Simple Harmonic Motion

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

Motion is oscillatory but not periodic and period is not defined. Particle moves from $-a$ to a.

OR

$x = a\cos(\alpha t)^2 \Rightarrow$ oscillatory periodic motion of small

$x(t) = a\cos(\alpha t)^2$

$x(t+T) = a\cos(\alpha t + T)^2$

$x(t+T) = a\cos\left(\alpha^2 t^2 + T^2 + 2\alpha t T\right)$

$x(t) \neq x(t+T)$

Hence, not periodic motion.

2. **Ans. (D)**

S-I : 4 + 6 sin ωt represents SHM about $m.p.$ = 4

S-II : $\dfrac{d^2x}{dt^2}$ is not proportional to x but to $(x - 4)$

3. **Ans. (A)**

$A = 5$

$T = \dfrac{2\pi}{\pi} = 2 \left(T = \dfrac{2\pi}{\omega}\right)$

4. **Ans. (C)**

$4\, a_y = -9_y$

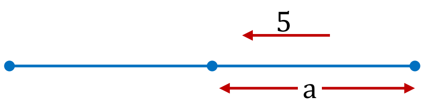
$a_y = -\left(\dfrac{3}{2}\right)^2 y$

$\therefore \quad \omega = \dfrac{3}{2} \left(a_y = -\omega^2 y\right)$

5. **Ans. (A)**

$x < 0$, $|x| = -5$

$\vec{a} = +5m/s^2 (t_1)$



$X > 0$, $|x| = 5$

$\vec{a} = -5m/s^2 (t_2)$

$\dfrac{1}{2} 5t_1^2 = a$

$\dfrac{1}{2} 5t_2^2 = a$

$T = 2t_1 + 2t_2$

$T = 4\sqrt{\dfrac{2a}{5}}$

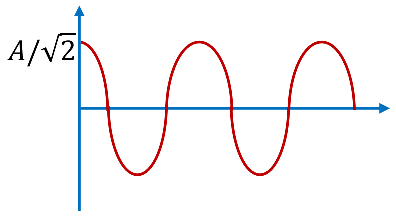
6. Ans. (A)

Amp. = $2A$

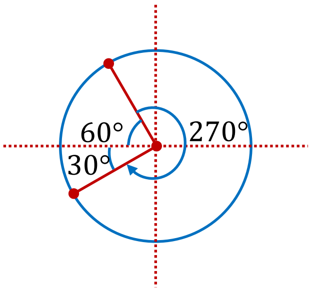
$$\phi = -\frac{\pi}{2}$$

$$\therefore \quad x = 2A \sin\left(\omega t - \frac{\pi}{2}\right)$$

7. Ans. (A)

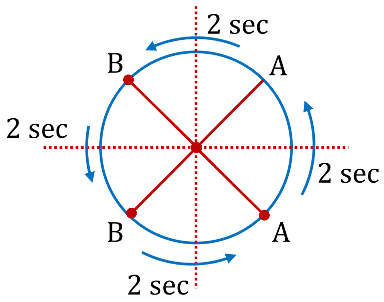


8. Ans. (A)



$-\frac{A}{2}$ away from *m.p.*

9. Ans. (B)

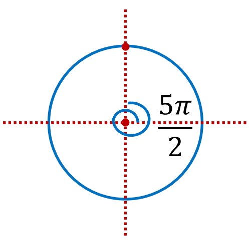


$T = 8$ sec

10. Ans. (C)

$\Delta\phi = 2.5\pi$

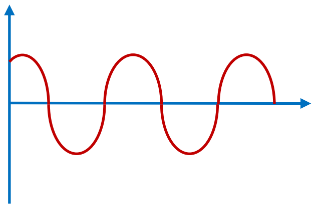
$$= \frac{5\pi}{2}$$

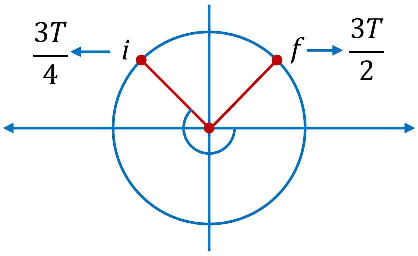


$\therefore$ distance travelled = $5A$

11. Ans. (B)

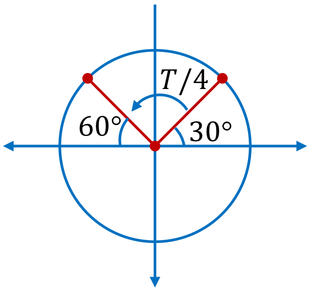
After $\frac{3T}{4}$ it would be at $\frac{A}{\sqrt{2}}$ and moving away from mean position



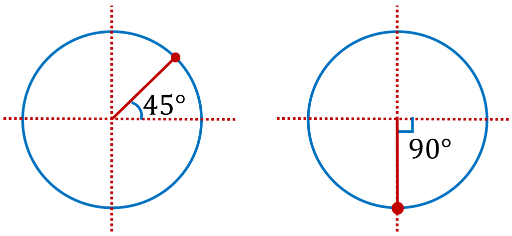


12. Ans. (A)

$$x = A\sin\left(\omega t + \frac{2\pi}{3}\right)$$



13. Ans. (C)



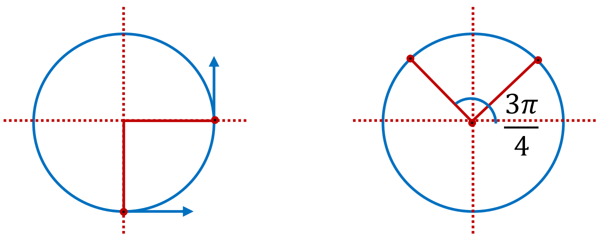
$\Delta\phi = 135°$

14. Ans. (A)

Time taken = $5T$ (LCM of time period)

$\therefore$ Oscillations of smaller pendulum = $\frac{5T}{T} = 5$

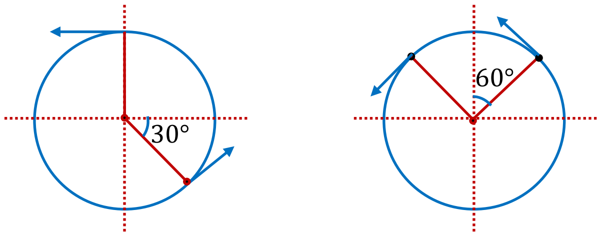
15. Ans. (B)



$$\Delta\phi = \frac{\pi}{4}$$

$$\Delta t = \frac{3T}{8}$$

16. Ans. (D)



$$\Delta\phi = \frac{\pi}{3}$$

$$\therefore \quad \Delta t = \frac{T}{6}$$

17. Ans. (C)

Maximum distance occurs when mean position lies between particles (midpoint) and velocity is in same direction.

$$\therefore \quad x \Rightarrow -\frac{A}{2} \quad \frac{A}{2}$$

$$\phi \Rightarrow -\frac{\pi}{6} \quad \frac{\pi}{6}$$

$$\therefore \quad \Delta\phi = \frac{\pi}{3}$$

18. Ans. (A)

Δt = 1 sec

$$\Delta\phi = \frac{\pi}{3}$$

$$v = \frac{v_{\max}}{2}$$

$$\therefore \quad v_{\max} = \frac{1}{2}$$

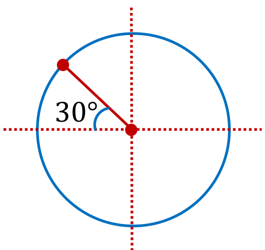
$$A = \frac{v_{\max}}{\omega} = \frac{3}{2\pi}$$

19. Ans. (C)

$$\Delta\phi = \frac{\pi}{3}$$

$$\therefore \quad \Delta t = \frac{T}{6}$$

$$\therefore \quad \vec{v} = \frac{3a}{T}$$



20. Ans. (A)

$d_{AB} = A \sin \omega t$

$$\therefore \quad v = \frac{A\sin\omega t}{t}$$

21. Ans. (C)

$x = \frac{A}{2}$

$v = \frac{\sqrt{3}}{2} A\omega$

$a = \frac{A\omega^2}{2}$

$\therefore \quad \omega = \sqrt{3}$

$f = \frac{\omega}{2\pi} = \frac{\sqrt{3}}{2\pi}$

22. Ans. (A)

$\omega = \frac{2\pi}{2} = \pi$

At critical case

$a = A\omega^2 = g$

$\therefore \quad A = \frac{g}{\omega^2} = \frac{10}{\pi^2}$

23. Ans. (B)

$v^2 = 108 - 9x^2$

$2v\frac{dv}{dx} = -9.2x \quad \left(a = 2v\frac{dv}{dx}\right)$

$\Rightarrow \quad a = -9x$

$\Rightarrow \quad a_3 = 0.27\ m/s^2$

$v = 9\sqrt{\left(\sqrt{12}\right)^2 - x^2}$

$A = \sqrt{12}$

24. Ans. (B)

$F = 6x^2 - 18x + 12$

$= 6(x-1)(x-2)$

$F' = 12x - 18$

As $F_{(\text{mean})} = 0$

$\therefore$ x may be 1 or 2

$U_{\min}$ must occur at mean

$F' > 0 \qquad\qquad x = 2$

$\therefore$ $U \approx 4\ J \rightarrow$ As for SHM amplitude is very small and hence is same for all positions.

25. Ans. (B)

$KE = 4J = \frac{1}{2} \times 2 \times v^2 \Rightarrow v = 2m/s$

and $A = 1\ m$

$\therefore \quad \omega = 2\ \text{rad}/s$

$\therefore \quad T = 3.14$

26. Ans. (B)

$KE = TE \propto pA^2\omega^2$

$PE = TE \propto qA^2$

$pA^2/\omega^2 = qA^2$

$$T = 2\pi\sqrt{\frac{p}{q}}$$

27. Ans. (B)

$$\frac{1}{2}k_1A_1^2 = \frac{1}{2}mv^2 = \frac{1}{2}k_2A_2^2$$

$$\therefore \quad \frac{A_1}{A_2} = \sqrt{\frac{k_2}{k_1}}$$

28. Ans. (C)

Let $k_A = k$

$\therefore \quad k_{sp} = 3k \qquad (\text{as } \ell' = \frac{\ell}{3})$

$\therefore \quad k_B = 9k$

$$\therefore \quad \frac{T_A}{T_B} = \sqrt{\frac{k_B}{k_A}} = 3$$

29. Ans. (A)

$$T = 2\pi\sqrt{\frac{m}{k_{eq}}}$$

$F_{\text{net}} = -(4k)(4x) = -16kx$

$\Rightarrow \quad k_{\text{eq}} = 16K$

30. Ans. (C)

$$T = \pi\left[\sqrt{\frac{M}{K}} + \frac{1}{2}\sqrt{\frac{M}{K}}\right] = \frac{3\pi}{2}\sqrt{\frac{M}{K}}$$

31. Ans. (B)

$$a_{\max} = \frac{kA}{m+m} = \frac{f}{m}$$

$$f_{\max} = \frac{kA}{2} \le \mu_s mg$$

32. Ans. (A)

$$E = -mg\ell(\cos\theta_0 - \cos\theta) + \frac{1}{2}\mu v^2_{\text{rel}}$$

33. Ans. (A)

$$\omega = \sqrt{\frac{g}{\ell}} = \pi$$

$\theta = \theta_A \sin(\omega t + \phi)$

$\Rightarrow \quad \theta = 3° \cos \pi t$

34. Ans. (A)

$$t = 2\pi\sqrt{\frac{\ell}{g}} = T$$

$$t' = \pi\sqrt{\frac{\ell}{g}} + \frac{\pi}{2}\sqrt{\frac{\ell}{g}} = \frac{3T}{4}$$

35. Ans. (C)

$y = 6t - 3.75\ t^2 s$

$a = -\ 7.5\ m/s^2$

$g_{eff} = 10-7.5 = 2.5\ m/s^2$

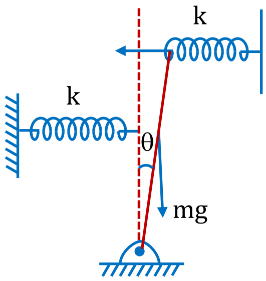
$$T = 2\pi\sqrt{\frac{\ell}{g}} = 2$$

$$T' = 2\pi\sqrt{\frac{\ell}{g/4}} = 2\times 2 = 4 \text{ sec.}$$

36. Ans. (C)

For SHM

Restoring Torque > Deforming Torque



$$k\ell\sin\theta\,\ell\cos\theta + K\frac{\ell}{2}\sin\theta\frac{\ell}{2}\cos\theta > mg\frac{\ell}{2}\sin\theta$$

For small value of θ $\sin\theta \approx \&\ \cos\theta \approx 1$

$$k\ell^2 + \frac{k\ell^2}{4} > \frac{mg\,\ell}{2}$$

$$k > \frac{2}{5}\frac{Mg}{\ell}$$

37. Ans. (A)

$$\frac{2c}{mR^2} = \frac{k}{m}$$

$$R = \sqrt{\frac{2c}{k}}$$

38. Ans. (C)

$I_A = MK^2 + M\ell_1^2$ but $\ell_1 = 50\ cm$ and $\frac{K^2}{\ell_1} = 30\ cm$ $K^2 = 30 \times 50\ cm^2$

