Center of Mass and Collision

SOLUTIONS

EXERCISE – O

1. Ans. (B)

$$x_{cm} = \frac{8m(a/2)-m.a}{8m-m}$$

$$= \frac{3a}{7}$$

Similarly, y_{cm} and z_{cm}

2. Ans. (A)

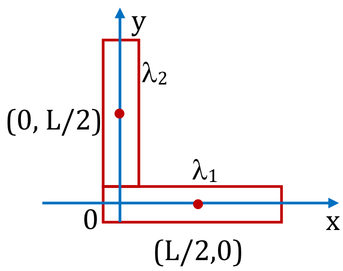
Distance of centre of mass $x_{cm} = \frac{\sum m_i x_i}{\sum m_i} = \frac{m\ell+2m(2\ell)+3m(3\ell)+\cdots..+(nm)(n\ell)}{m+2m+3m+\cdots..+nm}$

$$= \left(\frac{1+2^2+3^2+....+n^2}{1+2+3+....+n}\right)\ell = \frac{2n(n+1)(2n+1)}{6(n)(n+1)}\ell = \frac{(2n+1)}{3}\ell$$

3. Ans. (D)

$$x_{cm} = \frac{m_1.\frac{L}{2}}{m_1+m_2} = \frac{(\lambda_1.L).L/2}{(\lambda_1.L+\lambda_2L)} = \left(\frac{\lambda_1}{\lambda_1+\lambda_2}\right).\frac{L}{2}$$

$$y_{cm} = \frac{m_2.\frac{L}{2}}{m_1+m_2} = \left(\frac{\lambda_2.L}{\lambda_1.L+\lambda_2L}\right)\frac{L}{2} = \left(\frac{\lambda_2}{\lambda_1+\lambda_2}\right).\frac{L}{2}$$



$$x_{cm} + y_{cm} = \left(\frac{\lambda_1+\lambda_2}{\lambda_1+\lambda_2}\right).\frac{L}{2}$$

$$\Rightarrow \quad x_{cm} + y_{cm} = \frac{L}{2}$$

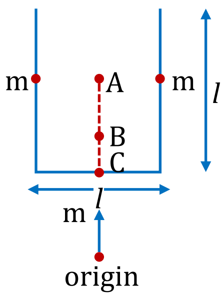
4. Ans. (B)

$$y_{cm} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{m\times0+m\times D+m\times2D+m\times D+m\times0}{m+m+m+m+m} = \frac{4D}{5}$$

5. Ans. (C)

$$x_{cm} = \frac{m\times\frac{L}{2}-\frac{mL}{2}}{3m} = 0$$

$$y_{cm} = \frac{\frac{m\ell}{2}+\frac{m\ell}{2}}{3m}$$



$$y_{cm} = \frac{\ell}{3}$$

Point (B)

6. **Ans. (A)**

$$Y_{CM} = \frac{m\left(\frac{2R}{\pi}\right)+m\left(\frac{-4R}{3\pi}\right)}{2m}$$

$$= \frac{2mR}{3\pi(2m)} = \frac{R}{3\pi}$$

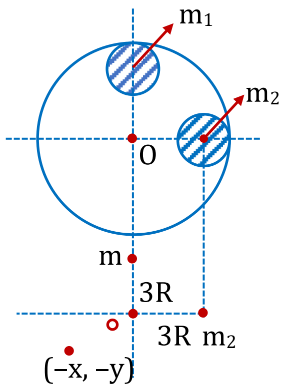
7. **Ans. (C)**

$m_2 . 3R = m_3.x$

$\Rightarrow \quad x = \left(\frac{m_2}{m_3}\right) . 3R = \frac{A_2}{A_3}(3R)$

$\Rightarrow \quad x = \left(\frac{\pi R^2}{\pi(4R)^2 - \pi R^2 \times 2}\right) . 3R$

$x = \frac{3R}{14}$



and $\quad m_1.3R = m_3.y \Rightarrow y = \left(\frac{m_1}{m_3}\right) . 3R$

$\Rightarrow \quad y = \left(\frac{\pi R^2}{\pi(4R)^2 - \pi R^2 \times 2}\right) . 3R$

$\Rightarrow \quad y = \frac{3R}{14}$

$\therefore \quad$ Co-ordinate of point $m_3 = -\frac{3R}{14}(\hat{\imath} + \hat{\jmath})$

8. **Ans. (B)**

$(v_{cm})_f = \frac{m.0+(m.v)\times 6}{7m} = \frac{6}{7}v$

$(v_{cm})_i = \frac{(mv)\times 7}{7m} = v$

$\therefore \quad (v_{cm})_f = \frac{6}{7}(v_{cm})_i$

9. **Ans. (C)**

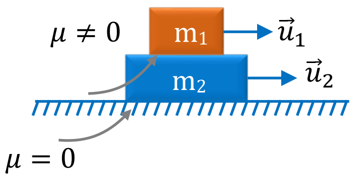
$F_{ext} = 0 \Rightarrow (V_{cm})_f = (V_{cm})_i$

10. **Ans. (B)**

$\vec{v}_{CM} = \frac{\vec{m}_1\vec{u}_1 + m_2\vec{u}_2}{m_1+m_2}$

u_1 and u_2 have same sign so

v_{COM} will never be zero.



11. **Ans. (A)**

$\bar{F}_{ext} \neq 0 \Rightarrow \bar{a}_{cm} \neq 0$

$\bar{v}_{cm}(t)$ may or may not be zero at any instant.

12. **Ans. (C)**

On sphere there is no external force in horizontal direction

$(a_{cm})_{\text{horizontal}} = 0 \Rightarrow (v_{cm})_{\text{horizontal}} = \text{constant} = 0$

and $\quad (f_{ext})_{\text{vertical}} \neq 0 \Rightarrow COM$ will fall vertically downward

$\therefore \quad$ Path of $COM \Rightarrow$ Straight line and vertical downwards

13. Ans. (A)

$$\text{Initial}\begin{cases} m_1 = 0.1kg \quad , \quad \bar{v}_1 = 20\hat{i} \\ m_2 = 0.2kg \quad , \quad \bar{v}_2 = 20\hat{j} \\ m_3 = 0.4kg \quad , \quad \bar{v}_3 = 20\hat{k} \end{cases}$$

$$\text{Final}\begin{cases} \bar{v}_3 = 0 \\ \bar{v}_2 = 10\hat{j} + 5\hat{k} \\ \bar{v}_1 = v_x\hat{i} + v_y\hat{j} + v_z\hat{k} \end{cases}$$

$\therefore \quad f_{ext} = 0$

$\Rightarrow \quad (\bar{v}_{cm})_f = (\bar{v}_{cm})_i$

$\Rightarrow \quad v_x)_{cm\,i} = (v_x)_{cm\,f}$

$\Rightarrow \quad \dfrac{0.1\times20}{0.7} = \dfrac{0.1\times v_x}{0.7}$

$\Rightarrow \quad v_x = 20m/s$

$\Rightarrow \quad (v_y)_{cm\,i} = (v_y)_{cm\,f}$

$\Rightarrow \quad \dfrac{0.2\times20}{0.7} = \dfrac{0.20\times10+0.1\times v_y}{0.7}$

$\Rightarrow \quad v_y = 20\ m/s$

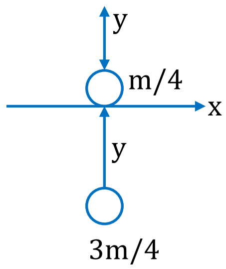
$(v_z)_{cm\,i} = (v_z)_{cm\,f}$

$\Rightarrow \quad \dfrac{0.4\times20}{0.7} = \dfrac{0.2\times5+0.1\times v_z}{0.7}$

$\Rightarrow \quad v_z = 70\hat{k}$

$\therefore \quad \vec{v}_1 = 20\hat{i} + 20\hat{j} + 70\hat{k}$

14. Ans. (A)



$0 = \frac{m}{4}\times15 + \frac{3m}{4}\times y \ \Rightarrow y = -5cm$

15. Ans. (B)

$m_1d_1 + m_2d_2 = 0$

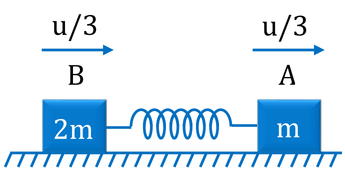
$\Rightarrow \quad \dfrac{1}{8}(0.14) + \dfrac{7}{8}d_2 = 0$

$\Rightarrow \quad d_2 = -0.02\ m$

16. Ans. (C)

$mu = 2mV + mV$

$V = u/3$



$\frac{1}{2}k\left(0^2 - x^2\right)$

$= \frac{1}{2}m(u/3)^2 + \frac{1}{2}2m(u/3)^2 - \frac{1}{2}mu^2$

17. **Ans. (i)- B, (ii)- C**

(i) $\vec{F}_{ext} = 0$

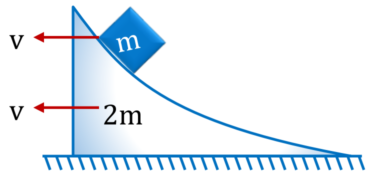
$\Rightarrow \vec{P}_{\text{system}} \Rightarrow$ conserved

(ii) It is also possible that momentum of individual particles is non-zero and might not be constant.

18. **Ans. (D)**

Work done by internal forces is equal to change in kinetic energy of the system.

19. **Ans. (C)**

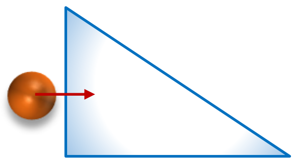


$3mv = mu \Rightarrow v = \frac{u}{3}$

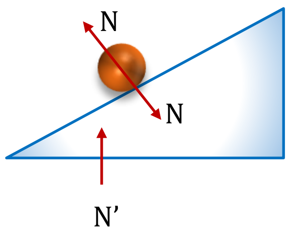
Also $\frac{1}{2}mu^2 = \frac{1}{2}(3m)\left(\frac{u}{3}\right)^2 + mgh$

$\Rightarrow \quad u = \sqrt{3gh}$

20. **Ans. (D)**



No impulsive force (external) either in $-ve\ x$ or is y-direction $P \Rightarrow$ constant



$N' \Rightarrow$ Impulsive $\Rightarrow f_r \Rightarrow$ Impulsive $\Rightarrow$ Momentum will change

21. **Ans. (C)**

$T = \int F.dt = F.t$

$T' = F.t' = F.2t = 2Ft = 2J$

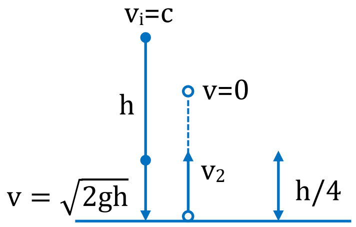
22. **Ans. (B)**

Change in momentum for m,

$\Delta\vec{p}_m = m(-2\hat{\imath} + \hat{\jmath}) - m(3\hat{\imath} + 2\hat{\jmath}) = m(-5\hat{\imath} - \hat{\jmath}) = \vec{J}_m$

But $\vec{J}_M = -\vec{J}_m = m(5\hat{\imath} + \hat{\jmath})$

23. Ans. (A)

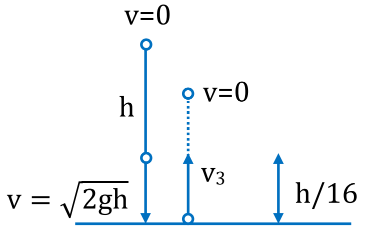


$O^2 = V_2^2 - 2g.h/4$

$\Rightarrow \quad V_2 = \sqrt{gh/2}$

$I_1 = P_f - P_i = m.\sqrt{\frac{gh}{2}} + m\sqrt{2gh}$

$I_1 = \frac{3m\sqrt{gh}}{\sqrt{2}}$



$0^2 = v_3^2 - 2g\frac{h}{16}$

$\Rightarrow \quad v_3 = \frac{\sqrt{gh}}{2\sqrt{2}}$

$\therefore \quad I_2 = m.\frac{\sqrt{2gh}}{4} + m\sqrt{2gh}$

$I_2 = \frac{5m\sqrt{2gh}}{4}$

Now $\frac{I_1}{I_2} = \frac{\frac{3}{\sqrt{2}}}{\frac{5}{4}\sqrt{2}} = \frac{6}{5}$

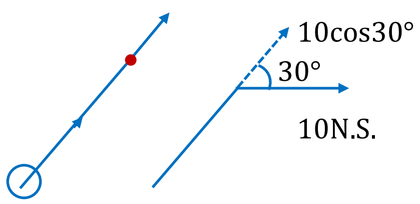
$\Rightarrow \quad 5I_1 = 6I_2$

24. Ans. (B)

$2mv = 10 \cos 30°$

$\Rightarrow \quad mv = \frac{5\sqrt{3}}{2}$

$\therefore \quad I_{tension} = (\Delta P)_1 = mv = \frac{5\sqrt{3}}{2}\ N\text{-}s$



25. **Ans. (B)**

Let velocity of $1kg$ after collision is v

$$\frac{1}{2}(1)(v^2) = \left[\frac{1}{2}(1)(1)\right]\frac{1}{4}$$

$$v = \frac{1}{2}m/s$$

Now, conserving momentum in collision

$$(1)\ (1) = (1)\left(\frac{1}{2}\right) + mv \Rightarrow mv = \frac{1}{2}$$

Also,

$$\frac{v-\frac{1}{2}}{1} = 1 \Rightarrow v = \frac{3}{2}m/s$$

So, $m = \frac{1}{3}kg$

26. **Ans. (B)**

Velocity after collision $mu = 2mv_0 \Rightarrow v_0 = u/2$

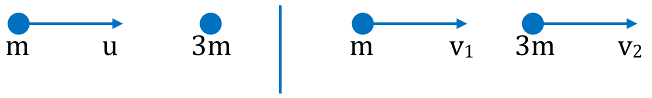
So final kinetic energy = $\frac{1}{2}(2m)\left(\frac{u}{2}\right)^2 = \frac{K}{2}$

Loss in kinetic energy $K - \frac{K}{2} = \frac{K}{2}$

27. **Ans. (C)**

Newton's third law.

28. **Ans. (A)**



Before collision　　　　After collision

$$e = \frac{V_2 - V_1}{u} \qquad ...(i)$$

$$mu = mv_1 + 3mv_2 \qquad ...(ii)$$

$$\frac{1}{3} \times \left(\frac{1}{2}mu^2\right) = \frac{1}{2}mv_1^2 + \frac{1}{2}3mv_2^2 \qquad ...(ii)$$

Solving, $v_1 = \frac{(1-3e)}{4}u,\ v_2 = \frac{(1+e)}{4}u$

Alternate : $\Delta KE = \frac{1}{2}\mu v_{rel}^2(1 - e^2)$ [μ = reduced mass, v_{rel} = initial relative velocity]

29. Ans. (A)

Height of projected body 'A' after 1 sec = $20 \times 1 - \frac{1}{2} \times 10 \times 1^2 = 15\ m$

$v_A = 20 - 10 \times 1 = 10\ m/s$

Now 2nd body 'B' released from height 20 m

$\therefore$ $S_{rel} = 5\ m$, $v_{rel} = 10\ m/s$, $a_{rel} = 0$

$\therefore$ Time after which the two body will meet

$= 1 + \frac{5}{10} = \frac{3}{2}$ sec.

