Kinematics

SOLUTIONS

Exercise-I (JEE-Main Pattern)

SECTION–A

1. **Ans. (1)**

$$\text{Average speed} = \frac{\text{Distance}}{\text{Time}} = \frac{\frac{2\pi}{3}R}{t} \qquad \text{...(i)}$$

$$\vec{r}_i = R\hat{j}$$

$$\vec{r}_f = R\left(\frac{\sqrt{3}}{2}\hat{i} - \frac{1}{2}\hat{j}\right)$$

$\Rightarrow$ Displacement = $|\vec{r}_f - \vec{r}_i|$

$$= R\left|\frac{\sqrt{3}}{2}\hat{i} - \frac{3}{2}\hat{j}\right|$$

$$= R\sqrt{3}$$

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{Time}}$$

$$= \frac{\sqrt{3}R}{t} \qquad \text{...(ii)}$$

$$\text{Ratio} = \frac{\text{Average speed}}{\text{Average velocity}} = \frac{2\pi}{3\sqrt{3}}$$

2. **Ans. (2)**

$$\frac{d^2\vec{r}}{dt^2} = \vec{a}$$

and $\vec{r}.\vec{a} = 0$

$\Rightarrow$ $t = 1$ sec

3. **Ans. (1)**

$\vec{r} = (t^2 - 2t - 3)\,\hat{k}$

$$\frac{d\vec{r}}{dt} = 2t - 2$$

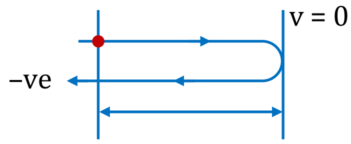
At $t = 1$ sec $|\vec{v}| = 0$; stand still

4. **Ans. (1)**

$u = 8$

$a = -4$

$t = 6$



$$s = 8 \times 6 + \frac{1}{2}(-4) \times (6)^2$$

$a = -4$

$u = 6 = -24$

Since displacement is it must have travelled in positive direction.

$v = 0$ at $t = 2$sec.

$s_1 = 8 \times 2 - \frac{1}{2} \times 4 \times 4 = 8$

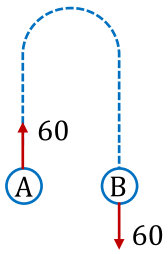
Hence distance is $24 + 8 + 8 = 40\ m$.

5. Ans. (3)

$v_{av} = \frac{v_1 + v_2}{2}$ for uniform acceleration/retardation is also velocity at mid time.

In first half time average velocity is higher, so the car travels more than half distance.

6. Ans. (3)



(A) will retrace (B) after coming down,

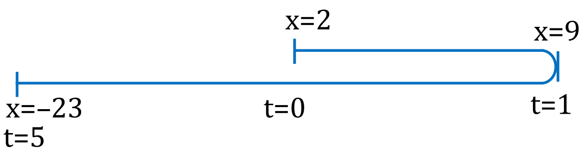
So difference of time is given by

$t = \frac{2u}{g} = \frac{2 \times 60}{10} = 12$ sec.

7. Ans. (2)

$v = 3t^2 - 18t + 15 = 3(t - 1)(t - 5)$

Motion diagram



8. Ans. (4)

For maximum height, $v_y = 0$

At $t = \frac{17}{3}\ s$, $v_y = 24\left(\frac{17}{3}\right) - 3\left(\frac{17}{3}\right) = \frac{119}{3}\ m/s$

After that, $a = -g = -10\ m/s^2$

And at $t = t_2$, $v_y = 0$

So $0 = \frac{119}{3} - 10\left(t_2 - \frac{17}{3}\right)$

Or $t_2 = \frac{289}{30} = 9.6$ sec.

9. Ans. (2)

Speeding up if v and a have same sign and speeding down if v and a have opposite sign.

10. **Ans. (3)**

$u_y = 40, a_y = -1$

$v_y = u_y + a_y t$

$\Rightarrow \quad 0 = 40 - t$

$\Rightarrow \quad t = 40\ s$

11. **Ans. (3)**

$$\frac{2u\sin\theta}{g} = 4.8$$

$2u \sin \theta = 48$

$u \sin \theta = 24$

$30 \sin \theta = 24$

$$\sin\theta = \frac{24}{30} = \frac{4}{5}$$

$\theta = 53°$

12. **Ans. (2)**

Range is maximum at 45°.

$\therefore$ Range increases from 30° to 45° and decreases from 45° to 60°.

Time of flight and maximum height depends on sinθ which increases with increase in θ.

$\therefore$ Both increases.

13. **Ans. (2)**

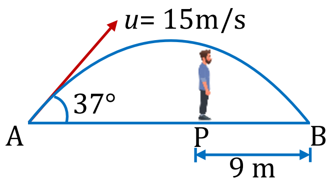
$H \propto T \propto u_y$

$\therefore$ Ball with maximum height have maximum time of flight.