$I_A = 2 \times (30 \times 50 \times 10^{-4}) + 2 \times (50 \times 10^{-2})^2 = 0.3 + 0.18 = 0.48\ kg\text{-}m^2$

39. Ans. (B)

$$2\pi\sqrt{\frac{m\left(\frac{\sqrt{3}L}{2}\right)^2}{mg\left(\frac{\sqrt{3}L}{3}\right)^2}} = 2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$$

40. Ans. (A)

k_3 can be replaced by k'_3 at centre of rod, such that

$$k_3\ell^2 = k'_3\frac{\ell^2}{4}$$

also k_2, k_3' are in series and their net in parallel with k_1

41. Ans. (C)

$$A=\sqrt{1+1+2\cdot\frac{1}{2}} = \sqrt{3}$$

42. Ans. (B)

$$A' = A\sqrt{2}$$

$$\therefore \quad E = \frac{1}{2}\cdot m\omega^2\cdot 2A^2 = m\omega^2A^2$$

43. Ans. (D)

$y = a \sin \omega t + b \cos \omega t$

$$= \sqrt{a^2+b^2}\sin\left(\omega t+\tan^{-1}\frac{b}{a}\right)$$

$\therefore$ It is SHM with $A=\sqrt{a^2+b^2}$

44. Ans. (C)

$$y_1 = 2A\frac{(1+\cos 2\omega t)}{2}$$

$$= A\,(1 + \cos 2\,\omega t)$$

$$y_2 = 2A\sin(\omega t)+\frac{\pi}{3}$$

$$y_2 = 2A\sin\left(\omega t+\frac{\pi}{3}\right)$$

$A_1 : A_2 = 1 : 2$

$V_1 : V_2 = 1 : 1$

$a_1 : a_2 = 2 : 1$

45. Ans. (A)

$$y = \sin\omega t+\sqrt{3}\cos\omega t$$

$$a = \frac{d^2y}{dt^2} = -\omega^2[\sin\omega t+\sqrt{3}\cos\omega t]$$

When acceleration $a = -g$ than mass just break of the plank

$$\omega^2\left[\frac{\sin\omega t}{2}+\frac{\sqrt{3}}{2}\cos\omega t\right]=g$$

$$\omega_{\min} = \sqrt{\frac{g}{2}}$$

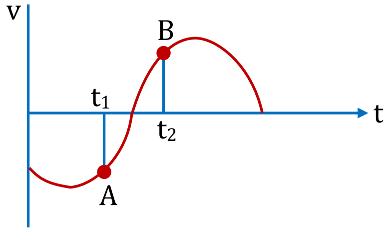
$$\sin(\omega t+60) = 1 = \sin(90°)$$

$$\omega t = \frac{\pi}{6} \qquad \Rightarrow \qquad t = \frac{\pi}{6\omega}$$

$$\Rightarrow \quad t = \frac{\pi}{6}\sqrt{\frac{2}{g}}$$

46. Ans. (BC)

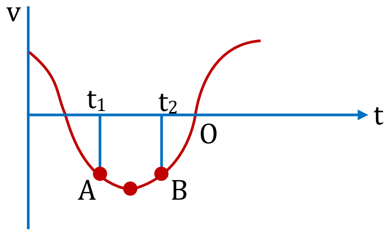
(A) If t_1 it is moving towards x_m



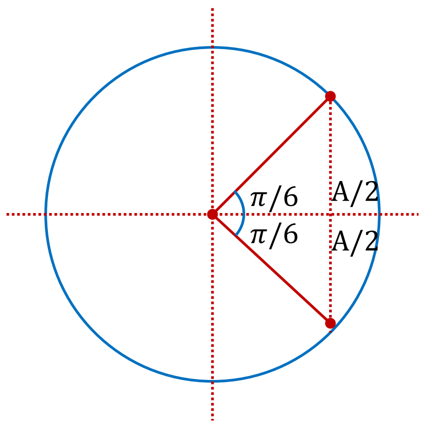
(B) At t_1 speed is decreasing

(C) At t_2 it lies between $-x_m$ and O

(D) $\Delta\phi$ lies between 0° and 180°



47. Ans. (AC)



$$\phi = \frac{\pi}{3}$$

They will collide after $\frac{T}{4}$ time

48. Ans. (ABCD)

(A) $y = \frac{A}{2}$ $\qquad v = \frac{\sqrt{3}v_{max}}{2} > \frac{v_{max}}{2}$

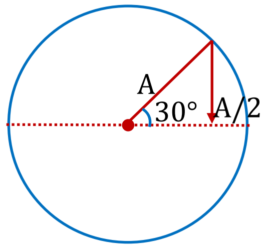
(B) $v = \frac{v_{max}}{2}$ $\qquad A = \frac{\sqrt{3}A}{2} > \frac{A}{2}$

(C) $t = \frac{T}{8}$ $\qquad y = \frac{A}{\sqrt{2}} > \frac{A}{2}$

(D) $y = \frac{A}{2}$ $\qquad t = \frac{T}{12} < \frac{T}{8}$

49. **Ans. (AB)**

$a = \frac{a_0}{2}$



$t = \frac{T}{12}$

$v = \frac{v_0\sqrt{3}}{2}$

50. **Ans. (ABC)**

By graph we can see that $V_{\max}$ = 10 cm/s and

A = 2.5cm and we know that

$$A_{\max} = A_{\omega}$$

$$\Rightarrow \quad 10 = 2.5\omega$$

$$T = \frac{2\pi}{\omega} = \frac{2\omega}{4} = \frac{3.14}{2} = 1.57s \Rightarrow A \text{ correct}$$

$$a_{\max} = A\omega^2 = 2.5\times 4^2 = 400cms^{-2} \Rightarrow B \text{ correct}$$

$$v(x=1cm) = w\sqrt{A^2 - X^2} = 4\sqrt{2.5^2 - 1^2} = 4\sqrt{5.25} = 4\sqrt{\frac{525}{100}} = 4\sqrt{\frac{21\times 25}{100}} = 2\sqrt{21}cm/s$$

$\Rightarrow$ C correct & D wrong.

51. **Ans. (BCD)**

Velocity is maximum at mean position & acceleration is maximum at exterme position. At mean position acceleration is zero.

52. **Ans. (AD)**

Maximum velocity for $C.M.$ = $\frac{v}{2}$ mechanical energy is equal to maximum $K.E.$

$$= \frac{1}{2}I\omega^2 \text{ (about } O\text{)}$$

$$= \frac{1}{2}(2mr^2)\left(\frac{v}{2r}\right)^2 = \frac{mv^2}{4}$$

and time period is given by = $2\pi\sqrt{\frac{I}{mgd}} = 2\pi\sqrt{\frac{2mr^2}{mgr}} = 2\pi\sqrt{\frac{2r}{g}}$

53. **Ans. (ACD)**

Motion is periodic and SHM in straight line

54. **Ans. (B)**

$mv_{\max} = 5kgm/s$ (m=1kg)

$v_{\max} = A\omega$ $\qquad$ (A = 10 m)

$\Rightarrow \quad \omega = \frac{5}{10} = \frac{1}{2} rad/\sec$

55. **Ans. (C)**

ωA = 5 [mass = 1 kg]

A = 10 $\boxed{\omega = 1/2}$

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(m\omega^2)A^2 = \frac{1}{2}\times 1\times\left(\frac{1}{2}\right)^2\times 10^2 \Rightarrow 12.5 \text{ joule}$$

56. Ans. (A)

$x = A\sin\omega t, \quad v = a\omega\cos\omega t$

$A = 2m,\ \omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{900}{3}} = 10\sqrt{3}\,rad/\sec$

$v = 20\sqrt{3}\cos\omega t$

When both the masses collide $x = 1\ m$ so $\sin\omega t = \frac{1}{2} \Rightarrow \cos\omega t = \frac{\sqrt{3}}{2}$

Just before the collision velocity of 3 kg mass = $v_1 = 20\sqrt{3}\left(\frac{\sqrt{3}}{2}\right) = 30\,m/s$

Common velocity of the blocks after collision = $\left(\frac{m_1v_1}{m_1+m_2}\right) = \frac{3\times30}{3+6} = 10\,m/s$

57. Ans. (C)

In the starting of resulting collision $v = 10\ m/s$

So $\omega = \sqrt{\frac{900}{9}} = 10\,rad/s$

We know $v = \omega\sqrt{A^2 - x^2} \Rightarrow 10 = 10\sqrt{A^2 - 1}$

$\Rightarrow A^2 = 2 \quad \Rightarrow A = \sqrt{2}m$

58. Ans. (A)

$\omega = \sqrt{\frac{2500}{100}} = 5\ rad/s$

59. Ans. (A)

$0 = \omega_0\sqrt{A^2 - x^2}$

$= \sqrt{\frac{2500}{25}} \times \sqrt{A^2 - \left(\frac{A}{2}\right)^2}$

$= \frac{\sqrt{3}}{2} \times 10 \times 0.1 = \frac{\sqrt{3}}{2}\ m/s$

From momentum conservation

$25 \times \frac{\sqrt{3}}{2} = 100 \times v \Rightarrow v = \frac{\sqrt{3}}{8}\ m/s$

$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$

$= \frac{1}{2} \times 2500 \times \frac{(0.1)^2}{4} + \frac{1}{2} \times 100 \times \frac{3}{64}$

$= \frac{25}{8} + \frac{75}{32} = \frac{175}{32}J$

60. Ans. (P-3,4), (Q-2,3,4), (R-1,3), (S-1,2,3,4)

$\vec{v}\cdot\vec{a} > 0$ possible for all 'x' other than mean position & extreme position

Velocity is negative possible at all 'x' except extreme positions Acceleration is negative for $0 < x \le A$ $\vec{v}\times\vec{a} = 0$ for all points.

EXERCISE – S

1. **Ans.** $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$

As $\omega = \frac{2\pi}{0.6\pi} = \frac{10}{3}$

$A = 2$

$\phi = -\frac{\pi}{2}$

$\therefore\ y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$

2. **Ans.** 0.33 sec

$v_{max} \Rightarrow$ m.p.

$2\pi t + \frac{\pi}{3} = \pi$

$t = \frac{1}{3} = 0.33\,\text{sec}$

3. **Ans.** 2 m/s

$d = 7A = 1.4\ m$

$\vec{v} = 2\ m/s$

4. **Ans.** $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

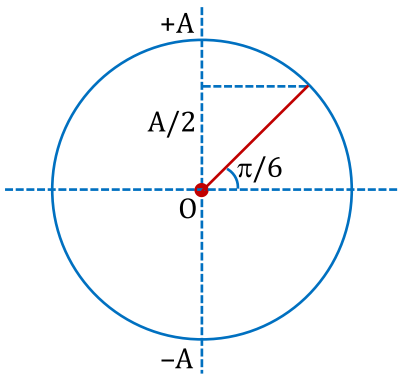
$\omega^2 = -\frac{\alpha}{x} = \frac{\beta}{\propto}$

$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$

5. **Ans.** (a) $T/12$, (b) $T/8$, (c) $T/6$, (d) $T/4$, (e) $T/8$

(a) $\theta = \frac{\pi}{6} = \frac{L\pi}{T}t$

$t = \frac{T}{12}$



(b) $\theta = \frac{\pi}{4} = \frac{2\pi}{T}t$

$t = \frac{T}{8}$

(c) $t = \frac{T}{4} - \frac{T}{12} = \frac{T}{6}$

(d) $t = \frac{T}{8} + \frac{T}{8} = \frac{T}{4}$

(e) $t = \frac{T}{8}$

6. **Ans.** $x = (10\ cm)\sin\left[\left(\frac{\pi}{3}s^{-1}\right)t+\frac{\pi}{6}\right],\ \frac{10}{9}\pi^2 \approx 11\ cm/s^2$

Given, $r = 10\ cm$

at $t = 0, x = 5\ cm$

$T = 6$ sec

So, $\omega=\frac{2\pi}{T}=\frac{2\pi}{6}=\frac{\pi}{3}\sec^{-1}$

At $t = 0, x = 5\ cm$

So, $5 = 10 \sin(\omega \times 0 + \phi) = 10 \sin\phi$

$\phi=\frac{\pi}{6}$

Equation of displacement $= x = 10cm\sin\left(\frac{\pi}{3}t+\frac{\pi}{6}\right)$

Also At $t = 4$ sec

$x=10\sin\left(\frac{\pi}{3}\times4+\frac{\pi}{6}\right)=10cm\left(\frac{8\pi+\pi}{6}\right)$

$=10\sin\left(\frac{3\pi}{2}\right)=-10$

acceleration $=-\omega^2x=-\omega^2x=-\left(\frac{\pi^2}{9}\right)\times(-10)=10.9cm/\sec = 11\ cm/s^2$

