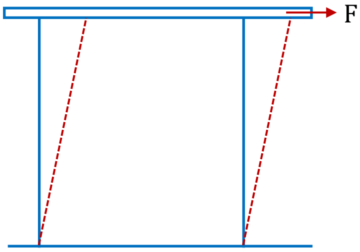
01 Fluid Mechanics

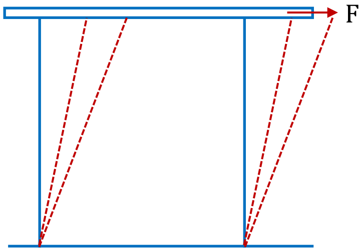
Fluid mechanics deals with the behaviour of fluids at rest and in motion. A fluid is a substance that deforms continuously under the application of shear (tangential) stress no matter how small the shear stress may be:

Thus, fluid comprise the liquid and gas (or vapour) phases of the physical forms in which matter exists.

We may alternatively define a fluid as a substance that cannot sustain a shear stress when at rest.



(a) Solid



(b) Fluid

Density of a Liquid

Density (ρ) of any substance is defined as the mass per unit volume or

$$\rho = \frac{\text{Mass}}{\text{Volume}}, \quad \rho = \frac{m}{V}$$

Relative Density (RD)

In case of a liquid, sometimes an another term relative density (RD) is defined. It is the ratio of density of the substance to the density of water at 4°C. Hence,

$$RD = \frac{\text{Density of substance}}{\text{Density of water at } 4°\text{C}}$$

RD is a pure ratio. So, it has no units. It is also sometimes referred as specific gravity.

Density of water at 4°C in CGS is 1 g/cm^3. Therefore, numerically the RD and density of substance (in CGS) are equal. In SI units the density of water at 4°C is 1000 kg/m^3.

Illustration 1:

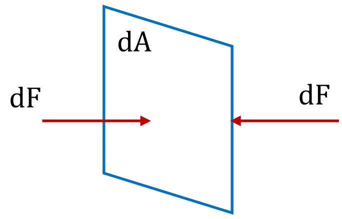
Relative density of an oil is 0.8. Find the absolute density of oil in CGS and SI units.

Solution:

Density of oil (in CGS) = $(RD) g/cm^3 = 0.8\ g/cm^3 = 800\ kg/m^3$

Pressure in a Fluid

When a fluid (either liquid or gas) is at rest, it exerts a force perpendicular to any surface in contact with it, such as a container wall or a body immersed in the fluid.



While the fluid as a whole is at rest, the molecules that makes up the fluid are in motion, the force exerted by the fluid is due to molecules colliding with their surrounding.

If we think of an imaginary surface within the fluid, the fluid on the two sides of the surface exerts equal and opposite forces on the surface, otherwise the surface would accelerate and the fluid would not remain at rest.

Consider a small surface of area dA centered on a point on the fluid, the normal force exerted by the fluid on each side is $dF_{\perp}$. The pressure P is defined at that point as the normal force per unit area, i.e.,

$$P = \frac{dF_{\perp}}{dA}$$

If the pressure is the same at all points of a finite plane surface with area A, then

$$P = \frac{F_{\perp}}{A}$$

where $F_{\perp}$ is the normal force on one side of the surface. The SI unit of pressure is pascal

where 1 pascal = $1Pa = 1.0\ N/m^2$

One unit used principally in meterology is the Bar which is equal to $10^5\ Pa$

$$1\ Bar = 10^5\ Pa$$

Note:

Fluid pressure acts perpendicular to any surface in the fluid no matter how that surface is oriented. Hence, pressure has no intrinsic direction of its own, its a **scalar**. By contrast, force is a vector with a definite direction.

Atmospheric Pressure (P_0)

It is pressure of the earth's atmosphere. This changes with weather and elevation. Normal atmospheric pressure at sea level (an average value) is $1.013 \times 10^5\ Pa.$

Absolute pressure and Gauge Pressure

The excess pressure above atmospheric pressure is usually called gauge pressure and the total pressure is called absolute pressure. Thus,

Gauge pressure = absolute pressure – atmospheric pressure

Absolute pressure is always greater than or equal to zero. While gauge pressure can be negative also.

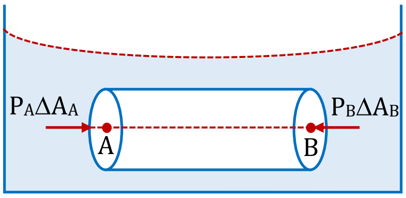
Variation of Pressure:

Variation of pressure in fluids can take place in several ways due to several reasons. We can classify them as follows :

Variation of Pressure

- (A) In Fluids at rest or at uniform speed
  - (1) In horizontal plane
  - (2) With height or depth
- (B) In Fluids under constant acceleration
  - (1) In horizontal direction
  - (2) In vertical direction
  - (3) In rotation

Variation of Pressure at two points in horizontal plane:



Consider two points A and B in the same horizontal line inside a fluid. Imagine small vertical areas ΔA_A and ΔA_B containing points A and B respectively. Consider the fluid contained in the horizontal cylinder bounded by ΔA_A and ΔA_B so that $\Delta A_A = \Delta A_B$. If pressure at A and B is P_A and P_B respectively then forces along the line AB are $P_A\,\Delta A_A$ towards right and $P_B\,\Delta A_B$ towards left. As the fluid remains in equilibrium,

$$P_A\,\Delta A_A = P_B\,\Delta A_B$$

$\Rightarrow$ $\quad P_A = P_B$

i.e., the pressure is same at two points in the same horizontal level.

Variation of pressure at two points in different height depth:

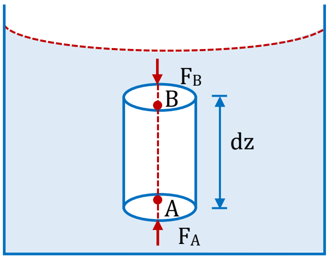
Consider two points A and B separated by a small vertical height dz. Imagine horizontal area ΔA_A and ΔA_B containing those points. Consider the fluid enclosed between the two surfaces ΔA_A and ΔA_B and the vertical boundary joining them, so that $\Delta A_A = \Delta A_B = \Delta A$. The vertical forces acting on this fluid are F_A vertically upward at ΔA_A, F_B vertically downward at ΔA_B and weight W of the cylindrical fluid vertically downward.

Thus, $\quad F_A = F_B + W$

Let - pressure at A is P and that at B is $P + dP$

then $\quad P\Delta A = (P + dP)\,\Delta A + \rho\,(\Delta A)\,(dz)g$; where ρ = density of fluid.

or $\quad dP = -\rho g\,dz$



i.e. as we move up through a height dz, the pressure decreases by $\rho g\,dz$.

Now consider two points at $z = 0$ and $z = h$ and if the pressure at $z = 0$ is P_1 and that at $z = h$ is P_2,

then, $\quad \int_{P_1}^{P_2} dP = \int_0^h -\rho g dz$

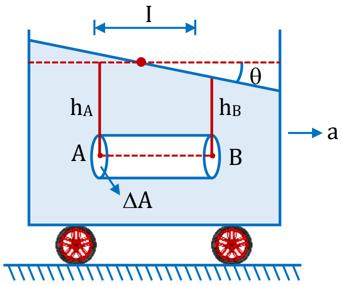
or $\quad P_2 - P_1 = -\rho gh$

or $\quad P_1 = P_2 + \rho gh$

i.e., if h be the variation in depth of two points then ρgh will be the variation in pressure at those points.

Variation of pressure at two points when fluid is in horizontal acceleration:

When a fluid is subjected to a constant acceleration, different elements of the fluid orient themselves so as to obtain a new equilibrium position with respect to the vessel. Since the fluid particles are relatively at rest with respect to one another, therefore, the laws of fluid statics are applicable but in the modified form to take into account the effect of acceleration.



Consider a horizontally accelerating container with liquid in it. Take two points A and B at a horizontal distance l (along the line of acceleration a). Take a horizontal cylinder within the two areas ΔA containing points A and B. Let pressure at A is P_A and at B is P_B.

The forces along the line AB are $P_A\,\Delta A$ towards right and $P_B\Delta A$ towards left. Under the action of these forces the element is accelerating,

thus, $\quad P_A\,\Delta A - P_B\,\Delta A = ma$

or $\quad (P_A - P_B)\,\Delta A = (\Delta A)\, l\,\rho a$

or $\quad P_1 - P_2 = l\,\rho a$

i.e., two points in same horizontal level do not have equal pressure for a fluid accelerating horizontally. If atmospheric pressure is P_0 then, $P_A = P_0 + h_A\,\rho g$ and $P_B = P_0 + h_B\,\rho g$, as there is no vertical acceleration. h_A and h_B are the depths of points A and B from free surface.

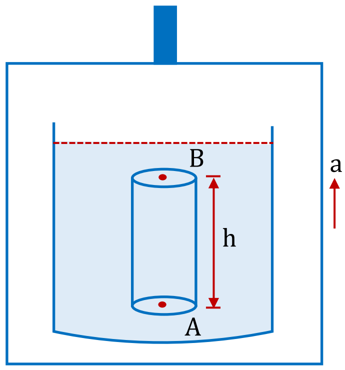
$\therefore \quad h_1\rho g - h_2\,\rho g = l\,\rho a$

or $\quad \dfrac{h_1 - h_2}{\ell} = \dfrac{a}{g}$

or $\quad \tan\theta = \dfrac{a}{g}$

where θ is the inclination of the free surface with the horizontal.

Variation of pressure at two points when fluid is in vertical acceleration:



Consider points A *and* B inside liquid with a vertical separation h where the liquid is accelerating upward. Consider a vertical cylinder of length h with boundaries at A and B. ΔS is the surface area of flat boundaries.

Let P_A and P_B are the pressures at A and B respectively. Thus, forces acting on the cylinder are $P_A\Delta S$ upward $P_B\Delta S$ downward and $W = (\Delta S)h\rho g$ downward. Under the action of these forces liquid cylinder is accelerating upward with acceleration *'a'*.

Thus, $\quad P_A\,\Delta S - P_B\,\Delta S - mg = ma$

or $\quad (P_A - P_B) = \rho\,(g + a)\,h$

Special cases :

(i) If a is (–) ve i.e. the vessel is accelerating downward then, $(P_A - P_B) = \rho\,(g - a)\,h$.

(ii) If a is greater than g then fluid occupies the upper part of the container.

Variation of pressure when fluid is under both horizontal and vertical acceleration:

When a fluid is accelerating in arbitrary direction then pressure varies both horizontally and vertical separately for horizontal and vertical component of acceleration respectively.

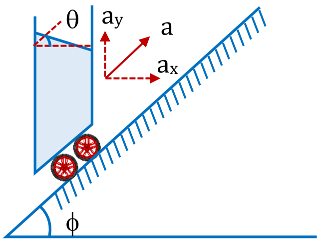
Here, $\vec{a} = a_x\hat{i} + a_y\hat{j}$ and $a_x = a\cos\phi$ and $a_y = a\sin\phi$ Now, difference in pressure at two points in same horizontal level at a separation l is given by

$$\Delta P = l\rho a_x = l\rho a\cos\phi$$

And similarly, for two points in same vertical line with a separation h, the pressure difference is

$$\Delta P = \rho(g + a_y)h = \rho(g + a\sin\phi)h$$

Inclination of the free surface with respect to the horizontal is given by



$$\tan\theta = \frac{a_x}{g+a_y}$$

or $$\tan\theta = \frac{a\cos\phi}{g+a\sin\phi}$$

Variation of pressure in a rotating fluid:

Consider the shape of the curved surface in the frame of container

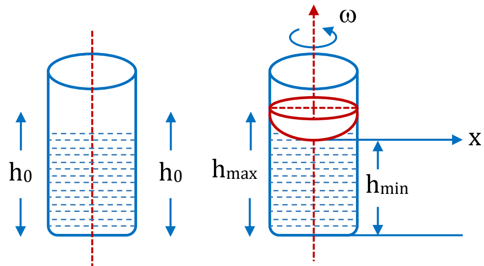


Figure-1 Figure-2

$$\frac{dy}{dx} = \tan\theta = \frac{m\omega^2 x}{mg} = \frac{\omega^2 x}{g}$$

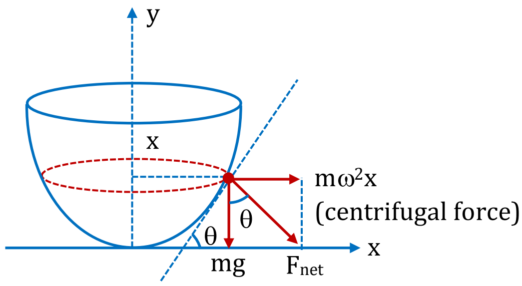
$$y = \frac{\omega^2 x^2}{2g}$$

The free surface of the liquid is the paraboloid of revolution

From figure (2)

$$h_{max} = h_{min} + h$$

$$h_{max} = h_{min} + \frac{\omega^2 x^2}{2g} \quad ...(1)$$



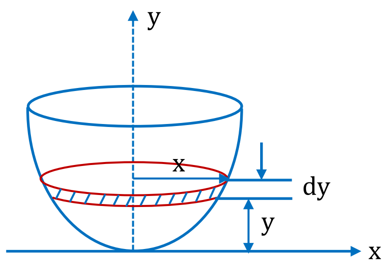
From volume conservation

$$h_0\pi R^2 = h_{max}\pi R^2 - V_P \quad ...(2)$$

where V_P = volume of paraboloid of revolution.

