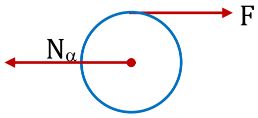
Rotational Motion

SOLUTIONS

EXERCISE - O

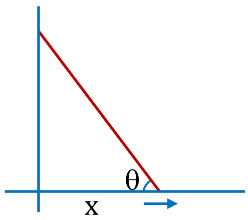
1. **Ans. (A)**



$F = N_\alpha$

2. **Ans. (A)**

$$\frac{x}{\cos\theta} = L$$



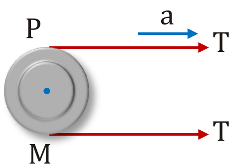
$\Rightarrow$ $v\cos\theta - x\sin\theta\,\omega = 0$

$v\cos\theta = x\sin\theta\,\omega$

$\therefore\ \omega = \frac{v}{x}\cot\theta$

$\therefore\ a = -\frac{v}{x}\operatorname{cosec}^2\theta\omega + \cot\theta\left(\frac{xa-v^2}{x^2}\right)$

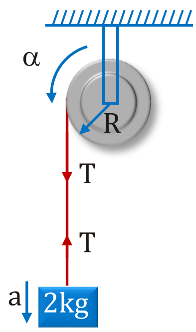
3. **Ans. (D)**



$2T = Ma \Rightarrow 2T = 2a \Rightarrow a = T$

Also for pulley : $T = 2\times\left(\frac{m\times m'}{m+m'}\right)g_{eff} = 2\times\frac{2}{3}\times(g-a)$ so $a = \frac{4}{3}(g-a) \Rightarrow a = \frac{4}{7}g$

4. **Ans. (C)**



For motion of block $2g - T = 2a$

For motion of pulley, $\tau = TR = I\alpha$

$$a = \alpha R \quad T = \frac{Ia}{R^2} \Rightarrow 2g - \frac{Ia}{R^2} = 2a \Rightarrow a = \frac{g}{1+\frac{I}{2R^2}}$$

$$\Rightarrow\ a = \frac{10}{1+\frac{0.32}{2\times0.2\times0.2}} = \frac{10}{1+4}\ \frac{10}{5} = 2\ ms^{-2}$$

5. **Ans. (A)**

$$MV_0a = \frac{2}{3}M(2a)^2\,\omega$$

$$\omega = \frac{3}{8}\frac{V_0}{a}$$

$$\frac{1}{2}I\omega^2 = Mg\frac{2a}{\sqrt{2}} - Mga = \sqrt{2}Mga - Mga$$

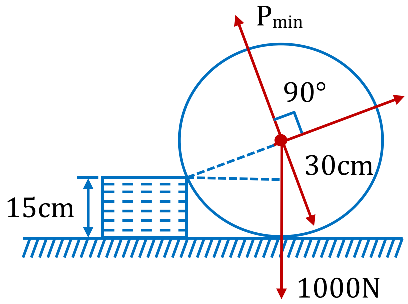
$$V_0 = \sqrt{\frac{16}{3}\left(\sqrt{2}-1\right)ga}$$

6. **Ans. (A)**

Moment of force = $F. \ell \sin\theta$

As θ increases $\sin\theta$ increases $\Rightarrow$ Moment of force increases.

7. **Ans. (A)**



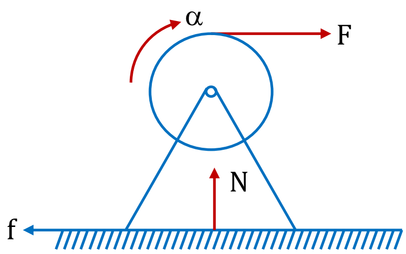
$P_{min} \times 30 = 1000 \times 15\sqrt{3}$

8. **Ans. (B)**

$N = (M + m)g$

$f_{max} = \mu(M + m)g$

Condition for sliding



$F > \mu\,(M + m)g$

$2t \geq \mu(M + m)g$

$$t \geq \frac{\mu(M+m)g}{2}$$

$$F(R) = \frac{mR^2\alpha}{2}$$

$$\alpha = \left(\frac{2F}{mR}\right)$$

$$\frac{d\omega}{dt} = \alpha = \left(\frac{4}{mR}\right)t$$

$$\int_0^{\omega} d\omega = \frac{4}{mR}\int_0^{\mu\left(\frac{M+m}{2}\right)g} t\,dt$$

$$\omega = \left(\frac{4}{mR}\right)\left(\frac{1}{2}\right)\left(\frac{\mu(M+m)g}{2}\right)^2 = \frac{[\mu(M+m)g]^2}{2mR}$$

$$KE = \frac{1}{2}I\omega^2 = \frac{1}{2}\frac{mR^2}{2}\frac{[\mu(M+m)g]^4}{4m^2R^2}$$

$$KE = \frac{1}{m}\left[\frac{\mu(M+m)g}{2}\right]^4$$

9. **Ans. (A)**

For small m

$$mg - T = ma$$

and $T \times R = \frac{MR^2}{2} \times \frac{a}{R}$ (No slipping)

$$mg - \frac{Ma}{2} = ma$$

$$a = \frac{2mg}{2m+M}$$

$\therefore$ Velocity at any time $v = \frac{2mgt}{(2m+M)}$

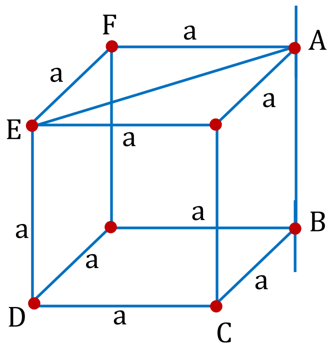
$\therefore$ $\omega = \frac{v}{R} = \frac{2mgt}{2m+M}$

10. **Ans. (D)**

$$\tau = F \cdot \frac{R}{2} + 2F \cdot R - F \cdot R$$

$$= \frac{3FR}{2}$$

11. **Ans. (C)**



$$I = \sum m_i r_i^2 = m_A r_A^2 + m_B r_B^2 + m_C r_c^2 + m_D r_D^2 + m_E r_E^2 + m_F r_F^2 + m_G r_G^2 + m_H r_H^2$$

$$= m\left[0 + 0 + a^2 + (\sqrt{2}a)^2 + (\sqrt{2}a)^2 + (a)^2 + a^2 + a^2\right]$$

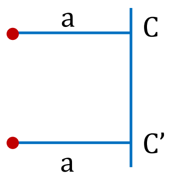
$$= m[8a^2] = 8ma^2$$

12. **Ans. (D)**

$$\frac{I_{BB}}{I_{AA}} = \frac{m_A \ell^2}{m_B \ell^2} = 3$$

$m_A = 3m_b$

13. **Ans. (C)**



$I_{CC'} = 2m\,a^2$

14. Ans. (C)

$I = 4\,mH^2 + 4m\,(H^2 + R^2)$

15. Ans. (D)

$I_C = \text{min}$

$\Rightarrow\ 4x - 12 = 0$

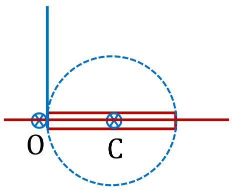
$\therefore\ \ x = 3$

16. Ans. (B)

By parallel axis theorem

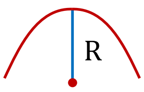
$$I_0 = I_C + m\left(\frac{\ell}{2}\right)^2$$

At points at distance of $\frac{\ell}{2}$ from C



17. Ans. (A)

$$I_1 = \frac{ML^2}{12}$$



$$MR^2 = \frac{ML^2}{\pi^2} = \frac{ML^2}{10} = I_2$$

18. Ans. (D)

$$\frac{I_Y}{I_X} = \frac{\frac{1}{2}m_yR_y^2}{\frac{1}{2}m_xR_x^2} = \frac{4^4}{4} = 4^3 = 64$$

19. Ans. (D)

$$\frac{3}{2}mR^2\ ;\quad 2\pi R = L$$

$$= \frac{3}{8}\frac{\rho L^3}{\pi^2}$$

20. Ans. (C)

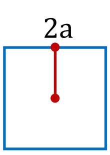
$I_A < I_B$, Q mass of A is closer is axis.

21. Ans. (B)

$$A = \frac{MR^2}{2}\ ;\ B = \frac{MR^2}{6}\ ;\ C = \frac{2}{5}MR^2$$

22. Ans. (D)

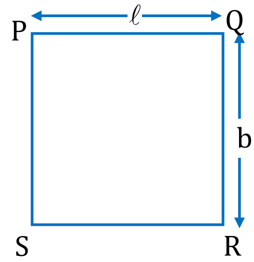
(a) $\frac{ma^2}{2}$; (b) ma^2 ; (c) $\frac{m(4a^2)}{6}$; (d)



$$4\times\left(\frac{m}{4}\frac{4a^2}{12} + \frac{ma^2}{4}\right) = \frac{ma^2}{3} + ma^2$$

23. Ans. (C)

Moment of Inertia PQRS about axis passing through centre ⊥ to plane



$I_{CM} = \frac{m}{12}(b^2 + \ell^2)$

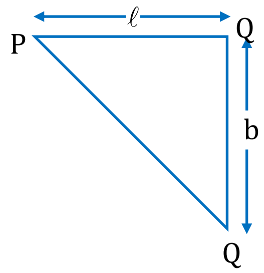
MI of $PQRS$ about axis passing through P, ⊥ to plane

$$I_{CMP} = \frac{m}{12}\left(b^2 + \ell^2\right) + m\left[\frac{\sqrt{\ell^2+b^2}}{2}\right]^2$$

$$= \frac{m}{3}\left(b^2 + \ell^2\right) = I$$

MI of inertia of ΔPQR about axis ⊥ to the plane passing through centroid of ΔPQR is

$$I_{CMT} = \left(b^2 + \ell^2\right)\frac{m'}{6} \text{ (here } m' = \frac{m}{2}\text{)}$$



If $p(-\ell,0), Q(0,0), R(0,b)$ then coordinates of centroids are $\left(-\frac{\ell}{3}, \frac{b}{3}\right)$

Distance of centroid from point P is d then

$$d = \sqrt{\left(-\ell + \frac{\ell}{3}\right)^2 + \left(0 - \frac{b}{3}\right)^2} = \sqrt{\frac{4\ell^2+b^2}{9}}$$

MI of PQR about axis passing through P ⊥to plane

$$= I_{CMT_P} = I_{CMT} + m'd^2$$

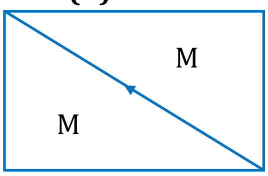
$$= \frac{m}{12}\left(b^2 + \ell^2\right) + \frac{m}{18}\left(4\ell^2 + b^2\right)$$

$$= \frac{m}{2}\left(\frac{11\ell^2+5b^2}{18}\right)$$

Since $\ell > b$

$$I_{CMT_P} = \frac{m}{2}\frac{11\ell^2+5b^2}{18} > \frac{m}{2}\frac{6a^2+6b^2}{18} > \frac{I}{2}$$

24. Ans. (B)



$$2I = \frac{2Ma^2}{6}$$

25. Ans. (A)

$$4I = \frac{4MR^2}{2}\ ;\ I = \frac{MR^2}{2}$$

26. Ans. (B)

Mass should be nearer to axis of rotation.

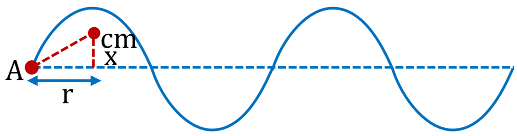
27. Ans. (D)

For the semicircular lamina of mass m, the moment of inertia about an axis through C is $I_C = \frac{1}{2}mr^2$

Let I_{CM} = moment of inertia about an axis through its centre of mass.

$$I_C = I_{CM} + md^2 = \frac{1}{2}mr^2 - m\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$$

28. Ans. (C)



$x = \dfrac{2r}{\pi}$, $m = \dfrac{2M}{4} = \dfrac{M}{2}$

Moment of inertia due to first semicircle

$I_1 = I_{cm} + mx^2$ But $I_1 = mr^2$

$I_{A1} = I_{cm} + m\,(x^2 + r^2) = mr^2 - mx^2 + m(x^2 + v^2) = 2mr^2$

Similarly

$I_{A2} = mr^2 + m(3r)^2 = 10mr^2$

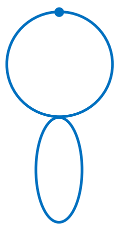
$I_{A3} = mr^2 + m(5r)^2 = 26mr^2$

$I_{A4} = mr^2 + m(7r)^2 = 50\,mr^2$

$I_A = 2mr^2 + 10mr^2 + 26mr^2 + 50mr^2 = 88\,mr^2 = 44Mr^2$

29. Ans. (D)

$I_{net} = 2mR^2 + \dfrac{MR^2}{2} + M(3R)^2$

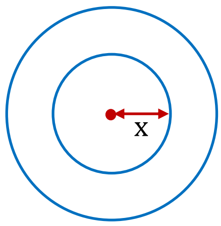


$I_{net} = \dfrac{5}{2}mR^2 + 9mR^2$

$I_{net} = \dfrac{23}{2}mR^2 = 11.5\,mR^2$

30. Ans. (C)

$= \dfrac{mR^2}{2} - \dfrac{\left(\dfrac{mx^2}{R^2}\right)\times\left(x^2\right)}{2}$



$= \dfrac{m}{2}\left[R^2 - \dfrac{x^4}{R^2}\right]$

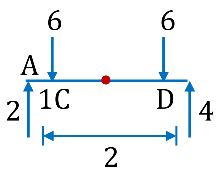
$= \dfrac{m}{2}\left[\dfrac{R^4 - x^4}{R^2}\right]$

$m' \Rightarrow \left(m - \dfrac{mx^2}{R^2}\right) = m\left(\dfrac{R^2 - x^2}{R^2}\right)$

$\dfrac{m}{R^2}\left(R^2 - x^2\right)\times k^2 = \dfrac{m}{2}\left[\dfrac{R^4 - x^4}{R^2}\right]$

$k^2 = \dfrac{\left(R^2 + x^2\right)}{2}$

31. Ans. (D)



$\tau_C \neq 0$

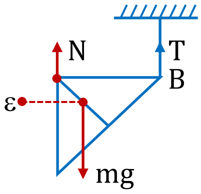
$\tau_D = 0$

32. Ans. (B)

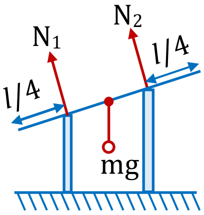
N only $\uparrow$ because no other Horizontal component

$\tau_B = 0 \Rightarrow N\ell = \dfrac{mg2\ell}{3}$

$\therefore \quad N = \dfrac{2mg}{3}$



33. Ans. (C)



$N_1 \dfrac{\ell}{4} = N_2 \dfrac{\ell}{4}$

$\therefore \quad N_1 = N_2$

34. Ans. (B)

Forces will be balanced if they are equal in magnitude and opposite in direction.

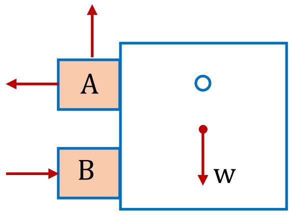
In equilibrium net force on body = 0

The net body in vertical direction = 0

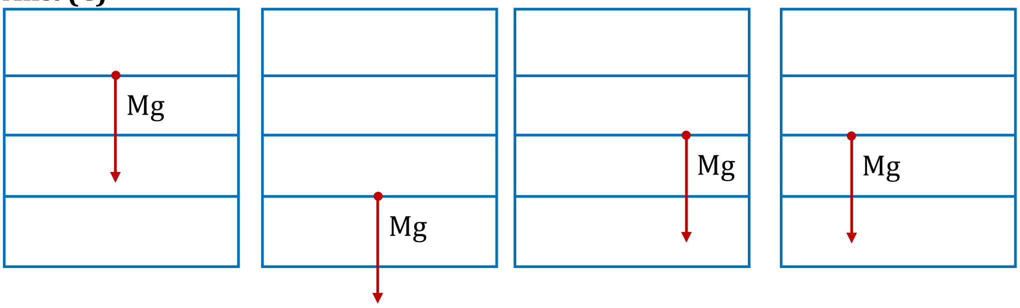
Since weight of the body is supported by the hinge A. So, A will be having equal force in upward direction.

So, there is a torque generated due to these two forces.

To oppose this torque normal force balances this and creates torque in opposite direction. So option B is correct.



35. Ans. (C)

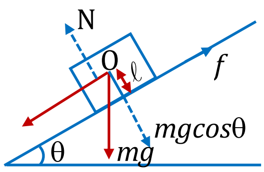


In C, mg will also help in toppling.

36. Ans. (A)

Since forces will balance out each other. There will be no pressure difference due to them.

Since friction is not acting on COM of brick, it will generate torque on the brick, to balance this torque normal force will shift towards left side and will generate torque in opposite direction. So due to this shift in point of action of normal force, forces in left part will be more, so left half will apply greater pressure on the plane.

