01 Newtons Laws of Motion and Friction

NEWTONS LAWS OF MOTION

Force

- A pull or push which changes or tends to change the state of rest or uniform motion or direction of motion of any object is called a force.
- Force is the interaction between the object and the source (providing the pull or push). It is a vector quantity.

Unit of force : Newton and $\frac{kg \cdot m}{s^2}$ (MKS System)

dyne and $\frac{g \cdot cm}{s^2}$ (CGS System)

1 Newton = 10^5 dyne

Kilogram force (kgf) : The force with which the Earth attracts a 1 *kg* body towards its centre is called kilogram force, thus

$$kgf = \frac{\text{Force in Newton}}{g} \qquad (g = 9.8\ m/s^2)$$

Dimensional Formula of force : $[MLT^{-2}]$

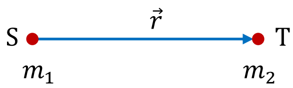
Fundamental Forces

All the forces observed in nature such as muscular force, tension, friction, elastic, weight, electric, magnetic, nuclear, etc., can be explained in terms of only following four basic interactions:

(A) Gravitational Force

The force of interaction which exists between two particles of masses m_1 and m_2, due to their masses is called gravitational force.

$$\vec{F} = -G\frac{m_1 m_2}{r^3}\vec{r}$$



$\vec{r}$ = Position vector of test particle '*T*' with respect to source particle '*S*'

G = Universal gravitational constant

$= 6.67 \times 10^{-11}\ Nm^2/kg^2$

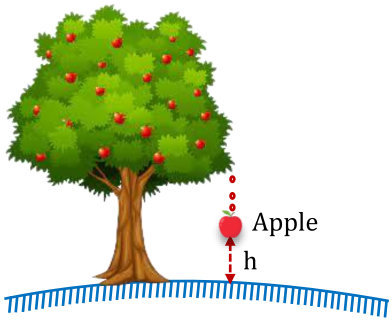
(i) It is the weakest force and is always attractive.

(ii) It is a long-range force as it acts between any two particles situated at any distance in the universe.

(iii) It is independent of the nature of medium between the particles.

An apple is freely falling as shown in figure, when it is at a height *h*, force between the Earth and apple is given by

$$F = \frac{GM_e m}{(R_e + h)^2}$$



where M_e (mass of earth), R_e (radius of earth). It acts towards Earth's centre. Now rearranging above result,

$$F = m\frac{GM_e}{R_e^2} \cdot \left(\frac{R_e}{R_e + h}\right)^2$$

$$F = mg\left(\frac{R_e}{R_e + h}\right)^2 \left\{g = \frac{GM_e}{R_e^2}\right\}$$

Here $h << R_e$, so $\frac{R_e}{R_e + h} = 1$

$\therefore \quad F = mg$

This is the force exerted by the Earth on any particle of mass m near the Earth surface. The value of $g = 9.81\ m/s^2 \simeq 10\ m/s^2 \simeq \pi^2\ m/s^2 \simeq 32\ ft/s^2$. It is also called acceleration due to gravity near the surface of the Earth.

(B) Electromagnetic Force

Force exerted by one particle on the other because of the electric charge on the particles is called electromagnetic force.

Following are the main characteristics of electromagnetic force

(a) These can be attractive or repulsive.

(b) These are long range forces

(c) These depend on the nature of medium between the charged particles.

(d) All macroscopic forces (except gravitational) which we experience as push or pull or by contact are electromagnetic, i.e., tension in a rope, the force of friction, normal reaction, muscular force, and force experienced by a deformed spring are electromagnetic forces. These are manifestations of the electromagnetic attractions and repulsions between atoms/molecules.

(C) Nuclear Force

It is the strongest force. It keeps nucleons (neutrons and protons) together inside the nucleus inspite of large electric repulsion between protons. Radioactivity, fission, and fusion, etc. result because of unbalancing of nuclear forces. It acts within the nucleus that too upto a very small distance.

(D) Weak Force

It acts between any two elementary particles. Under its action a neutron can change into a proton emitting an electron and a particle called antineutrino. The range of weak force is very small, in fact much smaller than the size of a proton or a neutron.

It has been found that for two protons at a distance of 1 Fermi :

$$\boxed{F_N : F_{EM} : F_W : F_G : \ :1:10^{-2}:10^{-7}:10^{-38}}$$

Classification of forces on the basis of contact :

(A) Field Force : Force which acts on an object at a distance by the interaction of the object with the field produced by other object is called field force.

Examples:

(a) Gravitational force (b) Electromagnetic force

(B) Contact Force : Forces which are transmitted between bodies by short range atomic molecular interactions are called contact forces. When two objects come in contact they exert contact forces on each other.

Examples : Normal force, frictional force.

Some standard forces:

Weight : Force experienced by a body due to gravity is known as weight. It is applicable to all bodies having mass but specifically for a body near the Earth's surface, we compute it as:

$$F = \frac{G \times \text{mass of the Earth} \times \text{mass of the body}}{(\text{radius of Earth})^2}$$

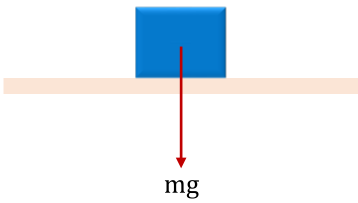
Where, $\frac{G \times \text{mass of the Earth}}{(\text{radius of the Earth})^2} = g(\text{constant})$

g = acceleration due to gravity of the Earth

The weight of an object of mass m near the surface of the Earth is mg directed towards the centre of the Earth.

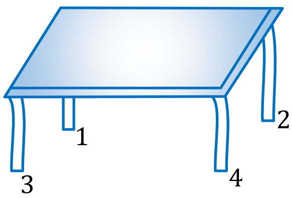
m = mass of block

g = acceleration due to gravity of the Earth

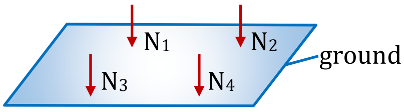


Normal Force (N) :

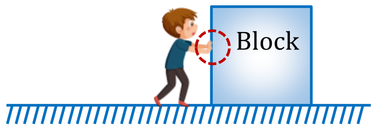
It is the component of contact force perpendicular to the surface. It measures how strongly the surfaces in contact are pressed against each other. It is the electromagnetic force. A table is placed on the Earth as shown in figure



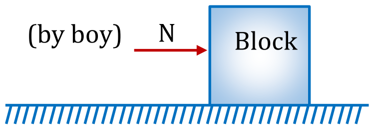
Here table presses the Earth so normal force exerted by four legs of table on the Earth are as shown in figure.



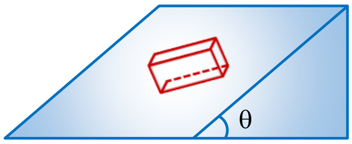
Now a boy pushes a block kept on a frictionless surface.



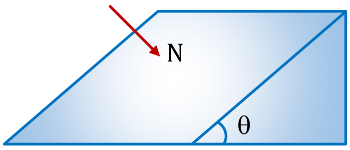
Here, force exerted by boy on block is electromagnetic interaction which arises due to similar charges appearing on finger and contact surface of block, it is normal force.



A block is kept on inclined surface. Component of its weight presses the surface perpendicularly due to which contact force acts between surface and block.



Normal force exerted by block on the surface of inclined plane is shown in figure.



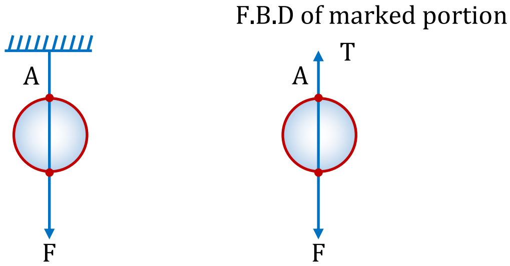
Force acts perpendicular to the surface.

Note :

Normal is a dependent force, it comes in role when one surface presses the other.

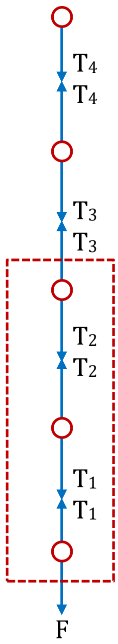
Tension :

Tension in a string is an electromagnetic force. It arises when a string is pulled. If a massless string is not pulled, tension in it is zero. A string suspended by rigid support is pulled by a force *'F'* as shown in figure, for calculating the tension at point *'A'* we draw F.B.D. of marked portion of the string; Here string is massless.

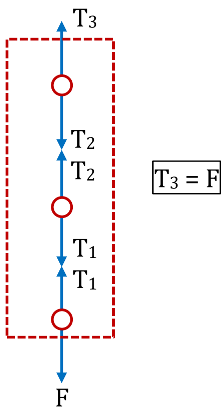


$\Rightarrow \qquad T = F$

String is considered to be made of a number of small segments which attracts each other due to electromagnetic nature as shown in figure. The attraction force between two segments is equal and opposite due to Newton's third law.



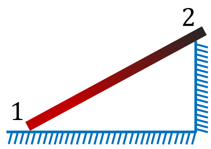
For calculating tension at any segment, we consider two or more than two parts as a system.



Here interaction between segments are considered as internal forces, so they are not shown in F.B.

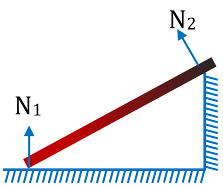
Illustration 1:

Draw normal forces on the massive rod at point 1 and 2 as shown in figure.



Solution:

Normal force acts perpendicular extended surface at point of contact.



(21 d)

Newton’s Laws of Motion

Newton's First Law of Motion

Every object will remain at rest or in uniform motion in a straight line unless compelled to change its state by the action of an external force.

Or

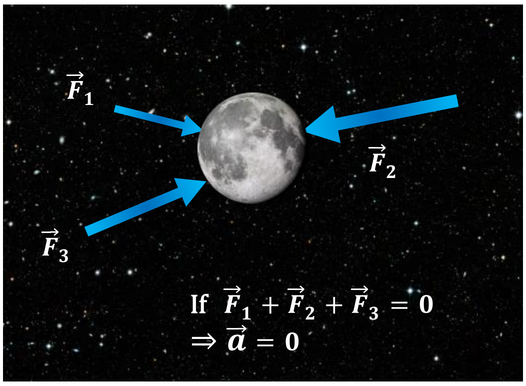
To change state of motion of a body a external force is necessary. There are two states of motion of a body.

(1) State of rest ($v = 0$, $a = 0$)

(2) State of uniform motion ($v \neq 0$, $a = 0$)

Or

If the vector sum of all the forces acting on a particle is zero, then and only then the particle remains unaccelerated (i.e., remains at rest or moves with constant velocity).



- First law is also known as the law of **inertia**.

Inertia: The resistance of a particle to change its state of rest or of uniform motion along a straight line. It could be of three types:

- Inertia of rest
- Inertia of motion
- Inertia of direction

Note:

1. **Force is the cause of changes in motion**

   Force does not cause motion. We can have motion in the absence of force, as described in Newton’s first law. Force is the cause of change in motion as measured by acceleration.

2. ***ma* is not a force**

   Equation 2 does not say that the product *ma* is a force. All forces on object are added vectorially to generate the net force on the left side of the equation. This net force is then equated to the product of the mass of the object and the acceleration that results from the net force.

Newton’s Second Law of Motion

The rate of change of linear momentum of a body is equal to **net** force acting on the body.

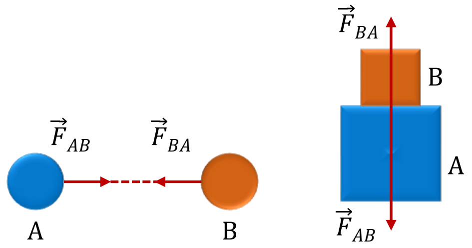
$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) \quad \text{(Mass is constant)} \quad ...(1)$$

$$\vec{F} = m\frac{d\vec{v}}{dt} = m\vec{a} \quad ...(2)$$

- Acceleration of a particle as measured from an inertial frame (**ground**) is given by the **vector sum** of all the **forces** acting on the particle divided by its **mass**.

