\ M_S)x_2$

$x_1 = 5x_2$ ...(ii)

From equation (i) and (ii)

$$x_1 = \frac{5d}{6}\ \&\ x_2 = \frac{d}{6}$$

Then for circular motion

$$\frac{G(11M_S)(2.2M_S)}{d^2} = \frac{(11M_S)V_B^2}{x_2}$$

$$\frac{G(2.2M_S)}{d^2} = \frac{V_B^2}{\left(\frac{d}{6}\right)}$$

and $$\frac{G(11M_S)(2.2M_S)}{d^2} = \frac{(2.2M_S)V_A^2}{x_1}$$

$$\frac{G(11M_S)}{d^2} = \frac{V_A^2}{\left(5\frac{d}{6}\right)}$$

$$\sqrt{\frac{G(55M_S)}{6d}} = V_A$$

then

$$\frac{L_B}{L_A} = \frac{m_BV_Br_B}{m_AV_Ar_A} = \frac{(11M_S)\left(\sqrt{\frac{2.2GM}{6d}}\right)\left(\frac{d}{6}\right)}{(2.2M_S)\sqrt{\frac{55GM}{6d}}\left(\frac{5d}{6}\right)}$$

$$\frac{L_B}{L_A}=\frac{1}{5}\Rightarrow\frac{L_A}{L_B}=5$$

then, $\frac{\text{Total angular momentum}}{\text{Angular momentum of } B}$

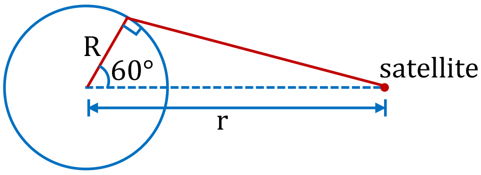
$$=\frac{L_A+L_B}{L_B}$$

$$=\frac{L_A}{L_B}+1$$

$= 5 + 1 = 6$

46. **Ans.** 2

$r\cos 60^{\circ}=R\Rightarrow r=2R$ orbital velocity $v_0=\sqrt{\frac{GM}{r}}=\sqrt{\frac{GM}{2R}}\Rightarrow\alpha=2$



47. **Ans.** 2

Energy in the orbit = $-\frac{GMm}{2r}$

Energy at the surface = $-\frac{GMm}{2R_e}$

Loss in energy = $\frac{GMm}{2}\left(\frac{1}{R_e}-\frac{1}{r}\right)$

Time required = $\frac{\text{Loss in energy}}{C}$

48. **Ans.** 3

Total mechanical energy

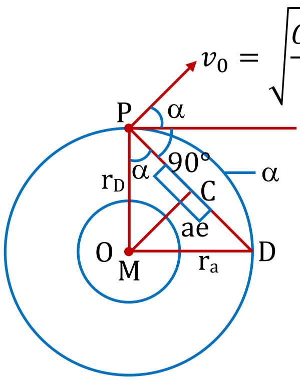
$$E=\frac{-GMm}{r_0}+\frac{1}{2}mv_0^2$$

$$=-\frac{GMm}{r_0}+\frac{1}{2}mv_0^2$$

$$=-\frac{GMm}{r_0}+\frac{GMm}{2r_0}$$

$$E=-\frac{GMm}{2r_0}$$

$$\frac{-GMm}{2a}=-\frac{GMm}{2r_0}$$



∴ Length of semi-major axis, $a = r_0$

Hence point 'P' must be an extreme

Point of the minor axis of the elliptical orbit

∵ $a\sin\alpha = ae$

$e = \sin\alpha = \sin 37° = 0.6$

Hence the eccentricity of the elliptical orbit

$e = 0.60$

49. **Ans.** (a) $\Delta E = \frac{GMm}{2r_1r_2}(r_2 - r_1)$ (b) $\Delta E_A = \frac{r_2\Delta E}{r_1 + r_2}$, $\Delta E_B = \frac{r_1}{r_1 + r_2}\Delta E$

For a satellite, orbiting in a circular orbit or radius r,

We have,

$$\frac{GMm}{r^2} = \frac{mv^2}{r} \text{ or } mv^2 = \frac{GMm}{r}$$

K.E. = $\frac{1}{2}\frac{GMm}{r}$ and $P.E. = -\frac{GMm}{r}$

Total Energy $E = -\frac{GMm}{2r}$

Total energy of the satellite orbiting in circular orbit of radius r_1 is given by

$$E_1 = -\frac{GMm}{2r_1}$$

Similarly, for an orbit or radius r_2

$$E_2 = -\frac{GMm}{2r_2}$$

So, additional energy imparted will be

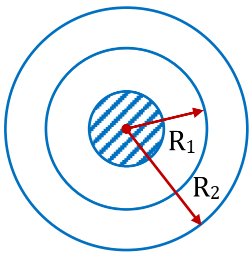
$$E_2 - E_1 = -\frac{GMm}{2r_2} - \left(-\frac{GMm}{2r_1}\right)$$

$$= \frac{GMm(r_2 - r_1)}{2r_1r_2}$$

50. **Ans.** 8

For astronaut $\sqrt{\frac{GM}{R_2}} = v_{ast}$

For body, $\sqrt{\frac{GM}{R_1}} = v_{body}$



Given, $v_{ast} - v_{body} = 5\ m/s$

$v_{ast} = 5000\ m/s$

$\therefore\ R_2 = \frac{GM}{(5000)^2}$, $R_1 = \frac{GM}{(5005)^2}$

$\therefore\ R_2 - R_1 = GM\left[\frac{1}{(5000)^2} - \frac{1}{(5005)^2}\right] \approx 32km$

51. **Ans.** 8

Conserving angular momentum $m(v_1 \cos 60°)\, 4R = mv_2R \Rightarrow \frac{v_2}{v_1} = 2$.

Conserving energy of the system $-\frac{GMm}{4R} + \frac{1}{2}mv_1^2 = -\frac{GMm}{R} + \frac{1}{2}mv_2^2$

$$\Rightarrow \frac{1}{2}v_2^2 - \frac{1}{2}v_1^2 = \frac{3}{4}\frac{GM}{R}$$

$$\Rightarrow v_1^2 = \frac{1}{2}\frac{GM}{R};\ v_1 = \frac{1}{\sqrt{2}}\sqrt{64\times10^6} = \frac{8000}{\sqrt{2}}m/s \Rightarrow X = 8$$

52. **Ans.** 2

$$v_{esc} = \sqrt{2gR} = 11$$

$$v'_{esc} = \sqrt{\frac{2g}{11} \times R'}$$

$$g = \frac{4}{3\pi} G\rho R$$

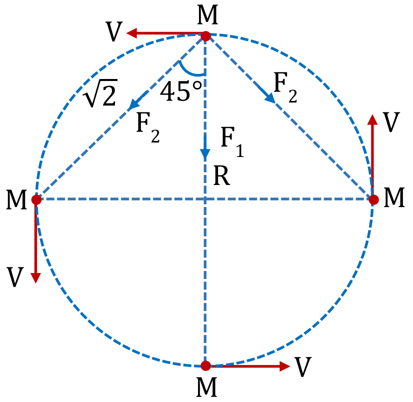
$$\frac{1}{11} \times \frac{4}{3} \pi G\rho R = \frac{g}{11} = \frac{4}{3} \pi G \frac{\rho}{4} R'$$

$$R' = \frac{4}{11} R$$

$$\Rightarrow \quad V'_{sec} = \sqrt{\frac{2g}{11} \times \frac{4R}{11}} = \frac{2}{11} \times 11 = 2km/s$$

EXERCISE - JEE (Main) PYQ

1. **Ans. (2)**



Net force on one particle

$$F_{net} = F_1 + 2F_2 \cos 45° = \text{Centripetal force}$$

$$\Rightarrow \quad \frac{GM^2}{(2R)^2} + \left[\frac{2GM^2}{(\sqrt{2}R)^2} \cos 45°\right] = \frac{MV^2}{R}$$

$$V = \frac{1}{2}\sqrt{\frac{GM}{R}(1+2\sqrt{2})}$$

2. **Ans. (4)**

By principle of superposition

$$V = -\frac{GM}{2R^3}\left[3R^2 - \frac{R^2}{4}\right] + \frac{3G\left(\frac{M}{8}\right)}{2\left(\frac{R}{2}\right)}$$

$$= \frac{-11\,GM}{8R} + \frac{3GM}{8R} = -\frac{GM}{R}$$

3. **Ans. (1)**

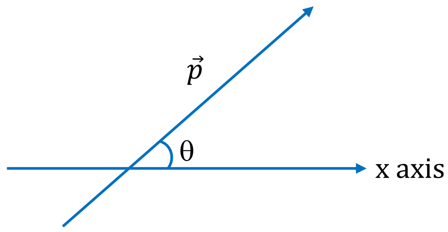
So from $\vec{\tau} = \vec{p} \times \vec{E}$

$$\tau\hat{k} - \tau\hat{k} = \left(p_x\hat{\imath} + p_y\hat{\jmath}\right) \times \left(E\hat{\imath} + \sqrt{3}E\hat{\jmath}\right)$$

$$= p_x \times \sqrt{3}E\ \hat{k} + p_y E\left(-\hat{k}\right)$$

$$0 = E\hat{k}\left(\sqrt{3}p_x - p_y\right)$$

$$\frac{p_y}{p_x} = \sqrt{3}$$



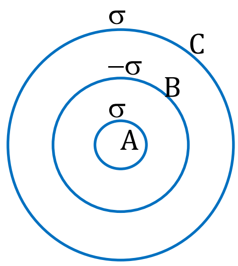
$\therefore \quad \tan\theta = \sqrt{3}$

$\theta = 60°$

4. **Ans. (1)**

$V_{\text{outside}} = \dfrac{KQ}{r}$, where r is distance of point from the centre of shell

$V_{\text{inside}} = \dfrac{KQ}{R}$, where '$R$' is radius of the shell



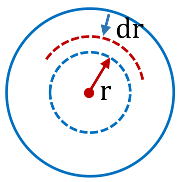
$$V_B = \frac{Kq_A}{r_b} + \frac{Kq_B}{r_b} + \frac{Kq_C}{r_c}$$

$$V_B = \frac{1}{4\pi\epsilon_0}\left[\frac{\sigma 4\pi a^2}{b} - \frac{\sigma 4\pi b^2}{b} + \frac{\sigma 4\pi c^2}{c}\right]$$

$$V_B = \frac{\sigma}{\epsilon_0}\left[\frac{a^2 - b^2}{b} + c\right]$$

5. **Ans. (4)**

$$Q = \int \rho dv = \int_0^R \frac{A}{r^2} e^{-2r/a}\left(4\pi r^2 dr\right)$$



$$= \int_0^R \frac{A}{r^2} e^{-2r/a}\left(4\pi r^2 dr\right) = 4\pi A\int_0^R e^{-2r/a} dr$$

$$= 4\pi A\left(\frac{e^{-2r/a}}{-\frac{2}{a}}\right)_0^R = 4\pi A\left(-\frac{a}{2}\right)\left(e^{-2R/a} - 1\right)$$

$$Q = 2\pi a A(1 - e^{-2R/a})$$

$$R = \frac{a}{2}\log\left(\frac{1}{1 - \frac{Q}{2\pi a A}}\right)$$

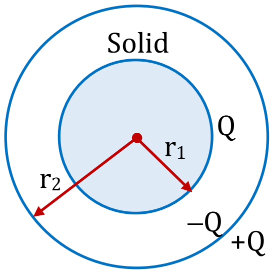
6. **Ans. (1)**

As given in the first condition :

Both conducting spheres are shown.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1}\right) - \left(\frac{kQ}{r_2}\right)$$

$$= kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$

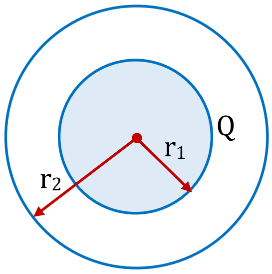


In the second condition :

Shell is now given charge $-4Q$.

$$V_{in} - V_{out} = \left(\frac{kQ}{r_1} - \frac{4kQ}{r_2}\right) - \left(\frac{kQ}{r_2} - \frac{4kQ}{r_2}\right)$$

$$= \frac{kQ}{r_1} - \frac{kQ}{r_2} \quad = kQ\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = V$$



Hence, we also obtain that potential difference does not depend on charge of outer sphere.