$\therefore$ Velocity of A before collision = $20 - \frac{3}{2} \times 10 = 5\ m/s$

$\therefore$ Velocity of B before collision = $\frac{1}{2} \times 10 = 5 m/s$

Collision is elastic $\Rightarrow$ velocity exchange [$\because$ masses are equal]

$\therefore$ $v_A = 5\ m/s$ downward

$v_B = 5\ m/s$ upward

30. Ans. (D)

Let velocity of ball after collision is V_1'

$V_1' = 2V_2 - V_1$ (V_2 is velocity of platform and V_1 is velocity of ball before collision)

$I = m(\vec{V}_1' - \vec{V}_1) = 30\ N-s$

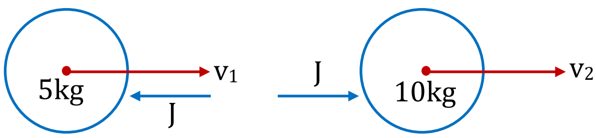
31. Ans. (C)

Impulse = $J = \int F dt$

= Area under curve

$= \frac{1}{2} \times 150 \times 0.3\ N-s$

$= \frac{45}{2}\ N\text{–}s$



$J = 5\ (4 - v_1)$ $J = 10\ (v_2 - (-0.5))$

$4 - v_1 = \frac{9}{2}$ $\frac{4.5}{2} - 0.5 = v_2$

$v_1 = 4 - 4.5 = -0.5 m/s$ $v_2 = 1.75\ m/s$

Velocity of separation = $1.75 + 0.5 = 2.25\ m/s$

Velocity of approach = $4 + 0.5 = 4.5\ m/s$

$e = \frac{2.25}{4.5} = \frac{1}{2}$

32. Ans. (C)

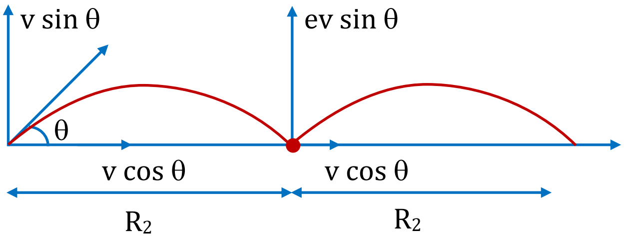
Speed with which the particle hit the ground

$v = \sqrt{u^2 + 2gh}$

As the particle just rises to height h, speed just after impact $v' = \sqrt{2gh}$

But $v' = ev \Rightarrow e = \frac{v'}{v} = \sqrt{\frac{2gh}{u^2+2gh}}$

33. Ans. (D)



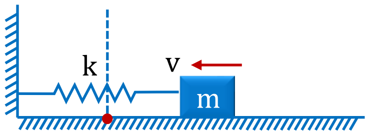
$R = R_1 + R_2$

$R = \dfrac{v^2 \sin 2\theta}{g} + \dfrac{ev^2 \sin 2\theta}{g}$

$R = \dfrac{v^2 \sin 2\theta(1+e)}{g}$

34. Ans. (A)

At maximum compression, a is maximum

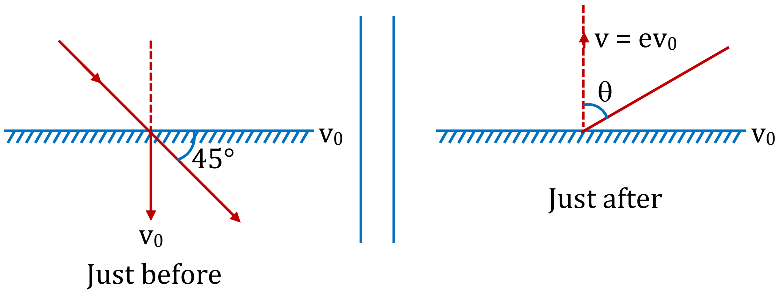


$\dfrac{1}{2}mv^2 = \dfrac{1}{2}kx_{max}^2$

$x_{max} = \sqrt{\dfrac{m}{k}} \times v$

$F = k = ma \quad \Rightarrow \quad a = \dfrac{k}{m} \times \sqrt{\dfrac{m}{k}}v = \sqrt{\dfrac{k}{m}}v$

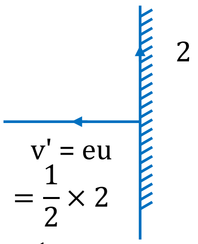
35. Ans. (B)



$\tan\theta = \dfrac{v_0}{ev_0} = \dfrac{1}{e} \qquad \because e \le 1$

$\tan\theta \ge 1 \Rightarrow \theta \ge 45°$

36. Ans. (B)



$v' = eu$

$= \dfrac{1}{2} \times 2$

$=1$

Smooth sphere $\Rightarrow f = 0 \Rightarrow v_y = 2\hat{j}\ m/s$ (unchanged)

$\therefore \quad \bar{v}_f = -\hat{i} + 2\hat{j}$

37. Ans. (A)

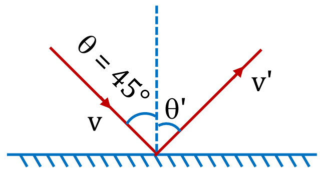
Since the floor exerts the force on the ball along the normal during collision so horizontal component of velocity remains same and only vertical component will change.

Therefore, $v' \sin \theta' = v \sin \theta = \dfrac{v}{\sqrt{2}}$

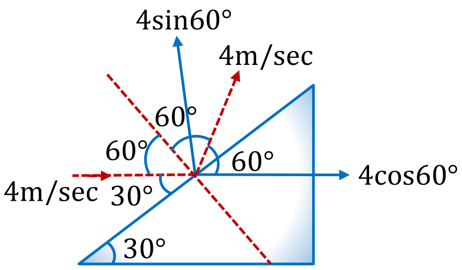
and $v' \cos \theta' = ev \cos \theta = \dfrac{1}{\sqrt{2}} v \times \dfrac{1}{\sqrt{2}} = \dfrac{v}{2}$

$\Rightarrow \quad v'^2 = \dfrac{v^2}{2} + \dfrac{v^2}{4} = \dfrac{3}{4}v^2 \Rightarrow v' = \dfrac{\sqrt{3}}{2}v$

and $\tan\theta' = \sqrt{2} \quad \Rightarrow \theta' = \tan^{-1}\sqrt{2}$



38. Ans. (C)



$\vec{v} = \left(4 \times \frac{1}{2}\right)\hat{\imath} + \frac{4\sqrt{3}}{2}\hat{\jmath} = (2\hat{\imath} + 2\sqrt{3}\hat{\jmath})\ m/\text{sec}$

39. Ans. (C)

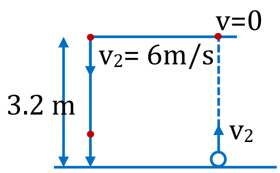
$0^2 = v_2^2 - 2 \times 10 \times 3.2$

$\Rightarrow \quad v_2 = 8\ m/s$

$v_1 = \sqrt{36 + 2 \times 10 \times 3.2} = \sqrt{100}$

$v_1 = 10\ m/ss$

$\therefore \quad e = \dfrac{8}{10} = 0.8$



40. Ans. (B)

Let the speeds of balls of mass m and $2m$ after collision be v_1 and v_2 as shown in figure. Applying conservation of momentum $mv_1 + 2mv_2 = mu$ and $-v_1 + v_2 = \frac{u}{2}$. Solving we get $v_1=0$ and $v_2 = \frac{u}{2}$

Hence the ball of mass m comes to rest and ball of mass $2m$ moves with speed $\frac{u}{2}$.

$t = \dfrac{2\pi r}{u/2} = \dfrac{4\pi r}{u}$

41. Ans. (D)

Just before collision

$WET \Rightarrow v_A = \sqrt{2gh} = \sqrt{2 \times 10 \times 0.45}$

$v_A = 3\ m/s$

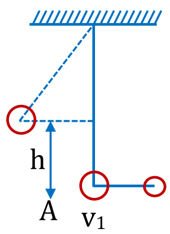
Velocity of ball B after collision is v'_B

$1 = \frac{1}{2} m_B v'^2_B$

$v'_B = 2m/s$

$1 = \dfrac{v'_B - v'_A}{v_A}$

$v_1' = 1\ m/s$ left



42. Ans. (A)

$\vec{F} = \frac{md\vec{v}}{dt} - \vec{v}_{rel}\frac{dm}{dt}$

$F = (M + \lambda x)a - (0 - v)\frac{dm}{dt}$

$= (M + \lambda x)a + \frac{vdm}{dx}\frac{dx}{dt}$

$F = (M + \lambda x)a + \lambda v^2$

43. Ans. (C)

$F_{thrust} = v_{Re\,l.}\frac{dm}{dt} = ma$

$2\frac{dm}{dt} = -m\frac{dv}{dt}$

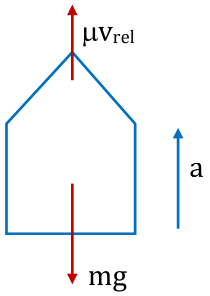
$\Rightarrow -2\int\frac{dm}{m} = \int dv$

$\Rightarrow -2[\ln m]_m^{m/2} = [v]_0^v$

$\Rightarrow 2\ln 2 = v$

$\Rightarrow v = 2\ln 2$

44. Ans. (C)



Initial $\Rightarrow$ mass = 4000

$\mu v_{rel} - mg = ma$

$\mu \times 980 - 4000 \times 9.8$

4000×19.6

$\Rightarrow \mu = 40 + 80 = 120\ kg/sec$

45. Ans. (BCD)

(B) Mass = $\sigma 4\pi r^2$, here σ = Mass per unit area

$m_C = \sigma 4\pi(R/2)^2 = m/4$

Mass of remaining part

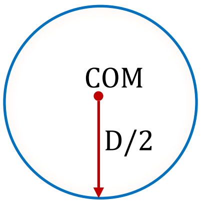
$= m - \frac{m}{4} = \frac{3m}{4}$

(C) $y = \frac{m\times 0 - \frac{m}{4}\times\frac{R}{2}}{m - \frac{m}{4}}$

$y = -\frac{R}{6}$

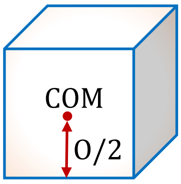
(D) Object is symmetric about y-axis.

46. Ans. (ABD)

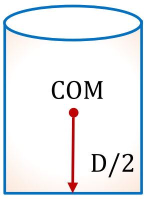


$U_P = mgR = \frac{4\pi}{3}\left(\frac{D}{2}\right)^3 \rho g \frac{D}{2} = \frac{\pi}{12}\rho . D^4 . g$

$U_Q = mg . \frac{D}{2} = D^3 . \rho . g . \frac{D}{2}$



$= \frac{1}{2}\rho . D^4 . g$



$U_R = mg . \frac{D}{4} = \frac{1}{3}\pi\left(\frac{D}{2}\right)^2 . D\rho \times g . \frac{D}{4} = \frac{\pi}{48}\rho . D^4 . g$

$U_S = mg.D/2 = \pi(D/2)^2 . D\rho g \frac{D}{2} = \frac{\pi}{8}\rho D^4 . g$

(A) $U_S > U_P$

(B) $\frac{1}{2} = 0.5, \frac{\pi}{8} = 0.4$

$\therefore \quad U_Q > U_S$

(C) $U_P < U_Q$

(D) $U_S > U_R$

47. Ans. (BC)

Since external force on the system ($A + B$ + plank) is horizontal direction is zero therefor centre of mass of the system remain at rest.

Let displacement of plank is x, than

$60(4 - x) - 50(4 + x) = 90°x$

$40 = 200x$

$x = 0.2\ m$

$d_{Ag} = d_{Ap} - d_{Pg} = 4.2\ m$

48. Ans. (ABC)

(A) No external force is acting so momentum conserved.

(B) In frame of centre of mass momentum is always zero.

(C) V_c is constant.

(D) KE in frame of centre of mass will keep on converting in spring potential energy.

49. Ans. (ABC)

$v_{cm} = \frac{3mv - 2mv}{5m}$, Initial velocity of A in COM frame $= -v - \frac{v}{5} = -\frac{6}{5}v$

Initial velocity of B in COM frame $= v - \frac{v}{5} = \frac{4}{5}v$

50. Ans. (ACD)

(A) $V_{CM} = \frac{mv_0}{M+m}$

(C) In the state of maximum compression, K.E. is equal to spring potential energy

$$K.E. = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \frac{1}{2}\frac{mM}{(m+M)}v_0^2 = \frac{1}{2}.kx^2$$

$$\therefore \quad x = v_0\sqrt{\frac{mM}{(m+M)}\frac{1}{K}}$$

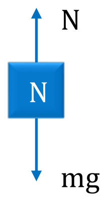
(D) At maximum compression both blocks move together so speed of centre of mass is also same.

51. Ans. (ABD)

(A) Since A has acceleration, so centre of mass will also be accelerating.

(B) At Q, acceleration of A is $\frac{V^2}{R}$ in frame of 'B' so acceleration of centre of mass will not zero.

(C) At Q



$$N - mg = \frac{mV^2}{R}$$

So net normal force will be greater than combined weight.

(D) When 'A' is at Q acceleration of B is zero.

52. Ans. (AC)

$F_{ext} = 0 \Rightarrow a_{cm} = 0 \Rightarrow v_{cm} = 0 \Rightarrow m_A v_A = m_B v_B$

$m_A < m_B$ (given)

$\Rightarrow \quad v_A > v_B$

53. Ans. (ABC)

$F_{ext} = 0 \Rightarrow P =$ conserved

54. Ans. (BC)

No external force in horizontal direction so acceleration in horizontal direction is zero.

Block will attain maximum height when its vertical speed is zero.

55. Ans. (AC)

Since kinetic energy of block decreases.

Since kinetic energy of wedge increases.

56. Ans. (ABD)

(A) No external force so no loss of energy

(B) Work done by hinge is zero so mechanical energy is conserved.

(C) Hinge will exert external force so linear momentum will not be conserved.

(D) No external force linear momentum is conserved.

57. Ans. (BCD)

$4 \times 4 + m \times (-2) = m \times 2 - 4 \times 1$ $[p_i = p_f]$

$\Rightarrow \quad 20 = 4m$

$\Rightarrow \quad m = 5kg$

During collision,

$4 \times 4 + 5 \times (-2) = 9v$

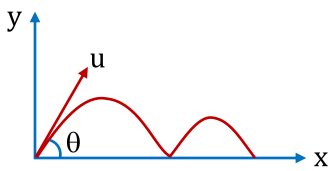
$\Rightarrow \quad v = \frac{6}{9} m/s = \frac{2}{3} m/s$

$\therefore \quad \Delta U_{max} = \frac{1}{2} \times 4 \times 4^2 + \frac{1}{2} \times 5 \times 4 - \frac{1}{2} \times 9 \times \frac{4}{9} = 40J$

Impulse of deformation = $9\left(4 - \frac{2}{3}\right) = 4 \times \frac{10}{3} = 13.33\ NS$

58. Ans. (AB)

$T = \frac{2u \sin\theta}{g} = \frac{2u_y}{g}$ or $T \propto u_y$



$\therefore \quad \frac{T_1}{T_2} = \frac{1}{e}$ Next $H = \frac{u^2 \sin^2\theta}{2g} = \frac{u_y^2}{2g}$

or $\quad H \propto u_y^2 \Rightarrow \frac{H_1}{H_2} = \frac{1}{e^2}$

Next $R = \frac{u^2 \sin 2\theta}{g} = \frac{2u \sin\theta u \cos\theta}{g} = \frac{2u_x u_y}{g}$

$\therefore \quad R \propto u_y$

$\Rightarrow \quad \frac{R_1}{R_2} = \frac{1}{e}$

59. Ans. (A)

$a = \left(\frac{10-2}{10+2}\right) g = \frac{2}{3} g$

$I_{cm} = \frac{10 \times \frac{2}{3} g - 2 \times \frac{2}{3} g}{10+2} = \left(\frac{4g}{9}\right) (\downarrow)$

60. **Ans. (C)**

Because the blocks are accelerating with acceleration 'a'

61. **Ans. (B)**

$-ve$ for 10 kg block because direction of tension and direction of motion is opposite.

$+ve$ for 2 kg block because direction of tension and direction of motion is same.