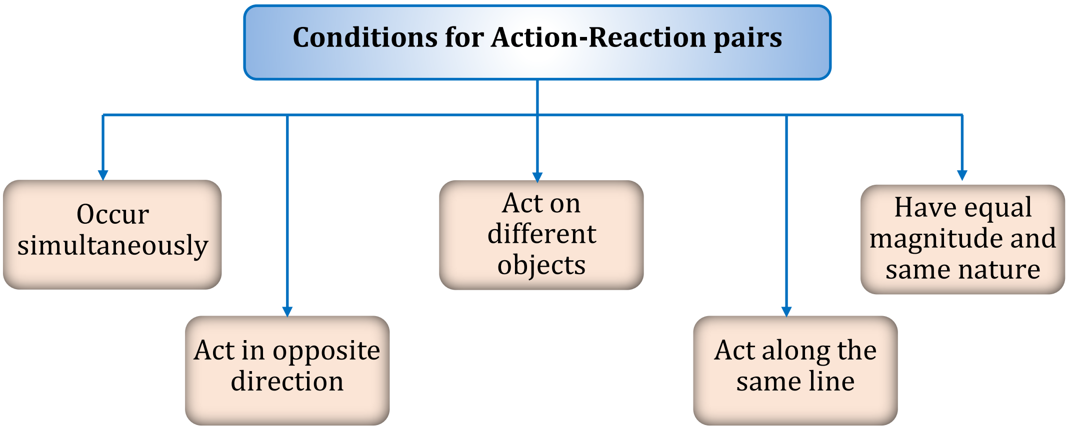
Newton's Third Law of Motion

If a body A exerts a force $\vec{F}$ on another body B, then B exerts a force $-\vec{F}$ on A, the two forces acting along the line joining the bodies.



These two forces in Newton's third law are known as Action-Reaction pairs.

Properties of action – reaction pairs



Third law of motion :

To every action, there is always an equal and opposite reaction. Newton's law from an 1803 translation from Latin as Newton wrote

Important points about the Third Law

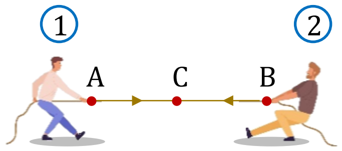
(a) The terms 'action' and 'reaction' in the Third Law mean nothing else but 'force'. A simple and clear way of stating the Third Law is as follows : Forces always occur in pairs. Force on a body A by B is equal and opposite to the force on the body B by A.

(b) The terms 'action' and 'reaction' in the Third Law may give a wrong impression that action comes before reaction i.e. action is the cause and reaction the effect. There is no such cause-effect relation implied in the Third Law. The force on A by B and the force on B by A act at the same instant. Any one of them may be called action and the other reaction.

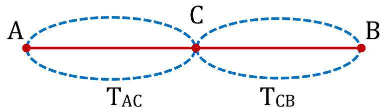
(c) Action and reaction forces act on different bodies, not on the same body. Thus, if we are considering the motion of any one body (A or B), only one of the two forces are relevant. It is an error to add up the two forces and claim that the net force is zero.

However, if you are considering the system of two bodies as a whole, F_{AB} (force on A due to B) and F_{BA} (force on B due to A) are internal forces of the system ($A + B$). They add up to give a null force. Internal forces in a body or a system of particles thus cancel away in pairs. This is an important fact that enables the Second Law to be applicable to a body or a system of particles.

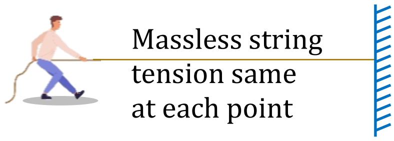
Example:



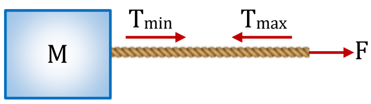
Tension at *'A'* is force applied by string on (1)
Tension at *'B'* is force applied by string on (2)



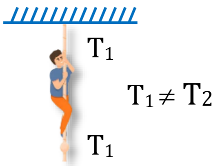
(d) String is assumed to be massless unless stated, hence tension in it everywhere remains the same and equal to applied force.



However, if a string has a mass, tension at different points will be different being maximum (applied force) at the end through which force is applied and minimum at the other end connected to a body. (eg.)



(e) Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break if the tension is increased beyond a certain limit. The maximum tension which a string can bear without breaking is called "breaking strength". It is finite for a string and depends on its material and dimensions.



What is System?

Any group of objects which we decide to study together can be taken as system.

Internal and external forces

If the action-reaction pair exists in the considered system, then it is known as **internal force,** otherwise it is known as **external force.**

Free-Body Diagram (FBD)

The diagrammatic representation of a body that is isolated from its surroundings, showing all the **external forces** acting on it, is known as the **free-body diagram (FBD).**

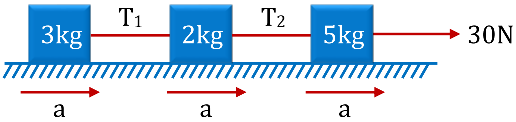
Steps for drawing the FBD

1. Isolate the free body.
2. Draw the external forces.
3. Choose the axes and resolve the forces.

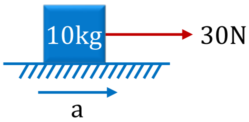
Illustration 2:

Draw the free body diagrams and find tension in each string and acceleration of the system.

Solution:

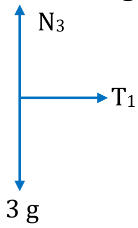


$$\boxed{\text{Acceleration} = \frac{\text{Net pulling force}}{\text{Total mass to be pulled}}}$$



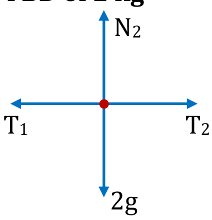
$$a = \frac{30}{10} = 3\ \text{m/s}^2$$

FBD of 3 kg



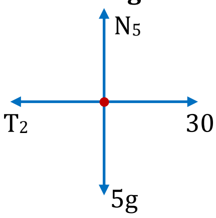
$T_1 = 3a$
$N_3 - 3g = 3 \times 0$
$N_3 = 30\ N$

FBD of 2 kg



$T_2 - T_1 = 2a$
$N_2 - 2g = 2 \times 0$
$N_2 = 20\ N$

FBD of 5 kg



$30 - T_2 = 5a$
$N_5 - 5g = 5 \times 0$
$N_5 = 50\ N$

Note:

Here N_{AB} and N_{BA} are the action–reaction pair (Newton's third law).

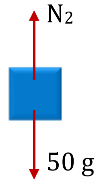
Illustration 3:

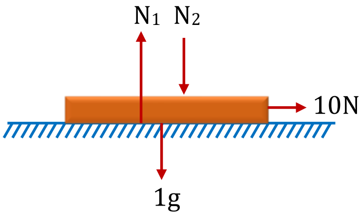
A block of mass 50 *kg* is kept on another block of mass 1 *kg* as shown in figure. A horizontal force of 10 *N* is applied on the 1*kg* block. (All surface are smooth). Find ($g = 10\ m/s^2$)

(a) Acceleration of block *A* and *B*.

(b) Force exerted by *B* on *A*.

Solution:





(a) **F.B.D. of 50 *kg***

$N_2 = 50\,g = 500\,N$

Along horizontal direction, there is no force $a_B = 0$

(b) **F.B.D. of 1 *kg* block :** along horizontal direction

$10 = 1\,a_A$

$a_A = 10\,m/s^2$

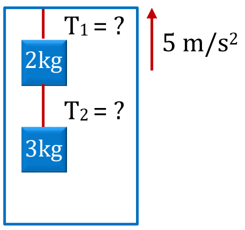
Along vertical direction

$\therefore\quad N_1 = N_2 + 1g$

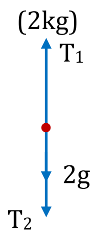
$= 500 + 10 = 510\,N$

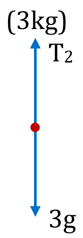
Illustration 4:

Find tension and acceleration.



Solution:





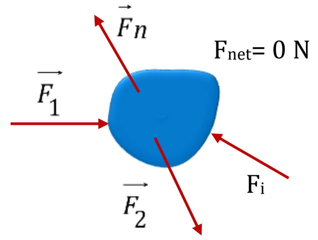
$T_1 - T_2 - 2g = 2 \times 5$	$T_2 - 3g = 3 \times 5$
$T_1 - T_2 = 30$	$T_2 = 15 + 30$
$T_1 - 45 = 30$	$T_2 = 45N$
$T_1 = 75$	

Translational Equilibrium

- A body in state of rest or moving with constant velocity is said to be in translational equilibrium. Thus, if a body is in translational equilibrium in a particular inertial frame of reference, it must have no linear acceleration.
- When it is at rest, it is in *static equilibrium*, whereas if it is moving at constant velocity it is in *dynamic equilibrium*.

Conditions for translational equilibrium

- For a body to be in translational equilibrium, no net force must act on it i.e. vector sum of all the forces acting on it must be zero.
- If several external forces $\vec{F}_1, \vec{F}_2$ ..... $\vec{F}_i$ ..... and $\vec{F}_n$ act simultaneously on a body and the body is in translational equilibrium, the resultant of these forces must be zero.



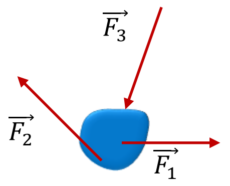
$\sum \vec{F}_i = \vec{0}$

If the forces

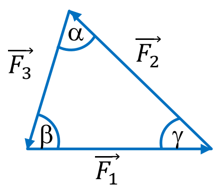
$\vec{F}_1, \vec{F}_2 ..... \vec{F}_i$ ..... and $\vec{F}_n$ are expressed in Cartesian components, we have :

$\sum F_{ix} = 0 \qquad \sum F_{iy} = 0 \qquad \sum F_{iz} = 0$

- If a body is acted upon by a single external force, it cannot be in equilibrium. If a body is in equilibrium under the action of only two external forces, the forces must be equal and opposite.
- If a body is in equilibrium under action of three forces, their resultant must be zero; therefore, according to the triangle law of vector addition they must be coplanar and make a closed triangle.



$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0} \Rightarrow$

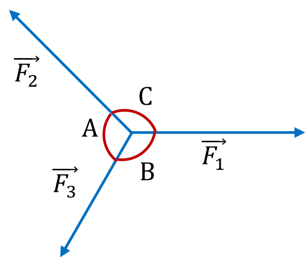


Lami's Theorem

Graphical method makes use of sine rule or Lami's theorem.

Sine rule : $\dfrac{F_1}{\sin\alpha} = \dfrac{F_2}{\sin\beta} = \dfrac{F_3}{\sin\gamma}$

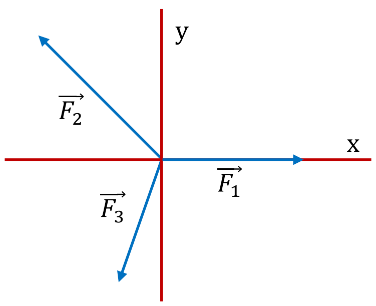
Lami's theorem : $\dfrac{F_1}{\sin A} = \dfrac{F_2}{\sin B} = \dfrac{F_3}{\sin C}$



Analytical method makes use of Cartesian components. Since the forces involved make a closed triangle, they lie in a plane and a two-dimensional Cartesian frame can be used to resolve the forces. As far as possible orientation of the *x-y* frame is selected in such a manner that angles made by forces with axes should have convenient values.

$$\sum F_x = 0 \Rightarrow F_{1x} + F_{2x} + F_{3x} = 0$$

$$\sum F_y = 0 \Rightarrow F_{1y} + F_{2y} + F_{3y} = 0$$



Problems involving more than three forces should be analyzed by analytical method. However, in some situations, there may be some parallel or anti-parallel forces and they should be combined first to minimize the number of forces. This may sometimes lead a problem involving more than three forces to a three-force system.