7. **Ans.** $0.06m$

$f=\frac{25}{\pi}$

$\omega = 50$

$R=\frac{0.2}{10}\times50\times50$

$= 500\ N/m$

$KE = 0.9 = \frac{1}{2}\times500\times A^2$

$\Rightarrow\ A^2=\frac{1.8}{500}=\frac{18}{5000}=\frac{9}{2500}$

$\therefore\ A = \frac{3}{50}=0.06m$

8. **Ans.** $y = 0.1\sin(4t + \pi/4)$

$KE_m = E_0 = \frac{1}{2}\ mw\pi$

$8\times10^{-3}=\frac{1}{2}\times0.1\times\omega^2\times0.01$

$\therefore\ \omega = 4$

$x = 0.1\ \sin\left(4t+\frac{\pi}{4}\right)$

9. **Ans.** (i) $x_0 = 2m$; (ii) $T = \sqrt{2}\,\pi$ sec.; (iii) $2\sqrt{3}$

(i) $v = x^2 - 4x + 3$

$F = -2x + 4$

$\therefore$ $m.p \Rightarrow x = 2$

(ii) $F = -2\,(x - 2)$

$$\omega^2 = \frac{2}{1}$$

$$\omega = \sqrt{2}$$

$$\therefore T = \frac{2\pi}{\sqrt{2}} = \sqrt{2}\pi \text{ s}$$

(iii) $\frac{1}{2} \times 2 \times A^2 = \frac{1}{2} \times 1 \times 24$

$$A = 2\sqrt{3}m$$

10. **Ans.** 3

$x = A\sin\omega t$

$$v = \frac{dx}{dt} = A\omega\cos\omega t$$

$$v_k = \frac{1}{2}mv^2 = \frac{1}{2}mA^2\omega^2\cos^2\omega t$$

$$v_p = \frac{1}{2}m\omega^2 A^2 \sin^2\omega t$$

$$\frac{v_k}{v_p} = \frac{\cos^2\omega t}{\sin^2\omega t} = \cot^2\omega t = \cot^2\left(\frac{2\pi}{T} \times \frac{T}{12}\right) = \cot^2\left(\frac{\pi}{6}\right) = \left(\frac{\sqrt{3}}{1}\right)^2 = 3:1$$

11. **Ans.** 3

After displacement x against spring C

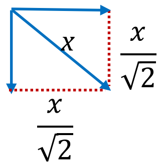


Figure-1

Deformation in springs ABC is shown in figure 1.

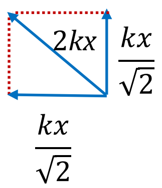


Figure-2

Forces applied by three springs ABC is shown in figure-2

$$F = \frac{kx}{\sqrt{2}}\frac{1}{\sqrt{2}} \times 2 + 2kx = 3\,kx = m\omega^2 x \Rightarrow \omega = \sqrt{\frac{3k}{m}}$$

$\therefore$ $N = 3$

12. **Ans.** 12

Let block is pushed down by x from its equilibrium position.

Let extension in springs be y_1 and y_2 as shown in figure.

so restoring force on block $F = K_2y_2$

Force in left spring is $2F$

From constraint motion

$$\Rightarrow\ 2 \times \frac{2F}{K_1} + \frac{F}{K_2} = x \Rightarrow\ F = \left(\frac{K_1K_2}{K_1+4K_2}\right)x = \omega^2 x$$

$$\text{So}\ \ \omega = \sqrt{\frac{1}{m}\left(\frac{K_1K_2}{K_1+4K_2}\right)}$$

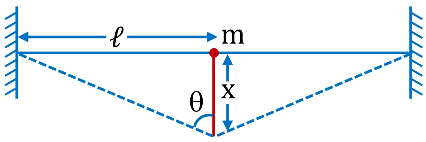
$$\Rightarrow\ T = 2\pi\sqrt{\frac{m\ (K_1+4K_2)}{K_1K_2}}$$

$$T = 2\pi\sqrt{\frac{m\ (K_1+4K_2)}{K_1K_2}} = \pi\sqrt{\frac{12m}{K}}$$

$\therefore\ \ n = 12$

13. **Ans.** $\omega = \sqrt{\dfrac{2T_0}{m\ell}}$

$x << \ell$



Net restoring force = $2T_0 \cos\theta$.

$\therefore$ we have,

$$\frac{2T_0 x}{\sqrt{\ell^2+x^2}} = ma$$

$$\Rightarrow\ a = \frac{2T_0}{m\ell}\cdot x\ = \omega = \sqrt{\frac{2T_0}{m\ell}}$$

14. **Ans.** $1/\sqrt{3}$

$$\frac{R_1}{R_2} \propto \frac{\ell_2}{\ell_1} = \frac{3}{1} \qquad \left(R \propto \frac{1}{\ell}\right)$$

$$\frac{T_1}{T_2} \propto \sqrt{\frac{R_2}{R_1}} = \frac{1}{\sqrt{3}} \qquad \left(T \propto \sqrt{\frac{m}{\ell}}\right)$$

15. **Ans.** $M = 1.6\ kg$

The time period T of the spring is $T = 2\pi\sqrt{\dfrac{M}{k}}$

$$\text{or, } 2 = 2\pi\sqrt{\frac{M}{k}} \qquad \text{...(i)}$$

In the second case

$$3 = 2\pi\sqrt{\frac{M+2}{k}} \qquad \text{...(ii)}$$

$$\frac{2}{3} = \sqrt{\frac{M}{M+2}} \Rightarrow \frac{4}{9} = \frac{M}{M+2} \Rightarrow M = 1.6\ kg$$

16. **Ans.** (a) $\frac{1}{T}=\frac{1}{2\pi}\sqrt{\frac{g}{L}}$, (b) $\frac{L}{3}$

(a) $f=\frac{\omega}{2\pi}=\frac{1}{2\pi}\sqrt{\frac{R}{m}}=\frac{1}{2\pi}\sqrt{\frac{g}{L}}$

(b) A = distance between v = 0 and a = 0

$=\frac{mg}{3R}$

$=\frac{L}{3}\left(As\frac{mg}{R}=\ell\right)$

17. **Ans.** 100 Nm^{-1}

$$\frac{2\pi}{3}\sqrt{\frac{0.9}{R}}=0.2$$

$$\frac{4\times10}{9}\times\frac{9}{10\times R}=\frac{4}{100}$$

$\Rightarrow\ R\approx100\ N/m$

18. **Ans.** $10/\pi\ Hz,\ \frac{5\sqrt{37}}{6}\ cm$

$$v_f=\frac{0.5\times3}{1.5}=1\ m/s$$

$$x=\frac{15}{600}-\frac{10}{600}=\frac{1}{120}$$

$$\omega=\sqrt{\frac{600}{1.5}}=20$$

$$I=20\sqrt{A^2-\left(\frac{1}{120}\right)^2}$$

$$=A^2=\frac{1}{20^2}+\frac{1}{120^2}$$

$$A^2=\frac{1}{400}\left(\frac{1}{1}+\frac{1}{36}\right)$$

$$\therefore\ A=\frac{\sqrt{37}}{120}m$$

$$\frac{5\sqrt{37}}{6}$$

19. **Ans.** 2

From work energy theorem maximum extension of spring is x_m

$mgx_m\sin\theta=\frac{1}{2}kx_m^2$ or $x_m=\frac{2mg\sin\theta}{k}$

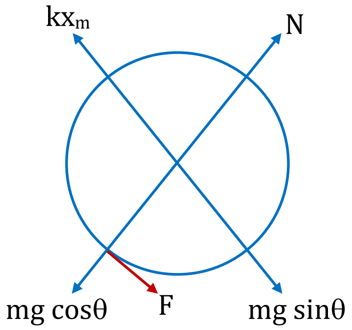
At lower extreme

$kx_m-mg\sin\theta-F=ma$ ...(i)

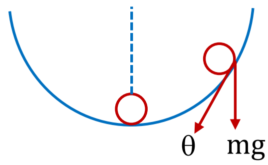
$a=R\alpha$ ...(ii)

$FxR=\frac{mR^2}{2}\alpha$ ...(iii)

Thus $F=\frac{mg\sin\theta}{3}$



20. **Ans.** 5



$F = mg \sin\theta \simeq mg \tan\theta$

$\tan\theta = \dfrac{dy}{dx} = 40\,x$

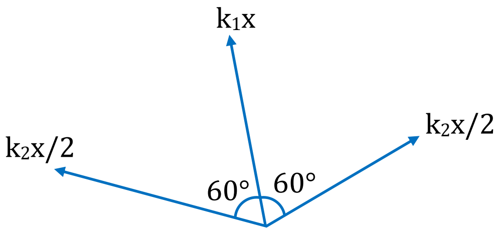
$-m\omega^2 x = -mg \times 40\,x$

$\omega = \sqrt{400} = 20$ rad/s

21. **Ans.** 1

$$f_a = \frac{1}{2\pi}\sqrt{\frac{\left(k_1 + \frac{k_2}{2}\right)}{m}}\ ,\ f_b = \frac{1}{2\pi}\sqrt{\frac{\left(k_1 + \frac{k_2}{2}\right)}{m}}$$

(a) In case (a) $k_{eq} = k_1 + \dfrac{k_2}{2}$



(b) $x' = x\cos 60°$

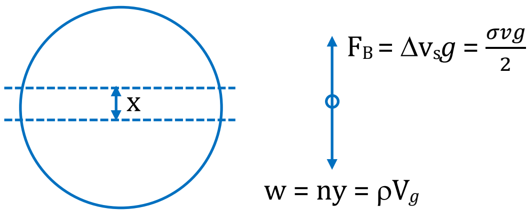
Thus making *FBD*

$$-\left(k_1 x + \frac{2k_2}{2}\cos 60°\right) = m\frac{d^2x}{dt^2}$$

In this case also $k_{eq} = k_1 + \dfrac{k_2}{2}$

So $f_b = f_a$

22. **Ans.** $f = \dfrac{1}{2\pi}\sqrt{\dfrac{3g}{2R}}$



$FB = w$

$\sigma \dfrac{V}{2} g = \rho V_g$

$\rho = \dfrac{\sigma}{2}$

$\Delta F = \sigma \Delta V_g = \pi R^2 g x$

$T = 2\pi\sqrt{\dfrac{m}{k}}$

$m = \left(\dfrac{4}{3}\pi R^3\right)\dfrac{r}{2}g$

$$T = 2\pi\sqrt{\frac{2\pi R^3\sigma}{3\sigma\pi R^2 g}} = 2\pi\sqrt{\frac{2R}{3g}}$$

$$f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$$

23. **Ans.** $\dfrac{5}{2\pi}$ *Hz*, 5 *cm*

$\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{100}{1}} = 10$

$\omega' = \sqrt{\dfrac{100}{4}} = 5$

$T' = \dfrac{2\pi}{\omega'} = \dfrac{2\pi}{5}$

$f' = \dfrac{1}{T'} = \dfrac{5}{2\pi}$

$v = \omega A = 10 \times \frac{1}{10} = 1 m/\sec$

$m'v' = mv$ ($m' = 4$ kg, $m = 1$ kg, $v = 1$ m/sec)

$4 \times v' = 1 \times 1$

$v' = \frac{1}{4}$

$\omega' A' = \frac{1}{4}$

$5A' = \frac{1}{4}$

$A' = \frac{1}{20}$

$A' = 5$ cm

24. **Ans.** 0.16 kg

$x = 25 = 0.25$ cm

$E = 5$ J, $f = 5$

$T = \frac{1}{5}\sec$

$P.E. = \frac{1}{2}kx^2 = \frac{1}{2} \times k(0.25)^2 = 5$

$k = 160$ N/m

$T = 2\pi\sqrt{\frac{m}{k}}$

$\frac{1}{5} = 2\pi\sqrt{\frac{m}{160}}$

$m = 0.16$ kg

25. **Ans.** 5

Let, in equilibrium position, compression of spring be x. Liquid of volume $\ell^2 x$ is displaced from its original position and level of liquid in tank rises as shown in figure.

This rise in level, $\Delta x = \frac{l^2 x}{A - l^2}$;

Where $A = 15$ $cm \times 20$ cm (Base area of tank)

or $\Delta x = 0.5x$

$\therefore$ Mass of water displaced by the block = $\ell^2(x + \Delta x)\rho = 15x$ kg

upthrust exerted by water = apparent weight of water displaced,

$\therefore$ Upthrust $F_1 = 15x(g - a) = 120.x$ Newton

Upward force exerted by spring

$F_2 = Kx = 280.x$

Considering free body diagram of the block, (Figure)

$mg - (F_1 - F_2) = ma$

Substituting values of F_1 and F_2, $x = 0.04$ $m = 4cm$

If the block is slightly pushed downward by dx, both F_1 and F_2 increase.

Increase in F_1 is $dF_1 = 120dx$

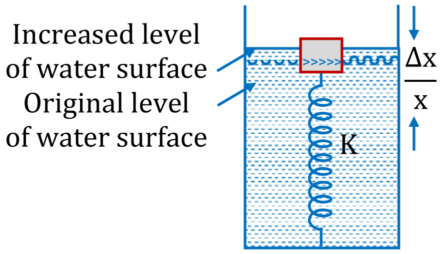
Increase in is $dF_2 = 280dx$

Restoring force on block = increase in F_1 + increase in F_2

$= dF_1 + dF_2 = (120dx + 280dx) = 400dx$

or Restoring acceleration

$$= \frac{400.dx}{m} = 200.dx = \omega^2(dx) \Rightarrow \omega = 10\sqrt{2}$$



Since, restoring acceleration ∝ displacement (dx)

Therefore, block performs SHM.

