Circular Motion and Work, Energy and Power

SOLUTIONS

EXERCISE – O

1. **Ans. (D)**

$$\omega_{rel} = \frac{(v_\perp)_{rel}}{R} = \frac{8\sin 30° + 6\sin 30°}{10}$$

$= 0.7$ rad/s

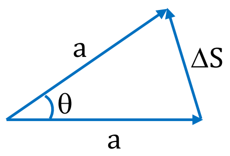
2. **Ans. (D)**

$v = \omega \cdot r$

$$= \frac{2\pi}{60} \times 6 = \frac{\pi}{5} = 2\pi\ mm/\text{sec}$$

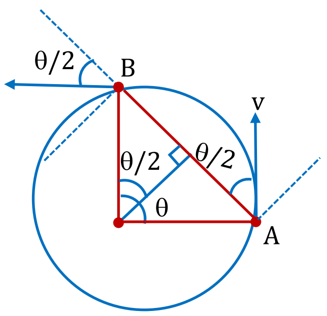
$$\Delta v = \sqrt{2}v$$

3. **Ans. (B)**



$$\Delta S = 2a\sin\frac{\theta}{2} = 2a\sin\frac{\omega t}{2}$$

4. **Ans. (C)**



$$\omega_{B,A} = \frac{\text{Velocity of } B \text{ w.r.t } A \text{ perpendicular to line } AB}{\text{Length of } AB}$$

$$\omega_{B,A} = \frac{2V\sin(\theta/2)}{2R\sin(\theta/2)}$$

$\omega_{B,A} = V/R$

5. **Ans. (B)**

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{10 \times 2\pi}{100} rad/s$$

6. Ans. (D)

$\omega_f = \omega_0 + \alpha t$

$$\alpha = \frac{(\omega_f - \omega_0)}{t_1}$$

$$\omega_f^2 = \omega_0^2 + 2\alpha\theta$$

$$\theta = \frac{(\omega_f^2 - \omega_0^2)}{2\alpha}$$

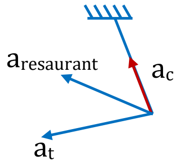
$$\theta = \frac{(\omega_f - \omega_0)(\omega_f + \omega_0)}{2\alpha}$$

$$2\pi n = \frac{(\omega_f - \omega_0)(\omega_f + \omega_0)}{2\alpha}$$

$$n = \frac{(\omega_f - \omega_0)(\omega_f + \omega_0)}{4\pi \times \left(\frac{\omega_1 - \omega_0}{t_1}\right)} = \; = \frac{(\omega_f + \omega_0)t_1}{4\pi}$$

7. Ans. (B)

$\vec{a} = \vec{a}_c + \vec{a}_t$



8. Ans. (D)

About centre of circle :

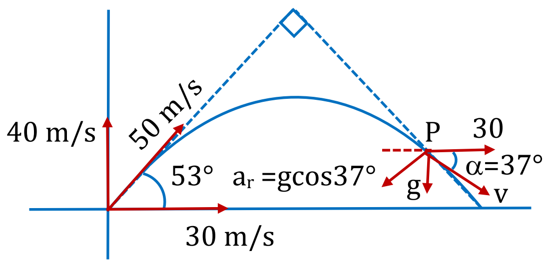
$\omega_C = 2\omega = 0.8$ rad/sec

$v = \omega_C \cdot R = 0.4\ m/s$

$a_C = \omega_C^2 R = 0.32\ m/sec$

9. Ans. (A)

Velocity at point P $v_x = 30$, $v_y = gt - 40$



$$V_x = v\cos\alpha = 30$$

$$v\cos 37 = 30$$

$$v = \frac{75}{2} m/\text{sec.}$$

$$v_y = \frac{75}{2} \times \sin 37° = \frac{45}{2} m/\text{sec.}$$

$$R_c = \frac{v^2}{a_\perp} = \frac{(V_x^2 + v_y^2)}{g\cos 37°}$$

$R_C = 175.78\ m$

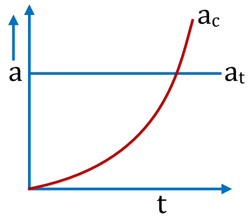
10. Ans. (D)

$v = \alpha\sqrt{x} \Rightarrow \dfrac{dx}{dt} = \alpha\sqrt{x} \Rightarrow x = \dfrac{\alpha^2 t^2}{4}$

$a_t = v\dfrac{dv}{dx} = \alpha\sqrt{x}\cdot\dfrac{\alpha}{2\sqrt{x}} = \dfrac{\alpha^2}{2}$ = constant

$a_C = \dfrac{v^2}{r} = \dfrac{\alpha^2 x}{r} = \dfrac{\alpha^2}{r}\times\dfrac{\alpha^2 t^2}{4}$

$a_C \propto t^2$



11. Ans. (B)

$\dfrac{\vec{a}.\vec{p}}{|\vec{a}||\vec{p}|} = \cos\theta$

$\Rightarrow$ $0 < \theta < 90°$

$\Rightarrow$ Accelerated circular motion.

12. Ans. (C)

$v = 2t$

$a_t = 2\ m/s^2$

$a_c = \dfrac{v^2}{r} = \dfrac{4t^2}{20} = \dfrac{t^2}{5} = \dfrac{25\times 3}{50} = \dfrac{3}{2} m/s^2$

So $a_{total} = \sqrt{4+\dfrac{9}{4}} = \sqrt{\dfrac{25}{4}} = 2.5$

13. Ans. (C)

$\vec{F} \perp \vec{v}$

Speed remains constant and hence kinetic energy is constant.

14. Ans. (A)

$\dfrac{mv^2}{r}$ is not actual force, other forces like mg, T and friction provides it according to situation.

15. Ans. (A)

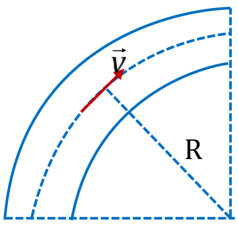
$\dfrac{mv^2}{R} \le \dfrac{mg}{10}$

$R \ge \dfrac{10v^2}{g}$

$v = 108.\dfrac{5}{18} = 30 m/s$

$R \ge 10\times\dfrac{(30)^2}{10}$

$R \ge 900m$



For R_{min} = 900 m

Min length = $\dfrac{2\pi R_{min}}{4}$

$= \dfrac{2\pi\times 900}{4}$ = 1413 m

Min. length = 1.413 km

16. **Ans. (A)**

$\omega = \alpha t \Rightarrow f_S = m\omega^2\ell$

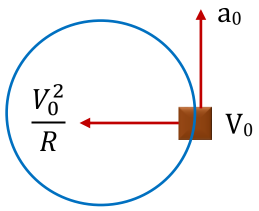
$N = m\alpha\ell \Rightarrow \mu m\alpha\ell = m\omega^2\ell$

$\mu\alpha = \alpha^2 t \Rightarrow t = \sqrt{\frac{\mu}{\alpha}}$

17. **Ans. (B)**

When it just start to slip

$$\left(\frac{V_0^2}{R}\right)^2 + a_0^2 = (\mu g)^2$$



$$\frac{V_0^4}{R^2} + a_0^2 = (\mu g)^2$$

$$\boxed{R = \frac{V_0^2}{\sqrt{(\mu g)^2 - a_0^2}}}$$

18. **Ans. (A)**

$N = m\alpha r$

$f_S = m\omega^2 r$

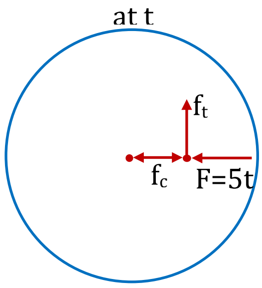
For static, $f_S \le f_{S,\max}$

$\Rightarrow \quad mr\omega^2 \le \mu\, mr\,\alpha$

$$\mu \ge \frac{\omega^2}{\alpha}$$

19. **Ans. (D)**

$f_t = ma_t = 3\ N$



$f_C = 5t - m\omega^2 R$

At $\quad t = 1$

$f_C = 9 - 5 = 4N$

$f_{net} = 5\ N = \mu mg$

$\mu = 0.5$

20. **Ans. (C)**

If force is constant than work done by a force on block is

$W = FS\cos\theta$

F = force

S = displacement of point of application of force.

21. **Ans. (C)**

$\vec{F} = \frac{30}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$, $\Delta\vec{r} = \hat{i} + \hat{j} + \hat{k} \Rightarrow W = 30\sqrt{3}J$

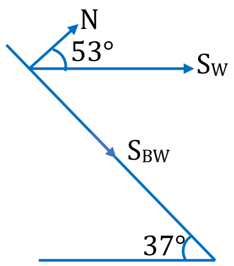
22. **Ans. (C)**

$$T - mg = -\frac{mg}{2} \Rightarrow T = \frac{mg}{2}$$

$$\Rightarrow \quad W = -T \cdot x = -\frac{1}{2}mgx$$

23. **Ans. (B)**

Displacement of the block in horizontal direction equals to the wedge displacement and also the displacement of block along inclined with respect to wedge



$$\vec{s}_B = \vec{s}_{BW} + \vec{s}_W$$

$\therefore \quad W_{normal} = NS_W \cos 53°$

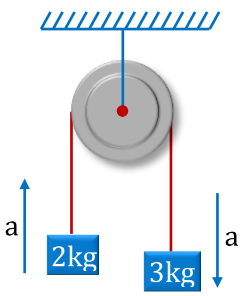
$$= 10 \times 10 \times \frac{4}{5} \times 10 \times 2 \times \frac{3}{5} = 960\,J$$

24. **Ans. (A)**

$$a = \frac{30-20}{5} = 2m/\sec^2$$

$$T = \frac{2\times6\times10}{5} = 24$$

$$\text{Displacement} = \frac{1}{2}\times2\times2^2 = 4$$



Work done by tension on

$3kg$ block $= -\,T \times d = -24 \times 4 = -96\,J$

25. **Ans. (A)**

$$\vec{F}.d\vec{r} = 0$$

$$\Rightarrow \quad (2\hat{i}+3\hat{j}).(dx\hat{i}+dy\hat{j}) = 0$$

$$\Rightarrow \quad (2\hat{i}+3\hat{j}).\left(dx\hat{i}+\left(-\frac{k}{3}dx\right)\hat{j}\right) = 0$$

$\Rightarrow \quad 2 - k = 0$

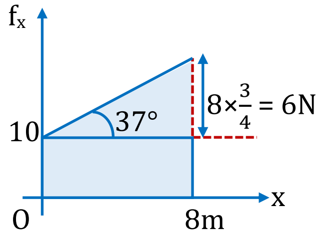
$\Rightarrow \quad k = 2$

26. **Ans. (A)**

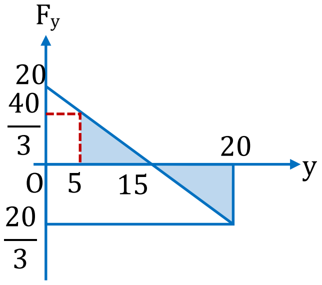
F is constant so it is conservative.

$W_1 = W_2$

27. Ans. (C)



Displacement along x is from 0 to 8 m



Work done = shaded area

$W_x = 80 + 24 = 104\,J$

Displacement along y is from 5 to 20

Work done = shaded area

$$W_y = \frac{1}{2}\times\frac{40}{3}\times10-\left(\frac{1}{2}\times\frac{20}{3}\times5\right)$$

$$=\frac{400}{6}-\frac{100}{6}=50J$$

Similarly for z

$$W_z=\frac{1}{2}\times16\times12=96J$$

$W = 104 + 50 + 96 = 250\,J$

28. Ans. (B)

The force acting $f(x,y)=x^2\hat{i}+y^2\hat{j}$

Thus we get $F_x = x^2$ and $F_y = y^2$

Now, $\frac{\partial F_y}{\partial x}=\frac{\partial y^2}{\partial x}=0$

Similarly, $\frac{\partial F_x}{\partial y}=\frac{\partial x^2}{\partial y}=0$

For the force to be a conservative force, $\frac{\partial F_y}{\partial x}-\frac{\partial F_x}{\partial y}=0$

$\Rightarrow$ $F(x, y)$ is conservative force.

Work done by a conservative force is zero for a closed loop.

$\Rightarrow$ Work done by force $F\,(x, y)$ around the path given is zero.