Tension in Rod or Massive String

Mass of Rod = M

Length of Rod = L

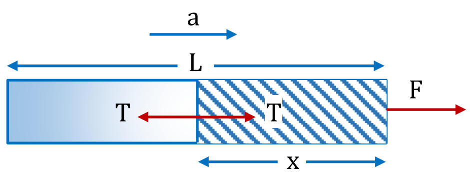
$F - T = ma$

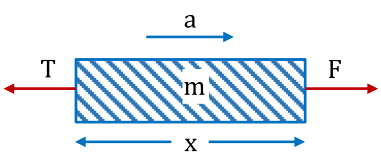
$\Rightarrow \quad T = F - ma$

$\Rightarrow \quad T = F - m\dfrac{F}{M}\left(\therefore a = \dfrac{F}{M}\right)$

$\because$ Mass of length $'L' = M \quad \therefore$ Mass of unit length $= \dfrac{M}{L}$

$\therefore$ Mass (m) of length $'x' = \dfrac{M}{L}x$





Put this value of 'm' in equation (1)

$$T = F - \left(\frac{M}{L}x\right) \times \frac{F}{M}$$

$$T = F - \frac{Fx}{L}; \quad \boxed{T = F\left(1 - \frac{x}{L}\right)}$$

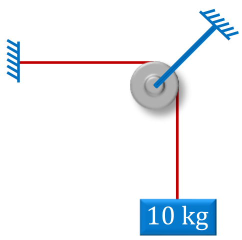
Pulley Systems

The only function of pulley (which has no friction on its axle to retard rotation) is to change the direction of the cord that joins the two blocks.

- Ideal pulley is considered weightless and frictionless.
- Ideal string is massless and inextensible.
- The pulley may change the direction of force in the string but not the tension.

Illustration 5:

Find magnitude of force exerted by string on pulley.



Solution:

F.B.D. of 10 kg block : $T = 10\,g = 100\ N$

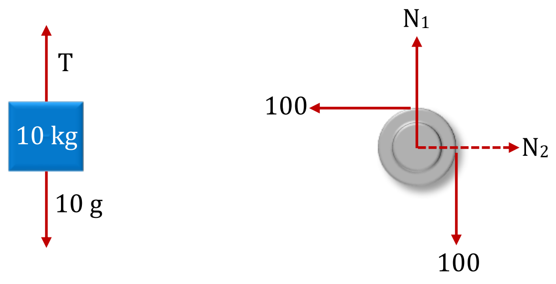
F.B.D. of pulley : Since string is massless, so tension in both sides of string is same.

Force exerted by string = $\sqrt{(100)^2 + (100)^2} = 100\sqrt{2}\ N$

Note:

Since pulley is in equilibrium position, so net forces on it is zero.

Hence force exerted by hinge on it is $100\sqrt{2}\ N$.



Some Cases of Pulley

Case I:

$m_1 = m_2 = m$

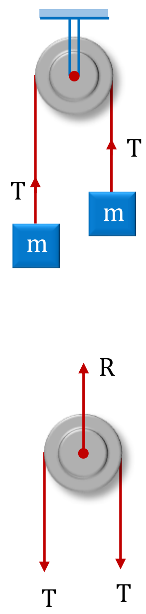
Tension in the string

$T = mg$

Acceleration $'a'$ = zero

Reaction at the suspension of the pulley

$R = 2T = 2\ mg$



$$\boxed{\text{Acceleration} = \frac{\text{Net pulling force}}{\text{Total mass to be pulled}}}$$

$$\boxed{\text{Tension} = \frac{2 \times \text{Product of the masses}}{\text{Sum of the two masses}} g}$$

Case II:

$m_1 > m_2$

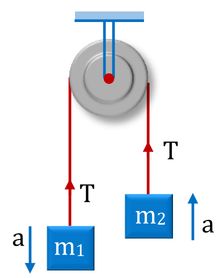
Now for mass m_1,

$m_1 g - T = m_1 a$ ...(i)

For mass m_2,

$T - m_2 g = m_2 a$ ...(ii)

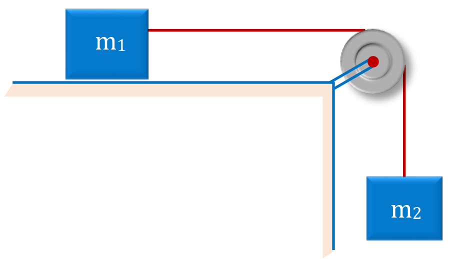
By (i) and (ii)

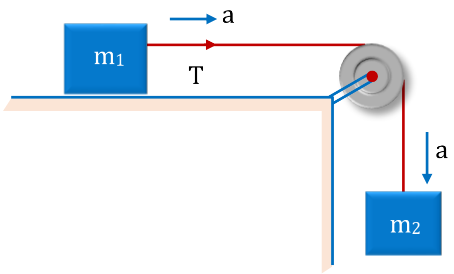


$$a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g \text{ and } T = \frac{2m_1 m_2}{(m_1 + m_2)} g$$

$$\boxed{\text{Reaction at the suspension of pulley } R = 2T = \left(\frac{4m_1 m_2}{m_1 + m_2}\right) g}$$

Case III:



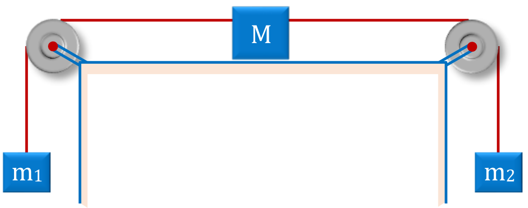


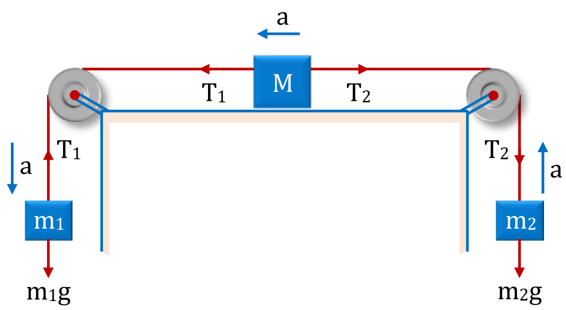
For mass $m_1 : T = m_1 a$

For mass $m_2 : m_2g - T = m_2 a$

acceleration $a = \dfrac{m_2 g}{(m_1 + m_2)}$ and $T = \dfrac{m_1 m_2}{(m_1 + m_2)} g$

Case IV : $(m_1 > m_2)$





$m_1g - T_1 = m_1a$ (i)

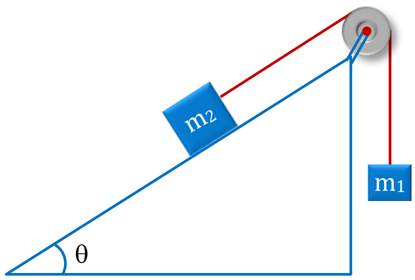
$T_2 - m_2g = m_2a$ (ii)

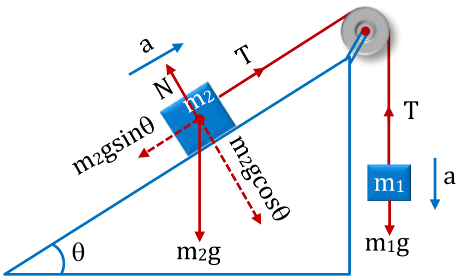
$T_1 - T_2 = Ma$ (iii)

By (i), (ii) and (iii)

$$a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)} g$$

Case V : Mass suspended over a pulley from another on an inclined plane.





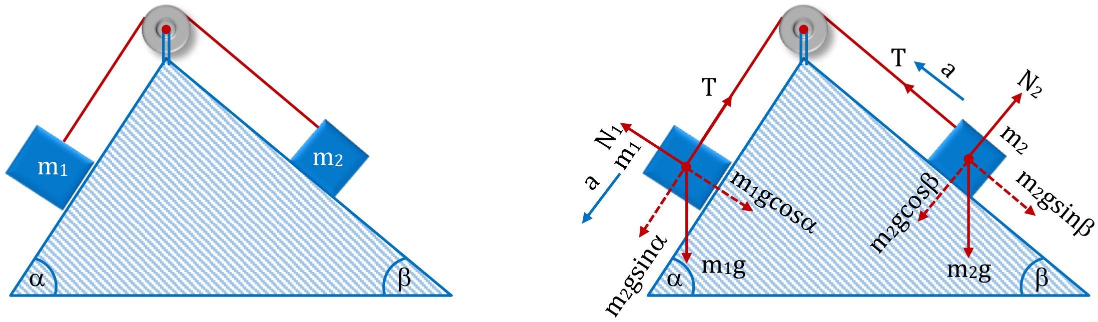
For mass $m_1 : m_1g - T = m_1 a$

For mass $m_2 : T - m_2g \sin\theta = m_2 a$

Acceleration $a = \dfrac{(m_1 - m_2 \sin\theta)}{(m_1 + m_2)} g$

$$T = \frac{m_1 m_2 (1 + \sin\theta)}{(m_1 + m_2)} g$$

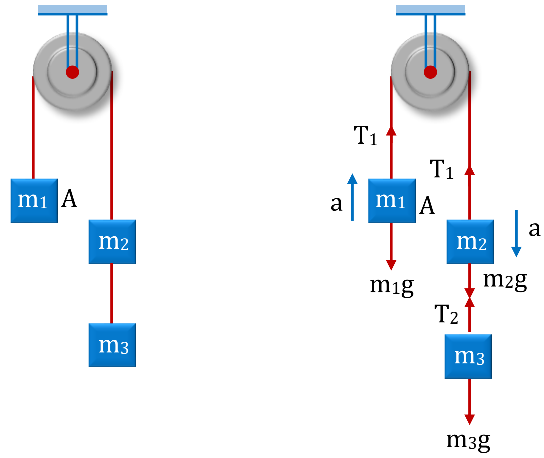
Case VI : Masses m_1 and m_2 are connected by a string passing over a pulley ($m_1 > m_2$)



Acceleration

$$a = \frac{(m_1 \sin\alpha - m_2 \sin\beta)}{(m_1 + m_2)} g$$

Tension, $T = \dfrac{m_1 m_2 (\sin\alpha + \sin\beta)}{(m_1 + m_2)} g$



Case VII : For mass m_1 : $T_1 - m_1 g = m_1 a$

For mass, m_2 : $m_2 g + T_2 - T_1 = m_2 a$

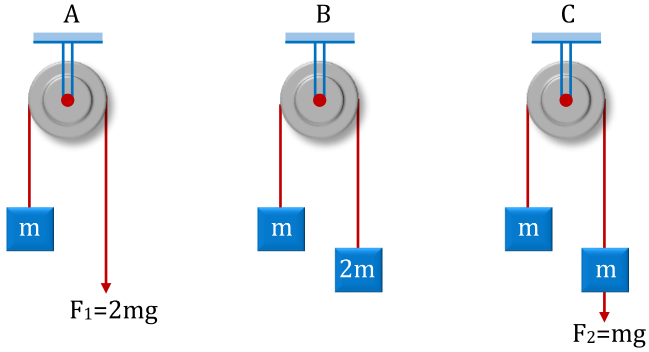
For mass, m_3 : $m_3 g - T_2 = m_3 a$

$$a = \frac{(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)} g$$

We can calculate tensions T_1 and T_2 from above equations.

Illustration 6:

In fig. *A*, *B* and *C*, each block has accelerations a_1, a_2 and a_3 respectively. F_1 and F_2 are external forces of magnitudes $2mg$ and mg respectively. Find the value of a_1, a_2 and a_3.