$\therefore$ P.d. remains same.

7. **Ans. (2)**

$U_i + K_i = U_f + K_f$

$$\frac{kq^2}{\sqrt{16a^2 + 9a^2}} + \frac{1}{2}mu^2 = \frac{kq^2}{3a}$$

$$\frac{1}{2}mu^2 = \frac{kq^2}{a}\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2kq^2}{15a}$$

$$u = \sqrt{\frac{4kq^2}{15ma}}$$

8. **Ans. (3)**

$$E\,4\pi a^2 = \frac{\int_0^a kr\,4\pi r^2\,dr}{\varepsilon_0}$$

$$E = \frac{k\,4\pi a^4}{4 \times 4\pi\varepsilon_0}$$

$$2Q = \int_0^R kr\,4\pi r^2\,dr$$

$$k = \frac{2Q}{\pi R^4}$$

$$QE = \frac{1}{4\pi\varepsilon_0}\,\frac{QQ}{(2a)^2}$$

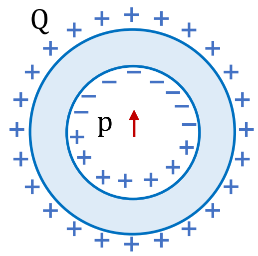
$R = a8^{1/4}$

9. **Ans. (1)**

Total charge of dipole = 0, so charge induced on outside surface = 0.

But due to non-uniform electric field of dipole, the charge induced on inner surface is non-zero and non-uniform.

So, for any observer outside the shell, the resultant electric field is due to Q uniformly distributed on outer surface only and it is equal to.



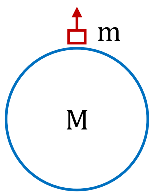
$$E = \frac{KQ}{r^2}$$

10. **Ans. (2)**

Minimum energy required (E) = – (Potential energy of object at surface of earth)

$$= -\left(-\frac{GMm}{R}\right) = \frac{GMm}{R}$$

Now $M_{\text{Earth}} = 64\ M_{\text{Moon}}$



$$\rho.\frac{4}{3}\pi R_e^3 = 64.\frac{4}{3}\pi R_m^3$$

$\Rightarrow \quad R_e = 4R_m$

Now $\dfrac{E_{\text{Moon}}}{E_{\text{Earth}}} = \dfrac{M_{\text{Moon}}}{M_{\text{Earth}}}.\dfrac{R_{\text{Earth}}}{R_{\text{Moon}}} = \dfrac{1}{64} \times \dfrac{4}{1}$

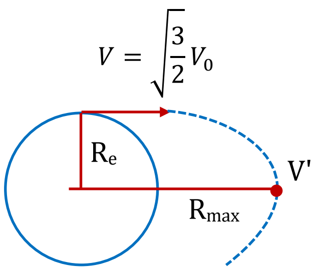
$\Rightarrow \quad E_{\text{Moon}} = \dfrac{E}{16}$

11. **Ans. (2)**

$$V_0 = \sqrt{\frac{GM}{R_e}}$$

$$\frac{-GMm}{R_e} + \frac{1}{2}mv^2 = \frac{-GMm}{R_{\max}} + \frac{1}{2}mv'^2 \quad \text{...(i)}$$

$VR_e = V'R_{max}$ ...(ii)

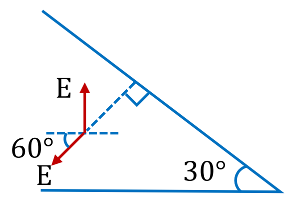


Solving (i) and (ii)

$$\boxed{R_{\max} = 3R_e}$$

12. **Ans. (2)**

Electric field due to each sheet is uniform and equal to $E = \dfrac{\sigma}{2\varepsilon_0}$



Now net electric field between plates

$$\vec{E}_{net} = E\cos 60°(-\hat{x}) + (E - E\sin 60°)(\hat{y})$$

$$= \frac{\sigma}{2\varepsilon_0}\left[-\frac{\hat{x}}{2} + \left(1 - \frac{\sqrt{3}}{2}\right)\hat{y}\right]$$

13. **Ans. (4)**

Fill the empty space with +ρ and −ρ charge density.

$$|E_A| = 0 + \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{R}{2}\right)^2} = k\rho\frac{4}{3}\pi\left(\frac{R}{2}\right)$$

$$|E_B| = \frac{k\rho.\frac{4}{3}\pi R^3}{R^2} - \frac{k\rho.\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}{\left(\frac{3R}{2}\right)^2}$$

$$= k\rho\frac{4}{3}\pi R - k\rho\frac{4}{3}\pi\frac{R}{18} = k\rho.\frac{4}{3}\pi\left(\frac{17R}{18}\right)$$

$$\frac{E_A}{E_B} = \frac{9}{17} = \frac{18}{34}$$

14. **Ans. (1)**

Now $Q_1 + Q_2 = Q'_1 + Q'_2 = 12\ \mu C - 3\ \mu C = 9\ \mu C$

and $V_1 = V_2 \Rightarrow \frac{KQ'_1}{\frac{2R}{3}} = \frac{KQ'_2}{\frac{R}{3}}$

$$Q'_1 = 2Q'_2 \Rightarrow 2Q'_2 + Q'_2 = 9\mu C$$

$\Rightarrow$ $Q'_2 = 3\ \mu C$

and $Q'_1 = 6\ \mu C$

15. **Ans. (1)**

$$\frac{kQq}{R} + mgy = \frac{kQq}{R+y} + \frac{1}{2}mv^2$$

$$v^2 = 2gy + \frac{2kQqy}{mR(R+y)}$$

16. **Ans. (226)**

From symmetry $\phi = \frac{1}{6}\left(\frac{q}{\varepsilon_0}\right)$

$$= \frac{12\times10^{-6}}{6\times8.85\times10^{-12}}$$

$$= 225.98 \times 10^3\ \frac{Nm^2}{s}$$

$$\simeq 226\times10^3\frac{Nm^2}{C}$$

17. **Ans. (640)**

$\phi = E_x A \Rightarrow \frac{2}{5} \times 4 \times 10^3 \times 0.4 = 640$

18. Ans. (2)

Energy is maximum when mass is split equally so $\frac{M}{m} = 2$.

19. Ans. (4)

$$T' = T\left(\frac{3R}{R}\right)^{3/2} = 3\sqrt{3}\,T$$

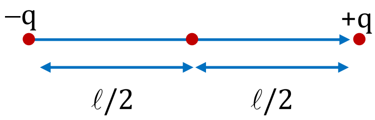
20. Ans. (1)

$$\frac{kq^2}{L^2} = \mu mg \Rightarrow L = \sqrt{\frac{k}{\mu mg}}q$$

21. Ans. (3)

Note: Dipole will rotate about COM but no option is matching with this calculation.

If we assume dipole to oscillate about midpoint



$$\tau = -PE\,.\,\theta$$

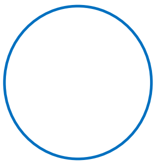
$$\Rightarrow \quad \left[m\frac{l^2}{4} + 2m\frac{l^2}{4}\right]\alpha = -qlE.\theta$$

$$\Rightarrow \quad \frac{3ml^2}{4}\alpha = -qlE.\theta \Rightarrow \alpha = -\frac{4qE}{3ml}\theta$$

$$\omega = \sqrt{\frac{4qE}{3ml}}$$

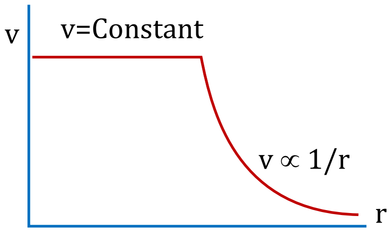
Correct Ans. is (3)

22. Ans. (1)



$$V_{inside} = \frac{kQ}{R}$$

$$V_{outside} = \frac{kQ}{r}$$



23. Ans. (4)

Planets revolve in elliptical paths around sun. Thus, their linear speed is not constant

24. Ans. (2)

$$\Rightarrow\ g' = \frac{gR^2}{r^2} = \frac{gR^2}{\left(R+\frac{R}{4}\right)^2} = \frac{16g}{25}$$

$$\therefore\ \text{Weight} = \frac{16}{25}\times 100 = 64N$$

EXERCISE - JEE (Advanced) PYQ

1. Ans. (B)

$$E_G = \frac{4\pi G r\rho}{3}$$

$dF = E_G\lambda dr$

$$F = \int_{\frac{4R}{5}}^{R} \frac{4\pi G\rho\lambda}{3} r\,dr = \frac{4\pi G\rho\lambda}{3}\left[\frac{r^2}{2}\right]_{\frac{4R}{5}}^{R}$$

$$= \frac{4\pi G\rho\lambda}{3\times 2}\left[R^2 - \frac{16R^2}{25}\right] = \frac{4\pi}{6}G\rho\lambda\times\frac{9}{25}R^2$$

$$F = \frac{4\pi}{6}G\times\frac{M}{\frac{4\pi}{3}R_e^3}\times\lambda\times\frac{9}{25}\times\frac{R_e^2}{100}$$

After solving $[F \Rightarrow 108\ N]$.