Hence, frequency, $f = \frac{5\sqrt{2}}{\pi}$ per second.

$a = 5$

26. **Ans.** 3

Suppose origin is at the equilibrium position and the direction of increasing x is toward the right. If the blocks are at the origin, the net force on them is zero. If the blocks are a small distance x to the right of the origin, value of the net force on them is $-4kx$. Applying Newton's second law to the two-block system gives

$$-4kx = 2ma$$

Applying Newton's second law to the lower block gives

$$k(x_1 - x) - f = ma$$

where x_1 = initial stretch and f is the magnitude of the frictional force.

$$f = k(x_1 + x)$$

The maximum value for x is the amplitude A and the maximum value for f is $\mu_s mg$. Thus, $\mu_s mg = k(x_1 + A_{max})$. Solving for A_{max} gives

$$A_{max} = \frac{\mu_s mg}{k} - x_1 = 3$$

27. **Ans.** 7

$$\omega = \frac{2\pi}{\pi} = 2$$

$2\mu\, mg = F_{max}$

$F_{max} - mg = m\omega^2 A$

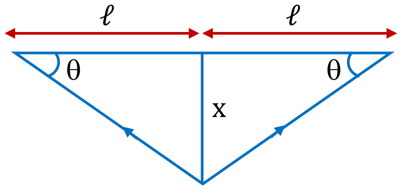
$F_{max} = mg + 4m = 14\, m$

$2\mu\, mg = 14\, m$

$\mu = 0.7$

28. **Ans.** 6

$F = -4F_0 \sin\theta = -\dfrac{4F_0 x}{\ell}$



29. **Ans.** $T = 2\sqrt{\ell/g}\ [\pi/2 + \sin^{-1}(\alpha/\beta)]$

The system can be assumed as an incomplete SHM.

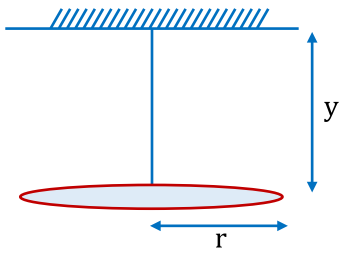
Thus, time period can be calculate as twice the (time taken by the bob to travel to mean position from extreme + time taken to travel from mean to wall)

$$\therefore\quad T_{net} = 2\left[\frac{T}{4} + \frac{\sin^{-1}\left(\frac{\alpha}{\beta}\right)T}{2\pi}\right] = 2\left[\frac{2\pi}{4}\cdot\sqrt{\frac{\ell}{g}} + \frac{\sin^{-1}\left(\frac{\alpha}{\beta}\right)}{2\pi}\cdot 2\pi\sqrt{\frac{\ell}{g}}\right]$$

$$= 2\sqrt{\frac{\ell}{g}}\left[\frac{\pi}{2} + \sin^{-1}\frac{\alpha}{\beta}\right]$$

30. **Ans.** $2\pi\sqrt{\dfrac{r\sqrt{2}}{g}}, r/\sqrt{2}$

$$T = 2\pi\sqrt{\frac{\frac{Mr^2}{2} + My2}{Mgy}} = 2\pi\sqrt{\frac{r^2 + 2y^2}{2y\,g}}$$



$$\frac{d}{dy}\left(\frac{r^2 + 2y^2}{2y}\right) = 0$$

$$\Rightarrow\quad 2y\frac{[0 + 4y] - (r^2 + 2y^2)^2}{4y^2} = 0$$

$$\Rightarrow\quad 8y^2 - 2r^2 - 4y^2 = 0$$

$$4y^2 = 2r^2$$

$$y = \frac{r}{\sqrt{2}}$$

$$T = 2\pi\sqrt{\frac{r^2 + 2\left(\frac{r^2}{2}\right)}{2\left(\frac{r}{\sqrt{2}}\right)g}}$$

$$= 2\pi\sqrt{\frac{2r}{\sqrt{2}g}} = 2\pi\sqrt{\frac{r\sqrt{2}}{g}}$$

31. **Ans.** $2\pi\sqrt{\dfrac{17L}{18g}}$

$$d_{cm} = \frac{3\ell}{4}$$

$$I = \frac{m\ell^2}{3} + \frac{m\ell^2}{12} + m\ell^2 = \frac{17m\ell^2}{12}$$

$$2mg\frac{3\ell}{4}\times\theta = \frac{17m\ell^2}{12}\times\omega^2\times\theta$$

$$\omega^2 = \frac{18g}{17\ell}$$

$$\therefore \quad T = 2\pi\sqrt{\frac{17\ell}{18g}} = \frac{2\pi}{3}\sqrt{\frac{17\ell}{2g}}$$

32. **Ans.** $\omega = \sqrt{\dfrac{k}{(m+6M)}}$

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + 2mv^2 + 2\times\frac{mR}{2}\times\left(\frac{v}{R}\right)$$

$$= \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + 3mv^2$$

$$\frac{dE}{dt} = kx\frac{dx}{dt} + mv\frac{dv}{dt} + 6mv\frac{dv}{dt} = 0$$

$\Rightarrow \quad kx = -(m + 6m)\,(-\omega^2 x)$

$$\Rightarrow \quad \omega = \sqrt{\frac{k}{m+6M}}$$

$$\therefore \quad f = \frac{1}{2\pi}\sqrt{\frac{k}{m+6M}}$$

33. **Ans.** (a) 2 sec, (b) $T = \dfrac{2}{5^{1/4}}$ sec

(a) $10\pi^2 \times \theta = 10 \times \omega^2 \times \theta$

$\omega = \pi$

$\therefore \quad T = 2$ sec

(b) $g_e = \sqrt{100+25} = 5\sqrt{5}$

$$\therefore \quad T = 2\pi\sqrt{\frac{5}{10\times5\sqrt{5}}} = \frac{2}{\sqrt[4]{5}}\text{sec}$$

34. **Ans.** (a) $T = mg + mgA^2\sin^2\sqrt{\dfrac{g}{\ell}}\,t$ (b) $t = (2n+1)\dfrac{\pi}{2}\sqrt{\dfrac{\ell}{g}}$; $n \in I$ $T_{max} = mg + mgA^2$

(a) $T = mg\cos\theta + m\omega^2\ell$

$$= mg\left(1-\frac{\theta^2}{2}\right) + m\times\frac{g}{\ell}\left(A^2-\theta^2\right)\times\ell = mg\left(A^2+1-\frac{3\theta^2}{2}\right)$$

(b) $T = mg + \dfrac{mg}{\ell}A^2\sin^2\sqrt{\dfrac{g}{\ell}}\times t\times\ell = mg\left(1+A^2\sin^2\sqrt{\dfrac{g}{\ell}}t\right)$

$T_{max} = mg\,(1 + A^2)$

$$t_{T_{max}} = \frac{(2n+1)}{2}\pi\sqrt{\frac{\ell}{g}}$$

35. **Ans.** (a) $\omega=\sqrt{\dfrac{g\ell}{\frac{1}{2}R^2+\ell^2}}$ (b) $\ell=\dfrac{R}{\sqrt{2}}$

$\dfrac{R}{\sqrt{2}}$

$$\omega=\sqrt{\frac{mgd}{I_P}}=\sqrt{\frac{mg\ell}{\frac{mR^2}{2}+m\ell^2}}=\sqrt{\frac{g\ell}{\frac{R^2}{2}+\ell^2}}$$

$$\omega^2=\frac{g\ell}{\frac{R^2}{2}+\ell^2}$$

$$\frac{d(\omega^2)}{d\ell}=0$$

$$\Rightarrow\ g\ell(2\ell)=g\left(\frac{R^2}{2}+\ell^2\right)$$

$$\ell^2=\frac{R^2}{2}$$

$$\Rightarrow\ \ell=\frac{R}{\sqrt{2}}$$

36. **Ans.** 2

From conservation of angular momentum

$$mv\frac{R}{2}+mv\frac{R}{2}+0=\frac{5}{2}mR^2W$$

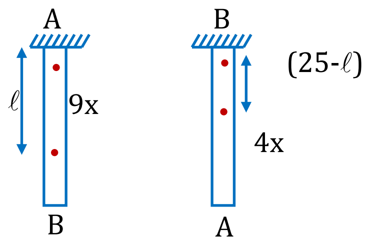
$$W=\left(\frac{2v}{5R}\right)$$

Angular freq. $W_0=\sqrt{\dfrac{C}{I}}=\sqrt{\dfrac{2C}{5mR^2}}$

$\therefore$ Angular Amplitude $(\beta)=\left(\dfrac{W}{W_0}\right)$

$$=\frac{2v}{5R}\sqrt{\frac{5mR^2}{2C}}=V\sqrt{\frac{2}{5}\frac{m}{C}}=20\sqrt{\frac{2}{5}\times\frac{2}{80}}=2\ \text{radian}$$

37. **Ans.** 20



T=3sec T=4sec

$$T=2\pi\sqrt{\frac{I}{mgl}}$$

Case-I

$$3 = 2\pi\sqrt{\frac{9x}{mgl}} \quad \text{...(i)}$$

Case-II

$$4 = 2\pi\sqrt{\frac{4x}{mg(25-l)}} \quad \text{...(ii)}$$

$$\frac{3}{4} = \sqrt{\frac{3x}{l}} \times \sqrt{\frac{(25-l)}{4x}} = \frac{3}{2}\sqrt{\frac{(25-l)}{l}}$$

$$\frac{1}{2} = \sqrt{\frac{25-l}{l}}$$

$$\frac{1}{4} = \frac{25-l}{l}$$

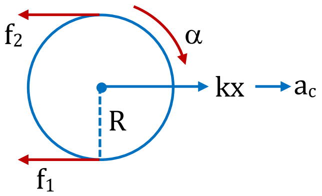
$\ell = 100 - 4\ell$

$5\ell = 100$

$\ell = 20\ cm$

38. **Ans.** 2

$f_2 = ma_p$ ... (1)



$k_x - f_1 - f_2 = ma_C$ ... (2)

$(f_1 - f_2)R = I_C\alpha = \frac{mR^2}{2}\alpha$ ... (3)

$a_C + \alpha R = a_P$ ... (4)

$a_C - \alpha R = 0$ ... (5)

Solving the equations we get

$$f_1 = \frac{5ma_C}{2}, f_2 = 2ma_C$$

$$\Rightarrow \quad kx - \frac{5ma_C}{2} = 3ma_C$$

$$\Rightarrow \quad kx = \left(\frac{5}{2} + 3\right)ma_C$$

$$\Rightarrow \quad kx = \frac{11}{2}ma_C$$

$$\Rightarrow \quad a_C = \frac{2kx}{11m} = \omega^2 x$$

$$\Rightarrow \quad \omega^2 = \frac{2k}{11m} \quad \Rightarrow \quad p^2 = \frac{2k}{11m} \quad \Rightarrow \quad \frac{11p^2m}{k} = 2$$

39. **Ans.** 1000

$$y = 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right) \;=\; 10\sin\left(3\pi t + \frac{\pi}{3}\right)$$

Thus, amplitude is 10 *m* or 1000 *cm*.

40. **Ans.** $-4\pi^2 \cos \pi t$

Equation of SHM $\Rightarrow r = A\sin\left(\omega t + \frac{\pi}{2}\right)$

$$A = 2\sqrt{2}$$

$$r = 2\sqrt{2}\cos\omega t$$

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi \qquad (T = 2\text{ sec})$$

$$r = 2\sqrt{2}\cos\pi t$$

$$x = r\cos 45° = 2\sqrt{2}\times\frac{1}{\sqrt{2}}\cos\pi t$$

$$a = -\,\omega^2 x$$

$$= -\,(\pi^2)\cdot 2\cos\pi t$$

$$F_x = ma_x = -4\pi^2\cos\pi t$$

EXERCISE - JEE (Main) PYQ

1. **Ans. (4)**

$x = A\cos\omega t$

Displacement in t time = $A - A\cos\omega t$

For $t = \tau$ए $A[1 - \cos\omega\tau] = a$

For $t = 2\tau$ए $A[1 - \cos 2\omega\tau] = 3a$

$$\frac{1-\cos\omega\tau}{1-\cos 2\omega\tau} = \frac{1}{3}$$

$$\frac{1-\cos\omega\tau}{2(1-\cos^2\omega t)} = \frac{1}{3}$$

Say $x = \cos\omega t$

$$\frac{1-x}{2\left(1-x^2\right)} = \frac{1}{3}$$

$$\Rightarrow \quad \frac{1}{2\left(1+x\right)} = \frac{1}{3}$$

$$\Rightarrow \quad 3 = 2 + 2x$$

$$\Rightarrow \quad x = \frac{1}{2} = \cos\omega\tau$$

$$A = 2a, \quad \omega\tau = \frac{\pi}{3}\text{ s}$$

$$\Rightarrow \quad \frac{2\pi}{T}\tau = \frac{\pi}{3}$$

$$\Rightarrow \quad T = 6\tau$$

2. Ans. (2)

K.E. is maximum at mean position, whereas P.E. is minimum.

At extreme position, K.E. is minimum and P.E. is maximum.