29. Ans. (D)

$\vec{F} = (3xy - 5z)\hat{j} + 4z\hat{k}$

$x^2 = y$

$\Rightarrow \quad x = y^{\frac{1}{2}}$

Substitute in above equation

$\vec{F} = (3y^{3/2} - 5z)\hat{j} - 4z\hat{k}$

$dw = \vec{F} \cdot \left(dx\hat{i} + dy\hat{j} + dz\hat{k}\right)$

In ($\hat{j}$) direction

$$W = \int_0^4 3y^{3/2} dy$$

$$\rightarrow \left[\frac{3y^{5/2}}{5/2}\right]_0^4$$

$$W = \frac{192}{5}\text{ Joule}$$

(in 'z-direction' work done is always zero)

30. Ans. (D)

Mass of man is M and his speed is u. speed of boy is v

$\Rightarrow \quad \frac{1}{2}Mu^2 = \frac{1}{2} \times \left(\frac{1}{2} \times \frac{M}{2}v^2\right) \quad$ ...(i)

and $\quad \frac{1}{2}M(u+1)^2 = \frac{1}{2} \times \left(\frac{M}{2}v^2\right) \quad$ ...(ii)

On solving equation (i) and (ii) we get

$$u = \left(\sqrt{2}+1\right)m/\text{sec}$$

31. Ans. (B)

$u = 0$, $v = 5(2)^{3/2}$

$\Rightarrow \quad W = \frac{1}{2}m\left(v^2 - u^2\right) = 50J$

32. Ans. (D)

$\frac{1}{2}mv^2 = mgh + \frac{1}{2}mu^2 = 150 + 5 = 155J$

33. Ans. (A)

$\frac{1}{2}mv_0^2 = mgh + \mu mgd$

$18 = 11 + 6d$

$D = \frac{7}{6}$

34. Ans. (D)

$W_F + W_{\text{spring}} = \Delta KE$

$$W_F - \frac{1}{2}\times 10\left((2)^2 - (5)^2\right) = \frac{1}{2}\times 2\times 5^2$$

$W_F = -80J$

35. Ans. (D)

$$k_f - k_i = W_g + W_f + W_{kx}$$

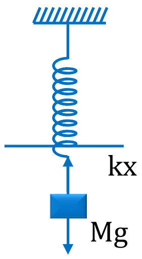
Let block travels x before coming to rest, then

$0 = 4J - 12\,x - 8x$

$\Rightarrow \quad x = 0.2 \Rightarrow K_f - 0 = (4 - 1.2 - 2.4)\,J$

$\Rightarrow \quad K_f = 0.4\,J$

36. Ans. (A)



Block will have max velocity at equilibrium

$$k \times \frac{1}{10} = 100$$

$k = 1000\ N/m$

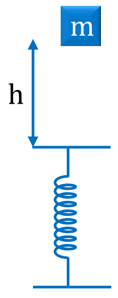
Apply $W.\,E.$ theorem

$$W_g + W_S = \frac{1}{2}\,mv^2$$

$$100 \times \frac{1}{10} - \frac{100}{2}\times\frac{1}{10} = \frac{1}{2}10v^2 = 5v^2$$

$v = 1m/s$

37. Ans. (B)



$mgx_0 = \frac{1}{2}k$

Velocity is maximum at equilibrium position

$kx_0 = mg$

$x_0 = \frac{mg}{k} = 5cm$

Height from surface of table = 20 – 5 = 15 *cm*

38. Ans. (A)

$y_B = 2y_A \Rightarrow y_A = \frac{1}{2}\ell$ and $u_B = 2u_A$

$mg\ell - \frac{1}{2}mg\ell = \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 \Rightarrow \frac{1}{2}mg\ell = \frac{5}{2}mv_A^2$

39. Ans. (B)

Maximum extension will be at the moment when both masses stop momentarily after going down. Applying *W-E* theorem from starting to that instant.

$k_f - k_i = W_{gr.} + W_{sp} + \left(-\frac{1}{2}Kx^2\right) W_{ten}.$

$0 - 0 = 2\,M.g.x + \left(-\frac{1}{2}Kx^2\right) + 0$

$x = \frac{4Mg}{K}$

System will have maximum *KE* when net force on the system becomes zero. Therefore

$2\,Mg = T$ and $T = kx$

$\Rightarrow \quad x = \frac{2Mg}{K}$

Hence *KE* will be maximum when $2M$ mass has gone down by $\frac{2Mg}{K}$.

Applying *W/E* theorem

$k_f - 0 = 2Mg.\frac{2Mg}{K} - \frac{1}{2}K.\frac{4M^2g^2}{K^2}$

$k_f = \frac{2M^2g^2}{K^2}$

40. **Ans. (C)**

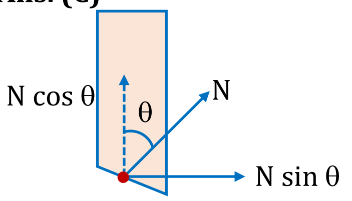
$$dw = \vec{F}\cdot d\vec{r} = \frac{A}{r^2}\hat{r}.dr\hat{r} = \frac{A}{r^2}dr$$

$$w = \int_1^5 \frac{A}{r^2}dr$$

41. **Ans. (B)**

Work done by conservative force = change in potential energy.

42. **Ans. (C)**



$N\cos\theta = mg$ ...(i)

$N\sin\theta = kx$ ...(ii)

From equation (i) and (ii)

$$\frac{kx}{mg} = \tan\theta$$

So potential energy = $\frac{1}{2}kx^2 = \frac{1}{2}k\left(\frac{mg\tan\theta}{k}\right)^2$

$$= \frac{m^2g^2\tan^2\theta}{2k}$$

43. **Ans. (C)**

$$U(at\ x=2) = 10\left(\frac{3.5-2}{3.5-1}\right) = 6J$$

Applying *COE*

$$6 + 0 = 2 + \frac{1}{2}mv^2$$

$v^2 = 4$

$\Rightarrow$ $v = 2m/\text{sec}$

44. **Ans. (B)**

$U = ax^3 - bx^2$

$$\frac{dU}{dx} = 3ax^2 - 2bx$$

$$\frac{d^2U}{dx^2} = 6ax - 2b$$

For equilibrium

$$\frac{dU}{dx} = 0 \Rightarrow 3ax^2 - 2bx = 0 \Rightarrow x = 0, \frac{2b}{3a}$$

For $x = \frac{2b}{3a}$, $\frac{d^2U}{dx^2} = 6a\times\frac{2b}{3a} - 2b = 2b > 0$

Therefore at $x = \frac{2b}{3a}$,

Potential energy is minimum.

This is a condition of stable equilibrium.

45. Ans. (A)

$$F = \frac{-dU}{dx}$$

$dU = -F.dx$

$$\int_0^U dU = \int_0^x -F.dx$$

$U = 8x - 2x^2 + 8$

46. Ans. (A)

Kinetic energy is (+*ve*) and total *ME* is 3 joule. So, *PE* cannot be 6*J*.

47. Ans. (A)

$dW = \vec{F}.\vec{dx}$ work to be maximum $\frac{dW}{dx}$

= 0 i.e. force must be zero

$F = Kx\,(x-\ell) = 0;\ x = 0;\ x = \ell$

48. Ans. (D)

$V = \sqrt{(10)^2 - 2\times10\times2} = \sqrt{60}$ $\Delta V = \sqrt{(60)+100} = \sqrt{160}$

49. Ans. (C)

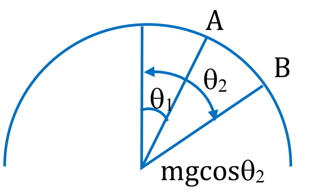
At *B*, normal becomes zero.

So,

$$V_B = \sqrt{gR\cos\theta} \quad \text{...(i)}$$

By *COE* $V_A = \sqrt{2gR(1-\cos\theta)}$

Applying *COE* at *A* and *B*



$$-mgR(1-\cos\theta_1) = -mgR(1-\cos\theta_2) + \frac{1}{2}mv_B^2$$

$$mgR\cos\theta_1 = mgR\cos\theta_2 + \frac{1}{2}mgR\cos\theta_2$$

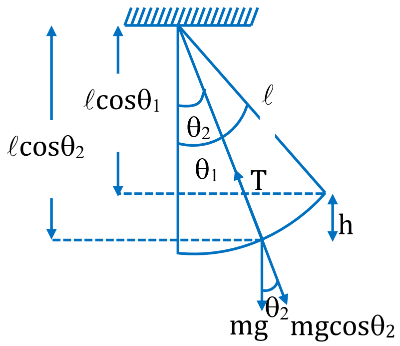
$$mgR\cos\theta_1 = \frac{3mgR\cos\theta_2}{2}$$

$$\Rightarrow \quad \cos\theta_2 = \frac{2}{3}\cos\theta_1$$

50. Ans. (A)

$h = \ell(\cos\theta_2 - \cos\theta_1)$

Applying conservation of mechanical energy at point *A* and *B*



$$\frac{1}{2}mv^2 = mgh$$

$$\Rightarrow \quad v = \sqrt{2gh} = \sqrt{2g\ell(\cos\theta_2 - \cos\theta_1)}$$

At B, $T - mg\cos\theta_2 = \frac{mv^2}{\ell}$

Where $v = \sqrt{2g\ell(\cos\theta_2 - \cos\theta_1)}$

$$\Rightarrow \quad T = mg\cos\theta_2 + \frac{m}{\ell}[2g\ell(\cos\theta_2 - \cos\theta_1)]$$

$$= mg(3\cos\theta_2 - 2\cos\theta_1)$$

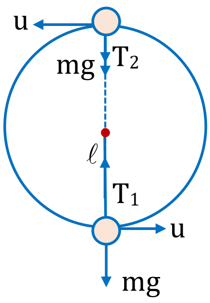
51. Ans. (D)

At lowest point,

$$T_1 - mg = \frac{mu^2}{\ell} \Rightarrow T_1 = \frac{mu^2}{\ell} + mg$$

At highest point,

$$T_2 + mg = \frac{mu^2}{\ell} \Rightarrow T_2 = \frac{mu^2}{\ell} - mg$$



52. Ans. (C)

$$mg(l - l\cos\theta) = \frac{1}{2}mv^2$$

$$v^2 = 2gl(1-\cos\theta)$$

$$T - mg = \frac{mv^2}{l}$$

$$T - mg = 2mg(1-\cos\theta)$$

$$T = \mu.4mg$$

$$\mu = \frac{3mg - 2mg\cos\theta}{4mg}$$

$$\mu = \frac{3mg - 2mg\frac{1}{2}}{4mg} = \frac{3-1}{4} = \frac{1}{2}$$

53. Ans. (D)

v_E must be $\sqrt{4gR}$

So by COME, $v_F = \sqrt{2gR}$, $v_B = \sqrt{3gR}$

$N_F = \frac{mv_F^2}{R}$, $N_E = mg = \frac{mv_E^2}{R}$

54. Ans. (B)

$$a = \frac{v_1}{t_1}, v = at = \frac{v_1 t}{t_1}$$

$$P = mav = \frac{mv_1^2 t}{t_1^2}$$

55. Ans. (B)

$F \propto v$

$P = F \cdot v \propto v^2$

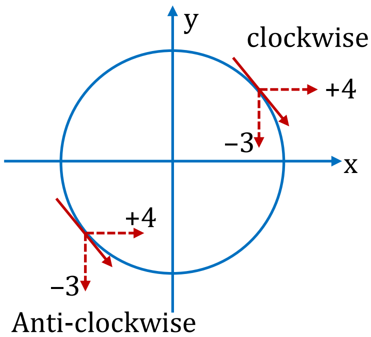
$\Rightarrow \quad v \propto \sqrt{P}$

56. Ans. (D)

$P = \vec{F}.\vec{v}$

Tension is always perpendicular to velocity.

57. Ans. (AC)



58. Ans. (ABCD)

$\ell = 1m$

$\omega = \frac{\theta}{t} = \frac{2\pi}{60} = \frac{\pi}{30} rad/\sec$ (B correct)

$v = \omega r$

$v = \frac{\pi}{30} \times 100 \frac{cm}{\sec} = \frac{10\pi}{3} cm/\sec$ (A correct)

Total change in velocity = 0

$a_{avg} = 0$

Instantaneous acceleration $a = \omega^2 r$

$= \frac{\pi^2}{(30)^2} \times 100$

$a = \frac{\pi^2}{9} cm/s^2$

59. Ans. (BCD)

$\left|\frac{d\vec{v}}{dt}\right|$ = magnitude of net acceleration

$\frac{d|\vec{v}|}{dt}$ = tangential acceleration.

60. Ans. (ABC)

$x^2 + v^2 = 1$

$\Rightarrow \quad 2x + 2v\frac{dv}{dx} = 0$

$\Rightarrow \quad v\frac{dv}{dx} = -\vec{x} = \vec{a}$

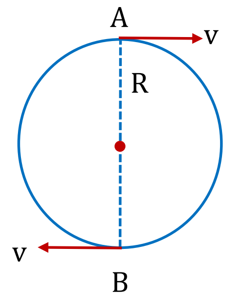
$\Rightarrow \quad Q\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) \Rightarrow x_Q = \frac{1}{\sqrt{2}} \Rightarrow |a| = \frac{1}{\sqrt{2}}$

61. Ans. (BD)

Both particles take same time *t*.

For *A*: $\theta = \omega t + \frac{1}{2}\alpha t^2$

$$\theta = \left(\frac{v}{R}\right)t \qquad ...(1)$$



For *B*: $\pi + \theta = \left(\frac{v}{R}\right)t + \frac{1}{2}\left(\frac{a}{R}\right)t^2 \qquad ...(2)$

From (1) and (2) : $\pi = \frac{1}{2}\left(\frac{a}{R}\right)t^2$

Time of collision

$$t = \sqrt{\frac{2\pi R}{a}}$$

$$\omega_B = \frac{v}{R} + \frac{a}{R}.t$$

Just before collision : $v_A = v$

$$v_B = v + \sqrt{2\pi a R}$$

$$v_B - v_A = \sqrt{2\pi a R} \quad v_{B/A} = \sqrt{2\pi a R}$$

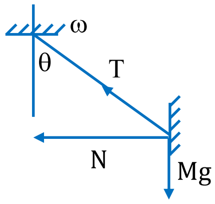
62. Ans. (ABC)

$$\vec{r} = ut\left[\cos\omega t\hat{i} + \sin\omega t\hat{j}\right]$$

$$\Rightarrow \quad \vec{v} = \frac{d\vec{r}}{dt} = u\left[\cos\omega t\hat{i} + \sin\omega t\hat{j}\right] + ut\left[-\omega\sin\omega t\hat{i} + \omega\cos\omega t\hat{j}\right]$$

$$|a_1| = \frac{d|\vec{v}|}{dt} = \frac{\omega^2 t u}{\sqrt{1+\omega^2 t^2}}$$

63. Ans. (ABC)



$T\cos\theta = mg \qquad ...(i)$

$N + T\sin\theta = \frac{mv^2}{r} \qquad ...(ii)$

64. Ans. (BCD)

$$\frac{\sqrt{3}}{2}(T_1 + T_2) = m\omega^2\ell\cos 30°$$

$T_1 + T_2 = m\omega^2\ell$ ...(i)

$T_1 \sin 30 - T_2 \sin 30 = mg$

$T_1 - T_2 = 2mg$ ...(ii)

When w increases, T_1 increases and T_2 decreases

$T_2 \geq 0$

$$\omega \geq \sqrt{\frac{2g}{\ell}}$$

65. Ans. (ABC)

$5 + f_1 = 1 \times \omega^2 \times \frac{1}{2}$

$5 + f_2 = 2 \times \omega^2 \times \frac{1}{2}$

Depending on sign of f_1 and f_2 they can be inward or outward.

