Wave on String and Sound Waves

SOLUTIONS

EXERCISE - O

1. **Ans. (C)**

Comparing with the equation $y = a \sin(\omega t + kx)$

$$a = 0.002\ mm, \omega = 50\ s^{-1}, k = 2m^{-1}$$

(A) $v = \frac{\omega}{k} = \frac{50}{2} = 25\ ms^{-1}$

(B) $\lambda = \frac{2\pi}{k} = \frac{2\pi}{2} = \pi\ m$

(C) $f = \frac{\omega}{2\pi} = \frac{50}{2\pi} = \frac{25}{\pi} Hz.$

(D) $a = 0.002\ mm$

2. **Ans. (D)**

Amplitude becomes double

Hence it becomes = 0.5

And frequency becomes half $f' = \frac{f}{2} \Rightarrow \lambda' = 2\lambda \Rightarrow k' = \frac{k}{2}$

Hence ω becomes = π and travelling in opposite direction

$\therefore \quad y = 0.5 \cos(\pi t + \pi x)$

3. **Ans. (B)**

Phase difference = $k \times 10 = [10\pi \times 0.01] \times 10$

4. **Ans. (A)**

$f(x, t) = A \exp\left(\frac{2abxt - a^2x^2 - b^2t^2}{c^2}\right)$

Velocity $= \frac{\omega}{k} = \frac{b}{a}$

5. **Ans. (D)**

$y = 3 \sin\frac{\pi}{2}(50\ t - x)$

Particle velocity $= \frac{\partial y}{\partial t} = 3\left(\frac{\pi}{2} \times 50\right) \cos\frac{\pi}{2}(50t - x)$

Maximum particle velocity = $75\ \pi\ m/s$

Wave velocity $v = \frac{\omega}{k} = \frac{50}{1} = 50 m/s$

Required ratio = $\frac{75\pi}{50} = \frac{3}{2}\pi$

OR

$\frac{(v_P)_{max.}}{V_{max.}} = \frac{A\omega}{\omega/k} = kA = \frac{\pi}{2} \times 3 = \frac{3\pi}{2}$

6. **Ans. (B)**

b is at extreme position and moving towards rightward therefore acceleration is max and directed towards downward.

7. **Ans. (C)**

$$\frac{\lambda}{2}=1 \Rightarrow \lambda=2 \Rightarrow v=\frac{\omega}{k} \Rightarrow \frac{\lambda}{T}=\frac{2}{2} \Rightarrow 1\,cm/\sec$$

$$v_p=\omega\sqrt{A^2-x^2}$$

$$=\frac{2\pi}{T}\sqrt{4^2-\left(2\sqrt{3}\right)^2}=\frac{2\pi}{2}\sqrt{16-12}$$

$$\frac{2\pi}{2}\times 2=2\pi\,cm/\sec$$

8. **Ans. (B)**

$\lambda = 9\ cm$. $T = 3$ sec, $f = 1/3$

$v_\omega = \lambda f$

$= 3\ cm/\text{sec}$

9. **Ans. (C)**

$$v_{wave}=\frac{\lambda}{T} \Rightarrow T=\frac{\lambda}{v_{\text{wave}}} \Rightarrow T=\frac{0.5}{0.1} \Rightarrow T=5\text{ sec}$$

$$v_P=\omega\sqrt{A^2-x^2}$$

$$v_P=\frac{2\pi}{T}\sqrt{A^2-x^2}$$

$$v_P=\frac{2\pi}{T}\sqrt{A^2-x^2}$$

$$v_P=\frac{2\pi}{5}\sqrt{100-25}\ cm/s$$

$$v_P=2\pi\sqrt{3}\ cm/s$$

$$v_P=\frac{\pi\sqrt{3}}{50}\hat{j}\ m/s$$

10. **Ans. (A)**

Equation of wave in $+x$ direction

$$Y=A\sin\left(\frac{x}{a}-\frac{t}{T}\right) \qquad \text{...(i)}$$

$$f(x,t)=A\sin\left(\frac{x^2}{a^2}\right) \qquad \text{...(ii)}$$

As $t = t - t_0$

Compare equation (i) and (ii)

$$f(x,t)=A\sin\left(\frac{x}{a}-\left(\frac{t-t_0}{T}\right)\right)^2$$

$$T=\frac{a}{v}$$

$$f(x,t)=A\sin\left(\frac{x}{a}-\frac{v(t-t_0)}{a}\right)^2$$

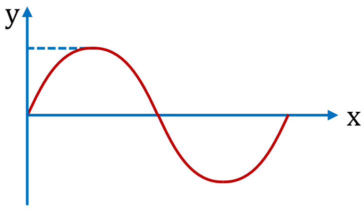
$$=A\sin\left(\frac{x-v(t-t_0)}{a}\right)^2$$

11. **Ans. (D)**

Wave moving in $+ve$ x direction

At $t = 0, x = 0, y = 0$

$\phi = 0, \pi$



$\because \dfrac{dy}{dt} < 0 \; ; \; \phi = \pi$

So $y = A \sin(\omega t - kx + \pi)$

$\sin(\theta + \pi) = -\sin\theta = \sin(-\theta)$

$y = A\sin(kx - \omega t)$

Wave is moving in $-ve$ x direction

$y = A\sin(\omega t + kx + \phi)$

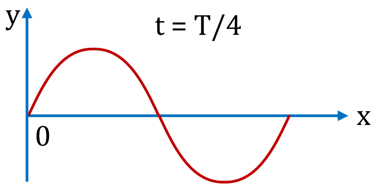
At $t = 0, x = 0, y = 0$

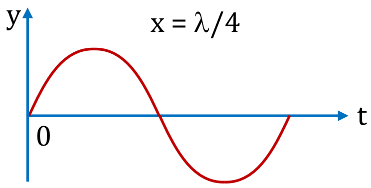
$\phi = 0, \pi$

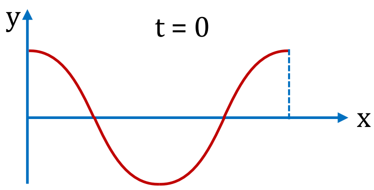
$\because \dfrac{dy}{dt} > 0 \; ; \; \phi = 0$

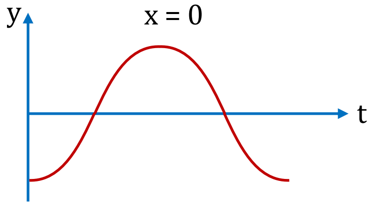
$y = A\sin(kx + \omega t)$

12. **Ans. (D)**









$y = A\sin(\omega t - kx + \pi/2)$

$y = A\sin(\omega t + kx + \pi/2)$

13. **Ans. (B)**

$y = A\sin\left[\omega t + \frac{\pi}{4}\right]$

$t \to t + \left(\frac{x - \lambda/2}{v}\right)$

$y = A\sin\left[\omega\left(t + \left(\frac{x-\lambda/2}{v}\right)\right) + \frac{\pi}{4}\right]$

$y = A\sin\left[\omega t + \frac{\omega x}{v} - \frac{\omega\lambda}{2v} + \frac{\pi}{4}\right]$

$y = A\sin\left[\omega t + kx - \pi + \frac{\pi}{4}\right]$

$y = A\sin\left[\omega t + kx - \frac{3\pi}{4}\right]$

14. Ans. (B)

$y = \frac{3}{2t^2+1}$ at $x = 0$

$v_w = 2\ m/s$ (towards $-ve\ x$ axis)

$y\,(x, t) = y\left(t + \frac{x}{v}\right)$

$y(x, t) = \frac{3}{2\left(t+\frac{x}{2}\right)^2+1}$

15. Ans. (B)

Speed of transverse wave in string, $v = \sqrt{\frac{T}{m}}$

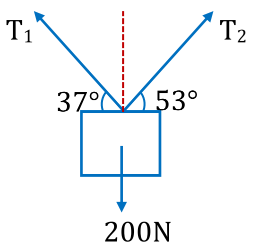
Where $T = 2g = 20N$ and $m = \frac{4.5\times10^{-3}}{2.25} = 2\times10^{-3}\ kg/m$

$\Rightarrow \quad v = \sqrt{\frac{20}{2\times10^{-3}}} = 100\ m/s$

Therefore, required time $= \frac{\ell}{v} = \frac{2}{100} = 0.02\ s$

16. Ans. (C)

$\frac{4T_1}{5} = \frac{3T_2}{5} \quad \Rightarrow T_1 = \frac{3}{4}T_2$



$v = \sqrt{\frac{T}{\mu}} \Rightarrow \frac{v_1}{v_2} = \sqrt{\frac{3}{4}}$

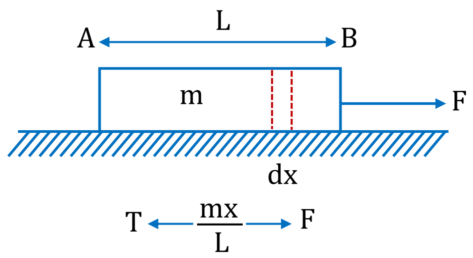
$\ell_1 = 10\times\frac{4}{5} = 8m$

$\ell_2 = 10\times\frac{3}{5} = 6m$

$\frac{t_1}{t_2} = \frac{l_1/v_1}{l_2/v_2} = \frac{8}{6}\sqrt{\frac{4}{3}} = \frac{4\sqrt{4}}{3\sqrt{3}}$

17. Ans. (B)

$F - T = \frac{mx}{L}\times\frac{F}{m}$



$T = F\left(1-\frac{x}{L}\right)$

Speed of transverse wave $= \sqrt{\frac{T}{\mu}}$

$$V = \sqrt{\frac{F\left(1-\frac{x}{L}\right)}{\frac{m}{L}}} = \sqrt{\frac{FL}{m}\left(1-\frac{x}{L}\right)}$$

$$\frac{dx}{dt} = \sqrt{\frac{FL}{m}\left(1-\frac{x}{L}\right)} \Rightarrow \int_0^L \frac{dx}{\sqrt{1-\frac{x}{L}}} = \int_0^T \sqrt{\frac{FL}{m}}\,dt$$

$$\left[2\sqrt{1-\frac{x}{L}}(-L)\right]_0^L = \sqrt{\frac{FL}{m}}\,[t]_0^T$$

$$2\ (-L)[0-1] = \sqrt{\frac{FL}{m}}\ (T-0)$$

$$2L\sqrt{\frac{m}{FL}} = T$$

$$4\frac{mL}{F} = T^2$$

$$4 = \frac{FT^2}{mL}$$

Value of $\dfrac{9\ FT^2}{mL} = 9 \times 4 = 36$

18. Ans. (A)

$P_0 = \frac{1}{2}A_0^2\omega_0^2\mu v$

$\dfrac{P_0}{2} = \dfrac{1}{2}A^2\ \omega^2\ \mu v$

$\therefore \quad 2 = \dfrac{{A_0}^2{\omega_0}^2}{A^2\omega^2}$

As $\quad \omega = \omega_0$ (Frequency remains the same)

$\therefore \quad A = \dfrac{A_0}{\sqrt{2}}$

19. Ans. (B)

$P = \mu A^2\omega^2 v\,cos^2(\omega t - kx)$

$= \dfrac{\mu\left(A^2\omega^2\,cos^2(\omega t - kx)\right)}{v}\dfrac{T}{\mu} = \dfrac{T}{v} \times v_p^2$

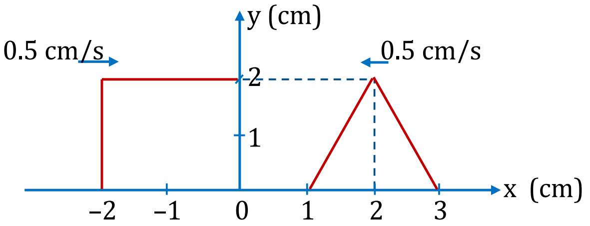
$\Rightarrow \quad 40 \times 10^{-3} = \dfrac{20}{20}v_p^2$

$v_p = 20\ cm/s$

20. Ans. (D)

At t = 1s, both pulses superimpose and cancel out each other.

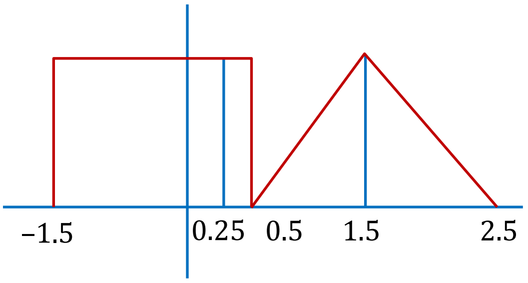
21. Ans. (B)



$x_1 = 0.5 \times t = 0.5 \times 1 = 0.5\ cm$

$x_2 = 0.5 \times t = 0.5 \times 1 = 0.5\ cm$

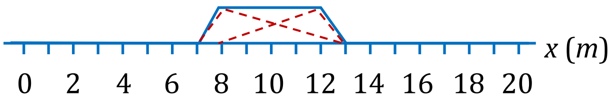
At, t = 1 sec



At $x = 0.25\ cm, y = 2\ cm$

22. Ans. (C)

The figure shows the two waves at $t = 6s$ and their superposition. The superposition is the point-by-point addition of the displacements of the two individual wave.



23. Ans. (B)

Superposition of waves does not alter the frequency of resultant wave and resultant amplitude

$\Rightarrow \quad a^2 = a^2 + a^2 + 2a^2 \cos\phi = 2a^2(1 + \cos\phi)$

$\Rightarrow \quad \cos\phi = -1/2 = \cos 2\pi/3$

$\therefore \quad \phi = 2\pi/3$

24. Ans. (B)

$$\frac{I_{max}}{I_{min}} = \left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right)^2 = \left(\frac{\sqrt{\frac{16}{9}}+1}{\sqrt{\frac{16}{9}}-1}\right)^2 = \frac{49}{1}$$

25. Ans. (B)

At $t = \frac{2L}{v}$, the wave will be back to the same place as it is directed free end. It will not get inverted while it will get inverted at the free end, therefore option B is answer.

26. Ans. (C)

At *'B'* shape of pulse gets inverted. The pulse will become erect after reflection at *'A'*. So, total distance travelled by wave will be 12 *m* to regain its original shape.

27. Ans. (B)

$y = 10 \sin 2\pi(100t - 0.02x) + 10 \sin 2\pi(100t + 0.02x)$

$y = 20 \sin(200\pi t) \cos 2\pi(0.02x)$

$A_{max} = 20$

Loop length $= \frac{\lambda}{2}$

$\lambda = \frac{2\pi}{k} = 50m$

Loop length $= 25m$

28. Ans. (B)

Check with all the options for condition of node at $x = 0$.

29. Ans. (A)

$y_1 = 8 \sin(\omega t + kx)$

$y_2 = 6 \sin(\omega t + kx + \pi/2)$

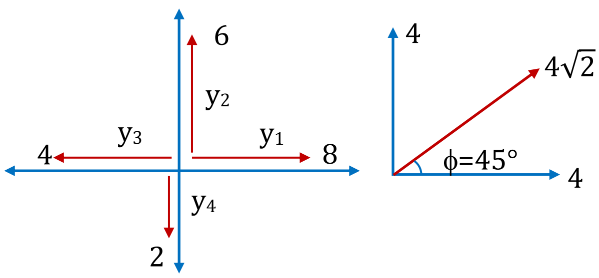
$y_3 = 4 \sin(\omega t + kx + \pi)$

$y_4 = 2 \sin(\omega t + kx + 3\pi/2)$

$y_5 = 4\sqrt{2} \sin\ (\omega t - kx + \pi/4)$

$y_1 + y_2 + y_3 + y_4$

$y' = 4\sqrt{2} \sin(\omega t + kx + \pi/4)$



$y_{net} = y' + y_5$

$= 4\sqrt{2}\,(\sin(\omega t - kx + \pi/4) + \sin(\omega t + kx + \pi/4)$

$= 4\sqrt{2}\,[2\sin(\omega t + \pi/4)\cos kx]$

$= 8\sqrt{2}\sin\left(\omega t + \frac{\pi}{4}\right)\cos k\,x$

30. Ans. (C)

$V_1 = \sqrt{\frac{T}{\mu}};\ V_2 = \sqrt{\frac{T}{4\mu}}$

$V_2 < V_1$

$\Rightarrow$ 2^{nd} is denser $\Rightarrow$ phase change of π

Wave reflected from denser medium

$$\Rightarrow \quad A_r = \frac{V_2 - V_1}{V_2 + V_1} \times 6 = \frac{\frac{V_1}{2} - V_1}{\frac{V_2}{2} + V_1} \times 6 = -2mm$$

$\Rightarrow$ eq^n $\Rightarrow$ – $(2mm)\sin(5t - 40x)$

31. Ans. (A)

$E_T = \frac{64}{100}E_i$

From energy conservation

$$E_r = E_i - E_T = \frac{36}{100}E_i$$

$$\Rightarrow \quad \frac{E_r}{E_i} = \frac{A_r^2}{A_i^2}$$

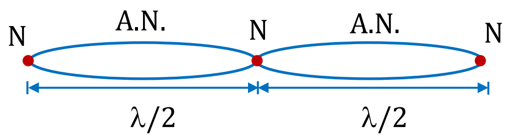
$$\Rightarrow \quad \frac{36}{100} = \frac{A_r^2}{100}$$

$\Rightarrow \quad A_r = 6\ cm.$

32. Ans. (A)

Waves A and B satisfied the conditions required for a standing wave.

33. Ans. (A)



Distance between two atoms = 1.21Å

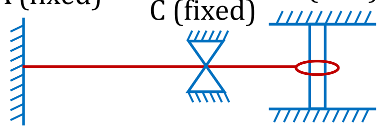
λ = 1.21 Å

34. Ans. (A)

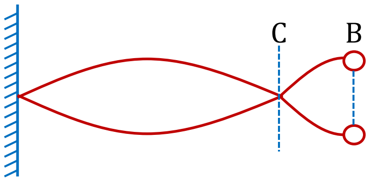
By comparing given equation with $y = a\sin(\omega t)\cos k\,x$

$$\Rightarrow \quad v = \frac{\omega}{k} = \frac{100}{0.01} = 10^4 m/s$$

35. Ans. (B)



Fundamental

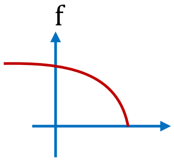


36. Ans. (A)

Fundamental frequency, $f = \frac{nv}{2L}$

$f \propto v$

$v \propto \sqrt{F}$ (F = Tension in wire)



$F = YA\left(\frac{\Delta L}{L}\right)$

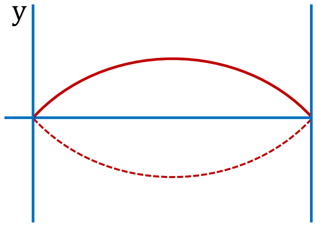
$\frac{\Delta L}{L} = \alpha \Delta T$

$F = (YA\,\alpha)\Delta T = (YA\,\alpha)\,(T-20°)$

$f \propto v \propto \sqrt{F} \propto \sqrt{(T-20)}$

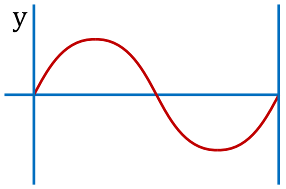
37. Ans. (C)

$y_1 = A \sin\left[\frac{\pi x}{L}\right] \sin \omega t$



$\lambda = 2L$

$y_2 = A \sin\left[\frac{2\pi x}{L}\right] \sin(2\omega t)$



$\lambda = L$

Energy per unit loop $= \frac{1}{8}\mu(\omega^2 A^2)\lambda$

$E_1 = 1 \times \left[\frac{1}{8}\mu\omega^2 A^2 (2L)\right]$

$E_2 = 2 \times \left[\frac{1}{8}\mu(2\omega)^2 A^2 (L)\right]$

$E_2 = 4E_1$

38. Ans. (C)

Here $A = 0.05\ m, \dfrac{5\lambda}{2} = 0.25 \Rightarrow \lambda = 0.1m$

Now standard equation of wave.