14. **Ans. (2)**

$u_x = 15 \times \frac{4}{5} = 20\ m/s$

$u_y = 15 \times \frac{3}{5} = 15\ m/s$



$u_{mb} = u_m - u_b$

$20 = u_m - 15$

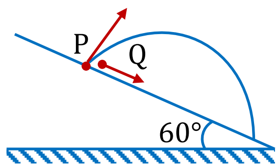
$u_m = 5\ m/s$

15. **Ans. (2)**

$t = 4s$

$$T = \frac{2u\sin\theta}{g\cos\alpha}$$

$$u = \frac{2u}{10\times\frac{1}{2}}$$



$$\frac{20}{2} = u$$

$u = 10\ m/s$

16. Ans. (4)

$\vec{v}_{m/T} = 1.5v\hat{\imath}$

$\vec{v}_{T/g} = v\hat{\imath}$

$x = L$

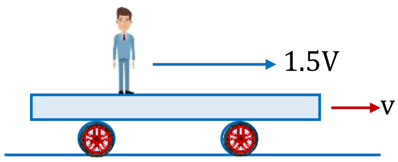
$u_{m/T} = -1.5v\hat{\imath}$

$L = ut + \frac{1}{2}at^2$

$t = \frac{2L}{1.5v}$

$D = vt$

$D = \frac{4L}{3}$



17. Ans. (3)

$\vec{v}_{L/g} = 2\hat{\jmath}$ $\qquad$ $\vec{v}_{b/g} = 2\hat{\jmath}$

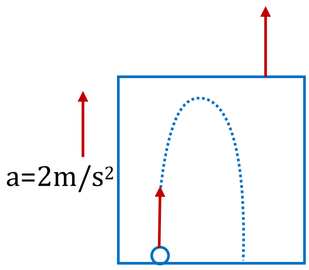
$\vec{a}_{L/g} = 2\hat{\jmath}$ $\qquad$ $\vec{a}_{b/g} = -10\hat{\jmath}$

$\vec{a}_{b/L} = -12\hat{\jmath}$

$\vec{v}_{b/c} = u$

$t = \frac{2u_{bL}}{a_y}$

$u_{bL} = 12\ m/s$



18. Ans. (3)

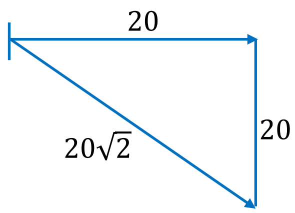
$\vec{v}_{F/g} = 20\hat{\jmath}$

$\vec{v}_{w/g} = 20\hat{\jmath}$

$\vec{v}_{F/w} = 20\hat{\jmath} - 20\hat{\imath}$

$\vec{v} = 20\sqrt{2}$

Direction of flag = south-east.



19. Ans. (2)

$\vec{v}_{w/g} = 2\hat{\imath}$

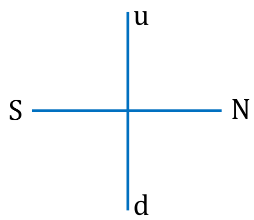
$\vec{v}_{r/w} = -v_0\hat{\jmath}$

$\vec{v}_r = 2\hat{\imath} - v_0\hat{\jmath}$

$\vec{v}_{m/a} = v\hat{\imath}$

$\vec{v}_{r/m} = (2 - v)\hat{\imath} - v_0\hat{\jmath}$

$v = 2\ m/s$



20. Ans. (2)

$\vec{v}_{r/a} = 5$

$t = 5$ sec

$D = 60\ m$

$\frac{60}{5} = u\cos\theta$

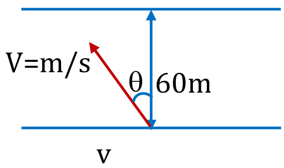
$u\cos\theta = 12$

Velocity in x is zero

$v_r - v\sin\theta = 0$

$v\sin\theta = 5$

$v = 13\ m/s$



SECTION–B

1. **Ans. (4)**

$v^2 = u^2 + 2\ as$

$v^2 = 40^2 + 2(2)\ (1000) = 5600$

$v'^2 = v^2 + 2gs$

$\Rightarrow \quad v'^2 = 5600 + 2 \times 10 \times 100$

$\Rightarrow \quad v' = 160 = 40\alpha$

$\Rightarrow \quad \alpha = 4$

2. **Ans. (5)**

$v_x = \frac{dx}{dt} = 3\ ,\ v_y = \frac{dy}{dt} = 4 - 12t$

At $t = 0, \vec{u} = u_x\hat{\imath} + u_y\hat{\jmath} = 3\hat{\imath} + 4\hat{\jmath}$

Speed of projection = $u = \sqrt{3^2 + 4^2} = 5\ m/s$

3. **Ans. (1200)**

Horizontal velocity remain constant.

$60 \cos 60° = v \cos 30°$

$v = 20\sqrt{3}$

$v^2 = 1200$

4. **Ans. (50)**

$H_{max} = u^2 \sin^2\theta/2g = u^2/2g$

$R_{max} = (u^2 \sin 2\theta/g)_{max} = u^2/g$

$2H_{max}$ = Range

$2H_{max} = 100$

$H_{max} = 50\ m$

5. **Ans. (20)**

$U = 100 \cos 30\hat{\imath} + 100 \sin 30°\hat{\jmath}$

$\vec{V} = 10 \cos 3\ 0\hat{\imath} + (100 \sin 3\ 0 - 10t)\hat{\jmath}$

$\vec{U}.\vec{V} = 0$

$\Rightarrow \quad \frac{100}{10\times\frac{1}{2}} = 20 \text{ sec}$

6. **Ans. (2)**

$t^2 = \frac{2h}{g} = 25$

$\Rightarrow \quad h = 125\ m$

$x = u_x \times t$

$h \cot 37° = u_x \times 5$

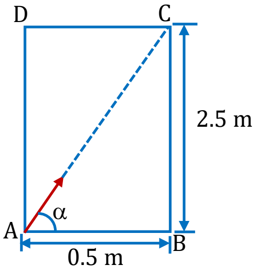
$u_x = 100/3$

7. **Ans. (6)**

$x = \sqrt{\frac{2h}{g}}\, v$

$\Rightarrow \quad v = \frac{x}{\sqrt{\frac{2h}{g}}} = \frac{6}{\sqrt{\frac{2\times5}{10}}} = 6\, m/s$