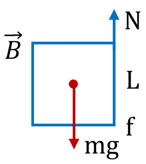


37. Ans. (C)

$FL = mg\dfrac{L}{2}$

$\therefore \quad F = \dfrac{mg}{2}$

$\tau = I\alpha$



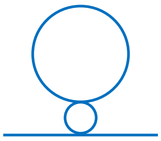
38. Ans. (C)

$\vec{\tau} = \vec{r} \times \vec{F}$

39. Ans. (C)

$\tau_{ICR} = FR = I_{ICR}\alpha = \frac{5}{7}\ mR^2\alpha$

$\Rightarrow\ a_n = \frac{5}{7}\frac{F}{m} = \frac{5}{7}\frac{at}{m}$



$F - f = ma \Rightarrow f = \frac{2at}{7}$

Slips $\Rightarrow 7\mu_s mg/2a$

Once it slips

$F - \mu mg = ma_n$

$at - \mu mg = ma_n$

$a_n = \frac{at}{m} - \mu g$

40. Ans. (C)

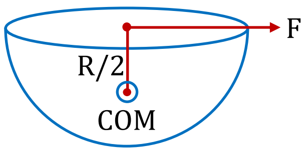
$Fr = I\alpha$

$F = \frac{1}{2}MR\alpha \quad = 3N$

41. Ans. (D)

Wrt COM of hemisphere

Torque = $I_{com}.\alpha$



$F \cdot \frac{R}{2} = I_{com}\alpha$

$F \cdot \frac{R}{2} = \left[\frac{2}{3}MR^2 - m\left(\frac{R}{2}\right)^2\right]\alpha$

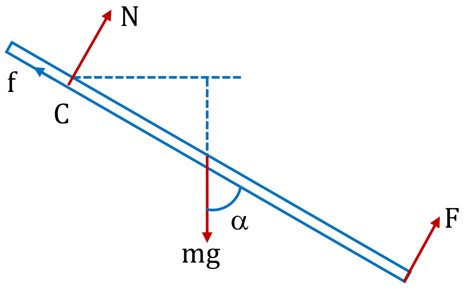
On solving

$\alpha = \frac{6F}{5MR}$

42. Ans. (A)

$(\vec{\tau}_{net})_C = 0$

$\frac{F3\ell}{4} - mg\frac{\ell}{4}\sin\alpha = 0$



$F = \frac{mg}{3}\sin\alpha$

$\mu N = f = mg\cos\alpha$

$N + F = mg\sin\alpha$

$\mu(mg\sin\alpha - F) = mg\cos\alpha$

$\mu = \frac{mg\cos\alpha}{mg\sin\alpha - F}$

$\mu = \frac{mg\cos\alpha}{mg\sin\alpha - \frac{mg\sin\alpha}{3}} = \frac{1}{\frac{2\tan\alpha}{3}}$

$\mu = \frac{9}{8}$

43. **Ans. (D)**

$AM = m\vec{r} \times \vec{v} = 2(+i + j) \times (2)(+i - j + k)$

$= -8\hat{k} - 4j + 4i$

$\therefore$ About z axis = –8

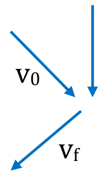
44. **Ans. (B)**

$L_i = mrv \sin(90 - \theta)$

$= mdv \cos\theta$

$L_f = -mdv \cos\theta$

$\therefore$ $\Delta L = 2\,mdv \cos\theta$



45. **Ans. (C)**

$I_1\omega_1^2 = I_2\omega_2^2$

$\therefore$ $\omega_1^2 = 4\omega_2^2$

$\therefore$ $\omega_1 = 2\omega_2$

$$\frac{L_{2R}}{L_R} = \frac{I_2\omega_2}{I_1\omega_1} = \frac{4}{1} \times \frac{1}{2} = 2$$

46. **Ans. (D)**

Angular momentum of particle remains constant as it moves along the line *BC.*

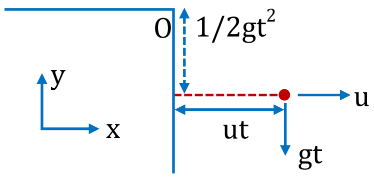
47. **Ans. (B)**

$\vec{L} = \vec{r} \times m\vec{u}$

$$= m\left[(v_0 \cos\theta\, t)\hat{\imath} + \left(v_0 \sin\theta\, t - \frac{1}{2}gt^2\right)\hat{\jmath}\right] \times [v_0 \widehat{\cos\imath} + (v_0 \sin\theta - gt)\hat{\jmath}]$$

$$= m\left[\frac{t^2 \cos\theta}{2} v_0\right](-\hat{k})$$

48. **Ans. (A)**



$$L_0 = mu\frac{1}{2}gt^2 - m(gt)ut = \frac{1}{2}mugt^2 - mugt^2 = -\frac{1}{2}mugt^2\hat{k}$$

49. **Ans. (B)**

Since no external torque is acting on the system $\tau_{ext} = 0 \Rightarrow \frac{dL}{dt} = 0$ here L is angular momentum.

$L = I\omega$ = constant.

So, total angular momentum is constant.

Since system is not having any energy exchange. Total energy is also conserved.

Because Beads are going down, distance from axis of rotation increasing, there I is increasing. Since $I\omega$ is constant, and I us increasing ω has to decrease.

So, Total angular momentum and total energy is constant.

50. **Ans. (B)**

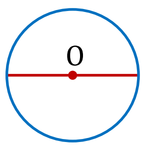
Angular momentum will be conserved about O

$I\omega$ = constant

$I\downarrow\uparrow$

$\therefore$ $\omega\uparrow\downarrow$

$\therefore$ $I = m|d - vt]^2 + I_{disk}$



51. Ans. (C)

Let ω is the angular speed of disk A after person jumps from it.

$$\frac{1}{2}MR^2\omega + m(u + R\omega)R$$

$$= \frac{1}{2}MR^2\omega_0 + m\omega_0 R^2$$

$$m(u + R\omega)R = \frac{1}{2}MR^2\omega_0$$

52. Ans. (A)

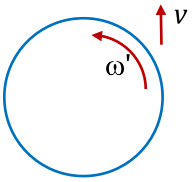
Moment of inertia decreases therefore angular velocity increases. KE also increases and therefore work done is positive.

53. Ans. (D)

$L_i = L_f$

$\Rightarrow \; m\omega R^2 + I\omega = mvR + I\omega'$

$$Q\ \omega' = \frac{(mR^2+I)\omega - mvR}{I}$$



54. Ans. (B)

Conservation of angular momentum $L_i = L_f \Rightarrow \omega_0 \times \frac{mR^2}{2} = \omega'\left[\frac{mR^2}{2} + \frac{m\ell^2}{3}\right] \Rightarrow \omega' = \frac{3\omega_0 R^2}{3R^2+2\ell^2}$

55. Ans. (D)

By conservation of angular momentum

$$\left(\frac{1}{2}MR^2 + mR^2\right)w_0 = \frac{1}{2}MR^2\omega$$

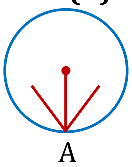
56. Ans. (C)

$I_1\omega_1 = I_2\omega_2$

$$\Rightarrow \omega_2 = \frac{I_1}{I_2}\,\omega_1 = \frac{mr^2/2}{mr^2/2 + \frac{m(2r)^2}{12}}\,\omega_1$$

$$\Rightarrow \omega_2 = \frac{1/2}{1/2+1/3} \times 2.4 = \frac{1/2}{5/6} \times 2.4 = 1.44 \text{ rev/sec.}$$

57. Ans. (A)



A is $ICOR$

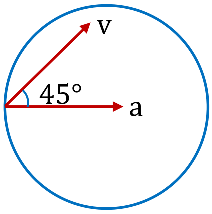
58. Ans. (A)

$a_1 = R\,\alpha - a^2$

$3 = a \times 3 - 6$

$\Rightarrow \; a = 3 \text{ rad}/s^2$

59. Ans. (B)



60. **Ans. (B)**

When the plank is pulled in right direction, the cylinder tents to move in the left direction and thus the frictional force between the cylinder & plank surface towards right direction along with the force on the plank.

Hence, Option B is correct.

61. **Ans. (A)**

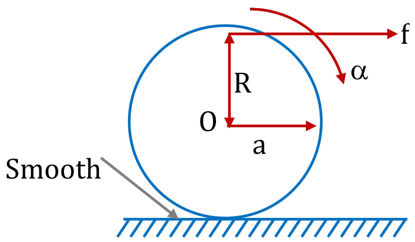
Rolling

$F = ma$ ...(1)

$\tau_0 = FR = \mathrm{d}\alpha$ ...(2)

for rolling

$a = r\alpha$ ...(3)



From (3) and (2)

$FR = I \cdot \frac{a}{R}$ but $F = ma$

$maR = I\frac{a}{R} \Rightarrow I = mR^2$

$\therefore$ Object is thin pipe.

62. **Ans. (C)**

Liquid has translation only,

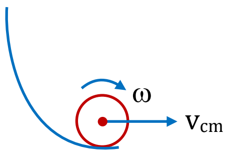
$$KE = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\frac{2}{3}mR^2\omega^2 + \frac{1}{2}mv^2 = \frac{4}{3}mv^2$$

$$AM = mvR + mvR + \frac{2}{3}mR^2\omega = \frac{8}{3}mvR$$

63. **Ans. (D)**

$$mg\ell = \frac{1}{2}mv_{cm}^2 + \frac{1}{2}\frac{2}{5}mR^2\omega^2 \Rightarrow \frac{7}{10}mv_{cm}^2$$

$$\therefore \; v_{cm} = \sqrt{\frac{10}{7}g\ell}$$



$$L_P = mv_{cm}\ell - \frac{2}{5}mR^2\omega$$

$$= mv_{cm}\ell - \frac{2}{5}mRv_{cm}$$

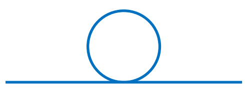
$$= mv_{cm}\left(\ell - \frac{2}{5}R\right)$$

$$L_P = M\sqrt{\frac{10}{7}gl}\left[l - \frac{2}{5}R\right]$$

64. **Ans. (C)**

L always conserved about O. Initial = 0

$\therefore$ Final = 0



65. **Ans. (D)**

$KE_{QO} = mg2h$

and $KQ_{PO} = mgh$

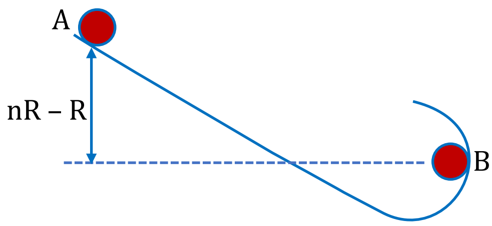
66. **Ans. (B)**

Using cons of energy from A to B

$$mg\,(nR - R) = \frac{1}{2}\cdot\frac{2}{5}MR^2.\ \frac{v^2}{R^2} + \frac{1}{2}mv^2$$

$$= \frac{Mv^2}{5} + \frac{Mv^2}{2} = \frac{7}{10}mv^2$$

$$v = \sqrt{\frac{10g(n-1)R}{7}}$$



67. **Ans. (A)**

AMC_1

$MV_0R = \text{i}\omega$ ...(1)

$$\omega = \frac{V}{R}$$

$MOI = MR^2 + MR^2 = 2MR^2$

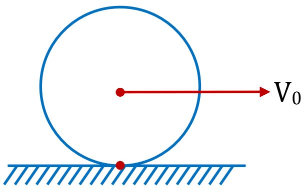
$I = 2MR^2$

$\therefore\ mV_0R = I\omega$

$$mV_0R = 2mR^2 \times \frac{V}{R}$$

$V_0 = 2V$

$$V = \frac{V_0}{2}$$



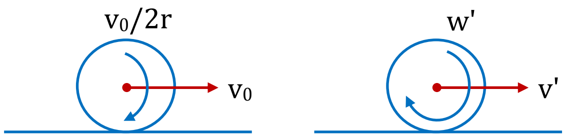
68. **Ans. (A)**

Angular momentum can be conserved about lowest point

$$mv_0r + \frac{2}{5}mr^2\frac{v_0}{2r} = mv'r + \frac{2}{5}mr^2\frac{v'}{r}$$

$$\frac{10mv_0r + 2Mv_0r}{10} = \frac{7mv'r}{5}$$

$$v' = \frac{6v_0}{7}$$



69. **Ans. (C)**

$L_i = L_f$

$$mvR + \frac{2}{3}mR^2\,.\frac{3v}{R} = m\,v_0R + \frac{2}{3}\,mR^2\,\frac{v_0}{R}$$

$$v + 2v = v_0 + \frac{2}{3}v_0$$

$$3v = \frac{5v_0}{3}$$

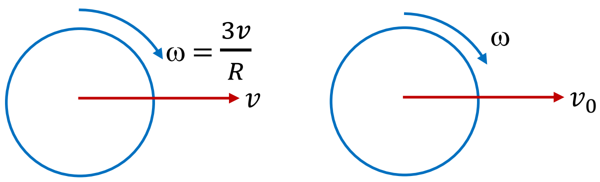
$$v_0 = \frac{9v}{5}$$

$$\frac{9v}{5} = v + \mu g \times t$$

$$\mu gt = \frac{9v}{5} - v$$

$$\mu gt = \frac{9v}{5}$$

$$t = \frac{9v}{5\mu g}$$



Velocity of point of contact is along positive x-axis so friction will be along negative x-axis.

70. Ans. (A)

$L_f = 0$

$\therefore \quad L_i = 0$

$$mv_0R - \frac{mR^2}{2}\omega_0 = 0$$

$$\therefore \quad \frac{v_0}{r\omega_0} = \frac{1}{2}$$

71. Ans. (C)

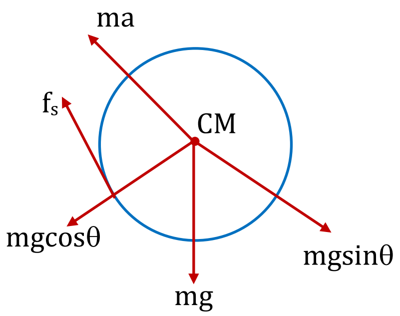
$mg\sin\theta - f_s = ma$

$f_sR = 3 = I\alpha$

$I = \frac{2}{5}mR^2$ and $a = R\alpha \quad \Rightarrow \quad f_s = \frac{2mg\sin\theta}{7}$

72. Ans. (D)

$f_s = mg\sin\theta - ma$



$$a = \frac{g\sin\theta}{1+\frac{K^2}{r^2}}$$

$$f_s = ma\left(1 + \frac{k^2}{r^2}\right) - ma$$

For solid uniform disc $k = \frac{r}{\sqrt{2}}$

$$f_s = ma\left(1 + \frac{1}{2} - 1\right)$$

$$f_s = \frac{1}{2}ma$$

73. Ans. (B)

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\frac{v^2}{R^2}$$

$$mgh = \frac{1}{2}m\frac{8gh}{7} + \frac{1}{2}I\frac{8gh}{7R^2}$$

$$m = \frac{8m}{14} + \frac{8I}{14R^2}$$

$$\frac{6m}{14} = \frac{8I}{14R^2}$$

$$I = \frac{3}{4}mR^2$$

74. Ans. (B)

$$mgl\sin\theta = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

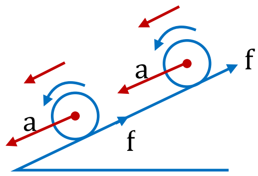
$$= \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\frac{v^2}{R^2} = \frac{3}{4}mv^2$$

$$\Rightarrow v = \sqrt{\frac{4gl\sin\theta}{3}}$$

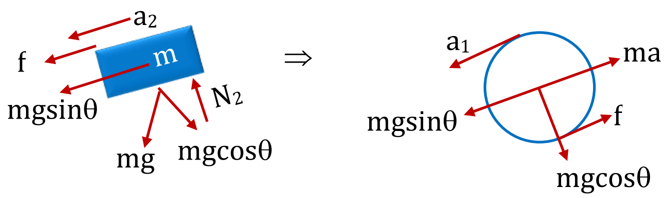
$$I = I_{cm}\omega + mvR = \frac{MR^2}{2}\frac{v}{R} + MvR = \frac{3MvR}{2}$$

$$= \frac{3}{2}mR\sqrt{\frac{4gl\sin\theta}{3}} = \sqrt{3m^2R^2gl\sin\theta}$$

75. Ans. (B)



76. Ans. (D)



$f = mg\sin\theta = ma_2$ ...(1) $\quad$ $mg\sin\theta - f = ma_1$ ...(2)

$f = \frac{2}{5}mr\alpha = \frac{2}{5}ma(a_1 - a_2)$ ...(5) $\quad$ $fR = \left(\frac{2}{5}mR^2\right)\alpha$ ...(3)

$a_1 - R\alpha = a_2$ ...(4)

For rolling

Now putting values of a_1 and a_2 from (1) and (2) into (5)

$$f = \frac{2}{5}m\left(g\sin\theta - \frac{f}{m} - \frac{f}{m} - g\sin\theta\right)$$

$$f = \frac{2}{5}m\left(\frac{2f}{m}\right) \Rightarrow f = -\frac{4f}{5} \Rightarrow \frac{9f}{5} = 0$$

$$f = 0$$

77. Ans. (C)

$$V_{cm} = \frac{I}{M} = \frac{Ft}{M} \text{ same}$$

78. **Ans. (B)**

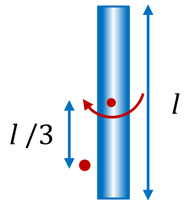
m_1 be mass of particle

$\frac{1}{2}I\omega^2 = \frac{1}{2}m_1v^2$

$L_i = L_f$

$I\omega = m_1 v \frac{\ell}{3}$

Get m_1



79. **Ans. (B)**

$$mv\frac{d}{2} = m\frac{\omega d}{2}\frac{d}{2} + \frac{6md^2}{12}\omega$$

$$\therefore \quad \omega = \frac{2v}{3d}$$

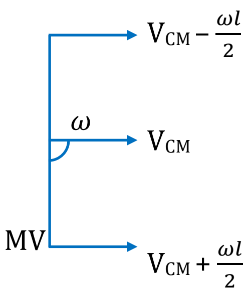
80. **Ans. (A)**

$L_i = L_f$

$$mv\frac{\ell}{2} = m\left(v_{cm} + \frac{\omega\ell}{2}\right)\frac{1}{2} + m\left(v_{cm} - \frac{\omega\ell}{2}\right)\frac{1}{2}$$

$$\therefore \quad \frac{v}{2} = \omega\ell\frac{1}{2}$$

$$\therefore \quad \omega = \frac{v}{\ell}$$



81. **Ans. (D)**

$$v_{cm} = \frac{J}{m}$$

$$J\frac{\ell}{2} = \frac{m\ell^2}{12}\omega$$

$$\therefore \quad \omega = \frac{6J}{m\ell}$$

82. **Ans. (B)**

$N_2 = f_1 = \mu N_1$

$N_1 + f_2 = mg$

$\frac{f_1}{\mu} + \mu N_2 = mg = 3f_1 + \frac{1}{3}f_1 = mg$

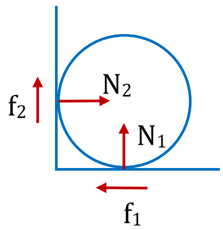
$\therefore \quad f_1 = \frac{3mg}{10}$

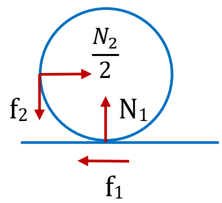
$N_2 = 0$

$\therefore \quad f_2 = 0$

$\therefore \quad f_1 = \mu N_1 = \mu mg = \frac{mg}{3}$

$\therefore \quad \frac{3}{10\times\frac{1}{3}} = \frac{9}{10}$

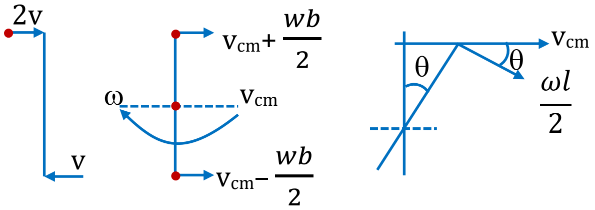




83. **Ans. (C)**

As the f_1 is acting to the right direction, so when f_1 is applied the centre of mass will move to the right.

