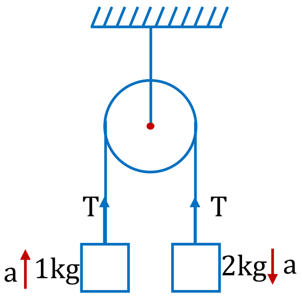
Heat and Thermodynamics

SOLUTIONS

EXERCISE - O

1. **Ans. (B)**



Here $T - g = a$ ...(1)

$2g - T = 2a$ ...(2)

$\Rightarrow 2g - T = 2(T - g)$

$\Rightarrow \boxed{T = \frac{4g}{3}}$

For minimum radius r :- $\frac{T}{\pi r^2}$ = Stress $\Rightarrow r = \sqrt{\frac{T}{\pi \times stress}}$

$\Rightarrow r = \sqrt{\frac{4g}{3\pi \times stress}} = \sqrt{\frac{40 \times 3\pi}{3\pi \times 40 \times 10^6}} = \frac{1}{1000} m = 1mm$

2. **Ans. (B)**

The strain developed in A is comparatively less as compared to B.

3. **Ans. (A)**

The area enclosed between deformation and restoration curve gives us the mechanical energy dissipated per unit volume of the given material.

4. **Ans. (D)**

$V = \pi r^2 \ell$

$$\frac{\Delta V}{V} \times 100 = 2\frac{\Delta r}{r} \times 100 + \frac{\Delta \ell}{\ell} \times 100$$

Stress $= Y\frac{\Delta \ell}{\ell}$

5. **Ans. (B)**

$T_S = 3mg$

$$\frac{3mg}{A_S} = y_s \times \frac{\Delta \ell_S}{\ell_S}$$

$$\frac{2mg}{A_b} = y_b \times \frac{\Delta \ell_b}{\ell_b}$$

$$\frac{\Delta \ell_S}{\Delta \ell_b} = \frac{3}{2} \times \frac{\ell_S}{\ell_b} \times \frac{A_b}{A_S} \times \frac{y_b}{y_S} = \frac{3}{2} \times \frac{a}{b^2 c}$$

6. **Ans. (D)**

Elongation in wire $\delta = \frac{F\ell}{AY}$

$$\delta = \frac{250 \times 100}{6.25 \times 10^{-4} \times 10^{10}}$$

$\delta = 4 \times 10^{-3}$ m

7. **Ans. (A)**

Elongation in first part with area $A = \dfrac{FL}{2AY}$

Elongation in second part with area $2A = \dfrac{FL}{4AY}$

Thus, total elongation $= \dfrac{3}{4}\dfrac{FL}{AY}$

8. **Ans. (C)**

$F_{\text{net}} = ma$

$2 = ma$

$a = \dfrac{m}{2}$

$5 - T = \left(\dfrac{M}{\ell}x\right) \times \dfrac{2}{M}$

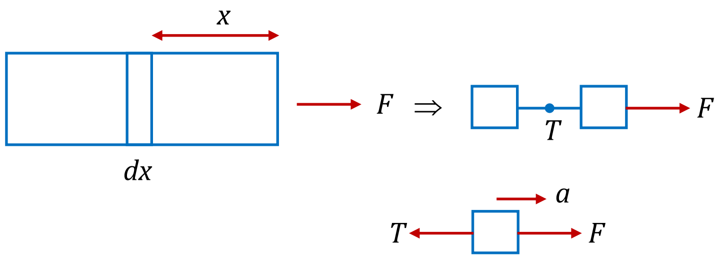
$T = 5 - \dfrac{2x}{\ell}$

$dy = \dfrac{F\ell}{AY}$

$\int_0^{\Delta\ell} dy = \int_0^{\ell}\left(5 - \dfrac{2x}{\ell}\right)\dfrac{dx}{AY}$

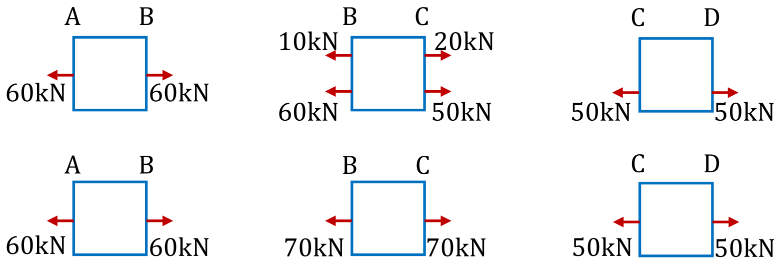
$\Delta\ell = \left(5L - \dfrac{\ell^2}{\ell}\right) \times \dfrac{1}{AY}$

$\boxed{\Delta\ell = \dfrac{4\ell}{AY}}$



9. **Ans. (A)**

The action of forces on each part of rod is shown in figure



We know that the extension due to external force F is given by

$$e = \frac{F\ell}{AY}$$

$$\therefore \quad e_{AB} = \frac{(60\times10^3)\times1.5}{1\times2\times10^{11}} = 4.5\times10^{-7}\,m$$

$$\therefore \quad e_{BC} = \frac{(70\times10^3)\times1}{1\times2\times10^{11}} = 3.5\times10^{-7}\,m \text{ and } e_{CD} = \frac{(50\times10^3)\times2}{1\times2\times10^{11}} = 5.0\times10^{-7}\,m$$

The total extension $e = e_{AB} + e_{BC} + e_{CD}$

10. **Ans. (A)**

$\sigma \propto T \propto (L^2 - x^2)$

11. **Ans. (B)**

$V = \dfrac{4}{3}\pi R^3$

$\dfrac{\Delta V}{V} = 3\dfrac{\Delta R}{R}$

$B = K = \dfrac{P}{\Delta V/V} \quad \Rightarrow \quad \dfrac{\Delta V}{V} = \dfrac{mg}{AK} = \dfrac{3\Delta R}{R}$

12. Ans. (C)

Modulus of rigidity (η) = $\dfrac{(F/bc)}{\phi}$

$\Rightarrow \ \phi = \dfrac{F}{bc\eta}$

13. Ans. (D)

Heat produced = $W_F - E.P.E.$

$= F\ell - \dfrac{F\ell}{2} = \dfrac{F\ell}{2}$

14. Ans. (A)

$Y = 7 \times 10^{10}\ N/m^2$

Strain $= \dfrac{0.04}{100}$

Energy $= \dfrac{1}{2}\left(\dfrac{YA}{l}\right)\Delta x^2$

Energy $= \dfrac{1}{2}YA\left(\dfrac{\Delta x}{l}\right)^2 \times l$

$\dfrac{E}{V} = \dfrac{1}{2} \times Y \times strain^2$

$= \dfrac{1}{2} \times 7 \times 10^{10} \times \dfrac{0.04 \times 0.04}{10^4} = 56 \times 10^2$

15. Ans. (D)

$\mu = -\dfrac{\Delta r/r}{\Delta\ell/\ell} \quad \Rightarrow \quad \dfrac{\Delta r}{r} = -\left(\dfrac{\pi}{10}\right) \times \dfrac{\Delta\ell}{\ell}$

$\dfrac{\Delta\ell}{\ell} = \dfrac{F}{AY}$

$\Delta r = -5 \times 10^{-5}\ mm$

$r_f - r_i = -5 \times 10^{-5}$

$r_f = r_i - 0.00005$

$= 1 - 0.00005$

$= 0.99995\ mm$

16. Ans. (A)

Let the original length be 'ℓ_0'

When $T_1 = 100\ N$, Extension = $\ell_1 - \ell_0$

When $T_2 = 120\ N$, Extension = $\ell_2 - \ell_0$

Then $100 = K(\ell_1 - \ell_0)$ ...(1)

And $120 = K(\ell_2 - \ell_0)$ ...(2)

$\dfrac{1}{2} \Rightarrow \dfrac{5}{6} = \dfrac{\ell_1 - \ell_0}{\ell_2 - \ell_0}$

$5\ell_2 - 5\ell_0 = 6\ell_1 - 6\ell_0$

$\ell_0 = 6\ell_1 - 5\ell_2$

$\ell_0 = 6\ell_1 - 5\left(\frac{11\ell_1}{10}\right)$

$\ell_0 = 6\ell_1 - \frac{11\ell_1}{2}$

$\ell_0 = \frac{\ell_1}{2}$

$\therefore x = 2$

17. **Ans. (D)**

$\Delta Q = mS\Delta T$

$\frac{\Delta T}{\Delta Q} = \frac{1}{mS} = \frac{1}{\text{Heat capacity}}$

18. **Ans. (D)**

$Q = 10 \times 80 + 20 \times 1 \times 100 + 10 \times 540 = 8200$

19. **Ans. (C)**

$\int dQ = \int_{20}^{40} 100 \times 10^{-3}(100T + 500)dT$

$= 10\left[\frac{1600-400}{2} + 5 \times 20\right] = 7000\,J$

20. **Ans. (C)**

$(5kg) \times L = 42\frac{kJ}{min} \times (10\ min)$

21. **Ans. (C)**

$m_1 s_1 = m_2 s_2$ ...(A)

$m_1 s_1 + m_2 s_2 = (m_1 + m_2)\,s$ ...(B)

$s = \frac{m_1 s_1 + m_2 s_2}{m_1 + m_2} = \frac{2m_1 s_1}{m_1 + m_2}\ (m_1 s_1 = m_2 s_2)$

$= \frac{2m_1 s_1}{m_1 + \frac{m_1 s_1}{s_2}} = \frac{2s_1 s_2}{s_1 + s_2}$

22. **Ans. (A)**

$P = \text{Power of heater} = \frac{(1) \times S_W \times 30}{10min.} \frac{(3) \times S_0 \times 30}{t}$ $(S_w = 2s_0)$

$t = 15$ min.

23. **Ans. (A)**

$C_A(T_A - T) + C_B(T_B - T) + C_C(T_C - T) = 0$

$\Rightarrow\ T = \frac{C_A T_A + C_B T_B + C_C T_C}{C_A + C_B + C_C}$

24. **Ans. (B)**

$\frac{dQ}{dt} = 2 \times 10^3 \times \frac{80}{100} = \frac{dm}{dt} \times S\Delta T = (100 \times 10^{-3})(4200)\Delta T$

$\Rightarrow\ \Delta T = 3.8°C$

So, $T_f = 13.8°C$

25. **Ans. (B)**

Heat liberated by water to attain $0°C = ms\Delta T = 10 \times 1 \times 40 = 400\ cal$

Amount of ice melted $= \frac{400}{800} = 5\ g$

Total amount of water $= 10 + 5 = 15\ gm$

26. **Ans. (C)**

$V_i - V_f = 1\ cm^3$

$\frac{m}{0.9} - \frac{m}{1} = 1cm^3$

$m = 9\ gm$

$Q = mL = 9 \times 80 = 720\ cal$

27. **Ans. (A)**

Let the total amount be = 100 unit

Let m solidity

$m \times L_f = (100 - m)L_V$

$m = 86.2$

28. **Ans. (D)**

The initial gravitational potential energy of the ball = mgh

$= m \times (10\ ms^{-2}) \times (6.2 \times 10^3\ m)$

$= m \times (6.2 \times 10^4\ m^2\ s^{-2}) = m \times (6.2 \times 10^4\ J\ kg^{-1})$.

All this energy is used to heat the ball as it reaches the ground with a small velocity. Energy required to take the ball from 30°C to 330°C is

$m \times (126\ J\ kg^{-1}{}^\circ C^{-1}) \times (300^\circ C)$

$= m \times 37800\ Jkg^{-1}$

and energy required to melt the ball at 330°C = mL

where L = latent heat of fusion of lead.

Thus,

$m \times (6.2 \times 10^4\ J\ kg^{-1}) = m \times 37800\ J\ kg^{-1} + mL$

or, $L = 2.4 \times 10^4\ J\ kg^{-1}$.

29. **Ans. (D)**

Volume of liquid = 500 cm^3

$\rho VS\Delta T = mgh$

$$h = \frac{1000 \times 5 \times 10^{-4} \times 4.2 \times 10^3 \times 15}{10 \times 10} = 315m$$

30. **Ans. (B)**

Isotropic photographic enlargement

31. **Ans. (B)**

$\ell = \ell_0(1 + \alpha\Delta T) \Rightarrow \ell - \ell_0 = \ell_0\alpha\Delta T \Rightarrow \frac{\ell_0\alpha_A\Delta T}{\ell_0\alpha_B\Delta T} = \frac{104-100}{106-100} = \frac{4}{6} \Rightarrow \frac{\alpha_A}{\alpha_B} = \frac{2}{3}$

32. **Ans. (A)**

The length of each rod increases by the same amount

$\Delta\ell_a = \Delta\ell_S$

$\ell_1\alpha_a\Delta T = \ell_2\alpha_S\Delta T$

$\frac{\ell_2}{\ell_1} = \frac{\alpha_a}{\alpha_S}$

$\Rightarrow \frac{\ell_2}{\ell_1} + 1 = \frac{\alpha_a}{\alpha_S} + 1$

$\Rightarrow \frac{\ell_1}{\ell_1 + \ell_2} = \frac{\alpha_S}{\alpha_a + \alpha_S}$

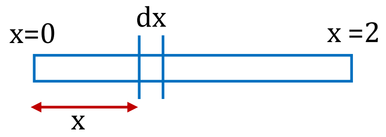
33. Ans. (C)

Take element of length dx at distance x from one end.

Let dL is the expansion of length dx then $dL = dx\alpha\Delta T$

Integrate from $x = 0$ to $x = 2$ for net expansion.

Length of the rod at 20°C



$$\frac{dL}{dx} = \alpha\Delta T$$

$$dL = \alpha\Delta T dx = (3x+2)\times 10^{-6}\times\Delta T dx$$

$$\int_{L_0}^{L} dL = \int_0^{L_0}(3x+2)dx\times 10^{-6}\times\Delta T$$

$$L - L_0 = \left(\frac{3L_0^2}{2}+2L_0\right)\times 10^{-6}\times\Delta T$$

$$L_0 = 2m = 200cm, \Delta T = 20°C$$

$$L = L_0 + \left(\frac{3L_0^2}{2}+2L_0\right)\times\Delta T\times 10^{-6}$$

$$L = 200 + \left(\frac{3\times 40000}{2}+2\times 200\right)\times 20\times 10^{-6}$$

$$L = 200 + 1.2 = 201.2cm$$

$$L = 2.012m$$

34. Ans. (B)

Stress = Y (strain) = $Y_S(\alpha_b - \alpha_s)\Delta T$ = (2×10^{11}) (0.6×10^{-5}) (100) = 1.2×10^{8} Nm^{-2}

35. Ans. (A)

Thermal Stress × Area = μMg.

$$(Y\propto\Delta T)S = \mu MG$$

$$\Delta T = \frac{\mu Mg}{\alpha YS}$$

36. Ans. (C)

$\Delta V = V\gamma t = \frac{4}{3}\pi r^3\gamma t = \frac{4}{3}\times 3.14\times (10)^3\times 5.4\times 10^{-5}\times 100 = 22.6\ cm^3$.

37. Ans. (B)

$$B = \frac{P}{\Delta V/V} \quad\Rightarrow\quad P = B\frac{\Delta V}{V} \qquad ...(i)$$

$$\frac{\Delta V}{V} = \gamma\Delta T = 3\alpha\Delta T \qquad ...(ii)$$

38. Ans. (C)

$C = \gamma_L - \gamma_C$ ...(i)

$S = \gamma_L - \gamma_S$ ...(ii)

$\gamma_S = 3\alpha_S$ ...(iii)

$C + \gamma_C = S + \gamma_S$

$$\gamma_S = \frac{C+\gamma_C-S}{3}$$

39. Ans. (D)

Coefficient of volume expansion $\gamma = \frac{\rho_1-\rho_2}{\rho_1\Delta T} = \frac{(10-9.7)}{10\times(100-0)} = 3\times 10^{-4}$

Hence, coefficient of linear expansion $\alpha = \frac{\gamma}{3} = 10^{-4}/°C$

40. Ans. (B)

When length of the liquid column remains constant, then the level of liquid moves down with respect to the container, thus γ must be less than 3α.

Now we can write $V = V_0(1 + \Delta T)$. Since $V = A\ell_0 = [A_0(1 + 2\alpha\Delta T)]\ell_0 = V_0(1 + 2\alpha\Delta T) \Rightarrow \gamma = 2\alpha$

41. Ans. (A)

$$\frac{dQ}{dt} = \frac{\Delta T}{R_{eq}}$$

$$s = \frac{\Delta T}{R_{eq_1}}(dt_1) = \frac{\Delta T}{R_{eq2}}(dt_2)$$

42. Ans. (B)

$$R_{(eq)_1} = \frac{\ell}{kA}$$

$$R_{(eq)_2} = \frac{\ell}{4kA}$$

$$\frac{dm}{dt_1}L = \frac{\Delta T}{R_{(eq)1}}$$

$$\frac{dm}{dt_2}L = \frac{\Delta T}{R_{(eq)2}}$$

$$\frac{dt_2}{dt_1} = \frac{R_{(eq)2}}{R_{(eq)1}} = \frac{1}{4}$$

43. Ans. (A)

$R_{eq} = R_1 + R_2 + R_3$

$$\frac{3L}{k_{eq}A} = \frac{2L}{kA} + \frac{L}{5kA} + \frac{L}{kA}$$

$$\Rightarrow \; k_{eq} = \frac{15k}{16}$$

44. Ans. (B)

$$\Delta T \propto \frac{1}{K}$$

K same $\Rightarrow \Delta T$ same

45. Ans. (D)

$$R_{H_2} = R_1 + R_2 = \frac{x}{kA} + \frac{4x}{2kA} = \frac{3x}{kA}$$

$$\frac{dQ}{dt} = \frac{\Delta T}{\frac{3x}{kA}} = \frac{(T_2 - T_1)kA}{3x} = \frac{1}{3}\left[\frac{A(T_2 - T_1)k}{x}\right]$$

46. Ans. (C)

$$\left(\frac{dm}{dt}\right)_{Ice} \times L = \frac{\Delta T}{R}$$

$$R = \frac{\ell}{kA}$$

47. Ans. (D)

$$\frac{dQ}{dt} = \frac{kA}{L}\Delta T$$

48. **Ans. (A)**

Both the rods are at steady state and temperature gradient constant and temperature drops linearly with the distance.

Temperature at $P = \dfrac{T_A+T_B}{2} \Rightarrow 50°C = T_P$

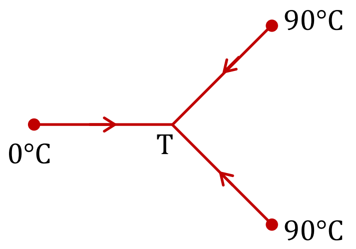
Temperature at $Q = \dfrac{T_C+T_D}{2} \Rightarrow 45°C = T_Q$

Since temperature at P more than at Q. So, heat flow from P to Q.