66. Ans. (ABCD)

$a_C = \frac{v^2}{r} = g\tan\theta$

$\tan\theta = \frac{v^2}{rg}$

$r = \ell \sin\theta$

$T\cos\theta = mg$

67. Ans. (AB)

If speed is less tendency of sliding is downward and friction up the plane for high speeds friction is down the plane.

68. Ans. (AD)

In static friction, there is no relative motion between surfaces.

69. Ans. (AB)

Work done by force is frame dependent quantity $W = \int \vec{F} \cdot \overrightarrow{dS}$

Displacement of point of application of force depends on reference frame.

Kinetic friction opposes the relative slipping between two surfaces.

70. Ans. (ACD)

If the body slips over a rough surface such that normal reaction of the surface has to balance only the normal component of weight of the body, then the energy lost against friction depends only upon the horizontal component of displacement and is equal to μmgx. It does not depend upon the shape of the surface.

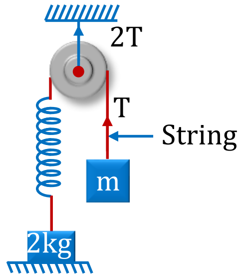
If the vertical component of displacement of the body is equal to y, increase in its gravitational potential energy will be equal to mgy.

Hence, total work done by the force will be equal to $\mu mgx + mgy$.

71. Ans. (BCD)

At the time at the time of maximum elongation velocity of mass *m* is zero.

Applying W.E.T.



$$\frac{1}{2}k\left(0-x^2\right)+mgx=0$$

$$2mg = kx$$

For 2*kg* block

$$kx = 20$$

⇒ When acceleration is zero

$$2mg = 20$$

$$mg = 10$$

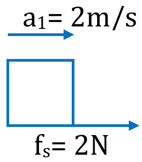
So tension in string is 10 *N* then tension in rod is 20.

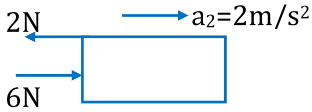
72. Ans. (ACD)

Work done by friction on $A \neq 0$

$$V = 2 \times 2 = 4\ m/s$$

$$K.E.\ A = \frac{1}{2}\times 1\times 4^2 = 8J$$





73. Ans. (ABC)

$$\vec F=\frac{3\vec r}{r^3}=\frac{3\hat r}{r^2}$$

$\vec F$ is a constant force and constant force is always conservative.

74. Ans. (ACD)

Change in *KE* is due to work done by all forces and not only by a single conservative force

75. Ans. (AC)

Force acting on the particle, $\vec F=-\frac{\partial U}{\partial x}\hat i-\frac{\partial U}{\partial y}\hat j=7\hat i-24\hat j$

Acceleration of particle = $\frac{F}{m}=\frac{\sqrt{7^2+24^2}}{5}=\frac{25}{5}$ = 5 ms^{-2}

Velocity of particle = $\vec v=\vec u+\vec a t$ + $\left(1.4\hat i-4.8\hat j\right)t$,

Speed at 4 sec. = $\sqrt{(14.4+5.6)^2+(19.2-4.2)^2}$ = 25 ms^{-1}

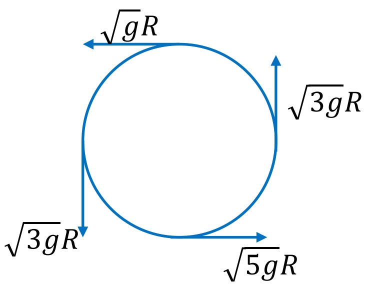
Now $\vec a.\vec u=\left(1.4\hat i-4.8\hat j\right)$. = 20.16 – 20.16 = 0

⇒ The acceleration of particle is normal to its initial velocity.

76. Ans. (ABCD)

Energy conservation and at highest point $T = 0$

When particle just completes vertical circle then



$$v_{top} = \sqrt{gR}$$

So $$a_{top} = \frac{v^2}{R} = \frac{\left(\sqrt{gR}\right)^2}{R} = g$$

$$v_{bottom} = \sqrt{5gR}$$

So $$a_{bottom} = \frac{v^2}{R} = 5g$$

When velocity is vertically up or vertically down

Then $v = \sqrt{3gR}$

$$a_t = g, \quad a_c = \frac{v^2}{R} = 3g$$

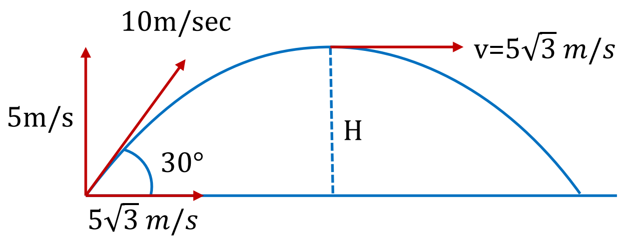
$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{10g^2} = g\sqrt{10}$$

77. Ans. (ABCD)

(A) $W = \frac{1}{2}m\left(v_f^2 - v_i^2\right)$

$$= W = \frac{1}{2} \times 2 \times (75 - 100) = -25\,J$$

(B) Change in KE = work done = 0



(C) $t = \frac{u\sin\theta}{g} = \frac{5}{10} = \frac{1}{2}$ sec

$$P_{avg} = \frac{W}{\text{time}} = \frac{-25}{\frac{1}{2}} = -50\ watt$$

(D) $$R_C = \frac{v_T^2}{a_e} = \frac{\left(5\sqrt{3}\right)^2}{10} = 7.5m$$

78. Ans. (ABCD)

Unit vector in the direction of displacement is $\dfrac{3\hat{i}+4\hat{j}+5\hat{k}}{5\sqrt{2}}$

$\therefore$ Velocity vector = $5\sqrt{2}\left(\dfrac{3\hat{i}+4\hat{j}+5\hat{k}}{5\sqrt{2}}\right)=3\hat{i}+4\hat{j}+5\hat{k}$

$$W=\vec{F}\cdot\vec{s}=\left(3\hat{i}+4\hat{j}\right)\cdot\left(3\hat{i}+4\hat{j}+5\hat{k}\right)=25J$$

$$P=\vec{F}\cdot\vec{v}=\left(3\hat{i}+4\hat{j}\right)\cdot\left(3\hat{i}+4\hat{j}+5\hat{k}\right)=25\text{ watt}$$

If velocity doesn't change $\Rightarrow$ net force = 0

$\Rightarrow$ Some other force must be present.

79. Ans. (ACD)

(A) $F = 0$ (equilibrium)

$$F=\frac{-dU}{dx}=0$$

and $\dfrac{d^2U}{dx^2}<0$

(C) At 1 and 3

$F=\dfrac{dU}{dx}=0$ (No power delivered)

80. Ans. (C)

$\Delta y_{\ell}=\dfrac{1}{2}a_0t_0^2$ in downward

$$W_g=+mg\Delta y$$

$$=\frac{1}{2}mga_0t_0^2$$

81. Ans. (D)

$N = m\,(g - a_0)$; $W_N = -N\Delta y_{\ell}=-\dfrac{Na_0t_0^2}{2}$

82. Ans. (B)

$W_{net} = W_g + W_N = \dfrac{1}{2}mga_0t_0^2-\dfrac{m\left(g-a_0\right)a_0t_0^2}{2}$

83. Ans. (D)

For observer A, displacement of mass is zero.

84. Ans. (B)

If the particle is released at the origin, it will try to go in the direction of force. Here $\dfrac{dU}{dx}$ is positive and hence force is in negative x-direction, as a result it will move towards –ve x-axis.

85. Ans. (B)

When the particle is released at $x=2+\Delta$ it will reach the point of least possible potential energy ($-15J$) where it will have maximum kinetic energy. $\therefore \frac{1}{2}mv_{\max}^2=25 \Rightarrow v_{\max}=5m/s$

86. Ans. (D)

For stable equilibrium $\frac{dU}{dx}=0$ & $\frac{d^2U}{dx^2}=\oplus ve$, For unstable equilibrium $\frac{dU}{dx}=0$ & $\frac{d^2U}{dx^2}=-ve$

87. Ans. (A-P,Q), (B-P,Q,S), (C-P,Q,R), (D-P,Q,R)

No slipping anywhere.

Net force is centripetal as v = constant.

88. Ans. (A-R), (B-S) (C-Q), (D-P)

(A) $a_t=0, a_C\neq 0$

(B) $a_t\neq 0, a_C\neq 0$

(C) $a_t\neq 0, a_C\neq 0$

(D) $a_t\neq 0, a_C=0$ $(v=0)$

89. Ans. (A-Q), (B-P), (C-Q), (D-S)

$y_A=2y_B \Rightarrow v_A=2v_B$

A moves $\downarrow$ and

B moves $\uparrow$ as

Tension on B is more than A.

$$mgh-\frac{mgh}{2}=\frac{1}{2}mv_A^2+\frac{1}{2}mv_B^2$$

$$\frac{gh}{2}=\frac{5}{2}v_B^2$$

$$\Rightarrow \quad v_B=\sqrt{\frac{5}{5}}=1, v_A=2$$

$mg-T=ma_A$

$2T-mg=ma_B$

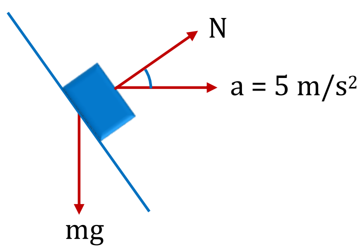
$a_A=2a_B$

90. Ans. (A-Q), (B-P), (C-Q), (D-P)

(A) $W_N+W_{mg}=\Delta KE$

$V_f=at=5\times 2=10\ m/s$

$W_N=\frac{1}{2}mV_f^2-0=\frac{1}{2}\times\frac{1}{2}\times 10^2=25J$



(B) $W_{mg}=0$ (No horizontal displacement)

(C) $W_{net}=\Delta KE=25J$

(D) $W.D_{\text{out}}=0(S=0)$

91. **Ans. (A-Q,S,T), (B-Q,S,T), (C-P), (D-R,T)**

For (A) $v_A = \sqrt{2gh} = \sqrt{5gR}$ circle will be completed and $N_A - N_B = 6mg$

$$N_C = \frac{mv_C^2}{R} = 3\,mg$$

For (B) $v_A = \sqrt{9gR}$ $\quad N_C = \frac{mv_c^2}{R} = 7mg$

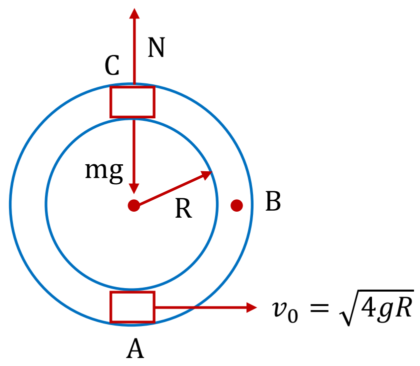
For (C) $v_A = \sqrt{2gR}$ $\quad v_C = 0,\ N_C = 0,\ F_{net}$ at $C = mg$

For (D) $v_A = \sqrt{4gR}$ $\quad$ Leaves the circle in region $B–C$

$$N_C = \frac{mv_c^2}{R} = 2mg$$

92. **Ans. (A-P,Q,R), (B-Q), (C-P,Q), (D-Q,S)**

(A)



$v_0 = \sqrt{4gR}$

$\Rightarrow$ Particle will just reached at highest point and come to at rest

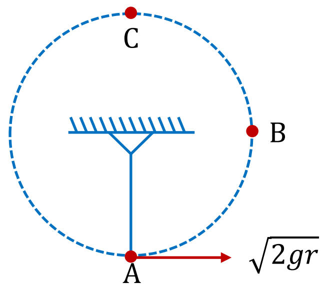
$\Rightarrow$ $V_c = 0$

(P) $N_C = mg$ (Upward)

(Q) $\vec{F} + \overrightarrow{mg} = m\overrightarrow{a_C}$ $\Rightarrow$ (at point 'A') Resultant of $\vec{F}$ and $\overrightarrow{mg}$ gives centripetal acceleration.

(R) As particle goes up speed decreases.

(B)

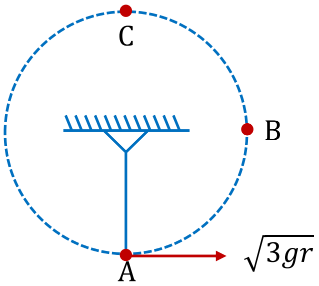


Particle will reach at 'B' and v_B become = 0. (By ME conservation)

At $B \Rightarrow v_B = 0,\ T = 0\left(\frac{T = mv_B^2}{R}\right)$

Particle first speed down then speed up.

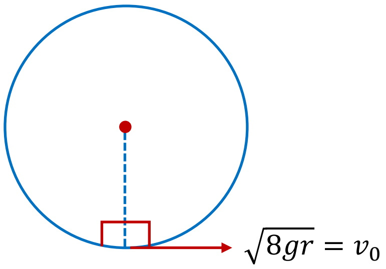
(C)



$\sqrt{2gR} < v_A < \sqrt{4gR}$

- Circle won't complete.
- Between B and C particle comes to at rest and then it will com back with increasing speed.
- At 'B', $T_B = \frac{mV_B^2}{R}$

(D)



Particle will complete circular motion.

- At 'B', $N_B = \frac{mV_B^2}{R}$
- Speed first decrease then increase.