$y = A \sin(\omega t - kx) \Rightarrow y = 0.05 \sin 2\pi (3300t - 10x)$

39. Ans. (D)

$P = -\beta \dfrac{dS}{dx}$

40. Ans. (B)

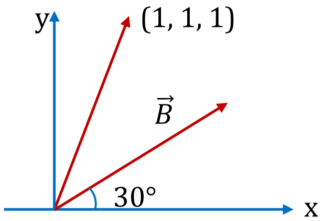
$\vec{B} = \frac{\sqrt{3}}{2}\hat{\imath} + \frac{1}{2}\hat{\jmath}$

$\vec{A} = \hat{\imath} + \hat{\jmath} + \hat{k}$

$\Delta\phi = \frac{2\pi}{\lambda} : \Delta x$

$\Delta x = \vec{A} \cdot \vec{B} = \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{\sqrt{3}+1}{2}$

$\Rightarrow \quad \Delta\phi = \dfrac{2\pi}{1} \times \dfrac{(\sqrt{3}+1)}{2}$



41. Ans. (D)

$p = -\beta \dfrac{ds}{dx}$

42. Ans. (B)

$v = \sqrt{\dfrac{\gamma RT}{M}}$

$\dfrac{v_{O_2}}{v_{H_2}} = \sqrt{\dfrac{M_{H_2}}{M_{O_2}}} = \sqrt{\dfrac{2}{32}} = \dfrac{1}{4}$

43. Ans. (B)

Bulk modulus $\beta = -\dfrac{\Delta P}{\frac{\Delta v}{v}} = \dfrac{100\times10^3}{\frac{5\times10^{-3}}{100}} = 2\times10^9$

Speed $v = \sqrt{\dfrac{\beta}{\rho}} = \sqrt{\dfrac{2\times10^9}{10^3}} \approx 1414\ m/s$

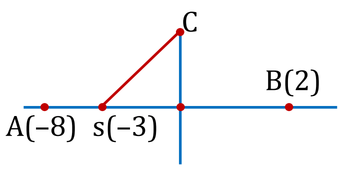
44. Ans. (A)

Source should be add mid for the wave front to reach at C,

$\therefore \quad x = -3$

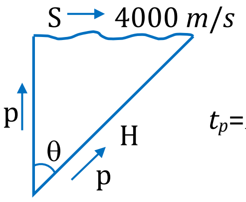
$5 = \sqrt{3^2 + y^2}$

$y = 4m$



45. Ans. (D)

Method 1:



$t_s = d/4000$

$t_p = d/7000$

$t_s - t_p = \frac{d}{4000} - \frac{d}{7000}$

$2 \times 60 = \frac{3d}{28000}$

$d = 28000 \times 40\ m = 1120\ km$

Method 2:

$t_s = \frac{H\cos\theta}{7000} + \frac{H\sin\theta}{4000} = t_p + 120$

Actually θ → 90° as depth of focus very small

$\Rightarrow \frac{H}{4000} = t_p + 120$

$\frac{H}{4000} = \frac{H}{7000} + 120$

$\frac{3H}{28000} = 120$

$H = 28000 \times 40\ m = 1120\ km$

46. Ans. (B)

$V \propto \sqrt{T}$

$\Rightarrow \quad \frac{V_2}{V_1} = \sqrt{\frac{T_2}{T_1}}$

$\Rightarrow \quad 2 = \sqrt{\frac{T_2}{(273+0)}}$

$\Rightarrow \quad T_2 = 273 \times 4 = 1092K = 819°C$

47. Ans. (A)

$v = \sqrt{\frac{\gamma P}{\rho}} \quad \Rightarrow \frac{v_{O_2}}{v_{H_2}} = \sqrt{\frac{\rho_{H_2}}{\rho_{O_2}}} = \sqrt{\frac{1}{16}} = \frac{1}{4}$

48. Ans. (A)

$V_T = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\pi r^2 \rho}}\ ;\ V_L = \sqrt{\frac{\gamma}{\rho}}$

$\Rightarrow \quad V_L = 100\ V_T$

$= 100\sqrt{\frac{T}{\pi r^2 \rho}} = \sqrt{\frac{\gamma}{\rho}}$

$\gamma = 10^4 \times \text{(Stress)}$

$\Rightarrow \quad \text{Stress} = \frac{\gamma}{10^4} = 1 \times 10^7\ N/m^2$

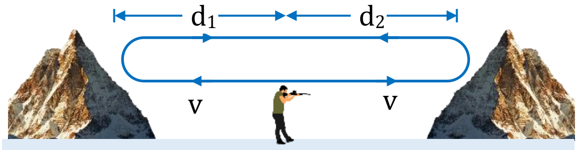
49. **Ans. (C)**

$2d_1 + 2d_2 = v \times t_1 + v \times t_2$

$\Rightarrow \quad 2(d_1 + d_2) = v\,(t_1 + t_2)$

$d_1 + d_2 = \frac{v(t_1+t_2)}{2} = \frac{340\times(1.5+3.5)}{2}$

$= 850\ m.$



50. **Ans. (D)**

Energy remains same

$\therefore \quad E_A = E_B$

$E \propto a^2n^2 \quad \Rightarrow \quad \frac{a_B}{a_A} = \frac{n_A}{n_B}$

$\frac{n_A}{n_B} = 8$

$a_B = 8a_A = 8A$

51. **Ans. (B)**

In option A, C, D the quality of sound is same.

But in option B pitch is same and quality of sound is different.

52. **Ans. (D)**

Loudness of sound $L = 10\log_{10}\frac{I}{I_0}$

$L' - L = 10\log_{10}\frac{I'}{I}$

The sound level attenuated by 20 dB.

i.e. $\quad L' - L = -\,20\ dB$

$-\,20 = 10\log_{10}\frac{I'}{I}$

$-2 = \log_{10}\frac{I'}{I}$

$I' = 10^{-2}I = \frac{I}{100}$

53. **Ans. (C)**

For a spherical wave amplitude $\propto \frac{1}{x}$, where x is distance of source from point of consideration.

So, $S \propto \frac{1}{x}$.

54. **Ans. (A)**

$Y_1 = 10\sin\omega \times t,$

$Y_2 = 4\sin(\omega \times t + \pi/2) = 4\cos\omega \times t,$

$Y_3 = 7\sin\ (\omega \times t + \pi) = -7\sin\omega \times t$

By the law of superposition of waves,

$$Y = Y_1 + Y_2 + Y_3 = 3\ \sin\ \omega \times t + 4\cos\omega \times t = 5\ \sin\ (\omega \times t + \phi)$$

Here,

$$\phi = \cos^{-1}3/5 = \sin^{-1}4/5$$

55. Ans. (B)

$\frac{I_1}{I_2}=\frac{4}{1}$ or $\sqrt{\frac{I_1}{I_2}}=\frac{2}{1}$

$\therefore \quad \frac{I_{max}}{I_{min}}=\left(\frac{\sqrt{\frac{I_1}{I_2}}+1}{\sqrt{\frac{I_1}{I_2}}-1}\right)^2=\left(\frac{2+1}{2-1}\right)^2=9$

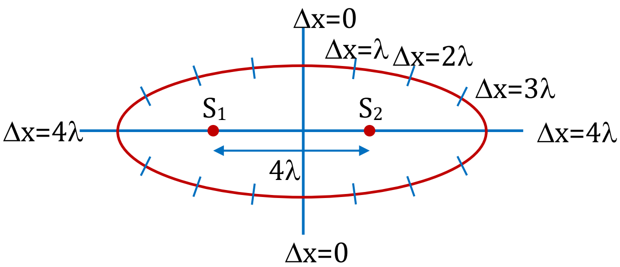
Loudness is measured by $\beta=10\,log\frac{I}{I_0}$

Now, change in loudness is

$\therefore \quad \beta_{max}-\beta_{min}=10\ \ log\frac{I_{max}}{I_0}-10\ \ log\ \frac{I_{min}}{I_0}$

$=10\ \ log\ \frac{I_{max}}{I_{min}}=10\ \ log(9)=20\ \ log\,3$

56. Ans. (A)



57. Ans. (A)

For the intensity to be maximum after interference, the path difference between the two waves must be an integral multiple of wavelength.

Hence, (5 + 5) – 6 = $n\lambda$

$\Rightarrow \quad \lambda=\frac{4}{n}$

This value is maximum for minimum value of n, which is 1.

Therefore, $\lambda_{max}=4$ meters

58. Ans. (C)

$\Delta\phi=2n\pi \quad \Rightarrow \quad \frac{\pi}{2}+\frac{2\pi}{\lambda}d\sin\theta=2n\pi;\ \frac{2\pi}{\lambda}d\sin\theta=\left(2n-\frac{1}{2}\right)\pi$

$\sin\theta=\frac{\lambda}{2d}\left(2n-\frac{1}{2}\right)=\frac{\lambda}{2d}\times\frac{1}{2}=\frac{\lambda}{2\times3\lambda}\frac{1}{2}\quad\Rightarrow\quad\frac{1}{12}=\frac{y}{100\lambda}$

$\frac{1}{12}=\frac{y}{(100\lambda)}$

$y=\frac{100\lambda}{12}=\frac{25}{3}\lambda$

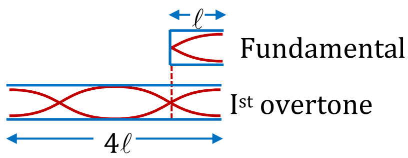
59. Ans. (B)

Closed Open

$\frac{v}{4\ell_1}=\frac{v}{\ell_2}$

$\ell_2=4\ell_1$

$\ell_1=\frac{\ell_2}{4}=\frac{50}{4}=12.5\ cm$



60. Ans. (B)

Second overtone of open pipe = $\frac{3V}{2\ell_1}$

Second overtone of closed pipe = $\frac{5V}{4\ell_2}$

Since, these frequencies are same

$\therefore \quad \frac{3V}{2\ell_1} = \frac{5V}{4\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{4\times3}{2\times5} = \frac{6}{5}$

Now, the ratio of fundamental frequencies:

$$\frac{f_1}{f_2} = \frac{\frac{V}{2\ell_1}}{\frac{V}{4\ell_2}}$$

$\Rightarrow \quad \frac{2\ell_2}{\ell_1} = 10 : 6 = 5 : 3$

61. Ans. (A)

$\frac{v}{4\ell_1} - \frac{v}{4\ell_2} = 5$

or $\quad \frac{v}{4}\left(\frac{1}{50} - \frac{1}{51}\right) = 5$

$\frac{v}{4} = \frac{5\times50\times51}{1}$

$\Rightarrow \quad f_1 = \frac{v}{4\ell_1} = \frac{5\times50\times51}{50} = 255\ Hz$

$\Rightarrow \quad f_2 = \frac{v}{4\ell_2} = \frac{5\times50\times51}{51} = 250\ Hz$

62. Ans. (D)

$\ell = n\frac{\lambda}{2} \quad \Rightarrow \quad \lambda = \frac{2l}{n}$

$f = \frac{nV}{2\ell}$

$400 = \frac{n\times350}{2\ell}$

$\ell = \frac{n\times350}{800} = \frac{7}{16}n$

$n = 1, 2, 3, \ldots\ldots$

63. Ans. (B)

$\frac{4L}{5} = \lambda$

$\Rightarrow \quad \lambda = 8cm$

Thus 2 cm corresponds to $\Delta\phi = \pi/2$

1 cm corresponds to $\Delta\phi = \pi/4$

So $\quad y = A \sin \pi/4 = 2 \times \frac{1}{\sqrt{2}} = \sqrt{2}mm$

64. Ans. (B)

$$\frac{\lambda}{4} = \ell_1 + e \qquad ...(1)$$

$$\frac{3\lambda}{4} = \ell_2 + e \qquad ...(2)$$

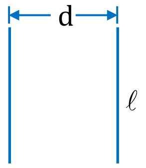
From (1) and (2)

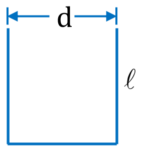
$e = 2\ cm$

65. Ans. (C)

$$\frac{\lambda_1}{2} = \ell + 0.6d,\ f_1 = \frac{V}{\lambda_1}$$

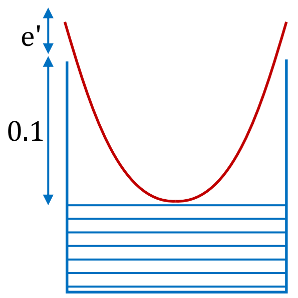
$$\frac{\lambda_2}{4} = \ell + 0.3d,\ f_2 = \frac{V}{\lambda_2}$$

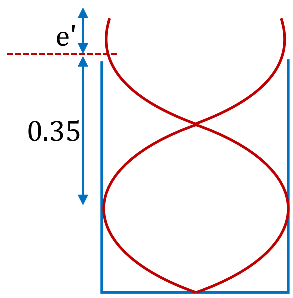




$$\frac{f_2}{f_1} = \frac{2\times(\ell+0.6d)}{4\times(\ell+0.3d)} = \frac{(\ell+0.6d)}{2\times(\ell+0.3d)}$$

66. Ans. (B)





$$\frac{\lambda}{4} = 0.1 + e$$

$\lambda = 0.4 + 4e \qquad ...(i)$

$$\frac{3\lambda}{4} = 0.35 + e$$

$3\lambda = 1.4 + 4e \qquad ...(ii)$

From (i) and (ii)

$1.2 + 12e = 1.4 + 4e$

$8e = 0.2$

$$e = \frac{0.2}{8} = 0.025\ m$$

67. Ans. (A)

$f_1\ell_1 = f_2\ell_2$

$\frac{f_1}{f_2} = \frac{5}{6}$ ...(i)

$f_2 - f_1 = 6\ Hz$

$\frac{f_1}{f_1+6} = \frac{5}{6}$ ...(ii)

$6f_1 = 5f_1 + 30$

$f_1 = 30Hz$

$f_2 = 36Hz$

$f_1 + f_2 = 66\ Hz$

68. Ans. (C)

$f_{ap.} = \left(\frac{v}{v-v_S}\right) f_{real}$, when source is moving towards observer

and $f_{ap.} = \left(\frac{v}{v+v_S}\right) f_{real}$, when source is moving away from observer

and $f_1{}' = \frac{330}{330+20} f = \frac{33}{35} f$

$f_2{}' = \frac{330}{330-20} f = \frac{33}{31} f$

$f_\omega = \frac{330}{330-20} f = \frac{33}{31} f$

So $f_1 = |f_1{}' - f_\omega|$

$f_2 = |f_2{}' - f_\omega|$

Hence $f_1 > f_2$.

69. Ans. (A)

Speed of motor cycle = V_m

Police van frequency observed by motor cycle $= 176\left(\frac{330-(V_m)}{330-22}\right)$

Stationary frequency observed by motor cycle $= 165\left(\frac{330+(V_m)}{330}\right)$

If motorcyclist does not observe any beat, then:

$$176\left(\frac{330-(V_m)}{330-22}\right) = 165\left(\frac{330+(V_m)}{330}\right)$$

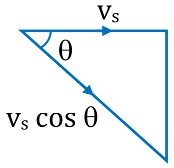
$$176\left(\frac{330-(V_m)}{308}\right) = \frac{330+(V_m)}{2}$$

$$\frac{16}{14}(330 - V_m) = 330 + V_m$$

$$8 \times 330 - 7 \times 330 = 7V_m + 8V_m$$

$$V_m = 22m/s$$

70. **Ans. (B)**



$$f_{\text{app}} = \frac{v}{v - v_S \cos\theta} f$$

$$= \frac{340}{340-16} \times 800 = 839.5$$

71. **Ans. (A)**

$$f' = f\left(\frac{v}{v - v_S}\right)$$

$f' = f$ as $v_S = 0$

and $f' \to \infty$ as $v_S = v$

So, graph (A) is the correct answer.

72. **Ans. (D)**

$V = r\omega = 80 m/s$

$$f_{max} = \left(\frac{320}{320-80}\right) \times 300$$

$$f_{max} = \frac{320}{240} \times 300 = 400\ Hz$$

$$f_{min} = \left(\frac{320}{320+80}\right) \times 300 = 240\ Hz$$

$240\ Hz\ \leq f \leq\ 400\ Hz$

73. **Ans. (B)**

Distance travelled by the wave relative to the down in $1\,\text{sec} = V + a - c$

$$\lambda' = \frac{V+a-c}{n}$$

$$f' = \frac{V+\ell-c}{\lambda'}$$

$$f' = \left(\frac{V+b-c}{V+a-c}\right) n$$

74. **Ans. (A)**

Speed of sound with respect to ground is $C + U$

$$V = f\ \lambda$$

$$\lambda\ = \frac{V}{f} = \frac{C+U}{f}$$

75. **Ans. (A)**

$\because$ Distance covered = $v.5 + \frac{1}{2} a.\, 5^2 = 5\ [\, v + (5a/2)]$

$\therefore$ Number of positive crests striking per second is same as the frequency

$$= \frac{\text{Distance}}{\lambda} = \frac{5\ [\, v + (5a/2)]}{\lambda}$$

76. **Ans. (D)**

$$\frac{\Delta\lambda}{\lambda} = \frac{v}{c}$$

Now $\Delta\lambda = \frac{0.5}{100}\lambda \quad \Rightarrow \quad \frac{\Delta\lambda}{\lambda} = \frac{0.5}{100}$ ----

$\therefore \quad v = \frac{0.5}{100} \times c = \frac{0.5}{100} \times 3 \times 10^8 = 1.5 \times 10^6 m/s$

Increase in λ indicates that the star is receding.