49. **Ans. (A)**

Slope $\propto \dfrac{1}{k}$

50. **Ans. (B)**



Let temperature of junction is T

$$\frac{kA(90-T)}{\ell}+\frac{kA(90-T)}{\ell}+\frac{kA(0-T)}{\ell}=0$$

$T = 60°C$

51. **Ans. (C)**

$$\left.\frac{dm}{dt}\right|_{Steam}=\frac{1}{16}\left.\frac{dm}{dt}\right|_{ice}$$

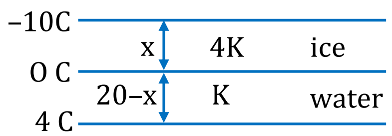
$$\left.\frac{dQ}{dt}\right|_{Steam}=\frac{T-100}{\left(\dfrac{2\ell}{3Ak}\right)}=\left(\frac{dm}{dt}\right)_{steam}L_v$$

$$\left.\frac{dQ}{dt}\right|_{Ice}=\frac{T-0}{\left(\dfrac{\ell}{3Ak}\right)}=16\left(\frac{dm}{dt}\right)_{steam}L_f$$

$$\frac{\dfrac{T-100}{2}}{\dfrac{T}{1}}=\frac{L_v}{16L_f}$$

$$\frac{(T-100)}{2T}=\frac{540}{16\times 80}\ ;\ \frac{T-100}{T}=\frac{540}{640}\ ;\ \frac{T-100}{2T}=\frac{540}{8\times 80}$$

52. **Ans. (B)**



The rate of heat flow is the same through water and ice in the steady state so

$$\frac{KA(4-0)}{20-x}=\frac{4KA[0-(-10)]}{x}\quad\Rightarrow\quad x=\frac{200}{11}m$$

53. **Ans. (C)**

using average rate of Newton's law of cooling

$$\frac{T_1 - T_2}{t} = K\left(\frac{T_1 + T_2}{2} - T_s\right)$$

Given $\frac{60-40}{7} = K(50-10)$ ... (i)

& $\frac{40-T}{7} = K\left(\frac{40+T}{2} - 10\right)$ ... (ii)

From (i) & (ii)

$T = 28°C$

54. **Ans. (A)**

Rate of cooling α Temperature difference

$$\frac{80-60}{5} = k\{70-20\} \text{ ... (i)}$$

$$\frac{60-40}{t} = k[50-20] \text{ ... (ii)}$$

$$\frac{4t}{20} = \frac{50}{30}$$

$$t = \frac{25}{3}\min = 500\sec$$

$\Rightarrow$ t = 500 seconds

55. **Ans. (A)**

The energy radiated per sec is given by

$E = e\sigma AT^4$

For same material e is same

$$\frac{E_1}{E_2} = \frac{T_1^4 A_1}{T_2^4 A_2} = \frac{T_1^4 (4\pi r_1^2)}{T_2^4 (4\pi r_2^2)} = \frac{(4000)^4 \times 1^2}{(2000)^4 \times 4^2} = \frac{1}{1}$$

56. **Ans. (B)**

$$\frac{dQ}{dt} = \sigma eA(T^4 - T_0^4)\mathbb{Z}$$

$$ms\frac{dT}{dt} = \sigma eA(T^4 - T_0^4) \quad \text{...(1)}$$

$m = 1\ kg$, $A = 6 \times 25 \times 10^{-4}$, $e = 1$

$S = 400\ J/kg\text{–}k$

$T = 227 + 273 = 500\ k$

$T_0 = 300\ k$

Put values in eq. (1)

$$\frac{dT}{dt} = 0.12°\text{C/s}$$

57. **Ans. (B)**

$$P \propto \frac{T^4}{d^2}$$

$$\frac{P}{P'} = \left[\frac{T}{2T}\right]^4 \left[\frac{2d}{d}\right] = \frac{1}{4} \Rightarrow P' = 4P$$

58. **Ans. (A)**

$$-\frac{dT}{dt} \propto e \qquad \text{...(i)}$$

From graph

$$\left[\frac{-dT}{dt}\right]_x > \left[\frac{-dT}{dt}\right]_y$$

So, $e_y > e_x$; $A_y > A_x$

59. **Ans. (B)**

$$\frac{dT}{dt} = \frac{\sigma e A}{ms}\left(T^4 - T_S^4\right)$$

$$\frac{dT}{dt} \propto \frac{A}{m} \qquad (m = \rho V)$$

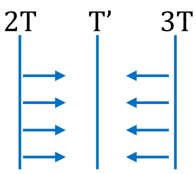
for disc (pizzas) $\quad m \propto A \quad$ so $\quad \frac{dT}{dt}$ is same for both

for Gulab jamuns (sphere) — Small (A, m); Big (4A, 8m)

$$\left(\frac{dT}{dt}\right)_{small} > \left(\frac{dT}{dt}\right)_{big} \qquad \text{(for Gulab jamun)}$$

60. **Ans. (C)**

Let temperature of middle plate = T'



$$k(2T)^4 + k(3T)^4 = 2k(T')^4$$

$$T' = \left(\frac{97}{2}\right)^{1/4} T$$

61. **Ans. (D)**

$$\frac{E_2}{E_1} = \frac{\sigma A_2 T_2^4}{\sigma A_1 T_1^4} = \frac{R_2^2 T_2^4}{R_1^2 T_1^4}$$

$R_2 = 2R$, $R_1 = R$

$T_2 = 2T$, $T_1 = T$

$$\Rightarrow \frac{E_2}{E_1} = \frac{(2R)^2 (2T)^4}{(R)^2 (T)^4} = 64$$

62. **Ans. (C)**

$$\frac{dT}{dt} \propto \frac{A}{m} \propto \frac{A}{V} \propto \frac{1}{r}$$

$$\frac{(dT/dt)_P}{(dT/dt)_Q} = \frac{3r_p}{r_p} = 3$$

63. **Ans. (B)**

$$\frac{(T_1 - T_2)}{t} = k\left(\frac{T_1 + T_2}{2} - T_S\right)$$

64. Ans. (D)

$10 = k(50 - 20)$ ...(i)

$\frac{0.2}{60} = \frac{k}{ms}(35-20)$ ...(ii)

from (i) and (ii)

ms = heat capacity = $1500\ J/°C$

65. Ans. (B)

Newtons law of cooling

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\Rightarrow \frac{d\theta}{(\theta-\theta_0)} = -kdt$$

Integrating

$$\ell n(\theta - \theta_0) = -kt + C$$

66. Ans. (C)

According to Newtons law of cooling, the temp goes on decreasing with time non-linearly.

$$\frac{dT}{dt} = -k(T - T_s)$$

$$\int \frac{dT}{(T-T_s)} = \int -kdt$$

$$\Rightarrow T = T_s + (T_i - T_s)e^{-kt}$$

67. Ans. (C)

Star B emit more power

$P = (\sigma e A T^4)$

So, it look more brighter.

68. Ans. (B)

$P \times V = \frac{4}{40} \times R \times T$

$P \times V = \frac{3.2}{40} R(T+50)$

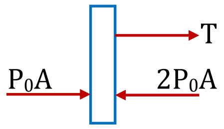
$T = 0.8\ (T + 50)$

$2T = 400$

$T = 200$

69. Ans. (C)

$P \to 2P$



70. Ans. (C)

$AB \to$ isobaric

$BC \to$ May be isothermal

71. **Ans. (C)**

$$\frac{(0.05A)76}{RT} = \frac{(P \times xA)}{RT}$$

$0.05 \times 76 = Px$ ...(i)

$P + (48 - x) = 76$

$P = 76 - 48 + x$

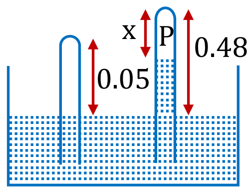
$P = 28 + x$ ...(ii)

Now $3.80 = (28 + x)x$

$x^2 + 28x - 3.8 = 0$

$x = 0.135$ and -28.13

$x = 0.135\ cm$.



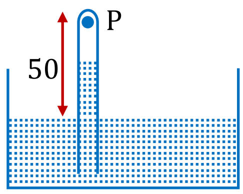
72. **Ans. (A)**

$P_{at.}\ 75\ rg = 10^5$

$P + 50 \times rg = 75\ rg$

$P = 25\ rg$

$= \frac{10^5}{3} = 33.3 \times 10^3\ Pa$



73. **Ans. (D)**

As $C_p - C_v = R$

For above equation, we can say that both C_p and C_v increase by same amount.

74. **Ans. (B)**

The pressure of the gas = atmospheric pressure + the pressure due to the difference in mercury levels. At 27°C, the pressure is 75 cm + 5 cm = 80 cm of mercury. At the liquid temperature, the pressure is 75 cm + 45 cm = 120 cm of mercury. Using $T_2 = \frac{P_2}{P_1}T_1$, the temperature of the liquid is

$T = \frac{120}{80} \times (27.0 + 273.15)\ K = 450.22\ K. = 177.07°C = 177°C$

75. **Ans. (A)**

Let ℓ_1 and ℓ_2 be the lengths of the upper part and the lower part of the cylinder respectively. Clearly, $\ell_1 = 50\ cm$ and $\ell_2 = 40\ cm$. Let the pressures in the upper and lower parts be p_1 and p_2 respectively.

Let the area of cross section of the cylinder be A. The temperature in both parts is $T = 300\ K$.

Consider the equilibrium of the piston. The forces acting on the piston are

(a) its weight mg

(b) p_1A downward, by the upper part of the gas and

(c) p_2A upward, by the lower part of the gas.

Thus, $p_2A = p_1A + mg$ ...(i)

Using $pV = nRT$ for the upper and the lower parts

$p_1\ell_1A = nRT$ ...(ii)

and $p_2\ell_2A = nRT$ ...(iii)

Putting p_1A and p_2A from (ii) and (iii) into (i),

$$\frac{nRT}{\ell_2}=\frac{nRT}{\ell_1}+mg.$$

Thus $m=\dfrac{nRT}{g}\left[\dfrac{1}{\ell_2}-\dfrac{1}{\ell_1}\right]$

$$=\frac{(0.1mol)(8.3JK^{-1}mol^{-1})(300K)}{9.8ms^{-2}}\left[\frac{1}{0.4m}-\frac{1}{0.5m}\right]=12.7\ kg$$

76. Ans. (C)

The rms speed of a gas molecule is

$$V_{RMS}=\sqrt{\frac{3RT}{M}}$$

$$V_{RMS}\ \alpha\ \sqrt{T}$$

77. Ans. (A)

$$C=\sqrt{\frac{\gamma RT}{M}}$$

$$C\propto\frac{1}{\sqrt{M}}$$

$$\frac{C_{H_2}}{C_{O_2}}=\sqrt{\frac{32}{2}}=4:1$$

78. Ans. (D)

$$\frac{3n_1}{N_A}\frac{f}{2}RT_1+\frac{n_2}{N_A}\frac{3}{2}fRT_2=\left(\frac{n_1+n_2}{N_A}\right)\left(\frac{3f}{2}R\right)T_f$$

79. Ans. (D)

For an ideal gas

$C_P - C_V = R$

If $C_P - C_V = 1.09\ R.$

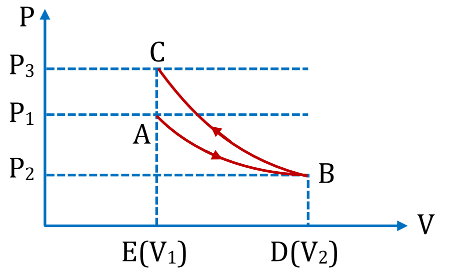
or $p_A > p_B$; $T_A < T_B$

Then gas will be real. Thus, pressure is high and temperature is low for real gas.

80. Ans. (C)

$$W=W_d+(-W_A)$$

$$\because\ W_A>W_d\ \Rightarrow W<0$$



81. Ans. (C)

Process AB :

$T \Rightarrow$ constant

$P\uparrow$, $V_f < V_i$, so work done is negative

Process BC :

$T_f > T_i$, so ΔT is negative

$\Delta U = nC_V\Delta T$ = increasing

Process CD :

$T \Rightarrow$ constant

Isothermal process.

PV = constant

$$P \propto \frac{1}{V}$$

$\Rightarrow$ $V_f > V_i$ = work done is positive

82. Ans. (C)

If temperature is constant then change in internal energy is zero.

83. Ans. (D)

$$\frac{\Delta Q}{\Delta W} = \frac{ms\Delta T}{P\Delta V}$$

84. Ans. (D)

Isobaric process

(A) $RT_0 \ell n\, V_f/V_i$

(B) $P\Delta V = P(\eta - 1)V_0 = 1RT_0(\eta - 1)$

(C) No change in temperature

(D) Initial and final pressure are same

85. Ans. (C)

Work done = Area under the curve

$$= -\left[\frac{1}{2} \times V_0 \times 2P_0 + P_0 \times V_0\right] = -2P_0V_0$$

86. Ans. (C)

$W.D.$ = π × Pressure Radius × volume Radius (area of ellipse)

$$W = \pi \left(\frac{P_2 - P_1}{2}\right)\left(\frac{V_2 - V_1}{2}\right) = \frac{\pi}{4}\ (P_2 - P_1)\ (V_2 - V_1)$$

87. Ans. (A)

$dW = 25 = nRdT$

$dU = 3nRdT = 75$

88. Ans. (B)

PV^{-1} = constant

$P \rightarrow 2P$

$\Delta U = 2 \times 3R \times 900 = 2700\ R$

$\Delta w = 2 \times R \times \frac{900}{2} = 2700\ R$

89. Ans. (B)

PV^{-2} = constant

$= \frac{R(T_2 - T_1)}{1-(-2)}$

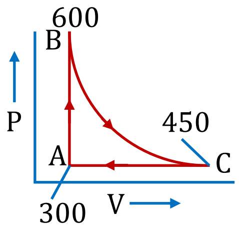
90. Ans. (C)

$dU = nC_V dT$ $\quad$ $dT_{AB} = 300$

$= 1 \times \frac{3}{2} R(\Delta T)$ $\quad$ $dT_{BC} = -150$

$T_{CA} = -150$

$dU_{cycle} = 0$



91. Ans. (A)

$\omega_{ADC} > \omega_{ABC}$

92. Ans. (A)

For AB : Pressure constant $\frac{1}{V}$ increases so V decreases

so $PV \rightarrow$ decreases

Temperature decreases

For BC :

V constant $\Rightarrow P$ increases so

PV increases $\Rightarrow$ Temp increase

For CA :

$P = \frac{m}{V} + C$

$PV = m + CV$

$nRT = m + CV$

$\frac{1}{V}$ decreases so V increase

T increase, work done by the gas.

93. Ans. (B)

$\frac{3}{2} nR\Delta T = 100$

$dQ = dU + dW$

For isobaric $dQ = n\ \frac{3}{2} R\Delta T + RndT$

$= \frac{5}{2}(nR\Delta T) = \frac{500}{3}$

$dQ = dU = \frac{500}{3}$

94. **Ans. (A)**

In case of free expansion work done by gas is zero.

95. **Ans. (B)**

$\Delta Q = -1200\ J$

$\Delta U = 0$ [Cyclic process]

$\Delta Q = \Delta U + W$

$W_{AB} + W_{BC} + W_{CA} = -1200\ J$

$W_{CA} = 0$

Process AB isobaric hence

$W_{AB} = nR\Delta T$

$nR\Delta T + W_{BC} + 0 = -1200\ J$

$2 \times 8.3 \times 200 + W_{BC} = -1200\ J$

$W_{BC} = -\ 4520\ J$

96. **Ans. (A)**

Here $T_1 = 400\ K$, $Q_1 = 100\ kcal$, $Q_2 = 70\ kcal$

(i) $\dfrac{Q_1}{Q_2} = \dfrac{T_1}{T_2}$ $\therefore T_2 = \dfrac{Q_2}{Q_1}\ T_1 = \dfrac{70}{100} \times 400 = 280\ K$

(ii) $\eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{280}{400} = 1 - 0{\cdot}7 = 0{\cdot}3$ or $\%\ \eta = 0{\cdot}3 \times 100 = 30\%$

97. **Ans. (B)**

$$1-\frac{T_H}{T_C}=\frac{W}{Q_H}$$

$$1-\frac{T_H}{T_C}=\frac{W}{Q_H}$$

$\Rightarrow$ $2 \times 10^6\ cal = W$

98. **Ans. (B)**

$$\beta=\frac{T_2}{T_1-T_2}=\frac{283}{293-283}=\frac{283}{10}$$

99. **Ans. (D)**

$$\beta=\frac{T_2}{T_1-T_2}=\frac{Q_2}{W}$$

$Q_2 = mL = 5 \times 10^3 \times 80 \times 4.2$

$W = \dfrac{Q_2}{\beta}$

$Q_1 = Q_2 + W$

$P_1 = \dfrac{Q_1}{t}$

100. **Ans. (A)**

$$COP=\frac{1}{\frac{373}{273}-1}=\frac{x\times10^3\times80}{1\times10^3\times540-x\times10^3\times80}$$

101. **Ans. (C)**

Max. possible efficiency $\eta=1-\dfrac{303}{423}=0.28\Rightarrow28\%$

So, Engine violet the second law of thermodynamics

102. Ans. (A)

Heat energy cannot be completely converted to work

103. Ans. (A)

Isochoric

$$\Delta S = nC_V \ell n\left(\frac{T_2}{T_1}\right)$$

Isobaric

$$\Delta S = nC_P \ell n\left(\frac{T_2}{T_1}\right)$$

104. Ans. (A)

Randomness of the system increases

105. Ans. (A)

Entropy is state function.

From 1 to 2, Q is positive so, entropy will increase.

Hence, $S_1 < S_2$

Similarly $S_2 < S_3 < S_4 < S_5$

106. Ans. (A)

$$\frac{dQ}{dt} = M.S.\frac{dT}{dt}$$

Change in entropy = $\int \frac{dQ}{T} = \frac{MS}{T} \cdot dT$

Specific change in entropy = $\frac{\Delta S}{M} = \int_{T_1}^{T_2} \frac{a + bT^2}{T} \cdot dT$

$$= \left[a \ell n T + \frac{bT^2}{2}\right]_{T_1}^{T_2} = a \ell n\left(\frac{T_2}{T_1}\right) + \frac{b}{2}\left[T_2^2 - T_1^2\right]$$

107. Ans. (A)

$\frac{W}{\Delta Q} = \frac{\Delta T}{T}$ For Carnot Cycle

$W = \frac{\Delta Q}{T} \cdot \Delta T$ = 1 × 10^3 × 4.2 × 100 = 420 kJ

108. Ans. (A)

Heat engine absorbs heat & produces work

$W = Q_H - Q_L$

$\therefore \oint dQ = Q_H - Q_L$ (for 1 cycle)

$$\oint dQ > 0$$

109. Ans. (AB)

Let x gm of steam at 100°C is converted to water at 100°C

= 50 × 0.5 × 10 + 50 × 80 + 51.5 × 1 × 100

= 9400

Heat lost = Heat gain

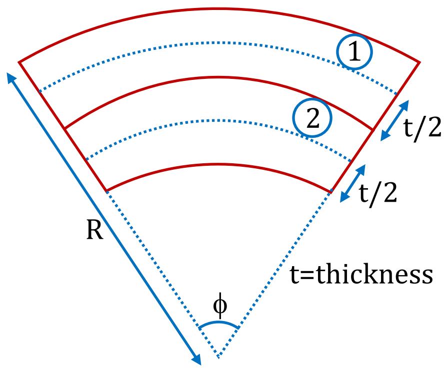
540 x = 9400

x = 17.4 gm

Water remaining = (50+17.4) = 67.4 gm

Steam remaining = 20 – 17.4 = 2.6 gm

110. Ans. (BD)



$L_1 = L_0(1 + \alpha_1\Delta T)$

$\left(R+\frac{t}{2}\right)\phi = L_0(1+\alpha_1\Delta T)$ ...(i)

$\left(R-\frac{t}{2}\right)\phi = L_0(1+\alpha_2\Delta T)$ ...(ii)

Solving

$$R \simeq \frac{t}{(\alpha_1-\alpha_2)\Delta T}$$

$\alpha_1 = \alpha_B$

$\alpha_2 = \alpha_C$

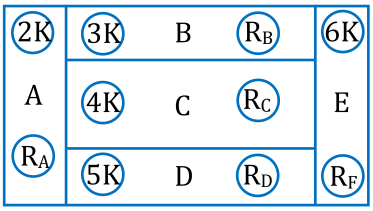
111. Ans. (BC)

$$E_1\frac{\Delta\ell_1}{\ell_1[1+\alpha_1 t]} = E_2\frac{\Delta\ell_1}{\ell_1[1+\alpha_1 t]} \quad ...(1)$$

$$\ell_1[1+\alpha_1 t] + \ell_2[1+\alpha_2 t] - \left[\Delta\ell_1+\Delta\ell_2\right] = \ell_1+\ell_2 \quad ...(2)$$

112. Ans. (ACD or ABCD)

$Q_A = Q_B + Q_C + Q_D = Q_E$



$$R = \frac{\ell}{kA}$$

Let width of slabs be W

$$R_A = \frac{L}{2k(4L)W} = \frac{1}{8kW}$$

$$R_B = \frac{4L}{3kLW} = \frac{4}{3kW}$$

$$R_C = \frac{4L}{4k(2LW)} = \frac{1}{2kW}$$

$$R_D = \frac{4L}{5k(LW)} = \frac{4}{5kW}$$

$$R_E = \frac{L}{6k(4LW)} = \frac{1}{24kW}$$

as $Q = \frac{\Delta T}{R}$ and $\Delta T_B = \Delta T_C = \Delta T_D$

$$Q_B : Q_C : Q_D = \frac{1}{R_B} : \frac{1}{R_C} : \frac{1}{R_D}$$

113. Ans. (AD)

Ist combination :-

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

$$R_1 = \frac{L\times 4}{k.\pi d^2},\ R_2 = \frac{L\times 4}{3k.\pi\left[4d^2 - d^2\right]} = \frac{4L}{9k\pi d^2}$$

$$R_{eq} = \frac{2L}{5k\pi d^2}$$

II[nd] combination

$R_{eq} = R_1 + R_2$

$R_1 = \dfrac{L}{kA}$

$R_2 = \dfrac{L}{4kA}$

$R_{eq} = \dfrac{L}{kA}\left[1+\dfrac{1}{4}\right] = \dfrac{5L}{4kA}$

114. Ans. (AC)

$m = 0.5\ Kg$

$P = 12N$

$$12 = ms\frac{dT}{dt} + K(T-15)$$

at $T \to 45°C \Rightarrow \dfrac{dT}{dt} = 0$

rate of heat lass = $K(45 - 15) = 12N$

$$K = \frac{12}{30} = \frac{2}{5} N/°C$$

at $T = 20°C$, rate of heat lass $= \dfrac{2}{5}(20-15) = 2N$

$$ms\int_{15}^{25} \frac{dT}{12-\frac{2}{5}(T-15)} = \int_0^{300} dt\ ;\ ms\int_{15}^{25}\frac{5dT}{90-25} = 300$$

$$\Rightarrow \frac{0.5\times s\times 5}{-2}\left[\ell n(90-2T)\right]_{15}^{25} = 120 \quad = -s\,\ell n\left(\frac{40}{66}\right) = 240$$

$$\Rightarrow s\,\ell n\left(\frac{3}{2}\right) = 240 \Rightarrow s = \frac{240}{\ell n\left(\frac{3}{2}\right)}$$

115. Ans. (CD)

No condition for greater and least speed

$V_R > V_a > V_P$

$$V_{rms} = \sqrt{\frac{3}{2}}V_P$$

$$V_{avg} = \frac{1}{2}mV_{rms}^2$$

$$KE = \frac{1}{2}m\left(\sqrt{\frac{3}{2}}V_P\right)^2 = \frac{3}{4}mV_P^2$$

116. Ans. (BC)

$V_{av} \propto \frac{1}{\sqrt{M_0}}$

$\therefore$ nitrogen molecule hits the wall with smaller average speed

Degree of freedom of N_2 is 5 and He is 3.

therefore nitrogen molecule hits the wall with greater average kinetic energy.