2. Ans. (C)

Point charge | Line charge | Infinite sheet

$$E_1(r_0) = \frac{Q}{4\pi\in_0 r_0^2} \qquad E_2(r_0) = \frac{\lambda}{2\pi\in_0 r_0} \qquad E_3(r_0) = \frac{\sigma}{2\in_0}$$

Given

$E_1(r_0) = E_2(r_0) = E_3(r_0)$

$$\frac{Q}{4\pi\in_0 r_0^2} = \frac{\lambda}{2\pi\in_0 r_0} = \frac{\sigma}{2\in_0} \qquad \text{...(i)}$$

So, $$\frac{Q}{4\pi\in_0 r_0^2} = \frac{\lambda}{2\pi\in_0 r_0} \Rightarrow Q = 2\lambda r_0$$

Now, $$E_1(r_0/2) = \frac{Q}{4\pi\in_0\left(\frac{r_0}{2}\right)^2} = \frac{Q}{\pi\in_0 r_0^2} = \frac{2\lambda r_0}{\pi\in_0 r_0^2} = \frac{2\lambda}{\pi\in_0 r_0}$$

$$E_2(r_0/2) = \frac{\lambda}{2\pi\in_0\frac{r_0}{2}} = \frac{\lambda}{\pi\in_0 r_0} = \frac{E_1(r_0/2)}{2} \neq 4E_3$$

From equation (i)

$$\frac{Q}{4\pi\in_0 r_0^2} = \frac{\sigma}{2\in_0} \Rightarrow Q = 2\sigma\pi r_0^2$$

Also, from equation (i)

$$\frac{\lambda}{2\pi\in_0 r_0} = \frac{\sigma}{2\in_0} \Rightarrow r_0 = \frac{\lambda}{\sigma\pi}$$

3. Ans. (C)

$$E_1 = \frac{KQ}{R^2}$$

$$E_2 = \frac{K(2Q)}{R^2}$$

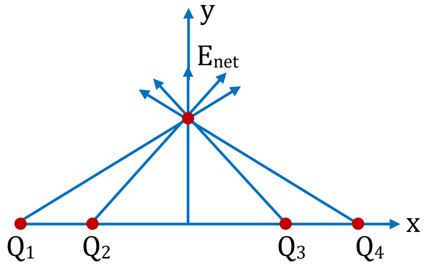
$$E_3 = \frac{K(4Q)}{(2R)^3} \times R = \frac{KQ}{2R^2}$$

$E_2 > E_1 > E_3$

4. Ans. (A)

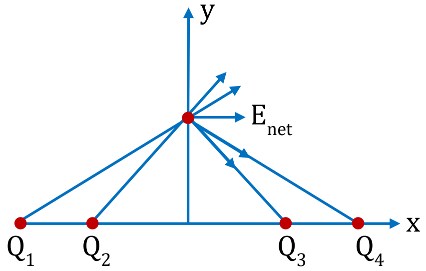
When all are positive net field is along $+y$ axis

$P \to 3$



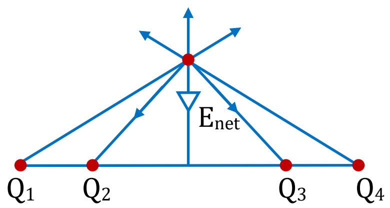
when Q_1, Q_2 positive Q_3, Q_4 negative

$Q \to 1$



When Q_1, Q_4 positive and Q_2, Q_3 negative

$R \to 4$

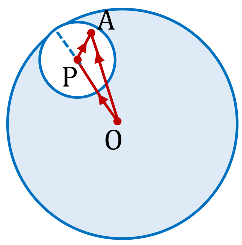


5. Ans. (D)

Considering any point, A inside the cavity electric field can be calculated at A by using super position principle.

$$\vec{E}_A = (\vec{E}_A)_{\text{Due to sphere without cavity}}$$
$$-(\vec{E}_A)_{\text{Due to sphere of radius } R_2 \text{ \& centre at } P}$$

$$\vec{E}_A = \frac{\rho}{3\epsilon_0} O\vec{A} - \frac{\rho}{3\epsilon_0}(P\vec{A})$$



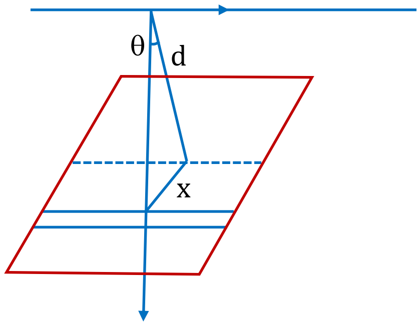
But $O\vec{P} + P\vec{A} = O\vec{P}$

$\therefore \quad O\vec{A} - P\vec{A} = O\vec{P}$

$$\therefore \quad \vec{E}_A = \frac{\rho}{3\in_0}(\vec{OP}) = \frac{\rho}{3\in_0}(\vec{a})$$

So E.F. is independent of location of A inside the cavity.

6. Ans. (6)



$$d\phi = \frac{\lambda}{2\pi\in_0\sqrt{d^2+x^2}} \cdot \frac{d}{\sqrt{d^2+x^2}} L dx$$

$$\phi = \frac{\lambda d L}{2\pi\in_0}\int_{-a/2}^{a/2}\frac{dx}{d^2+x^2}$$

$$= \frac{\lambda(d)(L)}{2\pi\in_0}\left(\frac{1}{d}\right)2\left[\tan^{-1}\frac{x}{d}\right]_0^{a/2} = \frac{\lambda L}{\pi\in_0}\left(\tan^{-1}\frac{a}{2.\frac{a\sqrt{3}}{2}}\right) = \frac{\lambda L}{6\pi\in_0}$$

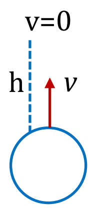
7. Ans. (C)

In first case if q is shifted towards one of the wires, its repulsive force will increase which will bring the charge back.

In second case, in same type of displacement attractive force due to wire increases which will make it move further towards wire.

8. Ans. (2)

$$g' = \frac{g}{\left(1+\frac{h}{R}\right)^2} = \frac{g}{4}$$



$$1 + \frac{h}{R} = 2$$

$\boxed{h=R}$ ...(i)

So, velocity of particle becomes zero at $h = R$

Given, $v_{esc} = v\sqrt{N}$

So, $\sqrt{\frac{2GM}{R}} = v\sqrt{N}$ ...(ii)

Applying conservation of energy

$$\frac{-GMm}{R} + \frac{1}{2}mv^2 = -\frac{GMm}{2R} + 0$$

On solving

$$v^2 = \frac{GM}{R}$$

So, $v = \sqrt{\frac{GM}{R}}$ putting in equation (ii)

$$\sqrt{\frac{2GM}{R}} = \sqrt{\frac{GM}{R}}\sqrt{N}$$

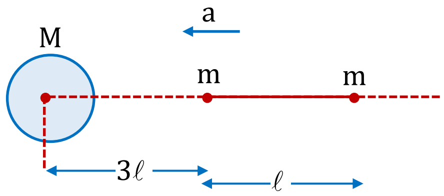
Comparing $N = 2$.

9. Ans. (7)

Due to gravitational interaction connected masses have some acceleration.

Let both small masses are moving with acceleration 'a' towards larger mass M.

$$\frac{GMm}{9\ell^2}-\frac{Gmm}{\ell^2}=\frac{GMm}{16\ell^2}+\frac{Gmm}{\ell^2}$$



Force on middle mass (m) = Force on mass (m) (right side)

$$\frac{7M}{144}=2m$$

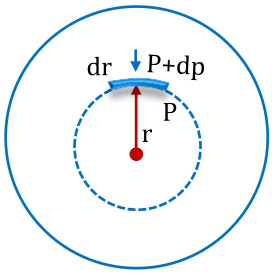
$\Rightarrow \quad m=\frac{7M}{288}=k\left(\frac{M}{288}\right)$

$\Rightarrow \quad K = 7$

10. Ans. (BC)

Gravitational field at a distance r : $g=\frac{GM}{R^3}.r$

Taking a small element of surface area dA and thickness dr.



$$F=\left[P-(P+dP)\right].dA=\frac{GM}{R^3}.r.dm$$

$$-dP.dA=\frac{GM}{R^3}.r\left[\rho.dA.dr\right] \quad \{\because dM = \rho.dA.\ dr\}$$

$$\int_0^P -dP=\frac{GM\rho}{R^3}\int_R^r rdr$$

$$P=\frac{GM\rho}{2R^3}\left[R^2-r^2\right]$$

11. Ans. (BD)

We know,

$$F=\frac{1}{4\pi\varepsilon_0}\frac{q^2}{r^2} \qquad \Rightarrow \frac{q^2}{\varepsilon_0}=(Fr^2)4\pi$$

So, dimension

$$\frac{q^2}{\varepsilon_0}=\dim(Fr^2) = MLT^{-2}\times L^2 = ML^3T^{-2}$$

Similarly; $E=\frac{3}{2}K_BT$

$\Rightarrow \quad \dim(K_BT) = \dim(\text{Energy}) = ML^2T^{-2}$

(A) $\sqrt{\frac{nq^2}{\varepsilon k_BT}}=\sqrt{\frac{L^{-3}\times ML^3T^{-2}}{ML^2T^{-2}}}=\frac{1}{L}$

(B) $\sqrt{\frac{(E)\times vol}{Fr^2}}=\sqrt{\frac{ML^2T^{-2}\times L^3}{MLT^{-2}\times L^2}}=L$

(C) $\sqrt{\frac{Fr^2(vol)^{2/3}}{(K\varepsilon)}}=\sqrt{\frac{MLT^{-2}\times L^2\times L^2}{ML^2T^{-2}}}=\sqrt{L^3}=L^{3/2}$

(D) $\sqrt{\frac{Fr^2(vol)^{1/3}}{\text{Energy}}}=\sqrt{\frac{MLT^{-2}L^2\times L}{ML^2T^{-2}}}=L \quad [\therefore \text{dimension } n = \dim\left(\frac{1}{vol}\right)=L^{-3}]$

12. Ans. (AB)

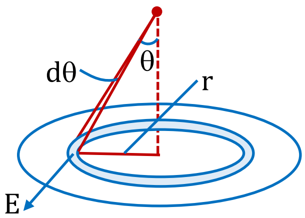
Every point on circumference of flat surface is at equal distance from point charge

Hence circumference is equipotential.

Flux passing through curved surface = – flux passing through flat surface.

$(d\phi)_{\text{through the ring}} = E\cos\theta . dA$

$$= \frac{1}{4\pi\epsilon_0}\frac{Q}{\left(\sqrt{r^2+R^2}\right)^2}\frac{R}{\sqrt{R^2+r^2}}.2\pi r dr$$



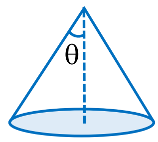
$$\therefore \quad \int d\phi = \frac{QR}{4\pi\epsilon_0}2\pi\int_0^R \frac{r dr}{\left(R^2+r^2\right)^{3/2}} = \frac{q}{2\epsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\therefore \quad \text{Flux through curved surface} = -\frac{q}{2\epsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

Note : Flux through surface can be calculated using concept of solid angle.