Calculation of V_P :

Volume of elemental disc = $dV = \pi x^2 dy$



$$y = \frac{\omega^2 x^2}{2g}$$

$$dy = \frac{\omega^2 x}{g}dx$$

$$dV = \frac{\pi\omega^2 x^3 dx}{g}$$

$$V_P = V = \int dV = \frac{\pi\omega^2}{g}\int_0^R x^3 dx$$

Volume of paraboloid of revolution = V_P

$$\frac{\pi\omega^2R^4}{4g}=\frac{\pi R^2}{2}\left(\frac{\omega^2R^2}{2g}\right) \qquad ...(3)$$

Putting (3) in (2)

$$h_0\pi R^2 = h_{max}\,\pi R^2 - \frac{\pi R^2}{2}\left(\frac{\omega^2R^2}{2g}\right)$$

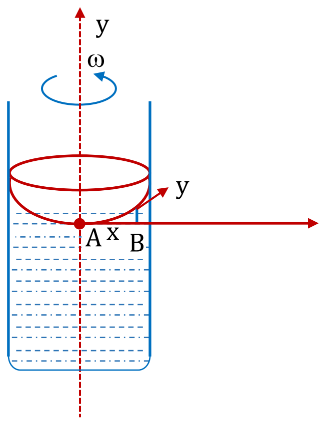
$$h_0 = h_{max} - \frac{\omega^2R^2}{4g} \qquad ...(4)$$

From (1) and (4)

$$h_{max} = h_{min} + \frac{\omega^2R^2}{2g}$$

$$h_{max} = h_0 + \frac{\omega^2R^2}{4g} \qquad ...(5)$$

$$h_{max} = h_0 - \frac{\omega^2R^2}{4g} \qquad ...(6)$$



From (5) and (6), it can be concluded that rise in the level of liquid at the periphery is equal to the fall in the level at the axis.

Pressure variation at any point inside the liquid.

$$P_B = P_0 + \rho g y$$

$$P_B = P_0 + \rho g\left(\frac{\omega^2x^2}{2g}\right)$$

$$P_B = P_0 + \frac{\rho\omega^2x^2}{2}$$

Illustration 2:

A beaker of circular cross-section of radius 4 *cm* is filled with mercury upto a height of 10 *cm*. Find the force exerted by the mercury on the bottom of the beaker. The atmospheric pressure = 10^5 N/m^2. Density of mercury = 13600 kg/m^3. Take g = 10 m/s^2.

Solution:

The pressure at the surface

= Atmospheric pressure

= 10^5 N/m^2

The pressure at the bottom

= 10^5 N/m^2 + $h\rho g$

= $10^5\ N/m^2 + (0.1\ m)\left(13600\frac{kg}{m^3}\right)\left(10\frac{m}{s^2}\right)$

= 10^5 N/m^2 + 13600 N/m^2

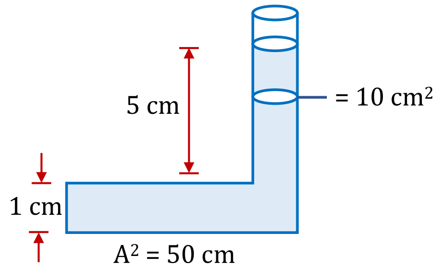
= 1.136×10^5 N/m^2

The force exerted by the mercury on the bottom

= $(1.136 \times 10^5\ N/m^2) \times (3.14 \times 0.04\ m \times 0.04\ m)$

= 571 N

Illustration 3:



The vessel shown in the adjoining fig. is filled with water of density

$\rho_w = 10^3\ kgm^{-3}$

(i) Find the net force acting at the base of the vessel.

(ii) What is the weight of water in the vessel?

Solution:

(i) Pressure at the base of the vessel is

$$P = \rho_w gh = (10^3)(10)(6 \times 10^{-2}) = 600\ Pa$$

$$\therefore\quad F = PA = (600)(50 \times 10^{-4}) = 3N$$

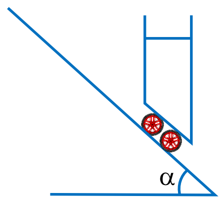
(ii) Weight of water in the tube is

$$W = \rho_w g\,(A_1 h_1 + A_2 h_2)$$

or $W = (10^3)(10)[(10 \times 10^{-4})(5 \times 10^{-2}) + (50 \times 10^{-4})(1 \times 10^{-2})]$

$$W = 1N$$

Illustration 4:



A trolley containing a liquid of density ρ slides down a smooth inclined plane of angle α with the horizontal. Determine the angle of inclination θ of the free surface with the horizontal.

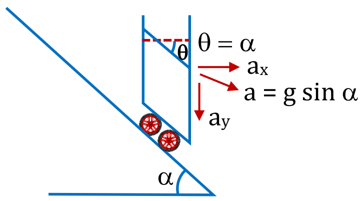
Solution:

The acceleration of the trolley along the inclined plane is

$$a = g \sin\alpha$$

$$a_x = a\cos\alpha = g\sin\alpha\cos\alpha$$

$$a_y = a\sin\alpha = g\sin^2\alpha$$



The inclination of the free surface is

$$\tan\theta = \frac{a_x}{g - a_y} = \frac{g\sin\alpha\cos\alpha}{g(1-\sin^2\alpha)} = \tan\alpha$$

or $\theta = \alpha$

Note:

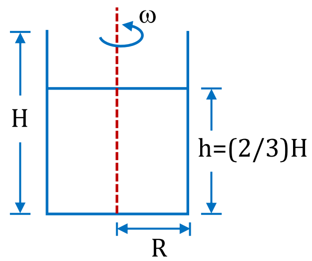
that the free surface becomes parallel to the inclined plane. It is because the net effective gravity inside the trolley is perpendicular to the inclined plane.

Illustration 5:

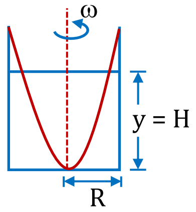
A cylinder of radius $R = 1\ m$ and height $H = 3\ m$, two-third filled with water, is rotated about its vertical axis, as shown in figure. Determine the speed of rotation when:

(i) The water just starts spilling over the brim.

(ii) The point at the centre of the base is just exposed.



Solution:



(i) We know that

$y_{max} = H = 3\ m\ ;\ h = \frac{2}{3}\ H = 2\ m$

$R = 1\ m;\ g = 10\ ms^{-2}$

Thus $\omega = \sqrt{\frac{4g(H-h)}{R^2}} = \sqrt{\frac{4(10)(3-2))}{1^2}}$

$= \sqrt{40}\ rad\ s^{-1}$

(ii) We know that

$y = \frac{\omega r^2}{2g} + y_0$, Here, $y = H = 3m;\ y_0 = 0\ ;\ r = R = 1m$

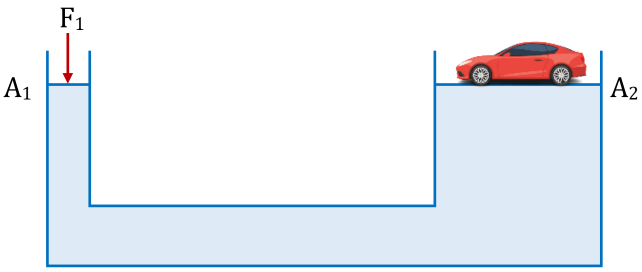
$g = 10\ m\ s^{-2}$

$\omega = \sqrt{\frac{2gH}{R^2}} = \sqrt{\frac{2(10)(3)}{1^2}} = \sqrt{60}\ \text{rad}\ s^{-1}$

Pascal's Law

It states that "pressure applied to an enclosed fluid is transmitted undiminished to every portion of the fluid and the walls of the containing vessel".

A well known application of Pascal's law is the hydraulic lift used to support or lift heavy objects. It is schematically illustrated in figure.



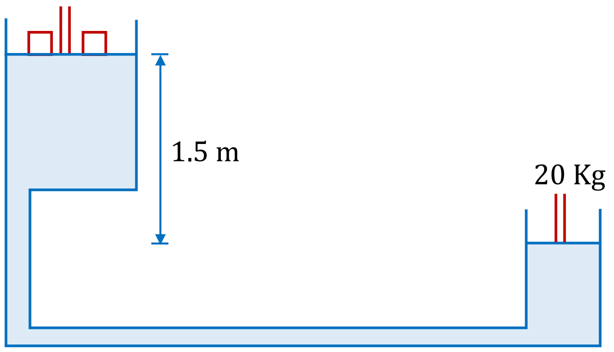
A piston with small cross section area A_1 exerts a force F_1 on the surface of a liquid such as oil. The applied pressure $P = \frac{F_1}{A_1}$ is transmitted through the connection pipe to a larger piston of area A_2. The applied pressure is the same in both cylinders, so

$$P = \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \text{or} \quad F_2 = \frac{A_2}{A_1}.F_1$$

Now, since $A_2 > A_1$, therefore, $F_2 > F_1$. Thus, hydraulic lift is a force multiplying device with a multiplication factor equal to the ratio of the areas of the two pistons. Dentist's chairs, car lifts and jacks, elevators and hydraulic brakes all are based on this principle.

Illustration 6:

Figure shows a hydraulic press with the larger piston of diameter 35 *cm* at a height of 1.5 *m* relative to the smaller piston of diameter 10 *cm*. The mass on the smaller piston is 20 *kg*. What is the force exerted on the load by the larger piston ? The density of oil in the press is 750 kg/m^3. (Take $g = 9.8 m/s^2$)



Solution:

Pressure on the smaller piston = $\dfrac{20\times 9.8}{\pi\times(5\times10^{-2})^2} N/m^2$

Pressure on the larger piston = $\dfrac{F}{\pi\times(17.5\times10^{-2})^2} N/m^2$

The difference between the two pressures = $h\rho g$

where $h = 1.5\,m$ and $\rho = 750\,kg/m^3$

Thus, $\dfrac{20\times 9.8}{\pi\times(5\times10^{-2})^2} - \dfrac{F}{\pi\times(17.5\times10^{-2})^2} = 1.5 \times 750 \times 9.8 = 11025$

$\Rightarrow \quad F = 1.3 \times 10^3\,N$

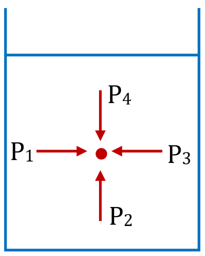
Note:

Atmospheric pressure is common to both pistons and has been ignored.

Important points in Pressure

1. At same point in a fluid, pressure is same in all directions. In the figure,

$P_1 = P_2 = P_3 = P_4$



2. Forces acting on a fluid in equilibrium have to be perpendicular to its surface. Because it cannot sustain the shear stress.

3. In the **same liquid** pressure will be same at all points at the same level. For example, in the figure:

$P_1 \neq P_2$

$P_3 = P_4$

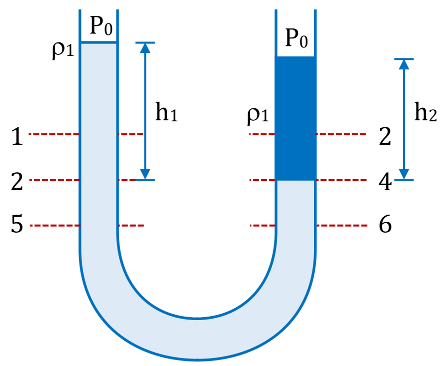
and $P_5 = P_6$

Further, $P_3 = P_4$

$\therefore$ $P_0 + \rho_1 gh_1 = P_0 + \rho_2 gh_2$

or $\rho_1 h_1 = \rho_2 h_2$

or $h \propto \dfrac{1}{\rho}$



4. **Torricelli Experiment (Barometer) :**

It is a device used to measure atmospheric pressure. In principle any liquid can be used to fill the barometer, but mercury is the substance of choice because its great density makes possible an instrument of reasonable size.