77. Ans. (A)

Doppler's shift is given by

$$\Delta\lambda = \frac{v\lambda}{c} = \frac{5000\times6000}{3\times10^8} = 0.1\text{Å}$$

78. Ans. (B)

Shifting towards ultraviolet region shows that Apparent wavelength decreased. Therefore, the source is moving towards the earth.

79. Ans. (B)

Due to expansion of universe, the star will go away from the earth thereby increasing the observed wavelength. Therefore, the spectrum will shift to the infrared region.

80. Ans. (C)

According to Doppler's principle $\lambda' = \lambda\sqrt{\frac{1-v/c}{1+v/c}}$

For $v = 0.8\,c$

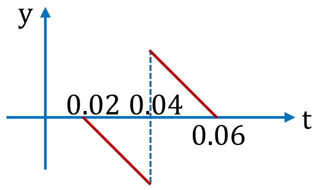
$$\lambda' = 5500\sqrt{\frac{(1-0.8)}{1+0.8}} = 1833.3$$

$\therefore$ Shift = 5500 – 1833.3 = 3167 $\mathring{A}$

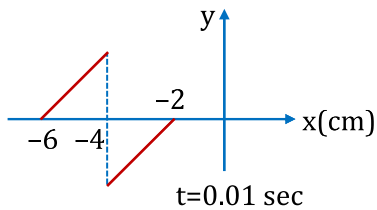
81. Ans. (ABCD)

$y = 10^{-4}\sin 2\pi\left(\frac{30t}{\pi} + \frac{x}{\pi}\right)$

82. Ans. (ACD)



x=2cm



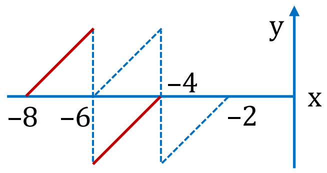
t=0.01 sec

Disturbance at $x = -4$ at t = 0.01sec and disturbance reaches $x = 2$ at t = 0.04 sec hence wave is travelling in positive x-direction.

$$v_w = \left(\frac{6cm}{0.03\,sec}\right)$$

= 200 cm/sec = 2m/sec

At t = 0 sec



83. Ans. (ABD)

$y = 0$ $\quad x < 0$

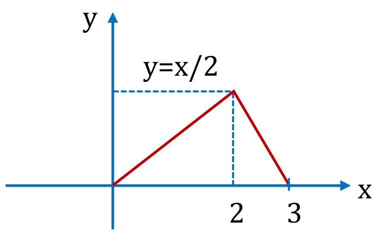
$y = x/2$ $\quad 2 > x \geq 0$

$y = 3 - x$ $\quad 3 > x \geq 2$

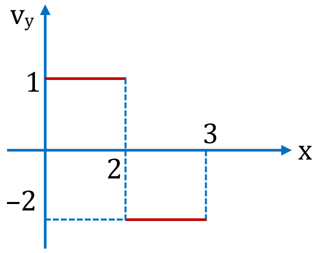
$y = 0$ $\quad x > 0$

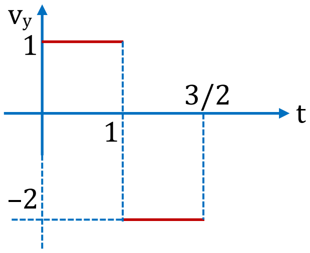
$[v_w = -2m/s]$

$$v_y = \frac{dy}{dt} = -v_w\frac{dy}{dt} = 2\frac{dy}{dx}$$



$$v_y = -\frac{2dy}{dx}\left[\begin{array}{ll} 0 & x\omega \\ +1 & 0 \le x < 2 \\ -2 & 2 \le x < 3 \\ 0 & 3 < x \end{array}\right.$$





84. Ans. (AC)

$I = \frac{1}{2}\rho\omega^2 A^2 v \Rightarrow I \propto A^2\omega^2$

Same speed $\Rightarrow \frac{50}{2} = \frac{100}{a} \Rightarrow a = 4$

85. Ans. (BCD)

Resonant frequency = 1001 Hz, 2639 Hz,

$1001 = 143 \times 7 = 11 \times 13 \times 7 = 11 f_0$

$2639 = 377 \times 7 = 7 \times 29 \times 13 = 29 f_0$

$\therefore$ Fundamental frequency = 13 × 7 = 91 Hz,

(A) String fixed at one end can produced only odd harmonics of 91 Hz, so 910 is not possible.

(B) 1911 = 91 × 21 Hz

(C) Fixed at both ends even harmonic possible

364 = 91 × 4 Hz

(D) 1001 Hz not fundamental frequency because 32 and 35 is not even or odd of 1001 Hz.

86. Ans. (ABD)

$\Delta x = (2n+1)\frac{\lambda}{2} = \frac{50}{100} m$

$\because \quad V = f\lambda$

$\left(\frac{2n+1}{2}\right)\frac{300}{f} = 0.5$

$f = (2n+1)300$

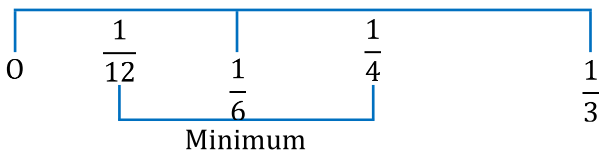
For $f = 300 \Rightarrow n = 0$

For $f = 900 \Rightarrow n = 1$

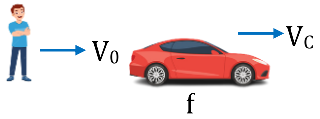
For $f = 2400 \Rightarrow n = \frac{7}{2}$ (not possible)

For $f = 4500 \Rightarrow n = 7$

87. Ans. (BD)



88. Ans. (BD)



The wavelength of sound reaching the hill is $\lambda = \frac{V-V_C}{f}$

$\Rightarrow \quad f_1 = f\left[\frac{V+V_0}{V+V_C}\right]$

$\Rightarrow \quad f_2 = f'\left[\frac{V+V_0}{V}\right] = \frac{V}{V-V_C}\ \frac{V+V_0}{V} = \frac{V+V_0}{V-V_C}$

Beat frequency = $f_2 - f_1$

$\Rightarrow \quad \left(\frac{V+V_0}{V-V_C} - \frac{V+V_0}{V+V_C}\right) f$

$\Rightarrow \quad (V + V_0)f\left[\frac{V+V_C-V+V_C}{V^2-V_C^2}\right]$

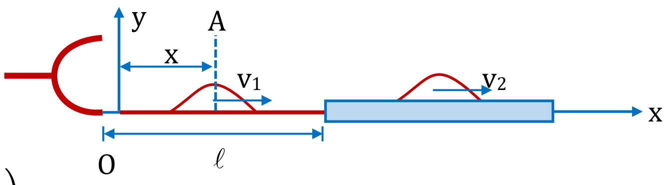
$\Rightarrow \quad \frac{2V_C\ (V+V_0)\ f}{V^2-V_C^2}$

89. Ans. (A)

Time to reach x distance = $\frac{x}{v_1}$

Displacement at A at time $t = t - \frac{x}{v_1}$

Displacement at A at time $t = a \sin \omega \left(t - \frac{x}{v_1}\right)$

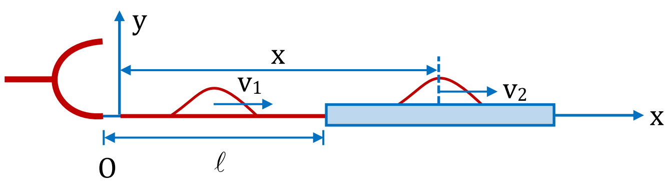


90. Ans. (C)

Time to reach x distance = $\frac{\ell}{v_1} + \frac{x-\ell}{v_2}$

Displacement at A at time $t = t - \frac{\ell}{v_1} - \frac{x-\ell}{v_2}$

$\therefore \quad y = a_t \sin \omega \ \left(t - \frac{\ell}{v_1} - \frac{x-\ell}{v_2}\right)$



91. Ans. (B)

$Y_i = a \sin \omega \left(t - \frac{x}{v_1}\right)$

Transmitted at $x = \ell$

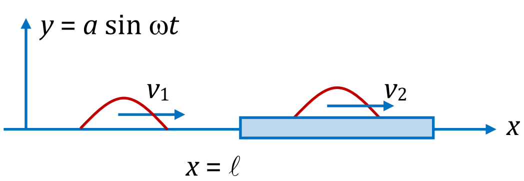
$Y_{(x=\ell)} = a \sin \omega \left(t - \frac{\ell}{v_1}\right)$

$Y_t = a_t \sin \omega \left[t - \frac{\ell}{v_1} - \left(\frac{x-\ell}{v_2}\right)\right]$

Reflected at $x = \ell$

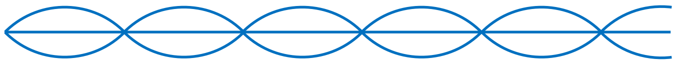
$y_{x=2\ell} = a \sin \omega \left(t - \frac{\ell}{v_1}\right)$

$y_{Reff} = a_r \sin \omega \left[t - \frac{\ell}{v_1} + \left(\frac{x-\ell}{v_1}\right) + \pi\right]$



92. Ans. (A)

Speed of wave $v = \sqrt{\frac{Y}{p}} = 4 \times 10^3$



$l = \frac{5\lambda}{2} + \frac{\lambda}{4} \quad \Rightarrow \quad \lambda = \frac{4l}{11}$

Frequency $f = \frac{v}{\lambda} = \frac{4\times10^3}{\frac{4}{11}\times1} = 11 \times 10^3 Hz,$

Wave number $k = \frac{2\pi}{\lambda} = \frac{11\pi}{2}$

Equation of standing wave in the rod $S = A \cos kx \sin(\omega t + \phi)$

where $A = 4 \times 10^{-6}\ m$

at $x = 0, t = 0 \Rightarrow S = A \Rightarrow A = A \cos k(0) \sin \phi \Rightarrow \sin \phi = 1 \Rightarrow \phi = \frac{\pi}{2}$

$S = 4 \times 10^{-6} cos\left(\frac{11\pi}{2}x\right) cos(22\pi \times 10^3 t)$

93. Ans. (B)

Strain $= \frac{ds}{dx} = -22\pi \times 10^{-6} sin \left(\frac{11\pi}{2}x\right) cos \ (22\pi \times 10^3 t) \because stress = Y \times strain$

$\Rightarrow$ Stress $= 140.8 \times 10^4 \cos(22\pi \times 10^3 t) \sin\left(\frac{11\pi}{2}x + \pi\right)$

94. Ans. (B)

Strain at $\left(x = \frac{l}{2} = \frac{1}{2}m\right) = \left|\frac{ds}{dx}\right|_{x=\frac{l}{2}, t=1} = 22\pi \times 10^{-6} \times sin\left(\frac{11\pi}{4}\right) = 11\sqrt{2}\pi \times 10^{-6}$

95. Ans. (A)

96. Ans. (B)

97. Ans. (C)

98. Ans. (D)

Solution for Questions No.95 to 98

From graph we can say that (frequency relation)

$f_1 > f_3 > f_2$ ...(i)

Now detector C is at rest w.r.t to source so frequency for detector c will be equal to frequency of source

$f_c = f_s$

Source is moving towards detector A so frequency for A will be greater than frequency of source.

$f_A > f_s$

For detector B source is moving away from the detector B so frequency of detector B is less than source.

$f_B < f_S$

$f_A > f_C > f_B$ ...(ii)

On comparing eq (i) and (ii) we can write

$f_A = f_1$

$f_B = f_2$

$f_C = f_3$

Now if source is moving along y axis then for detector B frequency will be less than frequency of source.

$f_2 < f < f_3$

99. **Ans. (A-Q,T), (B-P,S), (C-P), (D-S,T)**

(A) If source starts moving towards detector then frequency for detector will increase and wavelength of sound will decrease hence distance between any two wavefronts will decrease.

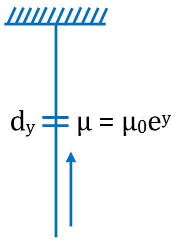
(B) If air starts moving right then velocity of sound in right direction will increase and frequency remain constant. Hence, wavelength will increase, so the distance between any two wavefronts will increase.

(C) Relative velocity between source and observer is zero so frequency remains constant but source is moving in left direction so wavelength in right side will increase so distance between wavefronts will increases.

(D) Velocity of sound and frequency both increases so time needed by sound to move from plane AA' to BB' will decrease.

100. **Ans. (A)**

Mass up to height y



$$m_y = \int_0^y \mu_0 e^y dy$$

$$= \left(\mu_0 e^y\right)_0^y$$

$$= \mu(e^y - 1)$$

$$v = \sqrt{\frac{T}{\mu}}$$

$$\mu v^2 = T$$

Tension at a height $y = m_y.g$

$$= \mu_0 g(e^y - 1)$$

$$\Rightarrow \quad v^2 = \frac{\mu_0 g(e^y - 1)}{\mu_0 e^y}$$

$$= g(1 - e^{-y})$$

101. **Ans. (A)**

$$P = \frac{1}{2}\mu A^2 \omega^2 V_\omega$$

$$= \frac{1}{2} \times 0.2 \times \left(\frac{8}{100}\right)^2 \times (4)^2 \times \sqrt{\frac{20}{0.2}} = 0.1024 \text{ watt}$$

50% heat transfer efficiency

Power transmit to bath = $\frac{0.1024}{2}$ = 0.0512 watt

102. **Ans. (B)**

$pt = ms\Delta T$

$.0512 \times t = 1000 \times 1 \times 4.2 \times 1$

$t = \frac{4200}{.0572}$ sec

$= 22.8hr$

103. **Ans. (B)**

If $\frac{\mu_1}{\mu_2}$ = 0 and ∞ No transmission.

If $\frac{\mu_1}{\mu_2}$ = 1, 100% energy transfer

So Ans. (B)

104. **Ans. (A)**

Difference between ν and ν – 1 is 1 Hz

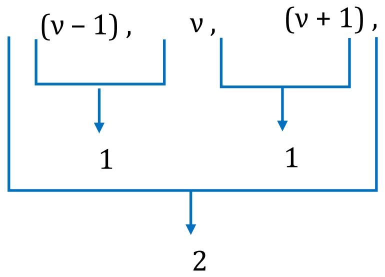
Difference between ν and ν + 1 is 1 Hz

Difference between ν – 1 and ν + 1 is 2 Hz

Let time period be T

So, beat for f_1 and f_2 will be heard at T

For f_3, beat will be heard at $\frac{T}{2}$ and T



So, neglecting one common time T, beats will be heard at $\frac{T}{2}$ and T so, 2 beats.

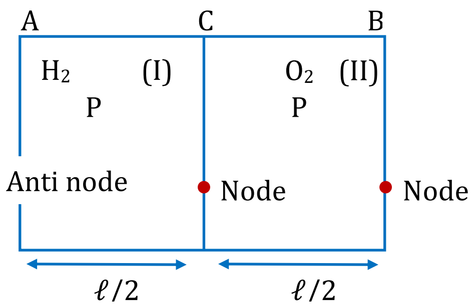
105. **Ans. (D)**

CO_2 = 330 m/s

In BC, $\frac{\ell}{2} = n\frac{\lambda_2}{2}$

In AC, $\frac{\ell}{2} = \left(m+\frac{1}{2}\right)\frac{\lambda_1}{2}$

now $n\frac{\lambda_2}{2} = \left(m+\frac{1}{2}\right)\frac{\lambda_1}{2}$



$2n\,\lambda_2 = (2m+1)\,\lambda_1$

$2n\frac{c_2}{f} = (2m+1)\frac{c_1}{f}$

$C_{sound} = \sqrt{\frac{\gamma RT}{M}}$

$\frac{C_1}{C_2} = \sqrt{\frac{M_2}{M_2}} = \sqrt{\frac{32}{2}}$ = 4

Now, $2n\,C_2 = (2m+1)\,C_1$

$2n \times C_2 = (2m+1)\,4\,C_2$

$2n = (2m+1)\,4$

$n = 4m + 2$

$m = 0$, $n = 2$

$f = \frac{nc}{2L}$

$= \frac{(2)\times 330}{2\times\frac{1}{2}}$ = 660 Hz

106. Ans. (AC)

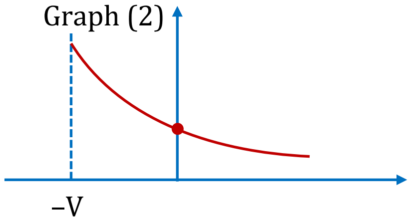
V = speed of sound wave

Case (i) = apparent f_{req}

$$f_R f_S = \left[\frac{V}{V \pm V_s}\right]$$

$V_s = -V$

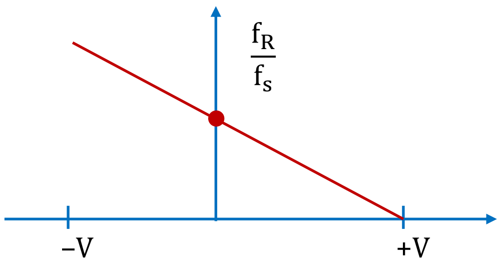
$f_R = \infty$



Graph (2) represents moving source

Case (II) Apparent f_{req}

$$f_R = f_S\left[\frac{V \pm V_R}{V}\right]$$



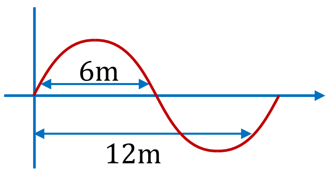
EXERCISE – S

1. Ans. π, 0

$f = 25\ Hz$, $\omega = 50\pi$ rad/s, $A = 2.5 \times 10^{-5}\ m$

$v = 300 m/s$

$\lambda = \frac{300}{25} = 12m$ $\qquad \Delta\phi = \left(\frac{\Delta x}{\lambda}\right)2\pi$



$\Delta x = 6m$

$\Delta\phi = \left(\frac{6}{12}\right) \times 2\pi$

$\Delta\phi = \pi$

$\Delta A = 0$

[Both point have same amplitude]

2. **Ans.** $0.2\cos[2\pi/5(x-10t)]$, (ii) $5n - (15/4)]$

$k = \frac{2\pi}{5}$

$y = 0.2 \sin\left[\frac{2\pi}{5}(x - vt) + \frac{\pi}{2}\right]$

$\rightarrow v_w$ + +ve

$T = \frac{1}{2}\text{sec}$

$v_w = \frac{\lambda}{T} = 10\ m/s$

$y = 0.2 \sin\left[\frac{2\pi x}{5} - 4\pi t + \frac{\pi}{2}\right]$

$= 0.2 \cos\left[\frac{2\pi x}{5} - 4\pi t\right]$

In figure (ii) $\left(\frac{dy}{dt} \rightarrow -ve\right)$

At $t = \frac{1}{4}, y = 0$

$0 = 0.02 \cos\left[\frac{2\pi}{5}x - \frac{4\pi}{1/4}\right]$

$= -\ 0.2 \cos\left[\frac{2\pi}{5}x\right]$

$\frac{2\pi}{5}x = (4n+1)\frac{\pi}{2}$

$x = \frac{5}{4}(4n+1)$

3. **Ans.** (a) 20 m/s; (b) 2 m; (c) $y(x, t) = 0.02\sin(\pi x - 20\pi t + \pi/6)$