62. **Ans. (D)**

$60(6 - x) = 120x$

$x = 2\ m$

So, distance between raft and pier = 2 + 0.5 = 2.5 m

63. **Ans. (B)**

$60\ (3 - v) - 120\ (v) = 0$

$180 - 60\ v - 120\ v = 0$

$v = 1\ m/s$

$$W_m = \frac{1}{2}60(2)^2 + \frac{1}{2}120(1)^2$$

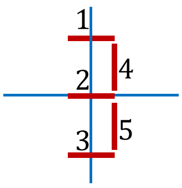
$$= (120 + 60)\ J$$

$W_m = 180\ J$

64. **Ans. (A-P), (B-S), (C-Q,R), (D-T)**

If each stick of length 'l' and mass 'm' then

For P :



$$x = \frac{x_1 m + x_2 m + x_3 m + x_4 m + x_5 m}{5m}$$

$$= \frac{0 + 0 + 0 + \frac{l}{2}m + \frac{l}{2}m}{5m} = \frac{l}{5} > 0$$

$$y = \frac{lm + 0 + (-l)m + \left(\frac{l}{2}\right)m + \left(-\frac{l}{2}\right)m}{5m} = 0$$

$x > 0$ and $y = 0$ i.e. (A)

Similarly for Q : $x = \frac{l}{8} > 0$ and $y = \frac{l}{8} > 0$ i.e. (C)

For R: $x = \frac{l}{12} > 0$ and $y = \frac{l}{12} > 0$ i.e. (C)

For S : $x = 0$ and $y = \frac{l}{6} > 0$ i.e. (B)

For T : $x = 0$ and $y = 0$ i.e. (D)

65. Ans. (A-Q), (B-P,Q), (C-R), (D-S)

(A) $(F_{\text{ext}})_{\text{horizontal}} = 0 \Rightarrow$ COM is same in horizontal
But $(F_{\text{ext}})_{\text{vertical}} \neq 0$ (gravity)
So, COM is shifts downwards

(B) $(F_{\text{ext}})_{\text{horizontal}} \neq 0 \Rightarrow$ COM is shifts towards right
$(F_{\text{ext}})_{\text{vertical}} \neq 0 \Rightarrow$ COM shifts towards downward

(C) Both Block and monkey goes upward $\Rightarrow$ COM is shifts upward

(D) since both blocks are of some mass and one is moving down then other will automatically move up therefore,
$(F_{\text{ext}})_{\text{vertical+horizontal}} = 0$

$$\text{velocity of COM} = \frac{mu - mu}{2m} = 0$$

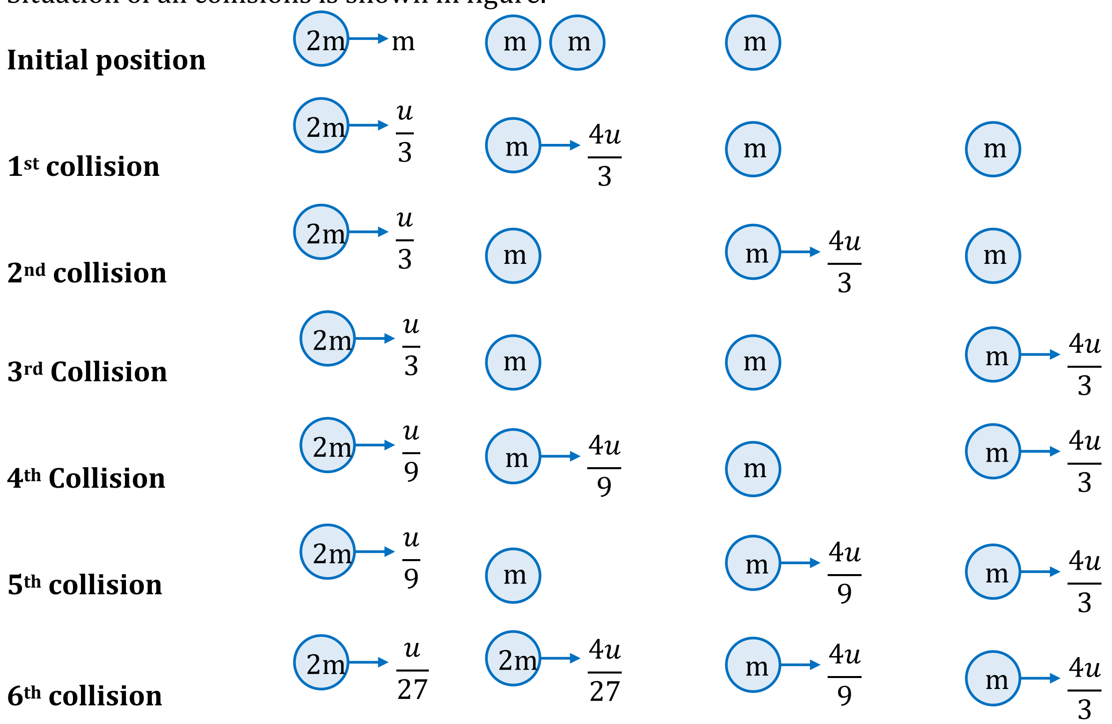
$\Rightarrow$ COM does not shift

66. Ans. (A-S), (B-Q), (C-P), (D-R)

In 1st collision between A and B

$2mu = 2mv_A + mv_B$ and $e = 1 = \dfrac{v_B - v_A}{u}$, $v_A = \dfrac{u}{3}$, $v_B = \dfrac{4u}{3}$

Situation of all collisions is shown in figure.



For (A) Total impulse on $A = 2m\left(u - \frac{u}{27}\right) = \frac{52}{27}mu$

For (B) Total impulse on $B = m\left(\frac{4u}{27} - 0\right) = \frac{4}{27}mu$

For (C) Total impulse on $C = m\left(\frac{4u}{9} - 0\right) = \frac{4}{9}mu$

For (D) Total impulse on $D = m\left(\frac{4u}{3} - 0\right) = \frac{4}{3}mu$

67. Ans. (A-R), (B-T), (C-S), (D-Q)

For (A) : During collision forces are action - reaction pair. So they are in same magnitude.

For (C) : $e = -\left(\frac{v_2 - v_1}{u_2 - u_1}\right) \Rightarrow \left|\frac{v_2 - v_1}{u_2 - u_1}\right| < 1$; $|v_2 - v_1| < |u_2 - u_1|$. So (S) is correct not (P).

EXERCISE - S

1. **Ans.** 1/7, 23/14

$$x_{cm} = \frac{(5\times1)+(3\times(-1))+(2\times2)+(4\times(-1))}{14} = \frac{1}{7}m$$

$$y_{cm} = \frac{(5\times6)+(3\times5)+(2\times-3)+(4\times-4)}{14} = \frac{23}{14}m$$

2. **Ans.** $\sqrt{13}\ m, \left(\frac{14}{5}, \frac{19}{5}\right)$

$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = 2\hat{\imath} - 3\hat{\jmath}$

$\therefore \quad \ell = |\Delta\vec{r}| = \sqrt{4+9} = \sqrt{13}\ m$

$$x_{cm} = \frac{3\times2+2\times4}{5} = \frac{14}{5} = 2.8\ m$$

$$y_{cm} = \frac{3\times5+2\times2}{5} = \frac{19}{5} = 3.8\ m$$

3. **Ans.** 3 m, 1 m, 8 m

$$\frac{6(\hat{\imath}+2\hat{\jmath}+3\hat{k})+5(-\hat{\imath}+3\hat{\jmath}-2\hat{k})+5\vec{r}}{16} = (\hat{\imath} + 2\hat{\jmath} + 3\hat{k})$$

$\vec{r} = (3\hat{\imath} + \hat{\jmath} + 8\hat{k})$

$x_{cm} = 1$	$m_1 = 6\ kg$	$m_2 = 5kg$	$m_3 = 5kg$
$y_{cm} = 2$	(1, 2, 3)	(–1, 3, –2)	(x, y, z)
$z_{cm} = 3$			

$$x_{cm} = \frac{6\times1+5\times(-1)+5x}{16} = 1\ m$$

$\Rightarrow \quad x = 3\ m$

$$y_{cm} = \frac{6\times2+5\times3+5y}{16} = 2 \Rightarrow y = 1\ m$$

$$z_{cm} = \frac{6\times3-5\times2+5z}{16} = 3\ m$$

$\Rightarrow \quad z = 8\ m$

$\therefore \quad (x, y, z) = (3, 1, 8)$

4. **Ans.** $4R$ from O

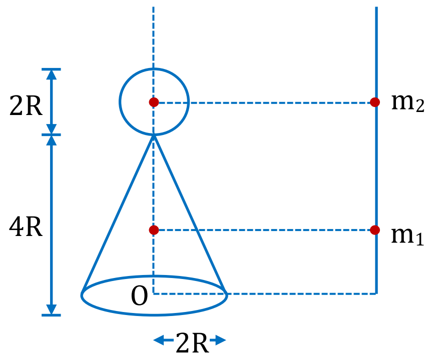
$x_{cm} = 0$

$$y_{cm} = \frac{m_1.\left(\frac{4R}{4}\right)+m_2\times5R}{m_1+m_2} = \left[\frac{V_1\rho_1+5V_2\rho_2}{V_1\rho_1+V_2\rho_2}\right]R$$

$$m_1 = \frac{1}{3}\times\pi\times(2R)^2\times4R\times\rho = \frac{16}{3}\pi R^3\rho$$

$$m_2 = \frac{4\pi}{3}.R^3\times12\rho = 16\pi R^3\rho$$

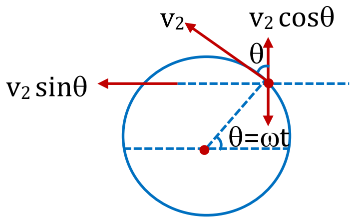
$$\therefore \quad y_{cm} = \frac{\frac{16}{3}\pi R^3\rho R+16\pi R^3 5R\rho}{\frac{16}{3}\pi R^3\rho+16\pi R^3\rho} = \frac{R\left(\frac{1}{3}+5\right)}{\left(\frac{1}{3}+1\right)} = 4R$$



$y_{cm} = 4R$

5. **Ans.** $\vec{P}_{PM} = m\bar{v}_{PM} = -mv_2 \sin\omega t\hat{\imath} + m(v_2 \cos\omega t - v_1)\hat{\jmath}$

$\omega = \frac{v_2}{R}$



$\vec{v}_{rel} = (-v_2 \cdot \sin\theta)\hat{\imath} + (v_2\cos\theta - v_1)\hat{\jmath}$

$= (-v_2 \cdot \sin\omega t)\hat{\imath} + (v_2 \cos\omega t - v_1)\hat{\jmath}$

$\vec{P}_{rel} = m \cdot \vec{v}_{rel}$

$= -mv_2 \sin\omega t\hat{\imath} + m(v_2\cos\omega t - v_1)\hat{\jmath}$

6. **Ans.** 150 kg

Momentum conservation

$10 \times 50 = (50 + M) \times 2.5$

$\Rightarrow M = 150\ kg$

7. **Ans.** $\frac{\sqrt{13}}{2} v_0$

Gravity free

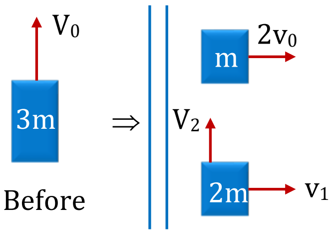
$F_{ext} = 0$

$\Rightarrow P \Rightarrow$ conserved

Along $x \Rightarrow 0 = m \,.\, 2v_0 + 2mv_1$

$\Rightarrow v_1 = -v_0$

Along $y \Rightarrow 3mv_0 = 2mv_2$



$\Rightarrow v_2 = \frac{3v_0}{2}$

$\therefore\ \vec{v}_{2m} = -v_0\hat{\imath} + \frac{3v_0}{g}\hat{\jmath}$

$\therefore\ |\vec{v}_{2m}| = \sqrt{v_0^2 + \frac{9v_0^2}{4}} = \frac{\sqrt{13}}{2} v_0$

8. **Ans.** 0.3

$m \times 6 + 2m \times 3 = 3m \times V_C$

$V_C = 4$

with respect to P

$0 = 3^2 - 2 \times (3\mu g/2) \times L$

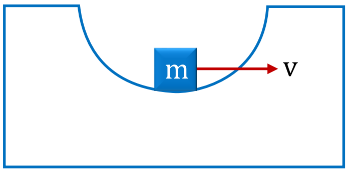
$\mu = 0.3$

9. **Ans.** 4

$-60 \times 2 + 80 \times 1 + 100\ V = 0$

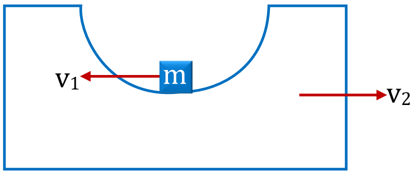
$V = \frac{4}{10} m/s$

10. Ans. 2



$\frac{1}{2} \times mv^2 = mg \times 0.2$

$v^2 = 20 \times 0.2 = 4$



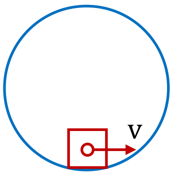
$mv_2 - mv_1 = m \times 2$ ...(i)

$\frac{1}{2} \times m(v)^2 = \frac{1}{2} \times mv_1^2 + \frac{1}{2} \times mv_2^2$ ...(ii)

$v_1 = 0$

$v_2 = 2m/s$

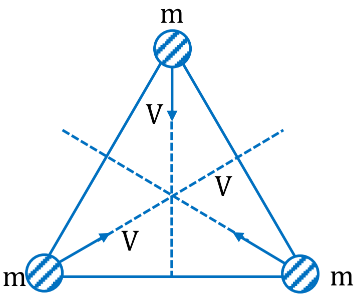
11. Ans. (i) $v_0/3$, (ii) $3\sqrt{5gR}$



(i) $mv_0 + 0 = 3mv \quad \Rightarrow v = \frac{v_0}{3}$

(ii) For successful motion $\Rightarrow v = \sqrt{5gR} \quad \Rightarrow \frac{v_0}{3} = \sqrt{5gR} \quad \Rightarrow v_0 = 3\sqrt{5gR}$

12. Ans. $\vec{v}_C = -\vec{v}_B$



As all taken as system :

$\vec{F}_{ext} = 0 \Rightarrow \vec{P}_i = \vec{P}_f$

$m\vec{V}_A + m\vec{V}_B + m\vec{V}_c = P_i = 0$

$m_A(0) + m_B \times v + m_c V_C = 0$

$mv = -mv_C$

$v_C = -v$

13. **Ans.** 2

After covering distance of 1.8m, velocity of block will be,

$$v=\sqrt{2gh}=\sqrt{2\times 10\times 1.8}$$

$$v = 6\ m/s$$

From impulse equation, $\int Tdt = mv - 0$

$$-\int Tdt = mv - 6m$$

$$-2v = v - 6$$

$$3v = 6$$

$$\Rightarrow \quad v = 2m/s$$

14. **Ans.** (i) $\sqrt{\frac{2ag}{3}}$ (ii) $\frac{3v}{g}$ (iii) $\frac{2v}{g}$

(i) When a strikes $\Rightarrow A$ move a down & B move a P

$$WET \Rightarrow 2mga - mga = \frac{1}{2}.2mv^2 + \frac{1}{2}mv^2$$

$$\Rightarrow \quad mga = \frac{3}{2}mv^2 \Rightarrow v = \sqrt{\frac{2}{3}ga}$$

(ii) For time between release of black A to just before a strikes.