117. Ans. (BD)

Temp increases from A to C then decrease from C to B

118. Ans. (ACD)

$P_1V_1^{\gamma} = P_2V_2^{\gamma}$ & $V_1 + V_2 = V_0$

$\Rightarrow P_1V_1^{\gamma} = P_2(V_0 - V_2)^{\gamma}$

$\Rightarrow \left(\frac{P_1}{P_2}\right)^{\frac{1}{\gamma}} = \frac{V_0 - V_1}{V_1} = \frac{V_0}{V_1} - 1 \Rightarrow \frac{P_1^{\frac{1}{\gamma}} + P_2^{\frac{1}{\gamma}}}{P_2^{\frac{1}{\gamma}}} = \frac{V_0}{V_1}$

$\therefore V_1 = \frac{P_2^{\frac{1}{\gamma}} V_0}{P_1^{\frac{1}{\gamma}} + P_2^{\frac{1}{\gamma}}}$ & $V_2 = \frac{P_1^{\frac{1}{\gamma}} V_0}{P_1^{\frac{1}{\gamma}} + P_2^{\frac{1}{\gamma}}}$

(C) since whole chamber & separator wall is adiabatic so heat given to left part is zero

(D) Let final common pressure be 'P' at equilibrium

$P_1V_1^{\gamma} + P_2V_2^{\gamma} = PV_0^{\gamma}$

At equilibrium : $V_1 = V_2 = \frac{V_0}{2}$

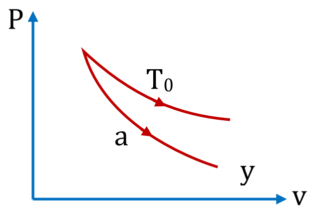
$P = \frac{P_1^{\frac{1}{\gamma}} + P_2^{\frac{1}{\gamma}}}{2}$

119. Ans. (BC)

(1) ΔU does not depends on path

(2) Work done depends area of PV graph

120. Ans. (ABC)



121. Ans. (B)

Material which is most ductile is easy to expand is used for making wire.

122. Ans. (C)

Material which breaks just after proportional point.

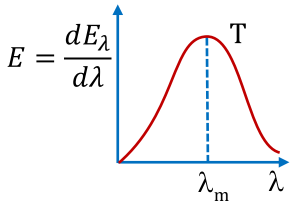
123. Ans. (D)

Material which retains permanent deformation.

124. Ans. (AB)

125. Ans. (ABCD)

Solution for Question No. 124 and 125



Explained by Plank's radiation law

for all λ, if $T\uparrow \Rightarrow E\uparrow$

area of curve = $\int E d\lambda = \sigma T^4 (stefan's\ law)$

= Total intensity of radiation at a particular time

126. Ans. (B)

For right chamber

PV^γ = constant

$P_0 V_0^{5/3} = (32P_0)(V_f)^{5/3}$

$V_f = \frac{V_0}{(32)^{3/5}} = \frac{V_0}{8}$

$V_{left} = 2V_0 - \frac{V_0}{8} = \frac{15V_0}{8}$

127. Ans. (A)

$W = -nC_V\Delta T$

$= \frac{3}{2}\left[P_0V_0 - (32P_0)\left(\frac{V_0}{8}\right)\right] = -\frac{9}{2}P_0V_0$

on the gas $\frac{9}{2}P_0V_0$

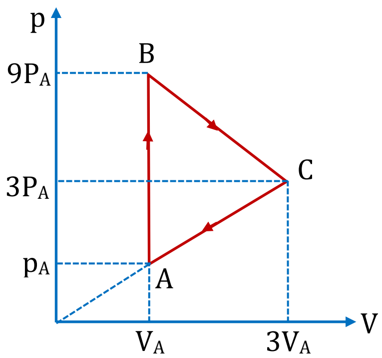
128. Ans. (B)

$\Delta U = nC_V\Delta T = \frac{3}{2}nR(T_f - T_i) = \frac{3}{2}\left[(32P_0)\left(\frac{15V_0}{8}\right) - P_0V_0\right] = \frac{3}{2}[59P_0V_0]$

$= \frac{177}{2}P_0V_0 \qquad \left(P_0V_0 = \frac{1}{2}RT_0\right)$

$\Delta U = \frac{177}{4}RT_0$

129. Ans. (D)



Area = $\frac{(8P_A)(2V_A)}{2} = 8P_AV_A$

130. **Ans. (D)**

AB is isochoric

AC is general process

$BC \rightarrow$ is general process

131. **Ans. (A-P,S,T), (B-P,S,T), (C-Q,R), (D-Q,S)**

(A) avg. force = $F = \frac{mg}{2}$

$$\therefore \quad \frac{Mg}{2A} = Y\frac{\Delta\ell}{L} \Rightarrow \Delta\ell = \frac{MgL}{2Ay}$$

Stress = $\frac{T}{A} \rightarrow$ varies $\Rightarrow \Delta\ell = \frac{MgL}{YA}$

stress is non uniform.

$$U = \frac{M^2g^2L}{6AY}$$

(B) Average force = $\frac{Mg}{2}$

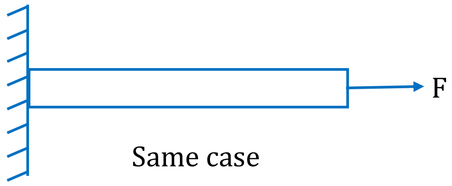
$\therefore$ all answers of (A) are valid

(C) Average force = $\frac{F+F}{2} = F = Mg$

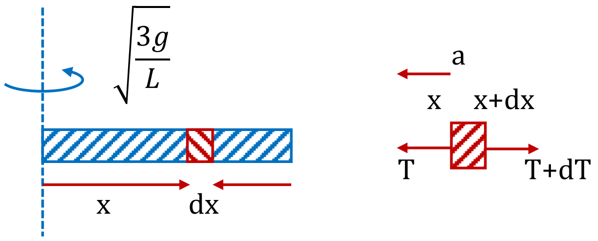
$$\frac{Mg}{A} = Y\frac{\Delta\ell}{L} \Rightarrow \Delta\ell = \frac{MgL}{YA}$$

$\sigma = \frac{F}{A}$ = Uniform

$$\frac{U}{v} = \frac{\sigma^2}{2Y} = \frac{M^2g^2}{2YA^2}AL = \frac{M^2g^2L}{2YA}$$



(D)



$- dT = (dM)\omega^2x$

$$-dT = \left(\frac{M}{L}dx\right)\omega^2 x$$

$$-\int_T^0 dT = \int_x^L \frac{M}{L}\omega^2 x dx$$

$$T = \frac{M}{L}\omega^2\left[\frac{L^2 - x^2}{2}\right] \Rightarrow T = \frac{M\omega^2}{2L}\left[L^2 - x^2\right]$$

$\sigma = \frac{T}{A}$ not uniform

$$\frac{T}{A} = Y\frac{\Delta\ell}{\ell}$$

$$\frac{M\omega^2}{2L}=\left[L^2-x^2\right]=Y\frac{\Delta\ell}{L}$$

$$\Delta\ell=\frac{M\omega^2}{2Y}\left[L^2-x^2\right]$$

132. Ans. (A-R), (B-P), (C-S), (D-Q)

$$\frac{dQ}{dt}=KA\frac{dT}{dx}$$

Thick cylindrical shell along the axis:

$$A=\pi(2R)^2-R^2=3\pi R^2$$

Length: $L = 3R$

$$\therefore \frac{dQ}{dt}=k_0\times 3\pi R^2\frac{(T_1-T_2)}{3R}=\pi k_0R(T_1-T_2)$$

Thick spherical shell, radial flow:

$$\frac{dQ}{dt}=-k_0\times 4\pi r^2\frac{dT}{dr}$$

$$\therefore -\frac{dQ}{dt}\times\int_R^{3R}\frac{dr}{r^2}=4\pi k_0\int_{T_1}^{T_2}dT$$

$$\therefore \frac{dQ}{dt}\times\frac{3R-R}{3R\times R}=4\pi k_0(T_1-T_2)$$

$$\therefore \frac{dQ}{dt}=6\pi k_0R(T_1-T_2)$$

Thick cylindrical shell radial flow:

$$\frac{dQ}{dt}=k_0\times 2\pi r(2R)\frac{dT}{dr}$$

$$\therefore \frac{dQ}{dt}\times\int_R^{2R}\frac{dr}{r}=2\pi(2R)k_0\int_{T_1}^{T_2}dT$$

$$\therefore \frac{dQ}{dt}\times\ln\left(\frac{2R}{R}\right)=2\pi(2R)k_0(T_1-T_2)$$

$$\therefore \frac{dQ}{dt}=\frac{4\pi k_0R}{\ln 2}(T_1-T_2)$$

Solid along the axis:

$$A=\pi(2R)^2-R^2=3\pi R^2$$

Length: $L = 3R$

$$\therefore \frac{dQ}{dt}=k_0\left(1+\frac{x}{3R}\right)\times 3\pi R^2\frac{(T_1-T_2)}{dx}$$

$$\therefore \frac{dQ}{dt}\times\int_0^{3R}\frac{dx}{1+\frac{x}{3R}}=3\pi R^2k_0(T_1-T_2)$$

$$\therefore \frac{dQ}{dt}\times 3R\ln 2=3\pi k_0R^2(T_1-T_2)$$

$$\therefore \frac{dQ}{dt}=\frac{\pi k_0R}{\ln 2}(T_1-T_2)$$

133. Ans. (A-P), (B-Q), (C-Q)

For (A)

$A \to C$ from circle $\Delta Q > 0$

between $A \to C$ direct adiabatic

$W\,(A \to C \text{ circle}) > W_{A \to C}$ direct

For (B)

$B \to C$

$W < 0$ (compresion)

$\Delta U < 0$ (PV product decreases)

For (C)

$C \to D$

$W < 0$ (compresion)

$\Delta U < 0$ (PV product decreases)

134. Ans. (I-P), (II-R), (III-T), (IV-Q)

(I) $\Delta U = \Delta Q - \Delta W$

$$= \left\{ \left(10^{-3} \times 2250\right) - \frac{10^5\left(10^{-3} - 10^{-6}\right)}{10^3} \right\} kJ$$

$= (2.25 - 0.0999)\ kJ$

$= (2.1501)\ kJ$

(II) $\Delta U = nC_V\,\Delta T$

$$= \frac{5}{2} nR\Delta T$$

$$\frac{5}{2}(0.2)(8)(1500-500)\,J = 4kJ$$

(III) $P_1 V_2^{\gamma} = P_2 V_2^{\gamma}$

$$\Rightarrow\ 2\left(\frac{1}{3}\right)^{5/3} = P_2\left(\frac{1}{24}\right)^{5/3}$$

$\Rightarrow\ P_2 = 64\ kPa$

$$\Delta U = nC_v\Delta T = \frac{3}{2}.\left(P_2V_2 - P_1V_1\right)$$

$$= \frac{3}{2}\left(64 \times \frac{1}{24} - 2 \times \frac{1}{3}\right) kJ\ = 3kJ$$

(IV) $\Delta U = nC_V\,\Delta T$

$$= n.\frac{7}{2}R\Delta T\ = \frac{7}{9}\Delta Q\ = 7kJ$$

EXERCISE – S

1. Ans. ℓ

Same material $\Rightarrow Y$ same

$Y_1 = Y_2$

$$\frac{f/\pi r^2}{\ell/L} = \frac{2f/4\pi r^2}{\ell'/2L} \Rightarrow \ell' = \ell$$

2. **Ans.** (a) $\theta = \frac{\pi}{2}$ (b) $\theta = \pi/4$.

$T.S = \frac{F\,sin^2\,\theta}{A}$

$S.S = \frac{F}{2A} sin\,2\,\theta$

$T.S_{max} = \frac{F}{A}$; $\left(\theta = \frac{\pi}{2}\right)$

$S.S_{max} = \frac{F}{2A}$, $\left(\theta = \frac{\pi}{4}\right)$

3. **Ans.** Strain in wire $A = 10^{-4}$ and strain in wire $B = 2 \times 10^{-4}$

The block R will descend vertically and the blocks P and Q will move on the frictionless horizontal table. Let the common magnitude of the acceleration be a. Let the tensions in the wires A and B be T_A and T_B respectively.

Writing the equations of motion of the blocks P, Q and R, we get,

$T_A = (3kg)a$ ...(i)

$T_B - T_A = (3kg)a$ ...(ii)

and $(3kg)g - T_B = (3kg)a$ ...(iii)

By (i) and (ii),

$T_B = 2T_A$.

By (i) and (iii),

$T_A + T_B = (3kg)g = 30\ N$.

or, $3T_A = 30N$

or, $T_A = 10\ N$ and $T_B = 20\ N$.

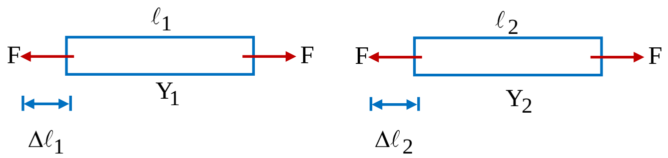
$$\text{Longitudinal strain} = \frac{\text{Longitudinal stress}}{\text{Young's modulus}}$$

$$\text{Strain in wire } A = \frac{10N/0.005cm^2}{2\times10^{11}\,N/m^2} = 10^{-4}$$

$$\text{and strain in wire } B = \frac{20N/0.005cm^2}{2\times10^{11}\,N/m^2} = 2\times10^{-4}$$

4. **Ans.** (i) 1.67, (ii) 2.4

(1) F.B.D of the figure



$\Delta\ell_1 = \frac{F\ell_1}{AY_1}$ $\Delta\ell_2 = \frac{F\ell_2}{AY_2}$

$$\text{Net elongation} = \frac{F\ell_1}{AY_1} + \frac{F\ell_2}{AY_2} = \frac{F}{A}\left(\frac{\ell_1}{Y_1} + \frac{\ell_2}{Y_2}\right)$$

$$\text{Now substituting the values} = \frac{2}{1}\left(\frac{1}{2\times10^{11}} + \frac{1}{3\times10^{11}}\right)$$

$$= 2\left(\frac{3+2}{6\times10^{11}}\right) = \frac{10}{6\times10^{11}} = 1.67 \times 10^{-11}\,m$$

Hence $x = 1.67$

(2) Now using the same $(\Delta\ell)_{eq.} = \Delta\ell_1 + \Delta\ell_2$

$$\frac{F.2\ell}{AY} = \frac{F\ell}{AY_1} + \frac{F\ell}{AY_2} \;=\; \frac{2}{Y} = \frac{1}{Y_1} + \frac{1}{Y_2}$$

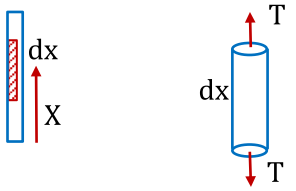
$$Y = \frac{2Y_1Y_2}{Y_1 + Y_2}$$

$$Y = \frac{2\times 2\times 10^{11}\times 3\times 10^{11}}{5\times 10^{11}}$$

$Y = 2.4 \times 10^{11}$

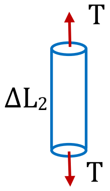
Hence $x = 2.4$

5. **Ans.** $\Delta L = \dfrac{3MgL}{2AY}$

$\Rightarrow$ 

$$T = \frac{M}{L}xg + Mg$$

$$\Delta L = \frac{FL}{AY}$$



$$\int dL = \int \frac{\left(\frac{M}{L}xg + Mg\right)}{AY}dx \quad \Rightarrow \quad \Delta L = \frac{Mg}{AY}\int\left(\frac{x}{L} + 1\right)dx$$

$$\Delta L = \frac{Mg}{AY}\left(\int_0^L \frac{x}{L}dx + \int_0^L 1dx\right) \quad \Rightarrow \quad \Delta L = \frac{Mg}{AY}\left[\frac{L}{2} + L\right]$$

$$\Delta L = \frac{Mg}{AY}\times\frac{3L}{2}$$

6. **Ans.** (i) 50 N, (ii) 0.045 J, 1.8×10^{-3} m (iii) 8.4×10^{-4} m, (iv) $x = 0.12$ m

(i) $2T = 100 \Rightarrow T = 50$

(ii) $Y = \dfrac{T/A}{\Delta\ell/\ell}$, $U = \dfrac{1}{2}\times$ stress × strain × volume

(iii) $Y = \dfrac{T/A}{\Delta\ell/\ell}$

(iv) $T_B + T_S = mg$

$T_Bx = T_S(L - x)$

$$Y_B = \frac{T_B/A_B}{\Delta\ell/\ell}$$

7. **Ans.** 25.5°C

$Q = \left(100\,\frac{cal}{s}\right)(240s) = 24000\ cal$

$Q = m_{al}S_{al}(20) + m_{ice}S_{ice}(20) + m_{ice}L_{ice} + m_{al}S_{al}(T) + m_wS_wT$

$\Rightarrow\ T = 25.5°C$

8. **Ans.** 1 : 1.26

Let ice = $x\ gm$

water = $(200 - x)gm$

Heat lost = Heat gain

$$\frac{30}{1000}\left(22.5\times10^5\right)+\frac{30}{1000}\times50\times4200 = \frac{200}{1000}\times50\times4200+\frac{100}{1000}\times50\times420 + \frac{x}{1000}\times\left(3.36\times10^5\right)$$

9. **Ans.** $5\alpha/3$

$L\,\alpha\,\Delta T + (2L)(2\alpha)\Delta T = 3L\alpha'\Delta T$

$\alpha' = \frac{5}{3}\alpha$

10. **Ans.** 5 sec slow

$T = 2\pi\sqrt{\frac{\ell}{g}}$

$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta\ell}{\ell}$

$\frac{\Delta T}{T} = \frac{1}{2}\alpha\Delta\theta = \text{time}\left(\text{lost/gained}\right)\text{ per sec}$

11. **Ans.** 10, 000 N

$F = YA\,\alpha\,\Delta T$

12. **Ans.** $5.8\times10^8\ Pa$

$\beta = \frac{P}{\frac{-\Delta V}{V}}$

Or $P = \beta\left(\frac{-\Delta V}{V}\right) = \beta\gamma\Delta\theta$

Or we know that $\gamma = 3\alpha$

$P = \beta(3\alpha)\Delta\theta$

$P = 1.6\times10^{11}\times10^{-6}\times3\times12\times10^{-6}\times100$

$P = 5.8\times10^8\ Pa$

13. **Ans.** $4\times10^{-6}\ m/^\circ C$

$\Delta y = \frac{1}{3}\left(\frac{\sqrt{3}}{2}\ell\right)\alpha\Delta\theta$

$\frac{\Delta y}{\Delta\theta} = \frac{1}{3}\left(\frac{\sqrt{3}}{2}\ell\right)\alpha$

14. **Ans.** $10cm$, $40cm$

$25\alpha_1\times100 = 0.05$

$40\alpha_2\times100 = 0.04$

$(\ell_1\alpha_1 + \ell_2\alpha_2)\times50 = 0.03$

$\ell_1 + \ell_2 = 50$

15. **Ans.** $F = \dfrac{(\alpha_1 L_1 + \alpha_2 L_2)t}{\left(\dfrac{L_1}{Y_1 A_1} + \dfrac{L_2}{Y_1 A_2}\right)}$

Let the first rod expand slightly (say by length δl) and the second rod get compressed by the same amount (since net elongation / compression of the rods is zero.)