$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{40}{0.1}} = 20\ m/s$ $(T = 0.1 \text{ sec}, f = 10\ Hz)$

$v = f\lambda$

$20 = 10\lambda$

$\lambda = 2m$ $\qquad k = \frac{2\pi}{\lambda} = \pi$

Equation

$y = 0.02 \sin[\omega t - \pi x + \phi]$

At $t = 0, x = 0$

$y = 0.01$

$\sin\phi = \frac{1}{2}$

$\frac{dy}{dt} \rightarrow$ negative

$w = 2\pi f = 20\pi$

$\phi = \dfrac{5\pi}{6}$

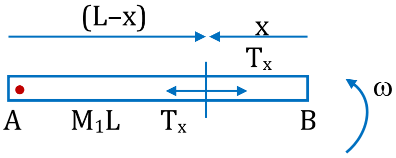
$y = 0.02 \sin\left(20\pi t - \pi x + \dfrac{5\pi}{6}\right) = 0.02 \sin\left(\pi x - 20\pi t + \dfrac{\pi}{6}\right)$

4. **Ans.** $\dfrac{\pi}{\sqrt{2}\omega}$

$T_x = \left(\dfrac{M}{L}\right) x\, \omega^2 \left[L - x + \dfrac{x}{2}\right]$

$T_x = \left(\dfrac{M}{L}\omega^2\right) x \left[L - \dfrac{x}{2}\right]$

$v_x = \sqrt{\dfrac{T_x}{(M/L)}} = \omega\sqrt{x\left(L - \dfrac{x}{2}\right)}$



$\Rightarrow \quad \dfrac{dx}{dt} = \omega\sqrt{x\left(L - \dfrac{x}{2}\right)}$

$\Rightarrow \quad \dfrac{dx}{\sqrt{x\left(L - \frac{x}{2}\right)}} = \omega dt$

Integrating the above equation

$\Rightarrow \quad \int_0^L \dfrac{dx}{\sqrt{x\left(L - \frac{x}{2}\right)}} = \int_0^t \omega dt \Rightarrow \dfrac{\pi}{\sqrt{2}} = \omega t$

$\Rightarrow \quad t = \dfrac{\pi}{\sqrt{2}\omega}$

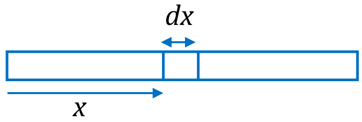
5. **Ans.** (a) $M = \dfrac{\kappa L^2}{2}$ (b) $t = \sqrt{\dfrac{8\,ML}{9F}}$

(a) $M = \int_O^L kx dx$

$\Rightarrow \quad M = \dfrac{\kappa L^2}{2}$

$\Rightarrow \quad L = \sqrt{\dfrac{2M}{\kappa}}$ ...(1)

(b) $v = \dfrac{dx}{dt}$ ...(2)



And $v = \sqrt{\dfrac{F}{\kappa x}}$ ...(3)

From 2 and 3

$\int_O^L \sqrt{x}\; dx = \int_O^t \sqrt{\dfrac{F}{\kappa}}\; dt$

$\Rightarrow \quad \left[\dfrac{2x^{3/2}}{3}\right]_O^L = \sqrt{\dfrac{F}{\kappa}} t$

$\Rightarrow \quad \dfrac{2L^{3/2}}{3} \sqrt{\dfrac{F}{\kappa}} . t$ ...(4)

$\Rightarrow$ From (1) and (4)

$t = \sqrt{\dfrac{8\,ML}{9F}}$

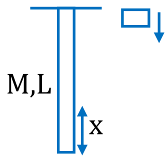
6. **Ans.** $L/9$

Initial speed of pulse

$v_i = \sqrt{\dfrac{Mg}{\mu}}$

$T_x = \left(\dfrac{M}{L}g\right)x$

$v_x = \sqrt{\dfrac{T_x}{\mu}} = \sqrt{gx}$ $\qquad s_w = ut + \dfrac{1}{2}gt^2 = \sqrt{gL}t - \dfrac{1}{2}\left(\dfrac{g}{2}\right)t^2$

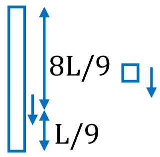


$v^2 = gx$

$s_2 = \dfrac{1}{2}gt^2$

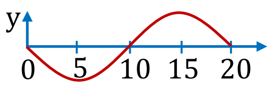
$2v\dfrac{dv}{dx} = g$ $\qquad s_2 = s_1$

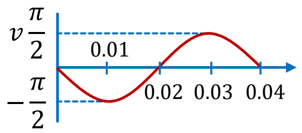
$v\dfrac{dv}{dx} = a = \dfrac{g}{2}$ $\qquad \dfrac{1}{2}gt^2 = \sqrt{g\ell}t - \dfrac{1}{2}\left(\dfrac{g}{2}\right)+2$



$t = \dfrac{4}{3}\sqrt{\dfrac{\ell}{g}}$

$s = \dfrac{1}{2}gt^2 = \dfrac{8L}{9}$

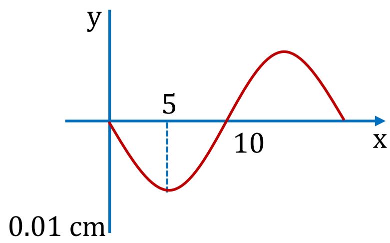
7. **Ans.** (i) 

x, (ii) $25 \times 10^{-6}\ W$, (iii) 

t , solved

(i) At $t = \dfrac{1}{50}$ sec

$y = (0.01\ cm)\sin\left[\dfrac{\pi x}{10} - \pi\right] = -0.01\sin\left[\dfrac{\pi x}{10}\right]$



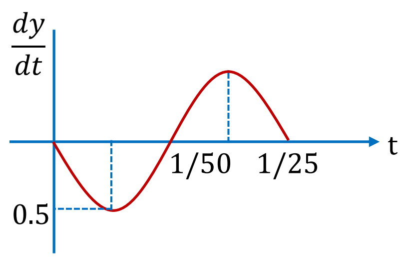
(ii) $<P> = \dfrac{1}{2}\mu v A^2\omega^2 = \dfrac{1}{2}\dfrac{\mu A^2\omega^2}{v}\left(\dfrac{T}{\mu}\right) = \dfrac{1}{2}\dfrac{TA^2\omega^2}{v}$

$= 25 \times 10^{-6}\ W$

(iii) y at $(x = 5)$ $\qquad y = 0.01\sin\left(\dfrac{\pi}{2} - 50\pi t\right)$

$\dfrac{dy}{dt} = 0.01\cos(50\pi t) \times -50$

$= -0.5\,\pi\sin(50\,\pi t)$



8. **Ans.** (a) Negative x ; (b) $y = 4\times10^{-3} \sin 100\pi \left(3t + 0.5x + \frac{1}{400}\right)$ (x , y in meter) ; (c) $144\pi^2 \times10^{-5} J$

(a) $v_P = - v_\omega \times$ slope

at P, $v_P \to +ve$

slope $\to +ve$

so, $v_\omega = -ve$ [wave travelling towards left]

(b) $y = 4 \sin \left(kx + \omega t + \frac{\pi}{4}\right)$

$\lambda = 4 \times 10^{-2}\, m \qquad k = \frac{2\pi}{4} \times 100 = 50\pi$

$v_P = -v_\omega \times$ slope

$\left(\frac{20\pi}{100}\right) m/s = -v_\omega \tan 6°$

$v_\omega = \frac{20\pi}{100} \times \frac{180}{6\pi} = 6\ m/s.$

$y = 4 \times 10^{-3} \sin (50x + 300\pi t + \frac{\pi}{4})$

(c) $E = \frac{1}{2}\mu A^2\omega^2 v$

$= \frac{1}{2} 50 \times 10^{-3} (4 \times 10^{-3})^2(300\,\pi)^2 \times 6$

$= 144\pi^2 \times 10^{-5} J$

9. **Ans.** 300

$P = T\left(\frac{\partial y}{\partial t}\right)\left(\frac{\partial y}{\partial x}\right)$

$y = A \sin(\omega t - kx + \phi)$

$\therefore \quad |P_{avg}| = \frac{1}{2}TA^2\omega k = \frac{1}{2}\sqrt{T\mu}A^2\omega^2$

$= \frac{1}{2}\sqrt{T\mu}A^2(2\pi f)^2 \Rightarrow f = 300Hz$

10. **Ans.** 6

Hint : $\frac{1}{2}\rho A^2\omega^2 \times \frac{\lambda}{4}$

11. **Ans.** $1 \times 10^9\ Nm^2$

Strain $= \frac{0.05}{40}$

Stress $= \frac{T}{10^{-6}}$

Fundament frequency $\Rightarrow \frac{1000}{64} = \frac{v}{2L}$

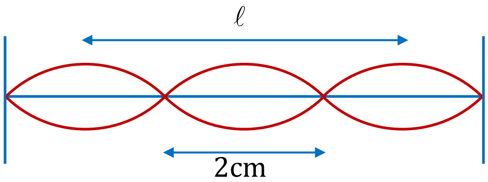
$L = 40\ cm,\ v = \sqrt{\frac{T}{\mu}}$

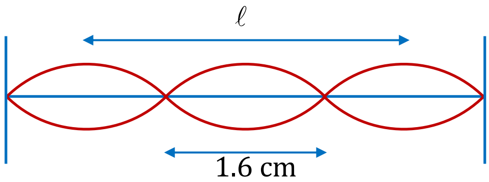
$\frac{1000}{64} = \frac{\sqrt{T/\mu}}{2\times40\times10^{-2}} \quad \mu = 3.2\ gm/40cm$

$T = 1.25\ N$

$Y = \frac{\text{Stress}}{\text{Strain}} = \frac{T}{10^{-6}} \times \frac{40}{0.05} = 10^9\ N/m^2$

12. **Ans.** 8.0 *cm*





$\lambda_1 = 2 \times 2 = 4cm$ $\qquad\qquad$ $\lambda_2 = 2 \times 1.6 = 3.2cm$

Length of wire is ℓ

In first case $\ell = \frac{n\lambda_1}{2}$

In second case $\ell = (n+1)\frac{\lambda_2}{2}$

$$\frac{n\lambda_1}{2} = (n+1)\frac{\lambda_2}{2}$$

$0.8n = 3.2$

$n = 4$

$\ell = \frac{n\lambda_1}{2} = 8cm$

13. **Ans.** (a) 30 *Hz* (b) 3rd, 5th and 7th (c) 2nd, 4th and 6th (d) 48 ms^{-1}

(a) Greatest common factor 30, so f = 30 *Hz*.

(b) $f_1 = 3f,\ f_2 = 5f,\ f_3 = 7f.$

(c) n^{th} overtone is $(n+1)^{th}$ frequency

So, $3f$ is 2^{nd} overtone and 3^{rd} harmonic.

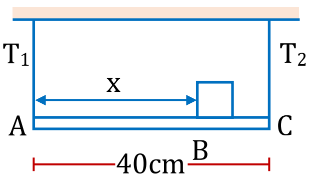
$5f$ is 4^{th} overtone and 5^{th} harmonic.

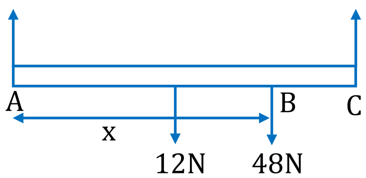
$7f$ = 6^{th} overtone and 7^{th} harmonic.

(d) $f_1 = \left(\frac{3}{2}\right)Lk$, where L is length = 80 *cm*, K is velocity so, $f_1 = \left(\frac{K}{\frac{2}{3}\ell}\right)$ K is velocity

$$90 = \left(\frac{3}{2} \times 80\right)K,\ K = 48m/s$$

14. **Ans.** 5 *cm* from the left end





L = 40 *cm* = 0.4 *m*

Let box is kept x list from the left end.

Given, $f_1 = 2f_2$

$$\therefore \quad \frac{1}{2l}\sqrt{\frac{T_1}{m}} = \frac{2}{2l}\sqrt{\frac{T_2}{m}}$$

$$\frac{T_1}{T_2} = 4 \qquad ...(i)$$

Now, $T_1 + T_2 = 60N$

$4T_2 + T_2 = 60N$

$\therefore \quad T_2 = 12\ N\ \&\ T_1 = 48\ N$

Now, Torque about Point A

$T_2 \times 0.4 = 48x + 12(0.2)$

$x = 0.05m = 5cm$ from the left

15. **Ans.** 180 Hz

$m_s = \rho_s A_s = 7.8 \times 10^{-3}\ kg/m$

$m_p = \rho_p A_p = 7.8 \times 10^{-3}\ kg/m$

Both are same

$\therefore$ Same per unit length mass

$\therefore$ Velocity is same

$$v = \left(\frac{T}{\mu}\right) = \sqrt{\frac{40}{7.8\times10^{-3}}} = 71.6m/s$$

For minimum frequency, max wavelength

For max wavelength minimum number of loops

Max distance of a loop = 20 cm.

$\Rightarrow \quad \lambda = 2 \times 20 = 40cm$

$$f = \frac{v}{\lambda} = \frac{71.6}{0.4} = 180\ Hz$$

16. **Ans.** (a) 80 Hz (b) 25 cm

Fundamental Frequency must be a factor of 240 and 320 Hz.

(A) So, 80 Hz.

(B) wave speed,

$V = 40\ m/s$

$$L \times 80 = \frac{1}{2} \times 40$$

$$L = \frac{1}{4}m = 25\ cm$$

17. **Ans.** $E = \dfrac{A^2\pi^2 T}{4l}$

Distance b/w 2 nodes = $\dfrac{\lambda}{2}$

$\Rightarrow \quad \ell = \dfrac{\lambda}{2} \quad \Rightarrow \quad \lambda = 2\ell$

And $\kappa = \dfrac{2\pi}{\lambda} = \dfrac{\pi}{\ell}$

$A = a \sin \kappa\ x = a \sin \dfrac{\pi}{\ell} x$

$\Rightarrow$ Mechanical Energy at x of length $dx = \dfrac{1}{2}(dm) A^2\omega^2$

$$dE = \frac{1}{2}\mu dx\ a^2\ \sin^2\frac{\pi}{\ell}x(2\pi v)^2$$

$$= \frac{1}{2}\mu\ a^2\ 4\pi^2\ v^2\ \sin^2\frac{\pi x}{\ell}dx \qquad \text{... (1)}$$

And $\nu = \frac{v}{\lambda} = \frac{\sqrt{T/M}}{2\ell}$

$\Rightarrow \quad \nu^2 = \frac{\frac{T}{\mu}}{4\ell^2}$

Substituting above value in (1)

$dE = 2\mu a^2\pi^2 \frac{T}{\mu 4\ell^2} sin^2\left(\frac{\pi x}{\ell}\right) dx$

$\Rightarrow \quad E = \frac{\pi^2 a^2 T}{2\ell^2} \int_o^\ell sin^2 \frac{\pi x}{\ell} dx$

$\Rightarrow \quad E = \frac{\pi^2 a^2 T}{4\ell}$

18. **Ans.** $\beta = 20dB$

We know that $\beta = 10 \log_{10}\left(\frac{l}{l_0}\right)$

$\beta_A = 10\, log \frac{l_A}{l_o}$, $\beta_B = 10\, log \frac{l_B}{l_o}$

$\Rightarrow \quad I_A/I_0 = 10^{(\beta_A/10)} \qquad \Rightarrow \quad I_B/I_o = 10^{(\beta_B/10)}$

$\Rightarrow \quad \frac{l_A}{l_B} = \frac{r_B^2}{r_A^2} = \left(\frac{50}{5}\right)^2 \Rightarrow 10^{(\beta_A \beta_B)} = 10^2$

$\Rightarrow \quad \frac{\beta_A - \beta_B}{10} = 2 \qquad \Rightarrow \quad \beta_A - \beta_B = 20$

$\Rightarrow \quad \beta_B = 40 - 20 = 20dB$

19. **Ans.** $\beta = 3\ dB$

We know that, $\beta = 10 \log_{10} I/I_0$

According to the questions

$\beta_A = 10\log_{10}(2I/I_0)$

$\Rightarrow \quad \beta_B - \beta_A = 10 \log (2I/I) = 10 \times 0.3010 = 3\ dB.$

20. **Ans.** 40 *cm*

If sound level = 120 *dB*, then I = intensity = 1 W/m^2

Given that, audio output = 2W

Let the closest distance be x.

So, intensity = $(2/4\pi x^2) = 1 \Rightarrow x^2 = (1/2\pi) \Rightarrow x = 0.4\ m = 40\ cm.$

21. **Ans.** 3.4 *kHz*

Distance between to maximum to a minimum is given by $\lambda/4 = 2.50\ cm$

$\Rightarrow \quad l = 10\ cm = 10^{-1}\ m$

We know, $V = nx$

$\Rightarrow \quad n = \frac{V}{\lambda} = \frac{340}{10^{-1}}$

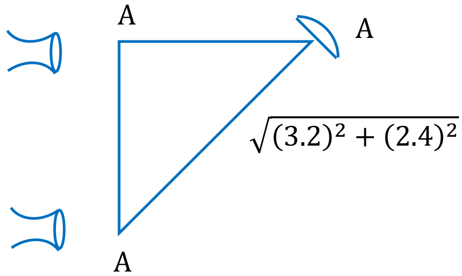
$= 3400\ Hz = 3.4\ kHz$

22. **Ans.** $f = 200\ (2n + 1)$, where $n = 1, 2, 3..., 49$

As shown in the figure the difference $2.4 = \Delta x = \sqrt{(3.2)^2 + (2.4)^2} - 3.2$

Again, the wavelength of the either sound waves = $\frac{320}{\rho}$

We know, destructive interference will be occur



If $\Delta x = \frac{(2n+1)\lambda}{2}$

$\Rightarrow \sqrt{(3.2)^2 + (2.4)^2 - (3.2)} = \frac{(2n+1)}{2}\frac{320}{\rho}$

Solving we get,

$\Rightarrow V = \frac{(2n+1)400}{2} = 200(2n + 1)$

where $n = 1.2.3$ …….. 49 (audible region)

23. **Ans.** 420 Hz

According to the given data

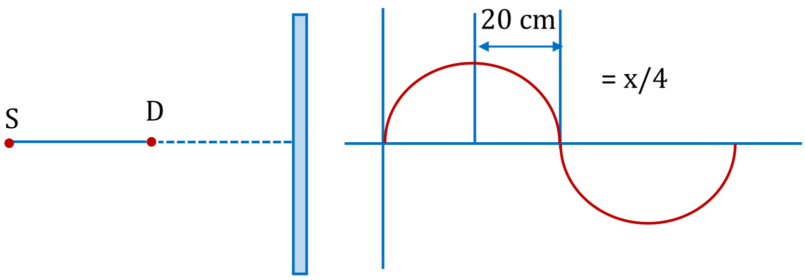
$v = 336\ m/s$

$\lambda/4$ = distance between maximum and minimum intensity

= (20 cm) $\Rightarrow \lambda = 80\ cm$

$\Rightarrow$ f = frequency

$= \frac{v}{\lambda} = \frac{336}{80 \times 10^{-2}} = 420Hz.$



24. **Ans.** 12.6 cm

According to the data

$\lambda = 20\ cm, S_1 S_2 = 20\ cm, BD = 20\ cm$

Let the detector is shifted to left for a distance x for hearing the minimum sound.