Solution:

$$a_1 = \frac{2mg - mg}{m} = g$$

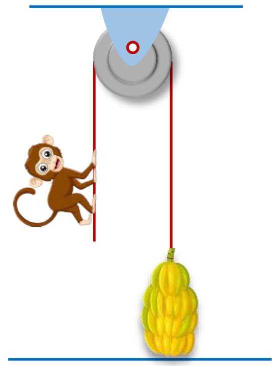
$$a_2 = \frac{2m - m}{2m + m} g = \frac{g}{3}$$

$$a_3 = \frac{mg + mg - mg}{2m} = \frac{g}{2}$$

Clearly $a_1 > a_3 > a_2$

Illustration 7:

A 12 *kg* monkey climbs a light rope as shown in fig. The rope passes over a pulley and is attached to a 16 *kg* bunch of bananas. Mass and friction in the pulley are negligible so that the pulley's only effect is to reverse the direction of the rope. What is the maximum acceleration the monkey can have without lifting the bananas? (Take $g = 10\ m/s^2$)



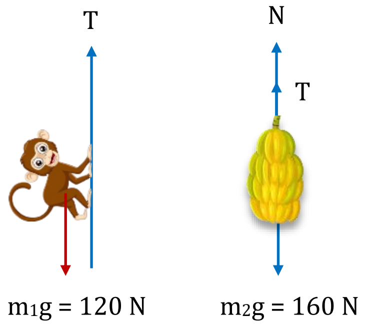
Solution:

For Monkey

$T - 120 = 12 \times a$ ...(i)

For Bananas

$160 - T = N$



Condition for just a loosing contact $N = 0$

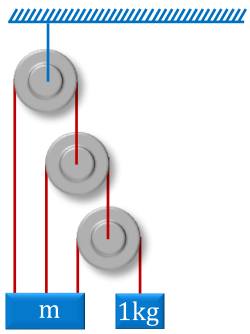
$160 - T = 0 \quad \Rightarrow T = 160$ ...(ii)

From equation (i) and (ii)

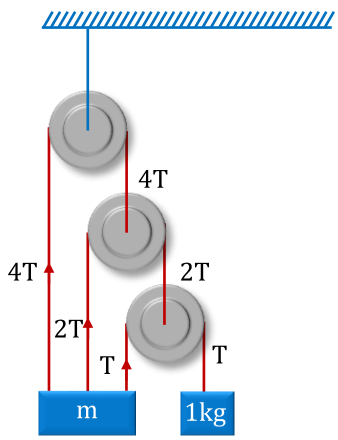
$160 - 120 = 12 \times a \Rightarrow a = 3.33\ m/s^2$

Illustration 8:

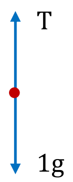
Find *M* for system at rest also find tension in each string.



Solution:



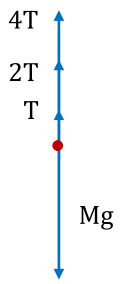
FBD of 1 kg



$T - g = 0$

$T = 0$

FBD of m



$7T - m \times 10 = 0$

$7 \times 10 = 10\ m$

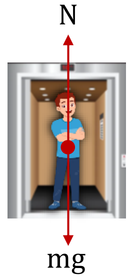
$m = 7\ kg$

Weighing Machine

A weighing machine does not measure the weight but measures the force exerted by object on its upper surface.

Effective or Apparent weight of a man in lift

Let us analyse a situation where a man is travelling in a lift.

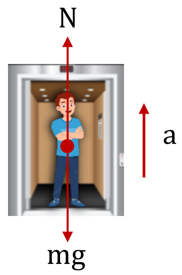


Case-I : If the lift is at rest or moving uniformly (a = 0), then

$$N = mg$$

So, $W_{app} = W_{actual}$ $\boxed{\begin{array}{l} W_{app} = N \\ W_{actual} = mg \end{array}}$

Case-II: If the lift is accelerated upwards, then - Net upward force on man = ma



$$N - mg = ma$$

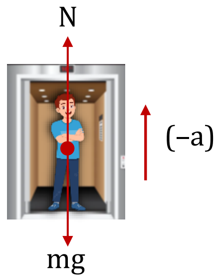
$$N = mg + ma$$

$$\therefore \quad N = m(g + a)$$

$$\boxed{W_{app} \text{ or } N = m(g+a)}$$

So, $W_{app} > W_{actual}$

Case-III: If the lift is retarding upwards, then



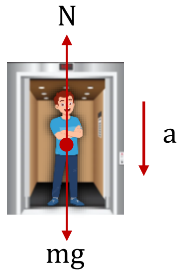
$$N - mg = m(-a)$$

$$N = mg - ma$$

$$\therefore \quad \boxed{W_{app} \text{ or } N = m(g-a)}$$

So, $W_{app} < W_{actual}$

Case-IV: If the lift is accelerated downwards, then –

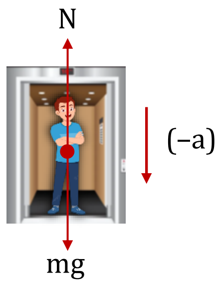


$$mg - N = ma$$

$$N = mg - ma$$

$$\boxed{W_{app} \text{ or } N = m(g-a)}$$

Case-V: If the lift is retarding downwards, then –



$$mg - N = m(-a)$$

$$mg - N = -\,ma$$

$$N = ma + mgs$$

$$\boxed{W_{app} \text{ or } N = m(g+a)}$$

So, $W_{app} > W_{actual}$

Two Special Cases of downward acceleration (IV Case)

I Special Case :- If the lift is falling freely, means its acceleration in the vertically downward direction is equal to acceleration due to gravity. i.e. $a = g$, then –

$$W_{app.} = m\,(g-a) = m\,(g-g) = m \times 0 = 0 \;\therefore\; W_{app.} = 0$$

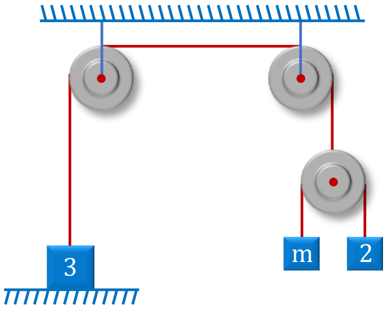
Means the man will feel weightless. This condition is known as Condition of Weightlessness.

The apparent weight of any freely falling body is always zero.

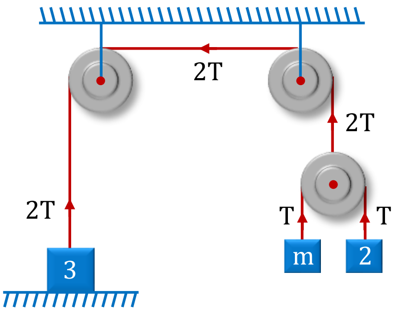
II Special Case :- If the lift is accelerating downwards with an acceleration which is greater than 'g', then the man will move up with respect to lift and he will stick to the ceiling.

Illustration 9:

Find min. 'm' to lift 3kg block.



Solution:



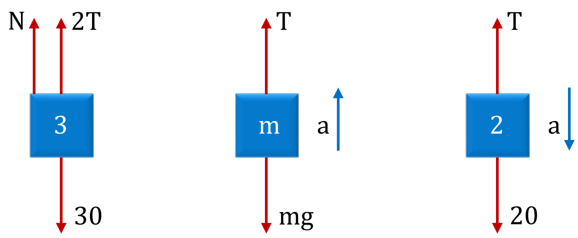
$$T = \frac{2\times 2\times m}{2+m}\times g$$

At the point of losing contact.

$N = 0$

$2T = 30$

$$\Rightarrow \quad 2\times\frac{4m}{m+2}\times g = 3\times g \Rightarrow m = \frac{6}{5}kg$$



Spring Force

It is the restoring force developed in a spring.

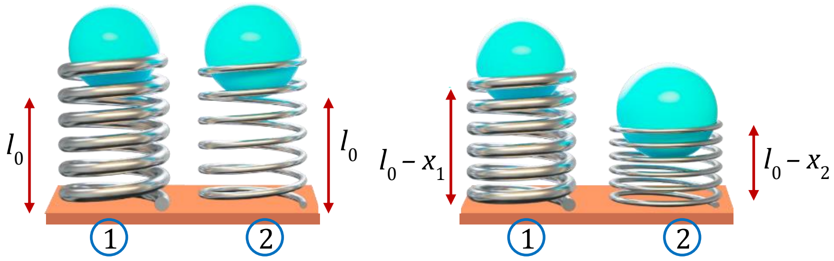
It is given by Hooke's law, $\vec{F} = -k\vec{x} = -kx\hat{x}$, where x is the change in length of the spring and k is the **spring constant.**

Spring force is always opposite to the direction of applied force or the direction of displacement. Hence, it is taken as negative.

For an ideal spring (massless) the restoring spring force, $F_{spring} = F_{ext}$

Spring constant (k): It denotes the **stiffness** of the spring which signifies how difficult it is to deform the spring. It is a measure of the inertia of spring to resist any kind of compression or extension.
In the figure, the same balls are kept on two springs of the same length, yet the change in length is different.

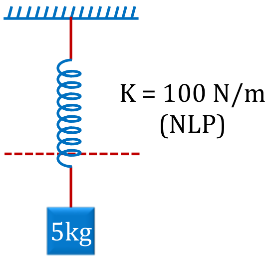
$$F_{\text{spring}} = F_{\text{ext}}$$



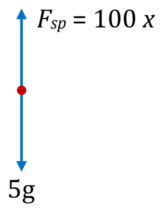
Thus, if the same force is applied on two springs of the same length, then the spring with higher spring constant will get elongated or compressed less than the spring with lower spring constant. Higher the spring constant k, more stiffer and robust the spring is, and more is the difficulty to compress or extend it.
Similarity between ideal spring and ideal string: Both are massless.
Difference between ideal spring and ideal string: Ideal string is inextensible, while due to helical nature of spring its length can be changed.

Illustration 10:

5 kg is in equilibrium and at rest find elongation in the spring.



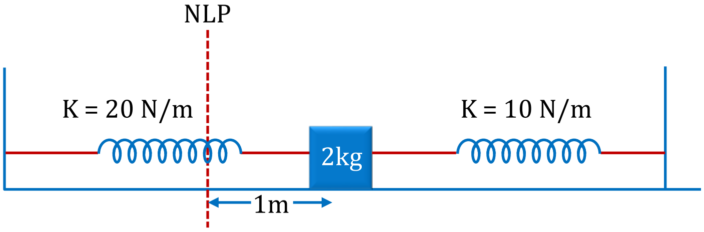
Solution:



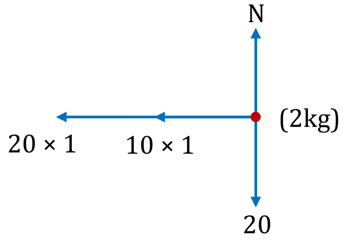
$100x = 50$
$x = 0.5\ m$

Illustration 11:

Find acceleration of block just after releasing?



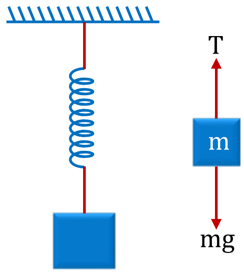
Solution:



$30 = 2a$

$a = 15\ m/s$

Spring balance



- It measures the restoring spring force which is equal to the tensile force on the spring. This tensile force is equal to the weight of the hanging body only under equilibrium condition.
- In the given figure, when the mass is hung to the vertical spring and the system comes in equilibrium, the tension in the string will be equal to the weight hung to it.
- It does not measure the weight. It measures the force exerted by the object at the hook.
- Symbolically, it is represented as shown in figure. A block of mass *'m'* is suspended at hook. When spring balance is in equilibrium, we draw the F.B.D. of mass *m* for calculating the reading of balance.

F.B.D. of *'m'*.

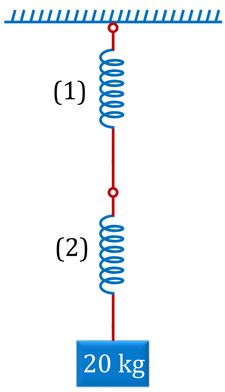
$mg - T = 0$

$T = mg$

Magnitude of T gives the reading of spring balance.

Illustration 12:

A block of mass 20 kg is suspended through two light spring balances as shown in figure. Calculate the



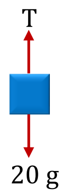
(1) reading of spring balance (1).

(2) reading of spring balance (2).

Solution:

For calculating the reading, first we draw F.B.D. of 20 *kg* block.

F.B.D of 20 *kg*.



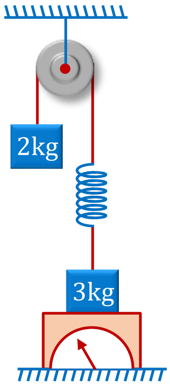
$mg - T = 0$

$T = 20\,g = 200\,N$

Since both balances are light so, both the scales will read 20 *kg*

Illustration 13:

System is in equilibrium. Find reading of both weighing machine and balance.