$\therefore$ Natural increase in length of the first rod (after being heated) when free to expand would have been $\alpha_1 L_1 t$. The expansion allowed is just δl (where $\delta l < \alpha_1 L_1 t$).

$\therefore$ Amount of elongation restricted = $\alpha_1 L_1 t - \delta l$

$\therefore$ Strain = $\dfrac{\text{elongation restricted}}{\text{original length}}$ = $\dfrac{\alpha_1 L_1 t - \delta \text{l}}{L_1\ (1 + \alpha_1 t)}$

Since $\alpha_1 t << 1$

$\therefore$ $1 + \alpha_1 t \simeq 1$

$\therefore$ Strain = $\dfrac{\alpha_1 L_1 t - \delta l}{L_1}$

$\therefore$ Stress = strain × Y = $\left(\dfrac{\alpha_1 L_1 t - \delta l}{L_1}\right) Y_1$.

or F = stress × A = $\dfrac{(\alpha_1 L_1 t - \delta l)}{L_1} Y_1 A_1$ ... (i)

Similarly, $F = \dfrac{(\alpha_2 L_2 t + \delta l)}{L_2} y_2 t_2$

or $\delta l = \alpha_1 L_1 t - \dfrac{F L_1}{Y_1 A_1} = \dfrac{F L_2}{Y_1 L_2} - \alpha_2 L_2 t$

or $F = \dfrac{(\alpha_1 L_1 + \alpha_2 L_2)t}{\left(\dfrac{L_1}{Y_1 A_1} + \dfrac{L_2}{Y_1 A_2}\right)}$

16. **Ans.** 7/2

Temp of B = 20°C

$$\frac{dQ}{dt} = \frac{kA}{\ell_{BD}}(90° - 20°) = \frac{kA}{\ell_{BC}}(20° - 0°)$$

$$\frac{\ell_{BD}}{\ell_{BC}} = \frac{7}{2}$$

17. **Ans.** (a) – 100°C (b) 1000 J

(a) Temperature gradient = $\dfrac{(T_2 - T_1)}{1} = \dfrac{(0 - 100)}{1} = -100°C/m$

(b) Steady state temp of element dx is = T = 100 (1 – x)

Heat absorbed by the element to reach steady state is dQ

$dQ = (dm)s\Delta T = (\lambda dx)s(T - 0)$

$\Rightarrow$ $dQ = 20[100\ (1 - x)]dx$

Total heat absorbed by the rod

$Q = \int dQ = 2000\int_0^1 (1-x)dx = 1000J$

18. **Ans.** 4/3ω

Case I: $\frac{1}{R_{eq}} = \frac{1}{2R} + \frac{1}{2R}$

$= R_{eq} = R$

Rate of heat flow = $\frac{\Delta T}{R_{eq}} = \frac{T_1 - T_2}{R}$

$= \omega$ ...(1)

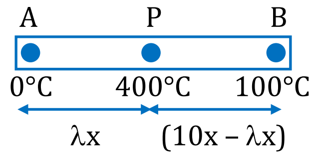
Case II: $\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{3R} \Rightarrow R_{eq} = \frac{4}{3R}$

Rate of heat flow = $\frac{\Delta T}{R_{eq}} = \frac{T_1 - T_2}{(3R/4)} = \frac{4}{3}\omega$

19. **Ans.** 9

For ice melting at 0°C

$80\frac{dm_1}{dt} = \frac{kA400}{\lambda x}$ ...(i)



For water evaporation at 100°C

$540\frac{dm_2}{dt} = \frac{kA(400-100)}{(10-\lambda)x}$ ...(ii)

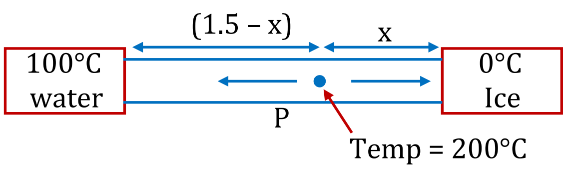
$\frac{dm_1}{dt} = \frac{dm_2}{dt}$

Divide equation (i) by (ii)

$\frac{540}{80} = \frac{300}{400} \times \frac{\lambda}{100-\lambda} \Rightarrow \lambda = 9$

20. **Ans.** 10.34 cm

$\frac{kA}{x}(200° - 0°) = 80\left(\frac{dm}{dt}\right)_{ice}$ ...(i)



$\frac{kA}{(1.5-x)}(200° - 100°) = 540\left(\frac{dm}{dt}\right)_{vapour}$ ...(ii)

$\left(\frac{dm}{dt}\right)_{ice} = \left(\frac{dm}{dt}\right)_{vapour}$

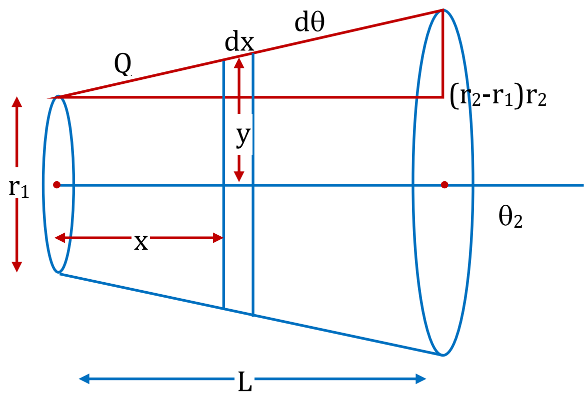
$\frac{kA \times 200}{80x} = \frac{kA \times 100}{540(105-x)} \quad \Rightarrow \quad x = 139.66cm$

Distance from steam end = (150 – x) cm

= (150 – 139.66)

= 10.34 cm

21. **Ans.** $\dfrac{K\pi r_1 r_2(\theta_2-\theta_1)}{L}$



$\text{Tan}\phi = \dfrac{r_2-r_1}{L}=\dfrac{(Y-r_1)}{x}$

$\Rightarrow\ xr_2-xr_1=yL-r_1L$

Differentiating *wr* to '*x*'

$\Rightarrow\ r_2-r_1=\dfrac{Ldy}{dx}-0$

$\Rightarrow\ \dfrac{dy}{dx}=\dfrac{r_2-r_1}{L}\Rightarrow dx=\dfrac{dyL}{(r_2-r_1)}$ ...(i)

Now, $\dfrac{Q}{T}=\dfrac{K\pi y^2 d\theta}{dx}\Rightarrow\dfrac{Qdx}{T}=k\pi y^2 d\theta$

$\Rightarrow\ \dfrac{QLdy}{r_2r_1}=K\pi y^2 d\theta$ from (i)

$\Rightarrow\ d\theta\dfrac{QLdy}{(r_2-r_1)K\pi y^2}$

Integrating both side

$\Rightarrow\ \int_{\theta_1}^{\theta_2}d\theta=\dfrac{QL}{(r_2-r_1)K\pi}\int_{r_1}^{r_2}\dfrac{dy}{y}$

$\Rightarrow\ (\theta_2-\theta_1)=\dfrac{QL}{(r_2-r_1)K\pi}\times\left[\dfrac{-1}{y}\right]_{r_1}^{r_2}$

$\Rightarrow\ (\theta_2-\theta_1)=\dfrac{QL}{(r_2-r_1)K\pi}\times\left[\dfrac{1}{r_1}-\dfrac{1}{r_2}\right]$

$\Rightarrow\ (\theta_2-\theta_1)=\dfrac{QL}{(r_2-r_1)K\pi}\times\left[\dfrac{r_2-r_1}{r_1+r_2}\right]$

$\Rightarrow\ Q=\dfrac{K\pi r_1 r_2(\theta_2-\theta_1)}{L}$

22. **Ans.** $1800J$

$\dfrac{d\theta}{dt}=\dfrac{60}{10\times 60}=0.1°C/\sec$

$\dfrac{dQ}{dt}=\dfrac{KA}{d}(\theta_1-\theta_2)$

$=\dfrac{KA\times 0.1}{d}+\dfrac{KA\times 0.2}{d}+.......+\dfrac{KA\times 060}{d}$

$=\dfrac{KA}{d}(0.1+0.2+.....+60)=\dfrac{KA}{d}\times\dfrac{600}{2}\times(2\times 0.1+599\times 0.1)$

$\left[\therefore a+2a+......+na=n/2\{2a+(n-1)a\}\right]$

$=\dfrac{200\times 1\times 10^{-4}}{20\times 10^{-2}}\times 300\times(0.2+59.9)=\dfrac{200\times 10^{-2}\times 300\times 60.1}{20}$

$3\times 10\times 60.1 = 1803J \approx 1800J$

23. **Ans.** $(T_2-T_1)e^{-\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1s_1m_2s_2}\right)t}$

$Q/t=KA(T_1-T_2)/L$

Fall in temp. in T_1 = $KA\,(T_1\text{-}T_2)/Lm_2s_2$

Rise in temp. in T_2 = > $KA\,(T_1 - T_2)\,/Lm_1s_1$

Final temp. $T_1 = T_1 - \dfrac{KA(T_1-T_2)}{Lm_1s_1}$

Final temp. $T_2 = T_2 + \dfrac{KA(T_1-T_2)}{Lm_1s_1}$

$$\frac{\Delta T}{dt}=T_1-\frac{KA(T_1-T_2)}{Lm_1s_1}-T_2-\frac{KA(T_1-T_2)}{Lm_2s_2}$$

$$\Rightarrow\ \frac{dT}{dt}=-\frac{KA(T_1-T_2)}{L}\left(\frac{1}{m_1s_1}+\frac{1}{m_2s_2}\right)$$

$$=(T_1-T_2)-\left[\frac{KA(T_1-T_2)}{Lm_1s_1}+\frac{KA(T_1-T_2)}{Lm_2s_2}\right]$$

$$\Rightarrow\ \frac{dT}{(T_1-T_2)}=-\frac{KA}{L}\left(\frac{m_2s_2+m_1s_1}{m_1s_1m_2s_2}\right)dt$$

$$\Rightarrow\ \ln\Delta t=-\frac{KA}{L}\left(\frac{m_2s_2+m_1s_1}{m_1s_1m_2s_2}\right)d+C$$

At time $t = 0$, $T = T_0$

$$\Rightarrow\ \ln\frac{\Delta T}{\Delta T_0}=-\frac{KA}{L}\left(\frac{m_2s_2+m_1s_1}{m_1s_1m_2s_2}\right)t$$

$$\Delta T=\Delta T_0\ \Rightarrow C=\ln\Delta T_0 \qquad \Rightarrow\ \frac{\Delta T}{\Delta T_0}=e^{-\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1s_1m_2s_2}\right)t}$$

$$\Rightarrow\ \Delta T=\Delta T_0e^{-\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1s_1m_2s_2}\right)t}=(T_2-T_1)e^{-\frac{KA}{L}\left(\frac{m_1s_1+m_2s_2}{m_1s_1m_2s_2}\right)t}$$

24. **Ans.** $\ln_2\dfrac{Lms}{2KA}$

$$\frac{\Delta Q}{\Delta t}=\frac{KA(T_1-T_2)}{L}$$

Rise in temperature in T_2 = > $(KA(t_1\text{-}t_2)/Lms$

Fall in temperature in $T_1 = \dfrac{KA(T_1-T_2)}{Lms}$

Final temperature in $T_1 = T_1 - \dfrac{KA(T_1-T_2)}{Lms}$

Final temperature $T_2 = T_2 + \dfrac{KA(T_1-T_2)}{Lms}$

Final $\frac{\Delta T}{dt}=T_1-\frac{KA(T_1-T_2)}{Lms}-T_2-\frac{KA(T_1-T_2)}{Lms}$

$$=(T_1-T_2)-\frac{2KA(T_1-T_2)}{Lms}=\frac{dT}{dt}=\frac{2KA(T_1-T_2)}{Lms}$$

$$\Rightarrow\ Ln\frac{(T_1-T_2)/2}{(T_1-T_2)}=\frac{-2KAt}{Lms}\Rightarrow \ln(1/2)=\frac{-2KAt}{Lms}$$

$$\Rightarrow \int_{(T_1-T_2)}^{(T_1-T_2)}\frac{dt}{(T_1-T_2)}=\frac{-2KA}{Lms}dt\ \Rightarrow \ln_2=\frac{2KAt}{Lms}\Rightarrow t=\ln_2\frac{Lms}{2KA}$$

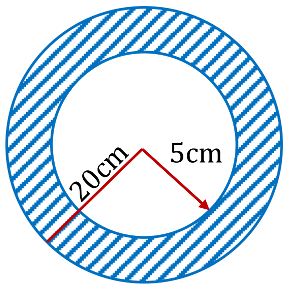
25. **Ans.** $3w/m-°C$

$a = r_1 = 5cm = 0.05m$

$b = r_2 = 20cm = 0.2m$

$\theta_1 = T_1 = 50°C,\ \theta_2 = T_2 = 10°C$

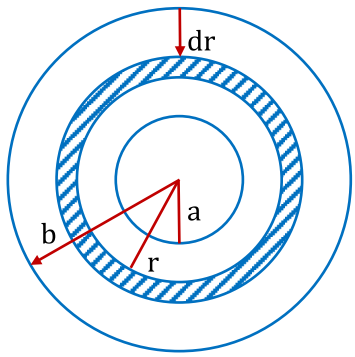
Now, considering a small strip of thickness 'dr' at a distance 'r'.



$A=4\pi r^2$

$H = -4\ \pi r^2 K\frac{d\theta}{dr}$ [(–)ve because with increase of r, θ decrease]

$=\int_a^b\frac{dr}{r^2}=\frac{-4\pi K}{H}\int_{\theta_1}^{\theta_2}d\theta$ On integration,



$$H=\frac{dQ}{dt}=K\frac{4\pi ab(\theta_1-\theta_2)}{(b-a)}$$

Putting the values we get

$$\frac{K\times4\times3.14\times5\times20\times40\times10^{-3}}{15\times10^{-2}}=100$$

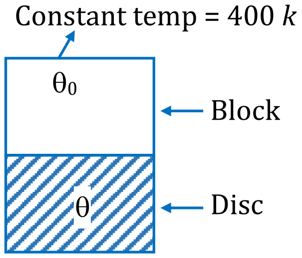
$$\Rightarrow\ K=\frac{15}{4}\times3.14\times4\times10^{-1}=2.985\approx3w/m-°C$$

26. **Ans.** 166.3 sec

θ = temp of disc

θ_0 = constant temp

Heat input to disc = $\frac{kA(\theta_0-\theta)}{L}$



Heat utilised by disc = $ms\frac{d\theta}{dt}$

S = Specific heat of disc

$$ms\frac{d\theta}{dt}=\frac{kA(\theta_0-\theta)}{L}$$

$$\Rightarrow\ \int_{300}^{350}\frac{d\theta}{(\theta_0-\theta)}=\frac{kA}{msL}\int_0^t dt\ \Rightarrow\ t=\frac{msL}{kA}\ell n\left(\frac{\theta_0-300}{\theta_0-350}\right)$$

$\Rightarrow\ t$ = 166.32 sec

27. **Ans.** 2 : 1

$$\frac{dQ}{dt} = ms\frac{dT}{dt}$$

$$\left[\frac{dQ}{dt}\right]_1 = m_1 s_1 \frac{dT}{dt}$$

$$\left[\frac{dQ}{dt}\right]_2 = m_2 s_2 \frac{dT}{dt}$$

as $\frac{dT}{dt} = \text{same}$

$$\frac{\left(\frac{dQ}{dt}\right)_1}{\left(\frac{dQ}{dt}\right)_2} = \frac{(\rho_1 V)S_1}{(\rho_2 V)S_2} = \frac{\rho_1 S_1}{\rho_2 S_2} = \frac{8}{1} \times \frac{1}{4} = \frac{2}{1}$$

28. **Ans.** $\left(\frac{6}{\pi}\right)^{1/3}$

$m \rightarrow$ same

$e \rightarrow$ same

$\rho \rightarrow$ same

$\Rightarrow$ volume same

$$\frac{4}{3}\pi R^3 = a^3 \Rightarrow a = \left(\frac{4}{3}\pi\right)^{1/3} R \qquad \text{...(i)}$$

$(4\pi R^2)R = 6a^2\left(\frac{a}{2}\right)$ from (i)

$$\frac{dT}{dt} \propto A$$

$$\frac{\left(\frac{dT}{dt}\right)_{cube}}{\left(\frac{dT}{dt}\right)_{sphare}} = \frac{A_1}{A_2} = \frac{6a^2}{4\pi R^2} = \frac{6}{4\pi} \times \left(\frac{4\pi}{3}\right)^{2/3} = \left(\frac{6}{\pi}\right)^{1/3}$$

29. **Ans.** 9

$\lambda_m T = b \quad \Rightarrow \quad \lambda_m \propto \frac{1}{T}$

$$\frac{T_A}{T_B} = \frac{\lambda_B}{\lambda_A} = \frac{1500}{500} = 3$$

Energy radiated = $E = \sigma A T^4$

$A = 4\pi R^2$

$$\frac{E_A}{E_B} = \left(\frac{T_A}{T_B}\right)^4 \left(\frac{R_A}{R_B}\right)^2 = 3^4 \left(\frac{6}{18}\right)^2 = 9$$

30. **Ans.** 1000 $J\ (C°)^{-1}$

$$\frac{d\theta}{dt} = ms\frac{dT}{dt}$$

ms = Heat capacity

$$\frac{d\theta}{dt} = (4 + 1.7 - 5.2)\ kw = 0.5$$

$$\frac{dT}{dt} = 0.5°Cs^{-1}$$

$$ms = \frac{0.5kw}{0.5°Cs^{-1}} = 10^3\ J/°C$$

31. **Ans.** 3025 K

$\lambda_m T$ = constant

$\Rightarrow$ (4753 × 6050) = T (9506)

T = 3025 k

32. **Ans.** 887J

$A = 1.6m^2$

$T = 37°C = 310\ K$

$\sigma = 6.0\times 10^{-8}\ w/m^2\text{–}k^4$

energy radiated per second is:

$A\sigma T^4 = 1.6\times6\times10^{-8}\ (310)4 = 8865801\times10^{-4} = 886.58 = 887\ J\ s$

33. **Ans.** $\left(\Delta T = \frac{mv_0{}^2}{3R}\right)$

$$\frac{1}{2}MV_0^2 = \frac{3}{2}\left(\frac{M}{m}\right)R\Delta T$$

34. **Ans.** 7

$\gamma = 1 + \frac{2}{f}$

$\therefore\ \ f = 7.$

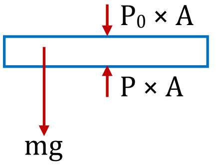
35. **Ans.** $\frac{3}{2}$

$$\gamma_{mix} = \frac{C_{Pmix}}{C_{Vmix}} = \frac{1\times\frac{5}{2}R + 1\times\frac{7}{2}R}{1\times\frac{3}{2} + 1\times\frac{5}{2}R} = \frac{3}{2}.$$

36. **Ans.** 327°C

$F = PA = mg + P_0A$

$P \times 0.1 \times 10^{-4} = 100 \times 10^{-3} \times 10 + 10^5 \times 10^{-4} \times 0.1$



$P = 2 \times 10^5$

$$\frac{P}{T} = \frac{nR}{V}$$

$$\frac{P}{T} = \frac{10^5}{300}$$

$T = 600$

37. **Ans.** 40

The molar mass of the gas is 40 *gm*, the number of degrees of freedom of the gas molecules is 6

$(0.2 - 0.15)\,M = 2$

$$\Rightarrow \; M = \frac{2}{.05} = 40$$

$C_V = 0.15 \times 40 = 6$

$$C_V = \frac{fR}{2}$$

38. **Ans.** $32°C$

Both are diatomic.