93. **Ans. (A-S), (B-R), (C-P), (D-Q)**

$F = mg \sin \theta + \mu_k mg \cos \theta$ and $p = \vec{F}.\vec{V}$

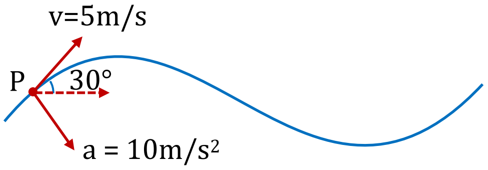
94. **Ans. (A)**

rate of speed = a_t

$a_t = 10 \cos 30°$

$= 5\sqrt{3} \text{m/s}^2$

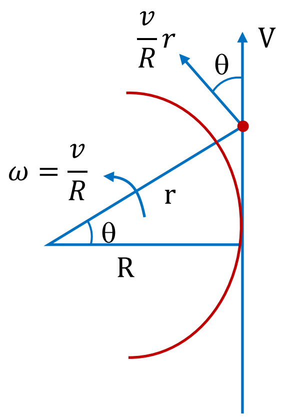
95. **Ans. (C)**



$$a_c = \frac{V^2}{R}$$

$$R = \frac{v^2}{a_c} = \frac{5 \times 5}{10 \times \frac{1}{2}} = 5m$$

96. Ans. (BD)



$$V\cos\theta = \frac{v}{R}.r \;\Rightarrow V.\frac{R}{r} = \frac{v}{R}.r$$

$$V = \frac{v}{R^2}.r^2 \;= \frac{v}{R^2}(R\sec\theta)^2 \;= \text{v Sec}^2\,\theta$$

$$\boxed{V = v\sec^2\left(\frac{vt}{R}\right)}$$

$$\boxed{a = \frac{2v^2}{R}\sec^2\left(\frac{vt}{R}\right)\tan\left(\frac{vt}{R}\right)}$$

97. Ans. (D)

$f_k = \mu_k N$

$$\boxed{f_k = \mu_k mg}$$

98. Ans. (A)

$w_{Net} = -(mg\sin\theta + \mu\, mg\cos\theta)\,dx + F_{ent}\,dx$

by WET

Δ k.ε. = w_{Net}

$\Rightarrow$ k.ε.$_f - 0 = w_{Net}$

For max. speed

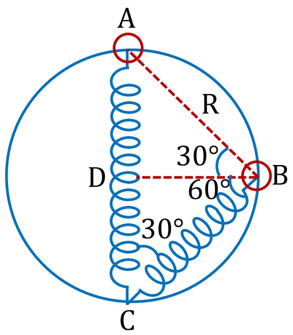
$$\frac{dk.\varepsilon._f}{dt} = 0 = \frac{dw_{Net}}{dt}$$

$$\Rightarrow \quad -(mg\sin\theta + \mu\, mg\cos\theta)\,\frac{dx}{dt} + F_{ext}\frac{dx}{dt} = 0$$

$$\Rightarrow \quad P = (mg\sin\theta + \mu\, mg\cos\theta)\, v$$

$$\Rightarrow \quad v = \frac{p}{mg\sin\theta + \mu\, mg\cos\theta}$$

99. Ans. (B)



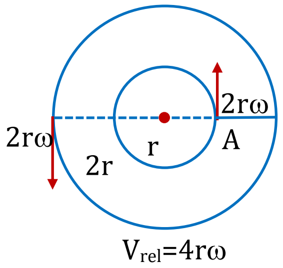
By *WET*

$$mg\,R\sin 30° + \frac{1}{2}\,k\,(2R - R\sqrt{3})^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad v = \sqrt{gR + \frac{kR^2}{m}(2-\sqrt{3})^2}$$

EXERCISE – S

1. **Ans.** 24



2. **Ans.** 4

$\omega_0 = 12$ rad/sec

$\Delta\theta = 80$ radian

$\omega = 28$ radian/sec

$\omega^2 = \omega_0^2 + 2\,\alpha\Delta\theta$ ...(i)

$\omega = \omega_0 + \alpha t$ ...(ii)

$$\left(\frac{\omega_0 + \omega}{2}\right) t = 80$$

3. **Ans.** $\frac{10}{\pi} ms^{-2}$

Change in velocity when particle complete the half revolution : $\Delta v = v_f - v_i = 2v$

Time taken to complete the half revolution $t = \frac{\pi R}{v}$

$$\text{Average acceleration} = \frac{\Delta v}{t} = \frac{2v}{\pi R/v} = \frac{2v^2}{\pi R}$$

$$= \frac{2\times 5^2}{\pi \times 5} = \frac{10}{\pi}\ m/s^2$$

4. **Ans.** (i) 2 rad/s^2

(ii) 12+2t for $t \le 2s$, 16 for $t \ge 2s$

(iii) $a = 169.01\ m/s^2$

(iv) 44 rad

(i) $\omega_f = \omega_i + 2t$

$$(2\pi)\left(\frac{980}{60\pi}\right) = 12 + \alpha \times 2$$

$a = 2$ rad/s^2

(ii) $v = R\omega = R\,(12 + 2 \times t)$

$v = 1(12 + 2t)$

After $t = 2$ sec it becomes constant.

(iii) At $t = 0.5$ sec

$v = 13\ m/\text{sec}$

$a_C = \dfrac{v^2}{R} = 169$

$a_t = R\alpha = 2$

$a_{\text{total}} = \sqrt{(169)^2 + (2)^2} \cong 169$

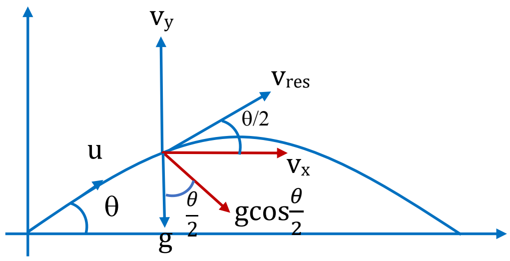
$a_{(t=3)} = a_{\text{total}} = a_C = \dfrac{v^2}{R} = \dfrac{16^2}{1} = 256\ m/s^2$

(iv) Angular displacement

$\theta = \omega t + \dfrac{1}{2}\alpha t^2$

Putting $t = 2$ sec we get ans.

5. **Ans.** $\dfrac{u^2\cos^2\theta}{g\cos^3\left(\theta/2\right)}$



$R = \dfrac{V^2}{g\cos\left(\theta/2\right)}$

$R = \dfrac{V_x^2 + v_y^2}{g\cos\left(\theta/2\right)} = \dfrac{v_x^2 + v_x^2\tan^2\left(\theta/2\right)}{g\cos\left(\theta/2\right)}$

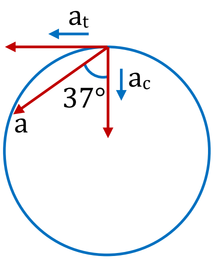
$R = \dfrac{v_x^2\left[1+\tan^2\left(\theta/2\right)\right]}{g\cos\left(\theta/2\right)}$

$= \dfrac{v_x^2\sec^2\left(\theta/2\right)}{g\cos\left(\theta/2\right)}$

$= \dfrac{v_x^2}{g\cos^3\left(\theta/2\right)} = \dfrac{u^2\cos^2\theta}{g\cos^3\left(\theta/2\right)}$

6. **Ans.** (i) $75 m/s^2$, (ii) $125 m/s^2$

$\tan 37° = \dfrac{a_t}{a_c}$



$\therefore\ \dfrac{3}{4} = \dfrac{(dv/dt)}{v^2/R}$

(i) $\dfrac{dv}{dt} = \left(\dfrac{3}{4}\right)\left(\dfrac{v^2}{R}\right)$

(ii) $a = \sqrt{\left(\dfrac{dv}{dt}\right)^2 + \left(\dfrac{v^2}{R}\right)^2}$

7. **Ans.** $\sqrt{2g}$ rad/s

$T = 2mg = mr\omega^2$

$\omega = \sqrt{\dfrac{2g}{r}} = \sqrt{2g}$ rad/s.

8. **Ans.** 5

$$\mu N = mg \quad \Rightarrow \quad \frac{\mu m v^2}{r} = mg \quad \Rightarrow \quad v = \sqrt{\frac{gr}{\mu}} = 25\ m/s$$

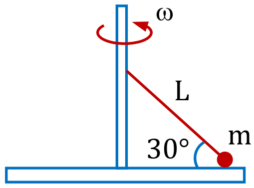
9. **Ans.** 5

$T \cos 30 = m(L \cos 30)\omega^2$ ...(i)

$T \sin 30 + N = mg$

$N = mg - T \sin 30$ ...(ii)

Putting value of T from equation in (ii) we get N.



10. **Ans.** $T = 6.6\ N$, $v_{max} = 35\ ms^{-1}$

$v = r\omega = (1.5)\ (2\pi)\left(\dfrac{40}{60}\right)$

$T = \dfrac{mv^2}{R} = \dfrac{0.25 \times 4\pi^2}{1.5} = 6.6N$

$200 = \dfrac{mv_{max}^2}{R}$, which gives $v_{max} = 35\ ms^{-1}$

11. **Ans.** $\dfrac{15\pi^2}{10} = 14.8N$, This force is exerted by blade of fan and equal force is exerted by particle on blade in same magnitude but opposite in direction.

$\omega = 2\pi n = \dfrac{2\pi \times 1500}{60}$ rad/sec

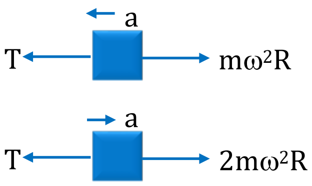
$r = \dfrac{d}{2} = 60\ cm = 0.6\ m$

$m = 1\ g = 10^{-3}\ kg$

$F = m\omega^2 r = 10^{-3} \times \left(\dfrac{2\pi \times 1500}{60}\right)^2 \times 0.6 = \dfrac{15\pi^2}{10} = 14.8$

This force is exerted by blade of fan and equal force is exerted by particle on blade in same magnitude but opposite in direction.

12. **Ans.** $\dfrac{4m\omega^2 R}{3}$



$T - m\omega^2 R = ma$ ...(i)

$2m\omega^2 R - T = 2ma$ ...(ii)

(i) + (ii)

$mw^2 R = 3ma$

$a = \dfrac{\omega^2 R}{3}$ Put in eq (i)

$T - m\omega^2 R = a = \dfrac{\omega^2 R}{3}$

$T = m\omega^2 R + \dfrac{m\omega^2 R}{3} = \dfrac{4m\omega^2 R}{3}$

13. **Ans.** Yes, $a_c = 8$, $\mu g = 1$

$v = \sqrt{\mu r g}$

So, required value of $\mu = \frac{v^2}{rg} = \frac{25}{3\times10} = \frac{5}{6} = 0.8$

$\mu = 0.1$

So, cyclist will slip.

14. **Ans.** $\frac{49}{80}$

Coefficient of friction $\mu \geq \frac{v^2}{rg}$

$\Rightarrow \quad \mu_{min} = \frac{(4.9)^2}{4\times9.8} = \frac{49}{80}$

15. **Ans.** $5\sqrt{2}\,ms^{-1}$

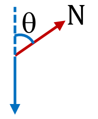
For safe driving, $v_{max} = \sqrt{\mu r g}$

$10 = \sqrt{\mu r g}$

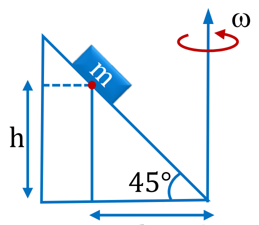
For wet road $v' = \sqrt{\frac{\mu}{2} rg} = \frac{10}{\sqrt{2}} = 5\sqrt{2}\, m/s$

16. **Ans.** 2.0

FBD of block in ground frame



$N \cos\theta = mg$



$N \sin\theta = m\omega^2 r$ [centripetal force]

$\Rightarrow \quad \tan\theta = \omega^2 r/g$

$\Rightarrow \quad \omega = \sqrt{\frac{g\tan\theta}{r}} = \sqrt{\frac{g}{h}}\,(\tan\theta) = 2 \times 1 = 2$ rad/s

17. **Ans.** $\theta = tan^{-1}\left(\frac{5}{18}\right)$

In case of banking $\tan\theta = \frac{v^2}{rg}$

Here $v = 60\ km/hr = 60 \times \frac{5}{18}\ ms-1$

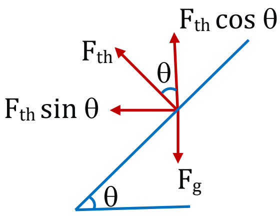
$= \frac{50}{3}\ ms^{-1}\ r = 0.1\ km = 100\ m$

So $\quad \tan\theta = \frac{\left(\frac{50}{3}\right)^2}{100\times10} = \frac{5}{18} \quad \Rightarrow \quad \theta = \tan^{-1}\left(\frac{5}{18}\right)$

18. **Ans.** 4

$F\cos\theta = mg$ ...(i)

and $F\sin\theta = \dfrac{mv^2}{R}$ ...(ii)



By solving equation (i) and (ii)

$R = 16\ km$

19. **Ans.** $\dfrac{\pi^2}{81}$

$n = 33\dfrac{1}{3}\ rev/min$

$$\omega = \frac{2\pi n}{60} = \frac{2\pi \times 33\frac{1}{3}}{60}\ \text{rad/sec.},$$

$r = 10\ cm = 0.1\ m.$

For no slipping,

$f \geq m\omega^2 r$

$\mu mg \geq m\omega^2 r$

$\mu \geq \dfrac{\omega^2 r}{g}$

$$\mu \geq \left(\frac{2\pi 100}{60\times 3}\right)^2 \frac{0.1}{10}$$

$\mu \geq \dfrac{\pi^2}{81}$

20. **Ans.** $\dfrac{2}{5}$

$N = \dfrac{mv^2}{r}$

Given $r = 5\ m$, $v = 5\sqrt{5}\ \ m/s$

for no slipping,

$f \geq mg$

$\mu_{min} N = mg$

$\mu_{min} = \dfrac{mg}{N} = \dfrac{rg}{v^2}$

$$\mu_{min} = \frac{5\times 10}{(5\sqrt{5})^2} = \frac{2}{5}$$

21. **Ans.** $\frac{2}{45}m$

v = 48 km/hr = 40/3 m/s

For safe turn without friction

$$\tan\theta = \frac{v^2}{rg} = \frac{h}{x}$$

Given $x \approx \ell$ = 1m

$$\Rightarrow \quad h = \frac{v^2}{rg} = \frac{(40/3)^2}{400\times 10} = \frac{2}{45}m$$

22. **Ans.** $\sqrt{\frac{50}{3}}\times\frac{18}{5} = 14.7\ km/h$

v = 36 km/hr = 10 m/s

$$\tan\theta = \frac{v^2}{Rg} = \frac{10\times 10}{20\times 10} = \frac{1}{2}$$

$$v_{max} = \sqrt{Rg\left(\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right)} = 15\ m/s = 54\ km/hr \quad \textbf{Ans.}$$

$$v_{min} = \sqrt{Rg\left(\frac{\tan\theta-\mu}{1+\mu\tan\theta}\right)} = \sqrt{\frac{20\times 10(0.5-0.4)}{(1+0.5\times 0.4)}}$$

$$= \sqrt{\frac{20\times 10\times 0.1}{1.2}} = \sqrt{\frac{50}{3}} \times \frac{18}{5}\ km/hr\ \textbf{Ans.}$$