$$a_A = a_B = \frac{2mg - mg}{3m} = \frac{g}{3}$$

$$\therefore \quad \text{For } A \Rightarrow a = b\ .\ \frac{g}{3}\ .\ 2\ t = \sqrt{\frac{6a}{g}}$$

$$= \sqrt{\left(\frac{2}{3}ag\right).\frac{9}{g^2}} = \frac{3}{g}.v$$

(iii) Just after impact $\Rightarrow v_B = v = \sqrt{\frac{2}{3}ga}$ upwards and $a_B = g$ downwards

$$0 = v - gt$$

$$\Rightarrow \ t = \frac{v}{g}$$

$$\therefore \ T = (\text{upwards + downwards})$$

$$T = \frac{2v}{g}$$

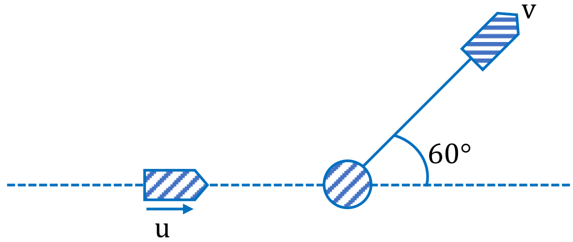
15. **Ans.** $m \times \sqrt{u^2 - uv + v^2}$

Impulse = $m\vec{v}_f - m\vec{v}_i = m\left|\vec{v}_f - \vec{v}_i\right|$

$$= m\sqrt{V_f^2 + V_r^2 - 2V_f V_i \cos\theta}$$

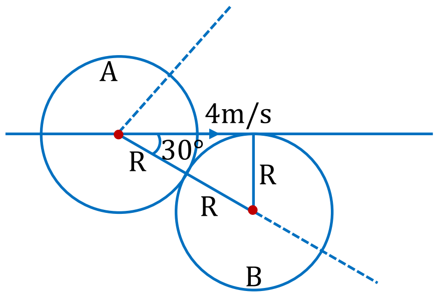
$$= m\sqrt{v^2 + u^2 - 2 \times v \times u \times \frac{1}{2}}$$

$$I = m\sqrt{v^2 + u^2 - uv}$$

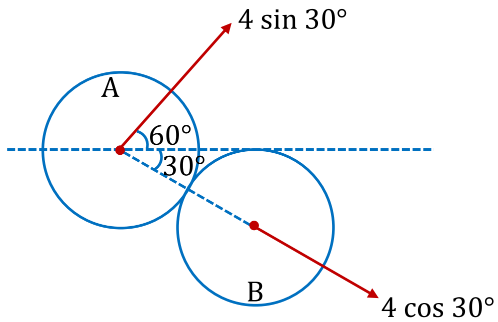


16. **Ans.** $(\hat{\imath} + \sqrt{3}\hat{\jmath})\ m/s$, $(3m\hat{\imath} - \sqrt{3}m\hat{\jmath})\ kg\text{-}m/s$

Before collision



After collision



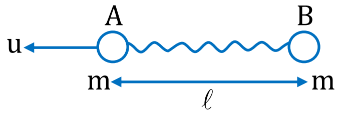
$v_A = 4\sin 30°[\cos 60\hat{\imath} + \sin 60\hat{\jmath}]$; $v_A = \hat{\imath} + \sqrt{3}\hat{\jmath}\ m/s$

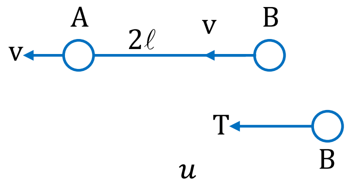
$\vec{J}_{A \text{ on } B} = m\left(\vec{V}_{B_f} - \vec{V}_{B_i}\right) = m[4\cos 30°(\cos 30°\hat{\imath} - \sin 30°\hat{\jmath}) - 0]$ $=(3mi - \sqrt{3}mj)\ kg\text{-}m/s$

There is a net loss in the kinetic energy of the two particle system in the collision.

17. **Ans.** (i) $\frac{u}{2}, \frac{mu}{2}$ (ii) $\frac{u\sqrt{13}}{8}, \frac{mu\sqrt{13}}{8}$ (iii) $\frac{u\sqrt{3}}{4}, \frac{mu\sqrt{3}}{4}$

(i)

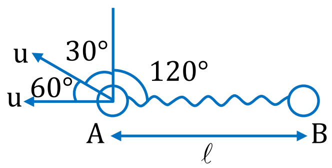


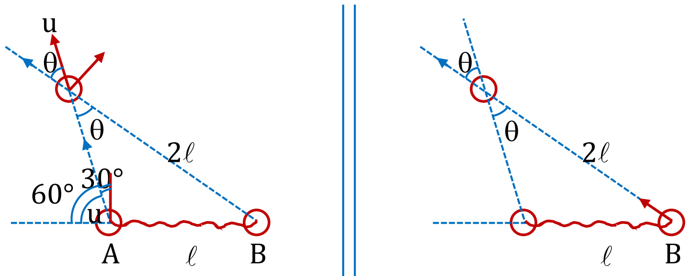


$2mv = mu \Rightarrow v = \frac{u}{2}$

$\therefore$ impulse of tension = $(\Delta P)_B = mv = mu/2$

(ii)





Just before string is tight

Just after string is tight

Sine rule $\Rightarrow \dfrac{\sin\theta}{\ell} = \dfrac{\sin 120^\circ}{2\ell} \Rightarrow \sin\theta = \dfrac{\sqrt{3}}{4}$

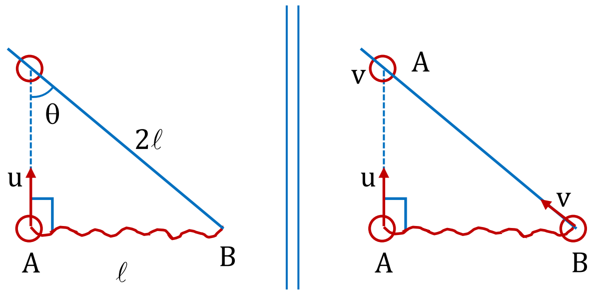
$\Rightarrow \quad \cos\theta = \dfrac{\sqrt{3}}{4}$

$2m.v = mu\cos\theta$

$v = \dfrac{u\cos\theta}{2} = \dfrac{u}{2} \times \dfrac{\sqrt{13}}{4} = \dfrac{u\sqrt{13}}{8}$

$\therefore$ Impulse of tension = $(\Delta P)_B = \dfrac{m.u\sqrt{13}}{8}$

(iii)



Just before

$\sin\theta = \dfrac{\ell}{2\ell} \Rightarrow \theta = 30^\circ$

From $\vec{P}$ conservation along string just after string is tight

$2mv = mu\cos\theta = mu\cos 30^\circ$

$v = \dfrac{u\sqrt{3}}{4}$

$\therefore$ Impulse of tension $=(\Delta P)_B = mv = \dfrac{mu\sqrt{3}}{4}$

18. **Ans.** (i) $v/2, v/2, 0$; (ii) $2mv^2/9$; (iii) $mv^2/72$; (iv) $x = \sqrt{\frac{m}{6k}}\,v$

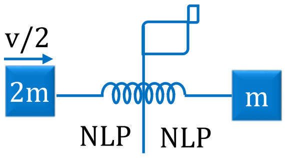
(i)

$mv = 2mv_1 \quad \Rightarrow \quad v_1 = v/2$

Just after collision $\Rightarrow v_{m_1} = v_{m_2} = v/2$

$v_{m_3} = 0$

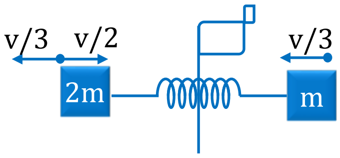
(ii)



COM

(rest)

$$v_{cm} = \frac{2m.\frac{v}{2}}{3m} = \frac{v}{3} \rightarrow (right)$$



$\therefore$ In COM frame

Range of velocity v_A (COM frame) $\left[\frac{v}{6}, \frac{5v}{6}\right]$, Range of velocity v_B $\left[-\frac{v}{3}, \frac{v}{3}\right]$

Range in (ground) A: $\left[\frac{v}{6} - \frac{v}{3}, \frac{5v}{6} - \frac{v}{3}\right]$ B: $\left[\frac{v}{6} - \frac{v}{3}, \frac{5v}{6} - \frac{v}{3}\right]$

$\therefore$ Max. velocity of m_3 during its motion

$\therefore \; (k_3)_{max} = \frac{1}{2}.m.\left(\frac{2v}{3}\right)^2$

$\therefore \; (k_3)_{max} = \frac{2}{9}mv^2$

(iii) According to option (ii)

$\left[\frac{v}{6}, \frac{v}{2}\right]$ min velocity of combined mass after collision = $\frac{v}{6}$

$\Rightarrow \; (v_2)_{min} = \frac{v}{6}$

(min of m_2)

$\therefore \; (k_2)_{min} = \frac{1}{2} \times m\left(\frac{v}{6}\right)^2 = \frac{mv^2}{72}$

(iv) Max. compression $\Rightarrow$ at max. compression $\Rightarrow$ $v_1 = v_2 = v_3\ v_c$

$$\frac{1}{2}uv_{rel2'}^2 = \frac{1}{2}mx^2$$

$$\Rightarrow \frac{1}{2}\times\frac{2m.m}{3m}\left(\frac{v}{2}\right)^2 = \frac{1}{2}kx^2$$

$$\Rightarrow x = v\sqrt{\frac{m}{6k}}$$

19. **Ans.** $1N$

Change in momentum of single bullet = $mv_f - m \times 0 = 10 \times 10^{-3} \times 100$

$\therefore$ Change in momentum of 60 bullets = 60 N_1 sec

$\therefore$ (DP)per sec = $\frac{60}{60} = 1 = \frac{dP}{dt}$

$\therefore$ Force exerted an bullet = $1N$

$\therefore$ Force on person = $1N$

20. **Ans.** $3\ Mgx/L$

Lets take a small, element at distance x of length dx, velocity with which element will strike the floor,

$$v = \sqrt{2gx}$$

Momentum transferred to floor is,

$$dp = dmv = \left[\frac{M}{L}xdx\sqrt{2gx}\right]$$

$$F_1 = \frac{dp}{dt}$$

$$v = \frac{dx}{dt} = \sqrt{2gx}$$

$$F_1 = \frac{M\sqrt{2gx}\times\sqrt{2gx}}{L}$$

$$F_1 = \frac{2mgx}{L}$$

Now, the force entered by length x due to its own weight is,

$$w = \frac{mgx}{L}$$

$$\text{Total force} = \frac{mgx}{L} + \frac{2mgx}{L}$$

$$F = \frac{3mgx}{L}$$

21. **Ans.** 1900 N

Area of roof, $A = 100\ m^2$

velocity of strike, $u = 20 m/s$

velocity of rebound, $v = 0$

strike rate in 100 m^2 area, n = 2000 hailstone/sec

Mass of hailstones, $m = \rho\left(\frac{4}{3}\pi r^3\right) = 900 \times \frac{4}{3}\pi(0.01)^3$

$m = 3.77 \times 10^{-3}\ kg$

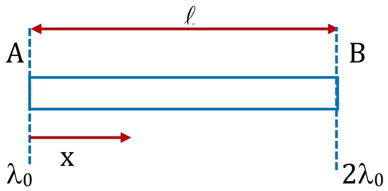
Force = (rate of mass strike) × (velocity change)

$= nm(v - u)$

$= 200 \times 3.77 \times 10^{-3} \times (20 - 0)$

Force = 150.8 N

22. **Ans.** (i) $\lambda(x) = \lambda_0 + \frac{\lambda_0 x}{\ell}$, (ii) $\frac{5}{9}\ell$



$\lambda(x) = \lambda_0 + \frac{(2\lambda_0 - \lambda_0)}{\ell}x = \lambda_0 + \frac{\lambda_0 x}{\ell}$

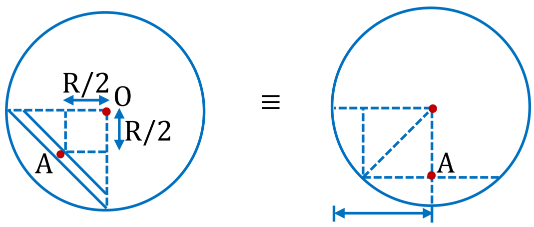
$\lambda = \lambda_0\left(1 + \frac{x}{\ell}\right)$

$x_{cm} = \frac{\int x dm}{\int dm} = \frac{\int x(\lambda dx)}{\int \lambda dx}$

$= \frac{\int_0^\ell x.\lambda_0\left(1+\frac{x}{\ell}\right)dx}{\int_0^\ell \lambda_0\left(1+\frac{x}{\ell}\right)dx} = \frac{\left(\frac{\ell^2}{2}+\frac{\ell^2}{3}\right)}{\ell + \frac{\ell}{2}}$

$= \frac{\frac{9}{6\ell^2}}{\frac{3}{\ell}} = \frac{5}{9}\ell$

23. **Ans.** $\frac{R}{4}$



Let displacement of sphere = x (right)

Displacement of rod = $x + R/2$

$x + R/2$ (right)

$\Sigma mx = 0$

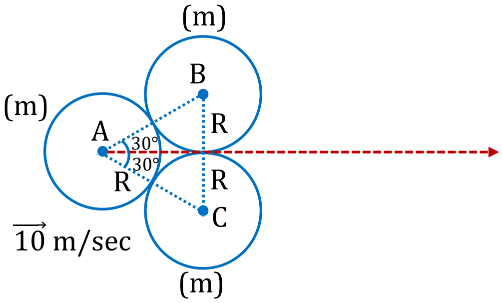
$\Rightarrow$ $mx + m(x + R/2) = 0$

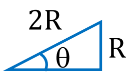
$\Rightarrow$ $x = -R/4 \Rightarrow R/4$ left

24. **Ans.** $-2 m/s$, $6.93\ m/s\ \angle 30°$

Let me name the balls as A, B and C.

Just before collision:



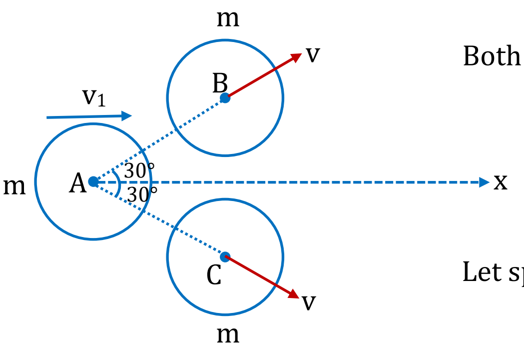


$$\sin\theta = \frac{R}{2R} = \frac{1}{2}$$

$$\theta = 30°$$

Each ball makes angle 30° with horizontal line x.

After collision, the condition is shown:



Both B & C will have same speed 'v' each

(Due to symmetry)

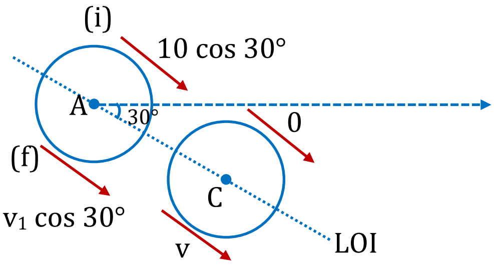
Let speed of ball 'A' is v_1 along x.

We apply momentum conservation along x :

$$P_i = P_f \Rightarrow\ m\times 10 + 0\times 0 = mV_1 + 2mV\cos 30°$$

$$10 = V_1 + \sqrt{3}V \qquad \text{... (1)}$$

Secondary we apply 'e' equation along LOI



$$e = \frac{V_{sep}}{V_{app}}$$

$$V - V_1\cos 30° = 1(10\cos 30 - 0)$$

$$V - V_1\frac{\sqrt{3}}{2} = 5\sqrt{3} \qquad \text{...(2)}$$

From equation (1) and (2) we get

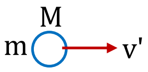
$$V = 4\sqrt{3}\ =\ 6.928 m/\sec$$

$$V_1 = -2\,m/\sec$$

EXERCISE - JEE (Main) PYQ

1. **Ans. (4)**

Energy loss will be maximum when collision will be perfectly inelastic



(By momentum) $mv = (m + M)\, v'$

Maximum energy loss = $K_i - K_f$

$$= \frac{1}{2}mv^2 - \frac{1}{2}(m + M)v'^2$$

$$= \frac{1}{2}mv^2 - \frac{1}{2}\,(m + M)\,\frac{m^2v^2}{(m+M)^2}$$

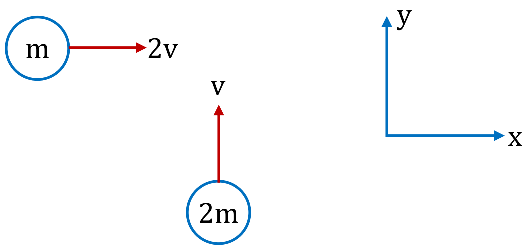
$$= \frac{1}{2}mv^2\left[1 - \frac{m}{m+M}\right]$$

$$= \left(\frac{M}{m+M}\right)\frac{1}{2}mv^2$$

Statement 1 is false.