$P_1 = P_2$

Here, P_1 = atmospheric pressure (P_0)

and $P_2 = 0 + \rho gh = \rho gh$

$P_0 = \rho gh$

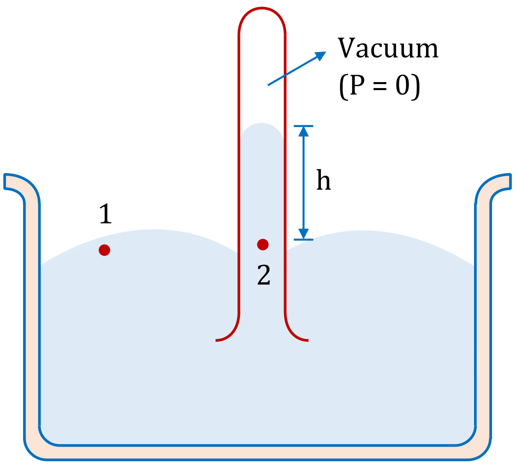
Here ρ = density of mercury

Thus, the mercury barometer reads the atmosheric pressure (P_0) directly from the height of the mercury column.

For example if the height of mercury in a barometer is 760 *mm*. then atmospheric pressure will be,

$P_0 = \rho gh = (13.6 \times 10^3)(9.8)(0.760)$

$= 1.01\times10^5\ N/m^2$



Manometer :

It is a device used to measure the pressure of a gas inside a container. The U-shaped tube often contains mercury

$P_1 = P_2$

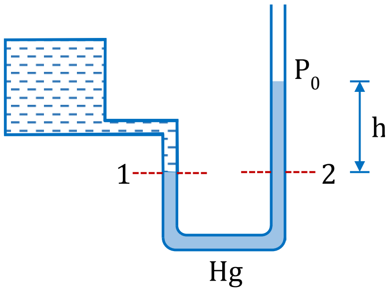
Here P_1 = pressure of the gas in the container (P)

and P_2 = atmospheric pressure (P_0) + ρgh

$P = P_0 + h\rho g$

This can also be written as

$P - P_0$ = gauge pressure = ρgh



Here, ρ is the density of the liquid used in *U*-tube.

Thus, by measuring h we can find absolute (or gauge) pressure in the vessel.

Archimedes Principle

If a heavy object is immersed in water, it seems to weigh less, than when it is in air. This is because the water exerts an upward force called buoyant force. It is equal to the weight of the fluid displaced by the body. **A body wholly or partially submerged in a fluid is buoyed up by a force equal to the weight of the displaced fluid.**

This result is known as **Archimedes principle.**

Thus, the magnitude of buoyant force (F) is given by, $F = V_i \rho_L g$

Here, V_i = immersed volume of solid

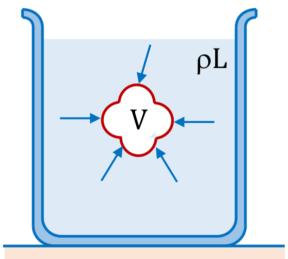
ρ_L = density of liquid

and g = acceleration due to gravity

Proof:

Consider an arbitrarily shaped body of volume V placed in a container filled with a fluid of density ρ_L. The body is shown completely immersed, but complete immersion is not essential to the proof. To begin with, imagine the situation before the body was immersed. The region now occupied by the body was filled with fluid, whose weight was $V_i \rho_L g$. Because the fluid as a whole was in hydrostatic equilibrium, the net upwards force (due to difference in pressure at different depths) on the fluid in region was equal to the weight of the fluid occupying that region.

Now, consider what happens when the body has displaced the fluid. The pressure at every point on the surface of the body is unchanged from the value at the same location when the body was on every point. This is because the pressure at any point depends only on the depth of that point the surface. Hence, the net force exerted by the surrounding fluid on the body is exactly the same as that exerted on the region before the body was present. But we now latter to be $V_i \rho_L g$, the weight of the displaced fluid Hence, this must also be the buoyant force exerted of the body. Archimedes' principle is thus proved.



Law of Floatation

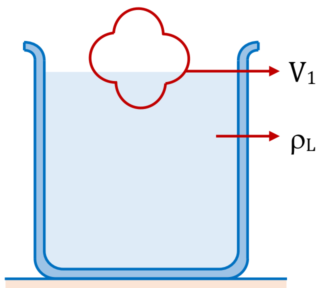
Consider an object of volume V and density ρ_s floating in a liquid of density ρ_L. Let V_i be the object immersed in the liquid.

For equilibrium of object,

Weight = Upthrust

$\therefore \quad V \rho_S g = V_i \rho_L g$

$\therefore \quad \dfrac{V_i}{V} = \dfrac{\rho_s}{\rho_L}$



This is the fraction of volume immersed in liquid.

Percentage of volume immersed in liquid = $\dfrac{V_i}{V} \times 100 = \dfrac{\rho_s}{\rho_L} \times 100$

Three possibilities may now arise.

(i) If $\rho_S < \rho_L$, only fraction of body will be immersed in the liquid. This fraction will be given by the above equation.

(ii) If $\rho_s = \rho_L$, the whole of the rigid body will be immersed in the liquid. Hence the body remains floating in the liquid wherever it is left.

(iii) If $\rho_s > \rho_L$, the body will sink.

Apparent Weight of a Body inside a Liquid

If a body is completely immersed in a liquid its effective weight gets decreased. The decrease in its weight is equal to the upthrust on the body. Hence,

$W_{app} = W_{actual} -$ Upthrust or $W_{app} = V\rho_s g - V\rho_L g$

Here, V = total volume of the body

ρ_s = density of body

and ρ_L = density of liquid

Thus, $W_{app} = Vg(\rho_s - \rho_L)$

If the liquid in which body is immersed, is water, then

$$\frac{\text{Weight in air}}{\text{Decrease in weight}} = \text{Relative density of body } (R.D)$$

This can be shown as under :

$$\frac{\text{Weight in air}}{\text{Decrease in weight}} = \frac{\text{Weight in air}}{\text{Upthrust}} = \frac{V\rho_s g}{V\rho_w g} = \frac{\rho_s}{\rho_w} = RD$$

Buoyant Force in Accelerating Fluids

Suppose a body is dipped inside a liquid of density ρ_L placed in an elevator moving with an acceleration $\vec{a}$.The buoyant force F in this case becomes,

$F = V\rho_L g_{eff}$

Here, $g_{eff} = |\vec{g} - \vec{a}|$

For example, if the lift is moving upwards with an acceleration a, value of g_{eff} is $g + a$ and if it is moving downwards with acceleration a, the g_{eff} is $g - a$. In a freely falling lift g_{eff} is zero (as $a = g$) and hence, net buoyant force is zero. This is why, in a freely filled with some liquid, the air bubbles do not rise up (which otherwise move up due to buoyant force).

Illustration 7:

Density of ice is $900 kg/m^3$. A piece of ice is floating in water of density $1000\ kg/m^3$. Find the fraction of volume of the piece of ice out side the water.

Solution:

Let V be the total volume and V_i the volume of ice piece immersed in water. For equilibrium of ice piece,

Weight = Upthrust

$\therefore$ $V\rho_i g = V_i\rho_w g$

Here ρ_i = Density of ice = $900\ kg/m^3$

and ρ_w = Density of water = $1000\ kg/m^3$

Substituting in above equation,

$$\frac{V_i}{V} = \frac{900}{1000} = 0.9$$

i.e, the fraction of volume outside the water,

$f = 1 - 0.9 = 0.1$

Illustration 8:

A piece of ice is floating in a glass vessel filled with water. How the level of water in the vessel change when the ice melts ?

Solution:

Let m be the mass of ice piece floating in water.

In equilibrium, weight of ice piece = upthrust

$$mg = V_i \rho_w g$$

or $$V_i = \frac{m}{\rho_w}$$

Here, V_i is the volume of ice piece immersed in water.

When the ice melt, let V be the volume of water formed by m mass of ice. Then, ρ_w

From equations (i) and (ii) we see that

$$V_i = V$$

Hence, the level of water will not change.

Illustration 9:

An ornament weighing 50 g in air weighs only 46 g in water. Assuming that some copper is mixed with gold to prepare the ornament. Find the amount of copper in it. Specific gravity of gold is 20 and that of copper is 10.

Solution:

Let m be the mass of the copper in ornament. Then mass of gold in it is $(50 - m)$.

Volume of copper $V_1 = \frac{m}{10}$ $\qquad \left(\text{Volume} = \frac{\text{Mass}}{\text{Density}}\right)$

and volume of gold, $V_2 = \frac{50-m}{20}$

when immersed in water ($\rho_w = 1\ g/cm^3$)

Decrease in weight = upthrust

$\therefore$ $$(50 - 46)g = (V_1 + V_2)\rho_w g$$

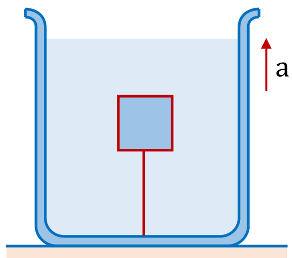
or $$4 = \frac{m}{10} + \frac{50-m}{20} \quad \text{or} \quad 80 = 2m + 50 - m$$

$\therefore$ $$m = 30g$$

Illustration 10:

The tension in a string holding a solid block below the surface of a liquid (of density greater then that of solid) as shown in figure is T_0 when the system is at rest. What will be the tension in the string if the system has an upward acceleration a ?

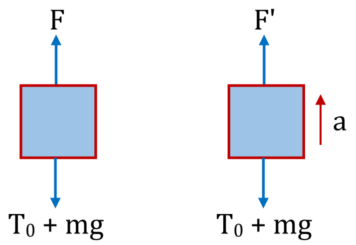
Solution:



Let m be the mass of block. Initially for the equilibrium of block,

$$F = T_0 + mg \qquad \text{...(i)}$$

Here, F is the upthrust on the block.



Where the lift is accelerated upwards, g_{eff} becomes $g + a$ instead of g.

Hence, $F' = F\left(\frac{g+a}{g}\right)$ ...(ii)

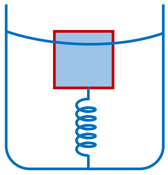
From Newton's second law, $F' - T - mg = ma$ ...(iii)

Solving equations (i), (ii) and (iii), we get

$$T = T_0\left(1+\frac{a}{g}\right)$$

Illustration 11:

A cubical block of plastic of edge 3 cm floats in water. The lower surface of the cube just touches the free end of a vertical spring fixed at the bottom of the pot. Find the maximum weight that can be put on the block without wetting it. Density of plastic = 800 kg/m^3 and spring constant of the spring = 100 N/m. Take g = 10 m/s^2.



Solution:

The specific gravity of the block = 0.8. Hence the height inside water = 3 cm × 0.8 = 2.4 cm. The height outside water = 3 cm – 2.4 = 0.6 cm. Suppose the maximum weight that can be put without wetting it is W. The block in this case is completely immersed in the water. The volume of the displaced water,

= Volume of the block = 27 × 10^{-6} m^3

Hence, the force of buoyancy

= (27 × 10^{-6} m^3) × 1(1000 kg/m^3) × (10 m/s^2) = 0.27 N

The spring is compressed by 0.6 cm and hence the upward force exerted by the spring

= 100 N/m × 0.6 cm = 0.6 N.

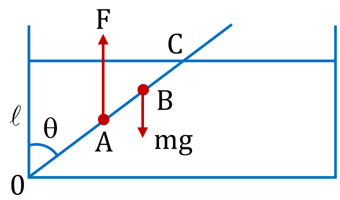
The force of buoyancy and the spring force taken together balance the weight of the block plus the weight W put on the block. The weight of the block is

W' = (27 × 10^{-6} m) × (800 kg/m^3) × (10 m/s^2) = 0.22 N.

Thus, W = 0.27 N + 0.6 N – 0.22 N = 0.65 N.

Illustration 12:

A wooden plank of length 2ℓ m and uniform cross-section is hinged at one end to the bottom of a tank as shown in figure. The tank is filled with water up to a height of ℓ m. The specific gravity of the plank is 0.5. Find the angle θ that the plank makes with the vertical in the equilibrium position. (Exclude the case θ = 0°)



Solution:

The forces acting on the plank are shown in the figure. The height of water level is ℓ. The length of the plank is 2ℓ. The weight of the plank acts through the centre B of the plank. We have $OB = \ell$. The buoyant force F acts through the point A which is the middle point of the dipped part OC of the plank.

We have $OA = \frac{OC}{2} = \frac{\ell}{2\cos\theta}$

Let the mass per unit length of the plank be ρ.

Its weight $mg = 2\ell\rho g$.

The mass of the part OC of the plank $= \left(\dfrac{\ell}{\cos\theta}\right)\rho$.

The mass of water displaced $= \dfrac{1}{0.5}\dfrac{\ell}{\cos\theta}\,\rho = \dfrac{2\ell\rho}{\cos\theta}$.

The buoyant force F is, therefore, $F = \dfrac{2\ell\rho g}{\cos\theta}$.

Now, for equilibrium, the torque of mg about O should balance the torque of F about O.