23. **Ans.** (a) 28 m/s (b) 38.19 m/s

(a) Optimum speed of car is

$$\tan\theta = \frac{v^2}{rg}$$

$$v^2 = rg\tan\theta$$

$$= 10 \times 300 \times \tan 15$$

$$v^2 = 780\ m/s$$

$$v = 28\ m/s$$

(b) Maximum permissible speed to avoid slipping is

$$v_m = \sqrt{Rg\left(\frac{\mu+\tan\theta}{1-\mu\tan\theta}\right)}$$

$$= \sqrt{300\times 10\left(\frac{0.2+\tan 15}{1-0.2\tan 15}\right)}$$

$$= \sqrt{1458.7}$$

$$= 38.19\ m/s$$

24. **Ans.** 2

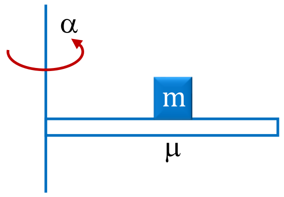
$\mu mg = m\sqrt{a_t^2 + a_n^2}$

$r\omega^2 = (\alpha t)^2 = x^2$

$(0.5 \times 10)^2 = x^4 + 9$

$x^4 = 16$

$x = 2$



25. **Ans.** $V = \left[\left(\mu^2 g^2 - a^2\right)R^2\right]^{\frac{1}{4}}$

$a_R = \sqrt{a_N^2 + a_t^2}$

$a_N = \dfrac{V^2}{R},\ a_t = \dfrac{dV}{dt} = a$

Friction, $f = ma_R$

$$\mu N = m\sqrt{\left(\frac{V^2}{R}\right)^2 + \left(\frac{dV}{dt}\right)^2}$$

$$\mu N = m\sqrt{\frac{V^4}{R^2} + a^2}$$

$$V = \left[\left(\mu^2 g^2 - a^2\right)R^2\right]^{\frac{1}{4}}$$

26. **Ans.** (a) $v = \sqrt{Rg}$, (b) $\dfrac{\pi R}{3}$, (c) $V = \sqrt{gR\cos\left(\dfrac{L}{2R}\right)}$

(a) At highest point

$$mg = \frac{mv^2}{R}$$

$$\Rightarrow v = \sqrt{Rg}$$

(b) $V = \dfrac{1}{\sqrt{2}}\sqrt{Rg}$

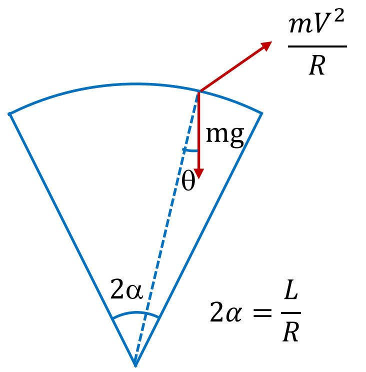
Losses contact at angle θ

$$mg\cos\theta = \frac{mV^2}{R}$$

(c) $\alpha = \dfrac{L}{2R}$

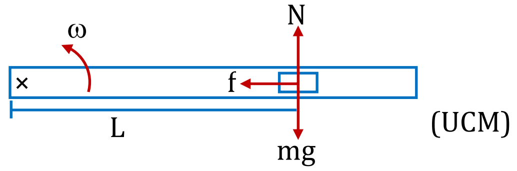
So, $\dfrac{mV^2}{R} = mg\cos\theta$

$$V = \sqrt{gR\cos\left(\frac{L}{2R}\right)}$$



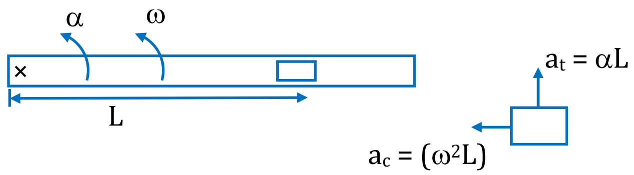
27. **Ans.** (A) $\omega_{max} = \sqrt{\frac{\mu g}{L}}$, (B) $\omega = \left(\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right)^{\frac{1}{4}}$

(A)



$$f_{max} = \mu N = (\mu mg)$$
$$f = (m\omega^2 L)$$
$$f \le f_{max}$$
$$m\omega^2 L \le \mu mg$$
$$\omega^2 \le \frac{\mu g}{L} \Rightarrow \omega \le \left(\sqrt{\frac{\mu g}{L}}\right)$$
$$\omega_{max} = \sqrt{\frac{\mu g}{L}}$$

(B)



$$a_{net} = \sqrt{a_C^2 + a_t^2} = \sqrt{(\omega^2 L)^2 + (\alpha L)^2}$$
$$f = ma_{net} = m\sqrt{(\omega^2 L)^2 + (\alpha L)^2}$$
$$f \le f_{max}$$
$$f = f_{max}$$
$$m\sqrt{(\omega^2 L)^2 + (\alpha L)^2} = \mu mg$$
$$\omega\left(\left(\frac{\mu g}{L}\right)^2 - \alpha^2\right)^{\frac{1}{4}}$$

28. **Ans.** (a) 975 N, 1025 N (b) 707 N (c) 683 N (d) μ = 1.037

(a) At B, $N = mg - \frac{mv^2}{R}$

At D, $N = mg + \frac{mv^2}{R}$

(b) Friction force is zero (no sliding tendency)
At C, $mg\sin\theta = F$

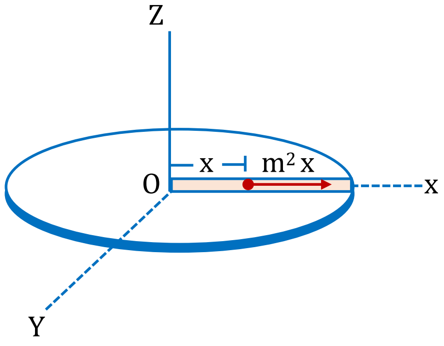
(c) Just before, $mg\cos\theta - N = \frac{mv^2}{R}$

(d) Minimum coefficient of friction
$\Rightarrow$ $\mu N = mg\sin\theta$

29. **Ans.** $v=\omega\sqrt{L^2-a^2}$

The situation is shown in figure.

Thus, the acceleration along the *X*-axis is



$$a=\frac{F}{m}=\frac{m\omega^2 x}{m}=\omega^2 x$$

or $$\frac{dv}{dt}=\omega^2 x$$

or $$\frac{dv}{dx}\cdot\frac{dx}{dt}=\omega^2 x$$

or $$\frac{dv}{dx}\cdot v=\omega^2 x$$

or $$vdv=\omega^2 xdx$$

or $$\int_0^v vdv=\int_a^L \omega^2 xdx$$

or $$\left[\frac{1}{2}v^2\right]_0^v=\left[\frac{1}{2}\omega^2 x^2\right]_a^L$$

or $$\frac{v^2}{2}=\frac{1}{2}\omega^2\left(L^2-a^2\right)$$

or $$v=\omega\sqrt{L^2-a^2}$$

30. **Ans.** $67.7\ J$

$W_1=\vec{F}_1.\vec{d}_1$ = (6)(4) = $24N$

$W_2=\vec{F}_2.\vec{d}_2$ where = 8 cos 30 i + 8 sin 30 j

$=\left(4\sqrt{3}\hat{i}+4\hat{j}\right)$, $\vec{s}=(4\hat{i}+4\hat{j})$

$\Rightarrow$ W_2 = 43.7 J therefore W_{net} = 67.7 J

31. **Ans.** (a) 7.5 J; (b) 15 J; (c) 7.5 J; (d) 30 J

W = area under the graph.

32. **Ans.** 3

$$W_1=\frac{1}{2}kx^2$$

$$W_2=\frac{1}{2}k\left[4x^2-x^2\right]=\frac{3}{2}kx^2$$

33. **Ans.** 0

Total work done = change in kinetic energy

$$=\frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2=\frac{1}{2}(2)\times(1)^2-\frac{1}{2}\times 2\times(1)^2=0$$

34. **Ans.** 4

$$mgh=\frac{1}{2}kx^2$$

$$1\times 10\times 5=\frac{1}{2}\times k\times\frac{1}{4}$$

k = 400

35. **Ans.** 10

By applying work energy theorem in wedge frame between point P and Q.

$$\Delta KE = W_g + W_{\text{pseudo force}}$$

$\Rightarrow$ $0 = -mgR + maR$

$\Rightarrow$ $a = g$

36. **Ans.** $\dfrac{mgl}{18}$

Mass of dx = $dx \times \dfrac{m}{L}$

$$dW = dmgh$$

$$dW = dx \times \frac{m}{L} gx$$

$$\int_0^W dW = \int_0^{L/3} \frac{m}{L} gxdx = \frac{mgL}{18}$$

37. **Ans.** $-\dfrac{2}{9}\mu MgL$

$$dM = \frac{M}{L} dx$$

$$\int_0^W dW = \int_0^{2L/3} -\mu \frac{M}{L} dxgx = \left(\frac{-2\mu MgL}{9}\right)$$

38. **Ans.** (i) $2 + 24t^2 + 72t^4$ J, (ii) $48\,t$ N, (iii) $48t + 288t^3$ W, (iv) 1248 J

$v = 1 + 6t^2$

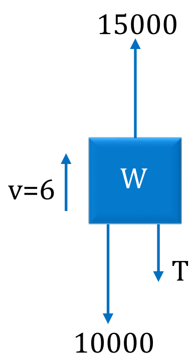
and $a = 12t$

$$KE = \frac{1}{2} m\left(1 + 36t^4 + 12t^2\right)$$

$$F = 4(12\,t) = 48t$$

$$P = F \cdot v$$

39. **Ans.** (a) -30 kW, (b) 19.5 kW



T = force applied by motor

(a) $P = F \cdot v = -T \times 6$

$= -5000 \times 6$

$= -30\ kW$

40. **Ans.** $46\,J$

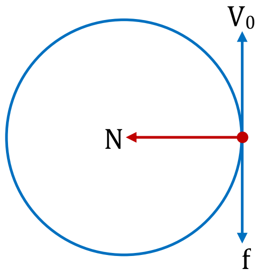
$$P = F \cdot v = mv \cdot \frac{dv}{dt}$$

$$\Rightarrow \quad \frac{1}{2}m\left(v^2 - u^2\right) = \int P dt$$

$$= \left[t^3 - t^2 + t\right]_2^4$$

41. **Ans.** (a) $\frac{mv^2}{R}$; (b) $\frac{\mu mv^2}{R}$; (c) $-\frac{\mu v^2}{R}$; (d) $v_0e^{-2\pi\mu}$

Top vies



(a) $F_c = N$

$F_t = f$

$N = \frac{mv^2}{R}$ and $f = \mu N$

(b) $f = \frac{\mu mv^2}{R}$

(c) $a_t = \frac{f_k}{m} = \left(\frac{\mu mv^2}{R}\right) . \frac{1}{m}$

$\vec{a}_t = -\frac{\mu v^2}{R}$ (acc. is opposite to velocity)

(d) $v\frac{dv}{ds} = -\frac{\mu v^2}{R}$

$v\frac{dv}{ds} = -\frac{\mu v^2}{R}$

$\frac{dv}{ds} = -\frac{\mu v}{R}$

$\int_0^{2\pi R} -\frac{\mu}{R} ds = \int_{v_0}^{v} \frac{1}{v} dv$

$-\frac{\mu}{R}(2\pi R) = \ln\left(\frac{v}{v_0}\right)$

$v = v_0e^{-2\pi\mu}$

42. Ans. (a) 0.2 *N*; (b) 30°

Given

$V = 36\ km/hr$

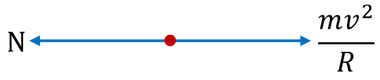
$= 36 \times \frac{5}{18} = 10\ m/s$

$R = 50\ cm$

mass of block = 100 gm = 0.1 kg

$\mu = 0.58$

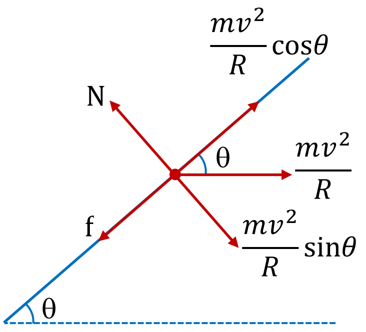
(a) Top view



$$N = \frac{mv^2}{R}$$

$$N = \frac{(0.1)(10)^2}{50} = 0.2\ newton$$

(b) In car frame (top view)



$$\frac{mv^2}{R}\cos\theta = f \qquad ...(1)$$

$$\frac{mv^2}{R}\sin\theta = N \qquad ...(2)$$

when slipping begins then

$f = \mu N$

$$\frac{mv^2}{R}\cos\theta = \mu.\frac{mv^2}{R}\sin\theta$$

$\cot\theta = \mu$

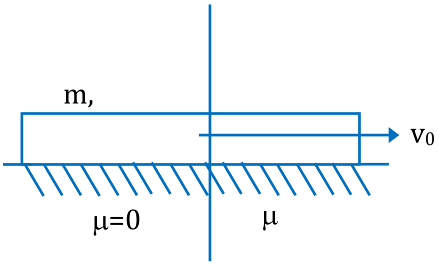
$\cot\theta = 0.58$

$\cot\theta = \frac{1}{\sqrt{3}}$

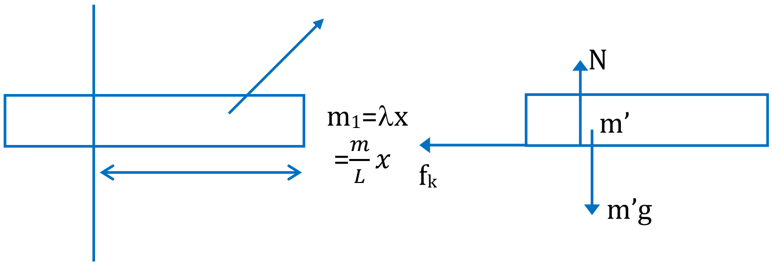
$\theta = 60°$ from horizontal

$\theta = 30°$ from vertical

43. **Ans.** (i) $f=\dfrac{\mu m}{L}xg$; (ii) $\sqrt{\mu g L}$; (iii) $\dfrac{5L}{2}$