So path difference $AI = BC - AB$

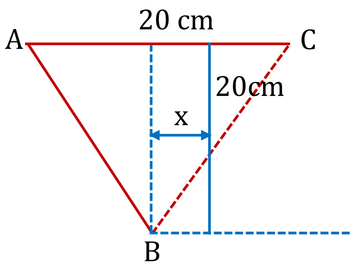
$= \sqrt{(20)^2 + (10 + x)^2} - \sqrt{(20)^2 + (10 - x)^2}$

So the minimum distance hearing for minimum

$= \frac{(2n+1)\lambda}{2} = \frac{\lambda}{2} = \frac{20}{2} = 10cm$

$\Rightarrow \sqrt{(20)^2 + (10 + x)} - \sqrt{(20)^2 + (10 - x)^2} = 10$ solving we get

$x = 12.6\ cm.$



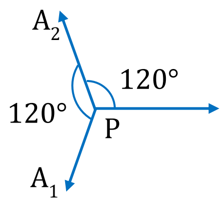
25. **Ans.** 0

Because the 3 sources have equal intensity, amplitude are equal

So, $A_1 = A_2 = A_3$

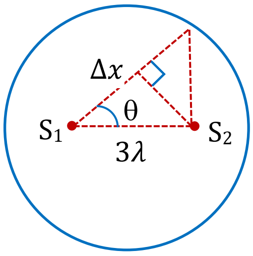
As shown in figure, amplitude of the resultant = 0 (Vector method)

So, the resultant, intensity at B is zero.



26. **Ans.** 0, $\frac{1}{3},\frac{2}{3}$.......... and so on

$\cos\theta = \frac{\Delta x}{3\lambda}$ for constructive interference.



$\cos\theta = \frac{n\lambda}{3\lambda}$ $(\Delta x = n\lambda)$

$\cos\theta = 0$

$\cos\theta = \frac{1}{3}$

$\cos\theta = \frac{2}{3}$ and so on

27. **Ans.** (a) $I_0/4$ (b) $I_0/4$

The two sources of sound S_1 and S_2 vibrate at same phase and frequency resultant intensity $P = I_0$

(a) Let the amplitude of the waves at S_1 and S_2 be r

when $\theta = 45°$ path difference = $S_1P - S_2P = 0$

$$S_1P = S_2P$$

So when source switched off intensity of sound P is $\frac{I_0}{4}$

(b) When $\theta = 60°$ path difference is also 0.

Similarly, it can be proved that, the intensity at p is $\frac{I_0}{4}$ when one switched off.

28. **Ans.** 2.5 kHz, 7.5 kHz

Given $L = 1m$, $Y = 2\times 10^{11}$ Pa

$\rho = 8gm/cm^3 = 8000\frac{kg}{m^3}$

Velocity of sound wave in steel rod, $v = \sqrt{\frac{Y}{\rho}}$

$v = \sqrt{\frac{2\times10^{11}}{8000}} = \frac{10^4}{2}m/s$

Frequency of fundamental mode i.e. $n = 1$,

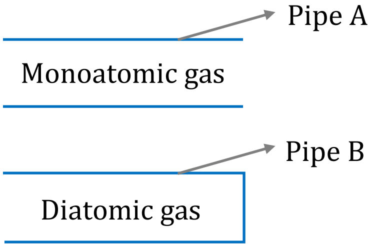
$f_1 = \frac{v}{2L}$

$f_1 = \frac{\frac{10^4}{2}}{2\times1} \Rightarrow f_1 = \frac{5}{2}kHz$

For 1st overtone $n = 3$

$\therefore f_3 = 7.5\ KHz$

29. **Ans.** (a) 2.116, (b) $\frac{3}{4}$



(a) $f_1 = \frac{2}{2L} \times \sqrt{\frac{5/3RT}{M_A}}$

$f_2 = \frac{3}{4L}\sqrt{\frac{7/5RT}{M_B}}$

$f_1 = f_2$ (given)

$$\frac{1}{L} \times \sqrt{\frac{5RT}{3M_A}} = \frac{3}{4L}\sqrt{\frac{7RT}{5M_B}}$$

$$\frac{5}{3M_A} = \frac{9}{16} \times \frac{7}{5M_B}$$

$$\frac{M_A}{M_B} = \frac{5\times5\times16}{3\times9\times7} = \frac{400}{189} = 2.116$$

(b) If pipe B is closed

$$f_B = \frac{1}{2L}\sqrt{\frac{7RT}{5M_B}}$$

$$f_A = \frac{1}{2L}\sqrt{\frac{5RT}{3M_A}}$$

$$\frac{f_A}{f_B} = \sqrt{\frac{5}{3} \times \frac{5}{7} \times \frac{M_B}{M_A}}$$

$$= \sqrt{\frac{25}{27} \times \frac{189}{400}}$$

$$= \frac{3}{4}$$

30. **Ans.** (a) $\frac{2\pi}{a}, \frac{b}{2\pi},$

(b) $y = -0.8\, A \cos(ax - bt)$ or $y = 0.8\, A \cos(ax - bt + \pi)$

(c) $1.8\, Ab$, 0

(d) $y = -1.6\, A \sin ax \sin bt + 0.2\, A \cos(ax + bt)$, $\left[n + \frac{(-1)^2}{2}\right]\frac{\pi}{a}$, $-X$ direction

OR

$y = 0.2\, A \cos(ax + bt) + 1.6\, A \cos ax \cos bt$, $x = \frac{n\pi}{a}$, $-X$ direction

(a) $y_1 = A \cos(ax + bt)$

$$k = a = \frac{2\pi}{\lambda} \quad \Rightarrow \quad \lambda = \frac{2\pi}{a}$$

$$\omega = \frac{2\pi}{T} = b \quad \Rightarrow \quad T = \frac{2\pi}{b}$$

(b) $I \propto A^2$

$$\frac{I_r}{I_i} = \left(\frac{A_r}{A_i}\right)^2 \quad \Rightarrow \quad A_r = 0.8A_i$$

Wave is reflected at $x = 0$

Hence there will phase difference of $\underline{\pi}$

$y_1 = A \cos(ax + bt)$ equation of particle

Equation of reflected wave

$y_2 = A_r \cos(ax - bt + \pi)$

$y_2 = -0.8\,A \cos(ax - bt)$

$\Rightarrow \quad y_2 = 0.8\,A \cos[ax - bt + \pi]$

(c) $y = y_1 + y_2$

$y = A \cos(ax + bt) - 0.8\,A \cos(ax - bt)$

$v = \frac{dy}{dt} = -Ab \sin(ax + bt) + 0.8Ab \sin(ax - bt)$

$= -Ab\,[\sin(ax + bt) + 0.8\sin(ax - bt)]$

$= -Ab\,[1.8 \sin ax \cos bt + 0.2\cos ax \sin bt]$

$\Rightarrow \quad |v_{max}| = 1.8\,Ab$

$\Rightarrow \quad |v_{min}| = 0$

(d) $y = y_1 + y_2 = A\cos(ax + bt) - 0.8A\cos(ax - bt)$

$y = -1.6 \sin ax \sin bt + 0.2A \cos(ax + bt)$

Antinode $\Rightarrow$ Amplitude is maximum

So, $|-1.6\sin ax| \Rightarrow$ max

$\Rightarrow \quad \sin ax = 1$

$ax = n\pi + (-1)^2 \times \frac{\pi}{2}$

$x = \left[n + \frac{(-1)^2}{2}\right]\frac{\pi}{a}$

direction of propagation of wave is in X – direction.

Similarly $y = y_1 - y_2$ gives $y = 0.2\,A \cos(ax + bt) + 1.6\,A \cos ax \cos bt$, $x = \frac{n\pi}{a}$, $-X$ direction

31. **Ans.** 33 *cm* and 13.2 *cm*

$$\frac{V_{air}}{2\,\ell_1} = \frac{V_{CO_2}}{4\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = \frac{2\,V_{air}}{V_{CO_2}} = \frac{2\times330}{264}$$

$$\ell_1 = \frac{330}{2\times500} = 33\ cm$$

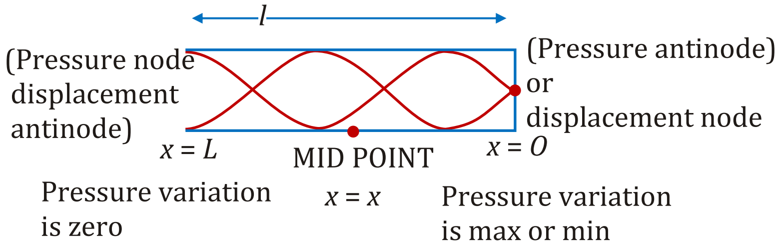
$$\ell_2 = \frac{33\times264}{2\times330} = 13.2\ cm$$

32. **Ans.** (a) $\frac{15}{16}m$, (b) $\frac{\Delta P_0}{\sqrt{2}}$, (c) equal to mean pressure, (d) $P_0 + \Delta P_0$, $P_0 - \Delta P_0$

(a) For second overtone as shown, $\frac{5\lambda}{4} = l \Rightarrow \lambda = \frac{4l}{5}$

Also, $V = \upsilon\lambda$

$$330 = 440 \times \frac{4l}{5} \quad \Rightarrow \quad l = \frac{15}{16}m.$$



(b) KEY CONCEPT: At any position x, the pressure is given by

$$\Delta P = \Delta P_0 \cos kx \cos \omega t$$

For $x = \frac{15}{2\times16} = \frac{15}{32}m\ (mid\ point)$

Amplitude $= \Delta P_0 \cos\left[\frac{2\pi}{(330/440)} \times \frac{15}{32}\right] = \frac{\Delta P_0}{\sqrt{2}}$

(c) At open end of pipe, pressure is always same i.e. equal to mean pressure

$\because \quad \Delta P = 0,\ P_{max} = P_{min} = P_0$

(d) At the closed end:

Maximum pressure $= P_0 + \Delta P_0$

Minimum Pressure $= P_0 - \Delta P_0$

33. **Ans.** 1650 Hz

As diaphragm C is a node and A and B are antinodes, each part behaves as closed organ pipe.

Thus fundamental frequency of the section are

$$n_H = \frac{v_H}{4l_H} = \frac{1100}{4\times0.5} = 550\ Hz$$

$$n_O = \frac{v_O}{4l_O} = \frac{330}{4\times0.5} = 150\ Hz$$

Since fundamental frequencies are unequal, we need to find those overtones when they are resonating

Thus $n = pn_H = qn_O$

$\Rightarrow \quad \frac{p}{q} = \frac{3}{11}$

Smallest integer value of p is 3.

Thus the minimum frequency of vibration of diaphragm C is

$$n = 3n_H = 1650 Hz$$

34. **Ans.** 3400 m/s

V_{rod} = ?

V_{air} = 340 m/sec

$L_r = 25\times10^{-2}\ m$

$d_2 = 5\times10^{-2}\ m$

$$\frac{V_r}{V_a} = \frac{2L_r}{d_2}$$

$$V_r = \frac{340\times25\times10^{-2}\times2}{5\times10^{-2}} = 3400\ m/sec$$

35. **Ans.** 478 Hz

If the frequency of the tunning fork = v

Then either $v - 476 = 2$ or $480 - v = 2$

Equating them

$v - 476 = 480 - v$

$2v = 956$

$v = 478\ Hz$

36. **Ans.** 252 Hz

Tunning fork produces 4 beat with a known tunning fork whole frequency = 256 Hz. So the frequency of unknown tunning fork = either 256 – 4 = 252

or 256 + 4 = 260 Hz

Now as the first one is load it $\frac{mass}{length}$ increase. So, its frequency decreases.

As it produce 6 beats now original frequency must be 252 Hz.

260 Hz is not possible as on decreasing the frequency the beat decreases which is not allowed here.

37. **Ans.** 0.39 cm

$L = 25\ cm = 25\times10^{-2}\ m$

By shortening the wire the frequency increases

$$f = \frac{1}{2\ell}\sqrt{\frac{T}{\mu}}$$

As the vibrating wire produces 4 beats with 256 Hz, Its frequency must be 252 Hz or 260 Hz. Its frequency must be 252 Hz, because beat frequency decreases by shortening the wire.

$$252 = \frac{1}{2\times25\times10^{-2}}\sqrt{\frac{T}{\mu}} \qquad ...(i)$$

Let length of the wire be L, after it is slightly shortened.

$$256 = \frac{1}{2L_1}\sqrt{\frac{T}{\mu}} \qquad ...(ii)$$

Dividing (i) by (ii) we get

$$\frac{252}{256} = \frac{L_1}{2\times25\times10^{-2}}$$

$L_1 = 0.2431m$

So, it should be shorten by (25-24.61) = 0.39 cm

38. **Ans.** 345, 341 or 349 Hz

The frequency of B can be 355 Hz or 345 Hz, When a wax is put on A its frequency decreases.

As the number of beats heard is decreasing, the new value of frequency of A must be closer to B, which implies frequency of B must be 345 Hz, and new frequency of A is 347 Hz.

The beats heard when B and C vibrates is 4 per second, which implies frequency of C must be 341 or 349 Hz.

The beats heard between A and C after A is waxed, is 6 Hz. Which implies frequency of C can be 341 Hz or 353 Hz.

Taking common of above two frequency of C is 341 Hz.

Hence frequency of B and C is 345, 341 Hz.

39. **Ans.** 11 $f/9$

In the ground frame, the wall appears to move towards the car with a speed of $\frac{c}{10} m/s$ and car is at rest w.r. t wind.

Let f be the frequency of the reflected sound of horn heard by the driver.

Given: Frequency of the horn = f

Doppler effect when wall act as an observer and car as source:

$$f' = f\left[\frac{V_{sound}+V_{car}}{V_{sound}}\right]$$

$$f' = f\left[\frac{c+\frac{c}{10}}{c}\right] = \frac{11}{10}f$$

Doppler effect when car act as an observer and wall as source:

$$f' = f\left[\frac{V_{sound}}{V_{sound}-V_{car}}\right]$$

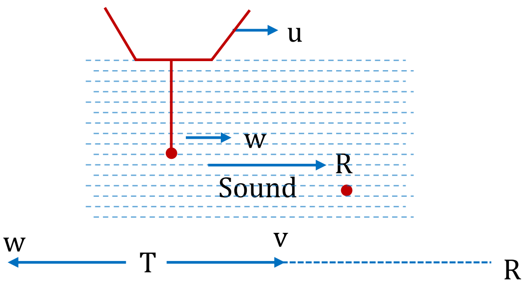
$$f' = \frac{11}{10}f \times \left[\frac{c}{c-\frac{c}{10}}\right] = \frac{11}{9}f$$

40. **Ans.** (a) 100696 Hz (b) 103038 Hz

(a) Speed of boat $u = 10\frac{m}{s}$

Speed of river stream $w = 2\frac{m}{s}$

Wavelength of sound wave inside water is 14.45 mm. speed of sound inside water is



$$v = \sqrt{\frac{B}{d}} = \sqrt{\left\{\frac{2.088\times10^{9}}{10^{3}}\right\}}$$

$$= \sqrt{2.088} \times 10^{3}\frac{m}{s} = 1445\frac{m}{s}$$

Frequency of wave inside water is

$$n = \frac{v}{\lambda} = \frac{1445}{14.45\times10^{-3}} = 10^{5} Hz$$

In this case source and medium both are in motion, so apparent frequency is given by

$$n' = \frac{(v+\omega)}{(v+\omega)-\mu}n = \frac{(1445+2)}{(1445+2)-10} \times 10^5$$

$$= \frac{1447}{1437} \times 10^5 Hz = 1.00696 \times 10^5\ Hz$$

(b) Speed of sound in air

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4\times 8.3\times 293}{28.8\times 10^{-3}}} = 344\ m/s$$

Speed of air (w) = 5 m/s

$$n' = \left(\frac{v-w}{v-w-10}\right) \times n$$

41. **Ans.** 5

$V_{\text{kink}} = R\omega$

= 1 × 100

= 100 m/s

42. **Ans.** 26.7 cm, 23.8 cm, 22.4 cm and 20.0 cm

$$196 = \frac{v}{2(0.3)}$$

$v = 2 \times 196 \times 0.3$

(i) $\ell_1 \frac{v}{2f} = \frac{2\times 196\times 0.3}{2\times 220}$ = 0.267 m = 26.7 cm

(ii) $\ell_2 = \frac{u}{2f} = \frac{2\times 196\times 0.3}{2\times 247}$ = 0.238 m = 23.8 cm

(iii) $\ell_3 = \frac{u}{2f} = \frac{2\times 196\times 0.3}{2\times 262}$ = 0.224 m = 22.4 cm

(iv) $\ell_4 = \frac{v}{2f} = \frac{2\times 196\times 0.3}{2\times 294}$ = 0.20m = 20 cm

43. **Ans.** 10800 Hz

$$\frac{T}{A} = Y\frac{\Delta\ell}{\ell} \qquad\qquad \Delta\ell = \frac{\pi R}{2}$$

$$T = \frac{AY\pi R}{2\ell}$$

$\mu = A\rho$

$$f = \frac{v}{2\ell} = \frac{1}{2\ell}\sqrt{\frac{Y\pi R}{2\rho\ell}}$$

$$= \frac{1}{2\times 0.06}\sqrt{\frac{20\times 10^{10}\times 3.14\times 2.5\times 10^{-3}}{2\times 7800\times 0.06}}$$

$= f \simeq 10800 Hz$

44. **Ans.** 17.5°

$f = \dfrac{4v}{2\ell}$ for 3rd overtone

$\dfrac{T}{\mu} = \dfrac{AY\alpha\Delta T}{A\rho}$

$200 = \dfrac{4}{2\times 0.25}\sqrt{\dfrac{2\times 10^{11}\times 10^{-5}\Delta T}{8000}}$