84. Ans. (C)



$2\, mv_{cm} = m\, 2v - mv$

$\therefore \quad v_{cm} = \dfrac{v}{2}$

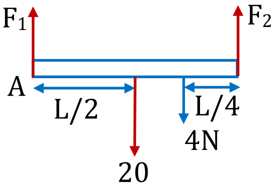
$$2vm\frac{b}{2} + mv\frac{b}{2} = m\left(v_{cm} + \frac{\omega b}{2}\right)\frac{b}{2} - m\left(v_{cm} - \frac{\omega b}{2}\right)$$

$\therefore \quad 3m\dfrac{vb}{2} = m\omega b\dfrac{b}{2}$

$\therefore \quad \omega = \dfrac{3v}{b}$

85. Ans. (AC)

Before hinge breaks

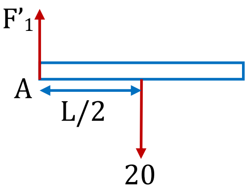


$F_1 + F_2 = 24$

$\tau_A = 0 \Rightarrow F_2 = 13N$

$\Rightarrow \quad F_1 = 11N$

After breaking hinge $\Rightarrow$

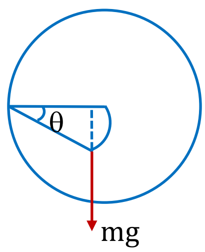


$$\tau_A \Rightarrow 20\frac{L}{2} = 2\times\frac{L^2}{3}.\alpha \Rightarrow \alpha = \frac{15}{L}$$

$$20 - F'_1 = 2 \times \ \alpha.\frac{L}{2}$$

86. Ans. (AC)

$mgR\cos\theta = 2\, mR^2\alpha$



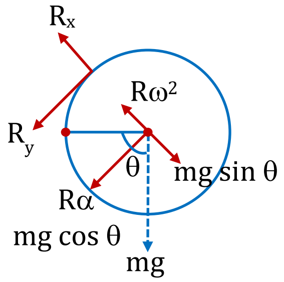
$\alpha = \dfrac{g\cos\theta}{2R}$

$mg\, R\sin\theta = \dfrac{1}{2}\times 2mR^2\omega^2$

$g\sin\theta = R\omega^2$

$\omega^2 = \dfrac{g\sin\theta}{R}$

$$mg\cos\theta + R_y = mR\left(\frac{g\cos\theta}{2R}\right) = \frac{mg\cos\theta}{2}$$



$$R_y = \frac{-mg\cos\theta}{2}$$

$$R_x - mg\sin\theta = mR \times \frac{g\sin\theta}{R}$$

$$R_x = 2mg\sin\theta$$

$$R_y^2 + R_x^2 = mg\sqrt{\frac{\cos^2\theta}{4} + 4\sin^2\theta}$$

$$= \frac{mg}{2}\sqrt{\cos^2\theta + 16\sin^2\theta}$$

$$= \frac{mg}{2}\sqrt{1 + 15\sin^2\theta}$$

87. **Ans. (ABC)**

$$2mg\left(\frac{\ell}{4}\right) = \frac{1}{2}\left[m\left(\frac{\ell}{4}\right)^2 + m\left(\frac{3\ell}{4}\right)^2\right]\omega^2$$

$$\Rightarrow\ \omega^2 = \frac{8}{5}\frac{g}{\ell}$$

$$F_{\text{net on B}} = m\omega^2\left(\frac{3\ell}{4}\right)$$

$$= m \times \frac{8}{5}\frac{g}{\ell} \times \left(\frac{3\ell}{4}\right)$$

$$F_{net} = \frac{6}{5}mg$$

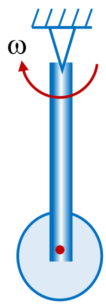
In the given position $\alpha = 0$ since $\Sigma\tau_{cm} = 0$

88. **Ans. (AC)**

$$mg\frac{\ell}{2} + mg\ell = \frac{1}{2}\left(\frac{m\ell^2}{3} + m\ell^2\right)\omega^2$$

$$\frac{3}{2}mg\ell = \frac{2ml^2}{3}\omega^2$$

$$\omega = \frac{3}{2}\sqrt{\frac{g}{\ell}}$$



Angular acceleration of disc at horizontal initial position is zero and it is zero at all time since it can't rotate it is performing translational motion only.

89. **Ans. (ABCD)**

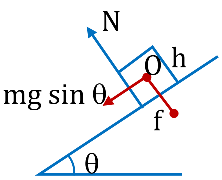
$I_1 = I_2 = I_3 = I_4$ and $I = I_1 + I_3$

90. **Ans. (AD)**

If $f\dfrac{h}{2} = N\dfrac{a}{2}$

$\therefore\ \ \mu Nh = Na$

$\therefore\ \ \mu_{min} = \dfrac{a}{h}$

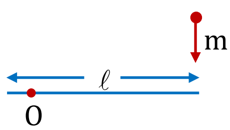


91. **Ans. (ACD)**

$L = mvl$

As $v\uparrow\ \therefore\ L\uparrow \Rightarrow \omega\uparrow$

τ same



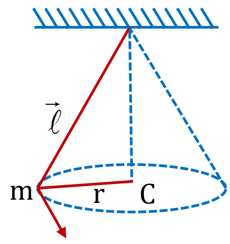
92. Ans. (BCD)

(A) Angular momentum about $O = \vec{\ell} \times \vec{v} = \vec{L}_0$, Since direction of angular momentum is changing at every point, $\vec{L}_0$ is not constant.

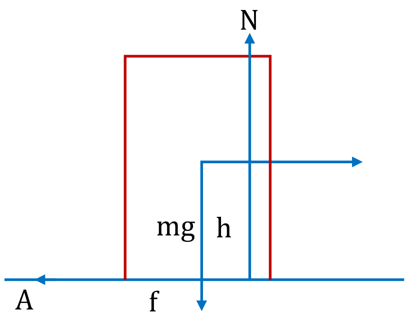
(B) Since direction of $\vec{r}$ and $\vec{v}$ both are changing, but, direction of $\vec{L}_C$ is always upwards and magnitude of $\vec{r}$ and $\vec{v}$ is always constant. $\vec{L}_C = \vec{r} \times \vec{v}$. So, angular momentum about C is constant.

(C) Since magnitude of $\vec{L}_0$ is not changing, because $|\vec{\ell}|$ and $|\vec{v}|$ both are constant. So vertical component of angular momentum about O is constant.

(D) As we have already discussed above, magnitude of $\vec{L}_0$ is constant.



93. Ans. (ABD)



AM not conserved

$\therefore$ Torque not 0

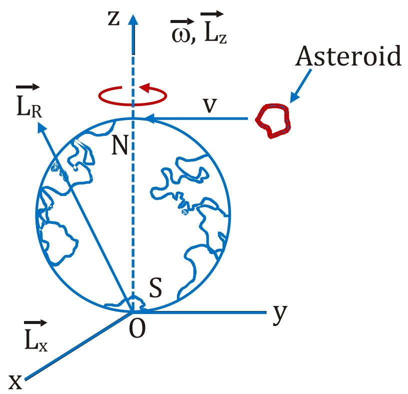
mg and *N* form Force couple

94. Ans. (BD)

$I\omega$ = constant

$\omega \propto \dfrac{1}{T}$

95. Ans. (BD)



Asteroid provides angular momentum about *x*-axis. Hence resultant angular Momentum.

$$\vec{L}_R = \vec{L}_x + \vec{L}_z$$

$\vec{L}_R$ is in *x-z* plane and due to gravitational force of real earth, model rotates about *z*-axis.

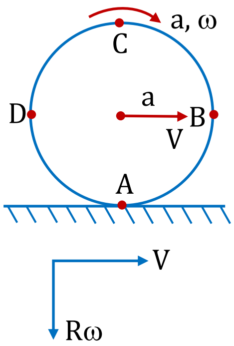
Since there in no external torque about *z*-axis

$L_z = I_z\,\omega_z \rightarrow I_z$ not same

96. Ans. (ABCD)

(A) For point B

$\Rightarrow \quad V_B = \sqrt{V^2 + (R\omega)^2}$

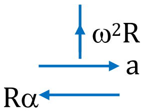


(B) For point C

$\rightarrow \quad V$

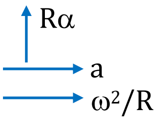
$\rightarrow \quad R\omega \Rightarrow V_C = V + R\omega$

(C) For point A



$a_A = \sqrt{(a - R\alpha)^2 + (\omega^2 R)^2}$

(D)



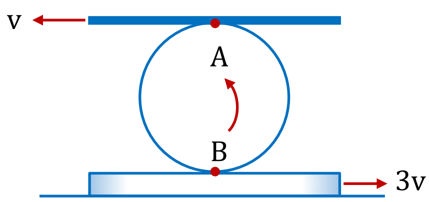
$\Rightarrow \quad a_D = \sqrt{(a + R\omega^2)^2 + (R\alpha)^2}$

97. Ans. (AC)

$v_A = \omega_0 R - v_0 = v$

$\omega_0 R - v_0 = v$ ...(i)

$v_B = \omega_0 R + v_0 = 3v$ ...(ii)



From equation (i) and (ii)

$2\omega_0 R = 4V$

$\Rightarrow \quad \omega_0 R = 2v$

$\omega_0 = \dfrac{2v}{R}$

From (1) $v_0 = v$

98. Ans. (BD)

In case of liquid motion is translational

$\Rightarrow \quad I = m\ell^2$

In case of solid motion is CRTM

$\Rightarrow \quad I = \frac{2}{5}mR^2 + m\ell^2$

99. Ans. (ACD)

$KE = \dfrac{1}{2}mV^2_e + \dfrac{1}{2}I c\omega^2$

$w = \dfrac{V}{4a}$ (For rod and disc since rigid body)

100. Ans. (BD)

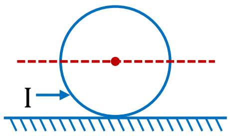
τ about point of contact on the ground = 0 $\Rightarrow \vec{L}_{\text{about point of contact}}$ is conserved

Initially slipping occurs, thus friction acts in forward (right) direction.

101. Ans. (AD)

$I = mv_0$

$I \times \frac{2R}{3} = \frac{2}{5} mR^2 \omega$



$\omega = \frac{5I}{3mR}$

$= \vec{v} = \left(\frac{I}{m} + \frac{5I}{3m}\right)\hat{\imath}$

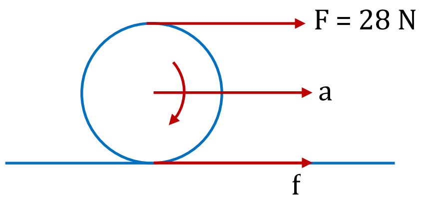
102. Ans. (AD)

$F + f = 10a$ ...(1)

$(F - f)\, R = \frac{2}{5} mR^2 \alpha$ ...(2)

$a = R\alpha$ ...(3)

$F - f = \frac{2}{5} \times 10\, a = 4a$ ...(4)



From eq. (1) and (4)

$F + f = 10a$ ...(1)

$F - f = 4a$ ...(4)

$2f = 14a$

$F = 7a$

$a = \frac{F}{7} = \frac{28}{7} = 4\, m/s^2$

$\therefore \quad f = 10a - 7a = 3a = 12$

Maximum friction available = 10

$\therefore \quad 28 \times R = \frac{2}{5}\, mR^2 \alpha$

$R \propto = 7$

$\therefore \quad a = \frac{28+10}{10} = \frac{38}{10} = 3.8\, m/s^2$ and the direction of friction is forward.

103. Ans. (AC)

Depends on friction.

104. Ans. (ABC)

Total work done by friction is 0.

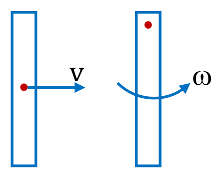
$\therefore$ It does not depend on frame of plank or sphere.

105. Ans. (ABC)

$Mv\frac{\ell}{2} = \frac{m\ell^2}{3}\omega$

$\omega = \frac{3v}{2\ell}$

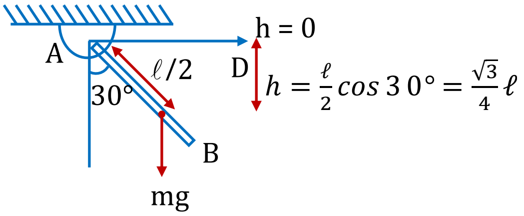
If rod has to complete the circle,



$\frac{1}{2} I\omega_{min}^2 = Mg\ell \Rightarrow \omega_{min} = \sqrt{\frac{2Mg\ell \times 3}{M\ell^2}}$

$\Delta KE = \frac{1}{2} Mv_0^2 - \frac{1}{2} \cdot \frac{M\ell^2}{3} \cdot \omega^2$

106. Ans. (A)



Initially the rod was at position *AD*. Let us assume *AD* as reference position, position of COM $h = 0$

At position *AB*, position of COM is $h = -\frac{\sqrt{3}}{4}\ell$

From energy conservation

$$0 = \frac{1}{2}I\omega^2 + mg\left(-\frac{\sqrt{3}}{4}\ell\right)$$

$$\Rightarrow \frac{\sqrt{3}}{2}mg\ell = \frac{1}{2}\left(\frac{2m\ell^2}{3}\omega^2\right)$$

$$\Rightarrow \omega = \left[\left(\frac{3\sqrt{3}}{2\ell}\right)g\right]^{\frac{1}{2}}$$

107. Ans. (B)

Let the force applied by the hinge on the rod along the rod is *N* then

$$N - mg\cos\theta = m\omega^2\frac{L}{2}$$

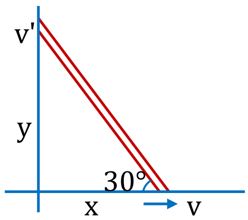
$$\Rightarrow N = m\left(\frac{3\sqrt{3}}{2\ell}g\right)\frac{\ell}{2} + mg\left(\frac{\sqrt{3}}{2}\right)$$

$$\Rightarrow N = mg\left[\frac{5\sqrt{3}}{4}\right]$$

108. Ans. (C)

$v'\cos 60° = v\cos 30°$

$\therefore\ v' = v\sqrt{3}$



$COM = \frac{x}{2}, \frac{v}{2}$

$\therefore\ v_{cm} = \left(\frac{v}{2}, \frac{v'}{2}\right) = \frac{v}{2}, \frac{\sqrt{3}}{2}$

$\therefore\ v_{cm} = \frac{v_2}{2} = v$

109. Ans. (A)

$-mg1 = \frac{1}{2}\left(\frac{7}{5}mv^2 - \frac{7}{5}m\frac{200}{7}\right)$

$\therefore\ -20 = \frac{7}{5}v^2 - 40$

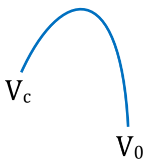
$\therefore\ \sqrt{\frac{100}{7}} = v$

110. Ans. (C)

Only v change and ω remains same

$$K_{\text{Rot}} = \frac{1}{2}\frac{2}{5}mR^2\omega^2 = \frac{1}{5}mv_c^2 = \frac{1}{5}m\frac{100}{7}$$

$$mg1 = \frac{1}{2}mv_D^2 + \frac{1}{2}\frac{2}{5}mR^2\omega^2 - \frac{1}{2}mv_c^2 - \frac{1}{2}\frac{2}{5}mR^2\omega^2$$



$$\therefore\ 20 = v_D^2 - \frac{100}{7}$$

$$\therefore\ K_T = \frac{1}{2}m\frac{240}{7} = \frac{120}{7}m$$

$$\therefore\ \text{ratio} = \frac{1}{6}$$

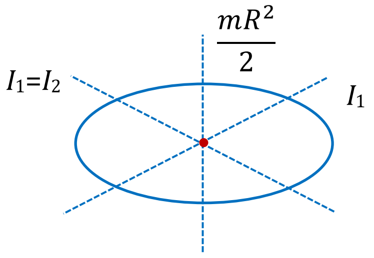
111. Ans. (B)

$$I_x = I_y = I_z = \frac{m\ell^2}{6}$$

$$\therefore\ \frac{3m\ell^2}{6} = 2\Sigma m_i r_i^2$$

$$\Rightarrow\ \text{Ans. is } \frac{m\ell^2}{4}$$

112. Ans. (C)



$$I_1 + I_2 + \frac{mR^2}{2} = 2mR^2$$

$$\Rightarrow\ I_1 = \frac{3}{4}mR^2$$

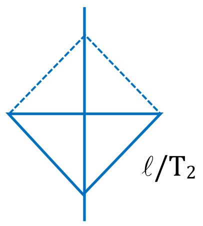
113. Ans. (C)

114. Ans. (A)

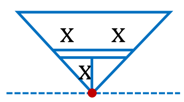
115. Ans. (C)

116. Ans. (C)

Solution for Question No. 113 to 116



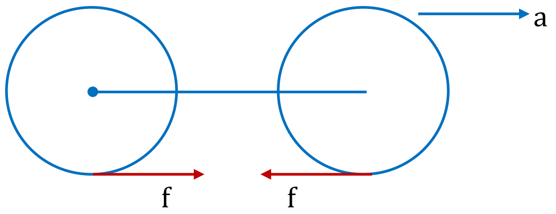
I about y axis $= \dfrac{m\ell^2}{24}$



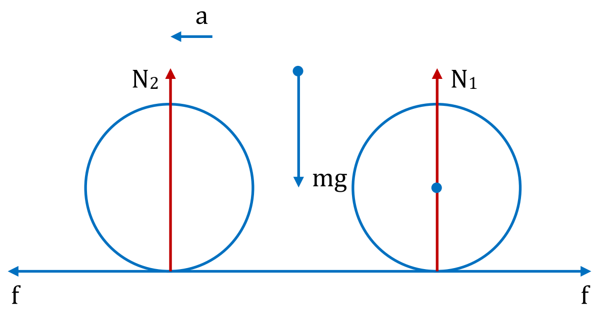
$$\int_0^{\frac{\ell}{2}} \frac{M}{\frac{\ell^2}{4}} 2x\,dx\ x^2 = \frac{m\ell^2}{8}$$

$$\therefore\ I_z = I_x + I_y = \left(\frac{1}{8} + \frac{1}{24}\right)m\ell^2 = \frac{1}{6}m\ell^2$$

117. Ans. (D)



118. Ans. (A)



$f_2 - f_1 = ma$ ...(1)

$N_1 - N_2 = mg$ ...(2)

$(N_2 - N_1)\dfrac{w}{2} + (f_2 - f_1)h = 0$ ...(3)

$$N_2 = \frac{mah}{w} + \frac{mg}{2}$$

$$N_1 = \frac{mg}{2} - \frac{mah}{w}$$

$$\mu (N_2 - N_1) = ma$$

$$\mu\frac{2mah}{w} = ma$$

$$\boxed{\mu = \frac{w}{2h}}$$

119. Ans. (B)

$$N_1 = \frac{mg}{2} - \frac{mah}{w}$$

$$0 = \frac{mg}{2} - \frac{mah}{w}$$

$$\boxed{a = \frac{gw}{2h}}$$

120. Ans. (A)

$\alpha = \frac{a}{R}$ $\qquad$ $S = \frac{1}{2}at^2$

$6.28 = \frac{1}{2} \times a \times (2)^2$

$a = 3.14$

$\alpha \Rightarrow \frac{3.14}{1.5}$

$\omega_f \Rightarrow \frac{3.14}{1.5} \times 1.5$

$\omega_f \Rightarrow \pi$

121. Ans. (B)

$S \Rightarrow ut$

$6.28 = u \times 2$

$\omega_0 = \frac{u}{r} \Rightarrow \frac{3.14}{1.5}$

$\omega_0 \Rightarrow \frac{3.14}{1.5} \Rightarrow \frac{2\pi}{3}$

122. Ans. (A)

$L = I\omega_1 + I\omega_2$

$= \frac{4\times(0.6)^2}{3}\left(\frac{2\pi}{0.7}+\frac{2\pi}{0.7}\right) = \frac{1.44\pi}{0.7} = \frac{14.4\pi}{7}$

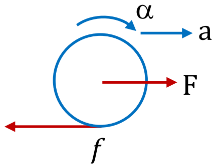
123. Ans. (D)

$L + \Delta L = 0$

$\frac{14.4\pi}{7} + 18\,\Delta\omega = 0$

$\Delta\omega = \frac{14.4\pi}{7\times180} = \frac{4\pi}{35}$ rad/s

124. Ans. (A)



125. Ans. (A)

Force is acting below the point of percussion

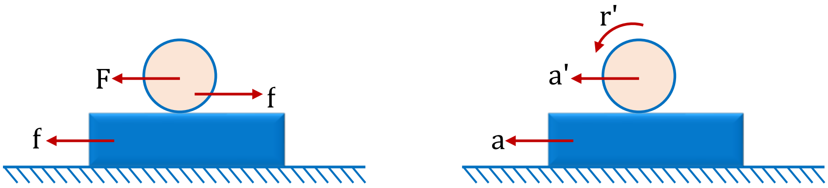
$\therefore$ Friction is backwards.