8. **Ans. (5)**



$\tan\alpha = \frac{AD}{AB}$

$\frac{2.5}{0.5}$

$\tan\alpha = 5$

9. **Ans. (6)**

$\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q$

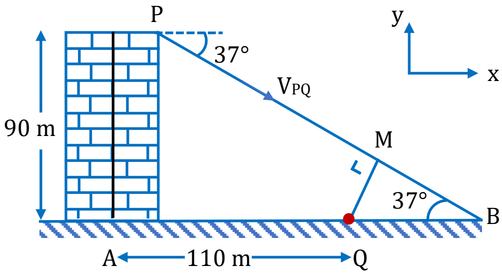
$= \left(\frac{10}{\sqrt{2}}\hat{\imath} + \frac{10}{\sqrt{2}}\hat{\jmath}\right) - \left(-\frac{70}{\sqrt{2}}\hat{\imath} + \frac{70}{\sqrt{2}}\hat{\jmath}\right)$

$= 40\sqrt{2}\hat{\imath} - 30\sqrt{2}\hat{\jmath}$

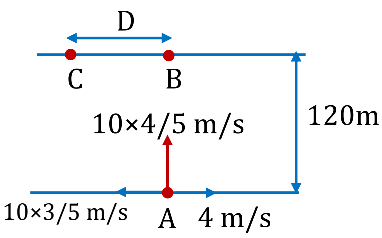
$AB = 90 \times \frac{4}{3} = 120m$

$\Rightarrow \quad QB = 10m$

$\Rightarrow \quad QM = 10\sin 37° = 10 \times \frac{3}{5} = 6m$



10. **Ans. (30)**



$t = \frac{120}{8} = 15$ sec

$\Delta = (6 - 4) \times 15 = 30m$

Exercise-II (JEE-Main PYQs)

1. **Ans. (2)**

$u = \hat{i} + 2\hat{j}$

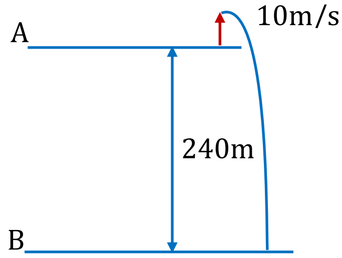
$u_x = 1 \; u_y = 2$

$\tan\theta = \frac{2}{1}$

$y = x\tan\theta - \frac{1}{2}g\frac{x^2}{u^2\cos^2\theta}$

$y = 2x - 5x^2$

2. **Ans. (1)**



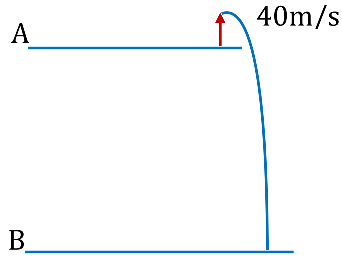
For particle 1

$-240 = +10t - \frac{1}{2}gt^2$

$5t^2 - 10t - 240 = 0$

$t_1 = 8$ sec

For particle 2



$-240 = 40t - \frac{1}{2}gt^2$

$5t^2 - 40t - 240 = 0$

$t_2 = 12$ sec

For $0 < t < 8$ sec $\rightarrow a_{rel} = 0$

Straight line x-t graph

For $8 < t < 12$ sec $\rightarrow a_{rel} = -g$

Downward parabola

For $t > 12$ sec $\rightarrow$ Both particles comes to rest.

3. **Ans. (2)**

$v = b\sqrt{x}$

$\frac{dv}{dt} = \frac{b}{2\sqrt{x}}\frac{dx}{dt}$

$a = \frac{bv}{2\sqrt{x}}$

$a = \frac{b\left(b\sqrt{x}\right)}{2\sqrt{x}}$

$\frac{dv}{dt} = a = \frac{b^2}{2}$

$v = \frac{b^2}{2}\tau$

4. **Ans. (3)**

$v_x = -a\omega \sin \omega t \quad \Rightarrow \quad v_y = a\omega \cos \omega t$

$v_z = a\omega \quad \Rightarrow \quad v = \sqrt{v_x^2 + v_y^2 + v_z^2}$

$v = \sqrt{2}a\omega$

5. **Ans. (4)**

$R = \dfrac{u^2 \sin 2\theta}{g}$

$A = \pi R^2$

$A \propto R^2$

$A \propto u^4$

$\dfrac{A_1}{A_2} = \dfrac{u_1^4}{u_2^4} = \left[\dfrac{1}{2}\right]^4 = \dfrac{1}{16}$

6. **Ans. (1)**

$\vec{S} = (5\hat{\imath} + 4\hat{\jmath})2 + \frac{1}{2}(4\hat{\imath} + 4\hat{\jmath})4$

$= 10\hat{\imath} + 8\hat{\jmath} + 8\hat{\imath} + 8\hat{\jmath}$

$\vec{r}_f - \vec{r}_i = 18\hat{\imath} + 16\hat{\jmath}$

$\vec{r}_f = 20\hat{\imath} + 20\hat{\jmath}$

$|\vec{r}_f| = 20\sqrt{2}$

7. **Ans. (2)**

$t = \dfrac{2 \times 2 \times \sin 15^\circ}{g \cos 30^\circ}$

$S = 2 \cos 15^\circ \times t - \dfrac{1}{2} g \sin 30^\circ t^2$

Put values and solve

$S \simeq 20cm$

8. **Ans. (1)**

Equation of trajectory is given as

$y = 2x - 9x^2$ ...(1)

Comparing with equation :