2. **Ans. (1)**

Before collision



Kinetic energy $= \frac{1}{2}m(2v)^2 + \frac{1}{2}2m(v)^2$

$= 3\ mv^2$

After collision

Applying momentum conservation for inelastic collision

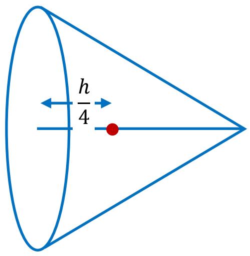
$$2mv\hat{j} + m2v\hat{i} = 3m\vec{v}_f$$

$$|\vec{v}_f| = \sqrt{\frac{8}{9}}v$$

$$K_f = \frac{1}{2} \times 3m \times \left(v_f^2\right) = \frac{4mv^2}{3}$$

$$\%\ \Delta K = \frac{K_i - K_f}{K_i} = \frac{5mv^2/3}{3mv^2} = \frac{5}{9} = 56\%$$

3. **Ans. (4)**



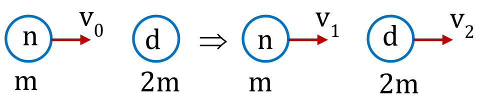
For solid cone c.m. is $\dfrac{h}{4}$ from base.

So $z_0 = h - \dfrac{h}{4} = \dfrac{3h}{4}$

4. **Ans. (4)**

Let initial speed of neutron is v_0 and kinetic energy is K.

1st collision : By momentum conservation



$$mv_0 = mv_1 + 2mv_2 \Rightarrow v_1 + 2v_2 = v_0$$

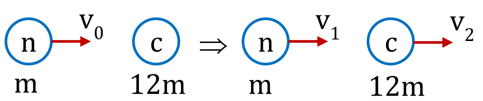
By $\quad e = 1 \qquad v_2 - v_1 = v_0$

$\Rightarrow \quad v_2 = \dfrac{2v_0}{3} \qquad v_1 = -\dfrac{v_0}{3}$

$$\text{Fractional loss} = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{v_0}{3}\right)^2}{\frac{1}{2}mv_0^2}$$

$$\Rightarrow \quad p_d = \frac{8}{9} \approx 0.89$$

2nd collision :



By momentum conservation

$$mv_0 = mv_1 + 12mv_2$$

$\Rightarrow \quad V_1 = V_0 - 12V_2$

By $\quad e = 1 \qquad v_2 - v_1 = v_0$

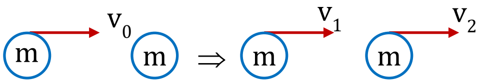
$V_2 = \dfrac{2V_0}{13} \qquad V_1 = \dfrac{-11V_0}{13}$

Now fraction loss of energy

$$p_C = \frac{\frac{1}{2}mv_0^2 - \frac{1}{2}m\left(\frac{11v_0}{13}\right)^2}{\frac{1}{2}mv_0^2} = \frac{48}{169} \approx 0.28$$

5. Ans. (1)

Initial



$$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$$

$$\Rightarrow \quad v_1^2 + v_2^2 = \frac{3}{2}v_0^2 \qquad ...(1)$$

From momentum conservation

$$mv_0 = m(v_1 + v_2) \qquad ...(2)$$

$$(v_1 + v_2)^2 = v_0^2$$

$$\Rightarrow \quad v_1^2 + v_2^2 + 2v_1v_2 = v_0^2$$

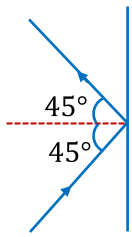
$$2v_1v_2 = -\frac{v_0^2}{2}$$

$$(v_1 - v_2)^2 = v_1^2 + v_2^2 - 2v_1v_2 = \frac{3}{2}V_0^2 + \frac{V_0^2}{2}$$

$$v_1 - v_2 = \sqrt{2}v_0$$

6. Ans. (4)

$$F = \frac{dp}{dt} = 2nmv\cos 45°$$



$$\text{Pressure} = \frac{F}{A} = \frac{2nmv\cos 45°}{Area}$$

$$= \frac{2\times10^{23}\times3.3\times10^{-27}\times10^{3}\times\left(\frac{1}{\sqrt{2}}\right)}{2\times10^{-4}}$$

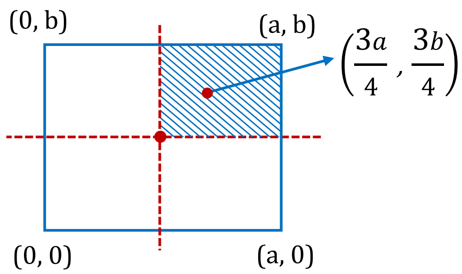
$$= 2.35 \times 10^3\ N/m^2$$

7. Ans. (4)

$$x = \frac{M\frac{a}{2} - \frac{M}{4}\times\frac{3a}{4}}{M - \frac{M}{4}}$$

$$= \frac{\frac{a}{2} - \frac{3a}{16}}{\frac{3}{4}} = \frac{\frac{5a}{16}}{\frac{3}{4}} = \frac{5a}{12}$$

$$y = \frac{M\frac{b}{2} - \frac{M}{4}\times\frac{3b}{4}}{M - \frac{M}{4}} = \frac{5b}{12}$$

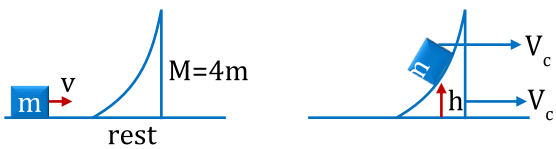


8. Ans. (3)

Applying Linear momentum conservation

$$mv = (m + M)v_c$$

$$v_c = \frac{v}{5}$$



Applying work energy theorem

$$-mgh = \frac{1}{2}(m + M)v_c^2 - \frac{1}{2}mv^2$$

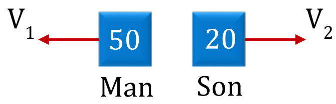
$$\text{Solve, } h = \frac{2v^2}{5g}$$

9. Ans. (1)

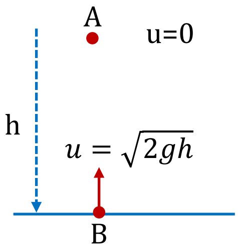
$\Rightarrow \quad 0 = 50V_1 - 20V_2$

and $\quad V_1 + V_2 = 0.7$

$\Rightarrow \quad V_1 = 0.20\ m/s$



10. Ans. (4)



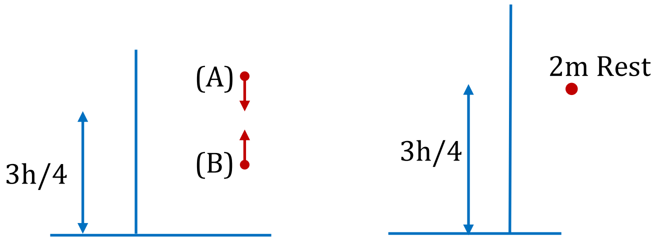
Particles will collide after time, $t_0 = \dfrac{h}{\sqrt{2gh}}$

At collision,

$v_A = gt_0 \qquad\qquad v_B = u_B - gt_0$

$\Rightarrow \quad v_A = -v_B$

Before collision $\qquad\qquad$ After collision



Time taken by combined mass to reach the ground

$$\text{Time} = \sqrt{\frac{2\times 3h/4}{g}} = \sqrt{\frac{3h}{2g}}$$

11. Ans. (2)

From momentum conservation

$$mu\hat{i} + mu\left(\frac{\hat{i}+\hat{j}}{2}\right) = (m+m)\,\vec{v}$$

$\Rightarrow \quad \vec{v} = \dfrac{3}{4}u\hat{i} + \dfrac{u}{4}\hat{j}$

$\Rightarrow \quad |v| = \dfrac{u}{4}\sqrt{10}$

Final kinetic energy $= \dfrac{1}{2}2m\left(\dfrac{u}{4}\sqrt{10}\right)^2 = \dfrac{5}{8}mu^2$

Initial kinetic energy

$$= \frac{1}{2}mu^2 + \frac{1}{2}m\left(\frac{u}{\sqrt{2}}\right)^2 = \frac{6}{8}mu^2$$

Loss in $K.E. = k_i - k_f = \dfrac{1}{8}mu^2$

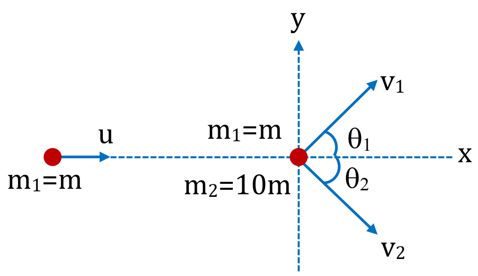
12. Ans. (10)

By momentum conservation along y :

$m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2$

i.e. $mv_1 \sin\theta_1 = 10mv_2 \sin\theta_2$

$\boxed{v_1 \sin\theta_1 = 10v_2 \sin\theta_2}$ ...(i)



$(k_f)_{m_1} = \frac{1}{2}(k_i)_{m1} i.e. \frac{1}{2}mv_1^2 = \frac{1}{2}\times\frac{1}{2}mu^2$

i.e. $\boxed{v_1 = \frac{u}{\sqrt{2}}}$ ...(ii)

Also collision is elastic : $k_i = k_f$

$\frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}10mv_2^2$

$\frac{1}{2}mu^2 = \frac{1}{2}\times\frac{1}{2}mu^2 + \frac{1}{2}10mv_2^2$

$\frac{1}{4}mu^2 = \frac{1}{2}\times 10\times mv_2^2$

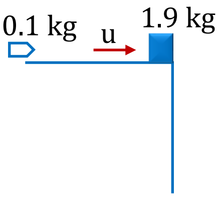
$\boxed{v_2 = \frac{u}{\sqrt{20}}}$ ...(iii)

Putting (ii) and (iii) in (i)

$\frac{u}{\sqrt{2}}\sin\theta_1 = 10.\frac{u}{\sqrt{20}}\sin\theta_2$

$\boxed{\sin\theta_1 = \sqrt{10}\sin\theta_2} \rightarrow$ Hence $n = 10$

13. Ans. (1)



$p_i = p_f \Rightarrow 0.1 \times 20 = 2v$

$\therefore \quad v = 1\ m/s$

$KE_f = mgh + \frac{1}{2}mv^2 = 21J$

14. Ans. (4)

$\vec{V}_{01} = (\sqrt{3}\hat{\imath} + \hat{\jmath})m/s$

$\vec{V}_{02} = 0\ m/s$

$m_1 = 2m_2$

After collision, $\vec{V}_1 = (\hat{\imath} + \sqrt{3}\hat{\jmath})m/s$

$\vec{V}_2 = ?$

Applying conservation of linear momentum,

$m_1\vec{v}_{01} + m_2\vec{v}_{02} = m_1\vec{v}_1 + m_2\vec{v}_2$

$2m_2(\sqrt{3}\hat{\imath} + \hat{\jmath}) + 0 = 2m_2(\hat{\imath} + \sqrt{3}\hat{\jmath}) + m_2\vec{v}_2$

$\vec{v}_2 = 2(\sqrt{3}\hat{\imath} + \hat{\jmath}) - 2(\hat{\imath} + \sqrt{3}\hat{\jmath})$

$= 2(\sqrt{3}\hat{\imath} - \hat{\jmath}) + 2(\hat{\imath} - \sqrt{3}\hat{\jmath})$

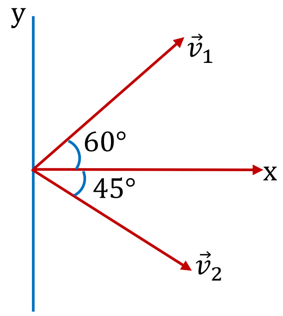
$\vec{v}_2 = 2(\sqrt{3} - 1)(\hat{\imath} - \hat{\jmath})$

For angle between $\vec{v}_1$ and $\vec{v}_2$,

$$\cos\theta = \frac{\vec{v}_1 . \vec{v}_2}{v_1 v_2} = \frac{2(\sqrt{3}-1)(1-\sqrt{3})}{2 \times 2\sqrt{2}(\sqrt{3}-1)}$$

$$\cos\theta = \frac{1-\sqrt{3}}{2\sqrt{2}} \Rightarrow \theta = 105°$$

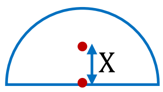
or



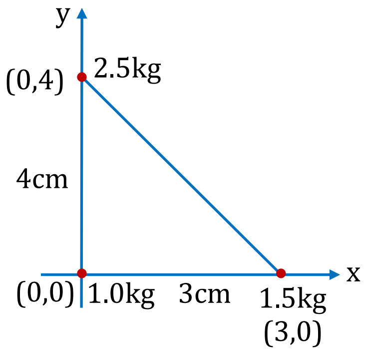
15. Ans. (3)

$x = \frac{3R}{8} = 3cm$

$x = 3$



16. Ans. (2)



Let 1 *kg* as origin and *x*-*y* axis as shown

$$x_{cm} = \frac{1(0)+1.5(3)+2.5(0)}{5} = 0.9\ cm$$

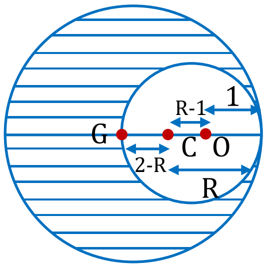
$$y_{cm} = \frac{1(0)+1.5(0)+2.5(4)}{5} = 2\ cm$$

17. Ans. (3)

By concept of *COM*

$$m_1R_1 = m_2R_2$$

Remaining mass × $(2-R)$ = cavity mass × $(R-1)$



$$\left(\frac{4}{3}\pi R^3\rho - \frac{4}{3}\pi 1^3\rho\right)(2-R) = \frac{4}{3}\pi 1^3\rho \times (R-1)$$

$$(R^3 - 1)(2 - R) = R - 1$$

$$(R^2 + R + 1)(2 - R) = 1$$

18. Ans. (1)

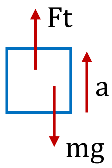
$$\frac{P_1^2}{2\times 4} = \frac{P_2^2}{2\times 16}$$

$$\frac{P_1}{p_2} = \frac{1}{2}$$

$\Rightarrow \quad n = 1$

19. Ans. (4)

$$F_{thrust} = \left(\frac{dm}{dt}\cdot V_{rel}\right)$$



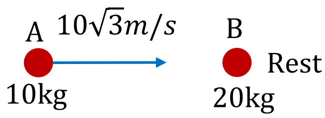
$$\left(\frac{dm}{dt}V_{rel} - mg\right) = ma$$

$$\Rightarrow \quad \left(\frac{dm}{dt}\right)\times 500 - 10^3\times 10 = 10^3\times 20$$