126. Ans. (D)

It is not clear if force is below or above the point of percussion.

$\therefore$ Friction direction can not be determined.

127. Ans. (C)



$F - f = ma'$; $a' - a'R = a$

$fR = \frac{mR^2}{2}\alpha'$

$f = ma$

$\therefore$ $R\alpha' = 2a$ $\quad\therefore$ $a' - 2a = a$ $\quad\therefore$ $a' = 3a$

$\therefore$ $F = m(a' + a) = 4ma$ $\quad\therefore$ $a = \frac{F}{4m}$

128. Ans. (B)

$f = \frac{F}{4}$

129. Ans. (C)

By conservation of linear momentum $mv_0 = 4mv_c \Rightarrow v_c = \frac{v_0}{4}$

130. Ans. (A)

By conservation of angular momentum $mv_0\left(\frac{L}{2}\right) = \frac{(4m)L^2}{12}\omega \Rightarrow \omega = \frac{3v_0}{2L}$

131. Ans. (D)

Time taken by rod in completing one revolution $= \frac{2\pi}{\omega} = \frac{4\pi L}{3v_0}$

Distance travelled by centre $=(v_c)\left(\frac{4\pi L}{3v_0}\right) = \left(\frac{v_0}{4}\right)\left(\frac{4\pi L}{3v_0}\right) = \frac{\pi L}{3}$

132. Ans. (ABC)

Theoretical

133. Ans. (C)

Theoretical

134. Ans. (BCD)

$mv_0r = mvr + mr^2\omega = 2mvr$

$\therefore$ $v = \frac{v_0}{2}$

and $\frac{v_0}{2} = v_0 - \mu gt$

$\therefore$ $t = \frac{\mu_0}{2\mu g}$

135. Ans. (ACD)

(A) $V_0t - \frac{1}{2}\mu gt^2 = \frac{v_0^2}{2\mu g} - \frac{1}{2}\mu g\frac{v_0^2}{4\mu g}$

(B) $\Delta KE = \frac{1}{2}mv^2 + \frac{1}{2}mR^2\omega^2 - \frac{1}{2}mV_0^2$

(D) $\frac{1}{2}mR^2\omega^2 = \frac{1}{2}m\frac{v_0^2}{4} = \frac{1}{8}mv_0^2$

136. Ans. (A-R), (B-P), (C-T), (D-Q,S)

C is $ICOR$ and speed of point at a distance r from C is $\omega r = \frac{v_{CM} r}{R}$.

137. Ans. (A-ST), (B-R), (C-PRT), (D-PQ)

For (A) : Hinge will apply external impulse hence momentum will not be conserved.
So (P) is not correct.
Mechanical energy reduce hence (S).
ω decreases by conservation of angular momentum hence (T).
For (B) **:** Hinge will apply external impulse hence momentum will not be conserved.
So (P) is not correct.
Mechanical energy will increase hence (R).
ω does not change by conservation of angular momentum hence (No 'T').
For (C) : No external horizontal impulse so momentum is conserved (P).
Mechanical energy will increase hence (R).
V will change so (T).
For (D) : There is no external impulse and no change in momentum as velocity of joker w.r.t. car is still same so P Not 'T '.
Mechanical energy remains same immediately after joker drops as he has no vertical velocity so (Q).

EXERCISE - S

1. Ans. $\frac{L}{3}$

$mg\frac{L}{2} = \frac{1}{3} mL^2\alpha$

$\therefore \quad \alpha = \frac{3g}{2L}$

and $\alpha x = g$

$\Rightarrow \quad \frac{3g}{2L} x = g$

$\therefore \quad x = \frac{2L}{3}$

$\therefore \quad$ Distance from $B = \frac{L}{3}$

2. Ans. (a)$\downarrow \frac{9g}{7}$ (b)$\uparrow \frac{4mg}{7}$

About hinge $mg\frac{L}{4} = I\alpha$

$I = \frac{ML^2}{12} + \frac{ML^2}{16} = \frac{7ML^2}{48}$

$\therefore \quad \frac{MgL}{4} = \frac{7ML^2\alpha}{48}$

$\therefore \quad \alpha = \frac{12g}{7L}$

Now $\alpha\frac{3L}{4} = \frac{12g}{7L}\frac{3L}{4} = \frac{9g}{7}\downarrow$

$a_{cm} = \frac{\alpha L}{4} = \frac{3g}{7}$

3. **Ans.** (a) $\omega = \sqrt{\frac{6g}{\ell}}$, (b) $T = \frac{11mg}{4}$

$$-mg\ell = -\frac{1}{2}\frac{m\ell^2\omega^2}{3}$$

$$\omega = \sqrt{\frac{6g}{\ell}}$$

$$T - \frac{mg}{2} = \frac{m}{2}\omega^2 \times \frac{3\ell}{4}$$

$$T = \frac{mg}{2} + \frac{3m\omega^2\ell}{8}$$

$$= \frac{mg}{2} + \frac{3m\ell}{8} \times \frac{6g}{\ell}$$

$$= \frac{2mg+9mg}{4}$$

4. **Ans.** 2

$$I_A = 3m\left(\frac{2}{3}\frac{\sqrt{3}}{2}\ell\right)^2 = m\ell^2$$

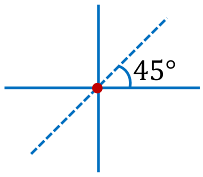
$$I_B = 2m\ell^2$$

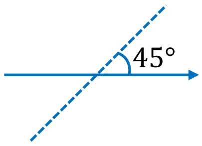
$\therefore$ Ratio = 2 : 1

5. **Ans.** $\frac{ml^2}{12}$

$$I_1 = \frac{M\ell^2}{12}\cos^2 45°$$

$$= \frac{M\ell^2}{12} \times \frac{1}{2}$$





$$I_1 = I_2$$

$$\therefore \quad I_1 + I_2 = \frac{M\ell^2}{12}$$

6. **Ans.** $\frac{a\ell^4}{36}$

$$\int dmx^2 = \int_0^\ell \lambda dx\, x^2 = \frac{a\ell^4}{4}$$

$$\text{Now } CM = d = \frac{\int_r^\ell \lambda dxx}{\int_0^\ell \lambda dx} = \frac{2\ell}{3}$$

$$\therefore \quad I_{CM} = I_0 - md^2 = \frac{a\ell^4}{4} - md^2$$

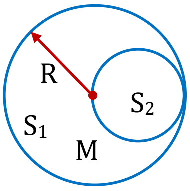
$$m = \int \lambda dx = \frac{a\ell^2}{2}$$

$$\therefore \quad \frac{a\ell^4}{4} - \left(\frac{a\ell^2}{2}\right)\frac{4}{9}\ell^2 = \frac{a\ell^4}{36}$$

7. **Ans.** $57/140MR^2$

$S_1 + S_2 = S$

$I_{S_1} + I_{S_2} = I_S$

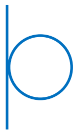


$M_2 = \rho\frac{4}{3}\pi\frac{R^3}{8} = \frac{M}{7}$

where $\rho = \dfrac{M}{\frac{4}{3}\pi R^3 - \frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$

$\rho = \dfrac{6M}{7\pi R^3}$

Now,



$I_{S_2} = \frac{2}{5}\frac{M}{7}\left(\frac{R}{2}\right)^2 + \frac{M}{7}\left(\frac{R}{2}\right)^2$

$= \frac{7}{5}\frac{M}{7}\frac{R^2}{4} = \frac{MR^2}{20}$

$I_S = \frac{2}{5}\left(M + \frac{M}{7}\right)R^2 = \frac{16M}{35}R^2$

$\therefore \quad I_{S_1} = I_S - I_{S_2} = \frac{16}{35}MR^2 - \frac{MR^2}{20}$

$\frac{(64-7)MR^2}{140} = \frac{57}{140}MR^2$

8. **Ans.** 8 kg

$16\ell_1 = m\ell_2$

$m\ell_1 = 4\ell_2$

$\frac{16}{m} = \frac{m}{4}$

$\therefore \quad m = 8\ kg$

9. **Ans.** $150(+\hat{i})$

$-d(i) \times 100(j) + (-aj) \times F(i) = 0$

$-100\ d\ \hat{k} + aF\ \hat{k} = 0$

$\therefore \quad F = \frac{100d}{a} = \frac{300}{2} = 150$

$\therefore \quad \vec{F} = 150\hat{i}$

10. **Ans.** 30°

$f_r = 75 \sin\theta$

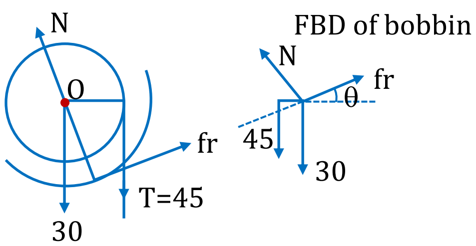
τ about O = 0

$\therefore \quad T \times 5 = fr \times 6$

$45 \times 5 = 75 \times 6 \times \sin\theta$

$\Rightarrow \quad \theta = 30°$

$\tau = I\alpha$



11. **Ans.** (a) 2.00 $N{\cdot}m$, (b) $\hat{k}$

$\vec{F} = 2\hat{\imath} + 3\hat{\jmath}$; $\vec{r} = 4i + 5\hat{\jmath}$

$$\vec{\tau} = \vec{r} \times \vec{F} = \begin{vmatrix} i & j & \hat{k} \\ 4 & 5 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \hat{k}(12 - 10) = 2\hat{k}$$

12. **Ans.** 10^{-3} rad/s^2 clockwise

$\tau = I\alpha$

$\tau = (-9 \times 30 - 10 \times 30 + 12 \times 5) \times 10^{-2}$

$= (-570 + 60) \times 10^{-2} = -510 \times 10^{-2}$

$\therefore \quad |\alpha| = \dfrac{510\times10^{-2}}{5100} = 10^{-3}\text{rad}/s^2$

13. **Ans.** 4

$f \times R + f \times R - 100 = 0$ ...(i)

$f = \mu N$ ...(ii)

N = Force applied by brakes

14. **Ans.** $\dfrac{\omega}{3}$

$I_1\omega_1 = I_2\omega_2$

$$\left(\frac{mR^2}{2} + 0\right)\omega = \left(\frac{mR^2}{2} + mR^2\right)\omega_2$$

$\therefore \quad \omega_2 = \dfrac{\omega}{3}$

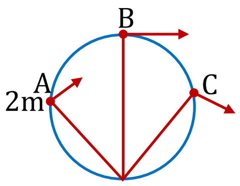
15. **Ans.** 6 mv_0^2

$\omega = \dfrac{v_0}{R}$

$\therefore \quad KE_{ring} = \dfrac{1}{2}I\omega^2 + \dfrac{1}{2}MV_{CM}^2$

$= \dfrac{1}{2}MR^2\omega^2 + \dfrac{1}{2}mv_0^2 = mv_0^2$

$= \dfrac{1}{2}2m\omega^2\left(\sqrt{2}R\right)^2 = m2v_0^2 \to KE_A$



$KE_{sys} = KE_{ring} + KE_A + KE_B + KE_C$

$= \dfrac{1}{2}m\omega^2(2R)^2 = 2mv_0^2 \to KE_B$

$= \dfrac{1}{2}m\omega\left(\sqrt{2}R\right) = mv_0^2 \to KE_C$

$KE_{sys} = 6mv_0^2$

16. **Ans.** (a) 6.68 m/s (b) 2.27 m

(a) $mg\,(1.5) = \frac{1}{2}mv^2 + \frac{1}{2} \times \frac{2}{5}m\omega^2 r^2$

$\therefore\ v_A^2 = \frac{10}{7} \times 10 \times 1.5$

Now $mg\,(1.2) = \frac{1}{2}mv_c^2 + \frac{1}{2}mr^2\omega^2 - \left(\frac{1}{2}mv_A^2 + \frac{1}{2}mr^2\omega^2\right)$

$\therefore\ 24 = v_c^2 - v_A^2$

$\Rightarrow\ v_c^2 = \left(24 + \frac{150}{7}\right)(.98)$

$V_C = (6.67)\ m/s$

(b) $\sqrt{\frac{2b}{g}}v_A = \sqrt{\frac{2 \times 1.2}{9.8}} \times \sqrt{\frac{10 \times 1.5 \times 9.8}{7}} = 2.27m$

17. **Ans.** (a) $\theta_C = \cos^{-1}(4/7)$, (b) $v = \sqrt{\frac{4}{7}gR}$, (c) $\frac{K_T}{K_R} = 6$

$mg\cos\theta_C = \frac{mv^2}{R}$

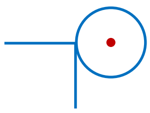
$mg\,R(1 - \cos\theta_C) = \frac{1}{2}mV_C^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$

$mg\,R(1 - \cos\theta) = \frac{3}{4}mV_1^2$

$\therefore\ \frac{1 - \cos\theta}{\cos\theta} = \frac{3}{4}$

$\therefore\ \frac{4r}{7} = \cos\theta_C$

(b) $V_c = \sqrt{\frac{4}{7}gR}\ \therefore\ \omega_C = \sqrt{\frac{4}{7}\frac{g}{R}}$

(c)  $mgR = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2\omega^2}{2}$

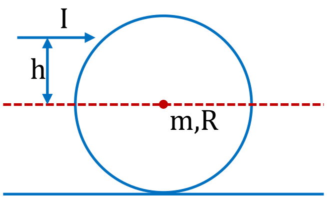
18. **Ans.** (a) $t = \frac{\pi Rm}{2I}$; (b) $s = \frac{\pi R}{2}$

(a) $I\frac{4R}{5} = \frac{2}{5}MR^2\omega$

$\frac{2I}{mR} = \omega$

$\omega t = \pi$

$t = \frac{\pi}{\omega} = \frac{\pi MR}{2I}$



(b) $v_{CM} = \frac{I}{m}$

$t \times v_{cm} = \frac{\pi R}{2}$

19. **Ans.** (a) $4\sqrt{\frac{3}{7}}\,m/s$, (b) $\frac{200}{7}N$

$T_1 r = \frac{mr^2}{2}\alpha_1$

$T_2 r - T_1 r = \frac{mr^2}{2}\alpha_2 = \frac{ma_2}{2}$

$\alpha_1 r = a_2 + \alpha_2 r = 2a_2$

$mg - T_1 - T_2 = ma_2$

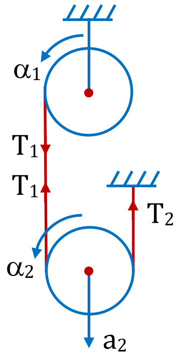
$a_2 = \alpha_2 r$

$\therefore\ T_1 = ma$

$T_2 - T_1 = \frac{ma}{2}$

$mg - T_1 - T_2 = ma$

$\therefore\ mg\ T_1 = \frac{5}{2}ma$



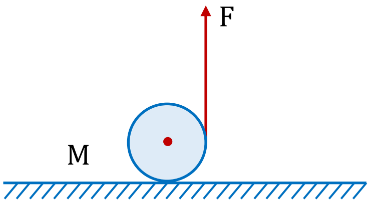
20. **Ans.** $\alpha_{max} = \frac{2g}{3r}$

$Fr - fr = \frac{mr^2}{2}\alpha = \frac{ma}{2}$

$f = ma$

$\therefore\ F = \frac{3}{2}ma$ $\qquad \therefore\ \frac{2F}{3mr} = \frac{a}{r}$

Now $F_{max} = mg$ $\qquad \therefore\ \frac{2g}{3r} = \alpha_{max}$



21. **Ans.** (i) $\frac{2V_0}{3}$, (ii) $t = \frac{v_0}{3\mu g}$, $W = \frac{1}{2}[3\mu^2 mg^2t^2 - 2\mu mg t v_0]$ $(t < t_0)$, $W = -\frac{1}{6}mv_0^2$ $(t > t_0)$