$y = x \tan \theta - \dfrac{g}{2u^2 \cos^2 \theta} . x^2$ ...(2)

We get;

$\tan \theta = 2$

$\therefore \quad \boxed{\cos \theta = \dfrac{1}{\sqrt{5}}}$

Also, $\dfrac{g}{2u^2 \cos^2 \theta} = 9$

$\Rightarrow \quad \dfrac{10}{2 \times 9 \times \left(\frac{1}{\sqrt{5}}\right)^2} = u^2$

$\Rightarrow \quad u^2 = \dfrac{25}{9}$

$\Rightarrow \quad \boxed{u = \frac{5}{3} m/s}$

9. Ans. (4)

$$t_2 = \frac{120+60}{110\times\frac{5}{18}}$$

$$t_1 = \frac{120+60}{50\times\frac{5}{18}}$$

$$\frac{t_1}{t_2} = \frac{110}{50} = \frac{11}{5}$$ **Ans.**

10. Ans. (4)

$\vec{r} = 15t^2\hat{\imath} + (4 - 20t^2)\hat{\jmath}$

$\vec{v} = \frac{d\vec{r}}{dt} = 30t\hat{\imath} + (-40t)\hat{\jmath}$

$\vec{a} = \frac{d\vec{v}}{dt} = 30\hat{\imath} - 40\hat{\jmath}$

$|\vec{a}| = 50m/s^2$

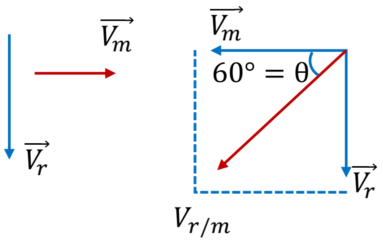
11. Ans. (4)

Rain is falling vertically downwards.

$\vec{v}_{r/m} = \vec{v}_r - \vec{v}_m$

$\tan 60° = \frac{v_r}{v_m} = \sqrt{3}$

$v_r = v_m\sqrt{3} = v\sqrt{3}$



Now, $v_m = (1 + B)v$

and $\theta = 45°$

$\tan 45 = \frac{v_r}{v_m} = 1$

$v_r = v_m$

$v\sqrt{3} = (1 + \beta)v$

$\sqrt{3} = 1 + \beta$

$\Rightarrow \quad \beta = \sqrt{3} - 1 = 0.73$

12. Ans. (3)

$x = u_x t + \frac{1}{2}a_x t^2$

$y = u_y t + \frac{1}{2}a_y t^2$

$32 = 0 \times t + \frac{1}{2}(4)(t)^2$

$t^2 = 16$

$t = 4$ sec

$x = 3 \times 4 + \frac{1}{2} \times 6 \times 4^2$

$= 12 + 48 = 60\ m$

$\therefore$ Correct answer (3).

13. Ans. (3)

Given $\vec{u} = 5\hat{j}\,m/s, \vec{a} = 10\hat{i} + 4\hat{j}$, final coordinate $(20, y_0)$ in time t

$$S_x = u_x t + \frac{1}{2} a_x t^2$$

$$20 - 0 = 0 + \frac{1}{2} \times 10 \times t^2$$

$$t = 2 \text{ sec}$$

$$S_y = u_y \times t + \frac{1}{2} a_y t^2$$

$$y_0 = 5 \times 2 + \frac{1}{2} \times 4 \times 2^2 = 18m$$

2 sec and 18 m.

14. Ans. (1)

For $0 \le x \le 200$

$$v = mx + C$$

$$v = \frac{1}{5} x + 10$$

$$a = \frac{vdv}{dx} = \left(\frac{x}{5} + 10\right)\left(\frac{1}{5}\right)$$

$$a = \frac{x}{25} + 2 \Rightarrow \text{Straight line till } x = 200$$

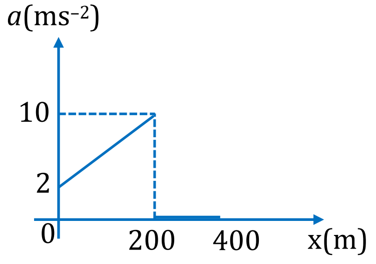
For $x > 200$

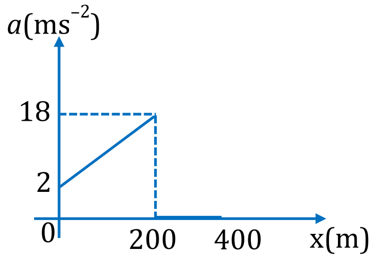
$$v = \text{constant}$$

$\Rightarrow$ $a = 0$

So, final a-x graph will be as follows-

$\Rightarrow$





Hence most appropriate option will be (1).

Otherwise it would be BONUS.

15. **Ans. (2)**

Option (2) represent correct graph for particle moving with constant acceleration, as for constant acceleration velocity time graph is straight line with positive slope and x-t graph should be an opening upward parabola.