No. of moles of $O_2 = n_1 = \frac{m}{M} = \frac{16}{32} = \frac{1}{2}$

No. of moles of $N_2 = n_2 = \frac{m}{M} = \frac{14}{28} = \frac{1}{2}$

$\therefore\ (n_1 + n_2)\,C_V.\,T = n_1\,C_V\,T_1 + n_2\,C_V\,T_2$ [By energy conservation]

$$= T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

$$= \frac{\left(\frac{1}{2}\right)310 + \left(\frac{1}{2}\right)300}{\frac{1}{2} + \frac{1}{2}}$$

$= 305\ K = 32°C$

39. **Ans.** $\left(\ell = \frac{21840}{673}\,cm\right)$

At equilibrium pressure on both side is same, for initial condition $PV = n_1\,RT$ and $PV = n_2\,RT$

here यn$_1 = n_1'$ and $n_2 = n_2'$

for $n_1 = n_1'$ $\Rightarrow \frac{pA \times 40}{R \times 300} = \frac{p'A \times x}{R \times 273}$ ...(1)

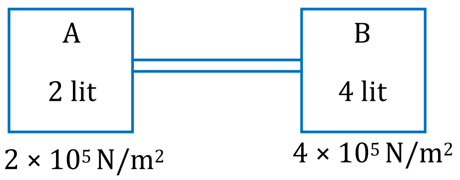
for $n_2 = n_2'$ $\Rightarrow \frac{pA \times 40}{R \times 300} = \frac{p'A \times (80 - x)}{R \times 400}$ ...(2)

from (1) and (2)

$$\frac{x}{273} = \frac{80 - x}{400} \Rightarrow x = \frac{80 \times 273}{673} = \frac{21840}{673}\ cm$$

40. **Ans.** $P = \frac{10}{3} \times 10^5\ N/m^2,\ T = \frac{10500}{31}\ K \simeq 338.71\ K$

From $PV = nRT$



$$n = \frac{PV}{RT}$$

$$\Rightarrow\ n_A = \frac{2 \times 10^5 \times 2 \times 10^{-3}}{\frac{25}{3} \times 300} = \frac{4}{25},$$

$$n_B = \frac{4 \times 10^5 \times 4 \times 10^{-3}}{\frac{25}{3} \times 350} = \frac{96}{175}$$

$U_1 + U_2 = U_{mix}$

$\Rightarrow$ $n_A C_V T_1 + n_B C_V T_2 = (n_A + n_B) C_V T$

$\Rightarrow$ $T = \dfrac{n_A T_1 + n_B T_2}{n_A + n_B}$

Putting values

$$P = \frac{(n_A + n_B)RT}{V_A + V_B}$$

Putting values

$$P = \frac{10}{3} \times 10^5 \; N/m^2$$

41. **Ans.** (a) $T_{\max} = \frac{2}{3}(p_0/R)\sqrt{p_0/3\alpha}$] (b) $T_{\max} = p_0/e\beta R$

(a) $pV = RT$

$$V = \frac{RT}{p}$$

$$p = p_0 - \alpha V^2$$

$$p = p_0 - \alpha\left(\frac{RT}{p}\right)^2$$

$$T = \frac{1}{R\sqrt{\alpha}}\sqrt{p_0 p^2 - p^3}$$

for maximum value T, $\sqrt{p_0 p^2 - p^3}$ should be maximum

$$\frac{d}{dP}(p_0 p^2 - p^3) = 0$$

$$p = \frac{2p_0}{3}$$

$$T = \frac{1}{R\sqrt{\alpha}}\sqrt{p_0 p^2 - p^3}$$

$$p = \frac{2p_0}{3}$$

$$T_{max} = \frac{2}{3}\,\frac{p_0}{R}\sqrt{\frac{p_0}{3\alpha}}$$

(b) $p = p_0 e^{-\beta V}$

or $\quad p = p_0\, e^{-\frac{\beta RT}{p}}$

for maximum value of T

$$\frac{dT}{dp} = 0$$

$$p = p_0\, e^{-\frac{\beta RT}{p}}$$

$$\ell n(p) = \ell n\, p_0 - \frac{\beta RT}{p}$$

$$\ell n\left(\frac{p}{p_0}\right) = -\beta\,\frac{RT}{p}$$

$$T = -\frac{p}{\beta R}\ \ell n\left(\frac{p}{p_0}\right)$$

for T_{max},

After solving $p = \frac{p_0}{e} \Rightarrow T_{max} = \frac{p_0}{e\beta R}$.

42. **Ans.** $m = (1 - e^{-Mgh/RT})\, p_0 S/g$

Barametric formula

$$P = P_o e^{-Mgh/RT}$$

$$\rho = \frac{PM}{RT}$$

at constant temperature.

$$\rho = \frac{MP_o}{RT} e^{-Mgh/RT}$$

Let us consider a mass element of the gas Contained in the column

$$dn = \rho(sdh) = \frac{MP_o}{RT} e^{-\frac{Mgh}{RT}} Sdh$$

$$m = \frac{MP_o S}{RT}\int_0^h e^{-Mgh/RT} dh$$

$$m = \frac{P_o S}{g}\left(1 - e^{-Mgh/RT}\right)$$

43. **Ans.** (a) 600 K, 900 K; (b) 10 J; (c) 14.9 J, 24.9 J; (d) 29.8 J

(a) As the process ab is isochoric.

V = constant

$\Rightarrow \frac{P}{T}$ constant

So, when pressure is double, temperature will be doubled.

$\Rightarrow T_b = 2 \times 300k = 600k$

bc is isobaric $\Rightarrow \frac{V}{T} = \text{constant}$

$\Rightarrow T_C = 1.5T_b$ = 900 k

(b) $\Delta W = \Delta W_{ab} + \Delta W_{bc}$

$= 0 + \left(200 \times 10^3 N/m^2\right)\left(50 \times 10^{-6} m^3\right)$

$= 10^5$

(c) $\Delta Q_{ab} = \Delta U_{ab} = \frac{f}{2} nR\Delta T = \frac{f}{2} V(\Delta P)$

$= \frac{3}{2} \times \left(100 \times 10^{-6} m^3\right)\left(100 \times 10^3 N/m^2\right)$ = 15 J

$\Delta Q_{bc} = \Delta U_{bc} + \Delta W_{bc}$

$= \left[\frac{3}{2}\left(200 \times 10^3\right)\left(50 \times 10^{-6}\right) + \left(200 \times 10^3\right)\left(50 \times 10^{-6}\right)\right] J$

= 15 J + 10 J = 25 J

(d) $\Delta U_{abc} = \frac{f}{2} nR(\Delta T)$

$$= \frac{f}{2}(\Delta PV) = \frac{3}{2}[P_c V_c - P_a V_a]$$

$$= \frac{3}{2}\left[\left(200 \times 10^{+3} \times 150 \times 10^{-6}\right) - \left(100 \times 10^{3} \times 100 \times 10^{-6}\right)\right] J$$

$$= \frac{3}{2} \times 20 = 30\, J$$

44. **Ans.** 1.5

$C_P \Delta T = 2500$

$100 \times 8.31 \times \frac{\gamma}{\gamma - 1} = 2500$

$\frac{\gamma}{\gamma - 1} = 3 \Rightarrow \gamma = 3\gamma - 3 \Rightarrow 3 = 2\gamma$

$\gamma = 1.5$

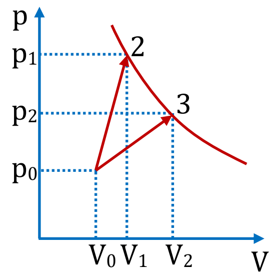
45. **Ans.** $-100\,\pi J$

work done = area under $P - V$ curve = Area of ellipse = π × Pressure radius × volume radius

$$= -\pi \frac{(30-10)}{2} \times 10^3 \times \frac{(30-10)}{2} \times 10^{-3} = -100\,\pi J.$$

46. **Ans.** $Q_1 < Q_2$

According to the first law of thermodynamics, the amount of heat ΔQ_1 received by a gas going over from state 1 (p_0, V_0) to state 2 (p_1, V_1) is



$\Delta Q_1 = \Delta U_1 + W_1$

where ΔU_1 is the change in its internal energy, and W_1 is the work done by the gas,

$$W_1 = \frac{(p_0 + p_1)(V_1 - V_0)}{2}$$

As the gas goes over from state 1 to state 3 (p_2, V_2) (points 2 and 3 lie on the same isotherm), the following relations are fulfilled :

$\Delta Q_2 = \Delta U_2 + W_2$,

$$W_2 = \frac{(p_0 + p_2)(V_2 - V_0)}{2}$$

Since the final temperature of the gas in states 2 and 3 is the same, $\Delta U_1 = \Delta U_2$. In order to find out in which process the gas receives a larger amount of heat, we must compare the works W_1 and W_2:

$$W_1 - W_2 = \frac{(p_0 + p_1)(V_1 - V_0)}{2} - \frac{(p_0 + p_2)(V_2 - V_0)}{2} = \frac{(p_0V_1 - p_0V_2) + (p_2V_0 - p_1V_0)}{2} < 0$$

47. **Ans.** $R/2$

$\frac{V}{1/T} = VT = \text{constant} = PV^2$

$\frac{3}{2}R + \frac{R}{1-2} = \frac{R}{2}$

48. **Ans.** (a) $2T_0$, $\frac{p_0}{2}$ in the vessel A and $\frac{T_0}{2^{\gamma-2}}$, $p_0/2^\gamma$ in vessel B, (b) $2T_0$, $P_0/2$

(a) For vessel A, Temperature is constant due to diathermic wall. For vessel B, no heat exchange due to adiabatically.

For vessel A

$P_1 V_1 = P_2 V_2$ $\quad (\therefore\ T = \text{const})$

$$P_2 = \frac{P_0(V_0/2)}{V_0} = \frac{P_0}{2}$$

Final temperature and pressure of vessel A is $2T_0$ and $\frac{P_0}{2}$

For vessel B adiabatic process.

$$P_2 = \frac{P_1V_1^\gamma}{V_2^\gamma} = \frac{P_0(V_0/2)^\gamma}{(V_0)^\gamma} = \frac{P_0}{2^\gamma}$$

$$T_2 = \frac{T_1V_1^{\gamma-1}}{V_2^{\gamma-1}} = \frac{2T_0(V_0/2)^{\gamma-1}}{(V_0)^{\gamma-1}} = \frac{T_0}{2^{\gamma-2}}$$

Final temperature and pressure of vessel B is

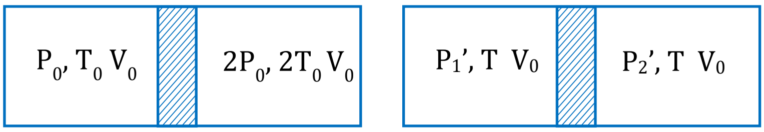
$\frac{T_0}{2^{\gamma-1}}$ and $\frac{P_0}{2^\gamma}$.

(b) After valve is open heat is flown to atmosphere from vessel A. So common temperature of both vessel is $2T_0$.

From mole conservation.

$$n_1 + n_2 = n_1' + n_2' = \frac{P_0(V_0/2)}{R2T_0} + \frac{P_0(V_0/2)}{R2T_0} = \frac{PV_0}{R2T_0} + \frac{PV_0}{R2T_0} \Rightarrow P = \frac{P_0}{2}$$

49. **Ans.** (a) zero (b) $\frac{3P_0}{2}$ on the left and $\frac{3P_0}{2}$ on the right (c) $\frac{3T_0}{2}$ (d) $\frac{3P_0V_0}{4}$



Initially finally

(a) As change in volume $\Delta V = 0$

$\therefore$ Work done by gas on right part = $\int p\,dV = 0$

(b) and (c) Due to adiabatic walls, no heat transfer from cylindrical tube.

$U_1 + U_2 = U_1 + U_2$

$$\frac{3}{2} n_1 RT_1 + \frac{3}{2} n_2 RT_2 = \frac{3}{2} n_1RT + \frac{3}{2} n_2 RT$$

$$T = \frac{n_1T_1 + n_2T_2}{n_1 + n_2}$$

$$= \frac{\frac{(P_0V_0)T_0}{RT_0} + \frac{2P_0(V_0)2T_0}{R(2T_0)}}{\frac{P_0V_0}{RT_0} + \frac{2P_0V_0}{R(2T_0)}} = \frac{3T_0}{2}$$

From mole conservation

$\frac{P_0(V_0)}{RT_0} = \quad \Rightarrow \quad p_1' = \frac{3P_0}{2}$

$\Rightarrow$ similarly $\frac{P_1'(V_0)}{RT} \quad p_2' = \frac{3P_0}{2}$

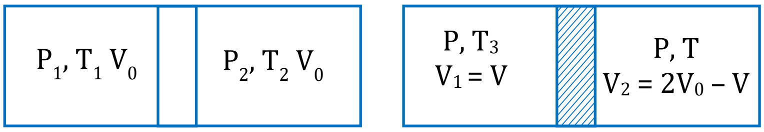
(d) Heat flow from the gas right to left.

$Q = \Delta U + W \qquad (\therefore\ W = 0)$

$Q = \Delta U = \frac{3}{2}\ n_1R(T - T_1)$

$= \frac{3}{2}\ \frac{P_0V_0R}{RT_0}\left(\frac{3T_0}{2} - T_0\right) = \frac{3P_0V_0}{4}$

50. **Ans.** (a) $\frac{2p_1^{1/\gamma}V_0}{A}$, $\frac{2p_2^{1/\gamma}V_0}{A}$, (b) zero, (c) $(A/2)^\gamma$ where $A = p_1^{1/\gamma} + p_2^{1/\gamma}$



(a) Initially finally

for left part $P_1\ (V_0)^\gamma = PV^\gamma$ ...(1)

for right part $P_2\ (V_0)^\gamma = P\ (2V_0 - V)^\gamma$ ...(2)

from eq. (1) and (2), we get

$V_1 = V = \frac{2V_0P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$

$V_2 = 2V_0 - V = \frac{2V_0P_2^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}$

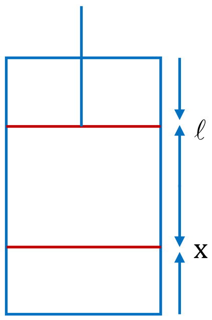
(b) heat given to the gas in left part is zero [As process is adiabatic]

(c) from eq (1)

$P_1\ (V_0)^\gamma = \left(\frac{2V_0P_1^{1/\gamma}}{P_1^{1/\gamma} + P_2^{1/\gamma}}\right)^\gamma\ P$

$\Rightarrow\ P = \left(\frac{P_1^{1/\gamma} + P_2^{1/\gamma}}{2}\right)^\gamma$

51. **Ans.** $\frac{nR}{P_aA}(T_S - T_0) + \left(1 - e^{-\frac{2KAt}{5nRx}}\right)$



$\Rightarrow \frac{n(5/2)RdT}{dt} = \frac{KA(T_S - T_0)}{x} \Rightarrow \frac{dT}{dt} = -\frac{2LA}{5nRx}(T_S - T_0)$

$\Rightarrow \frac{dT}{(T_S - T_0)} = -\frac{2KAdt}{5nRx} \Rightarrow \ln(T_S - T_0)_{T_0}^{T} = \frac{2KAdt}{5nRx}$

$\Rightarrow\ \ln\frac{T_S - T}{T_S - T_0} = -\frac{2KAdt}{5nRx} \Rightarrow T_s - T = (T_S - T_0)e^{-\frac{2KAt}{5nRx}}$

$\Rightarrow\ T = T_S - (T_S - T_0)e^{-\frac{2KAt}{5nRx}} = T_S + (T_S + T_0)e^{+\frac{2KAt}{5nRx}}$

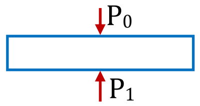
$\Rightarrow\ \Delta T = T - T_0 = (T_S - T_0) + (T_S - T_0)e^{+\frac{2KAt}{5nRx}} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{5nRx}}\right)$

$[p_a dv = nRdt]$ $\qquad$ $[p_a Al = nRdt]$ $\qquad$ $\left[dT = \frac{P_a AL}{nR}\right]$

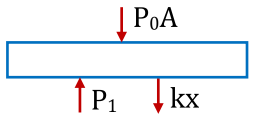
$$\Rightarrow \quad \frac{PaAL}{nR} = (T_S - T_0) + \left(1 + e^{+\frac{2KAt}{nR}}\right)$$

$$\Rightarrow \quad L = \frac{nR}{P_a A}(T_S - T_0) + \left(1 - e^{-\frac{2KAt}{5nRx}}\right)$$

52. **Ans.** (a) $P_i = P_a$ (b) $P_f = P_a + k(V - V_0)/A^2$ $V_0 = \ell A$ (c) $\Delta Q = P_a(V - V_0) + \frac{1}{2}k(V - v_0)^2 + C_v(T - T_0)$ $A = 1$

(A) 

$\boxed{A = 1}$

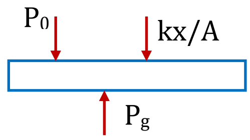
(B) 

$$P = P_0 + \frac{K(V_1 - V_0)}{A}$$

(C) $W = P_0(V_1 - V_0) + \frac{1}{2}K(V_1 - V_0)^2$

$$W = P_0 A x_0 + \frac{kx_0^2}{2} = P_0 A\left(\frac{V - V_0}{A}\right) + \frac{K}{2}\left(\frac{V - V_0}{A}\right)^2$$

53. **Ans.** 1.6 m, 364 K



$$P_g = P_0 + \frac{kx}{A} = P_0 + \frac{kh}{16A}$$

$$P_0(hA)^\gamma = P_g\left[\left(\frac{h}{2} + \frac{h}{16}\right)A\right]^\gamma$$

$$10^5(hA)^\gamma = \left[10^5 + \frac{3700 \times h}{16 \times 27 \times 10^{-4}}\right]\left[\frac{9h}{16}A\right]^\gamma$$

$$\frac{64}{27} = 1 + \frac{370 \times h}{16 \times 27}$$

$$\frac{37}{27} = \frac{370 \times h}{16 \times 27} \Rightarrow h = \frac{16}{10}$$

$PV = nRT$

$P_0 hA = nR \times 273$ (initial)

$$\left\{P_0 + \frac{kh}{16A}\right\}\left\{\frac{Ph}{16}A\right\} = nRT \quad \text{(final)}$$

divide

$$\left\{1 + \frac{kh}{16A \times P_0}\right\}\frac{9}{16} = \frac{T}{273}$$

$$T = 273 \times \frac{9}{16}\left[1 + \frac{370}{270}\right]$$

$$T = 273 \times \frac{4}{3} = 364k$$

54. **Ans.** 2

Given : $\frac{Q_h}{W_{in}} = (0.20)$ of max cop

$$cop = \frac{T_H}{T_H - T_L} = \frac{300}{30} = 10$$

$$\frac{Q_H}{W_{in}} = 0.20 \times 10 = 2$$

W_{in} = 1 J so $\boxed{Q_H = 2J}$

55. **Ans.** (A) 12·5% (B) 14·3%

For A engine T_1 = 400 K; T_2 = 350 K

$$\therefore \quad \eta_A = 1 - \frac{T_2}{T_1} = 1 - \frac{350}{400} = \frac{50}{400} = \frac{1}{8} \quad \Rightarrow \quad \%\ \eta_A = \frac{1}{8} \times 100 = 12{\cdot}5\%$$

For B engine T_1 = 350 K ; T_2 = 300 K

$$\therefore \quad \eta_B = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{350} = \frac{1}{7} \Rightarrow \%\ \eta_B = \frac{1}{7} \times 100 = 14{\cdot}3\%$$

56. **Ans.** 2400

W = 1200 J, T_1 = 800 K, T_2 = 400 K

$$\therefore \quad \eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} \Rightarrow 1 - \frac{400}{800} = \frac{1200}{Q_1} \quad \Rightarrow 0.5 = \frac{1200}{Q_1}$$

Heat energy supplied by source $Q_1 = \frac{1200}{0.5}$ = 2400 joule per cycle.

57. **Ans.** 5.7 J

T_2 = 260 K, T_1 = 315 K, W = 1 joule

Coefficient of performance of Carnot refrigerator $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\therefore \quad \frac{Q_2}{1} = \frac{260}{315 - 260} = \frac{260}{55} \Rightarrow Q_2 = \frac{260}{55} = 4.7\,J$$

$Q_2 + W = 5.7\,J$

58. **Ans.** $3.36 \times 10^6\,J$

Coefficient of performance (COP) = $\frac{T_2}{T_1 - T_2} = \frac{273}{300 - 273} = \frac{273}{27}$

$$W = \frac{Q_2}{COP} = \frac{mL}{COP} = \frac{100 \times 3.4 \times 10^5}{273/27} = \frac{100 \times 3.4 \times 10^5 \times 27}{273} = 3.36 \times 10^6\,J$$

59. **Ans.** (a) 19 J/K.mol); (b) 25 J/(K.mol)

For isochoric process

$$\Delta S = C_v \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = C_v \ln(n) = \frac{R}{\gamma - 1} \ln(n)$$

$$\Delta S = \frac{8.314}{0.3} \ln(2) = 19.2\,J/k\text{–mole}$$

$\approx 19\,J/k$-mole

For isobaric process

$$\Delta S = C_p \ln\left(\frac{T_f}{T_i}\right)$$

$$\Delta S = \frac{\gamma R}{\gamma - 1}\ln(n)$$

$$\Delta S = 24.96\frac{J}{k - mole}$$

$$\Delta S \approx 25\frac{J}{k - mole}$$

60. **Ans.** 4.4 J/K

Heat rejected by copper piece = Heat taken by water

$m_1c_1(T_1 - T) = m_2c_2(T - T_2)$

$m_1c_1T_1 + m_2c_2T_2 = (m_1c_1 + m_2c_2)T$

$$T = \frac{m_1c_1T_1 + m_2c_2T_2}{m_1c_1 + m_2c_2}$$

$$\Delta S = m_1c_1 \ln\left(\frac{T}{T_1}\right) + m_2c_2 \ln\left(\frac{T}{T_2}\right)$$

Here c_1 & c_2 = Specific heat capacity of copper & water.