$mv_0R = mv'R + \frac{mR^2}{2}\omega = \frac{3}{2}mv'$

$\therefore\ v' = \frac{2v_0}{3}$ $\qquad \therefore\ \frac{v_0 - v'}{\mu g} = t = \frac{v_0}{3\mu g}$

$a = \mu g$

$v = v_0 - \mu g t$

$\mu_0 mgR = \frac{mR^2}{2}\alpha$

$\therefore\ \alpha = \frac{2\mu_0 g}{R}$ $\qquad \therefore\ \omega = \frac{2\mu_0 g}{R}t$

$KE_f - KE_i = W_{fr}$

$\frac{1}{2}m(v_0 - \mu g t)^2 + \frac{1}{2}\frac{mR^2}{2}w^2$

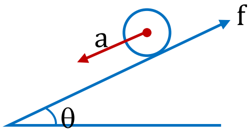
$= \frac{1}{2}m(v_0^2 + \mu^2 g^2 t^2 - 2\mu v_0 g t) + \frac{1}{2}\frac{mR^2}{2}\frac{4\mu_0^2 g^2 t^2}{R^2}$

After $t = t_0$, w_{fr} = constant $= -\frac{1}{6}mv_0^2$

22. **Ans.** 1

$$\frac{dL}{dt} = \frac{dm}{dt}(v_1 - v_2)R = 200 \times (5-2.5) \times 2 = 1000\,J$$

23. **Ans.** 1/2 *ma*



$a = \alpha r$

$$fr = \frac{1}{2}mr^2\alpha = \frac{1}{2}ma$$

24. **Ans.** 5

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$$

$$mgh = \frac{3}{4}mv^2$$

$$2\,mgh = \frac{1}{2}mv_f^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$$

$$2\,mgh = \frac{1}{2}mv_f^2 + mgh - \frac{1}{2}mv^2$$

$$mgh = \frac{1}{2}mv_f{}^2 - \frac{1}{2}mv^2$$

$$\frac{1}{2}mv_f{}^2 = \frac{5}{3}mgh$$

$$\therefore \quad \text{Ratio} = \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}\frac{mv^2}{2}} = \frac{\frac{5}{3}mgh}{\frac{1}{4}\times\frac{4}{3}mgh} = 5$$

25. **Ans.** 15

For a body

$mg\sin\theta - f = ma$ and $fr = I\alpha$, $a = \alpha r$

$$mg\sin\theta - \frac{I\alpha}{r} = ma$$

$$mg\sin\theta = \frac{Ia}{r^2} + ma$$

$$a = \frac{mg\sin\theta}{\left(\frac{1}{r^2}+m\right)} \Rightarrow a_{sc} = \frac{2}{3}\,g\sin\theta$$

and $a_{bc} = \frac{1}{2}g\sin\theta$

Relative acceleration $= \frac{2}{3}\,g\sin\theta - \frac{1}{2}\,g\sin\theta = \frac{1}{6}\,g\sin\theta = \frac{g}{12}$

Gap after time $t = 6$

$$t = 6,\ \frac{1}{2}\times\frac{g}{12}\times(6)^2 = \frac{1}{2}\times\frac{10}{12}\times 36 = 15m$$

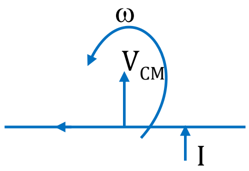
26. **Ans.** $\frac{l^2}{12x}$

$V_{CM} = \frac{I}{m}$

$I_x = \frac{m\ell^2}{12}\omega$

$v_{cm} - wy = 0$

$\Rightarrow \; y = \frac{v_{cm}}{\omega}$



27. **Ans.** (i) $\frac{J}{m}$ (ii) zero (iii) $\frac{J}{2m}$ (iv) $\frac{5}{2}\frac{J}{m}$

$J\frac{\ell}{4} = \frac{m\ell^2}{12}\omega$

$\Rightarrow \; \omega = \frac{3J}{m\ell}, v_{cm} = \frac{J}{m}$

$v_A = v_{cm} - \frac{\ell}{3}\omega = 0$

$v_{\text{upper}} = \left|v_{cm} - \frac{\ell}{2}\omega\right| = \frac{J}{2m}$

$v_{\text{lower}} = \left|v_{cm} + \frac{\ell}{2}\omega\right| = \frac{5}{2}\frac{J}{m}$

28. **Ans.** $\frac{mv}{4}$

Angular momentum conservation about O.

$\therefore \; mV\frac{\ell}{2} = 0 + \frac{m\ell^2}{3}\omega$

$\therefore \; \omega\ell = V\frac{3}{2}$

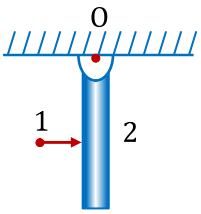
Now V_{CM} of rod $= \frac{\omega\ell}{2} = \frac{3V}{4}$

$\therefore \; I_1$ on (1) $= mv$

$I_1 - N_{imp} = m\frac{3v}{4}$

$\therefore \; mv - N_{Im} = \frac{3mv}{4}$

$\therefore \; N_{Im} = \frac{mv}{4}$



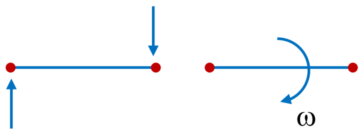
29. **Ans.** $\frac{2mv_0{}^2}{l}$

(V_{cm} of rod = 0 as $P_i = P_f$)

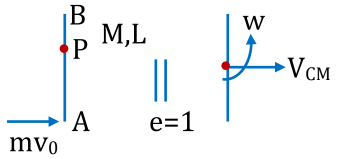
$L_i = L_f$

$\therefore \; \omega = \frac{2v_0}{\ell}$

$T = m\omega^2\frac{\ell}{2} = \frac{m4v_0^2}{\ell^2} \times \frac{\ell}{2} = \frac{2mv_0^2}{\ell}$



30. **Ans.** (a) $\frac{m}{M}=\frac{1}{4}$; (b) $x=\frac{2L}{3}$; (c) $\frac{v_0}{2\sqrt{2}}$



(a) $e=\frac{v'_2-v'_1}{v_1-v_2}=\frac{\frac{\omega L}{2}+v_{cm}}{v_0}$

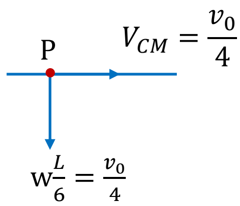
$v_{CM}=\frac{mv_0}{M}$ and $mv_0\frac{L}{2}=\frac{m_1L^2}{12}\omega$

$\omega=\frac{6mv_0}{ML}$

$\frac{\frac{3mv_0}{M}+\frac{mv_0}{M}}{v_0}=1$

$\Rightarrow\frac{m}{M}=\frac{1}{4}$

(b) $v_{cm}-\omega x=0$



$\therefore x=\frac{L}{6}$

$\therefore \frac{L}{2}+\frac{L}{6}=\frac{2L}{3}$

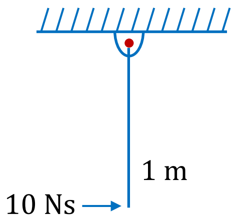
(c) $\omega\ell=\frac{3}{2}v_0$

$\omega t=\frac{\omega\pi\ell}{3v_0}$

$\frac{\pi\ell}{3v_0}\times\frac{3v_0}{2\ell}=\frac{\pi}{2}$

$\therefore$ After $\frac{\pi}{2}$

31. **Ans.** 75 J



$J=10\times1=I\omega-0$

$I=\frac{1}{3}m\ell^2$

$\therefore\ \omega=15$

$\therefore\ KE=\frac{1}{2}I\omega^2=\frac{1}{2}\times2\times\frac{1^2}{3}\times15^2=75J$

32. **Ans.** 48

Let V be the linear velocity of centre of rod which is struck by a horizontal force

For translational motion, $P_0=MV_{cm}$

For rotational motion, $P_0x=I_R\,\omega$

Angular velocity of rod $\omega=\frac{P_0x}{\frac{ML^2}{3}}=\frac{3P_0x}{ML^2}$

Initial speed of centre of mass of the rod $\frac{\omega L}{2}=\frac{3P_0x}{2ML}$

Impulse delivered by the pivot is $I=mV_{cm}-m\,\omega\,\frac{L}{2}$

Impulse is zero when $V_{cm}=\omega\frac{L}{2}$; $V_{cm}=\frac{P_0}{M}=\frac{3P_0x}{2ML}\Rightarrow x=\frac{2L}{3}$

33. **Ans.** 8

$$mv_0\frac{\ell}{4}=I_{cm}\omega$$

$$mv_0\frac{\ell}{4}=\frac{5}{24}m\ell^2\omega$$

$$\omega=\frac{6}{5}\frac{v_0}{\ell} \qquad ...(i)$$

$mv_0 = 2mv_c$

$$v_c=\frac{v_0}{2} \qquad ...(ii)$$

$v_P = v_C - \omega x = 0$

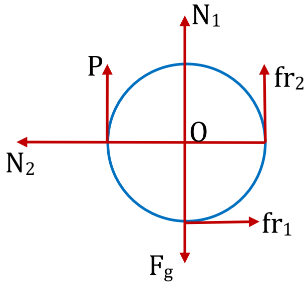
$$x=\frac{v_c}{\omega}=\frac{5\ell}{12}$$

Distance of P from A is $x_0 = x + \frac{\ell}{4}$

$$x_0=\frac{8\ell}{18}=80cm$$

$$\frac{x_0}{10}=8$$

34. **Ans.** (3/8) F_g



$N_1 + P + fr_2 = F_g$ ...(1)

$N_2 = fr_1 = \mu N_1$

$fr_1 = \mu N_1$ and $fr_2 = \mu N_2$

τ about $O = 0$

$-Pr + fr_1r + fr_2r = 0$

$\therefore\ P = fr_1 + fr_2 = \mu\,(N_1 + N_2)$ ...(2)

From (1) $N_1 + P + \mu N_2 = F_g$ ...(3)

$\therefore\ N_1\,(1 + \mu^2) = F_g - P$

$$\therefore\ N_1 = \frac{F_g-P}{(1+\mu^2)}$$

Putting in $P = \mu(N_1 + \mu N_1)$

$= \mu N_1(1 + \mu)$

$$P = \mu\frac{(F_g-P)(1+\mu)}{1+\mu^2}$$

$$\therefore\ P = \frac{\frac{1}{2}\left(1+\frac{1}{2}\right)}{1+\frac{1}{4}}\left(F_g - P\right)$$

$\Rightarrow\ 5P = 3F_g - 3P$

$$\therefore\ P = \frac{3F_g}{8}$$

35. **Ans.** (a) $\sqrt{3}\, ml\omega^2$, (b) $(F_{net})_x = -\frac{F}{4}$, $(F_{net})_y = \sqrt{3}\, ml\omega^2$

(a) $3mw^2\left(\frac{\sqrt{3}}{2}\ell\frac{2}{3}\right) = \sqrt{3}\, mw^2\ell$

(b) $F_y = \sqrt{3}\, m\ell\omega^2$

But for F_x

$$F\frac{\sqrt{3}}{2}\ell = 2\, m\ell^2\alpha$$

And $F - F_x = \frac{F}{4m}3m$

36. **Ans.** (a) $a_c = \frac{4F}{3m_1+8m_2}$, $a_p = 2a_c$

(b) Friction at the top of the cylinder = $3m_1F(3\, m_1 + 8\, m_2)$ towards right;

friction at the bottom = $m_1F/(3\, m_1 + 8\, m_2)$ towards right.

$F - f_1 = M_2(a + \alpha r) = m_2(2a)$

$f_1 - f_2 = m_1 a$

$(f_1 + f_2)\, r = \frac{m_1 r^2}{2}\alpha = \frac{m_1 a}{2}$

$f_1 = \frac{3}{4}m_1 a$

$\therefore\ F = 2m_2a + \frac{3}{4}m_1 a$

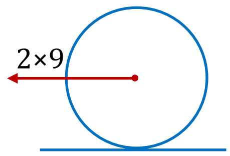
$\therefore\ a = \frac{4F}{8m_2+3m_1}$

37. **Ans.** $6N$, $-0.6\hat{j} \pm 0.6\hat{k}$

(a) $18 - f = 2a$

$f \times 0.1 = \frac{2(.1)^2}{2}\alpha$

$\therefore f = 6N$



(b) $\vec{r} = \left(-0.1\hat{j} - 0.1\hat{k}\right)$; $f = 6\hat{\imath}$

$\therefore -0.6(\hat{j}\times\hat{\imath}) - 0.6\left(\hat{k}\times\hat{j}\right) = 0.6\hat{k} - 0.6\hat{j}$

38. **Ans.** (a) 0.1 m, (b) 1 rad/s, (c) laminar plate will never come to rest.

For Laminar sheet

$I_{AB} = I_{CM} + my^2 = 1.2 + 30y^2$

Also impulse = ΔP

$\therefore\quad 6 = m\left(V_f - V_i\right) \Rightarrow 6 = 30\,(w_{fy} - w_i y)$

$\Rightarrow\quad 6 = 30\, y\left(w_f - w_i\right) \Rightarrow 1 = 5y\,(w_f - w_i)$ ...(1)

Also angular impulse = change in angular momentum

$\Rightarrow$ $6 \times 0.5 = I_{AB}\left(w_f - w_i\right)$ [About AB]

$\Rightarrow$ $w_f = \dfrac{3}{I_{AB}} - 1$ [Given $w_i = 1$ rad/s]

$\Rightarrow$ $w_f = \dfrac{3}{1.2+30_{y}z} - 1$ ...(2)

From (1) and (2)

$l = 5y\left[\dfrac{3}{1.2+30_{y}z}\right] \Rightarrow 30y^2 - 15y + 1.2 = 0$

$\therefore$ $y = 0.1$ or 0.4

$\therefore$ $w_f = 1$ rad/s for $y = 0.1$

$= -0.5$ rad/s for $y = 0.4$ (Not possible as laminar plate turns back)

$\therefore$ location of COM = 0.1 m

$w_f = 1$ rad/$s \Rightarrow$ Laminar plate will never stop.

Ans. (a) 0.1 m (b) 1 rad/s (c) laminar plate will never come to rest.

EXERCISE - JEE (Main) PYQ

1. **Ans. (4)**

$mg - T = ma$

$T \times R = mR^2 \times \dfrac{a}{R}$

$\Rightarrow$ $a = \dfrac{g}{2}$

2. **Ans. (3)**

$\vec{L} = \vec{r} \times \vec{P}$ or $\vec{L} = rp\sin\theta\,\hat{n}$

or $\vec{L} = r_{\perp}(P)\hat{n}$

For D to A

$$\vec{L} = \frac{R}{\sqrt{2}}mv(-\hat{k})$$

For A to B

$$\vec{L} = \frac{R}{\sqrt{2}}mv(-\hat{k})$$

For C to D

$$\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right)mv(\hat{k})$$

For B to C

$$\vec{L} = \left(\frac{R}{\sqrt{2}} + a\right)mv(\hat{k})$$

3. Ans. (3)

$$I=\frac{m\ell^2}{12}+\frac{mR^2}{4}$$

or $$I=\frac{m}{4}\left(\frac{\ell^2}{3}+R^2\right) \quad ...(1)$$

Also, $m = \pi R^2 \ell \rho$

$$\Rightarrow \quad R^2=\frac{m}{\pi\ell\rho}$$

Put in equation (1)

$$I=\frac{m}{4}\left(\frac{\ell^2}{3}+\frac{m}{\pi\ell\rho}\right)$$

For maxima and minima

$$\frac{dI}{d\ell}=\frac{m}{4}\left(\frac{2\ell}{3}-\frac{m}{\pi\ell^2\rho}\right)=0$$

$$\Rightarrow \quad \frac{2\ell}{3}=\frac{m}{\pi\ell^2\rho} \Rightarrow \frac{2\ell}{3}=\frac{\pi R^2\ell\rho}{\pi\ell^2\rho}$$

or $$\frac{2\ell}{3}=\frac{R^2}{\ell}$$

$$\Rightarrow \quad \frac{\ell^2}{R^2}=\frac{3}{2}$$

or $$\frac{\ell}{R}=\sqrt{\frac{3}{2}}$$

4. Ans. (1)

$$\frac{\text{Force}}{\text{Area}}=\frac{F}{\pi R^2}$$

Force on small area element $=\frac{F}{\pi R^2}(dA)$

Frictional torque on the element is $d\tau$

$$=\mu\left[\frac{F}{\pi R^2}\right](dA)r$$

On integrating

$$\tau=\frac{\mu F}{\pi R^2}(2\pi)\left(\frac{R^3}{3}\right)=\frac{2\mu FR}{3}$$

5. Ans. (4)

From dimension analysis

$$I_0=Kma^2\,(a=\text{side of triangle})$$

For small triangle.

$$I'=\frac{Km}{4}\left(\frac{a}{2}\right)^2=\frac{kma^2}{16}=\frac{I_0}{16}$$

So moment of inertia of remaining part

$$I-I'=\frac{15I_0}{16}$$

6. Ans. (3)

Angular impulse = change in angular momentum

$\tau \Delta t = \Delta L$

$$mg\frac{\ell}{2}\times .01 = \frac{m\ell^2}{3}\omega$$

$$\omega = \frac{3g\times 0.01}{2\ell}$$

$$= \frac{3\times 10\times .01}{2\times 0.3}$$

$$\frac{1}{2} = 0.5 = \text{rad}/s$$

Time taken by rod to hit the ground

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 5}{10}} = 1 \text{ sec.}$$

In this time angle rotate by rod

$\theta = \omega t = 0.5 \times 1 = 0.5$ radian

7. Ans. (4)

Kinetic energy, $KE = \frac{1}{2}I\omega^2 = k\theta^2$

$\Rightarrow \quad \omega^2 = \dfrac{2k\theta^2}{I}$

$\Rightarrow \quad \omega = \sqrt{\dfrac{2k}{I}}\,\theta \qquad ...(1)$

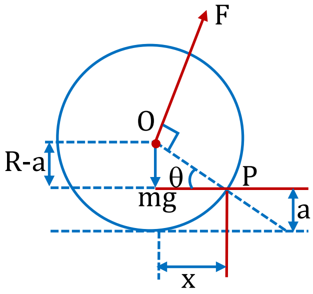
Differentiate (1) wrt time $\rightarrow$

$$\frac{d\omega}{dt} = \alpha = \sqrt{\frac{2k}{I}}\left(\frac{d\theta}{dt}\right)$$

$\Rightarrow \quad \alpha = \sqrt{\dfrac{2k}{I}} \cdot \sqrt{\dfrac{2k}{I}}\,\theta$ {by (1)}

$\Rightarrow \quad \alpha = \dfrac{2k}{I}\theta$

8. Ans. (4)



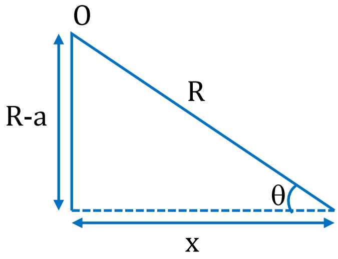
$(\tau)_P = 0$

$FR - mgx = 0$

$x = \sqrt{R^2 - (R-a)^2}$

$F = mg\dfrac{x}{R}$

$F = mg\sqrt{1-\left(\dfrac{R-a}{R}\right)^2}$



= Minimum value of force to pull.