16. **Ans. (3)**

In 4 sec. 1st drop will travel

$\Rightarrow \quad \frac{1}{2} \times (9.8) \times (4)^2 = 78.4\ m$

$\therefore$ 2nd drop would have travelled

$\Rightarrow \quad 78.4 - 34.3 = 44.1\ m$

Time for 2nd drop

$$\frac{1}{2}(9.8)t^2 = 44.1$$

$$t = 3 \text{ sec}$$

$\therefore$ Each drop have time gap of 1 sec

$\therefore$ 1 drop per sec

17. **Ans. (3)**

$u = \sqrt{2gh}$

Now, $S = \frac{h}{3}$, $a = -g$

$$S = ut + \frac{1}{2}at^2$$

$$\frac{h}{3} = \sqrt{2gh}t + \frac{1}{2}(-g)t^2$$

$$t^2\left(\frac{g}{2}\right) - \sqrt{2gh}t + \frac{h}{3} = 0$$

From quadratic equation

$$t_1, t_2 = \frac{\sqrt{2gh} \pm \sqrt{2gh - \frac{4g}{2}\frac{h}{3}}}{g}$$

$$\frac{t_1}{t_2} = \frac{\sqrt{2gh} - \sqrt{\frac{4gh}{3}}}{\sqrt{2gh} + \sqrt{\frac{4gh}{3}}} = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$$

18. **Ans. (4)**

$\vec{V}_{BW} = 4\sqrt{2}\cos 45\hat{\imath} + 4\sqrt{2}\sin 45\hat{\jmath}$

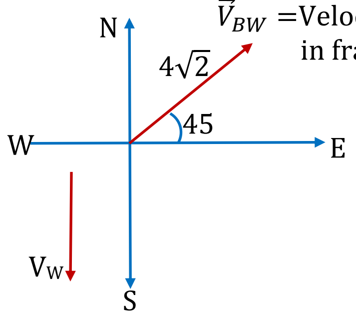
$\quad = 4\hat{\imath} + 4\hat{\jmath}$

$\vec{V}_W = -\hat{\jmath}$

$\vec{V}_B = \vec{V}_{BW} + \vec{V}_W = 4\hat{\imath} + 3\hat{\jmath}$

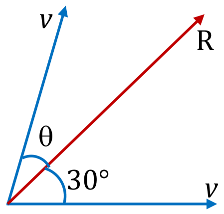
$\vec{S}_B = \vec{V}_B \times t = (4\hat{\imath} + 3\hat{\jmath}) \times 3 = 12\hat{\imath} + 9\hat{\jmath}$

$|\vec{S}_B| = \sqrt{(12)^2 + (9)^2} = 15m$



$\vec{V}_{BW}$ = Velocity of butterfly in frame of wind

19. Ans. (30)



Both velocity vectors are of same magnitude therefore resultant would pass exactly midway through them

$\theta = 30°$

20. Ans. (4)

$$R = \frac{V^2(2\sin\theta\cos\theta)}{g}$$

$$t = \frac{V\sin\theta}{g} \Rightarrow V = \frac{gt}{\sin\theta}$$

$$\Rightarrow \quad R = \frac{g^2t^2}{\sin^2\theta}\cdot\frac{2\sin\theta\cos\theta}{g}$$

$$\tan\theta = \frac{2gt^2}{R} = \frac{20t^2}{R}$$

$$\cot\theta = \frac{R}{20t^2}$$

21. Ans. (4)

$v = u + at$

$60 = 10 + 2t$

$t = 25$ sec.

Correct option (4)

22. Ans. (2)

$$R = \frac{u^2}{g}\sin 2\theta$$

R is maximum for $2\theta = 90°$.

23. Ans. (4)

$$R_1 = \frac{u_1^2\sin 2\theta_1}{g}; R_2 = \frac{u_2^2\sin 2\theta_2}{g}$$

$$\frac{R_1}{R_2} = \frac{u_1^2}{u_2^2}\frac{\sin 2\theta_1}{\sin 2\theta_2} = \frac{40^2\sin(2\times 30°)}{60^2\sin(2\times 60°)} = \frac{4}{9}$$

Exercise-III (JEE Advanced Pattern)

SECTION-I

1. Ans. (D)

Displacement along x-axis = 0

Displacement along y-axis = h

$\Rightarrow \quad v \sin\theta\, t - \frac{1}{2}at^2 = 0$

$\Rightarrow \quad t = \frac{2v\sin\theta}{a}$

and $\quad h = v\cos\theta t + \frac{1}{2}gt^2$

$= v\cos\theta\left(\frac{2v\sin\theta}{a}\right) + \frac{1}{2}g\left(\frac{2v\sin\theta}{a}\right)^2$

$= \frac{2v^2}{a}\sin\theta\left(\cos\theta + \frac{g}{a}\sin\theta\right)$

2. Ans. (C)

Displacement $t_y = -\ell\sin 53°$

$v_\perp = v_0 \sin 37°$

$-\ell\sin 53° = v_0\sin(37°)\,t - \frac{1}{2}g.t^2$

$\therefore \quad t = 2$ sec

3. Ans. (A)

$\vec{v}_{r/g} = 5\ m/s$

$u\sin 45° = 5$

$u = 5\sqrt{2}$

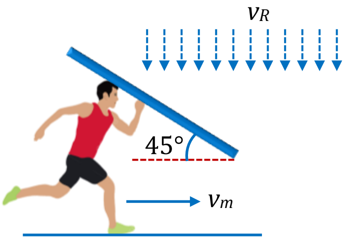
$v_{m/r} = 5\sqrt{2}$

$(v_{m/r})^2 - (v_{r/g})^2 = (v_{m/g})^2$

$50 - 25 = (v_{m/g})^2$

$\sqrt{25} = v_{m/g}$

$v_{m/g} = 5\ m/s$



4. Ans. (A)

Time to cross road

$t = \frac{2}{u\sin\theta}$ ...(1)

$8t = 4 + v\cos\theta t$

$8\left(\frac{2}{v\sin\theta}\right) = 4 + \frac{v\cos\theta \times 2}{v\sin\theta}$

$v = \frac{8}{2\sin\theta + \cos\theta}$

$\frac{dv}{d\theta} = 0 \quad \Rightarrow \quad \tan\theta = 2$

SECTION-II

5. Ans. (AD)

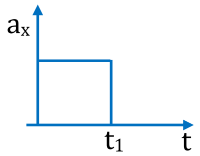
$(x, y) = (8t^2, 3)$ for $t \le t_1$

$= (8t\,t_1, 3)\ t > t_1$

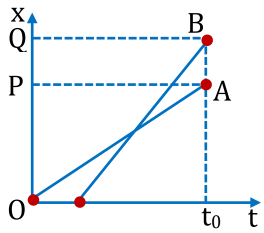
$x = 8t^2 \qquad y = 3 \qquad t \le t_1$

$x = 8t\,t_1 \qquad y = 3 \qquad t > t_1$

(A) Particle moves along a straight line parallel to x-axis.