$$\Omega = 2\pi(1-\cos\theta) = 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\therefore \quad \text{Solid angle subtended} = 2\pi\left(1-\frac{1}{\sqrt{2}}\right)$$



$$\phi \text{ for } 4\pi \text{ solid angle} = \frac{q}{\epsilon_0}$$

$$\therefore \quad \phi \text{ for } 2\pi\left(1-\frac{1}{\sqrt{2}}\right) \text{ solid angle} = \frac{q}{4\pi\epsilon_0}.2\pi\left(1-\frac{1}{\sqrt{2}}\right)$$

$$= \frac{q}{2\epsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$$

13. Ans. (C)

Given, $v_e = 11.2 km/\sec = \sqrt{\frac{2GM_e}{R_e}}$

From energy conservation

$$\frac{1}{2}mv_s^2 - \frac{GM_s m}{r} - \frac{GM_e m}{R_e} = 0 - 0$$

where r = distance of rocket from Sun

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2GM_s}{r}}$$

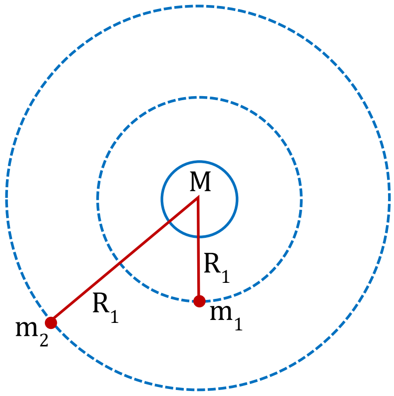
Given, $M_s = 3 \times 10^5 M_e$ and $r = 2.5 \times 10^4 R_e$

$$\Rightarrow \quad v_s = \sqrt{\frac{2GM_e}{R_e} + \frac{2G3\times10^5 M_e}{2.5\times10^4 R_e}} = \sqrt{\frac{2GM_e}{R_e}\left(1+\frac{3\times10^5}{2.5\times10^4}\right)} = \sqrt{\frac{2GM_e}{R_e}\times 13}$$

$$\Rightarrow \quad v_s \simeq 42 km/s$$

14. Ans. (B)

$$\left.\begin{array}{l}\dfrac{m_1}{m_2}=2\\[2mm]\dfrac{R_1}{R_2}=\dfrac{1}{4}\end{array}\right\}\text{given}$$



$$\frac{GMm_1}{R_1^2}=\frac{m_1v_1^2}{R_1}$$

$$v_1^2=\frac{GM}{R_1},\ v_2^2=\frac{GM}{R_2}$$

$$\frac{v_1^2}{v_2^2}=\frac{R_2}{R_1}=4$$

(P) $\dfrac{v_1}{v_2}=2$

(Q) $L = mvR$

$$\frac{L_1}{L_2}=\frac{m_1v_1R_1}{m_2v_2R_2}=2\times2\times\frac{1}{4}=1$$

(R) $K=\dfrac{1}{2}mv^2$

$$\frac{K_1}{K_2}=\frac{m_1v_1^2}{m_2v_2^2}=2\times(2)^2=8$$

(S) $T = 2\pi R/V$

$$\frac{T_1}{T_2}=\frac{R_1}{v_1}\times\frac{v_2}{R_2}=\frac{R_1}{R_2}\times\frac{v_2}{v_1}=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

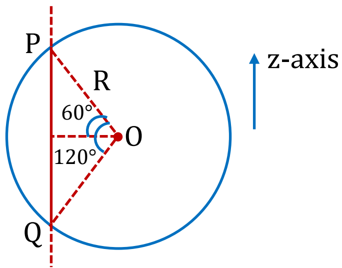
15. Ans. (AB)

Field due to straight wire is perpendicular to the wire and radially outward.

Hence $E_z = 0$

Length, $PQ = 2R\sin 60 = \sqrt{3}R$ According to Gauss's law

Total flux = $\oint \vec{E}.\vec{ds} = \dfrac{q_{in}}{\epsilon_0}=\dfrac{\lambda\sqrt{3}R}{\epsilon_0}$



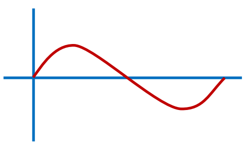
16. Ans. (2 [1.99, 2.01])

$n = 10^{-3}$ kg, $q = 1C$, at $t = 0$

$E = E_0 \sin \omega t$

Force on particle will be

$F = qE = qE_0 \sin \omega t$



At v_{max}, a, $F = 0$, $qE_0 \sin \omega t = 0$

$F = qE_0 \sin \omega t$

$$\frac{dv}{dt}=q\frac{E_0}{m}\sin\omega t$$

$$\int_0^v dv=\int_0^{\pi/\omega}\frac{qE_0}{m}\sin\omega t\,dt$$

$$v - 0 = \frac{qE_0}{m\omega}[-\cos\omega t]_0^{\pi/\omega}$$

$$v - 0 = \frac{qE_0}{m\omega}[(-\cos\pi)-(-\cos 0)]$$

$$v = \frac{1\times 1}{10^{-3}10^{3}}\times 2 = 2\ m/s$$

17. Ans. (B)

(i) $E=\frac{KQ}{d^2}\Rightarrow E\propto\frac{1}{d^2}$

(ii) Dipole

$$E = \frac{2kp}{d^3}\sqrt{1+3\cos^2\theta}$$

$E\propto\frac{1}{d^3}$ for dipole

(iii) For line charge

$$E=\frac{2k\lambda}{d}$$

$$E\propto\frac{1}{d}$$

(iv) $E=\frac{2K\lambda}{d-\ell}-\frac{2K\lambda}{d+\ell}\ =2K\lambda\left[\frac{d+\ell-d+\ell}{d^2-\ell^2}\right]$

$$E=\frac{2K\lambda(2\ell)}{d^2\left[1-\frac{\ell^2}{d^2}\right]}$$

$$E\propto\frac{1}{d^2}$$

(v) Electric field due to sheet

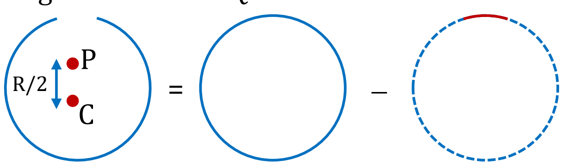
E is independent of r

18. Ans. (A)

Let charge on the sphere initially be Q.

$\therefore\quad \frac{kQ}{R}=V_0$

and charge removed = αQ

(A) 

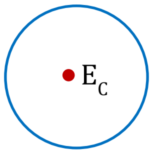
and $V_p = \frac{kQ}{R}-\frac{2K\alpha Q}{R}=\frac{kQ}{R}(1-2\alpha)$

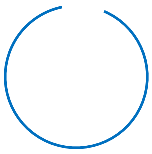
$$V_C = \frac{kQ(1-\alpha)}{R}$$

$$\therefore\ \frac{V_C}{V_p}=\frac{1-\alpha}{1-2\alpha}$$

(B) $(E_C)_{\text{initial}}$ = zero

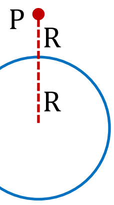
$(E_C)_{\text{final}} = \dfrac{k\alpha Q}{R^2}$

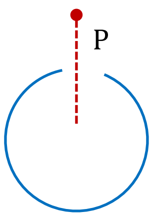




$\Rightarrow$ Electric field increases

(C) $(E_P)_{\text{initial}} = \dfrac{kQ}{4R^2}$

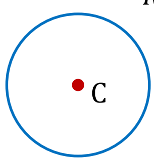


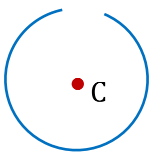


$(E_P)_{\text{final}} = \dfrac{kQ}{4R^2} - \dfrac{k\alpha Q}{R^2}$

$\Delta E_P = \dfrac{kQ}{4R^2} - \dfrac{kQ}{4R^2} + \dfrac{k\alpha Q}{R^2} = \dfrac{k\alpha Q}{R^2} = \dfrac{V_0\alpha}{R}$

(D) $(V_C)_{\text{initial}} = \dfrac{kQ}{R}$





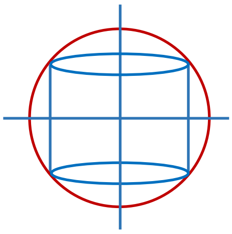
$(V_C)_{\text{final}} = \dfrac{kQ(1-\alpha)}{R}$

$\Delta V_C = \dfrac{kQ}{R}(\alpha) = \alpha V_0$

19. **Ans. (ABD)**

For option (A), cylinder encloses the shell, thus option is correct.

For option (B),

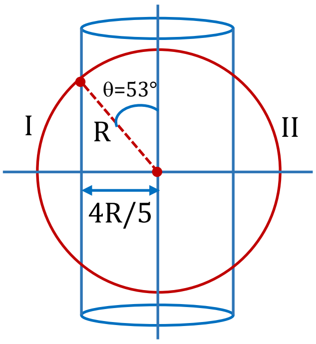


Cylinder perfectly enclosed by shell,

Thus $\phi = 0$, so option is correct.

For option (C)

$$\phi = \frac{2\times Q}{2\in_0}\left(1-\cos 53^\circ\right) = \frac{2Q}{5\in_0}$$



For option (D) :

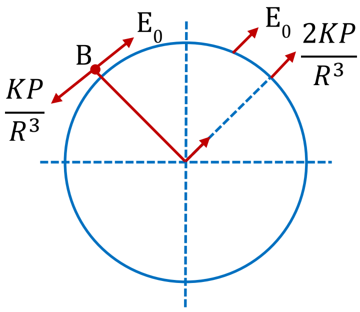
Flux enclosed by cylinder

$$= \phi = \frac{2Q}{2\in_0}\left(1-\cos 37^\circ\right) = \frac{Q}{5\in_0}$$

20. **Ans. (AD)**

(A) $\vec{P} = \frac{P_0}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

E.F. at *B* along tangent should be zero since circle is equipotential.



So, $E_0 = \frac{K|\vec{P}|}{R^3}$ and $E_B = 0$

So, $R^3 = \frac{KP_0}{E_0} = \left(\frac{P_0}{4\pi\in_0 E_0}\right)$

So $R = \left(\frac{P_0}{4\pi\in_0 E_0}\right)^{1/3}$

So, (A) is correct.

(B) Because E_0 is uniform and due to dipole *E.F.* is different at different points, so magnitude of total *E.F.* will also be different at different points.

So, (B) is incorrect

(C) $E_A = \frac{2KP}{R^3} + \frac{KP}{R^3} = 3\frac{KP}{R^3}\frac{P_0}{\sqrt{2}}\left(\hat{i}+\hat{j}\right)$

So, (C) is wrong

(D) $E_B = 0$

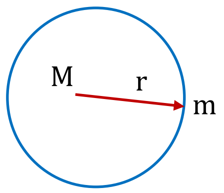
So, (D) is correct.

21. Ans. (D)

Let total mass included in a sphere of radius r be M.

For a particle of mass m,

$$\frac{GMm}{r^2}=\frac{mv^2}{r}$$



$$\Rightarrow \quad \frac{GMm}{r}=2K$$

$$\Rightarrow \quad M=\frac{2Kr}{Gm}$$

$$\therefore \quad dM=\frac{2Kdr}{Gm}$$

$$\Rightarrow \quad (4\pi r^2 dr)\rho=\frac{2K\,dr}{Gm}$$

$$\Rightarrow \quad \rho=\frac{K}{2\pi r^2 m^2 G}$$

$$\therefore \quad n=\frac{\rho}{m}=\frac{K}{2\pi r^2 m^2 G}$$

22. Ans. (BC)

$a_y = -400 \times 10^{10}$ $[qE_y = ma_y]$

$$R=5=\frac{40\times10^{12}\sin2\theta}{400\sqrt{3}\times10^{10}}\quad\left[R(\text{range})=\frac{u^2\sin2\theta}{a_y}\right]$$

$$\sin 2\theta=\frac{\sqrt{3}}{2}$$

$2\theta = 60°, 120° \Rightarrow \theta = 30°, 60°$

Time of flight,

$$T_1=\frac{2\times2\sqrt{10}\times10^6\times\frac{1}{2}}{400\sqrt{3}\times10^{10}}=\sqrt{\frac{5}{6}}\,\mu s\ \ (\text{for }\ \theta=30°)$$

Time of flight,

$$T_2=\frac{2\times2\sqrt{10}\times10^6\times\frac{\sqrt{3}}{2}}{400\sqrt{3}\times10^{10}}=\sqrt{\frac{5}{2}}\,\mu s\ \ (\text{for }\theta=60°)$$

Answer is (BC).