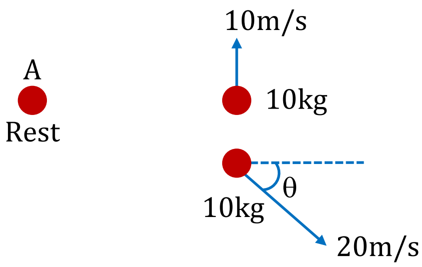
$$\frac{dm}{dt} = (60kg/s)$$

20. Ans. (30)

Before Collision



After Collision



From conservation of momentum along *x* axis;

$$\vec{P}_i = \vec{P}_f$$

$$10 \times 10\sqrt{3} = 200\cos\theta$$

$$\cos\theta = \frac{\sqrt{3}}{2}$$

$$\theta = 30°$$

21. Ans. (125)

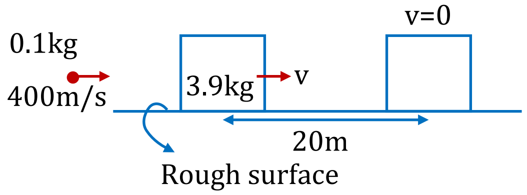
Kinetic energy of body = $\frac{p^2}{2m}$

Initial kinetic energy = $\frac{p_i^2}{2m}$

Final kinetic energy $= \frac{p_f^2}{2m} = \frac{(1.5p_i)^2}{2m} = \frac{2.25p_i^2}{2m}$

% increase in KE = $\frac{2.25\frac{p_i^2}{2m} - \frac{p_i^2}{2m}}{\frac{p_i^2}{2m}} \times 100 = 125\%$

22. Ans. (4)



$P_i = P_f$ (Collision)

$\Rightarrow$ (0.1) (400) = (0.1+3.9)v

$\Rightarrow$ $v = \frac{0.1 \times 400}{4} = 10m/s$

$a = \frac{\mu mg}{m} = \mu g$

Apply equation of motion,

$v^2 = u^2 + 2as$

$\Rightarrow$ $0 = (10)^2 - 2\mu g \times 20$

$\Rightarrow$ $40\mu g = 100$

$\Rightarrow$ $\mu = \frac{100}{2 \times 10 \times 20} = \frac{1}{4}$

23. Ans. (30)

Given

$M = 5\ kg$

$P_i = 10\ kg\ m/s$ (initial momentum)

Impulse = $F\Delta t = \Delta P = P_f - P_i$

$2 \times 5 = P_f - 10$

$P_f = 20\ kg\ m/s$ (final momentum)

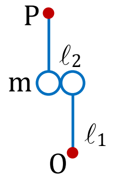
Increase in $KE = KE_f - KE_i$

$= \frac{P_f^2}{2m} - \frac{P_i^2}{2m}$

$= \frac{400}{2 \times 5} - \frac{100}{2 \times 5} = 40 - 10 = 30J$

EXERCISE - JEE (Advanced) PYQ

1. **Ans. (5)**



Speed at highest point = Initial speed

$$\sqrt{g\ell_1} = \sqrt{5g\ell_2} \Rightarrow \frac{\ell_1}{\ell_2} = 5$$

2. **Ans. (B)**

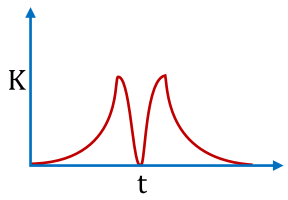
During fall, $v = 0 + gt$

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}m\,(gt)^2$$

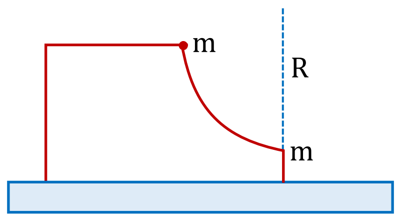
$KE \propto t^2$, so graph is upward parabola.

During collision, KE will decrease in compression and increase in reformation.

Finally, during going up KE will decrease.



3. **Ans. (AC)**

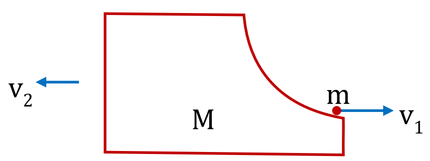


$$M\Delta x_{CM} = m(R - x) - Mx$$

$$\Delta x_{CM} = 0$$

$$Mx = m(R - x)$$

$$x = \frac{mR}{M+m} \text{ (A)}$$



$$0 = m\bar{v}_1 + M\bar{v}_2$$

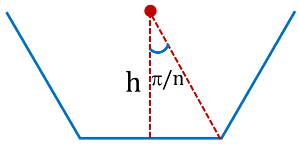
$$\bar{v}_2 = -\frac{m\bar{v}_1}{M}$$

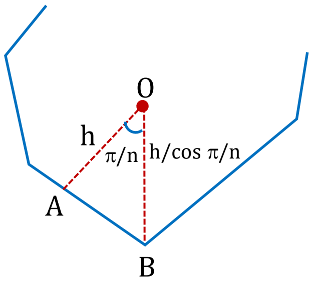
$$mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}Mv_2^2$$

$$mgR = \frac{1}{2}mv_1^2 + \frac{1}{2}M\left(\frac{mv_1}{M}\right)^2$$

$$\sqrt{\frac{2gR}{\left(1+\frac{m}{M}\right)}} = v_1 \text{ (C)}$$

4. Ans. (C)





$OA = h$

$OB = \dfrac{h}{cos\frac{\pi}{n}}$

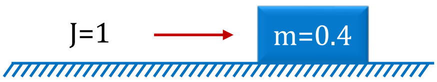
Initial height of $COM = h$

Final height of $COM = \dfrac{h}{cos\frac{\pi}{n}}$

$\therefore \quad \Delta = \dfrac{h}{cos\frac{\pi}{n}} - h$

$\Delta = h\left(\dfrac{1}{cos\frac{\pi}{n}} - 1\right)$

5. Ans. (6.3) [6.29, 6.31]



$v = v_0 e^{-t/\tau}$

$v_0 = \dfrac{J}{m} = 2.5\ m/s$

$v = v_0 e^{-t/\tau}$

$\dfrac{dx}{dt} = v_0 e^{-t/\tau}$

$\int_0^x dx = v_0 \int_0^\tau e^{-t/\tau} dt$

$\int e^{-x} dx = \dfrac{e^{-x}}{-1}$

$x = v_0 \left[\dfrac{e^{-t/\tau}}{-\frac{1}{\tau}}\right]_0^\tau$

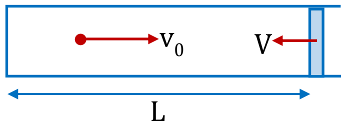
$x = 2.5\ (-4)\ (e^{-1} - e^0)$

$x = 25\ (-4)\ (0.37 - 1)$

$x = 6.30$ ans.

6. Ans. (BC)

(1) Average rate of collision = $\frac{2L}{v}$



(2) Speed of particle after collision = $2V + v_0$

Change in speed = $(2V + v_0) - v_0$

After each collision = $2V$

No. of collision per unit time (frequency) = $\frac{v}{2L}$

Change in speed in dt time = $2V$ × number of collision in dt time

$\Rightarrow$ $dv = 2V\left(\frac{v}{2L}\right).\frac{dL}{V}$

$\boxed{dv = \frac{vdL}{L}}$

Now, $dv = -\frac{vdL}{L}$ {as L decrease}

$\int_{v_0}^{v}\frac{dv}{v} = -\int_{L_0}^{L_0/2}\frac{dL}{L}$

$\Rightarrow$ $[lnv]_{v_0}^{v} = -[ln \quad L]_{L}^{L_0/2}$

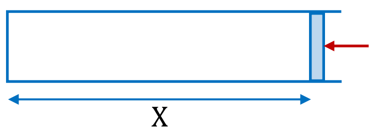
$\Rightarrow$ $v = 2v_0$

$\Rightarrow$ $KE_{L0} = \frac{1}{2}mv^2$ $\boxed{\frac{KE_{L_0/2}}{KE_0} = 4}$

$KE_{L0/2} = \frac{1}{2}m(2v_0)^2$

or $(dt)\left(\frac{v}{2x}\right)\frac{2mv}{dt} = F$

$F = \frac{mv^2}{x}$



$-m\,v\frac{dv}{dx} = \frac{mv^2}{x}$

$-\frac{dv}{v} = \frac{dx}{x}$

$\ln\left(-\frac{v_2}{v_1}\right) = ln\left(\frac{x_1}{x_2}\right)$

vx = constant

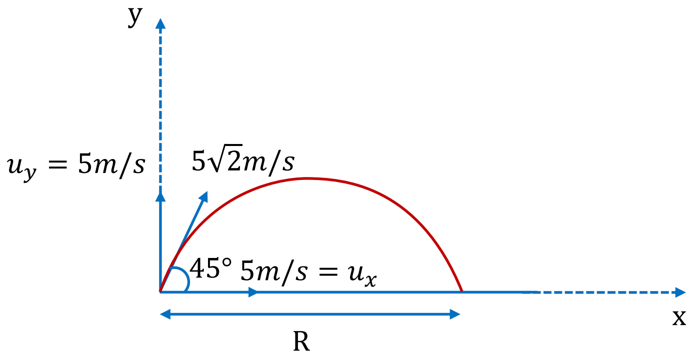
On decreasing length to half $K.E.$ becomes 4 times

$vdx + xdv = 0$

7. Ans. (0.50)

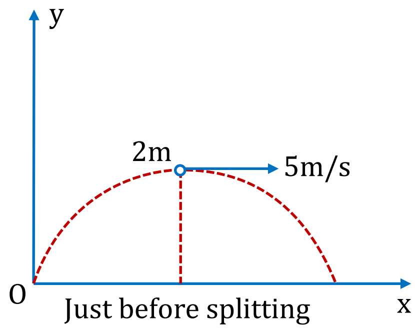
8. Ans. (7.50)

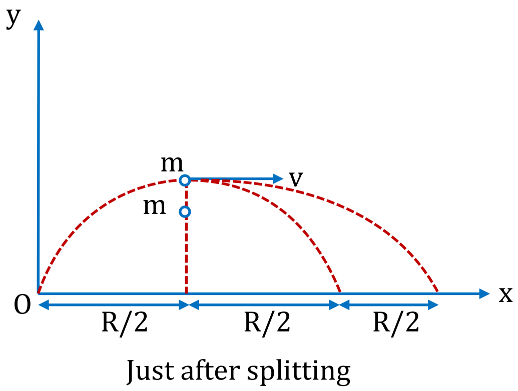
Solution for Question No. 7 and 8



Range $R = \frac{2u_x u_y}{g} = \frac{2\times5\times5}{10} = 5m$

Time of flight $T = \frac{2u_y}{g} = \frac{2\times5}{10} = 1\ sec$





$\because$ Time of motion of one part falling vertically downwards is = 0.5 sec = $\frac{T}{2}$

$\Rightarrow$ Time of motion of another part,

$t = \frac{T}{2}$ = 0.5 sec.

From momentum conservation

$\Rightarrow$ $P_i = P_f$

$2m \times 5 = m \times v$

$v = 10\ m/s$

Displacement of other part in 0.5 sec in horizontal direction = $v\frac{T}{2}$

= $10 \times 0.5 = 5\ m = R$

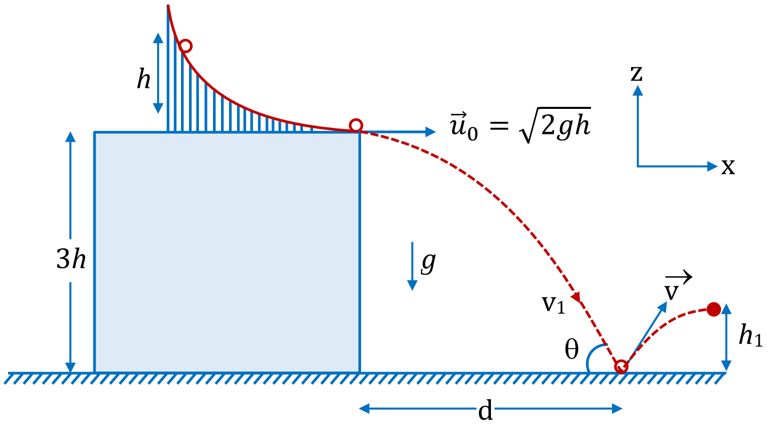
Total distance of second part from point 'O' is,

$x = \frac{3R}{2} = 3 \times \frac{5}{2}$

$x = 7.5\ m$

$\Rightarrow$ $t = 0.5$ sec.

9. Ans. (ACD)



$$\vec{v}_1 = \sqrt{2gh}\hat{i} - \sqrt{2g3h}\hat{k}$$

$$\vec{v} = \sqrt{2gh}\hat{i} + \sqrt{2g3h} \times \frac{1}{\sqrt{3}}\hat{k}$$

$$= \sqrt{2gh}\hat{i} + \sqrt{2gh}\hat{k}$$

$$\tan\theta = \frac{\sqrt{2g3h}}{\sqrt{2gh}} = \sqrt{3}, \quad \theta = 60°$$

$$h_1 = \frac{v_{1y}^2}{2g} = \frac{2gh}{2g} = h$$

$$d = v_x t = \sqrt{2gh} \times \sqrt{\frac{2 \times 3h}{g}}$$

$$= \sqrt{2gh}\sqrt{\frac{6h}{g}} = 2\sqrt{3}h$$

$$= \frac{d}{h_1} = 2\sqrt{3}$$

JEE (Main) Practice Paper

SECTION–A

1. Ans. (3)

Let the displacement of boat be x towards the shore

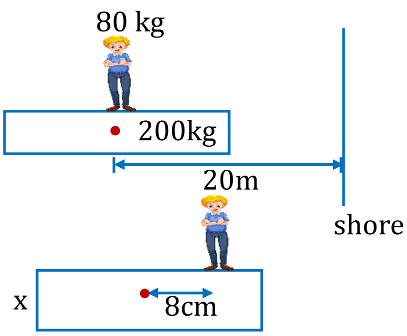
$$80(x + 8) + 200x = 0$$

$$\Rightarrow \quad x = \frac{-640}{280} = -\frac{16}{7}$$

$$\therefore \quad (\Delta x)_{min} = 8 - \frac{16}{7} = \frac{40}{7}$$

$\therefore$ New distance from shone

$$= 20 - \frac{40}{7} = \frac{100}{7} = 14.28m \approx 14.3\ m$$

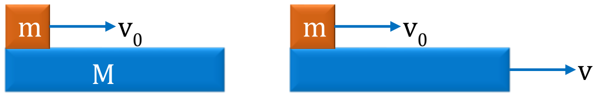


2. Ans. (3)

Momentum acquired = Area under curve = $5 \times 10 + \frac{1}{2} \times 4 \times 10$ = 50 + 20 = 70 N-s

3. Ans. (4)

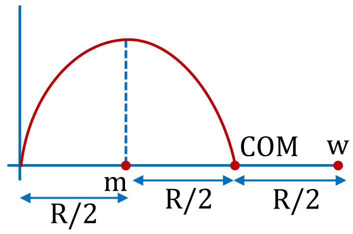
$mv_0 = (M + m)\, v \Rightarrow v = \dfrac{mv_0}{(M+m)}$



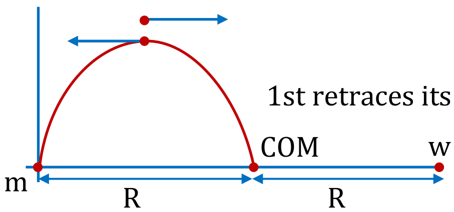
$(\Delta K)_{\text{system}} = \frac{1}{2}(M + m)v^2 - \frac{1}{2}mu_0^2$

$= \frac{1}{2}\left(\frac{m^2}{M+m}\right)v_0^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}mv_0^2\left[\frac{m}{M+m} - 1\right]$

4. Ans. (4)

(1) 

$\Rightarrow$ ↑st just falls down

(2) 

1st retraces its path

(3) $X_{cm} = \dfrac{mR+mR}{2m} = R$

$Y_{cm} = 0$

$Y_{cm} = \dfrac{-mR+mR}{2m} = 0$

$\therefore$ This is the possible poster

(4) $X_{cm} = \dfrac{2R.m+mB}{2m} = \dfrac{3R}{2}$

5. Ans. (2)

5 × 20 + m × 10 = 5 × 10 + 15m

5m = 50 $\Rightarrow$ m = 10

6. Ans. (4)

Final energy becomes more than initial energy.