(D) 

6. **Ans. (ABD)**



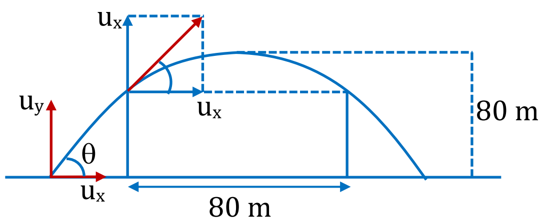
(A) A lives closer to school than B

(B) A starts from school earlier than B

(D) B overtakes A on way.

7. **Ans. (ABCD)**

Time between towers = $\frac{80}{u_x} = \frac{2u_x}{10}$



$u_x = 20$

$\therefore$ $t = 4s$

$\frac{u_y^2}{2g} = 80$

$\Rightarrow$ $u_y = 40\,; T_f = 8s$

$R = 160\ m$

$\theta = \tan^{-1}\left(\frac{40}{20}\right)$

8. **Ans. (BCD)**

Here total acceleration $a = \sqrt{g^2 + a_x^2}$ = constant so path may be parabolic or straight line.

9. **Ans. (BCD)**

$\vec{v}_{b/g} = 2\ m/s$

$\vec{v}_{\text{belt}/g} = -4\ m/s$

$u_y = at$

$\frac{6}{4} = a$

$s = 24 - \frac{1}{2} \times \frac{3}{2} \times 16$

$= 24 - 12 = 12\ m$

$s = ut + \frac{1}{2}at^2 = 2 \times 4 - \frac{1}{2} \times \frac{1}{2} \times 16$

$= 8 - 4 = 4m$

$v_y = u_y + at$

$4a = 2$

$a = \frac{1}{2}$

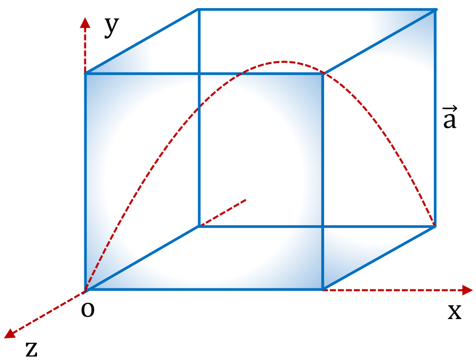
$s = 2 \times \frac{8}{3} - \frac{1}{2} \times \frac{6}{8} \times \frac{64}{9}$

$s = \frac{16-4}{3} = \frac{12}{3}$

(B) Magnitude of displacement w.r.t. ground in $4s$ is 4 m.

(C) Magnitude of displacement w.r.t belt is 12 m in 4 s.

10. Ans. (ABC)



$H = \frac{u_y^2}{2g} \quad \Rightarrow \quad u_y = 5m/s$

$T = \frac{24\sin\theta}{g} = \frac{2u_y}{g} = 1 \text{ sec}$

$L = v_1 t - \frac{1}{2} a t^2$ (as $\vec{a}$ is in x direction)

$\frac{5}{4} = v_1 \times 1 - \frac{1}{2} \times (0.5) \times 1^2, v_1 = 3/2\ m/\text{sec}$

Along z-axis,

$\frac{5}{2} = v_2 \times 1$

$\Rightarrow \quad v_2 = \frac{5}{4}\ m/\text{sec}$

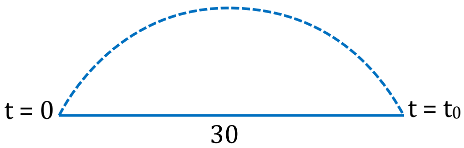
SECTION-III

11. Ans. (C)

12. Ans. (C)

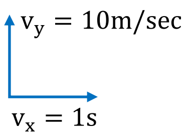
13. Ans. (B)

(Solution for Q.No. 11 to 13)



$15 \times t_0 = 30 \Rightarrow t_0 = 2$ sec

and



$\Rightarrow \quad \tan\theta = \frac{2}{3}$

$t_0 = \frac{2v_y}{g} \quad \Rightarrow \quad v_y = 10\ m/\text{sec}$

$v_x = 15$ and $H_{max} = 75 + \frac{(10)^2}{2g} = 80$

$$T = 2\sqrt{\frac{2H_{max}}{g}} = 2\sqrt{\frac{80 \times 2}{10}} = 8\,sec$$

$AB = v_x \times T = 8 \times 15 = 120\ m$

$v_{xrel} = (v_x)_{stone} - v_{car} = 0$

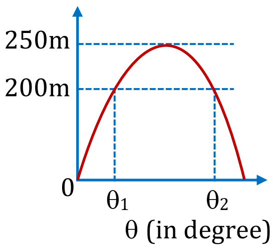
$\Rightarrow$ Car is just above the stone