3. Ans. (3)

$$v = \omega\sqrt{A^2 - \left(\frac{2A}{3}\right)^2}$$

$$v = \sqrt{5}\frac{A\omega}{3}$$

$v_{\text{new}} = 3v = \sqrt{5}A\omega$

So, the new amplitude is given by

$$v_{new} = \omega\sqrt{A_{new}^2 - x^2}$$

$$\Rightarrow \quad \sqrt{5}A\omega = \omega\sqrt{A_{new}^2 - \left(\frac{2A}{3}\right)^2}$$

$$A_{new} = \frac{7A}{3}$$

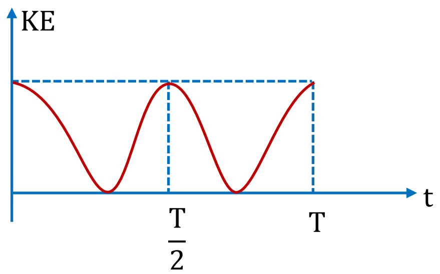
4. Ans. (1)

$x = A\sin(\omega t + \phi)$

$\phi = 0, \pi$

$x = \pm A\sin\omega t$

$$KE = \frac{1}{2}m\omega^2\left(A^2 - x^2\right) = \frac{1}{2}KA^2\cos^2\omega t$$



NOTE : But as per options given, best possible answer will be option (1).

5. Ans. (4)

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow k = 4\pi^2 m \times f^2$$

$$k = 4\pi^2 \times \frac{108\times10^{-3}}{6.02\times10^{23}} \times f^2$$

$$= \frac{4\times\pi^2\times108}{6.02} \times \frac{10^{24}\times10^{-3}}{10^{23}} = 7.1N/m$$

6. Ans. (2)

Frequency of torsional oscillations is given by

$$f = \frac{k}{\sqrt{I}}$$

$$f_1 = \frac{k}{\sqrt{\frac{M(2L)^2}{12}}}$$

$$f_2 = \frac{k}{\sqrt{\frac{M(2L)^2}{12} + 2m\left(\frac{L}{2}\right)^2}}$$

$$f_2 = 0.8f_1$$

$$\frac{m}{M} = 0.375$$

7. Ans. (4)

$v = \omega\sqrt{A^2 - x^2}$ ...(1)

$a = -\omega^2 x$ ...(2)

$|v| = |a|$ ...(3)

$\omega\sqrt{A^2 - x^2} = \omega^2 x$

$A^2 - x^2 = \omega^2 x^2$

$5^2 - 4^2 = \omega^2(4^2)$

$\Rightarrow 3 = \omega \times 4$

$T = 2\pi/\omega$

$$3 = \frac{2\pi}{T} \times 4$$

$$T = \frac{8\pi}{3}$$

8. Ans. (1)

Angular frequency of pendulum

$$\omega = \sqrt{\frac{g_{eff}}{\ell}}$$

$$\therefore \quad \frac{\Delta\omega}{\omega} = \frac{1}{2}\frac{\Delta g_{eff}}{g_{eff}} \qquad [\omega_s = \text{angular frequency of support}]$$

$$\frac{\delta\omega}{\omega} = \frac{1}{2} \times \frac{2A\omega_s^2}{10}$$

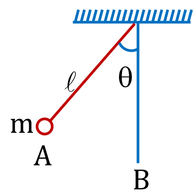
$$\frac{\delta\omega}{\omega} = 10^{-3}\, rad/s$$

9. Ans. (3)

Maximum kinetic energy at lowest point B is given by

$K = mgl\,(1 - \cos\theta)$

Where θ = angular amp.



$K_1 = mgl\,(1 - \cos\theta)$

$K_2 = mg(2l)\,(1 - \cos\theta)$

$K_2 = 2K_1$

10. Ans. (3)

$$k = \frac{1}{2}m\omega^2 A^2 \cos^2 \omega t$$

$$U = \frac{1}{2}m\omega^2 A^2 \sin^2 \omega t$$

$$\frac{k}{U} = \cot^2 \omega t = \cot^2 \frac{\pi}{90}(210) = \frac{1}{3}$$

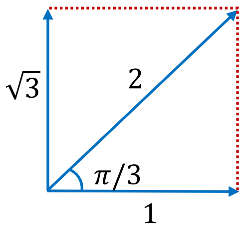
11. Ans. (4)

$y = 5[\sin(3\pi t) + \sqrt{3}\cos(3\pi t)]$

$= 10\sin\left(3\pi t + \frac{\pi}{3}\right)$

Amplitude = 10 cm

$T = \frac{2\pi}{\omega} = \frac{2\pi}{3\pi} = \frac{2}{3}\text{sec}$



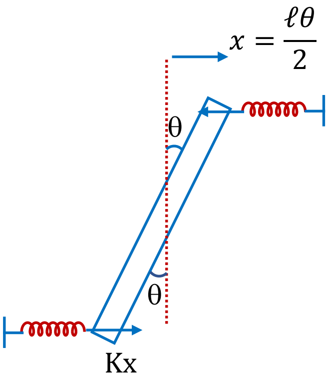
12. Ans. (1)

$\tau = -2Kx\frac{\ell}{2}\cos\theta$

$\Rightarrow \quad \tau = -\left(\frac{K\ell^2}{2}\right)\theta = -C\theta$

$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{C}{I}} = \frac{1}{2\pi}\sqrt{\frac{\frac{K\ell^2}{2}}{\frac{M\ell^2}{12}}}$

$\Rightarrow \quad f = \frac{1}{2\pi}\sqrt{\frac{6K}{M}}$



13. Ans. (1)

For a simple pendulum $T = 2\pi\sqrt{\frac{L}{g_{eff}}}$

Situation 1 : when pendulum is in air $\rightarrow g_{eff} = g$

Situation 2 : when pendulum is in liquid

$\rightarrow g_{eff} = g\left(1 - \frac{\rho_{\text{liquid}}}{\rho_{\text{body}}}\right) = g\left(1 - \frac{1}{16}\right) = \frac{15g}{16}$

So, $\frac{T'}{T} = \frac{2\pi\sqrt{\frac{L}{15g/16}}}{2\pi\sqrt{\frac{L}{g}}} \Rightarrow T' = \frac{4T}{\sqrt{15}}$

14. Ans. (4)

$\omega = \sqrt{\frac{k}{I}}$

$\omega = \sqrt{\frac{3k}{m\ell^2}}$

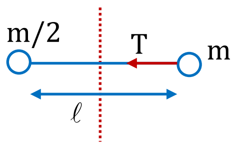
$\Omega = \omega\theta_0$ = Average velocity

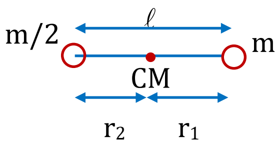
$T = m\Omega^2 r_1$

$T = m\Omega^2\frac{\ell}{3} = m\omega^2\theta_0^2\frac{\ell}{3} = m\frac{3k}{m\ell^2}\theta_0^2\frac{\ell}{3} = \frac{k\theta_0^2}{\ell}$

$I = \mu\ell^2 = \frac{\frac{m^2}{2}}{\frac{3m}{2}}\ell^2 = \frac{m\ell^2}{3}$

$\frac{r_1}{r_2} = \frac{1}{2} \Rightarrow r_1 = \frac{\ell}{3}$

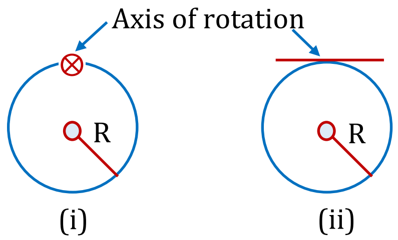




15. **Ans. (1)**

Moment of inertia in case (i) is I_1

Moment of inertia in case (ii) is I_2



$I_1 = 2MR^2$

$I_2 = \frac{3}{2}MR^2$

$T_1 = 2\pi\sqrt{\frac{I_1}{Mgd}}$; $T_2 = 2\pi\sqrt{\frac{I_2}{Mgd}}$

$$\frac{T_1}{T_2} = \sqrt{\frac{I_1}{I_2}} = \sqrt{\frac{2MR^2}{\frac{3}{2}MR^2}} = \frac{2}{\sqrt{3}}$$

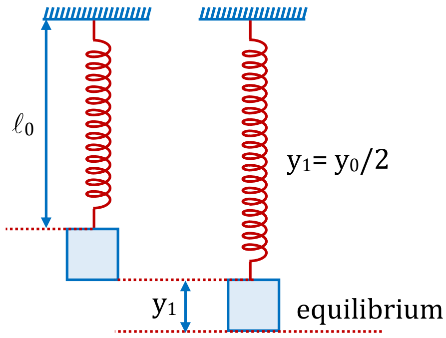
16. **Ans. (2)**

$y = y_0\sin^2\omega t$

$$y = \frac{y_0}{2}(1-\cos 2\omega t)$$

$$y - \frac{y_0}{2} = -\frac{y_0}{2}\cos 2\omega t$$

Amplitude : $\frac{y_0}{2}$



$$\frac{y_0}{2} = \frac{mg}{K}$$

$$2\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{2g}{y_0}}$$

$$\omega = \sqrt{\frac{g}{2y_0}}$$

Ans. (2)

17. **Ans. (3)**

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$g = \frac{4\pi^2\ell}{T^2}$$

$$\Rightarrow \quad \frac{\Delta g}{g} \times 100 = \left(\frac{0.1}{25} + \frac{2\times 1}{50}\right)\times 100 = 4.4\%$$

18. **Ans. (2)**

$$T_a = 2\pi\sqrt{\frac{M}{K}}$$

$$T_b = 2\pi\sqrt{\frac{M}{K/2}}$$

$$\frac{T_a}{T_b} = \sqrt{2} = \sqrt{x}$$

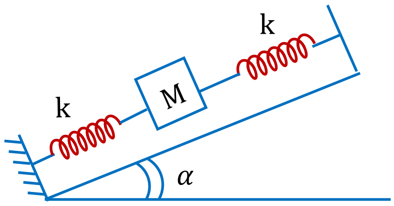
$$\Rightarrow \quad x = 2$$

19. Ans. (3)

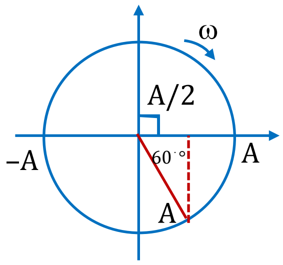
$K_{eq} = K_1 + K_2 = K + K = 2K$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{2K}}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$$ (option 3) is correct.



20. Ans. (3)



Initial phase $\frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

21. Ans. (4)

Time period $T = \frac{2\pi}{\omega'}$

$$\frac{\pi}{\omega} = \frac{2\pi}{\omega'}$$

$\omega' = 2\omega \rightarrow$ Angular frequency of SHM

Option (3)

$$\sin^2 \omega t = \frac{1}{2}\left(2\sin^2 \omega t\right) = \frac{1}{2}\left(1 - \cos 2\omega t\right)$$

Angular frequency of $\left(\frac{1}{2} - \frac{1}{2}\cos 2\omega t\right)$ is 2ω

Option (4).

Angular frequency of SHM

$3\cos\left(\frac{\pi}{4} - 2\omega t\right)$ is 2ω

So, option (3) and (4) both have angular frequency 2ω but option (4) is direct answer.

22. Ans. (4)

$$v^2 = \omega^2\left(A^2 - x^2\right)$$

$$A^2 = x_1^2 + \frac{v_1^2}{\omega^2} = x_2^2 + \frac{v_2^2}{\omega^2}$$

$$\omega^2 = \frac{v_1^2 - v_2^2}{x_2^2 - x_1^2}$$

$$T = 2\pi\sqrt{\frac{x_2^2 - x_1^2}{v_1^2 - v_2^2}}$$

23. Ans. (3)

From potential energy curve

$$U_{max} = \frac{1}{2}kA^2 \Rightarrow 10 = \frac{1}{2}k(2)^2$$

$\Rightarrow \quad k = 5$

Now $T_{Spring} = T_{pendulum}$

$$2\pi\sqrt{\frac{5}{5}} = 2\pi\sqrt{\frac{4}{g}}$$

$\Rightarrow \quad 1 = \sqrt{\frac{4}{g}} \Rightarrow g = 4\ m/s^2$ on planet

Option (3).