7. Ans. (2)

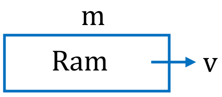
Impulse given by fan to sail through air = Impulse given by air to fan.

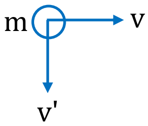
$\therefore$ Net impulse on system = zero

$\Rightarrow$ System remains at rest

8. **Ans. (4)**

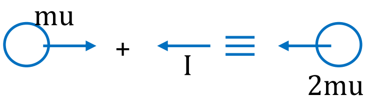
As snow falls mass of (trolly + snow) increases $\Rightarrow \frac{V}{P}$ decreases due to conservation.





No extra force due to Ram on snow in direction of v no extra force due to snow on ram $\Rightarrow$ Due throw of ice $\Rightarrow$ Velocity of trolly in forward direction is not changed.

9. **Ans. (2)**



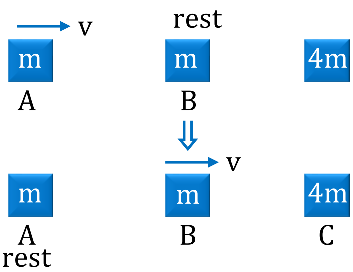
$\Rightarrow \quad mu - I = -2mu$

$\Rightarrow \quad I = 3mu$

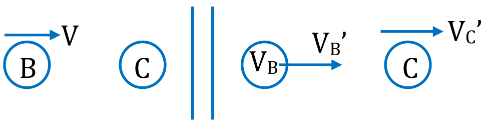
$$W = \Delta KE = \frac{1}{2}m(2u)^2 - \frac{1}{2}mu^2 = \frac{3}{2}mu^2 = \left(\frac{3}{2}mu\right)u$$

$$W = \frac{I}{2}u$$

10. **Ans. (1)**



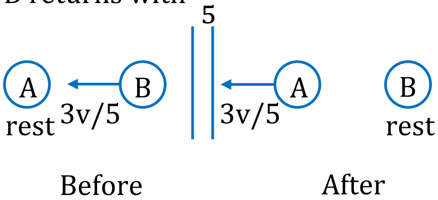
For $B - C$ collision



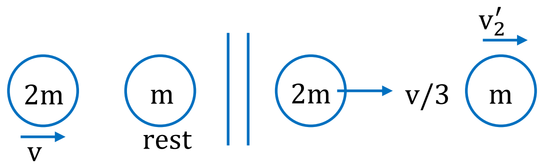
$$v_{com} = \frac{mv}{5m} = \frac{v}{5}$$

$$v'_B = 2v_C - v_1 = 2 \times \frac{v}{5} - v = -\frac{3v}{5}$$

B returns with $\frac{3v}{5}$



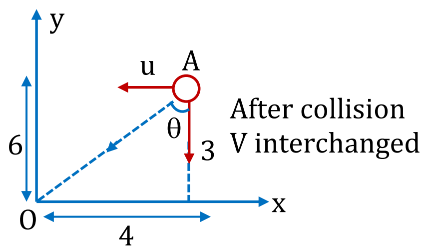
11. Ans. (2)



$v'_1 = (1 + e)\, v_C - v_1$

$$\frac{v}{3} = (1+e).\frac{2v}{3} - ev \Rightarrow e = 1$$

12. Ans. (2)

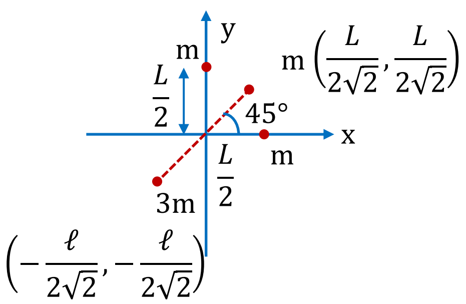


Resultant of 4 and 3 is along AO.

$$\tan\theta = \frac{4}{6} = \frac{4}{3} \Rightarrow u = 2\ m/s$$

$$\text{Striker } v = \begin{cases} u = 2 \Rightarrow \text{Before} \\ 0 \Rightarrow \text{After} \end{cases}$$

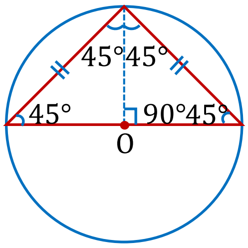
13. Ans. (1)

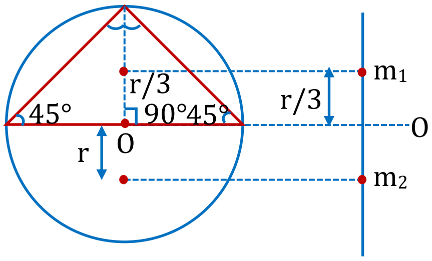


$$x_{cm} = 0 \Rightarrow m\frac{L}{2} + \frac{mL}{2\sqrt{2}} - 3m\frac{\ell}{2\sqrt{2}} = 0$$

$$\Rightarrow \frac{3\ell}{2\sqrt{2}} = \frac{L}{2}\left(1 + \frac{1}{\sqrt{2}}\right) \Rightarrow \ell = \frac{(\sqrt{2}+1)}{3}L$$

14. Ans. (1)





$R = a$

$m_1 \frac{R}{3}. = m_2 r$

$\Rightarrow \quad r = \frac{m_1}{m_2}\frac{R}{3}$

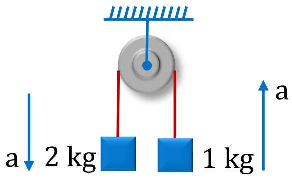
$\Rightarrow \quad r = \frac{A_1}{A_2} \cdot \frac{R}{3} = \left(\frac{\frac{1}{2}\times 2R\times R}{\pi R^2 - R^2}\right)\frac{R}{3}$

$\Rightarrow \quad r = \frac{R}{3(\pi - 1)}$

$\Rightarrow \quad (R = a)$

$\Rightarrow \quad \frac{a}{3(\pi - 1)}$

15. Ans. (2)



$a = \frac{2g - 1g}{3} = g/3$

$a_{cm} = \frac{2a - 1\times a}{3} = \frac{a}{3} = \frac{g}{9}$

16. Ans. (1)

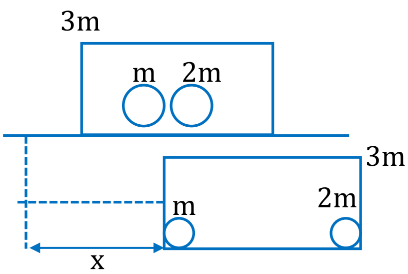
System [Box + two parts]

$3mx + m(x - 2L) + 2m(x + 2L) = 0$

$\Rightarrow \quad 6x + 2L = 0$

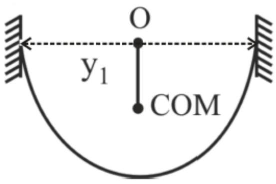
$\Rightarrow \quad x = -\frac{L}{3}$

$\Rightarrow \quad \text{Distance} = |x| = \frac{L}{3}$



17. Ans. (2)

Let ℓ = Length of chain



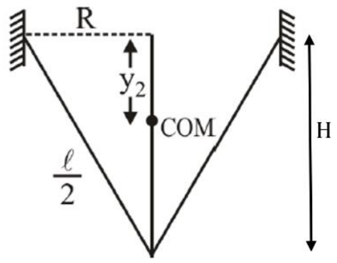
Initially,

$\pi R = \ell$

$R = \frac{\ell}{\pi}$

$y_1 = \frac{2R}{\pi} = \frac{2\ell}{\pi^2}$ (COM of a semi-circular ring is at $\frac{2R}{\pi}$)

Finally,



$H = \sqrt{\left(\frac{\ell}{2}\right)^2 - R^2}$

$y_2 = \frac{H}{2}$ (By similar triangles)

$y_2 = \frac{1}{2}\sqrt{\left(\frac{\ell}{2}\right)^2 - \left(\frac{\ell}{\pi}\right)^2}$

$y_2 < y_1 \Rightarrow$ COM goes up.

18. Ans. (2)

By conservation of linear momentum

(0.6) (4) + (0.4) (–2) = (0.6) (v_A) + (0.4) (3)

$\Rightarrow \quad v_A = \frac{2}{3}$ and $e = \frac{v_{sep}}{v_{app}} = \frac{3 - v_A}{4+2} = \frac{7}{18}$

19. Ans. (1)

(i) For B and $C \Rightarrow \vec{P}$ conserve momentum

$1 \times 1 + (-2) \times 2 = 3v$

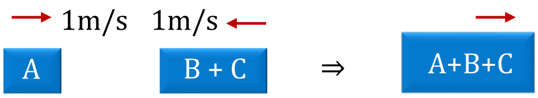
$\Rightarrow \quad v = -1 m/s$

$\therefore$ Loss of PE = Loss of KE ($\Delta PE = 0$)

$\Delta K = \left(\frac{1}{2} \times 1 \times 1^2 + \frac{1}{2} \times 2 \times 2^2\right) - \left(\frac{1}{2}(3)(v)^2\right) = \frac{1}{2} + 4 - \frac{3}{2} = 3J$

$\Delta K = 3J$

(ii)



$2 \times 1 - 3 \times 1 = 5v \Rightarrow v = -\frac{1}{5} m/s$

$\therefore \quad I_A = \Delta P_A = 2 \times \frac{1}{5} - (-2 \times 1) = \frac{12}{5} N\text{–}s$

20. Ans. (3)

From energy conservation,

$\begin{bmatrix} \text{after bullet get sembeded till the} \\ \text{system comes momentarily at rest} \end{bmatrix}$

$$(M + m)g\,h = \frac{1}{2}(M + m)v_1^2$$

[v_1 is velocity after collision]

$\therefore$ $v_1 = \sqrt{2gh}$

Applying momentum conservation, (just before and just after collision)

$$mv = (M + m)v_1$$

$$v = \left(\frac{M+m}{m}\right)v_1 = \frac{6}{10\times10^{-3}} \times \sqrt{2\times9.8\times9.8\times10^{-2}}$$

$$\approx 831.55 m/s$$

SECTION-B

1. Ans. (6)

$$(x_{cm})_i = \frac{1\times2+1\times8}{1+1} = 5m$$

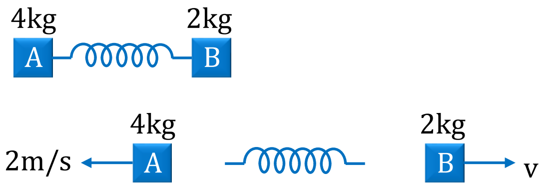
$$(u_{cm})_i = \frac{1\times5-1\times3}{1+1} = 1\ m/s$$

$\therefore$ $\vec{F}_{ext} = 0 \Rightarrow \vec{a}_{cm} = 0$

$\therefore$ At $t = 1$ sec

$(x_{cm})_f = (x_{cm})_i + (u_{cm})_i t + 0 = 5 + 1 \times 1 = 6m$

2. Ans. (4)



System $(A + B + \text{Spring})$

$\Rightarrow$ $F_{ext} = 0 \Rightarrow 2v - 2 \times 4 = 0$

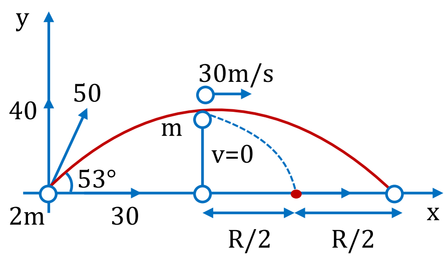
$\Rightarrow$ $v = 4\ m/s = v_B$

3. Ans. (108)

From $P_f = P_i$ in horizontal

$2m30 = mv$

$\Rightarrow$ $v = 60\ m/s$



Just before explosion,

$$K_i = \frac{1}{2}24v_x^2$$

$$= \frac{1}{2}\times 24 \times (30)^2 = \frac{1}{2}\times 21600J = 10800J$$

Just after explosion,

$$K_f = \frac{1}{2}mv^2 = \frac{1}{2}\times 12 \times 60^2 = 21600J$$

$\therefore$ Energy released $= K_f - K_i = 21600 - 10800 = 10800\ J = 100x$

$\therefore$ $x = 108$

4. **Ans. (30)**

By using reduced mass concept

$$\frac{1}{2}\mu\, u_{rel}^2 = \frac{1}{2} k x_{max}^2$$

$$\Rightarrow \quad x_{max} = u_{rel}\sqrt{\frac{\mu}{k}} = (2+1)\sqrt{\frac{\left(\frac{3\times6}{3+6}\right)}{200}} = 3\sqrt{\frac{2}{200}} = 0.3m = 30\ cm$$

5. **Ans.** 3

For collision in normal direction : $eu\cos 30° = v\cos 60°$

and in tangential direction $u\sin 30° = v\sin 60° \Rightarrow$ we get $e = \frac{1}{3}$

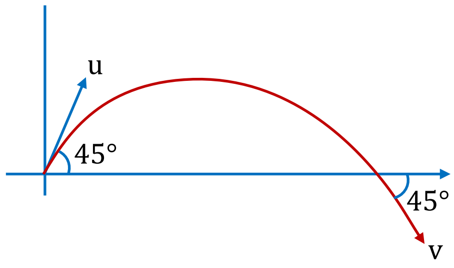
6. **Ans. (5)**

$|\vec{u}| = |\vec{v}|$ ...(1)

$\vec{u} = u\cos 45\hat{\imath} + u\sin 45\hat{\jmath}$ ...(2)

$\vec{v} = v\cos 45\hat{\imath} - u\sin 45\hat{\jmath}$ ...(3)

$|\overrightarrow{\Delta P}| = |\vec{m}(\vec{v} - \vec{u})|$ ...(4)



$\Delta P = 2\, mu \sin 45°$

$= 2\times5\times10^{-3}\times5\sqrt{2}\times\frac{1}{\sqrt{2}}$

$= 50\times10^{-3}$

$= 5\times10^{-2}\ kg\text{-}m/\sec$

7. **Ans. (25)**

$p_i = p_f$

$2\times4 = 2\times1 + m_2\times v_2$

$m_2 v_2 = 6$ ...(i)

By coefficient of restitution

$$1 = \frac{v_2 - 1}{4} \Rightarrow V_2 = 5m/s$$

By (i)

$m_2 \times 5 = 6$

$m_2 = 1.2$ kg

$v_{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$

$v_{cm} = \frac{2\times1 + 1.2\times5}{2+1.2} = \frac{8}{3.2} = \frac{25}{10}$

$\boxed{x = 25}$

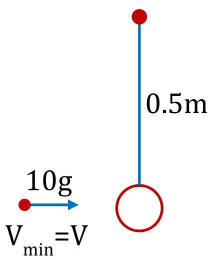
8. **Ans. (400)**

$V' = \sqrt{5gR} = \sqrt{5\times10\times0.5}$

$V' = 5\ m/s$

$m_1 V = m_2\times5 - m_1\times100$

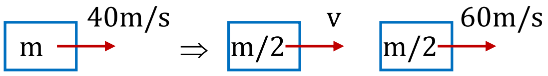
$V = 400\ m/s$



9. Ans. (1)

$P_i = P_f$

$m \times 40 = \frac{m}{2} \times v + \frac{m}{2} \times 60$



$40 = \frac{v}{2} + 30$

$\Rightarrow \quad v = 20\ m/s$

$(K.E.)_I = \frac{1}{2} m \times (40)^2 = 800m$

$(K.E.)_f = \frac{1}{2}\frac{m}{2}.(20)^2 + \frac{1}{2}.\frac{m}{2}(60)^2 = 1000\ m$