$25 = \sqrt{\dfrac{10^{3}\Delta T}{4}}$

$\Delta T = \dfrac{25\times 25\times 4}{1000}$ = 2.5 C

Hence T = 20 – 2.5 = 17.5°C

45. **Ans.** 27.04 N

String, ℓ = 0.25m, m = 0.0025 kg

Pipe ⇒ L = 0.4 m

1st overtone of string = $\dfrac{2V}{2\ell}$

$n_2 = \dfrac{1}{\ell}\sqrt{\dfrac{T}{\mu}} = \dfrac{1}{0.25}\sqrt{\dfrac{T}{0.0025/0.25}}$

$n_2 = 40\sqrt{T}$

For pipe

$n_1 = \dfrac{V}{4L} = \dfrac{320}{4\times 0.4}$ = 200 Hz

n_B = 8

∴ $n_2 - n_1 = 8$

$40\sqrt{T} - 200 = 8$

∴ T = 27.04 N

46. **Ans.** 5.9 sec

C = 340

$V_s = \dfrac{3}{4}C$

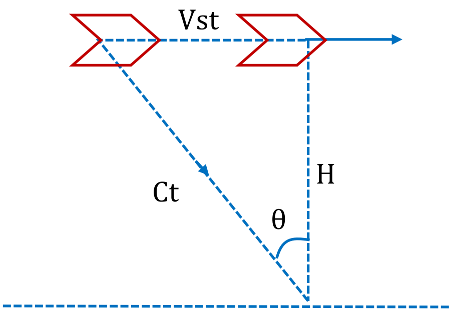
f_0 = 2 kHz

h = 1500 m

$f' = 2f_0$

$\sin\theta = \dfrac{V_s}{C}$

$\cos\theta = \dfrac{H}{Ct}$



$$\sqrt{1-\frac{v_s^2}{c^2}}=\frac{h}{Ct}$$

$$\sqrt{c^2-v_s^2}=\frac{H}{t}$$

$$t=\frac{H}{\sqrt{c^2-v_s^2}}$$

$$=\frac{1500}{\sqrt{c^2-\frac{9}{16}c^2}}=\frac{1500\times 4}{\sqrt{7}c}$$

$$=\frac{6000}{340\times\sqrt{7}}=5.9 \text{ sec}$$

EXERCISE - JEE (Main) PYQ

1. Ans. (3)

For closed organ pipe $f=\frac{(2n+1)v}{4\ell}$, $(n = 0,1,2......)$

$$\frac{(2n+1)v}{4\ell}<1250$$

$$(2n+1)<1250\times\frac{4\times 0.85}{340}$$

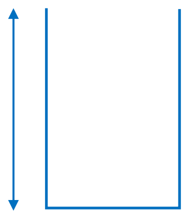
$(2n + 1) < 12.5$

$2n < 11.50$

$n < 5.25$

So $n = 0, 1, 2, 3.....5$

So we have 6 possibilities.

85 cm


2. Ans. (2)

$$f_{\text{before crossing}}=f_0\left(\frac{c}{c-v_S}\right)=1000\left(\frac{320}{320-20}\right)$$

$$f_{\text{after crossing}}=f_0\left(\frac{c}{c+v_S}\right)=1000\left(\frac{320}{320+20}\right)$$

$$\Delta f = f_0\left(\frac{2cv_S}{c^2-v_S^2}\right)$$

$$\frac{\Delta f}{f}\times 100\%=\frac{2\times 320\times 20}{300\times 340}\times 100=12.54\%\approx 12\%$$

3. Ans. (4)

$$v'=v\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}$$

$$v'=v\sqrt{\frac{1+\frac{1}{2}}{1-\frac{1}{2}}}=\sqrt{3}v$$

$v' = 10 \times 1.73 = 17.3\ GHz$

4. **Ans. (1)**

$$60 = \sqrt{\frac{Mg}{\mu}}$$

$$60.5 = \sqrt{\frac{M\left(g^2+a^2\right)^{1/2}}{\mu}}$$

$$\frac{60.5}{60} = \sqrt{\sqrt{\frac{g^2+a^2}{g^2}}}$$

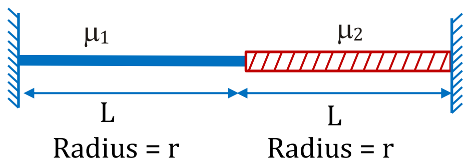
$$\left(1+\frac{0.5}{60}\right)^4 = \frac{g^2+a^2}{g^2} = 1+\frac{2}{60}$$

$$\Rightarrow \quad g^2+a^2 = g^2+g^2\times\frac{2}{60}$$

$$a = g\sqrt{\frac{2}{60}} = \frac{g}{\sqrt{30}} = \frac{g}{5.47} \approx \frac{g}{5}$$

5. **Ans. (4)**

Let mass per unit length of wires are μ_1 and μ_2 respectively.



Materials are same, so density ρ is same.

$$\therefore \quad \mu_1 = \frac{\rho\pi r^2 L}{L} = \mu$$

and $$\mu_2 = \frac{\rho 4\pi r^2 L}{L} = 4\mu$$

Tension in both are same = T, let speed of wave in wires are V_1 and V_2

$$V_1 = \sqrt{\frac{T}{\mu}} = V\,;$$

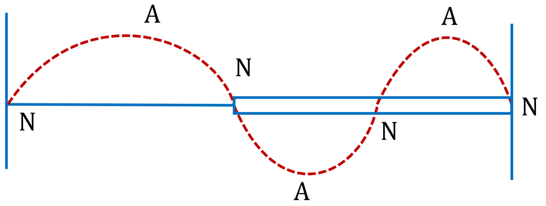
$$V_2 = \sqrt{\frac{T}{4\mu}} = \frac{V}{2}$$

So fundamental frequencies in both wires are

$$f_{0_1} = \frac{V_1}{2L} = \frac{V}{2L} \text{ and } f_{0_2} = \frac{V_2}{2L} = \frac{V}{4L}$$

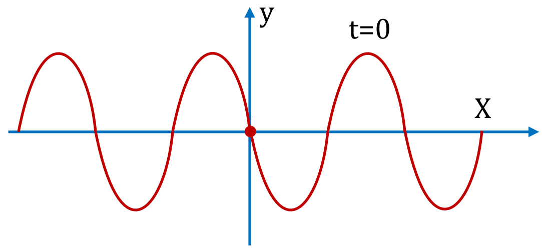
Frequency at which both resonate is L.C.M of both frequencies i.e. $\frac{V}{2L}$

Hence no. of loops in wires are 1 and 2 respectively.



So, ratio of no. of antinodes is 1 : 2.

6. Ans. (3)



$$y = A\sin(kx - wt + \phi)$$

at $x = 0, t = 0, y = 0$ and slope is negative $\Rightarrow \phi = \pi$

7. Ans. (4)

$$\frac{n}{2l}\sqrt{\frac{T}{\mu_1}} = \frac{m}{2L}\sqrt{\frac{T}{\mu_2}}$$

$$\therefore \quad \frac{n}{m} = \sqrt{\frac{\mu_1}{\mu_2}} = \frac{1}{2}$$

8. Ans. (3)

For closed organ pipe, resonance frequency is odd multiple of fundamental frequency.

$\therefore \quad (2n+1)f_0 \le 20{,}000$

(f_0 is fundamental frequency = 1.5 KHz)

$\therefore \quad n = 6$

$\therefore \quad$ Total number of overtone that can be heared is 6. (1 to 6).

9. Ans. (3)

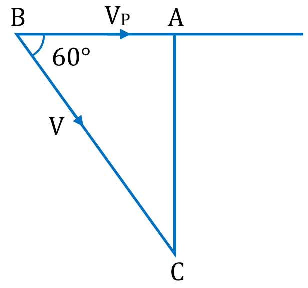
$AB = V_p \times t$

$BC = Vt$

$\cos 60° = \dfrac{AB}{BC}$

$\dfrac{1}{2} = \dfrac{V_p \times t}{Vt}$

$V_P = \dfrac{V}{2}$



10. Ans. (3)

$\Rightarrow \quad \lambda = 2(l_2 - l_1) \Rightarrow 2 \times (24.5 - 17)$

$\Rightarrow \quad 2 \times 7.5 = 15\ cm$

$\quad \& \ v = f\lambda \Rightarrow 330 = f \times 15 \times 10^{-2}$

$\quad f = \dfrac{330}{15} \times 100$

$\Rightarrow \quad f = 2200\ Hz$

11. Ans. (2)

$\Delta p = BkS_0$

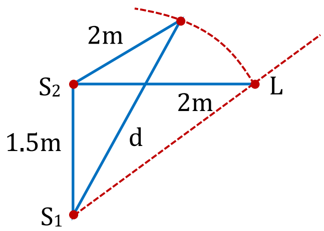
$\quad = \rho v^2 \times \dfrac{\omega}{v} \times S_0$

$\Rightarrow \quad S_0 = \dfrac{\Delta p}{\rho v \omega}$

$\quad \approx \dfrac{10}{1 \times 300 \times 1000} m$

$\quad = \dfrac{1}{30} mm \approx \dfrac{3}{100} mm$

12. Ans. (2)



Initially $S_2L = 2m$

$S_1L = \sqrt{2^2 + (3/2)^2}$

$S_1L = \frac{5}{2} = 2.5\ m$

$\Delta x = S_1L - S_2L = 0.5\ m$

Since $\lambda = 1m \quad \therefore\ \Delta x = \frac{\lambda}{2}$

So while listener moves away from S_1

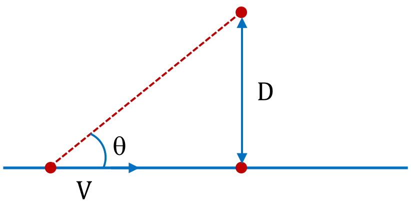
Then, $\Delta x = (S_1L - S_2L)$ increases

and hence, at $\Delta x = \lambda$ first maxima will appear.

$\Delta x = \lambda = S_1L - S_2L$

$1 = d - 2 \Rightarrow d = 3m$

13. Ans. (4)



$f_{\text{observed}} \Rightarrow \left(\frac{v_{sound}}{v_{sound} - v\cos\theta}\right) f_0$

Initially θ will be less $\Rightarrow \cos\theta$ more

$\therefore \quad f_{\text{observed}}$ more, then it will decrease.

$\therefore$ Ans. (4)

14. Ans. (7)

$y_1 = A_1 \sin k(x - vt)$

$y_1 = 12 \sin 6.28\ (x - vt)$

$y_2 = 5 \sin 6.28\ (x - vt + 3.5)$

$\Delta\phi = \frac{2\pi}{\lambda}(\Delta x)$

$= K(\Delta x)$

$= 6.28 \times 3.5 = \frac{7}{2} \times 2\pi = 7\pi$

$A_{net} = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi}$

$A_{net} = \sqrt{(12)^2 + (5)^2 + 2(12)(5)\cos(7\pi)}$

$= \sqrt{144 + 25 - 120}$

$= 7$

15. Ans. (1)

For node

$\cos(1.57cm^{-1})\,x = 0$

$(1.57cm^{-1})x = \frac{\pi}{2}$

$x = \frac{\pi}{2(1.57)} cm = 1cm$

Ans. 1.00

16. Ans. (2)

$v_s = \sqrt{\dfrac{\gamma RT}{M}}$

$v_{rms} = \sqrt{\dfrac{3RT}{M}}$

$\dfrac{v_s}{v_{rms}} = \sqrt{\dfrac{\gamma}{3}} = \dfrac{1}{\sqrt{2}} \Rightarrow \dfrac{\gamma}{3} = \dfrac{1}{2} \Rightarrow \gamma = \dfrac{3}{2}$

$\gamma = 1 + \dfrac{2}{f_{mix.}}$

$f_{mix.} = \dfrac{2\times3+n\times5}{n+2} = \dfrac{6+n\times5}{(n+2)}$

$\gamma = 1 + \dfrac{2(n+2)}{6+n\times5} = \dfrac{6+5n+2n+4}{6+5n}$

$\gamma = \dfrac{7n+10}{6+5n} = \dfrac{3}{2}$

$14n + 20 = 18 + 15n$

$n = 2$

17. Ans. (1)

$y = 2\sin(\omega t - kx)$

Maximum particle velocity = $A\,\omega$

Wave velocity = $\dfrac{\omega}{k}$

$\dfrac{\omega}{k} = A\,\omega$

$k = \dfrac{1}{A} = \dfrac{2\pi}{\lambda}$

$\lambda = 2\pi A$

$= 4\,\pi\, cm$

18. Ans. (2)

$v = \sqrt{\dfrac{T}{\mu}} = \sqrt{\dfrac{70}{70\times10^{-3}}} = 100\ m/s$

19. Ans. (2)

$f_1 = 300\left(\dfrac{330-0}{330-(-30)}\right) = 275$

$f_2 = 300\left(\dfrac{330-0}{330-(30)}\right) = 330$

$\Delta f = 330 - 275 = 55\ Hz.$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (ACD)**

$v = 100m/sec$

$f = \frac{(2n+1)v}{4\ell}$

(a) $n = 0$

$f = \frac{100}{12} = \frac{50}{6} = \frac{25}{3}$

$\omega = 2\pi f = \frac{50\pi}{3}$

$k = \frac{\omega}{v} = \frac{50\pi}{3\times100} = \frac{\pi}{6}$

$y(t) = A\, sin\frac{\pi}{6}x\, cos\frac{50\pi}{3}t$

(c) $n = 2$

$f = \frac{5}{12}\times 100 = \frac{250}{6} = \frac{125}{3}$

$\omega = 2\pi\times\frac{125}{3} = \frac{250\pi}{3}$

$k = \frac{250\pi}{3\times100} = \frac{5\pi}{6}$

$y(t) = A\, sin\frac{5\pi}{6}x\, cos\frac{250\pi t}{3}$

(d) $n = 7$

$f = \frac{\cancel{15}\times100}{4\times\cancel{3}} = \frac{250}{2} = 125$

$\omega = 125\times 2\pi = 250\pi$

$k = \frac{250\pi}{100} = \frac{5\pi}{2}$

$y(t) = A\, sin\frac{5\pi}{2}x\, cos\, 2\,50\pi t$

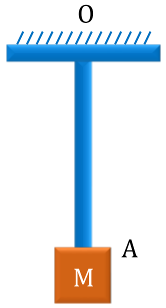
2. **Ans. (AD)**

(A) Speed of wave is property of medium so time taken to cross the string will be equal

(B) Speeds are same but velocity is vector, has opposite directions

(C) Wavelength $\lambda = \frac{v}{f} = \frac{1}{f}\sqrt{\frac{T}{\mu}}$ and $T_0 > T_A$

(D) Velocity of any pulse is $v = \sqrt{\frac{T}{\mu}}$ and it is property of medium.

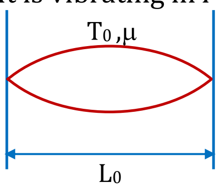


3. **Ans. (A)**

For string (1)

Length of string = L_0

It is vibrating in I^{st} harmonic i.e. fundamental mode.



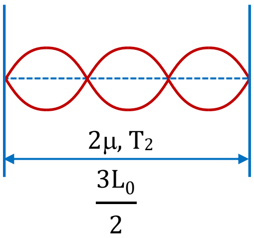
$f_0 = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow (P)$

For string (2)

Length of string = $\frac{3L_0}{2}$

It is vibrating in III^{rd} harmonic but frequency is still f_0.

$$f_0 = \frac{3v}{2L}$$



$$f_0 = \frac{3}{2\left(\frac{3L_0}{2}\right)}\sqrt{\frac{T_2}{2\mu}}$$

$$\Rightarrow \quad f_0 = \frac{1}{L_0}\sqrt{\frac{T_2}{2\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

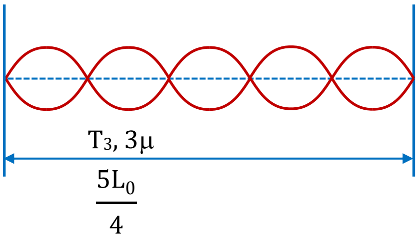
$$\Rightarrow \quad \boxed{T_2 = \frac{T_0}{2}} \Rightarrow (Q)$$

For string (3)

Length of string = $\frac{5L_0}{4}$

It is vibrating in $5th$ harmonic but frequency is still f_0.

$$f_0 = \frac{5V}{2L}$$



$$\Rightarrow f_0 = \frac{5}{2\left(\frac{5L_0}{4}\right)}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

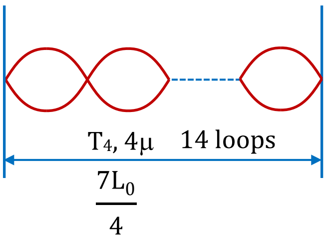
$$\Rightarrow \quad \frac{2}{L_0}\sqrt{\frac{T_3}{3\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$T_3 = \frac{3T_0}{16} \Rightarrow (T)$$

For string (4)

Length of string = $\frac{7L_0}{4}$

It is vibrating in 14^{th} harmonic but frequency is still f_0.



$$f_0 = \frac{14v}{2L}$$

$$\Rightarrow \quad f_0 = \frac{14}{2\left(\frac{7L_0}{4}\right)}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}}$$

$$\Rightarrow \quad \frac{4}{L_0}\sqrt{\frac{T_4}{4\mu}} = \frac{1}{2L_0}\sqrt{\frac{T_0}{\mu}} \Rightarrow \boxed{T_4 = \frac{T_0}{16}} \Rightarrow (U)$$

4. **Ans. (D)**

Given $\frac{\lambda}{4}$ = (0.350 + 0.005) m

$\Rightarrow$ λ = (1.400 ± 0.020) m

$f = 244\ Hz$

$\therefore$ $v = (341.60 \pm 4.88)\ m/s$

Now from the given option, $v = \sqrt{\frac{\gamma RT}{M}}$

$\therefore$ $v_{Ne} = \sqrt{\frac{1.67 \times RT}{20 \times 10^{-3}}} = 640 \times \sqrt{\frac{10}{20}} = 448\ m/s$

$v_{N_2} = \sqrt{\frac{1.40\ RT}{28 \times 10^{-3}}} = 590 \times \frac{3}{5} = 354\ m/s$

$v_{O_2} = \sqrt{\frac{1.40\ RT}{32 \times 10^{-3}}} = 331.875\ m/s$

$v_{Ar} = \sqrt{\frac{1.67\ RT}{36 \times 10^{-3}}} = 340\ m/s$

$\therefore$ Only velocity in Argon satisfies the range of velocity.