So, $\quad mg\,(OB)\sin\theta = F(OA)\sin\theta$

or, $\quad (2\ell\rho)\ell = \left(\dfrac{2\ell\rho}{\cos\theta}\right)\left(\dfrac{\ell}{2\cos\theta}\right)$ or $\cos^2\theta = \dfrac{1}{2}$ or $\cos\theta = \dfrac{1}{\sqrt{2}}$ or $\theta = 45°$

Illustration 13:

A cylindrical block of wood of mass m, radius r and density ρ is floating in water with its axis vertical. It is depressed a little and then released. If the motion of the block is simple harmonic. Find its frequency.

Solution:

Suppose a height h of the block is dipped in the water in equilibrium position. If r be the radius of the cylindrical block, the volume of the water displaced $= \pi r^2 h$. For floating in equilibrium,

$$\pi r^2 h\rho g = W \qquad \text{...(i)}$$

where ρ is the density of water and W the weight of the block.

Now suppose during the vertical motion, the block is further dipped through a distance x at some instant. The volume of the displaced water is $\pi r^2 (h + x)$. The forces acting on the block are, the weight W vertically downward and the buoyancy $\pi r^2(h + x)\rho g$ vertically upward.

Net force on the block at displacement x from the equilibrium position is

$$F = W - \pi r^2 (h + x)\rho g = W - \pi r^2 h\rho g - \pi r^2 \rho x g$$

Using (i) $F = -\pi r^2 \rho g x = -kx$,

where $k = \pi r^2 \rho g$.

Thus, the block executes SHM with frequency.

$$\nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{\pi r^2 \rho g}{m}}$$

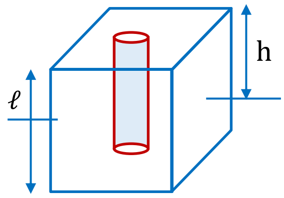
Illustration 14:

A large block of ice cuboid of height 'ℓ' and density $\rho_{ice} = 0.9\,\rho_w$, has a large vertical hole along its axis. This block is floating in a lake. Find out the length of the rope required to raise a bucket of water through the hole.

Solution:

Let area of ice-cuboid excluding hole = A

Weight of ice block = weight of liquid displaced



$$A\,\rho_{ice}\,\ell\,g = A\,\rho_w\,(\ell - h)\,g$$

$$\frac{9\ell}{10} = \ell - h$$

$$\Rightarrow \quad h = \ell - \frac{9\ell}{10} = \left(\frac{\ell}{10}\right)$$

Conditions of Ideal Fluid Flow

(i) Fluid is Incompressible which means density of fluid remains constant wrt time as well as wrt position.

(ii) Fluid is non-viscous which means dissipative forces are absent between the layers of fluid.

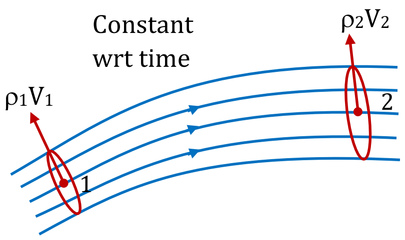
(iii) Fluid flow is Irrotational which means angular velocity of all fluid particles is zero wrt each other.

(iv) Fluid flow is steady (or streamlined).

Steady (Streamlines) Flow :

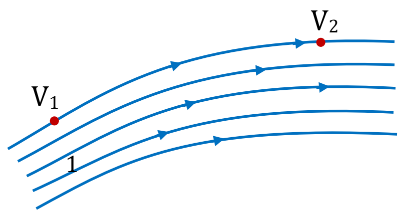
If during the flow, velocity and density of the fluid remains constant wrt time for any given point, then the flow is called steady flow.

Although density and velocity may vary wrt position but they must be constant wrt time in case of steady flow.

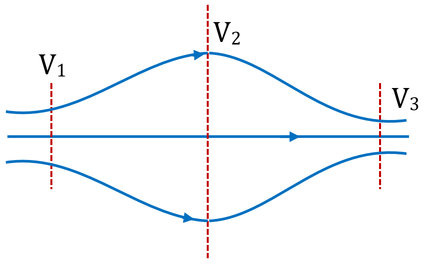


Stream lines:

These are the curves drawn into space tangent to which at any point gives the direction of velocity at that point.



Density of streamlines at any point is proportional to the magnitude of velocity at that point.

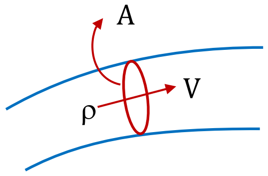


$V_3 > V_1 > V_2 \qquad V \rightarrow$ velocity

Two streamlines do not cross each other.

At a point there cannot be 2 direction of velocity.

Mass flow rate i.e. mass flowing through per unit time.



$$\boxed{\frac{dm}{dt} = \rho A V}$$

A = Area of cross-section

V = Velocity

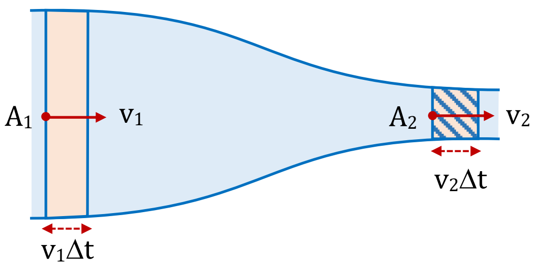
Volume flow rate i.e. vol. flowing through per unit time.

$$\boxed{\frac{dv}{dt} = AV}$$

Equation of Continuity

The continuity equation is the mathematical expression of the *law of conservation of mass in fluid dynamics*.

In the steady flow the mass of fluid entering into a tube of flow in a particular time interval is equal to the mass of fluid leaving the tube.



$$\frac{m_1}{\Delta t} = \frac{m_2}{\Delta t}$$

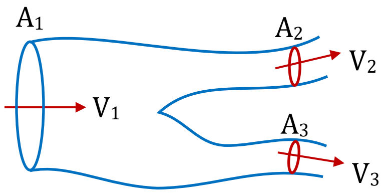
$\Rightarrow \quad \rho_1 A_1 v_1 = \rho_2 A_2 v_2 \; (\because \; \rho_1 = \rho_2)$

$\Rightarrow \quad A_1 v_1 = A_2 v_2 \;\; ; \;\; Av = \text{constant}$

(Here ρ = density of fluid, v = velocity of fluid, A = Area of cross–section of tube)

Therefore, the velocity of liquid is smaller in the wider parts of a tube and larger in the narrower parts.

Eg.



$$A_1V_1 = A_2V_2 + A_3V_3$$

Bernoulli's Theorem

Bernoulli's equation is mathematical expression of the law of mechanical energy conservation in fluid dynamics.

Bernoullis theorem is applied to the ideal fluids.

Characteristics of an ideal fluid are :

(i) The fluid is incompressible.

(ii) The fluid is non–viscous.

(iii) The fluid flow is steady.

(iv) The fluid flow is irrotational.

Every point in an ideal fluid flow is associated with three kinds of energies :

Kinetic Energy :

If a liquid of mass (m) and volume (V) is flowing with velocity (v) then

Kinetic Energy $= \frac{1}{2} mv^2$ and kinetic energy per unit volume $= \frac{\text{Kinetic Energy}}{\text{Volume}} = \frac{1}{2}\frac{m}{V}v^2 = \frac{\rho v^2}{2}$

Potential Energy :

If a liquid of mass (m) and volume (V) is at height (h) from the surface of the earth then its

Potential Energy $= mgh$ and potential energy per unit volume $= \frac{\text{Potential Energy}}{\text{Volume}} = \frac{m}{V}gh = \rho gh$

Pressure Energy :

If P is the pressure on area A of a liquid and the liquid moves through a distance (ℓ) due to this pressure then

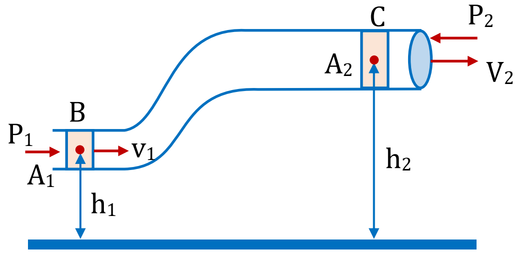
Pressure energy = Work done = Force × Displacement = Pressure × Area × Displacement = $PA\ell = PV$

[$\because A\ell$ = volume V]

Pressure energy per unit volume $= \frac{\text{Pressure energy}}{\text{Volume}} = P$

Theorem :

According to Bernoulli's Theorem, in case of steady flow of incompressible and non–viscous fluid through a tube of non–uniform cross–section, the sum of the pressure, the potential energy per unit volume and the kinetic energy per unit volume is same at every point in the tube, i.e., $P + \rho gh + \frac{1}{2}\rho v^2 =$ constant.



Consider a liquid flowing steadily through a tube of non–uniform area of cross–section as shown in figure. If P_1 and P_2 are the pressures at the two ends of the tube respectively, work done in pushing the volume ΔV of incompressible liquid from point B to C through the tube

$$W = P(\Delta V) = (P_1 - P_2)\Delta V \qquad \text{...(i)}$$

This work is used by the liquid in two ways :

(i) In changing the potential energy of mass Δm (in the volume ΔV) from

Δmgh_1 to Δmgh_2 i.e., $\Delta U = \Delta mg(h_2 - h_1)$ ...(ii)

(ii) In changing the kinetic energy from $\frac{1}{2}\Delta mv_1^2$ to $\frac{1}{2}\Delta mv_2^2$, i.e. $\Delta K = \frac{1}{2}\Delta m\left(v_2^2 - v_1^2\right)$

Now as the liquid is non–viscous, by conservation of mechanical energy,

$$W = \Delta U + \Delta K \text{ i.e., } (P_1 - P_2)\,\Delta V = \Delta mg(h_2 - h_1) + \frac{1}{2}\Delta m\left(v_2^2 - v_1^2\right)$$

$$P_1 - P_2 = \rho g(h_2 - h_1) + \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) \quad [\text{as } \rho = \Delta m / \Delta V]$$

$$P_1 + \rho gh_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho gh_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P + \rho gh + \frac{1}{2}\rho v^2 = \text{constant}$$

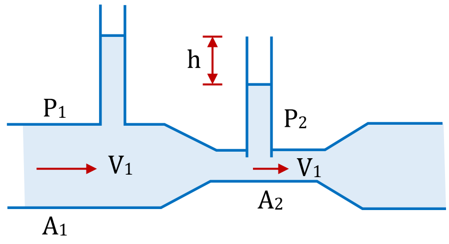
This equation is the Bernoulli's equation and represents conservation of mechanical energy in case of moving fluids.

Applications Based on Bernoulli's Equation

(a) Venturimeter

Figure shows a venturimeter used to measure flow speed in a pipe of non- uniform cross-section. We apply Bernoulli's equation to the wide (point 1) and narrow (point 2) parts of the pipe, with $h_1 = h_2$

$$P_1 + \frac{1}{2}\rho\, v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$



From the continuity equation $v_2 = \dfrac{A_1 v_1}{A_2}$

Substituting and rearranging, we get

$$P_1 - P_2 = \frac{1}{2}\rho v_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

The pressure difference is also equal to ρgh, where h is the difference in liquid level in the two tubes. Substituting in equation (i) we get

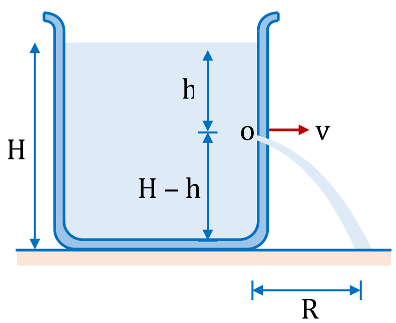
$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

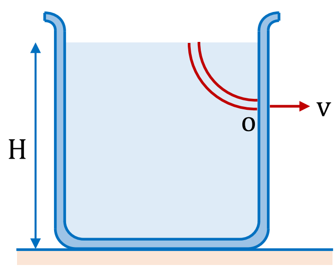
The discharge or volume flow rate can be obtained as,

$$\frac{dV}{dt} = A_1\, v_1 = A_1\sqrt{\frac{2gh}{\left(\frac{A_1}{A_2}\right)^2 - 1}}$$

(b) Speed of Efflux

Suppose, the surface of a liquid in a tank is at a height h from the orifice O on its sides, through which the liquid flows out with velocity v. The speed of the liquid coming called the speed of efflux. If the dimensions of the tank be sufficiently large, the velocity of the liquid at its surface may be taken to be zero and since the pressure there as well as at the orifice O is the same as atmospheric pressure so it plays no part in the flow of the liquid, which thus occurs purely in consequence of the hydrostatic pressure of the liquid itself. So that, considering a tube of flow, starting at the liquid surface and ending at the orifice, as shown in figure. Applying Bernoulli's equation we have





Total energy per unit volume of the liquid at the surface

$= KE + PE +$ pressure energy $= 0 + \rho gh + P_0$

and total energy per unit volume at the orifice

$$= KE + PE + \text{pressure} = \frac{1}{2}\rho v^2 + 0 + P_0$$

Since total energy of the liquid must remain constant in steady flow, in accordance with Bernoulli's equation we have

$$\rho gh + P_0 = \frac{1}{2}\rho v^2 + P_0$$

or $v = \sqrt{2gh}$

Evangellista Torricelli showed that this velocity is the same as the liquid will attain in falling freely through the height (h) from the surface to the orifice. This is known as Torricelli's theorem and may be stated as. "The velocity of efflux of a liquid flows out of an orifice is the same as it would attain if allowed to fall freely through the vertical height between the liquid surface and orifice.