9. Ans. (2)

Angular momentum conservation,

$$mvl = \frac{Ml^2}{3}\omega + ml^2\omega$$

$$\Rightarrow \quad \omega = \frac{1\times 6\times 1}{\frac{2}{3}+1} = \frac{18}{5}$$

Now using energy conservation,

$$\frac{1}{2}\left(M\frac{l^2}{3}\right)\omega^2 + \frac{1}{2}(ml^2)\omega^2 = (m+M)r_{cm}(1-\cos\theta)$$

$$= (m+M)\left(\frac{ml+\frac{Ml}{2}}{m+M}\right)g(1-\cos\theta)$$

$$\frac{5}{6}\times\left(\frac{18}{5}\right)^2 = 20(1-\cos\theta)$$

$$\Rightarrow \quad 1-\cos\theta = \frac{18}{5}\times\frac{3}{20}$$

$$\cos\theta = 1-\frac{27}{50}$$

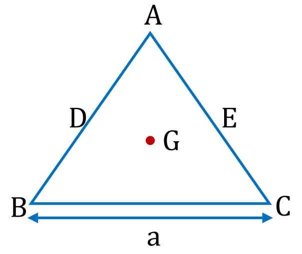
$$\cos\theta = \frac{23}{50}$$

$$\Rightarrow \quad \theta \approx 63^\circ$$

10. Ans. (11)

Let side of triangle is a and mass is m

MOI of plate ABC about centroid



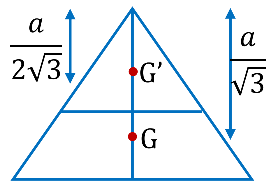
$$I_0 = \frac{m}{3}\left[\left(\frac{a}{2\sqrt{3}}\right)^2 \times 3\right] = \frac{ma^2}{12}$$

Triangle ADE is also an equilateral triangle of side $a/2$.

Let moment of inertia of triangular plate ADE about it's centroid (G') is I_1 and mass is m_1

$$m_1 = \frac{m}{\frac{\sqrt{3a^2}}{4}} \times \frac{\sqrt{3}}{4}\left(\frac{a}{2}\right)^2 = \frac{m}{4}$$

$$I_1 = \frac{m_1}{12}\left(\frac{a}{2}\right)^2 = \frac{m}{4\times 12}\frac{a^2}{4} = \frac{ma^2}{192}$$



$$\text{Distance } GG' = \frac{a}{\sqrt{3}} - \frac{a}{2\sqrt{3}} = \frac{a}{2\sqrt{3}}$$

So MOI of part ADE about centroid G is

$$I_2 = I_1 + m_1\left(\frac{a}{2\sqrt{3}}\right)^2 = \frac{ma^2}{192} + \frac{m}{4}.\frac{a^2}{12}$$

$$= \frac{5ma^2}{192}$$

Now MOI of remaining part

$$= \frac{ma^2}{12} - \frac{5ma^2}{192} = \frac{11ma^2}{12\times 16} = \frac{11I_0}{16}$$

$\Rightarrow$ $N = 11$

11. Ans. (1)

$\vec{F} = 4\hat{\imath} - 3\hat{\jmath}$

$\vec{r}_1 = 5\hat{\imath} + 5\sqrt{3}\hat{\jmath}$ and $\vec{r}_2 = -5\hat{\imath} + 5\sqrt{3}\hat{\jmath}$

Torque about 'O'

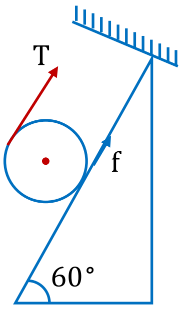
$$\vec{\tau}_O = \vec{r}_1 \times \vec{F} = (-15 - 20\sqrt{3})\hat{k} = (15 + 20\sqrt{3})(-\hat{k})$$

Torque about 'Q'

$$\vec{\tau}_Q = \vec{r}_2 \times \vec{F} = (-15 + 20\sqrt{3})\hat{k} = (15 - 20\sqrt{3})(-\hat{k})$$

12. **Ans. (3)**

Let's take solid cylinder is in equilibrium



$T + f = mg \sin 60$ ...(i)

$TR - fR = 0$ ...(ii)

Solving we get,

$$T = f_{req} = \frac{mg\sin\theta}{2}$$

But limiting friction < Required friction

$$\mu mg \cos 60° < \frac{mg\sin 60°}{2}$$

Hence cylinder will not remain in equilibrium

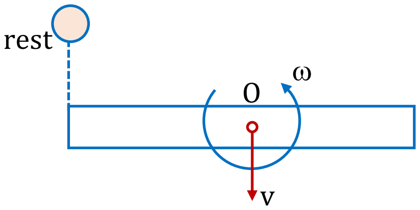
f = Kinetic

$= \mu_k N$

$= \mu_k mg \cos 60° = \dfrac{mg}{5}$

13. **Ans. (4)**

Just before collision



Just after collision

From momentum conservation, $P_i^0 = P_f$

$mu = Mv$ ...(i)

From angular momentum conservation about O,

$$mu \cdot \frac{L}{2} = \frac{ML^2}{12}\omega$$

$\Rightarrow \quad \omega = \dfrac{6mu}{ML}$ ...(ii)

From $e = \dfrac{R.V.S}{R.V.A}$

$$1 = \frac{V + \frac{\omega L}{2}}{u}$$

$$v + \frac{\omega L}{2} = u$$

$$v+\frac{3mu}{M}=u$$

$$\frac{mu}{M}+\frac{3mu}{M}=u$$

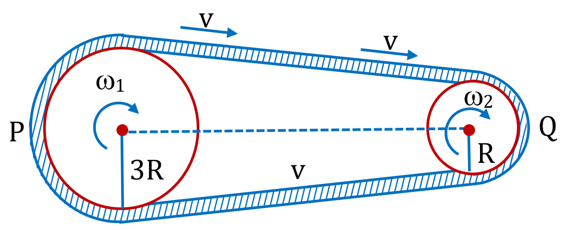
$$\frac{4mu}{M}=u$$

$$\frac{m}{M}=\frac{1}{4}$$

$x = 4$

14. Ans. (9)

$$\frac{1}{2}(I_1)(\omega_1)^2=\frac{1}{2}I_2(\omega_2)^2$$



$$I_1\left(\frac{v}{3R}\right)^2=I_2\left(\frac{v}{R}\right)^2$$

$$\frac{I_1}{I_2}=\frac{9}{1}$$

15. Ans. (3)

From conservation of angular momentum we get

$$I_1\omega_1+I_2\omega_2=(I_1+I_2)\omega$$

$$\omega=\frac{I_1\omega_1+I_2\omega_2}{I_1+I_2}$$

$$k_i=\frac{1}{2}I_1\omega_1^2+\frac{1}{2}I_2\omega_2^2$$

$$k_f=\frac{1}{2}(I_1+I_2)\omega^2$$

$$k_i-k_f=\frac{1}{2}\left[I_1\omega_1^2+I_2\omega_2^2-\frac{(I_1\omega_1+I_2\omega_2)^2}{I_1+I_2}\right]$$

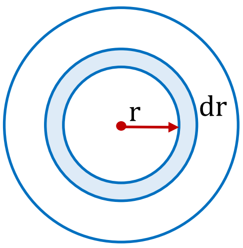
Solving above we get

$$k_i-k_f=\frac{1}{2}\left(\frac{I_1I_2}{I_1+I_2}\right)(\omega_1-\omega_2)^2$$

16. Ans. (1)

$dI = dmr^2$

$dI = \sigma 2\pi r\, dr\, r^2$



$dI = 2\pi(A + Br)\, r^3\, dr$

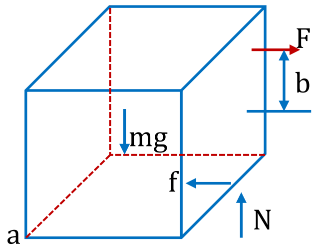
$$\int dI=2\pi\int_0^a(Ar^3+Br^4)dr$$

$$I = 2\pi a^4\left(\frac{A}{4}+\frac{Ba}{5}\right)$$

17. Ans. (75)

$F\left(b+\frac{a}{2}\right)=mg\ \frac{a}{2}$ ...(1)

$F = mg\mu$ ...(2)



$\mu mg\left(b+\frac{a}{2}\right)=mg\times\frac{a}{2}$

$\left(b+\frac{a}{2}\right)\mu=\frac{a}{2}$

$0.4 = \mu = \frac{a}{2b+a}$

$0.8b + 0.4a = a$

$0.8\ b = 0.6\ a$

$\frac{b}{a} = \frac{3}{4}$

$\Rightarrow \quad 100\times\frac{3}{4}=75$

18. Ans. (4)

Parallel axis theorem

$I = I_{CM} + Md^2$

$I=\frac{Mr^2}{2}+M\left(\frac{L}{2}\right)^2$

$2.7=M\frac{(0.2)^2}{2}+M\left(\frac{0.8}{2}\right)^2$

$2.7=M\left[\frac{2}{100}+\frac{16}{100}\right]$

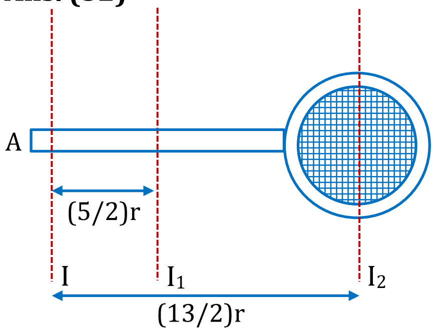
$M = 15kg$

$\Rightarrow \quad \rho=\frac{M}{\pi r^2 L}=\frac{15}{\pi(0.2)^2\times0.8}$

$= 0.1492 \times 10^3$

$= 1.49 \times 10^2\ kg/m^3$

19. Ans. (52)

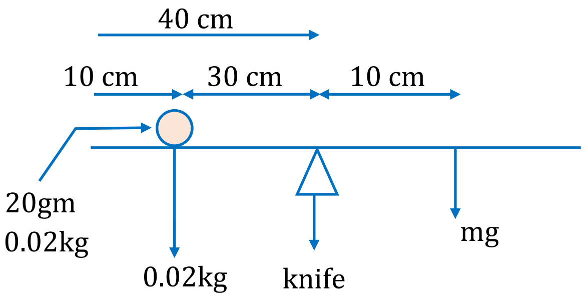


$I=\left[I_1+M\left(\frac{5}{2}r\right)^2\right]+\left[I_2+M\left(\frac{13r}{2}\right)^2\right]$

$=\left[\frac{M(36r^2)}{12}+\frac{M(25r^2)}{4}\right]+\left[\frac{Mr^2}{2}+\frac{169Mr^2}{4}\right] = 52\ Mr^2$

20. Ans. (6)

Let mass of meter scale be m.



Balancing torque about knife edge,

$(0.02g) \times (30 \times 10^{-2}) = mg \times (10 \times 10^{-2})$

$m = 0.06\ kg = 6 \times 10^{-2}\ kg$

21. Ans. (5)

For ring $I = mR_1^2 = mK_1^2$

$\therefore$ Radius of gyration $K_1 = R_1$

For solid sphere

$$I' = \frac{2}{5}m'R_2^2 = m'K_2^2$$

$\therefore$ Its radius of gyration $= K_2 = \sqrt{\frac{2}{5}}R_2$

$\therefore \quad K_1 = K_2$

$\therefore \quad R_1 = \sqrt{\frac{2}{5}}R_2$

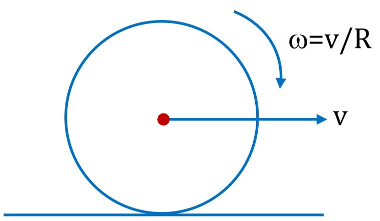
$\therefore \quad \frac{R_1}{R_2} = \sqrt{\frac{2}{5}}$

$\therefore \quad x = 5$

22. Ans. (2)

$$\frac{K_{rot}}{K_{Total}} = \frac{\frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{V}{R}\right)^2}{\frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{3}mR^2\right)\left(\frac{V}{R}\right)^2}$$

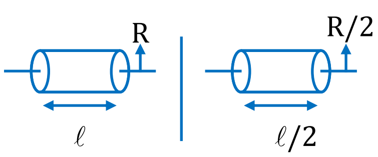
$$\Rightarrow \frac{x}{5} = \frac{2}{5} \quad \Rightarrow x = 2$$



23. Ans. (32)

$$I_1 = \frac{m_1R^2}{2} \qquad I_2 = \frac{m_2(R/2)^2}{2}$$

$$\frac{I_1}{I_2} = \frac{4m_1}{m_2} = \frac{4 \cdot \rho\pi R^2\ell}{\rho \cdot \frac{\pi R^2}{4} \times \frac{\ell}{2}} \Rightarrow \frac{I_1}{I_2} = 32$$



EXERCISE - JEE (Advanced) PYQ

1. Ans. (2)

Angular impulse = change in angular momentum

$\tau \Delta t = I\omega$

$3 \times F \times R \sin 30° \times \Delta t = I\omega$

$3 \times 0.5 \times 0.5 \times \frac{1}{2} \times 1 = \frac{1}{2} \times 1.5 \times 0.5 \times 0.5 \times \omega$

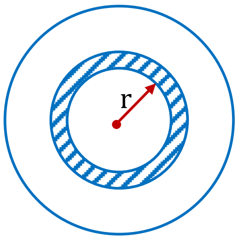
$\omega = 2$ rad/s

2. Ans. (6)

Consider a shell of radius r and thickness dr

$dI = \frac{2}{3}(dm)r^2$

$\therefore \quad dI = \frac{2}{3}\left[\left(\frac{kr}{R}\right)4\pi r^2 dr\right]r^2$



$\int dI = \left(\frac{8\pi}{3}\frac{k}{R}\right)\int r^5 dr$

$I_A = \left(\frac{8\pi k}{3R}\right)\left(\frac{R^6}{6}\right)$

$I_A = \left(\frac{8\pi k}{18}\right)R^5 \qquad$ ...(i)

Similarly,

$I_B = \frac{8k\pi}{3R^5}\int_0^R r^9 dr$

$\therefore \quad I_B = \frac{8\pi k}{30}R^5 \qquad$ ...(ii)

$\therefore \quad \frac{I_B}{I_A} = \frac{6}{10}$

So $\quad n = 6$

3. Ans. (D)

Force equation in x-direction,

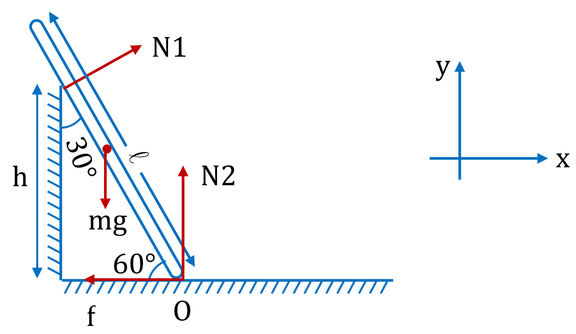
$N_1 \cos 30° - f = 0 \qquad$ ...(i)

Force equation in y-direction,

$N_1 \sin 30° + N_2 - mg = 0 \qquad$ ...(ii)

Torque equation about O, ...(iii)

Also, given $N_1 = N_2$ ...(iv)



$mg\frac{\ell}{2}\cos 60° - N_1\frac{h}{\cos 30°} = 0$

[Note taking reaction from floor as normal reaction only]

Solving (i), (ii), (iii) and (iv) we have

$\frac{h}{\ell} = \frac{3\sqrt{3}}{16}$ and $f = \frac{16\sqrt{3}}{3}$

4. Ans. (ABD)

$\vec{r} = \alpha t^3\hat{\imath} + \beta t^2\hat{\jmath}$

$\vec{v} = \dfrac{d\vec{r}}{dt} = 3\alpha t^2\hat{\imath} + 2\beta t\hat{\jmath}$

$\vec{a} = \dfrac{d^2\vec{r}}{dt^2} = 6\alpha t\hat{\imath} + 2\beta\hat{\jmath}$

At $t = 1$

(A) $\vec{v} = 3 \times \dfrac{10}{3} \times 1\hat{\imath} + 2 \times 5 \times 1\hat{\jmath} = 10\hat{\imath} + 10\hat{\jmath}$

(B) $\vec{L} = \vec{r} \times \vec{p}$

$\left(\dfrac{10}{3} \times 1\hat{\imath} + 5 \times 1\hat{\jmath}\right) \times 0.1\,(10\hat{\imath} + 10\hat{\jmath}) = -\dfrac{5}{3}\hat{k}$

(C) $\vec{F} = m \times \left(6 \times \dfrac{10}{3} \times 1\hat{\imath} + 2 \times 5\hat{\jmath}\right) = (2\hat{\imath} + \hat{\jmath})N$

(D) $\vec{\tau} = \vec{r} \times \vec{F} = \left(\dfrac{10}{3}\hat{\imath} + 5\hat{\jmath}\right) \times (2\hat{\imath} + \hat{\jmath}) = +\dfrac{10}{3}\hat{k} + 10(-\hat{k}) = -\dfrac{20}{3}\hat{k}$

5. Ans. (D)

Force on block along slot = $m\omega^2 r = ma = m\left(\dfrac{vdv}{dr}\right)$

$$\int_0^v vdv = \int_{R/2}^{r} \omega^2 r dr$$

$$\frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - \frac{R^2}{4}\right)$$

$$\Rightarrow \quad v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\Rightarrow \quad \int_{R/4}^{r} \frac{dr}{\sqrt{r^2 - \dfrac{R^2}{4}}} = \int_0^t \omega dt$$

$$\ell n\left(\frac{r + \sqrt{r^2 - \dfrac{R^2}{4}}}{\dfrac{R}{2}}\right) - \ell n\left(\frac{R/2 + \sqrt{\dfrac{R^2}{4} - \dfrac{R^2}{4}}}{\dfrac{R}{2}}\right) = \omega t$$

$$\Rightarrow \quad r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{\omega t}$$

$$\Rightarrow \quad r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2\omega t} + r^2 - 2r\frac{R}{2}e^{\omega t}$$

$$\Rightarrow \quad r = \frac{\dfrac{R^2}{4}e^{2\omega t} + \dfrac{R^2}{4}}{\mathrm{Re}^{\omega t}} = \frac{R}{4}\left(e^{\omega t} + e^{-\omega t}\right)$$

6. **Ans. (C)**

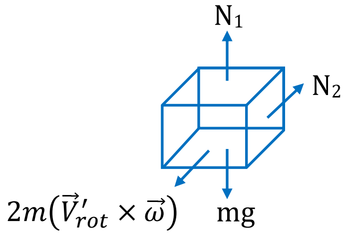
$\vec{N}_1 = mg\hat{k}$

$\vec{N}_2 = 2m(V'_{rot} \times \vec{\omega})\hat{j} = 2m\left[\frac{\omega R}{4}(e^{\omega t} - e^{-\omega t})\right]\omega\hat{j}$

$= \frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j}$

Total reaction on block = $\vec{N}_1 + \vec{N}_2$

$= \frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$



7. **Ans. (ABC)**

When the bar makes an angle θ; the height of its COM (mid point) is $\frac{L}{2}\cos\theta$

∴ Displacement = $\frac{L}{2} - \frac{L}{2}\cos\theta = \frac{L}{2}(1-\cos\theta)$

Since force on COM is only along the vertical direction, hence COM is falling vertically downward.