Solution:

$\therefore \quad SB = 2\,kg$

$T + N = 30$

$\Rightarrow \quad N = 10$

$\therefore \quad WM\text{ (Weigning machine)} = \dfrac{10}{g} = 1kg$

$T = 20\,N$

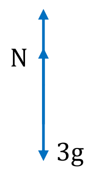
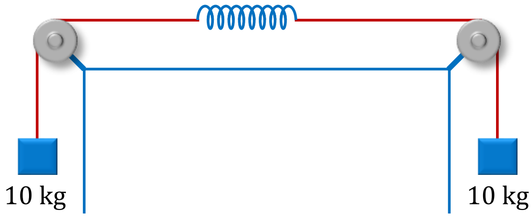
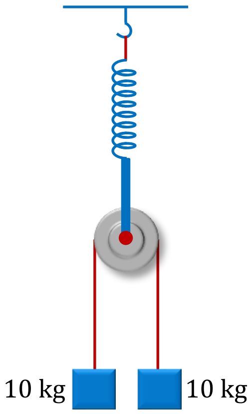


Illustration 14:

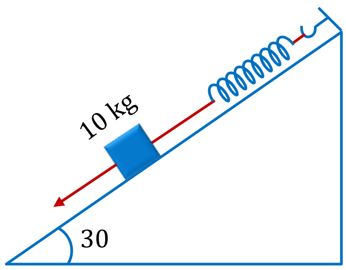
The system shown in fig. are in equilibrium. If the spring balance is calibrated in newtons, what does it record in each case? ($g = 10\ m/s^2$)



(A)

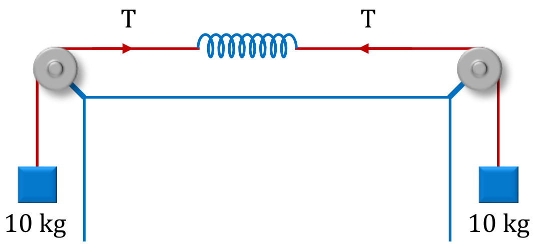


(B)



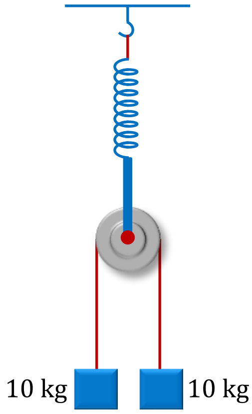
(C)

Solution:



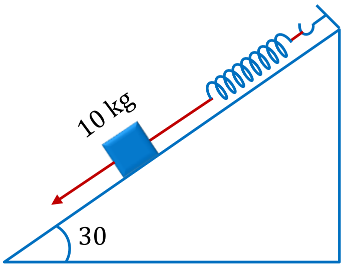
(A)

One weight act as support
another acts as weight
so tension $T = 10$ g
$= 100$ N



(B)

$$T' = 2T = 2\left[\frac{2\times10\times10\times g}{20}\right]$$
$$T' = 200N$$

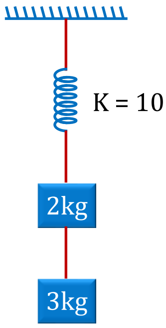


(C)

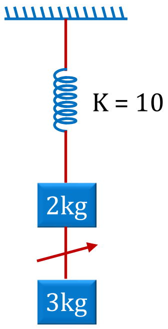
$T = 10 \times 10 \times \sin 30°$
$= 10 \times 10 \times ½ = 50$ N

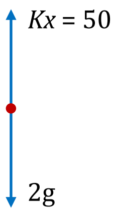
Illustration 15:

Assume system is in equilibrium. If string is cut then find acceleration of 2*kg* and 3*kg*.



Solution:





$kx = 50\ N$

$x = 5\ m \quad \therefore \quad kx - 2g = 2a \quad \therefore \quad 30 = 2a$

$\Rightarrow \quad a = 15\ ms^{-2}$

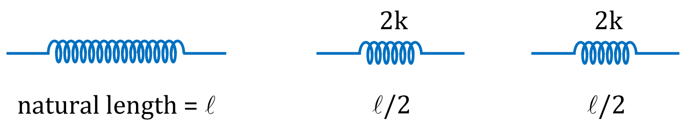
For 3 *kg* block

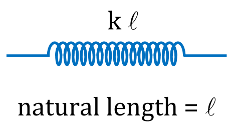
$3g = 3a$

$a = 10\ m/s^2$

Variation of *k* with natural length

$k\ell$ = constant





$k\ell = k_1\frac{\ell}{3}$; $k\ell = k_2\frac{2\ell}{3}$; $k_1 = 3k$ and $k_2 = \frac{3k}{2}$

Equivalent Spring Constant

- When Springs are connected in Parallel then we can replace them by single spring of spring constant k_e where

  $k_e = k_1 + k_2$

  For more than two spring $k_e = k_1 + k_2 + k_3 + ...$
- When Springs are connected in series then we can replace them by single spring of spring constant k_e where $1/k_e = 1/k_1 + 1/k_2$. As spring constants are not equal so extensions will not be equal, but total extension y can be written as sum of two extensions $y = y_1 + y_2$

  For more than two springs

  $$\frac{1}{k_e} = \frac{1}{k_1} + \frac{1}{k_2} + ...$$

Newton's law on a system

Selection of system

Consider $m_1, m_2, m_3, \ldots$ are the masses of the objects of the system and $\vec{a}_1, \vec{a}_2, \vec{a}_3 \ldots$ are the acceleration of the objects respectively.

We can balance the forces on individual body as:

$$\left(\sum \vec{F}_{ext}\right)_1 = m_1\vec{a}_1 \quad ...(i)$$

$$\left(\sum \vec{F}_{ext}\right)_2 = m_2\vec{a}_2 \quad ...(ii)$$

$$\left(\sum \vec{F}_{ext}\right)_3 = m_3\vec{a}_3 \quad ...(iii)$$

Adding all equations,

$$\left(\sum \vec{F}_{ext}\right)_{sys} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_3\vec{a}_3 + \ldots$$

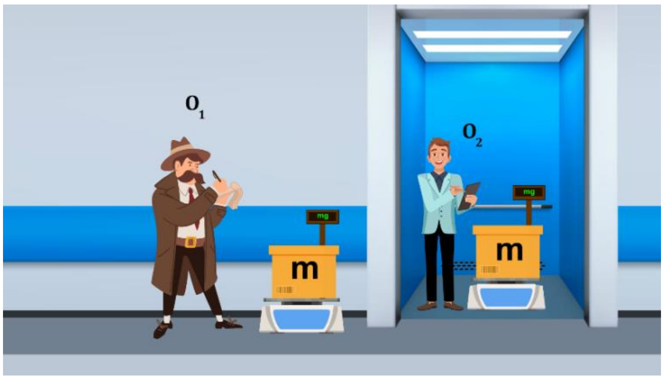
Thus, we can consider the group of masses as one system.

Pseudo Force

Consider a block of mass m kept on a weighing machine. Consider the system is kept in an elevator. Let observer O_1 observe this system from the ground frame. Let another observer O_2 be present inside the lift.

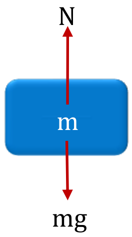
Case 1: Elevator is at rest

When the elevator is at rest, the reading on the weighing machine is same for both observers.



The forces acting on the block are weight (mg) and normal reaction (N).

Along vertical direction, $N = mg$



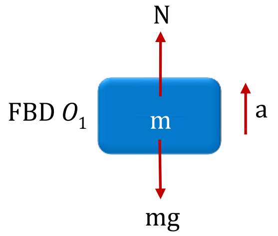
Case 2: Elevator is moving upwards with acceleration *a*

When the elevator starts moving upwards with an acceleration a, both observers observe different readings on the weighing machine than the reading when the elevator was at rest.

In order to understand why the reading is different in this case, they used concepts of physics as follows:

FBD of block for observer O_1 standing in ground frame:

This observer finds the system is moving upwards with acceleration a. Other forces acting on the block are its weight and normal reaction.



Along vertical direction,

$N - mg = ma$

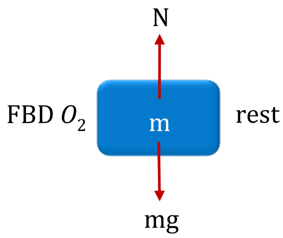
$N = mg + ma$

$N = m(g + a)$

As the weighing machine measures the normal reaction acting on it, the weighing machine will show a reading corresponding to $N = m(g + a)$.

FBD of block for observer O_2 standing in lift frame:

When the observer is in the elevator, the block is at rest with respect to him. So, for this case, the forces acting on the block are weight (mg) and normal reaction (N).



Along vertical direction,

$N = mg$

However, the observer O_2 sees the reading on the weighing machine is $m(g + a)$, and he couldn't explain his observation as his derivation was different. To find what is missing in this case, use the concept of relative motion.

Let $\vec{a}_{B,elevator}$ be the acceleration of the block with respect to elevator $\vec{a}_{B,G}$ be the acceleration of block with respect to ground and $\vec{a}_{elevator,G}$ be the acceleration of the elevator with respect to ground.

$$\vec{a}_{B,elevator} = \vec{a}_{B,G} - \vec{a}_{elevator,G}$$

Multiplying both sides by m,

$$m\vec{a}_{B,elevator} = m\vec{a}_{B,G} - m\vec{a}_{elevator,G}$$

The normal reaction is the $\left(m\vec{a}_{B,elevator}\right)$ sum of $\left(m\vec{a}_{B,G}\right)$ and $\left(-m\vec{a}_{elevator,G}\right)$ when the elevator is accelerating.

Here, $\left(m\vec{a}_{B,G}\right)$ is the weight of block and term $\left(-m\vec{a}_{elevator,G}\right)$ appears due to the accelerated frame. This can be written in terms of force as :

$$\left(\sum \vec{F}_{P_{elevator}}\right) = \left(\sum \vec{F}_{P\ Ground}\right) + \left(-m\vec{a}_{elevator,G}\right)$$

This new force term $\left(-m\vec{a}_{elevator,G}\right)$ appears due to the mass of block and acceleration of the elevator which does not make any sense physically.

Non-Inertial Frames

The change in the net force in the ground and elevator frame is owing/attributed to the acceleration of the elevator. If the elevator was moving with uniform velocity, this term would be zero.

Thus, it is not about movement of the elevator but acceleration of the elevator. Such frames that are accelerating are known as **non-inertial frames of reference.**

In **non-inertial frames of reference**, Newton's law seems to be failing. Thus, Newton's law is not valid in non-inertial frames of reference.

The extra term is unreal and known as **pseudo force.**

What is a pseudo force?

A pseudo force (also known as fictitious force, inertial force, or d'Alembert force) is an apparent force that acts on all the masses whose motion is described using a non-inertial frame of reference. The magnitude of the pseudo force is the mass of an object multiplied by the acceleration of the frame of reference.

$$\vec{F}_{pseudo} = -m\vec{a}_{elevator,G}$$

Thus, for non-inertial frames, Newton's law can be modified as,

$$\left(\sum \vec{F}_{ext\ elevator}\right) = \left(\sum \vec{F}_{ext}\right)_{Ground} + \vec{F}_{pseudo}$$

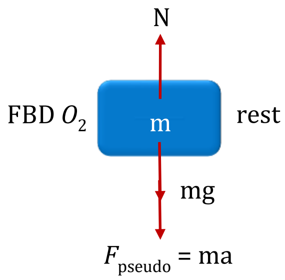
The pseudo forces are also known as **inertial forces**, although their need arises because of the use of **non-inertial frames.**

$$\vec{F}_{pseudo} = -m_{sys}\vec{a}_{non-inertial\ \text{frame}}$$

Properties of pseudo force:

- It's a fictitious force i.e., it will not have a reaction pair.
- It always acts in the direction opposite to the acceleration of the frame of reference.

Thus, the FBD for mass moving with acceleration in the elevator frame is modified as:



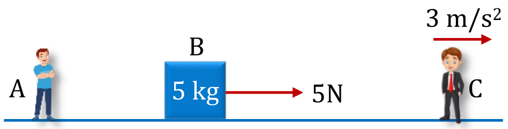
Balancing the forces in vertical direction,

$N = mg + ma$

$N = m(g + a)$

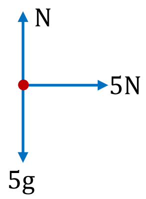
Illustration 16:

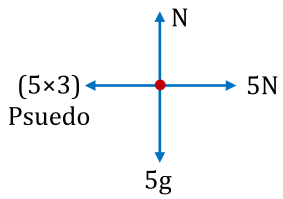
Find the acceleration of block with respect to A and C.