Range (R)

Let us find the range R on the ground.

Considering the vertical motion of the liquid,

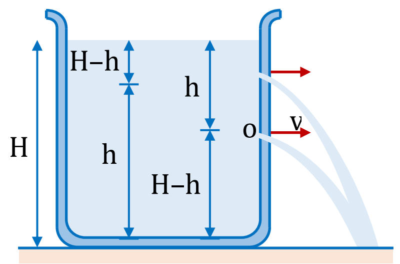
$$(H - h) = \frac{1}{2}gt^2 \qquad \text{or} \qquad t = \sqrt{\frac{2(H-h)}{g}}$$

Now, considering the horizontal motion, $R = vt$

$$R = \sqrt{2gh}\left(\sqrt{\frac{2(H-h)}{g}}\right) \quad \text{or} \quad R = 2\sqrt{h(H-h)}$$

From the expression of R, following conclusions can be drawn,

(i) $R_h = R_{H-h}$



As $R_h = 2\sqrt{h(H-h)}$ and $R_{H-h} = 2\sqrt{h(H-h)}$

This can be maximum at $h = \frac{H}{2}$ and $R_{max} = H$.

Proof : $R^2 = 4(Hh - h^2)$

For R to be maximum, $\frac{dR^2}{dh} = 0$

or $H - 2h = 0$ or $h = \frac{H}{2}$

That is, R is maximum at $h = \frac{H}{2}$ and $R_{max} = 2\sqrt{\frac{H}{2}\left(H - \frac{H}{2}\right)} = H$

Time taken to empty a tank

We are here interested in finding the time required to empty a tank if a hole is made at the bottom of the tank.

Consider a tank filled with a liquid of density ρ upto a height H. A small hole of area of cross section a is made at the bottom of the tank. The area of cross-section of the tank is A.

Let at some instant of time the level of liquid in the tank is y. Velocity of efflux at this instant of time would be

$$v = \sqrt{2gy}$$

Now, at this instant volume of liquid coming out of the hole per second is $\left(\frac{dV_1}{dt}\right)$.

Volume of liquid coming down in the tank per second is $\left(\frac{dV_2}{dt}\right)$.

To calculate time taken to empty a tank $\frac{dV_1}{dt} = \frac{dV_2}{dt}$

$\therefore \quad av = A\left(-\frac{dy}{dt}\right)$

$\therefore \quad a\sqrt{2gy} = A\left(-\frac{dy}{dt}\right)$

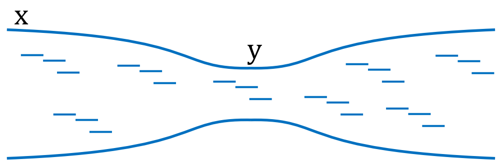
or $\quad \int_0^t dt = -\frac{A}{a\sqrt{2g}} \int_H^0 y^{-1/2}\, dy$

$\therefore \quad t = \frac{2A}{a\sqrt{2g}} [\sqrt{y}]_0^H$

$\therefore \quad t = \frac{A}{a}\sqrt{\frac{2H}{g}}$

Illustration 15:

Water flows in a horizontal tube as shown in figure. The pressure of water changes by 600 N/m^2 between x and y where the areas of cross-section are $3cm^2$ and $1.5cm^2$ respectively. Find the rate of flow of water through the tube.



Solution:

Let the velocity at $x = v_x$ and that at $y = v_y$.

By the equation of continuity, $\frac{v_y}{v_x} = \frac{3cm^2}{1.5cm^2} = 2.$

By Bernoulli's equation,

$$P_x + \frac{1}{2}\rho v_x^2 = P_y + \frac{1}{2}\rho v_y^2$$

or, $\quad P_x - P_y = \frac{1}{2}\rho(2v_y)^2 - \frac{1}{2}\rho v_y^2 = \frac{3}{2}\rho v_y^2$

or, $\quad 600\frac{N}{m^2} = \frac{3}{2}\left(1000\frac{kg}{m^3}\right)v_x^2$

or, $\quad v_x = \sqrt{0.4m^2/s^2} = 0.63\ m/s.$

The rate of flow = $(3\ cm^2)(0.63\ m/s) = 189\ cm^3/s.$

Illustration 16:

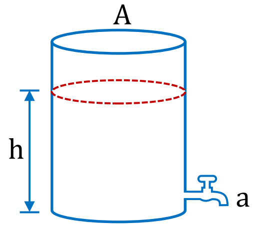
A cylindrical container of cross-section area, A is filled up with water upto height *'h'*. Water may exit through a tap of cross section area *'a'* in the bottom of container. Find out:

(a) Velocity of water just after opening of tap.

(b) The area of cross-section of water stream coming out of tap at depth h_0 below tap in terms of *'a'* just after opening of tap.

(c) Time in which container becomes empty.

(Given : $\left(\frac{a}{A}\right)^{1/2} = 0.02$, h = 20 cm, h_0 = 20 cm)



Solution:

(a) Applying Bernoulli's equation between (1) and (2) -

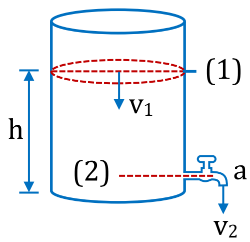
$$P_a + \rho gh + \frac{1}{2}\rho v_1^2 = P_a + \frac{1}{2}\rho v_2^2$$

Through continuity equation :

$$Av_1 = av_2,\ v_1 = \frac{av_2}{A}$$

$$\rho gh + \frac{1}{2}\rho v_1^2 = \frac{1}{2}\rho v_2^2$$

On solving - $v_2 = \sqrt{\dfrac{2gh}{1-\dfrac{a^2}{A^2}}} = 2m/sec.$ ...(1)



(b) Applying Bernoulli's equation between (2) and (3)

$$\frac{1}{2}\rho v_2^2 + \rho gh_0 = \frac{1}{2}\rho v_3^2$$

Through continuity equation –

$$av_2 = a' v_3$$

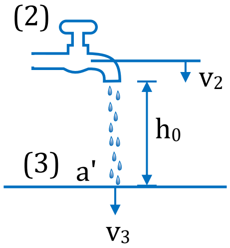
$$\Rightarrow v_3 = \frac{av_2}{a'}$$

$$\Rightarrow \frac{1}{2}\rho v_2^2 + \rho gh_0 = \frac{1}{2}\rho\left(\frac{av_2}{a'}\right)^2$$

$$\frac{1}{2} \times 2 \times 2 + gh_0 = \frac{1}{2}\left(\frac{a}{a'}\right)^2 \times 2 \times 2$$

$$\left(\frac{a}{a'}\right)^2 = 1 + \frac{9.8 \times .20}{2} \Rightarrow \left(\frac{a}{a'}\right)^2 = 1.98$$

$$\Rightarrow a' = \frac{a}{\sqrt{1.98}}$$



(c) From (1) at any height 'h' of liquid level in container, the velocity through tap,

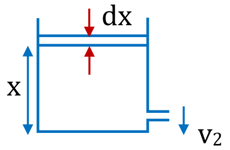
$$v = \sqrt{\frac{2gh}{0.98}} = \sqrt{20h}$$

we know, volume of liquid coming out of tap = decrease in volume of liquid in container.

For any small time, interval 'dt'

$$av_2 dt = -A \cdot dx$$

$$a\sqrt{20x}\,dt = -A\,dx \qquad \Rightarrow \int_0^t dt = -\frac{A}{a}\int_h^0 \frac{dx}{\sqrt{20x}}$$



$$t = \frac{A}{a\sqrt{20}}\left[2\sqrt{x}\right]_h^0 \qquad \Rightarrow t = \frac{A}{a\sqrt{20}}\,2\sqrt{h}$$

$$= \frac{A}{a} \times 2 \times \sqrt{\frac{h}{20}} = \frac{2A}{a}\sqrt{\frac{0.20}{20}} = \frac{2A}{a} \times 0.1$$

Given $\left(\frac{a}{A}\right)^{1/2} = 0.02$ or $\frac{A}{a} = \frac{1}{0.0004} = 2500$

Thus $t = 2 \times 2500 \times 0.1 = 500$ second.

Illustration 17:

A tank is filled with a liquid upto a height H. A small hole is made at the bottom of this tank. Let t_1 be the time taken to empty first half of the tank and t_2 is the time taken to empty rest half of the tank then find $\frac{t_1}{t_2}$.

Solution:

Substituting the proper limits in equation (i), derived in the theory, we have

$$\int_0^{t_1} dt = -\frac{A}{a\sqrt{2g}}\int_H^{H/2} y^{-1/2} dy$$

or $\quad t_1 = \frac{2A}{a\sqrt{2g}}\left[\sqrt{y}\right]_{H/2}^{H}$ or $t_1 = \frac{2A}{a\sqrt{2g}}\left[\sqrt{H} - \sqrt{\frac{H}{2}}\right]$ or $t_1 = \frac{A}{a}\sqrt{\frac{H}{g}}\left(\sqrt{2}-1\right)$

Similarly, $\int_0^{t_2} dt = -\frac{A}{a\sqrt{2g}}\int_{H/2}^{0} y^{-1/2}\,dy$

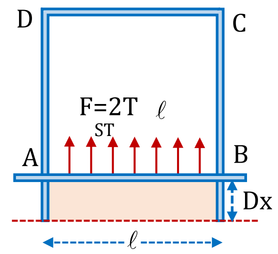
or $\quad t_2 = \frac{A}{a}\sqrt{\frac{H}{g}}$

We get, $\frac{t_1}{t_2} = \sqrt{2}-1$ or $\frac{t_1}{t_2} = 0.414$

Note:

From here we see that $t_1 < t_2$ This is because initially the pressure is high and the liquid comes out with greater velocity.

Surface Energy



According to molecular theory of surface tension the molecule in the surface have some additional energy due to their position. This additional energy per unit area of the surface is called ' Surface energy". Let a liquid film be formed on a wire frame and a straight wire of length ℓ can slide on this wire frame as shown in figure.

The film has two surfaces and both the surfaces are in contact with the sliding wire and hence, exert forces of surface tension on it. If T be the surface tension of the solution, each surface will pull the wire parallel to itself with a force $T\ell$.

Thus, net force on the wire due to both the surfaces is $2T\ell$. Apply an external force F equal and opposite to it to keep the wire is equilibrium. Thus, $F = 2T\ell$, now, suppose the wire is moved through a small distance dx, the work done by the force is, $dW = F\,dx = (2T\ell)\,dx$, but $(2\ell)\,(dx)$ is the total increase in area of both the surfaces of the film. Let it be dA.

Then, $dW = T\,dA \Rightarrow T = dW/dA$

Thus, the surface tension T can also be defined as the work done in increasing the surface area by unity. Further, since there is no change in kinetic energy, the work done by the external force is stored as the potential energy of the new surface.

$$T = \frac{dU}{dA} \quad [\text{as } dW = dU]$$

Splitting of Bigger Drop into Smaller Droplets

If bigger drop is spitted into smaller droplets then in this process volume of liquid always remain conserved. Let bigger drop has radius R. It is splitted into n smaller drops of radius r then by conservation of volume

(i) $$\frac{4}{3}\pi R^3 = n\left[\frac{4}{3}\pi r^3\right] \Rightarrow n = \left[\frac{R}{r}\right]^3 \Rightarrow r = \frac{R}{n^{1/3}}$$

(ii) Initial surface area = $4\pi R^2$ and final surface area = $n(4\pi r^2)$

Therefore, initial surface energy $E_i = 4\pi R^2 T$ and final surface energy, $E_f = n(4\pi r^2 T)$

Change in area $\Delta A = n4\pi r^2 - 4\pi R^2 = 4\pi\,(nr^2 - R^2)$

Therefore, the amount of surface energy absorbed i.e. $\Delta E = E_f - E_i = 4\pi T\,(nr^2 - R^2)$

$\therefore$ Magnitude of work done against surface tension i.e. $W = 4\pi\,(nr^2 - R^2)T$

$$W = 4\pi T\,(nr^2 - R^2) = 4\pi R^2\,T\,(n^{1/3} - 1) = 4\pi R^2 T\left[\frac{R}{r} - 1\right] \Rightarrow W = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right]$$

In this process temperature of system decreases as energy gets absorbed in increasing surface area.

$$W = J\,m\,s\,\Delta\theta = 4\pi R^3 T\left[\frac{1}{r} - \frac{1}{R}\right] \Rightarrow \Delta\theta = \frac{4\pi R^3 T}{\frac{4}{3}\pi R^3 J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right] = \frac{3T}{J\rho s}\left[\frac{1}{r} - \frac{1}{R}\right]$$

Where ρ = liquid density, s = liquid's specific heat

Thus, in this process area increasing, surface energy increasing, internal energy decreasing, temperature decreasing, and energy is absorbed.