24. Ans. (2)

$$x_2 = 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin 2\pi t + \frac{1}{\sqrt{2}}\cos 2\pi t\right)\sqrt{2}$$

$$= 10\sin\left(2\pi t + \frac{\pi}{4}\right)$$

$\therefore \quad \frac{A_2}{A_1} = \frac{10}{5} = 2$

25. Ans. (1)

$y_1 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right)$

$\Rightarrow \quad$ Amplitude = 10

$y_2 = 5\ (\sin 3\pi t + \sqrt{3}\ \cos 3\pi t)$

$$y_2 = 10\left(\frac{1}{2}\sin 3\pi t + \frac{\sqrt{3}}{2}\cos 3\pi t\right)$$

$$y_2 = 10\left(\cos\frac{\pi}{3}\sin 3\pi t + \sin\frac{\pi}{3}\cos 3\pi t\right)$$

$y_2 = 10 \sin\left(3\pi t + \frac{\pi}{3}\right) \Rightarrow$ Amplitude = 10

So ratio of amplitudes = $\frac{10}{10} = 1$

26. Ans. (2)

$V_{max} = \omega A$

$\Rightarrow \quad \frac{A_1}{A_2} = \frac{\omega_2}{\omega_1} = \sqrt{\frac{9}{2}\times\frac{1}{2}} = \frac{3}{2}$

27. Ans. (2)

$KE = PE + \frac{PE}{4}$

$KE = \frac{5}{4}PE$

$\frac{1}{2}m\omega^2 (A^2 - x^2) = \frac{5}{4} \times \frac{1}{2}m\omega^2x^2$

$[v = \omega\sqrt{A^2 - x^2}]$

$A^2 - x^2 = \frac{5}{4}x^2$

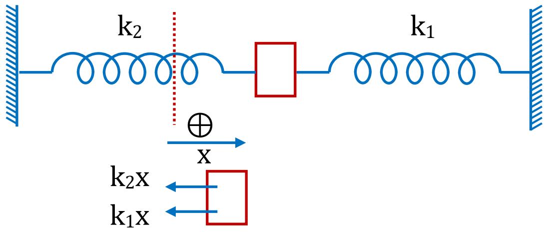
$\frac{9x^2}{4} = A^2$

$\boxed{x = \frac{2}{3}A}$

$\therefore \quad x = \frac{2}{3} \times 3cm$

$x = 2\ cm$

28. Ans. (3)



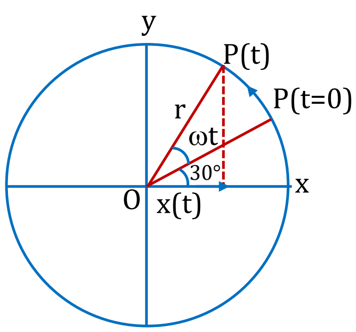
On displacing m to right by x

$F = -(k_1x + k_2x) = -(k_1 + k_2)x$

$$a = \frac{F}{m} = -\left(\frac{k_1 + k_2}{m}\right)x = \omega^2 x$$

$$\therefore \quad \omega = \sqrt{\frac{k_1 + k_2}{m}} \Rightarrow T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k_1 + k_2}}$$

29. Ans. (1)

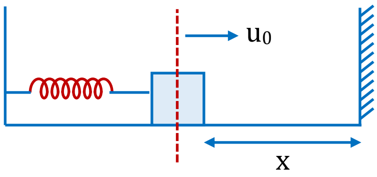


$x(t) = r\cos(\omega t + 30°)$

$x(t) = r\cos(\omega t + \pi/6)$

EXERCISE - JEE (Advanced) PYQ

1. Ans. (AD)



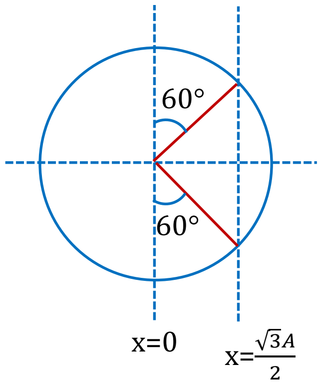
Collision is elastic so energy is conserve in given question so when it return to equilibrium, its speed is u_0

By $\quad v=\omega\sqrt{A^2-x^2}$

$$\frac{A\omega}{2}=\omega\sqrt{A^2-x^2}$$

$$\Rightarrow \quad x=\frac{\sqrt{3}A}{2}$$

Time period if collision not occurs $T=2\pi\sqrt{\frac{m}{k}}$



Now time to return for the first time at equilibrium = $\frac{\theta}{\omega}$

$$=\frac{2\pi}{3}\sqrt{\frac{m}{k}}$$

Time for option (C)

$$=\left(\frac{2\pi}{3}+\frac{\pi}{2}\right)\sqrt{\frac{m}{k}}=\frac{7\pi}{6}\sqrt{\frac{m}{k}}$$

Time for option (D) = $\left(\frac{2\pi}{3}+\pi\right)\sqrt{\frac{m}{k}}=\frac{5\pi}{3}\sqrt{\frac{m}{k}}$.

2. Ans. (BD)

$P_{1max}=ma\omega_1=b$

$P_{2max}=mR\omega_2=R$

$$\frac{\omega_1}{\omega_2}=\frac{1}{n^2}$$

$$\frac{\omega_2}{\omega_1}=n^2$$

$$E_1=\frac{1}{2}m\omega_1^2a^2$$

$$E_2=\frac{1}{2}m\omega_2^2R^2$$

$$\frac{E_1}{E_2}=\frac{\omega_1^2a^2}{\omega_2^2R^2}=\frac{\omega_1^2}{\omega_2^2}n^2=\frac{\omega_1^2\omega_2}{\omega_2^2\omega_1}$$

$$\frac{E_1}{E_2}=\frac{\omega_1}{\omega_2}$$

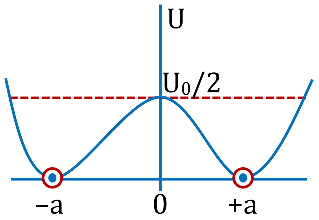
$$\frac{E_1}{\omega_1}=\frac{E_2}{\omega_2}$$

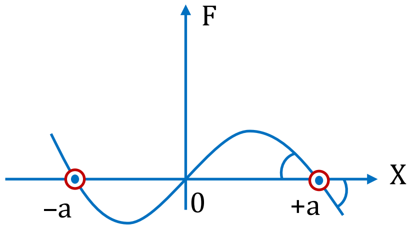
3. Ans. (A-P,Q,R,T), (B-Q,S), (C-P,Q,R,S), (D-P,R,T)

(A) $U_1(x) = \frac{U_0}{2}\left[1 - \frac{x^2}{a^2}\right]^2$

$F = -\frac{U_0}{2} 2\left[1 - \frac{x^2}{a^2}\right]\left[-\frac{2x}{a^2}\right]$

$= \frac{2U_0}{a^4}[a^2 - x^2]x$





$F = \frac{2U_0}{a^4} x(a-x)(a+x)$

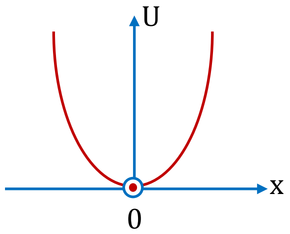
$F = 0$ at $x = 0, a, -a$

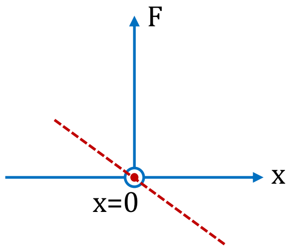
$\left.\begin{array}{l} x = -a, U = 0, \\ x = 0, U = \frac{U_0}{2} \end{array}\right\} \Rightarrow$ Oscillate when total ME is less then $\frac{U_0}{2}$

A $\to$ P,Q,R,T

(B) $U_0(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$

$F = -\frac{U_0 x}{a^2}$

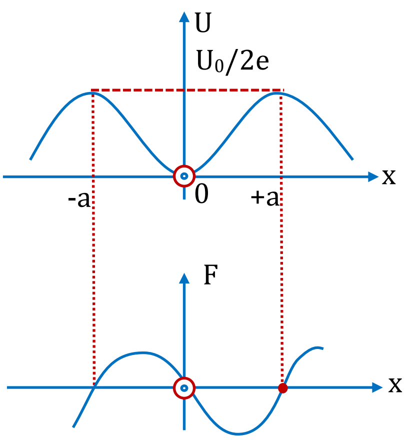




$F = 0, x = 0$

$B \to Q,S$

(C) $U_3(x) = \frac{U_0}{2}\left[\left(\frac{x}{a}\right)^2 exp\left(-\frac{x^2}{a^2}\right)\right]$



$$F = -\frac{U_0}{2}\left[\frac{2x}{a^2}exp\left(-\frac{x^2}{a^2}\right) + \frac{x^2}{a^2}exp\left(-\frac{x^2}{a^2}\right)\left(-\frac{2x}{a^2}\right)\right]$$

$$= -\frac{U_0}{2}\frac{2x}{a^4}exp\left(-\frac{x^2}{a^2}\right)[a^2 - x^2]$$

$$= \frac{U_0}{a^4}e^{-\frac{x^2}{a^2}}[x(x+a)(x-a)]$$

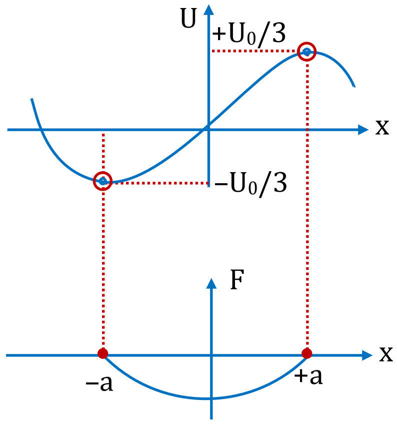
$x = 0$, $x = -a$, $x = +a$, $F = 0$

$x = 0$, $U = 0$ even function, hence minima

C $\rightarrow$ P,Q,R,S

(D) $U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{x^3}{3a^3}\right]$

$$F = -\frac{U_0}{2}\left[\frac{1}{a} - \frac{x^2}{a^3}\right] = \frac{U_0}{2a^3}[x^2 - a^2]$$



$x = +a$, $x = -a$, $F = 0$

$x = -a$, $U = -\frac{U_0}{3}$, $x = +a$, $U = +\frac{U_0}{3}$

Hence oscillates about $x = -a$ if $T.M.E < \frac{U_0}{3}$

D $\rightarrow$ P,R,T

4. **Ans. (ABD)**

$T_i = 2\pi\sqrt{\frac{M}{K}}, T_f = 2\pi\sqrt{\frac{M+m}{K}}$

Case I. $M(A\omega) = (M+m)V$

$\therefore$ Velocity decreases at equilibrium position.

By energy conservation

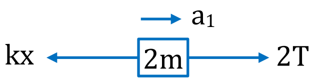
$$A_f = A_i\sqrt{\frac{M}{M+m}}$$

Case II. No energy loss, amplitude remains same

At equilibrium (x_0) velocity = $A\omega$.

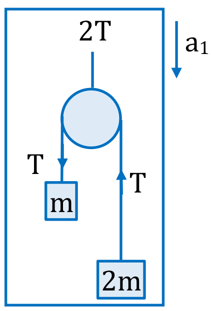
In both cases ω decreases so velocity decreases in both cases.

5. **Ans. (C)**



$2T - kx = 2ma_1$

(a) $T = \frac{2(2M)(M)}{3M}(g - a_1)$



$$= \frac{4M}{3}(g - a_1)$$

$$\frac{8M}{3}(g - a_1) - kx = 2Ma_1$$

$$\frac{8Mg}{3} - \frac{8Ma_1}{3} - kx = 2Ma_1$$

$$\frac{8Mg}{3} - kx = \frac{14Ma_1}{3}$$

$$\frac{8Mg-3kx}{14M}=a_1$$

$$a_1=\frac{8Mg-3kx}{14M}$$

$$\frac{vdv}{dx}=\left(\frac{8Mg}{14M}-\frac{3kx}{14M}\right)$$

$$\int vdv=\frac{1}{14M}\int(8Mg-3kx)dx$$

For max elongation

$$0=\frac{1}{14M}\int_0^{x_0}(8Mg-3kx)dx$$

$$=\frac{1}{14M}\left(8Mgx_0-\frac{3kx_0^2}{2}\right)$$

$$8Mgx_0=\frac{3kx_0^2}{2}$$

$$\boxed{x_0=\frac{16Mg}{3k}}$$

(b) At $x=\frac{x_0}{2}$

$$\int_0^v vdv=\frac{1}{14M}\int_0^{x_0/2}(8Mg-3kx)dx$$

$$\frac{v^2}{2}=\frac{1}{14M}\left(\frac{8Mgx_0}{2}-\frac{3kx_0^2}{2\times4}\right)$$

$$v^2=\frac{1}{7M}\left(\frac{8Mg}{2}\times\frac{16Mg}{3k}-\frac{3k}{8}\times\frac{16M^2g^2}{3k\times3k}\right)$$

$$=\frac{1}{7M}\left(\frac{64M^2g^2}{3k}-\frac{2M^2g^2}{3k}\right)$$

$$v^2=\frac{62Mg^2}{21k}$$

(c) For acc. $\boxed{2a_1=a_2+a_3}$ therefore $a_2-a_1=a_1-a_3$

(d) $a_1=\frac{8Mg-3kx_0/4}{14M}$

$$=\frac{8g}{14}-\frac{3kx_0}{14M\times4}$$

$$=\frac{8g}{14}-\frac{3k}{14M\times4}\times\frac{16Mg}{3k}$$

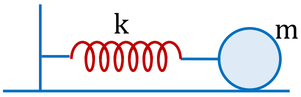
$$=\frac{8g}{14}-\frac{4g}{14}$$

$$=\frac{4g}{14}=\frac{2g}{7}$$

6. Ans. (6)

$\ell > \ell_0 \rightarrow k = k_1$

$\ell < \ell_0 \rightarrow k = k_2$



Time period of oscillation,

$$T = \pi\sqrt{\frac{m}{k_1}} + \pi\sqrt{\frac{m}{k_2}}$$

$$T = \pi\sqrt{\frac{0.1}{0.009}} + \pi\sqrt{\frac{0.1}{0.016}}$$

$$T = \frac{\pi}{0.3} + \frac{\pi}{0.4}$$

$\Rightarrow \quad T = \frac{0.7}{0.12}\pi \Rightarrow T = 5.83\pi$

$T \approx 6\pi$

So, $n = 6$

7. Ans. (C)

(I) $v_{BA}^2 = v_A^2 + v_B^2 - 2v_{AB}\cos\theta$

As $\omega_A = \omega_B, \theta = 90^\circ$ remains constant.