5. **Ans. (3)**

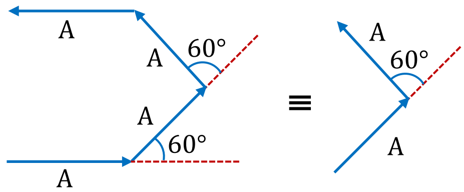
Let amplitude of individual wave is A.

then by using phasor method

So resultant amplitude $A_r^2 = A^2 + A^2 + 2A^2 \cos 60$

$\therefore$ $A_r^2 = 3A^2$

$\therefore$ $I_r = 3I_0$



6. **Ans. (ACD)**

$f_M = \frac{C + V\cos\theta}{C} f_1$

$f_N = \frac{C + V\cos\theta}{C} f_2$

$\Delta f = f_N - f_M$

$= \frac{C + V\cos\theta}{C}(f_2 - f_1)$

$\frac{d(\Delta f)}{dt} = -\frac{V}{C}(f_2 - f_1)\sin\theta \frac{d\theta}{dt}$

$\therefore$ $\frac{d(\Delta f)}{dt}$ is maximum when θ = 90°

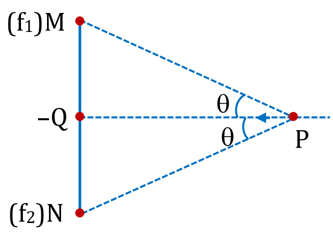
[$\therefore$ C is correct]

$v_P = \left(1 + \frac{V}{C}\cos\theta\right)\Delta f$

$v_Q = \Delta f$

$v_R = \left(1 - \frac{V}{C}\cos\theta\right)\Delta f$

$\therefore$ $v_P + v_R = 2v_Q$



7. Ans. (6)

Frequency of sound as received by large car approaching the source.

$$f_1 = \frac{C+V_0}{C} f_0 = \left(\frac{330+2}{330}\right) 492Hz.$$

This car now acts as source for reflected sound wave

$\therefore \quad f_{reflected} = f_1$

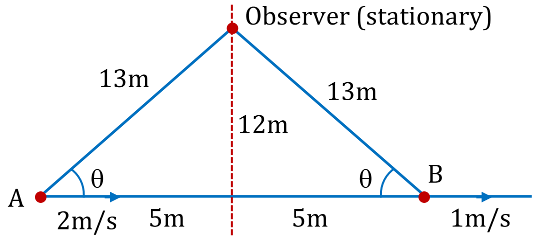
frequency of sound received by source,

$$f_2 = \left(\frac{C}{C-V_0}\right) f_{reflected} = \left(\frac{330}{330-2}\right) \times f_1 = \frac{330}{328} \times \frac{332}{330} \times 492Hz$$

$\therefore \quad$ Beat frequency $= |f_0 - f_2| = \left(\frac{332}{328} - 1\right) \times 492\ Hz$

$= 6\ Hz$

8. Ans. (5 [4.99, 5.01])



$$\cos\theta = \frac{5}{13}$$

$f_A = 1430\left[\frac{330}{330-2\cos\theta}\right] = 1430\left[\frac{1}{1-\frac{2\cos\theta}{330}}\right] = 1430\left[1+\frac{2\cos\theta}{330}\right]$ (By bionomial expansion)

$$f_B = 1430\left[\frac{330}{330+1\cos\theta}\right] = 1430\left[1-\frac{\cos\theta}{330}\right]$$

$$\Delta f = f_A - f_B = 1430\left[\frac{3\cos\theta}{330}\right] = 13\cos\theta$$

$$= 13\left(\frac{5}{13}\right) = 5.00Hz$$

9. Ans. (ABC)

Let n_1 harmonic is corresponding to 50.7 cm and n_2 harmonic is corresponding 83.9 cm.
since both one consecutive harmonics.

$\therefore \quad$ Their difference $= \frac{\lambda}{2}$

$\therefore \quad \frac{\lambda}{2} = (83.9 - 50.7)\ cm$

$\frac{\lambda}{2} = 33.2\ cm$

$\lambda = 66.4\ cm$

$\therefore \quad \frac{\lambda}{4} = 16.6\ cm$

Length corresponding to fundamental mode must be close to $\frac{\lambda}{4}$ and 50.7 cm must be closed to an odd multiple of this length as $16.6 \times 3 = 49.8\ cm$, therefore 50.7 is 3^{rd} harmonic.

If end correction is e, then

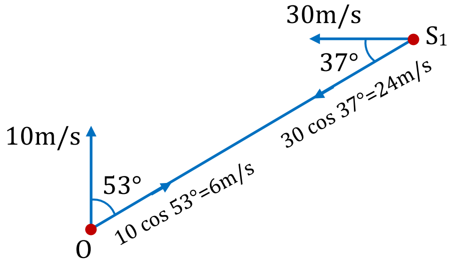
$e + 50.7 = \frac{3\lambda}{4}$

$e = 49.8 - 50.7 = -0.9\ cm$

Speed of sound, $v = f\lambda$

$\therefore \quad v = 500 \times 66.4\ cm/sec = 332.000\ m/s$

10. Ans. (8.12 to 8.13)



Frequency observed by O from S_2

$$f_2 = \frac{330+10}{330} \times 120 = \frac{340}{330} \times 120 = 123.63 Hz$$

Frequency observed by O from S_1

$$f_1 = \frac{330+6}{330-24} \times 120 = \frac{336}{306} \times 120 \approx 131.76 Hz$$

Beat frequency = 131.76 – 123.63 = 8.128 ≈ 8.12 to 8.13 Hz

11. Ans. (0.62 to 0.64)

$f \propto \frac{1}{\ell_1} \Rightarrow f = \frac{k}{\ell_1}$ ... (1)

($\ell_1 \Rightarrow$ initial length of pipe)

$\left(\frac{V}{V-V_T}\right) f = \frac{k}{\ell_2}$ {V_T Speed of tuning fork, ℓ_2 →new length of pipe} ...(2)

(1) ÷ (2)

$$\frac{V-V_T}{V} = \frac{\ell_2}{\ell_1}$$

$$\frac{\ell_2}{\ell_1} - 1 = \frac{V-V_T}{V} - 1$$

$$\frac{\ell_2-\ell_1}{\ell_1} = \frac{-V_T}{V}$$

$$\frac{\ell_2-\ell_1}{\ell_1} \times 100 = \frac{-2}{320} \times 100 = -0.625$$

Therefore, smallest value of percentage change required in the length of pipe is 0.625.

12. Ans. (AD)

$f = f_s\left(\frac{v}{v-u}\right)$

(A) $f = f_0\left(\frac{v}{v-0.8v}\right) = 5f_0$

(B) $f = 2f_0\left(\frac{v}{v-0.8v}\right) = 10f_0$

(C) $f = 0.5f_0\left(\frac{v}{v-0.8v}\right) = 2.5f_0$

(D) $f = 1.5f_0\left(\frac{v}{v-0.5v}\right) = 3f_0$

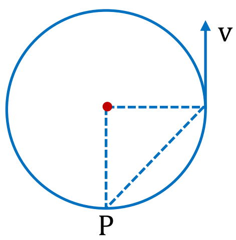
All odd harmonics are available in closed pipe therefore

correct Ans (A,D)

13. Ans. (648.00)

$$f' = \frac{C}{C + v\cos 45°} f$$

$$= \frac{324}{324 + 4\sqrt{2} \times \frac{1}{\sqrt{2}}} \times 656 = 648 \text{ Hz}$$



14. Ans. (8.00 to 8.40)

$f_{P\ from\ S_2} = 656 Hz$

$$f_{P\ from\ S_1} = \frac{C}{C-V} f = \frac{656 \times 324}{324-4} = 664.2$$

Δf = 664.2 – 656 = 8.2 Hz

JEE (Main) Practice Paper

SECTION–A

1. **Ans. (3)**

$y = Ae^{[-B(x-vt)^2]}$

$A = 1, B = 1, v = 2\ m/s.$

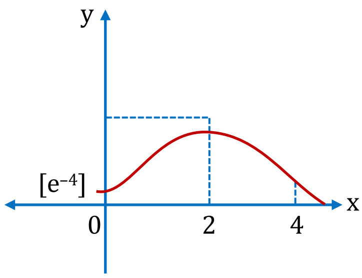
At $t = 1$sec.

$y = e^{[-(x-2)^2]}$

$y = e^{[4x-x^2-4]}$

At $t = 1$ sec

$y = 1$ at $x = 2$ point of maxima

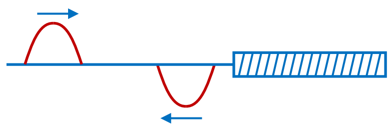


2. **Ans. (3)**

$Y = A\sin(kx - \omega t + 30°)$

Rellected wave will have opposite phase because it is reflected back from rigid end.

It will travel towards $-ve$ x axis.



$E_{reff} = 64\%$ of initial energy

$E \propto A^2$

$Y = A\sin(kx - \omega t + 30°)$

At $x = 0$

$Y = A\sin(-\omega t + 30°)$

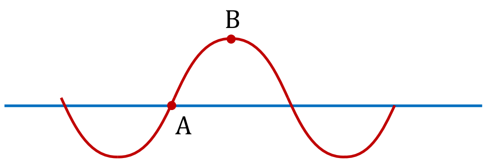
$Y = A\sin(\omega t + 150°)$

$Y_{ref} = (0.8A)\sin(kx + \omega t + 150° + 180°)$

$= 0.8\,A\sin(kx + \omega t + 330°)$

$= 0.8\,A\sin(kx + \omega t + 30°)$

3. **Ans. (2)**



Both KE and PE are maximum at $y = 0$ and minimum at $y = A$.

4. **Ans. (2)**

$y = A\sin(\omega t - kx)$

At $x = 4cm, t = T/6, y = A/2$

$$\frac{A}{2} = A\sin\left[\frac{2\pi}{T}\left(\frac{T}{6}\right) - K\left(\frac{4}{100}\right)\right]$$

$$\sin\left[\frac{\pi}{3} - K\left(\frac{4}{100}\right)\right] = \frac{1}{2}$$

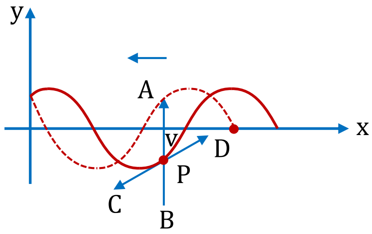
$$\frac{\pi}{3} - \frac{4k}{100} = \frac{\pi}{6}$$

$$\frac{4k}{100} = \frac{\pi}{6}$$

$$k = \frac{2\pi}{\lambda} = \frac{100\pi}{24}$$

$$\lambda = \frac{48}{100}m = 48\ cm$$

5. **Ans. (1)**



$\vec{v}$ in direction of $\overrightarrow{PA}$.

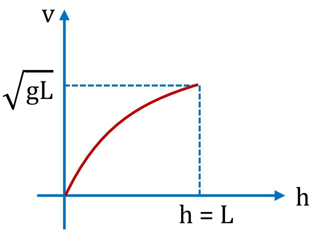
6. **Ans. (3)**

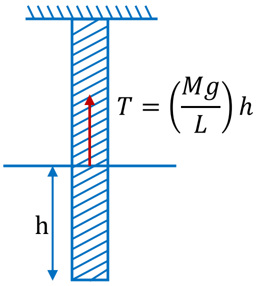
$y = [4t - 3x]^{-1}$ is not progressive wave because the function is not defined for all (x, t) such that $[4t - 3x = 0]$.

7. **Ans. (3)**

$$v_w = \sqrt{\frac{T}{u}} = \sqrt{\frac{Mgh \times L}{L \times M}}$$

$$= \sqrt{gh}$$





8. **Ans. (3)**

PE and KE both maximum at $y = 0$. [Simultaneously for two particles in phase]

Amplitude and frequency same for all particles.

There will be phase difference in motion of all particles.

9. **Ans. (2)**

After 2 sec the pulses will overlap completely. The string becomes straight and therefore does not have any potential energy and its entire energy must be kinetic.

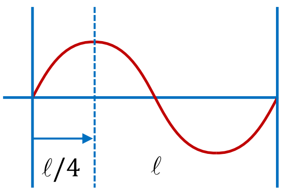
10. **Ans. (300)**

$$\frac{\ell}{4} = \frac{\lambda}{4}$$

$\lambda = \ell$

$f = 100$

$$100 = \frac{v}{\lambda}$$



$$f' = \frac{v}{\lambda'}$$

$$f' = \frac{v}{(\lambda/3)}$$

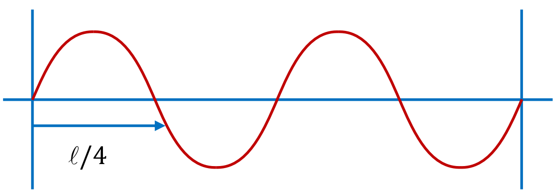
$f' = 3f$

$f' = 300\ Hz$

$$\frac{\ell}{4} = \frac{3\lambda'}{4}$$

$$\lambda' = \frac{\ell}{3}$$

$$\lambda' = \frac{\lambda}{3}$$



11. Ans. (2)

$V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{A\rho}} = \sqrt{\frac{T}{\pi r^2 \rho}}$

$\frac{f_1}{f_2} = \sqrt{\frac{T_1}{T_2} \cdot \frac{\rho_2}{\rho_1} \cdot \frac{r_2^2}{r_1^2}} = \sqrt{2 \times \frac{1}{2} \times \frac{1}{4}} = \frac{1}{2}$

12. Ans. (1)

When Tension increases, velocity of wave increases, so frequency increases

It least frequency is decreasing then $fo_2 > fo_1$

13. Ans. (2)

It difference between frequencies of wave is less then standing wave pattern is more compact.

14. Ans. (3)

$\ell_1 = \frac{\lambda}{4}, \ \ell_2 = \frac{3\lambda}{4}$

At top end there is displacement antinode, so distance is zero.

So, option 3

$\frac{\ell_2 - 3\ell_1}{2} = 0$

15. Ans. (2)

Distance between pressure node & pressure antinode is $\frac{\lambda}{2}$

16. Ans. (2)

According to graph at $x = 0$ pressure is maximum so, it is compression and at $x = L$ pressure is minimum, therefore it is rarefaction. Hence, option no. 2 is correct.

17. Ans. (2)

$S = S_0 \sin \ (kx \pm \omega t)$

$\Delta P = -B\frac{\partial S}{\partial x}$

For wave traveling towards Right $S = S_0 \sin (kx - \omega t)$

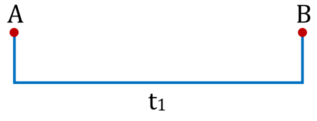
$\Delta P = BK \ S_0 \ \cos \ (kx - \ \omega t)$

$t = 0 \quad \Rightarrow \quad \Delta P = -BK \ S_0 \ \cos \ (kx)$

18. Ans. (4)

t_1 : time taken by sound to travel in water

t_2 : time taken by sound to travel in air.



$t_1 = \frac{d}{u}$

$t_2 = \frac{d}{v}$

$t_2 - t_1 = t$ (given)

$\frac{d}{v} - \frac{d}{u} = t$

$d\left[\frac{u-v}{uv}\right] = t$

$d = \frac{uvt}{u-v}$

19. **Ans. (1)**

$f_1 = 512 = \frac{V}{4\ell_1}$

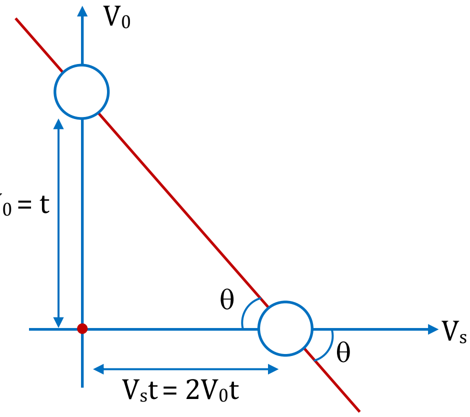
$f_2 = 512 = \frac{3V}{4\ell_2}$

$V = 512 \times 4 \times \frac{17}{100} = 348.16\ m/s$

The water reservoir was moved downward to obtain second Resonance.