23. Ans. (3.14)

$\Delta\ell \to x$

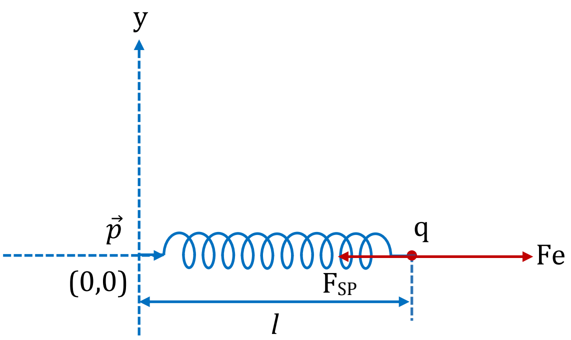
At ℓ : $Fe = F_{SP}$

$$k\ell=\frac{2kpq}{\ell^3}$$

$F_{net} = F_{sp} - F_e = k(\ell + x) - qE$

$$=k(x+\ell)-\frac{q(2kp)}{(\ell+x)^3}$$

$$=kx+k\ell-\frac{q(2kp)}{\ell^3(1+x/\ell)^3}$$



$$= kx + k\ell - q\left(\frac{2kp}{\ell^3}\right)\left(1-\frac{3x}{\ell}\right)$$

$$F_N = kx + k\ell\left(\frac{3x}{\ell}\right) = 4kx$$

$$k_{eq} = 4k\ ;\quad T = 2\pi\sqrt{\frac{m}{4k}} = \pi\sqrt{\frac{m}{k}}$$

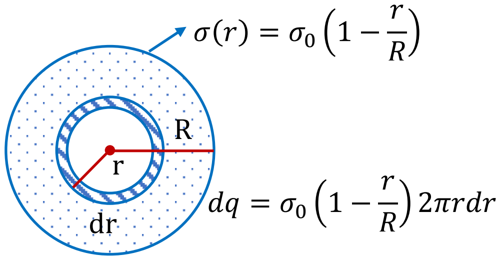
$$f = \frac{1}{\pi}\sqrt{\frac{k}{m}}$$

So $\delta = \pi = 3.14$

Answer is (3.14).

24. **Ans. (6.40)**

$$\phi_0 = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^R \sigma_0\left(1-\frac{r}{R}\right)2\pi r\,dr}{\varepsilon_0}$$



$$\phi = \frac{\int dq}{\varepsilon_0} = \frac{\int_0^{R/4} \sigma_0\left(1-\frac{r}{R}\right)2\pi r\,dr}{\varepsilon_0}$$

$$\therefore\quad \frac{\phi_0}{\phi} = \frac{\sigma_0 2\pi\int_0^R\left(r-\frac{r^2}{R}\right)dr}{\sigma_0 2\pi\int_0^{R/4}\left(r-\frac{r^2}{R}\right)dr}a = \frac{\frac{R^2}{2}-\frac{R^2}{3}}{\frac{R^2}{32}-\frac{R^2}{3\times 64}} = \frac{32}{5} = 6.40$$

Answer is (6.40).

25. **Ans. (2)**

$U_i = 0$

$$U_f = \frac{kqP}{\left(2\ell\sin\frac{\alpha}{2}\right)^2} + mgh \qquad \text{...(i)}$$

Now, from ΔOAB

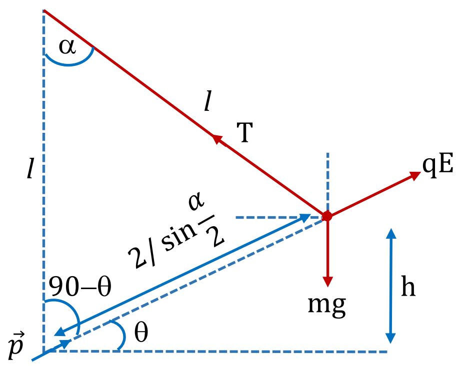
$\alpha + 90 - \theta + 90 - \theta = 180$

$\Rightarrow \quad \alpha = 2\theta$

From ΔABC: $h = 2\ell\sin\left(\frac{\alpha}{2}\right)\sin\theta$

$$h = 2\ell\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \quad h = 2\ell\sin^2\left(\frac{\alpha}{2}\right)$$



Now charge is in equilibrium at point *B*.

So, using sine rule

$$\Rightarrow \quad \frac{mg}{\sin\left[90+\frac{\alpha}{2}\right]}=\frac{qE}{\sin[180-2\theta]} \Rightarrow \frac{mg}{\cos\frac{\alpha}{2}}=\frac{qE}{\sin 2\theta}$$

$$\Rightarrow \quad \frac{mg}{\cos\frac{\alpha}{2}}=\frac{qE}{\sin\alpha} \Rightarrow \frac{mg}{\cos\frac{\alpha}{2}}=\frac{qE}{2\sin\frac{\alpha}{2}\cos\frac{\alpha}{2}}$$

$$\Rightarrow \quad qE=mg2\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \quad \frac{q2kp}{\left[2\ell\sin\frac{\alpha}{2}\right]^3}=mg2\sin\left(\frac{\alpha}{2}\right)$$

$$\Rightarrow \quad \frac{kpq}{\left[2\ell\sin\frac{\alpha}{2}\right]^2}=mgh$$

Substituting this in equation (i)

$$U_f=mgh+\frac{kpq}{\left[2\ell\sin\frac{\alpha}{2}\right]^2}$$

$\Rightarrow \quad U_f = 2mgh$

$W = \Delta U = Nmgh = N = 2$

26. Ans. (BC)

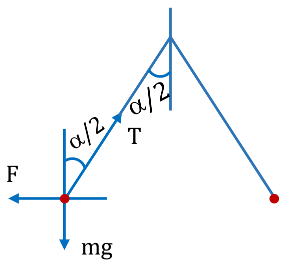
The net electric force on any sphere is lesser but by Coulomb law the force due to one sphere to another remain the same.

In equilibrium

$$T\cos\frac{\alpha}{2}=mg \quad \text{and} \quad T\sin\frac{\alpha}{2}=F$$

After immersed in dielectric liquid.

As given no change in angle α.



So $\quad T\cos\frac{\alpha}{2}=mg-V\rho g$

when ρ = 800 Kg/m^3

and $\quad T\sin\frac{\alpha}{2}=\frac{F}{e_r}$

$$\therefore \quad \frac{mg}{F}=\frac{mg-V\rho g}{\frac{F}{e_r}}$$

$$\frac{1}{e_r}=1-\frac{\rho}{d}$$

d = density of sphere

$$\frac{1}{21}=1-\frac{800}{d}$$

$d = 840$

27. Ans. (1.73)

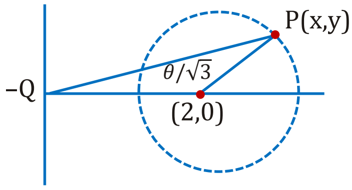
$V_P = 0$

$$\frac{-kQ}{\sqrt{x^2+y^2}} + \frac{kQ/\sqrt{3}}{\sqrt{(x-2)^2+y^2}} = 0$$

$3(x-2)^2 + 3y^2 = x^2 + y^2$

$(x-3)^2 + y^2 = \left(\sqrt{3}\right)^2$

$R = \sqrt{3} = 1.73$



28. Ans. (3)

Let a point P on circle

$$V_p = 0 = \frac{k(-Q)}{r_1} + \frac{kQ/\sqrt{3}}{r_2}$$

$$\frac{kQ}{r_1} = \frac{kQ/\sqrt{3}}{r_2}$$

$$\frac{1}{\sqrt{x^2+y^2}} = \frac{1}{\sqrt{3}\sqrt{(x-2)^2+y^2}}$$

$3(x-2)^2 + 3y^2 = x^2 + y^2$

$3(x^2+4-4x) - x^2 + 2y^2 = 0$

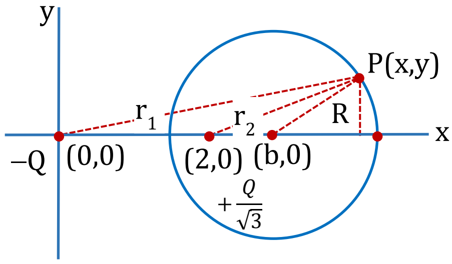
$2x^2 + 12 - 12x + 2y^2 = 0$

$x^2 = 6 - 6x + y^2$

$(x-3)^2 + y^2 = \left(\sqrt{3}\right)^2$

$R = \sqrt{3} = 1.73$

$b = 3$



29. Ans. (9)

Circular orbits

$$T = 2\pi\sqrt{\frac{R^2}{GM_S}}$$

Binary stars

$$nT = 2\pi\sqrt{\frac{(9R)^3}{G(3M_S+6M_S)}}$$

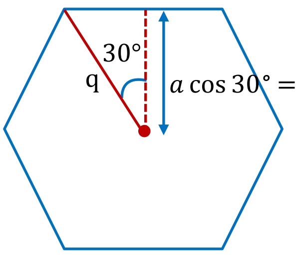
$$n\times 2\pi\sqrt{\frac{R^3}{GM_S}} = 9\times 2\pi\sqrt{\frac{R^3}{GM_S}}$$

$n = 9$

30. Ans. (ABC)

(A) Due to symmetry $\vec{E}_0 = 0$

(B)



$a\cos 30° = \frac{\sqrt{3}q}{2} = d$

$$E_{net} = \frac{kq}{(2d)^2} \times 2 = \frac{2q \times 4}{4\pi\varepsilon_0 \cdot 4 \cdot 3a^2} = \frac{q}{6\pi\varepsilon_0 a^2}$$

(C) $v = \frac{7kq}{2d} = \frac{7q}{4\pi\varepsilon_0 \cdot \sqrt{3}a} = \frac{7q}{4\sqrt{3}\pi\varepsilon_0 q}$

(D) $v = \frac{2kq}{2d} = \frac{2q}{4\pi\varepsilon_0 \cdot \sqrt{3}a} = \frac{q}{2\sqrt{3}\pi\varepsilon_0 q}$

31. Ans. (3)

From Gauss law,

$$\phi_{\text{hemisphere}} + \phi_{\text{Cone}} = \frac{q}{\varepsilon_0} \quad .....$$

Total flux produced from q in α angle

$$\phi = \frac{q}{2\varepsilon_0}[1 - \cos\alpha]$$

For hemisphere, $\alpha = \frac{\pi}{2}$

$$\phi_{\text{hemisphere}} = \frac{q}{2\varepsilon_0}$$

From equation (i)

$$= \frac{q}{2\varepsilon_0} + \phi_{cone} = \frac{q}{\varepsilon_0}$$

$$\phi_{\text{cone}} = \frac{q}{2\varepsilon_0}$$

$$\frac{4q}{6\varepsilon_0} = \frac{q}{2\varepsilon_0}$$

$n = 3$

Alternatively, $\phi \propto$ no of electric field lines passing through surface q is point charge which has uniformly distributed electric field lines thus half of electric field lines will pass through hemisphere and other half will pass through conical surface.