Also, $v_A = v_B = 1\ m/s$

So, $v_{BA} = \sqrt{2}m/s$

(II) $\vec{u}_A = \frac{5\pi}{2}\hat{i} + \frac{5\pi}{2}\hat{j}$

$$\vec{v}_A = \frac{5\pi}{2}\hat{i} + \left(\frac{5\pi}{2} - 10\cdot\frac{\pi}{3}\right)\hat{j}$$

$$= \frac{5\pi}{2}\hat{i} - \frac{5\pi}{6}\hat{j}$$

$$\vec{u}_B = -\frac{5\pi}{2}\hat{i} + \frac{5\pi}{2}\hat{j}$$

$$\vec{u}_B = -\frac{5\pi}{2}\hat{i} - \left(\frac{5\pi}{6} - 1\right)\hat{j}$$

$$\vec{v}_{BA} = -5\pi\hat{i} - \hat{j}$$

$$v_{BA} = \sqrt{25\pi^2 + 1}\ m/s$$

(III) $x_A = \sin t$

$$v_A = \cos t = \frac{1}{2} m/s$$

$x_B = \cos t$

$$v_B = -\sin t = -\frac{\sqrt{3}}{2} m/s$$

$$v_{BA} = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2} m/s$$

(IV) $\vec{v}_A \& \vec{v}_B$ are always perpendicular

So, $|\vec{v}_{BA}| = \sqrt{v_A^2 + v_B^2} = \sqrt{10}m/s$

Ans. (C), I-S, II-T, III-P, IV-R

8. Ans. (10)

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}}$

$\Rightarrow \omega = \sqrt{\frac{C}{I}}$

Where I = moment of inertia

$I = (30)(4)^2 + (20)(6)^2$

$= 1200\ gm\text{-}cm^2$

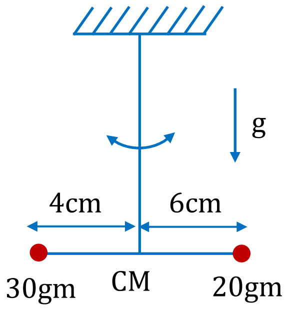
$= 1.2 \times 10^{-4}\ kg\text{-}m^2$

$\Rightarrow \omega = \sqrt{\frac{1.2 \times 10^{-8}}{1.2 \times 10^{-4}}}$

$\Rightarrow \omega = \sqrt{10^{-4}}$

$\omega = (10^{-2})$

$n \times 10^{-3} = 10^{-2} \Rightarrow n = 10$



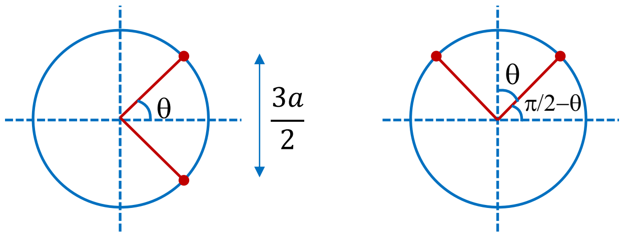
JEE (Main) Practice Paper

SECTION–A

1. Ans. (2)

$\sin\left(\frac{\pi}{2}-\theta\right) = \frac{\sqrt{7}}{4} = \frac{x}{a}$

$x = \frac{a\sqrt{7}}{4}$



2. Ans. (3)

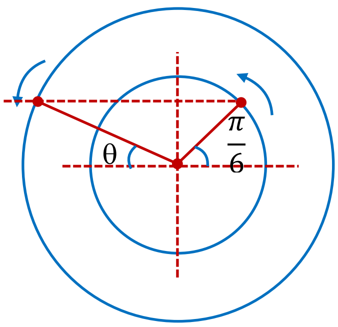
At $t = 0$, $x = +A$

$x = A \sin\left(\omega t + \frac{\pi}{2}\right)$

3. Ans. (2)

Since downward is positive $\Rightarrow$ initial phase will be 0° and at $t = 0$ particle is at mean position.

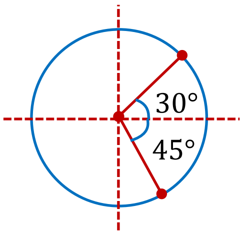
4. Ans. (1)



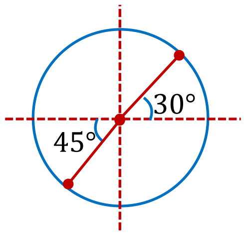
$\theta = \sin^{-1}\left(\frac{1}{4}\right)$

$\therefore \quad \Delta\phi = \pi - \sin^{-1}\left(\frac{1}{4}\right) - \frac{\pi}{6} = \frac{5\pi}{6} - \sin^{-1}\left(\frac{1}{4}\right)$

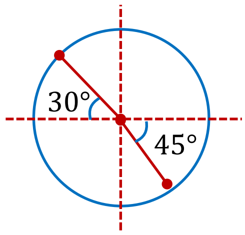
5. Ans. (3)



$\Delta\phi = 75°$(1)

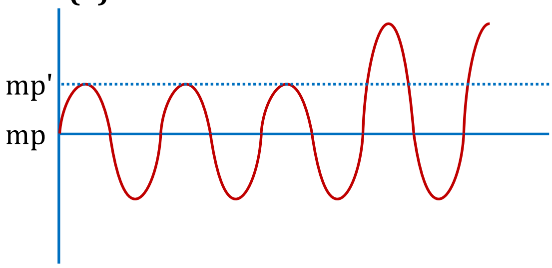


$\Delta\phi = 165°$ (2)



$\Delta\phi = 195°$ (4)

6. Ans. (4)



Now when gravity is switched off when box reaches its lowest position when its velocity is zero and only force pulling it upwards is due to spring. Therefore, at this instant gravity is off therefore, the force acting previously on box was $F_s - mg$ where F_s is spring force and force acting after switching off gravity is F_s. Therefore, Force increases in upward direction when velocity of box is zero. Therefore, $F_s - mg = kA_1$ before switching off gravity now, after switching gravity off $F_s = kA_2$.

We clearly see that $A_1 < A_2$.

While the lowest position of box remains same therefore, option (D).

7. Ans. (4)

t	0	1	2
ϕ	0	$\frac{\pi}{4}$	$\frac{\pi}{2}$
pos.	0	$\frac{A}{\sqrt{2}}$	A

$$d = \frac{\frac{1}{\sqrt{2}}}{1 - \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}-1} = \sqrt{2}+1$$

8. Ans. (4)

For maximum, mean position of the particle must be in mid. of time interval.

t	ϕ	x
0	$\frac{-\pi}{4}$	$\frac{-A}{\sqrt{2}}$
$\frac{T}{8}$	0	0
$\frac{T}{4}$	$\frac{\pi}{4}$	$\frac{A}{\sqrt{2}}$

$$\therefore \quad \vec{v} = \frac{\sqrt{2}A}{\frac{T}{4}} = \frac{4\sqrt{2}A}{T}$$

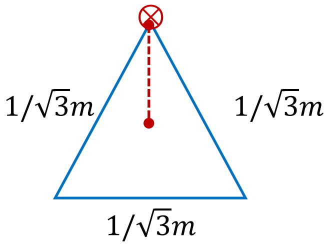
9. Ans. (4)

$$\tau = 3mg\frac{1}{3}\theta$$

$$I = 2\times\frac{m\ell^2}{3} + m\times\left(\frac{1}{2}\right)^2$$

$\therefore \quad \alpha = 2g\,\theta$

$$\Rightarrow \quad T = \frac{2\pi}{\sqrt{2g}} = \frac{\pi}{\sqrt{5}}$$



10. Ans. (1)

As E becomes twice.

A becomes $\sqrt{2}$ times.

No effect on period.

11. Ans. (4)

$$U = k\left(1 - e^{-x^2}\right) = k - ke^{-x^2}$$

$$F = +ke^{-x^2}\cdot(-2x)$$

$$= -2ke^{-x^2}\cdot x$$

(1) As $U(0) = 0 = U_{\min}$

Position is stable eq.

(2) $F = -2ke^{-x^2}$

F is discreted towards O.

(3) $ME = \dfrac{k}{2}$ $\qquad U = 0$

$\therefore \quad KE = \dfrac{k}{2} = KE_{\max}$

(4) $F = -2\,kx\,(1 - x^2)$

$\simeq -2kx$

$\therefore \quad F \propto -x$

Motion is SHM.

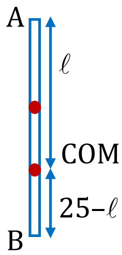
12. Ans. (4)

$$\left(\frac{T_1}{T_2}\right)^2 = \frac{I_1}{I_2}\cdot\frac{d_2}{d_1}$$

$$\Rightarrow \quad \frac{9}{16} = \frac{9}{4}\cdot\frac{25-\ell}{\ell}$$

$\Rightarrow \quad \ell = 100 - 4\ell$

$\ell = 20\ cm$



13. Ans. (3)

$$\omega = \sqrt{\frac{mgl_{cm}}{I_{support}}}$$

$$I_{support} = \left(\frac{mR^2}{2} + mR^2\right) = \frac{3}{2}mR^2$$

$$\omega = \sqrt{\frac{mgR}{\frac{3mR^2}{2}}} = \sqrt{\frac{2g}{3R}}$$

$$\therefore \quad \ell_{eq} = \frac{3R}{2} = 1.5m$$

14. Ans. (3)

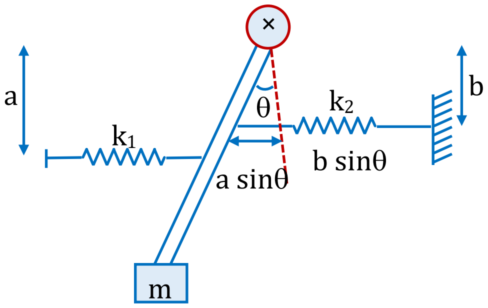
$$T' = 2\pi\sqrt{\frac{\ell}{g_{eff}}} = 2\pi\sqrt{\frac{\ell}{g+g/4}} = \frac{2}{\sqrt{5}}T$$

15. Ans. (1)

For small angular displacement (θ)

Net torque on body = $I\alpha$

$$= (k_1 a \sin\theta)a + (k_2 b \sin\theta)b = \left(mL^2 + \frac{ML^2}{3}\right)\alpha$$



For small θ

$$\Rightarrow \quad \alpha = \frac{k_1a^2 + k_2b^2}{mL^2 + \frac{ML^2}{3}}\theta$$

$$\Rightarrow \quad \text{Frequency} = \frac{1}{2\pi}\sqrt{\frac{k_1a^2+k_2b^2}{L^2(m+M/3)}}$$

16. Ans. (2)

Kinetic energy at mean position

$$K.E. = \frac{1}{2}KA^2$$

$$= \frac{1}{2}K\left(\frac{F}{K}\right)^2 = \frac{F^2}{2K}$$

17. Ans. (1)

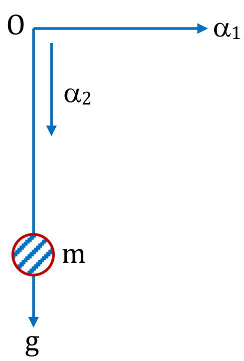
Point O is moving as shown

$$\vec{g}_{eff} = -\alpha_1\hat{i} + (\alpha_2 - g)\hat{j}$$

So $\quad g_{eff.} = \sqrt{\alpha_1^2 + (g-\alpha_2)^2}$

So time period

$$= 2\pi\sqrt{\frac{\ell}{(\alpha_1^2+(g-\alpha_2)^2)^{\frac{1}{2}}}}$$



18. **Ans. (1)**

$$T = 2\pi\sqrt{\frac{\frac{ma^2}{6}+m\left(\frac{a}{\sqrt{2}}\right)^2}{mg\frac{a}{\sqrt{2}}}}$$

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{\sqrt{8}a}{3g}}$$

19. **Ans. (1)**

$$2\pi\sqrt{\frac{L}{\sqrt{3}g}}, \frac{L}{2\sqrt{3}}$$

Moment of inertia about centre = $\frac{mL^2}{12} = mK^2$

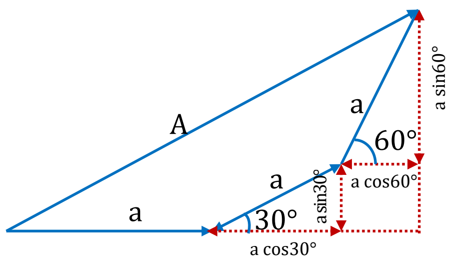
$\Rightarrow \quad K = \frac{L}{2\sqrt{3}} = \ell$; where ℓ is distance between centre and point of suspension.