$\Rightarrow$ Height = 75 m

14. Ans. (B)

Range = 200

$$\frac{u^2 \sin 2\theta}{g} = 200$$



$$\sin 2\theta = \frac{200}{250}$$

$h = 250$

$$\frac{u^2}{g} = 250$$

$2\theta = 53°$

$\theta_2 = 90 - \theta = 90 - 26.5$

$\theta_2 = 63.5$

15. Ans. (D)

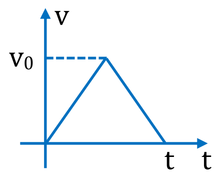
$$T = \frac{2u\sin\theta}{g}$$

$T \propto \sin\theta$

SECTION-IV

16. Ans. (400)

$u = 0$, $\alpha = 10\ cm/s^2$, $\beta = 20\ cm/s^2$



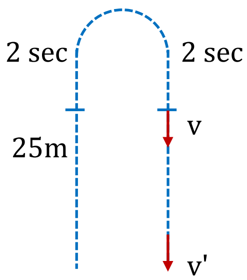
$$5\ km = \frac{1}{2} \times t \times v_m$$

$t = \alpha v_0 + \beta v_0$

17. Ans. (30)

$V = 10(2)$

$= 20\ m/s$



$v'^2 = 20^2 + 2 \times 10 \times 25$

$= 400 + 500$

$= 900$

$v' = 30\ m/s$ **Ans.**

SECTION-V

18. **Ans. (A-P,R), (B-Q), (C-S)**

Considering y-axis along the inclined plane

Time of flight = $\frac{2u \sin 37°}{g \sin 37°} = 2s$

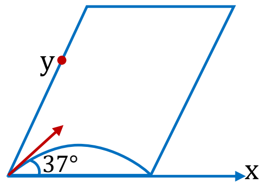
Speed is minimum at $t = 1s$

Velocity makes an angle 37° at $t = 2$

$y = 6t - \frac{1}{2} g \sin 37° \, t^2$

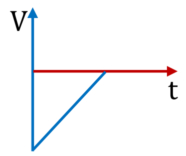
Substituting $y = 2.25$

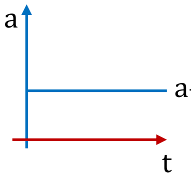
We get $t = 0.5$ and $1.5s$

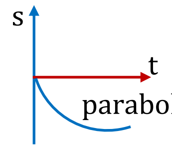


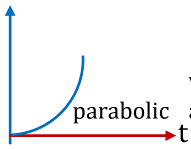
SECTION-VI

19. **Ans. (A-R,T), (B-T), (C-R,T), (D-S,T)**

(A)


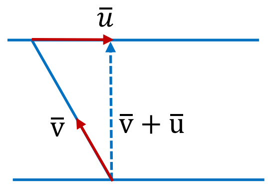
(B)


(C)


(D)


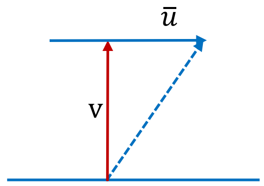
20. **Ans. (A-P,Q), (B-R), (C-S), (D-P,R)**

(A) $t = \frac{\ell}{|\bar{v}+\bar{u}|} = \frac{\ell}{\sqrt{v^2-u^2}}$



(B) $t = \frac{\ell_1+\ell_2}{|\bar{v}-(-\bar{u})|} = \frac{\ell}{|\bar{v}|+|\bar{u}|}$

(C) $t = \frac{\ell}{|\bar{v}|}$

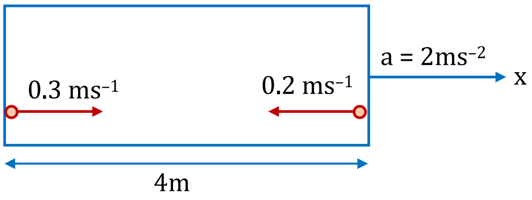


(D) $t = \frac{\ell}{|\bar{v}+\bar{u}|} = \frac{\ell}{|\bar{v}|+|\bar{u}|}$

Exercise-IV (JEE Advanced PYQs)

1. **Ans. (8 or 2)**

Assuming open chamber



$V_{\text{relative}} = 0.5\ m/s$

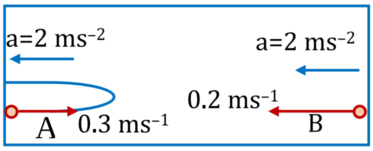
$S_{\text{relative}} = 4m$

Time $= \dfrac{4}{0.5} = 8\ m/s$

Alternate:

Assuming closed chamber

In the frame of chamber :



Maximum displacement of ball A from its left end is $\dfrac{u_A^2}{2a} = \dfrac{(0.3)^2}{2(2)} = 0.0225\ m$

This is negligible with respect to the length of chamber i.e. $4m$. So, the collision will be very close to the left end.

Hence, time taken by ball B to reach left end will be given by

$$S = u_B t + \frac{1}{2}at^2$$

$$4 = (0.2)\,(t) + \frac{1}{2}(2)\,(t)^2$$

Solving this, we get

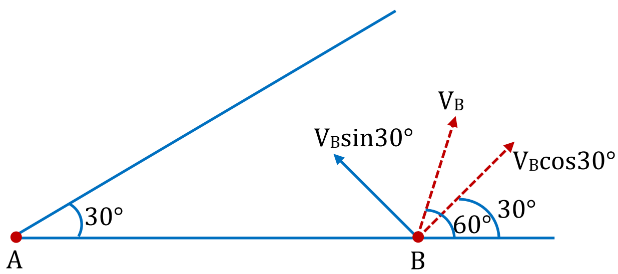
$$t \approx 2\ s$$

2. **Ans. (5)**

As observed from A, B moves perpendicular to line of motion of A. It means velocity of B along A is equal to velocity of A