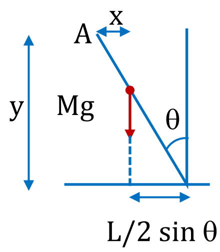
Instantaneous torque about point of contact is

$\tau = Mg \times \frac{L}{2}\sin\theta$

Now, $x = \frac{L}{2}\sin\theta$

$y = L\cos\theta$

$\frac{x^2}{(L/2)^2} + \frac{y^2}{L^2} = 1$



Path of A is an ellipse.

8. **Ans. (BC or C)**

If we assume that magnitude of force remain constant then option (C) would be correct and if we assume that force vector remain constant then option (BC) would be correct.

9. **Ans. (BC)**

$V = \frac{kr^2}{2}$

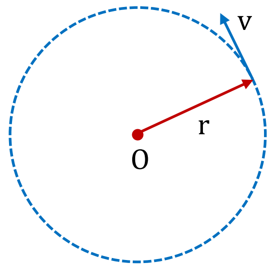
$F = -kr$ (towards centre) $\left[F = -\frac{dV}{dr}\right]$

At $r = R$,

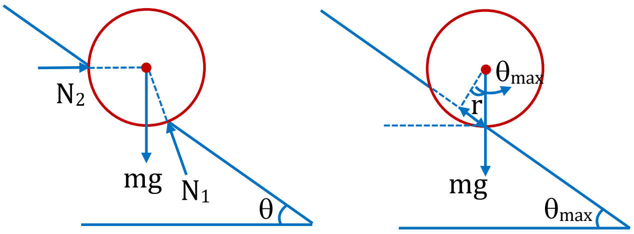
$kR = \frac{mv^2}{R}$ [Centripetal force]

$v = \sqrt{\frac{kR^2}{m}} = \sqrt{\frac{k}{m}}R$

$L = m\sqrt{\frac{k}{m}}R^2 = \sqrt{mk}R^2$



10. Ans. (A)



For θ_{max}, the football is about to roll, then $N_2 = 0$ and all the forces (Mg and N_1) must pass through contact point

$\therefore \quad \cos(90° - \theta_{max}) = \frac{r}{R}$

$\Rightarrow \quad \sin\theta_{max} = \frac{r}{R}$

11. Ans. (B)

For no slipping at the ground,

$V_{centre} = \omega R$ (R is radius of roller)

$\therefore$ Velocity of scale = $(V_{center} + \omega r)$ [r is radius of axle]

Given, $V_{center} \cdot t = 50\ cm$

$\therefore$ Distance moved by scale = $(V_{center} + \omega r)t$

$$= \left(V_{center} + \frac{V_{center} r}{R}\right)t = \frac{3V_{center}}{2} \cdot t$$

$= 75\ cm$

12. Ans. (25.60)

Initially

$N_1 + N_2 = Mg$

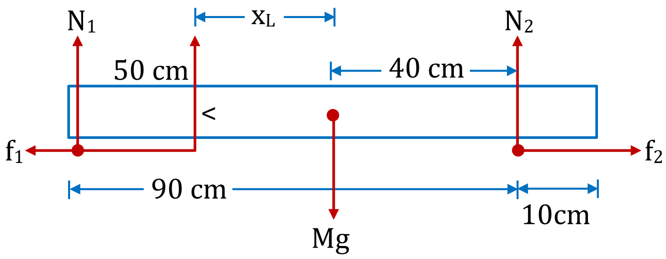
$(\tau_N = 0)$ about centre

$N_1(50) = N_2(40)$

$5N_1 = 4N_2$

$N_1 = \frac{4Mg}{9}$

$N_2 = \frac{5Mg}{9}$



$f_{1_K} = \mu_K N_1$ $\quad f_{1_L} = \mu_S N_1$

$f_{1_K} = 0.32\ N_1$ $\quad f_{1_L} = 0.4\ N_1$

$f_{2_K} = 0.32\ N_2$ $\quad f_{2_L} = 0.4\ N_2$

Suppose x_L = distance of left finger from centre when right finger starts moving

$(\tau_n = 0)_{\text{about centre}}$

$\Rightarrow \quad N_1 x_L = N_2(40)$

$f_{K_1} = f_{L_2}$

$\Rightarrow \quad 0.32\ N_1 = 0.40\ N_2$

$4N_1 = 5N_2$

$N_1 x_L = \frac{4N_1}{5}(40)$

$x_L = 32$

Now x_R = distance when right finger stops and left finger starts moving

$(\tau_n = 0)_{\text{about centre}}$

$\Rightarrow$ $N_1 x_L = N_2(x_R)$

$f_{L_1} = f_{K_2}$

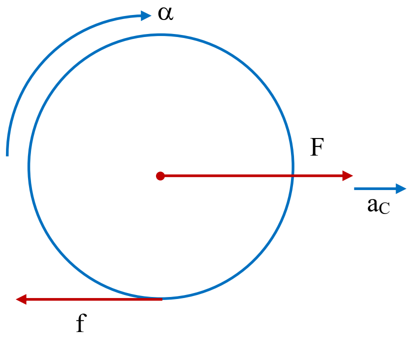
$\Rightarrow$ $0.4\, N_1 = 0.32\, N_2$

$5N_1 = 4N_2$

$$\frac{4N_2}{5}(32) = N_2 x_R$$

$$x_R = \frac{128}{5} = 25.6\ cm$$

13. **Ans. (BD)**



$$F - f = ma_C$$

$$fR = I_C\alpha$$

$$a_C - \alpha R = 0$$

$$F - I_C\frac{\alpha}{R} = ma_C$$

$$a_C = \frac{F}{\frac{I_C}{R^2} + m}$$

$$f = \frac{I_C\alpha}{R} = \frac{I_C}{R^2}a_C = \frac{I_C}{R^2}\frac{F}{\left[\frac{I_C}{R^2} + m\right]}$$

$$f = \frac{F}{\left(m + \frac{I_C}{R^2}\right)}$$

Thin walled hollow cylinder,

$$I_C = mR^2$$

$$a_C = \frac{F}{2m}$$

$$fR = I_C\alpha = \frac{I_C a_C}{R}$$

$$f = \frac{I_C a_C}{R^2} \le \mu mg$$

$$a_C \le \frac{\mu mgR^2}{I_C}$$

For solid cylinder $I_C = \dfrac{mR^2}{2}$

$$a_C \le 2\mu g$$

$$(a_C)_{\max} = 2\mu g$$

14. Ans. (ABC)

$\vec{r}_A = -\hat{j}$

$S = \frac{1}{2}at^2 = 20m$

$\vec{\tau}_0 = \vec{r} \times \vec{F}$; $\vec{r}_B = 10\hat{\imath} - \hat{j}$

$\vec{F} = m\vec{a} = 0.2 \times 10\hat{\imath} = 2\hat{\imath}$

$\vec{\tau}_0 = (10\hat{\imath} - \hat{j}) \times (2\hat{\imath})$

$\vec{\tau}_0 = 2\hat{k}$

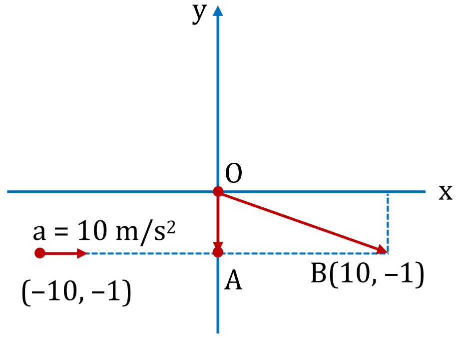
$\vec{L}_0 = \vec{r}_B \times \vec{p} = \vec{r}_B \times m\vec{v}$

$\vec{v} = \vec{a}t = 10\hat{\imath} \times 2 = 20\hat{\imath}$

$\vec{L}_0 = (0.2)[(10\hat{\imath} - \hat{j}) \times 20\hat{\imath}] = 4\hat{k}$

At point A(0, −1)

$\vec{\tau}_0 = \vec{r}_A \times \vec{F} = (-\hat{j}) \times 2\hat{\imath} = 2\hat{k}$



15. Ans. (49)

$$L = \frac{Ma^2}{3}\Omega + M\left(\frac{3a}{4}\right)^2\Omega + \frac{M\left(\frac{a}{4}\right)^2 4\Omega}{2}$$

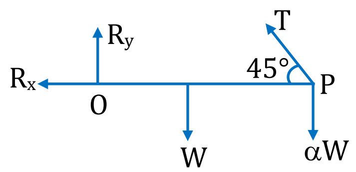
$$L = \frac{49}{48}Ma^2\Omega$$

$n = 49$

16. Ans. (ABD)

$$R_y + \frac{T}{\sqrt{2}} = W + \alpha W \quad ...(i)$$

$$R_x = \frac{T}{\sqrt{2}} \quad ...(ii)$$



Taking torque about 'O'

$$W\frac{\ell}{2} + \alpha W\ell = \frac{T}{\sqrt{2}}\ell$$

$$T = \sqrt{2}\left(\frac{W}{2} + \alpha W\right) \quad ...(iii)$$

$$R_x = \frac{T}{\sqrt{2}} = \left(\frac{W}{2} + \alpha W\right)$$

Taking torque about P

$$R_y\ell = W\frac{\ell}{2}$$

$$R_y = \frac{W}{2}$$

When $T = T_{max}$

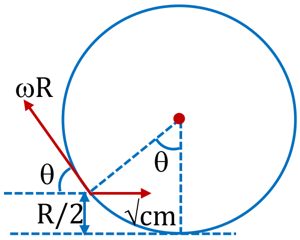
$$2\sqrt{2}W = \sqrt{2}\left(\frac{W}{2} + \alpha W\right)$$

We get $\alpha = \frac{3}{2}$

17. Ans. (0.52)

At $t = 0, \omega = 0$

At $t=\sqrt{\pi}$, $\omega=\alpha t=\frac{2}{3}\sqrt{\pi}, v=\omega r=\frac{2}{3}\sqrt{\pi}$

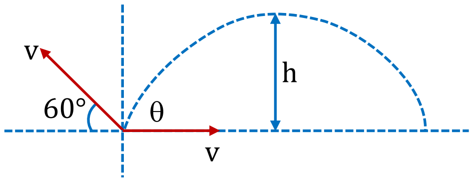


$$\theta=\frac{1}{2}\alpha t^2$$

$$\theta=\frac{1}{2}\times\frac{2}{3}\times\pi=\frac{\pi}{3}$$

$\theta = 60°$

$$v_y=v\sin 60=\frac{\sqrt{3}}{2}V$$



$$h=\frac{u_y^2}{2g}=\frac{\frac{3}{4}v^2}{2g}$$

$$h=\frac{\frac{3}{4}\times\frac{4}{9}\pi}{2g}$$

$$h=\frac{3\pi}{9\times 2g}=\frac{\pi}{6g}$$

Maximum height from plane,

$$H=\frac{R}{2}+h$$

$$H=\frac{1}{2}+\frac{\pi}{6\times 10}$$

$x=\frac{\pi}{6}$; $x = 0.52$

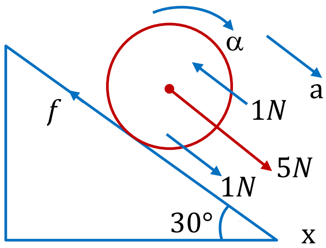
18. Ans. (2.85 to 2.86)

Solid sphere $1kg$, $1m$

$5 + 1 - 1 - f = 1a$

$5 - f = a$

About COM



$$f1 - 2(1(0.5)) = \frac{2}{5}Mr^2\alpha$$

$\Rightarrow$ $f-1=\frac{2}{5}a$

$\Rightarrow$ $f=1+\frac{2}{5}a$

$$5-a=1+\frac{2}{5}a$$

$\Rightarrow$ $4=\frac{7a}{5}$

$\Rightarrow$ $a=\frac{20}{7} = 2.86\ m/s^2$

19. Ans. (3)

Torque about origin is zero

So angular momentum about origin remains conserved.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{1}{\sqrt{2}} & \sqrt{2} & 0 \\ -\sqrt{2} & \sqrt{2} & \frac{2}{\pi} \end{vmatrix} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0.5 \\ v_x & v_y & \frac{2}{\pi} \end{vmatrix}$$

$$\hat{i}\left[\sqrt{2}\times\frac{2}{\pi}\right] - \hat{j}\left[\frac{\sqrt{2}}{\pi}\right] + \hat{k}[1+2]$$

$$= \hat{i}\left[\frac{y\times 2}{\pi} - 0.5v_y\right] - \hat{j}\left[\frac{x\times 2}{\pi} - 0.5v_x\right] + \hat{k}\left[xv_y - yv_x\right]$$

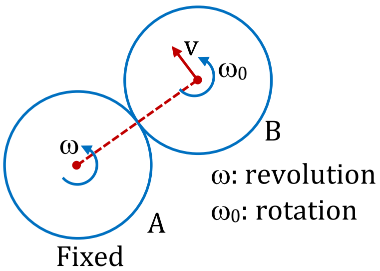
$x_{vy} - y_{vx} = 3$

20. Ans. (B)

$v = \omega\,(2R)$

$v = \omega_0 R$; no slipping

$\therefore \quad \omega_0 = 2\omega$



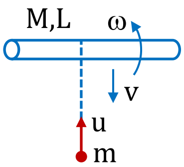
$$\vec{L} = m\vec{r}\times\vec{v}_c + I_c\omega_0$$

$$= M2Rv + \frac{1}{2}MR^2\omega_0$$

$$= 4MR^2\omega + \frac{1}{2}MR^2(2\omega) = 5MR^2\omega$$

$\therefore \quad n = 5$

21. Ans. (A)



Applying angular momentum conservation about hinge

$$mv\frac{L}{2} + 0 = -mv\frac{L}{2} + \frac{ML^2}{3}\omega \qquad \text{...(i)}$$

Also, from equation of restitution

$$e = 1 = \frac{\omega\frac{L}{2}+V}{u} \Rightarrow u = \omega\frac{L}{2} + V \qquad \text{...(ii)}$$

Solving (i) and (ii)

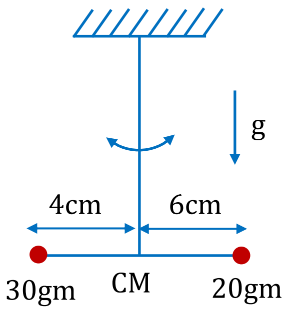
$\omega \approx 6.98$ rad/sec and $v = 4.30\ m/s$

Hence option (A)

22. Ans. (10)

$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{C}}$

$\Rightarrow \omega = \sqrt{\frac{C}{I}}$



Where I = moment of inertia

$I = (30)(4)^2 + (20)(6)^2$

$= 1200\ gm\text{-}cm^2$

$= 1.2 \times 10^{-4}\ kg\text{-}m^2$

$\Rightarrow \omega = \sqrt{\frac{1.2 \times 10^{-8}}{1.2 \times 10^{-4}}}$

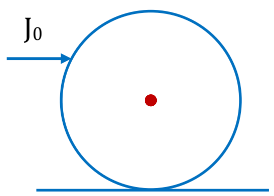
$\Rightarrow \omega = \sqrt{10^{-4}}$

$\omega = (10^{-2})$

$n \times 10^{-3} = 10^{-2} \Rightarrow n = 10$

23. Ans. (ABCD)

$J_0 = mv$ ...(1)



$J_0 h_m = I_c\omega$ ...(2)

$v = \omega R$ ...(3)

$\Rightarrow h_m = \frac{I_C}{mR}$

(A) If $a = 0$, $I_c = \frac{1}{2}mb^2$ and $R = b$ $\therefore h_m = \frac{b}{2}$

(B) If $a = b$, $I_C = mb^2$ and $R = b$ $\therefore h_m = b$

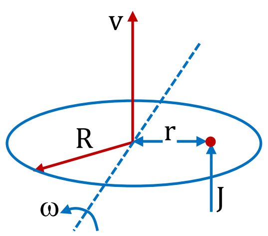
(C) $v = \frac{J_0}{m} \Rightarrow 100 = \frac{V}{R} = \frac{J_0}{mR}$

(D) Force is acting on COM $\therefore$ No rotation.