Minimum time period $T = 2\pi\sqrt{\frac{2k}{g}} = 2\pi\sqrt{\frac{L}{\sqrt{3}g}}$

20. **Ans. (1)**

$a\sqrt{4+2\sqrt{3}}$

From phasor diagram



$$A = \sqrt{\left(a+\frac{\sqrt{3}a}{2}+\frac{a}{2}\right)^2+\left(\frac{\sqrt{3}a}{2}+\frac{a}{2}\right)^2}$$

$$= \sqrt{\left(\frac{3}{2}a+\frac{\sqrt{3}a}{2}\right)^2+\left(\frac{\sqrt{3}a}{2}+\frac{a}{2}\right)^2}$$

$$A = \frac{a}{2}\sqrt{(3+\sqrt{3})^2+(\sqrt{3}+1)^2} = a\sqrt{4+2\sqrt{3}}$$

SECTION-B

1. **Ans. (40)**

Maximum acceleration = $\omega^2 A = g \Rightarrow A = \frac{g}{\omega^2} = 40$ cm

2. **Ans. (10)**

$T_a = 2\pi\sqrt{\frac{m}{\left(\frac{k_1k_2}{k_1+k_2}\right)}}$ $\qquad T_a = 2T_b$

$T_b = 2\pi\sqrt{\frac{m}{(k_1+k_2)}}$ $\qquad \frac{k_1+k_2}{k_1\times k_2} = 4\left(\frac{1}{k_1+k_2}\right)$

$(k_1+k_2)^2 = 4(k_1k_2)$

$(10+k_2)^2 = 4(10k_2) \Rightarrow k_2 = 10N/m$ Ans.

3. **Ans. (1)**

Here $T = \frac{2\pi}{\omega} = \frac{2\pi}{\pi} = 2\pi \qquad \sqrt{\frac{\ell}{g}} \Rightarrow \frac{\ell}{g} = \frac{1}{\pi^2}$ But $g = \pi^2$ therefore $\ell = 1\,m$

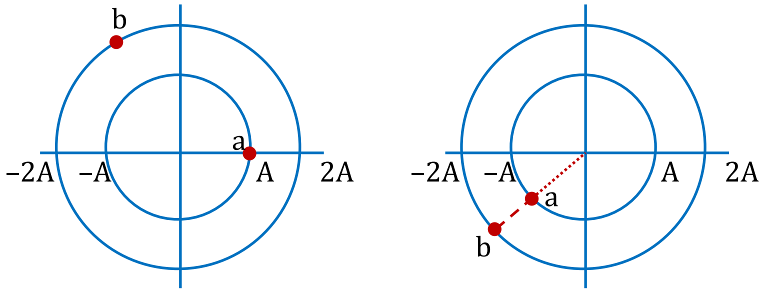
4. **Ans. (2)**

$$\frac{\pi}{3} = \omega t_1 \Rightarrow t_1 = \frac{\pi}{3\omega}$$

$$\frac{\pi}{6} = \omega t_2 \Rightarrow t_2 = \frac{\pi}{6\omega}$$

$$\frac{t_1}{t_2} = 2$$

5. **Ans. (3)**



$\theta_a = 120° \theta_b = 240°$

6. **Ans. (25)**

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\Rightarrow \quad \frac{2\pi}{2} = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\frac{1}{4} = \frac{\ell}{g}$$

$$\ell = \frac{g}{4} = 2.5\ m$$

7. **Ans. (100)**

$$KE_{max} = \frac{1}{2} \times 2 \times 10^6 \times 10^{-2} \times 10^{-2} J = 100J$$

8. **Ans. (200)**

$$.04 = \frac{1}{2} K \times \left(20 \times 10^{-3}\right)^2$$

$$\frac{.04 \times 2}{4 \times 1} \times 10^4 = 200$$

9. **Ans. (6)**

$$x = +\frac{A}{2}$$

$$\theta = \frac{\pi}{6}$$

$$\Delta\theta = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

$$t = \frac{\Delta\theta}{\omega} = \frac{T}{6}$$

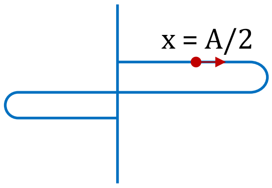
10. **Ans. (25)**

$$\frac{1}{2}kx^2 = \frac{1}{2}\mu v^2$$

$$\frac{1}{2} \times 1120 \times x^2 = \frac{1}{2} \times \frac{10}{7} \times 49 \Rightarrow x = 25cm$$

JEE (Advanced) Practice Paper

1. Ans. (A)



Initial phase $\phi = \sin^{-1}\left(\frac{A/2}{A}\right) = \frac{\pi}{6}$

2. Ans. (A)

Energy $= \frac{kA^2}{2}$

$$\frac{(k/4)A^2}{2} = \frac{k_{eq}A^2}{2}$$

$$\frac{1}{k_{eq}} = \frac{4}{k}$$

$$\frac{1}{k'} + \frac{1}{k} = \frac{4}{k}$$

$k' = k/3$

3. Ans. (A)

w.r.t stationary block

$$T = 2\pi\sqrt{\frac{\mu}{k}} \text{ (for oscillation)}$$

So time of contact $T/2$

$$t = \frac{T}{2} = \pi\sqrt{\frac{\frac{4\times4}{4+4}}{\pi^2}} = \sqrt{2} \text{ sec.}$$

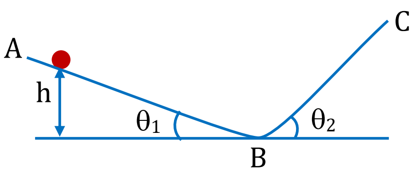
4. Ans. (C)

$$f = \frac{1}{2\pi}\sqrt{\frac{K_{eq}}{M}}$$

$$\frac{1}{k_{eq.}} = \frac{1}{k_1} + \frac{1}{k_2}$$

5. Ans. (B)

Time for $A \to B$ & $B \to A$ will be same & equals $t_0 = t_1$



$$\Rightarrow \quad \frac{h}{\sin\theta_1} = \frac{1}{2}\frac{g\sin\theta_1}{7/5}t_1^2 \Rightarrow t_1 = \sqrt{\frac{14h}{5g}}\sin\theta_1$$

Similarly

Time for $B \to C$ & $C \to B$ will be same & equals $t_0 = t_2$

$$\Rightarrow \quad \frac{h}{\sin\theta_2} = \frac{1}{2}\frac{g\sin\theta_2}{7/5}t_2^2$$

$$\Rightarrow \quad \sqrt{\frac{14h}{5g}}\sin\theta_2$$

Total time for one full oscillation

$$\Rightarrow \quad T = 2t_1 + 2t_2 = 2\sqrt{\frac{14h}{5h}}\left[\sin\theta_1 + \sin\theta_2\right]$$

6. **Ans. (B)**

$$T = 2\pi\sqrt{\frac{I_{\text{support}}}{mgI_{cm}}}$$

$$I_{\text{support}} = 2\left(\frac{ml^2}{3}\right) = \frac{2}{3}ml^2$$

$$I_{cm} = 1\cos 45° = \frac{1}{2}$$

$$T = 2\pi\sqrt{\frac{\frac{2m\ell^2}{3}}{2m\cdot g\cdot\frac{\ell}{2\sqrt{2}}}} = 2\pi\sqrt{\frac{2\sqrt{2}\ell}{3g}}$$

7. **Ans. (ABC)**

(A) $K.E. = 0.64\ E$

$P.E. = 0.36\ E$

$$\frac{x^2}{A^2} = 0.36$$

$\therefore\ x = 6\ cm$

(B) $P.E. = 0.25\ E$

$K.E. = 0.75\ E$

(C) $E = KE_{\text{max}} = PE_{\text{max}}$

(D) $v = \frac{v_m}{2}$ then $A = \frac{\sqrt{3}}{2}A_m$

8. **Ans. (ACD)**

(A) $\vec{r} = (1 + 2\cos 2\omega t)\hat{i} + \left(\frac{3}{2} - \frac{3}{2}\cos 2\omega t\right)\hat{j} + (3)\hat{k}$

$\therefore\ m_P = \left(1, \frac{3}{2}, 3\right)$

(B) It does not excute SHM about x-axis

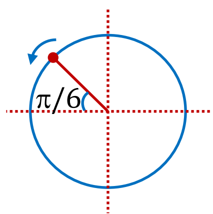
(C) $A = \sqrt{2^2 + \left(\frac{3}{2}\right)^2} = \frac{5}{2}$

(D) Direction of SHM is $\frac{2}{\frac{5}{2}}\hat{i} - \frac{\frac{3}{2}}{\frac{5}{2}}\hat{j} \Rightarrow \frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}$

9. **Ans. (BD)**

$a_{max} = \left(\frac{k}{m}\right)\times A = 15$ (B)

$= A\sin\left(\frac{2\pi}{T}t + \frac{\pi}{3}\right)$ (D)



10. Ans. (BCD)

$kx_0 = mg$

$\therefore x_0 - \frac{mg}{k} = \frac{1\times10}{500}$

$= 0.02\ m = 2\ cm$

So equilibrium is obtained after an extension of 2 *cm* of at a length of 42 *cm*. But it is releases from a length of 45 *cm*.

$\therefore\ A = 3\ cm = 0.3\ m$

(b) $v_{\max} = \sqrt{\frac{k}{m}}A$

$= 30\sqrt{5}\ cm/s$

(c) $a_{max} = \left(\frac{k}{m}\right)\times A = 15\ m/s^2$

(d) Mean position is at 42 *cm* length and amplitude is 3 *cm*. Hence, block oscillates between 45 *cm* and length 39 *cm*. Natural length 40 *cm* lies in between these two, where elastic potential energy = 0 .

11. Ans. (AB)

$T = 2\pi\sqrt{\frac{10}{360}} = \frac{\pi}{3}s$ (A)

$V_{\max} = \frac{\text{Impulse}}{\text{Mass}}\frac{1}{3}$ = 5 *m/s* (B)

$A = \frac{V_{\max}}{\omega} = \frac{5}{6}$ = 0.83 *m*

12. Ans. (ABCD)

$x_{eq} = \frac{2}{200}\times100 = 10cm$

$A = 1\ cm$ (A)

$\therefore\ x_{\max} = x_{eq} + A$

$= 2\ cm$ (B)

$f = \frac{1}{2\pi}\sqrt{\frac{200}{0.2}} = \frac{10\sqrt{10}}{2\pi}\times5Hz$ (C)

f is same as *m* and *k* are same (D)

13. Ans. (C)

$F = \frac{-dU}{dx}$

$\Rightarrow$ At mean position net force is 0 and it is at $x = 2\ m$.

So, $x = -3m$ is it's left extreme position and amplitude is $(|-3| + 2) = 5m$

14. Ans. (B)

Maximum *x* coordinate is $x = 7\ m$.

15. Ans. (A)

$x^2 + y^2 = A^2$ which represents a circle of radius *A* and starting point is $x = 0$ so answer will be (A).

16. Ans. (A)

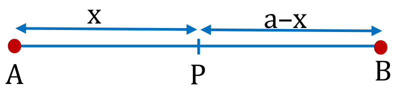
Time period for vertical S.H.M. is one third of horizontal S.H.M.

17. Ans. (D)

18. Ans. (B)

Solution for Question No. 17 and 18)

(i) Force on particle at point P



$$F = \frac{2mg(a-x)}{a} - \frac{mg(x)}{a}$$

$$F = \frac{mg}{a}(2a-3x)$$

$$F = \frac{-3mg}{a}(x-2a/3)$$

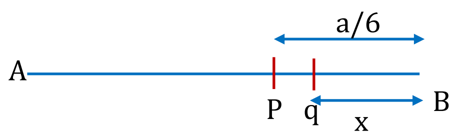
So this is equation of S.H.M ($F = m\omega^2 x$) so particle perform S.H.M with mean position $x - 2a/3 = 0$

So $\omega^2 = \frac{3g}{a} \Rightarrow \omega = \sqrt{3g/a}$

So Time period $T = 2\pi\sqrt{a/3g}$

And amplitude = $2a/3 - a/2 = a/6$

(ii) At point P velocity of particle = 0



and force = $\frac{2mgx}{a}$

(at point q)

acc. = $\frac{2gx}{a}$

$$\Rightarrow \int_0^v v\,dv = \int_0^{a/6} \frac{2gx\,dx}{a}$$

$$\Rightarrow V = \frac{\sqrt{2ga}}{6}$$

19. Ans. (5.00)

$$T = 2\pi\sqrt{\frac{I}{mg\ell}}, I = m\ell^2 + m(2\ell)^2 = 5m\ell^2$$

$$= 2\pi\sqrt{\frac{5m\ell^2}{2mg\frac{3\ell}{2}}} = 2\pi\sqrt{\frac{5\ell}{3g}} \quad \therefore Leq = \frac{5\ell}{3}.$$

20. Ans. (3)

Speed just before collision

$$v = \omega\sqrt{a^2 - \left(\frac{a}{2}\right)^2}$$

$$= \omega\frac{\sqrt{3}a}{2}$$

Speed just after collision

$$v' = \frac{v}{2} = \frac{\omega\sqrt{3}a}{4} = \sqrt{\frac{k}{m}}\frac{\sqrt{3}a}{4}$$

$$= a\sqrt{\frac{3k}{16m}}$$