(i) $\lambda = m/L$



$f_k = \mu_k N$

$= \mu\, m'g$

$$\boxed{f_k=\mu\frac{m}{L}gx}$$

(ii) To lie completely

$W_{net} = \Delta\, K\, E$

$$\cancelto{0}{W_{mg}} + \cancelto{0}{W_N} + W_{fk} = \cancelto{0}{KE_f} - KE_i$$

$$\int\left(\mu\frac{mg}{L}x\right)dx(-1)=0-\frac{1}{2}mv_0^2$$

$$-\frac{\mu mg}{L}\int_0^L x\ dx=-\frac{1}{2}mv_0^2$$

$$\frac{\mu mg}{L}\left[\frac{L^2}{2}\right]=\frac{1}{2}mv_0^2$$

$$v_0^2=\mu gL$$

$$\boxed{v_0=\sqrt{\mu gL}}$$

(iii) $\boxed{v = 2\sqrt{\mu g L}}$

$W_{Net} = \Delta KE$

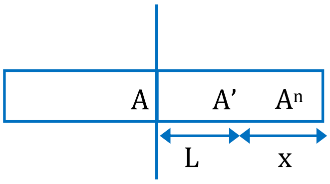
$$\cancelto{0}{W_{mg}} + \cancelto{0}{W_N} + W_{fk} = \Delta KE$$

$W_{fk} = KE_f - KE_i$

$$\int_0^L \left(\frac{\mu m g}{L}x\right)dx(-1) + [\mu m g]x(-1) = 0 - \frac{1}{2}m\left(2\sqrt{gL\mu}\right)^2$$

$$\frac{\mu m g}{L}\left[\frac{L^2}{2}\right] + \mu m g x = \frac{m \times 4}{2}(gL\mu)$$

$$\frac{L}{2} + x = 2L \;;\; \boxed{x = \frac{3L}{2}}$$



Total distance travelled

$$D = L + \frac{3L}{2} \;;\; \boxed{D = \frac{5L}{2}}$$

44. **Ans.** Graph-1 : For all x, Graph-2 : $x < a$ & $x > b$, Graph-3 : $-\frac{b}{2} < x < -\frac{a}{2}$ & $\frac{a}{2} < x < \frac{b}{2}$

E (total mechanical energy) = $KE + PE$

$\because \quad KE \geq 0$

$E - PE \geq 0$

$\boxed{E \geq PE}$

E must always greater than PE for particle to be exist

For graph –1 $\Rightarrow$ for $E < PE$

$x \in R$

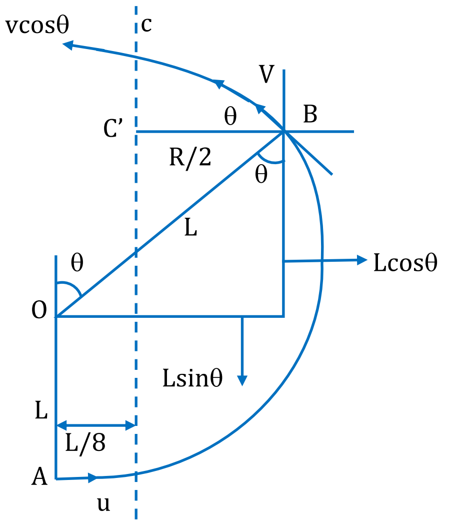
For graph –2 $\Rightarrow$ for $E < PE$

$x < a$ and $x > b$

For graph – 3 $\Rightarrow$ for $E > PE$

$$x \in \left(\frac{-b}{2}, \frac{-a}{2}\right) U \left(\frac{a}{2}, \frac{b}{2}\right)$$

45. **Ans.** $u = \sqrt{gL\left(2+\frac{3\sqrt{3}}{2}\right)}$



At point A, $v = u$

At point B, $v = v$ at θ with horizontal

$$\frac{1}{2}mu^2 = mg(L + L\cos\theta) + \frac{1}{2}mv^2$$

$u^2 = 2gL\,(1 + \cos\theta) + v^2$

$v^2 = u^2 - 2gL\,(1 + \cos\theta)$

∵ $\quad Range, R = \frac{v^2 \sin^2\theta}{g}$

Also, $\frac{R}{2} = L\sin\theta - \frac{L}{8}$ ...(1)

At $\quad B, T = 0$

$$mg\cos\theta = \frac{mv^2}{L}$$

$$\boxed{v^2 = Lg\cos\theta}$$

$u^2 - 2gL\,(1 + \cos\theta) = L\,g\cos\theta$

$u^2 = 2gL + 2gL\cos\theta + Lg\cos\theta$

$$\boxed{\frac{u^2 - 2gL}{3\,gL} = \cos\theta} \quad ...(2)$$

From eq (1)

$$\frac{R}{2} = L\sin\theta - \frac{L}{8}$$

$$\frac{2v^2\sin\theta\cos\theta}{2g} = L\sin\theta - \frac{L}{8}$$

$$\frac{(Lg\cos\theta)\sin\theta\cos\theta}{g}=L\sin\theta-\frac{L}{8}$$

$$L\sin\theta\cos^2\theta=L\sin\theta-\frac{L}{8}$$

$$\sin\theta\cos^2\theta=\sin\theta-\frac{1}{8}$$

$$\sin\theta(\cos^2\theta-1)=-\frac{1}{8}$$

$$+\sin\theta\times\sin^2\theta=+\frac{1}{8}$$

$$\sin^3\theta=\frac{1}{8}$$

$$\sin\theta=\left(\frac{1}{8}\right)^{\frac{1}{3}}=\frac{1}{2}$$

$$\boxed{\theta=30^\circ}$$

From eq (2)

$$\frac{u^2-2gL}{3gL}=\cos30^\circ$$

$$u^2-2gL=\frac{\sqrt{3}}{2}\times3gL$$

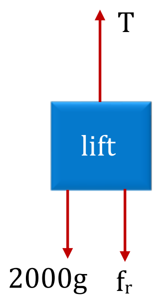
$$u^2=\left(\frac{3\sqrt{3}}{2}+2\right)gL$$

$$\boxed{u=\sqrt{gL\left(\frac{3\sqrt{3}}{2}+2\right)}}$$

EXERCISE - JEE (Main) PYQ

1. **Ans. (3)**

Let elevator is moving upward with constant speed V.



Tension in cable

$T = 2000\,g + f_r = 20000 + 4000$

$T = 24000\ N$

Power $P = TV$

$\Rightarrow$ $60 \times 746 = (24000)\ V$

$$V=\frac{60\times746}{24000}=1.865\approx1.9\ m/s.$$

2. Ans. (5)

Let the speed of bob at lowest position be v_1 and at the highest position be v_2.

Maximum tension is at lowest position and minimum tension is at the highest position.

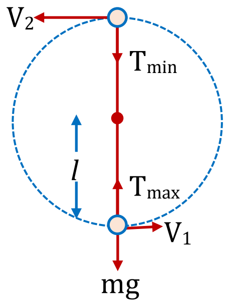
Now, using, conservation of mechanical energy,

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv_2^2 + mg\,2l$$

$\Rightarrow$ $v_1^2 = v_2^2 + 4gl$ ...(1)

Now $T_{max} - mg = \frac{mv_1^2}{l}$

$\Rightarrow$ $T_{max} = mg + \frac{mv_1^2}{l}$



And $T_{min} + mg = \frac{mv_2^2}{l}$

$\Rightarrow$ $T_{min} = \frac{mv_2^2}{l} - mg$

$$\frac{T_{max}}{T_{min}} = \frac{5}{1} \Rightarrow \frac{mg + \frac{mv_1^2}{l}}{\frac{mv_2^2}{l} - mg} = \frac{5}{1}$$

$\Rightarrow$ $mg + \frac{mv_1^2}{l} = \left[\frac{mv_2^2}{l} - mg\right]5$

$\Rightarrow$ $mg + \frac{m}{l}[v_2^2 + 4gl] = \frac{5mv_2^2}{l} - 5mg$

$\Rightarrow$ $mg + \frac{m_2^2}{l} + 4mg = \frac{5mv_2^2}{l} - 5mg$

$\Rightarrow$ $10mg = \frac{4mv_2^2}{l}$

$V_2^2 = \frac{10\times10\times1}{4}$

$\Rightarrow$ $v_2^2 = 25$

$\Rightarrow$ $v_2 = 5\ m/s$

Thus, velocity of bob at highest position is 5 m/s.

3. Ans. (6)

Let's say the compression in the spring be y.

So, by work energy theorem we have

$\Rightarrow$ $\frac{1}{2}mv^2 = \frac{1}{2}ky^2$

$\Rightarrow$ $y = \sqrt{\frac{m}{k}} \cdot v$

$\Rightarrow$ $y = \sqrt{\frac{4}{100}} \times 10$

$\Rightarrow$ $y = 2m$

Final length of spring

$= 8 - 2 = 6m$

4. **Ans. (1)**

$\frac{1}{2}kx^2 = \frac{1}{2}mv^2$

$kx^2 = mv^2$

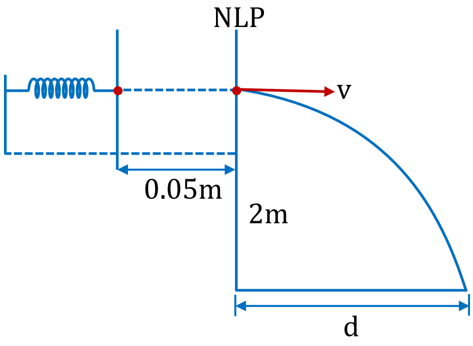
$v = x\sqrt{\frac{k}{m}} = 0.05\sqrt{\frac{100}{0.1}} = 0.05 \times 10\sqrt{10}$

$v = 0.5\sqrt{10}$

From $h = \frac{1}{2}gt^2$

$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2 \times 2}{10}} = \frac{2}{\sqrt{10}}$

$\therefore \quad d = vt = 0.5\sqrt{10} \times \frac{2}{\sqrt{10}} = 1m$



5. **Ans. (1)**

$K_2 = 4K_1$

$\frac{1}{2}mv_2^2 = 4\frac{1}{2}mv_1^2$

$v_2 = 2v_1$

$P = mv$

$P_2 = mv_2 = 2mv_1$

$P_1 = mv_1$

% change

$= \frac{\Delta P}{P_1} \times 100 = \frac{2mv_1 - mv_1}{mv_1} \times 100 = 100\%$

6. **Ans. (2)**

$W_{\text{Porter}} + W_{\text{mg}} = \Sigma K.E. = 0$

$W_{\text{Porter}} = -W_{\text{mg}} = -mgh$

$= -80 \times 9.8 \times .8 = -627.2\ J$

7. **Ans. (2)**

Given, $m = 0.5\ kg$ and $u = 20\ m/s$

Initial kinetic energy $(k_i) = \frac{1}{2}mu^2$

$= \frac{1}{2} \times 0.5 \times 20 \times 20 = 100\ J$

After deflection it moves with 5% of k_i

$\therefore \quad k_f = \frac{5}{100} \times k_i = \frac{5}{100} \times 100$

$\Rightarrow \quad k_f = 5\ J$

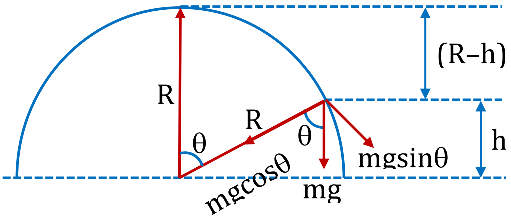
Now, let the final speed be 'v' m/s, then :

$k_f = 5 = \frac{1}{2}mv^2$

$\Rightarrow \quad v^2 = 20$

$\Rightarrow \quad v = \sqrt{20} = 4.47\ m/s$

8. Ans. (2)



$mg\cos\theta = \dfrac{mv^2}{R}$ ...(1)

$\cos\theta = \dfrac{h}{R}$

Energy conservation

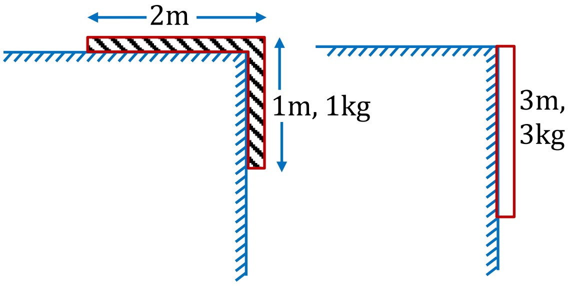
$mg\{R - h\} = \dfrac{1}{2}mv^2$ ...(2)

From (1) and (2)

$mg\left\{\dfrac{h}{R}\right\} = \dfrac{2mg\,\{R-h\}}{R}$

$h = \dfrac{2R}{3} = 2m$

9. Ans. (40)



From energy conservation

$K_i + U_i = k_f + U_f$

$0 + \left(-1 \times 10 \times \dfrac{1}{2}\right) = k_f + \left(-3 \times 10 \times \dfrac{3}{2}\right)$

$-5 = k_f - 45$

$k_f = 40\ J$

10. Ans. (2)

Given $W_A = W_B$

$F_A d\cos 45° = F_B d\cos 60°$

$F_A \times \dfrac{1}{\sqrt{2}} = F_B \times \dfrac{1}{2}$

$\dfrac{F_A}{F_B} = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$

$x = 2$

11. Ans. (1)

Work done = Change in kinetic energy

$$W_{mg} + W_{\text{air-friction}} = \frac{1}{2}m\left(.8\sqrt{gh}\right)^2 - \frac{1}{2}m(0)^2$$

$$W_{\text{air-friction}} = \frac{.64}{2}mgh - mgh = -0.68mgh$$

12. Ans. (4)

Statement I :

$$V_{max} = \sqrt{\mu Rg} = \sqrt{(0.2) \times 2 \times 9.8}$$

$v_{max} = 1.97\ m/s$

$7\ km/h = 1.944\ m/s$

Speed is lower than v_{max}, hence it can take safe turn.