Illustration 18:

A big drop is formed by coalescing 1000 small droplets of water. What will be the change in surface energy? What will be the ratio between the total surface energy of the droplets and the surface energy of the big drop?

Solution:

By conservation of volume $\frac{4}{3}\pi R^3 = 1000 \times \frac{4}{3}\pi r^3 \quad \Rightarrow \quad r = \frac{R}{10}$

Surface energy of 1000 droplets $= 1000 \times T \times 4\pi\left[\frac{R}{10}\right]^2 = 10\,(T \times 4\pi R^2)$

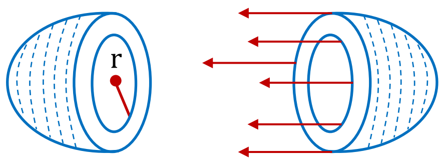
Surface energy of the big drop $= T \times 4\pi R^2$

Surface energy will decrease in the process of formation of bigger drop, hence energy is released.

$$\therefore \qquad \frac{\text{Total surface energy of 1000 droplets}}{\text{Surface energy of big drop}} = \frac{10(T \times 4\pi R^2)}{T \times 4\pi R^2} = \frac{10}{1}$$

Excess Pressure Inside a Liquid Drop and a Bubble

1. Inside a bubble : Consider a soap bubble of radius r. Let p be the pressure inside the bubble and p_a outside. The excess pressure $= p - p_a$. Imagine the bubble broken into two halves, and consider one half of it as shown in fig. Since there are two surface, inner and outer, so the force due to surface tension is



F = surface tension × length $= T \times 2$ (circumference of the bubble) $= T \times 2\,(2T\pi r)$ ...(1)

The excess pressure $(p - p_a)$ acts on a cross-sectional area πr^2, so the force due to excess pressure is

$\Rightarrow \qquad F = (p - p_a)\,\pi r^2$ ...(2)

The surface tension force given by equation (1) must balance the force due to excess pressure given by equation (2) to maintain the equilibrium , i.e. $(p - p_a)\,\pi r^2 = T \times 2(2\pi\, r)$

or $\qquad (p - p_a) = \frac{4T}{r} = p_{\text{excess}}$

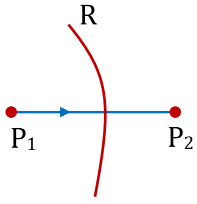
Above expression can also be obtained by equation of excess pressure of curve surface by putting $R_1 = R_2$.

2. Inside the drop: In a drop, there is only one surface and hence excess pressure can be written as

$$(p - p_a) = \frac{2T}{r} = p_{\text{excess}}$$

3. Inside air bubble in a liquid :

$$(p - p_a) = \frac{2T}{r} = p_{\text{excess}}$$

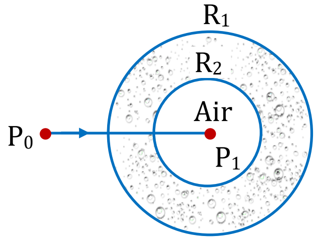


$$P_1 - \frac{2S}{R} = P_2$$

$P_1 > P_2$

Illustration 19:

Find the difference in pressure inside and outside the thick soap bubble shown in figure.

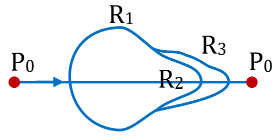


Solution:

$$P_0+\frac{2S}{R_1}+\frac{2S}{R_2}=P_1$$

Illustration 20:

Find the relation in T, P_0, R_1, R_2 and R_3 in the figure shown.



Solution:

$$P_0+\frac{4S}{R_1}-\frac{4S}{R_2}-\frac{4S}{R_3}=P_0$$

Illustration 21:

Prove that If two bubbles of radii r_1 and r_2 ($r_1 < r_2$) come in contact with each other then the radius of curvature of the common surface $r=\frac{r_1r_2}{r_2-r_1}$.

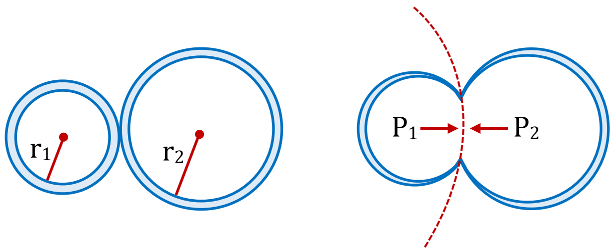
Solution:

$\because$ $r_1 < r_2$

$\therefore$ $P_1 > P_2$, small part of bubbles is in equilibrium.

$\Rightarrow$ $P_1(\Delta A) - P_2(\Delta A) = \frac{4T}{r}\Delta A$

$\Rightarrow$ $\frac{4T}{r_1}-\frac{4T}{r_2}=\frac{4T}{r}$



$\Rightarrow$ $r=\frac{r_1r_2}{r_2-r_1}$

Illustration 22:

Calculate the excess pressure within a bubble of air of radius 0.1 *mm* in water. If the bubble had been formed 10 *cm* below the water surface on a day when the atmospheric pressure was 1.013×10^5 *Pa*, then what would have been the total pressure inside the bubble? Surface tension of water = 73×10^{-3} *N/m.*

Solution:

Excess pressure $P_{excess}=\frac{2T}{r}=\frac{2\times73\times10^{-3}}{0.1\times10^{-3}}=1460\ Pa$

The pressure at a depth h, in liquid is $P = h\rho g$. Therefore, the total pressure inside the air bubble is

$$P_{in}=P_{atm}+h\rho g+\frac{2T}{r}=(1.013\times10^5)+(10\times10^{-2}\times10^3\times9.8)+1460$$

$$=101300+980+1460=103740=1.037\times10^5\ Pa$$

Illustration 23:

The limbs of a manometer consist of uniform capillary tubes of radii are 14×10^{-4} *m* and 7.2×10^{-4} *m*. Find out the correct pressure difference if the level of the liquid in narrower tube stands 0.2 *m* above that in the broader tube. (Density of liquid $=10^3\ kg/m^3$, Surface tension $= 72 \times 10^{-3}\ N/m$)

Solution:

If P_1 and P_2 are the pressures in the broader and narrower tubes of radii r_1 and r_2 respectively, the pressure just below the meniscus in the respective tubes will be $P_1 - \frac{2T}{r_1}$ and $P_2 - \frac{2T}{r_2}$.

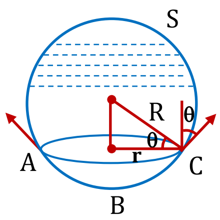
So that, $\left[P_1 - \frac{2T}{r_1}\right] - \left[P_2 - \frac{2T}{r_2}\right] = h\rho g$

or $\quad P_1 - P_2 = h\rho g - 2T\left[\frac{1}{r_2} - \frac{1}{r_1}\right]$

$$P_1 - P_2 = 0.2 \times 10^3 \times 9.8 - 2 \times 72 \times 10^{-3}\left[\frac{1}{7.2\times10^{-4}} - \frac{1}{14\times10^{-4}}\right] = 1960 - 97 = 1863\ Pa$$

Illustration 24:

Find the force of surface tension in the figure shown. Force of *ST* on on part *ABC* in *S*, *R*, *Q*?



Solution:

Force $= S \times 2\pi R$

But $S2\pi R \sin\theta$ cancels

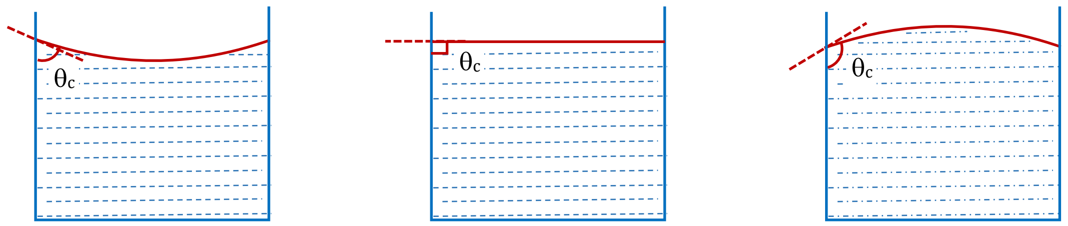
$\therefore \quad$ Force $= S\, 2\pi r \times \cos\theta$

$= S\, 2\pi R \cos\theta \times \cos\theta$

$= S\, 2\pi R \cos^2\theta$

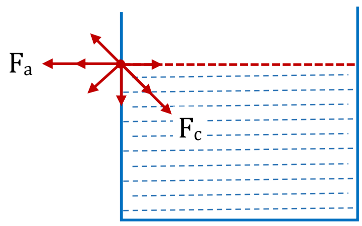
Contact Angle (θ_C) :

The surface of liquid near the plane of contact with another medium is in general curved. The angle between tangent to the liquid surface at the point of contact and solid surface (inside) the liquid is known as contact angle (θ_c).



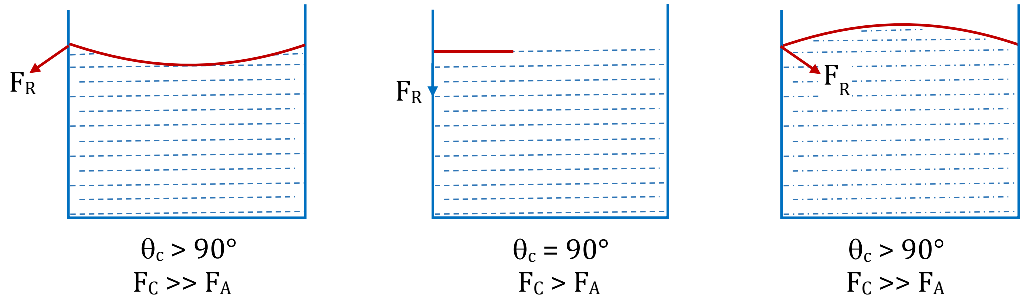
Glass vessel - water $\theta_c < 90°$ | Sipper vessel - water $\theta_c = 90°$ | Glass vessel – mercury $\theta_c > 90°$

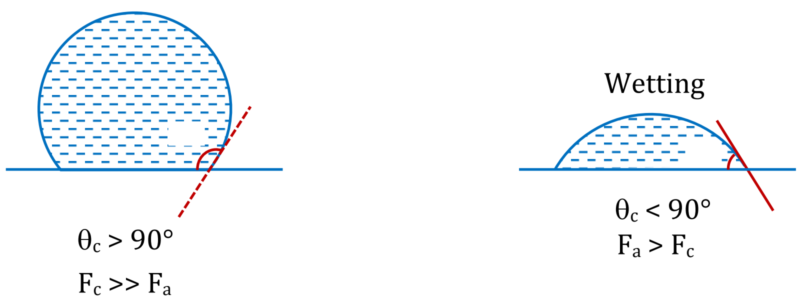
Force between same types of molecules is called cohesive force and force between diff. types of molecules is called adhesive force.



F_R i.e. resultant force will be resultant of F_a and F_c

Possibilities :

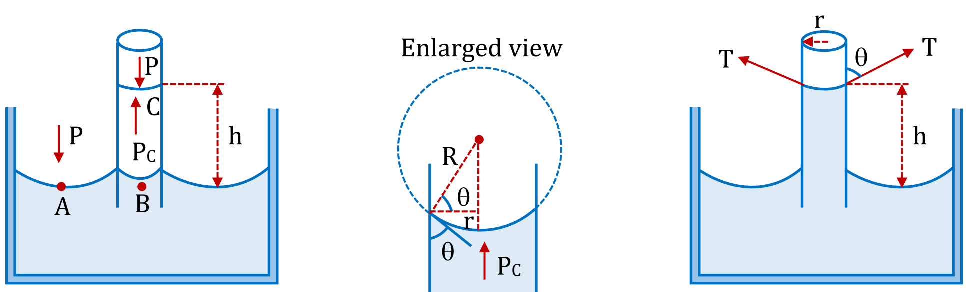




The value of θ_C determines whether a liquid will spread on the surface (wetting) or it will form droplets.