$|\Delta K.E| = |1000m - 800m| = 200m$

$\frac{\Delta K.E.}{(K.E.)_1} = \frac{200m}{800m} = \frac{1}{4} = \frac{x}{4}$

$x = 1$

10. Ans. (12)

Impulse = change in momentum

$= m[v - (-v)] = 2\ mv$

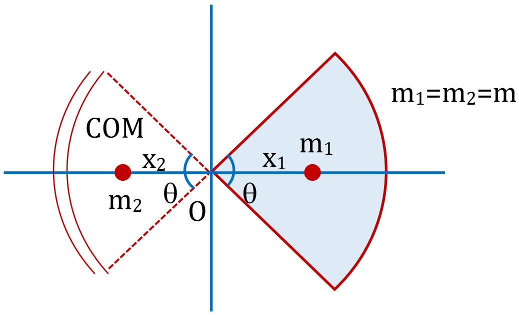
$= 2 \times 0.4 \times 15 = 12\ N$

JEE (Advanced) Practice Paper

1. Ans. (A)

$x_2 = \frac{2R_2}{\theta} sin\frac{\theta}{2}, x_1 = \frac{4R_1}{3\theta} sin\frac{\theta}{2}$

$m_1x_1 = m_2x_2$



$\Rightarrow \quad m.\frac{4R_1}{3\theta} sin\frac{\theta}{2} = m.\frac{2R_2}{\theta} sin\frac{\theta}{2}$

$\Rightarrow \quad R_1 = \frac{3R_2}{2}$

$\Rightarrow \quad R_2 = \frac{2R_1}{3}$

$\Rightarrow \quad R_2 = 2 \times \frac{12}{3} = 8\ cm$

2. Ans. (D)

$\Delta x = a - \left(\frac{a}{3} + \frac{a}{3}\right) = \frac{a}{3}$

During the process shift in of triangular (heads)

$(x_1) = -\frac{a}{2}$

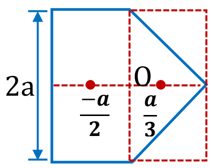


Figure-2

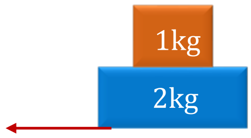
$(x_2) = \frac{a}{3}$

$x_{cm} = \frac{\frac{m}{2}\times\frac{a}{3} - \frac{m}{2}\times\frac{a}{2}}{m}$

$= -\frac{a}{12}$

So new COM is $\left(-\frac{a}{12}, 0\right)$.

3. **Ans. (D)**

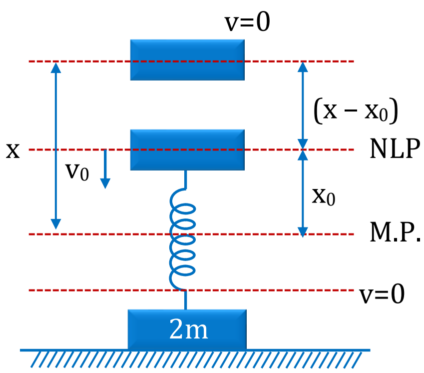


$F_{ext} = f_k = \mu N = 0.2 \times 30 = 6N$

$\therefore \quad a_{cm} = \frac{6}{3} = 2\ m/s^2$

4. **Ans. (B)**

(A)

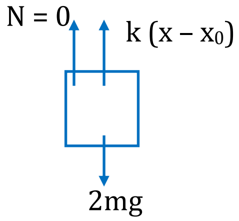


Just after

$kx_0 = mg$

$x_0 = \frac{mg}{k}$

Compression from MP = Elongation from $MP = x$



From W.E.T. $\Rightarrow mg(x + x_0) + \frac{1}{2}k\left[O^2 - (x + x_0)^2\right] = 0 - \frac{1}{2}mv_0^2 = -mgh$

$\Rightarrow \quad \frac{1}{2}k(x + x_0)^2 - mg(x + x_0) - mgh = 0 \Rightarrow (x + x_0)$

From (1) $kx - kx_0 = 2mg \Rightarrow kx - k.\frac{mg}{k} = 9mg$

$\Rightarrow \quad x = \frac{3mg}{k}$

$\therefore \quad x + x_0 = \frac{1}{2}k\left[O^2 - (x + x_0)^2\right] = 0 - \frac{1}{2}mv_0^2$

Now from W.E.T. $\Rightarrow$ from NLP to max compression

$mg\ (x + x_0) + \frac{1}{2}k\left[O^2 - (x + x_0)^2\right] = 0 - \frac{1}{2}mv_0^2$

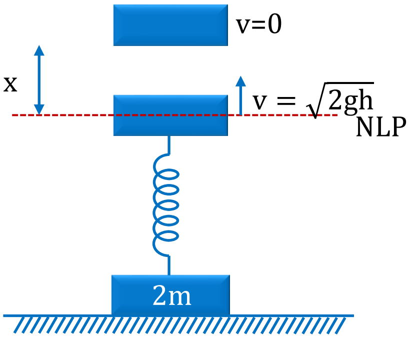
$$\Rightarrow \quad mg\,.\frac{4mg}{k} - \frac{1}{2}k.\frac{(4mg)^2}{k^2} = -\frac{1}{2}mv_0^2$$

$$\Rightarrow \quad -\frac{4(mg)^2}{k} = -\frac{1}{2}mv_0^2$$

$$\Rightarrow \quad \frac{8m^2g^2}{k} = m.\,2gh$$

$$\Rightarrow \quad h = \frac{4mg}{k}$$

Alternate Solution:



Just after

After collision with the ground and during upward journey of upper block, shown will be the condition.

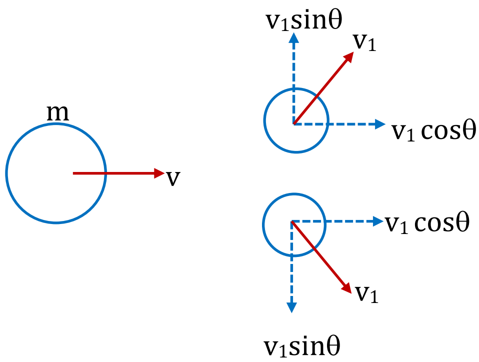
$$\frac{1}{2}m(2gh) = mgx + \frac{1}{2}kx^2 \qquad ...(i)$$

$$kx = 2mg \qquad ...(ii)$$

From (i) and (ii)

$$h = \frac{4mg}{k}$$

5. **Ans. (D)**

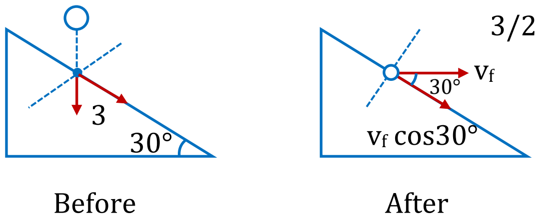


$$m\,.\,2v_1\cos\theta = mv$$

$$\Rightarrow \quad v_1 = \frac{v}{2\cos\theta}$$

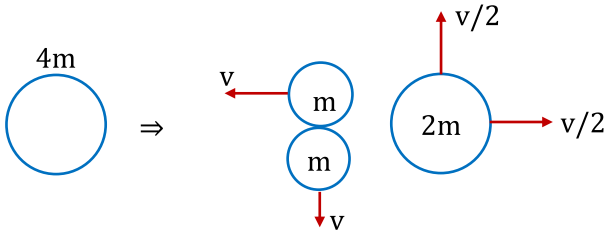
$$\because \quad \cos\theta < 1 \quad \Rightarrow v_1 > \frac{v}{2}$$

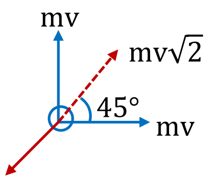
6. **Ans. (B)**



$v_f \cos 30° = \frac{3}{2} \quad \Rightarrow \quad v_f = \sqrt{3} m/s$

7. **Ans. (AC)**





Total mass = 0.5 *kg*

Mass of broken parts = $m, m, 2m$

Let smaller masses have velocity v.

Also, initial momentum $\vec{P}_{\text{initial}} = 0$

By momentum conservation –

$$0 = \vec{P}_1 + \vec{P}_2 + \vec{P}_3$$

$$0 = -mv\hat{\imath} - mv\hat{\jmath} + (2m)\vec{v}_3$$

$$\vec{v}_3 = \frac{1}{2}v(\hat{\imath} + \hat{\jmath})$$

$$|\vec{v}| = \frac{v}{\sqrt{2}}$$

Final K.E. of all the three particle are

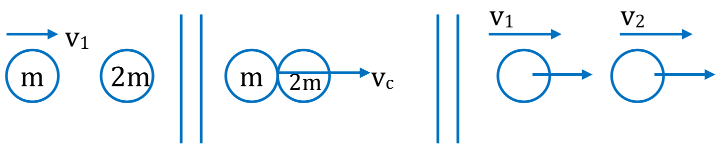
$$\frac{1}{2}mv^2, \frac{1}{2}mv^2, \frac{1}{2}(2m)\left(\frac{v}{\sqrt{2}}\right)^2 = \frac{1}{2}mv^2$$

$$v_3 = \frac{v}{\sqrt{2}} = v \sin\theta$$

$$\theta = 45°$$

$\Rightarrow$ Hence, angle between bigger mass 2m and each of smaller mass m is 135°.

8. **Ans. (ABD)**



Maximum compression ($RE \Rightarrow$ Max $\Rightarrow KE \Rightarrow$ Min)

$3mv_C = mv_1$

$\Rightarrow \quad v_C = \dfrac{v_1}{3}$

$\dfrac{1}{2}mv_1^2 = 3$

$(\mathrm{KE})_{\min} = \dfrac{1}{2} . 3mv_C^2$

$= \dfrac{3}{2}m\left(\dfrac{v_1}{3}\right)^2$

$= \dfrac{1}{6}mv_1^2$

$= \dfrac{1}{3} \times \left(\dfrac{1}{2}mv_1^2\right)$

$= \dfrac{1}{3} \times 3$

$\Rightarrow \quad (KE)_{\min} = 1J$

$(PE)_{\max} = 3 - 1 = 2J$

(C) $P \Rightarrow$ Always conserved

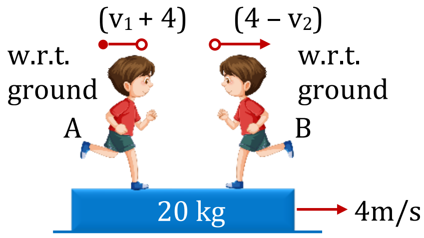
$KE \Rightarrow$ During collision changes

(D) During collision PE increases, becomes max. and then decreases.

$PE\uparrow$ and $KE\downarrow \Rightarrow \left(\dfrac{PE}{KE}\right)\uparrow \Rightarrow \dfrac{KE}{PE}\downarrow$

9. **Ans. (ABCD)**

Velocity of plank w.r.t. ground = $\dfrac{20}{5}$ = 4 m/s right



$P_f = 0$

$\Rightarrow \quad 40\,(v_1 + 4) + 40(4 - v_2) + 20 \times 4 = 0$

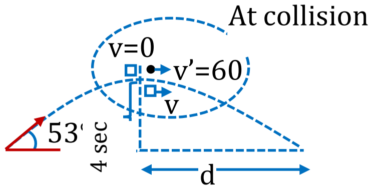
$\Rightarrow \quad 40\,(v_1 - v_2) + 320 + 80 = 0$

$\Rightarrow \quad v_1 - v_2 = -\,10$

10. Ans. (A)

11. Ans. (A)

Solution for Question No. 10 and 11



$$T_f = \frac{2u\sin\theta}{g} = \frac{2\times50\times4}{5\times10} = 8\ sec$$

$v = 50\cos 53 = 30\ m/s$

By momentum conservation

$$m \times 30 = \frac{m}{2} \times 0 + \frac{m}{2} \times v'$$

$v' = 60\ m/s$

The distance between the pieces

$d = v' \times t = 60 \times 4 = 240m$

Radius of curvature just before the explosion

$$\frac{mv^2}{r} = mg$$

$$r = \frac{v^2}{g} = \frac{900}{10} = 90m/s$$

Radius of curvature just after the explosion

$$\frac{mv'^2}{r} = mg$$

$$r' = \frac{v'^2}{g} = \frac{3600}{10} = 360\ m$$

Hence, $\frac{r}{r'} = \frac{90}{360} = \frac{1}{4}$

12. Ans. (B)

13. Ans. (C)

14. Ans. (D)

Solution for Question No. 12 to 14

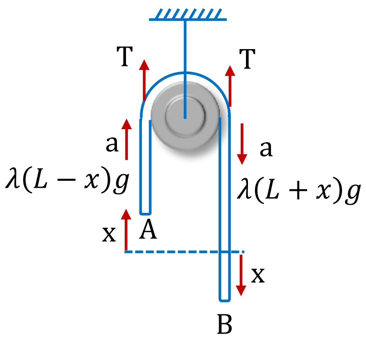
Maximum energy will be stored in the spring when blocks will be moving with same velocity.

$$v_c = \frac{8v_0 - 3v_0}{5} = v_0 \text{ towards left}$$

So, $U_{max} = \frac{1}{2} \times 2 \times 16v_0^2 + \frac{1}{2} \times 3 \times v_0^2 - \frac{1}{2} \times 5 \times v_0^2$

Velocity of centre of mass is v_0 towards left and both the blocks $2 kg$ *and* $3\ kg$ will perform simple harmonic motion with speed at mean position $3v_0$ and $2v_0$ respectively. So, maximum speed of 2 *kg* block will be $3v_0 + v_0 = 4v_0$ and maximum speed of 3 *kg* block will be $2v_0 + v_0 = 3v_0$.

15. Ans. (A)



$\lambda = \frac{M}{2L}$

$\lambda(L+x)g - T = \lambda(L+x)a$ ...(i)

$T - \lambda(L-x)g = \lambda(L-x)a$ ...(ii)

By solving (i) and (ii)

$$\left(a = \frac{x}{L}g\right)$$

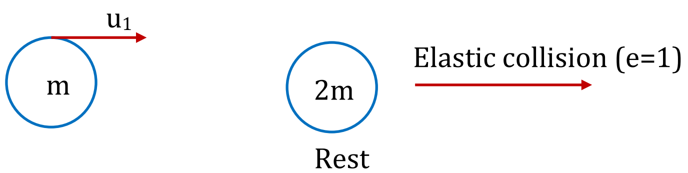
16. Ans. (C)

$v\frac{dv}{dx} = \frac{x}{L}g \Rightarrow \int_0^v v\,dv = \frac{g}{L}\int_0^L x\,dx$

$\frac{v^2}{2} = \frac{gL^2}{2L}$

$v = \sqrt{gL}$

17. Ans. (A-R), (B-T), (C-T), (D-S)



Given :

$m_1 = m, m_2 = 2m$

$u_1 = u_1,\ u_2 = 0$

$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_1}{m_1 + m_2}\right) u_2 = 0$ ...(i)

$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \left(\frac{2m_1}{m_1 + m_2}\right) u_1 = 0$ ...(ii)

$v_1 = \frac{-u_1}{3}$; $v_2 = \frac{2}{3} u_1$

$\Rightarrow$ Final momentum of first particle = $m_1 v_1 = -\frac{m_1 u_1}{3} = -\frac{P}{3}$

$\Rightarrow$ Final momentum of second particle = $m_2 v_2 = 2mv_2 = \frac{4P}{3}$

$\Rightarrow$ KE of first particle $K = \frac{1}{2} m_1 v_1^2 = \frac{1}{2} \frac{m_1 u_1^2}{9} = \frac{K}{9}$

$\Rightarrow$ KE of second particle $K = \frac{1}{2} m_2 v_2^2 = \frac{1}{2}(2m)\left(\frac{4}{9} u_1^2\right) = \frac{8}{9} K$