$$V_B \cos 30 = 100\sqrt{3}$$

$$V_B = 200$$



If A is observer A remains stationary therefore

$$t = \frac{500}{V_B \sin 30} = \frac{500}{100} = 5$$

3. **Ans. (C)**

Density of sphere is $\rho = \frac{m}{v} = \frac{3m}{4\pi R^3}$

Since, $\frac{1}{\rho}\frac{d\rho}{dt}$ = constant

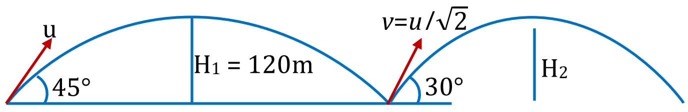
$\Rightarrow \quad \frac{4\pi R^3}{3m} \times \frac{3m}{4\pi}\left(\frac{-3}{R^4}\right)\frac{dR}{dt}$ = constant (Let K)

$\Rightarrow \quad \frac{-3}{R}\frac{dR}{dt} = K \quad \Rightarrow \quad \frac{dR}{dt} = \left(\frac{-K}{3}\right)R$

$\therefore \quad \frac{dR}{dt} \propto R$

Velocity v of any point on the surface of sphere is equal to $\frac{dR}{dt}$ (rate of change of radius of outer layer).

4. **Ans. (30 [29.60, 30.40])**



$H_1 = \frac{u^2 \sin^2 45}{2g} = 120$

$\Rightarrow \quad \frac{u^2}{4g} = 120$ ...(i)

When half of kinetic energy is lost $v = \frac{u}{\sqrt{2}}$

$H_2 = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30}{2g} = \frac{u^2}{16g}$ ...(ii)

From (i) and (ii)

$H_2 = \frac{H_1}{4}$ = 30 m on 30.00

5. **Ans. (4)**

Average velocity $= \frac{\text{Total displacement}}{\text{Total time}}$
<v>

Total time taken = $t_1 + t_2 + t_3 + ...$

$= t_1 + \frac{t_1}{\alpha} + \frac{t_1}{\alpha^2} + ...$

Total time = $\frac{t_1}{1-\frac{1}{\alpha}}$

Total displacement = $v_1t_1 + v_2t_2 + ...$

$= v_1t_1 + \frac{v_1}{\alpha}.\frac{t_1}{\alpha} + ...$

$= \frac{v_1t_1}{1-\frac{1}{\alpha^2}} = \frac{v_1t_1}{\left(1+\frac{1}{\alpha}\right)\left(1-\frac{1}{\alpha}\right)}$

On solving

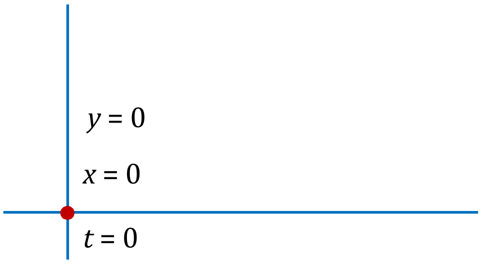
$\langle v \rangle = \frac{v_1\alpha}{\alpha+1} = 0.8v_1$

$\boxed{\alpha = 4.00}$

6. Ans. (ABCD)

$y = \frac{x^2}{2}$

At $t = 0$, $\left.\begin{array}{l} x = 0, y = 0 \\ u = 1 \end{array}\right\}$ given



$y = \frac{x^2}{2}$

$\frac{dy}{dt} = \frac{1}{2} . 2x \frac{dx}{dt} \quad \Rightarrow \quad v_y = xv_x$

Differentiate w.r.t. time

$a_y = \frac{dx}{dt} . V_x + xa_x$

$a_y = v_x^2 + xa_x$

Option:

(A) If $a_x = 1$ and particle is at origin

$(x = 0, y = 0)$

$a_y = v_x^2$

$a_y = 1^2 = 1$

At origin, at $t = 0$ sec

Speed = 1 (given)

(B) Option

$a_y = v_x^2 + xa_x$

Given in option B, $a_x = 0$

$\Rightarrow \quad a_y = v_x^2$

If $\quad a_x = 0$, v_x = constant = 1, (all the time)

$\Rightarrow \quad a_y = 1^2 = 1$ (all the time)

(C) At $\quad t = 0, x = 0, v_y = xv_x$

Speed = 1

$v_y = 0$

$v_x = 1$

(D) $a_y = v_x^2 + xa_x$

$v_y = xv_x$

$a_x = 0$ (given in D option)

$\Rightarrow \quad a_y = v_x^2$

If $\quad a_x = 0 \quad \Rightarrow V_x$ = constant initially $(v_x = 1)$

$\Rightarrow \quad a_y = 1^2 = 1$

At $\quad t = 1$ sec

$v_y = 0 + a_y \times t = 1 \times 1 = 1$

$\tan\theta = \frac{v_y}{v_x} = x$

($\theta \rightarrow$ angle with x axis)

$\tan\theta = \frac{v_y}{v_x} = \frac{1}{1} = 1$

$\theta = 45°$

7. Ans. (A)

$\vec{v} = \alpha(y\hat{x} + 2x\hat{y})$

$v_x = \alpha y \qquad\qquad v_y = 2\alpha x$

$\frac{dv_x}{dt} = \alpha\frac{dy}{dt} = 2\alpha^2 x \qquad\qquad \frac{dv_y}{dt} = 2\alpha v_x = 2\alpha^2 y$

$\therefore \quad \vec{F} = m\vec{a} = 2m\alpha^2(x\hat{x} + y\hat{y})$