61. **Ans.** Approx. 4 J/K

$$ds = \frac{dQ}{T} = \frac{dU + W}{T}$$

$$ds = nC_V \ln\left(\frac{T_2}{T_1}\right) + nR\ln\left(\frac{V_2}{V_1}\right)$$

$(T_1 = T_2)$

$$\Delta Q = W = nRT, \ln\left(\frac{V_2}{V_1}\right)$$

$$\Delta Q = (0.7)R \times T_1 \ln\left(\frac{10}{5}\right)$$

$$\frac{\Delta Q}{T_1} = 0.7R\ln(2) = 4.03J/k$$

4 J/k

EXERCISE - JEE (Main) PYQ

1. **Ans. (3)**

Thermal strain = $\alpha\Delta T$

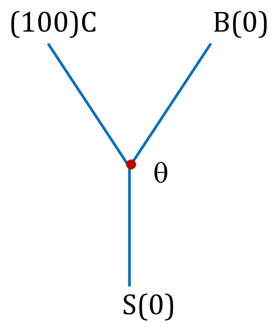
(by $\ell = \ell_0(1 + \alpha\,\Delta T)$)

$\Rightarrow$ Thermal stress in Rod (Pressure due to Thermal strain) = $Y\alpha\,\Delta T$

$= 2 \times 10^{11} \times 1.1 \times 10^{-5} \times 100$

$= 2.2 \times 10^{8}\ Pa$

2. **Ans. (1)**



$$\frac{100-\theta}{R_C} = \frac{\theta-0}{R_B} + \frac{\theta-0}{R_S}$$

Where, $R = \frac{\ell}{KA}$

On solving we get $\theta = 40$

Heat flow per unit time through copper rod

$$= \frac{(100-40)}{\ell_C}(K_C A_C) = \frac{60}{46} \times 0.92 \times 4 = 4.8\ cal/s$$

3. **Ans. (1)**

$$P = \frac{1}{3}\left(\frac{U}{V}\right)$$

$P \propto T^4$

using $PV = nRT$

$$\frac{1}{V} \propto T^3$$

$$\Rightarrow \quad T \propto \frac{1}{R}$$

4. **Ans. (2)**

$$T = 2\pi\sqrt{\frac{\ell}{g}}$$

$$\frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$$

When clock loses 12 sec

$$\frac{12}{T} = \frac{1}{2}\alpha\,(40-\theta) \qquad ...(1)$$

When clock gain 4 sec.

$$\frac{4}{T} = \frac{1}{2}\alpha\,(\theta-20) \qquad ...(2)$$

From equation (1) and (2)

$$3 = \frac{40-\theta}{\theta-20}$$

$3\theta - 60 = 40 - \theta$

$4\theta = 100$

$$\boxed{\theta = 25°C}$$

From equation (1)

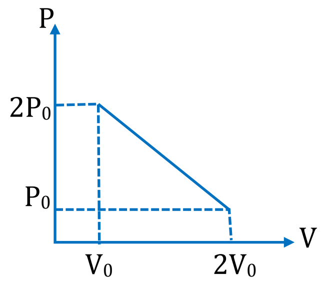
$$\frac{12}{T} = \frac{1}{2}\alpha(40-25)$$

$$\frac{12}{24 \times 3600} = \frac{1}{2}\alpha \times 15$$

$$\alpha = \frac{24}{24 \times 3600 \times 15}$$

$$\boxed{\alpha = 1.85 \times 10^{-5}\,/°C}$$

5. Ans. (2)



T will be max where product of PV is max.

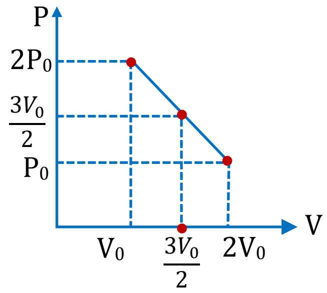
equation of line

$$P = \frac{-P_0}{V_0}V + 3P_0$$

$$PV = \frac{-P_0}{V_0}V^2 + 3P_0V = x \ \ (says)$$

$$\left.\begin{aligned}\frac{dx}{dV} = 0 \Rightarrow V = \frac{3V_0}{2} \\ \Rightarrow P = \frac{3P_0}{2}\end{aligned}\right\}\text{here } PV \text{ product is max.} \Rightarrow T = \frac{PV}{nR} = \frac{9}{4}\frac{P_0V_0}{nR}$$

Alternate



Since initial and final temperature are equal hence maximum temperature is at middle of line.

$PV = nRT$

$$\Rightarrow \frac{\left(\frac{3P_0}{2}\right)\left(\frac{3P_0}{2}\right)}{nR} = T_{max.} \Rightarrow \frac{9P_0V_0}{4nR} = T_{max}$$

6. Ans. (4)

Heat given = Heat taken

(100) (0.1)(T – 75) = (100)(0.1)(45) + (170)(1)(45)

10(T – 75) = 450 + 7650 = 8100

T – 75 = 810

T = 885°C

7. Ans. (1)

$C_P - C_V = R$

where C_P and C_V are molar specific heat capacities

As per the question

$$a = \frac{R}{2} \qquad\qquad b = \frac{R}{28}$$

$a = 14\,b$

8. Ans. (2)

In an adiabatic process

PV^{γ} = constant

and, $PV = nRT$, gives

$$\Rightarrow \quad V^{\gamma-1} \propto \frac{1}{T}$$

$$\left(\frac{V_1}{V_2}\right)^{\gamma-1} = \left(\frac{T_2}{T_1}\right)$$

$$T_2 = T_1 \left(\frac{V_1}{V_2}\right)^{\gamma-1}$$

monoatomic gas : $\gamma = \frac{5}{3}$

$$\Rightarrow \quad T_2 = (300\ K)\left(\frac{V}{2V}\right)^{\frac{5}{3}-1} = 189\ K \text{ (final temperature)}$$

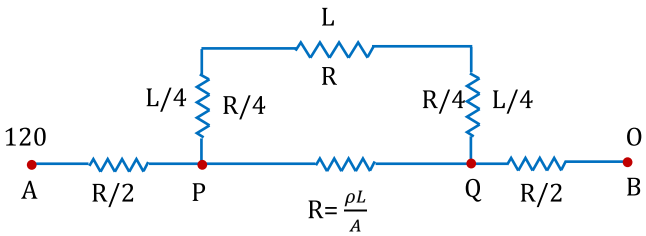
change in internal energy

$$\Delta U = n\frac{f}{2}R\ \Delta T = 2\left(\frac{3}{2}\right)\left(\frac{25}{3}\right)(-111) = -2.7\ kJ$$

9. Ans. (4)

$$\frac{\Delta T}{R_{eq.}} = I = \frac{(120)5}{8R} = \frac{120\times 5}{8R}$$

$$\Delta T_{PQ} = \frac{120\times 5}{8R}\times\frac{3}{5}R = \frac{360}{8} = 45°C$$

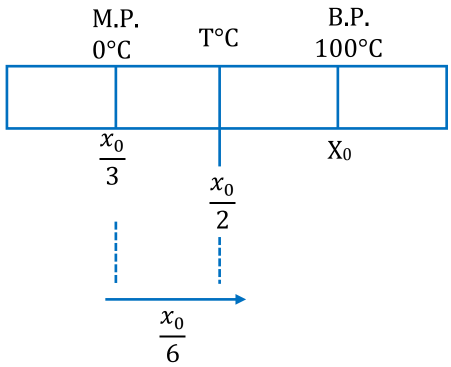


10. Ans. (2)

$$\Rightarrow T°C = \frac{x_0}{6}\ \&\left(x_0 - \frac{x_0}{3}\right) = (100-0°C)$$

$$x_0 = \frac{300}{2}$$

$$\Rightarrow T°C = \frac{150}{6} = 25°C$$



11. Ans. (4)

$$K_{eq} = \frac{K_1A_1 + K_2A_2}{A_1 + A_2}$$

$$K_{eq} = \frac{K_1(\pi R^2) + K_2(3\pi R^2)}{4\pi R^2}$$

$$K_{eq} = \frac{K_1 + 3K_2}{4}$$

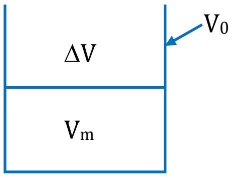
12. Ans. (2)

$$\frac{1}{2}mv^2 \times \frac{1}{2} = ms\Delta T$$

$$\Delta T = \frac{v^2}{4\times5} = \frac{210^2}{4\times30\times4.200}$$

$$= 87.5°C$$

13. Ans. (20)



$$\boxed{\Delta V = (V_0 - V_m)}$$

After increasing temperature

$$\Delta V' = (V'_0 - V'_m)$$

$$\Delta V' = \Delta V$$

$$V_0 - V_m = V_0(1+\gamma_b\Delta T) - V_m\,(1+\gamma_M\Delta T)$$

$$V_0\gamma_b = V_m\gamma_m$$

$$V_m = \frac{V_0\gamma_b}{\gamma_m} = \frac{(500)(6\times10^{-6})}{(1.5\times10^{-4})}$$

$$= 20\ CC$$

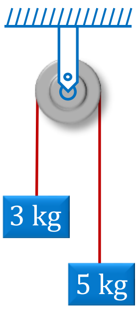
14. Ans. (1)

$$t \propto \frac{V}{\sqrt{T}} \qquad ...(1)$$

$$TV^{\gamma-1} = \text{constant} \qquad ...(2)$$

$$\therefore \quad t \propto V^{\frac{\gamma+1}{2}}$$

15. Ans. (3)



$$T = \frac{2m_1m_2g}{m_1+m_2} = \frac{2\times3\times5\times10}{8}$$

$$= \frac{75}{2}\,N$$

$$\text{Stress} = \frac{T}{A}$$

$$\frac{24}{\pi}\times 10^2 = \frac{75}{2\times\pi R^2}$$

$$R^2 = \frac{75}{2\times 24\times 100} = \frac{3}{8\times 24}$$

$\Rightarrow$ $R = 0.125\ m$

$R = 12.5\ cm$

16. Ans. (57)

By newton's law of cooling (with approximation)

$$\frac{\Delta T}{\Delta t} = -C\left(T_{avg} - T_s\right)$$

1st $\frac{-10°C}{5\min} = -C(70°C - 25°C)$

$\Rightarrow$ $C = \frac{2}{45}\min^{-1}$

2nd $\frac{T-65}{5\min} = -C\left(\frac{T+65}{2} - 25\right) = -\left(\frac{2}{45}\right)\left(\frac{T+15}{2}\right)$

$\Rightarrow$ $9\ (T - 65) = -\ (T + 15)$

$\Rightarrow$ $10T = 570$

$\Rightarrow$ $T = 57°C$

Alternate Solution :

Newton's law of cooling (without approximation)

$$T_P - T_S = (T_i - T_S)e^{-Ct}$$

1st $65 - 25 = (75 - 25)e^{-5C} \Rightarrow e^{-5C} = \frac{4}{5}$

2nd $T - 25 = (65 - 25)e^{-5C} = 40 \times \frac{4}{5} = 32$

$T = 57°C$

17. Ans. (1)

$$V_{RMS} = \sqrt{\frac{3RT}{M_W}}$$

At the same temperature $V_{RMS} \propto \frac{1}{\sqrt{M_w}}$

$\Rightarrow$ $V_H > V_O > V_C$

Option (1)

18. Ans. (3)

$B = 3 \times 10^{10}$

$$-\frac{\Delta V}{V} = 0.02$$

$$B = \frac{\Delta P}{-\frac{\Delta V}{v}} \Rightarrow \Delta P = -B\left(\frac{\Delta V}{V}\right)$$

$$= (3\times 10^{10})(0.02)$$

$$= 6\times 10^8\ N/m^2$$

19. Ans. (3)

$$\eta = \frac{WD}{Q_H}$$

$$\Rightarrow \quad WD = Q_H\left(1-\frac{T_L}{T_H}\right) = 5\times10^3\left(1-\frac{400}{1000}\right) = 3000\,kcal = 3000\times10^3\times4.18\,J = 12.6\times10^6\,J$$

20. Ans. (2)

$$\Delta Q = \Delta E + WD \Rightarrow Q = \Delta E + \frac{Q}{4}$$

$$\Rightarrow \quad n\frac{3R}{2}\Delta T = \Delta E = \frac{3Q}{4}$$

$$\therefore \quad n\Delta T = \frac{Q}{2R}$$

$$\therefore \quad C = 2R$$

21. Ans. (3)

Using average rate of Newton's law of cooling

$$\frac{T_1-T_2}{t} = K\left(\frac{T_1+T_2}{2}-T_s\right)$$

Given $\frac{60-40}{7} = K(50-10)$ ...(i)

And $\frac{40-T}{7} = K\left(\frac{40+T}{2}-10\right)$ ...(ii)

From (i) and (ii)

$T = 28°C$

22. Ans. (40)

Energy per unit volume = $\frac{1}{2}$ Stress × Strain

Energy $= \frac{1}{2}$ Stress × Strain × Volume

$$80 = \frac{1}{2}\times Y\times \text{Strain}^2\ A\times\ell$$

$$80 = \frac{1}{2}\times2\times10^{11}\times\frac{(2\times10^{-2})^2}{400}\times A\times20 \;\Rightarrow\; 20 = \frac{10^{+7}}{20}\times A \quad\Rightarrow\quad 40\times10^{-6}\,m^2 = A$$

$$A = 40\,mm^2$$

23. Ans. (4)

$$dQ = dU + dw$$

$$\frac{dU}{dt} = \frac{dQ}{dt} - \frac{dw}{dt}$$

$$\frac{dU}{dt} = 1000 - 200 = 800\,W$$

24. Ans. (1)

$$P = P_0 + \rho gh = 10^5\,Pa + 10^3\times10\times40 = 5\times10^5\,Pa$$

At T is constant

$$PV = P_0V_0 \Rightarrow 5\times10^5\,Pa\times1\,cm^3 = 10^5\,Pa\times V_0 \Rightarrow V_0 = 5\,cm^3$$

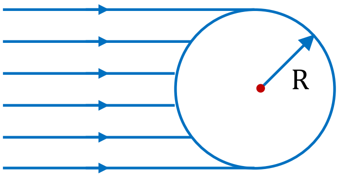
EXERCISE - JEE (Advanced) PYQ

1. Ans. (A)

Energy incident = $I\pi R^2 = 912 \times \pi R^2$

Energy emitted, assuming temp of the sphere at steady state to be T,

$\sigma \times 4\pi R^2 (T^4 - 300^4)$



At equilibrium,

$\sigma \times 4\pi R^2 (T^4 - 300^4) = 912 \times \pi R^2$

$\Rightarrow \quad T^4 - 300^4 = \dfrac{912}{5.7\times10^{-8}\times4} = 40 \times 10^8$

$\therefore \quad T^4 = (40 + 81) \times 10^8$

$\therefore \quad T = 331.66 \approx 330\ K$

2. Ans. (2)

$P = e\sigma A T^4$

λT = constant

$$\frac{P_A}{P_B} = \frac{e\sigma A_A T_A^4}{e\sigma A_B T_B^4} = \frac{A_A}{A_B}\frac{T_A^4}{T_B^4} = \frac{r_A^2}{r_B^2}\cdot\frac{\lambda_B^4}{\lambda_A^4}$$

$$10^4 = 400^2 . \left(\frac{\lambda_B}{\lambda_A}\right)^4$$

$$\frac{10^4}{16\times10^4} = \left(\frac{\lambda_B}{\lambda_A}\right)^4$$

$$\frac{\lambda_A}{\lambda_B} = \left(\frac{16}{1}\right)^{\frac{1}{4}} = 2$$

3. Ans. (AB)

Slope of this graph represents the reciprocal of Young's modulus.

Since, Slope of P > Slope of Q.

Hence, Y of P < Y of Q.

4. Ans. (B)

$$3000 - P = (120 \times 1)(4.2 \times 10^3)\ \frac{dT}{dt}$$

$$\frac{dT}{dt} = \frac{20}{60\times60\times3}$$

$P = 2067\ W$

5. Ans. (9)

$P = eA\sigma T^4$ where T is in kelvin

$$\log_2 \frac{eA\sigma(487+273)^4}{P_0} = 1 \qquad \text{...(i)}$$

$$\log_2 \frac{eA\sigma(2767+273)^4}{P_0} = x \qquad \text{...(ii)}$$

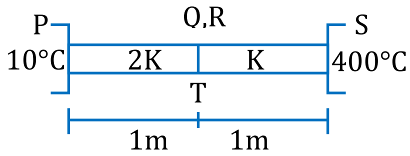
(ii) – (i)

$$\log_2\left(\frac{3040}{760}\right)^4 = x - 1$$

$\therefore \quad x = 9.$

6. **Ans. (A)**

Heat flow from P to Q



$$\frac{dQ}{dt}=\frac{2KA(T-10)}{1}$$

Heat flow from R to S

$$\frac{dQ}{dt}=\frac{KA(400-T)}{1}$$

At steady state heat flow is same in whole combination

$$\frac{2KA(T-10)}{1}=KA(400\text{-}T)$$

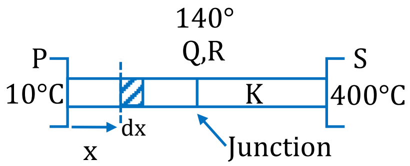
$2T - 20 = 400 - T$

$3T = 420$

$T = 140°C$

Temp of junction is 140°C

Temp at a distance x from end P is



$T_x = (130x + 10)$

Change in length dx is dy

$dy = \alpha dx(T_x - 10)$

$$\int_0^{\Delta y} dy=\int_0^1 \alpha dx(130x+10-10)$$

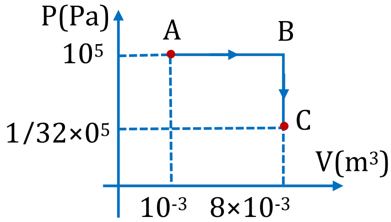
$$\Delta y=\left[\frac{\alpha x^2}{2}\times 130\right]_0^1$$

$\Delta y = 1.2 \times 10^{-5} \times 65$

$\Delta y = 78.0 \times 10^{-5}\ m = 0.78\ mm$

7. **Ans. (C)**

In adiabatic process



P^3V^5 = constant

$\Rightarrow$ $PV^{5/3}$ = constant

$\Rightarrow$ $\gamma=\frac{5}{3}\ \Rightarrow C_V=\frac{3}{2}R$ and $C_P=\frac{5}{2}R$

In another process

$\Delta Q = nC_P\Delta T + nC_V\Delta T$

$$=\frac{5}{2}nR(T_B-T_A)+\frac{3}{2}nR(T_C-T_B)$$

$$\Delta Q=\frac{5}{2}(P_BV_B-P_AV_A)+\frac{3}{2}(P_CV_C-P_BV_B)$$

Putting values

$\Delta Q = 587.5\ J \approx 588J$

8. **Ans. (C)**

Assumption : $e = 1$ (Black body radiation)

$P = \sigma A\,(T^4)$

(A) $P_{rad} = \sigma A T^4 = \sigma.1\,(T_0 + 10)^4$

$$= \sigma \cdot T_0^4\left(1+\frac{10}{T_0}\right)^4 \ (T_0 = 300 \text{ gives}) = \sigma\cdot(300)^4\cdot\left(1+\frac{40}{300}\right)\approx 460\times\frac{17}{15}\approx 520\,J$$

$\Rightarrow$ Energy lost in 1 second = 520 Joule

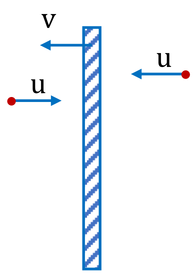
(B) $P = \sigma A\,(T^4)$

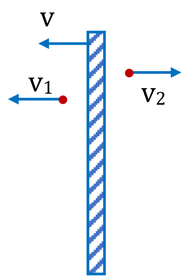
Since T = constant

P_{Radiated} = constant

Surface area decrease $\Rightarrow$ Energy radiation decreases

9. **Ans. (ABD)**





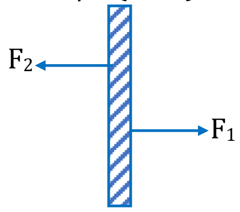
Just before the collision　　　　Just after the collision

$v_1 = u + 2v$　　　　$v_2 = (u - 2v)$

$\Delta v_1 = (2u + 2v)$　　　　$\Delta v_2 = (2u - 2v)$

$= F_1 = \frac{dp_1}{dt} = \rho A(u + v)\,(2u + 2v)$　　　　$= F_2 = \frac{dp_2}{dt} = \rho A(u - v)\,(2u - 2v)$

$= 2\rho A\,(u + v)^2$　　　　$= 2\rho A\,(u - v)^2$



$\Delta F = F_1 - F_2$　　(ΔF net force due to the air molecules on the plate)

$= 2\rho A\,(4uv) = 8\rho Auv$

$$P = \frac{\Delta F}{A} = 8\rho(uv)$$

$F_{net} = (F - \Delta F) = ma$　　(m is mass of the plate)

$F - (8\rho Auv) = ma$

10. **Ans. (B)**

Work (Column-I), process (Column-II) & corresponding graph (Column-III) are in this sequence.