32. Ans. (B)

$$q_1 = \int_0^1 kr 4\pi r^2 dr = \frac{4\pi k}{4} = \pi k$$

$$q_2 = \int_1^r \frac{2k}{r} 4\pi r^2 dr = \frac{8\pi k\left(r^2 - 1^2\right)}{2}$$

$q_2 = 4\pi k[r^2 - 1] = 4\pi kr^2 - 4\pi k$

$q_{\text{net}} = q_1 + q_2 = 4\pi kr^2 - 3\pi k$

$q_{\text{net}} = \pi k\,[4r^2 - 3]$

(A) $E_{net} = 0 \Rightarrow q_{net} = 0 \Rightarrow r = \frac{\sqrt{3}}{2}$

(B) $V = \frac{kQ_{net}}{r} = \frac{1}{4\pi\varepsilon_0} \frac{\pi k\left(4r^2 - 3\right)}{r}$

$$V=\frac{k}{4\varepsilon_0}\left[4r-\frac{3}{r}\right]=\frac{k}{4\varepsilon_0}\left[4\times\frac{3}{2}-\frac{3\times2}{3}\right]=\frac{k}{\varepsilon_0}$$

(C) $q_{net}=\pi k\left[4(2)^2-3\right]$ = $13\pi k$

(D) $$E_2=\frac{kQ}{r^2}=\frac{1}{4\pi\varepsilon_0}\frac{\pi k\left(4r^2-3\right)}{r^2}=\frac{k}{4\varepsilon_0}\left[\frac{4\left(\frac{5}{2}\right)^2-3}{(5/2)^2}\right]=\frac{k}{25\varepsilon_0}[25-3]=\frac{22}{25}\frac{k}{\varepsilon_0}$$

33. Ans. (ACD)

$$W_{el}+W_{ext}=k_f-k_i$$

$$qv_i-qv_f+W_{ext}=k_f-k_i$$

$$\frac{q\sigma}{2\in_0}\left[\sqrt{R^2+Z^2}-Z\right]-\frac{q\sigma R}{2\in_0}+CZ=k_f-0$$

$$C=\frac{q\sigma B}{2\in_0}$$

Substitute β and Z, calculate kinetic energy at z = 0.

If kinetic energy is positive, then particle will reach at origin.

If kinetic energy is negative, then particle will not reach at origin.

34. Ans. (2.30)

Given R_A = R_B = R

M_B = $2M_A$

Calculation of escape velocity for A:

Radius of remaining star = $\frac{R_A}{2}$.

Mass of remaining star = $\rho_A\frac{4}{3}\pi\frac{R_A^3}{8}=\frac{M_A}{8}$

$$\frac{-GM_{A/B}}{R_{A/2}}+\frac{1}{2}mv_A^2=0$$

$$\Rightarrow\quad v_A=\sqrt{\frac{2GM_{A/B}}{R_{A/2}}}=\sqrt{\frac{GM_A}{2R}}$$

Calculation of escape velocity for B

Mass collected over $B=\frac{7}{8}M_A$

Let the radius of B becomes r.

$$\therefore\quad\frac{4}{3}\pi(r^3-R_B^3)\rho_A=\frac{7}{8}\rho_A\frac{4}{3}\pi R_A^3$$

$$\Rightarrow\quad\pi^3=\frac{7}{8}R_A^3+R_B^3=\frac{(15)^{1/3}R}{2}$$

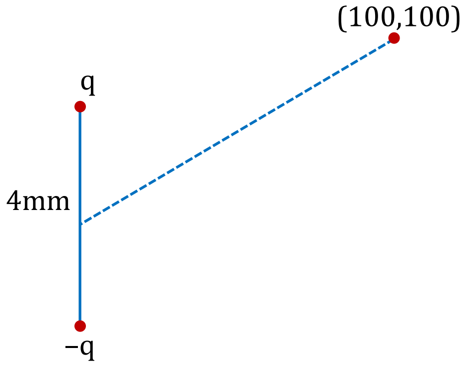
$$\therefore\quad\frac{V_B^2}{2}=\frac{23GM_A}{8\times15^{1/3}\frac{R}{2}}=\frac{23GM_A}{4\times15^{1/3}R}$$

$\therefore \quad V_B = \sqrt{\dfrac{23GM_A}{2\times 15^{1/3}R}}$

$\therefore \quad \dfrac{V_B}{V_A} = \sqrt{\dfrac{23}{15^{1/3}}} = \sqrt{\dfrac{10\times 2.30}{15^{1/3}}}$

$n = 2.30$

35. Ans. (B)



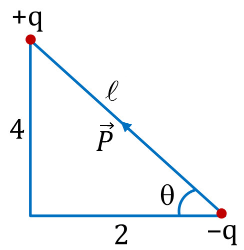
$P_1 = q(4)$

$\vec{P}_1 = P_1\hat{j}$

$\vec{r} = 100(\hat{i}+\hat{j})mm$

$v_0 = \dfrac{KP_1.\vec{r}}{r^3} = \dfrac{K(100P_1)}{(100\sqrt{2})^3}$

$\tan\theta = 2$



$\vec{P}_2 = P_2\left[-\cos\theta\hat{i}+\sin\theta\hat{j}\right]$

$\vec{r} = 100(\hat{i}+\hat{j})mm$

$P_2 = q\ell$

$v = \dfrac{K\vec{P}_2.\vec{r}}{r^3}$

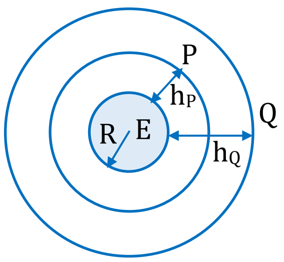
$v = \dfrac{K(100P_2)-(-\cos\theta+\sin\theta)}{(100\sqrt{2})^3}$

$\dfrac{v_0P_2}{P_1} = (-\cos\theta+\sin\theta)$

$v = v_0\dfrac{q\ell}{q(4)}[-\cos\theta+\sin\theta]$

$= \dfrac{v_0}{4}[-2+4] = \dfrac{v_0}{2}$

36. Ans. (A)



$\dfrac{g_P}{g_Q} = \dfrac{\dfrac{GM}{r_P^2}}{\dfrac{GM}{r_Q^2}} = \left(\dfrac{r_Q}{r_P}\right)^2$

$\dfrac{36}{25} = \left(\dfrac{r_Q}{r_P}\right)^2$

$\dfrac{r_Q}{r_P} = \dfrac{6}{5}$

$r_Q = \dfrac{6}{5}r_P$

$R + h_Q = \dfrac{6}{5}\left(R+\dfrac{R}{3}\right)$

$h_Q = \dfrac{24}{15}R - R = \dfrac{9}{15}R = \dfrac{3}{5}R$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (1)

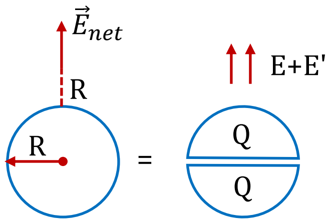
$E_A > E_B$ (Due to more field lines at A crossing a particular area).

$V_B > V_A$ (Because direction of field line from B to A).

2. Ans. (2)

Consider whole sphere

$$E_{net} = \frac{K(2Q)}{4R^2} = E + E',\ \rho = \frac{Q}{\frac{2}{3}\pi R^3}$$



$$E' = \frac{K(2Q)}{4R^2} - E$$

$$E' = \frac{1}{4\pi\varepsilon_0} \times \frac{1}{4R^2} \times 2 \times \left(\rho \times \frac{2}{3}\pi R^3\right) - E$$

$$= \frac{\rho R}{12\varepsilon_0} - E$$

3. Ans. (3)

At point P_1

E_{P_1} is vertically outward.

E_{P_2} is also vertically outward.

4. Ans. (2)

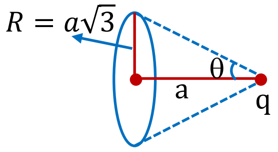
$$\frac{kqQ}{r} + 0 = \frac{kqQ}{2r} + \frac{1}{2}\left(\frac{p^2}{m}\right)$$

5. Ans. (1)

$$E = \frac{KQ}{R^2};\ E = \frac{K(Ze)}{R^2};\ V = \frac{KQ}{R};\ V = \frac{K(Ze)}{R}$$

6. Ans. (4)

We know that if a point charge q is placed on the apex of a cone of semi-vertex angle θ then electric flux through the base of the cone will be $\frac{q(1-\cos\theta)}{2\epsilon_0}$ here $\tan\theta = \frac{a\sqrt{3}}{a} = \sqrt{3} \Rightarrow \theta = 60°$.



Therefore $\phi = \frac{q(1-\cos 60°)}{2\epsilon_0} = \frac{q}{4\epsilon_0} \Rightarrow q = 4\epsilon_0\phi$

7. Ans. (4)

$$\oint EdS = \frac{q_{in}}{\varepsilon_0} \Rightarrow q_{in} = 2\times\left(200\times10^2\right)\times\varepsilon_0 = 35.4\times10^{-8}C$$

8. Ans. (3)

$$dU = \left(\frac{1}{2}\epsilon_0 E^2\right)dV$$

$$= \left(\frac{1}{2}\epsilon_0 (3y)^2\right)\left(a^2dy\right)$$

$$U = \int_{y=0}^{y=a} dU = \frac{3}{2}\epsilon_0 a^5$$

9. Ans. (1)

Electric field on surface of a uniformly charged sphere is given by $\frac{Q}{4\pi\varepsilon_0 R^3} = \frac{\rho R}{3\varepsilon_0}$

Electric field at outside point is given by $E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{\rho R^3}{3\varepsilon_0 r^2}$

$$|\bar{E}_r| = \frac{\rho r_0}{3\varepsilon_0} - \frac{\rho\left(\frac{r_0}{2}\right)^3}{3\varepsilon_0\left(\frac{3r_0}{2}\right)^2} = \frac{17\rho r_0}{54\varepsilon_0}$$

10. Ans. (1)

Field is perpendicular to equipotential surface field inside conductor is zero.