24. Ans. (30)

$J = mv$ ...(1)

$Jr = I_c\omega$ ...(2)



$J_C = \frac{1}{4}mR^2$ ...(3)

$t = \frac{2v}{g}$ ...(4)

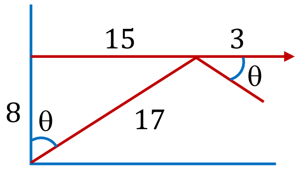
$\theta = 2\pi N = \omega t$ ...(5) $\therefore$ N = 30

JEE (Main) Practice Paper

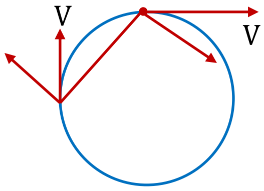
SECTION–A

1. Ans. (3)

$$\omega = \frac{3\cos\theta}{17} = \frac{3}{17} \times \frac{8}{17} = \frac{24}{289} rad/s$$



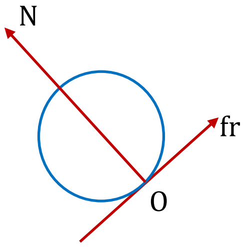
2. Ans. (2)



$$\omega = \frac{\frac{v}{\sqrt{2}} + \frac{v}{\sqrt{2}}}{\sqrt{2}R} = \frac{v}{R}$$

3. Ans. (3)

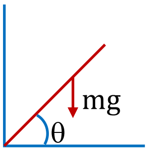
$\tau_0 = 0$, mg should pass through O



4. Ans. (4)

$$mg\frac{\ell}{2}\cos\theta = \frac{m\ell^2\alpha}{3}$$

$$\therefore \quad \alpha\ell = \frac{3g\cos\theta}{2}$$



5. Ans. (4)

Depends on v and ω.

6. Ans. (2)

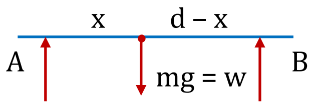
$$\frac{1}{2}MV_{cm}^2 + \frac{1}{2}MR^2\omega^2 = \frac{1}{2}MV_{cm}'^2 + \frac{1}{2}\frac{MR^2}{2}\omega'^2$$

7. Ans. (3)

$$\frac{\frac{1}{2}\frac{MR^2}{2}\omega^2}{\frac{1}{2}Mv^2} = \frac{1}{2}$$

8. Ans. (3)

$$N_A d = w(d-x)$$



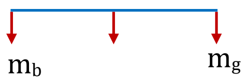
$$\therefore \quad N_A = \frac{w(d-x)}{d}$$

$$N_B = \frac{wx}{d}$$

9. Ans. (2)

Angular momentum same

10. Ans. (2)



$$m_a g\ell_1 + m_s x = m_b g\ell_2 \text{ and } m_a g\frac{\ell_1}{2} + m_s$$

$$x > \frac{m_b g\ell_2}{2}$$

11. Ans. (2)

$$P = \tau\omega = \tau\frac{\tau}{I}t$$

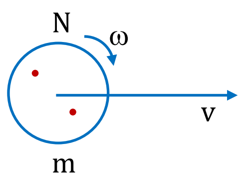
$$\therefore \quad P \propto t$$

12. Ans. (3)

$$\frac{K_B}{K_A} = \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}I\omega^2} = \frac{\frac{1}{2}mv_0^2}{\frac{1}{2}\times\frac{2}{3}mR^2\omega^2} = \frac{3}{2}$$

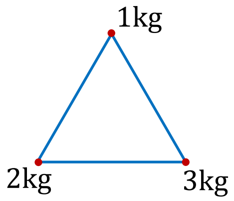
13. Ans. (4)

Different



14. Ans. (4)

$$\vec{r}_c = \frac{1\times1\left(\cos60\hat{i}+\sin60\hat{j}\right)+3\hat{i}}{6}$$



$$= \frac{3.5\hat{i}+\frac{\sqrt{3}}{2}\hat{j}}{6}$$

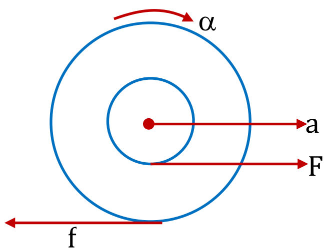
$I = I_C + mr_C^2 = 1 \times 12 + 3 \times 12$

$$I_C = 4 - \frac{6\times\left(\frac{49}{4}+\frac{3}{4}\right)}{6^2}$$

$$= 4 - \frac{52}{24}$$

$$= \frac{21}{6} kg-m^2 = 3.5\ kg\text{-}m^2$$

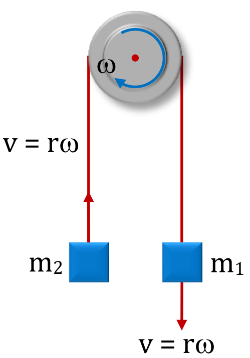
15. Ans. (2)



16. Ans. (3)

As the f_1 is acting to the right direction, so when f_1 is applied the centre of mass will move to the right.

17. Ans. (3)



$$L = m_1vr + m_2vr + I\omega$$

$$= (m_1+m_2)r^2\omega + I\omega$$

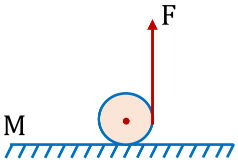
18. Ans. (2)

$F_r - f_r = \frac{mr^2}{2}\alpha = \frac{ma}{2}$

$f = ma$

$\therefore \quad F = \frac{3}{2}ma \qquad \therefore \quad \frac{2F}{3mr} = \frac{a}{r}$

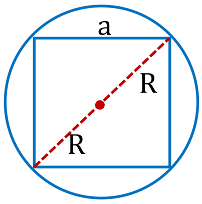
Now $F_{max} = mg \qquad \therefore \quad \frac{2g}{3r} = \alpha_{max}$



19. Ans. (1)

Let mass and side of cube be M' and $\sqrt{3}\,a = 2R$

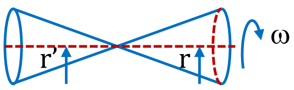
$M' = \frac{M}{\frac{4}{3}\pi R^3}a^3$



Moment of Inertia of cube = $\frac{M'a^2}{6}$

$= \left(\frac{M}{\frac{4}{3}\pi R^3}a^3\right)\frac{a^2}{6} = \boxed{\frac{4MR^2}{9\sqrt{3}\pi}}$

20. Ans. (2)



Say the distance of central line from instantaneous axis of rotation is r.

Then r from the point on left becomes lesser than that for right.

So, $v_{\text{left point}} = \omega r' < \omega r = v_{\text{right point}}$.

So, the roller will turn to left.

SECTION–B

1. Ans. (3)

$$\left(\frac{M_r\ell^2}{12} + 2m_p(0.1)^2\right)\omega_1 = \left(\frac{M_r\ell^2}{12} + 2m_p(0.2)^2\right)\omega_2$$

$$\Rightarrow \left(\frac{0.75\times0.4\times0.4}{12} + 2\times1\times(0.1)^2\right)30 = \left(\frac{0.75\times0.4\times0.4}{12} + 2\times1\times(0.2)^2\right)\omega_2$$

$\Rightarrow \quad \omega_2 = 10 rad/s$

$KE_i = KE_f$

$\Rightarrow \quad \frac{1}{2}I_1\omega_1^2 = \frac{I}{2}\omega_2^2 + 2\times\frac{1}{2}MV^2$

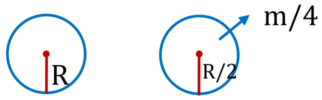
$v = 3m/s$

2. **Ans. (14)**

$w_{fr} = 0$

$w_g = KE_f$

$$mgR - \frac{m}{4}\frac{gR}{2} = \frac{1}{2}\frac{m}{4}v^2 + \frac{1}{2}\frac{\frac{m}{4}\left(\frac{R}{2}\right)^2}{2}w^2$$



$$\frac{7}{8}mgR = mv^2\left(\frac{1}{8}+\frac{1}{16}\right) = mv^2\frac{3}{16}$$

$$\therefore \quad v = \sqrt{\frac{14}{3}gR}$$

3. **Ans. (60)**

$$mg\frac{\ell}{2} = \frac{1}{2}\frac{m\ell^2}{3}\omega^2$$

$$\therefore \quad \omega^2 = \frac{3g}{\ell} \text{ at bottom}$$

$$W_g = \Delta KE$$

$$\Rightarrow \quad -\frac{m}{2}g\frac{\ell}{4}(1-\cos\theta) = 0 - \frac{1}{2}\left(\frac{m}{2}\right)\frac{\ell^2}{4}\times\frac{1}{3}\omega^2$$

$$\therefore \quad \frac{mg\ell}{8}(1-\cos\theta) = \frac{mg\ell}{16}$$

$$\therefore \quad \cos\theta = \frac{1}{2}$$

$$\therefore \quad \theta = 60°$$

4. **Ans. (15)**

$$ma\cos\theta\times\frac{\ell}{2} = I\alpha = \left(\frac{1}{3}m\ell^2\right)\alpha$$

$$\alpha = \frac{3a\cos\theta}{2\ell}$$

$$\frac{\omega d\omega}{d\theta} = \frac{3a\cos\theta}{2\ell} \Rightarrow \int_0^{\omega}\omega d\omega = \frac{3a}{2\ell}[\sin\theta]_0^{\frac{\pi}{2}}$$

$$\omega = \sqrt{\frac{3a}{\ell}}$$

$$v = \omega\ell = \ell\sqrt{\frac{3a}{\ell}} = \sqrt{3a\ell}$$

$$V = \sqrt{15}\ ft/s$$

5. **Ans. (2)**

$$\mu \times 2mg \times \ell = 2m\ell^2\alpha$$

$$\Rightarrow \quad \alpha = \frac{\mu g}{\ell}$$

$$\omega_0 = \left(\frac{\mu g}{\ell}\right)t$$

$$\Rightarrow \quad t = \frac{\omega_0\ell}{\mu g}$$

6. Ans. (5)

$KE_f = \frac{I}{2}I\omega^2$; $KE_i = 0$; $W_g = mg\left(\frac{2R}{4}\right) + mg2\times\left(\frac{5R}{4}\right)$

$I = m\left(\frac{5R}{4}\right)^2 + \left(\frac{MR^2}{4} + \frac{MR^2}{16}\right) = MR^2\left(\frac{25}{16} + \frac{1}{4} + \frac{1}{16}\right)$

$W_g = KE_f - KE_i$

Get $v = \omega\frac{5R}{4} = 5\ m/s$

7. Ans. (4)

$2mvR = \frac{M_1R^2}{2}\ \omega_d \Rightarrow \frac{v}{R} = \frac{3}{2}\omega_d$

$\therefore\quad \frac{\omega_m}{\omega_D} = \frac{3}{2}$

$\therefore\quad \omega_{m/D} = \frac{5\omega_D}{2}$

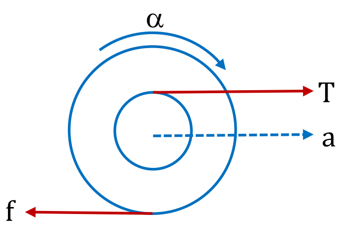
$\omega_{m/D}t = 2\pi$

$\omega_D t = 4\pi/5$

8. Ans. (16)

$T - f = ma$

$TR + f3R = mR^2\alpha$; $a = \alpha 3R$



Answer is $a + \alpha R$

9. Ans. (13)

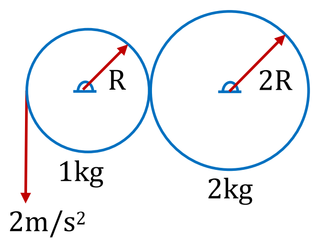
$KE = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$; get I by subtraction. ($I_{outer} - I_{inner}$).

$\omega = \frac{v}{2R}$

10. Ans. (2)

Now for (2) $f \times 2R = I_2\alpha_2$

$I_2 = \frac{2\times 4R^2}{2} = 4R^2$



Now $\alpha_2 2R = 2$ (a is same)

$f2R = 4R^2\ \frac{1}{R}$

$\therefore f = 2$

JEE (Advanced) Practice Paper

1. Ans. (B)

$N = mg + \dfrac{mv^2}{R}$

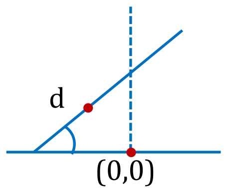
and $\dfrac{1}{2}mv^2 + \dfrac{1}{2}\dfrac{2}{5}mv^2 = mgR$

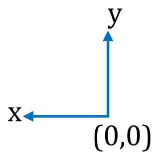
$\therefore \quad N = mg + \dfrac{10}{7}mg = \dfrac{17}{7}mg$

2. Ans. (B)

$x = \left(\dfrac{L}{2} - d\right)\cos\theta \quad$ and $\quad y = d\sin\theta$

$\therefore \quad \dfrac{x^2}{\left(\dfrac{L}{2}-d\right)^2} + \dfrac{y^2}{d^2} = 1$



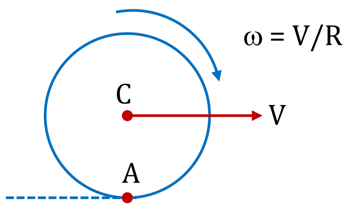


For circle $\dfrac{L}{2} - d = d \Rightarrow d = \dfrac{L}{4}$

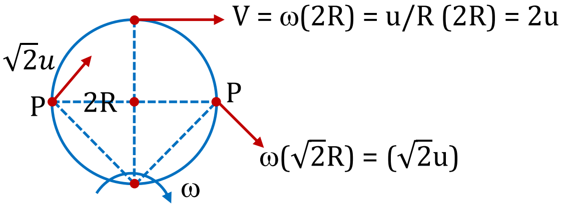
3. Ans. (ACD)

A ring is rolling on the horizontal surface.

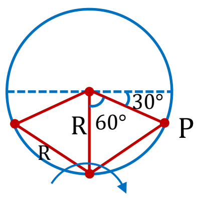
$\omega = \dfrac{u}{R}$



Point of contact of ring with ground surface is instantaneous centre of rotation. So we can say that body is performing fixed axis rotation about point A at that instant



$O \le V \le 2u$



Line CP. Makes an angle. Of 30° with horizontal.

4. Ans. (BCD)

$F - f = ma$

From A to B $\qquad 2\pi = \dfrac{1}{2}\alpha T^2 \qquad$ and $\quad \omega = \alpha T$

B to ahead $\qquad 2\pi = \omega t \qquad \therefore \quad t = \dfrac{T}{2}$

$\because \quad \omega$ will be constant.

5. **Ans. (BCD)**

If 2 lines of action pass through the centre of mass of the body and the force have different magnitude so the resultant not zero.

If 3 lines of action are not parallel, the body is in equilibrium. One line of action must be the result of other two lines of action, similarly if 3 lines of action are parallel.

If 4 lines of action are parallel and all the force have the same magnitude the body is in equilibrium the resultant must be zero.

6. **Ans. (A)**

All student use the which for 10 sec and lift same amount.

$\therefore$ Work by each is same.

$$W.D = \vec{F}.\vec{S}$$

$$= \vec{\tau}.\vec{\theta}$$

Statement-1: Now if W.D is same so if $\rightarrow x \uparrow \rightarrow F \downarrow F_1 \times x_1 = F_2 \times x_2$.

So statement-1 is correct.

7. **Ans. (D)**

Statement 2 : is incorrect because W.D is same for all.

Statement 3 : $P = \dfrac{dw}{dt}$

W.D. is same and time is same.

Power = Same

So incorrect.

8. **Ans. (D)**

$W.D = \vec{F}.\vec{S}$

So basically it is $\rightarrow x.y$ = constant

So, graph will be rectangular hyperbola.

9. **Ans. (B)**

$m\vec{r}_{cm} \times \vec{v}_{cm}$ down at 20°

10. **Ans. (B)**

Yes of mg and also direction of $\vec{L}$ is changing

$\therefore$ Horizontal

$\therefore$ Both $\vec{r}$ and mg is in vertical plane.

11. **Ans. (C)**

$\because \quad \tau = 0 \Rightarrow \dfrac{dL}{dt} = 0 \Rightarrow L = \text{constant}$

12. **Ans. (C)**

At ℓ_{max}, component of velocity along the length of spring is zero.

13. Ans. (B)

According to momentum conservation before elongation and after the elongation momentum will be equal so student A is right.

At maximum elongation $\frac{1}{2}k(\Delta\ell)^2 = \frac{1}{2}mv'^2$ so.

Student B is wrong.

According to energy conservation before elongation and after the elongation the total energy of the system will be equal i.e.

$$\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + \frac{1}{2}k\Delta\ell^2$$

So Student C is correct.

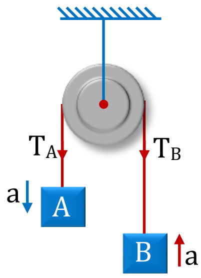
14. Ans. (B)

$mv'(\ell_0 + \Delta\ell) = mv_0\ell_0$ ...(1)

$\frac{1}{2}mv_0^2 = \frac{1}{2}mv'^2 + \frac{1}{2}k\Delta\ell^2$...(2)

By solving both equation we get $k = 210\ N/m$.

15. Ans. (5)



$40 - T_A = 4a$ ...(i)

$T_B - 20 = 2a$ ...(ii)

$(T_A - T_B) \times 0.1\ \frac{M(0.1)^2}{2}\alpha$ ...(iii)

$T_A - T_B = 2\alpha$ ...(vi)

$a = \alpha R \Rightarrow a = \frac{\alpha}{10}$ ...(iv)

Now (1) + (2) $\Rightarrow T_B - T_A + 20 = 6a$

$\therefore \quad -2\alpha + 20 = 6a$

$\therefore \quad -2\alpha + 20 = \frac{6\alpha}{10} \Rightarrow 20\ \frac{26\alpha}{10}$

$\therefore \quad \alpha = \frac{200}{26}$

$\therefore \quad \alpha = \frac{100}{13}$

$\therefore \quad a = \frac{\alpha}{10} = \frac{10}{13} m/s^2$

$$\frac{1}{2}\alpha t^2 = \frac{1}{2}\times\frac{100}{13}\times\frac{100}{2\pi} = \frac{5000}{26\pi} r\ell v$$

16. **Ans. (3)**

$F = ma \Rightarrow a = 5\ m/s^2$

$F(h-0.9) = mg \times 0.3$

$\therefore \quad 100(h-0.9) = 200 \times 0.3$

$\therefore \quad h = 1.5$

and $\quad 100(0.9-h) = 200 \times 3$

$\Rightarrow \quad h = 0.3$

$\therefore \quad 0.3 < h < 1.5$

17. **Ans. (3)**

$$\frac{mR^2}{2}\omega = \left(\frac{mR^2}{2} + mR^2\right)\omega'$$

$$\therefore \quad \omega' = \frac{\omega}{3}$$

18. **Ans. (A-R), (B-S), (C-P), (D-Q)**

(A) $V_{cm} = l$ and $\omega R = 1$

(B) $v_{rel\ 1/3} \neq 0 \quad \therefore$ Not a rigid body

(C) $v_2 = \hat{i}\,,\ v_3 = 0$

i.e. rotating at same position only (p).

(D) $V_i = -\hat{i} - \hat{j}$

$V_4 = 0$

(Lowest point) so (Q).

Important Notes

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