I $\rightarrow$ IV $\rightarrow$ Q

II $\rightarrow$ III $\rightarrow$ P

III $\rightarrow$ II $\rightarrow$ S

IV $\rightarrow$ I $\rightarrow$ R

Only "B" option follow the sequence.

11. Ans. (A)

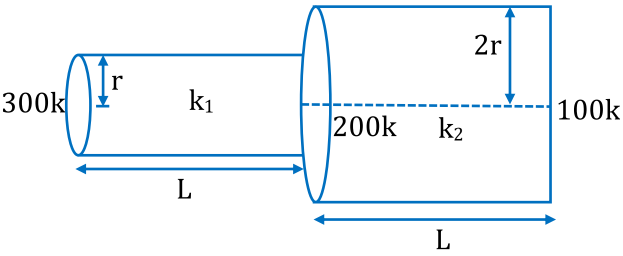
Only option "A" follow the sequence.

12. Ans. (D)

It is for an adiabatic process. Only option (D).

13. Ans. (4)

We have in steady state,

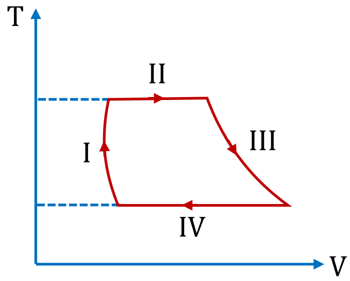


$$\left(\frac{\frac{200-300}{L}}{k_1\pi r^2}\right)+\left(\frac{\frac{200-100}{L}}{k_2\pi(2r)^2}\right)=0$$

$$\Rightarrow \quad \frac{k_1\pi r^2\times 100}{L}=\frac{100k_2\pi\times 4r^2}{L}$$

$$\Rightarrow \quad \frac{k_1}{k_2}=4$$

14. Ans. (BCD)



(A) Process-I is not isochoric, V is decreasing.

(B) Process-II is isothermal expansion

$\Delta U = 0$, $W > 0$

$\Delta Q > 0$

(C) Process-IV is isothermal compression,

$\Delta U = 0$, $W < 0$

$\Delta Q < 0$

(D) Process-I and III are NOT isobaric because in isobaric process $T \propto V$ hence isobaric T–V graph will be linear.

15. Ans. (C)

Process – I is an adiabatic process

$\Delta Q = \Delta U + W \qquad \Delta Q = 0$

$W = -\Delta U$

Volume of gas is decreasing $\Rightarrow W < 0$

$\Delta U > 0$

$\Rightarrow$ Temperature of gas increases.

$\Rightarrow$ No heat is exchanged between the gas and surrounding.

Process – II is an isobaric process

(Pressure remain constant)

$W = P\,\Delta V = 3P_0[3V_0 - V_0] = 6P_0V_0$

Process - III is an isochoric process

(Volume remain constant)

$\Delta Q = \Delta U + W$

$W = 0$

$\Delta Q = \Delta U$

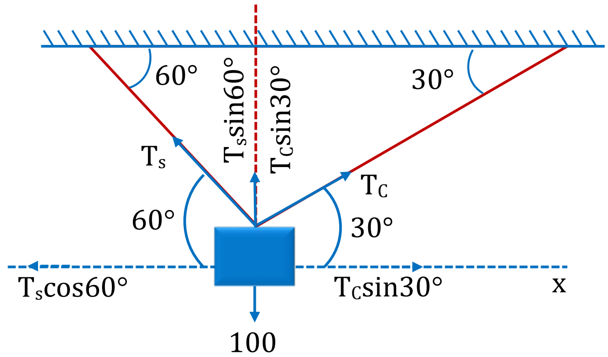
Process – IV is an isothermal process

(Temperature remains constant)

$\Delta Q = \Delta U + W$

$\Delta U = 0$

16. **Ans. (2.00)**



Let T_S = tension in steel wire

T_C = Tension in copper wire

In x direction

$$T_C \cos 30° = T_S \cos 60°$$

$$T_C \times \frac{\sqrt{3}}{2} = T_S \times \frac{1}{2} \quad \text{...(i)}$$

In y direction

$$T_C \sin 30° + T_S \sin 60° = 100$$

$$\frac{T_C}{2} + \frac{T_S\sqrt{3}}{2} = 100 \quad \text{...(ii)}$$

Solving equation (i) and (ii)

$$T_C = 50\ N$$

$$T_S = 50\sqrt{3}\ N$$

We know,

$$\Delta L = \frac{FL}{AY}$$

$$= \frac{\Delta L_C}{\Delta L_S} = \frac{T_C L_C}{A_C Y_C} \times \frac{A_S Y_S}{T_S L_S}$$

On solving above equation

$$\frac{\Delta L_C}{\Delta L_S} = 2$$

17. **Ans. (270.00)**

Let m = mass of calorimeter

x = specific heat of calorimeter

s = specific heat of liquid

L = latent heat of liquid

First $5g$ of liquid at 30° is poured to calorimeter at $110°C$

$\therefore \quad m \times x \times (110 - 80) = 5 \times s \times (80 - 30) + 5\ L$

$\Rightarrow \quad mx \times 30 = 250\ s + 5\ L \quad \text{...(i)}$

Now, $80g$ of liquid at 30° is poured into calorimeter at $80°C$, the equilibrium temperature reaches to $50°C$.

$\therefore \quad m \times x \times (80 - 50) = 80 \times s \times (50 - 30)$

$\Rightarrow \quad mx \times 30 = 1600s \quad \text{...(ii)}$

From (i) and (ii)

$250s + 5L = 1600\ s$

$\Rightarrow \quad 5L = 1350\ s$

$\Rightarrow \quad \frac{L}{s} = 270$

18. Ans. (ACD)

$n_1 = 5$ moles, $C_{V1} = \dfrac{3R}{2}$

$n_2 = 1$ mole, $C_{V2} = \dfrac{5R}{2}$

$$(C_V)_m = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2} = \frac{5\times\frac{3R}{2} + 1\times\frac{5R}{2}}{6} = \ = \frac{5R}{3}$$

$$(C_P)_m = \frac{5R}{3} + R = \frac{8R}{3}$$

$$\gamma_m = \frac{(c_P)_m}{(c_V)_m} = \frac{8}{5} \qquad \therefore \text{ Option 4 is correct}$$

$$P_0 V_0^{\gamma} = P\left(\frac{V_0}{4}\right)^{\gamma} \Rightarrow P = P_0(4)^{8/5}$$

(A) $\dfrac{3}{2}RT$ = 9.2 P_0 which is between $9P_0$ and $10P_0$

(B) Average K.E. = $5 \times 3\times\dfrac{RT}{2} + 1 \times \dfrac{5RT}{2} = 10RT$

To calculate T

$$\frac{P_0 V_0}{T_0} = 9.2P_0 \times \frac{V_0}{4\times T}$$

so $T = \dfrac{9.2}{4}T_0$

Now average $KE = 10\ R \times 9.2\ \dfrac{T_0}{4} = 23RT_0$

(C) $W = \dfrac{P_1V_1 - P_2V_2}{\gamma - 1} = \dfrac{P_0V_0 - 9.2P_0\times\frac{V_0}{4}}{3/5} = -13RT_0$

19. Ans. (D)

Process $1 \to 2$ is isothermal (temperature constant)

Process $2 \to 3$ is isochoric (volume constant)

(I) Work done in $1 \to 2 \to 3$

$W = W_{1\to 2} + W_{2\to 3}$

$$= nRT\ln\left(\frac{V_f}{V_i}\right) + W_{2\to3}$$

$$= \frac{RT_0}{3}\ln\left(\frac{2V_0}{V_0}\right) + 0$$

$$W = \frac{RT_0}{3}\ln 2 \ \Rightarrow \text{(P)}$$

(II) ΔU in $1 \rightarrow 2 \rightarrow 3$

$$\Delta U = \frac{f}{2} nR(T_f - T_i)$$

$$= \frac{3}{2} R\left(T_0 - \frac{T_0}{3}\right)$$

$$= \frac{3}{2} R\left(\frac{2T_0}{3}\right)$$

$$\boxed{\Delta U = RT_0} \Rightarrow \text{(R)}$$

(III) For any system, first law of thermodynamics

for $1 \rightarrow 2 \rightarrow 3$

$$\Delta Q = \Delta U + W$$

$$\Delta Q = RT_0 + \frac{RT_0}{3} ln2$$

$$\Delta Q = \frac{RT_0}{3}(3 + \ln 2) \Rightarrow \text{(T)}$$

(IV) For process $1 \rightarrow 2$ (isothermal)

$$\Delta Q = \Delta U + W$$

$$= \frac{f}{2} nR(T_f - T_i) + nRT \ln(V_f / V_i)$$

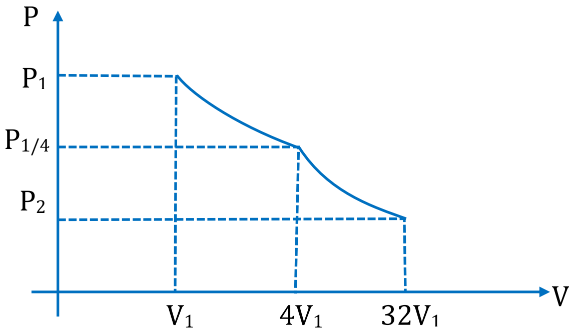
$$= 0 + R\left(\frac{T_0}{3}\right)\ln\left(\frac{2v_0}{v_0}\right)$$

$$\boxed{\Delta Q = \frac{RT_0}{3}\ln 2} \Rightarrow \text{(P)}$$

20. Ans. (1.77 to 1.78)

$$\frac{P_1}{4}(4V_1)^{5/3} = P_2(32v_1)^{5/3}$$

$$P_2 = \frac{P_1}{4}\left(\frac{1}{8}\right)^{5/3} = \frac{P_1}{128}$$



$$W_{\text{adi}} = \frac{P_1V_1 - P_2V_2}{\gamma - 1} = \frac{P_1V_1 - \frac{P_1}{128}(32V_1)}{\frac{5}{3} - 1}$$

$$= \frac{P_1V_1(3/4)}{2/3} = \frac{9}{8}P_1V_1$$

$$W_{\text{iso}} = P_1V_1 \ \text{In}\left(\frac{4V_1}{V_1}\right) = 2P_1V_1\ln 2$$

$$\frac{W_{iso}}{W_{adio}} = \frac{2P_1V_1\ln 2}{\frac{9}{8}P_1V_1} = \frac{16}{9}\ln 2 = f\ln 2$$

$$f = \frac{16}{9} = 1.7778 \approx 1.78$$

21. Ans. (BCD)

A = 64 mm^2, T = 2500 K (A = surface area of filament, T = temperature of filament, d is distance of bulb from observer, R_e = radius of pupil of eye)

Point source, d = 100 m

R_e = 3mm

(A) $P = \sigma A e T^4$

$= 5.67 \times 10^{-8} \times 64 \times 10^{-6} \times 1 \times (2500)^4$ (e = 1 black body)

= 141.75 w

Option (A) is wrong

(B) Power reaching to the eye

$$= \frac{P}{4\pi d^2} \times \left(\pi R_e^2\right)$$

$$= \frac{141.75}{4\pi \times (100)^2} \times \pi \times (3 \times 10^{-3})^2$$

$= 3.189375 \times 10^{-8}$ W

Option (B) is correct

(C) $\lambda_m T = b$

$\lambda_m \times 2500 = 2.9 \times 10^{-3}$

$\Rightarrow \quad \lambda_m = 1.16 \times 10^{-6}$

= 1160 nm

Option (C) is correct

(D) Power received by one eye of observer = $\left(\frac{hc}{\lambda}\right) \times \dot{N}$

$\dot{N}$ = Number of photons entering into eye per second

$\Rightarrow \quad 3.189375 \times 10^{-8}$

$$= \frac{6.63 \times 10^{-34} \times 3 \times 10^{8}}{1740 \times 10^{-9}} \times \dot{N}$$

$\Rightarrow \quad \dot{N} = 2.79 \times 10^{11}$

22. Ans. (8.33)

$$\frac{dQ}{dt} = \sigma e A(T^4 - T_0^4) \qquad \text{...(i)}$$

$$\frac{dQ}{Adt} = e\sigma(T_0 + \Delta T)^4 - T_0^4) = e\sigma T_0^4\left[\left(1 + \frac{\Delta T}{T_0}\right)^4 - 1\right]$$

$$= e\sigma T_0^4\left[\left(1 + 4\frac{\Delta T}{T_0}\right) - 1\right]$$

$$\frac{dQ}{Adt} = \sigma e T_0^3 \cdot 4\Delta T \qquad \text{...(ii)}$$

Now from equation (i)

$$ms\frac{dT}{dt} = \sigma e A(T^4 - T_0^4)$$

$$\frac{dT}{dt}=\frac{\sigma eA}{ms}[(T_0+\Delta T)^4-T_0^4] = \frac{\sigma eA}{ms}T_0^4\times\left[\left(1+\frac{\Delta T}{T_0}\right)^4-1\right]$$

$$\frac{dT}{dt}=\frac{\sigma eA}{ms}T_0^3\cdot 4\Delta T$$

$$\frac{dT}{dt}=K\Delta T\ ;\ \left(K=\frac{4\sigma eAT_0^3}{ms}\right)$$

$$\Rightarrow\quad 4\sigma eAT_0^3=\frac{K}{A}(ms)$$

From equation (i)

$$\frac{dQ}{Adt}=e\sigma T_0^3\cdot 4\Delta T$$

$700 = (K/A)\ (ms)\ \Delta T$

$$\therefore\quad \Delta T=\frac{700\times5\times10^{-2}}{10^{-3}\times4200}=\frac{50}{6}=\frac{25}{3}$$

$\Delta T = 8.33$

23. Ans. (9)

$$\sigma AT^4=-ms\frac{dT}{dt}$$

$$\int_{200}^{100}\frac{dT}{T^4}=-\int_0^{t_1}kdt$$

$$\left.\frac{1}{3T^3}\right|_{200}^{100}=kt_1$$

$$\frac{1}{3}\left(\frac{1}{100^3}-\frac{1}{200^3}\right)=kt_1$$

$$\left.\frac{1}{3T^3}\right|_{200}^{50}=kt_2$$

$$\frac{1}{3}\left(\frac{1}{50^3}-\frac{1}{200^3}\right)=kt_2$$

$$\frac{t_2}{t_1}=\left(\frac{200^3-50^3}{200^3-100^3}\right)\frac{100^3}{50^3}=9$$

24. Ans. (A)

Finally, $V_L=\dfrac{3V_0}{2},V_R=\dfrac{V_0}{2}$

$$C_v=\frac{R}{\gamma-1}=2R\Rightarrow\gamma-1=\frac{1}{2}$$

$$\gamma=\frac{3}{2}$$

For adiabatic process,

$$T_0V_0^{\gamma-1}=T_R\left(\frac{V_0}{2}\right)^{\gamma-1}\ \Rightarrow\frac{T_R}{T_o}=\sqrt{2}$$

25. Ans. (B)

Finally $V_L = \frac{3V_0}{2}, V_R = \frac{V_0}{2}$

$$C_V = \frac{R}{\gamma - 1} = 2R \Rightarrow \gamma - 1 = \frac{1}{2} \Rightarrow \gamma = \frac{3}{2}$$

$$T_0 V_0^{\gamma-1} = T_R \left(\frac{V_0}{2}\right)^{\gamma-1}$$

$$\frac{T_R}{T_0} = \sqrt{2}$$

$$\rho\left(\frac{V_0}{2}\right)^{\gamma} = P_0 V_0^{\gamma} \Rightarrow P = P_0 \times 2^{\frac{3}{2}}$$

$$\frac{PV}{T_L} = \frac{P_0 V_0}{T_0} \Rightarrow T_L = 2^{\frac{3}{2}} \times \frac{3}{2} T_0 = 3\sqrt{2} T_0$$

$$Q = nC_V \Delta T_1 + nC_V \Delta T_2 = 1 \times 2R \times \left(3\sqrt{2} - 1\right) T_0 + 1 \times 2R \times \left(\sqrt{2} - 1\right) T_0$$

$$\frac{Q}{RT_0} = 2\left(3\sqrt{2} - 1\right) + 2\left(\sqrt{2} - 1\right) = 8\sqrt{2} - 4 = 4\left(2\sqrt{2} - 1\right)$$

26. Ans. (B)

$$\frac{dm}{dt} = \rho_1 A_1 v_1 = 0.8\, kg/s$$

$$v_1 = \frac{0.8}{0.2 \times 0.1} = 40\, m/s$$

$g = 10\ m/s^2$

$\gamma = 2$

Gas undergoes adiabatic expansion,

$p^{1-\gamma}\, T^{\gamma}$ = Constant

$$\frac{P_2}{P_1} = \left(\frac{T_1}{T_2}\right)^{\frac{r}{1-\gamma}}$$

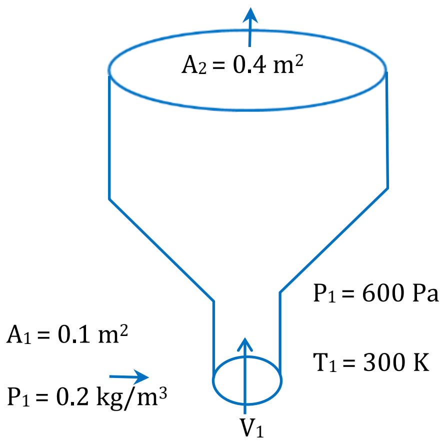
$$P_2 = \left(\frac{300}{150}\right)^{\frac{2}{-1}} \times 600$$

$$P_2 = \frac{600}{4} = 150 Pa$$

Now $\rho = \frac{PM}{RT} \Rightarrow \rho \propto \frac{P}{T}$

$$\frac{\rho_1}{\rho_2} = \left(\frac{P_1}{P_2}\right)\left(\frac{T_1}{T_2}\right) = \left(\frac{150}{600}\right)\left(\frac{300}{150}\right) = \frac{1}{2}$$

$$\rho_2 = \frac{\rho_1}{2} = 0.1\ kg/m^3$$



Now $\rho_2 A_2 v_2 = 0.8 \Rightarrow v_2 = \dfrac{0.8}{0.1\times 0.4} = 20 m/s$

Now $W_{\text{on gas}} = \Delta K + \Delta U$ + (Internal energy)

$$P_1A_1\Delta x_1 - P_2A_2\Delta x_2 = \frac{1}{2}\Delta m V_2^2 - \frac{1}{2}\Delta m V_1^2 + \Delta mgh + \frac{f}{2}(P_2\Delta V_2 - P_1\Delta V_1)$$

$$\Rightarrow \quad 2P_1\frac{\Delta V_1}{\Delta m} - 2P_2\frac{\Delta V_2}{\Delta m} = \frac{V_2^2 - V_1^2}{2} + gh$$

$$\Rightarrow \quad \frac{2\times 600}{0.2} - \frac{2\times 150}{0.1} = \frac{20^2 - 40^2}{2} + 10h$$

$h = 360\ m$

27. **Ans. (BCD)**

For adiabatic process (A $\rightarrow$ B)