11. Ans. (3)

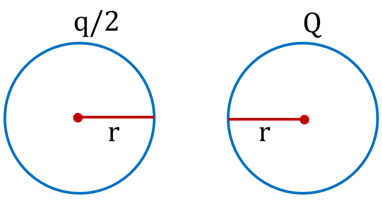
$$PA = F = kx \Rightarrow \frac{\sigma^2}{2\epsilon_0}\pi R^2 = kx \Rightarrow x = \frac{\sigma^2\pi R^2}{2\varepsilon_0 k}$$

12. Ans. (3)

Let the Q charge on sphere

$$\frac{Kq/2}{d} + \frac{KQ}{r} = 0$$

$$Q = -\frac{qr}{2d}$$



13. Ans. (3)

At equilibrium $\sigma_1 = -\sigma_2$

$$E_A = \frac{\sigma_1}{2\varepsilon_0}\hat{i} - \frac{\sigma_2}{2\varepsilon_0}\hat{i}$$

$$= \frac{\sigma_1}{\varepsilon_0}\hat{i} \text{ or } \frac{-\sigma_2}{\varepsilon_0}\hat{i}$$

14. Ans. (4)

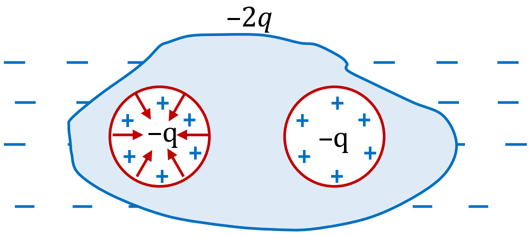
(1) If $Q = 2q$ net charge on outer surface becomes zero

$\Rightarrow E_{\text{outside}} = 0$

$\Rightarrow$ Potential $= \int E.dr = 0$

(2) External field affects outer charge distribution.

(3) Electric field direction inside cavity is towards centre and we know potential decreases in direction of field.



15. Ans. (3)

$$F = G\frac{m_1m_2}{r^2} = 6.675\times\frac{1\times1}{1^2}\times10^{-11} = 6.675\times10^{-11}\,N$$

16. Ans. (3)

$$gh = g\left(1+\frac{h}{R}\right)^{-2} \approx g\left(1-\frac{2h}{R}\right),$$

Fractional change $\dfrac{\Delta g}{g} = \dfrac{g_h - g}{g} = 1 - \dfrac{2h}{R} - 1$

$= -\dfrac{2h}{R}$

17. Ans. (4)

$$\frac{1}{2}m\times\frac{GM}{R} - \frac{GMm}{R} = -\frac{GMm}{R+h}$$

$h = R$

18. Ans. (2)

We know that, $g = \dfrac{GM}{R^2}$

On the planet, $g_p = \dfrac{GM/7}{R^2/4} = \dfrac{4g}{7} = \dfrac{4}{7}g$

Hence weight on the planet $= 700\times\dfrac{4}{7} = 400\ gmwt$

19. Ans. (1)

If we consider ring $5M$ and mass M to be a system we can apply momentum & energy conservation as $F_{ext} = 0$

$$Mv_{mass} = 5MV_{Ring}$$

$$Mv = 5MV \qquad ...(1)$$

$$U_i + k_i = U_f + k_f$$

$$-\frac{GM5M}{2R} = \frac{-G5MM}{R} + \frac{1}{2}mv^2 + \frac{1}{2}5MV^2$$

$$\frac{5M5M}{2R} = \frac{1}{2}Mv^2 + \frac{1}{2}5M\left(\frac{v}{5}\right)^2 = \frac{v^2}{2} = \frac{v^2}{2}\left(1+\frac{1}{5}\right)$$

$$\frac{5GM}{2R} = \frac{6}{5}\times\frac{v^2}{2}$$

$$\sqrt{\frac{25GM}{6R}} = v$$

20. Ans. (2)

$v_e = \sqrt{2}v_0 = 1.414\,v_0$

Fractional increase in orbital velocity $\left(\frac{\Delta v}{v}\right)$

$$= \frac{v_e - v_0}{v_0} = 0.414$$

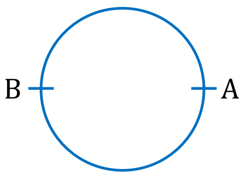
$\therefore$ Percentage increase = 41.4%.

SECTION-B

1. Ans. (6)

After reaching at B, it will complete the circle

Conservation of energy at A & B



$$\frac{KQ}{4a} = V_A$$

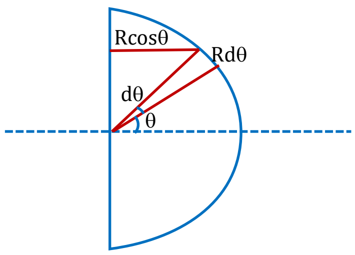
$$\frac{Kq(Q)}{4a} + \frac{1}{2}mv^2 = \frac{KqQ}{a}$$

$$\frac{1}{2}mv^2 = \frac{KqQ}{a}\left(\frac{3}{4}\right)$$

$$\frac{1}{2}mv^2 = 3Vq$$

$$V = \sqrt{\frac{6Vq}{m}}$$

2. Ans. (8)



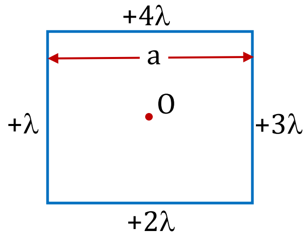
$$\int dV = 2\int_0^{\pi/2} E(R\cos\theta)(\lambda R d\theta) = 2ER^2\frac{\pi}{\pi R}\times 1 = 2\times 2\times 2 = 8J$$

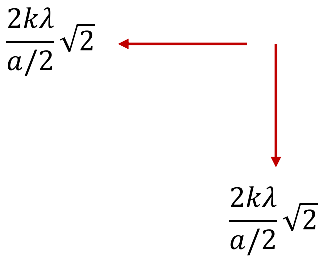
3. Ans. (2)

$(\Delta U)_{gain} = (\Delta K)_{loss}$; $q\Delta V = \frac{1}{2}mv^2 - 0$; $qkQ\left(\frac{1}{\sqrt{a^2+0^2}} - \frac{1}{\sqrt{a^2+3a^2}}\right) = \frac{1}{2}mv^2$

$$v = \sqrt{\frac{kQq}{ma}} = \sqrt{\frac{(\lambda 2\pi a)q}{4\pi\in_0 ma}} = \sqrt{\frac{\lambda q}{2m\in_0}}$$

4. **Ans. (8)**

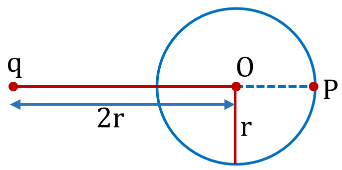




E_{net} at $O \Rightarrow \frac{8k\lambda}{a}$

5. **Ans. (3)**

$V_0 = \frac{kq}{2r} = 9$



$V_P = \frac{kq}{3r} + V_{ind} = V_0$

$9 = 6 + V_{ind}$

6. **Ans. (5)**

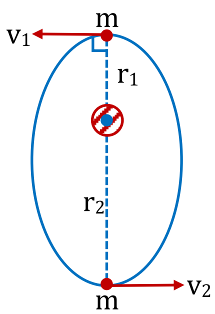
$E = \frac{\sigma}{2\epsilon_0} = 5$

7. **Ans. (2)**

$0 = g - R\omega^2 \cos^2 45°$

$\frac{R\omega^2}{2} = g$ or $\omega = \sqrt{\frac{2g}{R}}$ 1

8. **Ans. (2)**



By conservation of angular momentum

$mv_1r_1 = mv_2r_2$

$\frac{v_1}{v_2} = \frac{r_2}{r_1} = \frac{144\times10^6}{72\times10^6} = 2$

9. Ans. (4)

$$\frac{dA}{dt}=\frac{\pi R^2}{T}$$

$$T^2=\frac{4\pi^2}{GM}R^3$$

$$T=\frac{2\pi}{\sqrt{GM}}R^{3/2}\Rightarrow\frac{dA}{dT}=\frac{\pi R^2}{\frac{2\pi}{\sqrt{GM}}R^{3/2}}$$

$$\frac{\frac{dA_1}{dt}}{\frac{dA_2}{dt}}=\sqrt{\frac{R_1}{R_2}}=n\Rightarrow\frac{R_1}{R_2}=n^2$$

10. Ans. (3)

$$\frac{GM_1M_2}{(4r)^2}=\frac{M_2M_1\omega_S^2(4r)}{(M_1+M_2)}$$

$$\omega_e^2=\left(\frac{v}{r}\right)^2=\frac{GM_S}{r^3}$$

JEE (Advanced) Practice Paper

1. Ans. (C)

$V_A = V_B$

$r_A \geq r_B$

So, charge density at A > charge density at B

$E_A > E_B$

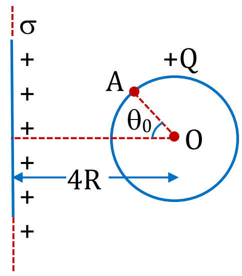
flux = 0

2. Ans. (A)

Whatever the charge distribution of Q on sphere.

$$V=K\int\frac{dQ}{R}=\frac{KQ}{R}$$

The potential due to Q is $\frac{KQ}{R}$ at centre



$V_A = V_{\text{due to }Q} + V_{\text{due to sheet}}$

$V_O = V_{\text{due to }Q} + V_{\text{due to sheet}}$

$V_A = V_O$

$V_{QA} + V_{\text{sheet }A} = V_{QO} + V_{\text{sheet, }O}$

$V_{QA} + V_{\text{sheet }A} = \frac{KQ}{R} + V_{\text{sheet }O}$

$VQ_A = \frac{KQ}{R} + (V_{\text{sheet, }O} - V_{\text{sheet, }A})$

$= \frac{KQ}{R}-\frac{\sigma}{2\varepsilon_0}\left(\frac{R}{2}\right)$

$\frac{KQ}{R}-\frac{\sigma R}{4\varepsilon_0}$

$$V=-\int_A^O E\cdot dr$$

3. **Ans. (D)**

Acceleration above the surface = $\dfrac{g}{\left(1+\dfrac{h}{R}\right)^2}$

Acceleration below the surface = $g\left(1-\dfrac{h}{R}\right)$

Equating them will give answer.

4. **Ans. (A)**

Let's take strip of length 'dx' at length x, from (0, 0).