In Frame (A) In Frame (C)

Solution:



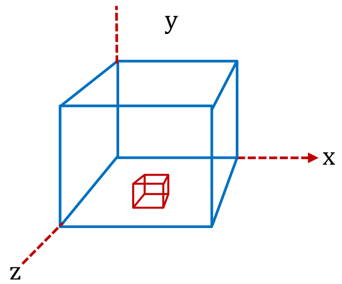


$5 = 5\ a_{B/A}$ $15 - 5 = 5\ a_{B/C}$

$a_{B/A} = 1\ m/s^2$ $a_{B/C} = 2\ m/s^2$

Illustration 17:

A block of mass 2 *kg* is kept at rest on a big box moving with velocity $2\hat{i}\ m/s$ and having acceleration $-3\hat{i}+4\hat{j}\ m/s^2$. Find the value of 'Pseudo force' acting on block with respect to box.



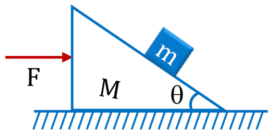
Solution:

$$\vec{F} = -m\vec{a}_{frame} = -2\left(-3\hat{i}+4\hat{j}\right)$$

$$F = 6\hat{i} - 8\hat{j}\ N$$

Illustration 18:

All surfaces are smooth in the adjoining figure. Find *F* such that block remains stationary with respect to wedge.

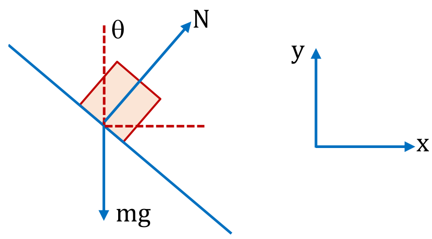


Solution:

Acceleration of (block + wedge) is $a = \dfrac{F}{(M+m)}$

Let us solve the problem by using both frames.

From inertial frame of reference (Ground)



F.B.D. of block w.r.t. ground (Apply real forces):

with respect to ground block is moving with an acceleration 'a'.

$\therefore$ $\quad \sum F_y = 0 \Rightarrow N\cos\theta = mg$ ...(i)

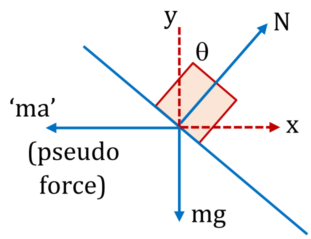
And $\quad \sum F_x = ma \Rightarrow N\sin\theta = ma$ ...(ii)

From equations (i) and (ii)

$$a = g\tan\theta$$

$\therefore \quad F = (M+m)\,a = (M+m)\,g\tan\theta$

From non–inertial frame of reference (Wedge) :



F.B.D. of block w.r.t . wedge (real forces + pseudo force)

w.r.t. wedge, block is stationary

$\therefore$ $\quad \sum F_y = 0 \Rightarrow N\cos\theta = mg$ ...(iii)

$\quad \sum F_x = 0 \Rightarrow N\sin\theta = ma$ ...(iv)

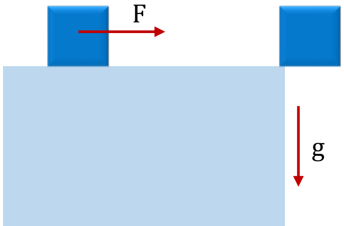
From equations (iii) and (iv), we will get the same result

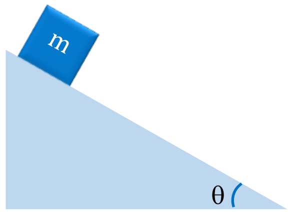
i.e. $\quad F = (M+m)\,g\tan\theta$

Constrained Motion

Constrained motion results when an object is forced to move in a restricted way.

Here, the object is constrained to move along the surface of the block due to the force and then falls vertically downwards due to acceleration due to gravity.





Similarly, here the block is constrained to slide along an inclined plane. The force due to which the body is kept under a constraint is known as **constraint force**.

There are two primary constraints:

1. String/Rod constraints
2. Wedge constraints

String Constraints

Consider objects connected through a string that has the following properties:

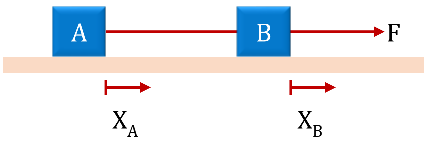
- The length of the string remains constant, i.e., string is inextensible.

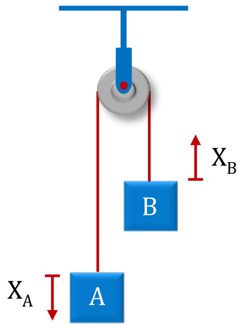
  Which means that if one block moves *x* distance, the other block also moves the same distance.

  It can be mathematically written as,

  $x_A = x_B$

  Where x_A is displacement of block *A* and x_B is displacement of block *B*.





- It always remains tight and does not slack.

  Parameters of the motion of such objects along the length of the string and in the direction of extension have a definite relation between them.

  Here, the parameters of motion means the displacement, velocity, and acceleration.

  Objects mean the bodies that are directly connected to string.

  Definite relation means the constraint relation, such as $x_A = x_B$

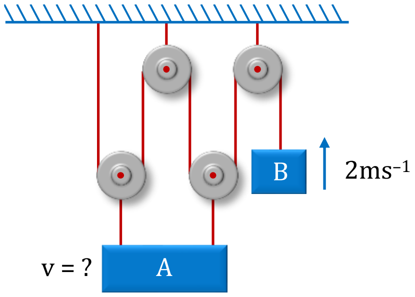
  For a system involving ideal pulleys and ideal strings we can write,

  $$\sum \vec{T}.\vec{S} = 0 \text{ and } \sum \vec{T}.\vec{v} = 0$$

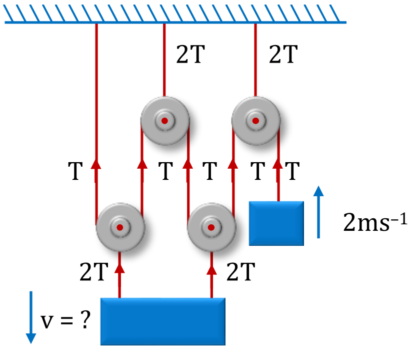
  and if acceleration of blocks are collinear with their velocity then, $\sum \vec{T}.\vec{a} = 0$

Illustration 19:

Find the velocity of block A.



Solution:



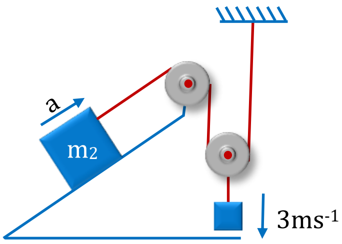
$v(4T)(-1) + (T)(2) = 0$

$2 = 4v$

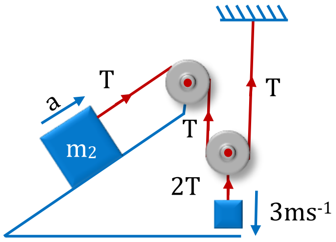
$\therefore \quad v = 1/2\ ms^{-1} \downarrow$

Illustration 20:

Find the velocity (a) of block.



Solution:

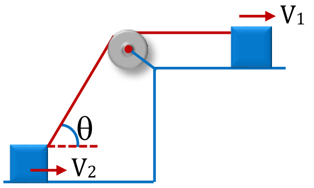


$(2T)(3)(-1) + (a)(T) = 0$

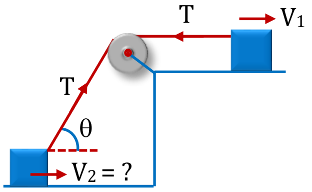
$a = 6\ ms^{-1} \uparrow$

Illustration 21:

Find relation between V_1 and V_2 for the given figure.



Solution:



$-(T)(V_1) + T\,(V_2 \cos\theta) = 0 \;\therefore\; V_2 = V_1/\cos\theta \; ms^{-1} \rightarrow$

Steps to follow to solve problem involving constraint motion

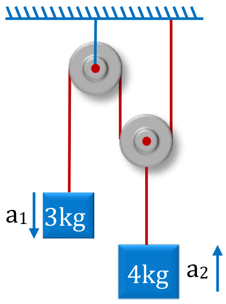
(i) Take acceleration of each block in some direction.

(ii) Make FBD of each block and write Newton's 2^{nd} law equation as per direction taken in Step-I

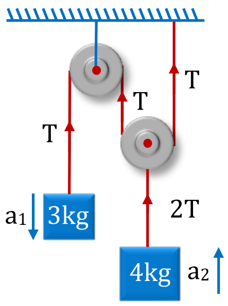
(iii) Write constraint equation and solve all equations to get acceleration of each block. If acceleration comes positive then the acceleration of the body is in the direction assumed in STEP-I and if acceleration comes negative then it is opposite to the direction assumed in STEP-I.

Illustration 22:

Find the acceleration (a_1 and a_2) of the blocks.



Solution:



$3g - T = 3a_1$

$2T - 4g = 4a_3$

$T(a_1)(-1) + 2T(a_2)(1) = 0$

$2a_2 = a_1$

$3g - T = 3a_1$

$T - 2g = 2a_1$

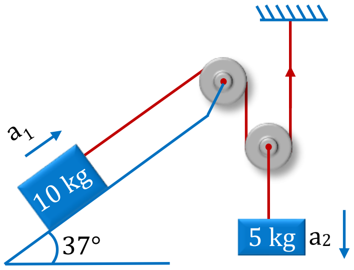
$g = 4a_1$

$\therefore \quad a_1 = 2.5\; ms^{-2}$

$a_2 = 1.25\; ms^{-2}$

Illustration 23:

Find acceleration of all the blocks.

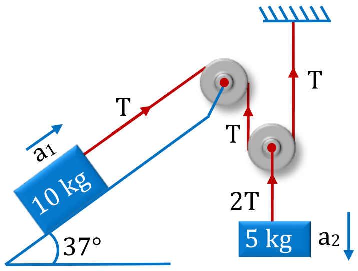


Solution:

$T - 10g\sin 37° = 10a_1$

$T - 60 = 10a_1$ ...(i)

$50 - 2T = 5a_2$ ...(ii)



From constraint relation,

$T(a_1) + 2(T)(-a_2) = 0$

$a_1 = 2a_2$

$2T - 120 = 40a_2$

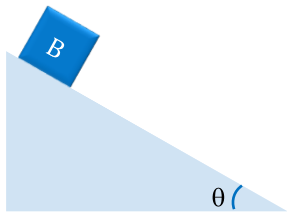
$$a_2 = \frac{-14}{9} m/s^2 \Rightarrow a_1 = \frac{-28}{9} m/s^2$$

Key point :

Constraint relation in string will also hold for string having mass provided it is inextensible.

Wedge Constraint

- Wedge constrained motion: When a body moves over a wedge, it follows certain sliding constraints known as wedge constraint.
- Wedge constraint says that component of velocity and acceleration perpendicular to the contact surface (interface) of two objects is always equal if there is no deformation and the objects remain in contact.

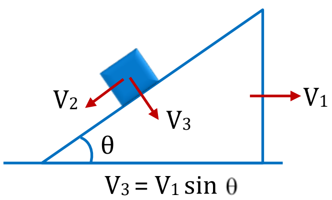


Conditions :

(i) There is a regular contact between two objects.

(ii) Objects are rigid.

The relative velocity perpendicular to the contact plane of the two rigid objects is always zero if there is a regular contact between the objects. Wedge constraint is applied for each contact.



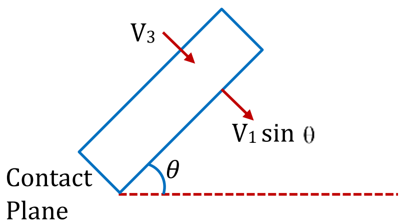
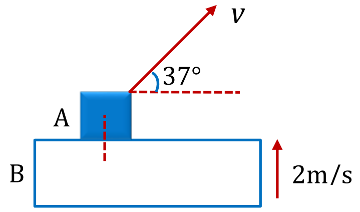


Illustration 24:

A and *B* are always is contact. Find *v* ?