Calculation of Capillary rise :

When a capillary tube is first dipped in a liquid as shown in the figure, the liquid climbs up the walls curving the surface. The liquid continues to rise in the capillary tube until the weight of the liquid column becomes equal to force due to surface tension. Let the radius of the meniscus is *R* and the radius of the capillary tube is *r*. The angle of contact is θ, surface tension is *T*, density of liquid is ρ and the liquid rises to a height *h*.



R = Radius of the meniscus

$R = \dfrac{r}{\cos\theta}$

R = Radius of the capillary tube

Now let us consider two points A and B at the same horizontal level as shown. By Pascal's law

$$P_A = P_B \quad \Rightarrow \quad P_A = P_C + \rho\, gh \qquad \Rightarrow \quad P_A - P_C = \rho\, gh \qquad (\because\ P_B = P_C + \rho gh)$$

Now, the point C is on the curved meniscus which has P_A and P_C as the two pressures on its concave and convex sides respectively.

$$\therefore \qquad P_A - P_C = \frac{2T}{R} = \frac{2T}{r/\cos\theta} \quad \Rightarrow \frac{2T}{r/\cos\theta} = \rho\, gh \quad \Rightarrow 2T\cos\theta = r\rho\, gh \quad \Rightarrow h = \frac{2T\cos\theta}{r\rho g}$$

Illustration 25:

Calculate the height to which water will rise in a capillary tube of diameter $1 \times 10^{-3}\ m$.

[Given : surface tension of water is $0.072\ N\ m^{-1}$, angle of contact is 0°, $g = 9.8\ m\ s^{-2}$ and density of water $= 1000\ kg\ m^{-3}$]

Solution:

$$\text{Height of capillary rise } h = \frac{2T\cos\theta}{r\rho g} = \frac{2\times 0.072\times\cos 0^0}{5\times 10^{-4}\times 1000\times 9.8}\, m = 2.94\times 10^{-2}\, m$$

Illustration 26:

Water rises to a height of 20 mm in a capillary. If the radius of the capillary is made one third of its previous value then what is the new value of the capillary rise?

Solution:

Since $h = \dfrac{2T\cos\theta}{r\rho g}$ and for the same liquid and capillaries of difference radii $h_1r_1 = h_2r_2$

$$\therefore \qquad \frac{h_2}{h_1} = \frac{r_1}{r_2} = \frac{1}{(1/3)} = 3 \text{, hence } h_2 = 3h_1 = 3\times 20\ mm = 60\ mm$$

II-Method

$$P_0 - \frac{2S}{R} + \rho gh = P_0$$

$$\boxed{h = \frac{2S}{\rho g R}}$$

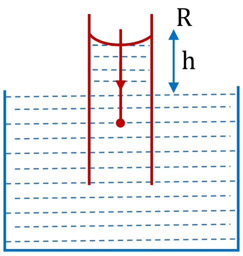


Illustration 27:

Find h of water in glass capillary of rod 1 mm. ($S = 0.075\ N/m$). $\theta_C = 0°$

Solution:

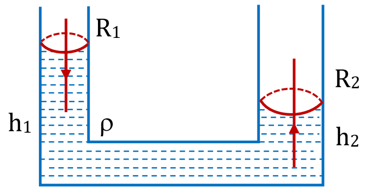
$$h = \frac{2S}{\rho g r}\cos\theta = \frac{2\times 75\times 1}{1000\times 10^3\times 10\times 1\times 10^{-3}} = \frac{150}{1000\times 10}\, m$$

$\Rightarrow \qquad 0.015\ m$

$\Rightarrow \qquad 1.5\ cm$

Illustration 28:

Find out surface tension of liquid.



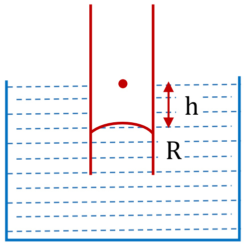
Solution:

$$P_0 - \frac{2S}{R_1} + \rho g h_1 - \rho g h_2 + \frac{2S}{R_2} = P_0$$

Illustration 29:

Find the fall in capillary tube.

Mercury glass



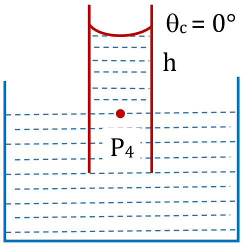
Solution:

$$P_0 + \frac{2S}{R} - \rho g h = P_0$$

$$h = \frac{2S}{\rho g R} \quad \text{(dip.)}$$

Illustration 30:

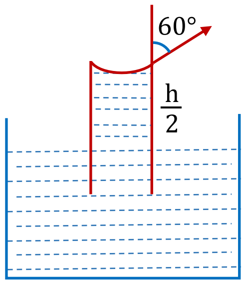
What will be contact angle between tube and water when the capillary tube is cut at $\frac{h}{2}$.



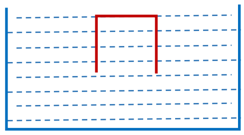
Capillary is cut

Solution:

$$h = \frac{2S}{\rho g R} \times \cos\theta_c \Rightarrow h = \frac{2S}{\rho g R} \times \cos 0°$$

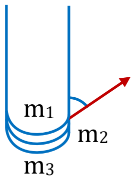


$$\frac{h}{2}=\frac{2S}{\rho gR}\times\cos\theta_C \quad \Rightarrow \frac{h}{2}=h\times\cos\theta_C' \quad \Rightarrow \theta_C' = 60°$$



$\theta_C = 90°$

Illustration 31:



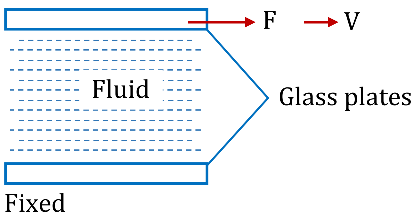
Solution:

$S\,2\pi r\cos\theta_C = mg$

as $m\uparrow$, $|\cos\theta_C|\uparrow$ till the limit

Viscosity

Viscosity is the property of the fluid (liquid or gas) by virtue of which it opposes the relative motion between its adjacent layers. It is the fluid friction or internal friction. The internal tangential force which try to retard the relative motion between the layers is called viscous force.



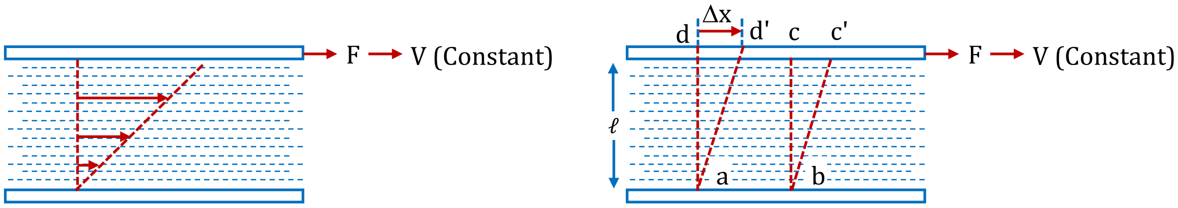
Assumption 1 : The fluid in contact with a surface has velocity as that of the surface.

Hence, in fig(1) top layer of the fluid which is in contact with upper plate, moves with velocity *'v'* and the bottom most layer of the fluid in contact with the fixed plate, is at rest.

Assumption 2 : Velocity of fluid layers increases uniformly from bottom most layer to the top layer.

For any layer of liq. its upper layer pulls forward while lower pulls it backward. This results force between the layers. This type of flow is known as laminar flow.

Newton's Law of Viscosity



Coefficient of viscosity (η)

$$\eta = \frac{\text{Shear stress}}{\text{Rate of change of shear strain}}$$

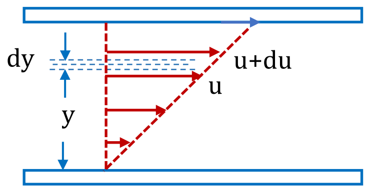
$$\eta = \frac{F/A}{\frac{d}{dt}\left(\frac{\Delta x}{\ell}\right)} \quad \Rightarrow \quad \eta = \frac{F/A}{\frac{v}{\ell}}$$

$$F = \eta \frac{Av}{\ell}$$

$F - F_{vus} = ma = 0$

Newtonian fluids, $F_{vis} = F = \eta \frac{Av}{\ell}$

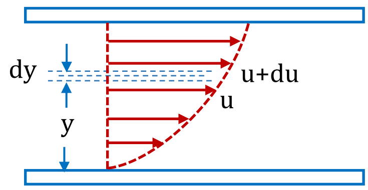
Newtonian Fluids



$\frac{du}{dy} = \frac{v}{\ell}$ = constant

$F_{vis.} = F = \eta \frac{Av}{\ell}$

Non-Newtonian Fluids



$\frac{du}{dy}$ is not constant

$F_{vis.} = F = \eta \frac{Adu}{dy}$

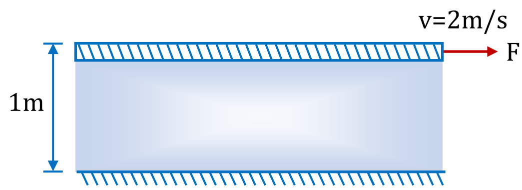
$\frac{du}{dy} \rightarrow$ velocity gradient

- **SI UNITS** : $\frac{N-s}{m^2}$ or deca poise
- **CGS UNITS** : $dyne\text{–}s/cm^2$ or poise (1 decapoise = 10 poise)
- **Dimension** : $M^1L^{-1}T^{-1}$

Illustration 32:

A plate of area 2 m^2 is made to move horizontally with a speed of 2 m/s by applying a horizontal tangential force over the free surface of a liquid. If the depth of the liquid is 1m and the liquid in contact with the plate is stationary. Coefficient of viscosity of liquid is 0.01 poise. Find the tangential force needed to move the plate.

Solution:



Velocity gradient = $\frac{\Delta v}{\Delta y} = \frac{2-0}{1-0} = 2 \frac{m/s}{m} = 2s^{-1}$

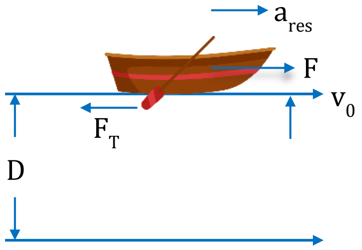
From, Newton's law of viscous force,

$$|F| = \eta A \frac{\Delta v}{\Delta y} = (0.01 \times 10^{-1})\,(2)\,(2) = 4 \times 10^{-3}\,N$$

Illustration 33:

A man is rowing a boat with a constant velocity v_0 in a river the contact area of boat is *'A'* and coefficient of viscosity is η. The depth of river is *'D'*. Find the force required to row the boat.

Solution:



$$F - F_T = m\,a$$

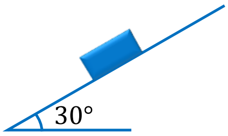
As boat moves with constant velocity $a = 0$ so $F = F_T$

But $\quad F_T = \eta A \dfrac{dv}{dz}$ but $\dfrac{dv}{dz} = \dfrac{v_0 - 0}{D} = \dfrac{v_0}{D}$

then $\quad F = F_T = \dfrac{\eta A v_0}{D}$

Illustration 34:

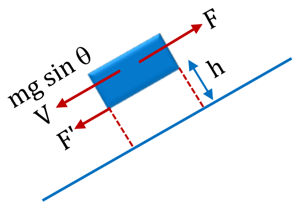
A cubical block (of side $2m$) of mass 20 kg slides on inclined plane lubricated with the oil of viscosity $\eta = 10^{-1}$ poise with constant velocity of 10 m/s. Find out the thickness of layer of liquid. ($g = 10\ m/s^2$)



Solution:

$$F = F'$$

$$\eta A \frac{dv}{dz} = mg \sin\theta$$



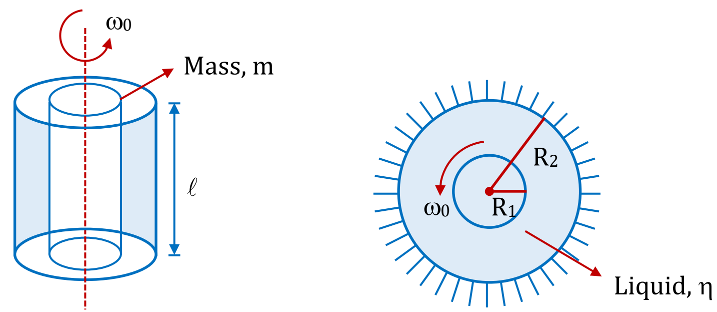
Here $\quad \dfrac{dv}{dz} = \dfrac{v}{h}$

$\Rightarrow \quad 20 \times 10 \times \sin 30° = \eta \times 4 \times \dfrac{10}{h}$

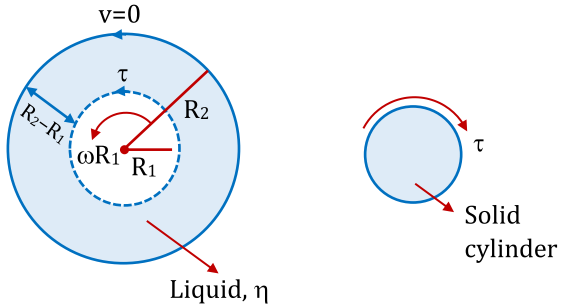
$\Rightarrow \quad h = 4 \times 10^{-3}\ m = 4\ mm$

Illustration 35:

As per the shown figure the central solid cylinder starts with initial angular velocity ω_0. Find out the time after which the angular velocity becomes half.