$P_A V_A^\gamma = P_B V_B^\gamma$

$10^5 \times (0.8)^{\frac{5}{3}} = 3\times 10^5 (V_B)^{\frac{5}{3}}$

$$\Rightarrow \quad V_B = 0.8\times\left(\frac{1}{3}\right)^{0.6} = 0.4\ m^3$$

Work done in process $A \rightarrow B$

$$W_{AB} = \frac{P_AV_A - P_BV_B}{\gamma - 1}$$

$$\Rightarrow \quad W_{AB} = \frac{10^5\times 0.8 - 3\times 10^5\times 0.4}{\frac{5}{3} - 1}$$

$\Rightarrow \quad W_{AB} = -60\ kJ = \Rightarrow |W_{AB}| = 60\ kJ$

Work done in process $B \rightarrow C$ (Isothermal process)

$$W_{BC} = nRT\ell n\frac{V_C}{V_B} = P_BV_B\ell n\frac{V_C}{V_B}$$

$$\Rightarrow \quad W_{BC} = 3\times 10^5\times 0.4\ell n\frac{0.8}{0.4}$$

$\Rightarrow \quad W_{BC} = 84\ kJ$

Work done in process $C \rightarrow A$

$W_{CA} = P\Delta V = 0 \quad (\because \Delta V = 0)$

So total work done in the process $A\rightarrow B\rightarrow C$

$W_{ABC} = W_{AB} + W_{BC} + W_{CA} = -60 + 84 + 0$

$W_{ABC} = 24\ kJ$

So correct options are (B,C,D)

28. Ans. (A)

$Y = c^{\alpha} h^{\beta} G^{\gamma}$

$ML^{-1}T^{-2} = (LT^{-1})^{\alpha}(ML^2T^{-1})^{\beta}(M^{-1}L^3T^{-2})^{\gamma}$

$1 = \beta - \gamma$ ...(1)

$-1 = \alpha + 2\beta + 3\gamma$ ...(2)

$-2 = -\alpha - \beta - 2\gamma$ ...(3)

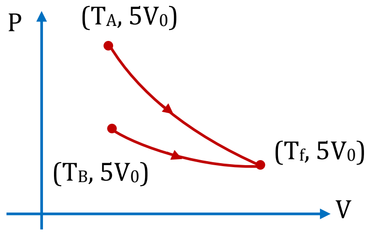
$-3 = \beta + \gamma$

$1 = \beta - \gamma$

$-2 = 2\beta \Rightarrow \beta = -1, \gamma = -2$

$-1 = \alpha - 2 - 6 \quad \therefore \alpha = 7$

29. Ans. (A)



$T_A V_0^{\gamma-1} = T_f (5V_0)^{\gamma-1}$

$\frac{T_A}{T_f} = 5^{\gamma-1} = \frac{T_A}{T_B}$

30. Ans. (121)

At constant pressure

$W = nR\Delta T = 66$

$\Delta U = n(C_V)_{mix}\Delta T$

$$(C_V)_{mix} = \frac{n_1 C_{V_1} + n_2 C_{V_2}}{n_1 + n_2}$$

$$(C_V)_{mix} = \frac{2 \times \frac{3}{2}R + 1 \times \frac{5}{2}R}{3}$$

$$(C_V)_{mix} = \frac{11}{6}R$$

$$\Delta U = \frac{11}{6}(nR\Delta T)$$

$$\Delta U = \frac{11}{6} \times 66 = 121J$$

31. Ans. (C)

$U = \frac{1}{2} f n r T = \frac{frT}{2}$

$\therefore$ A and B are wrong.

$$v_{\text{sound}} = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\left(\frac{2}{f} + 1\right)\frac{RT}{M}}$$

$\Rightarrow$ More 'f', less 'v'

$\therefore \quad v_5 > v_7$

32. **Ans. (2)**

$$\frac{W_I}{W_{II}}=\frac{4P_0V_0+8P_0V_0\ell n2-6P_0V_0-0}{4P_0V_0\ell n2-0-P_0V_0+0}$$

$$=\frac{8\ell n2-2}{4\ell n2-1}=2$$

JEE (Main) Practice Paper

SECTION–A

1. **Ans. (1)**

$$mgh=\frac{m}{5}L$$

2. **Ans. (2)**

$$Q=\int dQ=\int_{T=1K}^{T=2K} msdT \quad \int_1^2 aT^3dt=\frac{15a}{4}$$

3. **Ans. (3)**

$$Y=\frac{W/A}{\Delta\ell/\ell}$$

$$\frac{W}{\Delta\ell}=\frac{YA}{\ell}$$

Slope $\propto A$

4. **Ans. (1)**

> 100 *mm*

Since the coefficient of thermal expansion of steel is greater then metal *X*.

5. **Ans. (3)**

$\beta_{CDEH}=\alpha_x+\alpha_y$

6. **Ans. (1)**

Let *m kg* of steam condenses

1.12 × 1 × 65 = 540 × *m* + *m* × 1 × 20

Heat gain = Heat lost

7. **Ans. (4)**

$$\frac{dQ}{dt}=kA\frac{dT}{dx}$$

$$\frac{dT}{dx}\propto\frac{1}{kA}\Rightarrow \text{slope}\propto\frac{1}{k}$$

8. **Ans. (3)**

$$\frac{dQ}{dt}=\frac{2kA}{\ell}\ \Delta T_A$$

$$=\frac{kA}{\ell}\Delta T_B$$

9. Ans. (4)

$$\left(\frac{dm}{dt}\right)L = \frac{\Delta T}{R_{eq}}$$

10. Ans. (1)

$E = \sigma A(546)^4$ ...(i)

$E_{0°C} = \sigma A(273)^4$ ...(ii)

$$\frac{E}{E_{0°C}} = 2^4 \Rightarrow E_{0°C} = \frac{E}{16}$$

11. Ans. (4)

$$\lambda_0 T_0 = \frac{3}{4}\lambda_0 T_1 \Rightarrow T_1 = \frac{4}{3}T_0 \quad ...(i)$$

$$P_0 = \sigma A T_0^4 \quad ...(ii)$$

$$P_1 = \sigma A T_1^4 \quad ...(iii)$$

$$P_1 = \sigma A\left(\frac{4}{3}T_0\right)^4$$

12. Ans. (4)

$$\frac{E_2}{E_1} = \frac{\sigma A_2 T_2^4}{\sigma A_1 T_1^4} = \frac{R_2^2 T_2^4}{R_1^2 T_1^4}$$

$R_2 = 2R, R_1 = R$

$T_2 = 2T, T_1 = T$

$$\Rightarrow \quad \frac{E_2}{E_1} = \frac{(2R)^2(2T)^4}{(R)^2(T)^4} = 64$$

13. Ans. (1)

$P \times V = 2/m_1 \times R \times 298$

$1.5\, PV = (2/m_1 + 3/m_2)\, R \times 298 \Rightarrow 1 + \frac{m_1}{m_2} \times \frac{3}{2} = \frac{3}{2}$

14. Ans. (2)

$\frac{T^2}{V}$ = constant, $\frac{T_1}{T_2} = \sqrt{\frac{V_1}{V_2}}$

15. Ans. (1)

For point (i)

$P_0V_0 = nRT_1$

For point (ii)

$V_0 \times 4P_0 = nRT_2$

For point (iii)

$4V_0 \times P_0 = nRT_3$

16. Ans. (3)

$P(2V - V)$

$= PV = nRT$

17. Ans. (3)

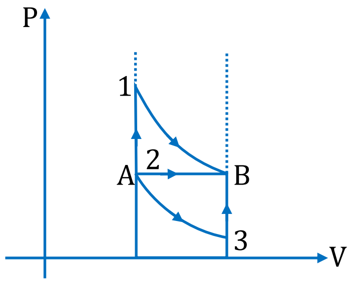
PV^{γ} = constant

$dPV^{\gamma} + (\gamma v^{\gamma-1}\, dv)P = 0$

$\Rightarrow \quad \frac{dP}{dV} = \frac{-\gamma P}{V}$

18. Ans. (1)

$W_1 = n\ RT_2 \ell n\left(\frac{v_2}{v_1}\right) \qquad \frac{v_1}{T_1} = \frac{v_2}{T_2}$



$= nRT_2\left(\ell n\left(\frac{T_2}{T_1}\right)\right)$

$W_2 = nR\Delta T = nR(T_2 - T_1) = nRT_2\left(1 - \frac{T_1}{T_2}\right)$

$W_2 = nRT_1 \ell n\left(\frac{v_2}{v_1}\right)$

19. Ans. (1)

$\Delta U_{Cycl} = 0 = U_{1-2} + U_{2-3} + U_{3-1} = 0 \qquad -\Delta U_{3-1} = W_{3-1}$

$0 + (-40) + U_{3-1} = 0 \qquad W_{3-1} = -40$

20. Ans. (4)

For gas A | For gas B

$\omega^2 = \frac{1}{3}\left(\frac{3RT}{M}\right) \qquad v^2 = \frac{3RT}{2M}$

$\frac{v^2}{\omega^2} = \frac{2}{3}$

SECTION-B

1. Ans. (3)

$Y = \frac{F/A}{\Delta\ell/\ell}$

Identical wires means (Y, A, L) same

$\Rightarrow \quad F \propto \Delta\ell$

$\frac{\Delta\ell_A}{\Delta\ell_B} = \frac{F_A}{F_B}$

$\Delta\ell_A = \Delta\ell_B \times \frac{F_A}{F_B} = 1.5 \times \frac{10g}{5g} = 3mm$

2. **Ans. (12)**

$100\, S_W(90 - 24) = m_S L_V + m_S S_W(100 - 90)$

$100 \times 1 \times 66 = m_S \times 540 + m_S \times 1 \times 10$

$m_S = 12\ gm$

3. **Ans. (65)**

$$\frac{dQ}{dt} = \frac{kA}{\ell}\Delta T$$

4. **Ans. (5)**

$\frac{dQ}{dt} = const.$ at steady state

$$\frac{kA}{\ell}(20^\circ - 10^\circ) = \frac{2kA}{\ell}(10^\circ - \theta^\circ)$$

$\theta = 5°C$

5. **Ans. (80)**

$$\frac{d\theta}{dt} = e\sigma A\left(T^4 - T_S^4\right) = ms\frac{dT}{dt}$$

6. **Ans. (10)**

$$\frac{(T_2 - T_1)}{t} = k\left[\frac{T_1 + T_2}{2} - T_S\right]$$

7. **Ans. (125)**

$$\frac{V}{T} = \frac{nR}{P} = \tan 53$$

$$\Rightarrow \quad P = \frac{2 \times 0.0821}{\tan 53}\ atm$$

8. **Ans. (30)**

$T \to 4T$

$$\left(\frac{3}{2}R\right) + 1\left(\frac{5}{2}R\right)\ (4 \times 300 - 300)$$

9. **Ans. (126)**

$6300 = nC_V \times 150 \qquad nC_V\, 300 = \frac{6300}{150} \times 300 = 12600J$

$8000 = nC_P 150$

10. **Ans. (416)**

$$2CV = \frac{2 \times 5}{2}R = 5R$$

JEE (Advanced) Practice Paper

1. **Ans. (A)**

Heat lost = Heat gain

$2kg\,Ice\ at -20°C \rightarrow 2kg\,Ice\,at\,0°C$

Heat gain = $2 \times 10^3 \times \frac{1}{2} \times 20 = 20\ Kcal$

5 Kg water a 20°C $\rightarrow$ 5 Kg water at 0°C

heat loss = 100 $Kcal$

Extra heat loss = 100 – 20 = 80 $Kcal$

Suppose x kg ice is melts upon supplying 80 $Kcal$ heat

80 $Kcal$ = $x \times 80\ cal/gm$

$x = 1\ Kg$

Final amount of water = 5 + 1 = 6 kg

2. **Ans. (B)**

Volume exp. = $\Delta V = V_0(\gamma_m - 3\alpha_g)T$

$\Delta V = A_f \times L = A_0(1 + 2\alpha_g T)L$

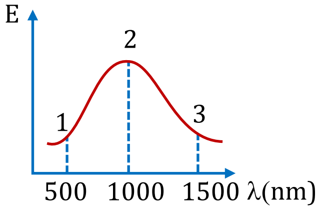
$\Rightarrow \quad L = \frac{V_0 T\left(V_m - 3\alpha_g\right)}{A_0\left(1 + 2\alpha_g T\right)}$

3. **Ans. (A)**

$\frac{dQ}{dt} \propto \frac{kA}{\ell} \propto \frac{r^2}{\ell}$

4. **Ans. (D)**

$\lambda_m T = b \ \Rightarrow \lambda = \frac{b}{T} = 1000nm$



$U_2 > U_1$, $U_2 > U_3$

5. **Ans. (C)**

$\eta_1 = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$

$Q_1 = \frac{Q_2}{1 - \eta_1}$

$\eta_2 = \frac{Q_2 - Q_3}{Q_2} = 1 - \frac{Q_3}{Q_2}$

$Q_3 = Q_2\,(1 - \eta_2)$

Now $\eta = \frac{Q_1 - Q_3}{Q_1} = 1 - \frac{Q_3}{Q_1} = 1 - \frac{Q_2\left(1 - \eta_2\right)}{Q_2}\left(1 - \eta_1\right)$

6. Ans. (D)

$PV^{\gamma} = C$

$$\ell nP + \gamma \ell nV = \ell nC$$

$\ell nP = -\gamma \ell nV + \ell nC$

$y = mx + C$

$m = -\gamma$

Slop of B > slope A

7. Ans. (D)

ΔU same

$16 - 20 = 9 - w$

$w = 9 + 4 = 13$

8. Ans. (C)

$PV = nRT$

$5 \times 4 = n_1RT$

$4 \times 4 = n_2RT$

Now – volume of vessel remain same.

Now – Temperature is same so KE same.

leakage = $\frac{n_1 - n_2}{n_1} \times 100 = \frac{4}{20} \times 100 = 20\%$

9. Ans. (AC)

$Y_S : Y_{Cu} = 2 : 1$

i.e. $Y_{Cu} = Y$

$Y_S = 2Y$

$$\frac{\Delta \ell_S}{\ell_S} = \frac{F}{2AY_S}$$

$$\Rightarrow \quad \Delta \ell_S = \frac{F\ell}{2AY_S} = \frac{F\ell}{2A(2Y)}$$

$$\Delta \ell_{Cu} = \frac{F(2\ell)}{AY}$$

& Stress in steel = $\frac{F}{2A}$

Stress in Cu = $\frac{F}{A}$

10. Ans. (ACD)

$PE = Mgl$

$EPE = ½\ Mgl$

as energy is conserved

$\therefore \quad PE$ = Heat energy + EPE

Heat energy = $Mgl - ½\ Mgl$

$= ½\ Mgl$

11. Ans. (AB)

$e_A = 0.01$, $e_B = 0.81$

Area = same for both

$\frac{dQ}{dt} = e\sigma AT^4$ = same for both

$\sigma e_A A T_A^4 = \sigma e_B A T_B^4$

$$\left(\frac{T_A}{T_B}\right)^4 = \frac{e_B}{e_A} = 81 = 3^4 \Rightarrow \qquad \frac{T_A}{T_B} = 3 \Rightarrow T_B = \frac{T_A}{3} = 1934$$

also $\lambda_B T_B = \lambda_A T_A$

$\lambda_B \times 1934 = (\lambda_A - 1)5802$

$\lambda_B = 3\lambda_B - 3$

$2\lambda_B = 3 \quad \Rightarrow \quad \lambda_B = 1.5\ \mu m$

12. Ans. (AC)

$m \propto V$

$$\frac{m_A}{m_B} = \frac{\frac{4}{3}\pi r_A^3}{\frac{4}{3}\pi r_B^3} = 4 = 2^2 \Rightarrow \frac{r_A}{r_B} = 2^{2/3}$$

e & σ is same for both

$$\frac{dQ}{dt} \propto \text{Area} \propto r^2$$

$$\frac{\left(\frac{dQ}{dt}\right)_A}{\left(\frac{dQ}{dt}\right)_B} = \left(\frac{r_A}{r_B}\right)^2 = 2^{4/3}$$

$$-\frac{dT}{dt} \propto \frac{\text{Area}}{\text{mass}}$$

$$\left|\frac{\left(\frac{dT}{dt}\right)_A}{\left(\frac{dT}{dt}\right)_B}\right| = \frac{2^{4/3}}{2^2} = 2^{-2/3}$$

13. Ans. (AB)

A $\rightarrow$ B

$$T_A^{\gamma} P^{1-\gamma} = T_B^{\gamma} P_B^{1-\gamma}$$

$$T_B = 1000\left(\frac{3}{2}\right)^{-2/5}$$

$$W_{AB} = -nC_V\Delta T = -2\times\frac{3}{2}\times R\left[T_B - T_A\right]$$

for B $\rightarrow$ C

$$\frac{P_B}{T_B} = \frac{P_C}{T_C} \Rightarrow T_C = T_B \times \frac{P_C}{P_B} = 2000\left(\frac{3}{2}\right)^{-2/5}$$

$$\Delta Q_{BC} = nC_V\Delta T = 2\times\frac{3}{2}R\left(T_C - T_B\right) = 3R\left(1000\times\left(\frac{3}{2}\right)^{-2/5}\right)$$

14. Ans. (AB)

$A \rightarrow B$ isobaric

$C \rightarrow D$ isochoric

in PT graph CD will be the line passing from origin

in VT graph AB will be the line passing from origin

15. Ans. (B)

$P_0[\ell A_2 + \ell A_1] = nRT_i$ ...(i)

$P_0V_0 = nRT$ ...(ii)

final

$P_0[(\ell + x)A_2 + (\ell - x)A_1] = nRT_f$ ...(iii)

from equation (iii) – (i)

$P_0x[A_2 - A_1] = nR(T_f - T_i)$

Now

$Q = P_0[A_2 - A_1]\,x + \frac{5}{2}nR\left(T_f - T_i\right)$ ...(iv)

$$Q = \frac{7}{2}nR\left(T_f - T_i\right)$$

from (ii)

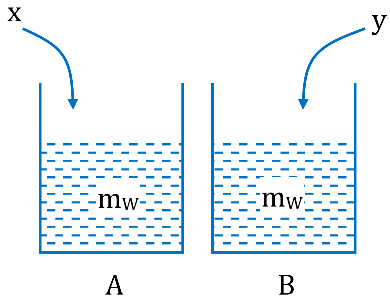
$$Q = \frac{7}{2}\frac{P_0V_0}{T_0}\left[T_f - T_i\right]$$

16. Ans. (A)

from (iv)

$$Pt = \frac{7}{2}P_0\left(A_2 - A_1\right)x$$

17. Ans. (27)



For (A)

$m_w S_w(22 - 20) = 5\, S_X(40 - 22)$ ...(i)

For (B)

$m_W S_W(23 - 20) = 5\, S_Y(40 - 23)$ ...(ii)

$$\frac{\text{equation (ii)}}{\text{equation (i)}} \Rightarrow \frac{3}{2} = \frac{S_Y}{S_x}\frac{17}{13} \Rightarrow S_Y = \frac{27}{85}$$

18. Ans. (0.75)

$$Y_S = \frac{F/A_S}{\Delta\ell/\ell_S}, Y_C = \frac{F/A_C}{\Delta\ell/\ell_C}$$

$$\frac{A_S}{A_C} = \frac{Y_C}{Y_S} \times \frac{\ell_S}{\ell_C} = \frac{7}{12} \times \frac{4.5}{3.5} = 0.75$$

19. Ans. (91)

$$\frac{dQ}{dt} = \frac{mL}{(30\,\text{min})} = ms\frac{dT}{dt}$$

$$\Rightarrow \quad \frac{S}{L} = \frac{1}{30 \times 3} = \frac{1}{90}$$

20. Ans. (10)

$$-\frac{d\theta}{dt} = k(\theta - \theta_0)$$

$$\int_{80}^{50} \frac{d\theta}{\theta - \theta_0} = -k\int_0^5 dt$$

$$\ell n\frac{50 - 20}{80 - 20} = -5k$$

$$k = -\frac{\ell n\left(\frac{1}{2}\right)}{5} \qquad \text{...(i)}$$

$$\text{then } \int_{60}^{30} \frac{d\theta}{(\theta - 20)} = \frac{\ell n\left(\frac{1}{2}\right)}{5} \times t$$

$$\ell n\frac{30 - 20}{60 - 20} = \frac{\ell n\left(\frac{1}{2}\right)}{5}t$$

$$5\ell n\left(\frac{1}{4}\right) = \ell n\left(\frac{1}{2}\right)t$$

$$5\ell n\left(\frac{1}{2}\right)^2 = \ell n\left(\frac{1}{2}\right)t$$

$\Rightarrow \quad t = 10$ min