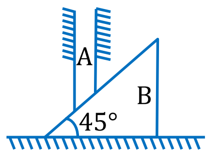
Solution:

$$v \sin 37° = 2 \quad ; \quad v = \frac{10}{3} m/s$$

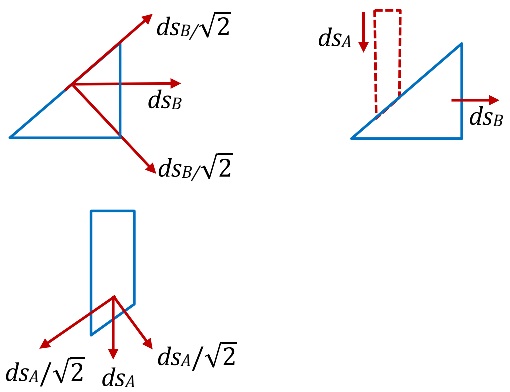
In other words, Components of velocity along perpendicular direction to the contact plane of the two objects is always equal if there is no deformations and they remain in contact.

Illustration 25:

Find the acceleration of rod *A* and wedge *B* in the arrangement shown in fig. if the mass of rod equal that of the wedge and the friction between all contact surfaces is negligible. Take angle of wedge as 45^o.



Solution:

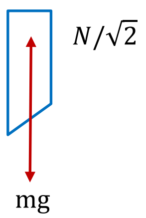


Perpendicular to the plane of contact displacement must be same.

$$\frac{ds_B}{\sqrt{2}} = \frac{ds_A}{\sqrt{2}}$$

$$ds_B = ds_A$$

Differentiating, $a_B = a_A$



$$mg - N/\sqrt{2} = ma \qquad ...(i)$$

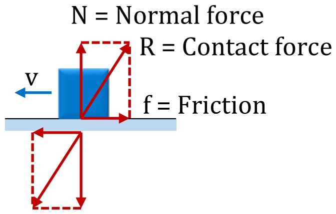
$$N/\sqrt{2} = ma \qquad ...(ii)$$

$$mg = 2ma$$

$\Rightarrow \quad a = g/2$

FRICTION

When two bodies are kept in contact, electromagnetic forces act between the charged particles (molecules) at the surfaces of the bodies. Thus, each body exerts a contact force on the other. The magnitudes of the contact forces acting on the two bodies are equal but their directions are opposite and therefore the contact forces obey Newton's third law.



The direction of the contact force acting on a particular body is not necessarily perpendicular to the contact surface. We can resolve this contact force into two components, one perpendicular to the contact surface and the other parallel to it (see in figure). The perpendicular component is called the normal contact force or normal force (generally written as N) and the parallel component is called friction (generally written as f).

Therefore, if R is contact force then $R = \sqrt{f^2 + N^2}$

Reasons for friction:

(i) Inter-locking of extended parts of one object into the extended parts of the other object. **(Old view)**

(ii) Bonding between the molecules of the two surfaces or objects in contact. **(Modern view)**

Static Friction Force

If there is a tendency of **relative slipping** (only tendency and not actual) between two surfaces in contact then the friction force acting between them is called static friction force.

It is a variable force whose value is equal to requirement to stop relative slipping till it reaches its **limiting value**.

For example consider a bed inside a room ; when we gently push the bed with a finger, the bed does not move. This means that the bed has a tendency to move in the direction of applied force but does not move as there exists static friction force acting in the opposite direction of the applied force.

Laws of static friction:

1st law : Static friction force acting between two surfaces in contact does not depend on area of contact.

2nd law : Maximum value of static friction (Limiting friction) is directly proportional to normal force acting between the two surfaces.

$(f_s) \propto N$

$\boxed{(f_s)_{\text{max.}} = \mu_s N}$ $\qquad f_s$ = Limiting friction ; μ_s = coefficient of static friction

Direction of static friction force :

The static friction force on an object is opposite to its impending motion relative to the surface.

Following steps should be followed in determining the direction of static friction force on an object.

(i) Draw the free body diagram with respect to the other object on which it is kept.

(ii) Include pseudo force also if contact surface is accelerating.

(iii) Decide the resultant force and the component parallel to the surface of this resultant force.

(iv) The direction of static friction is opposite to the above component of resultant force.

Note:

Here once again the static friction is involved when there is no relative motion between two surfaces.

Kinetic Friction Force

→ Kinetic friction comes into picture when relative **slipping occurs**.

→ It acts in direction opposite to **relative velocity**.

→ Its value is **constant**.

Laws of kinetic friction:

1st law : Kinetic friction acting between two surfaces in contact does not depend on area of contact.

2nd law : Value of kinetic friction force is directly proportional to the normal force acting between the two surfaces in contact.

$f_k \propto N$

$\boxed{f_k = \mu_k N}$ μ_k = coefficient of kinetic friction

Note:

$\mu_k = \mu_s = \mu$ (if not mentioned separately)

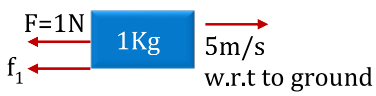
Illustration 26:

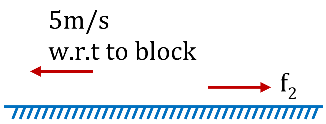
Find the direction of kinetic friction force

(a) on the block, exerted by the ground.

(b) on the ground, exerted by the block.

Solution:

(a) 

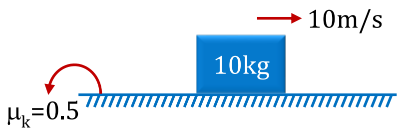
(b) 

where f_1 and f_2 are the friction forces on the block and ground respectively.

Also note that the direction of kinetic friction has nothing to do with applied force *F*.

Illustration 27:

Find out the distance travelled by the blocks shown in the figure before it stops.



Solution:

$N - 10\,g = 0$

$N = 100\ N$

$f_k = \mu_k N$

($\mu = \mu_s = \mu_k$ when not mentioned)

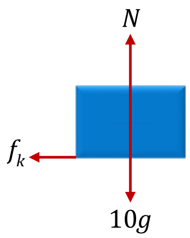
$f_k = 0.5 \times 100 = 50\ N$

$F_x = ma$

$50 = 10\,a \Rightarrow a = 5\ m/s^2$

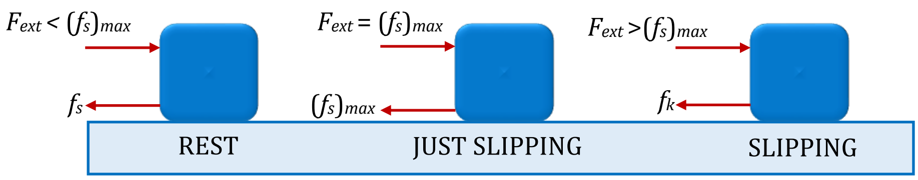
$\therefore$ $v^2 = u^2 + 2as$

$0^2 = 10^2 + 2\,(-5)\,(S)$ $\therefore$ $S = 10\ m$



Limiting Friction

- The maximum possible friction force between the two surfaces before sliding begins is known as **limiting friction**.
- If the applied external force exceeds the value of the limiting friction {(f_s)max}, then the slipping starts and the nature of friction changes to kinetic friction.



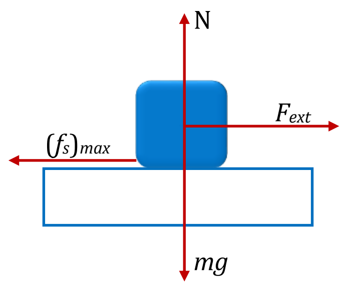
- The magnitude of limiting friction is proportional to normal reaction acting on the contact surface.

  Limiting Friction $\propto$ Normal Reaction

  $(f_s)_{max} \propto N$

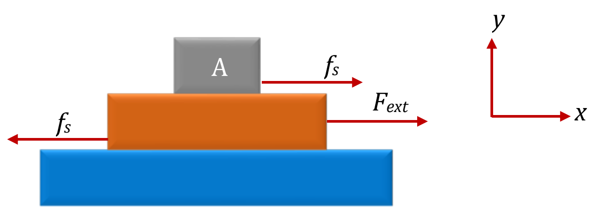
  $(f_s)_{max} = \mu_s N$

  where μ_s is the coefficient of static friction.

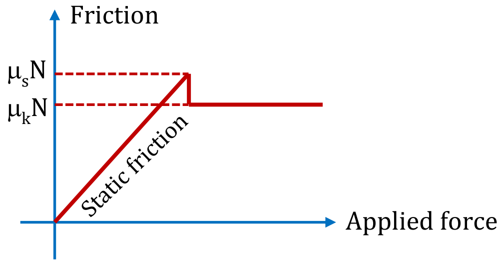


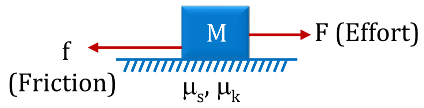
- Direction of static friction is always opposite to the tendency of relative motion.

  In the given example, blocks A and B move together in positive x-direction when external force is applied, tendency of motion of A with respect to B is in negative x-direction. Hence, the direction of static friction on block A will be in positive x - direction.



Graph of Friction v/s Applied Force





Few Important Points

(1) Value of μ_k is always less than μ_s ($\mu_k < \mu_s$) from experimental observation.

(2) If only coefficient of friction (μ) is given in a problem then $\mu_s = \mu_k = \mu$

(3) Value of μ_s and μ_k is independent of surface area it depends only on surface properties of contact surface.

(4) μ_k is independent of relative speed.

(5) μ_s and μ_k are properties of a given pair of surfaces i.e. for wood to wood combination μ_1 , then for wood to iron μ_2 and so on.

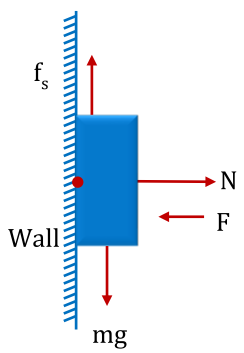
Frictional forces exist everywhere in nature and result in loss of energy that is primarily dissipated in form of heat. Wear and tear of moving bodies is another unwanted result of friction. Therefore, sometimes, we try to reduce their effects – such as in bearings of all types, between piston and the inner walls of the cylinder of an *IC* engine, flow of fluid in pipes, and aircraft and missile propulsion through air.

Though these examples create a negative picture of frictional forces, yet there are other situations where frictional forces become essential and we try to maximize the effects. It is the friction between our feet and the earth surface, which enables us to walk and run. Both the traction and braking of wheeled vehicles depend on friction.

Illustration 28:

A book of 1 *kg* is held against a wall by applying a perpendicular force *F*. If $\mu_S = 0.2$, then what is the minimum value of *F* ? (Take $g = 9.8\ m/s^2$)

Solution:



The situation is shown in fig. The forces acting on the book are–

For book to be at rest it is essential that, $Mg = f_s$

But $f_{s\,max} = \mu_S N$

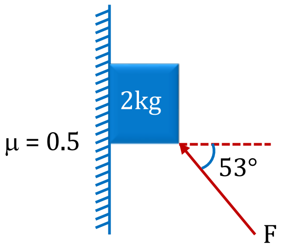
and $N = F$

$Mg = \mu_S F$

$$\Rightarrow \quad F = \frac{Mg}{\mu_s} = \frac{1\times 9.8}{0.2} = 49\ N$$

Illustration 29:

Find acceleration and friction force in the situation shown.