Its mass = $dm = \rho dx = (a+bx^2)dx$

Force due to this strip on

$$'m' = dF = \frac{Gm}{x^2}\ dm = Gm\ \frac{a+bx^2}{x^2}\ dx$$

Force on 'm' due to

$$dm = dF = \frac{Gm}{x^2}\ dm = Gm\ \frac{a+bx^2}{x^2}\ dx$$

Total force,

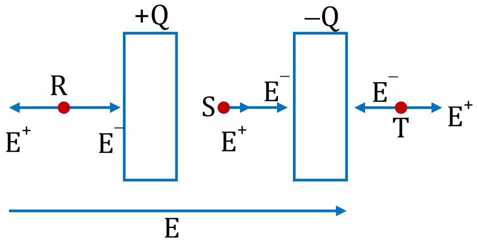
$$F = \int dF = \int_{\alpha}^{\alpha+\ell} Gm \frac{a+bx^2}{x^2} dx$$

$$= Gm \int_{\alpha}^{\alpha+\ell} \left(\frac{a}{x^2}+b\right) dx$$

$$= Gm \left\{\frac{a}{\alpha} - \frac{a}{\alpha+\ell} + b\ell\right\}$$

$$= Gm \left\{a\left(\frac{1}{\alpha} - \frac{1}{\alpha+\ell}\right) + b\ell\right\}$$

5. **Ans. (AD)**



6. **Ans. (AB)**

$\phi = \vec{E}.\vec{A}$

$\phi_{in} = -ve$ and $\phi_{out} = +ve$

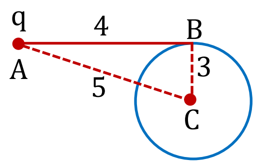
$Q_{net} = \dfrac{q_{in}}{\varepsilon_0}$

$\phi_1 = 0, \phi_2 = E_0a^3, \phi_3 = E_0a^3, \phi_4 = 0, \phi_5 = 0, \phi_6 = 0$

$Q_{net} = \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 = 2E_0a^3 \Rightarrow 2E_0a^3 = \dfrac{q_{in}}{\epsilon_0} \Rightarrow q_{in} = 2\varepsilon_0 E_0 a^3$

7. Ans. (AC)

$V = \dfrac{kq}{AC} = \dfrac{9\times10^{9}\times1\times10^{-6}}{5}$ = 1.8 × 103 = 1.8 kV



$V_B = (V_B)_{\text{due to } q} + (V_B)_i$

Here $(VB)i$ = Potential at B due to induced charge

$\therefore \quad 1.8\times10^3 = \dfrac{kq}{AB} + (V_B)_i$

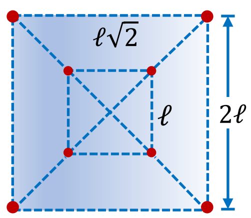
$\Rightarrow \quad 1.8\times10^3 = 2.25\times10^3 + (V_B)_i$

$\Rightarrow \quad (V_B)_i = -0.45\ kV$

8. Ans. (AC)

Since, the situation is having symmetry, particle moves symmetrically along the diagonal shown. The energy conservation law is to be used.

$$U_i = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q^2}{\ell}+\frac{2q^2}{\sqrt{2}\ell}\right],\ U_f = \frac{1}{4\pi\varepsilon_0}\left[\frac{4q^2}{2\ell}+\frac{2q^2}{\sqrt{2}\times2\ell}\right]$$



$$U_i = U_f + \frac{4mv^2}{2}$$

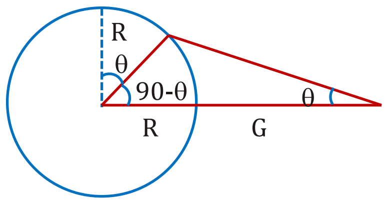
$$\Rightarrow \quad 2mv^2 = \frac{1}{4\pi\varepsilon_0}\left[\frac{2q^2}{\ell}+\frac{q^2}{\ell\sqrt{2}}\right] \Rightarrow v = \left[\frac{q^2}{8\pi\varepsilon_0 m\ell}\left[2+\frac{1}{\sqrt{2}}\right]\right]^{1/2}$$

9. Ans. (ABC)

$\left[\sin\theta = \dfrac{R}{R+G}\right]$

$\sin\theta = \dfrac{R}{R+G} \qquad \cos\theta' = \cos(90-\theta) = \dfrac{R}{R+G}$

$\theta = \sin^{-1}\left(\dfrac{R}{R+G}\right) \qquad \theta' = \cos^{-1}\left(\dfrac{R}{R+G}\right)$



Area covered

$= 2\pi R^2(1-\cos(90-\theta))$

$= 2\pi R^2(1-\sin\theta)$

Area escaped

$= 4\pi R^2 - 2\pi R^2(1-\sin\theta)$

$= 2\pi R^2(1+\sin\theta)$

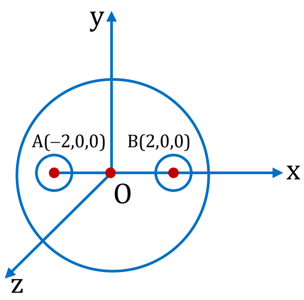
10. Ans. (ACD)

Let

$\vec{F}_A$ = gravitational field due to sphere A (if it were present at that position)

$\vec{F}_B$ = gravitational field due to sphere B (if it were present at that position)

$\vec{F}_R$ = gravitational field due to remaining portion after the cavities are made.



Then from superposition principle, we can see that $\vec{F}_A + \vec{F}_B + \vec{F}_R = 0$, as the force due to entire, sphere is zero at centre.

Now since due to symmetry.

Hence = 0, hence option (i) is correct

Now at B.

Field due to entire sphere is given by .

Then from superposition principle, we can see that $\vec{F}_A + \vec{F}_B + \vec{F}_R = 0$, as the force due to entire sphere is zero at centre.

Now since $\vec{F}_A + \vec{F}_B = 0$ due to symmetry.

Hence $\vec{F}_R$, hence option (i) is correct

Now at B,

Field due to entire sphere is given by

$$\vec{F} = \frac{GM}{R^3} r = \frac{GM}{64} 2 = \frac{GM}{32}$$

whereas $\vec{F}_A = \dfrac{Gm}{4^2} r = \dfrac{Gm}{16} = \dfrac{GM}{16\times64} = \dfrac{GM}{1024}$,

where m = mass of sphere $A = M/64$ and $\vec{F}_B = 0$

From superposition principle, we have

$$\vec{F}_A + \vec{F}_B + \vec{F}_R = \vec{F}$$

$$\Rightarrow \quad \vec{F}_R = \vec{F} - \vec{F}_A = \frac{GM}{32}\hat{\imath} - \frac{GM}{1024}\hat{\imath} = \frac{31GM}{1024}\hat{\imath} \neq 0$$

Hence option (ii) is not correct.

Regarding potential at a point on $y^2 + z^2 = 36$, we can see that the radius of the circle is 6 units, however, all the points on it are symmetrically located from remaining sphere. Hence potential must be same at every point on this circle. Same logic holds for the circle $y^2 + z^2 = 4$ also, though this circle lies inside the remaining sphere.

Hence options (iii) and (iv) are also correct.

Hence options (i), (iii) and (iv) are correct.

Note, we can use superposition principle to calculate the potential at these points.

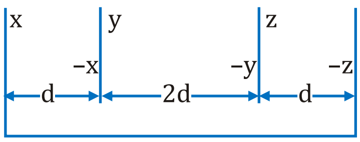
In option (iii) it will be equal to $-\dfrac{GM}{2}\left(\dfrac{1}{3} - \dfrac{1}{32\sqrt{10}}\right)$

In option (iv) it will be equal to $-\dfrac{GM}{64}\left(22 - \dfrac{1}{\sqrt{2}}\right)$

11. **Ans. (B)**

By KVL

$$-\frac{x}{C} - \frac{y}{\frac{C}{2}} - \frac{z}{C} = 0$$



$x + 2y + z = 0$ ...(1)

$y - x = Q$ ...(2)

$-y + z = 0$ ...(3)

Solving (1) (2) (3)

$$y = z = \frac{Q}{4}$$

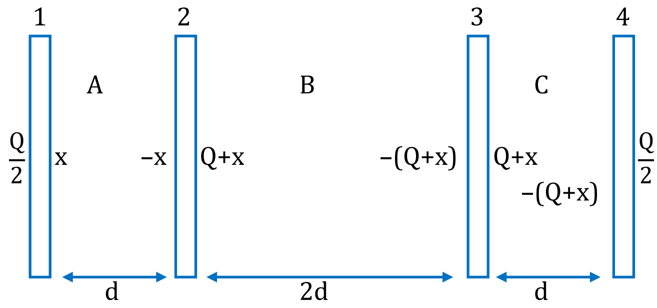
$$x = \frac{-3Q}{4}$$

12. **Ans. (D)**

Charge apply on right side of plate 4 = $\frac{Q}{2}$.

Let start from plate 1 and assume potential of plate 1 = U_1.
Also, potential of plate 4 = U_1 because both are connected.
Electric field in region (A)

$$\frac{x}{2A\varepsilon_0}\times 2=\frac{x}{A\varepsilon_0}$$



Electric field in region (B)

$$\frac{Q+x}{2A\varepsilon_0}\times 2=\frac{Q+x}{A\varepsilon_0}$$

Electric field in region (C)

$$\left(\frac{Q+x}{2A\varepsilon_0}\right)\times 2=\frac{Q+x}{A\varepsilon_0}$$

Now $\quad U_1-\frac{x}{A\varepsilon_0}\times d-\frac{Q+x}{A\varepsilon_0}\times 2d-\frac{Q+x}{A\varepsilon_0}d=U_4$

$U_1 = U_4$

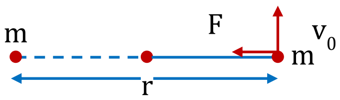
So, $\quad \frac{d(x+2Q+2x+Q+x)}{A\varepsilon_0}=0$

$$x=\frac{3Q}{4}$$

13. **Ans. (C)**

$$V_1-V_2=\frac{3Qd}{4\pi\varepsilon_0}$$

14. **Ans. (C)**



$$\frac{Gm^2}{r^2}=\frac{m\,v_0^{\,2}}{(r/2)} \qquad \text{...(i)}$$

$$2\left(mv_0\frac{r}{2}\right) = nJ_0 \qquad \text{...(ii)}$$

From (i) and (ii)

$$r = n^2\left(\frac{2J_0^{\,2}}{Gm^3}\right)$$

15. **Ans. (D)**

$$v_0 = \frac{Gm^2}{2n\,J_0}$$

$\Rightarrow \quad K = \frac{1}{2}mv_0^2 = \frac{1}{4}\frac{Gm^2}{r} = -\frac{U}{4}$, from (i)

$\therefore \quad E = U + K = -4K + K + K = -2K$

$$= -\frac{1}{n^2}\left(\frac{1}{4}\frac{m^5G^2}{J_0^{\,2}}\right)$$

16. **Ans. (3)**

Net flux in x–direction = 0;

Net flux in y–direction = $A\left[3\left(1^2+2\right)\right]-A\left[3\left(0^2+2\right)\right] \Rightarrow q = -3\in_0 \ (as\ A=1m^2)$

17. **Ans. (1)**

Using superposition method,

$$E_{net} = E_{sheet} - E_{disk}$$

18. **Ans. (2)**

Energy in the orbit = $-\dfrac{GMm}{2r}$

Energy at the surface = $-\dfrac{GMm}{2R_e}$

Loss in energy = $\dfrac{GMm}{2}\left(\dfrac{1}{R_e}-\dfrac{1}{r}\right)$

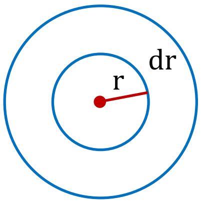
Time required = $\dfrac{\text{Loss in energy}}{C}$

19. **Ans. (4)**

$dm = kr4pr2dr$

$m = kpr4$

$E = \dfrac{Gm}{r^2}$



$-dP \times 4pr2 = G\dfrac{m}{r^2}dm$

$-dP = \dfrac{Gk\pi r^4 \times k4\pi r^3 dr}{4\pi r^4}$

$-dP = Gk^2pr^3dr$

$-\int_{P=0}^{P} dP = Gk^2\pi\int_{r=R}^{r=r} r^3 dr$

$-P = Gk^2\pi\dfrac{r^4-R^4}{4}$

$P = G\pi k^2\left(\dfrac{R^4-r^4}{4}\right)$

20. **Ans. (A-P,S), (B-P,Q,S), (C-P,R), (D-P)**

$V = \sqrt{\dfrac{2GM}{r}}$ escape velocity

$V = \sqrt{\dfrac{Gm}{r}}$ orbital velocity