Statement II :

$$v_{max} = \sqrt{Rg\left[\frac{\tan\theta+\mu}{1-\mu\tan\theta}\right]}$$

$$= \sqrt{2 \times 9.8\left[\frac{1+0.2}{1-0.2}\right]} = 5.42m/s$$

$18.5\ km/h > 5.42\ m/s$

Speed is lower than v_{max}, hence it can take safe turn.

13. Ans. (728)

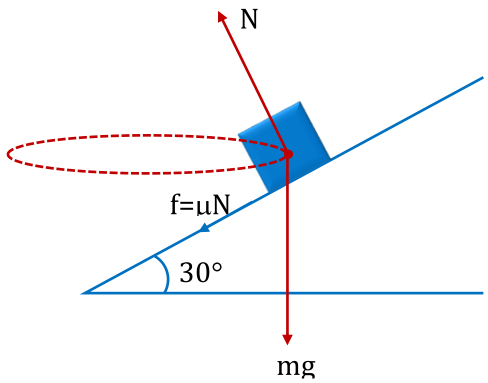
We know, $\theta = \left(\frac{\omega_1+\omega_2}{2}\right)t$

Let number of revolutions be N

$\therefore \quad 2\pi N = 2\pi\left(\frac{900+2460}{60\times2}\right) \times 26$

$N = 728$

14. Ans. (1)



At v_{max}, f will be limiting in nature.

$\therefore \quad$ Balancing force in vertical direction,

$N\cos 30° - mg - \mu N\cos 60° = 0$

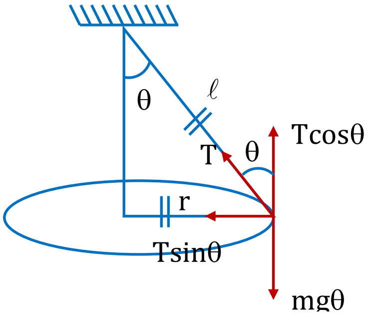
$\Rightarrow \quad N[\cos 30° - \mu\cos 60°] = mg$

$\therefore \quad N = \frac{800\times10}{(0.87-0.1)} \approx 10.2 \times 10^3 kgm/s^2$

Hence option 1.

15. Ans. (1)

Conical pendulum



$sin\,\theta = \frac{r}{\ell} = \frac{1}{\sqrt{2}}$

θ = 45°

$T\,sin\,\theta = \frac{mv^2}{r}$

$T\cos\theta = mg$

$tan\theta = \frac{v^2}{rg} \Rightarrow v = \sqrt{rg}$

Ans. 1

16. Ans. (2)

$R = \frac{\ell}{\theta}$

Time = $\frac{4\times 2\pi R}{v} = \frac{4\times 2\pi}{v}\left(\frac{\ell}{\theta}\right)$

Put $\ell = 4.4 \times 9.46 \times 10^{15}$

$v = 8 \times 1.5 \times 10^{11}$

$\theta = \frac{4}{3600} \times \frac{\pi}{180}$ rad

We get time = 4.5×10^{10} sec.

17. Ans. (3)

$T = M\omega^2 R$

$T = 80\,N \quad M = 0.1 \quad \omega = ? \quad R = 2m$

$80 = 0.1\,\omega^2(2)$

$\omega^2 = 400$

$\omega = 20$

$2\pi f = 20$

$f = \frac{10}{\pi}\frac{rev}{s} = \frac{600}{\pi}\frac{rev}{min}$

18. Ans. (3)

$\frac{-10A}{r^{11}} + \frac{5B}{r^6} = 0$

$r^5 = \frac{10A}{5B} = \frac{2A}{B}$

19. Ans. (3)

$F = 4x\hat{\imath} + 3y^2\hat{\jmath}$

$WD = \Delta KE$

$W = \int \vec{F}.(dx\hat{\imath} + dy\hat{\jmath})$

$= \int_1^2 4xdx + \int_2^3 3y^2\,dy$

$= (2x^2)_1^2 + (y^3)_2^3$

$= (8-2) + (27-8)$

$= 6 + 19 = 25J$

20. Ans. (120)

By energy conservation

$PE = KE$

$$mg\left(H+\frac{H}{2}\right)=\frac{1}{2}kx^2\left(x=\frac{H}{2}\right)$$

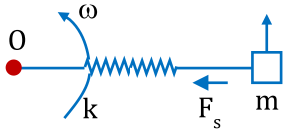
$$0.100\times 10\times\frac{3}{2}(0.10)=\frac{1}{2}k(0.05\times 0.05)$$

$$k=\frac{3\times 0.10}{0.05\times 0.05}=\frac{3\times 1000}{25}=120N/m$$

21. Ans. (3)

Natural length = L_0

Extension = x



$$kx=m(L_0+x)\omega^2$$

$$\Rightarrow 12.5x=\frac{1}{5}(L_0+x)25$$

$$\Rightarrow 1.5x=L_0$$

$$\Rightarrow \frac{x}{L_0}=\frac{2}{3}$$

22. Ans. (40)

$$v\frac{dv}{dx}=\frac{v^2}{R}\Rightarrow\int_{15}^{v}\frac{dv}{v}=\frac{1}{R}\int_0^x dx$$

$$v=15e^{x/R}$$

$$\frac{dx}{dt}=15e^{x/R}$$

$$\int_0^{\frac{\pi R}{2}}e^{-x/R}dx=15\int_0^{t_0}dt$$

$$t_0=40\left(1-e^{-\pi/2}\right)$$

23. Ans. (100)

$$\vec{F}=t\hat{i}+3t^2\hat{j}$$

$$\frac{md\vec{v}}{dt}=t\hat{i}+3t^2\hat{j}$$

$$m=1kg,\ \int_0^{\vec{v}}dv=\int_0^t tdt\hat{i}+\int_0^t 3t^2dt\hat{j}$$

$$\vec{v}=\frac{t^2}{2}\hat{i}+t^3\hat{j}$$

Power = $\vec{F}\cdot\vec{V}=\frac{t^3}{2}+3t^5$

At t = 2, power= $\frac{8}{2}+3\times 32=100$

24. Ans. (1)

Work done (W) = $Fs \cos\theta$

(A) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is positive.

(B) Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

(C) Work done by friction on a body sliding down an inclined plane is negative.

(D) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity in positive.

(E) Work done by the air resistance on an oscillating pendulum in negative.

EXERCISE - JEE (Advanced) PYQ

1. Ans. (D)

$\vec{F} = k\left[\frac{\vec{r}}{r^3}\right] \Rightarrow$ force is radial.

So, $W = 0$.

2. Ans. (5)

$P = F \cdot v \Rightarrow 0.5 = 0.2v\frac{dv}{dt} \Rightarrow v^2 = 5t$

3. Ans. (A)

4. Ans. (B)

Solution for Q. No. 3 and 4

$mg\,R \sin 30° - 150 = \frac{1}{2}mv^2$

$\Rightarrow \quad 200 - 150 = \frac{1}{2}v^2$

$v = 10\ m/sec$

$N - mg\cos 60° = \frac{mv^2}{R}$

$N - 5 = 2.5 \Rightarrow N = 7.5$

5. Ans. (5)

$K_f - K_i = W_{all}$

$K_f = W_{ext} + W_{gr}$

$= 18 \times 5 - 1 \times 10 \times 4 = 50 = 5 \times 10$

6. **Ans. (D)**

By circular motion equation

$$mg\cos\theta - \frac{mv^2}{R} = N$$

by conservation of energy

$$\frac{1}{2}mv^2 = mgR\left(1-\cos\theta\right)$$

$$\frac{mv^2}{R} = 2mg(1-\cos\theta)$$

So, $mg\cos\theta - 2mg(1-\cos\theta) = N$

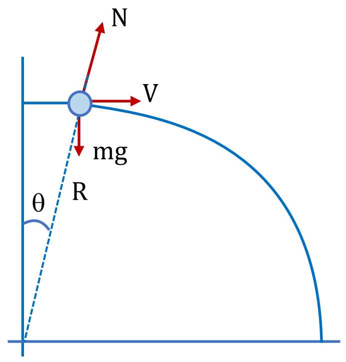
$\Rightarrow$ $N = mg(3\cos\theta - 2)$

For $0 \le \theta < \cos^{-1}\left(\frac{2}{3}\right)$, N, N is positive

So, N on ball is outward and its reaction force on wire is inward.

For $\theta > \cos^{-1}\left(\frac{2}{3}\right)$, N is negative,

So, N on ball is inward, and its reaction force on wire is outward.



7. **Ans. (D)**

Force on block along slot = $m\omega^2 r = ma = m\left(\frac{vdv}{dr}\right)$

$$\int_0^v vdv = \int_{R/2}^r \omega^2 rdr$$

$$\frac{v^2}{2} = \frac{\omega^2}{2}\left(r^2 - \frac{R^2}{4}\right) \Rightarrow v = \omega\sqrt{r^2 - \frac{R^2}{4}} = \frac{dr}{dt}$$

$$\Rightarrow \int_{R/4}^r \frac{dr}{\sqrt{r^2 - \frac{R^2}{4}}} = \int_0^t \omega dt$$

$$\ell n\left(\frac{r+\sqrt{r^2-\frac{R^2}{4}}}{\frac{R}{2}}\right) - \ell n\left(\frac{R/2+\sqrt{\frac{R^2}{4}-\frac{R^2}{4}}}{\frac{R}{2}}\right) = \omega t$$

$$\Rightarrow r + \sqrt{r^2 - \frac{R^2}{4}} = \frac{R}{2}e^{\omega t}$$

$$\Rightarrow r^2 - \frac{R^2}{4} = \frac{R^2}{4}e^{2\omega t} + r^2 - 2r\frac{R}{2}e^{\omega t}$$

$$\Rightarrow r = \frac{\frac{R^2}{4}e^{2\omega t} + \frac{R^2}{4}}{Re^{\omega t}} = \frac{R}{4}(e^{\omega t} + e^{-\omega t})$$

8. **Ans. (C)**

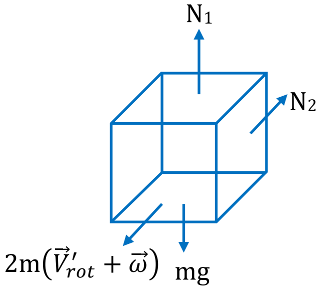
$\vec{N}_2 = mg\hat{k}$

$\vec{N}_2 = 2m(V'_{rot} \times \vec{\omega})\hat{j} = 2m\frac{\omega R}{4}[e^{\omega t} - e^{-\omega t}]\omega\hat{j}$

$= \frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j}$

Total reaction on block = $\vec{N}_1 + \vec{N}_2$

$= \frac{1}{2}m\omega^2 R(e^{\omega t} - e^{-\omega t})\hat{j} + mg\hat{k}$



9. Ans. (ABD)

$\frac{dk}{dt} = \gamma t$ as $k = \frac{1}{2}mv^2$

$\therefore \quad \frac{dk}{dt} = mv\frac{dv}{dt} = \gamma t$

$\therefore \quad m\int_0^v vdv = \gamma\int_0^t tdt$

$\frac{mv^2}{2} = \frac{\gamma t^2}{2}$

$v = \sqrt{\frac{\gamma}{m}}t$ ...(i)

$a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}} =$ constant

Since $F = ma$

$\therefore \quad F =$ constant

10. Ans. (0.75)

$\vec{F} = (\alpha y\hat{\imath} + 2\alpha x\hat{\jmath})$

$W_{AB} = (-1\hat{\imath})\cdot(1\hat{\imath}) = -1J$

$\begin{bmatrix} \vec{F} = -1\hat{\imath} + 2\alpha x\hat{\jmath} \\ \vec{S} = 1\hat{\imath} \end{bmatrix}$

Similarly,

$W_{BC} = 1J$

$W_{CD} = 0.25J$

$W_{DE} = 0.5\,J$

$W_{EF} = W_{FA} = 0\,J$

$\therefore$ New work in cycle = 0.75 J

11. Ans. (AD)

By the energy conservation (ME) between bottom point and point Y

$\frac{1}{2}mv_0^2 = mgh + \frac{1}{2}mv_1^2$

$\therefore \quad v_1^2 = v_0^2 - 2gh$ ...(i)

Now at point Y the centripetal force provided by the component of mg

$\therefore \quad mg\sin 30° = \frac{mv_1^2}{R}$

$\therefore \quad v_1^2 = \frac{gR}{2}$

From (i)

$\frac{gR}{2} = v_0^2 - 2gh$

At point x and z of circular path, the points are at same height but less then h. So, the velocity more than a point y.

So, required centripetal $\frac{mv^2}{r}$ is more.

JEE (Main) Practice Paper

SECTION–A

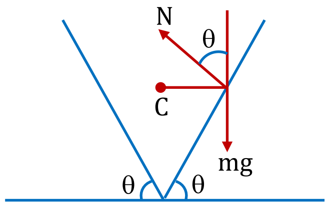
1. **Ans. (1)**

For same sense : $5\omega t = \omega t + 2\pi$

2. **Ans. (3)**

F = pseudo force or centrifugal force. T is resultant of centripetal force and weight so it should be larger than both.

3. **Ans. (3)**



$N \cos \theta = mg$

$N \sin \theta = \frac{mv^2}{r}$

4. **Ans. (2)**

If N is the normal force on the wings.

$N \cos \theta = mg$, $N \sin \theta = \frac{mv^2}{R}$

which give $R = \frac{v^2}{g \tan \theta} = \frac{200 \times 200}{10 \times \tan 15°} = 15km$

5. **Ans. (4)**

$a_n = \frac{v^2}{R}$

$R = \frac{v_0^2 \cos^2 \theta}{g} = \frac{v_0^2}{2g}$

6. **Ans. (2)**

$\omega_A = \frac{20 \times \frac{4}{5}}{40} = \frac{2}{5}\text{rad/sec}$

$\omega_B = \frac{10}{20} = \frac{1}{2}\text{rad/sec}$

The apparent velocity depends on the angular velocity observed by us.