(i) $F = 20\ N$ (ii) $F = 100\ N$ (iii) $F = 10\ N$

Solution:

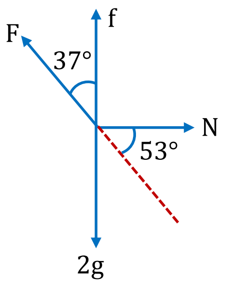
Assumption = No relative slipping

(i) $N = F \sin 37°$

$$N = 20 \times \frac{3}{5}$$

$N = 12\ N$

$\therefore\ f + F\cos 37° = 2g$



$$f + 20 \times \frac{4}{5} = 20$$

$f = 4N$

$(f_s)_{max.} = 0.5 \times 12 = 6\ N$

$\therefore\ f < (f_s)_{max.}$

Assumption is correct

$\therefore\ a = 0$

$f = 4N$

(ii) $N = F \sin 37°$

$$N = 100 \times \frac{3}{5}$$

$N = 60\ N$

$(f_s)_{max.} = 0.5 \times 60 = 30\ N$

$\therefore\ f + F\cos 37° = 20$

$$f + 100 \times \frac{4}{5} = 20$$

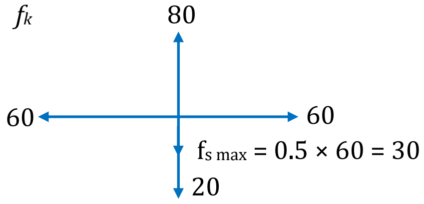
$f = -60\ N$

$\therefore\ f$ acts downward $= 60\ N$

$f > (f_s)_{max.}$

$\therefore$ Assumption is wrong

$\therefore\ f_k$



$f_k = \mu \times N = 0.5 \times 60 = 30\ N$

Since applied force is greater hence block will move in upward direction.

$\therefore$ $80 - 30 - 20 = 2a$

$80 - 50 = 2a$

$a = 15\ m/s^2$

(iii) $N = 10 \times \sin37° = 10 \times \frac{3}{5} = 6N$

$\therefore$ $(f_s)_{max.} = 0.5 \times 6 = 3\ N$

$f + F\cos 37° = 20$

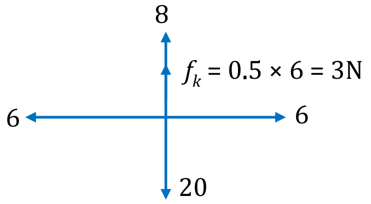
$f + 10 \times \frac{4}{5} = 20$

$f = 12\ N$

$f > (f_s)_{max.}$

$\therefore$ Assumption is wrong and hence block will move.

$\therefore$



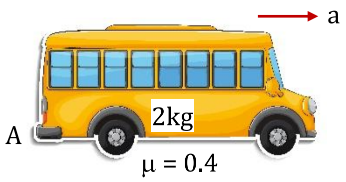
$\therefore$ $20 - 3 - 8 = 2a$

$9 = 2a$

$a = 4.5\ m/s^2$.

Illustration 30:

Find $a_{max.}$ such that block do not slip w.r.t. bus.



Solution:

$f - 2a = 0$

$f = 2a$

$\mu 2g = 2a$

$a = \mu g$

$a = 4\ m/s^2$

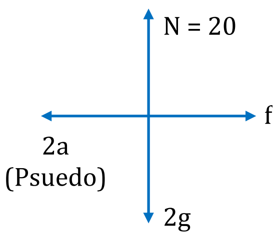
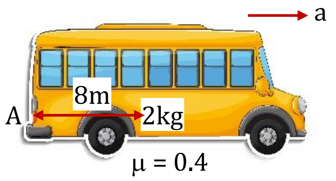


Illustration 31:

If $a = 5m/s^2$; find time after which block will reach to surface A ?



Solution:

$f_{req.} = 10\ N > (f_s)_{max.} = 8N$

$10 - 8 = 2a$

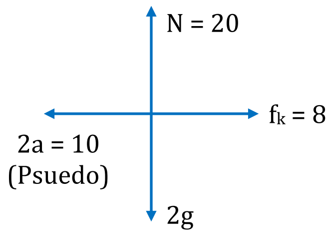
$a_{rel.} = 1\ m/s^2$

$$s = \frac{1}{2} \times a \times t^2$$

$$8 = \frac{1}{2} \times 1 \times t^2$$

$t^2 = 16$

$t = 4\ sec.$

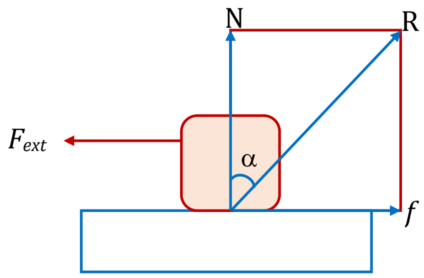


Angle of Friction (α)

It is the angle made by the resultant of the normal reaction force and friction force, with the normal reaction.

$$\tan\alpha = \frac{f}{N}$$

$$\therefore \qquad \tan\alpha_{max} = \frac{(f_s)_{max}}{N} = \frac{\mu_s N}{N} = \mu_s$$



Angle of Repose (θ)

It is the minimum angle (θ) made by an inclined plane with the horizontal such that an object placed on the inclined surface just begins to slide.

$$N = mg\cos\theta$$

$$(f_s)_{max} = \mu_s N = \mu_s mg\cos\theta$$

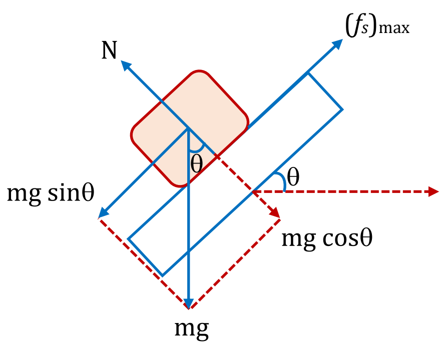
When the object just start to slide,

$$(f_s)_{max} = mg\sin\theta$$

$$\mu_s mg\cos\theta = mg\sin\theta$$

$$\tan\theta = \mu_s$$

$\therefore$ Angle of Repose, $\theta = \tan^{-1}(\mu_s)$



Two Block Problems

Method of solving

Step 1 : Make force diagram.

Step 2 : Show static friction force by f because value of friction is not known.

Step 3 : Calculate separately for two cases.

Case 1 : Move together

Step 4 : Calculate acceleration.

Step 5 : Check value of friction for above case.

Step 6 : If required friction is less than available it means they will move together else move separately.

Step 7(a) : Above acceleration will be common acceleration for both.

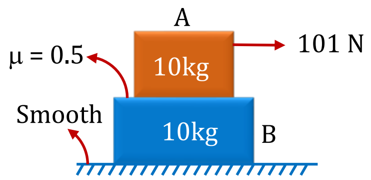
Case 2 : Move separately

Step 7(b) : If they move separately then kinetic friction is involved. Whose value is μN.

Step 8 : Calculate acceleration for above case.

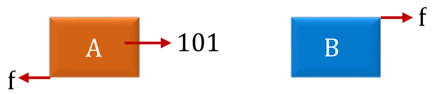
Illustration 32:

Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?



Solution:

$f_{max} = 50\ N$ $\therefore f \le 50$ N



(i) If they move together $a = \dfrac{101}{20} = 5.05\ m/s^2$

(ii) Check friction on B

$f = 10 \times 5.05 = 50.5$ (required)

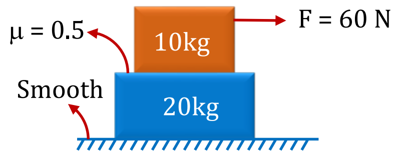
50.5 > 50 (therefore required friction > available friction). Hence they will not move together.

(iii) Hence, they move separately so kinetic friction is involved.

$\therefore$ For $a_A = \dfrac{101-50}{10} = 5.1\ m/s^2 \Rightarrow a_B = \dfrac{50}{10} = 5\ m/s^2$

Also, $a_A > a_B$ as force is applied on A.

Illustration 33:



Find the acceleration of the two blocks. The system is initially at rest and the friction coefficient are as shown in the figure?

Solution:

Move Together	**Move Separately**
$a = \dfrac{60}{30} = 2\ m/s^2$	No need to calculate.

Check friction on 20 *kg*.

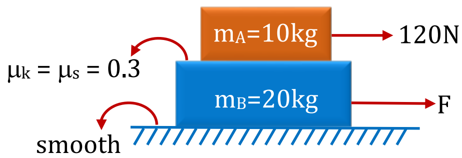
$f = 20 \times 2$

$f = 40$ (which is required)

$40 < 50$ (therefore required friction < available friction)

$\therefore$ Will move together.

Illustration 34:



In the figure given below force F applied horizontally on lower block, is gradually increased from zero. Discuss the direction and nature of friction force and the accelerations of the block for different values of F (Take $g = 10\ m/s^2$).

Solution:

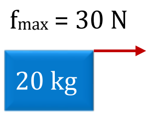
In the above situation we see that the maximum possible value of friction between the blocks is $\mu_s m_A g = 0.3 \times 10 \times 10 = 30\ N$.

Case (i) : When $F = 0$.

Considering that there is no slipping between the blocks the acceleration of system will be

$$a = \frac{120}{20+10} = 4\ m/s^2$$

But the maximum acceleration of B can be obtained by the following force diagram.

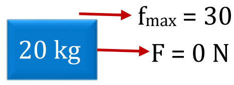


$$a_B = \frac{30}{20} = 1.5\ m/s^2$$

($\because$ only friction force by block A is responsible for producing acceleration in block B)

Because $4 > 1.5\ m/s^2$ we can conclude that the blocks do not move together.

Now drawing the F.B.D. of each block, for finding out individual accelerations.



$$a_A = \frac{120-30}{10} = 9\ m/s^2 \text{ towards right}$$

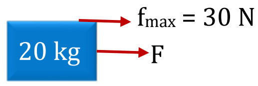
$$a_B = \frac{30}{20} = 1.5\ m/s^2 \text{ towards right.}$$

Case (ii) F is increased from zero till the two blocks just start moving together.

As the two blocks move together the friction is static in nature and its value is limiting. *FBD* in this case will be

$$a_A = \frac{120-30}{10} = 9\ m/s^2 \qquad \Rightarrow a_B = \frac{F+30}{20} = a_A \Rightarrow \frac{F+30}{20} = 9$$

$\therefore \quad F = 150\ N$

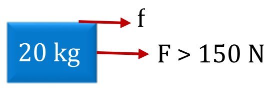


Hence when $0 < F < 150\ N$ the blocks do not move together, and the friction is kinetic. As F increases acceleration of block B increases from $1.5\ m/s^2$.

At $F = 150\ N$ limiting static friction start acting and the two blocks start moving together.

Case (iii) When F is increased above $150\ N$.

In this scenario the static friction adjusts itself so as to keep the blocks moving together. The value of static friction starts reducing but the direction still remains same. This happens continuously till the value of friction becomes zero. In this case the *FBD* is as follows:



$$a_A = a_B = \frac{120-f}{10} = \frac{F+f}{20}$$

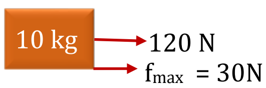
$\therefore$ When friction force f gets reduced to zero the above accelerations become

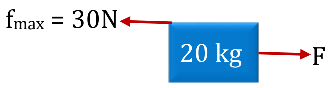
$$a_A = \frac{120}{10} = 12\ m/s^2 \qquad \Rightarrow a_B = \frac{F}{20} = a_A = 12\ m/s^2$$

$\therefore \quad F = 240\ N$

Hence when $150 \le F \le 240\ N$ the static friction force continuously decreases from maximum to zero at $F = 240\ N$. The accelerations of the blocks increase from $9\ m/s^2$ to $12\ m/s^2$ during the change of force F.

Case (iv) When F is increased again from $240\ N$ the direction of friction force on the block reverses but it is still static. F can be increased till this reversed static friction reaches its limiting value. *FBD* at this junction will be





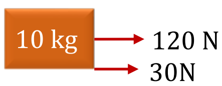
The blocks move together therefore.

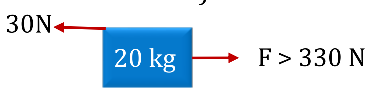
$$a_A = \frac{120+30}{10} = 15\ m/s^2 \quad \Rightarrow a_B = \frac{F-30}{20} = a_A = 15\ m/s^2$$

$$\therefore \quad \frac{F-30}{20} = 15\ m/s^2$$

Hence $F = 330\ N$.

Case (v) When F is increased beyond $330\ N$. In this case the limiting friction is achieved and slipping takes place between the blocks (kinetic friction is involved).





$\therefore \quad a_A = 15\ m/s^2$ which is constant

$$a_B = \frac{F-30}{20}\ m/s^2 \text{ where } F > 330\ N.s$$