20. **Ans. (2)**

Speed of source (V_s) = 2 speed of observer (V_0)



$tan\ \theta = \frac{1}{2}$

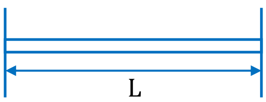
$f = f_0 \left[\frac{V - V_0 cos(90-\theta)}{V + Vs\, cos\,\theta}\right]$

Independent of time & less than f_0

SECTION-B

1. **Ans. (70)**

$\Delta T = -20°C$



$L' = L(1 + \alpha\Delta T)$

$\Delta L = L[\alpha\Delta T]$

$\frac{\Delta L}{L} = \alpha\Delta T = 1.7 \times 10^{-5} \times 20$

$= 3.4 \times 10^{-4}$

$\frac{F}{A} = (3.4 \times 10^{-4})Y$

$F = YA[\alpha\Delta T]$

$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{YA\alpha\Delta T}{(M/L)}}$

$= \sqrt{\frac{Y\alpha(\Delta T)}{(M/LA)}}, \frac{M}{LA} \rightarrow$ Density

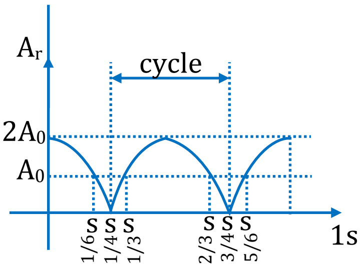
$v = 70\ m/s$

2. **Ans. (8)**

$y_1 = A \sin \omega_1 t$

$y_2 = A \sin \omega_2 t$

$y_r = 2A \cos\left\{\left(\frac{\omega_2 - \omega_1}{2}\right)\right\} t \left\{\sin\left(\frac{\omega_2+\omega_1}{2}\right)\right\} t$



Resultant amplitude $A_r = 2A_0 |\cos(\Delta\omega)t/2|$

$$(\Delta\omega)\ \frac{t}{2} = \frac{\pi}{2} \Rightarrow t = \frac{1}{4} s$$

$$(\Delta\omega)\frac{t}{2} = \frac{\pi}{3} \Rightarrow t = \frac{1}{6} s$$

In one cycle of intensity of 1/2s, the detector remain idle for

$$2\left(\frac{1}{4} - \frac{1}{6}\right) s = \frac{1}{6} sec$$

$\therefore$ In ½ sec cycle, active time is $\left(\frac{1}{2} - \frac{1}{6}\right) = 1/3\ sec$

$\therefore$ In 12 sec interval, active time is $12 \times \frac{(1/3)}{(1/2)} = 8$ sec

3. **Ans. (16)**

$y = 200 \times 10^9\ Pa$; $\alpha = 10^{-5}/°C$

$T_i = 20°C$ density $= \delta = 8\ g/cm^3 = 8000\ kg/m^3$

$f_0 = 200\ Hz$

For 3rd overtone $f = 200\ Hz = \frac{4V}{2L}$

$$\Rightarrow \quad V = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{\Delta\ LY}{L\rho}} \begin{Bmatrix} \Delta\ L = \frac{FL}{Ay} \\ F = \frac{\Delta L\ AY}{L} \end{Bmatrix}$$

$$\Rightarrow \quad \frac{1}{4} \times 100 = \sqrt{\frac{10^{-5} \Delta T \ \ 200 \times 10^9}{8 \times 10^3}}$$

$$\frac{10}{4} = \frac{\Delta T}{4} \times 10^3$$

$$\Delta T = \frac{10}{4} = 2.5$$

$$\Rightarrow \quad 20 - T = 2.5$$

$$\Rightarrow \quad T = 17.5°C$$

4. Ans. (3)

$\frac{\lambda}{4} = 24.1 + e$

$\frac{3\lambda}{4} = 74.1 + e$

$3(24.1 + e) = 74.1 + e$

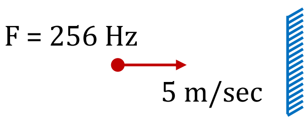
$2e = 74.1 - 72.3$

$e = 74.1 - 72.3$

$e = 0.3\ D$

$\Rightarrow \quad D = \frac{74.1 - 72.3}{2 \times 0.3} = 3$

5. Ans. (8)



$f_{wall} = f\left(\frac{v}{v - v_S}\right)$

$f_{recived} = f_{wall}\left(\frac{v + v_0}{v}\right)$

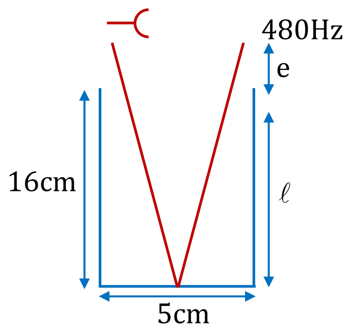
$f_{recived} = f\left(\frac{v + v_0}{v - v_S}\right)$, $v_0 = v_s = 5\ m/s$

$f_{\text{recived}} - f = \text{beats}$

$f\left(\frac{v + v_0}{v + vs}\right)$

$f\left(\frac{v_0 + v_S}{v - v_S}\right) - f = 8$ Beats

6. Ans. (336)



$\frac{v}{4(\ell + 0.3d)} = 480$

$\frac{v}{4(16 + 0.3 \times 5)} = 480$

$v = 480 \times 4 \times 17.5$

$v = 336 m/\,sec$

7. Ans. (1)

$f = k\sqrt{T}$

$f^2 = kT$

$\ell n f^2 = \ell n k + \ell n$

$2\frac{\Delta f}{f} = \frac{\Delta T}{T}$

$\frac{2 \times 1 \times 100}{200} = \frac{\Delta T}{T}\%$

$1\% = \frac{\Delta T}{T}\%$

8. Ans. (3)

$\ell = \frac{1}{10} m \qquad \mu = \frac{1gm}{10cm}$

$\frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = f \qquad \mu = \frac{10^{-3}}{10\times10^{-2}} kg/m$

$\frac{10}{2}\sqrt{64\times100} = f \qquad \mu = \frac{1}{100} kg/m$

$5\times80 = f$

$400Hz = f$

$400 - f\left(\frac{v}{v-v_s}\right) = 1$

$400 - 400\left(\frac{300}{300+v_s}\right) = 1$

$400\left[1 - \frac{300}{300+v_s}\right] = 1$

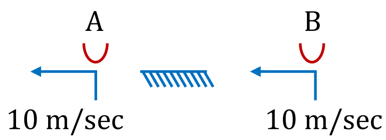
$\frac{400\times v_s}{300+v_s} = 1$

$400\ v_s = 300 + v_s \ \Rightarrow\ v_s \approx 0.75 = \frac{v}{4} \Rightarrow\ v = 3\ m/s$

9. Ans. (5)

$f_{0A} = f\left(\frac{V-V\omega}{V-V\omega+vs}\right) = 85\left(\frac{330}{340}\right)$

$f_{0B} = f\left(\frac{V+V\omega}{V+V\omega-vs}\right) = 85\left(\frac{330}{340}\right)$



Beats $= \frac{85\times20}{340}$

Beats = 5

10. Ans. (6)

Beats $= f\left(\frac{v_0+v_s}{v-v_s}\right)$

Beats $= 200\left(\frac{5+5}{342-5}\right)$

Beats $= \frac{200\times10}{337} = 5.93 = 6$

JEE (Advanced) Practice Paper

1. Ans. (C)

$P = P_0 \cos\left(\frac{3\pi x}{2}\right)\ \sin\ (300\pi t)$

At $x = 0 \quad \cos\frac{3\ \pi\ (0)}{2} = 1$ Hence pressure us maximum

At $x = 2 \quad \cos\frac{3\pi(2)}{2} = -1$ Hence pressure us maximum

So length of pipe is 2 m

2. **Ans. (A)**

$y = \frac{1}{\sqrt{1+x^2}}$ at $t = 0$

$y = \frac{1}{\sqrt{x^2-2x+2}}$ at $t = 1$

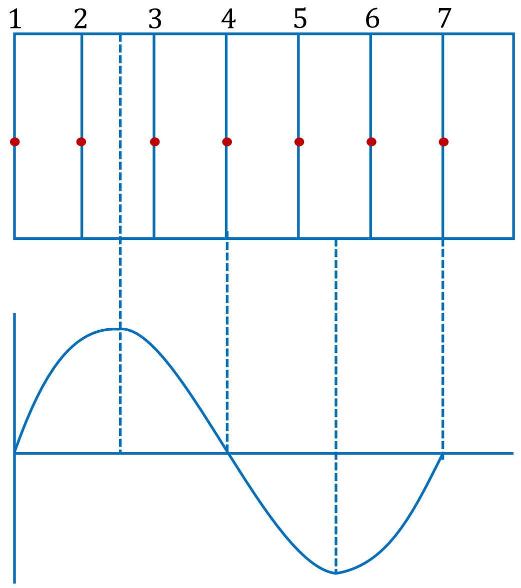
$= \frac{1}{\sqrt{(x-1)^2+1}}$

$\because \quad y = \frac{1}{\sqrt{1+(x-t)^2}}$

$v = 1\ m/s$ towards the x-axis.

3. **Ans. (BCD)**

For figure (B), S-x graph becomes



For (4), $\frac{\partial S}{\partial x} < 0 \Rightarrow V_P > 0$ $\left[\because \begin{array}{c} v_P = -v_\omega\left(\frac{\partial S}{\partial x}\right) \\ v_w > 0 \end{array}\right]$

For 3 & 5 $\frac{\partial S}{\partial x} < 0 \Rightarrow V_P > 0$

For 7 $\frac{\partial S}{\partial x} > 0$ $\Rightarrow V_P < 0$

4. **Ans. (AB)**

$P = \frac{P_0}{2}\left[sin\left(203\pi t + \frac{5\pi}{6}\right) + sin\left(197\pi t + \frac{\pi}{6}\right)\right]$

$f_1 = \frac{203\pi}{2\pi} > 100$

$f_2 = \frac{197\pi}{2\pi} < 100$

$f_{beat} = \frac{203\pi}{2\pi} - \frac{197\pi}{2\pi} = 3$

5. Ans. (AC)

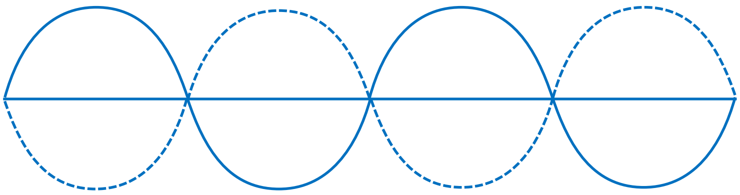
$y_1 = 0.02\, sin\left[400\pi\left(\frac{x}{330} - t\right)\right]$

$y_2 = 0.02\, sin\left[404\pi\left(\frac{x}{330} - t\right)\right]$

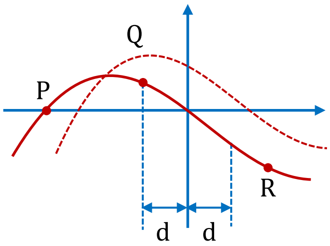
$y_1 + y_2 = 2(0.02)\, sin\left[404\pi\left(\frac{x}{330} - t\right)\right] cos\left[2\pi\left(\frac{x}{330} - t\right)\right]$

$= 0.04\, sin\left(\frac{402}{330}\pi x - 402\pi t\right) cos\left(\frac{2\pi x}{330} - 2\pi t\right)$

6. Ans. (BC)



7. Ans. (CD)



$v_P \rightarrow \downarrow, v_Q \rightarrow \uparrow, v_R \rightarrow \uparrow, v_Q = v_R$

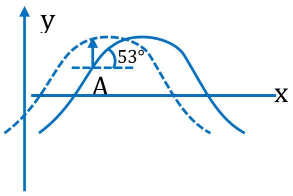
$\because$ [cos(–θ) = cos θ]

$v_P > v_Q$ [speed max. at $y = 0$]

8. Ans. (BC)

As shown in the curve, if wave is moving along –x axis, V_P is positive.

$\frac{V_p}{V_w} = -\tan\theta$; $|-\tan\theta| > 1 \therefore V_p > V_w$.



9. Ans. (BD)

Given $\omega = 3\pi$ $\quad\therefore f = \frac{\omega}{2\pi} = 1.5$

Also $\Delta x = 1.0\, cm$

Given, $\phi = \frac{2\pi}{\lambda}\Delta x \quad \Rightarrow \frac{\pi}{8} = \frac{2\pi}{\lambda} \times 1$

$\Rightarrow \quad \lambda = 16\, cm \Rightarrow v = f\lambda = 16 \times 1.5 = 24\, cm/sec$

$y = a\, sin\, \frac{2\pi}{\lambda}(vt - x)$

$y = 2\, sin\, \frac{2\pi}{\lambda}(24t - x)$

for $t = 1, x = 4\, cm$

$y = 2 sin\left(\frac{2\pi}{16} \times 20\right) = 2\, sin\left(2\pi + \frac{\pi}{2}\right) = y = 2\, cm.$

10. Ans. (BC)

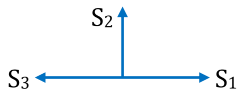
For S_1 & S_2

$$I = I_0 + I_0 + 2I_0\cos\theta_1$$

For S_2 & S_3,

$$I = I_0 + I_0 + 2I_0\cos\theta_2$$

Thus,



11. Ans. (BC)

$$f = \frac{5}{2L}\sqrt{\frac{T}{\mu}}$$

(a) no. of nodes =6 (incorrect)

(b) $\frac{5\lambda}{2} = \ell$

$$\frac{2\pi}{\lambda} = 62.8$$

$$\lambda = \frac{\cancel{2\times 3.14}\times 10}{\cancel{62.8}\times 100} = \frac{1}{10}$$

$$\ell = \frac{5}{2}\times\frac{1}{10} = \frac{1}{4} = 0.25m$$

(c) $2A = 0.01m$

(d) fundamental frequency $= \frac{1}{2\ell}\sqrt{\frac{T}{\mu}} = \frac{1}{2\ell}.V$

$$V = \frac{\omega}{K} = \frac{\cancel{628}\times 10}{\cancel{62.8}} = 10$$

$$f = \frac{1\times\cancel{100}\times 10}{2\times\cancel{25}} = 20Hz \text{ (incorrect)}$$

12. Ans. (C)

13. Ans. (A)

14. Ans. (B)

Solution for Question No. 12 to 14

$y_1 = A\sin[\omega t - kx]$

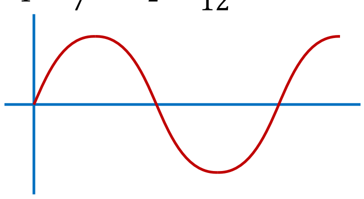
$y_1 = A\sin(\omega t + kx + \pi/2]$

$A = 2mm,\ \omega = 4\pi,\ k = 2\pi$

$$y_{Res.} = 2A\sin\left(\omega t + \frac{\pi}{6}\right)\cos\left(kx + \frac{\pi}{6}\right)$$

$$= 2A\cos\left(kx + \frac{\pi}{6}\right)\sin\left(\omega t + \frac{\pi}{6}\right)$$

$x_1 = \frac{1}{7},\quad x_2 = \frac{5}{12}$



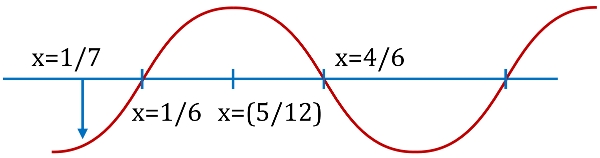
Nodes : $y = 0$

$\cos\left(kx+\frac{\pi}{6}\right)=0$

$kx+\frac{\pi}{6}=(2n+1)\frac{\pi}{2}$

$x_0=\frac{\pi}{3k}=\frac{\pi}{3\times 2\pi}=\frac{1}{6}$

$x_1=\frac{4\pi}{3k}=\frac{4k}{3\times 2\pi}=\frac{4}{6}$



$\Delta\phi=\pi$

Antinodes

$\cos\left(kx+\frac{\pi}{6}\right)=1$

$kx=\frac{\pi}{6}=n\pi$

$x=\left(n-\frac{1}{6}\right)\frac{\pi}{k}$

$x=\left(n-\frac{1}{6}\right)\frac{\pi}{k}=\frac{1}{2}\left(n-\frac{1}{6}\right)$

$x=\frac{5}{12},\frac{11}{12},\frac{17}{12},\frac{23}{12}\ldots..$

Location of max. potential energy, are nodes

$x=\frac{1}{6}$

15. **Ans. (A-Q,S), (B-P,R,T), (C-Q,T), (D-P,R,S)**

(A) $y=A\sin(\omega t-kx)$

$v_p=\frac{dy}{dt}=\omega A\cos(\omega t-kx)$

At $x=\frac{\lambda}{4}$ (point A) $v=(\omega A)\cos(\omega t-\pi/2)$

At $t=0$ $v_p=(\omega A)\cos(-kx)$

(B) $y=A\sin(\omega t-kx+\pi)$

$v_P=\frac{dy}{dt}=\omega A\cos(\omega t-kx+\pi)$

At (point A) $v=(\omega A)\cos(\omega+-\pi/2)=\omega A\sin\omega t$

At $x=\frac{\lambda}{4}$ $t=0$ $v=\omega A\cos(\pi-kx)$

$=-\omega A\cos kx$

(C) $y=A\sin(\omega t+kx)$

$v_p=\omega A\cos(\omega t+kx)$

At $x=\lambda/4$ $v=\omega A\cos(\pi/2-\omega t)$

$=\omega A\sin\omega t$

At $t=0$ $v=\omega A\cos k\pi$

(D) $y = A \sin(\omega t + kx + \pi)$

$v_P = \omega A \cos(\omega t + kx + \pi)$

At $x = \lambda/4$ $\quad v = \omega A \cos(3\pi/2 - \omega t)$

$= \omega A \sin\omega t$

At $t = 0$ $\quad v = \omega A \cos(\pi + k\pi)$

$-\omega A \cos kx$

16. Ans. (A-P,Q,S,T), (B-R), (C-Q,T), (D-R)

Velocity of wave = $v_\omega = +ve$

Velocity of Particle = $v_\omega = \frac{ds}{dx}$

$\Delta P = -B\frac{ds}{dx}$

Strain = $\frac{\partial s}{\partial x}$

A is a point of minimum compression so, density will be minimum

17. Ans. (A-P), (B-R,S), (C-Q,T), (D-P,R)

$\Delta P = -B\frac{ds}{dx}$

$V_p = -v_w\frac{ds}{dx} = \frac{v_\omega}{B} \quad \Delta P$

$\because$ $V\omega$ is $+ve$ & B is $+ve$

Therefore sign Vp & ΔP is same

(A) Velocity of those Points is towards right whose

$\Delta P \;\; is \;\; +ve \;\; i.e. \;\; Point'P'$

(B) Velocity of those Points is towards is towards Left whose

$\Delta P \;\; is \;\; -ve \;\; i.e. \;\; Point\ 'R'\&\ 'S'$

(C) Velocity of those Points is zero whose

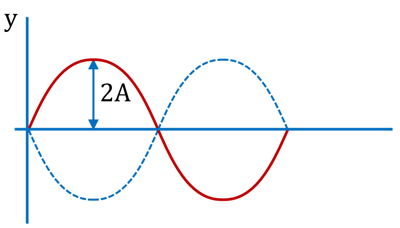
ΔP is zero i.e. points 'Q' & 'T'

(D) Speed of those points is maximum whose

$|\Delta P| \;\; is \;\; maximum$ $\quad$ i.e. Points 'P' & 'R'

18. Ans. (A-P,R,S,T), (B-S,T), (C-Q,S,T), (D-P,R,S,T)

$y = 2A \sin kx \cos \omega t$



(A) At $t = 0$

$y = 2A \sin kx$

Energy per unit length = $\frac{1}{2}n((2A)\omega)^2$ss

$= 2\mu A^2\omega^2$

Power transmission is zero at nodes and antinodes because energy is trapped between nodes and antinodes in standing waves

So, at $\quad x = \frac{\lambda}{4} \quad P_t = 0 \quad$ (Antinode)

$x = \lambda \quad P_t = 0 \quad$ (Node)

(B) At $t = \frac{T}{8}$

$y = 2A \sin kx \cos\left(\frac{\pi}{4}\right)$

$= \left(\sqrt{2}A\right) \sin k\,x$

$E = \frac{1}{2}\mu\left(\sqrt{2}A\omega\right)^2$

$= u\,A^2\omega^2$

At $x = \lambda$(Node), $\lambda/4$ (Antinode) Power transmitted = 0.

(C) At $t = T/4$

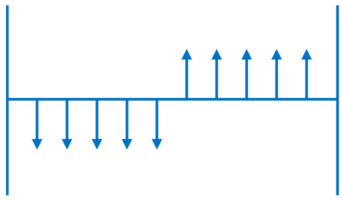
$y = 2A \sin kx\, cos(\pi/2)$

$y = 0$

$dk = \frac{1}{2}(udx)v_p^2$

$\frac{dk}{dx} = \frac{1}{2}u\left(\frac{dy}{dt}\right)^2 = \frac{1}{2}u(2A\omega)^2 (\sin kx \sin \omega t)^2$

$= 2\,uA^2\omega^2 (\sin^2 kx)\sin^2\omega t$



At $x = 0$ $\frac{dk}{dx} = 0$

At $x = 0$ $2\mu A^2\omega^2$

(D) Same as (A)