$\omega = \frac{v_\perp}{r}$

7. **Ans. (2)**

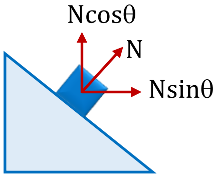
$W = \frac{1}{2}k(x_2^2 - x_1^2) = \frac{1}{2} \times 800 \left(\frac{225}{10000} - \frac{25}{10000}\right) = 8\,J$

8. **Ans. (3)**

$N \sin \theta = ma = 10$

$\Delta x = \frac{1}{2}at^2 = 15$

$W = N \sin \theta \cdot \Delta x = 150\, J$



9. **Ans. (1)**

Gravitational force is conservative force.

10. **Ans. (1)**

$\omega = 300 \times \frac{2\pi}{60} = 10\ \pi \text{rad/sec}$

$V = R\omega = 10\pi \times 1 = 10\ \pi$ rad/sec

$KE = \frac{1}{2} \times 5 \times 100\pi^2 = 250\pi^2$ Joule

11. **Ans. (1)**

$\frac{1}{2}m\left(v^2 - u^2\right) = -\frac{3}{4} \times \frac{1}{2}mu^2$ and $a = -\mu g$

$v = u + at \Rightarrow \frac{v_0}{2} = v_0 - \mu g t_0$

12. **Ans. (2)**

$ky_0 = mg \Rightarrow y_0 = a$

$KE = mga - \frac{1}{2}ka^2 = \frac{1}{2}mga$

13. **Ans. (1)**

For mas m :

$mg \sin \theta + f_{s,\ max} = kx$

$mg \sin \theta + \mu mg \cos \theta = kx$

for mass M :

$Mgx = \frac{1}{2}kx^2 \Rightarrow kx = 2mg$

$\Rightarrow \quad M = \frac{m(\sin\theta + \mu\cos\theta)}{2}$

14. **Ans. (1)**

$a = -kx \Rightarrow \frac{dv}{dx} \cdot v = -kx \quad \Rightarrow \quad \frac{v^2 - u^2}{2} = -\frac{1}{2}kx^2$

15. **Ans. (3)**

$\frac{1}{2}mv^2 = mgh$

$\Rightarrow \quad v = \sqrt{2gh}$

$a = g \sin\theta$

$\theta_2 > \theta_1$

$\sin \theta_2 > \sin \theta_1$

$a_2 > a_1$

$t_2 < t_1$

16. Ans. (4)

To reach at C, particle must reach at B first.

$$\frac{1}{2}mu^2 = mgh$$

$u^2 = 2 \times 10 \times 320$

$u = 80\ m/\text{sec.}$

17. Ans. (1)

$dU = -F \cdot dx$

$dU = -kxdx$

$$U = -\frac{1}{2}kx^2$$

18. Ans. (4)

Maxima of potential energy curve gives position of unstable equilibrium

So, at x_2, there is unstable equilibrium.

19. Ans. (4)

$\frac{dU}{dx}$ is zero for BC

$$F = -\frac{dU}{dx}$$

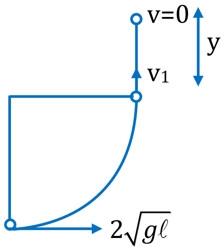
$\frac{dU}{dx}$ is maximum for CD.

20. Ans. (3)

By energy conservation

$v_1 = \sqrt{2g\ell}$ and $y = \frac{(2g\ell)}{2g} = \ell$

distance $= \frac{\pi\ell}{2} + \ell = \frac{14}{9}\left[\frac{11}{7} + 1\right] = 4m$



SECTION-B

1. Ans. (6)

$$\alpha = \frac{\omega d\omega}{d\theta}$$

$$\int \omega d\omega = \int \alpha d\theta$$

$\frac{\omega^2}{2}$ = Area under α vs θ graph $= \frac{1}{2}(9 \times 4)$

$\omega = \sqrt{36} = 6\ \text{rad}/s$

2. Ans. (3)

At $t = 1$ sec

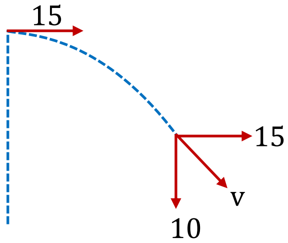
$\vec{v} = 15\hat{i} - 10\hat{j}$

$\vec{a} = -10\hat{j}$

$$a_t = \frac{\vec{a} \cdot \vec{v}}{v} = \frac{20}{\sqrt{13}}$$

$$a_N = \sqrt{a^2 - a_t^2} = \frac{30}{\sqrt{13}}$$

$$\frac{a_N}{a_T} = \frac{30}{20} = 1.5 = x$$



3. **Ans. (12)**

$W = F_Z \cdot \Delta Z = 3 \times 4 = 12\ J$

4. **Ans. (8)**

Method I: $W_{all} = \Delta KE = \frac{1}{2} m\left(v_2^2 - v_0^2\right)$

$v = \frac{dx}{dt} = 2t + 2 = 2(t+1)$

$v_2 = 6\ m/s$, $v_0 = 2\ m/s$

$W_{all} = \frac{1}{2}\left(\frac{1}{2}\right)(6^2 - 2^2) = 8J$

Method II : $x = t^2 + 2t$

$a = \frac{d^2x}{dt^2} = 2\ m/s^2$ = constant

so $F = m(A) = 1\ N$ = constant

so, $W = F.S = (1)(8-0) = 8J$

5. **Ans. (4)**

$W_f + W_g = 0 \Rightarrow W_f = -W_g = -mgh = -0.2 \times 10 \times 2 = -4\ J$

6. **Ans. (4)**

$F_x = -\frac{dU}{dx} = -4x$

7. **Ans. (2)**

$ky = mg$

$m_1 gy = \frac{1}{2} ky^2$

$2m_1 g = mg$

$m_1 = \frac{m}{2}$

8. **Ans. (6)**

$U = 4x^2 + x^4$

$F = -\frac{dU}{dx} = -8x - 4x^3$

At $x = 1$, $F = -8 - 4 = -12\ N$

$\Rightarrow \quad a = \frac{F}{m} = \frac{-12}{2} = -6m/s^2$

9. **Ans. (4)**

$N = mg \Rightarrow v_f = 0$

$mg\frac{R}{2} = \frac{1}{2} mv_i^2 \Rightarrow v = \sqrt{gR} = 4m/s$

10. **Ans. (18)**

$mg = \frac{mv^2}{r}$ ...(i)

$v^2 = rg = 360$

From energy conservation $mgh = \frac{1}{2} mv^2$

JEE (Advanced) Practice Paper

1. **Ans. (C)**

$a_B = \frac{v_B^2}{R} = 3g$

$v_B^2 = 3gR$

Applying COE at A & B

$mgR + \frac{1}{2}mv_A^2 = \frac{1}{2}mv_B^2 = \frac{1}{2}m \times 3gR$

$\frac{1}{2}mv_A^2 = \frac{mgR}{2}$

$v_A = \sqrt{gR}$

2. **Ans. (A)**

$P = F \cdot v = mav = mv^2 \cdot \frac{dv}{ds}$

$\int Pds = \int mv^2 dv$

$\Rightarrow \quad 30 = \frac{10}{7} \times \frac{1}{3}[v^3]_1^v$

$\Rightarrow \quad 63 = v^3 - 1 \qquad \Rightarrow \quad v = 4\ m/s$

3. **Ans. (C)**

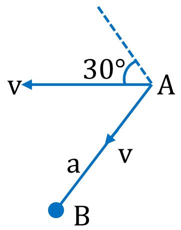
Here centripetal force is provided by normal reaction of vertical wall.

(i) Normal reaction of floor $N_F = mg$

(ii) Normal reaction of vertical wall $N_W = \frac{mv^2}{r}$.

4. **Ans. (ABCD)**

w.r.t B :



$\omega_{A,B} = \frac{v\cos 30°}{a}$

5. **Ans. (AC)**

displacement would be different for both observers.

6. **Ans. (BCD)**

F_1 is a central force

So it is conservative

W by $F_1 = 20 \times 6\sqrt{2}$

$= 120\sqrt{2}J$

7. Ans. (ABCD)

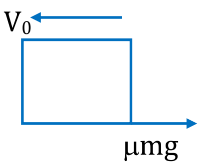
$\overrightarrow{OA} = \hat{\imath} + \hat{k}, \overrightarrow{OB} = 2\hat{\imath} + \hat{\jmath} + \hat{k}, \overrightarrow{OC} = \hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\Rightarrow \quad \overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \hat{\imath} + \hat{\jmath}, \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \hat{\jmath} - \hat{\imath}, \overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = 2\hat{\jmath}$

$W_{AB} = \vec{F}.\overrightarrow{AB} = 7, W_{BC} = \vec{F}.\overrightarrow{BC} = 1, W_{AC} = \vec{F}.\overrightarrow{AC} = 8$

A constant force is always a conservative force.

8. Ans. (A)



In the frame of belt, $t = \dfrac{v_0}{\mu_k g}$

Displacement of belt = $v_0 \times \dfrac{v_0}{\mu_k g}$

9. Ans. (B)

Work done by friction on block in belt frame,

$$W = \Delta KE = -\frac{1}{2}Mv_0^2$$

10. Ans. (C)

Applying WET

$$-mg(h+R) = \frac{1}{2}mv_B^2 - \frac{1}{2}mv_0^2 \qquad \text{... (i)}$$

Considering motion from B to C

$$v_B \times \sqrt{\frac{2R}{g}} = d \quad \Rightarrow \quad v_B = d\sqrt{\frac{g}{2R}}$$

Putting in equation (i) we get option (C).

11. Ans. (A)

Particle should not loose contact any where on the circular track.

Considering $N = 0$ at highest point.

$$\frac{mv^2}{R} = mg \quad \Rightarrow \quad d = \sqrt{2}R$$

12. Ans. (B)

$v_0^2 - v_A^2 = 2gh$

$F_A = \dfrac{mv_A^2}{R} = \dfrac{m(v_0^2 - 2gh)}{R}$

13. Ans. (25)

$kx_0 = mg \Rightarrow$ k = 2000 N/m [x_0 = 1 cm in equilibrium]

to lift 3 kg mass, extension in the spring must be, $y = \frac{30}{2000} = \frac{3}{200}m$

Let further compression in spring is x.

$$W_s + W_{mg} = \Delta KE \Rightarrow -\frac{1}{2}k\left[y^2 - \left(x + \frac{1}{100}\right)^2\right] - mg\left[x + \frac{1}{100} + y\right] = 0$$

$$\Rightarrow \quad -1000\left[\frac{9}{40000} - \left(x + \frac{1}{100}\right)^2\right] = 20\left[\frac{3}{200} + \left(x + \frac{1}{100}\right)\right]$$

$$\Rightarrow \quad -\frac{9}{800} + 50\left(x + \frac{1}{100}\right)^2 = \frac{3}{200} + \left(x + \frac{1}{100}\right)$$

14. Ans. (25)

By using work energy theorem

$$W = \Delta KE$$

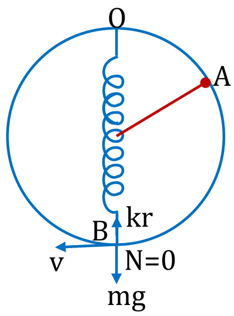
$$\Rightarrow \quad \frac{1}{2}kx^2 - 0 - W_f = 0$$

$$\Rightarrow \quad \frac{1}{2}kx^2 = \mu mgd$$

$$\Rightarrow \quad d = \frac{1}{2}\frac{kx^2}{\mu mg} = \frac{1}{2}\frac{(100)(0.5)^2}{(0.05)(1)(10)} = 25m$$

15. Ans. (500)

Initial length of spring = r (relaxed)



$$kr - mg = \frac{mv^2}{r} \qquad \text{...(i)}$$

COE at A and B

$$mgr\,(1 + \cos 60°) - \frac{1}{2}kr^2 = \frac{1}{2}mv^2$$

$$\Rightarrow \quad mv^2 = 3\,mgr$$

From equation (i)

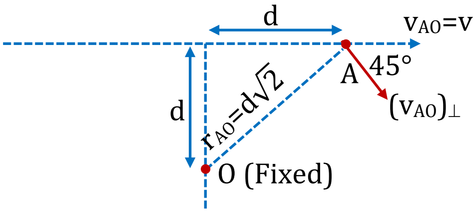
$$kr - mg = 3mg$$

$$kr = 4\,mg$$

$$k = \frac{2mg}{r} = 500$$

16. Ans. (1)

Angular velocity of A with respect to O is ;



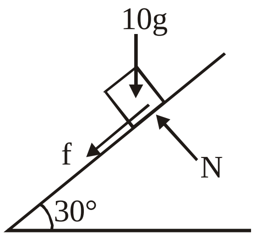
$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}}$$

$$v_{AO} = v_1 (v_{AO})_\perp = \frac{v}{\sqrt{2}}$$

$$r_{AO} = d\sqrt{2}$$

$$\omega_{AO} = \frac{(v_{AO})_\perp}{r_{AO}} = \frac{v/\sqrt{2}}{d\sqrt{2}} = \frac{v}{2d}$$

17. Ans. (300)



Since the car is moving with maximum safe speed. Therefore, friction will be at its maximum value.

$N \cos 30° = f \sin 30° + Mg$; $f = \mu N$ cos30°

18. Ans. (A-Q,S), (B-P,S), (C-R,S), (D-P,S)

For block, $W_N + W_g = \frac{1}{2} m v_b^2$

$\Rightarrow \quad W_N + mgh = \frac{1}{2} m v_b^2$

$\Rightarrow \quad W_N = \frac{1}{2} m v_b^2 - mgh$

For wedge, $W_N = \frac{1}{2} m v_W^2$

By $C.O.M.E$, $mgh = \frac{1}{2} m v_b^2 + \frac{1}{2} M v_W^2$

$\Rightarrow \quad \left(\frac{1}{2} m v_b^2 - mgh\right) + \frac{1}{2} M v_W^2 = 0$

W_N, block + $W_{N,\text{ wedge}} = 0$