Solution:



$$F = \eta A \frac{dv}{dz}\text{ , where } \frac{dv}{dz} = \frac{\omega R_1 - 0}{R_2 - R_1}$$

$$F = \eta \frac{(2\pi R_1 \ell)\omega R_1}{R_2 - R_1} \quad \text{and} \quad \tau = FR_1 = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$$

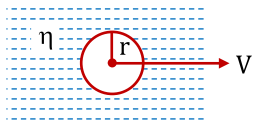
$$I\alpha = \frac{2\pi\eta R_1^3 \omega\ell}{R_2 - R_1}$$

$$\Rightarrow \quad \frac{mR_1^2}{2}\left(-\frac{d\omega}{dt}\right) = \frac{2\pi\eta R_1^3 \omega \ell}{R_2 - R_1}$$

or $$-\int_{\omega_0}^{\omega_0/2} \frac{d\omega}{\omega} = \int_0^t \frac{4\pi\eta R_1 \ell}{m(R_2 - R_1)} dt$$

$$\Rightarrow \quad t = \frac{m(R_2 - R_1)\ell n2}{4\pi\eta\ell R_1}$$

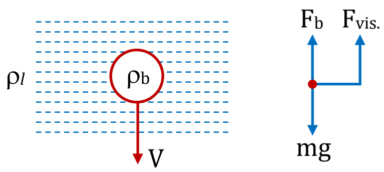
Stokes law :



Viscous force acting on a sphere of radius *'r'* moving with velocity *'V'* (relative to fluid) in a fluid of viscosity η is given by

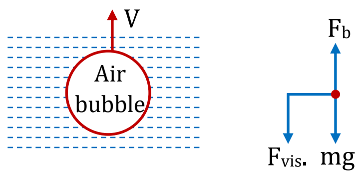
$\boxed{F_{Vis} = -6\pi\eta r V}$ (F_{vis} is opp. to V.)

eg.



$\rho_b > \rho_l$

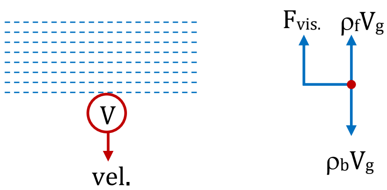
eg.



$\rho_{air} < \rho_l$

Illustration 36:

Consider a body of density ρ_b in a fluid of density ρ_f, if it released from rest then find terminal velocity?



Solution:

At terminal vel. i.e. V_T ; acc = 0

$$ma = mg - F_b - F_{visc}$$

$$ma = \rho_b V_g - \rho_f V_g - 6\pi\eta r V_T$$

$$V_T = \frac{(\rho_b - \rho_f)4\pi r^3 g}{6\pi\eta r \times 3}$$

$$\boxed{V_T = \frac{2}{9}\frac{(\rho_b - \rho_f)r^2 g}{\eta}}$$

eg. Solid object $\rho_b >> \rho_f = \rho_{air}$

$$\therefore \; V_T \approx \frac{2\rho_b r^2 g}{9\eta}$$

eg. Air bubble in water

$$\Rightarrow \; \rho_b = \rho_{air} << \rho_w = \rho_f$$

$$V_T \approx \frac{-2}{9}\frac{\rho_w r^2 g}{\eta}$$

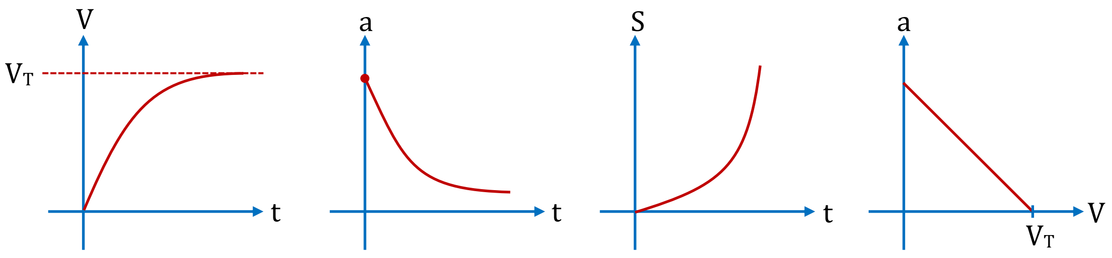


Illustration 37:

If velocity of fluid is zero at $t = 0$ then find velocity (v) as a function t ?

Solution:

$$\frac{dV}{dt} = a = \underbrace{\frac{mg}{m} - \frac{F_b}{m}}_{K} - \underbrace{\frac{6\pi\eta r}{m}}_{C}V$$

$$\frac{dV}{dt} = K - CV$$

$$\int_0^V \frac{dV}{K - CV} = \int_0^t dt$$

$$\left[\ell n(K - CV)\right]_0^V = -Ct$$

$$\frac{\ell n(K - CV)}{K} = -Ct$$

$$K - CV = Ke^{-Ct}$$

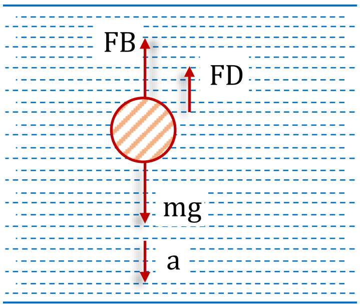
$$\boxed{V = \frac{K}{C}(1 - e^{-Ct}) = V_T(1 - e^{-Ct})}$$

Stoke's Law and Terminal Velocity :

Stoke showed that if a small sphere of radius r is moving with a velocity v through a homogeneous medium (liquid or gas), coefficient of viscosity η then the viscous force acting on the sphere is $F_D = 6\pi\eta rv$. It is called Stoke's Law.

$$mg - F_B - F_D = ma$$

$$a = g - \frac{F_B}{m} - \frac{F_D}{m} \qquad \Rightarrow a = g\left(1 - \frac{\rho_L}{\rho_S}\right) - \frac{6\pi\eta r v}{\frac{4}{3}\pi R^3 \rho_S}$$



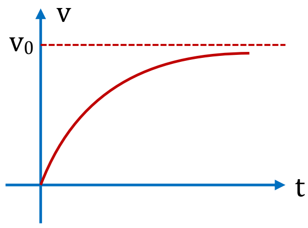
$$\frac{dv}{dt} = a = K_1 - K_2 v$$

$$\int_0^v \frac{dv}{K_1 - K_2 v} = \int_0^t dt \qquad \Rightarrow v = \frac{K_1}{K_2}\left(1 - e^{-K_2 t}\right)$$

$$v = v_0\left(1 - e^{-K_2 t}\right)$$

$$t \to \infty, \quad v \to \frac{K_1}{K_2}$$

Terminal velocity $\boxed{v_0 = \frac{2}{9}\frac{r^2 g}{\eta}(\rho_L - \rho_S)}$



- **Some applications of Stoke's law :**
  (a) The velocity of rain drops is very small in comparison to the velocity of a body falling freely from the height of clouds.
  (b) Descending by a parachute with lesser velocity.
  (c) Determination of electronic charge with the help of Milikan's experiment.

Illustration 38:

A spherical ball is moving with terminal velocity inside a liquid. Determine the relationship of rate of heat loss with the radius of ball.

Solution:

Rate of heat loss = power = $F \times v = 6\pi\eta r v \times v = 6\pi\eta r v^2 = 6P\eta r\left[\frac{2}{9}\frac{gr^2(\rho_0 - \rho_\ell)}{\eta}\right]^2$

Therefore, rate of heat loss $\propto r^5$.

Illustration 39:

A drop of water of radius 0.0015 *mm* is falling in air. If the coefficient of viscosity of air is 1.8×10^{-5} $kg\ m^{-1}\ s^{-1}$. What will be the terminal velocity of the drop? Density of air can be neglected.

Solution:

$$v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} = \frac{2 \times \left[\frac{15 \times 10^{-4}}{1000}\right]^2 \times 10^3 \times 9.8}{9 \times 1.8 \times 10^{-5}} = 2.72 \times 10^{-4}\ \text{m/s}$$

Reynolds Number (Re) :

The type of flow pattern (laminar or turbulent) is determined by a non–dimensional number called Reynolds number (R_e). Which is defined as $R_e = \frac{\rho v d}{\eta}$ where ρ is the density of the fluid having viscosity η and flowing with mean speed v, d denotes the diameter of obstacle or boundary of fluid flow.

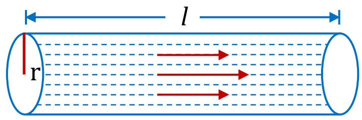
Although there is not a perfect demarcation for value of R_e for laminar and turbulent flow but some authentic references take the value as

R_e	<1000	>2000	Between 1000 to 2000
Type of flow	laminar	often turbulent	may be laminar or turbulent

on gradually increasing the speed of flow at certain speed transition from laminar flow to turbulent flow takes place. This speed is called **critical speed.** For lower density and higher viscosity fluids laminar flow is more probable.

Poiseuille's Formula :

Poiseuille studied the stream-line flow of liquid in capillary tubes. He found that if a pressure difference (P) is maintained across the two ends of a capillary tube of length 'l' and radius r, then the volume of liquid coming out of the tube per second is



(i) Directly proportional to the pressure difference (P).

(ii) Directly proportional to the fourth power of radius (r) of the capillary tube

(iii) Inversely proportional to the coefficient of viscosity (η) of the liquid.

(iv) Inversely proportional to the length (l) of the capillary tube.

i.e. $V \propto \frac{Pr^4}{\eta l}$ or $V = \frac{KPr^4}{\eta l}$

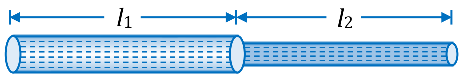
$\therefore$ $V = \frac{\pi P r^4}{8\eta l}$ [Where $K = \frac{\pi}{8}$ is the constant of proportionality]

This is known as Poiseuille's equation. This equation also can be written as,

$$V = \frac{P}{R} \text{ where } R = \frac{8\eta l}{\pi r^4}$$

R is called as liquid resistance.

Series combination of tubes :



(i) When two tubes of length l_1, l_2 and radii r_1, r_2 are connected in series across a pressure difference P,

Then, $P = P_1 + P_2$ ...(i)

Where P_1 and P_2 are the pressure difference across the first and second tube respectively.

(ii) The volume of liquid flowing through both the tubes i.e. rate of flow of liquid is same.

Therefore $V = V_1 = V_2$

i.e., $V = \dfrac{\pi P_1 r_1^4}{8\eta l_1} = \dfrac{\pi P_2 r_2^4}{8\eta l_2}$ ...(ii)

Substituting the value of P_1 and P_2 from equation (ii) to equation (i) we get

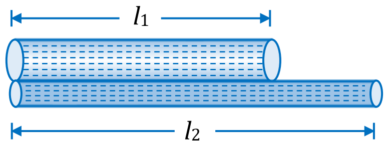
$$P = P_1 + P_2 = V\left[\frac{8\eta l_1}{\pi r_1^4} + \frac{8\eta l_2}{\pi r_2^4}\right]$$

$$\therefore \quad V = \frac{P}{\left[\dfrac{8\eta l_1}{\pi r_1^4} + \dfrac{8\eta l_2}{\pi r_2^4}\right]} = \frac{P}{R_1 + R_2} = \frac{P}{R_{eff}}$$

Where R_1 and R_2 are the liquid resistance in tubes.

(iii) Effective liquid resistance in series combination, $R_{eff} = R_1 + R_2$.

Parallel combination of tubes



(i) $P = P_1 = P_2$

(ii) $V = V_1 + V_2 = \dfrac{P\pi r_1^4}{8\eta l_1} + \dfrac{P\pi r_2^4}{8\eta l_2} = P\left[\dfrac{\pi r_1^4}{8\eta l_1} + \dfrac{\pi r_2^4}{8\eta l_2}\right]$

$$\therefore \quad V = P\left[\frac{1}{R_1} + \frac{1}{R_2}\right] = \frac{P}{R_{eff}}$$

(iii) Effective liquid resistance in parallel combination is

$$\frac{1}{R_{eff}} = \frac{1}{R_1} + \frac{1}{R_2}$$

or $\quad R_{eff} = \dfrac{R_1 R_2}{R_1 + R_2}$